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ABSTRACT

This paper develops a model to study the aggregate effects of labor market frictions in an open economy through their impact on the growth and investment decisions of …rms. The model features interactions between firms' dynamic fixed investments in exporting and search frictions with job-to-job mobility. Search frictions induce slow firm growth and are a source of dispersion in firm size and export status. Job-to-job transitions are a crucial ingredient of the analysis, as in their absence search frictions do not affect outcomes per worker. The model is tractable for general-equilibrium analysis and accommodates several extensions which are useful for quantitative work. A calibration to Argentina's economy suggests that frictions in job-to-job mobility may have considerable effects on firm growth and aggregate income, and that barriers to worker mobility across firms may be relevant to measure the gains from international trade.

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1 Introduction

This paper develops a model to study the aggregate effects of labor market frictions in an open economy through their impact on the growth and investment decisions of firms. The novel aspect of the model is the interaction between firms' dynamic fixed investments in exporting and search frictions with job-to-job mobility. The model is tractable for general-equilibrium analysis and accommodates several extensions which are useful for quantitative work. A calibration to Argentina's economy suggests that frictions in job-to-job mobility may have considerable effects on firm growth and aggregate income, and that they may have measurable impact on the macro gains from international trade.

A recent view in macroeconomics and international trade stresses that to understand aggregate outcomes it is necessary to study the allocation of resources across firms. In Melitz (2003), firm heterogeneity impacts export decisions and shapes aggregate outcomes, while in Hsieh and Klenow (2009) distortions in the allocation across producers result in aggregate productivity losses. More recently, Hsieh and Klenow (2012) have documented that the life cycle of plants is important for misallocation. In India and Mexico, low aggregate productivity is associated with slow growth and productivity gains over the life cycle of plants relative to the United States. What forces generate heterogeneity across firms and deter them from growing? Assessing distinctive mechanisms underlying these phenomena is relevant to inform future theories, empirical studies and policy.

Labor market frictions are a natural candidate to explain dispersion in firm size and productivity. Firms must spend time and resources to attract workers, and when firms operate in more rigid labor markets these costs and delays of hiring workers increase.[1] This may cause aggregate income losses because many important investments have a fixed-cost component and can be profitably undertaken only by producers with sufficient scale. I focus the analysis on investments in export capacity, as exporting is a clear discrete investment that requires scale and is generally associated with increases in measured productivity per worker.[2] Firms' export status is also commonly observed in the data, and for the calibration I can use data from Argentina on firm growth, worker transitions between firms and export dynamics that are well suited for the analysis.

I build upon a standard model of a labor market with search frictions and job-to-job transitions in the tradition of Burdett and Mortensen (1998). Frictions in job-to-job transitions are a key ingredient of the analysis because they have an asymmetric impact on firms with different size and export status. Larger firms are more likely to export and rely more on job-to-job hiring to grow. Hence, lower frictions in job-to-job transitions increase income per worker through faster growth of the more productive firms and higher incentives to invest. If job-to-job transitions are not allowed in the model, search frictions do not have general-equilibrium effects on outcomes per worker.

[1] Recent cross-country evidence and studies of policy reforms relate firm-level employment adjustment to institutional features of the labor market. Caballero et al. (2004) and Haltiwanger et al. (2008) show that firm-level job flows and employment adjustments are lower in more rigid labor markets, while Kugler (2007) summarizes a body of evidence based on reform episodes that affected specific groups of firms in Italy, Spain, Germany and the United States. As a general finding, a tightening in employment protection reduces net employment adjustments for the affected firms.

[2] See Bernard et al. (2007) and Das et al. (2007) for evidence on exporter premia and fixed costs of exporting.

In the model, ex-ante symmetric firms match with workers, who learn about job opportunities both when unemployed and on-the-job. As in Postel-Vinay and Robin (2002), firms engage in Bertrand competition for workers. Job-finding rates may vary by worker's employment status and capture the extent of labor-market frictions.

I extend this framework, first, by allowing for firm dynamics. Firms are born and die continuously due to exogenous shocks. Surviving firms contact potential employees slowly and discount future profits at positive rate. Second, firms can make a one-off investment in exporting. For this, the extended labor search model is embedded in a two-country trade model with monopolistic producers, as in Krugman (1980) and Melitz (2003). The slow labor adjustment created by labor market frictions implies that the market-size expansion offered by trade leads to an increase in revenue per worker; in contrast, Krugman (1980) and Melitz (2003) are frictionless models so that firms instantaneously adjust to their preferred size and all firms share the same equilibrium level of revenue per worker regardless of export status. As a result, labor market frictions impact on aggregate outcomes through the interaction between investing in a high-revenue activity and firm growth.

I start in Section 2 by setting up the benchmark model and characterizing the investment decision in partial equilibrium. Firms are born small, accumulate workers slowly, and invest in exporting when they are sufficiently large. The timing of entry into exporting is the key outcome of the model. Firms have incentives to delay export entry to save on sunk costs, but they also have incentives to invest earlier to obtain greater revenues on their current workforce. In addition, a firm that invests earlier has incentives to hire workers more aggressively from firms who offer jobs of lesser value, growing faster as a result. Flexibility in job-to-job transitions determine the magnitude of this complementarity between the investment and firm growth.

Section 3 fully characterizes the general-equilibrium impact of labor market frictions. In general equilibrium, the timing of each firm's investment depends on the distribution of competitors, summarized by their entry and investment decisions; in addition, the market size in one country impacts the incentives to export in the trading partner. Because workers flow from younger and smaller firms into older and larger firms who are more likely to invest, smaller frictions in transitions between jobs induce earlier investment, generating income and exports increases at the aggregate level. In contrast, frictions in transitions out of unemployment have neither effects on outcomes per employed worker nor on the timing of export entry because in general equilibrium their impact is absorbed by firm entry or exit. The model also implies a complementarity between the labor market policies of trading partners. When job-to-job transitions become easier in one country, exports and real income must also increase in the trading partner.

Section 4 presents several extensions to the model. The baseline theory builds on Postel-Vinay and Robin (2002), inheriting from that framework the assumptions that contact rates between workers and firms are exogenous (but allowed to vary between employed and unemployed workers) and that workers have no bargaining power against firms. I show that the main results carry through when endogenous matching rates are allowed, and that key outcomes of the model admit a tractable characterization when workers are allowed to have bargaining power. Additionally, I characterize the investment decision when firms can sequentially engage in multiple fixed investments of varying size, as if for example several export markets are available. In this case, the timing of entry to each market is characterized by a simple recursive structure, a feature that I exploit in the calibration.

Section 5 presents the calibration and counterfactuals. The natural challenge in the parametrization is to discipline the degree of frictions in worker transitions, as the theory shows that these frictions are key for aggregate outcomes. For that, the model is calibrated to Argentina's economy, where, in addition to summary statistics from the firm size, age, and export distributions, I observe the rates at which firms with different export status hire workers directly from other firms. These rates of job-to-job hiring by export status serve to discipline the extent of frictions in job-to-job mobility in the model.

The calibrated model replicates well the empirical pattern of firm growth and suggests that frictions in job-to-job transitions may have sizeable effects on firm growth and aggregate income. For example, 40-year-old Argentinean firms are 3.8 times larger than 5-year-old firms, a ratio that is closely replicated by the calibrated model. To have a sense of the importance of frictions, I perform the counterfactual experiment of lowering frictions in the model to match the rates of job-to-job hiring observed in the U.S.. In that case, the ratio of firm sizes between ages 40 and 5 increases to 6.2, coming close to the Hsieh and Klenow (2012) finding for the U.S. that plants at age 40 are on average 8 times larger than at age 5. This reduction in frictions also leads to a 35% increase in real income per worker.

Finally, the application to international trade aims to speak to a broader discussion about the role of micro features in shaping the macro gains from trade. In an influential study, Arkolakis et al. (2012) show that, in a widely used class of trade models, aggregate trade shares and an aggregate trade elasticity suffice to measure the aggregate gains from increasing an economy's exposure to international trade.[3] Several features, such as firm-level dynamics and frictions in resource allocation, set my model outside of that class. Therefore, it is meaningful here to ask if frictions in job-to-job mobility matter for the model-implied gains from trade. I find numerically that, conditioning on the aggregate trade share, the gains from a reduction in trade costs are smaller when frictions in job-to-job transitions are larger. This suggests that modelling barriers to worker mobility across firms and accounting for their magnitude may be important to measure the gains from international trade.

The article is related to the search literature with job-to-job transitions in the spirit of Burdett and Mortensen (1998). Most studies in this tradition restrict their analysis to the static decisions of firms in their long-run scale, but here the nature of the fixed-cost investment problem shifts the focus to the dynamic aspect in the firms' decision. Burdett and Menzio (2013) is formally a closely related paper. They characterize the optimal stopping time of a firm that must decide when to change its price in the presence of fixed menu costs and search frictions in product markets. Kaas and Kircher (2011) study a frictional labor market with multi-worker firms and decreasing returns to scale. The distinguishing feature of my analysis is the dynamic fixed investment decision, which, as in that paper, creates a relationship between job values and firm size even though here

[3] Costinot and Rodriguez-Clare (2013) review the recent international trade literature on this subject.

firms operate constant-returns-to-scale technologies. Moscarini and Postel-Vinay (2011) and Schaal (2012) study frictional reallocation of workers across heterogeneous firms over the business cycle.

This paper is of course not the first to investigate how features of the labor market impact aggregate outcomes. The distinctive feature of my approach is the interaction between frictions in job-to-job mobility, firm growth, and fixed investments. As such, it complements papers in the spirit of Hopenhayn and Rogerson (1993), who embed labor taxes in a model with firm dynamics. The impact of labor market frictions on firms' investment decisions is also explored by Acemoglu and Shimer (2000) in a directed search framework with single-worker firms. Lagos (2006) presents a model with a dependence of TFP on search frictions.

The paper also complements studies that embed labor market imperfections in trade models with heterogeneous-productivity firms, such as Helpman and Itskhoki (2010) or Amiti and Davis (2011). The distinguishing aspect of my approach is that frictions induce slow growth and a firm life cycle, while these are static setups. Recent quantitative assessments of models with search frictions in open economy, such as Coşar et al. (2010), and Coşar (2011), do not allow for job-to-job mobility, which are a key feature of my analysis. Recent models with firm dynamics and exporting include Atkeson and Burstein (2010) and Arkolakis (2009). From an empirical standpoint my approach is distinguished by specific predictions regarding the composition of new hires by export status, and by the impact of labor market frictions on export dynamics and aggregate outcomes.

The paper is structured as follows. Section 2 studies the partial-equilibrium problem of a firm that decides the timing of investment. Section 3 moves to general-equilibrium analysis. Section 4 presents the extensions, including the one to multiple investment options. Section 5 calibrates the model to Argentina's economy and performs the counterfactuals. Finally, Section 6 concludes. I relegate all proofs to the appendix.

2 The Model

I develop an open economy model where firms expand their workforce slowly and can pay a sunk cost to enter foreign markets. Labor market frictions determine the ease of hiring employed or unemployed workers, affect the timing of export entry, and, through these channels, impact the aggregate outcomes of the economy. In closed economy, the investment can be viewed as the decision to use a high-productivity technology.

2.1 Preferences

The world economy consists of two countries, home and foreign.[4] In each country there is mass of identical workers of measure one. Time is continuous. Workers have linear utility for consumption of a final non-tradable good and they discount the future at rate ρ:

$$U = \int_0^\infty e^{-\rho t} c_t dt.$$

[4] In the extensions and in the calibration I consider multi-country environments.

I focus on a steady state in which aggregate variables are constant, so that the flow value of aggregate utility equals consumption of the final good, c.

The trade environment shares the central features of Krugman (1980). Monopolistically competitive firms sell varieties of a differentiated good. These varieties are internationally traded subject to an iceberg cost $\tau \geq 1$, and then aggregated in each country into the final non-tradable good with a constant elasticity of substitution (CES) $\sigma > 1$ across varieties.

2.2 Revenues of Exporters and Domestic Producers

In the home market, an endogenous mass of firms of measure M produces the differentiated varieties using a constant-returns-to-scale technology with labor as the only factor of production. All firms are born identical and they can choose between exporting to the foreign market subject to a sunk cost or remaining domestic. Firms enter the market with no workers and grow subject to their contacts in the labor market, as I describe below. Firms suffer a shock that forces them to exit at rate μ, and there is continuous re-entry to replace exiting firms.

A known feature of the CES demand structure is that product differentiation leads to downward sloping demand and concave revenue-functions because, as firms expand their supply, consumers derive a progressively lower marginal utility from a particular variety. However, to incorporate a frictional labor market with job-to-job mobility into the model, it is convenient to operate with linear revenue functions, as in Burdett and Mortensen (1998) and many others. To resolve this tension, I extend the standard CES structure with a simple quality choice by firms whereby firms may shift their demand curves outwards as they grow in size. The key feature of this quality choice is that, when workers are perfect substitutes between producing quality or quantity, the reduction in marginal utility due to increased supply and the increase in marginal utility due to increased quality exactly compensate as the firm expands. As a result, revenue per worker is independent from firm size and only depends on firm export status. I explain in detail this quality choice in Section A of the appendix.

Let $j = D, X$ respectively indicate that a firm is a domestic producer (i.e., non-exporter) or an exporter. Real revenues of each type of firm, measured in terms of the final non-tradable good, as function of its number of workers n are

$$r_j(n) = y_j n \quad \text{for} \quad j = D, X, \tag{1}$$

where y_j for $j = D, X$ denotes the real revenue per worker in a type-j firm. Letting Y denote real income per capita in the home economy, the results from Section A of the appendix imply that the real revenue per worker in each type of firm is

$$y_D = Y^{1/\sigma}, \tag{2}$$

$$y_X = \Gamma y_D, \tag{3}$$

where the *revenue premium of exporters,* Γ, is determined endogenously and depends on the relative size of both economies,

$$\Gamma = \left[1 + \tau^{-(\sigma-1)} (P^*)^{\sigma} \frac{Y^*}{Y}\right]^{\frac{1}{\sigma}}. \quad (4)$$

In this expression, P^* denotes the price index in the foreign country relative to the price index at home, which is normalized to one, and Y^* is real income per capita in the foreign market. Since $\Gamma > 1$, when a firm sinks the fixed cost of entry into a foreign market it can earn more revenue than a firm that sells only at home.[5] From the perspective of an individual firm, the relative size of the foreign economy increases due to less competition (higher P^*), larger market size (higher Y^*/Y) or lower trade costs (lower τ). As in Melitz (2003), the revenue premium reflects that, by exporting, firms can sell to consumers with higher willingness to pay for its products.[6]

This structure is useful because it preserves a key aspect of monopolistically competitive models of trade, namely the increase in revenue associated with export activity, without compromising tractability once the frictional labor market with job-to-job transitions is introduced. By making the marginal valuation for a new worker independent from firm size, the value of a match in a domestic firm will only depend on how long a domestic firm plans to wait until upgrading revenues to y_X. This will imply a simple pattern of transitions between jobs, with workers moving from younger to older firms.

2.3 Labor Market Environment

Labor markets are subject to a standard search friction whereby workers learn of jobs when unemployed or employed according to a random process. The Poisson rate at which a worker makes contact with some firm is λ_u for unemployed workers and λ_e for employed workers. In reduced form, these parameters capture institutional features of the labor market that affect worker mobility and are interpreted as summary measures of labor market rigidity.[7]

Following Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002) I assume that the rates λ_u and λ_e are exogenous. Section 4.1 examines endogenous matching rates.

In addition to the transitions between jobs to be described below, jobs are terminated at an exogenous rate γ. Taking into account the chance μ of firm exit, this means that every employee moves into the pool of unemployed workers at rate $\delta = \gamma + \mu$.[8] The steady-state rate of unemployment is readily given by parameters: $u = \delta / (\lambda_u + \delta)$. Therefore, the theory is focused on

[5] In the terminology of Redding and Venables (2004), Γ is the "market access" and $PY^{1/\sigma}$ is the "market capacity" of the home country. McGrattan and Prescott (2008) derive an expression similar to Γ in an open economy setting with perfectly competitive product markets and decreasing returns in production. In their case, the productivity increase results from diversification of resources across destinations by multinational firms.

