NBER WORKING PAPER SERIES

GOVERNMENT DEBT AND BANKING FRAGILITY: THE SPREADING OF STRATEGIC UNCERTAINTY

Russell Cooper
Kalin Nikolov

Working Paper 19278
http://www.nber.org/papers/w19278

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2013

We are grateful to seminar participants at the Federal Reserve Bank of Kansas City, the Cornell-PSU Fall 2013 meeting, McGill University, the International Macroeconomics Conference at the Federal Reserve Bank of Atlanta, the University of Pittsburgh, the Riksbank and the Guanghua School of Management at Peking University for helpful comments and questions. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty
Russell Cooper and Kalin Nikolov
NBER Working Paper No. 19278
August 2013, Revised July 2015
JEL No. E44,G33,H12,H63

ABSTRACT

This paper studies the interaction of government debt and financial markets. Both markets are fragile: excessively responsive to fundamentals and prone to strategic uncertainty. This interaction, termed a diabolic loop, is driven by government willingness to bail out banks and the resulting incentives for banks not to self-insure through equity buffers. We provide conditions such that the diabolic loop is a Nash Equilibrium of the interaction between banks and the government arising from instability in debt markets and financial arrangements.

Russell Cooper
Department of Economics
The Pennsylvania State University
611 Kern
State College, PA 16802
and NBER
russellcoop@gmail.com

Kalin Nikolov
European Central Bank
Financial Research Department
Kaiserstrasse 29
Frankfurt-am-main
Germany
Kalin.Nikolov@ecb.int

Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty*

Russell Cooper† Kalin Nikolov‡

This Version: July 24, 2015
First Version: August 9, 2013

Abstract

This paper studies the interaction of government debt and financial markets. Both markets are fragile: excessively responsive to fundamentals and prone to strategic uncertainty. This interaction, termed a 'diabolic loop', is driven by government willingness to bail out banks and the resulting incentives for banks not to self-insure through equity buffers. We provide conditions such that the 'diabolic loop' is a Nash Equilibrium of the interaction between banks and the government arising from instability in debt markets and financial arrangements.

1 Introduction

The following quote is from a 2012 speech by IMF Director Christine Lagarde:

> We must also break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks. This works both ways. Making banks stronger, including by restoring adequate capital levels, stops banks from hurting sovereigns through higher debt or contingent liabilities. And restoring confidence in sovereign debt helps banks, which are important holders of such debt and typically benefit from explicit or implicit guarantees from sovereigns.[1]

*We are grateful to seminar participants at the Federal Reserve Bank of Kansas City, the Cornell-PSU Fall 2013 meeting, McGill University, the International Macroeconomics Conference at the Federal Reserve Bank of Atlanta, the University of Pittsburgh, the Riksbank and the Guanghua School of Management at Peking University for helpful comments and questions.

†Department of Economics, Pennsylvania State University, USA, email: russellcoop@gmail.com

‡Research Department, European Central Bank, Kaiserstrasse 29, Frankfurt-am-Main, email: kalin.nikolov@ecb.int

[1] This entire speech by Christine Lagarde, Managing Director, IMF, is available at `https://www.imf.org/external/np/speeches/2012/012312.htm`

Following the Greek sovereign debt write-down in 2011, the four largest Greek banks made losses of more than 28 billion euros (or 13% of GDP).[2] This was enough to wipe out almost all of their combined equity capital. In 2010, the Irish government ran an unprecedented peace-time deficit, reaching 32% of GDP as it bailed out its banking system. Under the weight of nationalized banks' losses, Ireland was forced to seek financial support from the IMF and the EU in November 2010.

These are two recent examples of a 'diabolic loop' between banks and sovereigns. In the case of Greece, banks that were otherwise solvent, were made insolvent by the default of their sovereign whose debt they were holding.[3] In the case of Ireland, a government which had previously had one of the lowest levels of debt to GDP in Europe, suffered a withdrawal of funding as markets became concerned about the contingent liabilities involved in bailing out its large, insolvent banking system. Throughout the rest of southern Europe, this 'diabolic loop' has operated in a less dramatic fashion but has nevertheless contributed to ongoing strains in sovereign and bank debt markets.

This paper models the channels that transmit fragility in the valuation of government debt onto the banking system. The framework combines the canonical model of sovereign debt fragility (Calvo (1988)) with the canonical model of banking instability (Diamond and Dybvig (1983)). Put differently, the framework studies the interaction of strategic complementarities in debt and financial markets.

Sovereign debt fragility arises due to a strategic complementarity between the buyers of government bonds and the government default decision, as in Calvo (1988). Since the government's ability to repay debt depends inversely on the real interest rate it has to pay, this opens up the possibility of self-fulfilling pessimistic equilibria in which the high interest rate needed to compensate bond holders for high expected default risk weakens the government's solvency and validates the pessimistic default expectations.

Banks are fragile due to liquidity and solvency risks as they provide liquidity insurance to their depositors while holding risky assets such as government debt. The collapse of the intermediation process leads to large welfare costs to the economy.

Motivated from the European experience, we consider two channels whose interactions complete the 'diabolic loop'. The first is the strong tendency by banks to hold (their own) government debt both as a long-term investment and as a source of liquidity. The second channel arises due to the explicit (via deposit insurance) or implicit guarantees that governments provide to their banking systems. One of the contributions of the paper is to provide conditions for governments to provide guarantees.

Our model economy highlights fragility in debt markets arising from multiple self-fulfilling valuations of government debt, building on the interaction of domestically held debt and government guarantees. In this setting, if the government debt market switches to a pessimistic (high interest

[2]National Bank of Greece, Alpha Bank, Pireus and Eurobank.

[3]The term 'diabolic loop' was evidently coined by Markus Brunnermeier in a presentation on the Euro Crisis at the July 2012 NBER Summer Institute.

rate, high default risk) equilibrium, government bond prices fall and the banks holding the bonds suffer losses. At this point (due to the high output costs of bank defaults), governments are forced to intervene and bail their banks out, further increasing government debt at precisely the point when high interest rates are making repayment difficult. The result is a further decline in government debt prices, leaving a deeper hole in bank balance sheets and requiring a larger bailout. This is the 'diabolic loop' between government debt and the banking system.

The case of strategic uncertainty emanating from debt markets is the focus of the analysis. However, once the diabolic loop is established, the model is used to study the possibility of bank runs and the interaction between fragility in financial arrangements and debt markets.

There is ample evidence for the interaction we study in the paper. Acharya, Drechsler, and Schnabl (2014) and Hannoun (2011) show that European sovereign and bank CDS prices exhibit positive co-movement over the crisis period while showing little correlation pre-crisis. When government and bank balance sheets become closely intertwined, their default probabilities become highly correlated too. Our focus on bailouts as an important linkage between banks and sovereigns is supported by Pagano (2014) who provides evidence that European governments have shown a greater willingness to provide assistance to financial institutions compared to the US and UK. [4]

In this paper we examine the policy options of the crisis-prone country in isolation of other members of a currency or economic union. We consider two ways in which policymakers and private agents can (in Christine Lagarde's words) 'break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks'.

On the banking side, equity cushions can break the adverse feedbacks between banks and sovereigns. Banks that hold adequate capital against potential sovereign risks can absorb losses from government default thus becoming completely insulated from developments in debt markets. This severes a key channel of crisis transmission from governments to the banking system. However, when banks expect bailout assistance to be provided *ex post*, the incentive for them to self-insure by building up equity buffers against losses disappears. Worse than this, banks overinvest in risky government debt, putting further contingent liabilities on an already fragile sovereign. Battistini, Pagano, and Simonelli (2014) provide compelling evidence for this behaviour. They show that southern european banks increased their holdings of domestic sovereign debt in response to an increase in country-specific risk - a finding they attribute to moral hazard.[5] A key policy conclusion from our analysis is that the zero weight on sovereign debt in the Basel III capital adequacy regime must be reformed in order to avoid the development of a sovereign-banking loop in the future.

[4]Pagano (2014) examines the difference in banks' 'standalone' credit ratings with the ratings they receive when potential government support is taken into account. He shows that government support reduces banks' funding costs by 60 bps in the EU as compared to 10-20 bps for the US and UK.

[5]Yet another possibility is 'moral suasion' by domestic governments who put pressure on their banks to buy domestic sovereign debt. Ongena, Popov, and Van Horen (2015) find evidence for this during the Euro Area debt crisis.

On the sovereign side, we examine a key policy which affects the 'diabolic loop' - the *ex post* choice of whether to provide bailout assistance to the banking system during a crisis. We argue that if the collapse of the financial system is very costly for the real economy, then governments always provide a bailout *ex post*, thus removing the need for banks to self-insure *ex ante* by issuing equity. However, a sovereign with the power to commit *ex ante* would choose not to do so, leaving it to the banks to protect depositors through equity buffers. This highlights a key part of the analysis: a necessary ingredient for the 'diabolic loop' is limited commitment by the government.

The interaction between sovereign and bank balance sheets has been the subject of a growing literature. Acharya, Drechsler, and Schnabl (2014) also model the balance sheet linkages between banks and sovereigns but take banks' sovereign debt holdings as given and do not consider how anticipated bailouts affect banks' incentives to invest in government bonds and/or issue equity to guard against sovereign exposures.[6]

Uhlig (2013) appeals to moral hazard in order to explain banks' tendency to hold large quantities of government debt. In Uhlig (2013), inadequate collateral haircuts imposed by the central bank in a monetary union allow weak country banks to profitably default on the central bank when economic fundamentals deteriorate. Farhi and Tirole (2014) examine the effects of fundamental shocks in creating feedback effects between banks and sovereigns. Leonello (2014) analyses strategic complementarities between bank depositors and sovereign debt holders and uses a 'global games' framework to endogenize the crisis probability.

Gennaioli, Martin, and Rossi (2013) offer a more positive view of banks' government debt holdings. They show that banks' domestic debt exposures can serve as a government commitment device against strategic sovereign default. Defaulting on debt held by banks causes large credit contractions so when selective default is impossible, this increases the amount of debt the government can credibly promise to repay. In that model, a bailout is exogenous and has no implications for the value of government debt.

Broner, Erce, Martin, and Ventura (2014) show using a different mechanism, that banks' holdings of government debt can become a source of multiple equilibria in the sovereign debt market by crowding out lending for productive purposes during fiscal crises.[7] Our mechanism for generating multiplicity is complementary to theirs and relies on the feedbacks between government bailouts of banks and the probability of sovereign default.

Our analysis differs from these other studies in a number of ways. First, our analysis stresses the importance of banks' equity buffers for the existence and severity of the 'diabolic loop'. As we show subsequently, significant investments in government bonds are not a problem *per se* as long as banks hold significant capital buffers against sovereign exposures. But, anticipating a bailout of

[6]Bolton and Jeanne (2011) consider international spillovers through cross-country sovereign debt holdings. Lucas, Schwaab, and Zhang (2014) present evidence for such cross-country spillovers.

[7]Gennaioli, Martin, and Rossi (2014) and Popov and Van Horen (2015) provide empirical evidence of such effects in the european sovereign debt crisis.

banks, this equity buffer will not be established. In this respect our paper is also related to the work of Admati and Hellwig (2013) which stresses the importance of adequate equity buffers in order to make banking safe without resorting to bailouts.

Second, our paper places multiple equilibria at the heart of the analysis.[8] The diabolic loop is instrumental here in two dimensions. The link from the debt market to banks can amplify the effects of strategic uncertainty associated with the valuation of government liabilities. Further, a credible promise to support the banks can by itself become the source of multiplicity.

Finally, our analysis ties the bank resolution mechanism to debt fragility. If the breakdown of the intermediation process is not very costly, the government's will not have a big incentive to support the banking system. This leaves depositors vulnerable to any banking instability, leading to self-insurance via equity buffers. In this case, paradoxically, the probability of government default is lower since the banks are not supported. Further, for some specifications of the model, as long as banks are not supported, the diabolic loop does not exist.[9]

Though the paper is not intended to provide policy guidance, it does lend support to regulatory interventions which strengthen capital requirements. Further, policies that provide incentives for bank holding of domestic debt, such as the Basel III regime, only strengthen the 'diabolic loop'. Interventions to increase capital requirements and reduce bank reliance on domestic debt holdings would weaken the 'diabolic loop', consistent with the policy goals of IMF Director Christine Lagarde.

The paper is structured as follows. Section 2 outlines the baseline model. The multiplicity of valuations of government debt is highlighted in Section 3. Section 4 characterizes the Nash equilibria of the interactions between the banks and the government. Section 5 adds instability emanating from the banking sector into the model, complementing the earlier emphasis on debt fragility. Section 6 concludes.

2 Framework

Time lasts for three periods: 0, 1 and 2. The model has two principal components. The first is a banking relationship between intermediaries and depositors, following Diamond and Dybvig (1983). The second component is the pricing of government debt, following Calvo (1988) and others.[10]

The intermediation process and pricing of government debt are linked in a couple of ways. First,

[8] Yet another approach to analyzing strategic complementarities would be to use the global games approach of Carlsson and Van Damme (1993) and Morris and Shin (1998) to obtain uniqueness and replace a response to sunspots with amplification of fundamental shocks. As, for example, Vives (2014) has argued in the context of a banking fragility problem, uniqueness actually obtains under very restrictive assumptions on the interplay of private and public signals. See De Grauwe and Ji (2012) for a discussion of empirical evidence.

[9] This contrasts with Gennaioli, Martin, and Rossi (2013). In their setting, a costly bank resolution mechanism is an important commitment device to limit default.

[10] There are now a number of papers building on Calvo (1988), including Cole and Kehoe (2000) and, more recently, Corsetti and Dedola (2012), Roch and Uhlig (2012), Cooper (2012) and many others.

the value of the government debt held by the banks affects their solvency. Second, the potential and realized needs to bailout the financial sector influences the value of government debt. These interactions can be activated by either fundamental shocks or self-fulfilling expectations influencing the value of government debt.

There are four types of agents: households, banks, investors and the government. We discuss the choices and objectives of these agents and then characterize the equilibria.

Ultimately the uncertainty in the model will come from self-fulfilling variations in investors beliefs about government debt repayment. That is, we will study debt fragility as part of a sunspot equilibrium. In framing the choice problems for agents, let s denote the state of the economy. The state is linked to investor beliefs in the characterization of a sunspot equilibrium in section 4.

2.1 Households

Households are of size 1. They have an endowment of goods d at $t = 0$ with preferences

$$V_0^H = \pi u\left(c_1 + \beta c_2\right) + (1 - \pi)\, u\left(\beta c_1 + c_2\right)$$

where β is less than 1. With probability π they are *early* consumers who prefer consuming at $t = 1$ and with probability $1 - \pi$ they are *late* consumers who prefer consuming at $t = 2$. The shares of early consumers at the aggregate level is fixed at π. We assume $u(\cdot)$ is strictly increasing, strictly concave and $u(0)$ is finite.

