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THE OUTPUT AND WELFARE EFFECTS OF GOVERNMENT SPENDING SHOCKS
OVER THE BUSINESS CYCLE

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The Output and Welfare Effects of Government Spending Shocks over the Business Cycle
Eric Sims and Jonathan Wolff
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ABSTRACT

How does the output response to a change in government spending vary over the business cycle? What are the welfare effects of spending shocks? This paper studies the state-dependence of the output and welfare effects of shocks to government purchases in a DSGE model with real and nominal frictions and a rich fiscal financing structure. Both the output multiplier (the change in output for a one dollar change in government spending) and the welfare multiplier (the consumption equivalent change in welfare for the same change in spending) move significantly across states. The magnitudes of the output and welfare multipliers tend to move opposite from one another across states of the business cycle. In an historical simulation, the output multiplier is countercyclical (correlation with detrended output of -0.4) and strongly negatively correlated with the welfare multiplier (correlation between multipliers of -0.9).

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1 Introduction

The recent “Great Recession” has led to renewed interest in fiscal stimulus as a tool to fight recessions. There nevertheless seems to be a lack of consensus concerning some fundamental questions. How large are fiscal output multipliers? Do fiscal multipliers vary in magnitude over the business cycle? What are the welfare implications of fiscal stimulus? This paper quantitatively analyzes the state-dependence of the output and welfare effects of government spending shocks in an estimated medium scale dynamic stochastic general equilibrium (DSGE) model.

To first develop some intuition for the output and welfare effects of spending shocks across states of the business cycle, we begin in Section 2 with a highly stylized economy in which labor is the only factor of production and consumption equals output less government spending. A representative household receives flow utility from consumption, leisure, and government purchases. We consider a first order approximation of flow utility about a point and derive an expression for the “utility multiplier,” or the change in flow utility for a one unit change in government spending. The utility multiplier equals the “output multiplier” (the change in output for a one unit change in government spending) times a term measuring static inefficiency, plus the difference between the marginal utilities of government spending and private consumption. The term measuring static inefficiency is closely related to the “labor wedge” (Chari, Kehoe, and McGrattan, 2007).

In an efficient allocation, the first term in the expression for the utility multiplier will be zero for any value of the output multiplier, leaving only the second term. This means that the utility multiplier would be procyclical – for a given level of government spending, depressed output implies that the marginal utility of private consumption is high relative to the marginal utility of government spending. Some reduced form empirical evidence and simple theoretical reasoning suggests that the output multiplier is high when output is depressed. To the extent to which this is the case, this would mean that the utility and output multipliers would co-move negatively with one another across states of the business cycle in an efficient allocation. In a distorted economy, however, the term in the approximation measuring static inefficiency will be positive and potentially time-varying. Since this term multiplies the output multiplier, this would tend to make the utility multiplier move in the same direction as the output multiplier. In a general setting, it is thus theoretically ambiguous how the utility multiplier of government spending shocks ought to vary with the state of the economy, and in turn how the utility multiplier ought to vary with the output multiplier.

To further explore the co-movements of the output and welfare effects of government spending shocks over the business cycle in a quantitative setting, in Section 3 we present a fairly standard “medium scale” DSGE model along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and others. Monopolistic competition in price- and wage-setting give rise to a steady state distortion. In addition, the model features price and wage stickiness, capital accumulation, several real frictions, and a number of shocks. A representative household receives flow utility from consumption and leisure as well as a utility flow from government spending. The government finances itself with a rich array of tax instruments and can borrow through a non-state

contingent bond. We estimate the model parameters using Bayesian maximum likelihood. The estimated parameter values are sensible relative to the existing literature, and the model produces second moments that are in line with their empirical counterparts.

In Section 4, we use the estimated parameters and solve the model via a second order approximation. This is the minimum higher order approximation which will allow us to investigate the state-dependence of fiscal multipliers. The output multiplier is defined as above, while we define the “welfare multiplier” as the one period consumption equivalent change in the present discounted value of flow utility for a one unit change in government spending (one can think of the welfare multiplier as the present discounted value of the utility multipliers mentioned above; the conversion to consumption equivalent terms transforms the multiplier into interpretable units). We calculate these multipliers by simulating impulse responses to a persistent government spending shock starting from different initial positions of the state vector.

In the estimated model, the output multiplier is 1.2 evaluated at the model’s steady state, but ranges from about 1 to 1.5 across states. The output multiplier is estimated to be strongly countercyclical – it is higher than average in a typical recession, and lower than average in a robust expansion. The welfare multiplier moves significantly across states, and is about an order of magnitude more volatile than the output multiplier. It displays the opposite cyclicity from the output multiplier – it tends to be relatively high in an expansion, and lowest in a recession.¹ In spite of the fact that our model features significant time-varying distortions relative to the first best, this means that the intuition for the cyclicity of the welfare multiplier from an efficient allocation seems to quantitatively dominate. In an historical simulation based on smoothed retrospective states from our Bayesian estimation, the output and welfare multipliers have a correlation coefficient of -0.9. The output multiplier peaked near the end of the Great Recession period, but the welfare multiplier was estimated to be low. Our basic results that the welfare multiplier is procyclical and strongly negatively correlated with the output multiplier are qualitatively robust to different fiscal financing regimes, different monetary policy regimes, and different parameter values.

Our paper is related to a growing literature on fiscal policy multipliers. There is a large empirical literature that seeks to estimate fiscal output multipliers using reduced form techniques. Using orthogonality restrictions in estimated vector auto-regressions (VARs), Blanchard and Perotti (2002) identify shocks by “ordering” government spending first in a recursive identification, and report estimates of spending multipliers between 0.9 and 1.2. Mountford and Uhlig (2009) use sign restrictions in a VAR and find a multiplier of about 0.6. Ramey (2011) uses narrative evidence to construct a time series of government spending “news,” and reports multipliers in the range of 0.6-1.2. This range aligns well with a number of papers that make use of military spending as an instrument for government spending shocks in a univariate regression framework (see, e.g. Barro,

¹The absolute magnitude of the welfare multiplier is impossible to identify, since this depends on where government spending is set relative to its optimal level. In our baseline parameterization, government spending is set inefficiently low in the steady state, so that the welfare multiplier is positive. When government spending is set to a higher or low fraction of steady state GDP, the magnitude and sign of the welfare multiplier can be significantly different, but its overall state-dependence is qualitatively unaffected. For this reason, we focus on the state-dependence of the welfare multiplier, not its absolute magnitude.

1981; Hall, 1986 and 2009; Barro and Redlick, 2009; Ramey and Shaprio, 1988; Eichenbaum and Fisher, 2005; and Nakamura and Steinsson, 2014). The bulk of this empirical literature suggests that spending multipliers are approximately 1. Because these estimates are based on full sample averages, they cannot speak to any form of state-dependence, and given non-observability of utility, these empirical papers are silent concerning welfare.

There is also a limited but growing literature that seeks to estimate state-dependent multipliers using econometric techniques. A drawback of this approach is that there are limited time series observations, particularly during periods of economic slack. Auerbach and Gorodnichenko (2012) estimate a regime-switching VAR model and find that output multipliers are highly countercyclical and as high as 3 during recessions and as low as 0 during expansions. Bachmann and Sims (2012) and Mitnik and Semmler (2012) use similar methods and reach similar conclusions. Owyang, Ramey, and Zubairy (2013) use newly constructed historical data and Jorda’s (2005) “local projection” technique to study state-dependent multipliers. For the US, they find no evidence of countercyclical output multipliers, while for Canada they do. Nakamura and Steinsson (2014) consider a regression model that allows the multiplier to vary with the level of unemployment, and find that the multiplier is substantially larger when unemployment is high.

Another strand of the literature, closer to the current paper, looks at the magnitude of fiscal output multipliers within the context of DSGE models. Baxter and King (1993) is an early contribution. Zubairy (2013) estimates a medium-scale DSGE model similar to the one presented in the current paper and finds the output multiplier to be about 1.1. Coenen, et al (2012) calculate fiscal multipliers in seven popular DSGE models, and conclude that the output multiplier can be far in excess of one. Cogan, Cwik, Taylor and Wieland (2010) and Drautzberg and Uhlig (2011) conclude, in contrast, that the multiplier is likely less than unity. Leeper, Traum, and Walker (2011) use Bayesian prior predictive analysis not to produce a point estimate for the output multiplier, but rather to provide plausible bounds on it in a fairly general DSGE model. As noted by Parker (2011), almost all of the DSGE work, including that cited here, is based on linear approximations, which necessarily cannot address the state-dependence of multipliers.

A third strand of the literature looks at output multipliers and their interaction with the stance of monetary policy. In particular, there is a growing consensus that output multipliers can be substantially larger than normal under “passive” monetary policy regimes, such as the recent zero lower bound period. Early contributions in this regard include Christiano (2004) and Eggertson and Woodford (2003). Woodford (2011) conducts analytical exercises in the context of a conventional New Keynesian model without capital to study the fiscal output multiplier, both inside and outside of a zero lower bound episode. Most recently, Christiano, Eichenbaum, and Rebelo (2011) analyze the consequences of the zero lower bound for government spending multipliers in DSGE models similar to the ones in the current paper, and find multipliers in excess of 2. Though Christiano, Eichenbaum, and Rebelo (2011) are mostly focused on the output multiplier, they do briefly examine welfare, and find that it is optimal to substantially increase government spending at the zero lower bound. We also find that both output and welfare multipliers can be substantially larger when the zero lower bound binds. As their analysis is based on linearization, they do not discuss other state-

dependence of government spending shocks on welfare. Nakata (2013) reaches a similar conclusion that it is optimal to increase government spending when the zero lower bound binds. His is one of the only papers of which we are aware which makes use of non-linear solution techniques in the context of studying fiscal multipliers, though he does not look at the type of state-dependence that we do.