[6] In Melitz (2003) the revenue advantage created by exporting is instantaneously offset by the firm's increase in size, so that in equilibrium all firms share the same equilibrium level of revenue per worker regardless of export status. Here, in contrast, the slow labor adjustment implies that the market-size expansion offered by trade leads to an equilibrium difference in revenue per worker between exporters and non-exporters.

[7] Empirical estimates of job-finding rates of unemployed and employed workers are typically higher in countries with more flexible labor markets. E.g., Hobijn and Sahin (2009) report a considerably larger job-finding rate in the U.S. than in Western European countries. See also Bontemps et al. (2000) and Jolivet et al. (2006).

[8] Firm exit is necessary to induce an invariant distribution of ages. Exogenous separations serve to bound the size of surviving firms.

explaining how employment is distributed across firms and takes the unemployment rate as given. To save notation later, I define the normalized contact rate on the job, $\kappa_e = \lambda_e/\delta$.

2.4 Value of Jobs

Revenue per worker changes throughout the life of firms. In equilibrium, exporting firms with revenue y_X do not switch back into y_D, but domestic firms generating y_D units of revenue per worker may intend to upgrade at some point. Therefore, the value of jobs offered to prospective workers depends on how long a firm expects to wait until starting to export. Let x indicate this "time until exporting" for a given firm. Across the economy there are (potentially) three classes of firms: $x = 0$ denotes firms that have already started to export; $x \in (0, \infty)$ denotes firms that will start exporting in x periods from now if they survive for that long; and $x = \infty$ denotes firms that will never export no matter how long they survive.[9]

Let $v(x)$ represent the total value of a job held by a firm whose time until exporting, if it does not suffer an exit-inducing shock before then, is x. This value reflects the joint surplus of a match shared by the firm and the worker. When a new relationship is formed, the partners divide the surplus according to the game posited by Postel-Vinay and Robin (2002): firms observe the current status of contacted workers, tender take-it-or-leave-it offers, and commit to the value promised to the worker. As a consequence, when an unemployed worker meets a firm, the offer leaves the worker indifferent between the job and the value of unemployment, w_u, and is accepted.

The present discounted sum of future expected profits generated in firm x by a worker who enters the firm from unemployment equals the total value of a job held by this firm, net of the amount necessary to lure the worker, namely

$$J_u(x) = v(x) - w_u. \tag{5}$$

In contrast, when an employed worker meets a new firm, the current employer hears the job offer and makes a counter-offer. The outcome is similar to Bertrand competition: the firm offering the job of greater total value obtains the worker, offering in exchange a value equal to what the worker could obtain in the alternative employment. Since transitions are efficient, I conjecture that workers flow from firms with higher x into firms with lower x. Therefore, when a worker transits from a firm x_0 to a firm $x < x_0$, firm x captures a present discounted value of profits of

$$J(x_0, x) = v(x) - v(x_0). \tag{6}$$

Both $J_u(x)$ and $J(x_0, x)$ denote present discounted sums of expected profits captured by a firm from one particular worker when the worker enters the firm. After that moment, the worker might leave due to an exogenous shock or make contact with another firm, triggering a renegotiation or a quit. These possible events are included in the computation of $J(x_0, x)$.

[9] Since firms are homogeneous and will all choose the same outcome, the equilibrium will either feature firms who never invest ($x = \infty$) or firms who invest at some point ($0 \leq x < \infty$), but not both. The calibration of Section 5 allows for heterogeneity, so that both types can coexist.

The following Lemma shows that the value of a match admits a simple characterization.

Lemma 1 (Value of a New Job) *The total value of a job held by a firm whose time until switching is x is*

$$v(x) = \frac{y_D + (y_X - y_D) e^{-(\rho+\delta)x} + \delta w_u}{\rho + \delta}. \tag{7}$$

This expression is key for what follows. The value of a job consists of the expected revenues generated by the worker throughout the duration of the match plus the value of unemployment obtained by the worker when the match is dissolved. This value increases as the firm approaches the time of exporting (i.e., it decreases with x), confirming the conjecture that workers move from high-x to low-x firms, but not vice-versa.[10]

It is also worth noting that the rate of contact on the job λ_e and the distribution of job offers faced by employed workers do not appear in $v(x)$. This is a consequence from assuming that workers have no bargaining power against the hiring firm, and it helps to make the model especially tractable. Section 4.2 shows how to extend the formula in (7) with bargaining power for workers, in which case both λ_e and the distribution of job offers would appear in $v(x)$.

2.5 Value of Firms, Stock Effect and Timing of Export Entry

As anticipated, firms can choose between the alternative export status y_D and y_X. Firms enter the market with no workers and grow subject to their contacts in the labor market while facing the exit risk. At birth, they are endowed with y_D, but they can choose at any time to make a once-and-for-all investment to start exporting and upgrade revenues to $y_X = \Gamma y_D$. This investment entails a sunk cost with flow-equivalent value of f_X in units of the final good. This decision is similar to a stopping time in which firms decide the optimal time to switch into a superior technology.[11]

A firm has perfect foresight about the evolution of its stock of employees, facing no uncertainty beyond the exit probability.[12] As a result, firms choose an age h to start their export activity. Firms commit to the timing of exporting when they are born.

The export entry decision is made on the basis of the flow of workers obtained in each period and the valuation attached to each. At any moment, a firm makes contact with

$$\left(\frac{s}{\bar{s}M}\right)\left[\lambda_u u + \lambda_e (1-u)\right] \tag{8}$$

workers, where $s/\bar{s}$ is the search effort exerted by the firm to find workers relative to average search activity in the economy, and M is the measure of firms. Until the quantitative analysis of Section

[10] In the data workers also move in the opposite direction; for example, there are transitions from exporters ($x = 0$ in the model) into non-exporters ($x > 0$). The model can be reconciled with these (relatively uncommon) flows adding heterogeneity in firm productivity. Section C.2 in the appendix describes the general equilibrium of an extended model with heterogeneity in firm productivity.

[11] Burdett and Menzio (2013) study a formally related problem. They characterize the optimal stopping time of a firm that must decide when to change its price in the presence of fixed menu costs and search frictions in product markets.

[12] Since I treat the stock of workers in the firm as a continuous set, the individual contact and exit rates equal the fraction of workers who experience these shocks. Since growth is deterministic, it is equivalent to cast the firm problem in terms of fixed costs f_X per period.

5, s is assumed to be common to all firms. As a result, a worker who hears of an opening has the same probability of being matched with any firm. Differences in the rate at which firms accumulate workers arise solely from the ability to attract workers away from other firms.

Due to the linearity of the revenue function, firms wish to grow as large as possible. Therefore, every match with an unemployed worker results in a hire. In contrast, out of all contacts made with employed workers, a firm with time until export entry of x only attracts those workers employed in firms offering jobs of lesser value, i.e. in firms at $x_0 > x$. Let $G(x)$ be the share of employment in firms whose time until exporting is less than x; this distribution may have mass points at 0 or at ∞ that measure employment in exporting firms or in firms that will never export, respectively. The fraction of new hires out of all workers contacted by a firm with time until exporting x is:[13]

$$\frac{1+\kappa_e\left[1-G(x)\right]}{1+\kappa_e}. \tag{9}$$

The number of firms M in (8) and the distribution of employment across firms with different time until export $G(x)$ in (9) reflect competition in the labor market and will be determined in general equilibrium.

The present discounted value of profits generated by all workers who are hired by a firm in state x is the sum of the values generated by each of these workers individually:

$$\pi(x)=\frac{\lambda_u u}{M}J_u(x)+\frac{\lambda_e(1-u)}{M}\int_x^\infty J(x_0,x)\,dG(x_0). \tag{10}$$

The first term in this sum is the present value of profits generated by workers attracted from the pool of unemployment, and the second term corresponds to profits from workers attracted from other firms, drawn from the employment distribution G.[14] A firm whose time until exporting is x attracts all workers who are contacted from firms whose time until exporting is $x_0 > x$. Using this expression, the value *at entry* of a firm who will start exporting at some generic age h is

$$\Pi(h)=\int_0^h e^{-(\rho+\mu)a}\pi(h-a)\,da+e^{-(\rho+\mu)h}\left[\frac{\pi(0)-f_X}{\rho+\mu}\right]. \tag{11}$$

This value of a firm at entry captures the following life cycle: a new firm starts with no workers; at age $a = h - x < h$, incoming workers generate average expected profits with present discounted value of $\pi(h-a)$; and after h the firm obtains $\pi(0)$ from new workers for the rest of its expected life but it must pay the sunk cost with flow-equivalent value f_X. The effective rate of time discount, $\rho + \mu$, takes into account the probability of firm exit.[15]

[13] To obtain this expression, first write the measure of new hires by a firm with time until switch of x, $(s/\bar{s}M)\{\lambda_u u+\lambda_e(1-u)[1-G(x)]\}$, and normalize by the measure of contacted workers in (8). Using the values of u and κ_e yields (9).

[14] More precisely, $\lambda_u u/M$ is the flow of workers hired from unemployment, while $\frac{\lambda_e(1-u)}{M}\int_x^\infty dG(x_0)$ is the flow of workers hired from other firms. The job values $J_u(x)$ and and $J(x_0,x)$ correspond to the present discounted value of profits generated in a firm at x by each worker attracted from unemployment and by each worker attracted from a firm at x_0, respectively. When a firm at $x = 0$ is contacted by a worker employed in another firm at $x = 0$, workers are indifferent between switching or not. I assume that in this case workers switch jobs with 50% chance.

[15] Equation (70) in the appendix shows the law of motion for the number of workers within the firm. Note that

Firms choose the age h to start exporting when they are born to maximize $\Pi(h)$. To understand this decision, consider a firm that starts to export at age h. If that firm delays export entry, it gives up revenues by delaying the increase in sales per worker. In addition, it gives up growth by reducing the inflow of workers at each age younger than h. As a result, $\pi(h-a)$ in (11) shifts down for all a. At the same time, by delaying the time of exporting, the firm saves the cost f_X.

In any positive solution, the first order condition for h that maximizes $\Pi(h)$ can, after some manipulation, be written as[16]

$$S(h) = f_X \text{ if } h < \infty, \tag{12}$$

$$S(h) \leq f_X \text{ if } h = \infty, \tag{13}$$

where

$$S(h) = \int_0^h e^{(\rho+\mu)x}\left[-\pi'(x)\right]dx. \tag{14}$$

I refer to $S(h)$ as the *stock effect* of a delay in h. It captures the opportunity costs of delaying the age of entry into exporting. The firm chooses the h where these marginal costs equal the flow-equivalent value of the sunk cost.

Since the firm grows over time, the longer it waits, the larger is the opportunity cost of not exercising the investment in exporting; this is reflected in that $S'(h) > 0$, which implies that the profit function is strictly concave. Furthermore, $S(0) = 0$, i.e. there is no stock effect at entry because there is no initial labor force; so it must be that $h > 0$ unless the sunk cost is zero, in which case exporting occurs at birth. Finally, $S(h)$ is bounded, which implies that the firm actually intends to invest, i.e. h is a finite number, if and only if the fixed cost is not too large.[17]

With simple manipulations, the value of a firm in (11) can be equivalently formulated as $\Pi(h) \equiv [\pi(h) - \Pi'(h)]/(\rho+\mu)$. This expression holds generically for any h. Letting $\Pi^e \equiv \max_h \Pi(h)$ be the value of the firm at entry when it chooses the export timing h optimally, this means that in an interior solution, where $\Pi'(h) = 0$, the value of a firm at entry is

$$\Pi^e = \frac{\pi(h)}{\rho+\mu}. \tag{15}$$

This expression is useful for the characterization of the general equilibrium.

To see how the different variables shape the timing of investment we note that the stock effect is stronger, which means that firms start to export earlier, the faster π grows with age. This growth in the value of new hires depends on two margins, the number of new hires and the expected discounted revenues that they generate. Substituting the expressions for the value of each match

(11) is the present discounted sum of the present discounted value of profits generated by the flow of all new hires at each age; as such, it already incorporates information about worker exit and on-the-job contact probabilities through $\pi(\cdot)$.

[16] I adopt the convention that $h = \infty$ denotes that the firm's optimal choice is to never invest.

[17] This follows from firm size being bounded; if $\gamma \to 0$ (no exogenous separations) then h is necessarily finite.

from (5) to (7) into $\pi(x)$ in (10), the integrand in the stock effect in (14) takes the form

$$e^{(\rho+\mu)x}\left[-\pi'(x)\right] = \mathcal{M}(\Gamma-1)\,e^{-\gamma x} \times \{1+\kappa_e[1-G(x)]\}, \tag{16}$$

where

$$\mathcal{M} \equiv \frac{\lambda_u u}{M} y_D \tag{17}$$

captures market size through y_D and competition in the labor market through the size of the unemployment pool and the number of firms. A larger return to the investment, Γ, naturally accelerates the timing, as does a lower tightness in the labor market, captured by a higher $\mathcal{M}$. Interactions among firms also take place through the employment distribution. A first-order shift in the employment distribution $G(x)$ towards low-x firms makes it more likely that a worker contacted from another job is employed in a firm that is close to investing. This reduces the share of meetings that translate into new hires, slowing down growth and delaying the investment.

I collect the relevant results from this section in the following proposition.

Proposition 1 (Timing of Investment in Partial Equilibrium) *In an interior solution, a firm chooses the unique h where (12) holds. The firm never invests at entry unless $f_X = 0$, but eventually does so if and only if f_X is below some finite threshold. At an interior solution, h is decreasing in Γ, λ_u and λ_e, and increasing in M, and a first-order shift in $G(x)$.*

For what follows, the main implication of this proposition is that lower frictions lead to earlier time of investment, while more competition, through either the measure of rival firms or the distribution of employment across them, delays export entry of an individual firm. To fully assess the impact of labor market frictions it is necessary to move on to general equilibrium, where competition is determined endogenously.

3 General-Equilibrium Impact of Labor Market Frictions

I now consider the general equilibrium with two countries. In each country, many firms interact based on the decisions of competitors. Countries may differ in labor market fundamentals $\{\lambda_u, \lambda_e, b\}$ and in relative fixed costs f_X/f_D. The revenue premium of exports, Γ, is taken as given in each country but is determined endogenously through trade balance. Foreign-country variables are denoted with a star.

Since all firms from the same country face the same problem for which, as shown in the previous section, there is a unique solution, in equilibrium they must all start to export at the same time after birth, H. This common timing for exporting corresponds one-to-one with a number of aggregate objects: the distribution across firms of the time until exporting $P(x)$, the share of exporting firms m_X, the share of employment in these firms e_X and income per employed worker y.

In equilibrium, these variables must be such that a number of conditions hold. First, each individual firm, taking $\{P(x), m_X, e_X, y\}$ as given, solves the problem in the previous section and optimizes over its choice of h. Second, firms must not have incentives to deviate from the common decision H. Third, a free entry condition must be satisfied. Finally, exports from each country must be such that trade is balanced. I proceed to define these aggregate variables, then I move to the definition and characterization of the equilibrium, and finally I show the comparative statics.

The growth of a firm depends on where it is located relative to other firms in terms of time until exporting. Across the economy, the share of firms that are less than x periods away from exporting equals the fraction of firms that have survived beyond age $H-x$. Since firms exit at constant rate μ, the share of firms that are at less than x periods from exporting is

$$P(x;H) = e^{-\mu(H-x)}, \text{ for } x \in [0,H]. \tag{18}$$

Workers, either employed or unemployed, who make contact with a potential new employer have a probability $P(x;H)$ of sampling one that is less than x periods away from switching into exporting. The pattern of transitions from high-x firms into low-x firms gives the steady-state cumulative distribution across employees of the time until investing of their employer:[18]

$$G(x;H) = \frac{(1+\kappa_e) P(x;H)}{1+\kappa_e P(x;H)}. \tag{19}$$

The shape of this distribution responds monotonically to first-order shifts in $P(\cdot)$; a change in the firm distribution towards stronger competitors naturally translates into a rise in $G(\cdot)$.

