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DO EQUILIBRIUM REAL BUSINESS CYCLE THEORIES EXPLAIN POST-WAR U.S. BUSINESS CYCLES?

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ABSTRACT

This paper presents and interprets some new evidence on the validity of the Real Business Cycle approach to business cycle analysis. The analysis is conducted in the context of a monetary business cycle model which makes explicit one potential link between monetary policy and real allocations. This model is used to interpret Granger causal relations between nominal and real aggregates. Perhaps the most striking empirical finding is that money growth does not Granger cause output growth in the context of several multivariate VARs and for various sample periods during the post war period in the U.S. Several possible reconciliations of this finding with both real and monetary business cycles models are discussed. We find that it is difficult to reconcile our empirical results with the view that exogenous monetary shocks were an important independent source of variation in output growth.

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1. Introduction

During the past decade there has been a resurgence of interest in equilibrium real business cycle theories. According to these theories, the recurrent fluctuation in outputs, consumptions, investments, and other real quantities-- what we shall refer to as the business cycle phenomenon-- is precisely what one should expect to emerge from industrial market economies in which consumers and firms solve intertemporal optimum problems under uncertainty. Moreover, fluctuations in real quantities are attributed to exogenous technological and taste shocks combined with various sources of endogenous dynamics including adjustment costs, time-to-build capital goods, and the non-time-separability of preferences (e.g., Kydland and Prescott (1982), Long and Plosser (1983), Kydland (1984), and Prescott (1986)). Common characteristics of these models are that there is a complete set of contingent claims to future goods and services, agents have common information sets, and the only "frictions" in the economy are due to technological factors. In particular, real business cycle (RBC) models abstract entirely from monetary considerations and the fact that exchange in modern economies occurs via the use of fiat money.

There are two interpretations of this modeling strategy. The first interpretation is that monetary institutions and monetary policy are assumed to be inherently neutral in the sense that real allocations are invariant to innovations in financial arrangements and monetary policy. Money is a veil regardless of how much the veil flutters. The second interpretation is that the market organizations and the nature of monetary policy in the sample period being examined are such as that an RBC model provides an accurate characterization of the real economy. Under this interpretation RBC models may be useful frameworks for examining the determination of real allocations for certain institutional environments and classes of monetary policies. Money may be a veil as long as the veil does not flutter too much. In our view, proponents of RBC theories are not claiming that monetary policy cannot or has never had a significant impact on the fluctuation of real output, investment, or consumptions. Rather we subscribe to the second interpretation of RBC analyses as investigations of real allocations under the assumption that, to a good approximation, monetary policy shocks have played an insignificant role in determining the behavior of real variables.

The assumption in equilibrium RBC models that monetary shocks have not been an important source of aggregate fluctuations is similar to the basic premise of certain early Keynesian models. According to these models, interest rates affected at most long-lived fixed investments, so the interest elasticity of aggregate demand was perceived as being low. Furthermore, the principal shocks impinging on the economy had their direct effects on aggregate demand. In particular, investment was thought to be influenced capriciously by "animal spirits." Thus, aggregate fluctuation was associated primarily with real shocks. Indeed, the economic environment was assumed to be such that changes in the money supply (both systematic and current updated shocks) had small impacts on real economic activity. The latter feature of these models led to the conclusion that fiscal policy was preferred to monetary policy for stabilizing output fluctuation.

More recent Keynesian-style models share some common features with both early Keynesian and equilibrium RBC models, but they are also fundamentally different in several important respects. Proponents of modern Keynesian models often argue that monetary policy shocks have not been a significant source of "instability" in the U.S. economy. For example Modigliani (1977, p.12) states that "..there is no basis for the monetarists' suggestion that our post war instability can be traced to monetary instability...". Thus on this feature of the economy, proponents of modern Keynesian models and of equilibrium RBC theories are often in agreement.

However, for at least two reasons, it would be misleading to argue that equilibrium RBC models and Keynesian models are simply different versions of business cycle models in which only (or primarily) real shocks matter for fluctuations in real quantities. First, the propogation mechanisms by which exogenous shocks impinge on the endogenous variables are typically very different in the two classes of models. As noted above, equilibrium RBC models focus on such technological frictions as gestation lags in building capital or intertemporal nonseparability of preferences in generating endogenous sources of dynamics. Contingent claims markets are assumed to be complete and goods and asset prices adjust freely in competitive markets. In contrast, Keynesian models typically emphasize various frictions that prevent perfectly flexible goods prices and wages (e.g., Modigliani 1958, Taylor 1980). The sources of these rigidities may be frictions associated with incomplete markets, contractual arrangements, or imperfect competition among firms. These differences in propogation mechanisms may imply very different reactions of the equilibrium RBC and Keynesian economies to the same type of shock.

Second, and closely related to this first consideration, the potential roles for stabilization in these economic environments may be very different. The inflexible prices and incomplete markets in Keynesian models have been used to justify activist fiscal <u>and</u> monetary policies designed to stabilize output fluctuation. Of particular relevance for our analysis is the important role often attributed to monetary policy in affecting the cyclical behavior of real variables in Keynesian models. While exogenous monetary policy shocks may not be important sources of aggregate fluctuations, activist monetary policies are often deemed to have had significant effects on the propogation of nonmonetary shocks. Hence, though the sources of uncertainty may be real, it seems clear that money plays a central role in modern Keynesian models. This interpretation of modern Keynesian models was made explicitly by Modigliani (1977, p.1): "Milton Friedman was once quoted as saying 'We are all Keynesians, now,' and I am quite prepared to reciprocate that 'we are all monetarists' - if by monetarism is meant assigning to the stock of money a major role in determining output and prices".

In contrast, equilibrium RBC models typically assume that there is a complete set of contingent claims markets and that prices are perfectly flexible. One implication of these assumptions in the models that have been examined to date is that real allocations are Pareto optimal. This, in turn, implies that there is no role for a central policy authority. Thus, equilibrium RBC theories assume both that exogenous monetary shocks have not been an important source of fluctuations, and that the policy rules followed by the monetary authorities have not had an important role in propogating nonmonetary shocks. For only under this joint hypothesis does it seem likely that an equilibrium RBC model would accurately characterize the economy.

This paper presents and interprets some new evidence on the validity of the RBC approach to business cycle analysis. Particular attention in given to: (1) the question of whether monetary policy shocks were an important determinant of real economic activity during the post war period, and (2) several potentially important pitfalls in interpreting the available evidence as supporting or refuting the validity of RBC models. The analysis of the first question bears on the validity of any business cycle model that assumes such shocks have been important. Certain aspects of our findings can be viewed as being consistent with both early Keynesian and equilibrium RBC models. However, the extreme assumptions that underlie the former models have been largely rejected in the recent literature (see, e.g., Modigliani 1977).[1] Accordingly, in discussing potential interpretations of our empirical findings we shall focus primarily on the properties of what we call equilibrium business cycles models-- monetary and nonmonetary models that are specified at the level of preferences and technology. This allows us to address directly the strengths and weaknesses of equilibrium RBC models as explanations of fluctuations in the U.S. economy during the post war period.

To date, empirical investigations of RBC models have typically proceeded by fitting these models to the data and evaluating the extent to which the cycles implied by the models match those exhibited by the data. Virtually without exception, studies of RBC models have not considered explicitly the conditions under which RBC models emerge approximately or exactly as special cases of monetary models of the business cycle. In contrast, we begin our analysis in Section 3 by setting forth an explicit monetary business cycle model. The presentation of this model serves three purposes. First, we are able to make precise one potential link between monetary policy actions and real allocations in an equilibrium model. Money matters in this model because agents face a cash-in-advance constraint. Therefore the money-output linkages in our model are very much in the spirit of those in the recent models of Lucas and Stokey (1983, 1984), Townsend (1982), and Svensson (1985) among others. Within the context of this model, we provide sufficient conditions for an RBC model to provide an accurate characterization of real fluctuation in the monetary economy. The conditions discussed are, of course, specific to the model examined. Nevertheless, we feel they are suggestive of the type of strong restrictions on monetary policy rules that will be required for RBC models to accurately approximate other, more complicated, monetary models.

Second, we use this model to interpret Granger causal relations between nominal and real aggregates. Anticipating our results in Sections 4 and 5, we find little empirical support for the proposition that money growth or inflation Granger cause output growth. Interpreted within the context of the monetary model of Section 3, these results suggest that exogenous shocks to the monetary growth rate were not an important independent source of variation in output growth during the post war period in the U.S. Admittedly, this conclusion emerges from considering a model with very simple specifications of the production, monetary exchange technologies as well as a specific market structure. However, introducing more complicated market structures that lead to sticky prices and wages as in Fischer (1977) or overlapping nominal contracts as in Taylor (1980) would only increase the complexity of the interaction between monetary growth and real economic activity. This added complexity would make it more difficult to reconcile the absence of Granger causality of output growth by monetary growth with the belief that exogenous monetary policy shocks were an important source of variation in output. More generally, our empirical evidence suggests that monetary models which imply that money growth rates Granger cause output growth are inconsistent with post war U.S. data.

Third, for certain monetary policy rules, our monetary model implies that the failure to find Granger causality from money to output is not sufficient for an RBC model to accurately characterize the economic environment. More precisely, we argue that even if real shocks are the predominant source of variation in real quantities over the cycle, one may be seriously misled using an RBC model to evaluate the implications of alternative government policies. Essentially, monetary feedback policies can be set in a manner that affects the time series properties of output even though exogenous monetary shocks are not important determinants of output. This may be true even though RBC models appear to fit the data for the sample period by the usual criteria.

The remainder of the paper is organized as follows. Section 2 presents a brief review of the literature on equilibrium RBC models, and our equilibrium monetary business cycle model is presented in Section 3. In Section 4, the laws of motion for the quantity variables implied by an RBC model are compared to those of the monetary model. These two models are used to interpret bivariate vector autoregressive representations (VARs) of money and output. A more extensive set of empirical results are presented in Section 5. "Variance decompositions" for output are displayed for several multivariate time series models of output, monetary growth, inflation, and asset returns. Concluding remarks are presented in Section 6.

2. Overview of Equilibrium Business Cycle Models

A primary goal of business cycle analysis is to explain the recurrent fluctuations of real aggregate economic quantities about their trends and the co-movements among different aggregate economic time series. A key assumption of real business cycle theories is that monetary policy actions are unimportant for explaining either the amplitude or frequency of business cycles. In the words of King and Plosser (1984), RBC theories view "business cycles as arising from variations in the real opportunities of the private economy, which include shifts in government purchases or tax rates as well as technical and environmental conditions (p.363)." In this section we briefly summarize some of the sources of variations in private market economies that have received the most attention in the literature on RBC models.

In modeling aggregate fluctuations, RBC theorists often distinguish between the exogenous sources of uncertainty impinging on economic decisions and the endogenous propagation mechanisms for the exogenous shocks. Adopting this taxonomy, we shall consider, first, the various exogenous sources of uncertainty that have typically been emphasized in the recent literature. In modern economies, many types of shocks impinge on the decisions of agents in different sectors. For the purpose of aggregate business cycle analyses, it is typically assumed implicitly or explicitly that idiosyncratic sector or agent-specific shocks "average out" and have no effect on aggregate quantities. On the other hand, the common components of individual shocks that remain after aggregation are interpreted as the aggregate sources of uncertainty in business cycle models. The number of these aggregate disturbances is typically assumed to be small; often the number does not exceed the number of real quantity variables appearing in the model.

Furthermore, when using RBC models to describe economic data, attention has been restricted almost exclusively to exogenous shifts in the production technologies of goods. See, for example, Kydland and Prescott (1982), Long and Plosser (1983), King and Plosser (1984), Altug (1985), and Hansen (1985). Notably absent from these models are aggregate shocks to preferences and to the fiscal policy rules of governments. The omission of shocks to preferences seems to be simply a matter of taste. Apparently, an implicit assumption in many of the RBC theories is that large technology shocks are more likely than large aggregate stochastic shifts in tastes. We relax this assumption in constructing our illustrative monetary and real business cycle models in Sections 3 and 4.

