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ABSTRACT

The European debt crisis shares features of the historical episodes of outright default on domestic public debt identified by Reinhart and Rogoff (2008) as a forgotten research subject. This paper proposes a theory of domestic sovereign default in which a government chooses debt and default optimally, responding to distributional incentives affecting the welfare of risk-averse agents who are heterogeneous in wealth. Equilibria with debt do not exist if the government is utilitarian and default is costless. Adding an exogenous default cost, the model supports equilibria with debt exposed to default risk in which debt falls as wealth inequality rises. A quantitative experiment calibrated to Europe shows that, in the observed range of inequality in bond holdings, the model accounts for 1/3rd of the median debt ratio at a default probability of 0.9%, and the debt is sharply lower than in the absence of default risk. Equilibria with debt also exist if, instead of default costs, a political bias leads the government to weigh its creditors' welfare by more than their wealth share. Quantitatively, combining default costs and political bias yields debt ratios similar to Europe's at low default probabilities.

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1 Introduction

The innovative study by Reinhart and Rogoff (2008) identified 68 episodes in which governments defaulted outright (i.e. by means other than inflation) on their domestic creditors in a cross-country database going back to 1750. These domestic defaults occurred via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspensions of payments. Reinhart and Rogoff also documented that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about $2/3^{\text{rds}}$ on average), and that domestic defaults were associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses and full-blown economic crises. Despite of these striking features, they also found that domestic sovereign default is a "forgotten history" that remains largely unexplored in economic research.

The ongoing European debt crisis also highlights the importance of studying domestic sovereign default. In particular, four features of this crisis make it more akin to a domestic default than to the typical external default that dominates the literature on public debt default. First, countries in the Eurozone are highly integrated, with the majority of their public debt denominated in their common currency and held by European residents. Hence, from an European standpoint, default by one or more Eurozone governments means a suspension of payments to "domestic" agents, instead of external creditors. Second, as Table 1 shows, domestic public debt-GDP ratios are high in the Eurozone in general, and very large in some of its members, particularly in the countries at the epicenter of the crisis (Greece, Ireland, Italy, Spain and Portugal). Third, the Eurozone's common currency and common central bank rule out the possibility of individual governments resorting to inflation as a means to lighten their debt burden without an outright default. Fourth, and perhaps most important from the standpoint of the theory proposed in this paper, European-wide institutions such as the ECB and the European Commission are weighting the interests of both creditors and debtors in assessing the pros and cons of sovereign defaults by individual countries, and both creditors and debtors are aware of these institutions' concern and of their key role in influencing expectations and the potential for repayment or default.[1]

Table 1 also shows that the Eurozone fiscal crisis has been characterized by rapid increases in public debt ratios and sovereign spreads that coincided with increases in government expenditure ratios. The Table documents these facts using data for seven key Eurozone countries (the countries at the center of the crisis plus France and Germany). While there are clear differences

[1]The analogy with a domestic default is imperfect, however, because the Eurozone is not a single country, and in particular there is no fiscal entity with tax and debt-issuance powers over all the members. Still, the situation resembles more a domestic default than an external default in which debtors are not concerned for the interests of creditors.

in the figures for France and Germany v. the other countries, the direction of the changes from pre-crisis averages to the crisis peaks is the same. The mean and median debt ratios of the seven countries rose by nearly 30 percentage points of GDP, while the mean and median of government purchases rose about 3 percentage points of GDP. The average of the spreads rose from 0.22 to 7.7 percent, and the median increased from 0.17 to 5.67 percent.

The Table also shows that wealth is unevenly distributed in the seven Eurozone countries listed, with mean and median Gini coefficients of around 2/3rds. This is, however, a relatively low degree of wealth inequality relative to other countries. For example, both the world-wide and U.S. Gini coefficients are estimated at 0.8.[2] Thus, the very high Eurozone 2011 debt ratios of as much as 133 percent of GDP were attained in economies with a relatively low degree of wealth inequality.

Table 1: Euro Area: Key Fiscal Statistics and Wealth Inequality

	Gov. Debt		Gov. Exp.		Spreads		Gini
Moment (%)	Avg.	2011	Avg.	"crisis peak"	Avg.	"crisis peak"	Wealth
France	34.87	62.72	23.40	24.90	0.08	1.04	0.73
Germany	33.34	52.16	18.80	20.00	-	-	0.67
Greece	84.25	133.09	18.40	23.60	0.37	21.00	0.65
Ireland	14.07	64.97	16.10	20.50	0.11	6.99	0.58
Italy	95.46	100.22	19.40	21.40	0.27	3.99	0.61
Portugal	35.21	75.83	20.00	22.10	0.20	9.05	0.67
Spain	39.97	45.60	17.60	21.40	0.13	4.35	0.57
Avg.	48.17	76.37	19.10	21.99	0.22	7.74	0.64
Median	35.21	64.97	18.80	21.40	0.17	5.67	0.65

Note: Author's calculations based on OECD Statistics, Eurostat, ECSB and Davies et. al. (2009). "Gov. Debt" refers to Total General Government Net Financial Liabilities (avg 90-07); "Gov. Exp." corresponds to government purchases in National Accounts (avg 00-07); "Sov Spreads" correspond to the difference between interest rates of the given country and Germany for bonds of similar maturity (avg 00-07). For a given country i, they are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$. "Crisis Peak" refers to the maximum value observed during years 2008-2012 (i.e. during the current financial crisis) as reported by Eurostat. Gini Wealth as reported for each country for year 2000 by Davies et. al. (2009) Appendix V.

Further and more systematic empirical analysis of the relationship between wealth inequality and public debt shows a statistically significant, bell-shaped relationship: Debt is increasing in wealth inequality when inequality is low, until the Gini coefficient reaches about 0.75, and then becomes decreasing in wealth inequality. This finding follows from estimating a cross-country panel regression using the international database of Gini coefficients for wealth produced by

[2]These wealth Gini coefficient are 2000, PPP-adjusted estimates taken from the work of Davies et al. (2009).

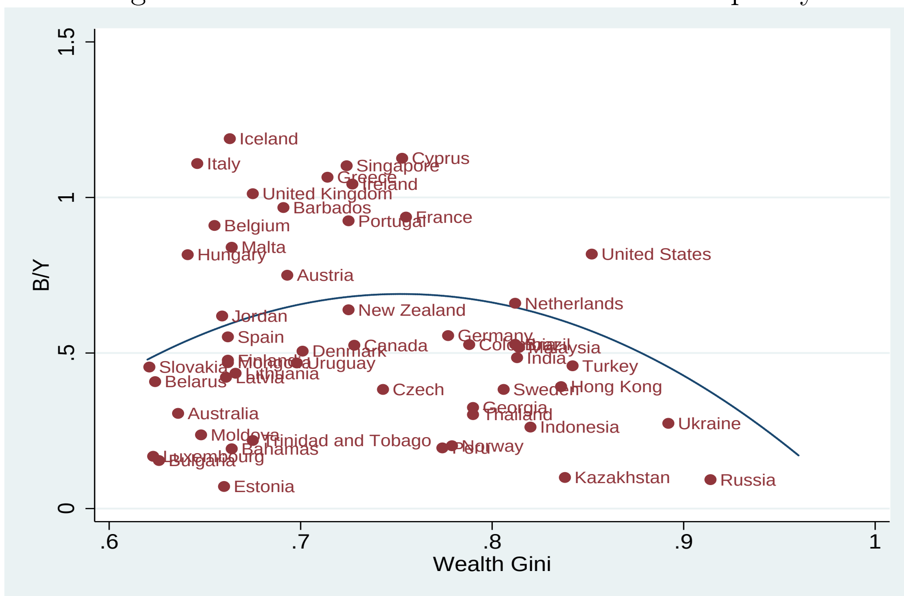
Davies et. al. (2009) and Davies et al. (2012), together with data on public debt-output ratios and control variables (i.e. the size of the government, proxied by the ratio of government expenditures to GDP, and country fixed effects). The results of panel regressions of the debt ratio on the Gini coefficient, the square of this coefficient, and the controls (using the observations for 2000 and 2012) are shown in Table 2, and Figure 1 shows a plot of the 2012 data and the fitted regression curve. The regressions only explain about a tenth of the cross-country variations on debt ratios, but the linear and quadratic effects of the wealth Gini coefficient are statistically significant and opposite in sign.

Table 2: Panel Regressions of Public Debt Ratios and Wealth Inequality

	Panel Country Fixed Effect	
	Dep. Var: Debt to GDP	
Wealth Gini	17.65**	18.11**
s.e.	7.06	7.22
Wealth Gini^2	-17.65**	-12.03**
s.e.	4.76	4.85
G/Y	-	-0.010
s.e.	-	0.012
avg. country FE	-5.92**	-5.95**
s.e.	2.60	2.64
years	2000-2012	2000-2012
R^2	0.118	0.136
# obs.	139	139

Note: Author's calculations based on WDI, Davies et. al. (2009) and Davies et. al. (2012). Debt to GDP refers to Central Government Debt as share of GDP (source WDI). Wealth Gini as reported for each country for year 2000 and 2012 by Davies et. al. (2009) and Davies et. al. (2012). G/Y corresponds to government final consumption as a share of GDP (source WDI). Coefficients and standard errors reported are derived from a panel linear regression with Country Fixed Effects. The sample contains 87 countries with data for years 2000 and 2012. If debt to GDP or government expenditure ratio is not available for year 2000 or 2012 data for year 1999 or 2011 is used.

Figure 1: Public Debt Ratios and Wealth Inequality



Note: Author's calculations based on WDI, Davies et. al. (2009) and Davies et. al. (2012).
Fitted line from results presented in Table 2

Taken together, the empirical facts documented by Reinhart and Rogoff (2008) and the evidence for Europe and from the cross-country analysis reviewed above pose two important questions: What explains the existence of relatively high mean domestic debt ratios exposed to small but positive risk of default on average? And, is wealth inequality an important determinant of domestic debt exposed to default risk?[3]

This paper aims to answer these questions by proposing a framework for explaining domestic sovereign defaults driven by distributional incentives. This framework is motivated by the key fact that a domestic default entails substantial redistribution across domestic agents, with all of these agents presumably entering in the payoff function of the sovereign. This is in sharp contrast with what standard models of external sovereign default assume, particularly those based on the classic work of Eaton and Gersovitz (1981). Models in this class approach default as a decision made by a government with a payoff given by the utility of a representative home agent, and assuming risk-neutral foreign lenders who are disconnected from the economy they lend to (except for the fact that they bought the home government's debt taking a risk neutral bet on the possibility of default). In these models, the effects of a government default on the welfare of creditors are irrelevant for the sovereign making the default decision, and both the costs and benefits of default affect all domestic agents in the same way (since the economy is

[3]A third important question relates to explaining the transitional dynamics leading to a debt crisis (i.e. the rapid increases in debt and spreads). In this paper, however, we will study a two-period economy which is not suitable for addressing this issue.

inhabited by a representative agent).

These observations suggest that standard models of sovereign default face serious limitations in explaining the forgotten history of domestic debt. In actual domestic defaults and in the European crisis, governments and institutions making default decisions take into account the implications of the default choice for the welfare of government creditors, and evaluate the different costs and benefits of default on various groups of domestic agents. Hence, a theoretical framework aiming to explain domestic default needs to reformulate the government's strategic incentives so as to take into account default effects on both creditors and debtors, which in turn implies that agent heterogeneity also needs to be taken into account.

We propose an analytically tractable two-period model with heterogeneous agents and non-insurable aggregate risk in which domestic default can be optimal for a government that responds to distributional incentives. A fraction γ of agents start as low-wealth (L) agents and a fraction $1-\gamma$ are high-wealth (H) agents, depending on the size of their initial holdings of government bonds. The government finances the gap between exogenous stochastic expenditures and endogenous lump-sum taxes by issuing non-state-contingent debt, retaining the option to default. In the first environment that we study, the government has a utilitarian social welfare function, so it assigns the weights γ and $1-\gamma$ to the welfare of L and H agents respectively. We also study a second environment, in which the government's payoff function has ad-hoc weights that are biased in favor of one group of agents or the other, which can be justified by political economy considerations.

Private agents choose optimally their bond holdings in the first period, taking as given bond prices and the probability that a default may occur in the second period. The government chooses how much debt to issue taking into account its inability to commit to repay: First, it evaluates the agents' payoffs under repayment and default given their optimal savings plans and the government budget constraints. Second, it uses those payoffs to formulate a default decision rule for the second period that depends on how much debt is issued, the realization of government expenditures, and the degree of wealth concentration as measured by γ. Third, it chooses optimally how much debt to issue in the first period to maximize its lifetime social welfare function internalizing how the debt choice affects default incentives, default risk and the price of bonds.

In this model, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Default is optimal when the taxes needed to repay the debt hurt relatively poor agents more than defaulting hurts relatively rich agents, and this happens when, for a given amount of debt and wealth concentration, the realization of government expenditures is high enough. This is necessary but not sufficient, however, for the model to support an equilibrium with debt subject to default risk. A second necessary condition is that the debt market in the first period can be sustained. This requires that the government finds it optimal to sell debt that is left exposed to default risk in the second period, even after internalizing this risk and the response of the equilibrium price of bonds to the amount of debt issued, and that private agents are willing to buy that debt at the market price.

If the government is a utilitarian social planer and default is costless, the model cannot support an equilibrium with debt. This is because for any given level of debt that could have been issued in the first period, the government always attains the second period's socially efficient levels of consumption allocations and redistribution by choosing to default, and if default at period two is certain the debt market collapses in the first period.

An equilibrium with debt under a utilitarian social welfare function exists if default entails an exogenous cost in terms of disposable income. When default is costly, if the amount of period-two consumption dispersion that the competitive equilibrium with repayment supports yields higher welfare than the default equilibrium net of default cost, repayment becomes optimal. In the variant of the model with political bias in the government's payoff function, debt can be supported even without default costs, if the government's weight on H-type agents is higher than the actual fraction of these agents in the wealth distribution.

Quantitative results based on a calibration to European data are used to illustrate the model's key predictions. The model with utilitarian social welfare displays bond prices and default risk that are increasing in the level of wealth concentration. Hence, lower public debt is sustainable as γ rises. Because of default risk, the sustainable debt is significantly lower than what the same model supports at the same levels of wealth inequality and with the same government expenditure shocks but assuming that the government commits to repay (i.e. in the absence of default risk). Most of the optimally chosen debt positions over a wide range of values of γ are exposed to a positive probability of default, except the ones chosen when γ is very low, which have zero default risk. Debt is convex and decreasing in the level of wealth inequality. In the range of empirically relevant ratios of the fraction of agents who own government bonds in Europe, the model supports debt ratios about 1/3rd of the median European debt ratio at spreads close to 40 basis points and default probabilities of about 0.9 percent. Qualitatively these results are robust to changes in the initial levels of government debt and expenditures and the calibration of default costs, but quantitatively they vary.

In the model with political bias in the social welfare function, for given government weights pinning down its preference for redistribution, the debt is an *increasing* function of observed wealth inequality, instead of *decreasing* as in the utilitarian case. This is because the incentives to default get weaker as the government's weight on L-type agents falls increasingly below γ.

The lower the weight capturing the government's preference for redistribution, the higher the debt that can be supported at every value of γ. These debt amounts can easily exceed those supported in the utilitarian case by wide margins, and can be of similar magnitude as the European median with spreads again around 40 basis points and default probabilities around 1 percent.

This work is related to various strands of the large literature on public debt. First, studies on public debt as a self-insurance mechanism and a vehicle that alters consumption dispersion in heterogeneous agents models without default (e.g. Aiyagari and McGrattan (1998), Golosov and Sargent (2012)). Second, the literature on external sovereign default in the line of the Eaton-Gersovitz model (e.g. Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2012), Yue (2010)) but with the important differences noted earlier.[4] Third, another important strand of the external default literature that focuses on the role of secondary markets and discriminatory v. nondiscriminatory default (e.g. Broner, Martin and Ventura (2010) and Gennaioli, Martin and Rossi (2013)).[5] Fourth, the literature on political economy and sovereign default, which also largely focuses on external default (e.g. Amador (2003), Dixit and Londregan (2000), D'Erasmo (2011) Guembel and Sussman (2009), Hatchondo, Martinez and Sapriza (2009) and Tabellini (1991)), but includes studies like those of Alesina and Tabellini (1990) and Aghion and Bolton (1990) that focus on political economy aspects of government debt in a closed economy, and the work of Aguiar, Amador, Farhi and Gopinath (2013) on optimal policy in a monetary union subject to self-fulfilling debt crises.

The rest of this paper is organized as follows: Section 2 describes the payoff functions and budget constraints of households and government. Section 3 characterizes the model's equilibrium and provides an illustration of the mechanism that drives optimal default as a policy for redistribution. Section 4 presents the benchmark calibration and the quantitative results for the utilitarian social welfare function. Section 5 discusses the results of a sensitivity analysis and the political economy extension. The last Section provides conclusions.

2 Economy with Utilitarian Social Welfare

Consider a two-period economy inhabited by a continuum of agents with aggregate unit measure. Agents differ in their initial wealth position, which is characterized by their holdings of

[4] See also Panizza, Sturzenegger and Zettelmeyer (2009), Aguiar and Amador (2013), and Wright (2013) for detailed reviews of the sovereign debt literature.

[5] Default in our setup is also non-discriminatory, because the government cannot discriminate across different types of agents when it defaults. Our setup differs in that the default decision is driven by the distribution of debt among domestic agents and the incentives of the government to use debt optimally as redistributive policy.

government debt at the beginning of the first period. This initial distribution of wealth is exogenous, but the distribution at the beginning of the second period is endogenously determined by the agents' savings choices of the first period. The government is represented by a social planner with a utilitarian payoff who issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents. There is explicit aggregate risk in the form of shocks to government outlays, and also implicit in the form of default risk, and there is no idiosyncratic uncertainty.

2.1 Household Preferences & Budget Constraints

All agents have the same preferences, which are given by:

$$u(c_0) + \beta E[u(c_1)], \qquad u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0, 1)$ is the discount factor and c_t for $t = 0, 1$ is individual consumption. The utility function $u(\cdot)$ takes the standard CRRA form.

All agents receive a non-stochastic endowment y each period and pay lump-sum taxes τ_t, which are uniform across agents. Taxes and newly issued government debt are used to pay for government consumption g_t and repayment of outstanding government debt. The initial supply of outstanding government bonds at $t = 0$ is denoted B_0. Given B_0, the initial wealth distribution is defined by a fraction γ of households who are the L-type individuals with initial bond holdings b_0^L, and a fraction $(1-\gamma)$ who are the H-types and hold b_0^H, where $b_0^H = \frac{B_0 - \gamma b_0^L}{1-\gamma} \geq b_0^L \geq 0$. Hence, b_0^H is the amount consistent with market-clearing in the government bond market at $t = 0$.

The budget constraints of the two types of households in the first period are given by:

$$c_0^i + q_0 b_1^i = y + b_0^i - \tau_0 \quad \text{for } i = L, H. \tag{1}$$

Agents collect the payout on their initial holdings of government debt (b_0^i), receive endowment income y, and pay lump-sum taxes τ_0. This net-of-tax resources are used to pay for consumption and purchases of new government bonds b_1^i. Agents are not allowed to take short positions in government bonds, which is equivalent to assuming that bond purchases must satisfy the no-borrowing condition often used in heterogeneous-agents models: $b_1^i \geq 0$.