The firm and employment distributions evaluated at $x=0$ give, respectively, the share of exporting firms and the share of employment allocated to these firms:

$$m_X(H) \equiv P(0;H), \tag{20}$$

$$e_X(H) \equiv G(0;H). \tag{21}$$

The share of exporting firms is the fraction of firms that has survived beyond age H, and is sufficient to characterize the share of employment in exporting firms.

Real income per employed worker equals the employment-weighted average of revenue across firms:

$$y = [1-e_X(H)]\, y_D + e_X(H)\, y_X. \tag{22}$$

Hence, the timing of the investment H is a key outcome because it determines the employment share e_X, which in turn determines income per employed worker y. Misallocation is high when e_X is low. Aggregate real income is the number of employed workers times real income per employee, $Y=(1-u)\,y$. Using (2), (3), and (22), it is readily expressed as

$$Y = \{(1-u)[1+e_X(H)(\Gamma-1)]\}^{\sigma/(\sigma-1)}. \tag{23}$$

The take-it-or-leave structure implies that the flow value to workers who are unemployed, ρw_u, equals the income flow of unemployed workers. To determine this income flow I assume that the

[18] This is obtained by setting the expression describing the evolution of $G(x)$, $(1-u)\,dG(x) = \{\lambda_u u + \lambda_e (1-u)[1-G(x)]\} P(x) - \delta(1-u) G(x)$, equal to zero.

government levies a lump-sum tax to compensate each unemployed worker with a transfer equal to a fraction $b \in (0,1)$ of income per worker in the economy:

$$\rho w_u = by. \tag{24}$$

The distributions of firms and workers in (18) and (19) are function of H. Through its effect on these variables, H impacts the partial-equilibrium decision of firms characterized in Section 2. To make this dependency explicit, from now on I write the stock effect from (14) as $S(h;H)$. In an interior equilibrium, the first-order condition (12) is

$$S(h;H) = f_X. \tag{25}$$

This condition gives the age for exporting h chosen by an individual firm, taking the group of aggregate variables affected by H as given. In equilibrium this decision must be consistent across firms; i.e.,

$$h = H. \tag{26}$$

In addition, firms face entry or overhead expenses with flow-equivalent value of f_D units of the final good. Using the value of a new firm Π^e from (15), the free-entry condition implies that a potential entrant must be indifferent about entering,[19]

$$(\rho + \mu)\,\Pi^e = \pi(h;H) = f_D. \tag{27}$$

After imposing the optimality, consistency and free entry conditions, it is possible to compute aggregate output Y and aggregate investment in export entry and firm creation. The steady-state consumption level is obtained residually from market clearing in the final good. Finally, exports X in each country can be expressed as a function of the revenue premium Γ defined in (4) and the investment timing H. In equilibrium, relative market sizes must be such that trade balances, $X = X^*$.[20]

Now it is possible to define the equilibrium in the world economy.

Definition 1 *The equilibrium consists of a revenue premium Γ, labor market outcomes $\{h, H, M\}$, distributions $\{P(\cdot), G(\cdot)\}$, shares of exporting firms and employment in these firms $\{m_X, e_X\}$, output per worker y, consumption c and unemployment value w_u in each country such that:*
a) the first-order condition (25) from the firms' optimization problem holds;
b) the individual and the common age for switching are consistent, i.e. (26) holds;
c) the number of firms adjusts to satisfy free entry, i.e. (27) holds;
d) the firm and employment distributions are given by (18) and (19);
e) the shares of exporting firms and of employment in these firms are given by (20) and (21);
f) income per employed worker is given by (22);

[19] Since firms are continually exiting, a constant number of firms in steady state requires actual entry, so that the free-entry condition holds with equality.

[20] See (64) and (65) in Appendix 2.

g) the value of unemployment is given by (24);

h) goods market clear; and

i) international trade is balanced.

3.1 Equilibrium Existence and Uniqueness

My next step is to establish equilibrium existence and uniqueness. The structure of the equilibrium suggests a recursive solution. First, taking the revenue premia Γ and Γ^* as given, it is possible to solve for the time of export entry H and H^* in each country. Using these values it is then possible to generate the export functions $X(\Gamma)$ and $X^*(\Gamma^*)$ and impose trade balance to solve the model.

I start by characterizing a unique equilibrium value for H taking Γ as given. Broadly speaking, this can be interpreted as solving the equilibrium in a country whose size does not affect the revenue premium Γ. For that, it is useful to define the function $\Omega(h, H)$ as the ratio of the stock effect to the value of firms at birth. Using (25) and (27) we have that, in equilibrium, this adjusted stock effect equals the cost of exporting relative to entry costs,

$$\Omega(h, H) \equiv \frac{S(h; H)}{\pi(h; H)} = \frac{f_X}{f_D}. \tag{28}$$

Implicit in this equation is the reaction function of an individual firm, h, to the common exporting age H. In each country, an equilibrium consist of an H that satisfies $\Omega(H, H) = f_X/f_D$.

Uniqueness of H can be examined based on whether the incentive to export for each firm increases when other firms delay export entry. $\Omega(h, H)$, which captures all the forces that affect the export decision, simultaneously accounts for two margins: the stock effect and the value of firms at entry. As we know from Proposition 1, forces that increase the former lead to a lower h, while forces that increase the latter lead to more entry, increasing competition and delaying h. We must ask, then, how these two forces respond to changes in H. From (19), a larger H shifts the distribution of employment $G(x; H)$ towards firms that are further away from exporting; from Proposition 1, this strengthens the stock effect. At the same time, if firms take longer to invest, income y in (22) shrinks. The value of unemployment w_u in (24) shrinks as a consequence, increasing the value of a potential entrant. This induces entry and weakens the stock effect.

Summing up, a larger H affects h through one negative-feedback channel (distribution of competitors) and one positive-feedback channel (worker's value of unemployment). To make progress, the following regularity condition which ensures that the positive-feedback effect is weaker can be imposed:

$$\Gamma < \frac{1 + \kappa_e/b}{1 + \kappa_e}. \tag{29}$$

This condition requires that transfers to unemployed workers and the revenue premium are not too large relative to contacts made by employed workers.[21] When (29) holds, we can guarantee that

[21] Γ is an endogenous object, but it approaches 1 as $\tau \to \infty$ or $\sigma \to 1$. Therefore, a sufficiently large τ or small σ are sufficient to guarantee inequality (29) for $b \in (0, 1)$. When countries are symmetric, Γ is a function of τ and σ alone (see equation (33)).

the equilibrium H is unique.[22]

As for existence of an equilibrium with export entry, we have, as in partial equilibrium, that there is no stock effect at firm birth. Therefore, immediate exporting cannot be an outcome if $f_X > 0$. An alternative candidate for an equilibrium is that firms never export. Since the adjusted stock effect $\Omega(h, H)$ is bounded, firms invest if and only if

$$f_X/f_D < \lim_{H\to\infty} \Omega(H, H) = \frac{\rho+\delta}{\gamma}\frac{(1+\kappa_e)(\Gamma-1)}{1-b} \equiv \overline{f_X/f_D}, \tag{30}$$

i.e., whenever the sunk costs of exporting are not too large relative to the cost of creating new firms.[23] The results are summarized as follows.

Lemma 2 (Unique Timing of Investment given Γ) *For each value of Γ, if a finite investment age H exists it is unique. Firms never invest at entry if $f_X > 0$, but eventually invests if and only if $f_X/f_D < \overline{f_X/f_D}$.*

Lemma 2 establishes a unique value of H for each Γ. Once all equilibrium conditions are imposed, we reach an implicit solution for the timing H using $\Omega(H, H) = f_X/f_D$ in (28). From that solution, the free entry condition (27), $\pi(H, H) = f_e$, gives a closed-form solution for the number of firms, M.[24] With H and M at hand, characterizing all outcomes in one country given Γ is straightforward. In particular, we obtain the value of exports $X(\Gamma)$. Following similar steps we solve for outcomes in the foreign country to obtain $X^*(\Gamma^*)$. In general equilibrium, trade balance requires $X(\Gamma) = X^*(\Gamma^*)$. I work henceforth under the assumption that the relative fixed cost of exporting f_X/f_D is sufficiently small, but positive, or that the upper bound for these costs in (30) is sufficiently large. This ensures existence and uniqueness of the general equilibrium.

Proposition 2 (Uniqueness) *If $(\rho+\delta)(1+\kappa_e)/[\gamma(1-b)]$ is sufficiently large or f_X/f_D is sufficiently small, there exists a unique international trade equilibrium.*

3.2 Example: General Equilibrium without Job-to-Job Mobility

I make a brief detour to the case $\lambda_e = 0$, so that job-to-job transitions are not allowed. This will help demonstrate that job-to-job mobility is necessary for search frictions to have an effect on

[22] See proof of Lemma 2. Condition (29) depends on κ_e, b and Γ. Natural restrictions on their values can be imposed from readily available data to assess its validity. The share of GDP used to finance unemployment benefits in the model is $bu/(1-u)$, and from the OECD Social Expenditure Database, public spending on unemployment compensation as a fraction of GDP among OECD member countries was 1% on average between 1980 and 2000. In turn, Jolivet et al. (2006) estimate that the job-finding rate is strictly lower for employed than for unemployed workers in each of eleven OECD countries, implying $\lambda_e/\lambda_u < 1$. In addition, average unemployment in the OECD since 1980 has been 7.7%. Combining these three pieces of data, (29) determines an upper threshold for Γ of 5 when $\lambda_e/\lambda_u = 0.1$, approximately the value in Jolivet et al. (2006) for France and the U.K.. Mayer and Ottaviano (2010) find exporter value-added premia below this threshold, e.g. 2.7 in France and 1.3 in the U.K..

[23] Joint validity of (29) and (30) is guaranteed for small enough f_X/f_D.

[24] The solution is

$$M = \frac{\lambda_u u}{f_D}\frac{\{(1-u)[1+e_X(H)(\Gamma-1)]\}^{\frac{1}{\sigma-1}}}{\rho+\delta}\left\{1 - b\frac{1+[\Gamma(1+\kappa_e)-1]e^{-\mu H}}{1+\kappa_e e^{-\mu H}} + (\Gamma-1)e^{-(\rho+\delta)H}\right\}.$$

key aggregate outcomes. When $\lambda_e = 0$, the timing of entry H from (25) is readily given by

$$S(\cdot) = \mathcal{M}(\Gamma - 1)\frac{1 - e^{-\gamma H}}{\gamma} = f_X, \tag{31}$$

where $\mathcal{M}$ is the measure of labor- and product- market size defined in (17). This measure adjusts to satisfy free entry. After several manipulations, we can write the free-entry condition (27) as

$$\Pi^e(\cdot) = \mathcal{M}\frac{(\Gamma - 1)\left(e^{-(\rho+\delta)H} - be^{-\mu H}\right) + 1 - b}{\rho + \delta} = f_D. \tag{32}$$

Given Γ, an equilibrium without job-to-job mobility corresponds to the values $\{H, \mathcal{M}\}$ that solve (31) and (32). By inspection of these expressions, it is clear that the matching rate of unemployed workers, λ_u, has no effect on the timing H. From (17), changes in λ_u impact $\mathcal{M}$ directly, because firms find workers faster, and indirectly through the unemployment rate u and the revenue per worker y_D. But these effects are fully absorbed by changes in the number of firms, M.

Hence, labor market frictions, as captured by λ_u, do not matter for the outcomes per employee when job-to-job mobility is absent. Section 4.1 extends this result when the rates λ_e and λ_u are endogenously determined via a matching function.

3.3 Comparative Statics

Using (4), in a symmetric-countries case the revenue-premium of exporters is readily given by parameters,

$$\Gamma = \Gamma^* = \left(1 + \tau^{-(\sigma-1)}\right)^{1/\sigma}. \tag{33}$$

Since Γ can be treated as a parameter, the results for this case readily apply as well to a closed economy where the investment decision represents a discrete technology upgrading that increases productivity by Γ.

Proposition 3 (Effects of Frictions with Symmetric Countries) *In an equilibrium with symmetric countries, lower frictions in job-to-job mobility (i.e., higher λ_e), lead to a reduction in the age of entry into exporting H and to an increase in the export participation of firms m_X and the employment in exporting firms e_X. The contact rate from unemployment λ_u has no impact on these outcomes.*

A direct implication of these results is that trading partners gain from the joint implementation of labor market policies that facilitate transitions between jobs. Faster job-to-job transitions accelerate investment in high-revenue activities, increasing the real income per worker. In contrast, policies that ease transitions out of unemployment have no general-equilibrium effect on the timing of export entry, and do not change real income through this channel.

As it was partially discussed in Section 3.2, the irrelevance of λ_u for the time of exporting is a reflection of free entry. For an individual firm, a higher contact rate with unemployed workers results in a proportional impact on the stock effect and on firm value. The number of firms adjusts through free entry and competition heightens, offsetting this partial-equilibrium effect. In contrast, the frequency of contacts on the job λ_e changes the stock effect in different proportions for different types of firms. A lower λ_e benefits older and larger firms relatively more. It only strengthens the growth margin in (16) through the higher entry rate of workers from other jobs, so that variation in the number of firms cannot absorb this effect as with λ_u. As a result, the adjustment to lower λ_e occurs partly through the number of firms and partly through the common age for switching.

The specific role of frictions in job-to-job transitions in shaping the allocation also holds allowing for endogenous matching rates, as shown in Section 4.1. This feature of the model motivates the use of data on job-to-job mobility for the calibration.[25]

The model also yields implications for how international trade frictions, captured by τ, interact with frictions in job-to-job transitions, captured by λ_e. From the expression for real income per capita, (23), it is clear that τ has a direct impact on real income, via Γ, as well as an indirect impact via reallocations because it changes the export timing threshold H. The magnitude of the latter effect is mediated by frictions, as captured by the magnitude of the equilibrium response $e_X'(H)\frac{dH}{d\Gamma}$. In Section 5 I study this interaction numerically using a calibrated model.

Suppose next that countries are asymmetric, and consider how labor market or trade reforms in the foreign country affect outcomes at home. We can show that labor market reforms that favor export participation abroad have a positive impact on the home market.

Proposition 4 (Effects of Frictions with Asymmetric Countries) *In a trade equilibrium with asymmetric countries, lower frictions in job-to-job mobility in one country lead to an increase in firm export participation in both countries, and to an increase in income per worker in the trading partner.*

These results reflect a positive feedback between the income per worker of trading partners. Exporting firms are high-income firms, because they generate more value than non-exporters for the same amount of output. The prevalence of these firms depends on Γ, that captures the relative size of the foreign country.

In this context, when the foreign country implements policies that favor transitions between jobs, it promotes export participation. If this were the overall response, trade would not be balanced. However, at impact, this also raises income per worker in the foreign market, increasing the exporter revenue Γ at home. As a result, firms in the home country start to export faster and exported output adjusts up to the point that trade balances again. In the new equilibrium, both countries have a larger share of employment in the export sector, but there is a higher exporter premium in the domestic economy and a lower one in the foreign country. The latter outcome resembles the standard adverse response in the terms of trade faced by specialized countries that experience a productivity shock, common to many open-economy models.

These effects ultimately reflect that it takes time for firms to export. In Krugman (1980), where all firms are identical, an increase in the size of an economy is met with an increase in the

[25] Naturally, λ_u has a direct effect on market size because it changes the unemployment rate. Similarly, as implied by (21), λ_e has an additional direct effect on real income given H because it speeds transitions to exporters.

incentives to export to that country to balance trade. While in that model this occurs through the appreciation of the real wage in the economy experiencing the positive shock (i.e., the home market effect) to induce entry or exit of firms (all of whom are exporters), here the adjustment to a change in conditions occurs through the age of switching. This margin of adjustment shares the spirit of Melitz (2003), in that it derives from worker reallocations towards high-revenue firms and from firms switching export status.

