NBER WORKING PAPER SERIES

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Working Paper 19477
http://www.nber.org/papers/w19477

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2013

We acknowledge the support of the National Science Foundation under awards 1325122 and 1324740. The comments by participants at the Preconference for Sovereign Debt and Financial Crises Project of the NBER are also gratefully acknowledged. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Philadelphia, the Federal Reserve System, or the National Bureau of Economic Research.

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NBER Working Paper No. 19477
September 2013
JEL No. E44,E6,F34,H63

ABSTRACT

International historical records on public debt show infrequent episodes of outright default on domestic debt. Reinhart and Rogoff (2008) document these events and argue that they constitute a "forgotten history" in Macroeconomics. This paper develops a theory of domestic sovereign default in which distributional incentives, interacting with default costs, make default part of the optimal policy of a utilitarian social planner. The model supports equilibria with debt subject to default risk in which rising wealth inequality reduces the optimal debt and increases default probabilities and spreads. A quantitative experiment calibrated to European data shows that, in the observed range of inequality in the distribution of bond holdings, the model accounts for 1/3rd of the average debt and spreads of about 400 basis points. Default risk reduces sharply the sustainable debt, except when the weights in the government's payoff function value the utility of bond holders more than their share of the wealth distribution. If the former is sufficiently larger than the latter, the model supports debt ratios similar to European averages exposed to low default probabilities.

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1 Introduction

The innovative study by Reinhart and Rogoff (2008) identified 68 episodes in which governments defaulted outright (i.e. by means other than inflation) on their domestic creditors in a cross-country database going back to 1750. These domestic defaults occur via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspension of payments. Reinhart and Rogoff also documented that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about $2/3^{\text{rds}}$ on average), and that domestic default events were associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses and full-blown economic crises. Despite of these striking features, they also found that domestic sovereign default is a "forgotten history" that remains largely unexplored in the Macroeconomics literature.

The ongoing European debt crisis also highlights the importance of focusing research on domestic sovereign default, because four features of this crisis make it more akin to a domestic default than to the typical external default that dominates the literature on this subject. First, countries in the Eurozone are highly integrated in goods and asset markets, and the majority of their public debt is denominated in their common currency and held by European residents. Hence, from an European standpoint, default by one or more Eurozone governments means a suspension of payments to "domestic" agents, instead of external creditors. Second, domestic public debt-GDP ratios are high in the Eurozone as a whole, and very large in some of its members, particularly in the countries at the epicenter of the crisis (Greece, Ireland, Italy, Spain and Portugal). Third, the Eurozone's common currency and common central bank rule out the possibility of individual governments resorting to inflation as a means to lighten their debt burden without an outright default. Fourth, and perhaps most important from the standpoint of the theory developed in this paper, European-wide institutions such as the ECB and the European Commission are weighting the interests of both creditors and debtors in assessing the pros and cons of sovereign defaults by individual countries, and both creditors and debtors are aware of these institutions' concern and of their key role in influencing expectations and the potential for repayment or default.[1]

This paper proposes a framework for explaining domestic sovereign default motivated by the key fact that a domestic default entails substantial redistribution across domestic agents, with all of these agents presumably entering in the payoff function of the sovereign. This is in sharp contrast with what standard models of external sovereign default assume, particularly

[1]The analogy with a domestic default is imperfect, however, because the Eurozone is not a single country, and in particular there is no fiscal entity with tax and debt-issuance powers over all the members. Still, the situation resembles more a domestic default than an external default in which debtors are not concerned for the interests of creditors.

those based on the classic work of Eaton and Gersovitz (1981). Models in this class approach default as a decision made by a government with a payoff given by the utility of a representative home agent, and assuming risk-neutral foreign lenders who are completely disconnected from the economy they lend to (except for the fact that the lenders bought the home government's debt taking a risk neutral bet on the possibility of default). In these models, the effects of a government default on the welfare of creditors are irrelevant for the sovereign making the default decision, and both the costs and benefits of default affect all domestic agents in the same way (since the economy is inhabited by a representative agent).

These observations suggest that standard models of sovereign default face serious limitations in explaining the forgotten history of domestic debt. In actual domestic defaults and in the European crisis, governments and institutions making default decisions are taking into account the implications of the default choice on the welfare of government creditors, and evaluate the different costs and benefits of default on various groups of domestic agents. Hence, a theoretical framework aiming to explain domestic default needs to reformulate the government's strategic incentives so as to take into account default effects on both creditors and debtors, which in turn implies that agent heterogeneity also needs to be taken into account.

We propose a two-period model with heterogeneous agents and non-insurable aggregate risk in which domestic default can be optimal for a utilitarian government that responds to distributional incentives. A fraction γ of agents start as low-wealth (L) agents and a fraction $1-\gamma$ are high-wealth (H) agents, depending on the size of their initial holdings of government bonds. The government finances the gap between exogenous stochastic expenditures and endogenous lump-sum taxes by issuing non-state-contingent debt, retaining the option to default. This government evaluates the costs and benefits of default according to a utilitarian social welfare function, which uses γ and $1-\gamma$ to weight the welfare of L and H agents respectively. We also study an extension in which the government's payoff function has ad-hoc weights, which can be justified by political economy considerations.

Private agents choose optimally their bond holdings in the first period, taking as given bond prices and the probability that a default may occur in the second period. The government chooses how much debt to issue taking into account its inability to commit to repay: First, it evaluates the agents payoffs under repayment and default given their optimal savings plans and the government budget constraints. Second, it uses those payoffs to formulate a default decision rule for the second period that depends on how much debt is issued, the realization of government expenditures, and the degree of wealth concentration as measured by γ. Third, it chooses optimally how much debt to issue in the first period to maximize the lifetime utilitarian expected utility internalizing how the debt choice affects default incentives, default risk and the price of bonds. In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions.

Default is optimal when the taxes needed to repay the debt hurts relatively poor agents more than defaulting hurts relatively rich agents, and this happens when, for a given amount of debt and wealth concentration, the realization of government expenditures is high enough. This is necessary but not sufficient, however, for the model to support an equilibrium with debt subject to default risk. It is also necessary that the government finds it optimal to choose debt levels in the first period that are left exposed to default risk in the second period, even after the government internalizes that default risk and how the equilibrium price of bonds responds to the amount of debt being issued.

A third necessary condition to support an equilibrium with debt exposed to default risk is that default entails a cost in terms of disposable income. In particular, we show that with the weighted utilitarian social welfare function, the government will always default if default is costless. This is because the planner desires the lowest consumption dispersion that it can attain, and this is attained by choosing default. In contrast, when default is costly, if the amount of consumption dispersion that the competitive equilibrium with repayment supports yields higher welfare than the default equilibrium net of default cost, repayment becomes optimal. In contrast, we show in the political economy extension that debt can be supported even without default costs, if the government's weight on L-type agents is lower than the actual fraction of these agents in the wealth distribution.

Quantitative results based on a calibration to European data are used to illustrate the model's key predictions. The benchmark model with utilitarian social welfare displays default risk that is increasing in the level of wealth concentration. Hence, lower public debt is sustainable as γ rises. Because of default risk, the sustainable debt is significantly lower than what the same model supports at the same levels of wealth inequality and with the same government expenditure shocks but without default risk. Some of the optimally chosen debt positions over a range of values of γ exhibit zero default risk, and some are exposed to a positive probability of default. The latter are lower and tend to be associated with higher levels of wealth inequality. In the range of empirically relevant ratios of the fraction of agents who own government bonds, the model supports debt ratios about 1/3rd of the average European debt ratio at spreads close to 400 basis points. Qualitatively these results are robust to changes in the government's initial debt and initial level of expenditures, and the size of default costs, but quantitatively they vary.

In the political economy extension, for given government weights pinning down its preference for redistribution, the debt is an *increasing* function of observed wealth inequality, instead of *decreasing* as in the utilitarian benchmark. This is because the incentives to default get weaker as the government's weight on L-type agents falls increasingly below γ. The lower the weight capturing the government's preference for redistribution, the higher the debt that can be supported at every value of γ. These debt amounts can easily exceed those supported in the utilitarian benchmark by wide margins, and can be of similar magnitude as the European average with a default probability of about 5 percent.

This work is related to various strands of the large literature on public debt. First, studies on the role of public debt as a self-insurance mechanism and a vehicle that alters consumption dispersion in heterogeneous agents models without default (e.g. Aiyagari and McGrattan (1998), Golosov and Sargent (2012)). Second, the literature on external sovereign default using models in the line of the Eaton-Gersovitz model (e.g. Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2012), Yue (2010)) but with the important differences noted earlier.[2] Third, another important strand of the external default literature that focuses on the role of secondary markets and discriminatory v. nondiscriminatory default (e.g. Broner, Martin and Ventura (2010) and Gennaioli, Martin and Rossi (2013)).[3] Fourth, the literature on political economy and sovereign default, which also largely focuses on external default (e.g. Amador (2003), Dixit and Londregan (2000), D'Erasmo (2011) Guembel and Sussman (2009), Hatchondo, Martinez and Sapriza (2009) and Tabellini (1991)), but includes studies like those of Alesina and Tabellini (1990) and Aghion and Bolton (1990) that focus on political economy aspects of government debt in a closed economy.

The rest of this paper is organized as follows: Section 2 describes the payoff functions and budget constraints of households and government. Section 3 characterizes the model's equilibrium and provides an illustration of the mechanism that drives optimal default as a policy for redistribution. Section 4 presents the benchmark calibration and the quantitative results for the utilitarian social welfare function. Section 5 discusses the results of a sensitivity analysis and the political economy extension. The last Section provides conclusions.

2 Model Economy

Consider a two-period economy inhabited by a continuum of agents with aggregate unit measure. Agents differ in their initial wealth position, which is characterized by their holdings of government debt at the beginning of the first period. This initial distribution of wealth is

[2]See also Panizza, Sturzenegger and Zettelmeyer (2009), Aguiar and Amador (2013), and Wright (2013) for detailed reviews of the sovereign debt literature.

[3]Default in our setup is also non-discriminatory, because the government cannot discriminate across different types of agents when it defaults. Our setup differs in that the default decision is driven by the distribution of debt among domestic agents and the incentives of the government to use debt optimally as redistributive policy.

exogenous, but the distribution at the beginning of the second period period is endogenously determined by the agents' savings choices of the first period. The government is represented by a social planner with a utilitarian payoff who issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents. There is explicit aggregate risk in the form of shocks to government outlays, and also implicit in the form of default risk, and there is no idiosyncratic uncertainty.

2.1 Household Preferences & Budget Constraints

All agents have the same preferences, which are given by:

$$u(c_0) + \beta E[u(c_1)], \qquad u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0,1)$ is the discount factor and c_t for $t = 0, 1$ is individual consumption. The utility function $u(\cdot)$ takes the standard CRRA form.

All agents receive a non-stochastic endowment y each period and pay lump-sum taxes τ_t. Taxes and newly issued government debt are used to pay for government consumption g_t and repayment of outstanding government debt. We denote the initial supply of outstanding government bonds at $t = 0$ as B_0. Given B_0, the initial wealth distribution is defined by a fraction γ of households who are the L-type individuals with initial bond holdings b_0^L, and a fraction $(1-\gamma)$ who are the H-types and hold b_0^H, where $b_0^H = \frac{B_0 - \gamma b_0^L}{1-\gamma} \geq b_0^L \geq 0$. Hence, b_0^H is the level of bond holdings by H-type agents that is consistent with market-clearing in the government bond market at $t = 0$.

The agents' budget constraints take different form depending on whether the government defaults or not. If the government repays, the budget constraints are:

$$c_t^i + q_t b_{t+1}^i = y + b_t^i - \tau_t \quad \text{for } i = L, H. \tag{1}$$

In this case, agents collect the payout on their individual holdings of government debt (b_t^i), receive endowment income y, and pay lump-sum taxes τ_t, which are uniform across agents. This net-of-tax resources are used to pay for consumption and purchases of new government bonds b_{t+1}^i. If the government defaults, there is no repayment on the outstanding debt and the debt market closes. The agents' budget constraints are

$$c_t^i = (1 - \phi(g_t))y - \tau_t \quad \text{for } i = L, H. \tag{2}$$

As is standard in the sovereign debt literature, we assume that default imposes an exogenous cost that reduces income by a fraction ϕ. This cost ϕ is often modeled as a function of the realization of a stochastic endowment income, but since income is constant in this setup, we model it as a function of the realization of government expenditures g_t. In particular, the cost is an non-increasing, step-wise function: $\phi(g_t) \geq 0$, with $\phi'(g_t) \leq 0$ for $g_t \leq \mu_g$ (where μ_g is the average of government expenditures), $\phi'(g_t) = 0$ otherwise, and $\phi''(g_t) = 0$. This formulation is analogous to the step-wise default cost as a function of income proposed by Arellano (2008) and now widely used in the external default literature, and it also captures the idea of asymmetric costs of tax collection (see Barro (1979) and Calvo (1988)). Note, however, that for the model to support equilibria with debt, a linear default cost is sufficient. This non-linear formulation is useful for the quantitative analysis and for making it easier to compare the model with the standard external default models.[4]

2.2 Government

At the beginning of $t = 0$, the government has outstanding debt B_0 and can issue one-period, non-state contingent discount bonds $B_1 \in \mathcal{B} \equiv [0, \infty)$ at the price $q_0 \geq 0$. Each period it collects lump-sum revenues τ_t and pays for g_t. Since g_0 is known at the beginning of the first period, the relevant uncertainty with respect to government expenditures is for g_1, which is characterized by a well-defined probability distribution function with mean μ_g. We do not restrict the sign of τ_t, so $\tau_t < 0$ represents lump-sum transfers.

At equilibrium, the price of debt must be such that the government bond market clears:

$$B_t = \gamma b_t^L + (1-\gamma) b_t^H \quad \text{for } t = 0, 1. \tag{3}$$

This condition is satisfied by construction in period 0. In period 1, however, the price moves endogenously to clear the market.

The government has the option to default at $t = 1$. The default decision is denoted by $d_1 \in \{0, 1\}$ where $d_1 = 0$ implies repayment. The government evaluates the values of repayment and default as a benevolent planner with a utilitarian social welfare function. The benchmark case is one with a standard weighted utilitarian payoff $\gamma u(c_1^L) + (1-\gamma) u(c_1^H)$. Other government payoff functions can aggregate individual utilities with arbitrary weights, which could be justified by political economy considerations (see Section 5 for details), and can also be extended to incorporate egalitarian concerns. The government, however, cannot discriminate across the two

[4]In external default models, the non-linear cost makes default more costly in "good" states, which alters default incentives to make default more frequent in "bad" states and to support higher debt levels.

types of agents when setting taxation, debt and default policies.

At $t = 0$, the government budget constraint is

$$\tau_0 = g_0 + B_0 - q_0 B_1, \tag{4}$$

The level of taxes in period 1 is determined after the default decision. If the government repays, each household receives the payout on its corresponding b_1^i and taxes are set to satisfy the following government budget constraint:

$$\tau_1^{d_1=0} = g_1 + B_1. \tag{5}$$

Notice that, since this is a two-period model, equilibrium requires that there are no outstanding assets at the end of period 1 (i.e. $b_2^i = B_2 = 0$ and $q_1 = 0$). If the government defaults, taxes are simply set to pay for government purchases:

$$\tau_1^{d_1=1} = g_1. \tag{6}$$

3 Equilibrium

The analysis of the model's equilibrium proceeds in three stages. First, we characterize the households' optimal savings problem and determine their payoff (or value) functions, taking as given the government debt, taxes and default decision. Second, we study how optimal government taxes and the default decision are determined. Third, we examine the optimal choice of debt issuance that internalizes the outcomes of the first two stages. To characterize these problems, we take the values of the initial conditions (g_0, B_0, b_0^L) as exogenous parameters, thereby reducing the set of relevant states to three key variables: B_1, g_1 and γ. Hence, we can index the value of a household as of $t = 0$, before g_1 is realized, by the pair $\{B_1, \gamma\}$. Given this, the level of taxes τ_0 is determined by the government budget constraint once the equilibrium bond price q_0 is set. Bond prices are forward looking and depend on the default decision of the government in period 1, which will be given by the decision rule $d(B_1, g_1, \gamma)$.

3.1 Households' Problem

Given B_1 and γ, a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$v^i(B_1,\gamma) = \max_{b_1^i} \Big\{ u(y + b_0^i - q_0 b_1^i - \tau_0) + \beta E_{g_1}\Big[(1 - d_1(B_1, g_1, \gamma))u(y + b_1^i - \tau_1^{d_1=0}) + d_1(B_1, g_1, \gamma)u(y(1 - \phi(g_1)) - \tau_1^{d_1=1})\Big]\Big\}. \tag{7}$$

The term $E_{g_1}[.]$ in this maximization problem represents the expected payoff across the repayment and default states in period 1. Notice in particular that the payoff in case of default does not depend on the level of individual debt holdings (b_1^i), reflecting the fact that the government cannot discriminate across households when it defaults.

A key feature of the above optimization problem is that agents take into account the possibility of default in formulating their optimal choice of bond holdings. The first-order condition, evaluated at the equilibrium level of taxes, yields the following Euler equation:

$$u'(c_0^i) \geq \beta(1/q_0)E_{g_1}\left[u'(y - g_1 + b_1^i - B_1)(1 - d_1(B_1, g_1, \gamma))\right], \ = 0 \text{ if } \ b_1^i > 0 \tag{8}$$

In states in which, given (B_1, γ), the value of g_1 is such that the government chooses to default ($d_1(B_1, g_1, \gamma) = 1$), the marginal benefit of an extra unit of debt is zero.[5] Thus, conditional on B_1, a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes. This has important redistributive implications, because it implies that when choosing the optimal debt issuance, the government will internalize how by altering the bond supply it can affect the expected probability of default and the equilibrium bond prices. Note also that from the households' perspective, the individual bond decision has no marginal effect on $d_1(B_1, g_1, \gamma)$.