2.2 Banks

Following Diamond and Dybvig (1983), consumers can share liquidity risk through the banking system. Banks construct a portfolio for households which provides the needed liquidity while still taking advantage of longer term investment opportunities. In addition to providing liquidity, the bank provides insurance to households, both against their individual taste shock and government default risk. As is well understood, it is this interaction of liquidity needs and illiquid investment that can lead to fragility in the banking system. In our framework, by holding government debt as a means of meeting the liquidity needs of households, the bank is exposed to fluctuations in the value of government debt.

Banks are competitive. *Ex ante*, banks offer contracts to consumers. The contract specifies the levels of early, denoted $c^E(s)$, and late consumption, denoted $c^L(s, \mathbb{1}_G)$, dependent on the sunspot state s, realized in period 1, as well as the period 2 government repayment decision, where $\mathbb{1}_G = 1$ if the government defaults on its debt and $\mathbb{1}_G = 0$ if there is repayment. They raise deposits d from households in period 0. Investors also supply equity, denoted x_0, to the bank.

Banks invest in two types of assets in period 0. They can buy government bonds b_0 at price q_0. These bonds do not pay a coupon at the middle date but can be traded in the secondary market. Second, banks can make long term investments i_0 that return $R > 1$. These investments have a liquidation value at the middle date of $0 \leq \varepsilon \leq 1$. Banks can adjust their portfolios in the middle period, after s is realized.

The optimal contract between the banks and the households solves:

$$\max_{i_0,b_0,x_0,c^E(s),c^L(s,\mathbb{1}_G),\delta_2(s,\mathbb{1}_G),l_1(s),b_1(s),L_1(s)} E[\pi u\left(c^E(s)\right) + (1-\pi)\, u\left(c^L(s,\mathbb{1}_G)\right)] \tag{1}$$

such that

$$i_0 + q_0 b_0 \leq d + x_0 \tag{2}$$

$$\pi c^E(s) \leq q_1(s)(b_0 - b_1(s)) + \varepsilon l_1(s) + L_1(s)\, \forall s \tag{3}$$

$$(1-\pi)\, c^L(s,\mathbb{1}_G) \leq (1-\mathbb{1}_G) b_1(s) + R(i_0 - l_1(s)) - \delta_2(s,\mathbb{1}_G) - r^b L_1(s)\, \forall s \tag{4}$$

$$E\delta_2(s,\mathbb{1}_G) \geq R x_0. \tag{5}$$

In this problem, the expectation is taken over the distribution of the sunspot variable, s, and over the distribution of government default.

From (2), the total funding of the bank, $d + x_0$, is invested in illiquid investment, i_0 and government bonds, b_0, at a price q_0. The funding for the payment to the early households comes from three sources, as indicated by (3). First, the bank can sell some of the government debt it acquired in period 0 to the investors to obtain goods for early consumers. These sales occur at a state contingent price $q_1(s)$. Second, the bank could liquidate some of the illiquid investment, denoted $l_1(s)$ in (3). The liquidation of the illiquid technology is equivalent to having access to a storage technology with a return of ε between period 0 and 1. Finally, the bank could borrow from investors or other banks, denoted $L_1(s)$ in (3), at a rate r^b. We refer to this as a loan in the interbank market. This provides a second way for the bank to finance $c^E(s)$.

From (4), the state contingent consumption of late households is financed by the bonds held until the last period as well as the return on the illiquid investment that was not liquidated in the middle period. Further, the bank has the returns to investor loans made at the middle date.

The final constraint, (5) ensures that the expected return on equity is not less than the outside option of investing x_0 in the illiquid technology. Here $\delta(s,\mathbb{1}_G)$ is the state contingent payout of dividends to equity holders. The nature of this contingency and thus how the investors provide insurance to depositors is part of the equilibrium construction.

The potential risks to depositors should be clear from this optimization problem. First, there is uncertainty over the period 1 value of government debt. Second, there is sovereign default risk. The optimal contract will optimally allocate this risk between households and investors as well as provide liquidity to early households.

The first-order conditions for this problem are analyzed in Section 7.1 in the Appendix. These conditions are then used to characterize the equilibria in Section 4.

In the construction of equilibria, it will be necessary to describe the outcome of the banking arrangement when banks anticipate government support but, off the equilibrium path, it chooses not to provide it. This discussion of a 'resolution mechanism' is delayed until the characterization of equilibria.

2.3 Investors

Investors are risk neutral agents (of size 1) with endowments in periods t of A_t for $t = 0, 1, 2$. They consume in periods 1 and 2 with preferences given by $c_1 + \frac{c_2}{R}$. The assumption that investors discount at $\frac{1}{R}$ will determine the asset returns in equilibrium.

In the first period, investors allocate their endowment to the purchase of government debt (b_0^I), bank equity (x_0) and illiquid investments (i_0^I). Their budget constraint in period 0 is:

$$A_0 = q_0 b_0^I + x_0 + i_0^I. \tag{6}$$

Their budget constraint in period 1 is:

$$c_1^I(s) = A_1 + q_1(s)(b_0^I - b_1^I(s)) - L_1^I(s) \tag{7}$$

as the investor can purchase government debt of $b_1^I(s) - b_0^I$ and lend to banks, $L_1^I(s)$. The budget constraint in period 2 is:

$$c_2^I(s, \mathbb{1}_G) = (1 - \tau(\mathbb{1}_G))\, A_2(\mathbb{1}_G) + b_1^I\,(\mathbb{1}_G) + Ri_0^I + \delta_2(s, \mathbb{1}_G) + r^b L_1^I(s) \tag{8}$$

where τ is the tax rate on investor's endowment. In period 2, the endowment of the investor is augmented by the returns to bond holdings and the long term investments plus the repayments on bank loans. The government default decision influences investor consumption through the tax rate, the investors endowment (explained below) and the return on bonds.

The investors' endowment at the final date, A_2, serves as the tax base for debt service. Its value depends on the default choice of the government. Following Eaton and Gersowitz (1981) and the literature that follows, government default leads to output costs. This is reflected in the reduction in the $(1 - \gamma \mathbb{1}_G)$ term in the investors' endowment where $\mathbb{1}_G = 1$ if the government defaults. Specifically, the investor's endowment in the last period is given by:

$$A_2 = \bar{A}(1 - \gamma \mathbb{1}_G). \tag{9}$$

2.4 The Government

The government issues debt B_0 at price q_0 in period 0 to fund government expenditure G_0. This is two-period debt with repayment due in period 2. At the middle date, it issues additional debt to finance period 1 government expenditur G_1 and, if it chooses, make transfers to support the banking system. At the end of period 1, the debt outstanding is $B_1(q_1)$.

We assume that the size of time 0 government spending is smaller than the deposits of households in the bank. This makes it feasible for banks to buy the government debt stock which is convenient in the construction of the pessimistic equilibrium with government intervention.

Assumption 1. $d > G_0$.

2.4.1 The Dependence of B_1 on q_1

The dependence of the debt issuance in the middle period on q_1 is a key element of the analysis. In fact, $B_1(q_1)$ is a decreasing function of q_1.

A leading reason for $B_1(q_1)$ to be contingent on q_1 comes from government spending. Suppose the government is committed to spending $G_1 > 0$ in period 1. It must sell new debt of $\frac{G_1}{q_1}$ to finance this level of real spending.

A second reason for $B_1(q_1)$ to depend on q_1 comes from government support of the banking system through debt repurchases at above market values.[11] This support of the banking system is inversely related to the value of government debt in period 1. A reduction in q_1 can lead to the deterioration of bank balance sheets, bank failures and thus the provision of financial support for these intermediaries. By assumption, the government sells additional debt to finance these transfers.

Specifically, consider a scheme in which the government buys back debt from banks at a target price q_1^T. Therefore

$$T(q_1; q_1^T, B_0^B) = B_0^B(q_1^T - q_1) \tag{10}$$

where q_1^T is the buyback (target) price of debt and q_1 is the prevailing price of debt under pessimism.[12] Here B_0^B is the total amount of debt held by the banking system at the start of period 1. For notational convenience, denote by $T(q_1)$ the transfers to the banking system when the current price of debt is q_1, suppressing the dependence of the transfer on bank holdings and the target price. The debt outstanding at the end of period 1 is

$$B_1(q_1) = B_0 + \frac{G_1 + T(q_1)}{q_1}. \tag{11}$$

The analysis will generally use both of these channels that link debt issuance in the middle period to the state of the economy. In the construction of an equilibrium, the value of government debt in the middle period will be linked to the state s. Finally, we will allow the government to decide whether or not to support the financial system, thus making the dependence of B_1 on q_1 through this channel endogenous.

[11] Equivalently, the support could be in the form of deposit insurance.

[12] With this notation, we make explicitly the dependence of the transfer on the support price for debt, the second argument, and the level of debt held by the banks, the third argument.

2.4.2 Taxation and Default

The government taxes investors' endowments A_2 at the final date. The tax rate required to meet the total obligations of the government is equal to

$$\tau = \frac{B_1(q_1)}{A_2}.$$

By taxing investors' endowments, the government taxation does not directly impact the intermediation process. Any frictions that impinge on the deposit contract, such as sequential service, are irrelevant for the government's ability to collect taxes. However, the tax base does depend on the functioning of the intermediation process, as in (9).

To introduce the possibility of default into the analysis, assume the government's capacity to tax the endowment of the investors is random and drawn from a known probability distribution $F(\overline{\tau})$ with associated density $f(\overline{\tau})$.[13] The uncertainty about tax capacity, denoted $\overline{\tau}$, is realized at the final date. This naturally leads to the possibility of default due to bad fundamentals (as opposed to strategic default): a low realization of $\overline{\tau}$ could trigger government insolvency despite a large tax base (A_2).

If $\overline{\tau} < \frac{B_1(q_1)}{A_2}$, the government must default on its obligations. The probability of default is therefore equal to $F\left(\frac{B_1(q_1)}{A_2}\right)$ while the probability of repayment is given by $1 - F\left(\frac{B_1(q_1)}{A_2}\right)$. Once the government is forced to default, it defaults fully. But, if $\overline{\tau} > \frac{B_1(q_1)}{A_2}$, the government repays its debt obligation. No additional taxes are collected.

3 Debt Fragility

This section characterizes debt fragility in the economy. In particular, there can exist multiple valuations of government debt in period 1. This multiplicity will be used to construct sunspot equilibria.

In period 1, the debt is priced by risk neutral investors who discount the future at rate $\frac{1}{R}$. The price q_1 is determined by a no-arbitrage condition which will be specified later.

The analysis is structured to highlight two dimensions of the diabolic loop: (i) the amplification of shocks and (ii) the creation of strategic uncertainty. As the analysis develops, these two dimensions are linked to the presence of bank bailouts, $T(q_1)$, and the level of government purchases in period 1, G_1. Accordingly, some of the analysis will study the pricing of debt in the absence of bank bailout and in other cases government purchases are set to zero.

This section discusses these cases independently and then puts them together. Our analysis is ultimately based upon a setting with both bailouts and government purchases, allowing us to understand the role of these components in the creation and amplification of strategic uncertainty.

[13] Alternatively, A_2 could be random. In this formulation, these approaches are equivalent.

3.1 No Bank Bailout: $T(q_1) = 0$

In the case of no bailout, i.e. $T(q_1) = 0$, the arbitrage condition simplifies to:

$$\frac{1 - F\left(\frac{B_0 + (G_1/q_1)}{\bar{A}}\right)}{R} = q_1. \tag{12}$$

Even without bailouts, the dependence of B_1 on q_1 through the financing of G_1 can generate multiple solutions to this pricing equation. The key to the multiplicity is that both sides of (12) are increasing in q_1.

Assumption 2. *There are multiple solutions to* (12)*, including* $q_1 = \frac{1}{R}$ *and a locally stable solution with* $q_1 < \frac{1}{R}$.

Once $B_1(q_1)$ is decreasing in q_1, it is straightforward to construct multiple solutions of (12) since there is flexibility in the choice of the distribution of tax capacity. Implicitly this assumption is a restriction on $F(\cdot)$.

Assumption 2 also imposes a default free solution to the debt pricing equation. This establishes a useful benchmark but is not restrictive. The existence of a solution without default only simplifies the analysis: multiple solutions could exist even if there is no default free solution.

Figure 1 illustrate solutions to (12).[14] The function $[1 - F(\frac{B_0+(G_1/q_1)}{\bar{A}})]/R$ is the 'debt valuation equation' because it determines the price of government debt (as a function of itself). It is depicted as the black dashed curve. The points of intersection of this curve and the 45-degree line are solutions to (12).

From Assumption 2, there is a point labeled 'optimism' in Figure 1 where the default probability is zero so that $q_1 = \frac{1}{R}$. That is, $F\left(\frac{B_0+(G_1/q_1)}{\bar{A}}\right) = 0$. This corresponds to the valuation of government debt in the optimistic equilibrium without default.

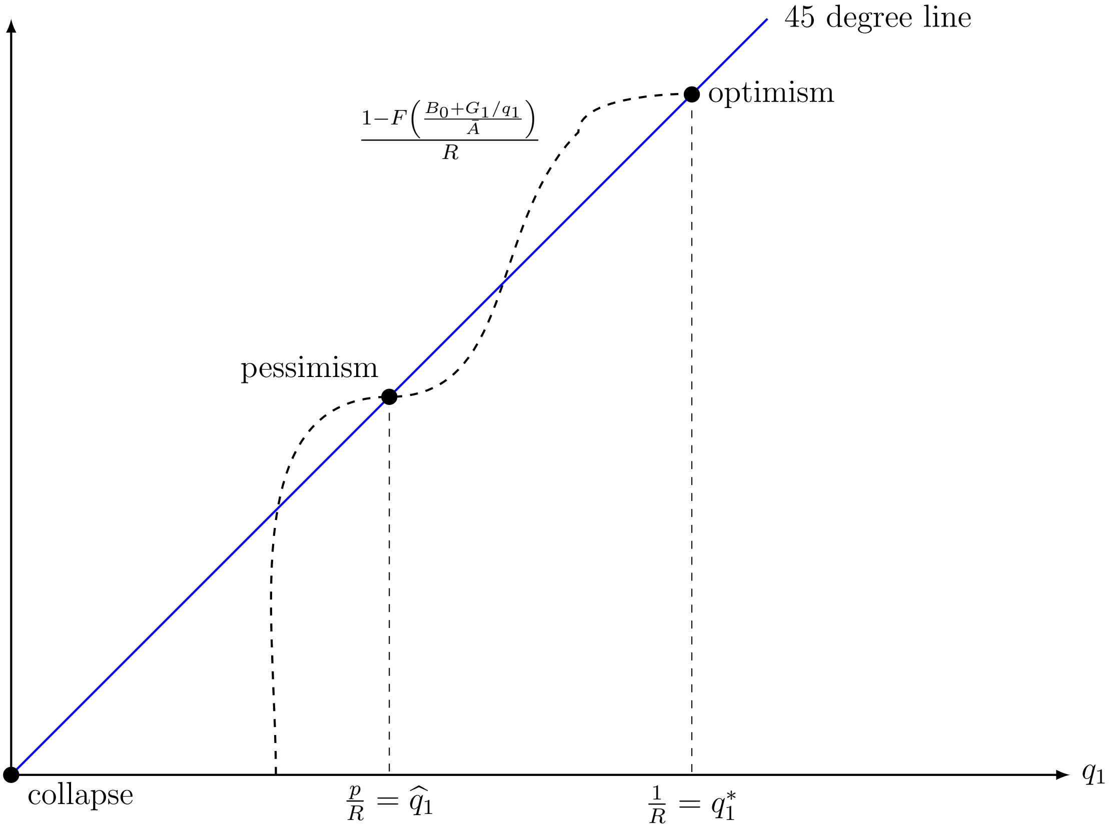
In addition, there are other equilibria in which the value of debt, q_1, is lower. These are labeled 'pessimism' and 'collapse' in the figure. The resulting higher debt obligation in period 2 generates a positive probability of default and thus lower values of q_1. By Assumption 2, there will exist a locally stable pessimistic solution.