3.4 Size distribution

In the model, workers transit from young and small firms to old and large firms. Firms are continually exiting and being replaced by small entrants. This process originates a distribution of firm sizes. In the data, a common feature of the size distribution of firms is a decreasing density in the upper tail. A reasonable requisite for the theory is therefore to be consistent with that feature. We can show that job-to-job transitions are necessary for the density not to be increasing in its entire domain, which would contradict this empirical evidence.

Proposition 5 (Shape of Size Distribution) *Suppose that the firm exit rate is lower than the rate of individual job termination ($\mu < \gamma$). Then, allowing for job-to-job transitions ($\lambda_e > 0$) is a necessary condition for the distribution of firm sizes to feature a decreasing density.*

To understand this, it is useful to consider a condition that holds whenever the density is decreasing. Let $N(h)$ be the size of firms of age h and $f(n)$ be the density of the distribution of firm sizes. Then, if $N(h) = n$,

$$f'(n) < 0 \text{ if and only if } \mu + \frac{N''(h)}{N'(h)} > 0. \tag{34}$$

This condition shows that there are two forces competing to determine the slope of $f(n)$: changes in net worker flows $N'(h)$ by firm age and the exit rate μ. Intuitively, if firm growth decelerates too fast and firms do not exit often, there is a tendency for firms to cluster at some point in the size distribution, resulting in an increasing density.[26] Without transitions between jobs ($\lambda_e = 0$), net flows slow down at the rate of job separations, $N''(h)/N'(h) = -\gamma$, implying from (34) that $f'(n) > 0$. When $\lambda_e > 0$, workers are attracted in each period from unemployment to any firm size, but as firms age they attract progressively more workers from other firms. The first effect dominates at firm entry and the second dominates when firms are large enough but still do not invest. Therefore, if firms invest at a sufficiently old age, there is a region in the distribution of firm sizes where the density is decreasing. By allowing for sequential investments as in the extension in 4.3, so that firms keep investing throughout their lifetime, the region where the density is decreasing can be made larger.

[26] Consider an extreme case with no firm exit where firms grow until a certain age and stop growing afterwards. The size distribution would collapse to a point at the size attained by firms at that age.

4 Extensions

4.1 Endogenous Matching Rates

The results highlight the importance of flexible job-to-job mobility for firm growth and export dynamics. In contrast, frictions in transition out of unemployment matter less for these outcomes. The baseline theory used to obtain these results builds upon Postel-Vinay and Robin (2002), inheriting from that paper the assumption of exogenous contact rates. A natural question is what would happen with endogenous matching rates.

To allow for endogenous matching rates I assume, as Mortensen (1998), that unemployed and employed workers are perfect substitutes in the matching process. Still, the two groups may search with different intensity. Now, let $\widetilde{\lambda}_i$ be the search intensity of workers with employment status $i = e, u$. If the aggregate matching function is homogeneous of degree one, the contact rates with a potential employer faced by employed and unemployed workers is

$$\lambda_i = \widetilde{\lambda}_i \upsilon \left(\frac{M}{\widetilde{\lambda}_u u + \widetilde{\lambda}_e (1-u)} \right) \text{ for } i = e, u, \tag{35}$$

where $\upsilon(\cdot)$ is an increasing and concave function. The model developed so far corresponds to the special case when $\upsilon \equiv 1$, so that $\lambda_i = \widetilde{\lambda}_i$.[27] Now, a labor market with higher frictions is represented by lower $\widetilde{\lambda}_e$ and $\widetilde{\lambda}_u$.

Proposition 6 (Effects of Frictions with Endogenous Matching Rates) *In an equilibrium with symmetric countries, or in a small open economy where Γ is given, higher frictions in transitions out of unemployment (lower $\widetilde{\lambda}_u$) affect the timing of investment only if job-to-job transitions are present (i.e., only if $\widetilde{\lambda}_e > 0$). When $\widetilde{\lambda}_e > 0$, changes in $\widetilde{\lambda}_u$ only affect the timing of investment through their effect on λ_e.*

The proposition states that, in a general equilibrium with endogenous matching rates, a higher search intensity for unemployed workers has no impact on the timing of investment when job-to-job mobility is absent (i.e., if $\widetilde{\lambda}_e = 0$). This result readily follows from a reasoning similar to Section 3.2. When job-to-job mobility is allowed (i.e., if $\widetilde{\lambda}_e > 0$), lower frictions in transitions out of unemployment may impact aggregate outcomes, but only through the equilibrium rate of job-to-job transitions λ_e. This confirms the specific role of frictions in job-to-job mobility in shaping the allocation.

4.2 Bargaining Power of Workers

The analysis so far also followed Postel-Vinay and Robin (2002) in assuming that workers have no bargaining power against a hiring firm. The main gain in terms of tractability stemming from this assumption is that the rate of contact on the job λ_e and the distribution of job offers faced by

[27] For example, if the aggregate matching function is Cobb-Douglas with share β on firms, we would have $\upsilon(x) = x^{\beta}$ and the baseline model would correspond to $\beta = 0$.

employed workers do not show up in the value of a new job $v(x)$ in (7). The model can be extended with positive bargaining power for workers, as in Cahuc et al. (2006), in which case both λ_e and the distribution of job offers would appear in $v(x)$. In that case, letting β be the bargaining power of workers, (7) would become:

$$v(x) = \frac{y_X + \delta w_u}{\rho + \delta} - (y_X - y_D) \int_0^x e^{-(\rho+\delta)x' - \lambda_e \beta \int_0^{x'} P(x'') dx''} dx'.$$

In this expression, $P(x)$ is the CDF of time until exporting across firms, and also the probability that a worker who contacts a firm finds a firm whose time to export is less than x. It can be verified that $\beta = 0$ corresponds to (7). Throughout the analysis and in the calibration I use the simpler case with $\beta = 0$.

4.3 Multiple Investment Options

The baseline model only includes one investment option. A natural extension is to allow for multiple fixed investments throughout the firms' life cycle. In my context this is interpreted as several export markets, although this can be equally interpreted as multiple technologies with varying levels of productivity and fixed costs. I use this extension in the calibration of the model.

Assume that firms may access a sequence of export markets $j = 1, .., K$ at different times, let y_k be the revenue per worker of a firm that exports to markets 1 to k, and let f_j be the fixed entry cost into market j.[28] The state of a firm is now the distribution of times until entry into each market, denoted by $\boldsymbol{x} \equiv \{x_1, x_2, .., x_K\}$, where $x_j \in [0, \infty]$.

The value of a new job from (7) retains the same structure as in the baseline model. From the proof of Lemma 1 in Appendix B, it is now

$$v(\boldsymbol{x}) = \frac{y_0 + \sum_{k=1}^{K} e^{-(\rho+\delta)x_k} (y_k - y_{k-1}) + \delta w_u}{\rho + \delta}. \tag{36}$$

As in (7), the flow value of a new job is the expected discounted value of revenues generated by a worker plus the value to the worker in unemployment if the job is terminated. Using (36) and similar arguments to those in Section 2, we can define $\pi(\boldsymbol{x})$ as the present discounted value of profits generated by all workers who are hired by a firm with times until investment $\boldsymbol{x}$.[29]

Let h_j be the time elapsed between a firm's entry to markets j and $j+1$. I.e., conditional on surviving, a firm enters market 1 at age h_1, market 2 at age $h_1 + h_2$, and market k at age $\sum_{i=1}^{k} h_i$. The decision of a firm is to choose the distribution of entry times $h_1, h_2, .., h_K$.

As in the baseline model, this problem can be characterized by the stock effect. Similarly to (14), we can define $S_1(h_1, h_2, .., h_K)$ as the change, after a delay in the time of entry into market 1, in the present discounted value of all workers attracted between ages 0 and h_1; and, generically, $S_j(h_j, h_{j+1}, .., h_K)$ as the change, after a delay in the age of entry into market j, in the value of all

[28] In the baseline model from sections 2 and 3 we only had y_D and y_X to denote non-exporters and exporters. Now, y_0 corresponds to non-exporters, y_1 to exporters to the first market, and y_k to exporters to markets 1 to $k \leq K$. Equations (47) and (48) in Appendix A characterize y_j explicitly as function of foreign market sizes and trade costs.

[29] See the proof of Proposition 7 in Appendix B for a explicit formulation of $\pi(\boldsymbol{x})$ with multiple investments.

workers attracted between ages $\sum_{i=1}^{j-1} h_i$ and $\sum_{i=1}^{j} h_i$:

$$S_j\left(h_j, h_{j+1}, .., h_K\right) \equiv \int_0^{h_j} e^{(\rho+\mu)x_j}\left[-\pi_j\left(\mathbf{x}_j\right)\right] dx_j \text{ for } j > 1,$$

where $\pi_j(\cdot)$ denotes the partial derivative of $\pi(\boldsymbol{x})$ with respect to its j^{th} argument, and where the i^{th} argument of $\mathbf{x}_j$ is the time until entry to market $i = 1, .., K$ for a firm that has entered market $j-1$ but is still x_j periods away from entering market j.

The following proposition characterizes the solution to the sequence of entry times $\{h_j\}_{j=1}^K$.

Proposition 7 (Investment Times with K Investment Options) *Suppose that a firm sequentially enters markets $j = 1, .., K$. Then, it chooses the times of entry $h_1, h_2, .., h_K$ that satisfy*

$$S_1\left(h_1, h_2, .., h_K\right) = f_1, \tag{37}$$

$$S_j\left(h_j, h_{j+1}, .., h_K\right) = f_j - e^{-\gamma h_j} \frac{y_j - y_{j-1}}{y_{j-1} - y_{j-2}} f_{j-1} \text{ for } j = 2, .., K. \tag{38}$$

These conditions characterize the timing and ordering of multiple investments in the presence of search frictions and job-to-job transitions. The time of entry to market j depends on conditions in other markets. Firms enter faster in market j the lower is its relative cost f_j/f_{j-1} or the higher is its relative revenue gain $\left(y_j - y_{j-1}\right)/\left(y_{j-1} - y_{j-2}\right)$. Therefore, changes in conditions in one market impact on the distribution of entry times to subsequent markets. This solution also gives a necessary condition such that the ordering $1, .., K$ is indeed chosen. Because an interior solution requires a positive stock effect S_j at $h_j = 0$, the sequence $1, ..., K$ cannot be an outcome if $f_j/f_{j-1} < \left(y_j - y_{j-1}\right)/\left(y_{j-1} - y_{j-2}\right)$ for some j. If that were the case, market j would be more attractive than market $j-1$ and the firm would prefer to revert their ordering.[30]

5 Calibration

5.1 Data and Quantitative Model

I match the model to summary statistics from official tax records of the manufacturing sector of Argentina from 2003 to 2007. The dataset reports the number of firms by cells of employment size, age, and export status. Export status includes three categories: non-exporters, exporters to 5 countries or less, and exporters to more than 5 countries. These two export categories roughly correspond to firms that export only to South America, and to firms that export to both South America and to other destinations such as the European Union or the U.S.

Importantly, the dataset also includes information about worker transitions between firms. In particular, it includes the share of total hires that enter firms with different export status, sizes,

[30] The proposition also suggests a simple numerical solution algorithm for the model. Because the first-order condition to enter market j only depends on the times $h_j, .., h_K$, these conditions define a triangular system. Hence, it can be easily solved starting with the solution for h_K from $S_K\left(h_K\right) = f_K - e^{-\gamma h_K} \frac{y_K - y_{K-1}}{y_{K-1} - y_{K-2}} f_{K-1}$ and iterating backwards. I exploit this to calibrate the model.

or ages directly from other jobs in consecutive months, rather than being hired straight from non-employment. These statistics were extracted from microdata that include the universe of formal employment in manufacturing in Argentina for the period 2003 to 2007. Appendix C.1 describes in more detail the summary statistics used in the calibration and the underlying dataset from which they were extracted.[31]

To match moments from these data, the quantitative model includes some extensions with respect to the benchmark model developed so far. I now describe the extensions, while Section C in the appendix formally develops the extended model.

First, using the results from Section 4.3, I allow for two export markets, $k = 1, 2$. This allows to match that the patterns of age, size and intensity of job-to-job hiring clearly depend the number of export destinations. Also, as discussed in Section 3.4, the inclusion of several investments helps to generate a size distribution of firms with realistic shape.

Second, firms are now allowed to make a variable effort to adjust their size and partly overcome labor market frictions by choosing the hiring intensity s in (8). Following Bertola and Caballero (1994), among others, I assume a generic convex cost function, $c(s) = s^{\zeta}$ with $\zeta > 1$. This also captures frictions in firm-level adjustment not included in the theory.

Finally, I also allow for heterogeneity in firm-level fixed export costs. Without this, the model yields a relationship between the age and the share of firms with different export status that is too tight. I assume that a share ω of firms have high fixed costs and never export, while the remaining firms might export at some age.[32] While the model is prepared to accommodate more general forms of heterogeneity, this binomial distribution is sufficient to match the targets of the calibration.[33]

Patterns of job-to-job transitions in this extended model are more intricate than in the benchmark model. Now, workers may flow from large and old high-cost firms into small and young low-cost firms. As a result, general-equilibrium objects such as the distribution of employment are not as straightforward as in Section 3, where the aggregate investment age was sufficient statistic to characterize them.[34] Section C of the appendix lays out this extended model, presenting the equilibrium conditions and the numerical algorithm that were used to solve it.

5.2 Calibration Strategy

The model includes 14 parameters: preferences $\{\rho, \sigma\}$, labor markets characteristics $\{\mu, \gamma, \lambda_u, \lambda_e, b\}$, adjustment costs ζ, heterogeneity in fixed costs ω, entry costs $\{f_0, f_1, f_2\}$, and foreign market capacities $A_k \equiv \tau_k^{1-\sigma} P_k^{\sigma} Y_k$ for $k = 1, 2$. Given the size of the domestic market, $\{A_1, A_2\}$ map to the

[31] The years 2003 to 2007 chosen for the calibration feature rapid economic growth after a recession. Table $A1$ in Appendix C.1 shows that the summary statistics matched in the calibration are stable over the recession years $1998 - 2002$ and the expansion years $2003 - 2007$. Therefore, the calibration is robust to choosing this particular time period.

[32] Heterogeneity in fixed export costs are typically included in quantitative models of trade. E.g., see Eaton et al. (2011).

[33] In the spirit of Melitz (2003), the model also accomodates heterogeneity in firm-level productivity. The equilibrium and the numerical solution method described in Appendix C include this margin.

[34] See equations (18) to (23).

revenue premia $\{\Gamma_1, \Gamma_2\}$.[35]

Table $A2$ in Appendix C.4 lists the parameters set without solving the model. The parameters $\{\mu, \gamma, \lambda_u, \rho\}$ match direct empirical counterparts from my data. The exit rate of firms is $\mu = 0.075$ to fit the density of the firm age distribution. The job separation rate $\gamma = 0.15$ matches the probability in the data that workers employed in non-exiting firms move into the unemployment pool. These parameters give the rate of worker transition into unemployment, $\delta = \mu + \gamma$. The contact rate for unemployed workers, λ_u, is readily given by the unemployment rate, $u = \frac{\delta}{\lambda_u + \delta}$, equal to 10% on average as reported by the Argentine institute of statistics. The rate of time discount ρ generates an interest rate of 6% to match the average rate on deposits at the fourth quarter according to the Argentine Central Bank. The elasticity of demand σ is set equal to 2.98 following Eaton et al. (2011).

Table $A3$ in Appendix C.4 lists the remaining 9 parameters, $\{f_0, f_1, f_2, \lambda_e, b, \zeta, \omega, A_1, A_2\}$. They are chosen to minimize the sum of square residuals between the model prediction and the empirical values of the 10 targets listed in Table $A4$: average firm size, the share of firms that export to each number of markets and their shares of employment, the average ages of exporters and non-exporters, and the shares of job-to-job transitions in the total number of new hires within firms that export to different numbers of destinations.