The budget constraints in the second period differ depending on whether the government defaults or not. If the government repays, the budget constraints take the standard form:

$$c_1^i = y + b_1^i - \tau_1 \quad \text{for } i = L, H. \tag{2}$$

If the government defaults, there is no repayment on the outstanding debt, and the agents' budget constraints are:

$$c_1^i = (1 - \phi(g_1))y - \tau_1 \quad \text{for } i = L, H. \tag{3}$$

As is standard in the sovereign debt literature, we can allow for default to impose an exogenous cost that reduces income by a fraction ϕ. This cost is often modeled as a function of the realization of a stochastic endowment income, but since income is constant in this setup, we model it as a function of the realization of government expenditures in the second period g_1. In particular, the cost is a non-increasing, step-wise function: $\phi(g_1) \geq 0$, with $\phi'(g_1) \leq 0$ for $g_1 \leq \overline{g}_1$, $\phi'(g_1) = 0$ otherwise, and $\phi''(g_1) = 0$. Hence, $\overline{g}_1$ is a threshold high value of g_1 above which the marginal cost of default is zero. This formulation is analogous to the step-wise default cost as a function of income proposed by Arellano (2008) and now widely used in the external default literature, and it also captures the idea of asymmetric costs of tax collection (see Barro (1979) and Calvo (1988)). Note, however, that for the model to support equilibria with debt under a utilitarian government all we need is $\phi(g_1) > 0$. The additional structure is useful for the quantitative analysis and for making it easier to compare the model with the standard external default models.[6]

2.2 Government

At the beginning of $t = 0$, the government has outstanding debt B_0 and can issue one-period, non-state contingent discount bonds $B_1 \in \mathcal{B} \equiv [0, \infty)$ at the price $q_0 \geq 0$. Each period it collects lump-sum revenues τ_t and pays for g_t. Since g_0 is known at the beginning of the first period, the relevant uncertainty with respect to government expenditures is for g_1, which is characterized by a well-defined probability distribution function with mean μ_g. We do not restrict the sign of τ_t, so $\tau_t < 0$ represents lump-sum transfers.

At equilibrium, the price of debt issued in the first period must be such that the government bond market clears:

$$B_1 = \gamma b_1^L + (1 - \gamma) b_1^H \quad \text{for } t = 0, 1. \tag{4}$$

This condition is satisfied by construction in period 0. In period 1, however, the price moves

[6]In external default models, the non-linear cost makes default more costly in "good" states, which alters default incentives to make default more frequent in "bad" states, and it also contributes to support higher debt levels.

endogenously to clear the market.

The government has the option to default at $t = 1$. The default decision is denoted by $d_1 \in \{0, 1\}$ where $d_1 = 0$ implies repayment. The government evaluates the values of repayment and default as a benevolent planner with a social welfare function. In the rest of this Section we focus on the case of a standard weighted utilitarian social welfare function: $\gamma u(c_1^L) + (1-\gamma) u(c_1^H)$. In Section 5 we consider an alternative formulation with arbitrary weights, which can be justified by political economy considerations, and the analysis could also be extended to incorporate egalitarian concerns. The government, however, cannot discriminate across the two types of agents when setting taxation, debt and default policies.

At $t = 0$, the government budget constraint is

$$\tau_0 = g_0 + B_0 - q_0 B_1. \tag{5}$$

The level of taxes in period 1 is determined after the default decision. If the government repays, taxes are set to satisfy the following government budget constraint:

$$\tau_1^{d_1=0} = g_1 + B_1. \tag{6}$$

Notice that, since this is a two-period model, equilibrium requires that there are no outstanding assets at the end of period 1 (i.e. $b_2^i = B_2 = 0$ and $q_1 = 0$). If the government defaults, taxes are simply set to pay for government purchases:

$$\tau_1^{d_1=1} = g_1. \tag{7}$$

3 Equilibrium

The analysis of the model's equilibrium proceeds in three stages. First, we characterize the households' optimal savings problem and determine their payoff (or value) functions, taking as given the government debt, taxes and default decision. Second, we study how optimal government taxes and the default decision are determined. Third, we examine the optimal choice of debt issuance that internalizes the outcomes of the first two stages. We characterize these problems as functions of B_1, g_1, and γ, keeping the initial conditions (g_0, B_0, b_0^L) as exogenous parameters. Hence, we can index the value of a household as of $t = 0$, before g_1 is realized, as a function of the pair $\{B_1, \gamma\}$. Given this, the level of taxes τ_0 is determined by the government budget constraint once the equilibrium bond price q_0 is set. Bond prices are forward looking and depend on the default decision of the government in period 1, which will be given by the decision rule $d(B_1, g_1, \gamma)$.

3.1 Households' Problem

Given B_1 and γ, a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$\begin{aligned} v^i(B_1,\gamma) &= \max_{b_1^i} \Big\{ u(y + b_0^i - q_0 b_1^i - \tau_0) + \\ &\quad \beta E_{g_1}\Big[(1 - d_1(B_1, g_1, \gamma)) u(y + b_1^i - \tau_1^{d_1=0}) + d_1(B_1, g_1, \gamma) u(y(1 - \phi(g_1)) - \tau_1^{d_1=1}) \Big] \Big\}, \end{aligned} \tag{8}$$

subject to $b_1^i \geq 0$. The term $E_{g_1}[.]$ represents the expected payoff across the repayment and default states in period 1. Notice in particular that the payoff in case of default does not depend on the level of individual debt holdings (b_1^i), reflecting the fact that the government cannot discriminate across households when it defaults.

A key feature of the above optimization problem is that agents take into account the possibility of default in formulating their optimal choice of bond holdings. The first-order condition, evaluated at the equilibrium level of taxes, yields the following Euler equation:

$$u'(c_0^i) \geq \beta(1/q_0) E_{g_1}\left[u'(y - g_1 + b_1^i - B_1)(1 - d_1(B_1, g_1, \gamma)) \right], \; = \; \text{if} \;\; b_1^i > 0 \tag{9}$$

In states in which, given (B_1, γ), the value of g_1 is such that the government chooses to default ($d_1(B_1, g_1, \gamma) = 1$), the marginal benefit of an extra unit of debt is zero.[7] Thus, conditional on B_1, a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the expected marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes. This has important redistributive implications, because it implies that when choosing the optimal debt issuance, the government will internalize how by altering the bond supply it can affect the expected probability of default and the equilibrium bond prices. Note also that from the households' perspective, the individual bond decision has no marginal effect on $d_1(B_1, g_1, \gamma)$.

The agents' Euler equation has three other important implications: First, the premium over a world risk-free rate (defined as q_0/β, where $1/\beta$ can be viewed as a hypothetical opportunity cost of funds for an investor, analogous to the role played by the world interest rate in the standard external default model) generally differs from the default probability for two reasons: (a) because the agents are risk averse, instead of risk-neutral as in the standard model, and (b) because in the

[7]Utility in the case of default equals $u(y(1 - \phi(g_1)) - g_1)$, and is independent of b_1^i.

repayment state agents face higher taxes, whereas in the standard model investors are not taxed to repay the debt. For agents with positive bond holdings, the above optimality condition implies that the premium over the risk-free rate is $E_{g_1}\left[u'(y-g_1+b_1^i-B_1)(1-d_1(B_1,g_1,\gamma))/u'(c_0^i)\right]$.[8] Second, if the Euler equation holds with equality for $H-$type agents (i.e. $b_1^H>0$) then $L-$type agents are *credit constrained*. This implies that $H-$type agents are the marginal investor and their Euler equation can be used to derive the equilibrium price. Third, as we confirm numerically in Section 4, there are sufficiently high threshold values of (B_1,γ) such the government chooses $d_1(B_1,g_1,\gamma)=1$ for all g_1. In these cases, the expected marginal benefit of purchasing government bonds vanishes from the agents' Euler equation, and hence the equilibrium for that B_1 does not exist, since agents would not be willing to buy debt at any finite price.[9]

The equilibrium bond price is the value of $q_0(B_1,\gamma)$ for which, as long as consumption for all agents is non-negative and the default probability of the government is less than 1, the following market-clearing condition holds:

$$B_1=\gamma b_1^L(B_1,\gamma)+(1-\gamma)b_1^H(B_1,\gamma), \tag{10}$$

where B_1 in the left-hand-side of this expression represents the public bonds supply, and the right-hand-side is the aggregate government bond demand. If $b_0^L=0$ and $b_1^H>0$, the Euler equation for the $L-$types implies that $b_1^L=0$ (since they are credit constrained), and hence the market clearing condition becomes

$$B_1=(1-\gamma)b_1^H(B_1,\gamma). \tag{11}$$

It is instructive to analyze further the households' problem assuming logarithmic utility ($u(c)=\log(c)$) and $b_0^L=0$, because under these assumptions we can use Euler equation (9) and the bond market-clearing condition to solve for the "equilibrium" bond price in closed form, and use the solution to establish some important properties of bond prices and the associated default risk spreads. Note, however, that this is an equilibrium price only in the sense that it clears the bond market at date 0 for a given supply of debt, not in the sense of the model's full equilibrium as defined later in this Section.

Assuming log utility and $b_0^L=0$, we show in the Appendix that the equilibrium bond price

[8]With log utility, the debt pricing function with default risk provided in the Appendix can be used to show that this premium starts *lower* than the default probability at low default probabilities, and eventually grows much larger as the probability of default approaches 1.

[9]This result is reminiscent of a similar result in standard models of external default, in which rationing emerges at t for debt levels so high that the government would choose default at all possible income realizations in $t+1$.

is:

$$q_0(B_1,\gamma) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \Pi(B_1,\gamma)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \Pi(B_1,\gamma),} \tag{12}$$

where $\Pi(B_1,\gamma) \equiv E_{g_1}\left[\frac{1-d(B_1,g_1,\gamma)}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1}\right]$ is the expected marginal utility of $H-$type agents for the second period, which as explained earlier weights only non-default states because the marginal benefit of debt is zero in default states. Since, as also shown in the Appendix, $\frac{\partial \Pi(B_1,\gamma)}{\partial B_1} < 0$, it follows that $\frac{\partial q_0(B_1,\gamma)}{\partial B_1} < 0$ for $c_0^H > 0$. Moreover, since bond prices are decreasing in B_1, it follows that the "revenue" the government generates by selling debt, $q_0(B_1,\gamma)B_1$, behaves like the familiar *debt Laffer curve* of the Eaton-Gersovitz models derived by Arellano (2008).[10] This Laffer curve will play a key role later in determining the government's optimal debt choice. In particular, the government internalizes that higher debt eventually produces decreasing revenues, and that in the decreasing segment of the Laffer curve revenues fall faster as the debt increases, and much faster as default risk rises sharply.

The above equilibrium price of debt can be easily compared with the price that would arise in a model with full commitment. In particular, if the government is committed to repay, the equilibrium price is

$$q_0^{ND}(B_1,\gamma) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \Pi^{ND}(B_1,\gamma)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \Pi^{ND}(B_1,\gamma)} \tag{13}$$

where $\Pi^{ND}(B_1,\gamma) \equiv E_{g_1}\left[\frac{1}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1}\right]$. Since $\Pi^{ND}(B_1,\gamma) \geq \Pi(B_1,\gamma)$ it follows that $q_0^{ND}(B_1,\gamma) \geq q_0(B_1,\gamma)$. Hence, for given (B_1,γ) the equilibrium price without default risk is the upper bound of the price with risk of default, and as a result the risk premium is a non-negative, non-linear function of B_1. More specifically, if we define the default risk spread as $S(B_1,\gamma) \equiv [1/q(B_1,\gamma)] - [1/q^{ND}(B_1,\gamma)]$, the spread reduces to the following expression:

$$S(B_1,\gamma) = \left(\frac{1}{\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right)}\right)\left[\frac{1}{\Pi(B_1,\gamma)} - \frac{1}{\Pi^{ND}(B_1,\gamma)}\right] \tag{14}$$

[10] It is straightforward to show that revenue $R(B_1) = q_0(B_1,\gamma)B_1$ follows a Laffer curve in the $[0, B_1^{max}]$ interval, where B_1^{max} is the upper bound of debt such that the government chooses default for any realization of g_1 and thus $q_0(B_1^{max},\gamma) = 0$. Since $R(0) = 0$ with $R'(0) = q(0,\gamma) > 0$, and $R(B_1^{max}) = 0$ with $R'(B_1^{max}) = q_0'(B_1^{max},\gamma)B_1^{max} < 0$, it follows by Rolle's theorem that $R(B_1)$ has at least one local maximum in $(0, B_1^{max})$.

Clearly, since $\Pi(B_1,\gamma) \leq \Pi^{ND}(B_1,\gamma)$ the spread is non-negative, and it is strictly positive if there is default at equilibrium. The spread is increasing in B_1, because as the debt rises default is chosen optimally in more of the possible realizations of g_1 and hence $\Pi(B_1,\gamma)$ falls further below $\Pi^{ND}(B_1,\gamma)$, so that the gap between the reciprocals of these two terms widens. Note also that the spread is a multiple of the gap between these reciprocals, with the multiple given by $1/\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right)$. As a result, the total date-0 resources available for consumption of the H-types $(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0)$ have a first-order *negative* effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. The are also second order effects, because the equilibrium allocation of B_1 also depends on that income measure, and thus $\Pi(B_1,\gamma)$ and $\Pi^{ND}(B_1,\gamma)$ vary with it as well, but these are not considered here.

In terms of the effect of changes in γ on $S(B_1,\gamma)$, notice that there are two effects. First, there is a negative *demand composition* effect, because higher γ means that, for a given B_0, the resources available for date-0 consumption of H types increase, since fewer H type agents need to demand enough initial bonds to clear the bond market, which means that per-capita each of the H types hold more date-0 bonds and have more bond income. Second, there is a positive effect because rising γ strengthens default incentives as the welfare of the wealthy is valued less, and hence default is optimally chosen in more states, which increases $\left[1/\Pi^D(B_1,\gamma) - 1/\Pi(B_1,\gamma)\right]$. Thus, in principle the response of the spread to increases in inequality is ambiguous. The weaker the response of the default probability to changes in γ, however, the more likely it is that the first effect will dominate and the spreads will be a decreasing function of inequality.

3.2 Government's Problem

As explained earlier, we analyze the government's problem following a backward induction strategy by studying first the default decision problem in the final period $t = 1$, followed by the optimal debt issuance choice at $t = 0$.

3.2.1 Government Default Decision at $t = 1$

At $t = 1$, the government chooses to default or not by solving this optimization problem:

$$\max_{d\in\{0,1\}} \left\{W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1, \gamma)\right\}, \tag{15}$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(B_1, g_1, \gamma)$ denote the values of the social welfare function at the beginning of period 1 in the case of repayment and default respectively. Using the government budget constraint to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government's utilitarian payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1) + (1-\gamma)u(y - g_1 + b_1^H - B_1) \tag{16}$$

and

$$W_1^{d=1}(g_1, \gamma) = u(y(1-\phi(g_1)) - g_1). \tag{17}$$

Combining these payoff functions, if follows that the government defaults if this condition holds:

$$\begin{aligned} &\gamma \left[u(y - g_1 + \overbrace{(b_1^L - B_1)}^{\leq 0}) - u(y(1-\phi(g_1)) - g_1) \right] + \\ &(1-\gamma) \left[u(y - g_1 + \overbrace{(b_1^H - B_1)}^{\geq 0}) - u(y(1-\phi(g_1)) - g_1) \right] \leq 0 \end{aligned} \tag{18}$$

Notice that all households lose g_1 of their income to government absorption regardless of the default choice. Moreover, debt repayment reduces consumption and welfare of L types and rises them for H types, whereas default implies the same consumption and utility for both types of agents.

The distributional effects of a default are implicit in condition (18). Given that debt repayment affects the cash-in-hand for consumption of low- and high-wealth agents according to $(b_1^L - B_1) \leq 0$ and $(b_1^H - B_1) \geq 0$ respectively, it follows that, for a given B_1, the payoff under repayment allocates (weakly) lower welfare to L agents and higher to H agents, and that the gap between the two is larger the larger is B_1. Moreover, since the default payoffs are the same for both types of agents, this is also true of the *difference* in welfare under repayment v. default: It is higher for H agents than for L agents and it gets larger as B_1 rises. To induce default, however, it is necessary not only that L agents have a smaller difference in the payoffs of repayment v. default, but that the difference is negative (i.e. they must attain lower welfare under repayment than under default), which requires $B_1 > b_1^L + y\phi(g_1)$. This also implies that taxes under repayment need to be necessarily larger than under default, since $\tau_1^{d=0} - \tau_1^{d=1} = B_1$.

Since we can re-write the consumption allocations under repayment as $c_1^L = y - \tau_1^{d=0} + b_1^L$ and $c_1^H = y - \tau_1^{d=0} + b_1^H$, the distributional effects of default can also be interpreted in terms of how the changes in taxes and wealth caused by a default affect each agent's consumption (and hence utility). First, since $b_1^H > b_1^L$, default has a larger effect on the net worth of H agents than L agents (or no effect on the latter if $b_1^L = 0$), thus reducing the welfare of the former more than the latter. Second, with regard to taxes, we established above that for default incentives to make default optimal it is necessary that $\tau_1^{d=0} > \tau_1^{d=1}$. This still has distributional implications, because, even tough both types of agents face the same tax, marginal utility is higher for L agents, and thus they suffer more if taxes rise under repayment. Since repayment requires higher taxes than default, default is always preferable than repayment for L agents.

The distribution of wealth determines the weights the utilitarian planner assigns to the gains and losses that default imposes on the different agents. As γ increases, the fraction of L agents is larger, and thus the value of repayment for the government decreases because it weights more the welfare loss that L agents endure under repayment. Note that differences in γ also affect the date-0 bond decision rules b_1^L and b_1^H and hence the market price of bonds q_0, even for an unchanged supply of bonds B_1.

The distributional mechanism determining the default decision can be illustrated more clearly by means of a graphical tool that compares the utility levels associated with the consumption allocations of the default and repayment states with those that would be generally efficient. To this end, it is helpful to express the values of optimal debt holdings as $b_1^L = B_1 - \epsilon$ and $b_1^H(\gamma) = B_1 + \frac{\gamma}{1-\gamma}\epsilon$, for some hypothetical decentralized allocation of debt holdings given by $\epsilon \in [0, B_1]$. Consumption allocations under repayment would therefore be $c_1^L(\epsilon) = y - g_1 - \epsilon$ and $c_1^H(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon$, so ϵ also determines the decentralized consumption dispersion.

The efficient dispersion of consumption that the social planner would choose is characterized by the value of ϵ^{SP} that maximizes social welfare under repayment, which satisfies this first-order condition:

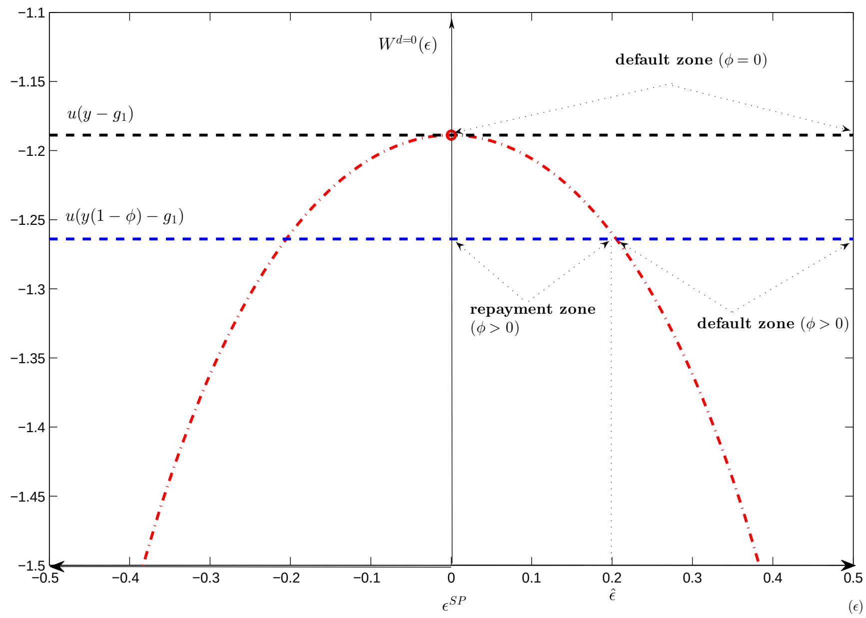
$$u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right) = u'\left(y - g_1 - \epsilon^{SP}\right). \tag{19}$$

Hence, the efficient allocations are characterized by zero consumption dispersion, because equal marginal utilities imply $c^{L,SP} = c^{H,SP} = y - g_1$, which is attained with $\epsilon^{SP} = 0$.

