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ABSTRACT

Traditional economic models of vaccination assume that agents free-ride on the vaccination decision of others. These models show that private vaccination rates are always below the social optimal and even large subsidies cannot achieve disease eradication. In this paper, we build a model where in addition to the desire to free-ride, agents have a desire to conform to the vaccination decisions of their peers. In this model privately optimal vaccination rates can be higher or lower than the social optimal and thus subsidies for vaccination are not always optimal. However, in certain cases, even small subsidies can achieve disease eradication.

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1. Introduction

Vaccinations are one of the most cost effective ways of preventing disease and promoting health. Yet, vaccination rates for several diseases remains low despite significant government intervention to promote vaccination coverage. Although some of the low rates of vaccinations especially in developing countries might be explained by restricted supply or poor availability of health care, lack of demand for vaccinations also plays an important role (Datar et al. (2007)). Recent vaccine scares and subsequent drops in vaccination uptake highlight the importance of this issue in the US and other developed countries. There is also emerging evidence that individuals might not fully appreciate the costs and benefits of vaccinations when deciding whether or not to vaccinate (Gu and Sood (2011), Santibanez et al. (2002), Clark et al. (2009), Van Essen et al. (1997), O'Reilly et al. (2005), Winston et al. (2006), Johnson et al. (2008), Logan (2009)). In addition, in our increasingly networked world with instant access to information and the opinions of others, individuals do not operate in a vacuum; friends', family, and even strangers' decisions might influence our behavior. Therefore, it is imperative to understand how individuals make decisions regarding vaccinations and the implications of alternate decision models or processes on the design of efficient public health policy to maximize vaccination coverage and reduce the burden of vaccine preventable diseases.

In this paper, we consider two alternate models of the decision to vaccinate. The models differ in how individuals decision to vaccinate are influenced by the decision of peers to vaccinate. In particular, we consider two types of peer effects. In the first, rational agents desire to free-ride on the vaccination decisions of their peers. For example, as an individual sees the overall vaccination coverage of her peers increasing, she has less desire to vaccinate herself, as there is less and less chance that she will herself be infected. In this case, peer effects are *non-conforming* – an increase in the vaccination coverage by peers leads to a decrease in an individual's probability of vaccinating. In the second type of peer effect, agents desire to copy what their peers are doing, through the simple desire to avoid being different. For example, consider an agent surrounded by peers who choose not to vaccinate, believing that the vaccine in question carries a very high risk. Such an agent could face an enormous amount of peer pressure to conform. As a result, one would expect the desire to copy others' behavior to play a large role in the vaccination decision-making process. In this case, peer effects are *conforming* – an increase in vaccination by peers leads to an increase in an individual's probability of vaccinating, and vice versa.

The economics literature on standard models of decision making and discussions of vaccination decisions consider the positive and normative implications of non-conforming peer effects or free-riding (Philipson (2000)). However, the role of conforming peer effects has largely been ignored despite a vast literature documenting the existence of conforming peer effects in a variety of contexts, such as unhealthy behaviors, academic achievement and productivity (Sacerdote (2001), Lundborg (2006), Evans et al. (1992), Gaviria and Raphael (2001)). In particular, some recent studies have documented the presence of conforming peer effects in vaccination decisions; see (Hanratty et al. (2000), Henderson et al. (2008), May and Silverman (2003), Parker et al. (2006), Rao et al. (2007), Schmid et al. (2008), Stewart-Freedman and Kovalsky (2007)) for details. In particular, Rao et al. (2007) looked at flu vaccination decisions made by undergraduates at a large private university and examined the role of the social network in health beliefs and vaccination choices. The authors determine that social effects play a large role in changing people's perceptions of the benefits of immunization. Taking advantage of the random assignment of students to housing, they were further able to show that the clustering of decisions in a social network were not simply due to homophily, but rather due to positive peer effects on individuals' decisions.

In this paper, we develop a theoretical model based on the standard economic models of decision-making and incorporate both non-conforming and conforming peer effects. Using this model, we examine how the introduction of peer effects affects our understanding of the decision to vaccinate and the role of public health policy in vaccination markets. We note two important related papers here: Fu et al. (2010), Bauch (2005), both of which model how imitation influences the dynamics of epidemics and vaccination uptake. In Fu et al. (2010), individuals estimate the costs and benefits of vaccination by learning from others in the population. As agents imitate successful strategies, overall vaccination coverage drops below even the individual optimum. In Bauch (2005), the authors propose a dynamic model in which individuals adopt strategies by imitating others while considering the current disease prevalence. This model leads to regimes in which the vaccination uptake oscillates, as is often seen in vaccine scares. The model we propose in this paper explicitly examines the role of conforming and non-conforming peer effects in determining individually optimal strategies. We further build on these papers and others mentioned above by looking at the role of these peer effects in the effectiveness of various public health policies.

Overall, our results demonstrate that adding conforming peer effects to the traditional model of vaccination decisions can have important implications. In the traditional economic model, agents free ride on the decisions of others and as a result the privately optimal vaccination rate is always below the socially optimal vaccination rate. In contrast, in the model with conforming peer effects privately optimal vaccination rates can be above or below the social optimal. In the fact the model produces several evolutionary stable equilibria including no vaccination coverage, full vaccination coverage and a mixed strategy equilibrium. Traditional models also imply that vaccine subsidies are always optimal and even large subsidies cannot achieve disease eradication. In contrast, in the model with conforming peer effects subsidies for vaccination are not always optimal. However, in certain cases, depending on disease and vaccine parameters, even small subsidies can achieve disease eradication.

To give a brief overview of this paper, in Section 2, we develop a standard model of vaccination decisions, where rational economic agents maximize expected utility or payoffs. We carefully examine the difference between the individually optimal strategy and the socially optimal level of vaccination coverage, showing how the parameters of the model will affect the gap between them. We also highlight the effect of government subsidies on vaccination uptake and how their effectiveness depends on the cost and risk of the vaccine and disease in question. In Section 3 we add conforming peer effects to the standard model and describe the changes in the individually optimal strategy. With the addition of conforming peer effects, the individually optimal strategy may lead to a higher level of vaccination coverage that what is socially optimal – we discuss the implications of this result and its effect on public policy in the second half of Section 3.

2. Standard economic model with non-conforming peer effects

In this section we develop a standard economic model of vaccination decisions, where rational economic agents maximize expected utility or payoffs, based on the models in (Bauch, 2004, Bauch et al., 2003). Vaccination confers immunity against an infectious disease but also may have adverse health side effects as well as monetary costs. In this model individual vaccination decisions are linked to decisions of the group as the benefit of vaccination depends on the prevalence of the infectious disease, which in turn depends on the group's likelihood of vaccination. For example, an increase in vaccination rate among peers would reduce disease prevalence which in turn would reduce individual incentives to vaccinate. Thus, in the standard economic model, peer effects are *non-conforming* – individual decisions are inversely related to group decisions. In Section 3 we add *conforming peer effects* to the standard model, where an increase in the group's likelihood to vaccinate leads to an increase in the individual's likelihood to vaccinate. Next, we contrast the normative and positive implications of the two models.

2.1. Payoffs

We start with a model of risk-neutral agents with additively separable utility in health and consumption. Under this model the expected utility from vaccination is given by

$$E_{vac} = h(H - d_v) + u(C - m) \tag{1}$$

where H is an individual's health endowment, C is consumption, d_v is the morbidity cost of side effects, and m is the marginal cost of producing the vaccine. If agents are risk-neutral, then the functions $h(\cdot)$ and $u(\cdot)$ are linear and the payoff from vaccination can be expressed as

$$E_{vac} = H - d_v + \theta(C - m) \tag{2}$$

where θ is the marginal utility of consumption in health units.

In the standard economic model with non-conforming peer effects the payoff for not vaccinating varies only with the infection probability, which depends on the total vaccination coverage. If agents are risk-neutral, the payoff from not vaccinating can be expressed as follows,

$$E_{nv}(p) = (H - d_i)w(p) + H(1 - w(p)) + \theta C \tag{3}$$

$$= H - d_i w(p) + \theta C \tag{4}$$

where d_i represents the morbidity cost of infection and $w(p)$ is the probability of being infected when the vaccination coverage is p. We assume that $w(p)$ is strictly decreasing in p for all $p \leq p_{crit}$. For $p \geq p_{crit}$, $w(p) = 0$, that is, p_{crit} is the critical vaccination threshold above which herd immunity is achieved and the disease eradicated. Note that in this model, the cost of not vaccinating only involves a cost of infection; individuals are fully insured against medical expenses related to treatment of vaccine preventable disease and face no other monetary or psychological costs of not vaccinating.