A key parameter in the model is the job-finding rate of employed workers, λ_e. Matching the share of job-to-job transitions in new hires is important to determine this parameter.[36] On average, 40% of all new hires in firms that export to more than 5 countries enter from jobs in the formal employment sector, in contrast to 27% in firms that export to 5 destinations or less and to 16% in non-exporters. Given the job-finding rate for unemployed workers, λ_u, the calibration gives a value for λ_e/λ_u close to 10%. This relation between the job finding rate of employed and unemployed workers is comparable in magnitude to results from structural estimations that use micro data such as Bontemps et al. (2000) and Jolivet et al. (2006). The latter estimate values of λ_e/λ_u between 6% and 19%. The remaining labor market parameter, b, implies a share of GDP spent on unemployment transfers close to other countries at similar stage of development than Argentina.[37]

5.3 Counterfactuals

A natural check on the calibrated model before using it for counterfactuals is to verify that it yields reasonable rates of firm growth. To assess this, the left panel of Figure 1 shows the pattern of employment growth by age for exporters (in grey) and non-exporters (in black) in the data (dashed

[35] See (83) to (85) in Appendix C.2. Throughout the calibration and counterfactuals, foreign-market capacities $\{A_1, A_2\}$ are treated as parameters, but $\{\Gamma_1, \Gamma_2\}$ adjust endogenously.

[36] The share of new hires entering from other jobs in a firm that exports to the first set of markets is

$$jtj_1(a) = \frac{(\lambda_e/\delta)\, G(v_X^*(a))}{1 + (\lambda_e/\delta)\, G(v_X^*(a))}.$$

This share is increasing in λ_e. In the calibration, $jtj_1(a)$ is averaged across the ages of all exporters to the first set of markets to obtain the share of job-to-job hires in total hires for that group. Therefore a lower empirical value of that share implies a lower λ_e.

[37] E.g. 0.1% in Turkey, 0.3% in Slovak Republic, 0.4% in Greece, and 0.5% in Hungary according to the OECD Social Expenditure Database.

lines) and in the model (solid lines).

The calibrated model replicates well the pattern of employment by firm age and export status from the data. To have a sense of magnitudes, Hsieh and Klenow (2012) report that, in the U.S., plants are on average 8 times larger at age 40 than at age 5, while in Mexico they are twice as large. In Argentina, where I observe the size of firms rather than plants, that ratio is 3.8. In turn, the $40 - 5$ ratio predicted by the calibrated model is 3.6. The model also matches well the growth of exporting firms. It is worth noting that the calibration does not target these rates of firm growth.[38]

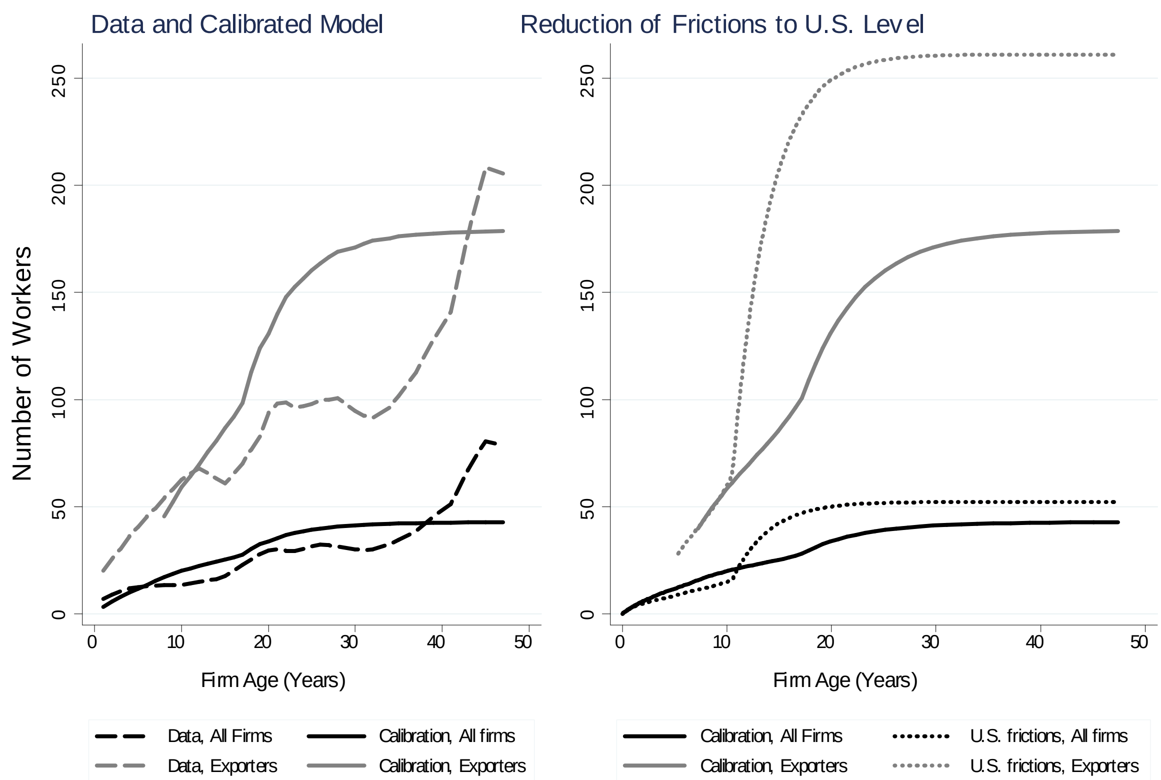

Figure 1: Firm Size by Age (Data, Calibrated Model, and Counterfactual with Lower Frictions)

Can frictions in job-to-job transition generate sizeable differences in firm growth as those observed between Argentina and the U.S.? The right panel of Figure 1 shows the profiles of size by age from the calibrated model (solid lines) and the profiles corresponding to lower frictions (higher λ_e), keeping the other calibrated parameters constant. To discipline the magnitude of the shock, λ_e is increased from its calibrated value so that, in the new equilibrium, the rate of new hires entering firms directly from other jobs matches the U.S. average of 36% reported by Fallick and Fleishman (2004).[39] Table 1 lists the aggregate effects of this reduction in frictions.

[38] In an alternative restricted calibration where job-to-job transitions are shut down in the model and the empirical rates of job-to-job hiring by export status are not targeted, the predicted firm-size ratio between ages 40 and 5 falls to 2.9. In this restricted calibration, the parameters $\{f_0, f_1, f_2, A_1, A_2, \omega\}$ are calibrated to match all the moments used in the baseline calibration except for the rates of job-to-job hiring, while the remaining parameters are set at the values of the baseline calibration.

[39] This fraction is the average ratio between employment-to-employment flows and the sum of unemployment-

	Calibration	Lower λ_e
Share of job-to-job hires in total number of new hires	17%	36%
Average Firm Size Ratio between Ages 40 and 5	3.7	6.2
Age of Entry into Market 1	7.0	5.2
Age of Entry into Market 2	17.5	10.5
Real Income Per Employee Relative to Calibration	1	1.35

Table 1: Effect of reducing λ_e to match U.S. rates of job-to-job hiring

As a result of this change in frictions, firms grow considerably faster, reach a larger long-run size, and invest earlier in their life cycle. The ratio of firm size between ages 40 and 5 increases to 6.2. Hence, changing frictions in job-to-job mobility in Argentina to reach U.S. levels of labor market flexibility goes a long way in generating a rate of firm growth close to the Hsieh and Klenow (2012) figure for U.S. plants. From the right panel of Figure 1 it also follows that the growth effect is more pronounced for exporters, because these firms rely more strongly on job-to-job hiring to grow. At age 40, exporters have 178 workers in the calibrated economy and 260 when frictions are set at the U.S. level.

This faster firm growth and quicker investment rates result in 35% real income growth. These gains occur because the calibrated model features large exporter revenue premia, $\Gamma_1 = 2.4$ and $\Gamma_2 = 3.1$, to match the large relative size of exporters; i.e., exporters are between two and three times more productive than non-exporters. Misallocation is large at the initial equilibrium, as a large share of workers is employed in non-exporters, who have low revenue per worker. The reduction in frictions leads to an increase in employment in the most productive firms from 28% to 66% both because workers move faster across firms and because firms invest earlier in their life cycle. For example, the age at which firms reach the top productivity level declines from 17 years to 10 years due to the reduction in frictions.

To conclude, I examine the interaction between trade and labor market frictions. Arkolakis et al. (2012) show that, in commonly used trade models, aggregate trade shares and an aggregate trade elasticity suffice to measure the impact of trade on the economy's real income. Several features, such as firm-level dynamics and frictions in resource allocation, set my model outside of that class of models. Therefore, it is meaningful here to ask if frictions in job-to-job mobility matter for the model-implied effects of lower trade costs. For that, I simulate a reduction in trade costs across economies that vary in labor market flexibility λ_e. These economies also vary in the fixed costs of exporting so as to start from the same aggregate foreign trade share. The trade elasticity may differ across counterfactuals due to differences in λ_e and in fixed costs.

Table 2 reports the change in real income and consumption corresponding to a 15% increase in foreign market sizes due to lower trade costs at the calibrated economy and in a counterfactual scenario where job-to-job transitions are not allowed, so that $\lambda_e = 0$. The trade liberalization in the

to-employment and non-employment-to-employment flows in the U.S. since 1994 using data from from http://www.federalreserve.gov/econresdata/researchdata/feds200434.html.

counterfactual scenario can be interpreted as the gains from trade reform that would be measured in an economy that looks identical to Argentina in every calibrated moment except for showing no job-to-job transitions.

As expected, the reduction in trade costs leads to an increase in real income and consumption. But the magnitude of this increase depends on frictions in job-to-job mobility. If job-to-job transitions are not allowed, the total effects on income and consumption are respectively 8% and 8.9%. But starting from the calibrated economy these magnitudes grow to respectively more than 10% and 11%. These numbers suggest that the aggregate effect of lower trade costs is larger when frictions are smaller, and that modelling barriers to worker mobility across firms, and appropriately accounting for their magnitude, may be important to measure the gains from international trade.

	Income Growth	Consumption Growth
Calibrated Economy	10.2%	11.6%
No job-to-job transitions ($\lambda_e = 0$)	8.0%	8.9%

Table 2: Effect of a Reduction in Trade Costs for Different Levels of Frictions

6 Conclusion

This paper developed a model to study the aggregate effects of labor market frictions in an open economy through their impact on the growth and investment decisions of firms. The model features interactions between firms' dynamic fixed investments in exporting and search frictions with job-to-job mobility, and is tractable for general-equilibrium analysis with multiple countries.

Using the model I demonstrated that frictions in worker mobility across heterogeneous firms is a key channel through which search frictions impact firm growth, fixed investments, and, through these outcomes, total income in the economy. In the absence of job-to-job transitions, frictions have no general-equilibrium impact on key aggregate outcomes.

I calibrated the model to Argentina's economy, where I can observe both firm export dynamics and the rates at which firms with different export status hire workers directly from other employers. The calibrated model replicates well the empirical pattern of firm growth and suggests that frictions in job-to-job transitions may have sizeable effects on aggregate income and firm growth. It also implies that barriers to worker mobility across firms may be relevant to measure the gains from international trade.

While I focused the analysis on investments in export capacity, the theory can be naturally applied to the interaction between labor market frictions and other fixed investments in open economy, such as technology choice, foreign direct investment, or number of products. The quantitative setup also lends itself to extensions not included in the calibration, such as an endogenous matching rate between workers and firms and more general forms of firm heterogeneity. These questions are left for future research.

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A Derivation of the Exporter Revenue-Premium Γ

I characterize the revenue premium in an environment with $K+1$ countries indexed by $j = 0, .., K$, where $j = 0$ denotes the home country. This environment can be specialized to the benchmark model, to the extension of section (4.3), and to the calibration. In each country j, aggregate output of the final good results from combining imported varieties i,

$$Y_j = \left(\int_{i \in I_j} z_j (i)^{1/\sigma} q_j (i)^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}, \tag{39}$$

where I_j is the set of varieties from any origin available in country j. Each differentiated variety is produced by a different firm. $z_j (i)$ and $q_j (i)$ respectively denote the quality and the quantity at which firm i sells its variety in country j.

To save notation, define the following country-specific measure of market size:

$$p_j \equiv P_j Y_j^{1/\sigma}, \tag{40}$$

where P_j is the price index in market j. Using (39), we obtain the inverse demand for each variety. From that inverse demand, the price in country j for a variety with quality $z_j (i) = z$ sold in quantity $q_j (i) = q_j$ is

$$\left(\frac{z}{q_j} \right)^{1/\sigma} p_j. \tag{41}$$

Consider the problem of a firm located in the home country that produces q units of output with quality z which are then shipped to markets 1 to $k \leq K$ in addition to the home market. The decision to export is characterized in the text, and here it can be taken as given. Given the set of export destinations $j = 1, 2, .., k$, the choice variable for the firm is the fractions s_j^k of total output q that is shipped to each of these markets. Because of iceberg trade costs, of each unit shipped to country j only a fraction $1/\tau_j \leq 1$ arrives. Therefore, the firm sells $\left(s_j^k/\tau_j\right) q$ units in market j, making revenues of $z^{1/\sigma} q^{1-1/\sigma} p_j \left(s_j/\tau_j\right)^{1-1/\sigma}$ in that market. Since we consider the problem of a firm located at $j = 0$, we have that $\tau_0 = 1$. Using (41), total revenues of this firm are

$$\widetilde{r}_k (z, q) = \max_{\left\{s_j^k\right\}_{j=0}^k} \left\{ z^{1/\sigma} q^{1-1/\sigma} \sum_{j=0}^k p_j \left(\frac{s_j^k}{\tau_j} \right)^{1-1/\sigma} \text{ s.t. } \sum_{j=0}^k s_j^k = 1 \right\} \tag{42}$$

$$= \left(\sum_{j=0}^k p_j^\sigma \tau_j^{1-\sigma} \right)^{1/\sigma} z^{1/\sigma} q^{1-1/\sigma}. \tag{43}$$

Equation (43) follows from evaluating the revenue function at the optimal shares of output directed to each destination that solves the maximization problem in (42),

$$s_j^k = \frac{p_j^\sigma \tau_j^{-(\sigma-1)}}{\sum_{j'=0}^k p_{j'}^\sigma \tau_{j'}^{-(\sigma-1)}}. \tag{44}$$

I assume that workers are perfect substitutes within the firm between production of quality z and quantity q. The number of workers in the firm, n, is determined over time through the process of labor search characterized in the text. A firm with n workers and productivity ψ that sells to markets $j = 0, .., k$

solves

$$\max_{z,q} \widetilde{r}_k (z, q) \text{ s.t. } (1/\sigma) z + (1 - 1/\sigma) q = \psi n, \tag{45}$$

where $\widetilde{r}_k (z, q)$ is given in (43). The introduction of the parameter σ in the constraint of this firm problem serves only the purpose of saving notation.

The optimal allocation of workers between producing quantity q and quality z that results from (45) implies that quantity and quality increase linearly with the stock of workers, regardless of the firm's export status:

$$z = q = \psi n. \tag{46}$$

Finally, using (46) into the revenue function (43) and letting $r_k (n) = \widetilde{r}_k / P_0$ be the total real revenues of a firm located in market $j = 0$ with n workers exporting to markets $j = 1, .., k$ we obtain

$$r_k (n) = y_k \psi n,$$

where y_k is the revenue per unit of output for a firm that exports to markets 1 to k,

$$y_k = \Gamma_k y_0, \tag{47}$$

where the revenue premium is given by

$$\Gamma_k = \left[1 + \sum_{j=1}^{k} \left(\frac{p_j}{p_0} \right)^{\sigma} \tau_j^{-(\sigma-1)} \right]^{1/\sigma}, \tag{48}$$

and where y_0 is the real revenue per unit of output of a firm who only sells domestically,

$$y_0 = Y_0^{1/\sigma} = p_0.$$

The revenue premium from the benchmark model in equation 4 corresponds to the case when $K = 1$, the home market is denoted by D, and the foreign market is denoted by X. Expressions (84) and (85) from the calibrated model correspond to $K = 2$.