The absence of shocks to government purchases or tax rates can be explained by the nature of the models that have been developed to date. Kydland and Prescott (1982), and many others since them, have examined economies in which there is a complete set of markets for contingent claims to future goods, there is no private information, and there are no public goods. Therefore, real allocations are Pareto optimal in these economies and there is no welfare improving role for a central government. The absence of incomplete insurance and market failures from these models can in part be attributed to the difficulties involved in analyzing aggregate fluctuations in general equilibrium when such "frictions" are present. However, in some cases, their absence also seems to be a manifestation of the belief that aggregate fluctuation in output that approximates that observed historically can be generated by equilibrium models without introducing these frictions. Long and Plosser (1983), for example, state, regarding their model, "We believe that major features of observed business cycles typically will be found in the kind of model economy outlined above (p.42)." A similar view has recently been expressed by Prescott (1986): "Given the people's ability and willingness to inter- and intra-temporally substitute consumption and leisure and given the nature of the changing production set, there would be a puzzle if the American economy did not display the business cycle phenomena. By display the business cycle phenomena, I mean that the amplitudes of fluctuations of the key economic aggregates and their serial correlation properties are close to that predicted by theory (p. 1)."

Although RBC models have relied on unobserved technology shocks to induce fluctuations in output, investment, consumptions, and hours worked, these models typically adopt quite parsimonious time series representations of these shocks. Parsimony is important if these models are to have refutable implications. It has long been known that low order stochastic difference equations can generate recurring irregular cyclical fluctuations not unlike those exhibited by aggregate time series. Thus, the covariogram of a given set of variables implied by a model can be matched to the sample covariagram of the data by specifying sufficiently rich laws of motion for the unobserved shocks in a model. But it is clear that profligately parameterized specifications of unobserved shock processes do not yield interesting explanations of business cycles. RBC models lead to overidentifying restrictions on the autocovariance functions by allowing for only a small number of shocks with parsimonious time series representations. More complicated patterns of autocorrelations and patterns of cross-correlations among the variables are then induced by endogenous sources of dynamics. It is these endogenous sources of dynamics (what King and Plosser (1985) refer to as the propagation mechanisms) that are the centerpieces of equilibrium RBC theories.

To date, RBC theorists have stressed the role of technology and agents' preferences in magnifying the response of economic systems to exogenous impulses. For example, Kydland and Prescott (1982) assume that it takes time to build capital goods (i.e., more than one period or decision interval). This time-to-build technology has an important effect on the time series properties of investment and output. They also assume that consumers' utility from leisure in the current period depends on past leisure decisions. Specifically, the marginal utility of an additional unit of leisure today is larger (smaller) the smaller (larger) is the amount of leisure consumed in previous periods. This specification of preferences leads to substantially more intertemporal substitution of leisure than the comparable time-separable specification of preferences. Since hours worked are more sensitive to a given change in the real wage rate with this specification of preferences, they are better able to capture the fluctuation in aggregate hours than the corresponding model with time-separable utility. Nevertheless, their model falls short of providing an adequate explanation of the time series behavior of hours worked. Efforts are currently underway to enrich the specification of the labor market in their model in an attempt to improve the fit of the model (Hansen 1985).

Although the models examined by Kydland and Prescott (1982) and Kydland (1984) fail to fully explain fluctuation in hours worked, it is nevertheless striking that they do so well in attempting to replicate the time series behavior of several of the economic aggregates considered. As the authors emphasize, their results are certainly encouraging for they show that fairly simple dynamic equilibrium models are capable of generating the types of cycles that are similar to those that have been observed historically. Moreover, these examples provide a useful reminder of the fact that fluctuation in real quantities per se need not reduce social welfare; real allocations are Pareto optimal in their model.

Those economists who believe firmly that monetary fluctuations and the actions of the monetary authority are central to an understanding of business cycles may be inclined to dismiss these models outright because of their omission of monetary considerations. Yet there seems to be both indirect and direct evidence that such a dismissal is difficult to support. Indirect evidence is provided by Kydland and Prescott (1982), Long and Plosser (1983), and Kydland (1984), among others. These authors have shown that, for plausible values of the parameters characterizing preferences and technology, the variances of the shocks to technology can be chosen such that equilibrium RBC models imply empirical second moments for certain real aggregates that approximately match the corresponding sample second moments.

More formal evidence is provided in the provocative studies by Sims (1980a, 1980b). He found that monetary shocks had little explanatory power for industrial production during the post World War II period, when lagged values of industrial production and nominal interest rates were included. Sims interpreted the contribution of nominal interest rates in predicting industrial production as capturing expectations about the future productivity of capital, which is very much in the spirit of real business cycle analysis. Litterman and Weiss (1985) corroborate Sims' findings and also give an RBC interpretation to their findings.

The models considered by Kydland and Prescott (1982) and Long and Plosser (1983) are representative agent models with complete markets so that credit does not enter prominently in the determination of real quantities. King and Plosser (1984) have discussed an extended version of an equilibrium RBC model in which there is a role for organized credit markets. Their model can be interpreted as a RBC model in which certain (perhaps informational) frictions in private markets lead to the creation of institutions that specialize in issuing credit. King and Plosser proceed to argue that, while credit may have an important influence on the nature of aggregate fluctuations, the actions of the Federal Reserve's Open Market Committee may not be an important independent source of fluctuations in real quantities and relative prices. As evidence for this proposition, they note that real output seems to be significantly correlated with inside money but is only weakly correlated with outside money.

More recently, critics of the view that RBC theories are accurate characterizatons of the post war experience have noted that these findings are also consistent with comparably simple business cycle models in which monetary policy plays an important role. For instance, McCallum (1983a) has shown that if the Federal Reserve followed an interest rate rule (or mixed interest rate-monetary aggregate rule) for most of the postwar period, then the interest rate innovation in Sims' regressions may be a proxy for the innovation in the policy rule of the Fed. For the same reasons, the analysis by King and Plosser (1984) may not accurately capture the effects of monetary policy shocks on real output (McCallum 1985).

Critics of RBC models have provided several additional challenges to the conclusion that the RBC models studied to date explain the post war experience in the U.S. Most of the studies purporting to provide evidence in support of RBC models have examined only a limited set of own- and cross-correlations of aggregate quantity variables. In particular, cross-correlations of relative prices and asset returns with aggreate real quantities have typically not been studied. There is no a priori reason why the restrictions examined should be given more weight than other restrictions on cross-correlations in evaluating the performance of these models. When cross-correlations of asset returns and output are examined there is substantially more evidence against RBC models (see, for example, Mehra and Prescott (1985)). Moreover, when particular RBC models are subjected to formal methods of estimation and inference which incorporate a fairly comprehensive set of moment restrictions, the results are not supportive of the models (e.g., Altug (1985), Eichenbaum, Hansen and Singleton (1985), and Mankiw, Rotemberg and Summers (1985)).

The acceptance or rejection of RBC models must be based in part on an assessment of the plausibility of the variances and autocorrelations of the technology shocks.[2] Kydland and Prescott (1982) take their structure of preferences and technology as given and then determine the values of the variances of the shocks to technology that are consistent with particular second moments of observed variables. Prescott (1986) provides an estimate of the standard deviation of the shock to technology in a simple growth model, but this measure is in effect the standard error of the residual from regressing the growth rate of output on the growth rates of capital and labor. This measure is justified by his assumption of a Cobb-Douglas production function defined over labor and capital and thus is clearly model dependent in same manner that the Kydland-Prescott estimates of standard deviations were model dependent. In a manner that attempts to account for measurement errors, Prescott estimates the ratio of the standard deviations of the percentage changes in the technology shock and output to be approximately .40. It seems difficult to assess the plausibility of this estimate and related estimates provided by Kydland and Prescott (1982), especially in light of the highly aggregated nature of their models.

Furthermore, we have little independent evidence about the absolute magnitudes of these second moments, since the theories do not lead us to specific sources for these disturbances. Hamilton (1983) provides some descriptive evidence that oil price shocks were an an important source of aggregate fluctuations in the U.S. during the post war period. Also, Miron (1985) argues that weather shocks are an important source of variation in aggregate consumption. This strategy of attempting to quantify the various shocks impinging on the economy may yield important insights into the nature of real shocks underlying business cycles.

A complementary strategy is to assess empirically the relative importance of alternative types of shocks in the context of models that accommodate various types of shocks. RBC theorists have not attempted to assess the relative importance of monetary versus technology shocks, for example. A comparison of the stochastic properties of these shocks may lead to more convincing conclusions about RBC models than statements about the absolute magnitudes of technology or preference shocks. In models that accommodate monetary shocks, it would be possible to decompose the variation in output into variation attributable to monetary and real shocks (assuming the model is identified). Then the theoretical problem of characterizing the classes of economies and monetary policies for which real allocations are well described by RBC models could be addressed. To our knowledge, King and Plosser (1984) have come closest to attempting such an exercise. However, they stop short of constructing a monetary model and evaluating the relative importance of alternative shocks. Constructing a model at a plausible level of generality and performing these calculations is admittedly a formidable task. Recent work is just beginning to provide the necessary tools needed to solve for the stochastic equilibria of monetary models (e.g., Lucas and Stokey 1984). Nevertheless, simple illustrative monetary models can be constructed, and it is this task to which we turn next.

3. An Equilibrium Business Cycle Model

In this section we present an equilibrium business cycle model and discuss some of its properties. The model is of a monetary economy in which anticipated monetary growth has real economic effects. Money is introduced into the model using the construct of a cash-in-advance constraint and this constraint is the only source of monetary nonneutrality in this model. In particular, we abstract from frictions that might lead to sticky prices or wages and agents are assumed to have rational expectations. The exclusion of these and other sources of frictions from the economic environment should not be interpreted as necessarily reflecting a view that other frictions are absent from modern economies. Rather, we are intentionally constructing an economic model that is simple to analyze and which allows us to discuss several pitfalls in interpreting the time series evidence that purportedly supports or refutes RBC models. Many of our observations are valid for a much larger class of real and monetary models than those examined explicitly.

The monetary business cycle model which we consider is an extended version of the RBC model discussed by Garber and King (1983) (which is closely related to the model in Long and Plosser (1983)). Suppose that a representative, competitive firm produces a nondurable consumption good y_t using the input x_t. The production function for y_t is given by

$$(3.1) \qquad y_t = x_t^{\alpha} \lambda_t, \qquad 0 < \alpha < 1,$$

where λ_t is a shock to the consumption good technology at date t. The intermediate good x_t depreciates at the rate of 100% when used in production. The firm buys x_t from consumers in a competitive market at the unit real price of w_t. Acquisitions of the input are made so as to maximize the expected discounted value of future profits. For our model, this optimum problem simplifies to the following static optimum problem: at each date t the firm chooses x_t to maximize

$$(3.2) \qquad x_t \lambda_t - w_t x_t.$$

Profits of the firm are returned each period to the shareholders in the form of dividends, d_t.

The intermediate good, x_t, is storable. The representative consumer has an initial endowment of the intermediate good of k_o. The law of motion for the consumer's holdings of the intermediate good at the beginning of period t is given by

$$(3.3) \qquad k_t = \theta_t [k_{t-1} - x_{t-1}].$$

In (3.3), θ_t represents a stochastic shock to the storage technology.

In addition to holdings of the intermediate good, the wealth of the representative consumer includes holdings of claims to the future cash flows of the firm and money holdings. We shall let z_t denote the number of shares in the firm held by the consumer and q_t^s (Q_t^s) denote the ex-dividend real (money) price per share in the firm.

There are a variety of ways in which to generate valued fiat money in theoretical models (see, e.g., the volume edited by Kareken and Wallace (1980)). Following Lucas (1980,1984), Townsend (1982), Lucas and Stokey (1984), and Svensson (1985), we adopt the construct of a cash-in-advance constraint. While this construct is known to have some undesirable characteristics, it is analytically convenient and can be interpreted as arising from a very special shopping time technology (e.g., McCallum 1983b). The timing conventions for monetary transactions in this economy are assumed to be as follows. The consumer enters the period t with a predetermined cash balance M_t, and a predetermined share, z_t, of claims to the dividends of the representative firm. He learns the realizations of all time t random variables and then chooses the quantity of intermediate goods (x_t) to sell to the firm and the amount of consumption goods (c_t) to purchase at the dollar price P_t. Payment for the sale of x_t to the firm is received in cash after the consumption decision. Therefore, the money received (if any) cannot be used to relax the cash-in-advance constraint in period t. It follows that goods purchases must obey the cash-in-advance constraint

(3.4) $P_t \; c_t \leq M_t.$

After the goods market is closed, the consumer receives in cash his share of dividends, $D_t z_t$, where D_t is the money value of dividends at date t. In addition, the consumer receives a lump sum monetary transfer from the policy authority in the amount J_t. Finally, at the end of period t, money and equity shares in the firm are traded. Thus, the evolution of the nominal money holdings by the consumer are described by the equation

(3.5) $M_{t+1} + Q_t^{\S} z_{t+1} \leq [M_t - P_t c_t] + [Q_t^{\S} + D_t] z_t + W_t x_t + J_t,$

where W_t is the per unit cash payment from the firm to the consumer for supplying the intermediate good. Dividing both sides of (3.4) and (3.5) by P_t yields the real relations

(3.6) $M_{t+1}/P_t + q_t^{\S} z_{t+1} \leq [m_t - c_t] + [q_t^{\S} + d_t] z_t + w_t x_t + \tau_t,$

(3.7) $c_t \leq m_t,$

where lower case letters denote real quantities and $\tau_t = J_t/P_t$. We shall henceforth assume that the constraint (3.7) is always binding. The consistency of this assumption with the specification of other aspects of the model will discussed subsequent to deriving the equilibrium law of motion for x_t.