The expected payoff for playing a mixed strategy P (vaccinating with probability P) when the vaccination coverage level is p is

$$\hat{E}[P, p] = H + \theta C - P[\theta m + d_v] - (1 - P)[d_i w(p)] \tag{5}$$

By defining the *relative cost* as $r = (\theta m + d_v)/d_i$ this expected payoff can be expressed as

$$E[P, p] = \frac{\hat{E}[P, p]}{d_i} = \frac{H + \theta C}{d_i} - rP - (1 - P)[w(p)] \tag{6}$$

where the multiplicative constant d_i will not make any difference in our proofs or calculations. With the assumption that $0 \leq \theta m + d_v \leq d_i$, we have $0 \leq r \leq 1$.

2.2. Equilibria

In the vaccination game with non-conforming peer effects, individuals seek to maximize their expected payoff given the current vaccination coverage p.

If $p \geq p_{crit}$, the expected payoff is clearly decreasing in P. As a result, if the current vaccination coverage is above the critical threshold, individuals will always choose to never vaccinate ($P = 0$). Assuming that the game is played repeatedly (or at least that individuals make decisions assuming that it is so), this will decrease the total vaccination coverage until $p < p_{crit}$ and the probability of infection becomes non-zero.

If $p < p_{crit}$, individuals' strategies will depend on the relative cost and the probability of infection. When $r > w(p)$ individuals will always choose to not vaccinate ($P = 0$), decreasing the total vaccination coverage p, and increasing $w(p)$ until the point p^* where the total vaccination coverage satisfies $w(p^*) = r$.[1] When $r < w(p)$, i.e., when the relative cost is sufficiently small, individuals will always choose to vaccinate ($P = 1$), increasing the vaccination coverage and increasing $w(p)$ until the point p^* where the total vaccination coverage satisfies $w(p^*) = r$. See Figure 1 for an illustration of this solution. This strategy is stable, as stated formally in Lemma 1; we leave the detailed proof of this lemma to Appendix B.

[1]Note that if $r > w(0)$, this process will continue to the point where nobody will

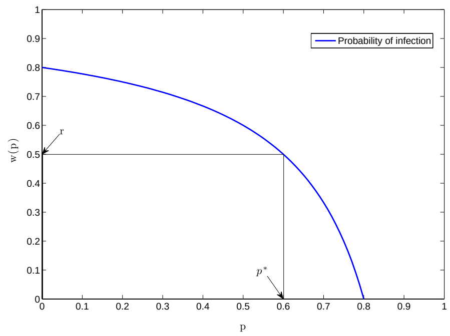


Figure 1: Illustration of stable mixed strategy in vaccination game with non-conforming peer effects.

Lemma 1. *The mixed strategy p^* that satisfies $w(p^*) = r$ is a weak Nash Equilibrium and an Evolutionarily Stable Strategy in the vaccination game with non-conforming peer effects if $r = \frac{\theta m + d_v}{d_i} < w(0)$. If $r > w(0)$, the pure strategy $P = 0$ is a strict Nash Equilibrium and Evolutionarily Stable Strategy.*

2.3. Social welfare and individually optimum strategies

In many games the equilibrium reached by rational agents may not be the socially optimal value. In our case, we define the social welfare as the normalized total utility of the population,

$$W(p) = pE_{vac} + (1-p)E_{nv}(p) \tag{7}$$

For a general infection probability function $w(p)$ with vaccination threshold p_{crit}, the socially optimal vaccination coverage p_{opt} is the vaccination level that maximizes the social welfare, i.e.,

$$p_{opt} = \underset{0 \leq p \leq 1}{\operatorname{argmax}} W(p) \tag{8}$$

The social welfare function is decreasing for $p > p_{crit}$; in this regime, increasing vaccination coverage reduces the social welfare, as the disease is

vaccinate, $p = 0$. This is an example of the classic "free-rider" problem, where individuals rationally choose a strategy where they benefit while not contributing to society, leading to the point where everyone follows the same strategy and nobody benefits.

already eradicated and increasing vaccinations provide no benefit but individuals incur the monetary costs of the vaccine. For $p < p_{crit}$, the social welfare function is increasing in vaccination coverage p under certain conditions.[2] If this condition is met, the maximum social welfare will be achieved at $p_{opt} = p_{crit}$, the point at which the disease is eradicated. If this condition is not met, we have $p_{opt} \leq p_{crit}$, since the social welfare function is always decreasing beyond p_{crit}.

Remark 1. Note that from a policy perspective, often the desired vaccination level is one that achieves herd immunity or disease eradication, so $p_{opt} = p_{crit}$, regardless of wherever the minimum of the social welfare function might be. For example, in 1977 the World Health Organization (WHO) successfully eradicated smallpox through a worldwide vaccination program (Fenner, 1988). The rationale is that disease eradication benefits not only the current generation but also future generations. The social welfare function we consider in this paper only models the welfare of the current generation and therefore within the context of our model p_{opt} can be lower than p_{crit}.

Remark 2. As a running example throughout this paper, we will consider the infection probability $w(p)$ as the steady-state infection probability in a SIR (Susceptible-Infected-Recovered) model with constant birth and death rate μ. In this case, $w(p) = 1 - \frac{1}{R_0(1-p)}$ where R_0 is the *reproduction ratio* of the disease in question. For a detailed description of this model and how to derive its infection probability function, see Appendix C. For this model, the disease will be eradicated if the vaccination level is at or above the critical vaccination threshold: $p_{crit} = 1 - 1/R_0$. In this example, the optimal vaccination coverage will be $p_{opt} = p_{crit}$, as the condition in (9) is satisfied for $w(p) = 1 - \frac{1}{R_0(1-p)}$, as long as $r < 1$, which is true by assumption.

However, if we assume that individuals are allowed to make their own vaccination decisions, disease eradication will not be possible, and often the optimal vaccination coverage is not achieved. The privately optimal strategy

[2] Differentiating the social welfare function with respect to p, it is easy to see that it is increasing in p for $p \leq p_{crit}$ if

$$r < w(p) - (1-p)\frac{\partial w(p)}{\partial p}. \tag{9}$$

p^* is always less than the critical vaccination threshold, as shown in Figure 1. Even in the case where the socially optimal vaccination rate is less than the critical threshold, the privately optimal strategy is still less than the social optimum. Leaving the proof to Appendix B, we state this result formally,

Theorem 1. *The private optimum $p^* \leq p_{opt}$ in the vaccination game with non-conforming peer effects.*

Exactly how much lower the social welfare is when individuals act selfishly from the optimal point depends on several factors. The social welfare at the socially optimal point, dividing by the constant d_i to obtain an expression in terms of the relative cost, is

$$\frac{W(p_{opt})}{d_i} = \frac{H + \theta C}{d_i} - w(p_{opt}) + p_{opt}[-r + w(p_{opt})]. \tag{10}$$

The social welfare at the private optimum, again dividing by d_i, is

$$\frac{W(p^*)}{d_i} = \frac{H + \theta C}{d_i} - w(p^*) \tag{11}$$

since $w(p^*) = r$. We can easily calculate the difference between the two:

$$\frac{W(p_{opt}) - W(p^*)}{d_i} = (1 - p_{opt})(r - w(p_{opt})) \tag{12}$$

again using $w(p^*) = r$. The above equations show that as the cost of the vaccine (m and d_v) increases and as the cost of infection (d_i) decreases, this gap in welfare will increase; fewer people will voluntarily choose to vaccinate and the social welfare will decrease.