Consider now the government's default decision when default is costless ($\phi(g_1) = 0$). Given that the only policy instruments the government can use, other than the default decision, are non-state contingent debt and lump-sum taxes, it is straightforward to conclude that default will always be optimal. This is because default produces identical allocations in a decentralized equilibrium as the socially efficient ones, since default produces zero consumption dispersion with consumption levels $c^L = c^H = y - g_1$. This outcome is invariant to the values of B_1, g_1, γ and ϵ (over their relevant ranges). Moreover, in this scenario default also yields the first-best outcome that attains maximum social welfare. This result also implies, however, that the model without default costs cannot support equilibria with domestic debt subject to default risk, because default is always optimal.

The above scenario is depicted in Figure 2, which plots the social welfare function under repayment as a function of ϵ as the bell-shaped curve, and the social welfare under default (which is independent of ϵ), as the black dashed line. Clearly, the maximum welfare under repayment is attained when $\epsilon = 0$ which is also the efficient amount of consumption dispersion ϵ^{SP}. Recall also that we defined the relevant range of decentralized consumption dispersion for $\epsilon > 0$, so welfare under repayment is decreasing in ϵ over the relevant range.

Figure 2: Default Decision and Consumption Dispersion



These results can be summarized as follows:

Result 1. *If $\phi(g_1) = 0$ for all g_1, then for any $\gamma \in (0,1)$ and any (B_1, g_1), the social value of repayment $W^{d=0}(B_1, g_1, \gamma)$ is decreasing in ϵ and attains its maximum at $\epsilon^{SP} = 0$ (i.e. when welfare equals $u(y - g_1)$). Hence, default is always optimal for any decentralized consumption dispersion $\epsilon > 0$, .*

The outcome is very different when default is costly. With $\phi(g_1) > 0$, default still yields zero consumption dispersion, but at lower levels of consumption and therefore utility, since consumption allocations in the default state become $c^L = c^H = (1 - \phi(g_1))y - g_1$. This does not alter the result that the first-best social optimum is $\epsilon^{SP} = 0$, but what changes is that default can no longer support the consumption allocations of the first best. Hence, there is now a threshold amount of consumption dispersion in the decentralized equilibrium, $\widehat{\epsilon}(\gamma)$, which varies with γ and such that for $\epsilon \geq \widehat{\epsilon}(\gamma)$ default is again optimal, but for lower ϵ repayment is now optimal. This is because when ϵ is below the threshold, repayment produces a level of social welfare higher than the one that default yields.

Figure 2 also illustrates this scenario. The default cost lowers the common level of utility of both types of agents, and hence of social welfare, in the default state (shown in the Figure as the blue dashed line), and $\widehat{\epsilon}(\gamma)$ is determined where the social welfare under repayment intersects social welfare under default. If the decentralized consumption dispersion with the debt market functioning (ϵ) is between 0 and less than $\widehat{\epsilon}(\gamma)$ then the government finds it optimal to repay. Intuitively, if dispersion is not too large, the government prefers to repay rather than default since the latter reduces the dispersion of consumption but imposes an income cost on households. Moreover, as γ increases the domain of $W_1^{d=0}$ narrows, and thus $\widehat{\epsilon}(\gamma)$ falls and the interval of decentralized consumption dispersions that supports repayment narrows. This is natural because a higher γ causes the planner to weight more L-types in the social welfare function, which are agents with weakly lower utility in the repayment state.

These results can be summarized as follows:

Result 2. *If $\phi(g_1) > 0$, then for any $\gamma \in (0,1)$ and any (B_1, g_1), there is a threshold value of consumption dispersion $\widehat{\epsilon}(\gamma)$ such that the payoffs of repayment and default are equal: $W^{d=0}(B_1, g_1, \gamma) = u(y(1-\phi(g_1)) - g_1)$. The government repays if $\epsilon < \widehat{\epsilon}(\gamma)$ and defaults otherwise. Moreover, $\widehat{\epsilon}(\gamma)$ is decreasing in γ.*

3.2.2 Government Debt Decision at $t = 0$

We are now in a position to study how the government chooses the optimal amount of debt to issue in the initial period. Before studying the government's optimization problem, it is important to emphasize that in this model debt is mainly a mechanism for altering consumption dispersion across agents both within a period and across periods. In particular, consumption dispersion in each period and repayment state is given by:

$$\begin{aligned}
c_0^H - c_0^L &= \frac{1}{1-\gamma}\left[B_0 - q(B_1,\gamma)B_1\right] \\
c_1^{H,d=0} - c_1^{L,d=0} &= \frac{1}{1-\gamma}B_1 \\
c_1^{H,d=1} - c_1^{L,d=1} &= 0.
\end{aligned}$$

These expressions make it clear that, given B_0, issuing at least some debt ($B_1 > 0$) reduces consumption dispersion at $t = 0$ compared with no debt ($B_1 = 0$), but increases it at $t = 1$ if the government repays (*i.e.*, $d = 0$). Moreover, the debt Laffer curve that governs $q_0(B_1, \gamma)B_1$ limits the extent to which debt can reduce consumption dispersion at $t = 0$. Starting from $B_1 = 0$, consumption dispersion in the initial period falls as B_1 increases, but there is a critical positive value of B_1 beyond which it becomes an increasing function of debt.

At $t = 0$, the government chooses its debt policy internalizing the above effects, including the dependence of bond prices on the debt issuance choice, and their implications for social welfare. To be precise, the government chooses B_1 so as to maximize the "indirect" social welfare function:

$$W_0(\gamma) = \max_{B_1} \left\{ \gamma v^L(B_1, \gamma) + (1-\gamma) v^H(B_1, \gamma) \right\}. \tag{20}$$

where v^L and v^H are the value functions obtained from solving the households' problems defined in the Bellman equation (9) taking into account the government budget constraints and the equilibrium pricing function of bonds.

We can gain some intuition about the solution of this maximization problem by deriving its first-order condition and re-arranging it as follows (assuming that the relevant functions are differentiable):

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta}{q(B_1, \gamma)\gamma} \left\{ \beta E_{g_1} \left[\Delta d \Delta W_1 \right] + \gamma \mu^L \right\} \tag{21}$$

where

$$\begin{aligned}
\eta &\equiv q(B_1, \gamma) / \left(q'(B_1, \gamma) B_1 \right) < 0, \\
\Delta d &\equiv d(B_1 + \delta, g_1, \gamma) - d(B_1, g_1, \gamma) \geq 0, \quad \text{for } \delta > 0 \text{ small}, \\
\Delta W_1 &\equiv W_1^{d=1}(g_1, \gamma) - W_1^{d=0}(B_1, g_1, \gamma) \geq 0, \\
\mu^L &\equiv q(B_1, \gamma) u'(c_0^L) - \beta E_{g_1} \left[(1 - d^1) u'(c_1^L) \right] > 0.
\end{aligned}$$

In these expressions, η is the price elasticity of the demand for government bonds, $\Delta d \Delta W_1$ represents the marginal distributional benefit of a default, and μ^L is the shadow value of the borrowing constraint faced by L-type agents.

If both types of agents could be unconstrained in their savings decisions, so that in particular $\mu_L = 0$, and if there is no change in the risk of default (or assuming commitment to remove default risk entirely), so that $E_{g_1} \left[\Delta d \Delta W_1 \right] = 0$, then the optimality condition simplifies to:

$$u'(c_0^H) = u'(c_0^L).$$

Hence, in this case the social planner would want to issue debt so as to equalize marginal utilities of consumption across agents at date 0, which requires simply setting B_1 to satisfy $q(B_1, \gamma) B_1 = B_0$.

As explained earlier, the above scenario cannot be sustained as an equilibrium outcome, because at equilibrium if H-type agents are unconstrained, then L-types are constrained (i.e. $\mu_L > 0$). In this case, and still assuming no change in default risk or a government committed to repay, the optimality condition reduces to:

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta\mu^L}{q(B_1,\gamma)}.$$

Since $\eta < 0$, this result implies $c_0^L < c_0^H$, because $u'(c_0^L) > u'(c_0^H)$. Thus, even with unchanged default risk or no default risk at all, the government's debt choice sets B_1 as needed to maintain an optimal, positive level of consumption dispersion, which is the one that supports an excess in marginal utility of L-type agents relative to H-type agents equal to $\frac{\eta\mu^L}{q(B_1,\gamma)}$. Moreover, since optimal consumption dispersion is positive, we can also ascertain that $B_0 > q(B_1,\gamma)B_1$, which using the government budget constraint implies that the government runs a primary surplus at $t = 0$. The government borrows resources, but less than it would need in order to eliminate all consumption dispersion (which requires zero primary balance).

The intuition for the optimality of issuing debt can be presented in terms of tax smoothing and savings: Date-0 consumption dispersion without debt issuance would be $B_0/(1-\gamma)$, but this is more dispersion than what the government finds optimal, because by choosing $B_1 > 0$ the government provides tax smoothing (i.e. reduces date-0 taxes) for everyone, which in particular eases the L-type agents credit constraint, and provides also a desired vehicle of savings for H types. Thus, positive debt increases consumption of L types (since $c_0^L = y - g_0 - B_0 + q(B_1)B_1$), and reduces consumption of H types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)(B_0 - q(B_1)B_1)$). But issuing debt (assuming repayment) also increases consumption dispersion a $t = 1$, since debt is then paid with higher taxes on all agents, while H agents collect also the debt repayment. Thus, the debt is being chosen optimally to trade off the social costs and benefits of reducing (increasing) date-0 consumption and increasing (reducing) date-1 consumption for rich (poor) agents. In doing so, the government internalizes the debt Laffer curve and the fact that additional debt lowers the price of bonds and helps reduce μ^L, which in turn reduces the government's optimal consumption dispersion.[11]

In the presence of default risk and if default risk changes near the optimal debt choice, the term $E_{g_1}[\Delta d \Delta W_1]$ enters in the government's optimality condition with a positive sign, which means the optimal gap in the date-0 marginal utilities of the two agents widens even more. Hence, the government's optimal choice of consumption dispersion for $t = 0$ is greater than

[11]Note, however, that without default risk the Laffer curve has less curvature than with default risk, because $q_0^{ND}(B_1,\gamma) \geqq_0 (B_1,\gamma)$.

without default risk, and the expected dispersion for $t = 1$ is lower, because in some states of the world the government will choose to default and consumption dispersion would then drop to zero. This also suggests that the government chooses a lower value of B_1 than in the absence of default risk, since date-0 consumptions are further apart. Moreover, the debt Laffer curve now plays a central role in the government's weakened incentives to borrow, because as default risk rises the price of bonds drops to zero faster and the resources available to reduce date-0 consumption dispersion peak at lower debt levels. In short, default risk reduces the government's ability to use non-state-contingent debt in order to reduce consumption dispersion.

In summary, the more constrained the $L-$types agents are (higher μ^L) or the higher the expected distributional benefit of a default (higher $E_{g_1}[\Delta d \Delta W_1]$), the larger the level of debt the government finds optimal to issue. Both of these mechanisms operate as pecuniary externalities: They matter only because the government debt choice can alter the equilibrium price of bonds which is taken as given by private agents.

3.3 Competitive Equilibrium with Optimal Debt & Default Policy

For a given value of γ, a **Competitive Equilibrium with Optimal Debt and Default Policy** is a pair of household value functions $v^i(B_1, \gamma)$ and decision rules $b^i(B_1, \gamma)$ for $i = L, H$, a government bond pricing function $q_0(B_1, \gamma)$ and a set of government policy functions $\tau_0(B_1, \gamma)$, $\tau_1^{d \in \{0,1\}}(B_1, g_1, \gamma)$, $d(B_1, g_1, \gamma)$, $B_1(\gamma)$ such that:

1. Given the pricing function and government policy functions, $v^i(B_1, \gamma)$ and $b_1^i(B_1, \gamma)$ solve the households' problem.

2. $q_0(B_1, \gamma)$ satisfies the market-clearing condition of the bond market (equation (10)).

3. The government default decision $d(B_1, g_1, \gamma)$ solves problem (15).

4. Taxes $\tau_0(B_1, \gamma)$ and $\tau_1^d(B_1, g_1, \gamma)$ are consistent with the government budget constraints.

5. The government debt policy $B_1(\gamma)$ solves problem (20).

4 Quantitative Analysis

In this Section, we study the model's quantitative predictions based on a calibration using European data. The goal is to show whether a reasonable set of parameter values can produce an equilibrium with debt subject to default risk, and to study how the properties of this equilibrium change with the model's key parameters. Since the two-period model is not well suited to account for the time-series dynamics of the data, we see the results more as an illustration of the potential relevance of the model's argument for explaining domestic default rather than as an evaluation of the model's general ability to match observed public debt dynamics.

We solve the model following a similar backward-recursive strategy as in the theoretical analysis. First, taking as given a set of values $\{B_1, \gamma\}$, we solve for the equilibrium pricing and default functions by iterating on (q_0, b_1^i) and the default decision rule d_1 until the date-0 bond market clears when the date-1 default decision rule solves the government's optimal default problem (15). Then, in the second stage we complete the solution of the equilibrium by finding the optimal choice of B_1 that solves the government's date-0 optimization problem (20). It is important to recall that, as explained earlier, for given values of B_1 and γ, an equilibrium with debt will not exist if either the government finds it optimal to default on B_1 for all realizations of g_1 or if at the givenB_1 the consumption of L types is non-positive. In these cases, there is no finite price that can clear the debt market.

4.1 Calibration

The model is calibrated to annual frequency, and most of the parameter values are set so as to approximate some of the model's predicted moments to those observed in the European data. The parameter values that need to be set are the subjective discount factor β, the coefficient of relative risk aversion σ, the stochastic process of government expenditures, the initial levels of government debt and expenditures (B_0 g_0), the level of income y, the fraction of households with low initial wealth γ, their initial wealth b_0^L and the default cost function $\phi(g_1)$. The calibrated parameter values are summarized in Table 3.

Table 3: Model Parameters

Parameter		Value
Discount Factor	β	0.96
Risk Aversion	σ	1.00
Avg. Income	y	0.79
Low household wealth	b_0^L	0.00
Avg. Gov. Consumption	μ_g	0.18
Autocorrel. G	ρ_g	0.88
Std Dev Error	σ_e	0.017
Initial Gov. Debt	B_0	0.79
Output Cost Default	ϕ_0	0.02

Note: Government expenditures, income and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain and Portugal.

The preference parameters are set to standard values: $\beta = 0.96$, $\sigma = 1$. We also assume for simplicity that L types start with zero wealth, $b_0^L = 0$.[12]

The stochastic process of g_1 is formulated as a discrete Markov process that approximates the cross-country averages of AR(1) models estimated with 1995-2012 data of the government expenditures-GDP ratio (in logs) for France, Germany, Greece, Ireland, Italy, Spain and Portugal. This AR(1) process has the standard form:

$$\log(g_{t+1}) = (1 - \rho_g)\log(\mu_g) + \rho_g \log(g_t) + e_t$$

where $|\rho_g| < 1$ and e_t is i.i.d. over time and distributed normally with mean zero and standard deviation σ_e. Given the parameter estimates for each country, we set μ_g, ρ_g and σ_e to the corresponding cross-country average. This results in the following values $\mu_g = 0.1812$, $\rho_g = 0.8802$ and $\sigma_e = 0.017$. Given these moments, we set $g_0 = \mu_g$ and use Tauchen's (1986) quadrature method with 45 nodes in $G_1 \equiv \{\underline{g}_1, \ldots, \overline{g}_1\}$ to generate the realizations and transition probabilities of the Markov process that drives expectations about g_1.[13]

We abstain from setting a calibrated value for γ and instead show results for $\gamma \in [0, 1]$. Note, however, that data for the United States and Europe suggest that the empirically relevant range for γ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of γ is useful we use $\gamma = 0.7$, which is the mid point of the plausible range. In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds but 50.4% have retirement accounts (which are very likely to own government bonds). These figures would suggest values of γ ranging from 50% to 88%. In Europe, comparable statistics are not available for several countries, but recent studies show that the wealth distribution is highly concentrated and that the wealth Gini coefficient ranges between 0.55 and 0.85 depending on the country and the year of the study (see Davies et al. (2009)).[14]

Average income y is calibrated such that the model's aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households' aggregate endowment must equal GDP net of fixed capital investment and net exports, since the latter two are not explicitly modeled. The average for the period 1970-2012 for the same set of countries used to estimate the g_1 process implies $y = 0.7883$. Note also that under this calibration of y and the Markov process of g_1, the gap $y - g_1$ is always positive, even for $g_1 = \overline{g}_1$, which in turn guarantees $c_1^H > 0$ in all repayment states.

[12] $\sigma = 1$ and $b_0^L = 0$ are also useful because, as we show in the Appendix, under these assumptions we can obtain closed-form solutions and establish some results analytically.

[13] As it is standard, we assume that G_1 is an equally spaced grid with $\log(\underline{g}_1)$ and $\log(\overline{g}_1)$ located -/+ 3 standard deviations from $\log(\mu_g)$ respectively.

[14] In our model, if $b_0^L = 0$, the Gini coefficient of wealth is equal to γ.

Setting the initial debt ratio is complicated because in this two-period model the consumption smoothing mechanism induces a reduction in the optimal B_1 relative to the initial condition B_0 even in a deterministic version of the model with stationary government purchases. Under these assumptions, the optimal debt choice is decreasing in γ and has an upper bound of $B_1 = B_0/(1+\beta)$ as $\gamma \to 0$.[15] Moreover, in the model with shocks to g_1 but without default risk (i.e. assuming that the government is committed to repay), the optimal debt choice is still decreasing in γ and has an upper bound lower than $B_0/(1+\beta)$, because of the disposable income risk associated with g_1 shocks. In addition, when default risk is introduced debt needs to be below the level that would lead the government to default in the second period with probability 1, and above the level at which either c_0^L or c_1^H become non-positive, otherwise there is no equilibrium. Given this inertia to reduce debt and restrictions on debt levels, we set $B_0 = 0.79$ so that at the maximum observed level of wealth inequality ($\gamma = 0.85$) there is at least one feasible level of B_1. With B_0 set at 0.79 we also obtain that solving the model without default risk and $\gamma = 0.7$ (the median in the European data of Table 1) the equilibrium debt is $B1 = 0.28$, which is close to the 1990-2007 median public debt ratio also shown in Table 1.[16]

The functional form of the default cost function is the following:

$$\phi(g_1) = \phi_0 + (\overline{g}_1 - g_1)/y$$

where $\overline{g}_1$ is calibrated to represent an "unusually large" realization of g_1 set equal to the largest realization in the Markov process of government expenditures, which is in turn set equal to 3 standard deviations from the mean (in the process in logs). This cost function shares a key feature of the default cost functions widely used in the external default literature to align default incentives so as to support higher debt ratios and trigger default during recessions (see Arellano (2008) and Mendoza and Yue (2012)): The default cost is an *increasing* function of disposable income $(y - g_1)$. In addition, this formulation ensures that households' consumption during a default never goes above a given threshold.

We calibrate ϕ_0 to match an estimate of the observed frequency of domestic defaults. According to Reinhart and Rogoff (2008), historically, domestic defaults are about 1/4 as frequent as external defaults (68 domestic v. 250 external in data since 1750). Since the probability of an external default has been estimated in the range of 3 to 5% (see for example Arellano (2008)), we estimate the probability of a domestic default at about 1%. The model is close to this default

[15]This occurs because as $\gamma \to 0$, the model in deterministic form collapses to a representative agent economy inhabited by H types where the optimal debt choice yields stationary consumption, $q_0 = 1/\beta$, and $B_1 = B_0/(1+\beta)$. In contrast, an infinite horizon, stationary economy yields $B_1 = B_0$ (see Appendix for details).