The agents' Euler equation has two other important implications: First, the default risk premium (defined as q_0/β, where $1/\beta$ can be viewed as a hypothetical opportunity cost of funds for an investor, analogous to the role played by the world interest rate in the standard external default model) generally differs from the default probability, because the agents are risk averse, instead of risk-neutral as in the standard model, and in the repayment state they are faced with higher taxation, whereas in the standard model investors are not taxed to repay the debt. For agents with positive bond holdings, the above optimality condition implies that the risk

[5] Utility in the case of default equals $u(y(1 - \phi(g_1)) - g_1)$, and is independent of b_1^i.

premium is equal to $E_{g_1}\left[u'(y-g_1+b_1^i-B_1)(1-d_1(B_1,g_1,\gamma))/u'(c_0^i)\right]$.[6] Second, it is possible, as we confirm numerically in Section 4, that for given values of B_1 and γ, the government chooses $d_1(B_1,g_1,\gamma)=1$ for all g_1. In this case, the expected marginal benefit of purchasing government bonds vanishes from the agents' Euler equation, and hence the equilibrium for that B_1 does not exist, since agents would not be willing to buy debt at any finite price. This result is also reminiscent of a similar result in standard models of external default, in which rationing emerges at t for debt levels so high that the government would choose default at all possible income realizations in $t+1$.

3.2 Government's Problem

Given that the government lacks the ability to commit to repay, we analyze the government's problem following a backward induction strategy by solving first the problem of the government in the final period $t=1$, when default is decided, followed by the optimal debt choice at $t=0$, when the debt issuance is decided. At $t=1$, the government chooses to default or not by solving:

$$\max_{d\in\{0,1\}}\left\{W_1^{d=0}(B_1,g_1,\gamma),W_1^{d=1}(g_1,\gamma)\right\}, \tag{9}$$

where $W_1^{d=0}(B_1,g_1,\gamma)$ and $W_1^{d=1}(B_1,g_1,\gamma)$ denote the values of the social welfare function at the beginning of period 1 in the case of repayment and default respectively. Using the government budget constraint to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, and using the weighted utilitarian social welfare function, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1,g_1,\gamma)=\gamma u(y-g_1+b_1^L-B_1)+(1-\gamma)u(y-g_1+b_1^H-B_1) \tag{10}$$

and

$$W_1^{d=1}(g_1,\gamma)=u(y(1-\phi(g_1))-g_1). \tag{11}$$

[6]With log utility, the debt pricing function with default risk provided in the Appendix can be used to show that the premium starts *lower* than the default probability at low default probabilities, and eventually grows much larger as the probability of default approaches 1.

Combining the above payoff functions, if follows that the government defaults if this condition holds:

$$\gamma\left[u(y-g_1+\overbrace{(b_1^L-B_1)}^{\leq 0})-u(y(1-\phi(g_1))-g_1)\right]+ \tag{12}$$
$$(1-\gamma)\left[u(y-g_1+\overbrace{(b_1^H-B_1)}^{\geq 0})-u(y(1-\phi(g_1))-g_1)\right]\leq 0$$

Notice that all households lose g_1 of their income to government absorption regardless of the default choice, and that utility under default is the same for all agents and given by $u(y(1-\phi(g_1))-g_1)$.

The distributional effects of a default are implicit in condition (12). Given that debt repayment affects the cash-in-hand for consumption of low- and high-wealth agents according to $(b_1^L-B_1)\leq 0$ and $(b_1^H-B_1)\geq 0$ respectively, it follows that, for a given B_1, the payoff under repayment allocates (weakly) lower welfare to L agents and higher to H agents, and that the gap between the two is larger the larger is B_1. Moreover, since the default payoffs are the same for both types of agents, this is also true of the *difference* in welfare under repayment v. default: It is higher for H agents than for L agents and it gets larger as B_1 rises. To induce default, however, it is necessary not only that L agents have a smaller difference in the payoffs of repayment v. default, but that the difference is negative (i.e. they must attain lower welfare under repayment than under default), which requires $B_1 > b_1^L + y\phi(g_1)$. This also implies that taxes under repayment need to be necessarily larger than under default, since $\tau_1^{d=0}-\tau_1^{d=1}=B_1$.

Since we can re-write the consumption allocations under repayment as $c_1^L = y-\tau_1^{d=0}+b_1^L$ and $c_1^H = y-\tau_1^{d=0}+b_1^H$, the distributional effects of default can also be interpreted in terms of how the changes in taxes and wealth caused by a default affect each agent's consumption (and hence utility). First, since $b_1^H > b_1^L$, default has a larger effect on the net worth of H agents than L agents (or no effect if $b_1^L=0$), thus reducing the welfare of the former more than the latter. Second, with regard to taxes, we established above that for default incentives to make default optimal, $\tau_1^{d=0} > \tau_1^{d=1}$. This still has distributional implications, because, even tough both types of agents face the same tax, marginal utility is higher for L agents, and thus they suffer more if taxes rise under repayment. Since repayment requires higher taxes than default, default is always preferable than repayment for L agents.

The distribution of wealth determines the weight the utilitarian planner assigns to the gains and losses that default imposes on the different agents. As γ increases, the fraction of L agents is larger, and thus the value of repayment for the government decreases because it weights more the welfare loss that L agents endure under repayment. Note that differences in γ also affect the date-0 bond decision rules b_1^L and b_1^H and hence the market price of bonds q_0, even for an unchanged supply of bonds B_1.

The distributional mechanism determining the default decision can be illustrated more clearly by means of a graphical tool that compares the utility levels associated with the consumption allocations of the default state with those that would be generally efficient. To this end, it is helpful to express the values of optimal debt holdings as $b_1^L = B_1 - \epsilon$ and $b_1^H(\gamma) = B_1 + \frac{\gamma}{1-\gamma}\epsilon$, for some hypothetical decentralized allocation of debt holdings determined by $\epsilon \in [0, B_1]$. Consumption allocations under repayment would therefore be $c_1^L(\epsilon) = y - g_1 - \epsilon$ and $c_1^H(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon$, so ϵ also determines the decentralized consumption dispersion.

The efficient dispersion of consumption that the social planner would choose is characterized by the value of ϵ^{SP} that maximizes social welfare under repayment, which satisfies this first-order condition:

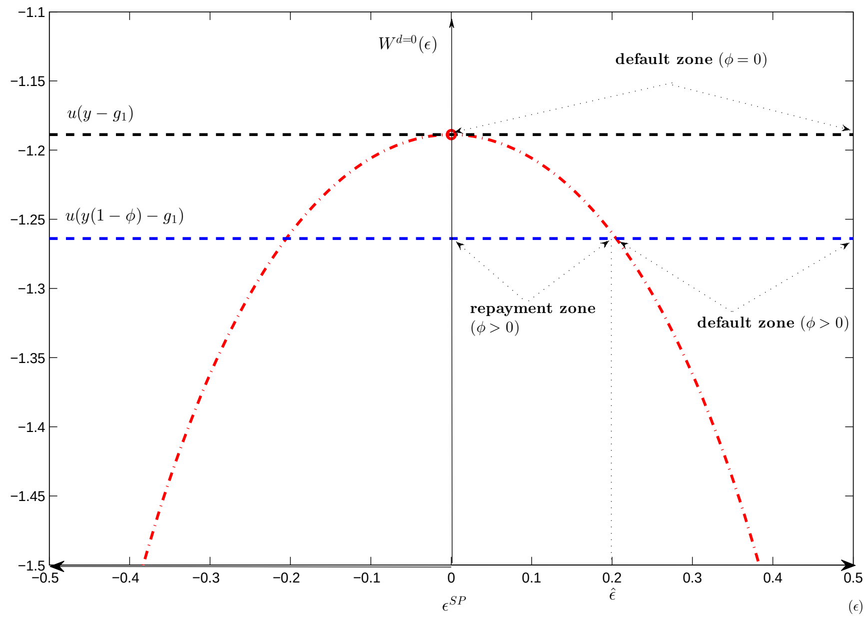
$$u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right) = u'\left(y - g_1 - \epsilon^{SP}\right). \tag{13}$$

Hence, the efficient allocations are characterized by zero consumption dispersion, because equal marginal utilities imply $c^{L,SP}=c^{H,SP} = y - g_1$, which is attained with $\epsilon^{SP} = 0$.

Consider now the government's default decision when default is costless ($\phi(g_1) = 0$). Given that the only policy instruments the government can use, other than the default decision, are non-state contingent debt and lump-sum taxes, it is straightforward to conclude that default will always be optimal. This is because default produces identical allocations in a decentralized equilibrium as the socially efficient ones, since default produces zero consumption dispersion with consumption levels $c^L=c^H = y - g_1$. This outcome is invariant to the values of B_1, g_1, γ and ϵ (over their relevant ranges). Moreover, in this scenario default also yields the first-best outcome that attains maximum social welfare. This result also implies, however, that the model without default costs cannot support equilibria with domestic debt subject to default risk, because default is always optimal.

The above scenario is depicted in Figure 1, which plots the social welfare function under repayment as a function of ϵ as the bell-shaped curve, and the social welfare under default (which is independent of ϵ), as the black dashed line. Clearly, the maximum welfare under repayment is attained when $\epsilon = 0$ which is also the efficient amount of consumption dispersion ϵ^{SP}. Recall also that we defined the relevant range of decentralized consumption dispersion for $\epsilon > 0$, so welfare under repayment is decreasing in ϵ over the relevant range.

Figure 1: Default Decision and Consumption Dispersion



These results can be summarized as follows:

Result 1. *If $\phi(g_1) = 0$ for all g_1, then for any $\gamma \in (0, 1)$ and any (B_1, g_1), the social value of repayment $W^{d=0}(B_1, g_1, \gamma)$ is decreasing in ϵ and attains its maximum at $\epsilon^{SP} = 0$ (i.e. when welfare equals $u(y - g_1)$). Hence, default is always optimal for any decentralized consumption dispersion $\epsilon > 0$, .*

The outcome is very different when default is costly. With $\phi(g_1) > 0$, default still yields zero consumption dispersion, but at lower levels of consumption and therefore utility, since consumption allocations in the default state become $c^L = c^H = (1 - \phi(g_1))y - g_1$. This does not alter the result that the first-best social optimum is $\epsilon^{SP} = 0$, but what changes is that default can no longer support the consumption allocations of the first best. Hence, there is now a threshold amount of consumption dispersion in the decentralized equilibrium, $\widehat{\epsilon}(\gamma)$, which varies with γ and such that for $\epsilon \geq \widehat{\epsilon}(\gamma)$ default is again optimal, but for lower ϵ repayment is now optimal. This is because when ϵ is below the threshold, repayment produces a level of social welfare higher than the one that default yields.

Figure 1 also illustrates this scenario. The default cost lowers the common level of utility of both types of agents, and hence of social welfare, in the default state (shown in the Figure as the blue dashed line), and $\widehat{\epsilon}(\gamma)$ is determined where the social welfare under repayment intersects social welfare under default. If the decentralized consumption dispersion with the debt market functioning (ϵ) is between 0 and less than $\widehat{\epsilon}(\gamma)$ then the government finds it optimal to repay. Intuitively, if dispersion is not too large, the government prefers to repay rather than default since the latter reduces the dispersion of consumption but imposes an income cost on households. Moreover, as γ increases the domain of $W_1^{d=0}$ narrows, and thus $\widehat{\epsilon}(\gamma)$ falls and the interval of decentralized consumption dispersions that supports repayment narrows. This is natural because a higher γ causes the planner to weight more L-types in the social welfare function, which are agents with weakly lower utility in the repayment state.

These results can be summarized as follows:

Result 2. *If $\phi(g_1) > 0$, then for any $\gamma \in (0,1)$ and any (B_1, g_1), there is a threshold value of consumption dispersion $\widehat{\epsilon}(\gamma)$ such that the payoffs of repayment and default are equal: $W^{d=0}(B_1, g_1, \gamma) = u(y(1-\phi(g_1)) - g_1)$. The government repays if $\epsilon < \widehat{\epsilon}(\gamma)$ and defaults otherwise. Moreover, $\widehat{\epsilon}(\gamma)$ is decreasing in γ.*

We are now in a position to study how the government chooses the optimal amount of debt to issue in the first period. At $t = 0$, the government chooses its debt policy internalizing the effects we described above and in the discussion of the households' problem. To be precise, the government chooses B_1 to maximize the "indirect" social welfare function:

$$W_0(\gamma) = \max_{B_1} \left\{ \gamma v^L(B_1, \gamma) + (1-\gamma) v^H(B_1, \gamma) \right\}. \tag{14}$$

where v^L and v^H are the value or indirect utility functions obtained from solving the households' problems.

We can provide some intuition about the solution of this maximization problem by rearranging its first-order condition as follows (assuming that the functions are differentiable):

$$\begin{aligned} & q_0 \left[\gamma u'(c_0^L) + (1-\gamma) u'(c_0^H))\right] - \beta E_{g_1}\left[(1-d_1)[\gamma u'(c_1^L) + (1-\gamma) u'(c_1^H)]\right] \\ & \qquad + \underbrace{\frac{\partial q_0}{\partial B_1}}_{\leq 0} \left[\gamma u'(c_0^L) \underbrace{(B_1 - b_1^L)}_{\geq 0} + (1-\gamma) u'(c_0^H) \underbrace{(B_1 - b_1^H)}_{\leq 0}\right] \\ & + \beta E_{g_1}\left\{\Delta d_1 \left[\gamma \underbrace{[u(y(1-\phi(g_1)) - g_1) - u(c_1^L)]}_{>\text{ or }<0} + (1-\gamma)\underbrace{[u(y(1-\phi(g_1)) - g_1) - u(c_1^H)]}_{<0}\right]\right\} \leq 0. \end{aligned}$$

This expression can be broken into four terms. The first two are analogous to those in the first-order condition of the households. Since they are evaluated from the perspective of the social planner, they represent the social marginal benefit and cost of one more unit of debt at a given debt price and default policy of the government. The planner takes into account that, as the level of debt increases, all agents pay less taxes today but pay more taxes in the following period, and agents who buy debt postpone more consumption today. The third term corresponds to the effect of issuing more debt on the equilibrium price of government bonds. Since L-type agents are net borrowers ($(b_1^L - B_1) \leq 0$) and H-type agents are net savers ($(b_1^H - B_1) \geq 0$), a lower value of q_0 has a differential effect, with net savers receiving a higher return as the total stock of government debt increases. This effect also represents a form of pecuniary externality: When the government is aiming to optimize the choice of resources it can use to redistribute resources across agents and over time, which are given by $q_0 B_1$, it internalizes how the price of bonds responds to its optimal borrowing decision, whereas private agents take bond prices as given. The fourth term captures the effect at $t+1$ of a change in the default policy of the government. A default is always costly for H-type agents who hold higher debt than the average and can be a benefit or a cost for L-type agents with low or zero debt (recall we showed earlier that repayment causes a welfare loss for L agents if $B_1 > b_1^L + \phi(g_1)y$, or equivalently if at $t = 1$ taxes in the repayment state are higher than in the default state).

If all households are unconstrained in their savings decisions, so that their Euler equations hold with equality, and $\Delta d_1 = 0$, then the above optimality condition simplifies to:

$$\frac{u'(c_0^L)}{u'(c_0^H)} = \frac{-(B_1 - b_1^H)(1-\gamma)}{(B_1 - b_1^L)\gamma},$$

Hence, under those assumptions we obtain the intuitive result that the social planner would want to issue debt at date 0 so as to equalize the ratio of date-0 marginal utilities of consumption of the two agent types with their weighted relative wealth positions, where relative wealth is defined as $B_1 - b_1^i$ for $i = L, H$. Moreover, if L agents are constrained (i.e. $b_1^L = 0$) but still $\Delta d_1 = 0$, then the optimality condition yields this result:

$$\underbrace{\gamma \left[q_0 u'(c_0^L) - \beta E \left[(1-d_1) u'(c_1^L) \right] \right]}_{>0} + \frac{\partial q_0}{\partial B_1} \left[\gamma u'(c_0^L) B_1 + (1-\gamma) u'(c_0^H)(B_1 - b_1^H) \right] = 0.$$

Thus, when the borrowing constraint is binding for some agents, the optimal level of debt issued by the government increases, because at the debt level consistent with the unconstrained optimality condition the planner would have $u'(c_0^L)/u'(c_0^H) > -(1-\gamma)(B_1 - b_1^H)/(\gamma B_1)$, and hence the marginal benefit of borrowing more to reduce τ_0 and allocate more consumption to L-type agents exceeds the cost of making H-type agents save more to buy the debt.

Finally, the equilibrium bond price is the value of $q_0(B_1, \gamma)$ for which, whenever the default probability of the government is less than 1, the following market-clearing condition holds:

$$B_1 = \gamma b_1^L(B_1, \gamma) + (1 - \gamma) b_1^H(B_1, \gamma), \tag{15}$$

where B_1 in the left-hand-side of this expression represents the public bonds supply, and the right-hand-side is the government bond demand.