Our paper seeks to bridge these somewhat disparate literatures. Even though there is some reduced-form empirical evidence about state-dependent output multipliers, there has been little or no attempt to connect this evidence with micro-founded models. We are aware of no other paper which looks at the dependence of output multipliers on the state of the economy in a DSGE context in the way that we do. We are also one of only a few papers to look at the welfare consequences of fiscal shocks, and the only, of which we are aware, that looks at how the welfare effects of fiscal shocks are related to the output effects. Indeed, we find that the welfare and output multipliers are strongly negatively correlated across states of the business cycle. To the extent to which one wants to use a micro-founded DSGE model for policy analysis, one ought to look at model-implied welfare effects of spending shocks, not the output effects. Our results suggest that the output multiplier is likely a poor measure of the welfare effects of a government spending shock.

2 Intuition in a Stylized Setting

To first build some intuition, consider a highly stylized economy. A representative household receives flow utility from private consumption, leisure (equal to one minus labor), and government expenditure:

$$U_t = U(C_t, 1 - N_t) + \Omega(G_t) \quad (1)$$

The utility function over consumption and leisure (equal to the normalized time endowment of 1 minus labor, $L_t = 1 - N_t$) has the usual properties: $U_C(\cdot) > 0$, $U_{CC}(\cdot) < 0$, $U_L(\cdot) > 0$, and $U_{LL}(\cdot) < 0$. The cross-partial can be positive, negative, or zero, so long as the overall function is concave. Utility from government purchases is assumed to be additively separable with $\Omega'(G_t) > 0$, and $\Omega''(G_t) < 0$. The resource constraint is $Y_t = C_t + G_t$ and output is an increasing and weakly concave function of labor input, $Y_t = F(N_t)$, with $F'(\cdot) > 0$ and $F''(\cdot) \leq 0$. Taking a first order approximation about a point denoted by $*$, and making use of the resource constraint and production function, one can derive an expression for the “utility multiplier” of government spending:

$$\frac{dU_t}{dG_t} = \left[U_C(Y^* - G^*, 1 - N^*) - \frac{U_L(Y^* - G^*, 1 - N^*)}{F'(N^*)} \right] \frac{dY_t}{dG_t} + \left[\Omega'(G^*) - U_C(Y^* - G^*, 1 - N^*) \right] \quad (2)$$

The utility multiplier, $\frac{dU_t}{dG_t}$, is equal to the output multiplier, $\frac{dY_t}{dG_t}$, times a term measuring static inefficiency, plus the difference between the marginal utilities of public expenditure and private

consumption. The term measuring static inefficiency, $U_C(\cdot) - \frac{U_L(\cdot)}{F'(\cdot)}$, is closely related to the “labor wedge” (Chari, Kehoe, and McGrattan, 2007, and Gali, Gertler, and Lopez-Salido, 2007).² In an efficient allocation $U_L(\cdot) = U_C(\cdot)F'(\cdot)$, so the first term would be zero and the utility multiplier would simply equal the difference between the marginal utility of public and private consumption. For a given value of government spending, the marginal utility of private consumption is high when output is low, which would make the utility multiplier relatively small. The reverse would be true when output is unusually high. This means that, in an efficient allocation, the utility multiplier ought to be positively correlated with the level of output.

The procyclicality of the utility multiplier is not guaranteed in a distorted economy, however. With monopoly price- and wage-setting, as characterizes most New Keynesian DSGE models, the term measuring static inefficiency in the approximation above will be positive. This follows since the economy is characterized by inefficiently low production. This means that the state-dependence of the utility multiplier is more complicated than in an efficient allocation. On the one hand, as in the previous paragraph, the marginal utility of private consumption will be high when output is low, which will tend to make the utility multiplier move in the same direction as output through the second term. On the other hand, the overall level of distortion in the economy may change over the business cycle (e.g. the magnitude of $U_C(\cdot) - U_L(\cdot)/F'(\cdot)$ could vary), and the output multiplier itself may also be state-dependent.

Some existing empirical research and simple theoretical reasoning suggests that the output multiplier is high when output is low (e.g. Auerbach and Gorodnichenko, 2012). Some other research on the labor wedge suggests that the economy appears relatively more distorted in recessions, so that $U_C(\cdot) - U_L(\cdot)/F'(\cdot)$ is bigger when output is lower than average (e.g. Chari, Kehoe, and McGrattan, 2007).³ This state-dependence in the output multiplier and the positive and countercyclical amount of inefficiency in the economy would tend to make the utility multiplier high when output is low, other things being equal. This would work against the procyclicality of the utility multiplier coming from the difference in the marginal utilities of public and private consumption. Which of these different channels would dominate in a model with time-varying distortions is theoretically unclear. It is therefore not possible to make general statements about how the welfare impact of government spending shocks varies over the business cycle, or how the welfare effect of spending shocks varies with the output effect. To answer those questions, we turn to a rich DSGE model with a number of real and nominal frictions.

²Chari, Kehoe, and McGrattan (2007) define the labor wedge as the residual from the static first order condition for the choice of labor in a social planner’s problem. Conceptualizing the labor wedge as a tax on labor income, the static first order condition for optimal labor using our notation is $U_L(\cdot) = U_C(\cdot)(1 - \tau)F'(\cdot)$. Re-arranging terms, $U_C(\cdot) - \frac{U_L(\cdot)}{F'(\cdot)} = \tau U_C(\cdot)$. Defining the labor wedge as the stand-in tax rate, τ , the term in our approximation measuring static inefficiency is equal to the labor wedge times the marginal utility of consumption.

³As shown in the previous footnote, $U_C(\cdot) - U_L(\cdot)/F'(\cdot) = \tau U_C(\cdot)$, so the term $U_C(\cdot) - U_L(\cdot)/F'(\cdot)$ could be high for two reasons: a high labor wedge (apparent tax on labor income, τ) and a high marginal utility of consumption, both of which we might expect to observe when output is low.

3 A DSGE Model

This section introduces and estimates a “medium scale” dynamic stochastic general equilibrium (DSGE) model, similar to the models in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model features both real and nominal frictions. It is comprised of several different actors, the problems and optimality conditions of whom we describe below. A government sets its spending exogenously, and finances its purchases through a combination of lump sum taxes and distortionary taxes on consumption, capital, and labor.

3.1 Household

There exists a representative household with preferences over consumption, leisure, and government spending. Welfare is the present discounted value of flow utility:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ \nu_t U(C_t - bC_{t-1}, 1 - N_t) + \Omega(G_t) \} \quad (3)$$

The discount factor is $0 < \beta < 1$, $b \in (0, 1)$ measures internal habit formation, and $1 - N_t$ is leisure, defining N_t as labor supply with the time endowment normalized to unity. We assume that utility is increasing and concave in both consumption and leisure, but allow for non-separability between consumption and leisure. This non-separability can play an important role in the magnitude of the output effect of government spending changes, as shown, for example, by Christiano, Eichenbaum, and Rebelo (2011). We assume that the household receives a utility flow from government spending which is additively separable from the other arguments of utility, with $\Omega'(G_t) > 0$ and $\Omega''(G_t) < 0$. The variable ν_t is a preference shock following an exogenous stochastic process to be specified below. We assume that it impacts the utility from consumption and leisure, but not the utility from government purchases.

Households can accumulate physical capital through the accumulation equation:

$$K_{t+1} = Z_t \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t + (1 - \delta) K_t \quad (4)$$

$S(\cdot)$ is a convex investment adjustment cost, with the properties that $S(1) = S'(1) = 0$ and $S''(1) > 0$. This specification of the adjustment cost in terms of the growth rate of investment follows Christiano, Eichenbaum, and Evans (2005). The depreciation rate of physical capital is $0 < \delta < 1$. The exogenous variable Z_t is an investment shock, as in Justiniano, Primiceri, and Tambalotti (2010). This shock measures the efficiency of transforming non-consumed output into productive capital, and can be interpreted as a reduced form for a shock to the financial sector. It obeys an exogenous stochastic process which we specify below.

We introduce nominal wage rigidity as in Schmitt-Grohe and Uribe (2006a). This framework differs slightly from the more common setup based on Erceg, Henderson, and Levin (2000), but permits non-separability in preferences between consumption and leisure.⁴ The household supplies

⁴As shown by Schmitt-Grohe and Uribe (2006b), to a first order approximation their setup produces an identical

labor to a continuum of labor markets, indexed by $h \in (0, 1)$. Each period, there is a fixed probability, $(1 - \theta_w)$ with $\theta_w \in (0, 1)$, that the wage charged in a given market can be re-optimized. Non-optimized wages may be indexed to lagged inflation at $\zeta_w \in (0, 1)$. The demand for labor in each market is:

$$N_t(h) = \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w} N_{d,t}, \quad \epsilon_w > 1 \quad (5)$$

The real wage charged in market h is $w_t(h)$, and $N_{d,t}$ is a measure of aggregate labor demand from firms. The aggregate real wage, w_t , is given by:

$$w_t^{1-\epsilon_w} = \int_0^1 w_t(h)^{1-\epsilon_w} dh \quad (6)$$

Combining (5) with a condition that total household labor supply equals the sum of labor supplied in each market, $N_t = \int_0^1 N_t(h) dh$, we get:

$$N_t = N_{d,t} \int_0^1 \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w} dh \quad (7)$$

The flow budget constraint facing the household is:

$$(1 + \tau_t^c)C_t + I_t + \Gamma(u_t) \frac{K_t}{Z_t} + \frac{B_t}{P_t} \leq (1 - \tau_t^k)r_t^k u_t K_t + (1 - \tau_t^n) \int_0^1 w_t(h) N_t(h) dh + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t - T_t \quad (8)$$

The variables τ_t^c , τ_t^n , and τ_t^k are potentially time-varying distortionary tax rates on consumption, labor income, and capital income. The rental rate on capital services, which is the product of utilization, u_t , and the physical capital stock, K_t , is r_t^k . $\Gamma(u_t)$ represents a resource cost of capital utilization, and satisfies $\Gamma(1) = 0$, $\Gamma'(1) = \frac{1}{\beta} - (1 - \delta)$, and $\Gamma''(1) > 0$.⁵ This resource cost comes out of physical capital; the division by Z_t expresses the cost in terms of consumption goods. B_t is the nominal stock of bonds with which a household enters period $t + 1$, and i_t is the nominal interest rate known at t that pays off in $t + 1$. Lastly, Π_t denotes real dividends distributed from firms and T_t is a lump sum tax/transfer from the government.