[16]This is a pre-crisis median that used data up to 2007, the year before the surge in debt and government expenditures associated with the 2008 global financial crisis.

frequency on average when solved over the empirically relevant range of γ's ($\gamma \in [0.55, 0.85]$) if we set $\phi_0 = 0.02$. Note, however, that this calibration of ϕ_0 to target the default probability and the calibration of B_0 to the target described early needs to be done jointly by repeatedly solving the model until both targets are well approximated.

4.2 Results

We examine the quantitative results in the same order in which the backward solution algorithm works. We start with the second period's utility of households under repayment and default, the government's default decision, and the associated tax policy for given ranges of values of B_1, g_1 and γ. We then move to the first period and examine the households' decision rules for demand of government bonds, the equilibrium bond prices and taxes for the same ranges of B_1, g_1 and γ. Finally, we examine solutions of the full competitive equilibrium including the optimal government debt issuance B_1 for a range of values of γ.

4.2.1 Second period default decision and taxes for given (B_1, g_1, γ)

Using the agents' optimal choice of bond holdings, we compute the equilibrium utility levels they attain at $t = 1$ under repayment v. default for different triples (B_1, g_1, γ). The differences in these payoffs are then converted into cardinal measures by computing compensating variations in consumption that equate utility in the two scenarios. This is analogous to the calculations typically done to compute welfare effects in representative agent models. In particular, we compute the individual utility gain of a default $\alpha^i(B_1, g_1, \gamma)$ as the percent increase in consumption that renders an agent $i \in \{L, H\}$ indifferent between the government repayment and default options for different triples (B_1, g_1, γ). Since we are looking at the last period of a two-period model, these compensating variations reduce simply to the percent changes in consumption across the default and no-default states of each agent:[17]

$$\alpha^i(B_1, g_1, \gamma) = \frac{c_1^{i,d=1}(B_1, g_1, \gamma)}{c_1^{i,d=0}(B_1, g_1, \gamma)} - 1 = \frac{(1 - \phi(g_1))y - g_1}{y - g_1 + b_1^i - B_1} - 1$$

A positive (negative) value of $\alpha^i(B_1, g_1, \gamma)$ implies that agent i prefers government default (repayment) by an amount equivalent to an increase (cut) of $\alpha^i(\cdot)$ percent in consumption.

[17] These calculations are straightforward given that in the equilibria we solve for $b_1^L = 0$, and hence $b_1^H = B_1/(1-\gamma)$. The same formula would apply, however, even if these conditions do not hold, using instead the policy functions $b_1^i(B_1, g_1, \gamma)$ that solve the households' problems for any given pair of functions $d_1(B_1, g_1, \gamma)$ and $q_0(B_1, \gamma)$, including the ones that are consistent with the government's default decision and equilibrium in the bond market.

The individual welfare gains of default are aggregated using γ to obtain the utilitarian representation of the social welfare gain of default:

$$\overline{\alpha}(B_1, g_1, \gamma) = \gamma\alpha^L(B_1, g_1, \gamma) + (1-\gamma)\alpha^H(B_1, g_1, \gamma).$$

A positive value indicates that default induces a social welfare gain and a negative value a loss.

Figure 3: Social Welfare Gain of Default $\overline{\alpha}(B_1, g_1, \gamma)$

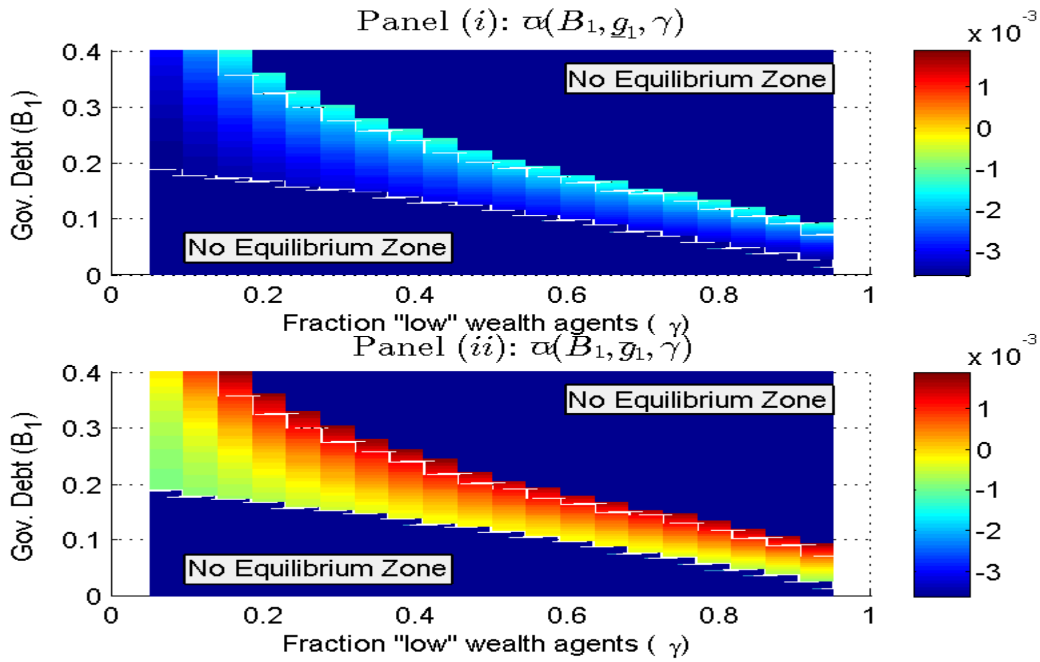


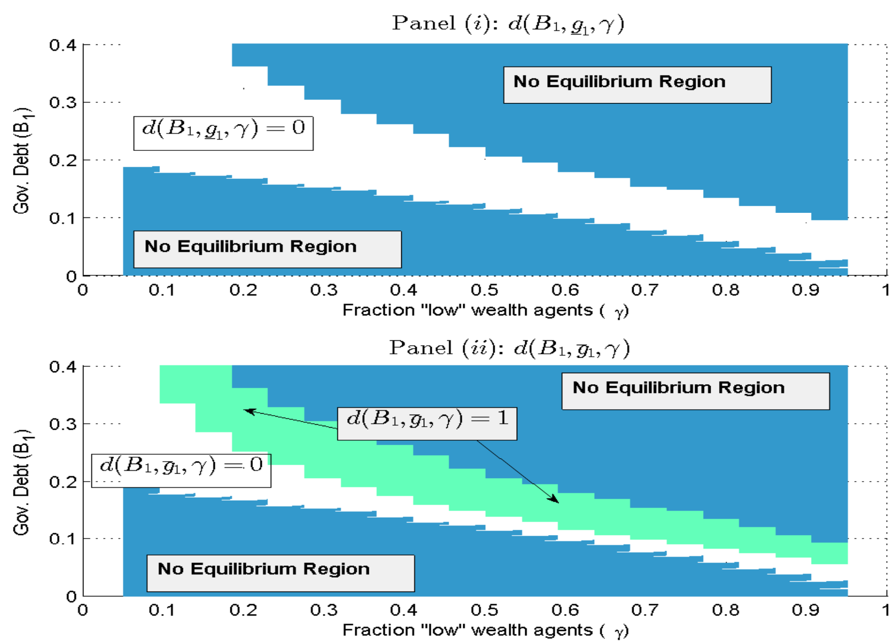
Figure 3 shows two intensity plots of the social welfare gain of default for the ranges of values of B_1 and γ in the vertical and horizontal axes respectively. Panel (i) is for a low value of government purchases, $\underline{g}_1$, set 3 standard deviations below μ_g (in the process in logs), and panel (ii) is for a high value $\overline{g}_1$ set 3 standard deviations above μ_g (in the process in logs). The intensity of the color or shading in these plots indicates the magnitude of the welfare gain according to the legend shown to the right of the plots. The regions shown in dark blue and marked as "No Equilibrium Zone", represent values of (B_1, γ) for which the debt market collapses and no equilibrium exists. In the zone in the upper right, there is no equilibrium because at the given γ the government chooses to default on the given B_1 for all values of g_1. In the zone in the lower left, there is no equilibrium because the given (B_1, γ) would yield $c_0^L \leq 0$, and so the government would not supply that particular B_1.[18]

[18]Note that to determine if $c_0^L \leq 0$ at some (B_1, γ) we also need $q_0(B_1, \gamma)$, since combining the budget constraints of the L types and the government yields $c_0^L = y - g_0 - B_0 + q_0 B_1$. Hence, to evaluate this condition

The area in which the social welfare gains of default are well defined in these intensity plots illustrates two of the key mechanisms driving the government's distributional incentives to default: First, fixing γ, the welfare gain of default is higher at higher levels of debt, or conversely the gain of repayment is lower. Second, keeping B_1 constant, the welfare gain of default is also increasing in γ (i.e. higher wealth concentration increases the welfare gain of default). This implies that lower levels of wealth dispersion are sufficient to trigger default at higher levels of debt.[19] For example, for a debt of 20% of GDP ($B_1 = 0.20$) and $g_1 = \overline{g}_1$, social welfare is higher under repayment if $0 \leq \gamma \leq 0.25$ but it becomes higher under default if $0.25 < \gamma \leq 0.6$, and for higher γ there is no equilibrium because the government prefers default not only for $g_1 = \overline{g}_1$ but for all possible g_1. If instead the debt is 40% of GDP, then social welfare is higher under default for all the values of γ for which an equilibrium exists.

The two panels in Figure 3 differ in that panel (ii) displays a well-defined transition from a region in which repayment is socially optimal ($\overline{\alpha}(B_1, g_1, \gamma) < 0$) to one where default is optimal ($\overline{\alpha}(B_1, g_1, \gamma) > 0$) but in panel (i) the social welfare gain of default is never positive, so repayment is always optimal. This reflects the fact that higher g_1 also weakens the incentives to repay.

Figure 4: Government default decision $d(B_1, g_1, \gamma)$



we take the given B_1 and use the H types Euler equation and the market clearing condition to solve for $q_0(B_1, \gamma)$, and then determine if $y - g_0 - B_0 + q_0 B_1 \leq 0$, if this is true, then (B_1, γ) is in the lower no equilibrium zone.

[19]Note that the cross-sectional variance of initial debt holdings is given by $Var(b) = B^2 \frac{\gamma}{1-\gamma}$ when $b_0^L = 0$. This implies that the cross-sectional coefficient of variation is equal to $CV(b) = \frac{\gamma}{1-\gamma}$, which is increasing in γ for $\gamma \leq 1/2$.

Figure 4 shows two panels with the optimal default decision organized in the same way as the panels of Figure 3. The plots separate the regions where the government chooses to repay ($d(B_1, g_1, \gamma) = 0$ shown in white), where it chooses to default ($d(B_1, g_1, \gamma) = 1$ in green) and where the equilibrium does not exist (in blue).

These plots illustrate the implications of the mechanisms highlighted in Figure 3 for the default decision. The repayment region ($d(B_1, g_1, \gamma) = 0$) corresponds to the region with $\overline{\alpha}(B_1, g_1, \gamma) < 0$. Hence, the government defaults at higher B_1 the lower a given γ, or at higher γ the lower a given B_1. Moreover, the two plots show that when $g_1 = \overline{g}_1$ the government defaults for combinations of γ and B_1 for which it repays when $g_1 = \underline{g}_1$. Thus, default occurs over a wider set of (B_1, γ) pairs at higher levels of government expenditures, and thus it is also more likely to occur.

We examine further the behavior of the default decision by computing the threshold value of γ such that the government is indifferent between defaulting and repaying in period for a given (B_1, g_1). These indifference thresholds ($\hat{\gamma}(B_1, g_1)$) are plotted in Figure 5 against debt levels ranging from 0 to 0.4 for three values of government expenditures $\{\underline{g_1}, \mu_g, \overline{g}_1\}$. For any given (B_1, g_1), the government chooses to default if $\gamma \geq \hat{\gamma}$.

Figure 5: Default Threshold $\hat{\gamma}(B_1, g_1)$

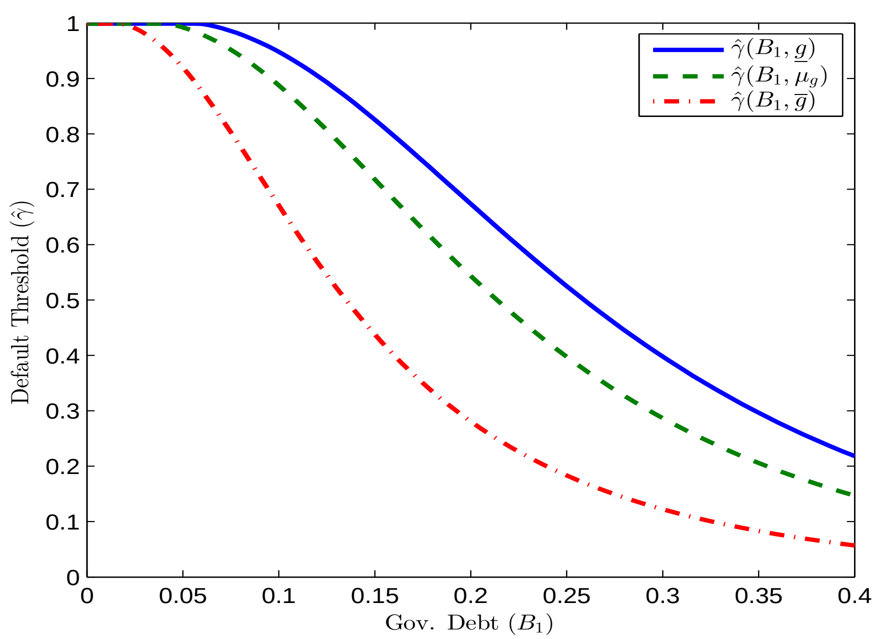
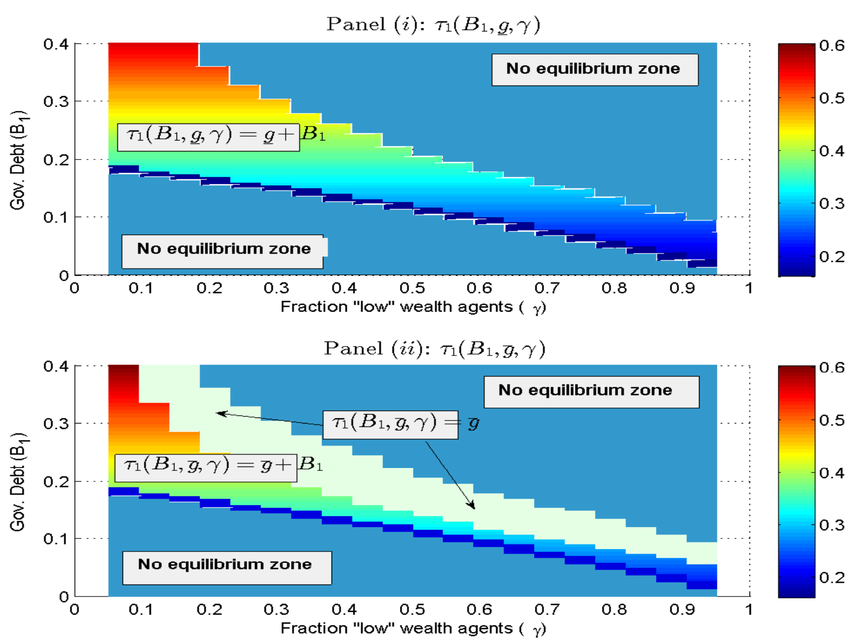


Figure 5 shows that the default threshold is decreasing in B_1. Hence, the government tolerates higher debt ratios without defaulting only if wealth concentration is sufficiently low. Also, default thresholds are decreasing in g_1, because the government has stronger incentives to default when government expenditures are higher (i.e. the threshold curves shift inward).[20] This last feature of $\hat{\gamma}$ is very important to determine equilibria with debt subject to default risk. If, for a given value of B_1, γ is higher than the curve representing $\hat{\gamma}$ for the lowest realization in the Markov process of g_1 (which is also the value of $\underline{g}_1$), the government defaults for sure and, as explained earlier, there is no equilibrium. Alternatively, if for a given value of B_1, γ is lower than the curve representing $\hat{\gamma}$ for the highest realization of g_1 (which is the value of $\overline{g}_1$), the government repays for sure and debt would be issued effectively without default risk. Thus, for the model to support equilibria with debt subject to default risk, the optimal debt chosen by the government in the first period for a given γ must lie between these two extreme threshold curves. We show that this is the case later in this Section.

Figure 6 shows intensity plots of the equilibrium tax functions in period 1 organized in the same way as in Figure 3.

Figure 6: Equilibrium Tax Functions $\tau_1^{d(B_1,g_1,\gamma)}$



Putting together these plots with those of the default decision in Figure 4 illustrates the model's distributional incentives to default from the perspective of tax policy. If the government defaults, the tax is $\tau_1 = g_1$, but if it repays the tax is $\tau_1 = g_1 + B_1$. Since all agents pay the same taxes but L types do not collect bond repayments, the lower taxes under default provide

[20] $\hat{\gamma}$ approaches zero for B_1 sufficiently large, but in Figure 5 B_1 reaches 0.40 only for exposition purposes.

a distributional incentive to default that is larger when wealth is more concentrated. Figure 6 shows that, for given g_1, the repayment scenarios with higher taxes are more likely when a large fraction of households hold debt (low γ) and thus benefit from a repayment, or when the debt is low so that the distributional incentives to default are weak. Moreover, equilibria with higher taxes are more likely to be observed at low than at high levels of government expenditures, because default is far likely with the latter.

4.2.2 First period taxes, bond prices and decision rules for given $(B_1, \gamma$

Figure 7 shows equilibrium bond prices and a comparison with the prices from the model with the government committed to repay. This figure presents $q_0(B_1, \gamma)$ as a function of γ for three values of B_1 ($B_L < B_M < B_H$ with B_M set to the value that we show later corresponds to the optimal debt choice of the government when $\gamma = 0.5$, which is denoted $B_1^*(\gamma = 0.5)$). As explained earlier, at high enough values of γ, for a given B_1, the government is certain to default for all realizations of g_1 and the model cannot support an equilibrium with debt. The bond price functions are truncated when this is the case.

Figure 7: Equilibrium Bond Price

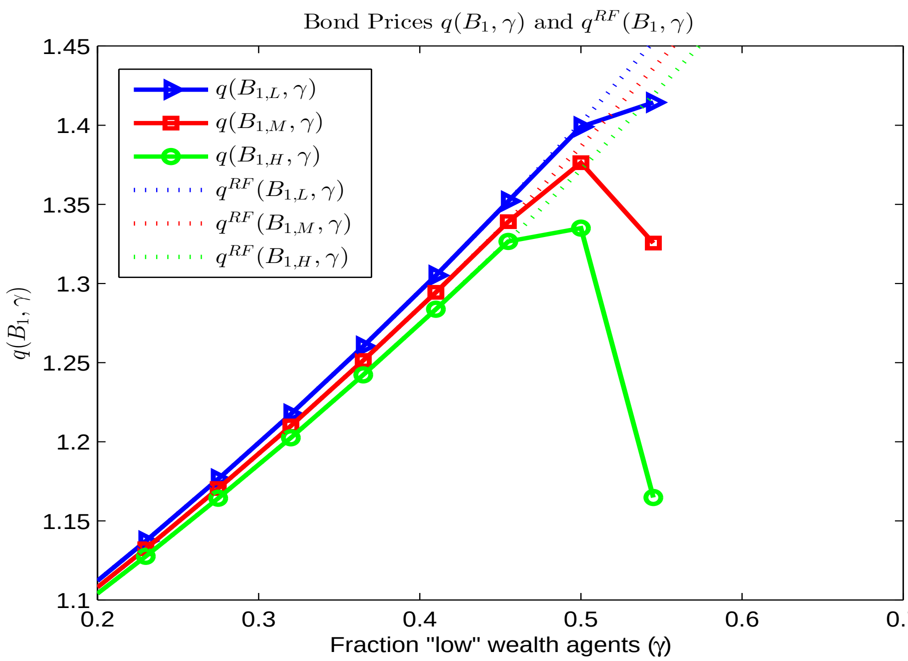


Figure 7 illustrates three key features of public debt prices discussed in Section 3:

(**i**) *The equilibrium price is decreasing in B_1 for given γ* (the pricing functions shift downward as B_1 rises). This follows from a standard demand-and-supply argument: For a given γ, as the government borrows more, the price at which the H types are willing to demand the additional debt falls and the interest rate rises. As explained in Section 3, rhe equilibrium price must be consistent with the demand for bonds implied by their Euler equation, which implies that, for any b_0^H, as B_1 increases, a lower q_0 is needed to induce the rise in b_1^H that clears the bond market. This effect is present whether there is risk of default or not, and even without uncertainty (see the Appendix for proofs showing that $q\prime(B_1) < 0$ in all these scenarios for the log utility case).