[14]The slope of the debt valuation equation is given by $-f\left(\frac{B_1}{A_2}\right)\frac{B_1'(q_1)}{A_2}$ where $f(\cdot)$ is the density associated with the distribution function $F(\cdot)$. This expression is zero at high levels of q_1 when government debt is very far from the default point. This is the case when the density of the tax capacity random variable $f\left(\frac{B_1}{A_2}\right) = 0$.

The curve crosses the x-axis at the point at which the price of government debt becomes so low that the government is insolvent with probability 1. The location of this intersection point depends on the support of the distribution of the tax capacity shock.

At the 'pessimism' point, the debt valuation curve is assumed to have a slope less than unity implying that the pessimistic equilibrium would be stable under a dynamic adjustment of private beliefs.

Figure 1: Debt Fragility



3.2 No Government Purchases: $G_1 = 0$

Suppose there are no government purchases in period 1 but there are transfers to the banking system, i.e. $T(q_1) > 0$ for all q_1. In this setting the debt arbtrage condition becomes:

$$\frac{1 - F\left(\frac{B_0+T(q_1)/q_1}{A}\right)}{R} = q_1 \tag{13}$$

As argued earlier, $B_1(q_1)$ is decreasing in q_1 due to transfers to support the banking system. Hence the left side of (13) is increasing in q_1. As q_1 increases, the amount of debt outstanding decreases and the probability of repayment increases with q_1. Figure 1 also describes this case (where we merely replace G_1 with $T(q_1)$)

Thus the strategic complementarity induced by government spending in period 1 is present as well if government support of the banking system is decreasing in q_1. In this case, the multiplicity is created by the government bailout policy. That is, if there are no transfers to banks, then the multiplicity is eliminated.

3.3 Transfers and Government Purchases: $G_1 > 0, T(q_1) > 0$

The combination of government purchases and bailouts is the most general case and is therefore the central case of our analysis. In this case the multiplicity is based upon the need to finance period 1 government purchases. It is then amplified by the government bailout policy. By bailing out banks, the resulting increase in amount of debt outstanding reduces the value of the debt and hence makes banks even more precarious.

The interaction of government purchases and bailouts is illustrated in Figure 2. The solid curve assumes no bailouts while the dashed one allows bailouts. For illustration, the dashed curve is drawn for a case where the structure of the set of equilibria is not affected by the government buyback policy.[15] By continuity, this will be the case for sufficiently small buyback programs. The following assumption on $T(q_1)$ focuses on this case. The analysis will study buyback policies that satisfy this assumption and those that do not.

Assumption 3. *For every locally stable solution to* (13) *with* $T(q_1) \equiv 0$*, there exists a locally stable solution to* (13) *with debt buyback,* $T(q_1) \neq 0$.

By Assumption 2, there are multiple crossings of the solid curve and the 45 degree line. As $T(q_1) > 0$, the dashed curve is below the solid curve for all q_1. By Assumption 3, debt prices in the locally stable pessimistic equilibria are shown as $\widehat{q}_1$ and $\tilde{q}_1$. Due to local stability, the pessimistic debt price without a bailout is higher than that with one.

To summarize, this section constructs multiple solutions to the valuation of government debt, (13), in period 1. The multiplicity relies upon an inverse relationship between the amount of debt issued in period 1 and the value of government debt. The analysis provides two mechanisms for this inverse relationship: (i) financing of government spending and (ii) support of the banking system.

In the next section we show that banks' exposure to sovereign debt and the government's decision to bailout the banks under pessimism will be the result of privately optimal decisions.

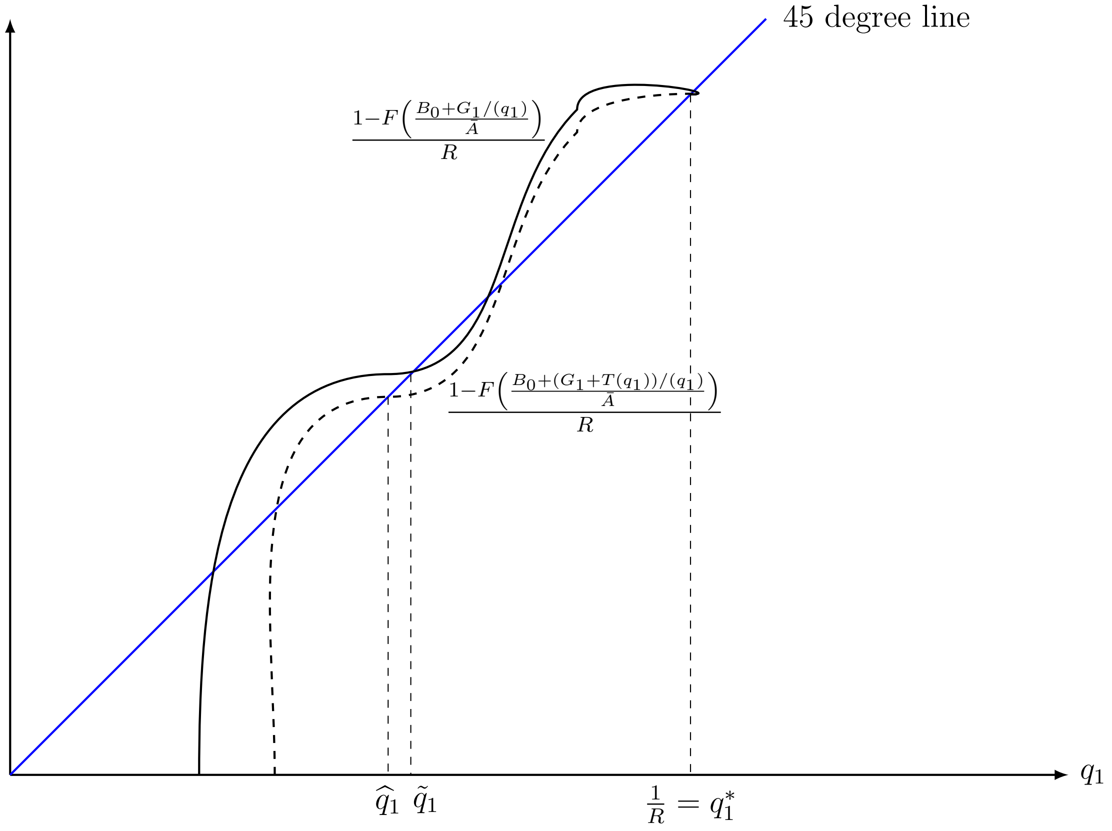
4 The 'Diabolic Loop' as a Nash Equilibrium

This section characterizes sub-game perfect Nash Equilibria for our economy. The players are the banks, the households and the government. The banks simultaneously and independently move first, setting contracts with households and deciding on their portfolio, including the amount of equity financing. These contracts are set in period 0, recognizing the possibility of strategic uncertainty influencing the valuation of government debt in period 1 as well as any government support.

We construct sunspot equilibria as a randomization between two solutions to (13). One is the no default solution labeled 'optimism' in Figure 1. The other solution is labeled 'pessimism' in Figure 1 for the case in which there are no bailouts. If there are bailouts, the outcome with pessimism is

[15] In other words, the locally stable pessimistic equilibrium still exists under bailouts.

Figure 2: Debt Fragility: The Impact of Bailouts



The solid curve displays the case in which there are no bailouts. The dashed curve allows bailouts $T(q_1)$ to support the banking system.

a locally stable solution to (13) with a valuation of debt less than $\frac{1}{R}$. An example of this is shown in Figure 2 as the point associated with the debt price of $\widehat{q}_1$. This crossing exists if Assumption 3 holds. In the analysis, we study situations in which this assumption holds and when it fails.

Given the choices of the banks, after the sunspot is realized, the government will, in period 1, either support the banks or not. This decision is a key part of the analysis. Along the equilibrium path, the expectations underlying the choices of the investors, depositors and the banking contract are fulfilled.

Definition 1. *A* ***Sub-game Perfect Nash Equilibrium*** *(SPNE) is a set of bank equity issuance and debt purchase strategies, a set of government bailout provision strategies and a set of realizations of government debt prices as a function of the debt sunspot realizations such that: (i) Individual banks solve* (1) *given the government's bank bailout strategy, the exogenous probabilities of government debt sunspot shock realizations and the prices of government debt at these sunspot realizations, (ii) the government chooses whether or not to bailout the banks in order to maximize social welfare taking bank government debt exposures as given, and (iii) the government debt markets clear at each sunspot realization.*

The SPNE will depend on the government's ability to commit to a bank bailout policy. Our approach is to study two cases. At one extreme, a **committed government** chooses *ex ante*, i.e. in period 0, whether to bailout the banks.[16] At the other extreme, a **weak government** is incapable of any kind of commitment and decides whether or not to bailout a financial institutions in period 1 to maximize *ex post* social welfare.

We study whether the diabolic loop exists in all these cases. If the government lacks commitment and the cost of default (particularly the disruption of the intermediation process) is large enough, then the government will be led to support the banking system. Anticipating this, banks will issue no equity and are vulnerable to the strategic uncertainty emanating from the debt market.

But, if the government is able to commit not to provide financial support, then the banking system is immune from debt fragility. Anticipating no government support, the banks are led to issue enough equity to shield depositors from variations in the price of government debt.

4.1 Optimistic Equilibrium

Before exploring equilibria with variations in debt prices due to strategic uncertainty, we establish a benchmark equilibrium in which sunspots do not matter. This equilibrium is interesting in its own right because debt markets can function perfectly well in our economic environment. We will also use this equilibrium as a basis for welfare comparisons. The analysis that follows builds upon the optimistic equilibrium by introducing the strategic uncertainty.

This analysis uses Assumption 2 to construct a risk free equilibrium with $q_0 = q_1(s) = \frac{1}{R}$ for all s.[17] Markets clear at these prices, given the solution of the bank contracting problem.

4.1.1 Optimal Contract

Given debt prices $q_0 = q_1 = \frac{1}{R}$, the optimal contract between the households and the banks solves (1) subject to the constraints as described in section 2.2. This problem generates a demand for government debt by the banking system. In an optimistic equilibrium, neither the banks nor the depositors anticipate variations in the price of government debt as sunspots, by construction, do not matter.

The banks hold a portfolio of government debt and long-term illiquid investment. They provide for the consumption of early households by selling government debt to investors in period 1. When the liquidation value of the illiquid investment, ε, is less than one, trading government debt strictly dominates liquidating the long-term investment. At $\varepsilon = 1$, the bank is indifferent between liquidation and the selling of government debt and we assume there is no liquidation in this case either.

[16] Here the government is limited to choosing bailout or no bailout, including the imposition of a tax on investors to finance these flows. We do not consider other *ex ante* tools for redistribution.

[17] As we are constructing an equilibrium without sunspots, the notation s is eliminated.

Proposition 1. *In the optimal banking contract with no default risk and* $q_0 = q_1 = \frac{1}{R}$*: (i)* $c^L > c^E$*, (ii)* $l_1 = 0$ *and (iii)* $x_0 = 0$*.*

Proof. See Appendix, Section 7.3.1. ∎

In the subsequent discussion, let (c^{*E}, c^{*L}) denote the optimal contract characterized in Proposition 1. We will refer to this as the first best contract. The property that $c^{*L} > c^{*E}$ implies that depositors have an incentive to reveal their true taste types.[18]

From (1), there are other elements of the bank's problem to determine. To implement the optimal contract, it is sufficient that debt holdings of the bank satisfy: $(b_0 = \frac{\pi c^{*E}}{q_1}, i_0 = \frac{(1-\pi)c^{*L}}{R})$. Further, $(b_1 = L_1 = 0)$ as trades in period 1 are not needed in the case of optimism. In an optimistic equilibrium, bank equity, x_0 is irrelevant to the allocation of the households. Thus for convenience, we set $x_0 = 0$ in the construction of an optimistic equilibrium. Equity will play a more important role in the sunspot equilibrium later on in the paper.

4.1.2 Equilibrium

Given the banking contract, the last step in constructing an equilibrium is to guarantee market clearing. There are three markets to consider: (i) the period 0 market for government debt, (ii) the period 1 market for government debt and (iii) the interbank loan market. Let (q_0^*, q_1^*) denote the values of the debt prices in an optimistic equilibrium.

Proposition 2. *There exists an optimistic rational expectations equilibrium with* $q_0^* = q_1^* = \frac{1}{R}$*,* $r^b = R$ *and the banking contract given by* (c^{*E}, c^{*L})*.*

Proof. See Appendix, Section 7.3.2. ∎

We refer to the allocation characterized by Proposition 2 as the **first best allocation**. In this equilibrium, risks are shared efficiently between the risk averse household and investors through the banking system. Further, there are no resources lost due to default and/or disruptions of the intermediation process. In this way, this allocation will serve as a benchmark for *ex ante* comparisons of other allocations.

We assume that at this allocation, a government with the ability to redistribute the endowments of households and investors in period 0 would have no incentive to do so. This implicitly defines a welfare weight for investors in period 0, ω, such that $u'(c^{*E}) = \omega$.[19]

[18] As is well understood, there may also exist a bank runs equilibrium in this environment. That is not the focus of this analysis and is initially left aside to focus on crises emanating from uncertainty over government debt repayment. We return to this in Section 5.

[19] This comes from a planner's problem allocating the deposit of households between illiquid investment and government bonds that yield, as in an optimistic equilibrium, a return of unity between period 0 and period 1.

This is a benchmark equilibrium for this economy in which there is no strategic uncertainty and no default. The existence of an equilibrium without default requires Assumption 2, which we maintain throughout this discussion.

