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CAPITAL REALLOCATION AND AGGREGATE PRODUCTIVITY

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ABSTRACT

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Capital Reallocation and Aggregate Productivity*

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May 9, 2014

Abstract

This paper studies the productivity implications of the cyclical reallocation of capital. Frictions in the reallocation process are a source of factor misallocation. Cyclical movements in these frictions lead to variations in the degree of reallocation and thus in productivity. These frictions also impact the capital accumulation decision. The effects are quantitatively important in the presence of fluctuations in adjustment frictions and/or the cross sectional variation of profitability shocks. These fluctuations depend on the joint distribution of capital and plant-level productivity rather than mean values alone. Even without aggregate productivity shocks, the model has quantitative properties that resemble those of a standard stochastic growth model: (i) persistent shocks to the Solow residual, (ii) positive comovement of output, investment and consumption and (iii) consumption smoothing.

1 Motivation

Frictions in the reallocation of capital and labor are important for understanding aggregate productivity. With heterogenous plants, the assignment of capital, labor and other inputs across production sites impacts directly on aggregate productivity. Frictions in the reallocation process thus lead to the misallocation of factors of production (relative to a frictionless benchmark). This point lies at the heart of the analysis of productivity across countries in Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013) and Restuccia and Rogerson (2008).[1]

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[1] More specific differences with these and other studies are discussed below.

In this paper we consider the **cyclical dimension of reallocation** in the presence of capital adjustment costs. In important empirical contributions, Eisfeldt and Rampini (2006) and Kehrig (2011) show that capital reallocation is pro-cyclical and that the cross-sectional productivity dispersion behaves counter-cyclically.[2] This not only underlines the importance of heterogeneity in the production sector but also suggests that frictions in the adjustment to capital may produce cyclical effects on output over the business cycle. One contribution of this paper is to specify a dynamic equilibrium model to further understand these findings about cyclical reallocation and dispersion in productivities.

Not properly taking cross-sectional heterogeneity into account will also lead to a mis-measurement of total factor productivity (TFP). We are interested in the cyclical component of the output loss resulting from frictions in the adjustment process which will be reflected in a mis-measured TFP. This relates to the question how micro-frictions like physical adjustment costs translate into aggregate outcomes. **We find that if the only shocks in the economy are to aggregate TFP, then the productivity loss from costly reallocation has no cyclical element.** This is consistent with results on the aggregate implications of lumpy investment, as in Thomas (2002), Khan and Thomas (2003) and Gourio and Kashyap (2007). If an aggregate model behaves as if there were no non-convexities at the plant-level, then the distortions in the allocation of capital across plants with different productivities will matter only for aggregate *levels*. As a result, the distribution over plants' capital stock and idiosyncratic productivity can be extremely well approximated by its first moment.

In addition to shocks to TFP, we also study shocks to plants' investment opportunities as in Eisfeldt and Rampini (2006), together with shocks to the distribution of idiosyncratic productivity as in Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Gilchrist, Sim, and Zakrajsek (2013), or Bachmann and Bayer (2013). **Those shocks create cyclical movements in reallocation and productivity as well as time-varying productivity dispersion. Cross-sectional heterogeneity now plays an important role for shaping aggregate dynamics.** In the presence of those shocks, reallocation is correlated with measured aggregate productivity. The cross-sectional joint distribution over plants' capital stock and idiosyncratic productivity is a slow-moving object in this environment and tracking its evolution only by its first moment is insufficient: higher order moments are needed to characterize the outcome of the planner's problem, in particular the covariance of the cross-sectional distribution between plants' capital stocks and profitability.

[2] Eisfeldt and Rampini (2006) use dispersion in firm level Tobin's Q, dispersion in firm level investment rates, dispersion in total factor productivity growth rates, and dispersion in capacity utilization. Kehrig (2011) constructs dispersion measures based on TFP estimates using the estimation strategy in Olley and Pakes (1996).

Importantly these features of our model are interrelated. The fact that the covariance matters as a moment for determining the optimal allocation is indicative of the significance of reallocation effects. If this covariance did not matter for describing optimal allocations, for example because it is constant over time or perfectly correlated with the mean, then it could not have a cyclical effect on aggregate output. Thus the covariance that matters from the perspective of the Krusell and Smith (1998) approach is precisely the moment that reflects gains to capital reallocation.

This last point is worth stressing. Studies following Krusell and Smith (1998) routinely find that only first moments of distributions are needed to summarize cross sectional distributions. In our economy, the covariance of the cross sectional distribution between a plant's capital and its profitability is needed in the state space of the problem. When there are shocks either to the capital adjustment process or to the cross sectional distribution, this covariance evolves in response to these shocks. In the presence of such shocks the approximate solution to the planner's problem using only average capital fails: the solution requires higher order moments.

As a final exercise, we study the business cycle properties of an economy driven by shocks to adjustment rates and to the cross sectional distribution of idiosyncratic shocks assuming constant aggregate total factor productivity.[3] This exercise provides a basis for "adverse" aggregate productivity shocks and the serial correlation of the Solow residual. **The aggregate moments produced by this economy are very similar to the moments of the standard stochastic growth model.** In particular: (i) the Solow residual is pro-cyclical and positively serially correlated, (ii) consumption, investment and output are positively correlated, (iii) consumption is smoothed, (iv) reallocation is pro-cyclical and (v) the standard deviation of productivity across plants is counter-cyclical. The first three properties match those of the standard RBC model. The last two properties match those stressed by Eisfeldt and Rampini (2006) and Kehrig (2011). In our setting a reduction in the Solow residual comes from variations in the distribution of shocks, not an adverse shock to total factor productivity.

2 Frictionless Economy

To fix basic ideas and notation, start with an economy with heterogeneity and no frictions. The planner maximizes

$$V(A,K) = max_{K',k(\varepsilon)} \;\; u(c) + \beta E_{A'|A} V(A',K') \tag{1}$$

[3]This analysis shares some features with Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann and Bayer (2013). Differences and similarities are made clear in the next sections.

for all (A, K). The constraints are

$$c + K' = y + (1-\delta)K, \tag{2}$$

$$\int_{\varepsilon} k(\varepsilon) f(\varepsilon) d\varepsilon = K, \tag{3}$$

$$y = A \int_{\varepsilon} \varepsilon k(\varepsilon)^{\alpha} f(\varepsilon) d(\varepsilon). \tag{4}$$

The objective function is the lifetime utility of the representative household. The state vector has two elements: A is aggregate TFP and K is the aggregate stock of capital. There is a distribution of plant specific productivity shocks, $f(\varepsilon)$ which is fixed and hence omitted from the state vector.

There are two controls in (1). The first is the choice of aggregate capital for the next period. The second is the assignment function, $k(\varepsilon)$, which allocates the given stock of capital across the production sites, indexed by their current productivity.

At the beginning of the period, A as well as the idiosyncratic productivity shocks ε realize. While aggregate capital K requires one period time-to-build, the reallocation of existing capital takes place instantaneously and is given by $k(\varepsilon)$.

The resource constraint for the accumulation of aggregate capital is given in (2). The constraint for the allocation of capital across production sites in given in (3), where $f(\cdot)$ is the distribution function for ε.

From (4), total output, y, is the sum of the output across production sites. The production function at any site is

$$y(k, A, \varepsilon) = A\varepsilon k^{\alpha} \tag{5}$$

where k is the capital used at the site with productivity ε.[4] The idiosyncratic productivity ε is persistent, parameterized by $\rho_{\varepsilon} \in [0, 1]$. We assume $\alpha < 1$ as in Lucas (1978).[5] In this frictionless environment, a plants' optimal capital stock is entirely determined by ε.

[4]Labor and other inputs are not made explicit. One interpretation is that these inputs have no adjustment costs and are optimally chosen each period, given the state. In this case, the marginal product of labor (and other inputs) will be equal across production sites. This does not imply equality of the marginal products of capital. Adding labor adjustment, perhaps interactive with capital adjustment, would be a natural extension of our model. Presumably, adding labor frictions would enhance our results. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) include labor adjustment costs while Bachmann and Bayer (2013) assume flexible labor.

[5]As in Cooper and Haltiwanger (2006), estimates of α are routinely below unity. This is interpreted as reflecting both diminishing returns to scale in production and market power due to product differentiation. For simplicity, our model ignores product differentiation and treats the curvature as reflecting diminishing returns. The analysis in Kehrig (2011) includes product differentiation at the level of intermediate goods.

The assumption of diminishing returns to scale, $\alpha < 1$, implies that the allocation of capital across production sites is non-trivial. There are gains to allocating capital to high productivity sites but there are also gains, due to $\alpha < 1$, from spreading capital across production sites.

Let $k(\varepsilon) = \xi(\varepsilon)K$, so that $\xi(\varepsilon)$ is the fraction of the capital stock going to a plant with productivity ε. Then (4) becomes:

$$y = AK^{\alpha} \int_{\varepsilon} \varepsilon \xi(\varepsilon)^{\alpha} f(\varepsilon) d(\varepsilon) = AK^{\alpha}(\mu + \phi) \tag{6}$$

where $\mu = \bar{\varepsilon} \int_{\varepsilon} \xi(\varepsilon)^{\alpha} f(\varepsilon) d(\varepsilon)$ and $\phi = Cov(\varepsilon, \xi(\varepsilon)^{\alpha})$.[6] As is well understood from the Olley and Pakes (1996) analysis of productivity, aggregate output will depend on the covariance between the plant-level productivity and the factor allocation.

In the frictionless economy with time invariant distribution $f(\varepsilon)$ and costless reallocation of capital, this covariance is constant so that the joint distribution of plant-specific capital and ε is not part of the state vector. As this analysis progresses, this will not always be the case.

2.1 Optimal Choices

Within a period, the condition for the optimal allocation of capital across production sites is given by $\alpha A \varepsilon k(\varepsilon)^{\alpha-1} = \eta$ for all ε, where η is the multiplier on (3). This condition is intuitive: absent frictions, the optimal allocation equates the marginal product of capital across production sites.

Working with this condition,

$$k(\varepsilon) = \frac{\eta}{\alpha A \varepsilon}^{\frac{1}{\alpha-1}}. \tag{7}$$

Using (3),

$$\eta = A\alpha K^{\alpha-1} \left(\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon \right)^{1-\alpha}. \tag{8}$$

The multiplier is the standard marginal product on an additional unit of capital times the effect of the ε distribution on productivity.