2.4. Effect of government subsidies

If the government offers subsides of the monetary cost of the vaccine, individuals' expected payoff for vaccinating becomes a function of the subsidy:

$$E_{vac}(s) = H + \theta(C - m(1 - s)) - d_v \tag{13}$$

where s represents the percentage of the marginal production cost of the vaccine that the government is subsidizing. The private optimum, as a function of the subsidy,[3] becomes

$$p^*(s) = w^{-1}\left(\frac{\theta m(1-s)+d_v}{d_i}\right). \tag{14}$$

Note that this function is strictly decreasing in its argument, since the original probability of infection function $w(p)$ is also strictly decreasing. In general, the effectiveness of the subsidy is inversely related to the morbidity cost of the disease, d_i. That is, vaccine subsidies are less effective for more deadly diseases. However, the effectiveness of the subsidy is directly related to the morbidity (and monetary) cost of the vaccine, d_v – the vaccine subsides are more effective for more dangerous vaccines. Looking at (14) more closely, we see that it is an increasing function of s and d_i, but decreasing in m and d_v. Intuitively, as the government subsidy increases and lowers the monetary cost of the vaccine (or as the danger of infection increases), more people will be inclined to buy the vaccine. Similarly, as the cost and risk of the vaccine increases, less people will be inclined to vaccinate.

Continuing with our running example, we examine the private optimum as a function of the subsidy for a specific infection probability function $w(p) = 1 - \frac{1}{R_0(1-p)}$. In this case, the private optimum becomes

$$p^*(s) = 1 - \frac{1}{R_0\left(1 - \frac{\theta m(1-s)+d_v}{d_i}\right)} \tag{15}$$

Inspecting the rate of change of the individually optimal strategy with the subsidy s, we can see that the effectiveness of the subsidy will be *higher* for a higher cost and higher risk (larger m and d_v) vaccine, whereas for a more dangerous disease (larger d_i) the subsidy will not be as effective.

If the goal is to eradicate the disease, we need $p^*(s) = p_{crit}$. The subsidy that achieves disease eradication even when individuals behave selfishly is

$$s_{opt} = 1 + \frac{d_v}{\theta m} \tag{16}$$

Note that the optimal subsidy here is greater than one – the subsidy must compensate individuals for more than just the monetary cost of the vaccine

[3] Recall that the private optimum when subsidies are not present is $p^* = w^{-1}\left(\frac{\theta m + d_v}{d_i}\right)$.

in order to eradicate the disease. If we impose the constraint that $0 \leq s \leq 1$, the optimal subsidy will be exactly $s_{opt} = 1$.

The social welfare function, as a function of the subsidy, is

$$W(p^*(s)) = p^*(s)[H+\theta(C-m)-d_v]+(1-p^*(s))[H+\theta C-d_i w(p^*(s))] \quad (17)$$

When $s = s_{opt}$, this simplifies to

$$W(p^*(s_{opt})) = H + \theta(C - p_{crit}m) - p_{crit}d_v. \quad (18)$$

In contrast, if we look at the social welfare function when there is no subsidy and individuals behave selfishly, we have

$$W(p^*) = H + \theta(C - m) - d_v \quad (19)$$

which is always less than the social welfare at the optimum. Further, it is easy to show that $W(p^*(s)) > W(p^*)$ for any $s > 0$.

3. Standard Economic Model with Conforming and Non-conforming Peer Effects

In this section we add conforming peer effects to the standard model in Section 2, demonstrating the interaction between the desire to behave rationally and the desire to conform to what others are doing. We follow the same format as in the previous section, first describing the payoffs and equilibria of the model and then discussing the differences between the individually optimal strategies and the socially optimal vaccination coverage, as well as the effect of government subsides in this new, more realistic model of human decisions.

3.1. Payoffs

We begin by defining payoff functions for following each strategy: vaccinating and not vaccinating. We use a linear combination of the payoff functions from Section 2 and new payoff functions capturing the reward one gets by conforming:

$$E_{vac}(p) = \gamma' f(p) + H - d_v + \theta(C - m) \tag{20}$$
$$E_{nv}(p) = \gamma' g(p) + H - d_i w(p) + \theta C \tag{21}$$

where $\gamma' \in (0, \infty]$ measures the strength of the desire to conform, $f(p)$ is a strictly increasing function representing the desire to conform to the vaccinating strategy, and $g(p)$ is a strictly decreasing function representing the desire to conform to the non-vaccinating strategy. Note that this general formulation can capture bias; for example, a given population might put more weight on conforming to the non-vaccinating, rather than vaccinating, strategy. All other variables are the same as defined in Section 2.

We can also describe a simpler game with symmetric linear payoff functions for conformity, as follows. Let $f(p) = p$ and $g(p) = 1 - p$, so that

$$E_{vac}(p) = \gamma' p + H - d_v + \theta(C - m) \tag{22}$$
$$E_{nv}(p) = \gamma'(1 - p) + H - d_i w(p) + \theta C \tag{23}$$

The expected payoff for playing a mixed strategy P (vaccinating with probability P) when the vaccination coverage level is p is

$$\hat{E}[P, p] = P[\gamma' f(p) + H + \theta(C - m) - d_v] + (1 - P)[\gamma' g(p) + H + \theta C - d_i w(p)] \tag{24}$$
$$= H + \theta C - P[\theta m + d_v - \gamma' f(p)] - (1 - P)[d_i w(p) - \gamma' g(p)] \tag{25}$$

Again dividing through by d_i and using the relative cost of the vaccine, we can express the expected payoff as

$$E[P,p] = \frac{\hat{E}[P,p]}{d_i} = \frac{H+\theta C}{d_i} - P[r-\gamma f(p)] - (1-P)[w(p)-\gamma g(p)] \quad (26)$$

where $\gamma = \frac{\gamma'}{d_i}$ is just a scaled constant measuring the strength of the desire to conform. Note that we retain the assumption that $0 \leq r \leq 1$. For convenience, let $h(p) = g(p) - f(p)$, a useful strictly decreasing summary function. We assume

$$h(0) = \alpha \quad (27)$$
$$h(1) = -\beta. \quad (28)$$

For reference, we also define the *pure conformity game*, where payoffs are only a function of the desire to conform to others, and there are no non-conforming peer effects.

$$E_{vac}(p) = \gamma' f(p) \quad (29)$$
$$E_{nv}(p) = \gamma' g(p) \quad (30)$$

The pure conformity game has two pure strict NE (and ESS's): $P = 0$ and $P = 1$, which can easily be shown to always exist. It further has a weak NE at $P = p^*$ where p^* is the solution to $0 = \gamma' h(p^*)$, but this equilibria is not an ESS. If we use the symmetric linear conformity payoff functions as described in (22) and (23), we have

$$E_{vac}(p) = \gamma' p \quad (31)$$
$$E_{nv}(p) = \gamma'(1-p) \quad (32)$$

This simple pure conformity game will always have two pure strict NE (and ESS's): $P = 0$ and $P = 1$. It also has a weak NE at $P = p^* = 1/2$, where p^* is the solution to $0 = h(p^*) = \gamma'(1-2p^*)$, but this equilibria will not be an ESS. To illustrate this result, imagine the conformity game with exactly half of the population vaccinating. As soon as the fraction vaccinating slightly increases (or decreases), the majority strategy is no longer 50–50, and the population will converge to the strict NE $P = 1$, always vaccinating (or $P = 0$, never vaccinating).

3.2. Equilibria

Just as described in Section 2.2, individuals here will seek to maximize their expected payoff given the current vaccination coverage p. However, when the game also includes conforming peer effects, individual strategies become more complicated, reflecting the tension between the desire to conform and to free-ride on others' decisions to vaccinate. In this section, we show that the vaccination game with both conforming and non-conforming peer effects can have multiple stable equilibria whose existence and stability depend on the disease and cost parameters of the model, in contrast to the vaccination game with only non-conforming peer effects, which only has one stable equilibrium.