Default could arise in equilibrium because of fundamentals. That is, if Assumption 2 did not hold, then an equilibrium without default would not exist. Instead, even in the absence of strategic uncertainty, low realizations of the tax capacity would trigger a default.

An alternative way to understand default is through the power of investors' beliefs. Under Assumption 2, there may be other solutions to the debt valuation equation. We now consider sunspot equilibria arising from debt fragility.

4.2 Discretionary Government

Under pessimism, the banks could be insolvent and the government will decide whether to support them or not. A discretionary government is unable to commit not to bailout the banks. Instead, *ex post* it decides whether to engage in a debt buyback program or not.

Since the government lacks commitment, it is necessary to specify what happens in the event it chooses, off the equilibrium path, not to engage in a debt buyback scheme. As banks anticipated this bailout, if it is not provided they are insolvent: i.e. their liabilities to depositors exceed their assets.

We consider a particular resolution mechanism which resolves the banks in an efficient manner. It is formally described in Section 7.2.[20] If the government does not bailout a bank, then the insolvent bank is liquidated allowing assets, including the liquidated long-term investments, to be used to pay off depositors in an optimal way. This involves no government help or sovereign debt issuance and is simply a reallocation of existing bank assets and liabilities.

The following proposition provides conditions such that a government will choose, *ex post*, to support the banking system through a full debt buy back, i.e. $q_1^T = q_1^* = \frac{1}{R}$.

Proposition 3. *Suppose Assumption 3 holds and the government lacks commitment. If*

1. *either (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large, there will exist a SPNE with a government debt buyback at a price of $q_1^T = \frac{1}{R}$ in the pessimistic sunspot state. The first best banking contract is offered to households supported by a bailout in the pessimistic state. No equity is issued by the bank.*

2. *the default cost, γ, is sufficiently large and the cost of disrupting the intermediation process, ψ, is sufficiently small, there will not exist a SPNE with a government debt buyback at a price of $q_1^T = \frac{1}{R}$ in the pessimistic sunspot state.*

Proof. See Appendix, Section 7.3.3. ∎

[20] The procedure is related to that studied in Ennis and Keister (2009) and Cooper and Kempf (2013).

Absent commitment, the government will choose in period 1 whether to bailout the banks or not. There are three factors influencing the bailout decision which are made explicit in the proof. First, relative to an allocation without a bailout, there are gains to redistribution from investors to depositors. This motivates a bailout. Second, if bankruptcy costs ψ are high, then there are gains to bailout from protecting the banking sector. Third, as the bailout is debt financed, it may increase the probability of default. The magnitude of this cost depends on the size of γ as well as the sensitivity of the probability of debt to changes in the amount of debt outstanding. In our model, this last effect will depend on the shape of $F(\cdot)$ in the neighborhood of a pessimistic equilibrium.

The sufficient conditions for bailout, given in the first part of the proposition, reflect these tradeoffs. If γ is low, then bailout is provided because redistribution through government support is desired and saving the financial sector is important. Even if there are costs of sovereign default, as long as ψ is large enough, bailout will be desired.

From the proposition, banks anticipate the bailout and thus choose not to self-insure through equity buffers. In fact, banks become the natural holders of risky government debt, buying the entire stock. This creates further contingent liabilities for the sovereign, thus activating the diabolic loop.

The second part of the proposition provides sufficient conditions for a full bailout not to occur. Given the tradeoff between the default cost and the gains from saving the banking system, if the default cost is large enough relative to the cost of disrupting the intermediation process, then the bailout equilibrium will not exist.

Assumption 3 is used in Proposition 3 to guarantee that a pessimistic equilibrium exists under a full debt buyback scheme. For a sufficiently small transfer to the banking system, Assumption 2 along with continuity implies a pessimistic solution to (13). Assumption 3 is not needed. But for a large enough transfer to the banks, the continuity argument fails.

Without Assumption 3, there is no SPNE with full bailout as there is no pessimistic equilibrium under the debt buyback scheme specified in (10). The government is simply not able to borrow enough to finance the transfers to the banks which are needed for a full debt buyback.

There is a maximal level of debt the government could incur while maintaining a positive probability of repayment. This is illustrated in Figure 3. Thus if a partial bailout, defined by lower buyback price, $q_1^T < \frac{1}{R}$ was feasible, it might be provided even if a full bailout was not possible.

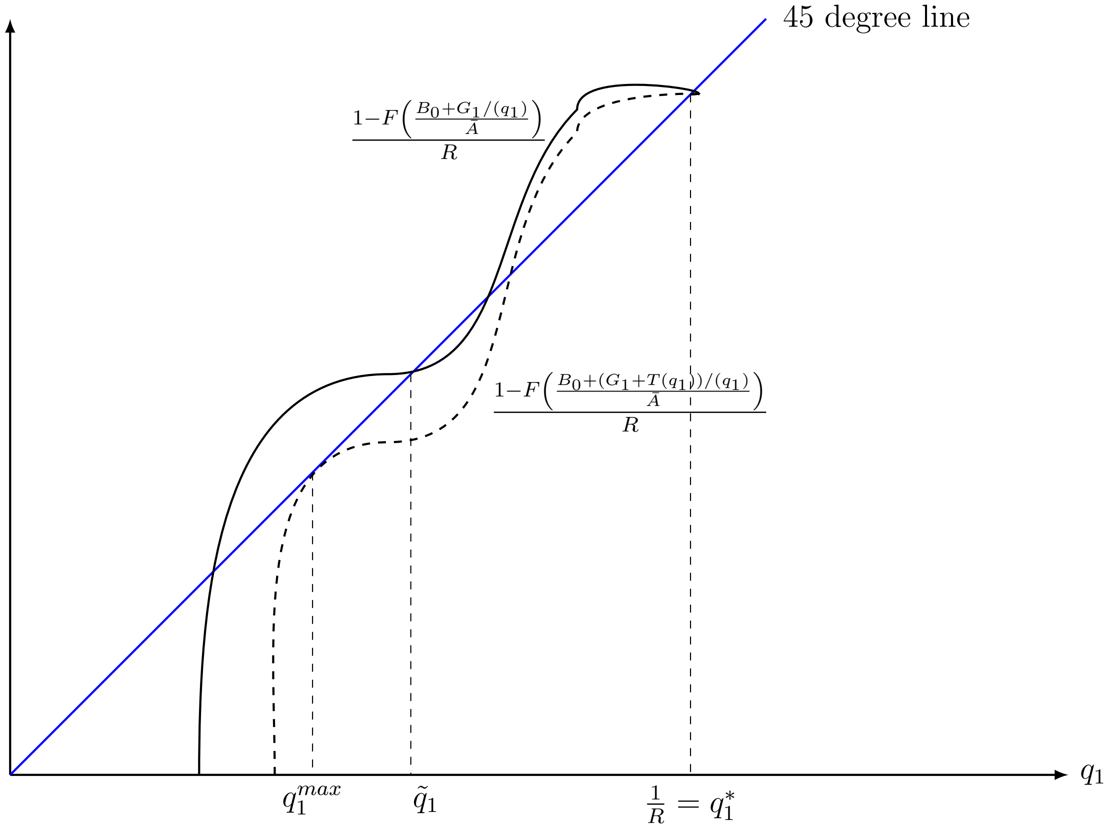
To evaluate the choice of the government, it is necessary to be clear on the solution to the contracting problem of the bank in the presence of risky government debt. Given the risk aversion of depositors and the risk neutrality of investors, in the face of fiscal risk, the investors insure depositors through the intermediary. Formally,

Proposition 4. *When government debt is risky, the bank can fully insure its depositors by issuing equity. The first best banking contract is offered to depositors.*

Proof. See Appendix, Section 7.3.4. ∎

Proposition 4 demonstrates that the bank has the options at its disposal in order to implement

Figure 3: Partial Bailout



The solid curve displays the case in which there are no bailouts. The dashed curve allows the maximal level of bailouts to support the banking system.

the first best contract even when the government does not offer full bailout assistance. However, we will see in subsequent analysis that the bank will minimize the amount of self-insurance it does when it anticipates government help. As we saw in Proposition 3, the bank will issue no equity at all, knowing that the government will come to the rescue. At most, as Proposition 5 below demonstrates, the bank will engage in partial equity issuance in order to insure itself from risks when the government is unable to offer full insurance *ex post*.

Proposition 5. *Suppose Assumption 3 does not hold and the government lacks commitment. If*

1. *either (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large, there will exist a SPNE with a government debt buyback at a price of $q_1^{\max} < \frac{1}{R}$ in the pessimistic sunspot state, where $q_1^{\max}$ is the maximum buyback price such that a pessimistic equilibrium exists. The first best banking contract is offered to households, supported by a combination of investor equity and a partial bailout in the pessimistic state.*

2. *the default cost, γ, is sufficiently large and the cost of disrupting the intermediation process,*

ψ, is sufficiently small, there will not exist a SPNE with a government debt buyback at a price of $q_1^{\max}$ in the pessimistic sunspot state.

Proof. See Appendix, Section 7.3.5. ∎

Finally, it is useful to return to the earlier discussion of the role of $G_1 > 0$ for these results. The above results characterize the consequences of the resulting multiple equilibria, building upon Assumption 2. In this setting, the bailout of the banks serves to magnify the fragility of debt markets through the diabolic loop.

If $G_1 = 0$, then the multiplicity of equilibria requires $T(q_1) \neq 0$: if there are no transfers to banks, the equilibrium in the debt market is unique. In this setting, the bailout of the banks is the source of the multiplicity of equilibria: without bailout, there is no debt fragility.

4.3 Commitment

Without commitment, if the default cost (γ) is small, and the intermediation sector is crucial for economic activity (ψ is large), the government will be induced to support the banking system. In this case, sunspot driven fluctuations in the value of government debt are ultimately absorbed by the investors who provide the taxes to support the banks.

This form of risk sharing through the government is costly. Insofar as a bailout is financed by the issuance of new government debt, the higher debt burden increases the likelihood of sovereign default.

There is another, more efficient, way to share the risk associated with debt fragility: through bank equity. Suppose that the government is able to commit in period 0 to a bailout policy. It does so understanding that the banks will respond to the government's choice in the design of the banking contract.

As we demonstrate, the government will choose to commit **not to** bailout the banks. In response, the banks will issue enough equity to insure households against fluctuations in the value of government debt. As a result, if the government is able to commit to its bailout policy, the banking system is immune to strategic uncertainty: there is no diabolic loop.

Proposition 6. *A* ***committed government*** *will choose not to bailout the banks. In the SPNE, banks self-insure through equity issuance and provide the first best contract to households.*

Proof. See Appendix, Section 7.3.6. ∎

Proposition 6 shows that government discretion is a necessary ingredient for the existence of the 'diabolic loop'. A committed government that withholds bailouts *ex post* will induce banks to self-insure in a way that obviates the need for government assistance. In this case, when Assumption 2 holds, strategic uncertainty remains in the government debt market but it does not spill over to the banking system.

The resulting allocation is not identical to the optimistic equilibrium (first best allocation) characterized in Proposition 2. Households indeed receive the same consumption allocations under the two equilibria because banks always offer the first best banking contract. However, the optimistic equilibrium has no default while the equilibrium in Proposition 6 entails a positive probability of default in pessimistic sunspot states. Thus there is an expected loss in investors' consumption from default when $\gamma > 0$.

Propositions 3 and 6 demonstrate how bank risk taking in the sovereign debt market grows and the joint fragility of banks and sovereigns worsens with diminished commitment. If governments can commit not to bail out, bank and sovereign balances sheets become disconnected and there is no sovereign-banking loop. When no commitment is possible, we get full moral hazard: banks over-invest in government debt and issue no equity. The probability of government default in the pessimistic equilibrium is higher under discretion than in the case of the government with commitment.

In both of these equilibria the households suffer no losses from the onset of pessimism in the debt market. But this happens in very different ways. In the equilibrium with commitment, the banks self insure through equity. So when there are variations in the value of government debt, the banks have a buffer.

In the equilibrium without commitment, there is no equity. The banks are insolvent following a pessimistic sunspot. The government steps in to protect depositors: this is a banking crisis, but not one that entails depositor losses. Because of the bailout, government debt is higher, thus increasing the chance of a costly default (when $\gamma > 0$). This is the source of the welfare loss without commitment.

Absent the ability to commit, the government may take actions *ex ante* to make more credible a pledge of "no bailouts" *ex post*. As suggested by Proposition 3, *ex ante* actions to increase the costs of default along with measures to reduce the vulnerability of the economy to disruptions in the intermediation process, will reduce incentives for *ex post* bailout. In our model, these actions reduce the value of ψ. Recent efforts to limit the importance of banks who otherwise might be "too big to fail" would be one example. Another is the increased reliance on explicitly 'bailinable' debt instruments in addition to insured deposits in banks' liability structure. Indeed, when bailouts are avoided and $G_1 = 0$, fragility in the sovereign debt market disappears. Even if $G_1 > 0$, multiple equilibria remain but do not infect the banking system.

5 The Coexistence of Debt and Banking Fragility

The existence of the diabolic loop has been illustrated based upon an initial bout of pessimism in debt markets. Yet the banking sector, along the traditional lines of Diamond and Dybvig (1983), can also be a source of pessimism in the form of a bank run, spreading to the debt market through government support of banks.

This section illustrates this dimension of the diabolic loop and highlights the interaction of debt fragility and bank runs.[21] To focus on the interaction of debt and banking fragility, we consider an environment with several key features.

First we assume a Diamond and Dybvig (1983) style bank: (i) there is no equity ($x_0 = 0$), (ii) in the event a bank fails, it is unable to borrow against its illiquid investment and (iii) the banking contract is not contingent on the sunspot determining depositors' choice to withdraw or not.

Second, we assume that the size of the government debt stock is large enough that if banks held all of it, they would be so liquid in the optimistic equilibrium so as to stop being vulnerable to runs. The incentive for banks to be very liquid was argued in Proposition 3. The proof of the following proposition makes clear precise how much government debt must be held to prevent runs.

In order to study the conditions for the existence of bank runs, we focus on the case when full bailout assistance is not provided by the government. Therefore, we modify the bank resolution mechanism in order to allow a partial bailout to be provided for reasons of risk sharing. This possibility was excluded in the proof of Proposition 3 because the use of the bank resolution mechanism occurred off the equilibrium path. Here this mechanism is used along the equilibrium path when there is pessimism in both debt markets and the banks. For this reason, as detailed in the proof of Proposition 7, the government offers banks a partial bailout inside the bank resolution mechanism.

The following proposition characterizes a SPNE for the case of sunspots in both the debt market and the banking arrangement. This is a natural extension of the SPNE, as in Definition 1, where the sunspot captures beliefs in both debt markets and among depositors. Importantly, the bank contract is restricted, as described above, to follow Diamond and Dybvig (1983).