Putting these two conditions together,

$$k(\varepsilon) = K \frac{\varepsilon^{\frac{1}{1-\alpha}}}{\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon}. \tag{9}$$

Substituting into (4) yields

[6] This uses $E(XY) = EX \times EY + cov(X, Y)$, where $\bar{\varepsilon}$ is the mean of the plant-specific shock.

$$y = AK^{\alpha}\left(\int_{\varepsilon}\varepsilon^{\frac{1}{1-\alpha}}f(\varepsilon)d\varepsilon\right)^{1-\alpha}. \tag{10}$$

This is a standard aggregate production function, AK^{α}, augmented by a term that captures a "love of variety" effect from the optimal allocation of capital across plants. With a given distribution $f(\cdot)$ the idiosyncratic shocks magnify average aggregate productivity as the planner can reallocate inputs to the more productive sites.

The condition for **intertemporal optimality** is $u'(c) = \beta EV_K(A', K')$ so that the marginal cost and expected marginal gains of additional capital are equated. Using (1), this condition becomes

$$u'(c) = \beta Eu'(c')\left[(1-\delta) + A'\alpha K'^{\alpha-1}\left(\int_{\varepsilon}\varepsilon^{\frac{1}{1-\alpha}}f(\varepsilon)d\varepsilon\right)^{1-\alpha}\right]. \tag{11}$$

The left side is the marginal cost of accumulating an additional unit of capital. The right side is the discounted marginal gain of capital accumulation. Part of this gain comes from having an extra unit of capital to allocate across production sites in the following period. The productivity from these production sites depend ons two factors, the **future** values of: aggregate productivity, A' and the cross sectional distribution of idiosyncratic shocks, $f(\varepsilon)$.

The choice of k for each plant within a period is independent of the choice between consumption and saving. The planner optimally allocates capital to maximize the level of output and then allocates output between consumption and capital accumulation. Clearly, once we allow for limits to reallocation, the capital accumulation decision will depend upon the future allocation of capital across production sites. In this way, variations in the distribution of $f(\cdot)$ can impact on the capital accumulation choice.

2.2 Aggregate Output and Productivity

For this economy, there is an interesting way to represent total output. This is seen from defining

$$\tilde{A} \equiv A\int_{\varepsilon}\varepsilon k(\varepsilon)^{\alpha}f(\varepsilon)d\varepsilon \tag{12}$$

so that

$$y = \tilde{A}K^{\alpha}. \tag{13}$$

from (4).

Researchers interested in measuring TFP from the aggregate data will typically uncover $\tilde{A}$ rather than A. This is the mis-measurement referred to earlier. As the discussion progresses, we will refer to $\tilde{A}$ as the Solow residual, as distinct from aggregate TFP.[7] There are three

[7] Thanks for Susanto Basu for urging us to make these terms clear.

factors which influence $\tilde{A}$. The first one is A. The influence of A, aggregate TFP, on $\tilde{A}$, measured TFP, the Solow residual is direct and has been central to many studies of aggregate fluctuations. Second, the distribution $f(\varepsilon)$. Variations in $f(\varepsilon)$ influence $\tilde{A}$ because variations in the cross sectional distribution of the idiosyncratic shocks lead to different marginal productivities of plants and thus changes in the Solow residual. Finally, there is the allocation of factors, k. If factors are optimally allocated, then the distribution of capital over plants does not have an independent effect on $\tilde{A}$. However, the existence of frictions may imply that, in a static sense, capital is not efficiently allocated. In that case, even with $f(\varepsilon)$ fixed, the reallocation process will lead to variations in $\tilde{A}$.[8]

Starting with Olley and Pakes (1996), many researchers have recognized the dependence of aggregate productivity on factor allocation. In many studies the underlying frictions are due to policies which influence steady state productivity across countries.[9] Our analysis differs from these studies in a couple of important ways. We next focus on (i) frictions through adjustment costs to capital, (ii) dynamic inefficiency brought about through the adjustment process so that the magnitude of the inefficiency and thus aggregate productivity are endogenous and (iii) the behavior of aggregate productivity over business cycles.

3 Capital Adjustment Costs

The allocation of capital over sites with heterogeneous idiosyncratic productivity has important effects on measured total factor productivity. In a frictionless economy there are no cyclical effects of reallocation on productivity. However, there is ample evidence in the literature for both non-convex and convex adjustment costs. Introducing these adjustment costs will enrich the analysis of productivity and reallocation.[10]

There are two distinct frictions to study, corresponding to the two dimensions of capital adjustment. The first is "costly reallocation" in which the friction is associated with the allocation of capital across the production sites. The second is "costly accumulation" in which the adjustment cost refers to the cost of accumulating rather than allocating capital.

Our focus here lies on studying the presence of costs to the reallocation (assignment) process. We introduce a special type of adjustment costs that is very tractable, although

[8]This decomposition of productivity taken from Olley and Pakes (1996) highlights the interaction between the distribution of productivity and factors of production across firms. Gourio and Miao (2010) use a version of this argument, see their equation (45), to study the effects of dividend taxes on productivity. Khan and Thomas (2006) study individual choice problems and aggregation in the frictionless model with plant specific shocks. Basu and Fernald (1997) also discuss the role of reallocation for productivity in an aggregate model.

[9]Bartelsman, Haltiwanger, and Scarpetta (2013) discuss these other studies in their analysis of productivity differences over 24 economies.

[10]In contrast to Midrigan and Xu (2010), there are no borrowing frictions. They argue that these frictions do not create large losses in aggregate productivity.

not very informative about the source of the friction. Following Calvo (1983) and more recently adopted to study investment decisions by Sveen and Weinke (2005), assume that each period a Bernoulli draw determines the fraction $\pi \in [0, 1]$ of plants the planner can costlessly reallocate capital between. This represents a stochastic investment opportunity. The remaining fraction of plants $1 - \pi$ produces with its beginning-of-period capital stock. This structure of adjustment costs captures the fact that plants adjust their capital stock infrequently. Applying a law of large numbers, the plant-specific shocks ε are assumed to be equally distributed over the fractions π and $1 - \pi$ of adjustable and non-adjustable plants. The two distributions of plants will be referred to as F^a and F^n. This also implies that $E(\varepsilon)$ is time-invariant and the same across adjustable and non-adjustable plants.

By assumption, π is not dependent on the state of the plant. This simplification makes our analysis tractable. At the same time it does not preclude a role for the cross sectional distribution in the state space of the problem. Besides tractability, there are other arguments for this specification.

First, a model with just non-convex adjustment costs, or a mixture of non-convex and quadratic adjustment costs, as in Cooper and Haltiwanger (2006), captures inaction and bursts of investment but misses small adjustments. While not as elegant as the state dependent adjustment model, the constant hazard structure does generate inaction, bursts of investment as well as smaller adjustment rates. A similar point about price adjustment is used in Midrigan (2011) to justify a constant adjustment rate specification[11]

Second, the focus of our analysis is on (12): the impact of the cross sectional distribution of profitability shocks on the Solow residual and thus output. The constant hazard assumption allows us to isolate the effects of the cross sectional distribution through its effects on the allocation of capital and hence output rather than through adjustment costs alone. This does not deny the significance of adjustment costs but rather focuses solely on the output effects of the cross sectional distribution. There is an important cost to this specification: there is no option value of waiting. In a model with non-stochastic fixed costs, if adjustment is not made in the current period, it is available for sure in the next one. Once adjustment costs are stochastic, the option value of waiting is reduced.

3.1 The Planner's Problem

For the dynamic program of the planner in the presence of adjustment costs, the state vector contains aggregate productivity A, the aggregate capital stock K, and Γ. The high-dimensional object Γ describes the joint distribution over capital (at the start of the period) and productivity shocks across plants. Γ is needed in the state vector because the presence of adjustment costs implies that a plant's capital stock may not reflect the current draw of ε. As

[11]See also Costain and Nakov (2013).

noted above, there is time variation in the probability of adjustment π. Furthermore, there are shocks to the variance of idiosyncratic productivity shocks, parameterized by λ. Changes in the variance of the cross-sectional idiosyncratic productivity, as recently highlighted in Bloom (2009) and Gilchrist, Sim, and Zakrajsek (2013), have an effect on output. Such changes can be interpreted as variations in uncertainty. Consider a mean-preserving spread (MPS) in the distribution of ε. In a frictionless economy such a spread would incentivize the planner to carry out more reallocation of capital between plants because capital can be employed in highly productive sites. Let $s = (A, K, \Gamma, \lambda, \pi)$ denote the vector of aggregate state variables. Note the assumed timing: changes in the distribution of idiosyncratic shocks are known in the period they occur, not in advance.[12] The adjustment status of a plant is given by $j = a, n$, where a stands for 'adjustment', while n stands for 'non-adjustment'.

Given the state, the planner makes an investment decision K' and chooses how much capital to reallocate across those plants whose capital stock can be costlessly reallocated, $(k, \varepsilon) \in a$. Let $\tilde{k}_j(k, \varepsilon, s)$ for $j = a, n$ denote the capital allocation to a plant that enters the period with capital k and profitability shock ε in group j after reallocation. The capital of a plant in group $j = a$ is adjusted and is optimally set by the planner to the level $\tilde{k}_a(k, \varepsilon, s)$. The capital of a plant in group $j = n$ is not adjusted so that $\tilde{k}_n(k, \varepsilon, s) = k$.

The choice problem of the planner is:

$$V(A, K, \Gamma, \lambda, \pi) = max_{\tilde{k}_a(k,\varepsilon,s),K'} \; u(c) + \beta E_{[A',\Gamma',\lambda',\pi'|A,\Gamma,\lambda,\pi]} V(A', K', \Gamma', \lambda', \pi') \tag{14}$$

subject to the resource constraint (2) and

$$y = \int_{(k,\varepsilon)\in F^a} A\varepsilon \tilde{k}_a(k, \varepsilon, s)^\alpha d\Gamma(k, \varepsilon) + \int_{(k,\varepsilon)\in F^n} A\varepsilon \tilde{k}_n(k, \varepsilon, s)^\alpha d\Gamma(k, \varepsilon), \tag{15}$$

which is simply (4) split into adjustable and non-adjustable plants. Here F^j is the set of plants in group $j = a, n$. The fraction of plants whose capital stock can be adjusted is equal to π

$$\int_{(k,\varepsilon)\in F^a} f(\varepsilon) d\varepsilon = \pi \tag{16}$$

and the amount of capital over all plants must sum to total capital K:

$$\pi \int_{(k,\varepsilon)\in F^a} \tilde{k}_a(k, \varepsilon, s) d\Gamma(k, \varepsilon) + (1 - \pi) \int_{(k,\varepsilon)\in F^n} \tilde{k}_n(k, \varepsilon, s) d\Gamma(k, \varepsilon) = K. \tag{17}$$

[12] Other models, such as Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), include future values of λ in the current state as a way to generate a reduction in activity in the face of greater uncertainty about the future. We include the implications of this alternative timing as part of the results below.