(**ii**) *Default risk reduces the price of bonds below the risk-free price and thus induces a risk premium.* This again was shown in Section 3 in comparing the prices with and without risk of default (equations (11) and (12)) and deriving the risk spread expression (eq. (13)). In Figure 7, the (B_1, γ) pairs for which $d(B_1, g_1, \gamma) = 0$ for all realizations of g_1, the default probability is zero, and hence altough the government is not committed to repay, since it chooses always to repay the prices with and without default risk are identical and there is no risk spread. Similarly, if the default probability is low, the spread is negligible and the prices with and without default risk are very similar. Quantitatively, the prices are either identical or nearly identical, and the spreads zero or negligible, for all three levels of debt at levels of inequality such that $\gamma \leq 0.45$. As γ increases above 0.45, however, default risk becomes nontrivial and bond prices subject to default risk fall sharply below the risk free prices.

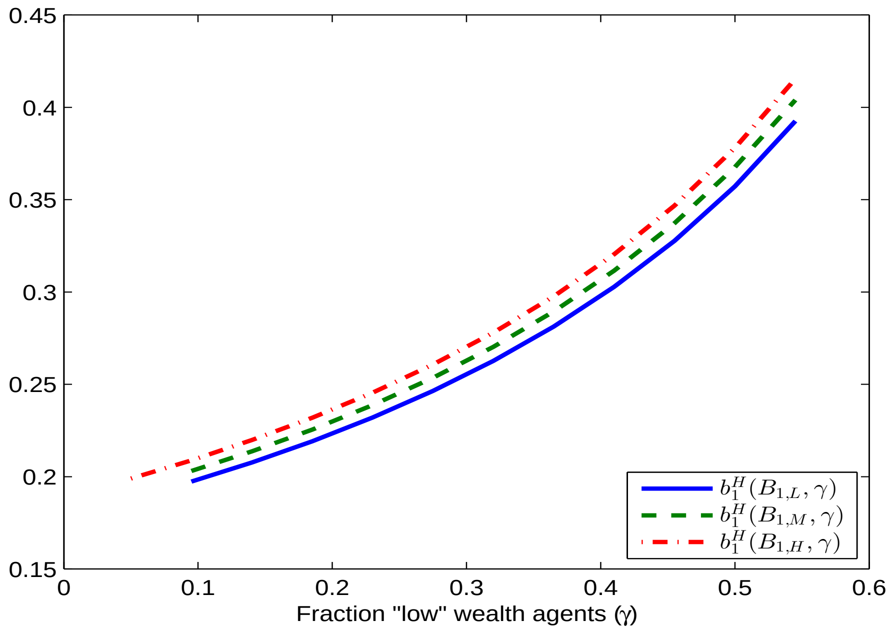
(**iii**) *Bond prices are a non-monotonic function of wealth dispersion*: When default risk is sufficiently low, bond prices are increasing in γ, but eventually they become a steep decreasing function of γ. Whether default risk is increasing or decreasing in γ depends on the relative strength of the composition effect mentioned earlier, due to how changes in γ affect the per-capita demands for government debt, v. the risk of default. Higher γ implies a more dispersed wealth distribution, so that H-type agents become a smaller fraction of the population, and hence they must demand a larger amount of debt per capita in order to clear the bond market (i.e. b_1^H increases with γ), which pushes bond prices up. While default risk is low this demand composition effect dominates and thus bond prices rise with γ, but as γ increases and default risk rises (since higher wealth dispersion strengthens default incentives), the growing risk premium becomes the dominating force (at about $\gamma > 0.5$) and produces bond prices that fall sharply as γ increases.

Figure 8 plots the bond demand decision rules (for given B_1) of the H types in the same layout as the bond prices (i.e. as functions of γ for three values of B_1).[21] This plot validates the intuition provided above about the properties of these agents' bond demand function. In particular, as γ increases, the demand for bonds of H types grows at an increasing rate, reflecting the combined effects of higher per-capita demand by a smaller fraction of H-type agents and a

[21] We do not include the bond decision rules for L types because they are credit constrained (i.e. their Euler condition holds with inequality) and choose $b_1^L = b_0^L = 0$.

rising default risk premium. Thus, the convexity of these bond decision rules reflects the effects of wealth dispersion on demand composition and default risk explained earlier.

Figure 8: Equilibrium HH bond decision rules b_1^i

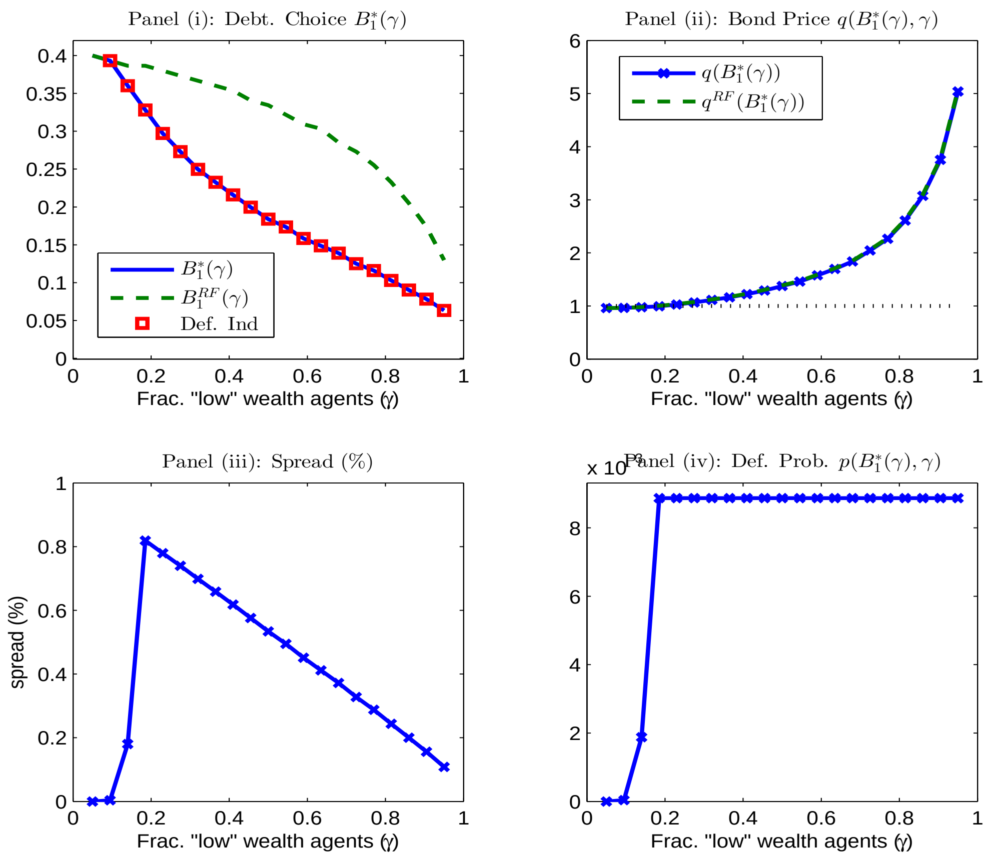


4.2.3 Full Equilibrium with Optimal Debt Choice

Given the solutions for household decision rules, tax policies, bond pricing function and default decision rule, we finally solve for the government's optimal choice of debt issuance in the first period (i.e. the optimal B_1 that solves problem (20)) for a range of values of γ. Moreover, given this optimal debt, we go back and identify the equilibrium values of the rest of the model's endogenous variables that are associated with optimal debt choices.

Figure 9 shows the four main components of the equilibrium: Panel (i) plots the optimal first-period debt issuance in the model with default risk, $B_1^*(\gamma)$, and in the case when the government is committed to repay so that the debt is risk free, $B_1^{RF}(\gamma)$; Panel (ii) shows the equilibrium debt prices that correspond to the optimal debt of the same two economies; Panel (iii) shows the default spread (the difference in the inverses of the bond prices); and Panel (iv) shows the probability of default. Since the government that has the option to default can still choose a debt level for which it prefers to repay in all realizations of g_1, we identify with a square in Panel (i) the equilibria in which $B_1^*(\gamma)$ has a positive default probability. This is the case for all but the smallest value of gamma considered ($\gamma = 0.05$), in which the government sets $B_1^*(\gamma)$ at 40% of GDP with zero default probability.

Figure 9: Equilibrium Optimal Government Debt Policy



Panel (i) shows that optimal debt falls as γ increases in both the economy with default risk and the economy with a government committed to repay. This occurs because, as explained in Section 3, in both cases the government seeks to reallocate consumption across agents and across periods by altering the product $q(B_1)B_1$ optimally, and in doing this it internalizes the response of bond prices to its choice of debt. As γ rises, this response is influenced by stronger default incentives and a stronger demand composition effect. The latter dominates in this quantitative experiment, because panel (ii) shows that the equilibrium bond prices always rise with γ. Hence,the government internalizes that as γ rises the demand composition effect strengthens demand for bonds, pushing bond prices higher, and as a result it can actually attain a higher $q(B_1)B_1$ by choosing lower B_1. This is a standard Laffer curve argument: In the upward sloping segment of this curve, increasing debt increases the amount of resources the government acquires by borrowing in the first period.

Although the Laffer curve argument and the demand composition effect explain why both $B_1^*(\gamma)$ and $B_1^{RF}(\gamma)$ are decreasing in γ, default risk is not innocuous. As Panel (i) shows, the optimal B_1 choices of the government that cannot commit to repay are sharply lower than those of the governemt that can, and the negative relationship between B_1 and γ changes from concave for the former to convex for the latter. The first result reflects the fact that the government optimally chooses smaller debt levels once it internalizes the effect of default risk on the debt Laffer curve and its distributional implications. The second result is also due to default risk, because at lower values of γ a higher γ strengthens default incentives and the demand composition effect is weak (i.e. en Panel (ii) the pricing function is relatively flat), whereas in the absence of default risk only the demand composition effect is at play. Hence, optimal debt declines faster at relatively low values of γ with default risk than without it.

The negative relationship between B_1 and γ is in line with the empirical evidence on the negative relationship between public debt ratios and wealth Gini coefficients at relatively high levels of inequality documented in the Introduction. The model cannot, however, account for the positive relationship observed at lower levels of inequality. Moreover, in this quantitative exercise the relationship is conditional on the structure of the model and all the parameters held constant as γ varies in Figure 9, while in the empirical analysis we conditioned the relationship in a non-structural fashion by introducing country fixed effects and the size of the government as control variables.

In the range of empirically relevant values of γ, optimal debt ratios range from 20 to 32% of GDP without default risk and from 8 to 15% with default risk. Since the median in the European data was 35%, these ratios are relatively low, but still they are notable given the simplicity of the two-period setup, with the inertia to reduce debt levels and constraints on feasible equilibrium debt positions discussed earlier. In particular, the model lacks income- and tax-smoothing effects and self-insurance incentives that are likely to be strong in a longer time horizon (see Aiyagari and McGrattan (1998)), and, as explained earlier, the model has an upper bound on the optimal debt choice for $\gamma = [0, 1]$ lower than $B_0/(1+\beta)$ (which is the upper bound as $\gamma \to 0$ in the absence of default risk).

Panel (*ii*) shows that bond prices of the optimal debt range from very low to very high as the value of γ rises, including prices sharply above 1 that imply large negative real interest rates on public debt. In fact, equilibrium bond prices are similar and increasing in γ with or without default risk (recall that the pricing equations (11) and (12) yield nearly identical prices when $\Pi(B_1, \gamma) \approxeq \Pi^{ND}(B_1, \gamma)$). This is because at equilibrium, the government chooses optimal debt positions for which default risk is low (see panel (iv)), and thus the demand composition effect that strengthens as γ rises dominates and yields bond prices increasing in γ and similar with or without default risk.

Panels (iii) and (iv) show that, in contrast with standard models of external default, in this model the default spread is neither similar to the probability of default nor does it have a monotonic relationship with it.[22] Both the spread and the default probability start at zero for $\gamma = 0.05$ because $B_1^*(0.05)$ has zero default probability. As γ increases up to 0.2, both the spread and the default probability of the optimal debt choice are similar in magnitude and increase together, but for $\gamma > 0.2$ the spread falls with γ while the default probability remains unchanged around 0.9%. For $\gamma = 0.95$ the probability of default is 9 times larger than the spread (0.9% v. 0.1%).

These results are in line with the findings of the theoretical analysis in Section 3. As noted there, the expression we derived for the default spreads in eq. (13) implies that the response of the spreads to increases in γ has an ambiguous sign, because it is influenced by opposing effects: First, the demand composition effect for the given initial debt B_0 induces an increase in the initial per capita wealth of H types, which has a first-order negative effect on spreads. Second, increasing γ affects the gap of the inverse date-1 marginal utilities of H types with and without default risk ($\left[\frac{1}{\Pi(B_1,\gamma)} - \frac{1}{\Pi^{ND}(B_1,\gamma)}\right]$). This gap depends explicitly on γ and implicitly via the effects of γ on the default decision and the optimal debt choices $B_1^*(\gamma)$ and $B_1^{RF}(\gamma)$. The constant and low default probability, and the nearly linear decline in the spreads as γ rises above 0.2 suggest, however, that the first effect dwarfs the second under the calibrated parameters. Decomposing the spread into these two effects we confirmed that indeed the approximately linear, decreasing spread for $\gamma \geq 0.2$ is driven by the first effect. The second effect pushes for increasing spreads as γ rises but its much smaller than the first.

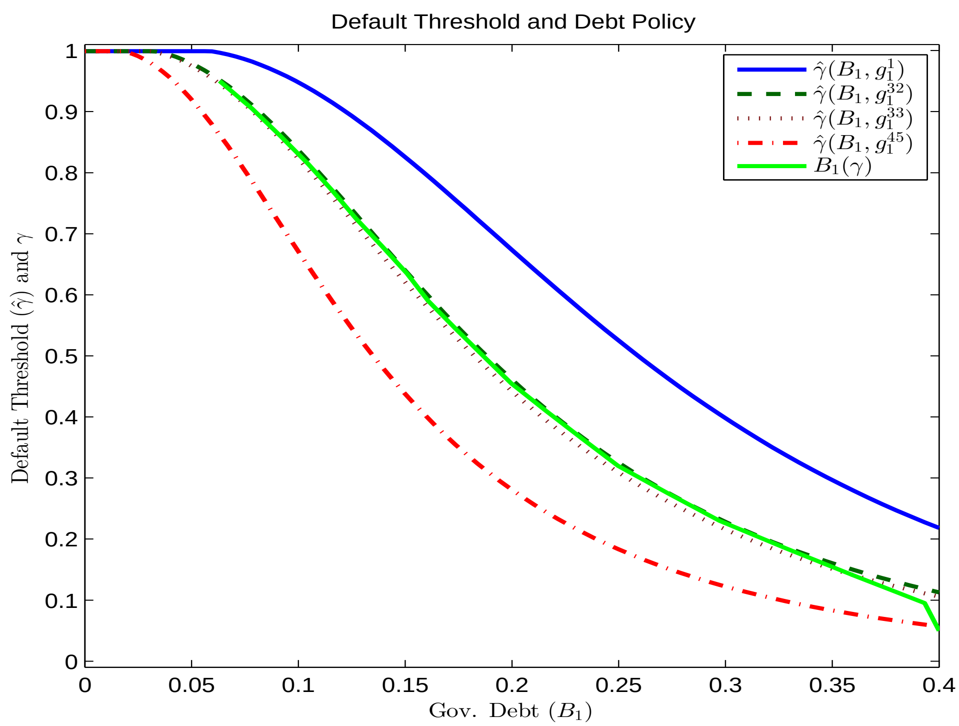
Why is the default probability of the optimal debt position constant as γ rises above 0.2, even though $B_1^*(\gamma)$ falls sharply? The answer is illustrated in Figure 10. This figure shows the optimal debt choice $B_1^*(\gamma)$ plotted in Figure 9 (with the axis inverted) together with curves for the wealth concentration default thresholds $\hat{\gamma}(B_1, g_1)$ corresponding to four values of g_1 (the lowest and highest realizations, as in Figure 5, and the 32nd and 33rd realizations, g^{32} and g^{33}, which are 1.23 and 1.36 standard deviations above the mean of g_1 respectively). As Figure 10 shows, for $\gamma \geq 0.2$ the government's optimal debt choice is below the threshold curve for g^{32} but above the one for g^{33}. The default probability is equal to $Pr[g_1 > g^{32}] = .009$ in the Markov process of g_1 as long as the debt choice is below the threshold curve for g^{32} but above the one for g^{33}.[23] This is because under these conditions the government defaults for $g_1 \geq g^{33}$, since the optimal debt lies above the default threshold, and repays for $g_1 \leq g^{32}$, since the opposite is true. The default probability would be higher if the optimal debt choice were above the threshold

[22]In the standard models, the two are similar and a monotonic function of each other because of the arbitrage condition of a representative risk-neutral

[23]With a continuous probability process for g_1, the default probability would in fact vary slightly as the optimal debt choice changes within the interval of the threshold curves for g^{32} and g^{33}.

curve for g^{32}, or lower if the optimal debt choice were below the threshold curve for g^{33} (as it occurs, for example, when γ falls to 0.15 and below, where panel (iv) of Figure 9 shows the default probability dropping).

Figure 10: Default Threshold, Debt Policy and Equilibrium Default

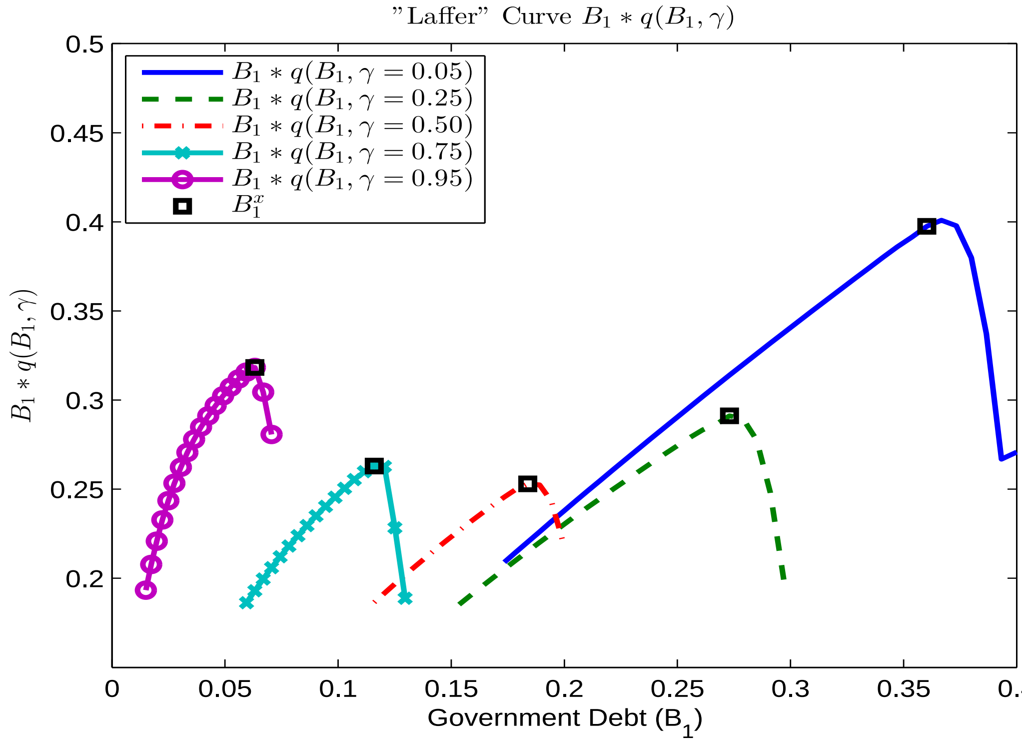


Note: g^1 and g^{45} are the smallest and largest possible realizations of g_1 in the Markov process of government expenditures, which are set to -/+ 3 standard deviations off the mean respectively. g^{32} and g^{33} are the 32nd and 33rd realizations, which are equal to 1.23 and 1.36 standard deviations off the mean. When the optimal debt choice remains between the contiguous threshold curves for g^{32} and g^{33}, the default probability is constant and equal to $\Pr[g_1 > g^{32}] = 0.009$.