The consumer is assumed to choose contingency plans for c_t, x_t, M_{t+1} and z_{t+1} so as to maximize the logarithmic intertemporal objective function

$$(3.8) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \nu_t \, \ell n c_t,$$

subject to the constraints (3.3) and

$$(3.9) \qquad M_{t+1}/P_t + q_t^s z_{t+1} = [q_t^s + d_t] z_t + w_t x_t + \tau_t.$$

In (3.8), E_t denotes the expectation conditioned on agents' information set at date t and the disturbance ν_t is a taste shock, which is assumed to follow the process

$$(3.10) \qquad \nu_t = \nu_{t-1}^a \, \epsilon_t, \qquad |a| \le 1,$$

where $\ell n \epsilon_t$ is normally distributed with mean zero and variance σ_ϵ^2.

The first-order conditions for this optimum problem are

$$(3.11) \quad E_t \left\{ \frac{\beta \, \nu_{t+1}}{M_{t+1}} \right\} + \xi_t / P_t = 0,$$

$$(3.12) \qquad -w_t \xi_t + \beta \, E_t \{ w_{t+1} \, \Theta_{t+1} \, \xi_{t+1} \} = 0,$$

$$(3.13) \qquad -q_t^s \xi_t + \beta \, E_t \{ \xi_{t+1} \, (q_{t+1}^s + d_{t+1}) \} = 0,$$

where ξ_t is the Lagrange multiplier associated with the constraint (3.9). Substituting (3.11) into (3.12) gives

$$(3.14) \qquad E_t \{ W_t \, \nu_{t+1} / M_{t+1} \} = E_t \{ \beta \, W_{t+1} \, \nu_{t+2} \, \Theta_{t+1} / M_{t+2} \}.$$

Using (3.4), an equivalent way of writing (3.14) is

$$(3.14') \quad E_t \left[\frac{W_t \nu_{t+1}}{P_{t+1} c_{t+1}} \right] = E_t \left[\frac{\beta W_{t+1} \theta_{t+1} \nu_{t+2}}{P_{t+2} c_{t+2}} \right],$$

which has the following interpretation. By selling a unit of the intermediate good to the firm at date t the consumer receives W_t dollars, which can be used to acquire the consumption good at date t+1. Therefore, the benefits from providing x_t are evaluated using the marginal utility of consumption at date t+1 (ν_{t+1}/c_{t+1}). On the other hand, postponement of the sale of x for one periods yields the physical rate of return of θ_{t+1} from storage. At date t+1 the consumer can then sell x for θ_{t+1} W_{t+1} dollars, and these dollars can be used to purchase consumption goods at date t+2. Equation (3.14') states that in equilibrium the consumer is indifferent between these two strategies. Notice that the timing convention of our model implies that, in supplying the intermediate good to the firm at date t for a per unit nominal price W_t, consumers are effectively contracting for an uncertain per unit real price of W_t/P_{t+1} at date t+1.

Equation (3.14) can be used to determine the equilibrium law of motion for x_t, once a money supply rule has been specified. We assume that M_t follows the process

$$(3.15) \quad M_{t+1} = M_t f(s_t),$$

where s_t is a vector of state variables that are known at date t. (Recall that M_{t+1} is known at the end of date t and M_t is determined at date t-1.) Substituting (3.15) into (3.14) and using the cash-in-advance constraint leads to

$$(3.16) \quad E_t \left\{ \frac{w_t \nu_{t+1}}{f(s_t) c_t} \right\} = E_t \left\{ \frac{\beta w_{t+1} \theta_{t+1} \nu_{t+2}}{f(s_{t+1}) c_{t+1}} \right\}.$$

In equilibrium, $z_t = 1$ (all equity claims are held) and $c_t = y_t$ (consumption equals output). Also, the first-order conditions to the firm's optimum problem imply that the real price of the intermediate good equals the marginal product of the good in production:

$$(3.17) \quad w_t = \alpha\, x_t^{\alpha-1}\, \lambda_t.$$

Imposing these equilibrium conditions and using (3.16) gives

$$(3.18) \quad E_t\left\{\frac{\nu_{t+1}}{x_t\ f(s_t)}\right\} = E_t\left\{\frac{\beta\ \theta_{t+1}\ \nu_{t+2}}{x_{t+1}\ f(s_{t+1})}\right\}.$$

Given that $f(s_t)$ is in agents' information set at date t, it is straightforward to verify that

$$(3.19) \quad x_t = \beta\ \exp[\tfrac{1}{2}\sigma_\epsilon^2]\ x_{t-1}\ \nu_t^{a-1}\ \theta_t\ f(s_{t-1})/f(s_t)$$

satisfies (3.18).[3] Therefore, (3.19) is the equilibrium law of motion for x_t. Taking logarithms of both sides of (3.19) gives

$$(3.20) \quad \ell n x_t = \tfrac{1}{2}\sigma_\epsilon^2 + \ell n[\beta\ \nu_t^{a-1}\ x_{t-1}\ \theta_t] + (\ell n f(s_{t-1}) - \ell n f(s_t)).$$

More general linear time series representations for $\ell n\nu_t$ could easily be accommodated; they would only complicate the manner in which preference shocks enter (3.20).

Before proceeding with our discussion of (3.20), it is instructive to discuss briefly whether the assumption that the cash-in-advance constraint is always binding is consistent with our assumptions about the distributions of the exogenous shocks. In the context of our model, the cash-in-advance constraint is binding if and only if the nominal interest rate on a one-period pure discount bond that pays off one dollar is positive. Thus, to check for consistency we must verify that, having imposed the constraint, the nominal rate implied by the model is in fact positive. The rate of return on a one-period nominal bond in this environment is given by[4]

$$r_{t+1}^{b} = \nu_t^{1-a}\ f(s_t)\ \exp\{-\tfrac{1}{2}\sigma_\epsilon^2\}/\beta - 1.$$

In the absence of preference shocks, a necesary and sufficient condition for r_{t+1}^b to be positive is that the growth rate of money, $f(s_t)$, exceed β. We used the growth rate of M1 as a measure of $f(s_t)$ in our empirical analysis. Interestingly, for the sample period 1949:1 through 1983:6, this measure of $f(s_t)$ exceeds .995 (a plausible value of β for monthly decision intervals) in all but five months. More generally, the assumption that $r_{t+1}^b > 0$ imposes restrictions on the joint distribution of the taste shock, ν_t, and $f(s_t)$.[5]

From (3.15) it is seen that the growth rate in the money supply, $(\ell n M_{t+1} - \ell n M_t)$, is equal to $\ell n f(s_t)$. It follows that changes in the growth rate of the money supply affect the consumer's decision rule for supplying the intermediate good. The dependence of x_t on monetary growth, in turn, implies that equilibrium output and dividends are also affected by monetary policy. As is typical of models with cash-in-advance constraints, increases in the growth rate of money decrease output. A similar property often emerges in corresponding models in which money is introduced directly as an argument of the agent's utility function or through a shopping time transactions technology. Lucas (1985) has conjectured that a positive relation between money growth and output growth may obtain with the addition of informational imperfections to models with cash-in-advance contraints. The qualitative nature of the major conclusions drawn subsequently from (3.20) and our empirical findings are not sensitive to whether the sign of the effect of money on output in positive or negative. What is crucial is that money affect real activity through more than just the current innovation in money.

Notice, that real allocations in our monetary economy are unaffected by permanent and proportional increases in either the level or the growth rate of the money supply. Thus, this monetary model displays the property of super neutrality. The result that a once and for all change in the level of the money stock has no effect on ouput was derived in a more general setting by Lucas (1984). The stronger result that money is super neutral will typically not obtain in models in which consumers make non-trivial labor supply decisions as well as consumption decisions.

Examination of the logarithms of (3.1), (3.17), and (3.19) reveals several interesting characteristics of the model set forth above:

$$(3.21) \quad \ell n x_t = \ell n \beta + \tfrac{1}{2}\sigma_\epsilon^2 + \ell n x_{t-1} + (a-1)\ell n \nu_t + \ell n \theta_t + \ell n f(s_{t-1}) - \ell n f(s_t),$$

$$(3.22) \quad \ell n c_t = \alpha \ell n x_t + \ell n \lambda_t,$$

$$(3.23) \quad \ell n w_t = \ell n \alpha + (\alpha-1)\ell n x_t + \ell n \lambda_t.$$

Notice, first of all, that output ($\ell n c_t$) is a function of both θ_t (through x_t) and λ_t. This is an illustration of the more general principle that sectoral shocks associated with the production of intermediate goods will have a cumulative effect on final output. Furthermore, if these shocks are positively correlated, then the variance of the sum will exceed the sum of the own variances. That the "aggregate" technological shock impinging on final output represents such a combination of shocks may affect what is perceived to be a plausible value for the variance of the shock to final output.

Equations (3.21)-(3.23) also illustrate the well known, but often ignored, fact that trends in the endogenous variables may be intimately related. Simply put, this is because all of the endogenous variables are functions of subsets of the same set of taste and technology shocks and trends enter the model primarily through these shocks. It turns out that many of our statistical results using VARs are not insensitive to the assumptions about the nature of trends in the variables examined. The sensitivity of estimates from multivariate autoregressive representations to the method of detrending has been noted previously by Kang (1985) and Bernanke (1985). In Section 5 we use equations (3.21)-(3.23) to interpret the sensitivities of our results to the specifications of trends.

Having deduced the equilibrium laws of motion for the real quantities in our monetary model, we turn next to an investigation of conditions under which an RBC model provides an accurate approximation to this economy.

4. Interpreting Bivariate VARs using Monetary and RBC Models

As background information for comparing the properties of the monetary and RBC models, it is instructive to examine some empirical evidence. Accordingly, we begin this section with a discussion of the findings from estimating bivariate autoregressive time series representations of output and money growth. A much more comprehensive set of empirical results is discussed in Section 5.

The Granger causality relations between output and money were investigated using monthly data for the U.S. economy over the sample period February 1949 through June 1983. Output was measured by the Industrial Production Index constructed by the Federal Reserve Board. Money was measured by M1 and was obtained from the CITIBASE data tape. Table 4.1 displays the results for the growth rate of output (the difference in the logarithm of industrial production) and the second difference in the logarithm of M1. The second difference of the money stock was used, because this is the empirical counterpart to the construct appearing in the expression for $\Delta \ell ny_t$ in Section 3. Both $\Delta \ell ny_t$ and $\Delta^2 \ell nM_{t+1}$ were assumed to be covariance stationary stochastic processes. (The issue of trends is discussed in more detail in Section 5.) All VARs included twelve lags of each variable and a constant. In none of the five sample periods considered does $\Delta^2 \ell nM_{t+1}$ Granger cause $\Delta \ell ny_t$ at the one percent level. And only in the sample period 1959,2-1983,6 does $\Delta^2 \ell nM_{t+1}$ Granger cause $\Delta \ell ny_t$ at the five percent level. Though not reported in Table 4.1, the corresponding results obtained with $\Delta \ell nM_{t+1}$ in place of $\Delta^2 \ell nM_{t+1}$ were virtually identical.

These results could plausibly be interpreted as evidence in favor of the view that RBC models provide accurate characterizations of the behavior of real quantity variables for this sample period. The remainder of this section explores ways in which this interpretation of the evidence can be reconciled with economic theory and ways in which advocates of various models can be misled by this evidence. In Subsection 4.A we study a version of the model from Section 3 in which no cash-in-advance constraint is imposed. In addition, we deduce conditions under which real allocations from this RBC model correspond to those in the cash-in-advance economy. Subsection 4.B explores the possibility of RBC models providing an accurate, though not exact, approximation to the monetary economy.