3.3 Competitive Equilibrium with Optimal Debt & Default Policy

For a given value of γ, a **Competitive Equilibrium with Optimal Debt and Default Policy** is a pair of household value functions $v^i(B_1, \gamma)$ and decision rules $b^i(B_1, \gamma)$ for $i = L, H$, a government bond pricing function $q_0(B_1, \gamma)$ and a set of government policy functions $\tau_0(B_1, \gamma)$, $\tau_1^{d\in\{0,1\}}(B_1, g_1, \gamma)$, $d(B_1, g_1, \gamma)$, $B_1(\gamma)$ such that:

1. Given the pricing function and government policy functions, $v^i(B_1, \gamma)$ and $b_1^i(B_1, \gamma)$ solve the households' problem.
2. $q_0(B_1, \gamma)$ satisfies the market-clearing condition of the bond market (equation (15)).
3. The government default decision $d(B_1, g_1, \gamma)$ solves problem (9).
4. Taxes $\tau_0(B_1, \gamma)$ and $\tau_1^d(B_1, g_1, \gamma)$ are consistent with the government budget constraints.
5. The government debt policy $B_1(\gamma)$ solves problem (14).

4 Quantitative Analysis: Benchmark Case

In this Section, we study the model's quantitative predictions based on a calibration using European data. The goal is to show whether a reasonable set of parameter values can produce an equilibrium that supports domestic public debt subject to default risk, and to study how the properties of this equilibrium change with the model's key parameters. We conduct this exercise acknowledging that, as we explain in discussing the results, the simplicity of the model comes at the cost of limitations that hamper its ability to account for important features of the data. Hence, we see the results more as an illustration of the potential relevance of the model's argument for explaining domestic default rather than as an evaluation of the model's general ability to match the observed empirical regularities of domestic debt.

We solve the model with a backward-recursive method. First, taking as given a set of values of $\{B_1, \gamma\}$, we solve for the equilibrium pricing and default functions by iterating on q_0, b_1^i and the default decision rule d_1 until the date-0 bond market clears when the date-1 default decision rule solves the government's optimal default problem (9). Then, in the second stage we complete the solution of the equilibrium by finding the optimal choice of B_1 that solves the government's date-0 optimization problem (14). It is important to recall that, as explained earlier, for given values of B_1 and γ, an equilibrium with debt will not exist if the government finds it optimal to default on B_1 for all realizations of government expenditures g_1. In these cases, there is no finite price that clears the government debt market and its expected return is zero.

4.1 Calibration

The model is calibrated to annual frequency. The parameter values that need to be set are those for the subjective discount factor β, the coefficient of relative risk aversion σ, the stochastic process of government expenditures, the initial levels of government debt B_0 and government expenditures g_0, the level of income y, the fraction of households with initial low wealth γ, their initial wealth b_0^L and the default cost function $\phi(g_1)$.

The calibrated parameter values are summarized in Table 1.

Table 1: Model Parameters

Parameter		Value
Discount Factor	β	0.96
Risk Aversion	σ	1.00
Output Cost Default	ϕ_0	0.0075
Avg. Gov. Consumption	μ_g	0.18
Autocorrel. G	ρ_g	0.88
Std Dev Error	σ_e	0.017
Large G/Y shock	$\overline{g}$	0.205
Prob. $\overline{g}$	$p_{\overline{g}}$	0.05
Avg. Income	y	0.79
Initial Gov. Debt	B_0	0.935
Low household wealth	b_0^L	0.00

Note: Government expenditures, income and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain and Portugal.

We set preference parameters to standard values ($\beta = 0.96$, $\sigma = 1$), and to simplify the analysis we set $b_0^L = 0$.[7] The remaining parameters are set so as to approximate some of the model's predicted moments to those of a subset of European countries.

[7] $\sigma = 1$ is also useful because, as we show in the Appendix, in the log-utility case we can obtain closed-form solutions and establish some results analytically.

The stochastic process of g_1 has two components. The first component is a *regular* shock, defined by a standard set of realizations and transition probabilities chosen to match those of a Markov process that approximates an AR(1) model estimated with 1995-2012 data for the government expenditures-GDP ratio (in logs) in France, Germany, Greece, Ireland, Italy, Spain and Portugal. This AR(1) process has the standard form:

$$\log(g_{t+1}) = (1 - \rho_g)\log(\mu_g) + \rho_g \log(g_t) + e_t$$

where $|\rho_g| < 1$ and e_t is i.i.d. over time and distributed normally with mean zero and standard deviation σ_e. Given the parameter estimates for all countries, we set μ_g, ρ_g and σ_e to the corresponding cross-country average. This results in the following values $\mu_g = 0.1812$, $\rho_g = 0.8802$ and $\sigma_e = 0.017$. Given these moments, we set $g_0 = \mu_g$ and use Tauchen's (1986) quadrature method with 25 nodes to approximate the Markov realizations and transition probabilities that drive expectations about g_1.

The second component of the g_1 shocks is an *unusually large* i.i.d. component $\overline{g}$ that occurs with a low probability $p_{\overline{g}}$. Defining unusually large shocks as those larger than three standard deviations in our European dataset of government expenditures-GDP ratios, we identified one event and set $\overline{g}$ to match it. This event measured 3.35 standard deviations (of the process in logs) and coincided with the 2008 global crisis. This implies setting $\log(\overline{g}) = \log(\mu_g) + 3.35\sigma_g = \log(0.2051)$, where σ_g is the standard deviation of $\log(g_t)$ (equal to $\sigma_e/(1-\rho_g^2)^{1/2}$). The probability of this shock is set to match its observed frequency, which was one observation per country in the 18-year sample, $p_{\overline{g}} = 0.05$.

This large g_1 shocks captures the fact that the European debt crisis did follow after an unusually large increase in government expenditures, triggered by financial stabilization policies in response to the 2008 crisis and expansionary fiscal policies in the aftermath. As we show below, the expectation of a relatively large g_1 realization even with a low probability has important implications for default incentives. In particular, for any given debt ratio, it makes default more likely at lower wealth concentration levels (lower γ), and hence allows the model to produce equilibria with debt exposed to default risk even if default costs are very small.[8]

Regarding the initial wealth distribution, we show results for $\gamma \in [0, 1]$. Note, however, that data for the United States and Europe suggest that the empirically relevant range for γ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of γ is needed we use $\gamma = 0.7$, which is in the middle of the plausible range. In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds but 50.4% have

[8]Default costs cannot be removed completely because, as shown in the previous Section, without them the government always defaults.

retirement accounts (which are likely to own sovereign bonds). These figures would suggest values of γ ranging from 50% to 88%. In Europe, comparable statistics are not available for several countries, but recent studies show that the wealth distribution is highly concentrated and that the Gini coefficient ranges between 0.55 and 0.85 depending on the country and the year of the study (see Davies et al. (2009)).[9]

Average income y is calibrated such that the model's aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households' aggregate endowment must equal GDP net of fixed capital investment and net exports, since the latter two are not explicitly modeled. The average for the period 1970-2012 for the same set of countries implies $y = 0.7883$.

Setting the initial debt ratio is complicated because in this two-period model the consumption smoothing mechanism induces a reduction in the optimal government debt relative to the initial condition B_0 even in a deterministic version of the model with stationary consumption. In the deterministic case, with $g_0 = g_1$, the optimal debt choice is decreasing in γ and has an upper bound of $B_1 = B_0/(1+\beta)$ as $\gamma \to 0$.[10] Moreover, in the model with shocks to g_1 but without default risk (i.e. assuming that the government is committed to repay) the optimal debt choice is still decreasing in γ and has an upper bound lower than $B_0/(1+\beta)$ because of risk. Given this inertia to reduce debt from its initial condition, we set the value of B_0 so that the optimal choice $B_1(\gamma)$ in the absence of default risk with $\gamma = 0.70$ matches the median of government net financial liabilities across the European countries in our dataset for the 1990-2007 period, which is 35% of GDP.[11] This yields $B_0 = 93.35\%$ as a share of GDP.

The default cost function is formulated in a similar manner as in the recent quantitative external sovereign default literature by letting $\phi(g_1)$ take the following form:

$$\phi(g_1) = \begin{cases} \phi_0 + (\mu_g - g_1)/y & \text{if } g_1 \leq \mu_g \\ \phi_0 & \text{otherwise} \end{cases}$$

As in Arellano (2008), this functional form implies that households' consumption during a default never goes above a given threshold. In this case, consumption never goes above $y(1-\phi_0) - \mu_g$ when the government defaults.

We calibrate ϕ_0 to match an estimate of the observed frequency of domestic defaults. Ac-

[9] In our model, if $b_0^L = 0$, the Gini coefficient of wealth is equal to γ.

[10] This occurs because as $\gamma \to 0$, the model in deterministic form collapses to a representative agent economy inhabited by H types where the optimal debt choice yields stationary consumption, $q_0 = 1/\beta$, and $B_1 = B_0/(1+\beta)$. In contrast, an infinite horizon, stationary economy yields $B_1 = B_0$ (see Appendix for details).

[11] We use 2007 because it is the year just before the large surge in debt and government expenditures started with the 2008 crisis.

cording to Reinhart and Rogoff (2008), historically, domestic defaults are about 1/4 as frequent as external defaults (68 domestic v. 250 external in data since 1750). Since the probability of an external default has been estimated in the range of 3 to 5% (see for example Arellano (2008)), we estimate the probability of a domestic default at about 1%. The model matches this default frequency on average when solved over the empirically relevant range of γ's ($\gamma \in [0.55, 0.85]$) if we set $\phi_0 = 0.0075$. Hence, if default occurs when $g_1 > \mu_g$, the cost of default is 0.75% of income and consumption falls to $(1-\phi_0)y - g_1$, which decreases with g_1. On the other hand, if default occurs when $g_1 \leq \mu_g$, consumption falls to a constant level given by $(1-\phi_0)y - \mu_g$ independent of g_1.

4.2 Results

We examine the quantitative results in the same order in which the backward solution algorithm works. We start with the second period's utility of households under repayment and default, the government's default decision, and the associated tax policy for a given range of values of B_1 (not just the B_1 chosen optimally by the government in the first period) and also ranges of g_1 and γ. We then move to the first period and examine the households' decision rules for demand of government bonds, the equilibrium bond prices and taxes also for given ranges of values of B_1, g_1 and γ. Finally, we examine solutions of the full competitive equilibrium including the optimal government debt issuance B_1 for a range of values of γ, and show how the model produces equilibria with debt exposed to domestic default risk.

4.2.1 Second period default decision and taxes (for given B_1)

Using the agents' optimal choice of bond holdings, we compute the equilibrium utility levels they attain at $t = 1$ under repayment v. default for different triples (B_1, g_1, γ). The differences in these payoffs are then converted into cardinal measures by computing compensating variations in consumption that equate utility in the two scenarios. This is analogous to the calculations typically done to compute welfare effects in representative agent models. In particular, we compute the individual utility gain of a default on domestic public debt $\alpha^i(B_1, g_1, \gamma)$ as the percent increase in consumption that renders an agent $i \in \{L, H\}$ indifferent between the repayment and the default options for different triples (B_1, g_1, γ):

$$\alpha^i(B_1, g_1, \gamma) = \left[\frac{u(y - g_1 + b_1^i - B_1)}{u((1-\phi(g_1))y - g_1)}\right]^{\frac{1}{1-\sigma}} - 1.$$

A positive (negative) value of $\alpha^i(B_1, g_1, \gamma)$ implies that agent i prefers the default (repayment) option by an amount equivalent to an increase (cut) of $\alpha^i(\cdot)$ percent in consumption. Notice the above formula uses the optimal $b_1^i(B_1, g_1, \gamma)$ that solves the household's problem, which in turn is obtained using $d_1(B_1, g_1, \gamma)$ to compute expected utility at $t = 1$. Hence, in principle the calculation can be done for any arbitrary $d_1(.)$ function, but we use the one that is consistent with the optimal plans of the households and the associated equilibrium price.

The individual utility gains of default are aggregated using γ to obtain a utilitarian representation of the social welfare gain of default:

$$\overline{\alpha}(B_1, g_1, \gamma) = \gamma\alpha^L(B_1, g_1, \gamma) + (1-\gamma)\alpha^H(B_1, g_1, \gamma).$$

A positive value indicates that default induces a social welfare gain and a negative value a loss.

Figure 2 shows intensity plots of the social welfare gain of default for different values of B_1 and γ. Panel (i) is for $g_1 = \underline{g}_1$ set 1 standard deviation below μ_g, and panel (ii) is for $g_1 = \overline{g}_1$ set 1 standard deviation above μ_g. The intensity of the color or shading in these plots indicates the magnitude of the welfare gain according to the legend shown to the right of the plots. The region shown in dark blue and marked as "No Equilibrium Region", represents values of (B_1, γ) for which no equilibrium exists for that particular B_1. This is because, as we explained earlier, at the given value of γ the government chooses to default on B_1 for all values of g_1, and thus the debt market collapses. In this region, the value of $\overline{\alpha}(B_1, g_1, \gamma)$ is not defined.

These intensity plots illustrate two of the key mechanisms driving the government's distributional incentives to default: First, fixing γ, higher levels of debt imply higher $\overline{\alpha}(B_1, g_1, \gamma)$. That is, the welfare gain of default is higher at higher levels of debt, or conversely the gain of repayment is lower. Second, keeping B_1 constant, $\overline{\alpha}(B_1, g_1, \gamma)$ increases with γ, or conversely the welfare gain of repayment is decreasing in γ. Hence, a higher concentration of wealth increases the welfare gain of default. This implies that lower levels of wealth dispersion are necessary in order to trigger default at higher levels of debt.[12] For example, when the debt equals 20% of GDP ($B_1 = 0.20$) and $g_1 = \overline{g}$, if $0.15 < \gamma < 0.35$ households are better off (in terms of utilitarian social welfare) if the government defaults, since debt repayment would result in higher taxes (for $\gamma \geq 0.35$ there is no equilibrium at this level of B_1).

The bottom panel in Figure 2 also displays a well-defined transition from a region in which repayment is optimal ($\overline{\alpha}(B_1, g_1, \gamma) < 0$) to one where default is optimal ($\overline{\alpha}(B_1, g_1, \gamma) > 0$) before moving into the no-equilibrium region. This differs from the top panel, in which the welfare

[12]Note that the cross-sectional variance of initial debt holdings is given by $Var(b) = B^2\frac{\gamma}{1-\gamma}$ when $b_0^L = 0$. This implies that the cross-sectional coefficient of variation is equal to $CV(b) = \frac{\gamma}{1-\gamma}$, which is increasing in γ for $\gamma \leq 1/2$.

gain of default is never positive, so repayment is always optimal. This reflects clearly the fact that higher levels of government expenditures also weaken the incentives to repay.

Figure 2: Social Welfare Gain of Default $\overline{\alpha}(B_1, g_1, \gamma)$

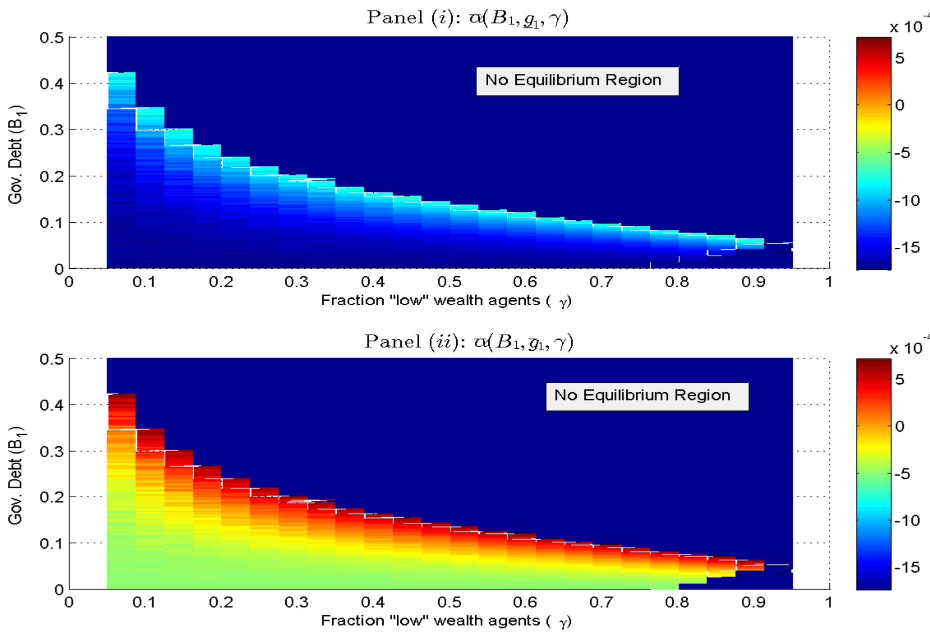
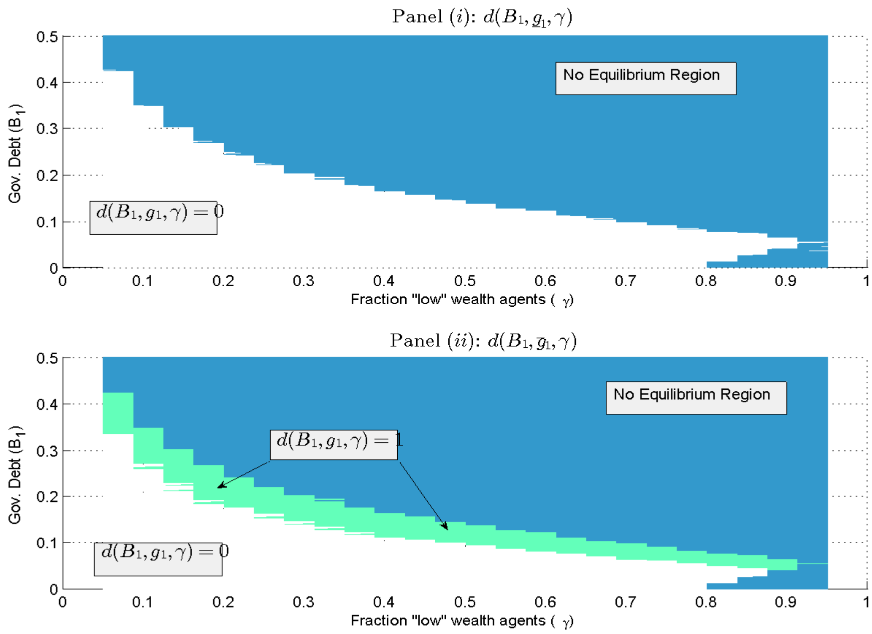


Figure 3 shows two panels with the optimal default decision organized in the same way as the two panels of Figure 2. The plots separate the region where the government chooses to repay ($d(B_1, g_1, \gamma) = 0$ shown in white), where it chooses to default ($d(B_1, g_1, \gamma) = 1$ in green) and where the equilibrium does not exist (i.e. the region where the government would choose to default for all levels of g_1, in blue).