It is important to note that the optimal debt choice $B_1^*(\gamma)$ lies in between the threshold curves for g^{32} and g^{33} for all values of $\gamma \geq 0.2$ that we considered. The reason is again that the government is choosing debt responding to its incentives to reallocate consumption across agents and across periods internalizing the dependence of the debt Laffer curve on its borrowing decision. In fact, whenever $B_1^*(\gamma)$ is exposed to nontrivial risk of default at equilibrium, $B_1^*(\gamma)$ yields the maximum resources to the government that the Laffer curve allows. Figure 11 illustrates this result. It shows debt Laffer curves for five values of γ in the [0.05,0.95] range.[24]

[24] Each curve is truncated at values of B_1 in the horizontal axis that are either low enough for $c_0^L \leq 0$ or high enough for default to be chosen for all realizations of g_1, because as noted before in these cases there is no equilibrium.

Figure 11: Debt Laffer Curve



In all but one case, $B_1^*(\gamma)$ is located at the maximum of the corresponding Laffer curve. In these cases, setting debt higher than at the maximum of the Laffer curve is suboptimal because default risk reduces bond prices sharply, moving the government to the downward sloping region of the Laffer curve, and setting it lower is also suboptimal because then default risk is low and extra borrowing generates more resources since bond prices change little, leaving the government in the upward sloping region of the Laffer curve. Thus, if the optimal debt has a nontrivial probability of default, the government's debt choice exhausts its ability to raise resources by borrowing. The exception is the case with $\gamma = 0.05$, in which $B_1^*(\gamma)$ has zero default probability. In this case, the government's optimal debt is to the left of the maximum of the Laffer curve, and thus the debt choice does not exhaust the government's ability to raise resources by borrowing. This also happens when the default probability is positive but negligible. For example, when $\gamma = 0.15$ the default probability is close to zero and the optimal debt choice is again slightly to the left of the maximum of the corresponding Laffer curve.

Each Laffer curve is much steeper in its decreasing segment than in its increasing segment. This is because the decreasing segment is driven by the steeply declining bond prices as default risk increases, while the increasing segment is nearly linear, because bond prices change slightly while the incentives to default remain weak. The demand composition effect does not play a role here because each Laffer curve maintains γ fixed. Note also that, while the Laffer curves shift to the left as γ rises, reflecting the reduced borrowing capacity of the government due to stronger default incentives, the maximum value of the Laffer curves follows a U-shaped path (it falls first as γ rises, and then increases with γ). This result does reflect the tension of the opposing forces of default risk and demand composition on bond prices.

5 Sensitivity Analysis & Extensions

This Section presents the results of a set of counterfactuals that shed more light on the workings of the model and also show some results for the case in which the social welfare function has biased weights that do not correspond to the fractions of L and H types in the economy.

5.1 Sensitivity Analysis

The sensitivity analysis studies how the main results discussed in the previous Section are affected by changes in the initial debt B_0, initial government expenditures g_0, and the constant in the default cost function ϕ_0.

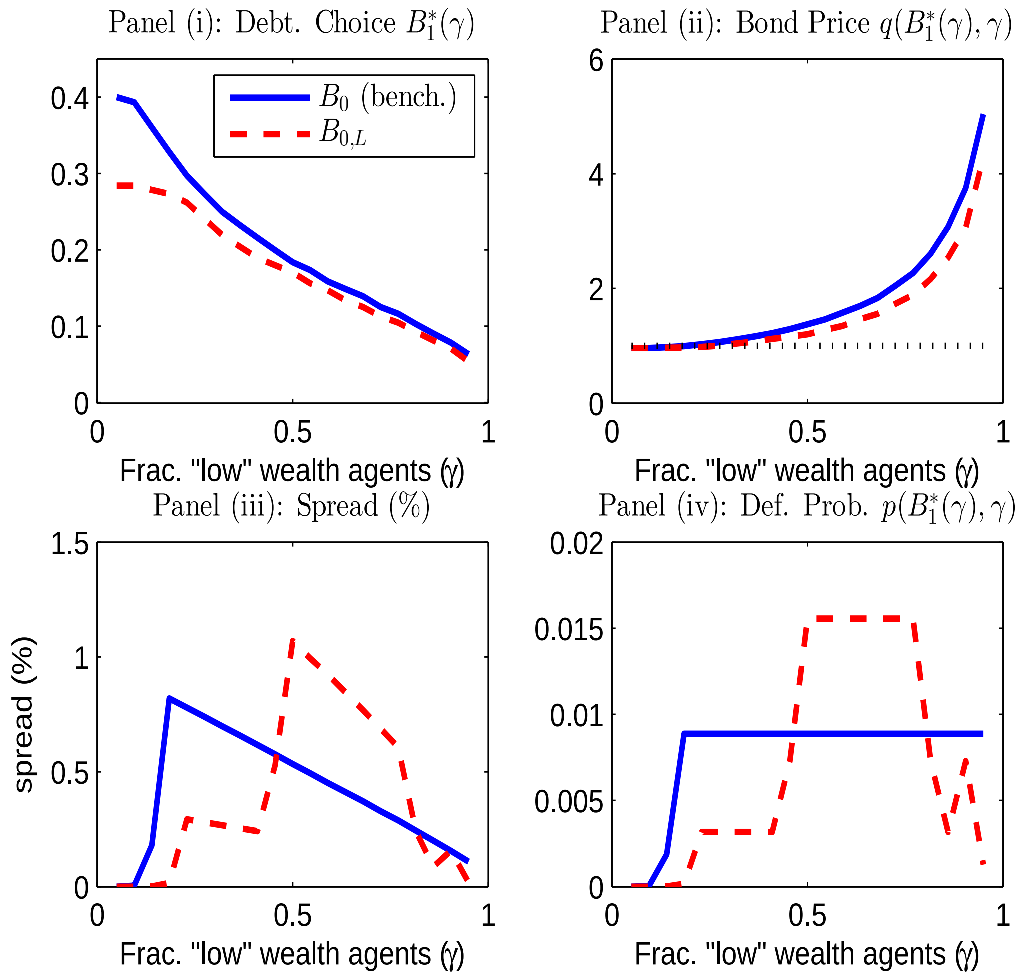
5.1.1 Lower Initial Debt Level B_0

Figure 12 uses the same layout of Figure 9 to compare the optimal government debt and associated equilibrium bond prices, spreads and default probability under the original calibration with $B_0 = 0.79$ and a value that is 20% lower ($B_{0,L} = 0.624$).

Panel (i) shows that the optimal debt choice is always lower for the lower B_0, but the difference narrows as γ rises, and the optimal debt is about the same for the two levels of B_0 for $\gamma \geq 0.2$. This occurs because at low γ default risk plays a negligible role, and the demand composition effect implies lower per-capita demand for bonds from H type agents and a smaller fraction of credit-constrained L types, so the government wants to sell less debt. But once default risk becomes relevant, the optimal debt choice is about the same.

The optimal debt choice is similar for $\gamma \geq 0.2$ under the two values of B_0 even tough the behavior of bond prices, spreads and default probabilities is different. With the lower B_0, bond prices are uniformly lower albeit slightly (panel (ii)), spreads are increasing as γ rises for a wider range of γ and attain a higher maximum (panel (iii)), and the same is true for default probabilities (panel (iv)).

Figure 12: Changes in Initial Government Debt B_0



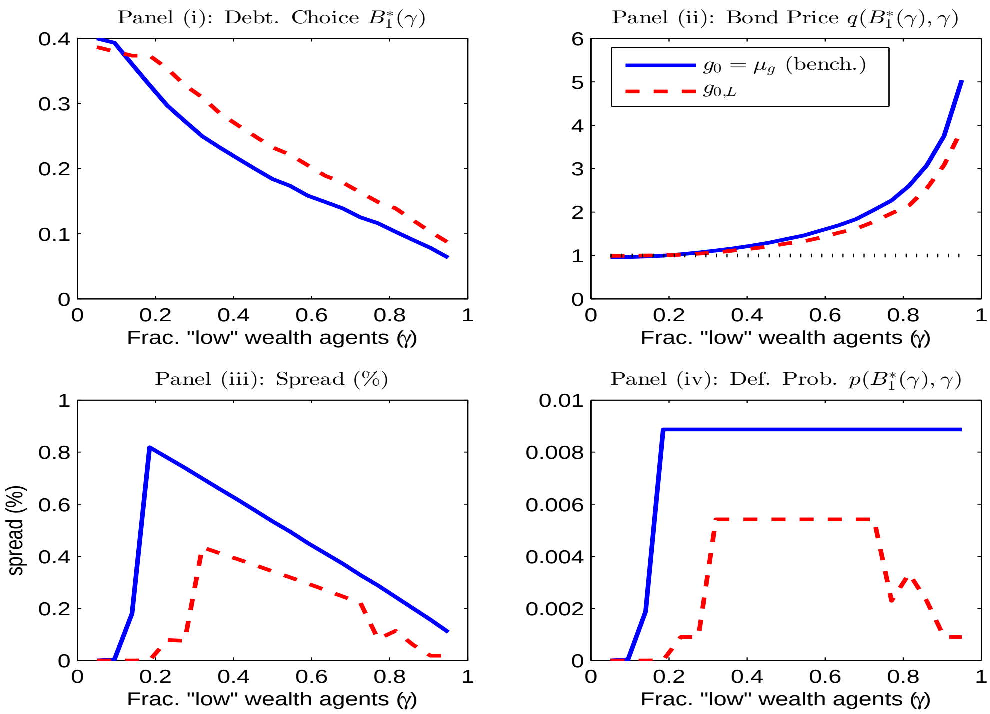
Bond prices are lower with lower initial debt because for a given γ this implies lower initial wealth of H types (b_0^H), and in turn this requires a lower bond price to clear the market. This effect is stronger than two other effects that push bond prices in the opposite direction: First, the slightly lower debt B_1 that the government finds optimal to supply at lower levels of B_0. Second, the higher disposable income of households resulting from the lower date-0 taxes needed to repay lower levels of B_0, which increases demand for bonds.

In terms of the implications for the empirically relevant range of γ in the European data, these results show that at the lower B_0 the model continues to predict debt ratios of about 8 to 15% as before, but now at spreads that are about twice as large (80 v. 40 basis points) and at default probabilities around 1.5 percent instead of 0.9 percent. Moreover, these results also show that spreads and default probabilities can display richer patterns than the ones found in the initial calibration. In particular, non-negligible default probabilities display significantly more variation with the lower value of B_0.

5.1.2 Lower Initial Government Expenditures g_0

Figure 13 compares the model's equilibrium outcomes under the calibrated value of initial government expenditures, $g_0 = \mu_g = 0.181$, and a scenario in which g_0 is 1.5 standard deviations below the mean, $g_{0,L} = 0.171$.

Figure 13: Changes in Initial Government Expenditures g_0



Like the reduction in B_0, the reduction in g_0 increases date-0 disposable income via lower date-0 taxes. They differ, however, in two key respects: First, changes in g_0 affect the expected level of government expenditures for $t = 1$, as reflected in changes in the transition probabilities which are conditional on g_0. Second, changes in g_0 do not affect the aggregate wealth of the economy and the initial bond holdings of H types.

Panel (i) shows that the optimally debt is slightly higher with lower g_0, unless γ is below 0.15. This reflects the fact that the lower g_0 allows the government to issue more debt in the initial period, because the likelihood of hitting states with sufficiently high g_1 for optimal default to occur in the second period is lower. This also explains why in panels (ii)-(iv), despite the higher optimal debt with the lower g_0, bond prices, default probabilities and spreads are lower.

Moreover, optimal debt is about the same with lower g_0 as in the initial calibration for $\gamma \leq 0.15$ because at this low level of wealth concentration default risk is not an issue and the mechanism that we just described is irrelevant.

5.1.3 Fixed Cost of Default ϕ_0

Panels $(i) - (iv)$ in Figure 14 compare the equilibrium outcomes for the initial calibration of the default cost parameter ($\phi_0 = 0.02$) with scenarios with higher and lower values ($\phi_{0,L} = 0.01$ and $\phi_{0,H} = 0.03$ respectively).

Figure 14: Changes in Cost of Default ϕ_0

Qualitatively, the changes in the equilibrium are in the direction that would be expected. Higher (lower) default costs increase (reduce) the optimal debt and reduce (increase) bond prices, spreads and default probabilities. Quantitatively, however, the changes in optimal debt and bond prices are small for both higher and lower default costs, while the changes in spreads and default probabilities are significantly larger when lowering default costs than when rising them. Thus, again in terms of the comparison vis-a-vis the empirically relevant range of γ, the model continues to predict debt ratios of about 8 to 15% as in the initial calibration, but small variations in ϕ_0 result in these debt ratios co-existing with spreads of up to 150 basis points and default probabilities of up to 3 percent, compared with 40 basis points and 1.5 percent respectively in the initial calibration.[25]

5.2 Biased Welfare Weights

We consider now a scenario in which the weights of the government's payoff function differ from the utilitarian weights γ and $1-\gamma$. This can be viewed as a situation in which, for political reasons or related factors, the government's welfare weights are biased in favor of one group of agents. The government's welfare weights on L- and H-type households are denoted ω and $(1-\omega)$ respectively, and we refer to ω as the government's political bias.

5.2.1 A Government with Political Bias

The government's default decision at $t=1$ is determined by following the following optimization problem:

$$\max_{d\in\{0,1\}} \left\{W_1^{d=0}(B_1, g_1, \gamma, \omega), W_1^{d=1}(g_1)\right\}, \tag{22}$$

where $W_1^{d=0}(B_1, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ denote the government's payoffs in the cases of no-default and default respectively. Using the government budget constraints to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma, \omega) = \omega u(y - g_1 + b_1^L - B_1) + (1-\omega)u(y - g_1 + b_1^H - B_1) \tag{23}$$

and

$$W_1^{d=1}(g_1) = u(y(1-\phi(g_1)) - g_1). \tag{24}$$

Combining the above payoff functions we get a similar default condition as before but with

[25] We also experimented introducing a variable cost of default coefficient (ϕ_1) using the form $\phi(g_1) = \phi_0 + \phi_1(\overline{g}_1 - g_1)/y$. This is the same as our current setup if we set $\phi_1 = 1$. Using $\phi_1 = 0.5$ or 2 we found very similar effects on debt and bond prices as those produced by changing ϕ_0. The effects on spreads and default probabilities were qualitatively the same but smaller in magnitude.

ω in the place of γ:

$$\omega\left[u(y-g_1+\overbrace{(b_1^L-B_1)}^{\leq 0})-u(y(1-\phi(g_1))-g_1)\right]+$$
$$(1-\omega)\left[u(y-g_1+\overbrace{(b_1^H-B_1)}^{\geq 0})-u(y(1-\phi(g_1))-g_1)\right] \quad \leq \quad 0$$

We follow the same approach as before to characterize the optimal default decision graphically. The parameter ϵ is used again to represent the dispersion of hypothetical decentralized consumption allocations under repayment: $c^L(\epsilon)=y-g_1-\epsilon$ and $c^H(\gamma,\epsilon)=y-g_1+\frac{\gamma}{1-\gamma}\epsilon$. Under default the consumption allocations are again $c^L=c^H=y(1-\phi(g_1))-g_1$. Recall that under repayment, the dispersion of consumption across agents increases with ϵ, and under default there is zero consumption dispersion. The repayment government payoff can now be rewritten as:

$$W^{d=0}(\epsilon,g_1,\gamma,\omega)=\omega u(y-g_1+\epsilon)+(1-\omega)u\left(y-g_1+\frac{\gamma}{1-\gamma}\epsilon\right).$$

As before, the socially efficientplanner chooses its optimal consumption dispersion ϵ^{SP} as the value of ϵ that maximizes the above expression. Since as of $t=1$ the only instrument the government can use to manage consumption dispersion relative to what the decentralized allocations support is the default decision, it will repay only if doing so allows it to get closer to ϵ^{SP} than by defaulting.

The planner's optimality condition is now:

$$\frac{u'(c_1^H)}{u'\left(c_1^L\right)}=\frac{u'\left(y-g_1+\frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y-g_1-\epsilon^{SP}\right)}=\left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right). \tag{25}$$

This condition implies that optimal consumption dispersion for the planner is zero only if $\omega=\gamma$. For $\omega>\gamma$ the planner likes consumption dispersion to favor L types so that $c_1^L>c_1^H$, and the opposite holds for $\omega<\gamma$.

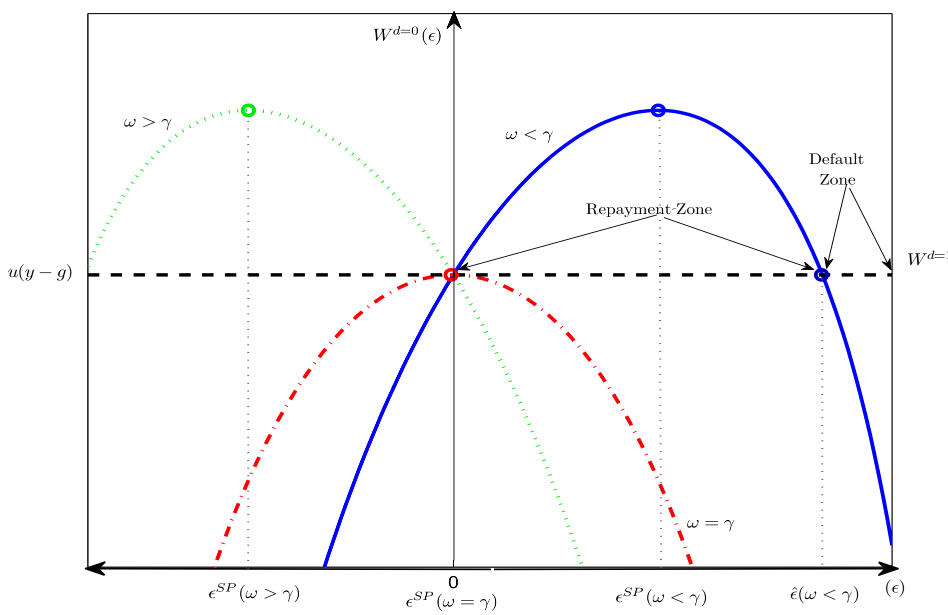
The key difference in this model with political bias v. the model with a utilitarian government is that it can support equilibria with debt subject to default risk even without default costs. Assuming $\phi(g_1)=0$, there are two possible scenarios depending on the relative size of γ and ω. First, if $\omega\geq\gamma$, the planner again always chooses default as in the setup of Section 2. This is because for any decentralized consumption dispersion $\epsilon>0$, the consumption allocations feature $c^H>c^L$, while the planner's optimal consumption dispersion requires $c^H\leq c^L$, and hence ϵ^{SP}

cannot be implemented. Default brings the planner the closest it can get to the payoff associated with ϵ^{SP} and hence it is always chosen.