The SPNE has a couple of key properties. First, there is debt fragility, as in Proposition 3. Second, there are no bank runs in isolation. This follows from an interesting implication of debt fragility. As shown in Section 4, when banks anticipate a bailout in the event of pessimism in debt markets, they hold a lot of government debt and become very liquid. Under the conditions of the proposition, this liquidity allows the banks to meet demands of early households and thus prevent a bank run. Thus the simple bank contract outlined above is optimal in the face of either debt fragility or a bank run in isolation.

Third, the proposition establishes the existence of a SPNE with pessimism in debt markets and a bank run. In this case, a full bailout is not feasible and thus the government is unable to provide full assistance to the bank.[22] However, we allow the government to provide a partial bailout in the context of the bank resolution mechanism and show that it will choose to do so. Nevertheless the outcome of the partial bailout leaves late households with relatively low consumption. As a result, a pessimistic debt market equilibrium exists along with a bank run.

[21] The interaction of strategic uncertainty in sovereign debt and banking markets has been investigated by Leonello (2014) within a global games framework.

[22] Here, and made formal in the proof of Proposition 7, a full bailout is not feasible if there does not exist a solution to the debt pricing equation, (13), in which the transfers to banks fully insure households.

Proposition 7. *Assume a Diamond-Dybvig style bank. Suppose the government lacks commitment and (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large and (iii) government debt outstanding is large. If households are not too risk averse, and a full bailout is not feasible, then there exists a SPNE with: (i) debt fragility and (ii) bank runs iff there is pessimism in debt markets.*

Proof. See Appendix, Section 7.3.7. ∎

The key to this result is that under pessimistic expectations about debt repayment, the government cannot completely support the banks. As a consequence, households are exposed to the risk in debt markets. The optimal resolution of the banks incorporates the resource loss of banks due to insolvency, ψ. This loss leads to a low consumption allocation for late households in the bank resolution mechanism, supporting a bank run.

This proposition assumed a Diamond-Dybvig style bank. In the event of either pessimism in debt markets or among depositors, this contract along with the assumption of no equity is sufficient to support the first-best consumption allocation. That is, depositors consumption is protected in both situations. It is only in the (less likely) event of pessimism in both markets, that the Diamond-Dybvig style bank fails to support the first-best allocation and bank runs occur.

6 Conclusions

This paper builds a model of the feedback loops between banks and sovereigns in Europe. From Diamond and Dybvig (1983) and Calvo (1988), banks as well as sovereign debt markets are individually subject to powerful sources of strategic uncertainty, which can lead to multiple Pareto-ranked equilibria. Our paper characterizes a 'diabolic loop' that links these markets and thus propagates and amplifies the impact of strategic uncertainty emanating from debt markets.

Bank solvency is affected by sovereign bond market turmoil because the financial system holds a large amount of (largely domestic) government debt. In turn, government solvency is affected due to the implicit or explicit guarantees extended by governments to their banking systems. These interactions amplify the impact of pessimism in the government debt market. The initial decline in government debt prices reduces bank solvency and causes the implicit government promises to its banks to turn into explicit debt issuance at precisely the time when the government is least able to issue debt on favorable conditions. The higher debt issuance then pushes government debt prices even lower, completing the diabolic loop which has been rocking a number of European economies since 2010. The impact of these feedbacks is to make sovereign-banking crises much more severe than they otherwise would have been.

While we study the effects of strategic uncertainty in debt markets as the initial shock, the model is general enough to accommodate other sources and types of uncertainty. As is well understood, the Diamond and Dybvig (1983) model often has a bank run equilibrium which itself could influence debt valuation through the cost of a government fulfilling obligations to banks. In fact, even if the bank run could be avoided through government intervention, the costs of those actions through debt valuation would be an important part of the calculus concerning the *ex post* provision of deposit insurance. Moreover, fundamental shocks to either the banking system or the government fiscal situation would be magnified and propagated through the mechanisms identified in our model.

Having built a model of the crisis, we can consider a number of simple remedies for cutting the diabolic loop. One often suggested policy is just to let the banking system fail, imposing losses on depositors. Such a policy, if credible, would have multiple benefits. First, it would reduce the need for bailout assistance to add to government debt during a sovereign crisis. This, taken in isolation, would diminish the crisis amplification mechanisms we study in our paper. Second, when banks know they will not be bailed out, they will issue equity which will absorb losses from sovereign bond holdings without needing government assistance. Hence, bank solvency becomes completely decoupled from government solvency, severing a key linkage that has amplified the financial crisis in a number of EU countries.

The problem with such a commitment to let the banking system fail is that it is not credible. Governments, acting with discretion after a crisis, prefer (under some plausible conditions) to bailout the banking system rather than incur the deadweight losses associated with a breakdown of the intermediation process.

In turn, banks, anticipating that government assistance will be provided, have little incentive to issue equity. To the benefit of depositors, they take advantage of the 'heads-I-win, tails-you-lose' nature of the financial safety net. If the economy finds itself in an optimistic equilibrium, banks profit from high *ex post* bond returns. When the economy finds itself in the pessimistic equilibrium, the bank expects the government to bail it out in order to protect household deposits. This strategy extracts a transfer from taxpayers to bank depositors which makes the latter better off. As a result, banks rationally prefer to remain exposed to a sovereign debt crisis.

This moral hazard by banks might be corrected by regulatory interventions which impose capital requirements on banks' sovereign debt holdings until they become insulated from shocks in the debt market. In the light of this finding, it is puzzling that the new Basel III regime continues to favor domestic government debt over other assets by assigning it a zero capital weight. Moreover, domestic government debt continues to be exempt from large exposure limits creating exactly the kinds of incentives for banks to become overexposed to it described in our paper.

In our paper, capital requirements on sovereign exposures implement the first best allocation. Holding large equity buffers is not costly and insulates banks from strategic uncertainty in the sovereign debt market without distorting the banking contract. The only reason it is not privately implemented is due to moral hazard. In reality, of course, large capital buffers may come with their own costs such as higher tax costs or weaker incentives (as discussed in Mendicino, Nikolov and Suarez (2015)). While beyond the scope of this paper, adding a richer analysis of government capital regulation seems to us like a interesting avenue for future research.

Future work will analyze the international dimension of the European twin crisis. Financial stability policy in Europe is undergoing major reform with the establishment of the Single Supervisory Mechanism (SSM) and with the use of Outright Monetary Transactions (OMT) by the European Central Bank. We intend to embed our single country model into a multi-country setting and consider the union-wide policies which can limit the economic damage done by the 'diabolic loop'.

7 Online Appendix

This is intended as an online appendix.

7.1 Banking Problem

The bank solves:

$$\max_{i_0,B_0^B,x_0,c^E(s),c^L(s,\mathbb{1}_G),\delta_2(s,\mathbb{1}_G),l_1(s),B_1^B(s),L_1(s)} E[\pi u\left(c^E(s)\right)+(1-\pi)\,u\left(c^L(s,\mathbb{1}_G)\right)] \tag{A.1}$$

such that

$$i_0+q_0B_0^B\leq d+x_0 \tag{A.2}$$

$$\pi c^E(s)\leq q_1\left(s\right)\left(B_0^B-B_1^B\left(s\right)\right)+\varepsilon l_1\left(s\right)-L_1\left(s\right),\forall s \tag{A.3}$$

$$\left(1-\pi\right)c^L(s,\mathbb{1}_G)\leq(1-\mathbb{1}_G)b_1\left(s\right)+R\left(i_0-l_1\left(s\right)\right)-\delta_2\left(s,\mathbb{1}_G\right)-r^bL_1\left(s\right),\forall s \tag{A.4}$$

$$E\delta_2(s,\mathbb{1}_G)\geq Rx_0 \tag{A.5}$$

The first order conditions to the contracting problem in (A.1) with respect to $(c^E(s),c^L(s,\mathbb{1}_G),B_0^B,i_0,x_0,\delta_2(s,\mathbb{1}_G),l_1(s),B_1^B(s),L_1(s))$ are:

$$\nu(s)u'\left(c^E(s)\right)-\lambda^E(s)=0 \tag{A.6}$$

$$p(s)\nu(s)u'\left(c^L(s,\mathbb{1}_G=0)\right)-\lambda^L(s,\mathbb{1}_G=0)=0 \tag{A.7}$$

$$(1-p(s))\nu(s)u'\left(c^L(s,\mathbb{1}_G=1)\right)-\lambda^L(s,\mathbb{1}_G=1)=0 \tag{A.8}$$

$$q_0\phi=\sum_s q_1(s)\lambda^E(s) \tag{A.9}$$

$$\phi=R\sum_{s,\mathbb{1}_G}\lambda^L(s,\mathbb{1}_G) \tag{A.10}$$

$$\left(\phi-R\chi\right)x_0=0 \tag{A.11}$$

$$\left(p(s)\nu(s)\chi-\lambda^L(s,\mathbb{1}_G=0)\right)\delta_2(s,\mathbb{1}_G=0)=0 \tag{A.12}$$

$$\left((1-p(s))\nu(s)\chi-\lambda^L(s,\mathbb{1}_G=1)\right)\delta_2(s,\mathbb{1}_G=1)=0 \tag{A.13}$$

$$(\varepsilon\lambda^E(s) - R\sum_{\mathbb{1}_G}\lambda^L(s,\mathbb{1}_G))l_1(s) = 0. \tag{A.14}$$

$$(\lambda^E(s)q_1(s) - \lambda^L(s,\mathbb{1}_G))B_1^B(s) = 0. \tag{A.15}$$

$$(\lambda^E(s) - r^b\sum_{\mathbb{1}_G}\lambda^L(s,\mathbb{1}_G))L_1(s) = 0 \tag{A.16}$$

where $\nu(s)$ is the probability of state s, ϕ is the multiplier on (A.2), $\lambda^E(s)$ is the multiplier on (A.3), $\lambda^L(s,\mathbb{1}_G))$ is the multiplier on (A.4), for all s and default choices, and χ is the multiplier on (A.5). Here, $p(s)$ is the probability of sovereign debt repayment in period 2 when the sunspot state in period 1 was s. These necessary conditions will be used in the subsequent proofs.

7.2 Bank Resolution Mechanism

Assume that the government does not bailout the banks. This is off the equilibrium path as the banks had anticipated a bailout. As a consequence, the banks are insolvent and the banking system is shut down. At this point, the banks re-optimize given their existing assets and liabilities without government involvement. This re-optimization occurs with a cost ψ which is proportional to the size of the bank's balance sheet. This is a standard bankruptcy cost. The bankruptcy cost comes out of liquid assets but this is not important due to the presence of the interbank market which allows the bank to borrow against the value of the long term technology if the payment of the bankruptcy cost creates a shortage of liquidity in period 1. The bank solves the following problem:

$$\begin{aligned}
&\max_{c^E,c^L(\mathbb{1}_G),B_1^B\geq 0,L_1} \pi u\left(c^E\right) + (1-\pi)\left[p^{NB}u\left(c^L(\mathbb{1}_G=0)\right) + \left(1-p^{NB}\right)u\left(c^L(\mathbb{1}_G=1)\right)\right] \\
&+\lambda^E\left(q_1^{NB}\left(B_0^B - B_1^B\right) - \psi d + L_1 - \pi c^E\right) \\
&+\lambda^L\left(\mathbb{1}_G=0\right)\left(R\left(i_0 - L_1\right) + B_1^B - (1-\pi)\,c^L(\mathbb{1}_G=0)\right) \\
&+\lambda^L\left(\mathbb{1}_G=1\right)\left(R\left(i_0 - L_1\right) - (1-\pi)\,c^L(\mathbb{1}_G=1)\right)
\end{aligned} \tag{A.17}$$

In these expressions, $\mathbb{1}_G = 0$ is a state of debt repayment by the government and $\mathbb{1}_G = 1$ denotes default. The probability of repayment is p^{NB}. Here the bank chooses the consumption levels of the two types of households, with the consumption of late households contingent on the government default decision in the next period. The bank can sell debt to finance the consumption of early households, $(B_0^B - B_1^B)$, as well as borrow from investors in period 1, L_1. While the bank could also liquidate the illiquid investment, this is dominated by borrowing from investors at an interest rate of R, the marginal rate of substitution for investors, and is not considered. We establish later that this is the equilibrium rate in the interbank market i.e. $r^b = R$.

For this problem, the price of debt, q_1^{NB} is taken as given as individual banks are small. Further, there is no fiscal operation associated with the bank resolution. The first order conditions are:

$$u'\left(c^E\right) - \lambda^E = 0 \tag{A.18}$$

$$p^{NB} u' \left(c^L \left(\mathbb{1}_G = 0\right)\right) - \lambda^L \left(\mathbb{1}_G = 0\right) = 0 \tag{A.19}$$

$$\left(1 - p^{NB}\right) u' \left(c^L \left(\mathbb{1}_G = 1\right)\right) - \lambda^L \left(\mathbb{1}_G = 1\right) = 0 \tag{A.20}$$

$$\left(-q_1^{NB} \lambda^E + \lambda^L \left(\mathbb{1}_G = 0\right)\right) B_1^B = 0 \tag{A.21}$$

$$\lambda^E - R \sum_{\mathbb{1}_G} \lambda^L \left(\mathbb{1}_G\right) = 0 \tag{A.22}$$

Let $\hat{c}^E$ and $\hat{c}^L(\mathbb{1}_G)$ denote the optimal consumption allocations when the bank gets 'resolved' in this manner. Substituting (A.22) into (A.21), implies:

$$\left(\lambda^L(\mathbb{1}_G = 0)(1 - Rq_1^{NB}) - Rq_1^{NB} \lambda^L(\mathbb{1}_G = 1)\right) B_1^B = 0. \tag{A.23}$$

Since $q_1^{NB} = p^{NB}/R$, this implies that:

$$\left(\lambda^L(\mathbb{1}_G = 0)(1 - p^{NB}) - p^{NB} \lambda^L(\mathbb{1}_G = 1)\right) B_1^B = 0. \tag{A.24}$$

Using (A.19) and (A.20), we get:

$$\left(u' \left(c^L \left(\mathbb{1}_G = 0\right)\right) - u' \left(c^L \left(\mathbb{1}_G = 1\right)\right)\right) B_1^B = 0. \tag{A.25}$$

As long as $B_1^B > 0$, households suffer losses when the government defaults and $u' \left(c^L \left(\mathbb{1}_G = 0\right)\right) - u' \left(c^L \left(\mathbb{1}_G = 1\right)\right) < 0$. Thus for (A.23) to hold, $B_1^B = 0$.

When $B_1^B = 0$, the resources to finance late consumption are independent of government default, $\hat{c}^L(\mathbb{1}_G = 0) = \hat{c}^L(\mathbb{1}_G = 1)$. From the first three first-order conditions and (A.22):

$$u'(\hat{c}^E) = Ru'(\hat{c}^L). \tag{A.26}$$

Thus the marginal rate of substitution is the same without bailout as it is with bailout. But the solution to the problem without bailout must generate lower welfare to depositors than the optimistic outcome since $q_1^{NB} < \frac{1}{R}$.