4.A. An RBC Model

Consider the following version of the economy of Section 3 in which agents do not face a cash-in-advance constraint. Suppose that agents face the period budget constraint

$$(4.1) \qquad c_t + q_t^s\, z_{t+1} = (d_t + q_t^s)\, z_t + w_t\, x_t .$$

In this economy the consumption good is the numeraire and agents are paid in consumption goods for supplying the intermediate good to the firm. Payments during period t are available for immediate consumption during period t. Also suppose that the representative consumer chooses contingency plans for c_t, x_t, and z_{t+1} so as to maximize the logarithmic intertemporal objective function (3.8) subject to the constraints (3.3) and (4.1). In equilibrium, $z_t=1$ (share market clears), $y_t=c_t$ (consumption equals output),

$$(4.2) \qquad w_t x_t + d_t = y_t$$

(dividends plus factor costs exhaust output), and the supply and demand of the intermediate good are equilibrated, so that (3.17) holds. Imposing these equilibrium conditions and using arguments similar to those leading to (3.19) yields the following laws of motion for the quantity variables:

$$(4.3)\qquad x_t = \beta \exp[\tfrac{1}{2}\sigma_\epsilon^2]\, x_{t-1}\, \nu_{t-1}^{a-1}\, \theta_t,$$

$$(4.4)\qquad c_t = x_t^\alpha \lambda_t,$$

$$(4.5)\qquad d_t = (1-\alpha)\, x_t^\alpha \lambda_t,$$

where the last expression follows from (4.2).

The equilibrium laws of motion for x_t in the monetary and non-monetary economies, given by (3.19) and (4.3) respectively, differ by the multiplicative term $[f(s_{t-1})/f(s_t)]$ and the dating of the preference shock ν. The latter difference arises because cash payments received at date t from the sale of the intermediate good to the firm cannot be used to acquire the consumption good until date t+1 in the monetary model.

When will the real allocations be identical in the monetary and non-monetary models described above? The difference between the dating of the preference shock in the law of motion for x_t will trivially be inconsequential if preference shocks are absent from the model. Most of the RBC models that have been studied to date do exclude preference shocks. Alternatively, if the preference shock follows a logarithmic random walk (a=1), then again this shock will not appear in the law of motion for x_t. Additionally, the monetary term in (3.19) must be zero. This will be the case if the monetary authorities follow a constant growth rate rule. Under these circumstances, (3.19) and (4.3) give the same equilibrium laws of motion for x_t. This, in turn, implies that the real equilibrium for the real and monetary models are identical, since d_t and c_t are proportional to x_t in both economies. Given that a=1, a constant monetary growth rate is both necessary and sufficient for the real allocations to be identical in these economies. In addition, it can be shown that formulas for asset prices and relative prices of goods are identical in the monetary and RBC models under these assumptions.

4.B Comparing Monetary and Real Business Cycle Models

Now, in fact, the monetary growth rate has not been constant over the postwar period. In our view, this observation alone is not sufficient to dismiss an RBC explanation of aggregate fluctuations. The extreme case of monetary policy having no impact on real allocations, what we will refer to as the strong RBC hypothesis, is presumably not what most proponants of RBC theories have in mind when arguing that RBC models fit the data. Instead, such claims are more plausibly interpreted as statements about the insensitivity of real allocations to the current structure of financial institutions, and the insignificance of exogenous shocks to the money supply relative to exogenous real shocks. We refer to this latter interpretation as the weak RBC hypothesis. Initially we discuss conditions under which measures of monetary growth are not likely to Granger cause the growth rate of output under the weak RBC hypothesis. Then, circumstances under which measures of monetary growth Granger cause the growth rate of output are discussed.

In our illustrative monetary model, the equilibrium quantities will be approximately unaffected by exogenous money supply shocks if the change in the growth rate of money is not too variable relative to preference and technology shocks. Under this condition, the last term in expression (3.21),

$$\ell n x_t = \ell n\beta + \tfrac{1}{2}\sigma_\epsilon^2 + \ell n x_{t-1} + (a-1)\ell n\nu_t + \ell n\theta_t - \Delta^2\ell n M_{t+1} \quad , \tag{4.6}$$

can be ignored. The corresponding expression from the RBC model set forth in Section 4.A is

$$(4.7) \qquad \ell n x_t = \ell n\beta + \tfrac{1}{2}\sigma_\epsilon^2 + \ell n x_{t-1} + (a-1)\ell n\nu_{t-1} + \ell n\theta_t.$$

Comparing (4.6) and (4.7) it is seen that when technology and taste shocks predominate the two laws of motion are nearly identical, the only difference being the dating of the preference shock. Thus, whether or not a=1, the RBC model will provide a reasonably good approximation to the monetary economy under this assumption about monetary growth. Furthermore, for the sample sizes typically considered in macroeconomic time series analyses, money may not Granger cause output in this system due to statistical power considerations.

It seems difficult to reconcile the absence of Granger causality from money to output with the opposing view that monetary policy shocks contributed significantly to the variability of output during our sample period. Within the model of Section 3, one such reconciliation is achieved by assuming that $\Delta^2 \ell nM_{t+1}$ is serially uncorrelated (monetary growth follows a random walk). Then the last term in (4.6) is serially uncorrelated. Consequently, money will not Granger cause output in the bivariate system $[\Delta\ell ny_t, \Delta^2\ell nM_{t+1}]$. This will be the case even though money shocks affect output and may have a large variance relative to real shocks. The assumption that the change in the monetary growth rate is serially uncorrelated is counterfactual, however (see Table 4.1).

More generally, suppose that the monetary growth rate is chosen by the policy authorities to follow the process

$$(4.8) \qquad \begin{aligned} \Delta\ell nM_{t+1} &= \gamma_1\ \ell ny_{t-1} + \gamma_2\ \ell ny_{t-2} + \gamma_3\ \Delta\ell nM_t + \psi_t, \\ &= (1-\gamma_3 L)^{-1}[\gamma_1 \ell ny_{t-1} + \gamma_2 \ell ny_{t-2}] + (1-\gamma_3 L)^{-1}\ \psi_t \end{aligned}$$

where $|\gamma_3|<1$, ψ_t is a random shock to the policy rule in period t that is independent of the preference and technology shocks, and L is the lag operator. Taking the first difference of the output equation (3.22) and substituting (4.8) for $\Delta^2 \ell nM_{t+1}$ and ignoring constant terms gives the following equations:

$$(4.9) \qquad \Delta\ell ny_t = \alpha[(a-1)\ell n\nu_t + \ell n\theta_t - (1-\gamma_3 L)^{-1}\{\gamma_1 \Delta\ell ny_{t-1} + \gamma_2 \Delta\ell ny_{t-2}\}] + \Delta\ell n\lambda_t - \alpha(1-\gamma_3 L)^{-1} \Delta\psi_t,$$

$$(4.10) \qquad \Delta^2 \ell nM_{t+1} = (1-\gamma_3 L)^{-1}[\gamma_1 \Delta\ell ny_{t-1} + \gamma_2 \Delta\ell ny_{t-2} + \Delta\psi_t].$$

In this example, $\Delta^2 \ell nM_{t+1}$ will not Granger cause $\Delta\ell ny_t$ only under the special assumptions that $\gamma_3=0$ in (4.8) and the composite shock $[(a-1)\ell n\nu_t + \ell n\theta_t + \Delta\ell n\lambda_t + \Delta\psi_t]$ is serially uncorrelated. The assumption that the projection of $\Delta^2 \ell nM_{t+1}$ onto its own history and the past history of $\Delta\ell ny_t$ is not a function of $\Delta^2 \ell nM_{t-j}$, $j\geq 0$, is counterfactual. Also, we are not aware of any compelling scientific evidence which strongly suggests that either the changes or levels of the taste and technology shocks are white noise processes.

Of course, if the variance of the monetary policy shocks are small relative to the variances of the real shocks, then money may not Granger cause output even though $\Delta^2\ell nM_{t+1}$ is highly variable. In particular, suppose that the variance of $(1-\gamma_3 L)^{-1} \Delta\psi_t$ is small relative to the variances of the terms involving the other shocks, so that the monetary policy shock is not an important independent source of variation of output in this model. Then inspection of (3.22), (3.23), and (4.9) reveals that monetary growth may fail to Granger cause output in the mulivariate system $[\ell nx_t, \ell nc_t, \ell nw_t, \Delta^2 \ell nM_{t+1}]$. This scenario presumably motivates RBC analyses.

While it is necessarily true that real shocks should be the focal point of business cycle analyses if monetary shocks are relatively unimportant, as the bivariate VARs suggest, it is premature to conclude that RBC models correctly represent the structure of the economy. The economic environment leading to the use of fiat money as a medium of exchange may be very different than the economic environment underlying RBC models. That is, modern institutional arrangements are associated with very different market structures than the structures adopted in RBC models and hence monetary models may have very different sources of endogenous dynamics.

To illustrate this point, consider again the output equation (4.9) associated with the money supply rule (4.8). Suppose that the linear combination of the taste and technology shocks in (4.9) is serially uncorrelated. The persistence in output is then due entirely to the fact that the monetary policy rule feeds back on lagged output growth; there is nothing technological about this persistence. Yet an RBC theorist might explore non-time-separable specifications of technology and preferences in order to induce autocorrelations similar to those implied by (4.9). It seems likely that RBC models could be developed with sufficiently rich specifications of preferences and technology to match the autocorrelations of output quite closely. For, as noted previously, low order stochastic difference equations are often acceptable representations of economic aggregates. But inferences about the structure of the economy using an RBC model could be very misleading. It follows that policy analysis using the RBC formulation of the economy could lead to misguided policy prescriptions.

As a second example of this phenomenon, suppose that $\Delta^2 \ell n M_{t+1}$ is set so as to substantially attenuate (or completely offset) the impact of certain shocks on $\Delta \ell n y_t$. Then the time series properties of $\Delta \ell n y_t$ will be determined by the remaining shocks in (4.6) and money growth may not Granger cause output growth. Nevertheless, the properties of $\Delta \ell ny_t$ are clearly affected by the particular feedback rule adopted by the monetary authorities. We suspect that a version of this scenario could be developed in which lagged values of $\Delta^2 \ell nM_{t+1}$ also were useful in predicting $\Delta^2 \ell nM_{t+1}$.

Up to this point we have focused on situations in which money growth does not Granger cause output growth. In anticipation of empirical results using linearly detrended data reported in Section 5, we briefly explore structural interpretations of a Granger causal relation from money to output. The most straightforward interpretation of such a statistical relation is that in fact monetary policy actions are important determinants of aggregate real activity. On the other hand, a statistical relation between money and output may reflect the fact that money is proxying for unobserved shocks to tastes and technologies. That is, if there are more aggregate shocks to preferences and technology than real variables included in the VAR, then in general $\Delta^2 \ell nM_{t+1}$ will Granger cause the real variables. This is the interpretation of money-output correlations adopted by Litterman and Weiss (1985).

To illustrate a signaling role for money, consider a trivariate VAR of $\Delta \ell ny_t$, $\Delta \ell nx_t$, and $\Delta^2 \ell nM_{t+1}$. In our model, when $a \neq 1$, there are three real shocks so if monetary growth follows a feedback rule like (4.8), then $\Delta^2 \ell nM_{t+1}$ will convey information about $[\Delta \ell ny_t, \Delta \ell nx_t]$ that is not embodied in the past histories of the quantity variables. Thus, money will Granger cause output in this setting even if monetary policy shocks are absent from the model. On the other hand, when $a=1$, there are only two real shocks so that the vector of quantities will not be Granger caused by monetary growth if the variance of the monetary shock is small.

In concluding this section, we examine the question of what proportion of the variance of output is accounted for by innovations in nominal aggregates. Attempts at answering this question raise additional questions regarding the interpretation of VARs. We begin by describing a procedure proposed by Sims (1980b) for calculating variance decompositions and then review an important objection to the procedure which has been raised by Blanchard and Watson (1984) and Bernanke (1985), among others. These issues are intimately connected with the set of circumstances under which Granger-causality tests can be used to shed light on structural, rather than statistical, issues.

Let $S_t' = [S_{1t}', S_{2t}']$, where S_{1t} is a j x 1 vector of real variables and S_{2t} is a (k-j) x 1 vector of nominal variables. We suppose that S_t has the vector autoregressive representation,

$$(4.11) \qquad G(L)S_t = \epsilon_t ,$$

where G(L) is a k x k matrix polynomial in the lag operator L, $I_0 + G_1 L + G_2 L^2 + \ldots$, which satisfies the invertibility conditions, and ϵ_t is a k x 1 vector of serially uncorrelated disturbances which may be contemporaneously correlated. In general the elements of the vector ϵ_t will be nonlinear functions of the innovations to agents' preferences, productivity shocks and nominal disturbances. Suppose that the analyst estimates the parameters of G(L) and the contemporaneous covariance of ϵ_t,

$$(4.12) \qquad E\epsilon_t\epsilon_t' = \Sigma_\epsilon .$$

The analyst wishes to partition the variance of the T-step ahead forecast error of a particular element of S_t into the portions attributable to innovations in the different components of S_t.