Some extra variables are used in the general equilibrium of the model and in the Proof of Proposition 2 below. Combining (44) and (48) implies $s_j^k = \left(\frac{p_j}{p_0} \right)^{\sigma} \tau_j^{-(\sigma-1)} \Gamma_k^{-\sigma}$, so that the share of output that is sold domestically by an exporter to k markets is

$$s_0^k = \Gamma_k^{-\sigma}. \tag{49}$$

Finally, let p_j^k be the price set in market j by a firm from the home country that exports to k different markets. To find this, use (46) together with (41) to get $p_j^k = \left(\tau_j / s_j^k \right)^{1/\sigma} p_j$, where p_j is defined in (40). Then, replace for s_j^k from (44) to obtain

$$p_j^k = p_0 \Gamma_k \tau_j. \tag{50}$$

Therefore, the price at which a firm that exports to k markets sells domestically is $p_0^k = p_0 \Gamma_k$, and the price at which a non-exporter sells domestically is $p_0^0 = p_0$. This structure has the immediate implications that firms set higher prices in larger markets, and that firms who export to more destinations set higher prices in every market. Both features are consistent with evidence from Manova (2012).

B Proofs

Proof of Lemma 1

I present a general solution for the value of a new job, $v(x)$. This solution includes a case with many countries and with firms that may differ in productivity and fixed costs. Using the general solution, it is easy to specialize to (7) in Section 2 and to (36) in Section 4.3.

Let i and j be two arbitrarily chosen firms in the economy, and consider the value of a new job created in firm j when a worker is hired from firm i. By definition, the value of a job equals the sum of values obtained by the worker and the firm. When a worker moves from firm i to firm j, the values obtained by the worker and by firm j are denoted by $W_{i,j}$ and $J_{i,j}$, respectively. The bargaining process from Postel-Vinay and Robin (2002) implies that the splitting of the total surplus in the hiring firm j, v_j, occurs as if the worker used the total value in the previous employment, v_i, as outside option in a bilateral bargaining with j in which the new firm has monopsony power:

$$W_{i,j} = v_i, \tag{51}$$

$$J_{i,j} = v_j - v_i. \tag{52}$$

Line (51) says that, at the moment of the transition, the worker obtains the total value in the previous job. The second line says that the hiring firm obtains the difference between the value of the new job and the value of the old job.

At the moment of transiting from i to j, the value obtained by the worker, $W_{i,j}$, satisfies

$$\left(\rho + \delta + \lambda_e P_{k:v_i \le v_k}\right) W_{i,j} = \omega_{i,j} + \delta w_u + \lambda_e \left(\int_{k:v_i \le v_k \le v_j} W_{k,j} dP_k + \int_{k:v_j < v_k} W_{j,k} dP_k \right) + dW_{i,j}. \tag{53}$$

Except for the very last term in the right-hand side this expression follows Postel-Vinay and Robin (2002). The term in brackets on the left-hand side includes the exogenous rate δ at which the math is terminated. It also includes the rate $\lambda_e P_{k:v_i \le v_k}$ at which the worker meets a new employer k with value larger than the last employer. λ_e is the job-finding rate, and $P_{k:v_i \le v_k}$ is the probability of sampling a firm k such that $v_k \ge v_i$. In the right-hand side, $\omega_{i,j}$ is the flow transfer to the worker and δw_u is the value to the worker if the match is dissolved. The term within brackets is the value to the worker in the event of contacting firms offering jobs with value higher than the last employer, i. In the event of a contact with a firm k such that $v_i \le v_k \le v_j$, the worker stays in firm j but triggers a new negotiation that raises the value to the worker to $W_{k,j}$. In the event of a contact with a firm k such that $v_j < v_k$, the worker leaves firm j to firm k but triggers a new negotiation that raises the value to the worker to $W_{j,k}$.

Similarly, the value to firm j when it hires a worker from firm i, $J_{i,j}$, is given by

$$\left(\rho + \delta + \lambda_e P_{k:v_i \le v_k}\right) J_{i,j} = r_j - \omega_{i,j} + \lambda_e \int_{k:v_i \le v_k \le v_j} J_{k,j} dP_k + dJ_{i,j}. \tag{54}$$

In this expression, r_j is revenue per worker generated in firm j. This revenue per worker is allowed to change over time. In the model, the firm controls the process of r_j. Since r_j is allowed to change, both (53) and (54) include the dynamic terms $dW_{i,j}$ and $dJ_{i,j}$. These terms are absent in Postel-Vinay and Robin (2002) where firm's productivity is fixed.

We can follow steps similar to Postel-Vinay and Robin (2002) to solve for the value of a job. Let $P_v(v)$ be the probability of sampling a firm offering jobs with value less than v. Using (51) and (52), we can change the variable of integration in the term in brackets in (53) to express it as function of the distribution of job values, and then integrate by parts to obtain

$$(\rho+\delta) W_{i,j} = \omega_{i,j} + \delta w_u + \lambda_e \int_{v_i}^{v_j} \left[1 - P_v(v')\right] dv' + dW_{i,j}. \tag{55}$$

Next, suppose that a worker employed in j meets a firm j' whose total value is the same as in j, $v_j = v_{j'}$. In this instance, (51) and (52) lead to $J_{j',j} = 0$ and $W_{j',j} = v_j$. At the same time, the sum of the changes in value obtained by firm and worker add up to the change in the value of the job: $dW_{i,j} + dJ_{i,j} = dv_j$. Evaluating (55) and (54) at $i = j'$ and summing over these equations gives

$$(\rho+\delta) v_j = r_j \; + \delta w_u + dv_j. \tag{56}$$

Thus, the total value of a job in firm j, v_j, is characterized by a differential equation that depends on the process r_j for current revenue per worker in firm j.

Equation (56) is a differential equation that characterizes the value of a new job in a firm j for any process for revenue per worker, r_j. In this paper, I study a specific process where r_j is a step function. A firm j has constant physical productivity per worker ψ but enters sequentially in multiple markets $k = 1, .., K$ at ages $\{H_k\}_{k=1}^K$. The revenue per unit of output generated by a firm who has entered in the first k is y_k, defined in (47) in section A of the Appendix. Then, we can write revenue per worker in firm j, r_j, as a step function of its age a_j,

$$r_j = r(a_j) = \psi \sum_{k=1}^{K} 1_{(H_k \le a_j < H_{k+1})} y_k, \tag{57}$$

where $1_{(H_k \le a_j < H_{k+1})}$ is an indicator of age a_j being between H_k and H_{k+1}. Therefore, when the firm is older than H_k but younger than H_{k+1}, the revenue per worker is ψy_k.

Using (57) into (56) we obtain a simple linear differential equation for the job value over firm age a independently from the firm's identity j,

$$(\rho+\delta) v(a) = \psi r(a) + \delta w_u + v'(a).$$

We can express the solution for $v(a)$ as function of of the time until entry into each market,

$$x_k \equiv \max\left[H_k - a, 0\right],$$

to obtain the value of a new job in a firm with productivity ψ that has age a and sequentially invests at ages $\{H_k\}_{k=1}^K$:

$$(\rho+\delta) v(x_1, x_2, .., x_K) = \psi \left[y_0 + \sum_{k=1}^{K} e^{-(\rho+\delta)x_k} (y_k - y_{k-1}) \right] + \delta w_u. \tag{58}$$

Setting $\psi = 1$ this corresponds to (36) in Section 4.3. Further setting $K = 1$ and letting $y_0 \equiv y_D$, $y_1 \equiv y_X$, and $x_1 \equiv x$, we reach the solution for $v(x)$ in (7).

Proof of Lemma 2

First, write the function $\Omega(h, H)$ explicitly. Replacing $G(x)$ from (19) in (62), and using $\pi(x; H)$ from (10) we have

$$\Omega(h, H) \equiv \frac{S(h; H)}{\pi(h; H)} = \frac{(\Gamma - 1)(1+\kappa_e) \int_0^h \frac{e^{-\gamma x}}{1+\kappa_e e^{-\mu(H-x)}} dx}{\widetilde{J}_u(h; H) + \kappa_e \int_h^H \widetilde{J}(x, h)\, dG(x)}, \tag{59}$$

where $\widetilde{J}_u(h, H) \equiv J_u(h, H)/y_D$ are $\widetilde{J}(x, h) \equiv J(x, h)/y_D$ the values of new jobs defined in (5) and (6) normalized by the revenue of domestic producers, y_D. In turn, let $\Omega_0(H) \equiv \Omega(H,H)$. In an interior equilibrium, the consistency and free-entry conditions, (26) and (27), imply $\Omega_0(H) = f_X/f_D$.

In what follows, I use the notation $\Omega_1(h,H) \equiv \frac{\partial\Omega(h,H)}{\partial h}$ and $\Omega_3(h,H) \equiv \frac{\partial\Omega(h,H)}{\partial H}$. By inspection of $\Omega(h,H)$, $J_u(h,H)$ and $J(x,h)$, we have that $\Omega_1(h,H) > 0$ for all H. Therefore, if $\Omega_2(H,H) > 0$, we have that $\Omega_0'(H) > 0$, implying that if an interior equilibrium exists, it must be unique. To show that this is the case, using $J_u(x)$, $J(x_0,x)$ and $v(x)$ from (5) to (7) we can rewrite $\Omega(h,H)$ in 59 as $A(h,H)/[B(h,H)+C(h,H)]$, where

$$\begin{aligned} A(h,H) &= (\Gamma-1)(1+\kappa_e)\int_0^h \frac{e^{-\gamma x}}{1+\kappa_e e^{-\mu(H-x)}}dx, \\ B(h,H) &= \frac{1-\frac{\rho w_u}{y_D}+(\Gamma-1)e^{-(\rho+\delta)h}}{\rho+\delta}, \\ C(h,H) &= \frac{\kappa_e(\Gamma-1)}{\rho+\delta}\int_h^H \left[e^{-(\rho+\delta)h}-e^{-(\rho+\delta)x}\right]dG(x), \end{aligned}$$

where, in $B(h,H)$:

$$\frac{\rho w_u}{y_D} = \frac{by}{y_D} = b[1+e_X(H)(\Gamma-1)] = b\frac{1+[\Gamma(1+\kappa_e)-1]e^{-\mu H}}{1+\kappa_e e^{-\mu H}}. \tag{60}$$

The first equality above follows from (24), the second from (22) and the third from (21). Next, note that $C(H,H) = C_2(H,H) = 0$, implying that $\Omega_2(H,H) = \widetilde{\Omega}_2(H,H)$, where $\widetilde{\Omega}(h,H) \equiv A(h,H)/B(h,H)$. Hence, $\Omega_2(H,H) > 0$ if and only if $\widetilde{\Omega}_2(H,H) > 0$. Using $A(h,H)$ and $B(h,H)$, multiplying numerator and denominator of $\widetilde{\Omega}(h,H)$ by $\left(1+\kappa_e e^{-\mu H}\right)(\rho+\delta)$, and changing the variable of integration in $A(h,H)$ to $h_0 = H - x$ gives:

$$\widetilde{\Omega}(h,H) = \frac{(\rho+\delta)(\Gamma-1)(1+\kappa_e)\int_{H-h}^H\left(e^{-\gamma H}+\kappa_e e^{-\delta H}\right)\left(e^{-\gamma h_0}+\kappa_e e^{-\delta h_0}\right)^{-1}dh_0}{1-b+\{\kappa_e-b[\Gamma(1+\kappa_e)-1]\}e^{-\mu H}+(\Gamma-1)\left(1+\kappa_e e^{-\mu H}\right)e^{-(\rho+\delta)h}}. \tag{61}$$

From assumption (29), the denominator is decreasing in H. To prove that $\widetilde{\Omega}_2(H,H) > 0$ it suffices to show that the numerator increases with H. After some manipulations, we can show that when $h = H$ this is the case if

$$LHS(H) \equiv \int_0^H \frac{1}{e^{-\gamma h_0}+\kappa_e e^{-\delta h_0}}dh_0 < \frac{1}{\gamma e^{-\gamma H}+\delta\kappa_e e^{-\delta H}}\left(1-\frac{e^{-\gamma H}+\kappa_e e^{-\delta H}}{1+\kappa_e}\right) \equiv RHS(H).$$

To prove this inequality it suffices to show that $LHS'(H) < RHS'(H)$ for all H. Computing these expressions and some manipulation implies that this holds if and only if $\left(e^{-\gamma H}+\kappa_e e^{-\delta H}\right)/(1+\kappa_e) < 1$, which holds for $H > 0$.

For existence of the interior equilibrium, $\Omega_0(0) = 0$ implies that $H > 0$. On the other hand, if $\lim_{H\to\infty}\Omega_0(H) \equiv \overline{f_X/f_D} \le f_X/f_D$, where $\overline{f_X/f_D}$ is defined in (30) in the text, then no interior equilibrium exists. In the other direction, if $H = \infty$ is an equilibrium, then it must be that no firm invests when no firm invests, i.e. $\lim_{h\to\infty}\lim_{H\to\infty}\Omega(h,H) \equiv \overline{f_X/f_D} \le f_X/f_D$. Therefore, $H = \infty \longleftrightarrow \overline{f_X/f_D} \le f_X/f_D$.

Finally, we can use (59) to show that in the unique interior equilibrium we have

$$\Omega(H,H) = \frac{(\rho+\delta)(\Gamma-1)(1+\kappa_e)\int_0^H \frac{e^{-\gamma x}}{1+\kappa_e e^{-\mu(H-x)}}dx}{1-b\left\{1+[\Gamma(1+\kappa_e)-1]e^{-\mu H}\right\}\left(1+\kappa_e e^{-\mu H}\right)^{-1}+(\Gamma-1)e^{-(\rho+\delta)H}} = \frac{f_X}{f_D}.$$

The only two endogenous objects in this equation are Γ and H. Therefore, for each value of Γ, there is a unique value of H that satisfies the interior equilibrium conditions of the model.

Proof of Proposition 1

The first order condition in the firm problem is:

$$\begin{aligned} \Pi'(h) &= e^{-(\rho+\mu)h}\left[f_X - S(h)\right] \leq 0 \text{ if } h = 0, \\ &= e^{-(\rho+\mu)h}\left[f_X - S(h)\right] = 0 \text{ if } h > 0. \end{aligned}$$

where, replacing the expression in (16) into (14),

$$S(h) = (\Gamma - 1)\, y_D\, (\lambda_u u/M) \int_0^h e^{-\gamma x} \{1 + \kappa_e [1 - G(x)]\}\, dx. \tag{62}$$

This implies: (i) $S(0) = 0$; (ii) $S'(h) > 0$; and (iii) if $G(x) = 0$, $\lim_{h\to\infty} S(h) = (\Gamma - 1)\, y_D\, (\lambda_u u/M)(1 + \kappa_e)/\gamma$. (i) and the first order condition imply that $h > 0$ if $f_X > 0$. From (ii), there is a unique interior solution to the firm problem. From (iii), if $f_X > (\Gamma - 1)\, y_D\, (\lambda_u u/M)(1 + \kappa_e)/\gamma$ then $S(h) < f_X$ for all h, and the first order condition implies that $h = \infty$. This proves the first part of the proposition. Comparative statics follow from the interior solution $S(h) = f_X$, inspection of the change in $S(h)$ with respect to each parameter, and (ii).