The first order necessary conditions over consumption, utilization, investment, future capital, labor supply, and bonds are, respectively:

$$(1 + \tau_t^c)\mu_{1,t} = \nu_t U_C(C_t - bC_{t-1}, N_t) - \beta b E_t \nu_{t+1} U_C(C_{t+1} - bC_t, 1 - N_{t+1}) \quad (9)$$

$$(1 - \tau_t^k)r_t^k = \frac{\Gamma'(u_t)}{Z_t} \quad (10)$$

wage Phillips Curve to the Erceg, Henderson, and Levin (2000) setup, albeit with different wage rigidity parameters. The Schmitt-Grohe and Uribe (2006a) framework also simplifies welfare analysis, as there is a representative household, and one therefore does not need to specify a social welfare function to aggregate the utility of heterogeneous agents as in Erceg, Henderson, and Levin (2000).

⁵The restriction that $\Gamma'(1) = \frac{1}{\beta} - (1 - \delta)$ insures that the steady state normalization of $u^* = 1$ is consistent with the first order conditions below.

$$1 = q_t Z_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \frac{\mu_{1,t+1}}{\mu_{1,t}} q_{t+1} Z_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left[\frac{I_{t+1}}{I_t} \right]^2 \quad (11)$$

$$q_t = \beta E_t \frac{\mu_{1,t+1}}{\mu_{1,t}} \left[(1 - \tau_{t+1}^k) r_{t+1}^k u_{t+1} - \frac{\Gamma(u_{t+1})}{Z_{t+1}} + (1 - \delta) q_{t+1} \right] \quad (12)$$

$$U_L(C_t - bC_{t-1}, 1 - N_t) = \mu_{3,t} \quad (13)$$

$$\mu_{1,t} = \beta E_t \mu_{1,t+1} (1 + i_t) (1 + \pi_{t+1})^{-1} \quad (14)$$

Above, $\mu_{1,t}$ is the multiplier on the flow budget constraint, (8), and $\mu_{3,t}$ is the multiplier on (7), the constraint that household labor supply meets total demand. The variable q_t is the ratio of the multiplier on the capital accumulation equation (4), $\mu_{2,t}$, to the multiplier on the budget constraint, e.g. $q_t = \frac{\mu_{2,t}}{\mu_{1,t}}$. The optimal reset wage for the subset of wages that can be re-optimized in period t will be common across labor markets, which allows us to drop h subscripts. We denote this common reset wage by $w_t^\#$. After some algebraic manipulations, the optimality condition over re-optimized wages can be written recursively as follows:

$$w_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{F_{1,t}}{F_{2,t}} \quad (15)$$

$$F_{1,t} = U_L(C_t - bC_{t-1}, 1 - N_t) w_t^{\epsilon_w} N_{d,t} + \theta_w \beta E_t (1 + \pi_t)^{-\epsilon_w \zeta_w} (1 + \pi_{t+1})^{\epsilon_w} F_{1,t+1} \quad (16)$$

$$F_{2,t} = \mu_{1,t} (1 - \tau_t^n) w_t^{\epsilon_w} N_{d,t} + \theta_w \beta E_t (1 + \pi_t)^{\zeta_w (1 - \epsilon_w)} (1 + \pi_{t+1})^{\epsilon_w - 1} F_{2,t+1} \quad (17)$$

3.2 Production

Production is split into two sectors. A competitive representative final good firm aggregates a continuum of intermediate goods, indexed by $j \in (0, 1)$, into a final good available for consumption and investment according to a CES technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \quad \epsilon_p > 1 \quad (18)$$

Profit maximization implies a downward-sloping demand curve for each intermediate good and an aggregate price index:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t, \quad \forall j \quad (19)$$

$$P_t^{1 - \epsilon_p} = \int_0^1 P_t(j)^{1 - \epsilon_p} dj \quad (20)$$

Intermediate firms produce output using both capital services, $\widehat{K}_t \equiv u_t K_t$, and labor, $N_{d,t}$, and face a common productivity shock, A_t . The production function is:

$$Y_t(j) = A_t \widehat{K}_t(j)^\alpha N_{d,t}(j)^{1 - \alpha}, \quad 0 < \alpha < 1 \quad (21)$$

These firms are price-takers in input markets. Cost-minimization implies all intermediate goods firms hire capital services and labor in the same ratio, which is in turn equal to the aggregate capital services to labor ratio, and have the same real marginal cost:

$$\frac{\widehat{K}_t}{N_{d,t}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} \quad (22)$$

$$mc_t = \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{A_t} (1-\alpha)^{\alpha-1} \alpha^{-\alpha} \quad (23)$$

Price rigidity is introduced via the Calvo (1983) sticky price assumption in a way similar to wage rigidity. Each firm faces a constant probability, $(1 - \theta_p)$ with $\theta_p \in (0, 1)$, of being able to adjust its price in any period. This probability is independent of when the price was last adjusted and is also equal to the fraction of intermediate goods prices that are able to be adjusted in a period. Non-updated prices can be indexed to lagged inflation at $\zeta_p \in (0, 1)$. The pricing problem of an updating firm in period t is given by:

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{\mu_{1,t+s}}{\mu_{1,t}} \left(\prod_{m=1}^s (1 + \pi_{t+m-1})^{\zeta_p(1-\epsilon_p)} P_t(j)^{1-\epsilon_p} P_{t+s}^{\epsilon_p-1} Y_{t+s} - \prod_{m=1}^s (1 + \pi_{t+m-1})^{-\zeta_p \epsilon_p} P_t(j)^{-\epsilon_p} mc_{t+s} P_{t+s}^{\epsilon_p} Y_{t+s} \right)$$

The first order condition is an optimal reset price that will be the same for all updating firms, $P_t^\#$. It can be written recursively in terms of aggregate variables, where $\pi_t^\# = \frac{P_t^\#}{P_{t-1}} - 1$:

$$\frac{1 + \pi_t^\#}{1 + \pi_t} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1,t}}{X_{2,t}} \quad (24)$$

$$X_{1,t} = mc_t \mu_{1,t} Y_t + \theta_p \beta E_t (1 + \pi_t)^{-\zeta_p \epsilon_p} (1 + \pi_{t+1})^{\epsilon_p} X_{1,t+1} \quad (25)$$

$$X_{2,t} = \mu_{1,t} Y_t + \theta_p \beta E_t (1 + \pi_t)^{\zeta_p(1-\epsilon_p)} (1 + \pi_{t+1})^{\epsilon_p-1} X_{2,t+1} \quad (26)$$

3.3 Government

We do not model an explicit Ramsey problem for the government. Rather, we postulate the existence of simple rules for both monetary and fiscal policy. Monetary policy is set according to a standard Taylor-type rule in which the interest rate reacts to deviations of inflation from an exogenous target, π^* , and to output growth. The exogenous variable $e_{i,t}$ is a shock to the monetary policy rule and follows a standard normal distribution, with s_i the standard deviation of the shock. The steady state nominal interest rate is $i^* = \beta^{-1}(1 + \pi^*)$:

$$i_t = (1 - \rho_i) i^* + \rho_i i_{t-1} + (1 - \rho_i) (\phi_\pi (\pi_t - \pi^*) + \phi_y (\ln Y_t - \ln Y_{t-1})) + s_i e_{i,t} \quad (27)$$

On the fiscal side, the government budget constraint is:

$$G_t + i_{t-1} \frac{B_{g,t-1}}{P_t} = \tau_t^c C_t + \tau_t^n w_t N_{d,t} + \tau_t^k r_t^k \widehat{K}_t + T_t + \frac{B_{g,t} - B_{g,t-1}}{P_t} \quad (28)$$

$B_{g,t-1}$ is the stock of debt with which the government enters period t . Government expenditure plus interest payments on outstanding debt must equal tax collections plus issuance of new debt. The tax instruments follow AR(1) processes with a non-negative response to the deviation of government debt from an exogenous long run target level, B_g^* . We assume that there are no other exogenous shocks to the tax rates. Some or all of the tax instruments must react sufficiently to debt so as to satisfy a condition that the path of government debt be non-explosive:

$$\tau_t^c = (1 - \rho_c)\tau^c + \rho_c\tau_{t-1}^c + (1 - \rho_c)\gamma_c(B_{g,t-1} - B_g^*) \quad (29)$$

$$\tau_t^n = (1 - \rho_n)\tau^n + \rho_n\tau_{t-1}^n + (1 - \rho_n)\gamma_n(B_{g,t-1} - B_g^*) \quad (30)$$

$$\tau_t^k = (1 - \rho_k)\tau^k + \rho_k\tau_{t-1}^k + (1 - \rho_k)\gamma_k(B_{g,t-1} - B_g^*) \quad (31)$$

$$T_t = T + \gamma_T(B_{g,t-1} - B_g^*) \quad (32)$$

τ^c , τ^n , τ^k , and T are the steady state values of the tax rates. The autoregressive parameters are restricted such that $0 \leq \rho_l < 1$, for $l = c, n, k$, and the coefficients on lagged debt must be non-negative, $\gamma_l \geq 0$, for $l = c, n, k$. Because the exact timing of lump sum taxes is irrelevant, it is without loss of generality to not include an AR(1) term in the process for lump sum taxes. The response of lump sum taxes to lagged debt must also be non-negative, $\gamma_T \geq 0$. Government spending is assumed to follow a stationary AR(1) process in the log, with G^* the steady state level of spending. The variable $e_{g,t}$ a shock drawn from a standard normal distribution and with standard deviation s_g :

$$\ln G_t = (1 - \rho_g)G^* + \rho_g \ln G_{t-1} + s_g e_{g,t}, \quad 0 \leq \rho_g < 1 \quad (33)$$