Lemma 2. *The pure non-vaccinating strategy $P = 0$ is a strict Nash Equilibrium and Evolutionarily Stable Strategy of the vaccination game with conforming and non-conforming peer effects if $r > w(0) - \gamma\alpha$.*

Proof. Using Definition 1 from Appendix A, the pure strategy $P = 0$ (never vaccinating) is a strict Nash Equilibrium (and thus an evolutionarily stable strategy) if $E(P, P) - E(Q, P) > 0$. Calculating this, we have

$$E(P,P) - E(Q,P) = (P - Q)(E_v(P) - E_{nv}(P)) \tag{33}$$

$$= -Q(E_v(0) - E_{nv}(0)) \tag{34}$$

$$= -Q(-\theta m - d_v + d_i w(0) - \gamma'\alpha) \tag{35}$$

Thus, $E(P, P) > E(Q, P)$ and $P = 0$ is a strict NE and ESS if

$$-\theta m - d_v + d_i w(0) - \gamma'\alpha < 0, \tag{36}$$

or equivalently, if $r > w(0) - \gamma\alpha$. □

The above lemma states that if the vaccine is sufficiently costly or has large side effects relative to the mortality costs of infection, never vaccinating will be a stable equilibrium strategy. Alternatively, if the disease is sufficiently not infectious, that is, $w(0)$ is small, then never vaccinating will be a stable equilibrium strategy. Note that as $\gamma \to 0$ and the desire to conform goes away, this approaches the condition for never vaccinating when only non-conforming peer effects are present. However, when $\gamma \to \infty$, and the conforming strategy dominates, never vaccinating will always be a stable equilibrium strategy, as in the pure conformity game.

Lemma 3. *The pure vaccinating strategy $P = 1$ is a strict Nash Equilibrium and Evolutionarily Stable Strategy of the vaccination game with conforming and non-conforming peer effects if $r < \gamma\beta$.*

Proof. Using Definition 1 from Appendix A, the pure strategy $P = 1$ (always vaccinating) is a strict Nash Equilibrium (and thus an evolutionarily stable strategy) if $E(P, P) - E(Q, P) > 0$. Calculating this, we have

$$E(P, P) - E(Q, P) = (P - Q)(E_v(P) - E_{nv}(P)) \tag{37}$$
$$= (1 - Q)(E_v(1) - E_{nv}(1)) \tag{38}$$
$$= (1 - Q)(-\theta m - d_v + \gamma'\beta) \tag{39}$$

since $w(1) = 0$. Thus, $E(P, P) > E(Q, P)$ and $P = 1$ is a strict NE and ESS if

$$-\theta m - d_v + \gamma'\beta > 0, \tag{40}$$

or equivalently, if $r < \gamma\beta$. □

The above lemma states that if the vaccine is sufficiently safe, always vaccinating will be a stable equilibrium strategy. Note that as $\gamma \to 0$ and the desire to conform goes away, always vaccinating will *never* be a stable equilibrium, as is the case for the vaccination game with only non-conforming peer effects. However, when $\gamma \to \infty$, and the conforming strategy dominates, always vaccinating becomes a stable equilibrium strategy, as in the pure conformity game.

Lemma 4. *The mixed strategy p^* satisfying $E_v(p^*) = E_{nv}(p^*)$ is a weak Nash Equilibrium and Evolutionarily Stable Strategy for the vaccination game with conforming and non-conforming peer effects if*

$$\frac{\partial w(p)}{\partial p} < \gamma \frac{\partial h(p)}{\partial p} \tag{41}$$

Proof. Consider a population following the mixed equilibrium strategy p^*, (vaccinating with probability p^*) where p^* is the solution to the equation

$$E_v(p^*) = E_{nv}(p^*) \tag{42}$$

This strategy is clearly a weak Nash Equilibrium, since

$$E(p^*, p^*) - E(Q, p^*) = (p^* - Q)(E_v(p^*) - E_{nv}(p^*)) = 0. \tag{43}$$

Using Definition 3 from Appendix A, p^* will be an evolutionarily stable strategy if

$$E(p^*, Q) > E(Q, Q) \iff (p^* - Q)(E_v(Q) - E_{nv}(Q)) > 0. \tag{44}$$

This definition states that vaccinating with probability p^* is preferable to some other level Q, given that the current vaccination coverage is Q – the equilibrium strategy p^* will be able to successfully "invade" a population with coverage Q. It turns out that p^* will be an ESS if $E_v(p) - E_{nv}(p)$ is strictly decreasing in the vaccination coverage p. To see this, consider first the case where $Q > p^*$, where the current vaccination coverage is greater than the equilibrium p^*. If $E_v(p) - E_{nv}(p)$ is strictly decreasing in the vaccination coverage p, then

$$E_v(Q) - E_{nv}(Q) < E_v(p^*) - E_{nv}(p^*) = 0 \tag{45}$$

and so we have $(p^* - Q)(E_v(Q) - E_{nv}(Q)) > 0$, and individuals vaccinating with probability p^* will obtain a higher expected payoff than the rest of the population when the coverage level Q is greater than p^*.

Now consider the case where $Q < p^*$, where the current vaccination coverage is less than the equilibrium p^*. Again, if $E_v(p) - E_{nv}(p)$ is strictly decreasing in the vaccination coverage p, then

$$E_v(Q) - E_{nv}(Q) > E_v(p^*) - E_{nv}(p^*) = 0 \tag{46}$$

and so we have $(p^* - Q)(E_v(Q) - E_{nv}(Q)) > 0$, and individuals vaccinating with probability p^* will obtain a higher expected payoff than the rest of the population.

Thus, individuals vaccinating with probability p^* solving $E_v(p^*) = E_{nv}(p^*)$ will have higher expected payoffs than the rest of the population when the vaccination coverage is at any level Q; i.e., p^* is not only a weak Nash Equilibrium, it is also an evolutionarily stable strategy if $E_v(p) - E_{nv}(p)$ is strictly decreasing in the vaccination coverage p. This condition is equivalent to

$$
\begin{aligned}
&\frac{\partial}{\partial p}[E_v(p) - E_{nv}(p)] < 0 \\
&\iff \frac{\partial}{\partial p}[\gamma' f(p) + H - d_v + \theta(C - m) - \gamma' g(p) - H + d_i w(p) - \theta C] < 0 \\
&\iff \gamma' \frac{\partial f(p)}{\partial p} - \gamma' \frac{\partial g(p)}{\partial p} + d_i \frac{\partial w(p)}{\partial p} < 0 \\
&\iff \frac{\partial w(p)}{\partial p} < \gamma \frac{\partial h(p)}{\partial p}. \qquad (47)
\end{aligned}
$$

For reference, we refer to (47) as the "mixed strategy ESS condition." □

We can explain the "mixed strategy ESS condition" above intuitively. Both the LHS and RHS of equation (47) are less than zero. The LHS shows how the probability of infection falls with an increase in vaccination coverage. The higher the absolute value of this gradient the greater the incentive to free-ride on others. The RHS shows how the payoff for conforming to the non-vaccination strategy relative to the vaccination strategy changes with an increase in vaccination coverage. The higher the absolute value of this gradient the greater the desire to conform to the majority vaccination strategy. If the desire to conform is relatively high then it will overpower the desire to free ride resulting in a corner solution with everyone following the same strategy, either 100% or 0% vaccinating. The equation shows that a mixed strategy ESS is only possible as long as the desire to conform does not completely offset the desire to free-ride.

Note that the vaccination game with conforming and non-conforming peer effects may have more than one weak Nash Equilibrium, in contrast to the game in Section 2.2, if (42) has more than one solution. However, in order for a given weak Nash Equilibrium to also be stable, the mixed strategy ESS condition in (47) must be satisfied at that point. Thus, depending on the disease and cost parameters of the model, the game here can have more than one stable equilibrium.