These plots illustrate the implications of the mechanisms highlighted in Figure 2 for the default decision. The repayment region ($d(B_1, g_1, \gamma) = 0$) corresponds to the region with $\overline{\alpha}(B_1, g_1, \gamma) < 0$. Hence, the government defaults at higher debt levels for a given value of γ, or at higher values of γ for a given debt level. Moreover, the two plots show that when $g_1 = \overline{g}_1$ the government finds it optimal to default for combinations of γ and B_1 for which it is still optimal to repay when $g_1 = \underline{g}_1$. Thus, default occurs over a wider set of (B_1, γ) pairs at higher levels of government expenditures, and thus it is also more likely to occur.

Figure 3: Government default decision $d(B_1, g_1, \gamma)$



We can examine further the behavior of the default decision by computing the value of γ such that the government is indifferent between defaulting and repaying in period 1 for a given pair (B_1, g_1). These indifference thresholds $(\hat{\gamma}(B_1, g_1))$ are plotted in Figure 4 against debt levels ranging from 0 to 0.50 for three values of government expenditures $\{\underline{g}, \mu_g, \overline{g}\}$. For any given (B_1, g_1), the government chooses to default if $\gamma \geq \hat{\gamma}$.

Figure 4: Default Threshold $\hat{\gamma}(B_1, g_1)$

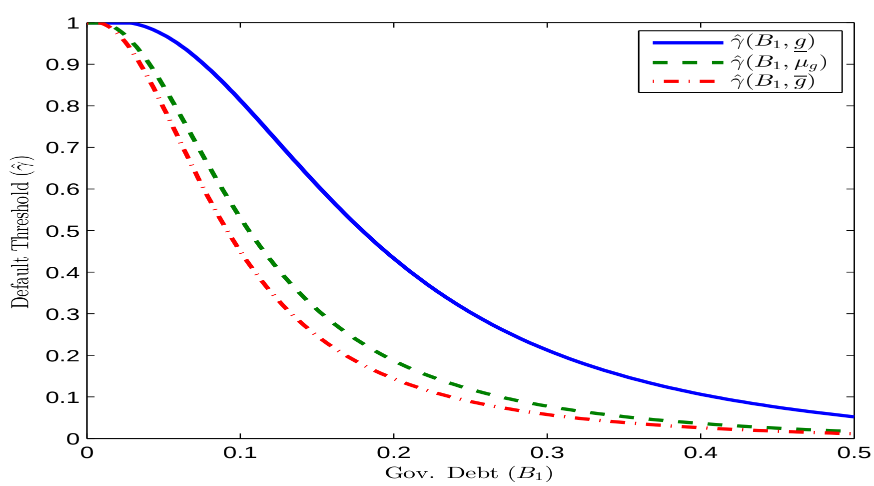


Figure 4 shows that the default threshold is decreasing in B_1. Hence, the government tolerates higher debt ratios without defaulting only if wealth concentration is sufficiently low. Also, default thresholds are decreasing in g_1, because the government has stronger incentives to default when government expenditures are higher.[13] This last feature of $\hat{\gamma}$ is very important to generate default at equilibrium. If, for a given value of B_1, the threshold $\hat{\gamma}$ is the same for all values of g_1, and γ is higher or equal than the threshold, the government chooses default for all realizations of g_1 at that B_1, and as noted already, in this case there is no equilibrium. On the other hand, if, for a given B_1, the threshold is different for different realizations of g_1, the probability of default will be positive but strictly less than 1 when the initial wealth distribution is in the interval $\gamma \in \left[\hat{\gamma}(B_1, \underline{g}), \hat{\gamma}(B_1, \overline{g})\right)$, so the price of government bonds can be positive and the government can borrow B_1 (i.e there is default with positive probability, but less than 1).

Figure 5: Equilibrium Tax Function $\tau_1^{d(B_1,g_1,\gamma)}$

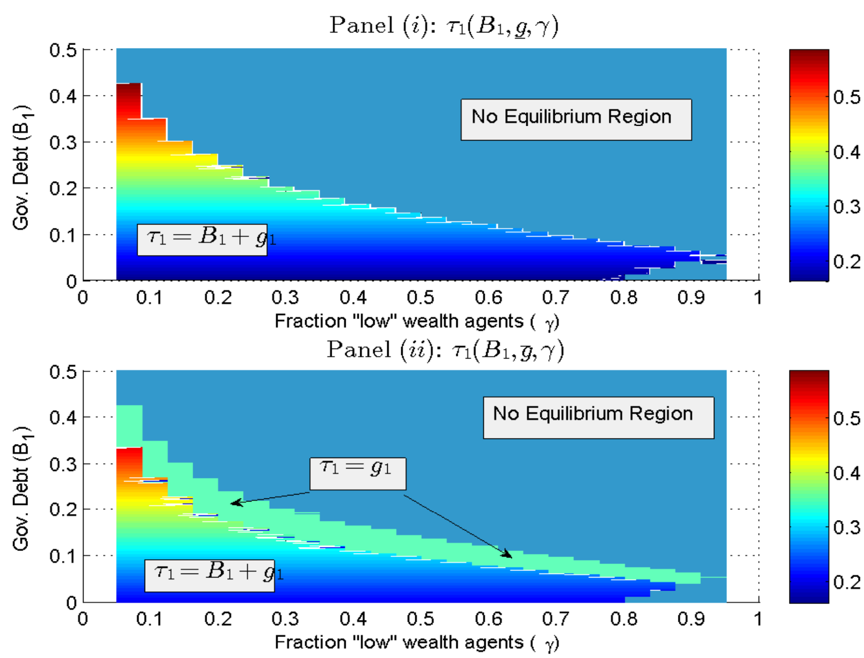


Figure 5 shows intensity plots of the equilibrium tax functions in period 1, also organized as the intensity plots of Figure 2. These plots together with those in Figure 3 show an additional distributional default incentive at work in the model via tax policy: Default entails lower taxation than repayment as long as debt service is low enough. When the government chooses to default, the tax is $\tau_1 = g_1$, while in the repayment states taxes are given by $\tau_1 = g_1 + B_1$. Since all agents pay the same taxes, the lower taxes under default add to the distributional incentives. Figure 5 shows that, for given g_1, the repayment scenarios with higher taxes are more likely when a large

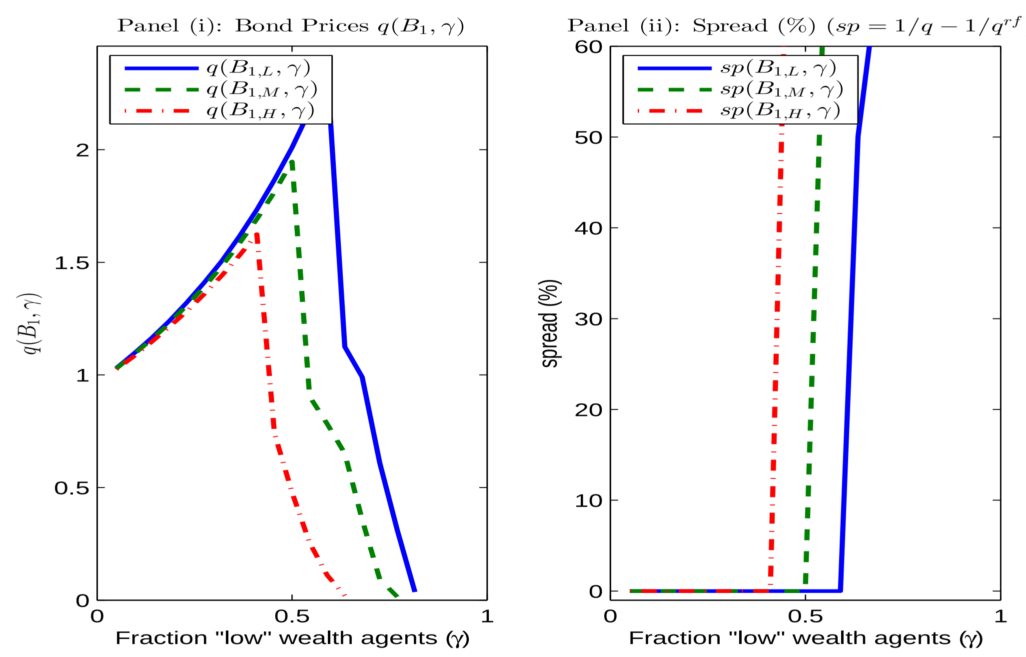
[13] $\hat{\gamma}$ approaches zero for B_1 sufficiently large, but in Figure 4 B_1 reaches 0.50 only for exposition purposes.

fraction of households hold debt (low γ) and thus benefit from a repayment, or when the debt is low so that the distributional incentives to default are weak. Moreover, equilibria with higher taxes are far more likely to be observed at low than at high levels of government expenditures, because default is far more like with the latter.

4.2.2 First period taxes, bond prices and decision rules for given B_1

Figure 6 shows equilibrium bond prices for given values of B_1 and their corresponding spreads. Panel (i) shows $q_0(B_1, \gamma)$ as a function of γ for three values of B_1 ($B_1 \in \{B_L, B_M, B_H\}$ with $B_L < B_M < B_H$ and B_M set to the value that we show later to be the optimal choice of the government when $\gamma = 0.5$, which is denoted $= B_1^*(\gamma = 0.5)$). Panel (ii) shows the associated default spreads or risk premia computed by defining the risk-free interest rate using equilibrium bond prices obtained solving the model when the government is not allowed to default, but g_1 remains stochastic. As explained earlier, at high enough values of γ, for a given B_1, the model cannot support an equilibrium with debt, and hence the bond price functions and spreads are truncated when this is the case.

Figure 6: Equilibrium Bond Price



The plots in Figure 6 illustrate three key features of public debt prices in the model:

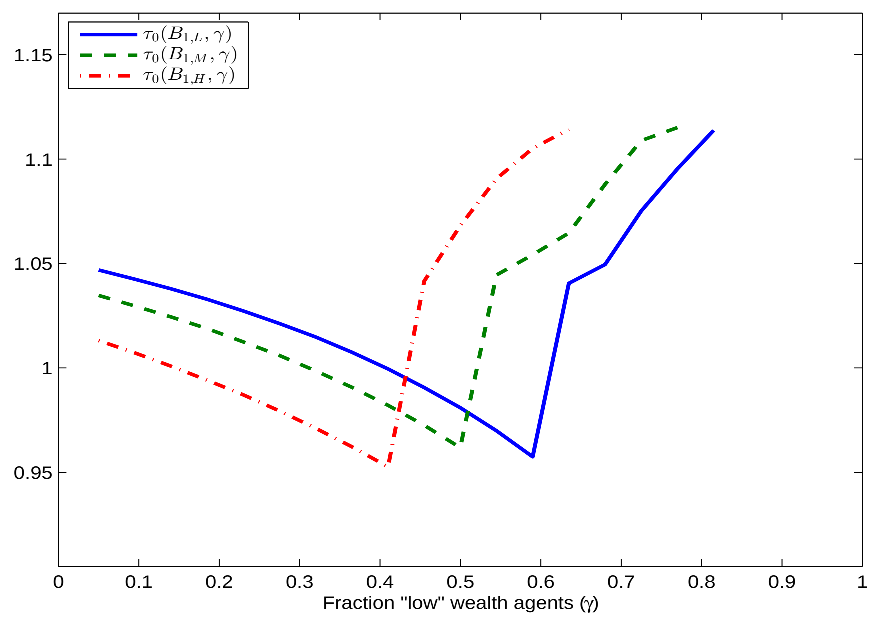
(**i**) *The equilibrium price is decreasing in* B_1 (the pricing functions shift downward as B_1 rises). This follows from a standard demand-and-supply argument: For a given γ, as the government borrows more, the price at which households are willing to demand more debt falls and the interest rate rises. Moreover, since as we show later $b_1^L = 0$ always holds in this calibration, the marginal investors determining the bond price are the type-H households. Hence, the equilibrium price must be consistent with the demand for bonds implied by their Euler equation, which implies that, for any b_0^H, as B_1 increases, a lower q_0 is needed to induce the rise in b_1^H that clears the bond market. Note that this effect is present even if we remove uncertainty from the model (see the Appendix for proofs showing that $q\prime(B_1) < 0$ with or without default risk in the case of logarithmic utility).

(**ii**) *Default risk reduces the price of bonds below the risk-free price and thus induces a risk premium* (see the Appendix again for a proof using logarithmic utility). This is reflected in the spreads shown in panel (ii). For example, when $B_{1,M}$ and $\gamma = 0.50$ the default probability equals 19.41% and the spread is about 40%. As noted earlier, spreads can be larger than default probabilities because the government's creditors are risk averse. Note that if for a given (B_1, γ) pair $d(B_1, g_1, \gamma) = 0$ for all realizations of g_1, so that the default probability is zero, there is no spread by definition, and also the spread is negligible and the prices with and without default risk are nearly identical if the default probability is low (see Appendix for details).

(**iii**) *Bond prices are a non-monotonic function of wealth dispersion*: When default risk is relatively low, bond prices are increasing in γ, but eventually they become a steep decreasing function of γ. This is a subtle effect driven by how changes in γ affect the demand for government debt v. the risk of default. Higher γ implies a more dispersed wealth distribution so that H-type agents become a smaller fraction of the population, and hence they must demand a larger amount of debt per capita in order to clear the bond market (i.e. b_1^H increases with γ), which pushes prices up. While default risk is low this *demand composition effect* dominates and thus bond prices rise with γ, but as γ increases and default risk rises (since higher wealth dispersion strengthens default incentives), the growing risk premium becomes the dominating force (at about $\gamma > 0.50$) and produces bond prices that fall sharply as γ increases.

Figure 7 plots the date-0 equilibrium taxes as a function of γ for three levels of B_1. Recall that these taxes are derived from the date-0 government budget constraint. Since g_0 and B_0 are being kept constant, all the variation in τ_0 reflects the negative of the changes in the amount of resources that access to the debt market provides to the government $(-q_0 B_1)$. Moreover, since we are plotting τ_0 for given B_1, the tax variation shown in each curve captures only the effect of the change in q_0 (i.e. each of the plotted tax functions is a reflection of the corresponding price function in Panel (i) of Figure 6 about the horizontal axis and stretched by a factor of B_1). As a result, taxes are decreasing in γ when bond prices are increasing and viceversa. The reasons for this non-monotonicity are the same behind the non-monotonicity of bond prices explained above. Also, these tax functions are truncated at sufficiently high values of γ for the same reason as the price functions (the model cannot support an equilibrium with debt).

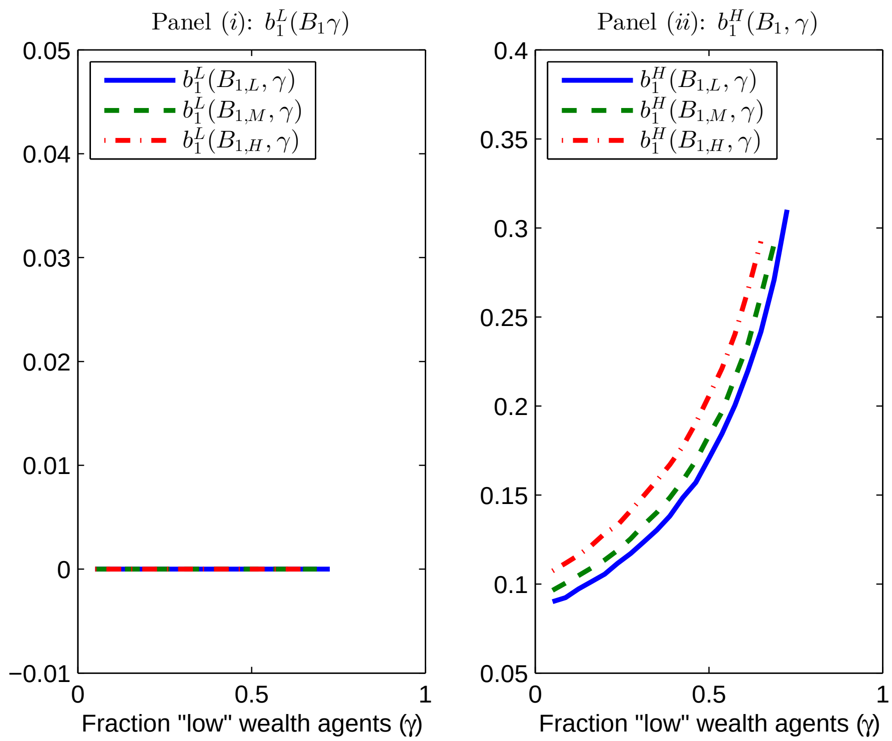
Figure 7: Equilibrium Tax Policy $\tau_0(B_1, \gamma)$



Interestingly, and in contrast with Figure 6, Figure 7 shows a non-monotonicity of taxes also with respect to the amount of debt. When wealth dispersion and default risk are relatively low ($\gamma \leq 0.40$), higher B_1 reduces τ_0. In this region, access to the public debt market allows the government to smooth taxation and redistribute, by lowering taxes for everyone, which effectively increases the relative cash-in-hand of L-type households more than for H-type households. As γ and default risk increase, however, taxes have to increase as government debt rises, because the available resources drawn from the debt market ($q_0(B_1, \gamma)B_1$) decrease with bond prices. The government would still like to issue government debt to smooth taxes at $t = 0$, but this option is restricted by the default choice the government cannot commit to avoid at $t = 1$. If the government could commit to repay, taxes in period 0 would be lower than those depicted in Figure 7.