3.4 Market-Clearing and Equilibrium

Neutral productivity and the investment shock follow stationary AR(1) processes in the log, with the non-stochastic means of the levels of the variables normalized to unity:

$$\ln A_t = \rho_a \ln A_{t-1} + s_a e_{a,t}, \quad 0 \leq \rho_a < 1 \quad (34)$$

$$\ln Z_t = \rho_z \ln Z_{t-1} + s_z e_{z,t}, \quad 0 \leq \rho_z < 1 \quad (35)$$

The preference shock also follows a stationary AR(1) in the log, where again we normalize the non-stochastic mean level to unity:

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + s_\nu e_{\nu,t}, \quad 0 \leq \rho_\nu < 1 \quad (36)$$

A competitive equilibrium is a set of prices and allocations for which households and firms behave optimally and all markets clear, given values of the exogenous processes and initial values of the endogenous states. That is, equations (9)-(17) for the household and (22)-(26) for intermediate

firms must simultaneously hold, given the monetary policy rule, (27); the process for the tax rates, (28)-(32); and the values of the exogenous variables A_t , Z_t , and G_t . In addition, the capital accumulation equation, (4), must hold. Market-clearing requires that government debt is held by the household, that total labor demand from firms be equal to total labor used in production, and that total capital services demanded by firms equals capital services supplied by the household:

$$B_{g,t} = B_t \quad (37)$$

$$N_{d,t} = \int_0^1 N_t(j) dj \quad (38)$$

$$\hat{K}_t = \int_0^1 \hat{K}_t(j) dj = u_t K_t \quad (39)$$

Combining the market-clearing conditions gives rise to the aggregate resource constraint:

$$Y_t = C_t + I_t + G_t + \Gamma(u_t) \frac{K_t}{Z_t} \quad (40)$$

The aggregate production function can be written:

$$Y_t = \frac{A_t \hat{K}_t^\alpha N_{d,t}^{1-\alpha}}{v_t^p} \quad (41)$$

Here v_t^p is a price dispersion term that arises due to staggered pricing. Given the the Calvo (1983) assumption, aggregate inflation and the price dispersion term can be written:

$$(1 + \pi_t)^{1-\epsilon_p} = (1 - \theta_p)(1 + \pi_t^\#)^{1-\epsilon_p} + \theta_p(1 + \pi_{t-1})^{\zeta_p(1-\epsilon_p)} \quad (42)$$

$$v_t^p = (1 + \pi_t)^{\epsilon_p} \left((1 - \theta_p)(1 + \pi_t^\#)^{-\epsilon_p} + \theta_p(1 + \pi_{t-1})^{-\epsilon_p \zeta_p} v_{t-1}^p \right) \quad (43)$$

The aggregate real wage evolves according to:

$$w_t^{1-\epsilon_w} = (1 - \theta_w) \left(w_t^\# \right)^{1-\epsilon_w} + \theta_w w_{t-1}^{1-\epsilon_w} (1 + \pi_t)^{\epsilon_w - 1} (1 + \pi_{t-1})^{\zeta_w(1-\epsilon_w)} \quad (44)$$

Aggregate labor supply can be expressed using (7) recursively as:

$$N_t = N_{d,t} v_t^w \quad (45)$$

Where v_t^w is a measure of wage dispersion:

$$v_t^w = (1 - \theta_w) \left(\frac{w_t^\#}{w_t} \right)^{-\epsilon_w} + \theta_w \left(\frac{w_t}{w_{t-1}} \right)^{\epsilon_w} \left(\frac{(1 + \pi_{t-1})^{\zeta_w}}{1 + \pi_t} \right)^{-\epsilon_w} v_{t-1}^w \quad (46)$$

Using this measure of wage dispersion, the value function of the representative household can be written recursively without reference to N_t :

$$V_t = U(C_t, 1 - N_{d,t}v_t^w) + \Omega(G_t) + \beta E_t V_{t+1} \quad (47)$$

In writing the value function with a subscript t , it is implicit that it is conditional on the realization of a particular state. Given initial values of the exogenous variables and endogenous states, equations (4), (9)-(17), (22)-(26), (27), (29)-(33), (34)-(36), (39)-(44), and (46)-(47) comprise an equilibrium in the variables $\{\mu_{1,t}, C_t, N_{d,t}, V_t, Y_t, v_t^p, G_t, T_t, B_{g,t}, \tau_t^c, \tau_t^n, A_t, \nu_t, w_t, mc_t, X_{1,t}, X_{2,t}, \pi_t, \pi_t^\#, i_t, w_t^\#, v_t^w, I_t, u_t, K_t, \hat{K}_t, F_{1,t}, F_{2,t}, \tau_t^k, r_t^k, q_t\}$.

3.5 Functional Forms

We assume that flow utility from consumption and leisure takes the following form:

$$U(C_t, 1 - N_t) = \frac{(C_t^\gamma (1 - N_t)^{1-\gamma})^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma > 0, \quad 0 < \gamma < 1 \quad (48)$$

This is the functional form used by Christiano, Eichenbaum, and Rebelo (2011). It is consistent with balanced growth for all permissible values of σ and γ . When $\sigma \rightarrow 1$, the utility function reverts to the popular log-log form of $\gamma \ln C_t + (1 - \gamma) \ln(1 - N_t)$. We assume that utility from government spending is logarithmic, with $\Omega(G_t) = \varphi \ln G_t$, with $\varphi > 0$.

The investment adjustment cost function takes the following form:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2, \quad \kappa \geq 0 \quad (49)$$

The resource cost of utilization takes the form:

$$\Gamma(u_t) = \Gamma_0 (u_t - 1) + \frac{\Gamma_1}{2} (u_t - 1)^2, \quad \Gamma_1 > 0 \quad (50)$$

Γ_0 must be restricted to equal $\frac{1}{\beta} - (1 - \delta)$, which insures that the normalization of steady state utilization to unity is consistent with the other optimality conditions.

3.6 Estimation

Our approach is to first calibrate a number of parameters that are closely tied to long run moments of the data and/or are difficult to estimate. We then estimate the remaining parameters via Bayesian maximum likelihood.

As a benchmark, we assume that all distortionary taxes are constant at zero, which implies that the exact mix between lump sum tax and bond finance is irrelevant. We can thus ignore the parameters governing the tax processes altogether.⁶ While this is undoubtedly unrealistic, it is fairly common to ignore distortionary tax rates in estimation of medium scale DSGE models. As we show in the robustness section, if distortionary taxes must react to finance spending increases, then

⁶In fact, without distortionary taxes it is without loss of generality to set $T_t = G_t$, so that none of the parameters of the tax processes need be given values.

the magnitudes of the output and welfare effects are altered, but their qualitative co-movements across states of the business cycle remain unaffected. Mountford and Uhlig (2009) have an extensive discussion on how different financing regimes can affect the magnitude of the output multiplier. We also assume zero trend inflation as a benchmark. The results which follow are not sensitive to this assumption.

Calibrated parameters include $\{\beta, \alpha, \delta, \epsilon_p, \epsilon_w, \varphi, \Gamma_1\}$. We set $\beta = 0.99$ and $\alpha = 1/3$. The elasticity of substitution among goods and labor are set to $\epsilon_p = \epsilon_w = 10$, which implies steady state price and wage markups of about 10 percent. We set $\delta = 0.025$. Together, this calibration implies a steady state investment-output ratio of 21 percent. We set $\varphi = 0.20$ and pick the value of steady state government spending to equal 20 percent of steady state output. The steady state share of government spending is close to its historical average value. The parameter φ is chosen somewhat arbitrarily; its value mainly affects the sign and magnitude of the welfare effect of spending shocks, but not the magnitude of the output response nor the movement of the welfare response to a spending shock across states. Finally, we calibrate $\Gamma_1 = 0.01$. This implies that the cost of utilization is nearly linear, which leads to significant amplification of shocks.

The observable variables in our estimation are the quarterly growth rates of output, investment, government spending, hours worked per capita, and the GDP implicit price deflator (e.g. inflation). Each variable covers the period 1985q1 - 2012q4. Investment is defined as the sum of expenditure on new durable consumption and private fixed investment, whereas output and government spending are the headline numbers from the NIPA tables. We use total hours worked in the non-farm business sector from the BLS as a measure of labor input, divided by the civilian non-institutionalized population aged 16 and over. These data series are all demeaned prior to estimation, since the model features no trend growth and no trend inflation. The prior and posterior distributions of the estimated parameters are given in Table 1. Overall the estimated parameter values are reasonable and are in line with the existing literature. The estimated model generates second moments that are close to their empirical counterparts. In terms of accounting for business cycle dynamics, the investment shock is by far the most important of the shocks, explaining about 50 percent of the unconditional variance of output growth. This is in line with the estimated importance of this shock in Justiniano, Primiceri, and Tambalotti (2010), among others. Shocks to the Taylor rule account for about 20 percent of the variance of output growth, neutral productivity shocks roughly 10 percent, and preference shocks about 15 percent. Government spending shocks account for the remaining 5 percent.