Proof of Proposition 2

The total output of exporting firms from the home country is $(1 - u)\, e_X$. From (49), each exporter from the home country exports a share

$$s_X(\Gamma) = 1 - \Gamma^{-\sigma} \tag{63}$$

of its output. Therefore, the total quantity exported by firms from the home country is $Q_X = (1 - u)\, e_X s_X$.In turn, from (50), exporters set the price

$$p_X = p\Gamma\tau.$$

where $p \equiv PY^{1/\sigma}$. Normalizing $P \equiv 1$ so that the final good from the home country is the numeraire, total exports from the home and foreign country are

$$\begin{aligned} X &= (p\Gamma\tau)(1 - u)\, e_X(H)\, s_X(\Gamma), \qquad (64) \\ X^* &= (p^*\Gamma^*\tau^*)(1 - u^*)\, e_X^*(H^*)\, s_X(\Gamma^*), \qquad (65) \end{aligned}$$

where $p^* \equiv P^* Y^{*1/\sigma}$. Therefore the trade balance condition, $X = X^*$, can be written

$$\frac{p^*}{p} = \frac{\tau\Gamma}{\tau^*\Gamma^*} \frac{(1 - u)\, e_X s_X}{(1 - u^*)\, e_X^* s_X^*}, \tag{66}$$

Note also that, from the definition of the exporter revenue premium in (4), an increase in the exporter premium in one country is associated with a reduction in the premium in the other country:

$$\frac{p^*}{p} = (\Gamma^\sigma - 1)^{\frac{1}{\sigma}}\, \tau^{1-\frac{1}{\sigma}} = \left[(\Gamma^{*\sigma} - 1)^{\frac{1}{\sigma}}\, \tau^{*1-\frac{1}{\sigma}}\right]^{-1}. \tag{67}$$

Using the first equality of (67) in (66) and combining with (63) we can write the trade balance condition as

$$\Gamma^* e_X^* s_X^* = \frac{\tau^{\frac{1}{\sigma}}}{\tau^*} \frac{(1-u)}{(1-u^*)} e_X s_X^{(\sigma-1)/\sigma}. \tag{68}$$

Since $de_X/d\Gamma \geq 0$ and $ds_X/d\Gamma > 0$, if $e_X > 0$ and $e_X^* > 0$ this gives an increasing relation between Γ and Γ^*. If $(\rho+\delta)(1+\kappa_e)/[\gamma(1-b)] \to \infty$ or $f_X/f_D \to 0$, we have from Proposition 1 that $e_X(1) = s_X(1) = 0$ and that $de_X/d\Gamma > 0$ if $e_X < 1$. The same applies in the foreign country. Therefore, (68) is satisfied with both sides equal to zero at $\Gamma = \Gamma^* = 1$ and each side is strictly increasing in its respective argument if $\Gamma > 1$ and $\Gamma^* > 1$. On the other hand, the second equality in (67) gives an hyperbole in the region determined by $\Gamma > 1$ and $\Gamma^* > 1$, with the property that $\Gamma^* \to \infty$ as $\Gamma \to 1$, and vice versa. This implies that only one point in the quadrant determined by $\Gamma > 1$ and $\Gamma^* > 1$ satisfies the equilibrium conditions.

Proof of Proposition 3

In an interior equilibrium with symmetric countries, the following expression holds in both countries:

$$\Omega(H,H) = \frac{(\rho+\delta)(\Gamma-1)(1+\kappa_e)\int_0^H e^{-\gamma x}\left[1+\kappa_e e^{-\mu(H-x)}\right]^{-1} dx}{1-b\{1+[\Gamma(1+\kappa_e)-1]e^{-\mu H}\}\left(1+\kappa_e e^{-\mu H}\right)^{-1}+(\Gamma-1)e^{-(\rho+\delta)H}} = \frac{f_X}{f_D} \tag{69}$$

Changes in parameters that increase $\Omega(H,H)$ given H lead to lower equilibrium H. $\partial\Omega(H,H)/\partial\lambda_u = 0$ holds by inspection. Multiplying numerator and denominator of $\Omega(H,H)$ by $\left(1+\kappa_e e^{-\mu H}\right)/(\Gamma-1)$ we obtain $\partial\Omega(H,H)/\partial\Gamma > 0$ if (29) holds. Finally, the numerator of $\Omega(H,H)$ is increasing in κ_e, while some manipulation shows that the denominator is decreasing because $e^{-\mu H} < 1$, implying $\partial\Omega(H,H)/\partial\kappa_e > 0$. By inspection of (20) to (22), the parameter changes that lead to a lower H also lead to an increase in m_X, e_X and y.

Proof of Proposition 4

Under the conditions for Proposition 2, from (68) we can implicitly write Γ as an increasing function of Γ^*. The equilibrium values for Γ and Γ^* correspond to the intersection between this function and (67). From the results in Proposition 3, $e_X^*(\Gamma^*)$ in (68) increases for each value of Γ^* with a rise in λ_e^*, hence the new equilibrium must have a larger Γ and a lower Γ^*. The increase in export participation in the home country follows from Proposition 3. For the increase in export participation in the foreign economy, we have that with a rise in λ_e the increase in Γ leads to an increase in the right hand side of (68). Since Γ^* decreases in the left hand side, so does $s_X(\Gamma^*)$, meaning that $e_X^*(\Gamma^*)$ and therefore $m_X^*(\Gamma^*)$ must increase.

Proof of Proposition 5

First I prove (34). Then, I use this condition to show that, if $\mu < \gamma$, then: (i) If $\lambda_e = 0$, $f'(n) > 0$ for all n; (ii) $f'(0) > 0$; (iii) If $h < H$, then $\lim_{h\to\infty} f'(N(h)) < 0$; and (iv) If $h > H$, $f'(N(h)) > 0$. These four properties imply the proposition.

Start by considering an equilibrium where every firm switches at age H and let $n = N(h)$ be the size of a firm of age h. The net flow of workers in a firm of age h is

$$N'(h) = \begin{cases} \left(\frac{\lambda_u u}{M}\right)[1+\kappa_e G_H(h)] - \{\gamma+\lambda_e[1-P_H(h)]\}N(h) & \text{if } h < H \\ \left(\frac{\lambda_u u}{M}\right)[1+\kappa_e G_H(H)] - \{\gamma+\varpi\lambda_e[1-P_H(H)]\}N(h) & \text{if } H \leq h \end{cases}, \tag{70}$$

where $P_H(h) = 1-e^{-\mu h}$ and $G_H(h) = P_H(h)/\{1+\kappa_e[1-P_H(h)]\}$ are the firm and employment distributions defined over age, instead of over time until investment as in (18) and (19), respectively. Workers in firms older than H who contact another firm older than H are indifferent about making a transition, in which case they move with exogenous probability $\varpi \in [0,1]$.

The rate at which workers leave the firm is weakly decreasing and the number of new hires is weakly increasing in h, so $N(h)$ is increasing. Letting $F(n)$ be the share of firms of size less than n, we have, from the exponential distribution of ages, that $F(n) = 1 - e^{-\mu N^{-1}(n)}$. This implies

$$f'(n)/f(n) = -\left[\mu + N''\left(N^{-1}(n)\right)/N'\left(N^{-1}(n)\right)\right]/N'\left(N^{-1}(n)\right),$$

implying (34). If $h < H$ and $\lambda_e = 0$ then $N''(h)/N'(h) = -\gamma$, and if $h > H$ then $N''(h)/N'(h) = -\{\gamma + \varpi\lambda_e[1 - P_H(H)]\}$, implying (i) and (iv) above. If $h < H$, from (70),

$$\frac{N''(h)}{N'(h)} + \mu = \frac{\left(\frac{\lambda_u u}{M}\right)\frac{1+\kappa_e}{1+\kappa_e e^{-\mu h}}\left(\mu - \gamma - \frac{\gamma + \lambda_e e^{-\mu h}}{\gamma + \mu + \lambda_e e^{-\mu h}}\lambda_e e^{-\mu h}\right) + \left[\left(\lambda_e e^{-\mu h} + \gamma\right)^2 - \mu\gamma\right]N(h)}{\left(\frac{\lambda_u u}{M}\right)\frac{1+\kappa_e}{1+\kappa_e e^{-\mu h}} - \left(\lambda_e e^{-\mu h} + \gamma\right)N(h)}. \tag{71}$$

At $h = 0$, (71) yields $N''(h)/N'(h) = -\gamma - \lambda_e(\gamma + \lambda_e)/(\gamma + \mu + \lambda_e) < 0$, implying (ii). Since the denominator in the right-hand side of (71) is positive we have that, as long as $h < H$, then $\lim_{h\to\infty} N''(h)/N'(h) + \mu > 0$ iff $\lambda_u u/\gamma M < \lim_{h\to\infty} N(h) = (1 + \kappa_e)(\lambda_u u/\gamma M)$, implying (iii).

Proof of Proposition 6

In an interior equilibrium of the model extended with endogenous matching rates, condition (69) from Proposition 3 must still determine the timing of investment H, where now $\{\lambda_e, \lambda_u\}$ are endogenously determined via (35). Given Γ, inspection of (69) implies that λ_u does not affect the timing of investment. Because every variable of (69) is exogenous except for $\kappa_e \equiv \lambda_e/\delta$, $\widetilde{\lambda}_u$ may only affect the equilibrium through λ_e. Therefore, when $\widetilde{\lambda}_e = 0$, changes in $\widetilde{\lambda}_e$ or $\widetilde{\lambda}_u$ do not affect H.

Proof of Proposition 7

I characterize the partial-equilibrium problem with K possible investments. For this, it is useful to define $P_v(v)$ as the probability of sampling a firm with value less than or equal to v. This yields the share of employment in firms with value of jobs below v,

$$G_v(v) = \frac{P_v(v)}{1 + \kappa_e[1 - P_v(v)]}.$$

Let $v_{\min}$ be the minimum of the support of the distribution of job values. The present discounted value of all workers attracted by a firm offering jobs with value $v \geq v_D$ is

$$\pi_v(v) = \frac{\lambda_u u}{M}(v - w_u) + \frac{\lambda_e(1-u)}{M}\int_{v_{\min}}^{v}(v - v_0)\,dG_v(v_0),$$

which after some manipulation can be written as $\pi_v(v) = \frac{\lambda_u u}{M}\pi_0(v)$, where

$$\pi_0(v) = (v - w_u) + \kappa_e\int_{v_{\min}}^{v} G_v(v_0)\,dv_0.$$

We define $\pi(\boldsymbol{x}) \equiv \pi_v(v(\boldsymbol{x}))$ as the present discounted value of all workers attracted by a firm with times until entry of $x = \{x_1, .., x_K\}$ for $x_j \in [0, \infty]$, where $v(\boldsymbol{x})$ is value of a new job defined in (58). We compute the partial derivative of $\pi(\boldsymbol{x})$ with respect to x_k to use it below,

$$\pi_k(\boldsymbol{x}) \equiv \frac{\partial \pi(\boldsymbol{x})}{\partial x_k} = -\left(\frac{\lambda_u u}{M}\right)\frac{1+\kappa_e}{1+\kappa_e\left[1-P_v\left(v(\boldsymbol{x})\right)\right]}\psi e^{-(\rho+\delta)x_k}\left(y_k - y_{k-1}\right), \tag{72}$$

and note that this implies

$$\frac{\pi_{k+1}(\boldsymbol{x})}{\pi_k(\boldsymbol{x})} = e^{-(\rho+\delta)(x_{k+1}-x_k)}\left(\frac{y_{k+1}-y_k}{y_k - y_{k-1}}\right). \tag{73}$$

In parallel to (11), let $\Pi(h_1, h_2, .., h_K)$ be the value at entry of a firm that enters to export markets $k = 1, 2, .., K$ at ages $H_k = \sum_{j=1}^{k} h_j$. To shorten notation, we define the length-K vectors

$$\mathbf{1}_i = \left\{\underbrace{0, ..., 0}_{i-1 \text{ times}}, 1, .., 1\right\}$$

and

$$\mathbf{h}_{i+1} = \left\{\underbrace{0, ..., 0}_{i \text{ times}}, h_{i+1}, h_{i+1}+h_{i+2}, h_{i+1}+h_{i+2}+h_{i+3}, .., h_{i+1}+h_{i+2}+...+h_K\right\}.$$

Using this notation, recursively define $\Pi_i(\mathbf{h}_{i+1})$ as the value of a firm at the moment of entry to market i,

$$\Pi_i(\mathbf{h}_{i+1}) = \int_0^{h_{i+1}} e^{-(\rho+\mu)(h_{i+1}-x_{i+1})}\pi\left(x_{i+1}\mathbf{1}_{i+1}+\mathbf{h}_{i+2}\right)dx_{i+1} + e^{-(\rho+\mu)h_{i+1}}\left(\Pi_{i+1}(\mathbf{h}_{i+2}) - \frac{f_{i+1}}{\rho+\mu}\right) \text{ for } i < K \tag{74}$$

where, at the next-to-last market,

$$\Pi_{K-1}(h_K) = \int_0^{h_K} e^{-(\rho+\mu)(h_K - x_K)}\pi\left(x_K\mathbf{1}_K\right)dx_K + e^{-(\rho+\mu)h_K}\frac{\pi(0,0,..,0)-f_K}{\rho+\mu}. \tag{75}$$

Therefore, the value at entry is $\Pi(\mathbf{h}_1) = \Pi_0(\mathbf{h}_1)$. After some manipulations (available upon request) we can express the derivative of the profit function with respect to each entry time as

$$\frac{\partial \Pi(\mathbf{h}_1)}{\partial h_1} = \sum_{i=1}^{K} e^{-(\rho+\mu)H_i}\left(\int_0^{h_i} e^{(\rho+\mu)x_i}\frac{d\pi\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right)}{dx_i}dx_i + f_i\right), \tag{76}$$

and

$$\begin{aligned}\frac{\partial \Pi(\mathbf{h}_1)}{\partial h_j} &= \sum_{i=1}^{j-1}\int_0^{h_i} e^{(\rho+\mu)(x_i - H_i)}\frac{d\pi\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right)}{dh_j}dx_i \\ &\quad + \sum_{i=j}^{K} e^{-(\rho+\mu)H_i}\left(\int_0^{h_i} e^{(\rho+\mu)x_i}\frac{d\pi\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right)}{dx_i}dx_i + f_i\right)\end{aligned} \tag{77}$$

for $j > 1$. Note, in addition, that

$$\frac{d\pi\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right)}{dh_j} = \sum_{m=j}^{K}\pi_m\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right) \text{ for } j > i, \tag{78}$$

$$\frac{d\pi\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right)}{dx_i} = \sum_{m=i}^{K}\pi_m\left(x_i\mathbf{1}_i+\mathbf{h}_{i+1}\right), \tag{79}$$

where $\pi_m(\cdot)$ denotes the partial derivative of $\pi(\cdot)$ with respect to its m^{th} argument. Combining (78) and (79) with (76) and (77), and imposing the first-order conditions $\frac{\partial \Pi(\mathbf{h}_1)}{\partial h_i} = 0$ for all $i = 1, .., K$, we reach the set of conditions:

$$\sum_{i=1}^{j} \int_0^{h_i} e^{(\rho+\mu)(x_i+H_j-H_i)} \pi_j \left(x_i \mathbf{1}_i + \mathbf{h}_{i+1}\right) dx_i + f_j = 0 \text{ for all } j \geq 1. \tag{80}$$

Finally, use (73) to get that, for $j > i$,

$$\frac{\pi_j \left(x_i \mathbf{1}_i + \mathbf{h}_{i+1}\right)}{\pi_{j-1} \left(x_i \mathbf{1}_i + \mathbf{h}_{i+1}\right)} = e^{-(\rho+\delta)h_j} \frac{y_j - y_{j-1}}{y_{j-1} - y_{j-2}}. \tag{81}$$

Evaluating (80) at j and at $j+1$ together with (81) gives the result.