Finally the net present value of the re-negotiated consumption promises to depositors in the resolved bank satisfies:

$$\pi \hat{c}^E + \frac{(1 - \pi)\, \hat{c}^L}{R} = q_1^{NB} B_0^B - \psi d + \frac{i_0}{R}. \tag{A.27}$$

Comparing (A.27) with the net present value of consumption allocations under the optimistic equilibrium, we can see that the difference arises due to two factors.

$$\pi \left(c^{*E} - \hat{c}^E\right) + \frac{(1 - \pi) \left(c^{*L} - \hat{c}^L\right)}{R} = \left(\frac{1}{R} - q_1^{NB}\right) B_0^B + \psi d, \tag{A.28}$$

These are the loss of wealth due to the decline in the government bond price as well as the resolution cost ψd.

7.3 Proofs

7.3.1 Proof of Proposition 1

Proposition 1. *In the optimal banking contract with no default risk and* $q_0 = q_1 = \frac{1}{R}$*: (i)* $c^L > c^E$ *(ii)* $l_1 = 0$ *and (iii)* $x_0 = 0$.

Proof. With $q_1(s) = \frac{1}{R}$, no sunspots and no default, the first-order conditions for the optimal contract become:

$$u'\left(c^E\right) - \lambda^E = 0 \tag{A.29}$$

$$u'\left(c^L\right) - \lambda^L = 0 \tag{A.30}$$

$$q_0\phi = \frac{\lambda^E}{R} \tag{A.31}$$

$$\phi = R\lambda^L \tag{A.32}$$

$$(\varepsilon\lambda^E - R\lambda^L)l_1 = 0. \tag{A.33}$$

Using $q_0 = \frac{1}{R}$, combining (A.31) and (A.32) implies $\phi = \lambda^E = R\lambda^L$. Substituting this into (A.29) and (A.30) implies

$$u'\left(c^E\right) = Ru'\left(c^L\right). \tag{A.34}$$

This condition implies property (i): $c^L > c^E$ for all $R > 1$ as $u(\cdot)$ is strictly concave. Using (A.33), l_1 is zero, and strictly so if $\varepsilon < 1$, as $\lambda^E = R\lambda^L$. Finally, because there is no uncertainty facing the bank, there is no need to issue equity. Hence $x_0 = 0$. ∎

7.3.2 Proof of Proposition 2

Proposition 2. *There exists an optimistic rational expectations equilibrium with* $q_0^* = q_1^* = \frac{1}{R}$, $r^b = R$ *and the banking contract given by* (c^{*E}, c^{*L}).

Proof. The equilibrium conditions are driven by the investors. We assume that the aggregate endowment of the investors is larger than the stock of government debt in period 0. The investors can either put their endowment directly in the illiquid technology and obtain R or purchase two period government debt. They are indifferent between these options if $q_0 = \frac{1}{R}$. If this condition holds, they are willing to purchase any of the government debt not held by the banking system. Since investors have linear utility of $c_1 + \frac{1}{R}c_2$, they are indifferent between consuming their period 1 endowment and buying one period government debt if $q_1 = \frac{1}{R}$. Assuming that investors' period 1 endowment is sufficiently large, if $q_1 = \frac{1}{R}$, the investors will purchase the debt sold by the banks in period 1 and the new debt issued by the government in period 1.

Thus, at these prices, all markets clear. The excess supply of government debt in period 0 is purchased by the investors. The stock of government debt held by bank is sold to the investors along with any new debt in period 1. The market for government debt clears in both periods. Given that $q_0^* = q_1^* = \frac{1}{R}$, the probability of government default is zero.

Further, at $r^b = R$, investors are indifferent both with respect to the timing of their consumption and the composition of their portfolio. This indifference guarantees market clearing in the interbank market at zero trade.

The result that the first best contract is provided in equilibrium comes from Proposition 1. In equilibrium, banks will hold enough debt to finance their payment to early consumers at the anticipated period 1 price: $b_0 q_1 = \pi c^{*E}$. The debt is sold to the investors for goods and those goods are transferred to the early consumers. There are no liquidations in an optimistic equilibrium. ∎

7.3.3 Proof of Proposition 3

Proposition 3. *Suppose Assumption 3 holds and the government lacks commitment. If*

1. *either (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large, there will exist a SPNE with a government debt buyback at a price of $q_1^T = \frac{1}{R}$ in the pessimistic sunspot state. The first best banking contract is offered to households supported by a bailout in the pessimistic state. No equity is issued by the bank.*

2. *the default cost, γ, is sufficiently large and the cost of disrupting the intermediation process, ψ, is sufficiently small, there will not exist a SPNE with a government debt buyback at a price of $q_1^T = \frac{1}{R}$ in the pessimistic sunspot state.*

Proof. The proof has three steps: (i) characterizing the optimal banking contract, (ii) determining the government's bailout choice and (iii) checking market clearing.

Step 1: Optimal Contract

To construct an equilibrium, suppose the banks anticipate a bailout by the government in the event of pessimism. The banking contract with expected bailouts solves (A.1) with the resource constraint for early consumers in the **pessimistic state** modified to reflect the government debt buyback program:

$$\pi c^E(s^p) \leq \frac{1}{R}\left(B_0^B - B_1^B\left(s^p\right)\right) + \varepsilon l_1\left(s^p\right) - L_1\left(s^p\right) \tag{A.35}$$

Here the government buys government bonds from the banks at the optimistic price of $q_1(s^p) = \frac{1}{R}$, making the return on government bonds independent of the sunspot. This is the form of anticipated government support in the pessimistic sunspot state.

The government debt support at $\frac{1}{R}$ implies that the contract between the bank and the households is immune from the sunspot. With $q_0 = \frac{1}{R}$, verified below, the bank problem is identical to that solved in the optimistic equilibrium, characterized in Proposition 1. Hence the banking contract is the first best one: (c^{*E}, c^{*L}).

Given that the households are insured through the government buyback, there is no gain to supplying equity to the bank. Hence $x_0 = 0$.

Step 2: Government's Choice

The government will choose between a buyback, denoted BB, and no bailout, denoted NB. Under a buyback scheme, the government will buy as much debt as banks supply at a price denoted $q_1^{BB} = \frac{1}{R}$. Throughout, let $(\mathbb{1}_G = 0)$ denote the states in which the government repays its debt and let $(\mathbb{1}_G = 1)$ denote default states. Given that the government debt market is in a state of pessimism, the analysis characterizes the payoffs in periods 1 and 2 to households and investors with and without a debt buyback.

Welfare under Full Bailout

First, suppose that the government supports the banks by purchasing debt at a price q_1^{BB}. This transfer is financed through the issuance of new debt.

Investors

If a debt buyback is provided, at the middle date investors consume the difference between their endowment, the bonds they buy from banks B_0^B and the amount they lend to banks L_1:[23]

$$c_1^{I,BB} = A_1 - q_1^{BB}\left(B_0^B + \left(G_1 + T\left(q_1^{BB}\right)\right)/q_1^{BB}\right) - L_1.$$

At the final date, investors consume all their net worth. Their consumption depends on the default decision of the government.

If the government repays its debt, consumption is given by:

$$\begin{aligned} c_2^{I,BB}(\mathbb{1}_G = 0) &= \bar{A} + B_1 + Ri_0^I + RL_1 - \tau\bar{A}. \\ &= \bar{A} + Ri_0^I + RL_1. \end{aligned} \tag{A.36}$$

where the second equality follows from the fact that the banking system sells its entire holding of government debt in the pessimistic equilibrium. Hence investors own all debt and pay the taxes to pay the debt obligation. The two cancel out ($B_1 = \tau\bar{A}$).

If the government defaults, investors' final period consumption is given by:

$$c_2^{I,BB}(\mathbb{1}_G = 1) = \bar{A}(1-\gamma) + Ri_0^I + RL_1. \tag{A.37}$$

Investors' welfare is affected by the default decision, in part, due to the output costs of default assumed in our specification of the final date endowment. Investors' expected welfare is:

$$W^{I,BB} = c_1^{I,BB} + E[\frac{1}{R}c_2^{I,BB}(\mathbb{1}_G)]$$

where the expectation is over the government default decision in period 2.

Depositors

[23] In the pessimistic equilibrium, banks sell their entire holdings of government debt (B_0^B) to investors in the middle period.

The utility of depositors when the government buys back debt is independent of its decision to default and is given by the utility delivered by the standard banking contract

$$W^{H,BB} = \pi u \left(c^{*E}\right) + (1-\pi)\, u \left(c^{*L}\right)$$

Welfare if a buyback is provided

Social welfare when a buyback is provided is:

$$\begin{aligned} W^{BB} &= \pi u \left(c^{E}\right) + (1-\pi)\, u \left(c^{L}\right) \\ &+\omega \left[A_1 - q_1^{BB}\left(B_0^B + \left(G_1 + T\left(q_1^{BB}\right)\right)/q_1^{BB}\right)\right] + \frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1 - p^{BB}\right)\gamma \bar{A}. \end{aligned} \tag{A.38}$$

With a buyback price of $\frac{1}{R}$, $T(q_1^{BB}) = B_0^B(\frac{1}{R} - q^{BB})$. Thus $q_1^{BB}\left(B_0^B + \left(G_1 + T\left(q_1^{BB}\right)\right)/q_1^{BB}\right) = G_1 + \frac{B_0^B}{R}$ so that investor consumption in period 1 is simply $A_1 - G_1 - \frac{B_0^B}{R}$.

In (A.38), p^{BB} is the probability that the government repays at the final date conditional upon a buyback being provided:

$$p^{BB} = 1 - F\left(\frac{B_0 + (G_1 + T(q_1^{BB}))/q_1^{BB}}{\bar{A}}\right). \tag{A.39}$$

In (A.38), q_1^{BB} denotes the price of government debt in the middle period if there is pessimism and a bailout is provided. This is determined from the investor's arbitrage condition of $\frac{p^{BB}}{q_1^{BB}} = R$ and (A.39). Finding the (p^{BB}, q_1^{BB}) that solves these two conditions is the same as finding a solution to (13), other than the optimistic equilibrium. By assumption, the economy is in a pessimistic solution to (13).

Welfare without a Bailout

Welfare if no buyback is provided

When no buyback is provided (when one had been anticipated), the banking system is insolvent and the government steps in to resolve the failing financial institutions. Full discussion of the bank resolution mechanism can be in section 7.2. Here we merely make use of the outcomes of the bank resolution mechanism to compute social welfare when no buyback is provided:

$$\begin{aligned} W^{NB} = & \quad \pi u \left(\hat{c}^{E}\right) + (1-\pi)\, u \left(\hat{c}^{L}\right) \\ + & \quad \omega \left[A_1 - q_1^{NB}\left(B_0^B + G_1/q_1^{NB}\right)\right] + \frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1 - p^{NB}\right)\gamma \bar{A}. \end{aligned} \tag{A.40}$$

In these expressions, the loans to banks made by the investors in period 1 cancel with the proceeds from those loans at the interbank loan rate of $r^b = R$.

Here p^{NB} is the probability that the government repays at the final date conditional upon no buyback being provided:

$$p^{NB} = 1 - F\left(\frac{B_0 + G_1/q_1^{NB}}{\bar{A}}\right). \tag{A.41}$$

Finding the (p^{NB}, q_1^{NB}) that solves (A.41) and the arbitrage condition is the same as finding a solution to (12), other than the optimistic equilibrium. By assumption, there exists a pessimistic solution to (12).

The Bailout Decision

The difference in the value of the social welfare function between bailout and no bailout is:

$$\begin{aligned}\Delta &\equiv W^{BB} - W^{NB} \\ &= \pi\left[u\left(c^{*E}\right) - u\left(\hat{c}^{E}\right)\right] + (1-\pi)\left[u\left(c^{*L}\right) - u\left(\hat{c}^{L}\right)\right] - \omega\left(\frac{1}{R} - q_1^{NB}\right)B_0^B \\ &+ \frac{\omega}{R}\left(p^{BB} - p^{NB}\right)\gamma\bar{A}.\end{aligned} \tag{A.42}$$

where p^{NB} is given by (A.41) and p^{BB} is given by (A.39).

From our analysis of the bank resolution mechanism in section (7.2), we know that the difference in the net present value of consumption promises under the optimistic contract and the bank resolution allocation is given by (A.28). Hence:

$$\omega(\frac{1}{R} - q_1^{NB})B_0^B = \omega\left(\pi[c^{*E} - \hat{c}^{E}] + (1-\pi)[\frac{c^{*L} - \hat{c}^{L}}{R}] - \psi d\right). \tag{A.43}$$

Using $\omega = u'(c^{*E}) = Ru'(c^{*L})$, the first term in (A.42) becomes

$$\begin{aligned}&\pi\left[u\left(c^{*E}\right) - u\left(\hat{c}^{E}\right)\right] + (1-\pi)\left[u\left(c^{*L}\right) - u\left(\hat{c}^{L}\right)\right] - \\ &u'\left(c^{*E}\right)\left(\pi[c^{*E} - \hat{c}^{E}] + (1-\pi)[\frac{c^{*L} - \hat{c}^{L}}{R}]\right) + u'\left(c^{*E}\right)\psi d\end{aligned} \tag{A.44}$$

By the strictly concavity of $u(\cdot)$ the first row of (A.44) is positive. This represents the insurance gain to redistribution through a debt buyback. The second row of (A.44) represents the gains due to the avoidance of the deadweight costs of bank default. It is rising in the size of deadweight losses from bank default ψd.

The second term in (A.42) is proportional to the difference in the expected output costs of default due to the provision of a bailout relative to no bailout.

$$\left(p^{BB} - p^{NB}\right)\gamma\bar{A} \tag{A.45}$$

This term is clearly negative because the probability of repayment under a bailout (p^{BB}) is always lower than the probability of repayment when no bailout is provided (p^{NB}). The term is rising in the size of deadweight losses from default $\gamma\bar{A}$.

$\Delta > 0$ and a bailout is provided whenever γ is sufficiently small and ψ is sufficiently large. In that case, the gains from redistribution and from avoiding costly bank failures outweight the increasein the expected costs of the bailout.

If markets clear at the presumed prices, then we have a sufficient conditions for $\Delta > 0$ and the provision of a bailout. Step 3 of the proof shows that markets clear at the conjectured prices.

Step 3: Market Clearing

We construct prices such that markets clear. All government debt is held by banks as they receive the benefits of the debt buyback. From Assumption 1, this is feasible because bank deposits are larger than the government debt stock at the initial date.

At $q_0 = \frac{1}{R}$, as assumed in the construction of the equilibrium, banks are indifferent between illiquid investment and the holding of government debt to finance the consumption of late households. The banks are the marginal holders of the government debt in period 0. Along the equilibrium path, banks sell all the risk debt to investors at date 1, i.e. $B_1^B = 0$.