As the capital is plant specific, it is necessary to specify transition equations at the plant level. Let $i = \frac{K'-K}{K}$ denote the gross investment rate so that $K' = (1-\delta+i)K$ is the aggregate capital accumulation equation. To distinguish reallocation from aggregate capital accumulation, assume that the capital at **all** plants, regardless of their reallocation status, have the same capital accumulation. The transition for the capital (after reallocation) this period and the initial plant-specific capital next period is given by

$$k_j'(k,\varepsilon,s) = (1-\delta+i)\tilde{k}_j(k,\varepsilon,s), \tag{18}$$

for $j = a, n$. Due to the presence of frictions $\tilde{k}_a(k,\varepsilon,s)$ is not given by (9). Notice that A affects unadjustable and adjustable plants in the same way. This implies that the optimal reallocation decision will occur independently of A. The shock to A will have an effect on the mis-measured part of TFP only in the presence of a capital accumulation problem, since the total amount of capital in adjustable and non-adjustable plants may differ.

The quantitative analysis will focus on reallocation of capital, defined as the fraction of total capital that is moved between adjustable plants within a period. Following a new realization of idiosyncratic productivity shocks, the planner will reallocate capital from less productive to more productive sites. Aggregate output is thus increasing in the amount of capital reallocation.

As $\tilde{k}_a(k,\varepsilon,s)$ denotes the post-reallocation capital stock of a plant with initial capital k, the plant-level reallocation rate would be $r(k,\varepsilon,s) = |\frac{\tilde{k}_a(k,\varepsilon,s)-k}{k}|$. Aggregating over all the plants who adjust, the aggregate reallocation rate is

$$R(s) \equiv 0.5 \int_{(k,\varepsilon)\in F^a} r(k,\varepsilon,s)d\Gamma(k,\varepsilon). \tag{19}$$

The multiplication by 0.5 is simply to avoid double counting flows between adjusting plants.

3.2 Joint Distribution of Capital and Productivity

In the presence of reallocation frictions, the state space of the problem includes the cross sectional distribution, Γ. Consequently, when making investment and reallocation decisions the planner needs to forecast Γ'. It is computationally not feasible to follow the joint distribution of capital and profitability shocks over plants, we represent the joint distribution by several of its moments. These forecast the marginal benefit of investment.

The right set of moments is suggested by the following expression for aggregate output, taken from (15)

$$y = \pi(\bar{\varepsilon}\mu_a + \phi_a) + (1-\pi)(\bar{\varepsilon}\mu_n + \phi_n), \tag{20}$$

where $\mu_j \equiv E(\tilde{k}_j(k, \varepsilon, s)^\alpha)$ and $\phi_j \equiv Cov(\varepsilon, \tilde{k}_j(k, \varepsilon, s)^\alpha)$, for $j = a, n$. Instead of Γ we retain μ_n and ϕ_n in the state vector of (14).

These two moments contain *all* the necessary information about the joint distribution of capital and profitability among non-adjustable plants. The information about capital in plants in F^A is not needed since capital in those plants can be freely adjusted, independently of their current capital stock. Together, μ_n and ϕ_n are sufficient to compute the output of those plants whose capital cannot be reallocated and thus to solve the planner's optimization problem. Note that by keeping μ_n and ϕ_n in the state space, we are not *approximating* the joint distribution over capital and productivity since the two moments can account for all the variation of the joint distribution. This feature of our choice of moments allows us to compare it with common approximation techniques in the spirit of Krusell and Smith (1998).

The covariance term ϕ_n is crucial for understanding the impact of reallocation on measures of aggregate productivity. If the covariance is indispensable in the state vector of the planner, then the model is not isomorphic to the stochastic growth model. That is, if the covariance is part of the state vector, then the existence of heterogeneous plants along with capital adjustment costs matters for aggregate variables like investment over the business cycle.

When either A or π is stochastic, it is possible to follow the evolution of these moments analytically.[13] The choice of $\tilde{k}_a$ for adjustable plants, along with the respective ε shocks at these plants, maps into values of the moments μ_a and ϕ_a. Together with the new realization of exogenous shocks at the beginning of the next period these map into the next period moments μ_n' and ϕ_n'. The laws of motion for the two states μ_n and ϕ_n are given by

$$\mu_n' = \pi' \mu_a + (1 - \pi')\mu_n \tag{21}$$

and

$$\phi_n' = \pi' \rho_\varepsilon \phi_a + (1 - \pi')\rho_\varepsilon \phi_n. \tag{22}$$

Together these laws of motion define the law of motion of the joint distribution Γ, allowing us to follow the evolution of this component of the aggregate state.[14] Equations (20)-(22) permit us to study the trade-off regarding the optimal allocation of capital across sites. The planner can increase contemporaneous output by reallocating capital from low- to high-productivity sites in F^a. This will increase the covariance between profitability and capital, ϕ_a, while at the same time decreasing μ_a because $\alpha < 1$. A fraction $1 - \rho_\pi$ of currently adjustable plants will not be able to adjust its capital stock tomorrow. The planner therefore has to trade off the higher instantaneous output from reallocation with the higher probability of a mismatch

[13]The analytics hold for the evolution of the mean, (21), but not the covariance, (22), when λ is stochastic.

[14]Note that $\phi' = Cov(k(\varepsilon)^\alpha, \varepsilon')$ is an expectation. The term ε' is made up of two components, one is the persistent part, and one is an i.i.d. part, denoted η. Rewrite $\varepsilon' = \rho_\varepsilon \varepsilon + (1 - \rho_\varepsilon)\eta$ to obtain $\phi' = Cov(k(\varepsilon)^\alpha, \rho_\varepsilon \varepsilon + (1 - \rho_\varepsilon)\eta) = \rho_\varepsilon \phi$.

between $\tilde{k}_n(k, \varepsilon, s) = k$ and the realization of ε' for plants in F^n tomorrow. This is captured in the laws of motion (21) and (22).

3.3 Stationary Equilibria

To fix ideas we can analyze the stationary economy where π and λ are not varying over time. In this environment a stationary distribution Γ^* exists. Using (21) it follows that $\mu_n = \mu_a = \mu^*$. Furthermore, stationary values ϕ_a^* and ϕ_n^* exist. Using (22) one can show that ϕ_n converges to

$$\phi_n^* = \phi_a^* \frac{\pi \rho_\varepsilon}{1 - (1 - \pi)\rho_\varepsilon}. \tag{23}$$

Hence (20) becomes

$$y = \bar{\varepsilon}\mu^* + \Lambda \phi_a^*, \tag{24}$$

where $\Lambda \equiv \frac{\pi}{1-(1-\pi)\rho_\varepsilon}$ is a function of parameters. Λ is (weakly) increasing in both π and ρ_ε.[15] Intuitively, an increase in π increases total output because more plants' capital stock can be costlessly adjusted. An increase in ρ_ε, the persistence of idiosyncratic productivity shocks, implies that the probability of a plant switching status and being non-adjustable with a mismatch between ε and k is decreased.[16]

Figure 1 shows equilibrium values of μ^* and ϕ_a^* in stationary economies for different values of π. As $\pi \to 0$ the planner reallocates less capital between plants. A value of $\mu^* = 1$ implies $\phi_a^* = 0$, because $k(\varepsilon) = 1$ for all sites, meaning that the capital level is independent of ε. On the other hand, as the fraction of adjustable plants increases, ϕ_a^* increases.

4 Quantitative Results

With exogenous movements in π and λ no stationary distribution of Γ exists and the two moments μ_n and ϕ_n become part of the state vector. This problem can no longer be solved analytically. This section presents quantitative results.

In the stationary economy, reallocation effects only mattered for aggregate **levels**. When are reallocation effects likely to play a role for aggregate **dynamics**? One key prerequisite is that the economy be subject to shocks that cause the distribution Γ to move over time. Without movements in Γ the benefits from reallocation are constant and the covariance term

[15] Formally, $\frac{\partial \Lambda}{\partial \pi} = \frac{1-\rho_\varepsilon}{[1-(1-\pi)\rho_\varepsilon]^2} \geq 0$, $\frac{\partial \Lambda}{\partial \rho_\varepsilon} = \frac{\pi(1-\pi)}{[1-(1-\pi)\rho_\varepsilon]^2} \geq 0$. The cross-derivatives are given by $\frac{\partial^2 \Lambda}{\partial \rho_\varepsilon \partial \pi} = \frac{\partial^2 \Lambda}{\partial \pi \partial \rho_\varepsilon} = \frac{1}{[1-(1-\pi)\rho_\varepsilon]^2} - \frac{2\pi}{[1-(1-\pi)\rho_\varepsilon]^3}$.

[16] In the extreme case of iid shocks to idiosyncratic productivity shocks the planner would be more reluctant to allocate large amounts of capital to high-productivity sites, decreasing aggregate output.

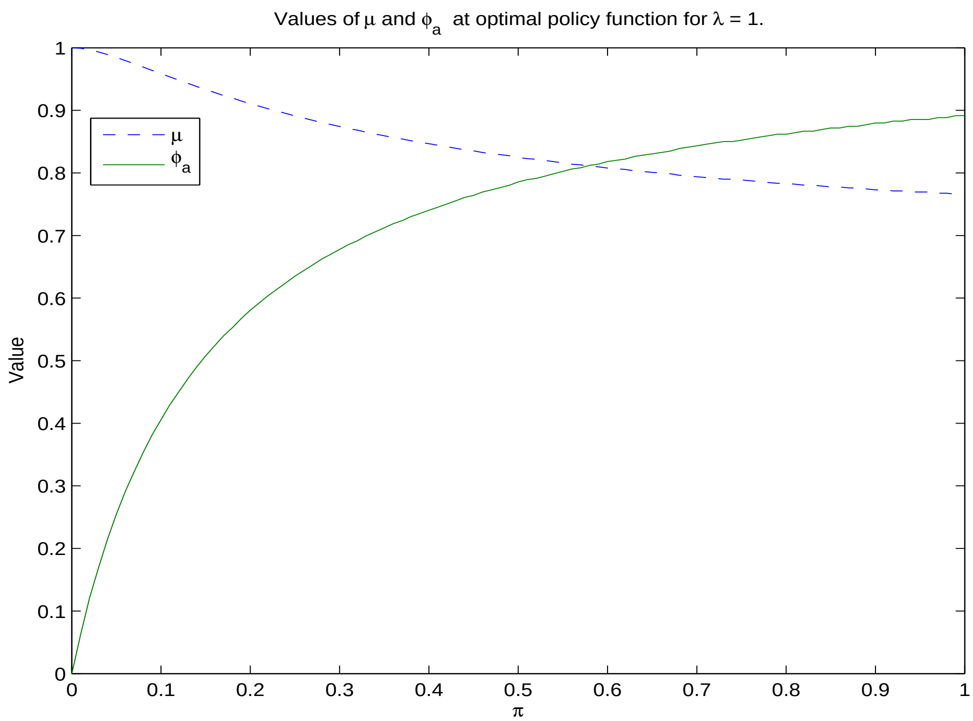

Figure 1: Values of μ and ϕ_a in stationary equilibrium for various π. Economy with $\lambda = 1$ and $\rho_\varepsilon = .9$

ϕ is not required to forecast Γ'. The reasons why Γ may vary and the implications of its variability will be clear as the analysis proceeds.