To illustrate the main results, we give here a specific example of a vaccination game with conforming and non-conforming peer effects. Assume symmetric linear conformity payoff functions:

$$f(p) = p \qquad (48)$$

$$g(p) = (1 - p). \qquad (49)$$

For these conformity functions, we have $h(0) = 1$ and $h(1) = -1$. The payoff functions become

$$E_{vac}(p) = \gamma' p + H - d_v + \theta(C - m) \tag{50}$$
$$E_{nv}(p) = \gamma'(1-p) + H - d_i w(p) + \theta C \tag{51}$$

and the expected payoff is

$$\hat{E}[P,p] = H + \theta C - P[\theta m + d_v - \gamma' p] - (1-P)[d_i w(p) - \gamma'(1-p)] \tag{52}$$

In terms of the relative cost, the expected payoff becomes

$$E[P,p] = \frac{\hat{E}[P,p]}{d_i} = \frac{H + \theta C}{d_i} - P[r - \gamma p] - (1-P)[w(p) - \gamma(1-p)] \tag{53}$$

For this example, let $w(p)$ be the SIR probability of infection: $w(p) = 1 - \frac{1}{R_0(1-p)}$, as described in Appendix C. The parameter R_0 is the basic reproduction ratio of the disease and will vary for different diseases. Using the equilibrium strategy analysis from earlier, never vaccinating ($P = 0$) will be a pure strict NE (and an ESS) when $r > 1 - 1/R_0 - \gamma$; i.e., when the vaccine is sufficiently risky. Similarly, always vaccinating ($P = 1$) will be a pure strict NE (and an ESS) when $r < \gamma$; i.e., when the vaccine is sufficiently safe. Note that by making the desire to conform sufficiently strong (increasing γ), we can achieve the same result. All solutions[4] to the equation

$$E_{vac}(p^*) = E_{nv}(p^*) \tag{54}$$

[4] There are three solutions to the equation in (54), two to the quadratic equation (55) and one to the linear equation which occurs when $p \geq p_{crit}$ and $w(p) = 0$. They are given by

$$q_{1,2} = \frac{\sqrt{R_0}(3\gamma + r - 1) \pm \sqrt{R_0(\gamma - r + 1)^2 - 8\gamma}}{4\sqrt{R_0}\gamma}$$
$$l = \frac{\gamma + r}{2\gamma}$$

Using the mixed ESS condition in (47) we see that the linear solution l will never be an ESS, and when the quadratic solutions exist, only q_1 will be an ESS.

will be weak Nash Equilibria for this example. For $p < p_{crit}$ this becomes

$$\gamma' p^* + H - d_v + \theta(C - m) = \gamma'(1 - p^*) + H - d_i \left[1 - \frac{1}{R_0(1 - p^*)}\right] + \theta C$$
$$-r = \gamma(1 - 2p^*) - \left(1 - \frac{1}{R_0(1 - p^*)}\right). \tag{55}$$

However, only one solution of (54) will satisfy the mixed strategy ESS condition in (47). This solution is given by

$$q^* = \frac{\sqrt{R_0}(3\gamma + r - 1) + \sqrt{R_0(\gamma - r + 1)^2 - 8\gamma}}{4\sqrt{R_0}\gamma}. \tag{56}$$

This solution will exist when

$$0 \leq \gamma \leq \frac{r}{1 - \frac{2}{R_0}} \tag{57}$$

or, in terms of r, when

$$\gamma\left(1 - \frac{2}{R_0}\right) \leq r \leq 1 + \gamma - 2\sqrt{2}\sqrt{\frac{\gamma}{R_0}}. \tag{58}$$

Note that as q^* is the solution to the quadratic equation, it will by necessity always be less than p_{crit}. We plot all of the evolutionarily stable strategies for this example as a function of γ in Figure 2, with $R_0 = 5$ and $r = 0.5$. The solid black horizontal line shows the vaccination coverage needed to eradicate the disease. Examining the figure, we see that for $\gamma = 0$, the private optimum is lower than the social optimum. As we introduce conforming peer effects and γ increases, the private optimum mixed strategy approaches the social optimum. However, increasing the strength of conforming peer effects also leads to the emergence of pure strategies with either no vaccination coverage ($P = 0$) or full vaccination coverage ($P = 1$) as evolutionary stable equilibria. This means that as conformity begins to dominate we could end up in a situation where we eradicate the disease but incur excess social costs due to over vaccination or a situation where no one vaccinates and we incur mortality and morbidity costs of high disease prevalence.

In Figure 3 we plot *all* Nash equilibria, both strict and weak. We note that the weak NE in this example will not be ESS's, but that they serve an important role in determining which ESS the system converges to. For

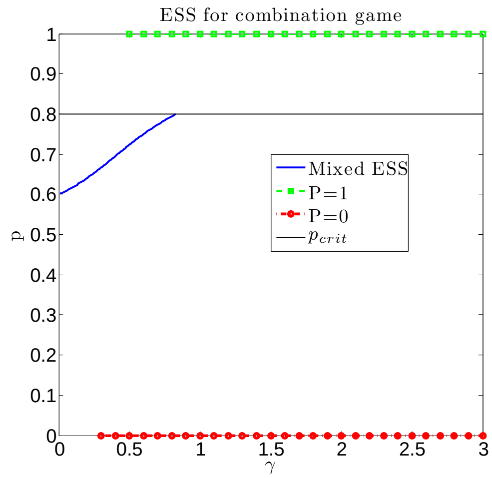


Figure 2: **ESS as a function of γ.** The y-axis represents the vaccination coverage p, while the x-axis represents the normalized conformity strength γ. The solid blue line is the mixed evolutionarily stable strategy, while the dashed red and dashed green lines represent the pure non-vaccinating and pure vaccinating strategies, respectively. Note that there is a small region of γ for which all three equilibria can exist simultaneously.

example, imagine a case with $\gamma = 0.5$ and the current vaccination coverage at 5%, below the weak NE. In this case, the system will converge to the pure non-vaccinator equilibrium, $P = 0$. However, if instead the current vaccination coverage is 40%, above the weak NE, the system will converge to mixed ESS, at approximately 70% coverage. Similar examples can be proposed for convergence to the pure vaccinator equilibrium at $P = 1$. We examine this effect in more detail in Section 3.4 in the context of government subsidies.

In summary, the game with only non-conforming peer effects, as described in Section 2, has only one unique evolutionarily stable strategy, either a mixed strategy below the social optimum, or the pure strategy with no vaccination coverage. In contrast, the game with both non-conforming and conforming peer effects has a much richer set of equilibria, admitting up to 3 evolutionarily stable strategies including a mixed strategy equilibrium and two pure strategy equilibria with either full or no vaccination coverage. The likelihood of observing pure strategy equilibria with full vaccination coverage increases with the strength of conforming peer effects and decreases with the relative

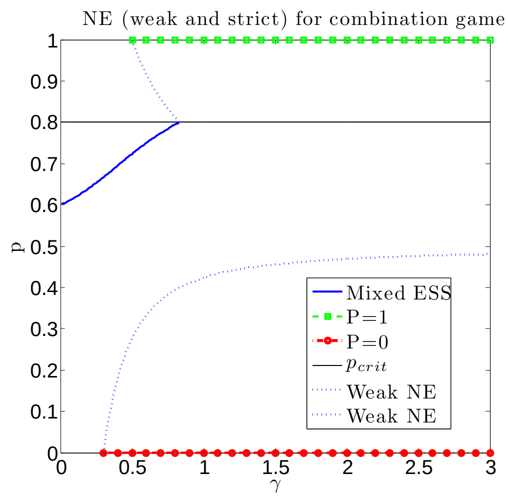


Figure 3: **All NE as a function of γ.** In this figure, the dotted blue lines represent the weak Nash Equilibria of the game with conforming and non-conforming peer effects, plotted as a function of the normalized conformity strength. Note that these dictate which of the stable equilibria (denoted by the solid blue and dashed red an green lines) will be observed.

cost of the vaccine. Similarly, the likelihood of observing pure strategy equilibria with no vaccination coverage increases with the strength of conforming peer effects and increases with the relative cost of the vaccine.

3.3. Social welfare and individually optimal strategies

For the game with both conforming and non-conforming peer effects, we consider the *same* social welfare as in Section 2, ignoring the additional utility given by the conformity functions. In other words, we assume that the value derived by individuals in conforming to a particular strategy does not have any social value. In theory, it is unclear whether the social value of utility derived from conforming is zero. We make this assumption as most policymakers or public health officials in charge of vaccination policy will likely discount the pure utility from conforming in making policy decisions. They are likely to only consider the public health impact and monetary costs of alternate policy options. As a result, we have the same social optimum as before, $p_{opt} = p_{crit}$, if the condition in (9) is met; otherwise $p_{opt} \leq p_{crit}$. For our running example with $w(p) = 1 - \frac{1}{R_0(1-p)}$, we have $p_{opt} = p_{crit}$, as discussed in Remark 2 earlier.