Suppose that the forecast error variance of interest is that of the first component of S_t, S_{1t}. Let L_1 be the 1 x k row vector with 1 in the first place and zero elsewhere. Also let $S_t = M(L)\epsilon_t$ denote the moving average representation of S_t where M(L) is a k x k matrix of polynomials in the lag operator which satisfies the invertibility conditions and $M(L)G(L) = I$, with I being the k x k identity matrix. Then the T-step ahead forecast error of S_{1t} can be written as

$$(4.13) \qquad E_tS_{1\,t+T} - S_{1\,t+T} = L_1\{-\epsilon_{t+T} - M_1\epsilon_{t+T-1}\ldots-M_{T-1}\epsilon_{t+1}\} \ .$$

The variance of the T-step forecast error variance of S_{1t}, is equal to,

$$(4.14) \qquad E\{[S_{1\,t+T} - E_tS_{1\,t+T}]^2\} = \sum_{\tau=0}^{T-1} \sum_{j=1}^{k} \sum_{\ell=1}^{k} M_\tau(1,j)\ M_\tau(1,\ell)\sigma_{j\ell}$$

where $M_\tau(i,j)$ is the ijth element of the matrix M_τ.

If Σ_ϵ is diagonal, then the percentage of the variance in the T-step forecast of S_{1t} due to innovations in the ℓ^{th} variable, $S_{\ell t}$, is

$$(4.15) \qquad \frac{100 \sum_{\tau=1}^{T-1} M_\tau^2(1,\ell)\sigma_\ell^2}{\sum_{\tau=0}^{T-1} \sum_{j=1}^{k} M_\tau^2(1,j)\ \sigma_j^2} \ .$$

As T approaches infinity the T-step ahead forecast error of S_{1t} converges to the unconditional variance of S_{1t}. Accordingly, the percentage of the unconditional variance of S_{1t} which is attributable to innovations in $S_{\ell t}$ is well approximated by calculating (4.15) for large values of T.

In general, the matrix Σ_ϵ is not diagonal, so that some set of normalizations or identifying restrictions must be imposed on the system before a decomposition of S_{1t} can be calculated. More precisely, we define a new set of error terms ζ_t, (4.16) $\zeta_t = F^{-1} \epsilon_t$.

with F chosen such that

(4.17) $E\epsilon_t\epsilon_t' = \Sigma_\epsilon = FE\zeta_t\zeta_t'F' = F\Sigma_\zeta F'$,

and Σ_ζ is a k x k diagonal positive definite matrix. Then (4.6) and

(4.18) $G(L)S_t = F\zeta_t$,

are observationally equivalent representations of S_t. In general there are an infinite number of matrices F which satisfy (4.17), so normalizations must be imposed before calculating variance decompositions.

The method suggested by Sims (1980b) for decomposing the vector ϵ_t into orthogonal components, (i.e., for choosing a particular matrix F) is to proceed with a particular ordering of the elements of S_t and restrict attention to the class of matrices F which are lower block triangular. This amounts to setting the ℓ^{th} off-diagonal elements of the j^{th} row of F^{-1} equal to minus the coefficient on $\epsilon_{\ell t}$ from the projection of the j^{th} element of ϵ_t on elements 1 through j-1 of ϵ_t, (ℓ=1,...,j-1). In general the matrix F depends on the ordering of the variables. Once the matrix F has been chosen we can substitute $\epsilon_t = F\zeta_t$ into (4.15) to achieve the desired decomposition. Notice that if Σ_ϵ is diagonal then there is a unique matrix F=I which satisfies (4.16) and (4.17). As Sims (1980b), Blanchard and Watson (1984), Bernanke (1985), and Cooley and LeRoy (1985) have noted this procedure for choosing the matrix F presumes that the structural model for S_t is recursive when Σ_ϵ is not diagonal.

In the context of the following example, we discuss the pitfalls of Sims' procedure when applied to our problem. Consider again the representation (4.9) for $\Delta\ell ny_t$. Relation (4.9) implies that the innovation to the growth rate of output will be a linear combination of the innovations to agents' preference shocks, sector specific technology shocks, and the change in the growth rate of money. In order to make our example as simple as possible we concentrate on a bivariate time series representation for $\Delta\ell ny_t$ and $\Delta^2\ell nM_{t+1}$, and simplify the stochastic structure of the model. Suppose that preference shocks follow a random walk (a=1), θ_t=1, and the law of motion for the technology shock is given by,

$$(4.19) \qquad \Delta\ell n\lambda_t = D(L)^{-1}\epsilon_{\lambda t},$$

where D(L) ≡ 1-Ld(L) is an invertible polynomial in the lag operator, d(L) = $(d_1 + d_2L + d_3L^2 + \ldots)$, and $\epsilon_{\lambda t}$ is a serially uncorrelated random variable. It is convenient to modify relation (4.6) by replacing the term $\Delta^2\ell nM_{t+1}$ by $z\Delta^2\ell nM_{t+1}$ where $z\epsilon\{0,1\}$. When z is equal to one, we obtain the monetary model of Section 3. When z is equal to zero, we obtain the RBC model discussed in Section 4.A which imposes the strong RBC hypothesis. Under these assumptions, the law of motion for $\Delta\ell ny_t$ becomes

$$(4.20) \qquad \Delta\ell ny_t = \Delta\ell n\lambda_t - z\alpha\mu_t ,$$

where $\mu_t \equiv \Delta^2\ell nM_{t+1}$. Substituting (4.19) into (4.20) and rearranging gives,

$$(4.21) \qquad \Delta\ell ny_t = d(L)\Delta\ell ny_{t-1} - z\alpha\mu_t + z\alpha d(L)\mu_{t-1} + \epsilon_{\lambda t} .$$

Next, suppose that the monetary authority sets the change in the monetary growth rate according to the feedback rule,

$$(4.22) \quad \mu_t = e(L)\Delta \ell ny_{t-1} + f(L)\mu_{t-1} + \epsilon_{\mu t} + x\epsilon_{\lambda t} ,$$

where $e(L)$ and $f(L)$ are scalar invertible polynomials in the lag operator L, x is a scalar constant, and $\epsilon_{\mu t}$ is a serially uncorrelated random variable. We assume that $\epsilon_{\lambda t}$ and $\epsilon_{\mu t}$ are contemporaneously uncorrelated.

Relations (4.21) and (4.22) imply that $\Delta \ell ny_t$ and μ_t have the bivariate VAR representation

$$(4.23) \quad \begin{bmatrix} \Delta \ell ny_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} 1 & -z\alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d(L) & z\alpha d(L) \\ e(L) & f(L) \end{bmatrix} \begin{bmatrix} \Delta \ell ny_{t-1} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

where

$$(4.24) \quad \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} = \begin{bmatrix} 1-z\alpha x & -z\alpha \\ x & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{\lambda t} \\ \epsilon_{\mu t} \end{bmatrix} .$$

Suppose the analyst estimates the VAR (4.23) and has in hand estimates of the three parameters of Σ_ϵ. For this economic model Σ_ϵ is a function of four parameters: $z\alpha$, x, σ_ϵ^2, and σ_λ^2, where the last two parameters are the variances of $\epsilon_{\mu t}$ and $\epsilon_{\lambda t}$, respectively. It follows that the parameters of the innovation covariance matrix cannot be identified separately from the parameters of the regression equation. This, in turn, implies that the methods proposed by Blanchard and Watson (1984), Bernanke (1985), and Sims (1985) for analyzing innovation covariance matrices are in general not applicable to dynamic economic models. Put differently, proponents of this approach are implicitly ruling out a large and important class of dynamic economic models as candidates for explaining business cycles.

Pursuing this observation, suppose that the empirical evidence suggests that $\Delta \ell ny_t$ is not Granger caused by μ_t. Within the context of (4.23), this implies that $z\alpha=0$ unless we are in the counterfactual case where d(L) is identically equal to f(L). Setting z=0 leads to the structural model

$$(4.25) \quad \begin{bmatrix} \Delta \ell ny_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} d(L) & 0 \\ e(L) & f(L) \end{bmatrix} \begin{bmatrix} \Delta \ell ny_{t-1} \\ \mu_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{\lambda t} \\ \epsilon_{\mu t} \end{bmatrix} .$$

The model (4.25) has three important features. First, it corresponds to the RBC model of Section 4.A. Second, this model exhibits a block recursive structure between the real and monetary sectors. Third, implementing Sims' procedure for orthogonalizing the covariance matrix of the innovations in a bivariate VAR by placing $\Delta \ell ny_t$ before μ_t would uncover the true orthogonal shocks of interest. It is worth emphasizing that the justification for the ordering of the variables in this example depends critically on the assumed pattern of Granger-causality results. For the RBC model in Section 4.A, imposing the Granger non-causality of output by money identifies the parameters of the matrix Σ_ϵ. There is no natural way to separate the identification of Σ_ϵ from the specification of the structure in this dynamic model.

Recall that the findings in Table 4.1 provide little evidence that $\Delta \ell ny_t$ is Granger caused by $\Delta^2 \ell nM_{t+1}$. Therefore, consistent with the previous discussion, we implemented Sims procedure for decomposing the variance of $\Delta \ell ny_t$ by placing $\Delta \ell ny_t$ before $\Delta^2 \ell nM_{t+1}$. All of the variance decompositions are for forty-eight month ahead forecast errors (T=48). Table 4.1 displays the decompositions and their associated standard errors.[6] Notice that for all the sample periods considered the innovation in $\Delta^2 \ell nM_{t+1}$ accounts for very little of the variance of $\Delta \ell ny_t$. In four of the five sample periods considered the percentage of the variance of $\Delta \ell ny_t$ explained by innovations to

$\Delta \ell ny_t$ is within one standard error of ninety five percent. Even in the sample period 1959,2-1983,6, where $\Delta^2 \ell nM_{t+1}$ Granger causes $\Delta \ell ny_t$ at the five percent level, the percentage of the variance of $\Delta \ell ny_t$ accounted for by innovations in $\Delta \ell ny_t$ is within one standard error of ninety percent. Viewed as a whole, the empirical results emerging from the bivariate time series analysis yield very little evidence against the weak RBC hypothesis.

In Section 5 we analyze VARs that include additional variables as a check on the robustness of the qualitative findings from the empirical analysis of the bivariate VARs. In decomposing the variance of the growth rate of output we proceed under the null hypothesis imposed by the strong RBC hypothesis that there is a block recursive structure between real and nominal variables.

5. Interpreting the Evidence from Vector Autoregressions

Much of the recent empirical evidence that has been used to support or refute RBC models has come from studying vector autoregressive or moving average representations of the variables comprising the models. See, for example, Sargent (1976), Sims (1980a,b), King and Plosser (1984), Altug (1985), and Bernanke (1985). In this section we examine the empirical evidence from multivariate VARs in light of the discussion in Section 4, using monthly data for the postwar U.S. economy.

At the outset of this discussion it is important to explore in detail the role of detrending in the empirical analysis. As noted in Section 3, several authors have documented the sensitivity of the results from estimating unconstrained VARs to the assumptions about trends. At the same time, previous studies have virtually ignored the fact that the trends are determined both by the specification of the underlying uncertainty in the economy <u>and</u> the structure of the economic model itself. Therefore, how one detrends is in general not independent of one's view about the structure of the economy. This observation represents an additional reason why it is not possible to embark on an empirical analysis of business cycle models using VARs without implicitly or explicitly restricting the class of models being investigated. We briefly illustrate the nature of these restrictions using the models of Section 3.

Different specifications of preferences and technologies, as well as the laws of motion for the exogenous shocks will in general lead one to consider different autoregressive representations of the data. For instance, had we started with quadratic preferences and technologies, a linear representation of the levels of the variables, instead of their logarithms, would have emerged from the model. The question of whether trends are deterministic or stochastic would then have been addressed to the levels of the variables. Rather than working out in detail some of the many possible time series representations that might emerge from alternative specifications of preferences and technologies, we continue to focus on the log-linear representation that emerges from the model in Section 3. Even holding fixed the specification of technology and preferences, it turns out that many possible trend specifications may arise in our model from alternative specifications of the structure of uncertainty.