Figure 8 shows the households' equilibrium bond demand decision rules (for given B_1) plotted in the same layout as the bond prices and taxes (i.e. as functions of γ for three values of B_1). This figure shows that in all cases L-type agents are credit constrained (in the sense that their Euler condition holds with inequality) and choose zero bond holdings. Hence, the equilibrium decision rules satisfy $b_1^L = 0$ and $b_1^H = \frac{B_1}{1-\gamma}$. This also means that the H-type households are the "marginal" investor for the pricing of government bonds. As a result, the convexity of their bond decision rules reflects the effects of wealth dispersion on demand composition and default risk explained earlier. In particular, as γ increases, the demand for bonds of H types grows at an increasing rate, reflecting the combined effects of higher per-capita demand by a smaller fraction of H-type agents and a rising default risk premium.

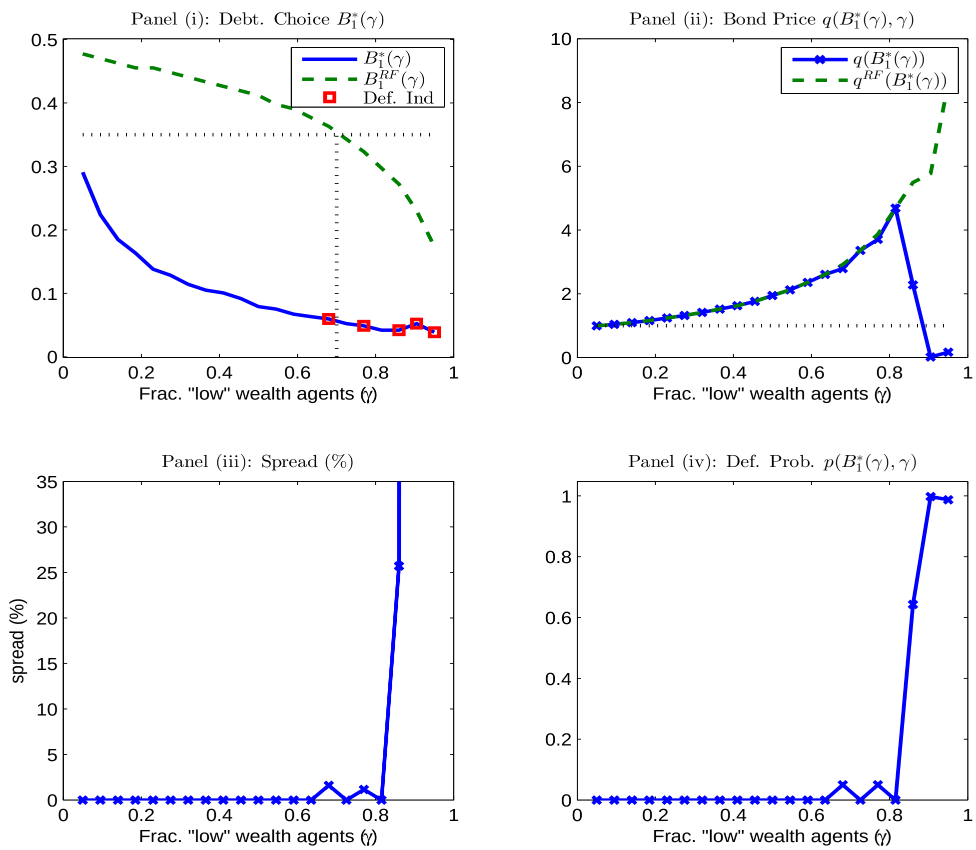
Figure 8: Equilibrium HH bond decision rules b_1^i



4.2.3 Full Equilibrium with Optimal Government Debt

Finally, given the solutions for household decision rules, tax policies, bond pricing function and default decision rule, we can solve for the government's optimal choice of debt issuance in period 0 (i.e. the optimal B_1 that solves problem (14)) for a range of values of γ. We can then go back and identify the equilibrium values of the rest of the model's endogenous variables that are associated with each optimal debt choice. Figure 9 shows four key components of this equilibrium: Panel (i) presents the optimal debt issuance in the benchmark model and in an alternative model with no default risk (i.e. a model where the government chooses risk-free debt $B_1^{RF}(\gamma)$ being committed to repay); Panel (ii) shows the prices at which the equilibrium debt levels in the benchmark and the risk-free alternative are sold; Panel (iii) shows the default spreads; and Panel (iv) shows the probability of default. Debt levels marked with a square in Panel (i) are those that correspond to equilibria with a positive default probability. This occurs for $\gamma \geq 0.65$, which includes the empirically relevant range $(0.55 < \gamma < 0.85)$.

Figure 9: Equilibrium Optimal Government Debt Policy



Panel (i) shows that optimal debt issuance falls as γ increases in both the benchmark model with default risk and the model with commitment to repay. This occurs because, as explained in Section 3, in both cases the government is seeking to reallocate consumption across agents and across periods by altering the product $q(B_1)B_1$ optimally, and in doing this it internalizes the response of bond prices to its choice of debt. Hence, the government knows that as γ rises and the demand composition effect strengthens demand for bonds, pushing for higher prices, it can actually attain a higher $q(B_1)B_1$ by choosing lower B_1, because doing so contributes to higher bond prices.

Panel (i) also illustrates that default risk has significant implications for the optimal debt choice. In particular, the risk of default reduces sharply the optimal choice of B_1, and changes the negative relationship between B_1 and γ from concave without default risk to convex with default risk. As expected, a higher level of wealth concentration strengthens default incentives and leads the government to optimally choose lower B_1, after taking into account the risk of domestic default and its distributional implications.

In the range of empirically relevant values of γ, optimal debt ratios range from 5 to 10%, which is relatively low compared with the calibrated debt ratio of 35 percent for $\gamma = 0.7$ in the model without default risk. Still, the predicted debt ratios about 1/7 to 1/3 of the data average are notable given the important limitations of this simple two-period setup in trying to match time-series dynamics of public debt and macro-aggregates. In particular, the model lacks income- and tax-smoothing effects and self-insurance incentives that are likely to be strong in a longer time horizon (see Aiyagari and McGrattan (1998)), and, as explained earlier, it has an upper bound on the optimal debt choice for $\gamma = [0, 1]$ lower than $B_0/(1+\beta)$ (which is the upper bound as $\gamma \to 0$ in the absence of default risk). Moreover, since the model was calibrated to produce $B_1 = 0.35$ without default risk and the risk of default lowers the optimal debt choice, it must be that for $\gamma \geq 0.7$ it predicts debt below 0.35.

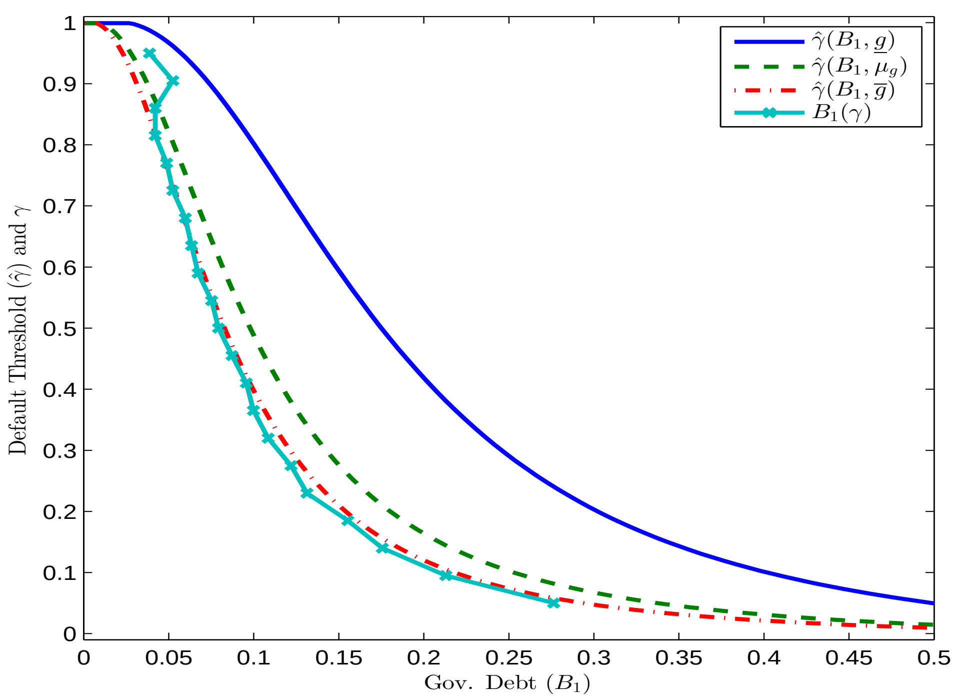
Panel (i) also illustrates an interesting testable empirical prediction of the model, indicating a negative conditional relationship between public debt ratios and inequality in the distribution of public debt holdings. This relationship is conditional on the structure of the model and the parameters held constant as γ varies in Figure 9.

Panel (ii) shows that bond prices of the equilibrium debt levels range from very low to very high as the value of γ rises (including even prices sharply above 1 that imply negative real interest rates on public debt). The behavior of these prices is consistent with the mechanisms driving bond prices described earlier: When default risk is low (for $\gamma < 0.8$), the prices rise with γ, because in this range an increase in γ triggers two effects that push for higher bond prices: the demand composition effect, as the per-capita demand for bonds of H-types rises with γ, and the sharply reduced supply of bonds that the government finds optimal to provide as default incentives strengthen, even tough it may optimally choose not to default at $t = 1$ for any realization of g_1. The latter can be observed in that for $\gamma < 0.65$ the default probability is zero (see Panel (iv)), which implies prices that are identical with and without default and zero spreads (see Panel (iii)).

In the range $\gamma \geq 0.8$ the bond prices at which the optimally chosen debt sells become a sharply decreasing function of γ, because here the default incentives are strong and make default risk high. Default probabilities are low in the two equilibria with positive default probability inside the empirically relevant range of γ, at about 4 percent for $\gamma = 0.65, 0.75$, and rise sharply to above 60 percent for $\gamma \geq 0.8$. The lower default probabilities produce prices slightly below the risk-free prices and spreads close to 4% in the plausible range of γ, while at the higher default probabilities the spreads become very large.

Figure 10 combines the wealth concentration default thresholds $\hat{\gamma}(B_1, g_1)$ of Figure 4 with the optimal debt choices $B_1(\gamma)$ plotted in Figure 9 (inverted) in order to illustrate how the model selects equilibria in which the optimal debt has also positive default probability. Consider first the default thresholds. Recall that these are decreasing in g_1, and in Figure 10 the lowest default threshold curve (shown as the red, dashed curve) is associated with the highest possible realization of g_1. It follows, therefore, that if for a given γ in the vertical axis the optimal debt choice curve (the light blue curve) lies below this lowest default threshold curve, the government chooses at $t = 0$ an optimal debt that it repays for sure at $t = 1$. This is because that given γ will be lower than the corresponding default threshold. As can be observed, this is the case for all $\gamma < 0.65$, which explains why in Panel (i) all the optimal debts chosen for $\gamma < 0.65$ have zero default risk.

Figure 10: Default Threshold, Debt Policy and Equilibrium Default



Consider now that as we move from the highest realization of g_1 to lower values, the default threshold curves shift out up to the furthest out curve shown in the Figure (shown in dark blue), which corresponds to the lowest realization of g_1. We can observe that as we move into the range $\gamma > 0.65$, the optimal debt choice moves into the area in between the threshold curves for the lowest and highest realizations of g_1. This is required for a positive default probability to exist. For example, for a $\gamma > 0.8$, the government has chosen a debt amount such that if the realization of g_1 is sufficiently high, it will imply that γ is higher than at least the lowest default threshold curve, and thus the government will default if that g_1 occurs. Moreover, the default probability will be higher the closer the optimally chosen debt is to the highest default threshold curve, because if it passes above it, it would imply that the government will choose to default at $t = 1$ for all possible realization of g_1, even the lowest. As the optimal debt curve approaches this boundary, default is becoming optimal for lower and lower values of g_1 and thus the probability of default is increasing as default is chosen for more of the support of the distribution of g_1.

To summarize: In order for the model to generate default in equilibrium at a given level of γ, the optimal debt choice $B_1(\gamma)$ needs to be located in between $\hat{\gamma}(B_1, \underline{g})$ and $\hat{\gamma}(B_1, \overline{g})$. If the γ that generates $B_1(\gamma)$ lies below $\hat{\gamma}(B_1, \overline{g})$, there is no equilibrium default risk, since the wealth dispersion is lower than required to obtain a default even at the highest value of g_1. On the other hand, we will never observe $B_1(\gamma)$ above the value of $\hat{\gamma}(B_1, \underline{g})$, because this means that the government defaults with certainty, even at the lowest g_1. As explained earlier, in this case the equilibrium does not exist. Values of $B_1(\gamma)$ that lie in between $\hat{\gamma}(B_1, \underline{g})$ and $\hat{\gamma}(B_1, \overline{g})$ correspond to cases where there is equilibrium default risk. In these cases, the default probability is higher than zero but lower than 1, the equilibrium bond price is well defined, and defaults can be observed in equilibrium for some values of g_1.

5 Sensitivity Analysis & Extensions

This Section presents the results of a set of counterfactuals that shed more light on the workings of the model and also the results of the political economy extension of the benchmark model. The counterfactuals focus on changes in the initial levels of government debt and expenditures and in the cost of default. The political economy extension introduces a non-utilitarian planner, with the aim of capturing some of the political economy mechanisms that could be at work in domestic defaults in a simple way.

5.1 Sensitivity Analysis

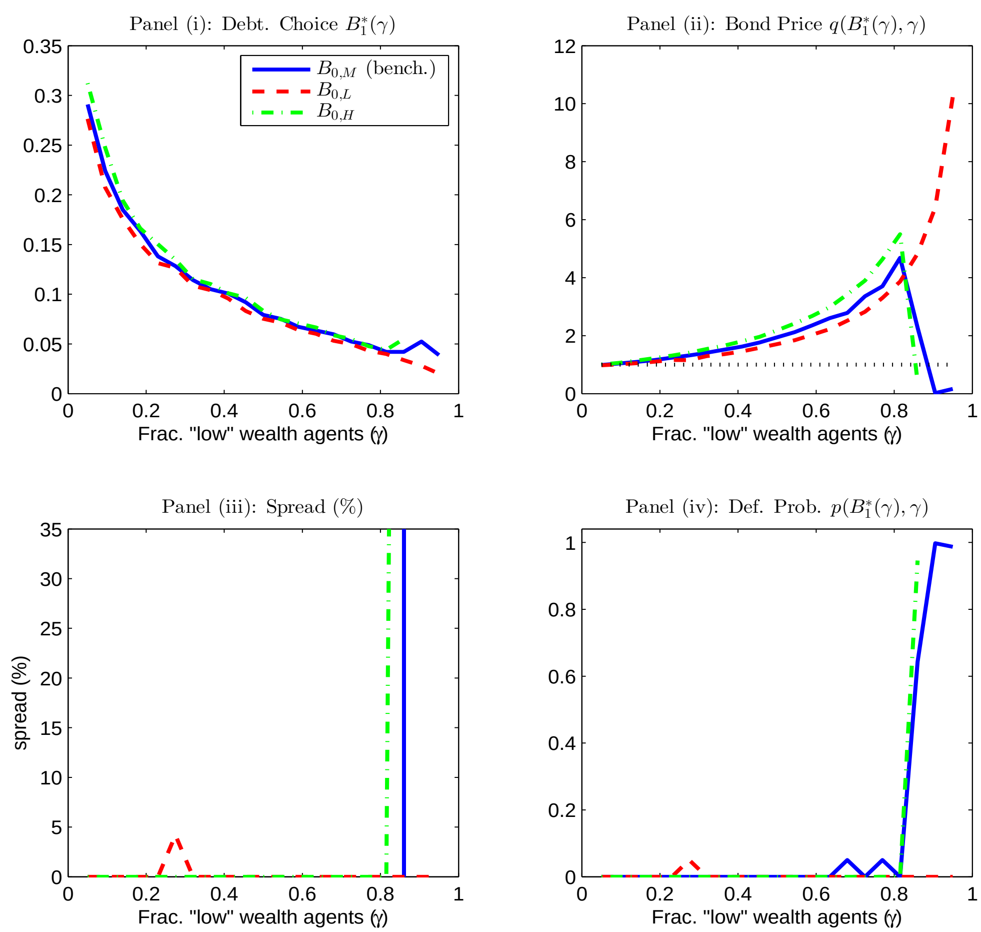
The sensitivity analysis studies how our main results are affected by changes in the initial debt B_0, initial government expenditures g_0, and the cost of default parameter ϕ_0.[14]

[14]We also experimented with changes in $p_{\overline{g}}$ but with omit them because they did not change significantly the benchmark results.

5.1.1 Initial Debt Level B_0

Figure 11 uses the same layout of Figure 9 to compare the optimal government debt and associated equilibrium bond prices, spreads and default probability under the benchmark initial debt ($B_0 = B_{0,M} = 0.935$) and values that are 20% lower and higher ($B_{0,L} = 0.748$ and $B_{0,H} = 1.12$ respectively).