In contrast to the model in Section 2, we note that when conforming peer effects are present in the standard economic model, the private vaccination level can be higher or lower than the social optimum. If the entire payoff function depends on conformity, there will only be two stable equilibria: one at everyone vaccinating and one at nobody vaccinating, while the social optimum remains constant at $p_{opt} = 1 - \frac{1}{R_0}$. When there is also a desire to behave "rationally," the regions where these pure strategy equilibria exist shrink, and a mixed strategy equilibria appears. This mixed stable strategy will always be less than the socially optimal level, as discussed in the derivation of the ESS's in the previous section, i.e., $q^* < p_{opt}$.

Define $p_{conform}$ as the weak NE (not ESS) of the pure conformity game, and $p_{non-conform}$ as the weak NE (ESS) of the vaccination game with non-conforming peer effects from Section 2.2. Also define $p_{combo} = q^*$ as the weak NE (ESS) of the vaccination game with both conforming and non-conforming peer effects from the previous section.

In terms of pure strategies, we have that if $p_{conform}$ decreases, we have a smaller α and a larger β, or *more pressure to vaccinate*. In this case, the range for which we have a pure (strict and ESS) NE at $P = 1$ will grow, while the range for which the pure strategy $P = 0$ is a strict NE (and ESS) shrinks. If we increase $p_{conform}$, however, increasing α and decreasing β, we will have *more pressure to not vaccinate*. In this case, the range for which we have a pure (strict and ESS) NE at $P = 1$ will shrink, while the range for which the pure strategy $P = 0$ is a strict NE (and ESS) grows.

In the case of mixed strategies, the advantage of having conformity will depend on the relative values of $p_{non-conform}$ and $p_{conform}$. We state the formal conditions for conformity to provide an advantage in the theorem below.

Theorem 2. *The Evolutionarily Stable Strategy (if it exists) of the vaccination game with both conforming and non-conforming peer effects will be higher than that of the game with only non-conforming peer effects if and only if the mixed strict Nash Equilibrium of the vaccination game with non-conforming peer effects is higher than the mixed weak Nash Equilibrium of the pure conformity game, i.e.,*

$$p_{non-conform} > p_{conform} \iff p_{combo} > p_{non-conform}.$$

Proof. For simplicity, let $p_{non-conform} = p_{nc}$ and $p_{conform} = p_c$.

$$
\begin{aligned}
& p_{nc} > p_c && (59)\\
\iff & \gamma' h(p_{nc}) < \gamma' h(p_c) && (60)\\
\iff & \gamma' h(p_{nc}) < 0 && (61)\\
\iff & \gamma' h(p_{nc}) - [d_i w(p_{nc}) - \theta m - d_v] < [\gamma' h(p_{combo}) - d_i w(p_{combo}) + \theta m + d_v] && (62)\\
\iff & \gamma h(p_{nc}) - w(p_{nc}) + r < \gamma h(p_{combo}) - w(p_{combo}) + r && (63)\\
\iff & p_{nc} < p_{combo} && (64)
\end{aligned}
$$

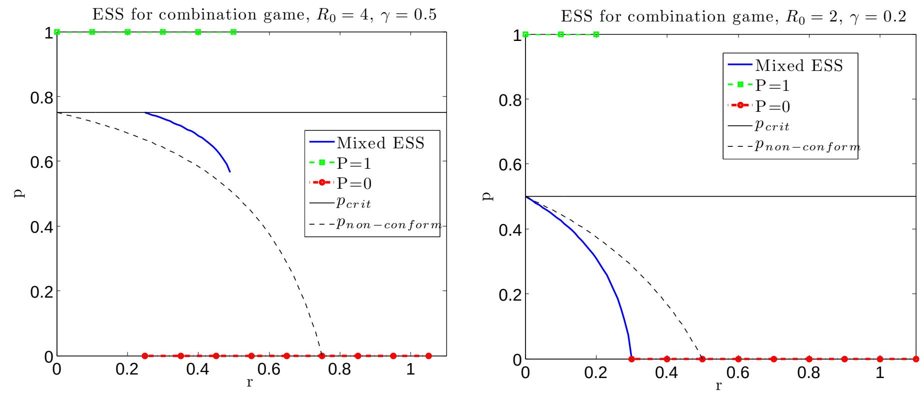
Line (60) come from the fact that $h(p)$ is decreasing in p; line (61) since $h(p_c) = 0$; and lines (62) and (63) use the equilibrium solutions of the vaccination games. The last line follows from the fact that, when p_{combo} exists, $\gamma h(p) - w(p)$ increases with p, according to mixed strategy ESS condition. □

We can see that under certain conditions, the private optimum achieved in the game with both conforming and non-conforming peer effects will be higher than that of the game with only non-conforming peer effects; i.e., under certain conditions, conformity "helps," bringing the private optimum closer to the socially optimal level. We illustrate this effect with our running example in Figure 4, plotting the ESS's as a function of r, for various γ and R_0. The solid horizontal black line plots the coverage required for disease eradication ($p_{opt} = 1 - \frac{1}{R_0}$), while the black curved dashed line represents the mixed equilibrium strategy for the game with only non-conforming peer effects (Section 2).

As r increases, either through an increase of its monetary cost or risk of serious side effects, the privately optimal strategy will drop below the socially optimal level. Depending on the disease and cost parameters, the mixed strategy may be higher or lower than in the game with only non-conforming peer effects – we see both cases illustrated in Figures 4a and 4b.

Comparing the mixed strategy ESS with the socially optimal strategy, we see that $p_{combo} \leq p_{opt}$. Exactly how far the individually optimal mixed strategy is from the social optimum can be easily calculated, as follows. The welfare at the private optimum, using the social welfare expression from (7) is

$$\frac{W(p_{combo})}{d_i} = \frac{H + \theta C}{d_i} - r - (1 - p_{combo})\gamma h(p_{combo}) \quad (65)$$



(a) $p_{combo} > p_{non-conform}$ (b) $p_{combo} < p_{non-conform}$

Figure 4: **ESS as a function of r.** In both figures, y-axis represents the vaccination coverage p and the x-axis the relative cost. The solid black line represent the critical vaccination threshold, p_{crit}, beyond which the disease is eradicated, and the dashed black line the mixed stable equilibrium from the game with only non-conforming peer effects. The solid blue line is the mixed evolutionarily stable strategy in the combination game, while the dashed red and dashed green lines represent the pure non-vaccinating and pure vaccinating strategies, respectively.

where we have taken advantage of the fact that $w(p_{combo}) = \gamma h(p_{combo}) + r$. Using this we have the difference between the social welfare at the optimum (using (10)) and private ESS as

$$\frac{W(p_{opt}) - W(p_{combo})}{d_i} = (1 - p_{opt})(r - w(p_{opt})) + (1 - p_{combo})\gamma h(p_{combo}). \tag{66}$$

In our running example, $p_{opt} = p_{crit}$, so the difference becomes

$$\frac{W(p_{opt}) - W(p_{combo})}{d_i} = (1 - p_{opt})r + (1 - p_{combo})\gamma h(p_{combo}). \tag{67}$$

Note that if $h(p_{combo}) > 0$, this implies that the difference between the social optimum and the private optimum will be less than in the game with only non-conforming peer effects and that $p_{combo} > p_{nc}$. In our specific example with linear symmetric conformity, this holds if $p_{combo} > 1/2$.

We note also that the major difference between this game and that with only non-conforming peer effects is the existence of the pure 100% vaccinator stable strategy. Lemma 3 states the formal conditions for this pure strategy to be stable. In this case, the classic "free-rider effect" does not hold, and the private optimum is in fact higher than the social optimum.

3.4. Effect of government subsidies

As before, if the government offers subsidies of the monetary cost of the vaccine, individuals' expected payoff for vaccinating becomes a function of the subsidy. However, in contrast to the earlier section, with conforming peer effects present, it will also be a function of the current coverage:

$$E_{vac}(p,s) = H + \theta(C - m(1-s)) - d_v + \gamma' f(p) \tag{68}$$

where s represents the percentage of the marginal production cost of the vaccine that the government is subsidizing. Unlike the game with only non-conforming peer effects, there is no simple way to write the private optimum as a function of the subsidy, so we instead focus on general effects and the optimal subsidy in the presence of conforming peer effects.