To see this, it is convenient to work with a special case of the monetary model given by (4.9) and (4.10). Suppose the monetary policy rule is given by

(5.1) $$\Delta^2 \ell nM_t = \rho \; \ell nx_{t-1} + \psi_t \; .$$

There is no deep economic significance to having monetary growth depend on the intermediate good. This specification allows us to make our points without extensive additional calculations using alternative specifications of the model. Substituting (5.1) into the equation (3.21) gives (ignoring constants):

(5.2) $$\ell nx_t = (1-\rho)\ell nx_{t-1} + (a-1)\ell n\nu_t + \ell n\theta_t - \psi_t$$

The expressions (3.22) and (3.23) for ℓnc_t and ℓnw_t, respectively, are unchanged. Notice that the coefficient on ℓnx_{t-1} in (5.2) is determined both by the specifications of the storage technology for the intermediate good and the monetary feedback rule. If $\rho<0$, then ℓnx_t will be an explosive process and so will ℓnc_t and ℓnw_t. In this case, neither the removal of a deterministic polynomial trend or first-differencing will render the logarithm of output stationary! Similar examples of potentially explosive processes for output are easily constructed by relaxing the assumption of independent taste and technology shocks. Also, we conjecture that an explosive output process may well emerge in models with increasing returns to scale in production. For instance, some specifications of learning-by-doing might generate such an increasing returns to scale technology.

If, on the other hand, $2>\rho>0$, then the trend in $\ell n x_t$ will be determined by the properties of $\ell n \nu_{t-1}$, $\ell n \theta_t$, and ψ_t. For instance, if a subset of these variables exhibit linear deterministic trends and the other variables are stationary stochastic processes, then the logarithms of output, consumption, and the price of the intermediate good will also exhibit linear trends. An assumption like this must implicitly underlie the common practice of removing linear trends. This result depends critically on the specification (5.1) of the money supply rule. If $\rho=0$, then removing linear trends from $[\ell n x_t, \ell n c_t, \ell n w_t]$ will not render these variable stationary (see below). It would be necessary in this model to adopt an entirely different specification of the storage technology to preserve the linear trend assumption.

If $2>\rho>0$ and $\ell n \nu_{t-1}$, $\ell n \theta_t$, ψ_t, and $\ell n \lambda_t$ are all stationary, then $\ell n y_t$ will also be stationary. At least for the postwar period, the evidence seems not to support this assumption about trends.

Finally, returning to the original specification of the model, if $\rho=0$, then there is a unit root in the process for $\ell n x_t$. If the technology shock $\ell n \lambda_t$ also has a unit root and $\ell n \nu_{t-1}$ and $\ell n \theta_t$ are stationary process, then $\Delta \ell n x_t$, $\Delta \ell n c_t$, and $\Delta \ell n w_t$ are all stationary stochastic processes. This case is of particular interest since Prescott (1986) has argued that the technology shock in his model approximately follows a random walk. Also, Altug (1985) found that the estimated autoregressive parameter in her time series representation of the technology shock was essentially unity. Thirdly, Nelson and Plosser (1982) have argued that many real macroeconomic time series must be differenced to induce stationarity. In our work we consider only the possible need to first difference output. However, the need for higher order differencing could be justified by introducing unit roots into the $\ell n\nu_t$ or $\ell n\theta_t$ processes.

With these observations in mind, we turn next to an empirical analysis of monthly data for the postwar period. Let S_t^* denote a k x 1 vector of variables observed by the econometrician. In light of our previous discussion regarding trends, we assume that S_t^* is related to the k x 1 covariance stationary stochastic process, S_t, via one of the following two relations:

$$(5.3a) \quad S_t^* = S_t + Ag(t) ,$$

or,

$$(5.3b) \quad A[S_t^* - S_{t-1}^*] = S_t ,$$

where g(t) is a scalar valued deterministic function of time and A is a k x k matrix of constants, and S_t has the autoregressive representation (4.11). As before, we partition the vector S_t as,

$$(5.4) \quad S_t' = [S_{1t}', S_{2t}'] ,$$

where S_{1t} is a j x 1 vector of real quantity variables and S_{2t} is a (k-j) x 1 vector of relative prices and nominal variables.

Initially, we address the question of whether nominal aggregates have predictive power for aggregate real output, once lagged outputs are accounted for. Trivariate VAR's including measures of aggregate real output, the rate of inflation, and the stock of money were examined. That is,

(5.5) $$S_t{*}' = [\ell n y_t,\ \pi_t,\ \ell n M_{t+1}]$$

where π_t denotes the rate of inflation between time t-1 and time t. (Recall that M_{t+1} is chosen at date t.) Similar VARs were studied by Sims (1980a) and Litterman and Weiss (1985). In order to allow comparisons with the latter study, the components of S_t^* were taken to be monthly versions of the data used by Litterman and Weiss (1985) over the sample period 1949,2-1983,6. The variable y_t was measured as the industrial production, M_{t+1} was measured as M1, and π_t was measured as the monthly change in the logarithm of the consumer price index less shelter. Seasonally adjusted versions of the money, price and output series were used.

It is worth pointing out that Sims (1980a) and Litterman and Weiss (1985) estimated their VARs using levels of variables. Sims (1980a) does not discuss the trend issue, while Litterman and Weiss (1985) note that their findings are largely insensitive to whether or not linear trends are removed. Their approach amounts to running a VAR using data on S_t^* rather than S_t. Here we investigate the sensitivity of Litterman and Weiss' Granger-causality findings to both changes in the sample period, and the removal of linear deterministic trends or first-differencing the data.

Five sample periods were considered: 1949,2-1983,6, 1952,1-1979,12, 1952,1-1983,6, 1959,2-1979,12 and 1959,2-1983,6. All VARs included twelve lagged values of each variable and a constant. Table 5.1 displays the test results for the null hypotheses that the coefficients on lagged values of π_t

and ℓnM_{t+1} (H_π and H_m, respectively) are zero in the context of the trivariate VAR for S_t^* (equation (5.5)). The null hypothesis H_M is rejected at the one percent level for the sample periods 1949,2-1983,6 and 1959,2-1983,6, but not for the other three sample periods. The null hypothesis H_π is rejected at the one percent level only for the period 1959,2-1979,12. There is substantially more evidence against the null hypothesis H_M at the five percent level; H_M is rejected at this level for all five sample periods.

Table 5.2 reports the corresponding tests for ℓny_t with S_t^* given by (5.3a). The matrix A was chosen to be the 3 x 3 identity matrix, and the function g(t) was given by

$$(5.6) \qquad g(t) = a + bt \ .$$

Thus the inputs into the VAR were the linearly detrended values of ℓny_t, π_t and ℓnM_{t+1}, denoted by $\ell ny_t'$, π_t', and $\ell nM_{t+1}'$, respectively. Let $H_M'(H_\pi')$ denote the null hypothesis that the coefficients on lagged values of $\ell nM_{t+1}'$ (π_t') in the $\ell ny_t'$ equation are zero. Interestingly, the strongest evidence against H_M' comes from the post-1959 sample periods. For these sub-periods, H_M' is rejected at the one percent level. The null hypothesis H_π' is not rejected at the five percent significance level for any of the subperiods.

The results of Table 5.2 are based on a decomposition of the movements in S_t^* into secular and cyclical components. Secular movements were modeled as deterministic functions of time. A number of authors like Nelson and Plosser (1982) have argued, on statistical grounds, that many macroeconomic time series display stochastic rather then deterministic time trends. Given the weak power of existing statistical tests for detecting the presence of unit roots in time series processes, it is not clear that these issues can be settled on purely statistical grounds (see for example the discussion of Nelson and Plosser (1982) in McCallum (1985)). However, we are sympathetic to the possibility that a variety of aggregate time series are members of what Nelson and Plosser (1982) call the class of difference stationary stochastic processes. Moreover, several of the models in Section 3 and 4 are consistent with the presence of a unit root in the autoregressive component of the time series representation for the logarithm of real output. Therefore, we re-estimated the trivariate VARs allowing for the possibility that the data display stochastic trends. However no attempt was made to decompose the movements in real output (or the other components of S_t^*) into secular and cyclical components. This is because we did not wish to impose the a priori restriction that nominal disturbances affect only the cyclical component of real output.

Table 5.3 displays the results from testing the null hypotheses that the coefficients on lagged values of $\Delta \ell n M_{t+1}$ and $\Delta\pi_t$ in the $\Delta \ell n y_t$ equation ($H_{\Delta M}$ and $H_{\Delta\pi}$, respectively) are zero. The trivariate VARs were estimated using observations on S_t with S_t and S_t^* related via (5.3b). The matrix A was chosen to be the 3 x 3 identity matrix so that S_t is simply equal to ΔS_t^*. A striking feature of the results is that neither $H_{\Delta M}$ or $H_{\Delta\pi}$ is rejected at the five percent level for any of the postwar sample periods. These results stand in sharp contrast to those reported in Tables 5.1 and 5.2.

To summarize this portion of our empirical analysis, the test results were most sensitive to changes in the sample period when the VARs were estimated using data which were not detrended in any way (as in Sims (1980a) and Litterman and Weiss (1985)) and least sensitive when the VAR's were estimated using first differenced data. In addition, the strongest evidence against the null hypothesis that real output is exogenous with respect to measures of inflation and the stock of money was obtained when the VARs were estimated using data which was not detrended. There is no evidence against this null hypothesis from VARs estimated using first-differenced data.

Next, we examine the consequences of adding asset returns to the joint time series representation for output, inflation and money. Sims (1980a) and Litterman and Weiss (1985) report that the role of money in VARs is very sensitive to the inclusion of rates of return on Treasury bill securities. For this reason, several VARs which included the ex post real value-weighted return on stocks listed on the New York Stock Exchange, r_t^s, and the ex post real rate of return on one month Treasury bills, r_t^b, were estimated. Real rates of return were studied instead of nominal returns, because the nominal rate of return on Treasury bills, R_t^b, displays a marked trend in the post war period. This trend is far less pronounced for the real rate of return, r_t^b.

The reasons for including stock returns in the empirical analysis are three-fold. First, we are sympathetic with the opinion of Fischer and Merton (1984) that macroeconomists have unduly neglected the role of the stock market as a determinant of aggregate output. Second, there are good theoretical reasons to believe that stock returns are useful statistical predictors of real output. Changes in stock prices reflect both revised expectations about future corporate earnings and revisions in the discount rate at which these expected earnings are capitalized. Revisions in both of these variables may be induced by the shocks impinging on output. Third, a number of authors including Doan, Litterman and Sims (1983), report that stock returns are, in fact, important indicators of movements in real output.

In estimating the VARs with asset returns, attention was restricted to the post 1959 period, because Tables 5.1 and 5.2 suggest that this period was the least likely to be consistent with RBC theories. Table 5.4 reports results from four-variate VARs including $\{\Delta \ell ny_t,\ r_t^b,\ \Delta\pi_t,\ \Delta \ell nM_{t+1}\}$ and $\{\Delta \ell ny_t,\ r_t^s,\ \Delta\pi_t,\ \Delta \ell nM_{t+1}\}$. In neither of the two sample periods can we reject at the five percent level the null hypotheses H_{r^b}, $H_{\Delta M}$, or $H_{\Delta\pi}$. However, the null hypothesis H_{r^s} can be rejected at the one percent level for the period 1959,2-1979,12.

Table 5.5 displays the results from four-variate VARs including $\{\ell ny_t',\ r_t^b,\ \pi_t',\ \ell nM_{t+1}'\}$ and $\{\ell ny_t',\ r_t^s,\ \pi_t',\ \ell nM_{t+1}'\}$. Somewhat surprisingly, only lagged values of $\ell ny_t'$ helped predict future values of $\ell ny_t'$ for the sample periods examined.

Table 5.6 displays results from five-variate VARs including $\{\Delta \ell ny_t,\ r_t^s,\ r_t^b,\ \Delta\pi_t,\ \Delta \ell nM_{t+1}\}$ and $\{\ell ny_t', r_t^s,\ r_t^b\ \pi_t',\ \ell nM_{t+1}'\}$. Only lagged values of $\Delta \ell ny_t$ help predict future values of $\Delta \ell ny_t$ at the one percent level. Similarly, only lagged values of $\ell ny_t'$ help predict $\ell ny_t'$ at the one percent level. However there is some evidence that ex post real stock returns have predictive power for both $\Delta \ell ny_t$ and $\ell ny_t'$ in the period 1959,2-1979,12. As with the trivariate VARs, there is somewhat more evidence that nominal aggregates help predict real variables when detrended data are used, but overall there is very little evidence of predictive power for nominal variables. Finally, Table 5.7 reports our results when real rates of return were replaced by first differences of nominal rates of return and linearly detrended nominal returns. The qualitative conclusions from these tests are essentially the same as those from using real returns.