Figure 11: Changes in Initial Government Debt B_0



Panel (i) shows that the optimal debt choice is slightly increasing in B_0. The government tries to smooth taxes by issuing more debt as the initial debt level raises, but in all cases the incentives to default continue to strengthen as γ rises, and this leads the government to choose lower debt levels. Smaller debts have to be chosen also because the risk of default also shrinks the set of debt amounts that can be supported at equilibrium as B_0 rises. For instance, for $\gamma > 0.85$ the equilibrium does not exist when $B_0 = B_{0,H}$ but it exists at lower values of B_0.

Panels (ii)-(iv) show that at the equilibrium debt level, bond prices are higher for higher B_0 as long as default risk is not too high (i.e. for values of γ in the range in which bond prices for the optimally chosen debt are increasing in γ). This is because higher initial debt increases the initial wealth of H-type agents (b_0^H), and in turn this requires a higher bond price to clear the market. This effect is stronger than two other effects that push bond prices in the opposite direction: First, the slightly higher optimal debt B_1 supplied at higher levels of B_0. Second, the lower disposable income of households resulting from the higher date-0 taxes needed to repay higher levels of B_0, which reduces demand for bonds. In contrast, at sufficiently high γ default risk becomes the main determinant of bond prices, making them fall sharply with $B_0 = B_{0,M}$ and $B_0 = B_{0,H}$. In the case with $B_0 = B_{0,L}$, however, default risk is always negligible, and hence equilibrium bond prices always rise with γ

5.1.2 Initial Government Expenditures g_0

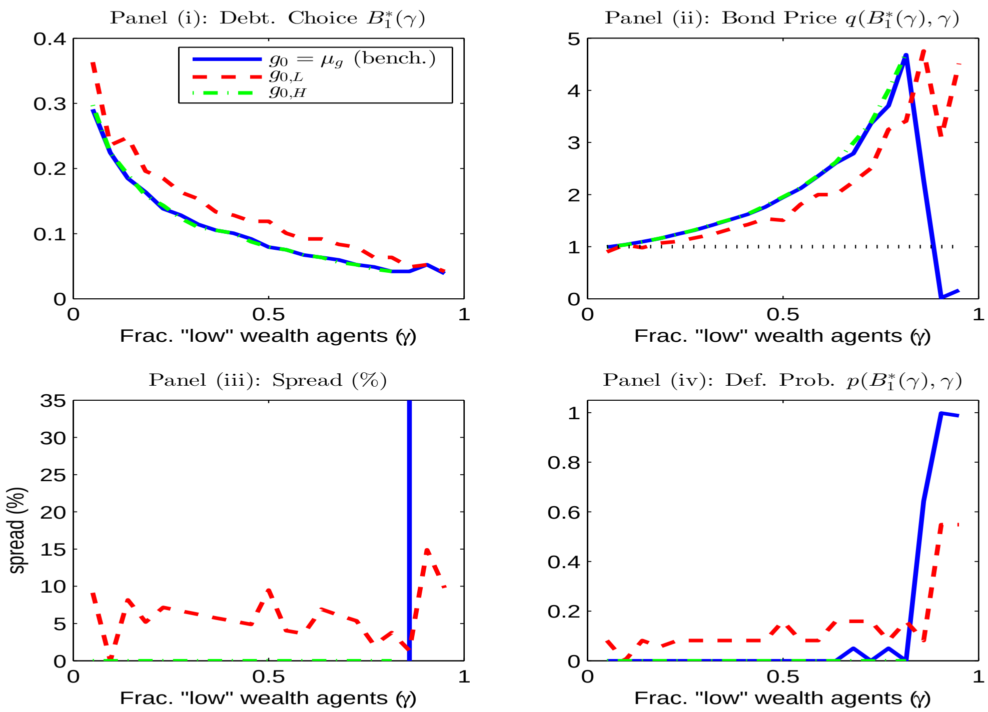
Figure 12 compares the optimal government debt and associated equilibrium bond prices, spreads and default probability under the benchmark initial value of $g_0 = \mu_g = 0.181$ and two alternatives that are 1.5 standard deviations above and below it, denoted $g_{0,L} = 0.171$ and $g_{0,H} = 0.192$ respectively. The curve for the benchmark spread in panel (iii) is truncated at the top so as to make visible the differences in spreads in these two scenarios.

Like changes in B_0, changes in g_0 affect date-0 disposable income via their effect on date-0 taxes. They differ, however, in two key respects: First, changes in g_0 affect the expected level of government expenditures for $t = 1$, as reflected in changes in the transition probabilities which are conditional on g_0. Second, changes in g_0 cannot affect the aggregate wealth of the economy and the initial bond holdings of H-type agents.

Figure 12 shows that the effects of changes in g_0 are not symmetric, even though we chose symmetric deviations above and below g_0. Increases in g_0 produce small differences in the optimally chosen debt $B_1(\gamma)$, except that as γ rises above 0.8 the benchmark case can support equilibria with debt but the scenario with $g_{0,H}$ cannot. Even tough optimal debt is similar with the benchmark and high values of g_0 in the range of γ in which both exist, the scenario with high $g_{0,H}$ does not support equilibria with positive default probability. In this scenario, for $\gamma \geq 0.8$ default is certain to occur and this prevents the government from issuing any debt for which spreads are positive, and for lower γ the government does not choose to default at any realization of g_1.

In the alternative scenario in which g_0 is lower than the mean, the optimally chosen debt is higher than in the benchmark. A reduction in g_0 allows the government to issue more debt in the initial period because the likelihood of hitting states in which default occurs in the second period is lower. As we observe in Panels (ii)-(iv), this increase in the optimal debt results in higher default probabilities, lower bond prices and higher spreads in the $g_{0,L}$ case relative to the benchmark, except for high values of γ where default risk has a stronger effect on bond prices in the benchmark case.

Figure 12: Changes in Initial Government Expenditures g_0

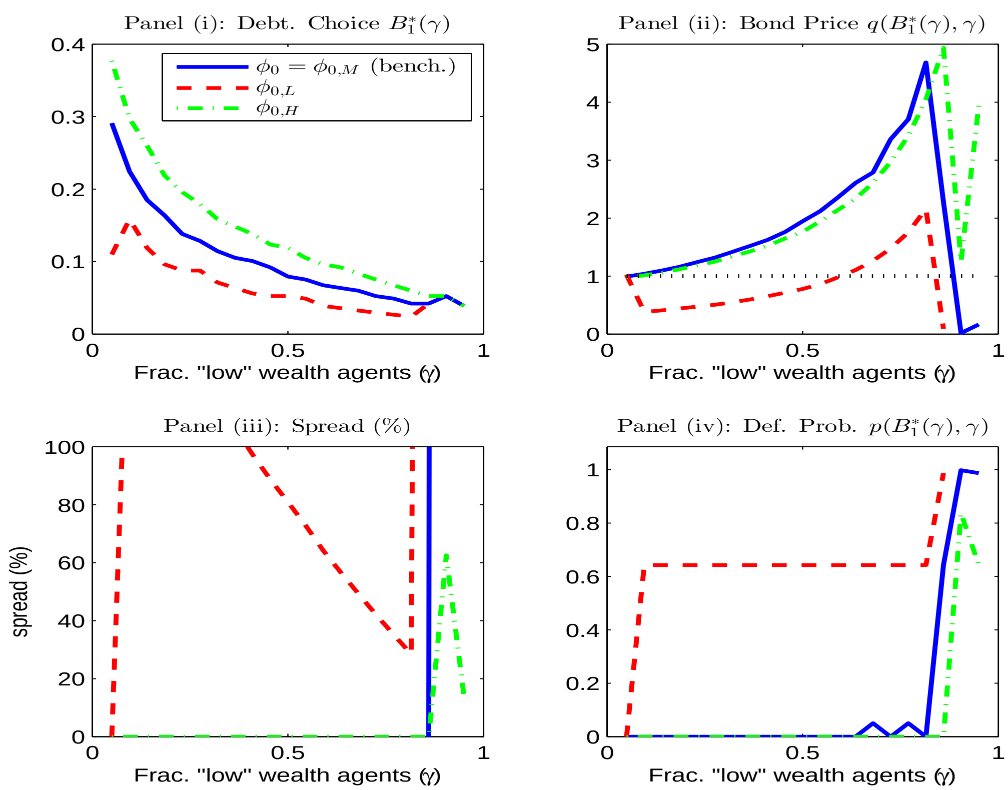


5.1.3 Cost of Default ϕ_0

Panels $(i) - (iv)$ in Figure 13 compare the optimal government debt, bond prices, spreads and default probability for the benchmark value of the default cost parameter ($\phi_0 = 0.0075$) with a scenario with $\phi_{0,L} = 0$ (i.e. no default cost for cases where $g_1 > \mu_g$) and a case in which the parameter is twice the size of the benchmark value ($\phi_{0,H} = 0.015$).

Panel (i) in Figure 13 shows that as the cost of default increases the government gains access to higher levels of optimal debt that allow it to smooth taxes across periods much better than in the benchmark (even for relatively high levels of γ). This increases in the level of debt are associated with slightly lower bond prices in the range of γ in which default risk is low and prices are increasing in γ(see Panel (ii)). On the other hand, for a lower cost of default, the government issues less debt, partly because the set of possible values of B_1 is constrained by default risk. Panel (ii) shows that despite this lower debt, bond prices are lower than in the benchmark. This is because with higher default costs, the lower optimally chosen debt levels carry higher default probabilities, as shown in in Panel (*iv*), which in turn are reflected in much larger spreads (see Panel (*iii*)). Interestingly, while the three different values of phi_0 produce different optimal debt levels, bond prices, default probabilities and spreads when γ is below 0.8, in all three scenarios default risk rises very rapidly for $\gamma > 0.8$ and produces sharply lower prices, higher default probabilities and large spreads.

Figure 13: Changes in Cost of Default ϕ_0



5.2 Political Economy Extension

In this extension of the model the weights of the government's payoff function do not necessarily match the wealth distribution. This can be viewed as a situation in which, for political reasons or related factors, the government does not maximize a standard social welfare function. The government's welfare weights on L- and H-type households are denoted ω and $(1-\omega)$ respectively, and we refer to ω as the government's preference for redistribution.

5.2.1 A Government with Preference for Redistribution

The government's default decision optimization problem at $t = 1$ is:

$$\max_{d\in\{0,1\}} \left\{W_1^{d=0}(B_1, g_1, \gamma, \omega), W_1^{d=1}(g_1)\right\}, \tag{16}$$

where $W_1^{d=0}(B_1, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ denote the government's payoffs in the cases of no-default and default respectively. Using the government budget constraints to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma, \omega) = \omega u(y - g_1 + b_1^L - B_1) + (1 - \omega)u(y - g_1 + b_1^H - B_1) \tag{17}$$

and

$$W_1^{d=1}(g_1) = u(y(1 - \phi(g_1)) - g_1). \tag{18}$$

Combining the above payoff functions we get a similar default condition as before but with ω in the place of γ:

$$\omega \left[u(y - g_1 + \overbrace{(b_1^L - B_1)}^{\leq 0}) - u(y(1 - \phi(g_1)) - g_1)\right] +$$
$$(1 - \omega) \left[u(y - g_1 + \overbrace{(b_1^H - B_1)}^{\geq 0}) - u(y(1 - \phi(g_1)) - g_1)\right] \leq 0$$

We follow the same approach as before to characterize the optimal default decision graphically. The parameter ϵ is used again to represent the dispersion of hypothetical decentralized consumption allocations under repayment: $c^L(\epsilon) = y - g_1 - \epsilon$ and $c^H(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon$. Under default the consumption allocations are again $c^L = c^H = y(1 - \phi(g_1)) - g_1$. Recall that under repayment, the dispersion of consumption across agents increases with ϵ, and under default there is zero consumption dispersion. The repayment government payoff can now be rewritten as:

$$W^{d=0}(\epsilon, g_1, \gamma, \omega) = \omega u(y - g_1 + \epsilon) + (1 - \omega)u\left(y - g_1 + \frac{\gamma}{1 - \gamma}\epsilon\right).$$

As in the model of Section 2, the planner chooses its optimal consumption dispersion ϵ^{SP} as the value of ϵ that maximizes (5.2.1). Since as of $t = 1$ the only instrument the government can use to manage consumption dispersion relative to what the decentralized allocations support is the default decision, it will repay only if doing so allows it to get closer to ϵ^{SP} than by defaulting.

The planner's optimality condition is now:

$$\frac{u'(c_1^H)}{u'\left(c_1^L\right)} = \frac{u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y - g_1 - \epsilon^{SP}\right)} = \left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right). \tag{19}$$

This condition implies that optimal consumption dispersion for the planner is zero only if $\omega = \gamma$. For $\omega > \gamma$ the planner likes consumption dispersion to favor L types so that $c_1^L > c_1^H$, and the opposite holds for $\omega < \gamma$.

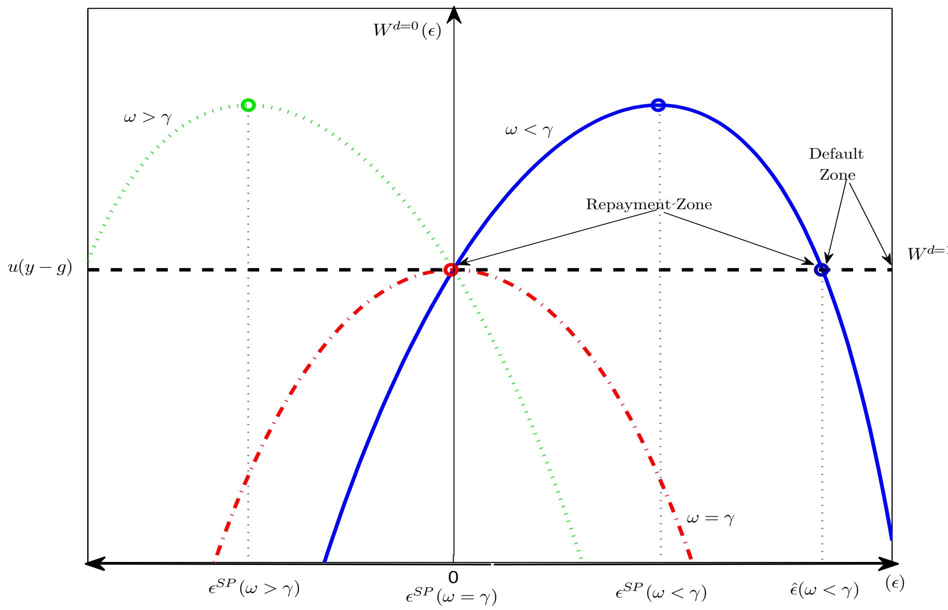
One key difference in this extension of the model is that it can support equilibria with debt subject to default risk even without default costs. Assuming $\phi(g_1) = 0$, there are two possible scenarios depending on the relative size of γ and ω. First, if $\omega \geq \gamma$, the planner still always chooses default as in the setup of Section 2. This is because for any decentralized consumption dispersion $\epsilon > 0$, the consumption allocations feature $c^H > c^L$, while the planner's optimal consumption dispersion requires $c^H \leq c^L$, and hence ϵ^{SP} cannot be implemented. Default brings the planner the closest it can get to the payoff associated with ϵ^{SP}.

In the second scenario $\omega < \gamma$. In this case, the model can support equilibria with debt even without default costs. In particular, there is a threshold consumption dispersion $\hat{\epsilon}$ such that default is optimal for $\epsilon \geq \hat{\epsilon}$, where $\hat{\epsilon}$ is the value of ϵ at which $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ intersect. For $\epsilon < \hat{\epsilon}$, repayment is preferable because $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W_1^{d=0}(g_1)$. Thus, without default costs, equilibria for which repayment is optimal require two conditions: (a) that the government's preference for redistribution be smaller than the fraction of L-type agents ($\omega < \gamma$), *and* (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Figure 14 illustrates the outcomes described above. This Figure plots $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ for $\omega \gtreqless \gamma$. The planner's default payoff and the values of ϵ^{SP} for $\omega \gtreqless \gamma$ are also identified in the plot. The vertical intercept of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ is always $W^{d=1}(g_1)$ for any values of ω and γ, because when $\epsilon = 0$ there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ follows from $u'(.) > 0, u''(.) < 0$.[15]

[15]Note in particular that $\frac{\partial W_1^{d=0}(\epsilon, g_1, \gamma, \omega)}{\partial \epsilon} \gtreqless 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \gtreqless (\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$. Hence, the planner's payoff is increasing (decreasing) at values of ϵ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $(\frac{\omega}{\gamma})(\frac{1-\gamma}{1-\omega})$.

Figure 14: Default decision with non-utilitarian planner ($\phi = 0$)



Take first a scenario with $\omega > \gamma$. In this case, the planner's payoff under repayment is the dotted bell curve shown in green. Here, $\epsilon^{SP} < 0$, because condition (19) implies that the planner's optimal choice features $c^L > c^H$. Since default is the only instrument available to the government, however, these consumption allocations are not feasible, and by choosing default the government attains $W^{d=1}$, which is the highest feasible government payoff for any $\epsilon \geq 0$. In contrast, in a scenario with $\omega = \gamma$, for which the planner's payoff function is the red, dashed bell curve, the planner would choose $\epsilon^{SP} = 0$, and default attains exactly the same payoff, so default is chosen. In short, if the fraction of L-type agents does not exceed the planner's preference for redistribution (i.e. $\omega \geq \gamma$), the government always defaults for any decentralized distribution of debt holdings determined by $\epsilon > 0$ and thus equilibria with debt cannot be supported.