In keeping with the distinction noted earlier between reallocation and accumulation, the initial quantitative analysis, presented in section 4.1 is for an economy with a fixed capital stock, thus highlighting reallocation. The economy is then enriched to allow for capital accumulation in section 4.2.

For each of these models, this section focuses on the effects of capital reallocation on aggregate productivity. In addition, we present evidence on whether higher order moments are needed in the solution of the planner's optimization problem in Section 5. As highlighted in the introduction, these two themes are connected: higher order moments are needed to follow the evolution of Γ precisely when capital reallocation matters for the cyclical movements in productivity.

We solve the model at a quarterly frequency, using these **baseline** parameters. Following the estimates in Cooper and Haltiwanger (2006), we set $\alpha = 0.6$.[17] We assume log-utility and a depreciation rate $\delta = 0.025$. Assuming an annual interest rate of 4% implies a discount factor $\beta = 0.987$. We set the mean of π to $\bar{\pi} = 0.5$. This implies that plants adjust their

[17]This curvature is 0.44 in Bachmann and Bayer (2013) and 0.4 in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012).

capital stock on average every two quarters. Sveen and Weinke (2005) treat changes in the capital stock of under 10% in absolute value as maintenance and hence use $\pi = 0.08$. In our setup, the choice of π mainly affects aggregate *levels*, not transitions. Aggregate profitability takes the form of an AR(1) in logs

$$\ln a_t = \rho_a \ln a_{t-1} + \nu_{a,t}, \qquad \nu_a \sim N(0, \sigma_a), \tag{25}$$

where $\rho_a = 0.9$ and $\sigma_a = 0.005$. Idiosyncratic profitability shocks are log-normal and evolve according to a law of motion with time-varying variance

$$\ln \varepsilon_t = \rho_\varepsilon \ln \varepsilon_{t-1} + \lambda_t \nu_{\varepsilon,t}, \qquad \nu_{\varepsilon,t} \sim N(0, \sigma_\varepsilon). \tag{26}$$

The parameters of the idiosyncratic shock process are $\rho_\varepsilon = 0.9$ and $\sigma_\varepsilon = 0.2$. The parameter λ governs the mean-preserving spread of the normal distribution from which idiosyncratic profitability ε is drawn. It has a mean of 1 and variance σ_λ

$$\lambda_t = \rho_\lambda \lambda_{t-1} + \nu_{\lambda,t}, \qquad \nu_{\lambda,t} \sim N(1, \sigma_\lambda). \tag{27}$$

We set $\rho_\lambda = 0.82$ as in Gilchrist, Sim, and Zakrajsek (2013). Finally, the process of π follows

$$\pi_t = \rho_\pi \pi_{t-1} + \nu_{\pi,t}, \qquad \nu_{\pi,t} \sim N(\bar{\pi}, \sigma_\pi), \tag{28}$$

with $\rho_\pi = 0.9$. In order to be able to compare the effect of different shocks, the standard deviations of the innovations, $\sigma_\pi = 0.03$ and $\sigma_\lambda = 0.014$ are set to generate the same amount of variation in output as shocks to A. Section 4.4 explores the sensitivity of our findings to this parameterization. The number of plants is set at 10,000 for these simulations. The computational strategy is discussed in further detail in the Appendix.

4.1 Capital Reallocation

Table 1 shows measures of the efficiency of the allocation of capital and the cyclicality of the Solow residual. These two aspects of the economy are inherently linked. Aggregate productivity is endogenous and responds to changes in the amount of capital reallocated.

The column labeled 'R/R^*' for 'Reallocation' measures the time series average of the cross-sectional reallocation of capital across plants as defined in (19), relative to the frictionless benchmark without adjustment costs. The column labeled $E_t(\sigma_i(arpk_{it}))$ measures the time series average of the cross sectional standard deviation of the average revenue product of capital. The column labeled G shows the output gap, defined as $G(s) = \frac{y^{FL}(s)-y(s)}{y^{FL}(s)}$, output in state s relative to the frictionless benchmark.[18] The column labeled $\sigma(\tilde{A}/A)$ reports

[18] The frictionless output $y^{FL}(s)$ is a function of s because changes in λ affect the output achieved in the frictionless case.

the standard deviation of the Solow residual relative to TFP. The columns $C(R, \tilde{A})$ and $C(\sigma_i(arpk_{it}), \tilde{A})$ show the correlation between the Solow residual and respectively capital reallocation and the standard deviation of the average revenue product of capital. These two columns provide a link back to the facts, noted in the introduction, about the cyclical behavior of reallocation and dispersion in productivity.

The first block of Table 1 reports results for the frictionless economy. The second block of results introduces capital adjustment costs.

Case	R/R^*	$E_t(\sigma_i(arpk_{it}))$	G	$\sigma(\tilde{A}/A)$	$C(R, \tilde{A})$	$C(\sigma_i(arpk_{it}), \tilde{A})$
		Frictionless				
nonstochastic	1 (0)	0 (0)	0 (0)	0 (0)	*na* (−)	*na* (−)
stochastic A	1 (0)	0 (0)	0 (0)	0 (0)	*na* (−)	*na* (−)
stochastic λ	1 (0)	0 (0)	0 (0)	0.083 (0.002)	0.950 (0.006)	*na* (−)
		Frictions				
nonstochastic	0.491 (0)	1.09 (0)	0.106 (0)	0 (0)	*na* (−)	*na* (−)
stochastic A	0.491 (0)	1.09 (0)	0.106 (0)	0 (0)	*na* (−)	*na* (−)
stochastic π	0.487 (0.008)	1.09 (0.007)	0.110 (0.001)	0.077 (0.004)	0.995 (0.004)	−0.977 (0.002)
stochastic λ	0.491 ($3.68e{-}06$)	1.09 (0.01)	0.106 ($7e{-}05$)	0.064 (0.001)	0.936 (0.005)	0.929 (0.006)
stochastic π, λ	0.492 (0.005)	1.09 (0.006)	0.108 (0.0008)	0.10 (0.003)	0.817 (0.012)	−0.194 (0.03)

Table 1: Capital Reallocation Model: Productivity Implications

Results from 100 simulations with T=1000, standard deviations in parentheses below. $\frac{R}{R^*}$ measures the time series average of the cross-sectional reallocation of capital across plants, relative to the frictionless benchmark, R^*. $E_t(\sigma_i(arpk_{it}))$ is the mean standard deviation of the average revenue product of capital. G refers to the output gap relative to the frictionless benchmark. The column $\sigma(\tilde{A}/A)$ shows the standard deviation of measured vs. real TFP. The last columns $C(R, \tilde{A})$ and $C(\sigma_i(arpk_{it}), \tilde{A})$ show the correlation between mismeasured TFP and respectively capital reallocation and the standard deviation of the average revenue product of capital. The "*na*" entry means that the correlation is not meaningful as one of the variables is constant.

4.1.1 Frictionless Economy

The first row of Table 1 shows the results for the frictionless economy, $\pi = 1$, without time series variations in TFP, the volatility of the idiosyncratic shocks λ, or the fraction of adjustable sites π.[19] This case serves as a benchmark. Without frictions, the marginal product of capital is equalized across plants and our measure of the inefficiency of the capital allocation, $E_t(\sigma_i(arpk_{it}))$, is zero. The first-best output is achieved. The mis-measurement of TFP is constant. The amount of capital reallocation is time-invariant and hence plays no role for aggregate productivity.

The second row, 'stochastic A' introduces variation in aggregate profitability. Variations in A have no effect on the reallocation of capital in this economy, because the planner reallocates capital across plants within a period. Consequently the amount of reallocation is the same as without variations in A. The allocation is efficient, $\tilde{A}$ varies only with A. The only difference with respect to the benchmark in the previous row is the variability of output, which is driven by changes in aggregate profitability. Since A enters total output multiplicatively all variation in output stems from variation in A. There is no endogenous propagation. As before, the amount of capital reallocation is time-invariant.

The third row 'stochastic λ' presents results for the frictionless economy with stochastic variance of idiosyncratic productivity shocks. The parameter λ is chosen to generate the same coefficient of variation of output as the previous case.[20] The resulting allocation has the same rate of reallocation as the benchmark and the cross sectional distribution of the average revenue product of capital is degenerate. Importantly, output and the Solow residual vary with λ, as shown in column $\sigma(\tilde{A}/A)$. This represents a pure reallocation effect through changes in $f(\varepsilon)$ and occurs even under constant A and π. The second to last column shows the high correlation between the amount of capital reallocation and output. The correlation is not equal to one because following a shock to λ, the subsequent change in the planner's chosen allocation of capital produces an overshooting of output. This is a result of the allocation of capital among non-adjustable plants.

This economy presents the simplest case where reallocation is the sole driver of business cycles. To some degree, it looks like an economy driven by exogenous TFP. Here the variations in productivity arise from the endogenous reallocation of capital. The following subsection studies environments where capital adjustment costs amplify this feature.

4.1.2 Costly Capital Reallocation

Setting $\pi < 1$ introduces capital adjustment costs to the frictionless economy, so that only a fraction of all plants' capital stocks can be adjusted within a given period. Costly capital

[19] In this abbreviated problem, the planner solves $V(\Gamma) = max_{k(\varepsilon)} \; u(c) + \; \beta EV(\Gamma')$ subject to the resource constraint (2) and total production given by (15).

[20] For this case, λ takes values between 0.966 and 1.0344. These values are chosen to generate the same amount of output volatility as direct shocks to A. Below we study the implications of larger variability in λ. Note that $\lambda > 1$ can imply that some values of the shock become negative. To avoid this, we apply the MPS to the underlying normal distribution and re-adjust its mean such that mean of the log-normal is preserved.

reallocation will have effects on measured productivity and its cyclical properties.

When π is non-stochastic and there are no other aggregate shocks, a stationary joint distribution Γ exists, with the moments (μ_n, ϕ_n) constant, as was shown in Section 3.3 above. Table 1 shows the results for this case in the row labeled 'nonstochastic'. In this economy the fraction of capital reallocated is far below the frictionless benchmark, as indicated in the second column. With $R < \pi$, the planner's chosen distribution of capital over adjustable plants is different from the distribution in the frictionless case. Although capital in a fraction π of plants could be costlessly reallocated, the reallocation rate is less than π. Instead, reallocation is lower indicating a reduced capital flow beyond the direct influence of $\pi < 1$.