The runs with differenced data provide little support for the view that monetary policy actions were important determinants of real output over the sample periods considered. The absence of Granger causality from money to output suggests that exogenous policy shocks were not a major source of output variability. Overall, the pattern of results from the VARs are not inconsistent with an RBC interpretation of the post war aggregate time series data.

There is perhaps one puzzling feature of these results. Recall from the discussion in Section 4.B that monetary growth might be expected to Granger cause output growth in an RBC model when the number of aggregate real shocks impinging on the economy is large relative to the number of real variables in S_{1t}. Here we have found little evidence that $\Delta \ell n M_{t+1}$ Granger causes $\Delta \ell n y_t$ in several multivariate VARs including bivariate representations. At the same time, stock returns evidently do Granger cause $\Delta \ell n y_t$, which suggests that there were at least two significant aggregate real shocks impinging on the post war economy. Money is evidently not proxying for unobserved taste or technology shocks in these VARs even though the number of real shocks exceeds the dimension of S_{1t}. These findings raise interesting questions about the links between money and output in the U.S. economy that warrant further investigation, and that are not easily addressed within the context of many recent monetary models or the model in Section 3.

Proceeding under the null hypothesis that a real business cycle view is correct, the variance decompositions of output were also calculated. Sims' method for orthogonalizing the error terms from the VAR was used, with real variables appearing before the nominal variables in the vector S_t. Under this ordering, the innovations in the real variables are interpreted as linear combinations of the innovations to agents' preference and technology shocks.

In all of the decompositions we chose an ordering of variables in which output appeared first. It would be equally consistent with the RBC model of Section 4.A to choose an ordering in which either of the two asset returns appeared prior to output. In practice, we encountered very little sensitivity to permutations of the orderings within the real block S_{1t}. In part this reflects the fact that monthly data was used. For monthly data, the contemporaneous correlations among the innovations in the VAR are small.

All reported variance decompositions are based on 48 month ahead forecast errors. Table 5.1 reports the decomposition of the variance of ℓny_t based upon trivariate VARs including ℓny_t, π_t and ℓnM_{t+1}. The reported variance decompositions of ℓny_t are sensitive to changes in the sample period. For example, in the sample period 1949,2-1979,12, the innovations to ℓnM_{t+1} and π_t account for 34 and 25 percent, respectively, of the error variance of ℓny_t. When the sample period is 1952,1-1983,6 these percentages are 9 and 64, respectively. Most dramatically, in the sample period 1959,2-1979,12, innovations in π_t account for 79 percent of the forecast error variance of ℓny_t, while monetary innovations account for only 9 percent of the forecast error variance in ℓny_t.

The decompositions of the variance of output in Table 5.2, which are based on trivariate VARs including $\ell ny_t'$, π_t' and $\ell nM_{t+1}'$, are also quite sensitive to changes in the sample period. Again innovations in the inflation rate account for the largest proportion of the variance of output in the post 1959 period. However, comparing Table 5.1 and 5.2 it is seen that innovations in the inflation rate play a relatively more important role in the variance decomposition of output based on VARs estimated with non-detrended data. Nevertheless, nominal variables accounted for fairly large proportions of the variance of $\ell ny_t'$ in all of the sample periods.

The decompositions of the variance of output reported in Table 5.3 are based on trivariate VARs which included $\Delta\ell ny_t$, $\Delta\pi_t$ and $\Delta\ell nM_{t+1}$. These results differ in two important respects from those reported in Tables 5.1 and 5.2. First, the results display very little sensitivity to changes in the sample period and the reported standard errors are smaller. Second, innovations in $\Delta\pi_t$ and $\Delta\ell nM_{t+1}$ account for only a small proportion of the error variance of $\Delta\ell ny_t$. Overall the most unfavorable sample periods from the point of view of RBC theories are the post 1959 periods.

Consider next the decompositions of the variance of output based on VARs which included asset return data. As before, attention is restricted to the post 1959 period. Table 5.4 displays the decomposition of the variance of output based on four-variate VARs including $\{\Delta\ell ny_t, r_t^b, \Delta\pi_t, \Delta\ell nM_{t+1}\}$ and $\{\Delta\ell ny_t, r_t^s, \Delta\pi_t, \Delta\ell nM_{t+1}\}$. As in Table 5.3, the reported decompositions are largely insensitive to changes in the sample period. In both cases the output innovations account for a large proportion of the variance of $\Delta\ell ny_t$. Innovations in r_t^b account for a very small proportion of the variance of $\Delta\ell ny_t$, while innovations in r_t^s account for approximately 25% of the variance of $\Delta\ell ny_t$.

Table 5.6 reports the variance decomposition of $\Delta\ell ny_t$ based on a five-variate VAR including $\{\Delta\ell ny_t, r_t^b, r_t^s, \Delta\pi_t, \Delta\ell nM_{t+1}\}$. The results are again insensitive to the choice of the sample period. Together, innovations in real variables account for over 90% of the variance of the growth rate in output. Innovations in $\Delta\pi_t$ and $\Delta\ell nM_{t+1}$ play an insignificant role in the decomposition. Both Tables 5.4 and Table 5.6 indicate that ex post real stock returns play a far more important role than ex post real Treasury bill returns in the time series behavior of $\Delta\ell ny_t$. This is reflected both in the Granger causality tests and the decomposition of the variance of $\Delta\ell ny_t$.

Finally, Table 5.6 displays the decomposition of the variance of output based on a five-variate VAR which included $\{\ell ny_t', r_t^b, r_t^s, \pi_t', \ell nM_{t+1}'\}$. Innovations in π_t' and $\ell nM_{t+1}'$ account for a significant proportion of the variance of $\ell ny_t'$. However, the results are sensitive to the choice in the sample period. For example, $\ell nM_{t+1}'$ accounts for 46% of the variance of $\ell ny_t'$ when the sample period is 1959,2-1983,6, but only 12% when the sample period is 1959,2-1979,12. A similar instability of the variance decompositions is displayed in Table 5.5 for the four-variate VARs including $\{\ell ny_t', r_t^b, \ell n\pi_t', \ell nM_{t+1}'\}$ and $\{\ell ny_t', r_t^s, \ell n\pi_t', \ell nM_{t+1}'\}$, and in Table 5.7 for the five-variate VARs which included $\{\ell ny_t', R_t^s{}', R_t^b{}', \pi_t', \ell nM_{t+1}'\}$. Not surprisingly standard errors are larger for the decompositions based on linearly detrended data than those based on the differenced data.

6. Concluding Remarks

A striking finding from our empirical analysis is that, for all sample periods considered and for various multivariate VARs, lagged values of the monetary growth rate are <u>not</u> helpful in predicting the current and future growth rates of output, after conditioning on the other variables in the VARs. Interpreted within the context of the monetary model of Section 3, these results suggest that exogenous shocks to the monetary growth rate were not an important independent source of variation in output growth during the post war period in the U.S. More precisely, statistical representations of the monetary growth rate suggest that the monetary policy rule involves feedback on both lagged monetary growth and output. This observation, combined with the dependence of output on monetary growth through our transactions technology, suggest that if monetary shocks were an important source of output variability, then monetary growth would have Granger caused the growth rate of output in VARs. Adding sticky wages and prices, overlapping nominal contracts, or entering money directly in agents' utility functions seems likely to reinforce the conclusion that monetary growth will Granger cause output growth if exogenous stocks to the money supply were an important source of business cycle fluctuation.

We hasten to add that the design of our empirical analysis is such that infrequent monetary shocks that had an important effect on output growth may not have been detected by our statistical procedures. All of the sample periods examined covered several different political administrations and, in several cases, there were significant changes in the structure of financial institutions. Additionally, specific events that are widely viewed as being monetary in nature occured during sample periods that may also have included numerous real shocks. Put differently, our results do not rule out the possibility that particular movements in output were largely due to monetary shocks. Our results only indicate that such shocks were not sufficiently frequent and large to be statistically significant over the entire sample period. More thorough studies of specific events seems worthwhile.

There are several other considerations that we feel make a real business cycle interpretation of our findings premature. First, the empirical results are not insensitive to the assumptions about the nature of trends. The virtual absence of Granger causality from money to output was obtained when both of these variables were first differenced. In contrast, when linear deterministic trends were removed from the logarithms of output and money, there was much more evidence that money Granger caused output. Even with linear trends, however, the evidence was mixed. For the sample period 1959,2 through 1983,6, and in a five-variate VAR including real stock and bond returns, money innovations accounted for 46% of the variation in detrended output. (Interestingly, this finding is not consistent with Sims' finding that monetary shocks explain little of output variation in the presence of interest rates.) For the sample period 1959,2 through 1979,12, monetary innovations accounted for 12% of the variation in detrended output. A comparison of the results for these two sample periods suggests that monetary shocks may have been important for output fluctuation during the early 1980's. But the standard errors for the estimated variance decompositions are quite large and detrended money did not Granger cause detrended output for the longer sample period. Also, recall that money accounted for an insignificant percentage of the variation in output over both sample periods when these series were time-differenced.

We wish to re-emphasize the potential importance of investigating the role of technological factors in generating business cycles within models that explicitly incorporate monetary exchange. Market structures in monetary economies are very different from the market structures that have been assumed in the literature on real business cycle models. Our example economies show that one could be seriously mislead in characterizing the structure of a monetary economy, despite obtaining a good statistical fit using a real business cycle model.

Finally, it is perhaps possible to reconcile our empirical findings with modern Keynesian or non-Keynesian monetary business cycle theories. However, to accomplish this reconciliation, these models must be formulated so as to be consistent with *both* the findings that output growth does not Granger cause money growth and that money growth depends on lagged output growth and money growth. We conjecture that in constructing such models researchers will be lead to re-examine the relative importance of various channels through which monetary factors affect real economic activity. In particular, the importance of financial institutions in the propogation of real shocks may be more pronounced in monetary models in which the structure of financial contracts emerges from a more thorough treatment of frictions.

Footnotes

[1]After we completed this paper, Bennett McCallum pointed out to us that an important implication of early Keynesian business cycle models with sticky prices and a horizontal LM function is that money should not Granger cause nominal income. This implication is, of course, counterfactual. The equilibrium real business cycle models that we examine do not have this implication.

[2]See McCallum (1985) for a related criticism of the Kydland-Prescott analysis.

[3]In solving for x_t we have followed the common practice of not imposing non-negativity constraints on the capital stock k_t. Whether or not this constraint is binding in practice will depend on the distribution of the shocks and the particular realizations in the current period. One way of obtaining a solution for which the constraint never binds is to assume that in equilibrium x_t is proportional to k_t with the proportionality factor being in the interval (0,1). This is the approach taken by Garber and King (1983) for their real economy. It turns out that the two approaches yield the same law of motion for x_t for the real economy in the absence of taste shocks, but they differ when taste shocks are present. In practice, neither solution may be strictly correct for the chosen distributions of the shocks under a non-negativity constraint. Fortunately, our key points about the properties of real versus monetary business cycle models emphasized subsequently are not sensitive to which solution is studied.

[4]This relation is derived by introducing one-period bond holdings into the consumer's budget constraint and deducing the first order conditions for this

modified optimum problem without assuming that the cash-in-advance constraint is always binding.

[5]Throughout this paper the assumption that $\ell n \epsilon_t$ is lognormally distributed is made only for convenience; none of our qualitative conclusions depend on this assumption. The expression for r^b_{t+1} implied by our model when the distribution of ϵ_t is left unspecified is identical to the expression in the text, except that $\exp\{\frac{1}{2}\sigma^2_\epsilon\}$ is replaced by $E_t\{\epsilon_{t+1}\}$.

[6]Standard errors of all variance decompositions reported in this paper were computed using the Monte Carlo procedure described in the RATS manual, page 17-3. Let b denote the estimated VAR coefficient vector and let V denote the estimated covariance matrix of the residuals in the VAR. Suppose that the VAR disturbances are i.i.d. and normally distributed. Then the posterior distribution of (b,V) is Normal-Wishart (see Zellner (1971)). Two hundred draws were taken from this distribution and the variance decomposition of the 48-month ahead forecast error implied by each draw was calculated. The reported standard error is the square root of the sample variance of the estimated variance decompositions from the two hundred simulations.