As part of the Bayesian estimation we use the Kalman smoother to form retrospective estimates of the states. We make use of these in an historical simulation in Section 4.3. Overall, the simulated states look quite reasonable. Consonant with the unconditional variance decomposition, negative shocks to the marginal efficiency of investment are estimated to be the primary culprit behind the three NBER-defined recessions in the estimation sample. There is a particularly large estimated decline in Z_t during the Great Recession period. To the extent to which this shock can be interpreted as a kind of proxy for a financial shock, this accords well with popular narratives of the recent downturn.

4 Quantitative Analysis

In this section we conduct quantitative analysis on the estimated model. We begin in Section 4.1 by briefly describing the solution methodology which permits us to analyze state-dependent effects of fiscal shocks. We also provide formal definitions of the output and welfare multipliers. In Section 4.2 we conduct some basic simulation analysis. We construct an historical simulation in Section 4.3 using the smoothed retrospective states from our Bayesian estimation to construct historical values of the output and welfare multipliers.

4.1 Solution Methodology and Multiplier Definitions

Given the estimated parameter values from the previous section, we solve for quantitative policy functions of the model using a second order approximation about the non-stochastic steady state. Let \mathbf{x}_t denote a stacked vector of all endogenous variables (states and controls) observed at time t , expressed in percent deviations from the non-stochastic steady state (or absolute deviations, in the case of variables already in percentage units, e.g. inflation and interest rates). The recursive expression for household welfare, V_t from (47), is included in the equilibrium conditions, which allows us to look at how welfare reacts to spending shocks. Let \mathbf{s}_t denote the vector of endogenous and exogenous state variables, also in deviation form. Let \mathbf{e}_t be a vector of shocks. The general form of the second order policy function is:

$$\mathbf{x}_t = \frac{1}{2}\mathbf{\Upsilon}_0 + \mathbf{\Upsilon}_1\mathbf{s}_{t-1} + \mathbf{\Upsilon}_2\mathbf{e}_t + \frac{1}{2}\mathbf{\Upsilon}_3(\mathbf{s}_{t-1} \otimes \mathbf{s}_{t-1}) + \frac{1}{2}\mathbf{\Upsilon}_4(\mathbf{e}_t \otimes \mathbf{e}_t) + \mathbf{\Upsilon}_5(\mathbf{s}_{t-1} \otimes \mathbf{e}_t) \quad (51)$$

The operator \otimes stands for the Kronecker product. In a more standard first order approximation all but $\mathbf{\Upsilon}_1$ and $\mathbf{\Upsilon}_2$ are matrices of zeros. The details of solving for the $\mathbf{\Upsilon}$ coefficient matrices can be found in Schmitt-Grohe and Uribe (2004).

We define the impulse response function as the change in the expected values of the endogenous variables conditional on the realization of a particular shock equal to one standard deviation in period t . In a higher order approximation the impulse responses to a shock depend on the initial value of the state, \mathbf{s}_{t-1} . Formally, the impulse response function to shock m is $\text{IRF}_m(h) = \{\mathbf{E}_t\mathbf{x}_{t+h} - \mathbf{E}_{t-1}\mathbf{x}_{t+h} \mid \mathbf{s}_{t-1}, \hat{e}_{m,t} = e_{m,t} + s_m\}$, where $h \geq 0$ is the forecast horizon. Numerically, we compute the impulse responses as follows. Given an initial value of the state, \mathbf{s}_{t-1} , we compute two sets of simulations of the endogenous variables using the same draws of shocks. In one simulation we add s_m to the realization of shock m in period t . We compute the simulations out to a forecast horizon of H , which we set to 20. We repeat this process T times, average over the realized values of the endogenous variables at forecast horizons up to H , and take the difference between the average simulations with and without the extra s_m shock in period t . We use a value of $T = 150$.⁷

The output multiplier is defined as the change in output for a one unit change in government spending. We compute the multiplier by taking the ratio of the impact response of output to the

⁷We use the pruning algorithm of Kim, Kim, Shaumburg, and Sims (2003) to ensure the stability of the simulations used to construct the impulse responses.

impact response of government spending (“impact” meaning $h = 0$). In the basic model, the impact response of output corresponds to the largest response to a spending shock at any forecast horizon. A natural way to define the welfare multiplier would be to take the ratio of the response of welfare, V_t , to the response of government spending to a spending shock on impact.⁸ A complication is that the units of welfare are not directly interpretable. We therefore define the welfare multiplier as the one period consumption equivalent change in welfare for a one unit change in government spending. To compute this, we divide the ratio of the impact response of V_t to the impact response of G_t by the steady state marginal utility of consumption; e.g. $\frac{dV_t}{dG_t} \frac{1}{U_C}$. This number gives the units of steady state consumption in the period of the shock that would yield an equivalent change in welfare to the spending shock. In making this transformation, we multiply by the marginal utility of consumption evaluated in steady state, even when the impulse responses are evaluated outside of steady state. This insures that the conversion to consumption units is constant across the state space, and that the consumption equivalent welfare multiplier is monotonically related to $\frac{dV_t}{dG_t}$.

4.2 Simulation Results

Given the approximated policy functions, we compute output and welfare multipliers by simulating impulse response functions from different values of the initial state, \mathbf{s}_{t-1} . First, we compute steady state multipliers, for which $\mathbf{s}_{t-1} = \mathbf{0}$ (since all variables are expressed in deviation form). Second, we compute multipliers from the state spaces in a typical “recession” and “expansion.” To do this, we simulate 10,000 periods of data using the estimated model. We define a “recession” as a period when output is in its lowest quintile, while a typical “expansion” is where output falls in the upper 20 percent of its simulated values. We then average over the simulated state spaces in recessions and expansions so defined, and use the average simulated state as the starting point for the impulse responses.

The results are summarized in Table 2. The table contains several rows focusing on different starting points for the initial state. The steady state output multiplier is 1.21. This is in line with, if not on the upper end of, multipliers estimated in DSGE models (see, e.g. Zubairy, 2013). Consumption and investment both decline slightly on impact in response to a surprise increase in G_t . The reason that this is compatible with a multiplier in excess of unity is that capital utilization increases, driving up expenditure on utilization, which appears on the right hand side of the resource constraint (40). The steady state welfare multiplier is 2.43. This means that the persistent increase in government spending generates an increase in welfare that is equivalent to the utility gain from a one period increase in consumption of 2.43 units per unit of government spending. The fact that the steady state welfare multiplier is positive reflects the fact that steady state government spending in the model is set lower than would be the case if the government picked spending to maximize welfare. A higher steady state share of government spending would result in a lower (or negative) welfare multiplier, while a lower government spending share would lead to higher welfare

⁸Note that the welfare multiplier defined in this section would be equal to the present discounted value of utility multipliers as defined in Section 2.

multipliers. As stressed in Footnote 1 of the Introduction, since it is impossible to know whether steady state government spending is inefficiently high or low in the real world, we do not want to make too much of the sign or magnitude of the welfare multiplier, but rather focus on how it varies across states of the business cycle.

The rows of Table 2 labeled “All Shocks” show output and welfare multipliers from simulated recessions and expansions. The output multiplier is substantially higher in a typical recession than in steady state (1.53 vs. 1.21), and lower in a typical expansion (1.02). The apparent countercyclicality of the output multiplier is consistent with empirical evidence using reduced form macroeconometric techniques in Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012), and Nakamura and Steinsson (2014). In contrast, the welfare multiplier is significantly lower in a recession and higher than steady state in an expansion. In other words, the welfare multiplier is procyclical and negatively correlated with the output multiplier. The welfare multiplier also appears to be substantially more volatile than the output multiplier. In particular, the welfare multiplier is 300 percent smaller in a recession and 200 percent bigger in an expansion relative to steady state, whereas the output multiplier is only about 25 percent larger in a recession and about 15 percent smaller in an expansion relative to steady state.

The remaining rows of Table 2 present multipliers in recessions or expansions generated by different shocks. To compute the states from which to calculate these multipliers, we again simulate 10,000 periods of data from the model, but set the variance of all but the listed shock to zero. We use the same definition of expansion and recession to construct the state from which to compute the multiplier. We find that the output multiplier is higher in a recession generated by neutral productivity, investment, or preference shocks, but is actually slightly lower in a recession caused by monetary policy shocks. The reverse pattern obtains in expansions. The welfare multiplier displays the opposite pattern of the output multiplier conditional on each shock – it is lower than steady state when the output multiplier is high, and higher than steady state when the output multiplier is low. In other words, even though the cyclicity of the output multiplier seems to be shock-dependent, the negative co-movement between the output and welfare multipliers seems to be a more general feature of the model.

Some care needs to be taken in interpreting the welfare results conditional on particular shocks. To focus on one particular shock as an example, one might think that periods when Z_t is low would be good periods to increase government spending, since private investment is relatively unproductive. But Z_t (or any other exogenous state) is not low in a vacuum. In recessions generated in the model, capital is low so labor is relatively unproductive, and consumption is low so that the marginal utility of consumption is relatively high, both of which make it a relatively bad time to increase government spending from the perspective of welfare.⁹ We present these results conditional

⁹Note that consumption is low even in a recession generated by shocks to Z_t . With the high estimated persistence of the stochastic process for Z_t , consumption actually falls on impact when Z_t declines, and continues to decline at longer forecast horizons. This fall in consumption is because of the strong wealth effect given the high estimated ρ_z . With a lower ρ_z , a substitution effect dominates and consumption and investment move in opposite directions on impact in response to a shock to Z_t , but at sufficiently long forecast horizons they both end up low after a negative shock to Z_t even when ρ_z is substantially smaller than in our baseline analysis.

on each shock only to emphasize that our results seem quantitatively fairly general, and are not driven by the parameterization of the shocks.