In general, when the monetary cost of the vaccine is lowered, the vaccination coverage will increase, as can be seen by decreasing r and looking at the mixed ESS in Figure 4. When only non-conforming peer effects are present, this produces a counter effect – as the vaccination coverage goes up, individuals' incentive to vaccinate goes down, as their probability of getting infected decreases with the coverage. So, the advantage gained by lowering the monetary cost of the vaccine is mitigated by the non-conforming peer effects, and subsides are less effective. However, when conforming peer effects are present, there is a third effect that can play a role – the individual desire to vaccinate less as more people vaccinate is balanced by the desire to conform, resulting in more effective subsidies than when only non-conforming peer effects are present.

To illustrate these concepts more concretely, we return to our running example, where $f(p) = p$, $g(p) = 1 - p$, and $w(p) = 1 - \frac{1}{R_0(1-p)}$. In this case, the optimal coverage is p_{crit}, and in order to achieve this level of vaccination, we solve for the optimal subsidy. It is easily verified (setting $E_{vac}(p_{crit}, s_{opt}) = E_{nv}(p_{crit})$ and solving for s_{opt}) that

$$s_{opt} = 1 + \frac{d_v - \gamma'\left(1 - \frac{2}{R_0}\right)}{\theta m}. \tag{69}$$

Comparing this optimal subsidy to that in Section 2.4, we see that if $R_0 > 2$

$$s_{opt}^{combo} < s_{opt}^{nc} \tag{70}$$

always – we require less subsidy to achieve the same level of vaccination coverage. Examining the optimal subsidy further, we look at when conformity "helps" and when it can "hurt." In the original game, the optimal subsidy was greater than one – individuals needed to be paid extra, not just have the cost subsidized, in order to eradicate the disease. When conforming peer effects are present, it is possible to avoid this problem. To see this, look at the case where $s_{opt} \leq 1$. Rearranging and solving for γ', we see that this is equivalent to

$$\gamma \geq \frac{d_v}{d_i\left(1 - \frac{2}{R_0}\right)} \tag{71}$$

In other words, if the conformity effect is strong enough, individuals do not need to be paid extra to achieve the social optimum. However, if the conformity effect is too strong, we might need to impose a "tax" to bring the coverage level down to the social optimum. Recall that in this combination game, a pure NE and ESS exists at $P = 1$; if the conformity is strong enough and a subsidy is used, it is possible that individuals would choose to always vaccinate. This would certainly lead to disease eradication, but the extra cost incurred would not make this a socially optimal strategy. Formally, $s_{opt} \leq 0$ if

$$\gamma \geq \frac{r}{1 - \frac{2}{R_0}}. \tag{72}$$

In other words, the optimal subsidy will be negative (a tax) if the conforming peer effects are too strong.

Figure 5 plots all Nash equilibrium (weak and strict) of the vaccination game with conforming and non-conforming peer effects as a function of r. Using this figure, we can again see the importance of the starting point and the weak NE, as discussed at the end of Section 3.2 in the context of γ. Here, we can see that the subsidy will also play a role. For example, assume that a new vaccine is being introduced, and so the starting point is at 0% coverage and suppose $r = 0.5$. In order to bump up the coverage to the social optimum, we need to reduce r to approximately $r_1 = 0.3$. However, if we use too large a subsidy and decrease r to $r_2 = 0.2$, the only equilibrium will be at 100% coverage, incurring too much extra cost. The weak NE come

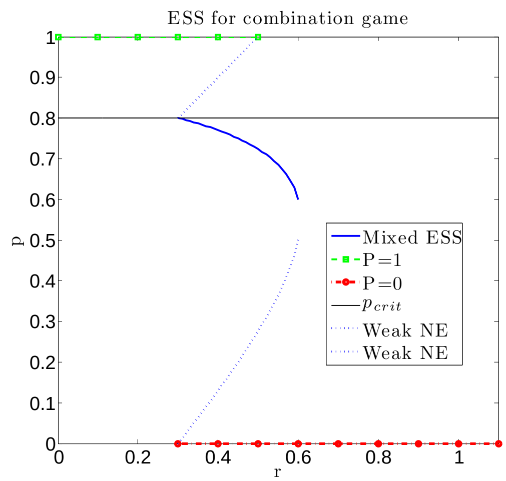


Figure 5: **All NE as a function of r.** In this figure, the dotted blue lines represent the weak Nash Equilibria of the game with conforming and non-conforming peer effects, plotted as function of the relative cost. Note that these dictate which of the stable equilibria (denoted by the solid blue and dashed red an green lines) will be observed.

into play if the starting point is somewhere between $0 - 100\%$, as discussed before. With conforming peer effects, we gain in that the required subsidy to achieve the social optimum is less, but there is now the possibility to over-subsidize, leading to over-vaccination. Knowing the current vaccination level and the value placed on conforming are both key to determining the appropriate subsidy.

4. Conclusions

In this paper we contrasted the positive and normative implications of two alternate models of vaccination decisions. In the first or traditional model, rational agents desire to free-ride on others' vaccination decisions. In the second model, agents have an additional desire to conform to their peers' vaccination decisions. We demonstrated that adding conforming peer effects to the traditional model can have important implications for understanding vaccination decisions and designing public health policy.

Adding conforming peer effects overturns several important results from traditional vaccination models. In most traditional models, privately optimal vaccination rates are always below the socially optimal rate. These models also produce a unique evolutionarily stable equilibrium. In contrast, in the model with conforming peer effects, privately optimal vaccination rates can be above or below the social optimum. In fact, the model produces several evolutionarily stable equilibria including no vaccination coverage, full vaccination coverage and a mixed strategy equilibrium. Since this model produces several equilibria vaccination rates, the final state not only depends on the vaccine and disease parameters but also on the initial conditions, implying that the effect of changes in the cost of vaccines or new side effect information might depend on the initial equilibrium vaccination rate.

Traditional models also imply that vaccine subsidies are always optimal since private vaccination rates are below the social optimum. Given the free-rider problem these models also imply that even when vaccines are free, coverage required to achieve disease eradication is impossible. In contrast, in the model with conforming peer effects, subsidies for vaccination are not always optimal as the privately optimal vaccination coverage might be above the social optimum. However, in certain cases, depending on the disease and vaccine parameters, even small subsidies can achieve disease eradication, but the effects of subsidies can also depend on initial vaccination rates.

Overall, these results suggest that conforming peer effects can have important implications for designing effective public health policy and understanding the effectiveness of interventions for improving vaccination coverage. Yet we know little about the magnitude of conforming peer effects and the extent to which these peer effects might vary across diseases, geography, and age group. We also know little about what factors influence peer effects in vaccination decisions and whether we can design interventions to change their magnitude. These are all fruitful avenues for future research.

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Appendix A. Definitions

Within the vaccination game with non-conforming peer effects, we are interested in equilibrium strategies to determine the behavior of rational agents. We define a Nash Equilibrium and an Evolutionarily Stable Strategy (ESS) as given in Hofbauer and Sigmund (1998).

Definition 1. A strategy P is a Nash Equilibrium if for all strategies $Q \neq P$

$$E(Q, P) \leq E(P, P). \tag{A.1}$$

Note that P is referred to a *strict NE* if the inequality is strict, or as a *weak NE* if the equality holds.

Definition 2. Let $p_{tot} = \epsilon Q + (1 - \epsilon)P$ represent the total vaccination coverage when ϵ-fraction of the population deviates from strategy P to Q. A strategy P is an Evolutionarily Stable Strategy (ESS) if for all $Q \neq P$

$$E(Q, p_{tot}) < E(P, p_{tot}) \tag{A.2}$$

holds for all $\epsilon > 0$ sufficiently small.

An alternate definition of an ESS is useful for proving when it exists:

Definition 3. A strategy P is an ESS $\iff$ for all strategies $Q \neq P$

$$\text{(i) } E(P, P) > E(Q, P), \text{ or} \tag{A.3}$$

$$\text{(ii) } E(P, P) = E(Q, P) \text{ and } E(P, Q) > E(Q, Q) \tag{A.4}$$

Note that the following relations hold for NE and ESS's:

$$\begin{aligned} \text{strict NE} &\implies \text{ESS} \\ \text{ESS} &\implies \text{NE} \end{aligned}$$

Appendix B. Proofs

In this section we present formal proofs not included in the main body of the paper.