Figure 2 plots capital reallocation as a function of π. The dashed line is the 45° line. The concave solid green line above it shows capital reallocation between adjustable plants (as a fraction of the frictionless benchmark). As $\pi \to 1$ it approaches the allocation derived in (9). For total capital reallocation (plotted as the red solid line beneath the 45° line) this implies that it approaches π as $\pi \to 1$.

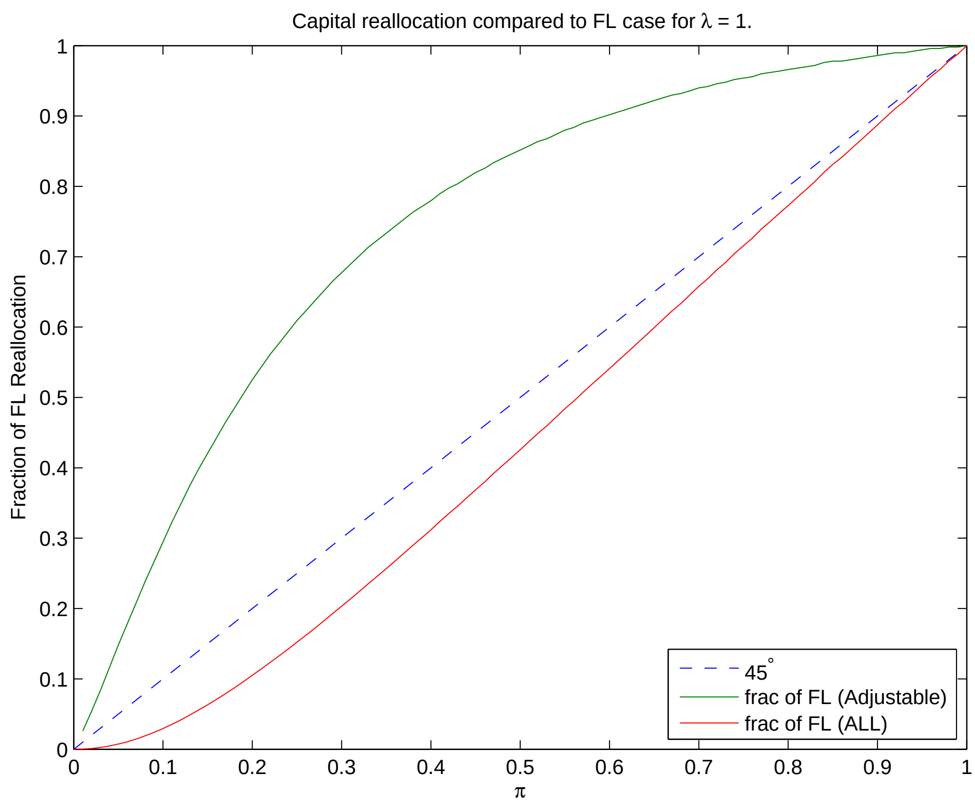

Figure 2: Capital Reallocation in adjustable and all plants as fraction of frictionless benchmark in stationary equilibrium for various π. Economy with $\lambda = 1$ and $\rho = .9$.

The inefficiency of the allocation when $\pi < 1$ is highlighted by the column labeled

$E_t(\sigma_i(arpk_{it}))$. This measure of the inefficiency of the allocation is larger than zero, reflecting frictions in the reallocation process that stem from two sources. First, the planner chooses not to equalize marginal products between adjustable plants, reflecting the tradeoffs discussed above. Secondly, the marginal products of capital among non-adjustable plants exhibit a high degree of heterogeneity due to the fact that their capital is fixed despite a new realization of idiosyncratic profitability. Because ϕ_n and μ_n converge to their steady-state values output does not vary in this economy. The output gap is positive, directly reflecting the impact of $\pi < 1$. Importantly, the mis-measurement in TFP is constant over time, we only obtain a level-effect.

The row labeled 'stochastic A' allows for randomness in aggregate productivity with constant π. As explained above, the amount of reallocation is independent of variations in A. Output and $\tilde{A}$ vary only with A. Because $\pi < 1$ the allocation is characterized by a positive standard deviation of average revenue products of capital and a positive output gap.

Variations in π create time series variation in the moments μ_n and ϕ_n, as shown in the row 'stochastic π'. Fluctuations in π lead to pro-cyclical capital reallocation patterns, as shown in column $C(R, \tilde{A})$. But this is not *simply* a correlation. In the presence of adjustment frictions, reallocation **causes** the observed time-variations in output. Variations in π therefore also lead to variations in (mis-measured) total factor productivity. The marginal products of capital are not equalized across plants, neither among the adjustable nor the unadjustable sites. This results in a positive output gap which varies with the evolution of μ_n and ϕ_n. This gap is about 11% of real GDP. Additionally, this economy exhibits counter-cyclical productivity dispersion, as seen in the last column. When π is low, less capital can be reallocated between adjustable plants. This decreases output and increases the standard deviation of marginal products between those plants. Though λ is held fixed, $\sigma_i(arpk_{it})$ nonetheless varies over time.

The row 'stochastic λ' of Table 1 studies the effects of time-variation in $f(\varepsilon)$ under costly capital reallocation. Due to the presence of adjustment costs, the marginal products of capital cannot be equalized over time. In addition, the variations in λ lead to changes in the optimal allocation decision by the planner and create considerable time-variation in μ_n and ϕ_n. The resulting fluctuations in output stem from different reallocation choices of the planner that show up in variations of the Solow residual. While variations in π affect output directly through the fraction of plants among which capital can be reallocated, the effect of changes in λ is less direct. Variations in λ induce different reallocation choices but a fraction of the effect on output comes from the fact that the marginal revenue product of capital is changed through productivity draws with larger or smaller tails. As the last two columns show, shocks to λ lead to pro-cyclical reallocation patterns. At the same time they produce a pro-cyclical dispersion in average revenue products of capital. A larger spread in the distribution of shocks leads to more reallocation of capital among adjustable plants by the planner and hence higher output. At the same time the increase in dispersion leads to a larger standard deviation of the marginal products of capital, both among adjustable and non-adjustable plants. This results is driven by the probability of a mismatch between k and ε' for plants in F^n.

The joint effects of changes in π and λ are presented in the last row of Table 1. Output varies significantly over time, with variations resulting directly from both shocks to π and λ. While $\pi < 1$ leads to a positive output gap the presence of a stochastic λ causes additional variation in this gap as was the case before. Notably, mis-measured TFP exhibits significantly more time variation than in the cases of varying λ or varying π alone. This is the result of changes in π and λ jointly affecting the slow-moving joint distribution Γ. Importantly, the correlation between capital reallocation and output is much lower in this environment. This comes about because mis-measured TFP reacts more strongly through changes in λ than π. On the other hand, both exogenous shocks affect the amount of reallocation. The effect of varying π on reallocation, however, is predominantly an extensive margin effect, as a changing fraction of plants can reallocate capital. The effect of λ is on the intensive margin: more capital is reallocated within a given fraction of adjustable plants. Together this explains the observed decrease in the correlation between reallocation and output.

Overall, adjustment frictions reduce reallocation, generating a non-degenerate distribution of average (and marginal) products of capital across plants. The cost is a reduction in output of about 11%, relative to the frictionless benchmark. In all of the experiments, reallocation is pro-cyclical. For these cases, measured variations in TFP are the consequence of reallocation rather than true variations in aggregate productivity. Variations in π lead to counter-cyclical productivity dispersion across firms.

The economy with variations in both π and λ mimic the patterns of pro-cyclical reallocation and counter-cyclical dispersion emphasized by Eisfeldt and Rampini (2006). This will be a leading case as the analysis proceeds.

4.2 Endogenous Capital Accumulation

With endogenous capital accumulation, solving (14), the capital *reallocation* process has significant interactions with the capital *accumulation* decision. The frictions exert a level effect on the optimal capital stock and induce different dynamics following an exogenous shock. As we saw above, reallocation behaves cyclically in the presence of time-series variation in π and/or λ. Variations in λ and π affect the instantaneous value of existing capital and, because of persistence, the expected future return to capital, too. This affects the planner's incentives to invest. Even absent any frictions to capital accumulation the dynamics of investment and consumption are considerably altered by the presence of exogenous shocks to reallocation or the variance of the idiosyncratic shock.

Adding endogenous capital accumulation does not alter the results on the reallocation process shown in Table 1. The reason parallels the argument for the independence of reallocation from A. From (10), total output is proportional to AK^{α}. Thus just as variations in A scale moments, so will variations in K. Consequently, the analysis focuses on the effects of frictions in reallocation on capital accumulation.

Table 2 summarizes results for the endogenous capital accumulation problem, using the baseline parameters, defined earlier. The aggregate capital stock is now endogenous and creates additional variation. The average capital stock (relative to the frictionless benchmark) is shown in the $\bar{K}/\bar{K}^*$ column. The other columns report correlations of reallocation with investment and output, $C(R,i)$ and $C(R,y)$ and the correlation of investment and the Solow residual, $C(\tilde{A},i)$.

Case	$\bar{K}/\bar{K}^*$	$C(R,i)$	$C(R,y)$	$C(\tilde{A},i)$
Frictionless				
nonstochastic	1 (0)	*na* (−)	*na* (−)	*na* (−)
stochastic A	1 (0)	*na* (−)	*na* (−)	*na* (−)
stochastic λ	1 (0)	0.94 (0.01)	0.90 (0.01)	0.99 (0.001)
Frictions				
nonstochastic	0.75 (0)	*na* (−)	*na* (−)	*na* (−)
stochastic A	0.75 (0)	*na* (−)	*na* (−)	0.955 (0.09)
stochastic π	0.75 (0.005)	0.97 (0.01)	0.91 (0.003)	0.97 (0.005)
stochastic λ	0.75 (0.0006)	0.93 (0.01)	0.88 (0.01)	0.979 (0.001)
stochastic π, λ	0.75 (0.003)	0.790 (0.01)	0.767 (0.02)	0.964 (0.01)

Table 2: Endogenous Capital Accumulation: Aggregate Moments

Results from 100 simulations with T=1000, N=10,000 are reported with standard deviations in parentheses below. Simulations with frictions were computed with a mean of π equal to 0.5, mean of $\lambda = 1$, a ρ of 0.6, N=10,000 plants. $\bar{K}/\bar{K}^*$ reports the average capital stock relative to the frictionless benchmark. $C(R,i)$ is the correlation between reallocation and investment, $C(R,y)$ is the correlation between reallocation and output, and $C(\tilde{A},i)$ is the correlation between mis-measured TFP and investment. The "*na*" entry means that the correlation is not meaningful as one of the variables is constant.