4.3 Historical Simulation

In this subsection, we use information available from the estimation to construct historical estimates of the actual output and welfare multipliers for the US. In particular, we use the Kalman smoother to construct retrospective “smoothed” estimates of the state variables over the estimation sample. Then, we simulate impulse responses to a government spending shock using the estimated values at each point in the estimated state space. This gives us a time series of output and welfare multipliers over the period 1985q1-2012q4.

Figure 1 plots the estimated output and welfare multipliers. The output multiplier is represented by the solid line and is measured on the left vertical axis, while the welfare multiplier corresponds to the dashed line and measured on the right vertical axis. Shaded gray regions represent recessions as defined by the NBER. It is visually apparent that the output and welfare multipliers both move significantly across time and are negatively correlated with one another. During each of the three recessions in the sample period, the output multiplier increases while the welfare multiplier declines. The output multiplier reaches its maximum value right at the end of the Great Recession, while it bottoms out at the height of the so-called “dot-com” boom in the late 1990s.

Table 3 provides summary statistics from the historical simulation. The mean output multiplier is 1.26 and the mean welfare multiplier 1.91. These are, respectively, slightly higher and lower than the steady state multipliers from Table 2. The welfare multiplier is an order of magnitude more volatile than the output multiplier. The output multiplier ranges from 1.05 to 1.46, and is negatively correlated with HP detrended real GDP as well as with first differenced log GDP. The welfare multiplier is positively correlated with both measures of the business cycle. Consonant with the first visual impression, the output and welfare multipliers are strongly negatively correlated with one another, with a correlation coefficient of -0.93.

5 Robustness

This section considers a number of robustness exercises. First, we analyze the sensitivity of our results to a passive monetary policy regime, such as the recent zero lower bound episode. Second, we consider robustness of our results to different methods of fiscal finance. Finally, we examine sensitivity to parameter values.

5.1 The Zero Lower Bound

Recent research has argued that fiscal multipliers may be substantially higher when the zero lower bound on interest rates binds (see, for example, Christiano, Eichenbaum, and Rebelo, 2011). While in our benchmark analysis we assume that the central bank obeys a standard Taylor rule, in this subsection we analyze the impact a binding zero lower bound for our results.

What matters for fiscal multipliers is not the zero lower bound on nominal interest rates binding per se, but rather a passive response of monetary policy to fiscal shocks. To simulate the effects of the zero lower bound, we therefore consider how the multipliers change under a deterministic interest rate peg. In particular, we construct impulse responses to a government spending shock in which the nominal interest rate is to be held fixed for H periods, $H \geq 0$. After this time, the central bank resumes following the Taylor type rule given in (28).¹⁰

Table 4 shows output and welfare multipliers for different peg lengths, evaluated in steady state as well as in typical simulated recessions and expansions. We use the same values of the state space to generate the recession and expansion multipliers as in Table 2. The steady state multiplier is larger under an interest rate peg than when the central bank obeys the Taylor rule, and is larger the longer is the peg. The steady state welfare multiplier is also larger under an interest rate peg than not. This follows from the expression for the utility multiplier, (2) in Section 2. That is, in a distorted economy (which this economy is because of monopoly power in price- and wage-setting), holding the state fixed, the utility multiplier from a government spending change ought to be increasing in the output multiplier. This is consistent with the analyses in Christiano, Eichenbaum, and Rebelo (2011) and Nakata (2013), who argue that it is welfare-improving to increase government spending at the zero lower bound. However, neither of these papers look at the kind of state-dependence of multipliers over the business cycle that we do. Focusing on the multipliers in recession and expansion, we see that the same kind of state-dependence emerges under an interest rate peg as in our baseline economy with a Taylor rule: the output multiplier is larger in recessions than in expansions, while the welfare multiplier is smaller in recessions and larger in expansions. Put differently, even though the levels of the output and welfare multipliers are larger under an interest rate peg, their co-movement across states and with each other is unaffected. The output multiplier remains countercyclical and the welfare multiplier procyclical.

Our benchmark historical simulation ignores the effects of the zero lower bound, which, given our analysis above, may be particularly relevant for the recent recession and recovery. Figure 2 shows the simulated values of the output and welfare multipliers over the shortened sample 2006-2012. The solid and dashed black lines show the estimated historical output and welfare multipliers, respectively, ignoring the zero lower bound. The solid and dashed light blue lines show the estimated multipliers taking the zero lower bound into account. In doing so, we assume that, starting in the final quarter of 2008 until the end of the sample, agents expected the nominal interest rate to remain pegged for the subsequent five quarters. While in retrospect the zero lower

¹⁰To implement this, we augment the monetary policy rule, (28), with anticipated policy shocks: $i_t = (1 - \rho_i)i_t^* + \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi(\pi_t - \pi^*) + \phi_y(\ln Y_t - \ln Y_{t-1})) + e_{i,t} + e_{i,1,t-1} + \dots + e_{i,H,t-H}$. In the period of a government spending shock, say period t , we numerically solve for a sequence of current and anticipated policy shocks, $e_{i,t}, e_{i,1,t}, \dots, e_{i,H,t}$ that will make the interest rate unresponsive, in expectation, to the spending shock for the current and next H periods. The effect of a government spending shock under a peg is therefore effectively the sum of the “direct” effect of the spending shock along with the effects of the current and anticipated policy shocks. The length of the peg, H , is assumed to be known by agents in the economy and is fixed. Erceg and Linde (2012) point out an issue that the length of the zero lower bound episode ought to be endogenous – for large enough fiscal stimulus, the expected duration of the zero lower bound will be lower, which works against larger fiscal multipliers under an interest rate peg. We ignore this issue.

bound episode has lasted significantly longer, in real time a five quarter peg seems reasonable. We see that both the output and welfare multipliers are higher in the simulation taking the zero lower bound into account, but otherwise follow a similar pattern to our baseline analysis. The second panel of Table 3 shows statistics for the estimated historical multipliers over the zero lower bound period (2008-2012) only. The multipliers are still negatively correlated with one another, with the output multiplier countercyclical and the welfare multiplier procyclical.

5.2 Fiscal Financing

In this subsection we move away from the unrealistic assumption that all fiscal financing comes via lump sum finance. The results are summarized in Table 5.

In the panel labeled “SS Tax,” we set the steady state tax rates on consumption, capital, and labor to 0.05, 0.10, and 0.20, respectively. However, we continue to assume that variable fiscal finance comes from lump sum taxes. This has very little effect on the magnitude of the output multiplier, evaluated either in steady state or in a recession or expansion. The magnitudes of the welfare multipliers are higher than in Table 2, though the movement across states is the same as in our benchmark analysis. The reason that the welfare multipliers are larger with positive tax rates is straightforward. With positive tax rates, the economy is even more distorted than in our benchmark case. Conditional on the initial state, a shock which leads to an output increase is therefore relatively more valuable from a welfare perspective.

The second panel of the table, labeled “SS Var. Tax 1,” continues to assume positive steady state tax rates, but now assumes that variable fiscal finance comes from changes in distortionary tax rates, not lump sum taxes. In particular, we set $\gamma_T = 0$ and $\gamma_k = \gamma_n = 0.25$, with $\gamma_c = 0$ and the autoregressive parameters in the tax processes all equal to zero as well. This means that tax rates on capital and labor will adjust in response to changes in government debt with a one period lag. We calibrate the steady state government debt target to equal 50 percent of steady state output. This setup results in a smaller steady state output multiplier and a substantially lower welfare multiplier. The reason for the lower output multiplier is straightforward – a spending increase leads to more debt, resulting in higher tax rates, which works against the stimulating effects of the spending increase. The welfare multiplier is lower for a similar reason – the spending shock triggers higher taxes, which results in even higher levels of distortion relative to the first best. The qualitative movements in the multipliers outside of steady state are the same as in our benchmark analysis: the output multiplier is larger in a recession and smaller in an expansion, with the welfare multiplier moving in the opposite direction. Interestingly, distortionary tax finance magnifies the state-dependence in both multipliers. For example, the output multiplier is about 50 percent larger in a recession compared to steady state (as opposed to a 25 percent difference in our baseline case).

The panel labeled “SS Var. Tax 2” is similar to “SS Var. Tax 1,” but assumes that $\rho_n = \rho_k = 0.90$. This means that capital and labor taxes both rise in response to a spending increase, but do so inertially and with a substantial lag. This results in a steady state output multiplier that is about the same as in the case with lump sum finance, but again works to magnify the state-dependence

of the output multiplier, with the output multiplier substantially higher in a recession and smaller in an expansion relative to steady state. The state-dependence of the welfare multiplier displays the same pattern as before. The welfare multipliers are higher in all states relative to “SS Var. Tax 2.” This results because the heightened distortions triggered by the increase in government spending occur farther off into the future when $\rho_n = \rho_k = 0.90$.

5.3 Parameter Values

Finally, we discuss the robustness of our results to different values of a subset of the parameters in the model. The results are summarized in Table 6. For these exercises we fix all non-listed parameter values at their benchmark estimated or calibrated values, and vary only the listed parameter.