Period 1 prices are $q_1(s^o) = \frac{1}{R}$ and $q_1(s^p) = \frac{p^{BB}}{R}$ where $p^{BB} < 1$ is the probability of repayment under bailout given by (A.39).

From the preferences of the investors, they are willing to lend as much as demanded in the interbank market if $r^b = R$. Thus this market will clear at that price with and without government debt buyback.

∎

7.3.4 Proof of Proposition 4

Proposition 4. *When government debt is risky, the bank can fully insure its depositors by issuing equity. The first best banking contract is offered to depositors.*

Proof. Let $q_1(s^o)$ be the price of government debt under optimism and $q_1(s^p)$ be the price of government debt under pessimism, with $q_1(s^o) > q_1(s^p)$. Using Assumption 2, the outcome under optimism is the no default solution to the debt pricing equation. So $q_1(s^o) = q_1^* = \frac{1}{R}$. $q_1(s^p) = \frac{p}{R}$ where $p < 1$ is the probability of government repayment in the pessimistic equilibrium. Let ν denote the probability of optimism.

The proof argues that there exists equity infusion x_0 such that (i) the contract with equity fully insulates depositors from risks and supports the first best allocation despite stochastic government debt prices and (ii) investors receive their required rate of return from the equity investment. We first determine the level of equity investment needed to support the first best contract, (c^{*E}, c^{*L}). We then argue that the return on this equity equals the outside option of the investors.

Let x_0 denote the period 0 investment of equity holders into the bank and let e_0 denote its market value. Because the first best contract delivers state-uncontingent allocations to consumers, we drop the dependence of consumption on the sunspot state or the government's default decision. In the first best contract, the expected net present value of promises to depositors equals the amount they deposit at the initial date:

$$\pi c^{*E} + \frac{(1-\pi)\, c^{*L}}{R} = d. \tag{A.46}$$

To support the first best contract, the bank must have sufficient resources to meet the contractual commitment to early consumers regardless of the realized government debt price:

$$\pi c^{*E} = q_1(s^p)B_0^B \tag{A.47}$$

where $q_1(s^p)$ is the period 1 price of government debt under pessimism. In this state, the bank sells its entire bond holding in order to pay off early consumers. Promises to late consumers are met through the illiquid investment:

$$(1-\pi)\, c^{*L} = Ri_0. \tag{A.48}$$

The cash flow for dividend payments to shareholders is only generated in the optimistic state. The bank rolls over its bond not needed to fund the early consumers:

$$\pi c^{*E} = q_1(s^o)\left(B_0^B - B_1^B(s^o)\right). \tag{A.49}$$

The rolled over bond holding is then used to pay dividends to shareholders at the final date:

$$\delta_2\,(s^o) = B_1^B(s^o).$$

Since there is no default under optimism, the payment to equity holders is not indexed by the government default decision. The value of the equity to the shareholder is the discounted expected value of this dividend:

$$e_0 = \frac{\nu B_1^B(s^o)}{R}. \tag{A.50}$$

For the equity investment to be undertaken in equilibrium it must be the case that this expected value equals the equity put into the bank by the investor, i.e. $x_0 = e_0$. Substituting (A.46), (A.47) and (A.48) into (A.2) yields $x_0 = q_0 B_0^B - \pi c^{*E}$. From this and from the definition of $q_1(s^o)$ and $q_1(s^p)$ the equity investment needed is:

$$x_0 = \nu \frac{(1-p)}{p}\pi c^{*E}. \tag{A.51}$$

Combining (A.47) with (A.49) we get:

$$B_1^B(s^o) = R\frac{(1-p)}{p}\pi c^{*E}.$$

Hence

$$e_0 = \nu \frac{(1-p)}{p}\pi c^{*E} = x_0.$$

There is a strict incentive for banks to issue equity to insure depositors against the strategic uncertainty created by $G_1 > 0$. This implements the first best contract. A bank offering any other contract would either not attract customers or would be unprofitable.

∎

7.3.5 Proof of Proposition 5

Proposition 5. *Suppose Assumption 3 does not hold and the government lacks commitment. If*

1. *either (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large, there will exist a SPNE with a government debt buyback at a price of $q_1^{\max} < \frac{1}{R}$ in the pessimistic sunspot state, where $q_1^{\max}$ is the maximum buyback price such that a pessimistic equilibrium exists. The first best banking contract is offered to households, supported by a combination of investor equity and a partial bailout in the pessimistic state.*

2. *the default cost, γ, is sufficiently large and the cost of disrupting the intermediation process, ψ, is sufficiently small, there will not exist a SPNE with a government debt buyback at a price of $q_1^{\max}$ in the pessimistic sunspot state.*

Proof. The proof has three steps: (i) characterizing the optimal banking contract, (ii) determining the government's bailout choice and (iii) checking market clearing.

Step 1: Optimal Contract

To construct an equilibrium when Assumption 3 does not hold, suppose the banks anticipate the maximum feasible bailout by the government in the event of pessimism. Let $q_1^{\max} < q_1^*$ denote the maximum buyback price at which the government is able to repurchase the sovereign bonds in the event of pessimism. This maximum buyback price is given by the tangency point of the debt valuation condition to the 45 degree line in Figure 3. A higher bailout price is not consistent with the existence of a pessimistic equilibrium.

The banking contract with expected bailouts solves (A.1) with the resource constraint for early consumers in the **pessimistic state** modified to reflect the government debt buyback program:

$$\pi c^E(s^p) \leq q_1^{\max}\left(B_0^B - B_1^B(s^p)\right) + \varepsilon l_1(s^p) - L_1(s^p) \tag{A.52}$$

Unlike in Proposition 3, here the government does not fully insulate the bank from the risk in government bonds.

This leaves the bank with a choice of whether to expose depositors to risk or whether to insure them fully by issuing equity. From Proposition 4 we know that equity issuance implements the first best banking contract. This contract gives the highest utility to a depositor bringing d units of deposits to the bank and, since it is feasible, the bank issues equity and offers this allocation to households.

The bank issues equity x_0 such that it is just solvent under pessimism provided that the government does buy back the debt at price $q_1^{\max}$. For the bank to be solvent in the pessimistic state, the net present value of liabilities (c^{*E}, c^{*L}) must be equal to the value of bank assets when a debt buyback is provided:

$$i_0 = \pi c^{*E} + \frac{(1-\pi)\, c^{*L}}{R} - q_1^{\max} B_0$$

Hence, using the period 0 budget constraint:

$$\begin{aligned} x_0\left(q_1^{\max}\right) &= q_0 B_0 + i_0 - d \\ &= \left(q_0 - q_1^{\max}\right) B_0 + \pi c^{*E} + \frac{(1-\pi)\, c^{*L}}{R} - d \\ &= \left(q_0 - q_1^{\max}\right) B_0 \end{aligned}$$

where the third equality follows from the fact that depositors receive claims whose net present value equals the funds they deposit in the bank. Equity is issued to protect the bank from potential losses on sovereign bond holdings. The lower the buyback price $q_1^{\max}$ relative to q_0 the more equity needs to be issued by the banks in order to protect their depositors from fluctuations in government bond values.

The bank has no incentive to issue more equity than this amount because it is insured by the government's buyback policy anyway. It will not want to issue less because, by the definition of $q_1^{\max}$ the government is unable (because Assumption 3 does not hold) to offer a larger bailout.

Step 2: Government's Choice

The proof proceeds in a parallel fashion to the proof of Proposition 3. The government will choose between the partial buyback, denoted PB, and no bailout, denoted NB. Under a buyback scheme, the government will buy as much debt as banks supply at a price of $q_1^{\max}$. Throughout, let $(\mathbb{1}_G = 0)$ denote the states in which the government repays its debt and let $(\mathbb{1}_G = 1)$ denote default states. Given that the government debt market is in a state of pessimism, the analysis characterizes the payoffs in periods 1 and 2 to households and investors with and without a debt buyback.

Welfare under the Partial Bailout

First, we examine the case when the government acts as expected by purchasing debt at a price $q_1^{\max}$. This transfer is financed through the issuance of new debt. Following similar derivations as in Proposition 5 social welfare when a buyback is provided is:

$$\begin{aligned} W^{PB} = \pi u\left(c^{*E}\right) + (1-\pi)\, u\left(c^{*L}\right) \\ +\omega\left[A_1 - G_1 - q_1^{\max} B_0^B\right] + \frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1 - p^{PB}\right)\gamma \bar{A}. \end{aligned} \tag{A.53}$$

In (A.53), p^{PB} is the probability that the government repays at the final date conditional upon a buyback at price $q_1^{\max}$ being provided:

$$p^{PB} = 1 - F\left(\frac{B_0 + (G_1 + T(q_1^{PB}))/q_1^{PB}}{\bar{A}}\right). \tag{A.54}$$

In (A.54), q_1^{PB} denotes the market price of government debt in the middle period if there is pessimism and a bailout is provided. This is determined from the investor's arbitrage condition of $\frac{p^{PB}}{q_1^{PB}} = R$ and (A.54). Finding the (p^{PB}, q_1^{PB}) that solves these two conditions is the same as finding a solution to (13), other than the optimistic equilibrium. By assumption, the economy is in a pessimistic solution to (13).

Welfare without a Bailout

When the government does not bailout the banks (as expected on the equilibrium path), they are insolvent. This triggers default costs up to the value of ψ fraction of the bank's balance sheet (ψd). At this point, the banks re-optimize given their existing assets and liabilities without government involvement.

As shown in the proof to Proposition 3, the bank solves (A.17) with a solution which delivers $\hat{c}^E < c^{*E}$ and $\hat{c}^L < c^{*L}$ to depositors such that $u'(\hat{c}^E) = Ru'(\hat{c}^L)$. The bank sells all its bonds to investors ($B_1^B = 0$) thus making the allocation to late investors independent of the sunspot shock. However all depositors receive a smaller allocation compared to the first best reflecting the decline in bank asset values as a result of the sunspot. The consumption allocation is increasing in the amount of equity issued by the bank at the initial date.

Welfare if no buyback is provided

Social welfare when no buyback is provided is:

$$\begin{aligned} W^{NB} = & \quad \pi u\left(\hat{c}^E\right) + (1-\pi)\, u\left(\hat{c}^L\right) \\ + & \quad \omega\left[A_1 - q_1^{NB} B_0^B + G_1\right] + \frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1 - p^{NB}\right)\gamma \bar{A}. \end{aligned} \tag{A.55}$$

In these expressions, the loans to banks made by the investors in period 1 cancel with the proceeds from those loans at the interbank loan rate of $r = R$.

Here p^{NB} is the probability that the government repays at the final date conditional upon no buyback being provided:

$$p^{NB} = 1 - F\left(\frac{B_0 + G_1/q_1^{NB}}{\bar{A}}\right). \tag{A.56}$$

Finding the (p^{NB}, q_1^{NB}) that solves (A.56) and the arbitrage condition is the same as finding a solution to (12), other than the optimistic equilibrium. By assumption, there exists a pessimistic solution to (12).

The Bailout Decision

The difference in the value of the social welfare function between buyback and not is:

$$\begin{aligned} \Delta & \equiv W^{PB} - W^{NB} \\ & = \pi\left[u\left(c^{*E}\right) - u\left(\hat{c}^E\right)\right] + (1-\pi)\left[u\left(c^{*L}\right) - u\left(\hat{c}^L\right)\right] - \omega\left(q_1^{\max} - q_1^{NB}\right) B_0^B \\ & + \frac{\omega}{R}\left(p^{PB} - p^{NB}\right)\gamma \bar{A}. \end{aligned} \tag{A.57}$$

where p^{NB} is given by (A.56) and p^{PB} is given by (A.54).

From the constraints of the banking problems,

$$\pi[c^{*E} - \hat{c}^E] + (1-\pi)[\frac{c^{*L} - \hat{c}^L}{R}] = (q_1^{\max} - q_1^{NB}) B_0^B - \psi d. \tag{A.58}$$

Using $\omega = u'(c^{*E}) = Ru'(c^{*L})$, the first term in (A.58) becomes

$$\pi \left[u\left(c^{*E}\right) - u\left(\hat{c}^{E}\right)\right] + (1-\pi)\left[u\left(c^{*L}\right) - u\left(\hat{c}^{L}\right)\right] - u'\left(c^{*E}\right)\left(\pi[c^{*E} - \hat{c}^{E}] + (1-\pi)\left[\frac{c^{*L} - \hat{c}^{L}}{R}\right]\right) \tag{A.59}$$

Due to the presence of deadweight losses from bank failure (ψd) and by the strictly concavity of $u(\cdot)$ this term is positive although it is smaller compared to (A.44) because the fact that the bank had issued some equity makes the consumption allocations under no bailout better compared to the one in Proposition 3. This term represents the net gain to the debt buyback ignoring its impact on debt fragility. The buyback redistributes between depositors and investors in a way that restores the bank to solvency and avoids the deadweight bankruptcy costs. It also removes the risks faced by depositors due to strategic uncertainty in the debt market. This term is larger, the larger the costs of disrupting intermediation ψ.

The second term represents the additional costs of the debt buyback in terms of higher bankruptcy costs. It is proportional to the change in expected bankruptcy costs due to the bailout:

$$\left(p^{PB} - p^{NB}\right)\gamma < 0. \tag{A.60}$$

and clearly negative due to higher probability of sovereign default under the bailout. It is clear that when ψ is sufficiently large and γ is sufficiently small, $\Delta > 0$.

If, as in the second part of the proposition, γ is sufficiently large and ψ is sufficiently small, then, a full bailout will not occur, i.e. $\Delta < 0$.

If markets clear at the presumed prices, then these are two sufficient conditions for $\Delta > 0$ and a sufficient condition for $\Delta < 0$. Step 3 of the proof shows that markets clear at the conjectured prices.

Step 3: Market Clearing

We construct prices such that markets clear. All government debt is held by banks as they receive the benefits of the debt buyback. From Assumption 1, this is feasible because bank deposits are larger than the government debt stock at the initial date.

When the bank knows it will receive a buyback price of $q_1^{\max}$ in the event of pessimism at the middle date and when it knows that the consumption allocations of depositors are independent of the sunspot (due to the combination of equity issuance and government bailout assistance), the bank is risk-neutral in its pricing of the bond. Let ν denote the probability of optimism. Hence at

$$q_0 = \nu \frac{1}{R} + (1-\nu)\, q_1^{\max} \tag{A.61}$$

as assumed in the construction of the equilibrium, banks are indifferent between illiquid investment and the holding of government debt to finance the consumption of late households. Since $q_1^{\max} > q_1^{BB}$ (above market price buybacks), the price (A.61) is above what investors are prepared to pay for government debt. Hence banks hold the entire stock of government debt in period 0 and investors hold all the debt after period 1 trades.