From Table 2, the interaction of costly reallocation and accumulation is evident in a number of forms. First, $\bar{K}$, which is the average capital for a particular treatment, depends on the nature and magnitude of the capital adjustment costs. Even in the absence of any aggregate shocks, the capital stock is around 25% lower when there are adjustment frictions compared to the frictionless case. This comparison of the average capital stocks with and without frictions stands regardless of the source of the shocks.

Second, the addition of the shocks increases the variability of capital. With shocks to both π and λ the standard deviation of the capital stock is considerably higher than when there are only exogenous productivity shocks.

Third, capital accumulation is positively correlated with both reallocation and the Solow residual. An increase in λ, for example, leads to an increase in investment, reallocation and output. The correlation of reallocation and investment, $C(R, i)$, is informative about the effects of frictions on the incentive to accumulate capital.[21] An increase in π say, will imply that more plants are able to adjust and for this reason alone reallocation will increase. With π correlated, it is likely that more plants will be able to adjust in the future, so investment increases too. The magnitude of this correlation is smaller when only λ is random. Though the same fraction of plants adjusts each period, the gains to adjustment are larger when λ is high. This generates a positive correlation between reallocation and investment.

Finally, reallocation is pro-cyclical in the presence of shocks to either π or λ. This returns to one of the themes of the paper. If variations arise from either changes in the fraction of adjusting plants, through π, or by a change in the spread of the shocks, through λ, output responds. The key to this response is reallocation: the effects on output of getting the right amount of capital into its most productive use. This is captured through $\tilde{A}$.

4.3 Impulse Response Functions

Figures 3 and 4 show impulse response functions for negative shocks to π and λ. The shocks occur in period $t = 5$. The x-axes show time, while the y-axes in panels 2-4 shows the % deviation from the unconditional mean. The drop in the exogenous shock of interest is plotted in the first panel, while all other exogenous shocks are set to their unconditional means.

We first discuss Figure 3. The second panel shows the evolution of the two moments μ_n and ϕ_n. The negative correlation between the two series is very high, as changes in π effect the evolution of μ_n and ϕ_n in very similar ways. The third panel illustrates the co-movement between reallocation 'R' and the Solow residual. Following the shock to π less capital can be reallocated between plants, which directly affects $\tilde{A}$. The effects on output and investment are negative, as the last panel shows. Consumption, though, increases in response to the innovation to π, as discussed further below.

Figure 4 shows the effects of a negative shock to λ. The second panel shows the evolution of the two moments μ_n and ϕ_n. The sharp drop in ϕ_n is a direct effect of the shock to λ,

[21] For the nonstochastic and stochastic A models, this correlation is not defined as capital reallocation is constant.

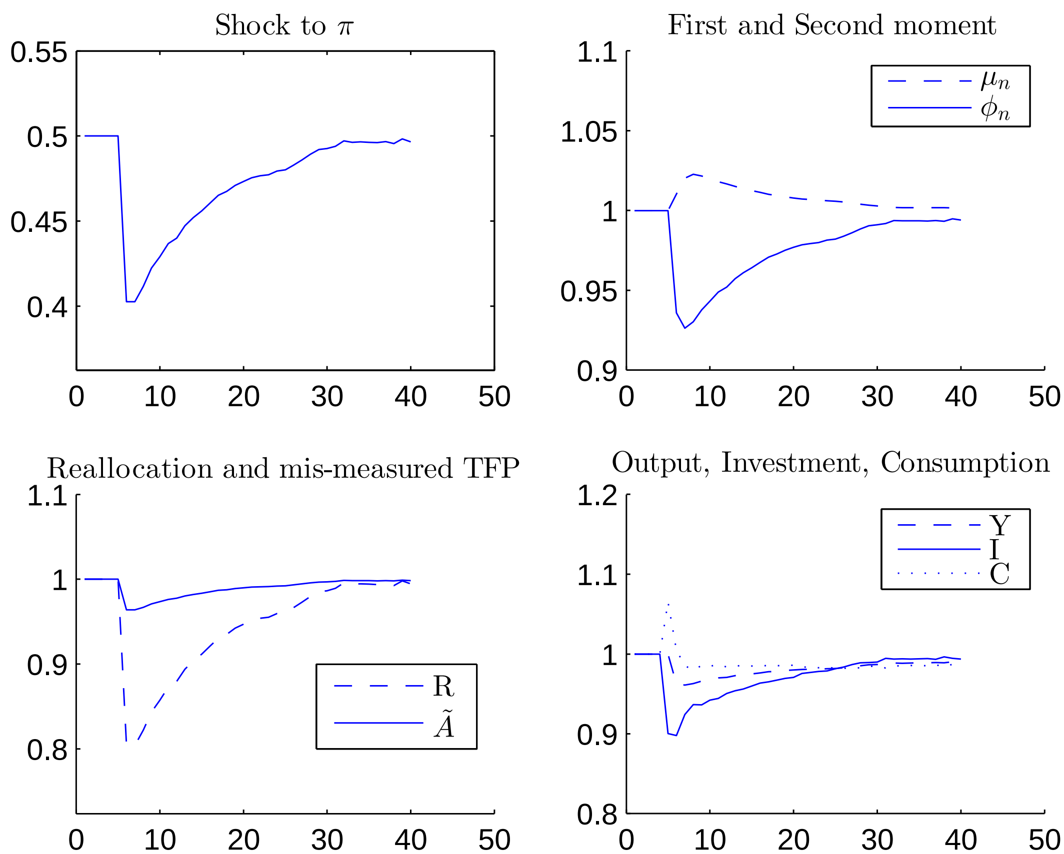

Figure 3: Variations in π: Impulse Response Functions. The y-axes show % deviations from unconditional means.

whereas the increase in μ_n reflects the effects of different reallocation choices. The panel highlights that Γ is a slow moving state variable, implying that μ_n and ϕ_n do not adjust immediately to their new values following a change in λ. Furthermore, the variations in λ have different effects on ϕ_n (direct) and μ_n (indirect), making the two moments imperfectly correlated. Variations in λ produce more cyclicality in ϕ_n than in μ_n.

Panel 3 shows the connection between mis-measured TFP and reallocation, which leads to a cyclical effect on output. In this economy with time-varying idiosyncratic uncertainty in the presence of adjustment costs there is a strong cyclical dimension of capital reallocation. Reallocation is driving time-variations in output.

Output and investment both fall in response to a negative shock to λ. The investment response is quite strong: when λ falls investment opportunities are reduced. Output falls as well due to the reduced dispersion in productivity across plants. These effects are driven by the "love of variety" aspect of the production technology. The large decrease in investment coupled with a smaller reduction in output implies that consumption increase at the time of

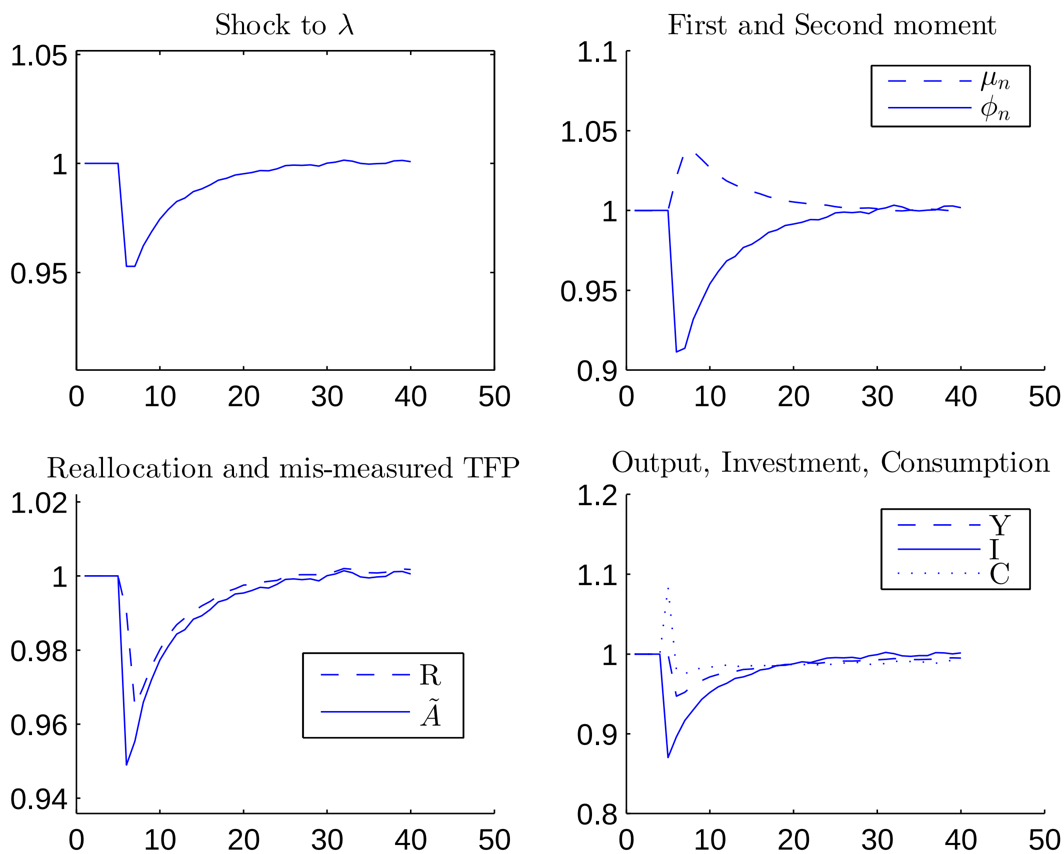

Figure 4: Negative shock to λ: Impulse Response Functions. The y-axes show % deviations from unconditional means.

the shock. We return to this point later.

These responses **do not** include the fall in output associated with an *increase* in the dispersion of shocks, as emphasized in Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and others. As noted above, this reflects a couple of features of our environment: (i) the timing of the shock to λ, (ii) the model of adjustment costs and (iii) the specification of the production function. Nonetheless, as indicated above, the model with both shocks, i.e. the stochastic (π, λ) case, is able to match the two key observations of pro-cyclical reallocation and a counter-cyclical dispersion in capital productivity.

4.4 Robustness

The previous results illustrated a couple of themes. First, variations in either π or λ are necessary to generate cyclical movements in reallocation, with resulting effects on mis-measured TFP. Second, evolution of the cross sectional distribution generated dynamics only in the stochastic π and/or λ cases. This is illustrated by the fact that higher order moments are relevant in the planner's optimization problem and the evolution of these moments are seen in the impulse response functions.