The first two panels consider different levels of steady state government spending. There are only modest effects on the magnitude and state-dependence of the output multiplier. The main effects are seen in the welfare multiplier. When government spending is a higher share of steady state output than in our baseline analysis, the welfare multipliers are all much smaller. In contrast, when government spending is a lower share of output, the welfare multipliers are significantly larger. Given our parameterization of φ , government spending is either above or below its welfare-optimal level in these two cases. This makes increasing government spending either less or more valuable from a welfare perspective. The state-dependence across recessions and expansions is otherwise similar.

The third panel of the table considers $\sigma = 1$, which corresponds to the case of additively separable log-log utility over consumption and leisure. This has surprisingly little effect on the value of the output multiplier, both in steady state as well as in recessions and expansions. The lower value of σ results in much higher welfare multipliers in all states, but otherwise similar state-dependence.¹¹ More rigid wages ($\theta_w = 0.75$, as opposed to 0.5 in our baseline) has very little effect, qualitatively or quantitatively, on either the output or welfare multipliers. The output multipliers are smaller in all states when prices are more flexible ($\theta_p = 0.5$ as opposed to 0.72), but the magnitude of the welfare multipliers and movements across states are similar to our baseline.

The panel labeled $\Gamma_1 = 100$ considers the case where capital utilization is (approximately) fixed. This results in significantly lower output multipliers in all states, with the steady state output multiplier less than unity (see our discussion above about how expenditure on utilization in our baseline analysis allows the output multiplier to exceed one). The output multiplier is still higher in a recession and lower in an expansion, but the movement in the multiplier across states is smaller than in our baseline case. The welfare multiplier is again low in a recession and high in an expansion, co-moving negatively with the output multiplier.

The panel labeled $\phi_\pi = 5$ corresponds to a more active monetary policy with a stronger response to inflation. This naturally results in lower output multipliers in all states, but the movement across states, as well as the state-dependence of the welfare multiplier, is the same as in our earlier

¹¹This obtains because the lower value of σ , given our specification of preferences, changes the welfare-optimal share of government spending, which significantly impacts the magnitude of the welfare multiplier.

analysis. This result is similar to our analysis of the zero lower bound, which can be thought of as an extremely inactive monetary policy rule. The final panel considers a lower value of ρ_g , the autoregressive parameter in the government spending process. This results in higher output multipliers in all states – this is because the lower persistence in the shock limits the negative wealth effect on consumption, which allows demand to rise by more. The welfare multipliers are smaller in all states, but otherwise display the same pattern of movement across states. The smaller value of the welfare multipliers naturally results when government spending shocks are less persistent because welfare is a forward-looking construct.

6 Conclusion

The principal contribution of this paper is to study the output and welfare effects of government spending shocks in a state-dependent context. Using a second order approximation to the equilibrium conditions of conventional DSGE models, we have documented a number of interesting results. First, the output multiplier tends to be strongly countercyclical, reaching high values in recessions and being relatively low in robust expansions. Second, the welfare multiplier, defined as the one period consumption equivalent change in welfare for a one unit change in government spending, tends to display the opposite pattern. Put differently, the output and welfare multipliers tend to move opposite one another across states of the economy. The degree of state-dependence is non-trivial – in an estimated historical simulation of a medium scale model, we find that the output multiplier varies from about 1 to 1.5 ignoring the zero lower bound, and rises above 2 during the recent zero lower bound period. The welfare multiplier is substantially more volatile across states than the output multiplier, and the two multipliers are strongly negatively correlated.

There are a number of potentially fruitful avenues for future research. First, there is a growing literature that seeks to empirically identify state-dependence in fiscal multipliers using reduced form techniques. To our knowledge, ours is one of the first papers to look at state-dependence (other than at the zero lower bound) in the context of reasonably conventional DSGE models. Better linking these two literatures seems likely to yield some useful insights, both in terms of how one models state-dependence in reduced form models and in terms of how we specify DSGE models. Second, our paper (and most of the literature) has focused on unproductive government expenditure, from which households receive utility. This is a reduced form way to motivate a desire for non-zero government purchases in the first place. It would be interesting to delve deeper and model the utility benefits of spending in a less ad-hoc way. It would also be useful to look at the effects of changes in productive government expenditure. Third, one could extend our analysis to the welfare effects of changes in tax policy. Finally, there has been substantial recent interest in the output effects of fiscal policies designed to reduce debt. In future work we plan to compare how the output and welfare effects of fiscal consolidation plans differ.

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Table 1: Estimated Model Parameters

Parameter	Dist.	Prior		Posterior		
		Mean	SE	Mode	Mean	SE
b	Beta	0.700	0.100	0.7100	0.7411	0.0875
σ	Normal	2.000	0.250	1.8906	1.9117	0.2416
γ	Beta	0.500	0.100	0.2775	0.2818	0.0237
κ	Normal	4.000	0.500	1.5703	1.9711	0.8614
θ_w	Beta	0.500	0.100	0.4959	0.5170	0.1068
θ_p	Beta	0.500	0.100	0.7166	0.7255	0.0584
ζ_w	Beta	0.500	0.200	0.4767	0.4739	0.2758
ζ_p	Beta	0.500	0.200	0.0623	0.0919	0.0461
ρ_i	Beta	0.900	0.050	0.8210	0.8055	0.0506
ϕ_π	Normal	1.500	0.100	1.7372	1.7387	0.0818
ϕ_y	Normal	0.125	0.100	0.1220	0.1195	0.0503
ρ_a	Beta	0.900	0.050	0.9858	0.9778	0.0109
ρ_z	Beta	0.900	0.050	0.9989	0.9983	0.0019
ρ_ν	Beta	0.900	0.050	0.9339	0.9363	0.0195
ρ_g	Beta	0.900	0.050	0.9747	0.9687	0.0148
s_a	Inv. Gamma	0.010	0.002	0.0051	0.0051	0.0004
s_z	Inv. Gamma	0.010	0.002	0.0200	0.0206	0.0020
s_ν	Inv. Gamma	0.010	0.002	0.0270	0.0316	0.0077
s_g	Inv. Gamma	0.010	0.002	0.0085	0.0086	0.0005
s_i	Inv. Gamma	0.002	0.002	0.0029	0.0036	0.0015

Note: The log-posterior density at the mode is -1966.17. The posterior is generated with 20,000 random walk Metropolis Hastings draws with an acceptance rate of approximately 20 percent. We use quarterly measures of output, investment, government spending, hours worked per capita, and inflation. Each variable covers the period 1985q1 - 2012q4. Priors in estimation are drawn from Christiano, Eichenbaum, and Rebello (2011) and are found to be in line with the data. See Section 3.6 for a full description of the parameterization process.

Table 2: Baseline Output and Welfare Multipliers

	Output Multiplier	Welfare Multiplier
Steady State	1.2119	2.4342
Recession		
All Shocks	1.5287	-4.9127
Neutral Prod.	1.2633	2.0226
Investment	1.5185	-5.2020
Preference	1.2759	2.1528
Monetary	1.2011	2.9666
Expansion		
All Shocks	1.0242	7.1115
Neutral Prod.	1.1509	3.6184
Investment	1.0452	6.8719
Preference	1.1630	3.5644
Monetary	1.2198	2.3485

Note: This table shows output and welfare multipliers for the estimated model at different points in the state space. The row labeled “Steady State” evaluates the multipliers at the non-stochastic steady state. To generate the state from which the multipliers are evaluated in “Recession” and “Expansion,” we simulate 10,000 periods of data from the model, and average over realizations of the state vector when output is in the lower 20th percentile (Recession) and upper 20th percentile (Expansion). We compute these simulations using all shocks in the appropriately labeled rows. In other rows we conduct the simulation conditional only on the labeled shock, setting the variance of the other shocks to zero. See Section 4.1 for a full description of the model simulation process used to generate the multipliers shown above.

Table 3: Historical Output and Welfare Multipliers
Summary Statistics

	Mean	Std. Dev.	Min	Max	corr($\ln Y_t^{hp}$)	corr($\Delta \ln Y_t$)	corr(Y mult)	corr(V mult)
Baseline								
Y mult	1.26	0.11	1.05 (98q1)	1.46 (09q3)	-0.40	-0.22	1.00	-0.93
V mult	1.91	1.06	-0.10 (91q1)	3.96 (99q2)	0.40	0.18	-0.93	1.00
ZLB								
Y mult	1.94	0.10	1.76 (12q4)	2.07 (09q2)	-0.91	-0.34	1.00	-0.73
V mult	2.00	0.61	1.01 (09q4)	3.05 (11q4)	0.64	-0.14	-0.73	1.00

Note: This table shows summary statistics on the time series of estimated output and welfare multipliers based on an historical simulation of the estimated model. The row labeled “baseline” corresponds to the historical simulation based on the baseline estimation. The section labeled “ZLB” calculate multipliers on the assumption that agents expect the interest rate to remain pegged for five subsequent quarters starting in the last quarter of 2008. The statistics for these rows are only calculated using data from the 2008-2012 period.

Table 4: Multipliers under Interest Rate Peg

	Output Multiplier	Welfare Multiplier
$H = 2$		
Steady State	1.3188	2.5309
Recession	1.6481	-4.7773
Expansion	1.1293	7.1864
$H = 3$		
Steady State	1.3910	2.6057
Recession	1.7308	-4.6695
Expansion	1.1987	7.2433
$H = 4$		
Steady State	1.5159	2.7413
Recession	1.8691	-4.4779
Expansion	1.3250	7.3518
$H = 5$		
Steady State	1.7922	3.0418
Recession	2.1514	-4.0794
Expansion	1.6395	7.6211

Note: This table is constructed in the same way as Table 2. However, we here use a passive interest rate rule in which the interest rate is pegged for H periods. The “Recession” and “Expansion” multipliers are computed in the same way as in Table 2 using all shocks in the simulations to generate the state space from which to compute the multipliers. See Section 4.1 for a full description of the model simulation process used to generate the multipliers shown above.