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Table 4.1

Decomposition of Variance of $\Delta \ln y_t$:*

	49,2-83,6	52,1-79,12	52,1-83,6	59,2-79,12	59,2-83,6
$\Delta \ln y_t$**	93 (4)	94 (4)	92 (4)	92 (5)	87 (6)
$\Delta^2 \ln M_{t+1}$	7 (4)	6 (4)	8 (4)	8 (5)	13 (6)
Probability Values: Granger causality of $\Delta \ln y_t$:•					
$\Delta \ln y_t$	0.00	0.00	0.00	0.00	0.00
$\Delta^2 \ln M_{t+1}$	.158	.189	.185	.230	.029
Probability values: Granger causality of $\Delta^2 \ln M_{t+1}$:•					
$\Delta \ln y_t$	.035	.024	.025	.079	.028
$\Delta^2 \ln M_{t+1}$	0.00	0.00	0.00	0.00	0.00

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Standard errors are displayed in parentheses.

**$\Delta \ln y_t$ denotes the growth rate of industrial output at time t and $\Delta^2 \ln M_{t+1}$ denotes the change in the growth rate of the stock of money at time t.

•Probability values for the null hypothesis that the lagged coefficients of the given variable in the regression equation for $\Delta \ln y_t$ are zero.

••Probability values for the null hypothesis that the lagged coefficients of the given variable in the regression equation for $\Delta^2 \ln M_{t+1}$ are zero.

Table 5.1

Decomposition of Variance of $\ln y_t$.*

	49,2-83,6	52,1-79,12	52,1-83,6	59,2-79,12	59,2-83,6
$\ln y_t$**	41 (13)	27 (11)	28 (13)	12 (8)	20 (11)
π_t	25 (14)	64 (14)	59 (16)	79 (12)	67 (14)
$\ln M_{t+1}$	34 (12)	9 (7)	13 (8)	9 (8)	13 (8)
Probability values for H_x:•					
$\ln y_t$	0.00	0.00	0.00	0.00	0.00
π_t	.414	.201	.578	.009	.127
$\ln M_{t+1}$	.003	.019	.017	.032	.006

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Standard errors are displayed in parentheses.

**$\ln y_t$ denotes the ln of industrial output at time t, π_t denotes the inflation rate between time t-1 and time t, and $\ln M_{t+1}$ denotes the ln of the stock of money at time t.

•Probability values for the null hypothesis that the coefficients of lagged values of variable x in the regression equation for $\ln y_t$ are zero.

Table 5.2

Decomposition of Variance of y_t:*

	49,2-83,6	52,1-79,12	52,1-83,6	59,2-79,12	59,2-83,6
$\ln y_t^{\sim}$**	83	45	62	29	42
	(10)	(12)	(12)	(9)	(11)
$\pi_t^{\sim}$	6	36	22	48	25
	(8)	(13)	(12)	(14)	(13)
$\ln M_{t+1}^{\sim}$	11	19	17	23	33
	(7)	(10)	(9)	(10)	(10)
Probability values for the null hypothesis $H_{x^{\sim}}$:•					
$\ln y_t^{\sim}$	0.00	0.00	0.00	0.00	0.00
$\pi_t^{\sim}$	.912	.246	.703	.090	.342
$\ln M_{t+1}^{\sim}$	.171	.010	.117	.004	.001

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Standard errors are displayed in parentheses.

**$\ln y_t^{\sim}$ denotes the detrended ln level of industrial output at time t, $\pi_t^{\sim}$ denotes the detrended rate of inflation between time t-1 and time t, and ln $M_{t+1}^{\sim}$ denotes the detrended ln level of the stock of money at time t.

•Probability values for the null hypothesis that the coefficients of lagged values of variable $x^{\sim}$ in the regression equation for $\ln y_t^{\sim}$ are zero.

Table 5.3

Decomposition of Variance of $\Delta \ln y_t$.*

	49,2-83,6	52,1-79,12	52,1-83,6	59,2-79,12	59,2-83,6
$\Delta \ln y_t$**	93 (3)	92 (4)	93 (4)	86 (6)	85 (6)
$\Delta \pi_t$	1 (2)	3 (3)	1 (2)	8 (5)	5 (3)
$\Delta \ln M_{t+1}$	6 (3)	5 (3)	6 (4)	6 (4)	11 (5)
		Probability values for $H_{\Delta x}$:•			
$\Delta \ln y_t$	0.00	0.00	0.00	0.00	0.00
$\Delta \pi_t$	.979	.937	.957	.197	.319
$\Delta \ln M_{t+1}$	.314	.259	.421	.238	.724

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Figures in parentheses are corresonding standard errors.

**$\Delta \ln y_t$ denotes the growth rate of industrial output at time t, $\Delta \pi_t$ denotes the growth rate in inflation rate at time t, and $\Delta \ln M_{t+1}$ denotes the growth rate in the stock of money at time t.

•Probability values for the null hypothesis that the coefficients on lagged values of variable Δx in the regression equation for $\Delta \ln y_t$ are zero.

Table 5.4

Decomposition of Variance of $\Delta \ln y_t$:*

	59,2-83,6	59,2-79,12		59,2-83,6	59,2-79,12
$\Delta \ln y_t$**	81 (6)	80 (7)	$\Delta \ln y_t$	70 (7)	66 (6)
r_t^b	3 (3)	11 (5)	r_t^s	24 (7)	27 (6)
$\Delta \pi_t$	12 (5)	3 (4)	$\Delta \pi_t$	3 (2)	4 (3)
$\Delta \ln M_{t+1}$	4 (3)	6 (4)	$\Delta \ln M_{t+1}$	3 (3)	3 (3)

Probability values for $H_{\Delta x}$:•

$\Delta \ln y_t$	0.00	0.00	$\Delta \ln y_t$	0.00	.005
r_t^b	.765	.608	r_t^s	.087	.004
$\Delta \pi_t$	.659	.970	$\Delta \pi_t$	.758	.628
$\Delta \ln M_{t+1}$	.108	.127	$\Delta \ln M_{t+1}$	.633	.282

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Figures in parentheses are corresponding standard errors.

**$\Delta \ln y_t$ denotes the growth rate of industrial output at time t, $\Delta \pi_t$ denotes the growth rate in inflation rate at time t, $\Delta \ln M_{t+1}$ denotes the growth rate in the ln of the stock of money at time t, r_t^b denotes the ex post real rate of return on Treasury securities at time t, and r_t^s denotes the real rate of return on stocks at time t.

•Probability values for the null hypothesis that the coefficients of lagged values of variable Δx in the regression equation for $\Delta \ln y_t$ are zero.

Table 5.5

Decomposition of Variance of $\ln y_t^{\sim}$*

	59,2-83,6	59,2-79,12		59,2-83,6	59,2-79,12
$\ln y_t^{\sim}$**	27 (8)	34 (8)	$\ln y_t^{\sim}$	27 (10)	23 (7)
r_t^b	4 (4)	27 (11)	r_t^s	14 (9)	17 (10)
$\ln \pi_t^{\sim}$	22 (11)	20 (9)	$\ln \pi_t^{\sim}$	7 (6)	21 (9)
$\ln M_{t+1}^{\sim}$	48 (12)	19 (11)	$\ln M_{t+1}^{\sim}$	52 (14)	39 (14)

Probability values for $H_{x^{\sim}}$:•

$\ln y_t^{\sim}$	0.00	0.00	$\ln y_t^{\sim}$	0.00	0.00
r_t^b	.111	.646	r_t^s	.148	.061
$\pi_t^{\sim}$	.096	.408	$\pi_t^{\sim}$	.612	.194
$\ln M_{t+1}^{\sim}$	.074	.060	$\ln M_{t+1}^{\sim}$	.224	.060

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Figures in parentheses are corresponding standard errors.

**$\ln y_t^{\sim}$ denotes the detrended ln level of industrial output at time t, $\pi_t^{\sim}$ denotes the detrended rate of inflation rate at time t, $\ln M_{t+1}^{\sim}$ denotes the detrended ln level of the stock of money at time t, r_t^b denotes the ex post real rate of return on Treasury securities at time t, and r_t^s denotes the real rate of return on stocks at time t.

•Probability values for the null hypothesis that the coefficients of lagged values of variable $x^{\sim}$ in the regression equation for $\ln y_t^{\sim}$ are zero.

Table 5.6

Decomposition of Variance of:

	$\Delta \ln y_t$			$\ln y_t'$	
	59,2-83,6	59,2-79,12		59,2-83,6	59,2-79,12
$\Delta \ln y_t$ **	66 (7)	64 (6)	$\ln y_t'$	18 (6)	23 (6)
r_t^s	24 (6)	24 (5)	r_t^s	18 (9)	26 (10)
r_t^b	3 (3)	6 (5)	r_t^b	3 (4)	23 (11)
$\Delta \pi_t$	4 (3)	4 (3)	π_t'	16 (9)	16 (7)
$\Delta \ln M_{t+1}$	3 (2)	3 (2)	$\ln M_{t+1}'$	46 (11)	12 (7)

Probability values for $H_{\Delta x}$:			Probability values for $H_{x'}$:		
$\Delta \ln y_t$	0.00	.006	$\ln y_t'$	0.00	0.00
r_t^s	.051	.011	r_t^s	.071	.013
r_t^b	.554	.800	r_t^b	.053	.217
$\Delta \pi_t$	.627	.857	π_t'	.049	.067
$\Delta \ln M_{t+1}$	.493	.266	$\ln M_{t+1}'$	.295	.047

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Figures in parentheses are standard errors.

**$\Delta \ln y_t$ denotes the growth rate of industrial output at time t, $\Delta \pi_t$ denotes the growth rate in inflation rate at time t, $\Delta \ln M_{t+1}$ denotes the growth rate in the stock of money at time t, y_t' denotes the detrended ln level of industrial output at time t, $\ln M_{t+1}'$ denotes the detrended ln level of the stock of money, π_t denotes the detrended rate of inflation at time t, r_t^b denotes the ex post real rate of return in Treasury securities at time t, π_t' denotes the detrended level of inflation, and r_t^s denotes the real rate of return on stocks at time t.

•Probability values for the null hypotheses that the coefficients of lagged values of the variables Δx and x' in the regression equations for $\Delta \ln y_t$ and $\ln y_t'$ respectively are zero.

Table 5.7

Decomposition of Variance of:

	$\Delta \ln y_t$			$\ln y_t^{\sim}$	
	59,2-83,6	59,2-79,12		59,2-83,6	59,2-79,12
$\Delta \ln y_t$**	75 (6)	72 (6)	$\ln y_t^{\sim}$	18 (6)	23 (6)
ΔR_t^s	10 (4)	12 (4)	$R_t^{s\sim}$	18 (9)	25 (8)
ΔR_t^b	7 (4)	7 (4)	$R_t^{b\sim}$	12 (9)	16 (8)
$\Delta \pi_t$	4 (3)	4 (3)	$\pi_t^{\sim}$	7 (6)	24 (9)
$\Delta \ln M_{t+1}$	3 (3)	4 (3)	$\ln M_{t+1}^{\sim}$	46 (13)	12 (8)
Probability values for $H_{\Delta x}$:			Probability values for $H_{x^{\sim}}$:•		
$\Delta \ln y_t$	0.00	0.00	$\ln y_t$	0.00	0.00
ΔR_t^s	.641	.037	$R_t^{s\sim}$	.080	.013
ΔR_t^b	.416	.348	$R_t^{b\sim}$	.049	.218
$\Delta \pi_t$	.463	.584	$\pi_t^{\sim}$	.213	.200
$\Delta \ln M_{t+1}$	.240	.238	M_{t+1}	.295	.047

*Entries give the percentage of the forecast error in the error variance accounted for by the orthogonalized innovations in the listed variables. The order of orthogonalization is as listed. Figures in parentheses are corresponding standard errors.

**$\Delta \ln y_t$ denotes the growth rate of industrial output at time t, $\Delta \pi_t$ denotes the growth rate in inflation rate at time t, $\Delta \ln M_{t+1}$ denotes the growth rate in the stock of money at time t, $\ln y_t^{\sim}$ denotes the detrended ln level of industrial output, $\pi_t^{\sim}$ denotes the detrended level of inflation, $\ln M_{t+1}^{\sim}$ denotes the detrended ln level of the stock of money, ΔR_t denotes the first difference of the monthly nominal return on Treasury securities at time t, $R_t^{b\sim}$ denotes the detrended monthly nominal rate of return on Treasury securities at time t, ΔR_t^s denotes the first difference of the nominal rate of return on stocks at time t and R_t^s denotes the monthly detrended nominal rate of return on stocks at time t.

•Probability values for the null hypotheses that the coefficients of lagged values of the variables Δx and $x^{\sim}$ in the regression equation for $\Delta \ln y_t$ and $y_t^{\sim}$, respectively, are zero.