C Appendix to Section 5: Calibration

C.1 Data Sources and Summary Statistics

I use summary statistics extracted from confidential microdata. Exports data comes from official customs data at the firm-year level. These data were merged with firm employment data from administrative records by the Employment and Business Dynamics Observatory (OEDE) of the Ministry of Labor and Social Security of Argentina. All firms are required to report their formal employees on a monthly basis. In each of six two-month periods within each year between 1998 and 2008, every formal worker aged 18 to 64 is linked to the firm where he/she is reported as earning the highest wage. Workers earning below the minimum wage are excluded. The data includes the universe of firms that report employment above the minimum wage in any period in these years.

Each firm-year observation is classified as exporter if the firm exports at least USD 10000, and, if exporter, further classified as exporting to 5 countries or less, or to more than 5 countries. Firm age is the difference between the current year and the year of birth in the tax record. The number of workers per firm is the average employment across periods within year. Industries are defined at the two-digit level.

A worker employed in a firm in a period is considered a new hire if he/she is not employed in the firm in the previous period. To compute the fraction of new hires entering from other formal jobs in any sector of the economy for each firm-year, the shares are first computed for each pair of consecutive periods within year, and then averaged across periods within firm-year. Similar steps are followed to compute the fraction of new hires from the manufacturing sector entering from jobs in exporting firms.

All figures are based on firms from the manufacturing sector. Exiting firms of any export status (i.e., firms present in a given year who do not report employment in the next) are excluded. Firms who do not report formal employment but who report exports are excluded, as well as industries with less than one-hundred firms in any year. The resulting sample represents, on average, 97% of the formal employment and 82% of all firms who either export or formally report the wages of their employees in the manufacturing sector between 1999 and 2007.

	Export Status	1999-2002	2003-2007
Firm Age	Non Exporters	12	12
	Exp., 5 destinations or less	19	19
	Exp., more than 5 destinations	29	28
Share of Job-to-job Transitions in New Hires	Non Exporters	15%	16%
	Exp., 5 destinations or less	23%	27%
	Exp., more than 5 destinations	33%	40%
Share of Employment	Non Exporters	47%	57%
	Exp., 5 destinations or less	23%	20%
	Exp., more than 5 destinations	30%	33%
Share of Firms	Non Exporters	90%	89%
	Exp., 5 destinations or less	9%	8%
	Exp., more than 5 destinations	2%	3%

Table $A1$: Summary Statistics

C.2 Full Description of the Calibrated Model

I fully describe the quantitative environment used in the calibration. Firms are distinguished by their productivity ψ and fixed cost ϕ. Firm type is denoted by $\varepsilon \equiv \{\psi, \phi\}$. The times until entry to each export markets is denoted by x_1, x_2, where $x_k \in [0, \infty]$. From (58), the value of a new job is

$$v(x_1, x_2; \psi) = \frac{\psi\left[y_0 + e^{-(\rho+\delta)x_1}(y_1 - y_0) + e^{-(\rho+\delta)x_2}(y_2 - y_1)\right] + \delta w_u}{\rho + \delta}. \tag{82}$$

Using (48) from Appendix A, and normalizing the domestic price index to 1, revenues per unit of output of domestic firms and each exporter type are given by

$$\begin{aligned} y_0 &= Y^{\frac{1}{\sigma}}, & (83)\\ y_1 &= \Gamma_1 y_0 = [Y + A_1]^{\frac{1}{\sigma}}, & (84)\\ y_2 &= \Gamma_2 y_0 = [Y + A_1 + A_2]^{\frac{1}{\sigma}}, & (85) \end{aligned}$$

where $Y = (1-u)\,y$ is income per capita in the domestic market and $A_k \equiv \tau_k^{-(\sigma-1)} P_k^{\sigma} Y_k$ for $k = 1, 2$ capture trade costs and foreign market sizes. In the numerical exercises the market size A_k are treated as parameters.

The benchmark model in Section 3 is structured around the observation that the time to switch into exporting is a sufficient statistic for the value of a new job. Now this no longer holds. Instead, we must use the distribution of employment across firms offering jobs with different value v, $G_v(v)$. Given this distribution, firms can compute the yield on their hiring effort.

Conveniently, the present discounted value of all workers attracted by a firm that offers jobs with value v, $\pi_v(v)$, is independent from firm type:

$$\pi_v(v) = \max_{s \geq 0}\left[\frac{\lambda_u u}{M\bar{s}}(v - w_u) + \frac{\lambda_e(1-u)}{M\bar{s}}\int_{v_{\min}}^{v}(v - v_0)\, dG_v(v_0)\right] s - c(s). \tag{86}$$

The term in square brackets is the return to search intensity s. Firms are subject to a convex adjustment cost $c(s)$. In each period the firm solves the static problem of how many workers to attract. With few manipulations, (86) can be written more compactly as

$$\pi_v(v) = \max_{s\geq 0} \frac{\lambda_u u}{M\bar{s}} \pi_0(v)\, s - c(s), \tag{87}$$

where $\pi_0(v) \equiv (v - w_u) + \kappa_e \int_{v_{\min}}^{v} G_v(v_0)\, dv_0$. From the solution to (87), a firm offering jobs with value v chooses

$$s(v) = \left[\frac{\lambda_u u}{M\bar{s}} \frac{\pi_0(v)}{\zeta}\right]^{1/(\zeta-1)}. \tag{88}$$

Using (82), we can define $\pi(x_1, x_2; \psi) \equiv \pi_v(v(x_1, x_2; \psi))$ as the present discounted value of all workers attracted by a firm with productivity ψ in state $\{x_1, x_2\}$. This is the equivalent to (10) in the baseline model, and it simply is given by

$$\pi(x_1, x_2; \psi) = (\zeta - 1)\, s(v(x_1, x_2; \psi))^{\zeta}. \tag{89}$$

Firms are born as domestic producers, but they can access markets $k = 1, 2$ by paying entry costs with flow equivalent values of ϕf_k. f_k is a component of entry costs in market k that is common across firms and ϕ is firm specific. Using (89), we can define $\Pi(h_1, h_2; \varepsilon)$ in parallel to $\Pi(h)$ in (11) as the value of a newborn firm of type ε that enters markets $k = 1, 2$ at ages h_1 and $h_1 + h_2$, respectively,

$$\Pi(h_1, h_2; \varepsilon) = \int_0^{h_1} e^{-(\rho+\mu)a} \pi(h_1 - a, h_1 + h_2 - a; \psi)\, da + e^{-(\rho+\mu)h_1} \left(\Pi_1(h_2; \varepsilon) - \frac{\phi f_1}{\rho + \mu}\right), \tag{90}$$

where $\Pi_1(h_2; \varepsilon)$ is the value of this firm at the moment of entry into market 1,

$$\Pi_1(h_2; \varepsilon) = \int_0^{h_2} e^{-(\rho+\mu)a} \pi(0, h_2 - a; \psi)\, da_2 + e^{-(\rho+\mu)h_2} \frac{\pi(0, 0; \psi) - \phi f_2}{\rho + \mu}. \tag{91}$$

Following similar steps to the general solution with multiple investment options from the proof of Proposition 7, in an interior solution the first order conditions can be written as[40]

$$S_1(h_1, h_2; \psi) \equiv \int_0^{h_1} e^{(\rho+\mu)x_1} \left[-\pi_1(x_1, x_1 + h_2; \psi)\right] dx_1 = \phi f_1, \tag{92}$$

$$S_2(h_2; \psi) \equiv \int_0^{h_2} e^{(\rho+\mu)x_2} \left[-\pi_2(0, x_2; \psi)\right] dx_2 = \left(f_2 - e^{-\gamma h_2} \frac{y_2 - y_1}{y_1 - y_0} f_1\right) \phi. \tag{93}$$

Using the expressions for $\pi_1(x_1, x_1 + h_2; \psi)$ and $\pi_2(0, x_2; \psi)$ that result from (89), as well as the expression for G_v from (99) below, the first-order conditions (92) and (93) can be explicitly written as

$$\left(\frac{\lambda_u u}{M\bar{s}}\right)^{\zeta/(\zeta-1)} \int_0^{h_1} \left(\frac{\pi_0(x_1, x_1 + h_2; \psi)}{\zeta}\right)^{1/(\zeta-1)} \frac{1 + \kappa_e}{1 + \kappa_e\left[1 - P(v(x_1, x_1 + h_2; \psi))\right]} e^{-\gamma x_1} dx_1 = \frac{\phi}{\psi} \frac{f_1}{y_1 - y_0}, \tag{94}$$

and

$$\left(\frac{\lambda_u u}{M\bar{s}}\right)^{\zeta/(\zeta-1)} \int_0^{h_2} \left(\frac{\pi_0(0, x_2; \psi)}{\zeta}\right)^{1/(\zeta-1)} \frac{1 + \kappa_e}{1 + \kappa_e\left[1 - P(v(0, x_2; \psi))\right]} e^{-\gamma x_2} dx_2 = \frac{\phi}{\psi} \left(\frac{f_2}{y_2 - y_1} - e^{-\gamma h_2} \frac{f_1}{y_1 - y_0}\right). \tag{95}$$

[40] I use the notation $\pi_1(a, b; \psi) = \partial \pi(x, y; \psi) / \partial x$ and $\pi_2(a, b; \psi) = \partial \pi(x, y; \psi) / \partial y$ evaluated at $(x, y) = (a, b)$.

Equations (94) and (95) solve for the entry times $\{h_1, h_2\}$ of a type-ε firm. The numerical solution of the model uses these two equations. The left-hand side of (95) is strictly increasing in h_2 and independent from h_1, while the the left-hand side of (94) is strictly increasing in both h_1 and h_2. This gives a unique interior solution to the firm problem. From these expressions, it also follows that more productive or lower-cost firms invest earlier in both markets.

Let $\{h_1(\varepsilon), h_2(\varepsilon)\}$ be the solution to (94) and (95) for a firm of type ε and note that, as in (15), the value of the firm at entry is

$$\Pi^e(\varepsilon) = \frac{\pi\left(h_1(\varepsilon), h_1(\varepsilon) + h_2(\varepsilon); \psi\right)}{\rho + \mu}. \tag{96}$$

Defining the equilibrium requires to identify the function $P_v(v)$ for the probability that a worker who samples a firm finds job with value below v; this is equivalent to $P(x)$ in the baseline model. To find this function, define first the equilibrium value of a job offered by firm ε over age a,

$$v^*(a;\varepsilon) \equiv v\left(\max\left[h_1(\varepsilon) - a, 0\right], \max\left[h_1(\varepsilon) + h_2(\varepsilon) - a, 0\right]; \psi\right).$$

Notice that $v^*(a;\varepsilon)$ is strictly increasing in a, as such having a well defined inverse denoted by $a^*(v;\varepsilon)$. That is, $a^*(v;\varepsilon)$ is the age at which firm type ε offers a job with value v. Using (88), define also the value of s chosen by firms of type ε and age a as,

$$s^*(a;\varepsilon) \equiv s\left(v^*(a;\varepsilon)\right). \tag{97}$$

The effective measure that a firm of age a and type ε has in the labor market is $s^*(a;\varepsilon)/\overline{s}$. Therefore, the probability that a worker samples a firm offering jobs with value lower than ν is

$$P_v(v) = \mathbb{E}_\varepsilon \int_0^{a^*(v;\varepsilon)} \left[\frac{s^*(a;\varepsilon)}{\overline{s}}\right] \mu e^{-\mu a} da, \tag{98}$$

where E_ε denotes the expectation over the distribution of firm types ε. This function readily yields the share of employment in firms with value of jobs below v,

$$G_v(v) = \frac{P_v(v)}{1 + \kappa_e\left[1 - P_v(v)\right]}. \tag{99}$$

The measure of firms M is determined by zero profits. Entry requires flow-equivalent fixed costs of f_0 in each period, so that the free entry condition is

$$\mathbb{E}_\varepsilon\left[\Pi^e(\varepsilon)\right] = f_0, \tag{100}$$

where $\Pi^e(\varepsilon)$ is given in (96).

Finally, aggregate income in the economy depends on both productivity and the distribution of switching ages. Let

$$y^*(a;\varepsilon) = 1_{(a<h_1(\varepsilon))} y_0 + 1_{(h_1(\varepsilon)\le a<h_1(\varepsilon)+h_2(\varepsilon))} y_1 + 1_{(h_1(\varepsilon)+h_2(\varepsilon)\le a)} y_2$$

be the revenue per unit of output in firm ε when it has age a. Output per employed worker y equals

$$y = \mathbb{E}\left[y^*(a;\varepsilon)\right]. \tag{101}$$

The expectation in 101 is taken with respect to the equilibrium distribution of employment over states (a,ε) directly induced by $P_v(v)$. As before, the value of unemployment is given by (24).

Definition 2 *A general equilibrium of the quantitative model consists of individual rules* $\{h_1(\varepsilon), h_2(\varepsilon), s^*(a;\varepsilon)\}$, *distributions* $\{G_v(v), P_v(v)\}$, *a number of firms* M, *output per worker* y, *consumption* c *and value of unemployment* w_u *such that:*
a) the first-order conditions from the firm problem, (37), (38) and (97), hold;
b) the individual decision rules are consistent with the aggregate distributions, (98) and (99);
c) the number of firms adjusts to satisfy free entry, (100);
d) output per worker is given by (101);
e) the value of unemployment is given by (24); and
f) goods markets clear.

C.3 Numerical Algorithm

The algorithm to solve the model consists of an outer loop on $\{P_v(v), y\}$ defined in (98) and (101), and an inner loop on the distribution of firm choices, $\{h_1(\varepsilon), h_2(\varepsilon), s(\varepsilon)\}$ and the number of firms M.

1. Start from a guess for $P_v(v)$ and income per worker y or from the last iteration outcome,
2. Guess a value for $S \equiv M\bar{s}$ or use the last iteration outcome
    (a) use (94) and (95) to solve for $\{h_1(\varepsilon), h_2(\varepsilon)\}$,
    (b) compute the value at entry for firm type ε, $\Pi^e(\varepsilon)$, using (96),
    (c) Adjust S so that the free-entry condition $E_\varepsilon[\Pi^e(\varepsilon)]$ holds,
    (d) Iterate on steps $2-a$ to $2-c$ until convergence of $\{h_1(\varepsilon), h_2(\varepsilon)\}$ and S,
3. Compute $\bar{s}$ using the rules $s^*(a;\varepsilon)$ from (97), and solve for M using the solution for S from step 2.
4. Compute the new distribution $P_v(v)$ and income per worker y using (98) and (101) and return to step 1.

C.4 Calibrated Parameters and Targets

Parameter		Value	Target/Source
Firm Exit Rate	μ	0.075	Slope of firm age distribution
Rate of Job Separation	γ	0.15	Probability of E-U transition
Contact Rate for Unemployed Workers	λ_u	2.025	Unemployment Rate
Rate of Time Discount	ρ	0.058	Interest Rate
Demand Elasticity	σ	2.98	Eaton et al. (2011)

Table $A2$: Parameters Set Without Using the Model

Parameter		Value
Firm entry cost	f_0	6.7
Entry cost to first market	f_1	79.9
Entry cost to second market	f_2	103.5
Job-finding rate of employed relative to unemployed	$\frac{\lambda_e}{\lambda_u}$	0.086
Share of GDP spent in unemployment transfers	$b * \frac{u}{1-u}$	0.0076
Convexity in Hiring Cost	ζ	1.77
Share of high fixed costs firms	ω	0.8
Foreign Market Capacity 1	A_1	29.7
Foreign Market Capacity 2	A_2	33.9

Table $A3$: Calibrated Parameters

Moment	Model	Data
Number of Workers per Firm	22	22
Share of firms exporting to 5 countries or less	6%	8%
Share of firms exporting to more than 5 countries	5%	3%
Share of employment in firms exporting to 5 countries or less	22%	20%
Share of employment in firms exporting to more than 5 countries	28%	33%
Average age of non-exporters	12	12
Average age of exporters	20	20
Share of job-to-job hires in total hires of non-exporters	14%	16%
Share of job-to-job hires in total hires of exporters to 5 countries or less	31%	27%
Share of job-to-job hires in total hires of exporters to more than 5 countries	40%	40%

Table $A4$: Matched Moments, Model and Data