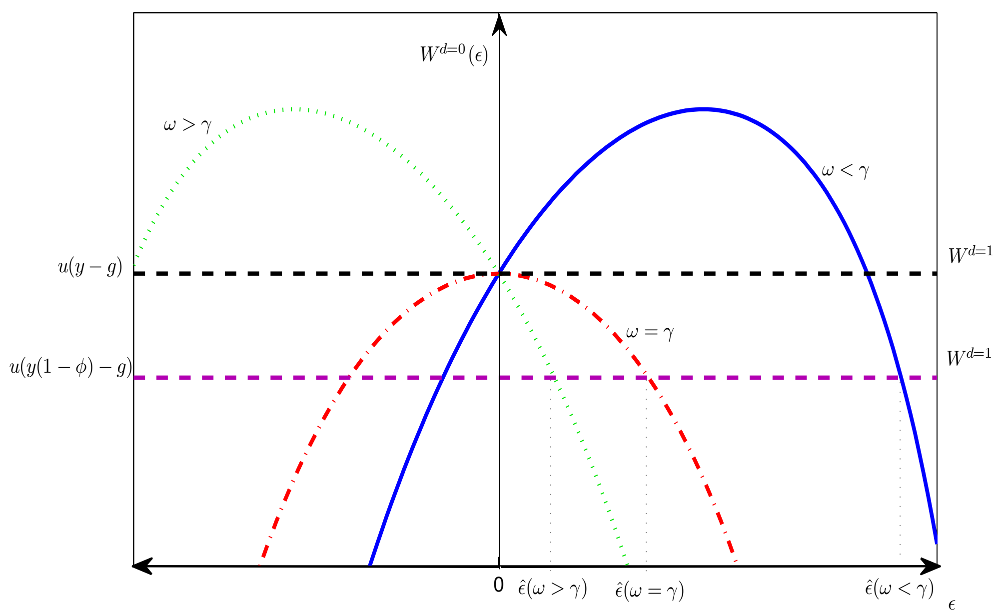
In the third scenario with $\omega < \gamma$, for which the planner's payoff is the blue bell curve, the intersection of the downward-sloping segment of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ with $W^{d=1}$ determines the default threshold $\hat{\epsilon}$ such that default is optimal only in the *default zone* where $\epsilon \geq \hat{\epsilon}$. Default is still a second-best policy for the planner, because with it the planner cannot attain $W^{d=0}(\epsilon^{SP})$, it just gets the closest it can get. In contrast, the choice of repayment is preferable in the *repayment zone* where $\epsilon < \hat{\epsilon}$,, because in this zone $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W^{d=1}(g_1)$.

Scenarios that would feature $\omega < \gamma$ are not difficult to visualize. Consider, for example, a social planner with weights that follow from the first-best, complete-markets equilibrium, in which the distribution of wealth matches the tail value of the agent's endowments priced with

Arrow securities. If both types of agents were ex-ante identical, then the social planner's weights that would support the first-best competitive equilibrium would be $\omega = 0.5$, which implies a uniform distribution of wealth and equal consumption for both agent types. On the other hand, the decentralized distribution of wealth γ that would result from having the same agents trade in some incomplete-markets environment (e.g. the stationary wealth distribution of a Bewley economy), would generally feature more L than H types ($\gamma > 0.5$). The planner of our example would then choose to support an optimal amount of consumption dispersion such that $c^L/c^H = \left[\left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right)\right]^{\frac{1}{\sigma}} < 1$.

Consider next the case with default costs, $\phi(g_1) > 0$. Here, it is possible to support repayment equilibria even when $\omega \geq \gamma$. As Figure 15 shows, there are thresholds value of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega >=< \gamma$, whereas in the previous Figure this was only the case for $\omega < \gamma$.

Figure 15: Default decision with non-utilitarian planner when $\phi(g_1) > 0$



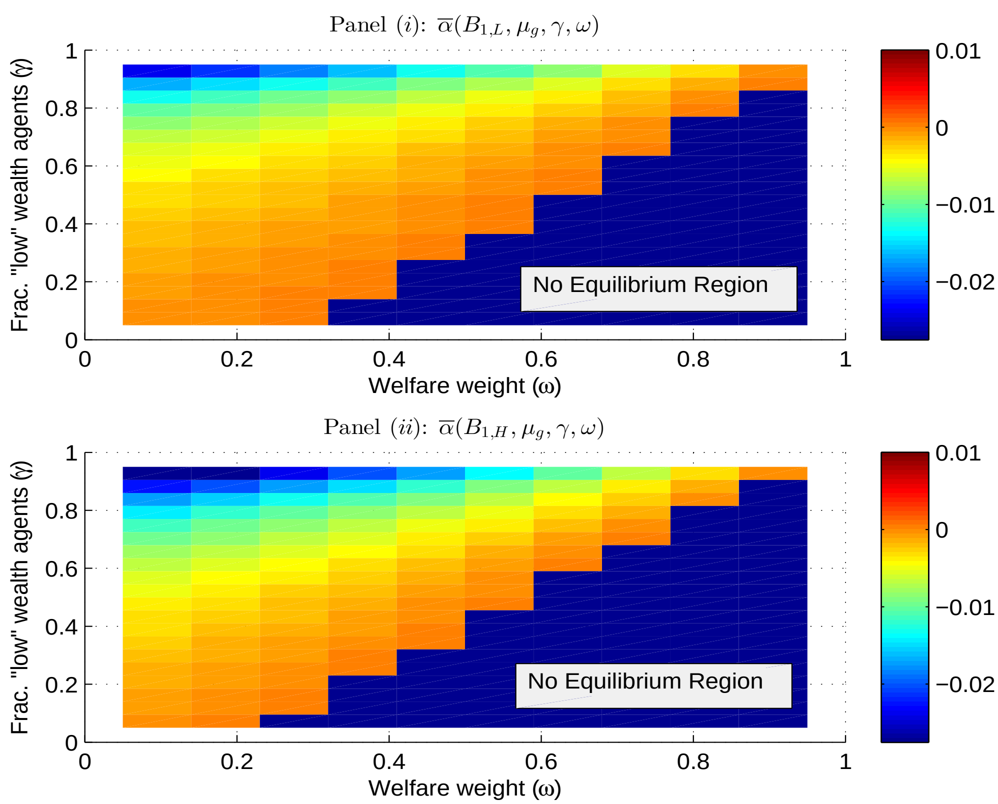
It is also evident in Figure 15 that the range of values of ϵ for which repayment is chosen widens as γ rises relative to ω. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in ϵ, do not produce consumption dispersion larger than the value of $\hat{\epsilon}$ associated with a given (ω,γ) pair. Intuitively, the consumption of H-type agents must not exceed that of L-type agents by more than what $\hat{\epsilon}$ allows. If it does, the preference for redistribution of the planner takes over, and default is optimal.

5.2.2 Quantitative Results

We discuss next a set of quantitative results for this extension of the model with the same set of calibrated parameter values shown in Table 1 and a range of values of ω. First we examine the planner's welfare gain of default ($\overline{\alpha}(B_1, g_1, \gamma, \omega)$), which is constructed in the same way as before, starting with the welfare gains of default for H- and L-type agents and aggregating them using the planner's payoff function. The only differences are in that now all the welfare measures are also functions of ω and the planner's payoff uses ω to weight the agents' utilities.

Figure 16 shows how the planner's welfare gain of default varies with ω and γ for two different levels of government debt ($B_{1,L} = 0.07$ and $B_{1,H} = 0.19$). The no-equilibrium region, which exists for the same reasons as before, is shown in dark blue. In this region, at the given values of γ and ω the government chooses to default on B_1 for all values of g_1 and thus the debt market collapses.

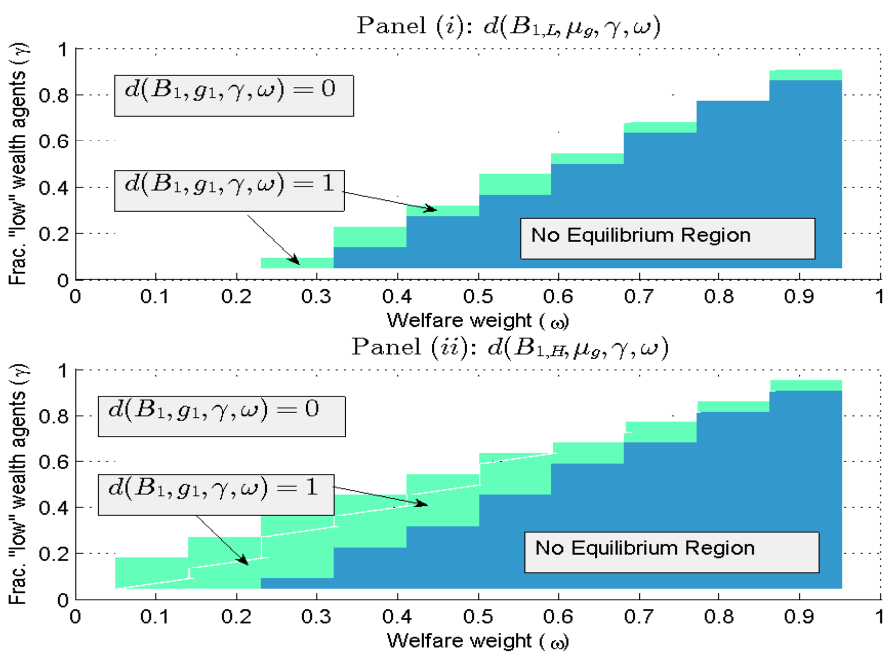
Figure 16: Planner's welfare gain of default $\overline{\alpha}(B_1, g_1, \gamma, \omega)$



In line with the previous discussion, within the region where the equilibrium is well defined, the planner's value of default increases monotonically as its preference for redistribution (ω) increases, keeping γ constant, and falls as actual wealth concentration (γ) rises, keeping ω constant. Because of this, the north-west and south-east corners in each of the panels present cases that are at very different positions on the preference-for-default spectrum. When ω is low, even for very high values of γ, the government prefers to repay (north-west corner), because the government puts relatively small weight on L-type agents. On the contrary, when ω is high, even for low levels of γ, a default is preferred. The two panels also show that for the two values of B_1 considered, the government prefers repayment instead of default when $\omega = \gamma$, because this exercise still has the default cost as calibrated in Section 4. It is also interesting to note that as we move from Panel (i) to Panel (ii), so that debt increases, the set of γ's and ω's such that the equilibrium exists or repayment is preferred (i.e. a negative $\overline{\alpha}(B_1, g_1, \gamma, \omega)$) shrinks. For example, for $\omega = 0.40$, the equilibrium exists for $\gamma > 0.10$ when $B_1 = B_{1,L}$, while it exists only for $\gamma > 0.25$ if $B_1 = B_{1,H}$.

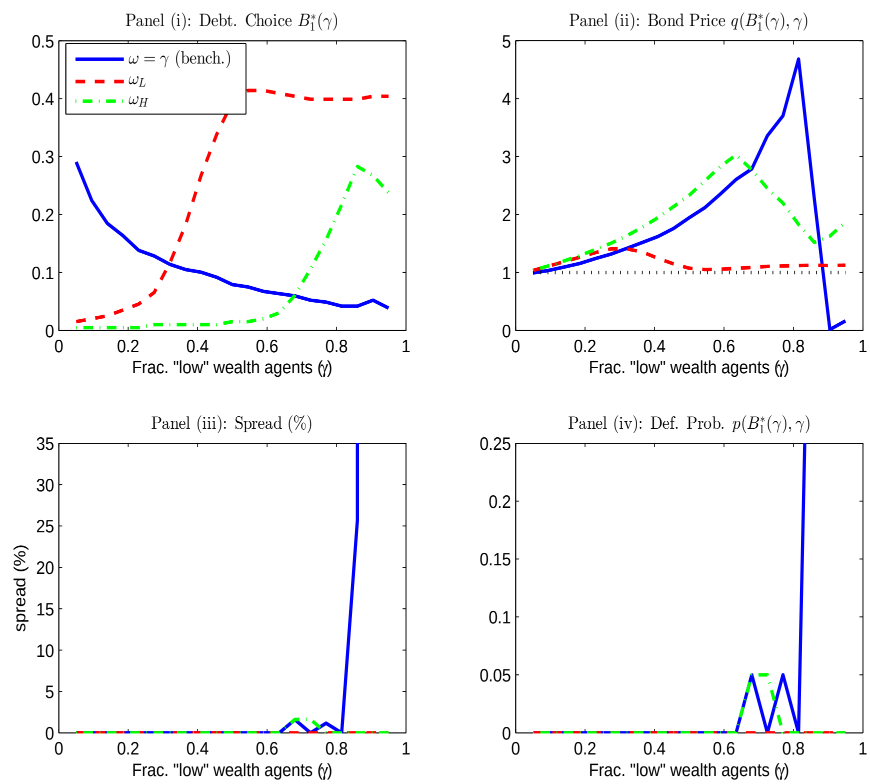
Figure 17 shows the default decision rule induced by the planner's welfare gains of default, again as a function of ω and γ for the same two values of B_1. The region in white corresponds to cases where $d(B_1, g_1, \gamma, \omega) = 0$, the green region corresponds to $d(B_1, g_1, \gamma, \omega) = 1$ and the blue region corresponds to cases in which there is no equilibrium.

Figure 17: Default Decision Rule $d(B_1, g_1, \gamma, \omega)$



In line with the pattern of the government's welfare gains of default presented in Figure 16, this Figure shows that when the government's preference for redistrbution is high enough, the government chooses default, and for a given ω the default region is larger the more dispersed is the wealth distribution (lower γ). Taxes and prices for given values of B_1 and ω are linked to the default decision and γ as in the benchmark model and the intuition behind their behavior is straightforward. Since the main novelty of this extension arises from how changes in ω affect government policies for given values of γ, we chose not show them here in order to keep the discussion brief.

Figure 18: Equilibrium Debt, Prices and Default Probability for different ω



Panels $(i) - (iv)$ in Figure 18 display three scenarios for the optimal debt levels chosen by the government in the first period and the associated equilibrium bond prices, spreads and default probabilities all as functions of γ. The scenario with $\omega = \gamma$, shown in blue, is just the baseline case of Section 4, and the other two scenarios consider cases in with the preference for redistribution is low ($\omega = \omega_L = 0.32$, shown in red) and high ($\omega = \omega_H = 0.68$, shown in green) to illustrate how the model's predictions are affected by arbitrary changes in planner's weights. It is important to note that along the benchmark case both ω and γ vary together because they are always equal, while in the other two scenarios ω is fixed and γ varies. For this reason, the line corresponding to the ω_L case intersects the benchmark solution when $\gamma = 0.32$, and the one for ω_H intersects the benchmark when $\gamma = 0.68$.

Figure 18 shows that the optimal debt level is increasing in γ until γ becomes larger than 0.5 for the ω_L case and 0.9 for the ω_H case. This is because, following the analysis illustrated in Figure 15, the incentives to default grow weaker and the repayment zone widens as γ increases for a fixed value of ω. Moreover, the demand composition effect of higher γ is still present, so along with the lower default incentives we still have the increasing per capita demand for bonds of H types. Together these effects drive the increase in the optimal debt choice of the government. Note, however, that the mechanism wears off around $\gamma = 0.5$, with a 40% debt ratio, for ω_L, and $\gamma = 0.9$, with a 30% debt ratio, for ω_H.

In this model with preference for redistribution, the government is still aiming to optimize debt focusing on the resources it can reallocate across periods and agents, which are determined by $q(B_1)B_1$, and internalizing the response of bond prices to debt choices.[16] This relationship, however, behaves very differently than in the benchmark model, because now *higher* optimal debt is carried at increasing equilibrium bond prices, which leads the planner internalizing the price response to choose higher debt, whereas in the benchmark model *lower* optimal debt was carried at increasing equilibrium bond prices, which led the planner internalizing the price response to choose lower debt. In the empirically relevant range of γ, and for values of ω lower than those in that γ range, this model can sustain significantly higher debt than the model with utilitarian payoff, and close to the observed European average. In the case with ω_H, if γ is near 0.8 the government chooses a 20 percent debt ratio that has a 5 percent probability of default. With that same γ, the planner with ω_L chooses a debt ratio of 40 percent with a negligible default probability.

Similar mechanisms to those explained above account for the fact that, for a given ω, bond prices are first increasing in γ and then decreasing, as the demand composition effect first dominates and then is dominated by the increasing bond supply and eventually by default risk. Spreads are increasing in ω and maintain the property that they are increasing in γ as in the benchmark.

6 Conclusions

This paper proposes a framework in which domestic sovereign default and public debt subject to default risk emerge as an equilibrium outcome. In contrast with standard models of sovereign debt, this framework highlights the role of wealth heterogeneity and the distributional effects of

[16]When choosing B_1, the government takes into account that higher debt increases disposable income for L-type agents in the initial period but it also implies higher taxes in the second period (as long as default is not optimal). Thus, the government is willing to take on more debt when ω is lower.

default across domestic agents in shaping the government's default incentives. These are features common to both historical domestic default events and the ongoing European debt crisis.

The framework we developed consists of a two-period model with high- and low-wealth agents, non-insurable aggregate uncertainty in the form of shocks to government expenditures and default risk, and a utilitarian government who sets debt, taxes and the default decision responding to distributional incentives. The government is aware of its inability to commit, and chooses how much debt to issue optimally, taking into account how the debt chosen in the first period influences the second period's default incentives and default probability, and the feedback of these two into the first period's equilibrium price of government bonds and default risk spreads.

In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Default is optimal when repaying hurts relatively poor agents more than defaulting hurts relatively rich agents, and this happens at optimally-chosen debt ratios if the ownership of public debt is sufficiently concentrated and government expenditures are relatively high. Under these conditions, the government values more the social costs implied by the increased taxation that is needed to both service the debt and pay for government expenditures than the costs associated with wiping out assets owned by the high-wealth agents. We also showed, however, that distributional incentives alone cannot support equilibria with debt, but that these can be supported introducing default costs or a planner with non-utilitarian welfare weights.