In the second scenario $\omega < \gamma$ (i.e. the political bias assigns more (less) weight to H (L) types than the fraction of each type of agents that actually exists). In this case, the model can support equilibria with debt even without default costs. In particular, there is a threshold consumption dispersion $\hat{\epsilon}$ such that default is optimal for $\epsilon \geq \hat{\epsilon}$, where $\hat{\epsilon}$ is the value of ϵ at which $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ intersect. For $\epsilon < \hat{\epsilon}$, repayment is preferable because $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W_1^{d=0}(g_1)$. Thus, without default costs, equilibria for which repayment is optimal require two conditions: (a) that the government's political bias favors bond holders ($\omega < \gamma$), *and* (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Figure 15 illustrates the outcomes described above. This Figure plots $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ for $\omega \gtreqless \gamma$. The planner's default payoff and the values of ϵ^{SP} for $\omega \gtreqless \gamma$ are also identified in the plot. The vertical intercept of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ is always $W^{d=1}(g_1)$ for any values of ω and γ, because when $\epsilon = 0$ there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ follows from $u'(.) > 0, u''(.) < 0$.[26]

Figure 15: Default decision with non-utilitarian planner ($\phi = 0$)



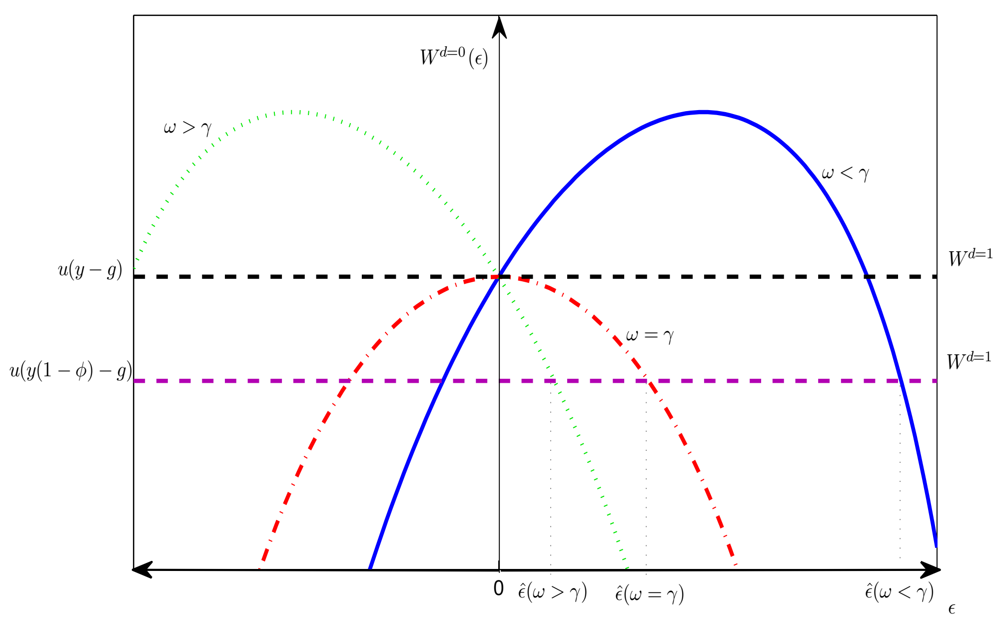
[26]Note in particular that $\frac{\partial W_1^{d=0}(\epsilon, g_1, \gamma, \omega)}{\partial \epsilon} \gtreqless 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \gtreqless (\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$. Hence, the planner's payoff is increasing (decreasing) at values of ϵ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $(\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$.

Take first a scenario with $\omega > \gamma$. In this case, the planner's payoff under repayment is the dotted bell curve shown in green. Here, $\epsilon^{SP} < 0$, because condition (25) implies that the planner's optimal choice features $c^L > c^H$. Since default is the only instrument available to the government, however, these consumption allocations are not feasible, and by choosing default the government attains $W^{d=1}$, which is the highest feasible government payoff for any $\epsilon \geq 0$. In contrast, in a scenario with $\omega = \gamma$, for which the planner's payoff function is the red, dashed bell curve, the planner would choose $\epsilon^{SP} = 0$, and default attains exactly the same payoff, so default is chosen. In short, if $\omega \geq \gamma$, the government always defaults for any decentralized distribution of debt holdings represented by $\epsilon > 0$, and thus equilibria with debt cannot be supported.

When $\omega < \gamma$, the planner's payoff is the blue curve. The intersection of the downward-sloping segment of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ with $W^{d=1}$ determines the default threshold $\hat{\epsilon}$ such that default is optimal only in the *default zone* where $\epsilon \geq \hat{\epsilon}$. Default is still a second-best policy for the planner, because with it the planner cannot attain $W^{d=0}(\epsilon^{SP})$, it just gets the closest it can get. In contrast, the choice of repayment is preferable in the *repayment zone* where $\epsilon < \hat{\epsilon}$,, because in this zone $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W^{d=1}(g_1)$.

Adding default costs to this political bias setup ($\phi(g_1) > 0$) makes it possible to support repayment equilibria even when $\omega \geq \gamma$. As Figure 16 shows, with default costs there are threshold values of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega >=< \gamma$.

Figure 16: Default decision with non-utilitarian planner when $\phi(g_1) > 0$



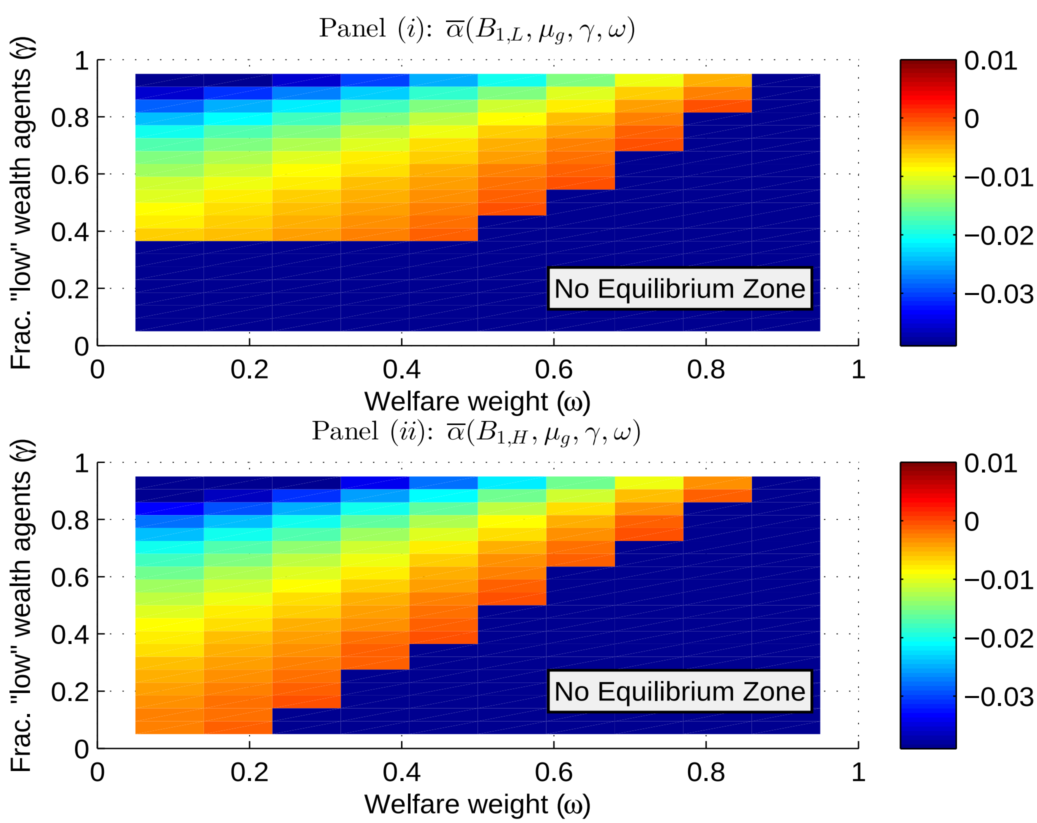
It is also evident in Figure 16 that the range of values of ϵ for which repayment is chosen widens as γ rises relative to ω. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in ϵ, do not produce consumption dispersion larger than the value of $\hat{\epsilon}$ associated with a given (ω,γ) pair. Intuitively, the consumption of H-type agents must not exceed that of L-type agents by more than what $\hat{\epsilon}$ allows, because otherwise default is optimal.

5.2.2 Quantitative Results

We discuss next a set of quantitative results for the model with political bias using same set of calibrated parameter values shown in Table 3 and a range of values of ω. First we examine the planner's welfare gain of default $(\overline{\alpha}(B_1, g_1, \gamma, \omega))$, which is constructed as before but using ω and $1-\omega$ to aggregate the individual welfare gains, instead of γ and 1γ.

Figure 17 shows how the planner's welfare gain of default varies with ω and γ for two different levels of government debt ($B_{1,L} = 0.143$ and $B_{1,H} = 0.185$). The no-equilibrium region, which exists for the same reasons as before, is shown in dark blue.

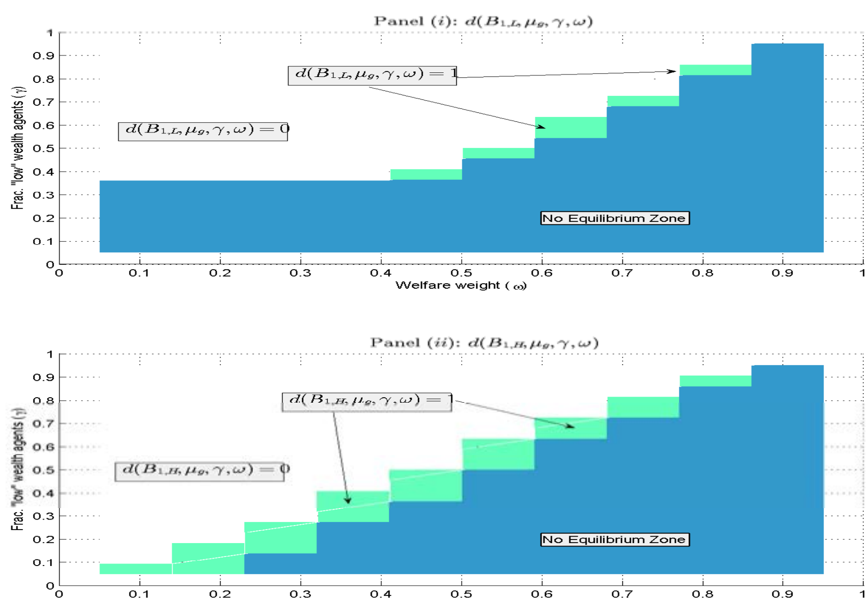
Figure 17: Planner's welfare gain of default $\overline{\alpha}(B_1, g_1, \gamma, \omega)$



In line with the previous discussion, within the region where the equilibrium is well defined, the planner's value of default increases monotonically as its preference for redistribution (ω) increases, keeping γ constant, and falls as actual wealth concentration (γ) rises, keeping ω constant. Because of this, the north-west and south-east corners in each of the panels present cases that are at very different positions on the preference-for-default spectrum. When ω is low, even for very high values of γ, the government prefers to repay (north-west corner), because the government puts relatively small weight on L-type agents. On the contrary, when ω is high, even for low levels of γ, a default is preferred. It is also interesting to note that as we move from Panel (i) to Panel (ii), so that government debt raises, the set of γ's and ω's such that the equilibrium exists or repayment is preferred (i.e. a negative $\overline{\alpha}(B_1, g_1, \gamma, \omega)$) expands. This is because as we increase the level of debt B_1, as long as the government does not choose to default for all g_1, the higher level of debt allows low-wealth households to attain positive levels of consumption (since initial taxes are lower).

Figure 18 shows the default decision rule induced by the planner's welfare gains of default, again as a function of ω and γ for the same two values of B_1. The region in white corresponds to cases where $d(B_1, g_1, \gamma, \omega) = 0$, the green region corresponds to $d(B_1, g_1, \gamma, \omega) = 1$ and the blue region corresponds to cases in which there is no equilibrium.

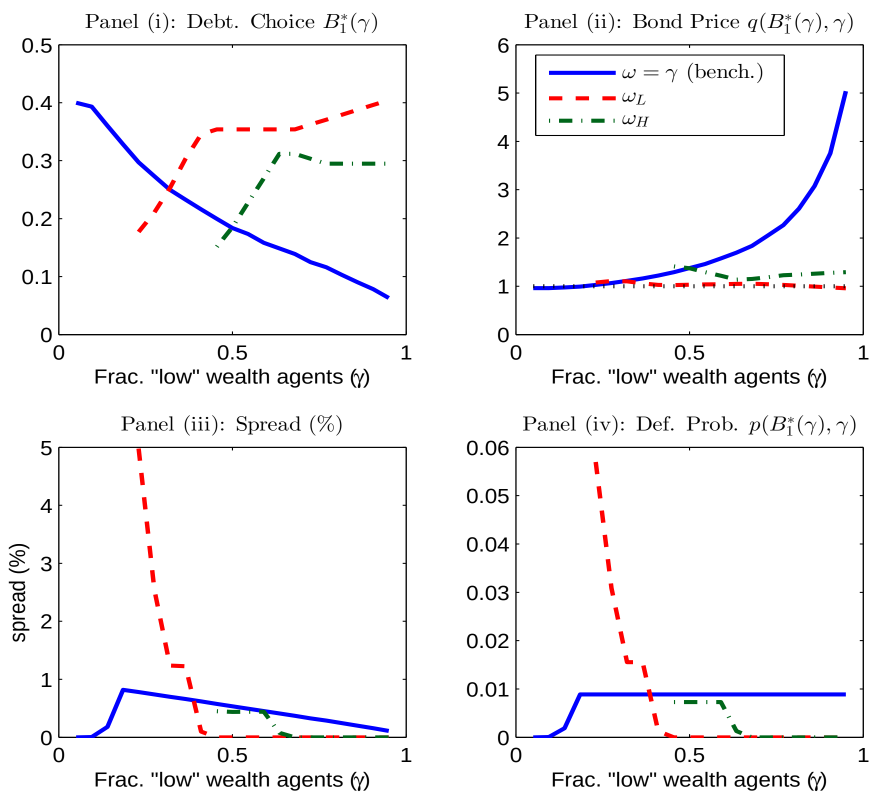
Figure 18: Default Decision Rule $d(B_1, g_1, \gamma, \omega)$



In line with the pattern of the government's welfare gains of default presented in Figure 17, this Figure shows that when the ω is low enough, the government chooses default, and for a given ω the default region is larger the lower is γ. Taxes and prices for given values of B_1 and

ω are linked to the default decision and γ as in the benchmark model and the intuition behind their behavior is straightforward.

Figure 19: Equilibrium of the Model with Political Bias for different values of ω



Panels $(i) - (iv)$ in Figure 19 display the model's equilibrium outcomes for the optimal debt chosen by the government in the first period and the associated equilibrium bond prices, spreads and default probabilities under three possible values of ω, all plotted as functions of γ. The scenario with $\omega = \gamma$, shown in blue corresponds to the utilitarian case of Section 4, and the other two scenarios correspond to high and low values of ω ($\omega_L = 0.32$ and $\omega_H = 0.50$ respectively). It is important to note that along the blue curve of the utilitarian case both ω and γ effectively vary together because they are always equal to each other, while in the other two plots ω is fixed and γ varies. For this reason, the line corresponding to the ω_L case intersects the benchmark solution when $\gamma = 0.32$, and the one for ω_H intersects the benchmark when $\gamma = 0.50$.

Figure 19 shows that the optimal debt level is increasing in γ. This is because, following the analysis illustrated in Figure 16, the incentives to default grow weaker and the repayment zone widens as γ increases for a fixed value of ω. Moreover, the demand composition effect of higher γ is still present, so along with the lower default incentives we still have the increasing per capita demand for bonds of H types. These two effects combined drive the increase in the optimal debt choice of the government. Note, however, that the mechanism wears off around $\gamma = 0.40$, with a 40% debt ratio, for ω_L, and $\gamma = 0.7$, with a 30% debt ratio, for ω_H. It is also interesting to note that in the ω_L and ω_H cases the equilibrium exists only for a small range of values of γ that are lower than ω. Without default costs each curve would be truncated exactly where γ equals either ω_H or ω_H, but since these simulations retain the default costs used in the utilitarian case, there can still be equilibria with debt for some lower values of γ (as explained earlier).

In this model with political bias, the government is still aiming to optimize debt by focusing on the resources it can reallocate across periods and agents, which are still determined by the debt Laffer curve $q(B_1)B_1$, and internalizing the response of bond prices to debt choices.[27] This relationship, however, behaves very differently than in the benchmark model, because now *higher* optimal debt is carried at increasing equilibrium bond prices, which leads the planner internalizing the price response to choose higher debt, whereas in the benchmark model *lower* optimal debt was carried at increasing equilibrium bond prices, which led the planner internalizing the price response to choose lower debt.

In the empirically relevant range of γ, and for values of ω lower than that range (since $\omega_L = 0.32$ and $\omega_H = 0.50$, while the relevant range of γ is $[0.55 - 0.85]$), this model can sustain significantly higher debt ratios than the model with utilitarian payoff, and those ratios are close to the observed European median. At the lower end of that range of γ, a government with ω_H chooses a debt ratio of about 25 percent with a 1 percent probability of default and a spread of 40 basis points, while a government with ω_L chooses a debt ratio of about 36 percent with a negligible default probability.

The behavior of equilibrium bond prices (panel (ii)) with either $\omega_L = 0.32$ or $\omega_H = 0.50$ differs markedly from the utilitarian case. In particular, the prices no longer display an increasing, convex shape, instead they are a relatively flat and non-monotonic function of γ. This occurs because the higher supply of bonds that the government finds optimal to provide offsets the demand composition effect that increases individual demand for bonds as γ rises.

At low values of γ the government chooses lower debt levels (panel (i)) in part because the default probability is higher (panel (iv)), which also results in higher spreads (panel (iii)). But as γ rises and repayment incentives strengthen (because ω becomes relatively smaller than *gamma*), the probability of default falls to zero, the spreads vanish, and debt levels increase. The price remains relatively flat because, again, the higher debt supply offsets the demand

[27]When choosing B_1, the government takes into account that higher debt increases disposable income for L-type agents in the initial period but it also implies higher taxes in the second period (as long as default is not optimal). Thus, the government is willing to take on more debt when ω is lower.

composition effect.

6 Conclusions

This paper proposes a framework in which domestic sovereign default and public debt subject to default risk emerge as an equilibrium outcome. In contrast with standard models of sovereign default on external debt, this framework highlights the role of wealth heterogeneity and the distributional effects of default across domestic agents in shaping the government's default incentives. These are features common to both the historical episodes of outright domestic default documented by Reinhart and Rogoff (2008) and the ongoing European debt crisis.

The framework we developed consists of a two-period model with high- and low-wealth agents, non-insurable aggregate uncertainty in the form of shocks to government expenditures and default risk, and a utilitarian government who sets debt, taxes and the default decision responding to distributional incentives. The government is aware of its inability to commit, and chooses how much debt to issue seeking to allocate resources optimally across agents and across periods. In choosing the optimal debt, the government takes into account how the debt chosen in the first period influences the second period's default incentives and default probability, and the feedback of these two into the first period's equilibrium price of government bonds and the resources that can be raised by borrowing, which are driven by a variant of the well-known debt Laffer curve of the external default literature.

In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Default is optimal when repaying hurts relatively poor agents more than defaulting hurts relatively rich agents, and this happens at optimally-chosen debt ratios if the ownership of public debt is sufficiently concentrated and government expenditures are relatively high. Under these conditions, the government values more the social costs implied by the increased taxation that is needed to both service the debt and pay for government expenditures than the costs associated with wiping out assets owned by the high-wealth agents. We showed, however, that distributional incentives alone cannot support equilibria with debt, because default is always optimal in the second period, and hence the debt market cannot function in the first period. We also showed that equilibria with debt can exist if we introduce either exogenous default costs or government preferences biased in favor of bond holders.