Period 1 government debt prices are $q_1(s^o) = \frac{1}{R}$ and $q_1(s^p) = \frac{p^{PB}}{R}$ where $p^{PB} < 1$ is the probability of repayment under bailout given by (A.54).

From the preferences of the investors, they are willing to lend as much as demanded in the interbank market if $r^b = R$. Thus this market will clear at that price with and without government debt buyback.

∎

7.3.6 Proof of Proposition 6

Proposition 6. *A* ***committed government*** *will choose not to bailout the banks. In the SPNE, banks self-insure through equity issuance and provide the first best contract to households.*

Proof. The proof has three steps: (i) characterizing the optimal banking contract, (ii) determining the government's bailout choice and (iii) checking market clearing.

Step 1: Optimal Contract

Under Assumption 2, sunspot equilibria exist even when the government does not bail out the banks and sovereign debt remains risky. Let $q_1(s^o)$ be the price of government debt under optimism and $q_1(s^p)$ be the price of government debt under pessimism, with $q_1(s^o) > q_1(s^p)$. Using Assumption 2, the outcome under optimism is the no default solution to the debt pricing equation. So $q_1(s^o) = q_1^* = \frac{1}{R}$. $q_1(s^p) = \frac{p}{R}$ where $p < 1$ is the probability of government repayment in the pessimistic equilibrium.

From Proposition 4 we know that, when the bank issues sufficient equity, depositors are fully insured against the uncertaintly in the government debt market and the first best banking contract can be offered.

Step 2: Government's Choice

If the government commits to no bailout, then the first-best contract is provided by the banks and households bear no risk from variability of debt prices. The intermediation process is never interrupted so that the welfare loss from $\psi > 0$ is avoided. There remains a positive probability of default given by:

$$p^{NB} = 1 - F\left(\frac{B_0 + G_1/q_1(s^p)}{\bar{A}}\right). \tag{A.62}$$

With $p^{NB} > 0$, the default cost of γ is borne with positive probability in the last period, conditional on a pessimistic sunspot in the middle period.

If, instead, the government commits to a bailout, it cannot improve upon the first-best allocation. The banks, as shown in the proof of Proposition 3, will continue to offer the first-best contract.

But the first-best allocation will not obtain. Under bailout, the government issues more debt. This will increase the probability of default directly and thus depress the valuation of government debt. Under pessimism, as seen in Figure 2, the outcome will be a lower value of government debt.

The reduction in the value of debt implies the government must raise even more debt to finance G_1 as well as any bailouts.

The probability of default under a bailout becomes

$$p^{BB} = 1 - F\left(\frac{B_0 + (G_1 + T(\widehat{q}_1))/\widehat{q}_1}{\bar{A}}\right). \tag{A.63}$$

where $\widehat{q}_1$ denotes the value of government debt under a bailout in the pessimistic sunspot state, as in Figure 2.

Comparing from (A.63) and (A.62), $p^{NB} > p^{BB}$ for two reasons. First, under bailout, the government issues more debt as $T(\widehat{q}_1) \geq 0$. Second, $\widehat{q}_1 < q_1(s^p)$, as seen in Figure 2. Hence the expected default cost, due to $\gamma > 0$, is higher under a bailout.

Step 3: Market Clearing

The construction of equilibrium prices such that markets clear parallels that in the proof of Proposition 2. In period 1, $q_1(s^0) = \frac{1}{R}$ as in an optimistic sunspot state, default probabilities are zero. Hence at this price, investors are indifferent between consumption in period 1 and purchasing government debt to finance consumption in period 2. This indifference along with the large endowment of investor is sufficient for them to purchase debt from banks and newly issued government debt at this price.

If there is pessimism in period 1, then $q_1(s^p) = \frac{p^{NB}}{R}$ where p^{NB} is the probability of debt repayment given in (A.62). At this price, investors are again willing to hold the excess supply of government debt.

In period 0, investors can either put their endowment directly in the illiquid technology and obtain R or purchase two period government debt. They are indifferent between these options if $q_0 = \frac{\nu+(1-\nu)p^{NB}}{R}$, where ν is the probability of optimism in period 1. If this condition holds, they are willing to purchase any of the government debt not held by the banking system. Thus, at these prices, all markets clear. The stock of government debt held by bank is sold to the investors along with any new debt in period 1. The market for government debt clears in both periods, in all states.

Further, at $r^b = R$, investors are indifferent both with respect to the timing of their consumption and the composition of their portfolio. This indifference guarantees market clearing in the interbank market at zero trades. ∎

7.3.7 Proof of Proposition 7

Proposition 7. *Assume a Diamond-Dybvig style bank. Suppose the government lacks commitment and (i) the default cost, γ, is sufficiently small or (ii) the cost of disrupting the intermediation process, ψ, is sufficiently large and (iii) government debt outstanding is large. If households are not too risk averse, and a full bailout is not feasible, then there exists a SPNE with: (i) debt fragility and (ii) bank runs iff there is pessimism in debt markets.*

Proof. Denote the vector of sunspots as $\overline{s} = \left[s^d, s^b\right]$ where s^d is the sunspot in the deposit market (i.e. a bank run) and s^b is the sunspot in the debt market (i.e. a debt run). Both sunspots take the values of o (for optimism) and p (for pessimism). For example, the price of debt when there is no bank run but there is pessimism in the sovereign debt market is $q_1(o, p)$.

Debt Fragility

The existence of an SPNE with debt fragility and optimism by depositors, i.e. $\overline{s} = [p, o]$, is established in Proposition 3. Anticipating a debt buyback, banks hold the entire stock of government debt:

$$B_0^B = B_0.$$

Bank Run

In this case, there is no debt fragility but pessimism by depositors, i.e. $\overline{s} = [o, p]$. The bank has enough liquidity to meet the demands of all households if its liquid assets are higher than liabilities at the optimistic equilibrium ($q_1(p, o) = \frac{1}{R}$):

$$c^{*E} < \frac{1}{R} B_0. \tag{A.64}$$

From section 7.1, at the optimal contract,

$$u'\left(c^{*E}\right) = Ru'\left(c^{*L}\right).$$

The intertemporal budget constraint of the bank is

$$\pi c^{*E} + \frac{(1-\pi) c^{*E}}{R} = d$$

From these conditions, c^{*E} will be low when the households are not too risk averse. When B_0 is large enough, the bank is liquid enough so that (A.64) holds and no bank run exists in isolation.

Debt Fragility and Bank Runs

In this case, there is debt fragility and pessimism by depositors, i.e. $\overline{s} = [p, p]$. The bailout required to save the banks is:

$$T(q_1(p, p)) = c^{*E} - q_1(p, p) B_0. \tag{A.65}$$

This includes the transfer required to provide banks with liquid funds against the run ($c^{*E} - \frac{1}{R} B_0$) as well as an additional transfer required to compensate them for losses on government debt holdings. Here $q_1(p, p)$ is the price of government debt.

A full bailout is feasible if there exists a $q_1(p, p)$ solving (13) with total transfers of $T(q_1(p, p))$ given by (A.65). If it is feasible for the government to provide a full bailout, the government will do so under the conditions of the proposition of a small γ and a large ψ. In this case, there is no bank run.

Else, a full bailout is not feasible, the bank is insolvent and enters the bank resolution mechanism which restructures the bank. In this case, under the bank resolution mechanism with partial bailout, a bank run and debt fragility will coexist.

The bank resolution mechanism solves the following problem:

$$\max_{c^E, c^L(\mathbb{1}_G), q_1^{PB}<q_1^*, B_1^B \geq 0, L_1 \leq 0} \pi u\left(c^E\right) + (1-\pi)\left[p^{PB} u\left(c^L(\mathbb{1}_G=0)\right) + \left(1-p^{PB}\right) u\left(c^L(\mathbb{1}_G=1)\right)\right]$$
$$+\omega\left[A_1 - q_1^{PB} B_0^B + G_1\right] + \frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1-p^{PB}\right)\gamma\bar{A} \tag{A.66}$$
$$+\lambda^E\left(q_1^{PB}\left(B_0^B - B_1^B\right) - \psi d + L_1 - \pi c^E\right)$$
$$+\lambda^L\left(\mathbb{1}_G=0\right)\left(R\left(i_0 - L_1\right) + B_1^B - (1-\pi)c^L(\mathbb{1}_G=0)\right)$$
$$+\lambda^L\left(\mathbb{1}_G=1\right)\left(R\left(i_0 - L_1\right) - (1-\pi)c^L(\mathbb{1}_G=1)\right)$$

where q_1^{PB} is the price at which the government buys debt from the bank within the bank resolution mechanism. The main difference between (A.17) and (A.66) is the fact that the buyback price q_1^{PB} is optimally chosen during the restructuring process and the probability of government default p^{PB} endogenously reflects the fiscal cost of bailout.

As long as the bank's bonds are sold to investors ($B_1^B = 0$) it can make the allocation to late investors independent of whether the government defaults or not ($\hat{c}^L\left(\mathbb{1}_G=0\right) = \hat{c}^L\left(\mathbb{1}_G=1\right) \equiv \hat{c}^L$). Since consumers are risk-averse, this is optimal. It is also feasible because the government can lend the proceeds of the bond sale to investors via L_1 in order to choose optimally the allocations to the early and late consumers conditional upon its pessimism asset value.

The first order conditions generate an allocation $\hat{c}^E < c^{*E}$ and $\hat{c}^L < c^{*L}$ to depositors such that $u'(\hat{c}^E) = Ru'(\hat{c}^L)$. The consumers receive less than promised under optimism because the value of the bank's bonds are below the price under optimism, i.e. $q_1(p,p) < q_1^* = 1/R$, and because the value of the bank's assets have been diminished by the bankruptcy cost ψd. However depositors are shielded from the risk of government default and hence, in what follows we drop the dependence of the government's default, $\mathbb{1}_G$.

Building on Ennis and Keister (2009) and Cooper and Kempf (2013), a necessary condition for the existence of a bank run is that late households obtain lower consumption than early households who are able to withdraw prior to the bank failing:

$$\beta c^{*E} > \hat{c}^L.$$

From the period 1 resource constraint in (A.66), this holds when ψ (the resource cost of bank default) is sufficiently large as stipulated in the proposition.

The value of government debt, $q_1(p,p)$, is determined jointly by (13) with total transfers given by $T(q_1(p,p)) = B_0(q_1^{PB} - q_1(p,p))$ The probability of default under partial bailout is $p^{PB} = 1 - F(\frac{B_0 + (T(q_1(p,p); q_1^{PB}, B_0)/q_1(p,p)}{\bar{A}})$. As the government chooses the debt buyback price, it internalizes the effects on both the probability of default and the equilibrium price of debt in period 1. The existence of a solution to (13) comes from the choice of q_1^{PB} by the government.

The result that $\beta c^{*E} > \hat{c}^L$ implies there will be a bank run when there is pessimism in debt markets. The existence of a pessimistic equilibrium was established in the proof of the first part of this proposition. ∎

References

ACHARYA, V. V., I. DRECHSLER, AND P. SCHNABL (2014): "A pyrrhic victory?-bank bailouts and sovereign credit risk," Journal of Finance.

ADMATI, A., AND M. HELLWIG (2013): Bankers' New Clothes: What is Wrong with Banking and What to do about it? Princeton University Press.

BATTISTINI, N., M. PAGANO, AND S. SIMONELLI (2014): "Systemic Risk, Sovereign Yields and Bank Exposures in the Euro Crisis," Economic Policy.

BOLTON, P., AND O. JEANNE (2011): "Sovereign Default Risk and Bank Fragility in Financial Integrated Economies," IMF Economic Review, 59(2), 162–194.

BRONER, F., A. ERCE, A. MARTIN, AND J. VENTURA (2014): "Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-Out Effects," Journal of Monetary Economics.

CALVO, G. (1988): "Servicing the public debt: The role of expectations," The American Economic Review, pp. 647–661.

CARLSSON, H., AND E. VAN DAMME (1993): "Global games and equilibrium selection," Econometrica.

COLE, H., AND T. KEHOE (2000): "Self-Fulfilling Debt Crises," Review of Economic Studies, 67(1), 91–116.

COOPER, R. (2012): "Fragile Debt and the Credible Sharing of Strategic Uncertainty," NBER Working Paper # 18377.

COOPER, R., AND H. KEMPF (2013): "Deposit Insurance and Orderly Liquidation without Commitment: Can we Sleep Well?," Working Paper 19132, National Bureau of Economic Research.

CORSETTI, G., AND L. DEDOLA (2012): "Fiscal Crises, Confidence and Default A Bare-bones Model with Lessons for the Euro Area," mimeo.

DE GRAUWE, P., AND Y. JI (2012): "Mispricing of Sovereign Risk and Multiple Equilibria in the Eurozone," mimeo.

DIAMOND, D., AND P. DYBVIG (1983): "Bank Runs, Deposit Insurance and Liquidity," Journal of Political Economy, 91, 401–19.

EATON, J., AND M. GERSOWITZ (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," Review of Economic Studies, 42(2), 289–309.

Ennis, H., and T. Keister (2009): "Bank runs and institutions: the perils of intervention," The American Economic Review, 99(4), 1588–1607.

Farhi, E., and J. Tirole (2014): "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops," mimeo.

Gennaioli, N., A. Martin, and S. Rossi (2013): "Sovereign Default, Domestic Banks and Financial Institutions," Journal of Finance.

——— (2014): "Banks, Government Bonds, and Default: What do the Data Say?," mimeo.

Hannoun, H. (2011): "Financial crises: the role of deposit insurance," Presentation, Bank for International Settlements.

Leonello, A. (2014): "Government Guarantees and the Two-Way Feedback between Banking and Sovereign Debt Crises," mimeo.

Lucas, A., B. Schwaab, and X. Zhang (2014): "Conditional Euro Area Sovereign Default Risk," Journal of Business and Economic Statistics.

Morris, S., and H. Shin (1998): "Unique equilibrium in a model of self-fulfillig currency attacks," American Economic Review.

Ongena, S., A. Popov, and N. Van Horen (2015): "Is there moral suasion in banking?," mimeo.

Pagano, M. (2014): "Lessons from the European Financial Crisis," Centre for Studies in Economics and Finance Working Paper No. 370.

Popov, A., and N. Van Horen (2015): "Exporting Sovereign Stress: Evidence from Syndicated Bank Lending During the Euro Area Sovereign Debt Crisis," Review of Finance.

Roch, F., and H. Uhlig (2012): "The Dynamics of Sovereign Debt Crises and Bailouts," `http://www.ecb.europa.eu/events/conferences/shared/pdf/researchforum7/Uhlig_Harald.pdf?090aa93a65cfbc331785bf7f09c79737`, mimeo.

Uhlig, H. (2013): "Sovereign Default and Banks in a Monetary Union," NBER Working Paper #19343.

Vives, X. (2014): "Strategic Complementarity, Fragility and Regulation," Review of Financial Studies.