Quantitative results based on a calibration to European data show that sustainable debt falls and default risk rises as the level of wealth concentration rises. Because of default risk, sustainable debt is much lower than when the government is committed to repay (at the same levels of wealth inequality). In the range of observed ratios of the fraction of agents who own government bonds, the model supports debt ratios of about 1/3rd of the average European debt ratio at spreads close to 400 basis points.

In what we labeled the political economy extension, a non-utilitarian planner displays a preference for redistribution reflected in the weight it gives to low-wealth agents. Sustainable debt becomes an increasing function of wealth inequality, instead of decreasing as in the utilitarian benchmark. This is because incentives to default are weaker when the government's preference for redistribution falls increasingly below wealth inequality. In this setup, optimal debt chosen in the observed range of wealth inequality measures can easily exceed those supported in the utilitarian benchmark by wide margins, and can be of similar magnitude as the European average with a default probability of about 5 percent.

We see this model as a simple blueprint for further research into models of domestic sovereign default driven by distributional incentives and their interaction with agent heterogeneity and incomplete insurance markets. The two-period environment is a very useful starting point because of the ease with which it can be analyzed and solved, but it also imposes limitations on attempts to bring the model to the data. In particular, self-insurance motives, which the literature shows can produce significant welfare benefits for the existence of public debt markets (see Aiyagari and McGrattan (1998)), are minimized by the two-period life horizon. In an infinite horizon model, this mechanism could produce a large, endogenous cost of default that may support the existence of public debt subject to default risk without exogenous costs of default and/or political economy considerations. In work in progress we are looking into this possibility.

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A1 Appendix: The Model without Default Risk and Logarithmic Utility

This Appendix derives solutions for a version of the model in which low-wealth (L) types do not hold any bonds and high-wealth (H) types buy all the debt. We cover first the fully deterministic case, without any shocks to income or government policies, and no default risk, but government expenditures may be deterministically different across periods. Government wants to use debt to relocate consumption across agents and across periods optimally given a utilitarian welfare function. Ruling out default on initial outstanding debt, the planner trades off the desire to use debt to smooth taxation for L types (reduce date-0 taxes by issuing debt) against the cost of the postponement of consumption this induces on H types who save to buy the debt. Log utility provides closed form solutions. The goal is to illustrate the mechanisms that are driving the model when default risk and stochastic government purchases are taken out. Later in the Appendix we derive some results for the model with stochastic government purchases, and make some inferences for the case with default risk.

A1.1 Fully deterministic model

A1.1.1 Households

A fraction γ of agents are L types, and $1-\gamma$ are H types. Preferences are:

$$\ln(c_0^i) + \beta \ln(c_1^i) \qquad for \quad i = L, H \tag{A.1}$$

Budget constraints are:

$$\begin{aligned} c_0^L &= y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - q b_1^H && \text{(A.2)} \\ c_1^L &= y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H && \text{(A.3)} \end{aligned}$$

Since L types do not save, the solution to their problem is trivial: they can only consume what their budget constraints allow. This is important because altering taxes affects disposable income, which will in turn affect the optimal debt choice of the government. For H types, the Euler equation is:

$$q = \beta \frac{c_0^H}{c_1^H} \tag{A.4}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, it must be the case that they are credit constrained (i.e. they would want to hold negative assets). That is, at the equilibrium price of debt their Euler equation for bonds would satisfy:

$$q > \beta \frac{c_0^L}{c_1^L} \tag{A.5}$$

A1.1.2 Government

The government budget constraints are:

$$\begin{aligned} \tau_0 &= g_0 + B_0 - qB_1 && \text{(A.6)} \\ \tau_1 &= g_1 + B_1 && \text{(A.7)} \end{aligned}$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma(\ln(c_0^L) + \beta \ln(c_1^L)) + (1-\gamma)(\ln(c_0^H) + \beta \ln(c_1^H)) \tag{A.8}$$

A1.1.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $b_1^H = B_1/(1-\gamma)$. By construction the same condition is assumed to hold for the initial conditions b_0^H and B_0.This implies that the initial wealth of H-types is given by $b_0^H = B_0/(1-\gamma)$.

Rewriting the Euler equation of H types using the budget constraint, the government budget constraints and the bond market-clearing conditions yields:

$$q = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q\left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \tag{A.9}$$

Hence, the equilibrium price of bonds for a given government supply is:

$$q(B_1) = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta) B_1} \tag{A.10}$$

Note that this price is not restricted to be lower than 1 (i.e. $q(B_1) > 1$ which implies a negative real rate of return on government debt can be an equilibrium outcome). In particular, as γ rises the per capita bond demand of H-types increases and this puts upward pressure on bond prices, and even more so if the government finds it optimal to offer less debt than the initial debt, as we showed numerically and explain further below. As $\gamma \to 1$, the limit of the equilibrium price goes to $q(B_1) = \frac{\beta}{1+\beta}\frac{B_0}{B_1}$ even tough market-clearing requires the demand of the infinitesimal small H type to rise to infinity.

After some simplification, the derivative of this price is given by:

$$q'(B_1) = \frac{-q(B_1)\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)}{\left[y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta)B_1\right]} \tag{A.11}$$

which at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$ (notice $c_1^H > 0$ implies that the denominator of this expression must be positive).

Consider now what happens to this equilibrium as the fraction of L-types vanishes. As $\gamma \to 0$, the economy converges to a case where there is only an H type representative agent, and the price is simply $q(B_1) = \beta\frac{y-g_0}{y-g_1}$, which is in fact independent of B_1 and reduces to β if government purchases are stationary. Trivially, in this case the planner solves the same problem as the representative agent and the equilibrium is efficient. Also, for an exogenously given B_0 and stationary g, the competitive equilibrium is stationary at this consumption level:

$$c^h = y - g + \left(\frac{\gamma}{1-\gamma}\right)\frac{B_0}{1+\beta} \tag{A.12}$$

and the optimal debt is:

$$B_1 = \frac{B_0}{1+\beta} \tag{A.13}$$

Hence, in this case consumption and disposable income each period are fully stationary, yet the optimal debt policy is always to reduce the initial debt by the fraction $1/(1+\beta)$. This is only because of the two-period nature of the model. With an infinite horizon, the same bond price would imply that an equilibrium with stationary consumption and an optimal policy that is simply $B_1 = B_0$. It also follows trivially that carrying no initial debt to start with would be first-best, using lump-sum taxation to pay for g.

A1.1.4 Optimal debt choice

The government's optimal choice of B_1 in the first period solves this maximization problem:

$$\max_{B_1}\left\{\begin{array}{c}\gamma\left[\ln\left(y - g_0 - B_0 + q(B_1)B_1\right) + \beta\ln\left(y - g_1 - B_1\right)\right] \\ +(1-\gamma)\left[\ln\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0 - q(B_1)\left(\frac{\gamma}{1-\gamma}\right)B_1\right) + \beta\ln(y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)B_1)\right]\end{array}\right\} \tag{A.14}$$

where $q(B_1) = \beta \frac{y-g_0+\left(\frac{\gamma}{1-\gamma}\right)B_0}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)B_1}$.

The first-order condition is:

$$\begin{aligned} &\gamma \left[u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta u'(c_1^L)\right] \\ &+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right)\left[-u'(c_0^H)\left[q'(B_1)B_1 + q(B_1)\right] + \beta u'(c_1^H)\right] = 0 \end{aligned} \tag{A.15}$$

Using the Euler equation of the H types and simplifying:

$$u'(c_0^L) + \left[\frac{u'(c_0^L)q(B_1) - \beta u'(c_1^L)}{q'(B_1)B_1}\right] = u'(c_0^H) \tag{A.16}$$

This expression is important, because it defines a wedge between equating the two agents' marginal utility of consumption that the planner finds optimal to maintain, given that the only instrument that it has to reallocate consumption across agents is the debt. Notice that, since as noted earlier for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation implies that at the equilibrium price: $u'(c_0^L)q(B_1) - \beta u'(c_1^L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L) > u'(c_0^H)$ or $c_0^H > c_0^L$, and notice that from the budget constraints this also implies $B_0 - q(B_1)B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$.

Comparison with no-debt equilibrium: Notice that since $B_0 - q(B_1)B_1 > 0$, the planner is allocating less utility to L type agents than those agents would attain without any debt. Without debt, and a tax policy $\tau_t = g_t$, all agents consume $y - g_t$ every period, but with debt L-types consume less each period given that $B_1 > 0$ and $B_0 - q(B_1)B_1 > 0$. Compared with these allocations, when the planner finds optimal to choose $B_1 > 0$ is because he is trading off the pain of imposing higher taxes in both periods, which hurts L types, against the benefit the H types get of having the ability to smooth using government bonds. Also, $B_0 - q(B_1)B_1 > 0$ highlights that there is a nontrivial role to the value of B_0, because if B_0 were zero B_1 would need to be negative which is not possible by construction. Hence, the model only has a sensible solution if there is already enough outstanding debt (and wealth owned by H type agents) that gives the government room to be able to improve the H type's ability to smooth across the two periods, which they desire to do more the higher is B_0.

Comparison with sub-optimal debt equilibrium: By choosing positive debt, the government provides tax smoothing for L types. Given B_0 and the fact that $B_0 - q(B_1)B_1 > 0$, positive debt allows to lower date-0 taxes, which increases consumption of L types (since

$c_0^L = y - g_0 - B_0 + q(B_1)B_1)$. The same policy lowers the consumption of H types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)(B_0 - q(B_1)B_1))$. Hence, debt serves to redistribute consumption across the two agents within the period. This also changes inter-temporal consumption allocations, with the debt reducing L types consumption in the second period and increasing H types consumption. Hence, with commitment to repay B_0, the debt will be chosen optimally to trade off these social costs and benefits of issuing debt to reallocate consumption atemporally across agents and intertemporally.

It is also useful to notice that the demand elasticity of bonds is given by $\eta \equiv q(B_1)/\left(q'(B_1)B_1\right)$, so the marginal utility wedge can be expressed as $\eta\left[u'(c_0^L) - \frac{\beta u'(c_1^L)}{q(B_1)}\right]$ and the planner's optimality condition reduces to:

$$1 + \eta\left[1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)}\right] = \frac{u'(c_0^H)}{u'(c_0^L)} \tag{A.17}$$

Hence, the planner's marginal utility wedge is the product of the demand elasticity of bonds and the L-type agents shadow value of being credit constrained (the difference $1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)} > 0$, which can be interpreted as an effective real interest rate faced by L-type agents that is higher than the return on bonds). The planner wants to use positive debt to support an optimal wedge in marginal utilities only when the demand for bonds is elastic AND L-type agents are constrained.

A1.2 Extension to Include Government Expenditure Shocks

Now consider the same model but government expenditures are stochastic. In particular, realizations of government purchases in the second period are given by the set $[g_1^1 < g_1^2 < ... < g_1^M]$ with transition probabilities denoted by $\pi(g_1^i|g_0)$ for $i = 1, ..., M$ with $\sum\limits_{i=1}^{M} \pi(g_1^i|g_0) = 1$.

A1.2.1 Households

Preferences are now:

$$\ln(c_0^i) + \beta\left(\sum_{i=1}^{M} \pi(g_1^i|g_0)\ln(c_1^i)\right) \qquad for \quad i = L, H \tag{A.18}$$

Budget constraints are unchanged:

$$c_0^L = y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - qb_1^H \tag{A.19}$$

$$c_1^L = y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H \tag{A.20}$$

We still assume that L types do not save, so they can only consume what their budget constraints allow. For H types, the Euler equation becomes:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left(\frac{c_0^H}{c_1^H} \right) \tag{A.21}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, their Euler equation for bonds must satisfy:

$$q > \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left(\frac{c_0^L}{c_1^L} \right) \tag{A.22}$$

A1.2.2 Government

The government budget constraints are unchanged:

$$\begin{aligned} \tau_0 &= g_0 + B_0 - qB_1 \\ \tau_1 &= g_1 + B_1 \end{aligned}$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma \left(\ln(c_0^L) + \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \ln(c_1^L) \right) + (1 - \gamma) \left(\ln(c_0^H) + \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \ln(c_1^H) \right) \tag{A.23}$$

A1.2.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $b_1^H = B_1/(1-\gamma)$ and the H-types Euler equation.

We can solve the model in the same steps as before. First, rewrite the Euler equation of H types using their budget constraints, the government budget constraints and the market-clearing conditions:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i | g_0) \left[\frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q \left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \right] \tag{A.24}$$

From here, we can solve again for the equilibrium price at a given supply of bonds:

$$q(B_1) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1}\right)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1}\right)} \tag{A.25}$$

As $\gamma \to 0$ we converge again to the world where there is only an H type representative agent, but now the pricing formula reduces to the standard formula for the pricing of a non-state-contingent asset $q(B_1) = \beta \left(\sum_{i=1}^{M} \pi(g_1^i|g_0) \frac{y-g_0}{y-g_1}\right)$. As $\gamma \to 1$ the equilibrium degenerates again into a situation where market clearing requires the demand of the infinitesimal small H type to rise to infinity.

The derivative of the price at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$. To show this, define $\Pi(B_1) \equiv \sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1^i + \left(\frac{\gamma}{1-\gamma}\right) B_1}$ which yields $\Pi'(B_1) = -\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y - g_1^i + \left(\frac{\gamma}{1-\gamma}\right) B_1\right)^2} < 0$. Then taking the derivative $q'(B_1)$ and simplifying we get:

$$q'(B_1) = \frac{\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \left[\Pi'(B_1) - \beta \left(\frac{\gamma}{1-\gamma}\right) (\Pi(B_1))^2\right]}{\left(1 + \beta \left(\frac{\gamma}{1-\gamma}\right) B_1 \Pi(B_1)\right)^2} \tag{A.26}$$

Since $\Pi'(B_1) < 0$ and positive c_0^H implies $y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 > 0$, it follows that $q'(B_1) < 0$.

We can also gain some insight into the implicit risk premium (the ratio $q(B_1)/\beta)$) and the related question of why the asset price can exceed 1 in this setup. Recall that in fact the latter was already possible without uncertainty when γ is large enough, because of the demand composition effect (higher γ implies by market clearing that the fewer H type agents need to demand more bonds per capita, so the bond price is increasing in γ and can exceed 1). The issue now is that, as numerical experiments show, an increase in the variance of g_1 also results in higher bond prices, and higher than in the absence of uncertainty, and again for γ large enough we get both $q(B_1) > 1$ and $q(B_1)/\beta > 1$. The reason bond prices increase with the variability of government purchases is precautionary savings. Government bonds are the only vehicle of saving, and the incentive for this gets stronger the larger the variability of g_1. Hence, the price of bonds is *higher* in this stochastic model than in the analogous deterministic model because of precautionary demand for bonds, which adds to the effect of demand composition (i.e. the price is higher with uncertainty than without at a given γ).

A1.2.4 Pricing Function with Default

We can also make an inference about what the pricing function looks like in the model with default risk, because with default we have a similar Euler equation, except that the summation that defines the term $\Pi(B_1)$ above will exclude all the states of g_1 for which the government chooses to default on a given B_1 (and also at a given value of γ). That is, the term in question becomes $\Pi^D(B_1) \equiv \sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1} \leq \Pi(B_1)$, and the pricing function with default risk is:

$$q^D(B_1) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \Pi^D(B_1)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \Pi^D(B_1)} \leq q(B_1) \tag{A.27}$$

Moreover, it follows from the previous analysis that this pricing function is also decreasing in B_1 ($q^{D\prime}(B_1) < 0$), and $\Pi^{D\prime}(B_1) = -\sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1\right)^2}$ is negative but such that $\Pi'(B_1) \leq \Pi^{D\prime}(B_1) < 0$. Also, it is clear from the above pricing functions that if the probability of default is small, so that are only a few values of i for which $d(B_1, g_1^i, \gamma) = 1$ and/or the associated probability $\pi(g_1^i|g_0)$ is very low, the default pricing function will be very similar to the no-default pricing function.

A1.2.5 Optimal debt choice

The government's optimal choice of B_1 solves again a standard maximization problem:

$$\max_{B_1} \left\{ \begin{array}{c} \gamma \left[\ln\left(y - g_0 - B_0 + q(B_1)B_1\right) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln\left(y - g_1 - B_1\right) \right] \\ +(1-\gamma) \left[\ln\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q(B_1)\left(\frac{\gamma}{1-\gamma}\right) B_1\right) \right. \\ \left. +\beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1) \right] \end{array} \right\} \tag{A.28}$$

where $q(B_1)$ is given by the expression solved for in the competitive equilibrium.

The first-order condition is:

$$\gamma \left[u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) u'(c_1^L) \right] \tag{A.29}$$

$$+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right) \left[-u'(c_0^H)\left[q'(B_1)B_1 + q(B_1)\right] + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) u'(c_1^H) \right] = 0$$

Using the stochastic Euler equation of the H types and simplifying:

$$u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta \sum_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L) = u'(c_0^H)q'(B_1)B_1 \tag{A.30}$$

$$u'(c_0^L) + \left[\frac{u'(c_0^L)q(B_1) - \beta \sum\limits_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L)}{q'(B_1)B_1}\right] = u'(c_0^H) \tag{A.31}$$

This last expression, compared with the similar expression of the planner without uncertainty, implies that in the planner's view, the government expenditure shocks only matter to the extent that they affect the shadow price of the binding credit constraint of the L types. As before, since for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation would imply that at the equilibrium price: $u'(c_0^L)q(B_1) - \beta \sum\limits_{i=1}^{M} \pi(g_1^i|g_0)u'(c_1^L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L) > u'(c_0^H)$ or $c_0^H > c_0^L$, and notice that from the budget constraints this implies again $B_0 - q(B_1)B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$. Thus the optimal debt choice again has an incentive to be lower than the initial debt.