Parameter changes	R/R^*	$E_t(\sigma_i(arpk_{it}))$	G	$\sigma(\tilde{A}/A)$	$C(R, \tilde{A})$	$C(\sigma_i(arpk_{it}), \tilde{A})$
Frictions						
Baseline	0.492 (0.005)	1.09 (0.006)	0.108 (0.0008)	0.10 (0.003)	0.817 (0.012)	−0.194 (0.03)
$\alpha = 0.8$	0.498 (0.008)	2.33 (0.02)	0.123 (0.0006)	0.25 (0.007)	0.52 (0.04)	0.475 (0.03)
$\bar{\pi} = 0.3$	0.283 (0.005)	1.23 (0.005)	0.207 (0.001)	0.145 (0.004)	0.945 (0.004)	−0.246 (0.04)
$\bar{\pi} = 0.9$	0.899 (0.005)	0.353 (0.01)	0.014 (0.0002)	0.088 (0.003)	0.54 (0.03)	−0.247 (0.05)
$\rho_\pi = 0.5$	0.492 (0.001)	1.09 (0.004)	0.107 (0.0003)	0.07 (0.001)	0.659 (0.01)	0.486 (0.025)
$\rho_\varepsilon = 0.5$	0.429 (0.008)	1.45 (0.01)	0.248 (0.002)	0.105 (0.004)	0.965 (0.002)	−0.487 (0.03)
$\sigma_\lambda = 0.1$	0.491 (0.006)	1.14 (0.02)	0.11 (0.002)	0.49 (0.001)	0.692 (0.01)	0.762 (0.02)
$\sigma_\lambda = 0.1$, $\rho_\lambda = 0.5$	0.491 (0.006)	1.14 (0.01)	0.112 (0.0006)	0.341 (0.006)	0.554 (0.019)	0.609 (0.02)
timing	0.498 (0.007)	0.930 (0.01)	0.106 (0.002)	0.10 (0.006)	0.84 (0.02)	−0.78 (0.02)

Table 3: Capital Reallocation: Robustness

Model with stochastic π and λ. Standard deviations in parentheses.

This section studies the robustness of these findings to alternative values of key parameters. Table 3 reports our findings. It has the same structure as Table 1. The first column indicates the model. The baseline is the case with adjustment costs and stochastic (π, λ) taken from Table 1.

In the second row we show the effects of moving α from 0.60 to 0.80. The increase in the curvature of the revenue function leads to a larger output gap and a higher degree of misallocation. This result is largely driven by the non-adjustable plants: The column R/R^* shows that reallocation among adjustable plants is higher than in the benchmark scenario.

The baseline model assumes $\bar{\pi} = 0.5$. The third and fourth rows of Table 3 study the implications of lower and higher adjustment rates. Not surprisingly, the reallocation rate is increasing in π, as frictions are lower. This is consistent with Figure 2. The correlation of reallocation and mis-measured TFP is positive, though lower than in the baseline at $\pi = 0.90$.

The standard deviation of actual to mismeasured TFP also varies with $\bar{\pi}$. When $\bar{\pi}$ is high, the response of the planner to a variation in λ is to reallocate capital so that $\sigma(\tilde{A}/A)$ is small compared to the case of low $\bar{\pi}$. This is reflected in the mean standard deviation of the average revenue product of capital.

The table includes two rows in which the serial correlation of shocks is set to 0.5, lower than their baseline values of $\rho_\pi = 0.9$ and $\rho_\varepsilon = 0.9$. Relative to the baseline, the reduction in the serial correlation of π leads to a reduction in the cyclicality of reallocation. With adjustment opportunities less correlated, the costs of reallocating resources that are subsequently mismatched with productivity is higher. Hence reallocation is less correlated with $\tilde{A}$. This will imply that the correlation of reallocation and investment is lower than in the baseline reflecting the costs of accumulating capital when future adjustment costs are less certain.

When ρ_ε is decreased, the planner has fewer incentives to reallocate capital among adjustable plants. Consequently, the amount of capital reallocation falls and the inefficiency of the solution becomes more pronounced. This can be seen in the larger standard deviation of the marginal products of capital and in a higher output gap.

The row labeled $\sigma_\lambda = 0.1$ increases the variability of λ relative to the baseline where $\sigma_\lambda = 0.014$. This spread is closer to that in Bloom (2009) and Gilchrist, Sim, and Zakrajsek (2013). Not surprisingly, this extra volatility in the spread of idiosyncratic shocks leads to much more volatility in $\tilde{A}$ relative to the baseline. Reallocation remains pro-cyclical though less compared to the baseline.

The next row shows how a reduction in the serial correlation of λ given the high variance of λ influences these moments. With a lower serial correlation of the shocks to λ, the correlation between reallocation and $\tilde{A}$, though still positive, is considerably lower than the baseline. With less persistent shocks, reallocation is less responsive to variations in λ and π.

The last row is a modification to the model that influences the extent of the "love of variety effect". The row labeled "timing" assumes that the planner knows of a change in the cross sectional distribution of the idiosyncratic shocks one period in advance. That is, the future value of λ is in the current state space. This is the timing used in Bloom (2009) as a way to emphasize the uncertainty effects of a change in the distribution. In our environment, the change in timing has some modest effects relative to the baseline. There is less dispersion in the average product of capital but this dispersion is more negatively correlated with $\tilde{A}$ compared to the baseline. With the alternative timing assumption the planner reallocates more capital when λ is known to remain high, and less capital when λ is known to remain low. This increases the counter-cyclicality of the dispersion and leads to an allocation of capital that is on average closer to the frictionless benchmark.

5 Approximation

The previous section showed that the covariance ϕ *matters* for determining the optimal capital allocation. The problem in (14) includes Γ, the joint distribution of (k, ε). Using the first two moments of this distribution, μ_n and ϕ_n, the evolution of Γ can be tracked perfectly. This is important for the planner, who has to forecast the expected future output from non-adjustable plants, y_n'. Variations in π and λ generate movements in Γ and hence in y_n. Capital reallocation is tightly linked to changes in the mis-measurement of TFP when stochastic shocks are present.

Movements in Γ may not be captured well by the first moment μ_n alone. In the frictionless case the two moments were perfectly correlated, but this perfect correlation is broken by the existence of time-variation in the adjustment probability π and/or λ. The impulse response functions above showed that both in the case of shocks to π or λ the two moments μ and ϕ were strongly correlated. However, different shocks imply different magnitudes of change in μ, ϕ, and output. A change in λ produces a stronger reaction in ϕ and a smaller reaction in μ compared to a shock in π. Output changes of the *same* magnitude can therefore occur at the same time as *different* changes in μ. This produces the reduced explanatory power of the first moment μ. The significance of reallocation effects is related to the forecasting power of ϕ_n.

Relative to the literature starting with Krusell and Smith (1998), this is an important finding. **In particular, this result is distinguished from preceding papers in that for our environment the approximation of the cross sectional distribution requires higher order moments.**

This section makes two points. First, it emphasizes the importance of including the higher order moments in the state vector. From this we can determine how well the evolution of Γ could be captured by different subsets of its moments under different cases of stochastic π and λ.

Second, we compare the aggregate outcome of the model against a standard stochastic growth model. This allows us to determine to what extent the reallocation effects influence cyclical properties of the model.

5.1 Goodness of Fit

Table 4 evaluates the importance of the higher order moments.[22] To understand this table, let "DGP" refer to a data set (and moments) created by solving the baseline model (with stochastic π and λ) using (μ, ϕ) in solving the planner's problem. In (14), the planner forecasts y_n', the output from non-adjustable plants next period. The correctly specified regression model including both moments is given by

$$y_{n,t}^{DGP} = \beta_0 + \beta_1\mu_{n,t} + \beta_2\phi_{n,t} + \beta_3 s_t + \varepsilon_t, \tag{29}$$

[22]Only the stochastic model with frictions is explored. The case of "stochastic A" is not of interest as the higher order moments did not matter. For these experiments, the shocks are held fixed to isolate the effects of the approximation.

where s_t includes π_t and λ_t. Estimation results in $\hat{\beta}_0 = 0$, $\hat{\beta}_1 = 1.6487 = \bar{\varepsilon}$, $\hat{\beta}_2 = 1$, and $\hat{\beta}_3 = 0$ with an $R^2 = 1$. The maximum forecast error (MCFE) is zero. As discussed in Den Haan (2010) a problem of R^2 measures to assess the approximation is that observations generated using the true law of motion are used as the explanatory variable. We construct a series $\hat{\hat{y}}_n$ which is using only the approximate law of motion. The forecast error is defined as $\hat{\hat{\varepsilon}}_{t+1} = |\hat{\hat{y}}_{n,t+1} - y_{n,t+1}|$, and the MCFE is the maximum of this series.

Below we study three cases (experiments). The first takes output of the non-adjusting plants from the DGP and regresses it on an intercept, the exogenous state, and the first moment only. Thus this exercise is about approximating the nonlinear solution with a linear representation. The regression model for the linear approximation is given by (29) where we force $\beta_2 = 0$. From Table 4, the linear representation is very accurate if only π is stochastic. When λ is random, the resulting movements in the distribution of shocks leads to much greater significance of the cross sectional distribution in forecasting (decisions do not change in this experiment).

The second case actually solves the planner's problem under the (false) assumption that the model is linear. The resulting decision rules and expectations are model consistent by construction, but not data consistent.[23] The goodness of fit measure is computed from a regression of the output of the non-adjusting plants in the DGP using the model consistent estimators from the linearized approximation. As before, the linear beliefs in the stochastic π case are approximately consistent with the outcome. Again this is not the case when λ is random. For this experiment, the linear forecast rule leads to very different allocative decisions by the planner. Consequently, the R^2 is quite low – movements in the cross sectional distribution are very important.

In the third case, the planner uses the DGP to obtain a linear approximation of the law of motion. With this representation, the planner solves the optimization problem. In this case, the expectations about the evolution of the state vector is consistent with the data, but not with the model. Here, none of the experiments generate a good fit. The planner is simply unable to capture the nonlinear movements in the economy with a linear approximation of the law of motion.

5.2 Comparison to the RBC Model

This section compares the aggregate properties of our model with those of the RBC model. There are two motivations for this exercise.

First, one of the key findings of Thomas (2002) and the literature that followed was the near equivalence between the **aggregate moments** of a model with lumpy investment

[23]The R^2 from the forecast of μ in the linearized version of the model typically exceeds 0.99. In this sense, the solution is internally consistent.

Case	R^2	MFCE
Truth, approximated		
Stochastic π	0.9907 (−)	0.031% (−)
Stochastic λ	0.966 (−)	1.37% (−)
Stochastic π, λ	0.94 (−)	2.5% (−)
Linear, consistent		
Stochastic π	0.9908 (−)	0.3954% (−)
Stochastic λ	0.6958 (−)	0.7289% (−)
Stochastic π, λ	0.7032 (−)	1.707% (−)
Linear using DG truth		
Stochastic π	0.94 (−)	1.52% (−)
Stochastic λ	0.82 (−)	1.339% (−)
Stochastic π, λ	0.948 (−)	1.78% (−)

Table 4: Different approximation strategies

The first column shows the R^2 of a regression of output from non-adjustable plants on an intercept and the first moment, μ only. The second column reports the maximum forecast error from such a regression.

and the aggregate implications of a real business cycle model with quadratic adjustment costs at the plant-level. This sub-section returns to that theme. Given that higher order moments matter in the planner's optimization problem, it is natural to conjecture that the non-convexities also matter for aggregate moments.