Proof. (Lemma 1) In the context of the vaccination game with only non-conforming peer effects, the Nash Equilibrium condition from Definition 1 can be rewritten as, for all $Q \neq P$,

$$
\begin{aligned}
& E(P,P) - E(Q,P) \geq 0 && \text{(B.1)}\\
\iff & P[E_{vac}] + (1-P)[E_{nv}(P)] - Q[E_{vac}] - (1-Q)[E_{nv}(P)] \geq 0 && \text{(B.2)}\\
\iff & (P-Q)[E_{vac} - E_{nv}(P)] \geq 0 && \text{(B.3)}\\
\iff & (P-Q)[-r + w(P)] \geq 0. && \text{(B.4)}
\end{aligned}
$$

Consider a population following the mixed equilibrium strategy p^* satisfying $w(p^*) = r$. This strategy is clearly a weak NE, since (using the rewritten NE condition from B.4)

$$E(p^*, p^*) - E(Q, p^*) = (p^* - Q)(-r + w(p^*)) = 0. \quad \text{(B.5)}$$

Using Definition 3, p^* will be an ESS if

$$E(p^*, Q) > E(Q,Q) \iff (p^* - Q)(-r + w(Q)) > 0. \quad \text{(B.6)}$$

Consider first the case where $Q > p^*$. In this case, we have $p^* - Q < 0$ and

$$-r + w(Q) < -r + w(p^*) = 0 \quad \text{(B.7)}$$

since by assumption the probability of getting infected $w(p)$ is a strictly decreasing function with vaccination coverage p (i.e., an individual's probability of getting infected goes down as more people choose to vaccinate). As a result, $(p^* - Q)(-r + w(Q)) > 0$.

Now consider the second case where $Q < p^*$. In this case, we have $p^* - Q > 0$ and

$$-r + w(Q) > -r + w(p^*) = 0 \quad \text{(B.8)}$$

again since the probability of getting infected $w(p)$ is a strictly decreasing function with vaccination coverage p. Note however that if $r > w(0)$, we have

$$-r + w(Q) < -w(0) + w(Q) < 0 \quad \text{(B.9)}$$

always. Thus, $(p^* - Q)(-r + w(Q)) > 0$ and the mixed equilibrium is an ESS only if $r < w(0)$, i.e., if the vaccine is sufficiently inexpensive and safe.[5] □

[5] In the case where $r > w(0)$ (when the relative cost of vaccination to being infected

Proof. (Theorem 1) Differentiating the social welfare function when $p < p_{crit}$, we have

$$\frac{\partial W(p)}{\partial p} = [-d_v - \theta m + d_i w(p)] + (1-p)\left[-d_i \frac{\partial w(p)}{\partial p}\right] \tag{B.11}$$

$$= \frac{\partial E[P,p]}{\partial P} + \underbrace{(1-p)\left[-d_i \frac{\partial w(p)}{\partial p}\right]}_{>0 \text{ always, since } \frac{\partial w(p)}{\partial p} < 0} \tag{B.12}$$

The first term captures the private benefit from increasing vaccination and the second term captures the societal benefit which arises as increasing vaccination reduces the probability of infection for the entire population. At the private optimum $p = p^*$, the social welfare will be increasing with p, since $\frac{\partial E[P,p]}{\partial P} = 0$ for $p = p^*$. Further, for $p < p^*$, the social welfare will also be increasing with p, as

$$-d_v - \theta m + d_i w(p - \epsilon) > -d_v - \theta m + d_i w(p^*) = 0 \tag{B.13}$$

where the inequality comes from the fact that $w(p)$ is decreasing in p. So, the social welfare function is increasing in p for all $p \leq p^*$, and as a result, since $p_{opt} = \text{argmax}\, W(p)$ and using the analysis above, we have

$$p^* \leq p_{opt} \leq p_{crit} \tag{B.14}$$

□

---

is smaller than the probability of being infected with zero coverage), there exists a pure strict NE and ESS at $P = 0$, nobody vaccinating. In this case,

$$E(P,P) - E(Q,P) = (P-Q)(E_v(P) - E_{nv}(P)) = -Q(E_v(0) - E_{nv}(0)) = -Q(-r + w(0)) \tag{B.10}$$

Thus, $E(Q,P) < E(P,P)$ if $w(0) < r$. If $r = 0$ (there is no cost or risk associated with the vaccine), any strategy $P \geq p_{crit}$ will be a weak NE, including the pure strategy $P = 1$, everyone vaccinating. However, none of these weak NE will be evolutionarily stable, since they are not resistant to a decrease in vaccination coverage. As a result, everyone will converge to the mixed strategy $p^* = p_{crit}$.

Appendix C. The SIR Model with constant population size and vaccination

Using the SIR model with constant population model (birth rate = death rate = μ) in Bauch (2004), often used to model childhood diseases, we have,

$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S \tag{C.1}$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I \tag{C.2}$$

$$\frac{dR}{dt} = \mu p + \gamma I - \mu R \tag{C.3}$$

where p is vaccination uptake, β is the mean transmission rate, $1/\gamma$ is the mean infectious period, and μ is the mean birth and death rate. We can reduce these equations to the following dimensionless form:

$$\frac{dS}{d\tau} = f(1-p) - R_0(1+f)SI - fS \tag{C.4}$$

$$\frac{dI}{d\tau} = R_0(1+f)SI - (1+f)I \tag{C.5}$$

where $\tau = t\gamma$ is time measured in units of the mean infectious period, $f = \mu/\gamma$ is the infectious period as a fraction of mean lifetime, and $R_0 = \beta/(\gamma+\mu)$ is the basic reproductive ratio (the average number of secondary cases produced by a typical primary case in a fully susceptible population). (From Anderson and May (1991), we have for childhood diseases, $f < .001$ and $R_0 \sim 5-20$.)

The predictions of the SIR model depend on the critical coverage level that eliminates the disease from the population, p_{crit}:

$$p_{crit} = \begin{cases} 0 & R_0 \leq 1 \\ 1 - \frac{1}{R_0} & R_0 > 1 \end{cases} \tag{C.6}$$

If $p \geq p_{crit}$, then the system converges to the disease-free state $(\hat{S}, \hat{I}) = (1-p, 0)$, whereas if $p < p_{crit}$, it converges to a stable endemic state given by

$$\hat{S} = 1 - p_{crit} \tag{C.7}$$

$$\hat{I} = \frac{f}{1+f}(p_{crit} - p) \tag{C.8}$$

Because S and I are constant in this situation, the probability that an unvaccinated individual eventually becomes infected can be expressed, using the above equations, as the proportion of susceptible individuals becoming infected versus dying in any unit time,

$$w(p) = \frac{R_0(1+f)\hat{S}\hat{I}}{R_0(1+f)\hat{S}\hat{I} + f\hat{S}} = 1 - \frac{1}{R_0(1-p)}. \tag{C.9}$$

Thus, we have our infection probability:

$$w(p) = \begin{cases} 1 - \frac{1}{R_0(1-p)} & 0 \leq p \leq p_{crit} \\ 0 & p_{crit} < p \leq 1 \end{cases} \tag{C.10}$$

Note that $w(p)$ is a decreasing function of p, as shown in Figure C.6.

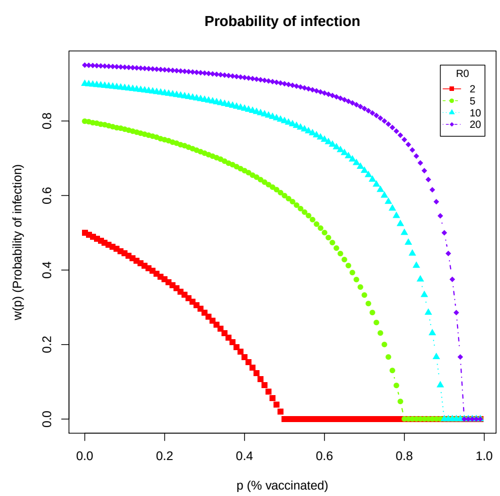

Figure C.6: Probability of infection