Table 5: Multipliers, Robustness to Fiscal Financing

	Output Multiplier	Welfare Multiplier
SS Tax		
Steady State	1.1918	7.2466
Recession	1.5028	1.1429
Expansion	1.0099	11.1237
SS Tax Var. Tax 1		
Steady State	1.0417	-10.7877
Recession	1.5404	-34.6252
Expansion	0.7922	2.2079
SS Tax Var. Tax 2		
Steady State	1.2201	-10.5138
Recession	1.7605	-32.8654
Expansion	0.9725	2.1142

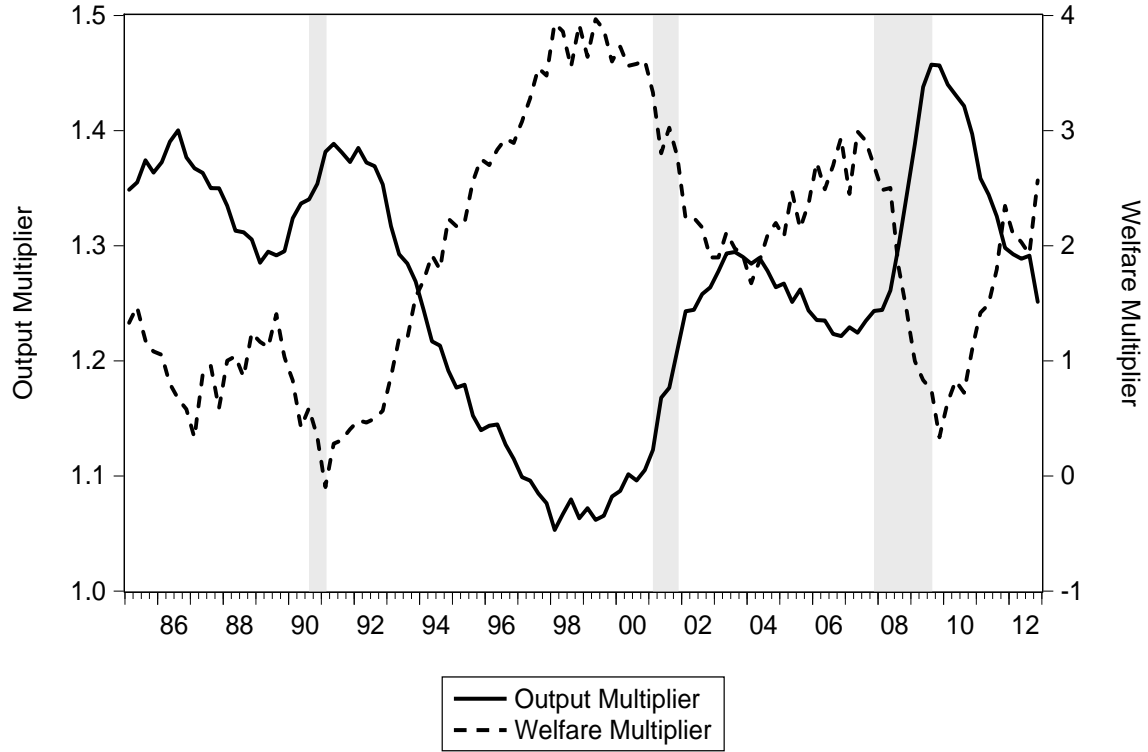
Note: This table is constructed in the same way as Table 2, but uses different assumptions about fiscal finance. The row “SS Tax” uses values of $\tau^c = 0.05$, $\tau^k = 0.10$, and $\tau^n = 0.20$, but assumes all variable finance comes through lump sum taxes. The row “SS Var. Tax 1” uses these same steady state values of tax rates, but has $g_n = g_k = 0.25$, with $g_T = 0.00$, along with $\rho_n = \rho_k = 0$. The row “SS Var. Tax 2” is the same as the former, but with $\rho_n = \rho_k = 0.90$. All rows assume a steady state debt-GDP ratio of 0.5. The “Recession” and “Expansion” multipliers are computed in the same way as in Table 2 using all shocks in the simulations to generate the state space from which to compute the multipliers.

Table 6: Multipliers, Robustness to Parameters

	Output Multiplier	Welfare Multiplier
$G/Y = 0.30$		
Steady State	1.2084	-8.6056
Recession	1.4859	-16.3930
Expansion	1.0411	-3.5791
$G/Y = 0.10$		
Steady State	1.2148	35.7108
Recession	1.5702	28.6966
Expansion	1.0067	40.1415
$\sigma = 1$		
Steady State	1.2051	28.2465
Recession	1.5360	22.6157
Expansion	1.0193	31.7796
$\theta_w = 0.75$		
Steady State	1.2088	2.4396
Recession	1.5269	-4.8779
Expansion	1.0210	7.0890
$\theta_p = 0.50$		
Steady State	1.1687	2.3819
Recession	1.4760	-4.9747
Expansion	0.9838	7.1126
$\Gamma_1 = 100$		
Steady State	0.8979	1.5814
Recession	1.1074	-6.5455
Expansion	0.7728	6.5258
$\phi_\pi = 5$		
Steady State	1.1681	2.3826
Recession	1.4755	-4.9887
Expansion	0.9834	7.1128
$\rho_g = 0.8$		
Steady State	1.3006	0.6057
Recession	1.6291	-0.7403
Expansion	1.0952	1.5189

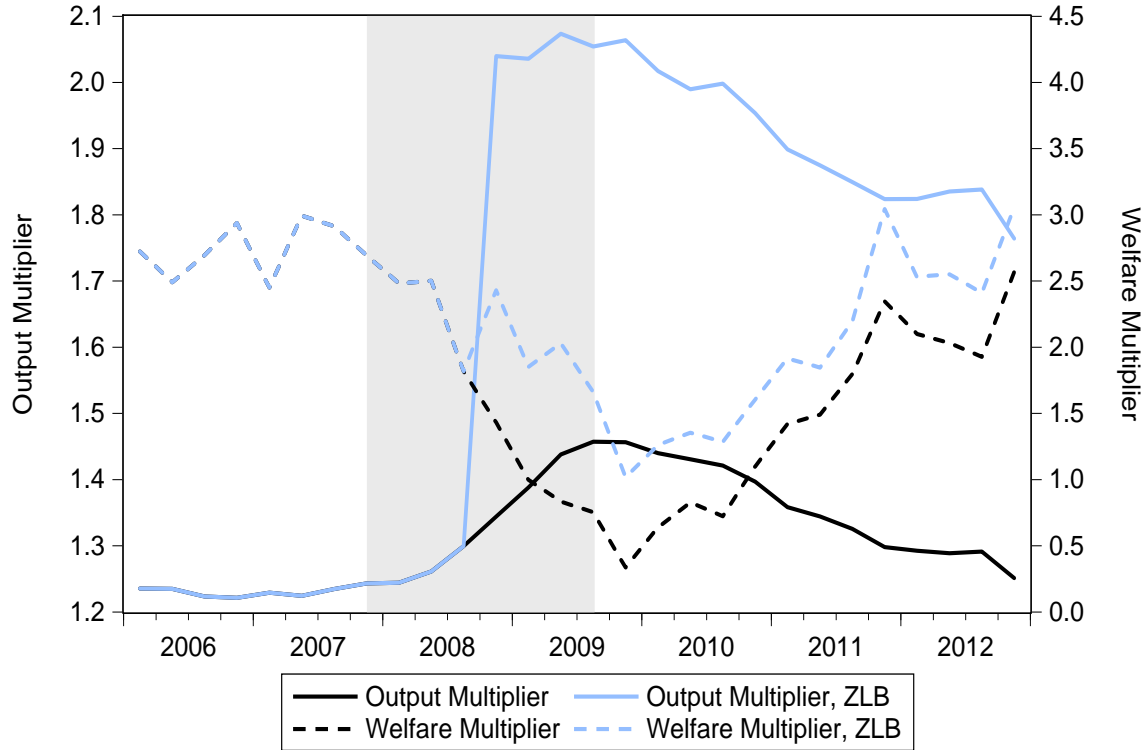
Note: This table is constructed using the baseline model laid out in Section 3 and the parameterization outlined in Section 3.6. The lone exception to the parameterization is the unique parameter we alter to generate each new set of multipliers as shown above. See Section 4.1 for a full description of the model simulation process used to generate the multipliers shown above.

Figure 1: Historical Output and Welfare Multipliers



Note: This figure plots the estimated historical output and welfare multipliers. Shaded gray regions are recessions as dated by the NBER. These simulations are constructed using the Kalman smoother from the estimated model to back out a history of states. At each point in the state space, we then compute the output and welfare multipliers. These lines show the output (solid) and welfare (dashed) multipliers, not taking the zero lower bound period into account. Summary statistics for the above figure are presented in Table 3. See section 4.3 for a full description of the simulation methodology.

Figure 2: Historical Output and Welfare Multipliers: Zero Lower Bound



Note: This figure plots the estimated historical output and welfare multipliers, taking the zero lower bound period into account. The shaded gray region is a recession as dated by the NBER. These simulations are constructed using the Kalman smoother from the estimated model to back out a history of states. At each point in the state space, we then compute the output and welfare multipliers. The black lines show the output (solid) and welfare (dashed) multipliers, not taking the zero lower bound period into account. The blue lines show the output (solid) and welfare (dashed) multipliers, taking the zero lower bound into account. To simulate the effects of the zero lower bound, we assume that agents expected the nominal interest rate to remain fixed for the subsequent five quarters at each point in time, starting in the fourth quarter of 2008. Summary statistics for the above figure are presented in Table 3. See section 4.3 for a full description of the simulation methodology.