Second, a standard criticism of the RBC model is technological regress: i.e. apparent reductions in total factor productivity. As emphasized in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) as well, model economies which induce variations in the Solow residual have the potential to explain technological regress and can potentially match other correlation patterns.

As we shall see, the aggregate moments of the model with stochastic (π, λ) share many of the characteristics of the RBC model. The Solow residual, driven by reallocation, has a serial correlation of nearly 0.92. Consumption, investment and output are positively correlated with the Solow residual and the model exhibits consumption smoothing. In our environment, the puzzle of "What causes a reduction in the Solow residual?" is easily resolved: measured productivity is low when reallocation is low, either due to lower adjustment rates or a contraction in the distribution of profitability shocks.

Our environment is different from Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) in a couple of important ways. First, our model includes shocks to both the distribution of idiosyncratic shocks and to adjustment costs. Second, as emphasized earlier, a mean preserving spread increases investment. This reflects the timing in our model as well as the structure of adjustment costs. In contrast to models with irreversibility and other forms of non-convexities, there is no option-to-wait in our model with Calvo style adjustment costs. Third, there are no adjustment costs to labor. Finally, as already emphasized, higher order moments matter for the planner and generate an underlying dynamic. In contrast, Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) exclude higher order moments in their approximation. As indicated earlier, there is a dynamic to these higher order moments that underlies the serial correlation in the Solow residual.

Case	$C(y,c)$	$C(y,i)$	$C(y,\tilde{A})$	$C(i,c)$	ρ_c	ρ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\sigma_y}$
				Frictions				
stochastic A	0.91 (0.01)	0.94 (0.01)	0.93 (0.01)	0.71 (0.02)	0.95 (0.02)	0.88 (0.02)	0.53 (0.03)	0.80 (0.03)
stochastic π	0.77 (0.04)	0.90 (0.01)	0.90 (0.002)	0.42 (0.04)	0.95 (0.01)	0.91 (0.01)	0.46 (0.06)	0.80 (0.05)
stochastic λ	0.72 (0.04)	0.93 (0.01)	0.89 (0.01)	0.42 (0.03)	0.97 (0.01)	0.82 (0.01)	0.34 (0.04)	0.66 (0.05)
stochastic π, λ	0.782 (0.02)	0.898 (0.008)	0.915 (0.003)	0.427 (0.02)	0.96 (0.003)	0.86 (0.006)	0.46 (0.03)	0.80 (0.03)
RBC	0.981 (0.002)	0.913 (0.01)	0.986 (0.002)	0.818 (0.01)	0.954 (0.01)	0.890 (0.013)	0.633 (0.04)	0.919 (0.02)

Table 5: Endogenous Capital Accumulation - Macroeconomic Moments

Results from 1000 simulations are reported with standard deviations in parentheses below. Here $C(x,y)$ are correlations, ρ_x is an autocorrelation and σ_x is a standard deviation. The variables are: output (y), consumption (c), investment (i) and the Solow residual (mis-measured TFP) ($\tilde{A}$).

Table 5 presents standard aggregate moments for a number of cases. These are the traditional macroeconomic moments: the correlations of output (y), consumption (c), investment (i) and TFP($\tilde{A}$). Here the TFP measure is the one constructed from the data as if plants were homogeneous, i.e. mis-measured TFP. The serial correlations of consumption and output as well as relative standard deviations are reported, too.

The rows are the various cases explored before, using the baseline parameters. The last row, "RBC" is the standard stochastic growth model with productivity shocks and without adjustment costs.[24] Here the productivity shocks come from fitting an AR(1) process to the mis-measured TFP series, $\tilde{A}$, generated by the stochastic (π, λ) case. We obtain an AR(1)

[24]The RBC moments are produced using our model without adjustment frictions. The only stochastic shocks occur to A.

parameter $\rho_{\tilde{A}} = 0.9183$ and standard deviation of the residual $\sigma_{\tilde{A}} = 0.0132$. This process is fed into the model without adjustment frictions to produce the "RBC" moments.

All of the models match the standard business cycle properties of positively correlated movements of consumption and investment with output. All of these variables are positively correlated with (mis-measured) TFP. So, in the case of shocks to λ, the Solow residual, investment and output all increase when there is a mean preserving spread in the distribution of shocks. The models exhibit consumption smoothing. The aggregate moments are all positively serially correlated.

Further, the models with stochastic π and/or λ create considerably lower comovement between consumption and investment compared to the RBC case. As in models with intermediation shocks, such as Cooper and Ejarque (2000), and discussed further for the case of stochastic λ in Bachmann and Bayer (2013), when returns to investment are large, say due to a high value of λ, consumption is reduced to finance capital accumulation.

The key to this lower correlation is the immediate inverse relationship between consumption and investment when there is a shock to λ. After the impact, consumption and investment move together in the transition dynamics. So, overall there is a positive correlation but one that is reduced due to the negative comovement in response to the innovation. This can be see in the impulse response functions for our model, Figures 3 and 4.

This effect appears in other models of shocks to the variance of productivity shocks. Looking at the impulse response functions in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Figures 7 and 8, and Bachmann and Bayer (2013), Figure 3, this negative comovement at impact is apparent. Further, though this negative comovement is not evident in unconditional data moments, it does appear in impulse response functions. In Figure 3 of Bachmann and Bayer (2013), the immediate response in the data to an increase in idiosyncratic risk is for output and investment to increase and consumption to fall.[25] Output and investment fall subsequently.

6 Conclusion

The goal of this paper was to understand the productivity gains from capital reallocation in the presence of frictions. To study this we have looked at the optimization problem of a planner facing frictions in capital accumulation and shocks to productivity, adjustment costs and the distribution of plant specific shocks.

The heterogeneity in plant-level productivity provides the basis for reallocation. The frictions in adjustment prevent the full realization of these gains. The model can generate

[25] These results are for German data. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) do not report impulse response functions to uncertainty shocks in US data.

cyclical movements in reallocation and in the cross sectional distribution of the average productivity of capital.

There are three key findings in this paper. The first is the cyclical behavior of reallocation and the distribution of capital productivity. When shocks to either adjustment frictions or the distribution of plant-level shocks are present, then reallocation is pro-cyclical. In fact, even if there are no direct shocks to TFP, the reallocation process creates fluctuations in output and investment. These effects are not present when the only shock is to TFP. Further the standard deviation of the cross sectional distribution of average capital productivity is counter-cyclical, as in Eisfeldt and Rampini (2006) and Kehrig (2011).

Second, in some, though not all environments, the plant-level covariance of capital and profitability shocks matters for characterizing the planner's solution. This is important for a few reasons. It is indicative of state dependent gains to reallocation and our economy is an example of one where moments other than means are needed in the planner's problem.

Third, the model with shocks to adjustment costs and the cross sectional distribution of productivity shocks can reproduce many features of the aggregate economy. A researcher would interpret the data as generated by a model with TFP shocks even though it is actually constant. That is, the researcher could certainly misinterpret the variations in the Solow residual driven by the reallocation of capital as variations in TFP.

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Appendix

The appendix describes our method of solving the planner's problem. The approach taken for characterizing the law of motion for the joint distribution, Γ, is described in the text. Here we focus on the planner's choice of capital in the reallocation process.

Any vector of capital allocated across adjustable plants $k(\varepsilon)$ will have associated values for μ_a and ϕ_a. Create a grid for potential vectors $k(\varepsilon)$. To do so, define two benchmarks for the planners decision regarding the allocation of capital across those plants that are in F^A. Define k^{MAX} as the vector where marginal products are equalized across plants. This vector was found in (9) for the frictionless benchmark case above. In the presence of Calvo adjustment costs, the planner will not reallocate more capital between plants than under the allocation rule k^{MAX}, but possibly less. The second benchmark will be called k^{MIN} and is simply the case where capital is equally distributed across adjustable plants (i.e. no reallocation). The idea behind this procedure is that the planner will choose a vector $k(\varepsilon)$ which is *between* k^{MAX} and k^{MIN}, meaning that the planner will reallocate some capital between plants, but not as much as under the frictionless benchmark. We consider convex combinations of k^{MAX} and k^{MIN}.

Define a variable m, that takes values between zero and one and determines a potential vector of $k(\varepsilon)$'s as follows: $k_m = m \cdot k^{\text{MAX}} + (1-m) \cdot k^{\text{MIN}}$. For each k_m compute $\mu_m = E(k_m(\varepsilon)^\alpha)$ and $\phi_m = Cov(\varepsilon, k_m(\varepsilon)^\alpha)$ characterizing this vector. This allows the calculation of output associated with m. The planner optimizes over m and this translates into μ_m, ϕ_m.

To check the robustness of this procedure start from a model with the baseline parameters without any exogenous shocks. It turns out that the planner chooses $m = 0.9508$, which means that the optimal vector $k(\varepsilon) = 0.9508 \cdot k^{\text{MAX}} + 0.0492 \cdot k^{\text{MIN}}$, so capital reallocation is about 5% lower compared to the frictionless benchmark. In order to see how good of an approximation the decision rule 'm' is, we apply the following procedure.

We work directly with the planner's value of the steady state (SS) allocation. The simplified version of the value function has only two states, μ_n and ϕ_n, so there will be a value $V(\mu_n^{SS}, \phi_n^{SS})$ associated to the steady state. This value is equal to forever receiving the output associated with the amount of reallocation 'm' times the fraction of adjustable plants, plus the output associated with the SS state vector times the fraction of non-adjustable plants.

$$V(\mu^{SS}, \phi^{SS}) = \frac{\int_{\varepsilon \in F^A} \varepsilon k(\varepsilon)^{\alpha} f(\varepsilon) d\varepsilon \ \ + (1-\pi)(E(\varepsilon)\mu^{SS} + \phi^{SS})}{1-\beta} \tag{30}$$

The planner can now choose any allocation of capital across plants. This allocation implies a mapping into the values of μ_n and ϕ_n. The planner will be allowed to choose the allocation that maximizes the expression for $V(\mu^{SS}, \phi^{SS})$ above. Being bound to the same grid, the resulting vector is identical to the one previously found. We now perturb this vector in order to find profitable deviations that keep the aggregate capital stock constant. The perturbation adds a random vector with mean zero to the k-vector that maximized (27) given the grid. If the resulting vector produces a higher lifetime utility, the k-vector is updated accordingly. This procedure is repeated 1,000,000 times. The results show that our grid for m comes extremely close to the optimal solution. Although profitable deviations *are* possible, they remain very small: the difference in output is around 0.01%.