Quantitative results based on a calibration to European data show that sustainable debt falls and default risk rises as the fraction of low-wealth individuals rises. Because of default risk, sustainable debt is much lower than when the government is committed to repay (at the same levels of wealth inequality). In the range of observed ratios of the fraction of agents who own government bonds, the model supports debt ratios of about 1/3rd of the average European debt ratio at spreads close to 40 basis points and a default probability around 0.9%.

In the variant of the model with government welfare weights biased in favor of bond holders, sustainable debt becomes an increasing function of wealth inequality, instead of decreasing as in the utilitarian benchmark. This is because incentives to default grow weaker as the government's weight on low-wealth agents falls increasingly below the actual fraction of the population that they represent. In this setup, optimal debt levels chosen in the empirically relevant range of wealth inequality measures exceed those supported in the utilitarian benchmark by wide margins, and can be of similar magnitude as the European media.

We see this model as a simple blueprint for further research into models of domestic sovereign default driven by distributional incentives and their interaction with agent heterogeneity and incomplete insurance markets. The two-period environment is a very useful starting point because of the ease with which it can be analyzed and solved, but it also imposes limitations on attempts to bring the model to the data. In particular, self-insurance motives, which the literature shows can produce significant welfare benefits for the existence of public debt markets (see Aiyagari and McGrattan (1998)), are minimized by the two-period life horizon. In an infinite horizon model, this mechanism could produce a large *endogenous* cost of default that may support the existence of public debt subject to default risk without exogenous costs of default and/or political biases in the government's preferences. In work in progress we are looking into this possibility.

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A1 Appendix: The Model without Default Risk and Logarithmic Utility

This Appendix derives solutions for a version of the model in which low-wealth (L) types do not hold any bonds and high-wealth (H) types buy all the debt. We cover first the fully deterministic case, without any shocks to income or government policies, and no default risk, but government expenditures may be deterministically different across periods. Government wants to use debt to relocate consumption across agents and across periods optimally given a utilitarian welfare function. Ruling out default on initial outstanding debt, the planner trades off the desire to use debt to smooth taxation for L types (reduce date-0 taxes by issuing debt) against the cost of the postponement of consumption this induces on H types who save to buy the debt. Log utility provides closed form solutions. The goal is to illustrate the mechanisms that are driving the model when default risk and stochastic government purchases are taken out. Later in the Appendix we derive some results for the model with stochastic government purchases, and make some inferences for the case with default risk.

A1.1 Fully deterministic model

A1.1.1 Households

A fraction γ of agents are L types, and $1-\gamma$ are H types. Preferences are:

$$\ln(c_0^i) + \beta \ln(c_1^i) \qquad for \quad i = L, H \tag{A.1}$$

Budget constraints are:

$$c_0^L = y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - q b_1^H \tag{A.2}$$

$$c_1^L = y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H \tag{A.3}$$

Since L types do not save, the solution to their problem is trivial: they can only consume what their budget constraints allow. This is important because altering taxes affects disposable income, which will in turn affect the optimal debt choice of the government. For H types, the Euler equation is:

$$q = \beta \frac{c_0^H}{c_1^H} \tag{A.4}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, it must be the case that they are credit constrained (i.e. they would want to hold negative assets). That is, at the equilibrium price of debt their Euler equation for bonds would satisfy:

$$q > \beta \frac{c_0^L}{c_1^L} \tag{A.5}$$

A1.1.2 Government

The government budget constraints are:

$$\tau_0 = g_0 + B_0 - qB_1 \tag{A.6}$$

$$\tau_1 = g_1 + B_1 \tag{A.7}$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma(\ln(c_0^L) + \beta \ln(c_1^L)) + (1-\gamma)(\ln(c_0^H) + \beta \ln(c_1^H)) \tag{A.8}$$

A1.1.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $b_1^H = B_1/(1-\gamma)$. By construction the same condition is assumed to hold for the initial conditions b_0^H and B_0.This implies that the initial wealth of H-types is given by $b_0^H = B_0/(1-\gamma)$.

Rewriting the Euler equation of H types using the budget constraint, the government budget constraints and the bond market-clearing conditions yields:

$$q = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q\left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \tag{A.9}$$

Hence, the equilibrium price of bonds for a given government supply is:

$$q(B_1) = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta) B_1} \tag{A.10}$$

Note that this price is not restricted to be lower than 1 (i.e. $q(B_1) > 1$ which implies a negative real rate of return on government debt can be an equilibrium outcome). In particular, as γ rises the per capita bond demand of H-types increases and this puts upward pressure on bond prices, and even more so if the government finds it optimal to offer less debt than the initial debt, as we showed numerically and explain further below. As $\gamma \to 1$, the limit of the equilibrium price goes to $q(B_1) = \frac{\beta}{1+\beta}\frac{B_0}{B_1}$ even tough market-clearing requires the demand of the infinitesimal small H type to rise to infinity.

After some simplification, the derivative of this price is given by:

$$q'(B_1) = \frac{-q(B_1)\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)}{\left[y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta)B_1\right]} \tag{A.11}$$

which at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$ (notice $c_1^H > 0$ implies that the denominator of this expression must be positive).

Consider now what happens to this equilibrium as the fraction of L-types vanishes. As $\gamma \to 0$, the economy converges to a case where there is only an H type representative agent, and the price is simply $q(B_1) = \beta\frac{y-g_0}{y-g_1}$, which is in fact independent of B_1 and reduces to β if government purchases are stationary. Trivially, in this case the planner solves the same problem as the representative agent and the equilibrium is efficient. Also, for an exogenously given B_0 and stationary g, the competitive equilibrium is stationary at this consumption level:

$$c^h = y - g + \left(\frac{\gamma}{1-\gamma}\right)\frac{B_0}{1+\beta} \tag{A.12}$$

and the optimal debt is:

$$B_1 = \frac{B_0}{1+\beta} \tag{A.13}$$

Hence, in this case consumption and disposable income each period are fully stationary, yet the optimal debt policy is always to reduce the initial debt by the fraction $1/(1+\beta)$. This is only because of the two-period nature of the model. With an infinite horizon, the same bond price would imply that an equilibrium with stationary consumption and an optimal policy that is simply $B_1 = B_0$. It also follows trivially that carrying no initial debt to start with would be first-best, using lump-sum taxation to pay for g.

A1.1.4 Optimal debt choice

The government's optimal choice of B_1 in the first period solves this maximization problem:

$$\max_{B_1}\left\{\begin{array}{c}\gamma\left[\ln\left(y - g_0 - B_0 + q(B_1)B_1\right) + \beta\ln\left(y - g_1 - B_1\right)\right] \\ +(1-\gamma)\left[\ln\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0 - q(B_1)\left(\frac{\gamma}{1-\gamma}\right)B_1\right) + \beta\ln(y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)B_1)\right]\end{array}\right\} \tag{A.14}$$

where $q(B_1) = \beta \frac{y-g_0+\left(\frac{\gamma}{1-\gamma}\right)B_0}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)B_1}$.

The first-order condition is:

$$\begin{aligned} &\gamma\left[u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta u'(c_1^L)\right] \\ &+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right)\left[-u'(c_0^H)\left[q'(B_1)B_1 + q(B_1)\right] + \beta u'(c_1^H)\right] = 0 \end{aligned} \tag{A.15}$$

Using the Euler equation of the H types and simplifying:

$$u'(c_0^L) + \left[\frac{u'(c_0^L)q(B_1) - \beta u'(c_1^L)}{q'(B_1)B_1}\right] = u'(c_0^H) \tag{A.16}$$

This expression is important, because it defines a wedge between equating the two agents' marginal utility of consumption that the planner finds optimal to maintain, given that the only instrument that it has to reallocate consumption across agents is the debt. Notice that, since as noted earlier for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation implies that at the equilibrium price: $u'(c_0^L)q(B_1) - \beta u'(c_1^L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L) > u'(c_0^H)$ or $c_0^H > c_0^L$, and notice that from the budget constraints this also implies $B_0 - q(B_1)B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$.

Comparison with no-debt equilibrium: Notice that since $B_0 - q(B_1)B_1 > 0$, the planner is allocating less utility to L type agents than those agents would attain without any debt. Without debt, and a tax policy $\tau_t = g_t$, all agents consume $y - g_t$ every period, but with debt L-types consume less each period given that $B_1 > 0$ and $B_0 - q(B_1)B_1 > 0$. Compared with these allocations, when the planner finds optimal to choose $B_1 > 0$ is because he is trading off the pain of imposing higher taxes in both periods, which hurts L types, against the benefit the H types get of having the ability to smooth using government bonds. Also, $B_0 - q(B_1)B_1 > 0$ highlights that there is a nontrivial role to the value of B_0, because if B_0 were zero B_1 would need to be negative which is not possible by construction. Hence, the model only has a sensible solution if there is already enough outstanding debt (and wealth owned by H type agents) that gives the government room to be able to improve the H type's ability to smooth across the two periods, which they desire to do more the higher is B_0.

Comparison with sub-optimal debt equilibrium: By choosing positive debt, the government provides tax smoothing for L types. Given B_0 and the fact that $B_0 - q(B_1)B_1 > 0$, positive debt allows to lower date-0 taxes, which increases consumption of L types (since

$c_0^L = y - g_0 - B_0 + q(B_1)B_1)$. The same policy lowers the consumption of H types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)(B_0 - q(B_1)B_1))$. Hence, debt serves to redistribute consumption across the two agents within the period. This also changes inter-temporal consumption allocations, with the debt reducing L types consumption in the second period and increasing H types consumption. Hence, with commitment to repay B_0, the debt will be chosen optimally to trade off these social costs and benefits of issuing debt to reallocate consumption atemporally across agents and intertemporally.

It is also useful to notice that the demand elasticity of bonds is given by $\eta \equiv q(B_1)/\left(q'(B_1)B_1\right)$, so the marginal utility wedge can be expressed as $\eta\left[u'(c_0^L) - \frac{\beta u'(c_1^L)}{q(B_1)}\right]$ and the planner's optimality condition reduces to:

$$1 + \eta\left[1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)}\right] = \frac{u'(c_0^H)}{u'(c_0^L)} \tag{A.17}$$

Hence, the planner's marginal utility wedge is the product of the demand elasticity of bonds and the L-type agents shadow value of being credit constrained (the difference $1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)} > 0$, which can be interpreted as an effective real interest rate faced by L-type agents that is higher than the return on bonds). The planner wants to use positive debt to support an optimal wedge in marginal utilities only when the demand for bonds is elastic AND L-type agents are constrained.

A1.2 Extension to Include Government Expenditure Shocks

Now consider the same model but government expenditures are stochastic. In particular, realizations of government purchases in the second period are given by the set $[g_1^1 < g_1^2 < ... < g_1^M]$ with transition probabilities denoted by $\pi(g_1^i|g_0)$ for $i = 1, ..., M$ with $\sum\limits_{i=1}^{M} \pi(g_1^i|g_0) = 1$.

A1.2.1 Households

Preferences are now:

$$\ln(c_0^i) + \beta\left(\sum_{i=1}^{M} \pi(g_1^i|g_0)\ln(c_1^i)\right) \qquad for \quad i = L, H \tag{A.18}$$

Budget constraints are unchanged:

$$\begin{aligned} c_0^L &= y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - qb_1^H && \text{(A.19)} \\ c_1^L &= y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H && \text{(A.20)} \end{aligned}$$

We still assume that L types do not save, so they can only consume what their budget constraints allow. For H types, the Euler equation becomes:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left(\frac{c_0^H}{c_1^H} \right) \tag{A.21}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, their Euler equation for bonds must satisfy:

$$q > \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left(\frac{c_0^L}{c_1^L} \right) \tag{A.22}$$

A1.2.2 Government

The government budget constraints are unchanged:

$$\begin{aligned} \tau_0 &= g_0 + B_0 - qB_1 \\ \tau_1 &= g_1 + B_1 \end{aligned}$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma \left(\ln(c_0^L) + \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \ln(c_1^L) \right) + (1-\gamma) \left(\ln(c_0^H) + \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \ln(c_1^H) \right) \tag{A.23}$$

A1.2.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $b_1^H = B_1/(1-\gamma)$ and the H-types Euler equation.

We can solve the model in the same steps as before. First, rewrite the Euler equation of H types using their budget constraints, the government budget constraints and the market-clearing conditions:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left[\frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q \left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \right] \tag{A.24}$$

From here, we can solve again for the equilibrium price at a given supply of bonds:

$$q(B_1) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1}\right)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1}\right)} \tag{A.25}$$

As $\gamma \to 0$ we converge again to the world where there is only an H type representative agent, but now the pricing formula reduces to the standard formula for the pricing of a non-state-contingent asset $q(B_1) = \beta \left(\sum_{i=1}^{M} \pi(g_1^i|g_0)\frac{y-g_0}{y-g_1}\right)$. As $\gamma \to 1$ the equilibrium degenerates again into a situation where market clearing requires the demand of the infinitesimal small H type to rise to infinity.

The derivative of the price at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$. To show this, define $\Pi(B_1) \equiv \sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1}$ which yields $\Pi'(B_1) = -\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1\right)^2} < 0$. Then taking the derivative $q'(B_1)$ and simplifying we get:

$$q'(B_1) = \frac{\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right)\left[\Pi'(B_1) - \beta\left(\frac{\gamma}{1-\gamma}\right)(\Pi(B_1))^2\right]}{\left(1 + \beta\left(\frac{\gamma}{1-\gamma}\right) B_1 \Pi(B_1)\right)^2} \tag{A.26}$$

Since $\Pi'(B_1) < 0$ and positive c_0^H implies $y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 > 0$, it follows that $q'(B_1) < 0$.

We can also gain some insight into the implicit risk premium (the ratio $q(B_1)/\beta)$) and the related question of why the asset price can exceed 1 in this setup. Recall that in fact the latter was already possible without uncertainty when γ is large enough, because of the demand composition effect (higher γ implies by market clearing that the fewer H type agents need to demand more bonds per capita, so the bond price is increasing in γ and can exceed 1). The issue now is that, as numerical experiments show, an increase in the variance of g_1 also results in higher bond prices, and higher than in the absence of uncertainty, and again for γ large enough we get both $q(B_1) > 1$ and $q(B_1)/\beta > 1$. The reason bond prices increase with the variability of government purchases is precautionary savings. Government bonds are the only vehicle of saving, and the incentive for this gets stronger the larger the variability of g_1. Hence, the price of bonds is *higher* in this stochastic model than in the analogous deterministic model because of precautionary demand for bonds, which adds to the effect of demand composition (i.e. the price is higher with uncertainty than without at a given γ).

A1.2.4 Pricing Function with Default

We can also make an inference about what the pricing function looks like in the model with default risk, because with default we have a similar Euler equation, except that the summation that defines the term $\Pi(B_1)$ above will exclude all the states of g_1 for which the government chooses to default on a given B_1 (and also at a given value of γ). That is, the term in question becomes $\Pi^D(B_1) \equiv \sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1} \leq \Pi(B_1)$, and the pricing function with default risk is:

$$q^D(B_1) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \Pi^D(B_1)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \Pi^D(B_1)} \leq q(B_1) \tag{A.27}$$

Moreover, it follows from the previous analysis that this pricing function is also decreasing in B_1 ($q^{D\prime}(B_1) < 0$), and $\Pi^{D\prime}(B_1) = -\sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1\right)^2}$ is negative but such that $\Pi'(B_1) \leq \Pi^{D\prime}(B_1) < 0$. Also, it is clear from the above pricing functions that if the probability of default is small, so that are only a few values of i for which $d(B_1, g_1^i, \gamma) = 1$ and/or the associated probability $\pi(g_1^i|g_0)$ is very low, the default pricing function will be very similar to the no-default pricing function.

If we define the default risk spread as $S(B_1, \gamma) \equiv [1/q^D(B_1, \gamma)] - [1/q(B_1, \gamma)]$, where γ has been added as an argument of the price functions because those prices also depend on variations in inequality, the spread reduces to the following expression:

$$S(B_1, \gamma) = \left(\frac{1}{\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right)} \right) \left[\frac{1}{\Pi^D(B_1, \gamma)} - \frac{1}{\Pi(B_1, \gamma)} \right]$$

Clearly since $\Pi^D(B_1, \gamma) \leq \Pi(B_1, \gamma)$ the spread is non-negative, and it is strictly positive if there is default at equilibrium. The spread is increasing in B_1, because as the debt rises default is chosen optimally in more of the possible realizations of g_1^i and hence $\Pi^D(B_1, \gamma)$ falls further below $\Pi(B_1, \gamma)$, so that the gap between the reciprocals of these two terms widens. Note also that the spread is a multiple of the gap between these reciprocals, with the multiple given by $1/\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right)$. As a result, the total date-0 resources available for consumption of the H-types ($y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0$) have a first-order *negative* effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. The are also second order effects, because the equilibrium allocation of B_1 also depends on that income measure, and thus $\Pi^D(B_1, \gamma)$ and $\Pi(B_1, \gamma)$ vary with it as well, but these are not considered here.

In terms of the effect of changes in γ on S, notice that there are two effects. First, there is a negative effect because higher γ means that, for a given B_0, the resources available for date-0 consumption of H types increase, since fewer H type agents need to demand enough initial bonds to clear the bond market, which means that per-capita each of the H types hold more date-0 bonds and have more bond income. Second, there is a positive effect because rising γ weakens default incentives as the welfare of the wealthy is valued more, and hence default is optimally chosen in more states, which increases $\left[1/\Pi^D(B_1,\gamma) - 1/\Pi(B_1,\gamma)\right]$. Thus, in principle the response of the spread to increases in inequality is ambiguous. The weaker the response of the default probability to changes in γ, however, the more likely it is that the first effect will dominate and the spreads will be a decreasing function of inequality.

A1.2.5 Optimal debt choice

The government's optimal choice of B_1 solves again a standard maximization problem:

$$\max_{B_1} \left\{ \begin{array}{c} \gamma \left[\ln\left(y - g_0 - B_0 + q(B_1)B_1\right) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln\left(y - g_1 - B_1\right) \right] \\ +(1-\gamma) \left[\begin{array}{c} \ln\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q(B_1) \left(\frac{\gamma}{1-\gamma}\right) B_1\right) \\ +\beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1) \end{array} \right] \end{array} \right\} \tag{A.28}$$

where $q(B_1)$ is given by the expression solved for in the competitive equilibrium.

The first-order condition is:

$$\begin{aligned} &\gamma \left[u'(c_0^L) \left[q'(B_1)B_1 + q(B_1)\right] - \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) u'(c_1^L) \right] \\ &+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right) \left[-u'(c_0^H) \left[q'(B_1)B_1 + q(B_1)\right] + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) u'(c_1^H) \right] = 0 \end{aligned} \tag{A.29}$$

Using the stochastic Euler equation of the H types and simplifying:

$$u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta \sum_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L) = u'(c_0^H)q'(B_1)B_1 \tag{A.30}$$

$$u'(c_0^L) + \left[\frac{u'(c_0^L)q(B_1) - \beta \sum\limits_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L)}{q'(B_1)B_1}\right] = u'(c_0^H) \tag{A.31}$$

This last expression, compared with the similar expression of the planner without uncertainty, implies that in the planner's view, the government expenditure shocks only matter to the extent that they affect the shadow price of the binding credit constraint of the L types. As before, since for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation would imply that at the equilibrium price: $u'(c_0^L)q(B_1) - \beta \sum\limits_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L) > u'(c_0^H)$ or $c_0^H > c_0^L$, and notice that from the budget constraints this implies again $B_0 - q(B_1)B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$. Thus the optimal debt choice again has an incentive to be lower than the initial debt.