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GOVERNMENT DEBT AND BANKING FRAGILITY: THE SPREADING OF STRATEGIC UNCERTAINTY

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ABSTRACT

This paper studies the interaction of government debt and interbank markets. Both markets are known to be fragile: excessively responsive to fundamentals and prone to strategic uncertainty. The goal is to understand the channels that link these markets and to evaluate policy measures for their stability.

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August 5, 2013

Abstract

This paper studies the interaction of government debt and interbank markets. Both markets are known to be fragile: excessively responsive to fundamentals and prone to strategic uncertainty. The goal is to understand the channels that link these markets and to evaluate policy measures for their stability.

1 Introduction

> We must break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks. This works both ways. Making banks stronger, including by restoring adequate capital levels, stops banks from hurting sovereigns through higher debt or contingent liabilities. And restoring confidence in sovereign debt helps banks, which are important holders of such debt and typically benefit from explicit or implicit guarantees from sovereigns. (Christine Lagarde, April 17, 2012)

Following the Greek sovereign debt write-down in 2011, the four largest Greek banks made losses of more than 28 billion euros (or 13% of GDP).[1] This was enough to wipe out almost all of their combined equity capital. In 2010, the Irish government ran an unprecedented peace-time of deficit,

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[1] National Bank of Greece, Alpha Bank, Pireus and Eurobank.

reaching 32% of GDP as it bailed out its banking system. Under the weight of nationalized banks' losses, Ireland was forced to seek financial support from the IMF and the EU in November 2010.

These are two recent examples of a 'diabolic loop' between banks and sovereigns. In the case of Greece, banks that were otherwise solvent, were made insolvent by the default of their sovereign whose debt they were holding.[2] In the case of Ireland, a government which had previously had one of the lowest levels of debt to GDP in Europe, suffered a withdrawal of funding as markets became concerned about the contingent liabilities involved in bailing out its large, insolvent banking system. Throughout the rest of southern Europe, this 'diabolic loop' has operated in a less dramatic fashion but has nevertheless contributed to ongoing strains in sovereign and bank debt markets.

In this paper we build a model of the channels that transmit fragility between debt and banking markets and evaluate policy measures for the stabilization of these markets. The framework combines the canonical model of sovereign debt fragility (Calvo (1988)) with the canonical model of banking instability (Diamond and Dybvig (1983)). Put differently, the framework studies the interaction of strategic complementarities in debt and financial markets.

Sovereign debt fragility arises due to a strategic complementarity between the buyers of government bonds as in Calvo (1988). Since the government's ability to repay debt depends inversely on the real interest rate it has to pay, this opens up the possibility of self-fulfilling pessimistic equilibria in which the high interest rate needed to compensate bond holders for high expected default risk weakens the government's solvency and validates the pessimistic default expectations. Banks face liquidity and solvency risks as they provide liquidity insurance to their depositors while holding risky assets such as government debt. The collapse of intermediation (either because of 'runs' or fundamental shocks to solvency) then leads to large output and welfare costs to the real economy.

The key contribution of our paper is to examine the interactions between these two sources of financial fragility. Motivated from the European experience, we consider two channels whose interactions complete the 'diabolic loop'.

The first is the strong tendency by banks to hold (their own) government debt both as a long-term investment and as a source of liquidity. Table 1 shows data on European banks' government debt holdings which was released as part of the EBA stress test conducted in 2011.[3] The table focuses on the so-called 'peripheral Eurozone countries' whose debt had come under pressure during the sovereign debt crisis which began in 2010.

Two things are immediately apparent from the table. First of all, the exposure of southern European banks to EEA sovereign debt is very high.[4] The average GIIPS, (Greece, Ireland, Italy,

[2] The term 'diabolic loop' was evidently coined by Markus Brunnermeier in a presentation on the Euro Crisis at the July 2012 NBER Summer Institute.

[3] European Banking Authority.

[4] European Economic Area. This includes the 27 EU countries as well as Norway and Switzerland.

Portugal and Spain), bank holds 15.8% of risk-weighted assets in EEA government securities. The second fact highlighted in the table is that banks are heavily invested in the debt of their own government.

Table 1: European Holding of Sovereign Debt

	All GIIPS	Greece	Spain	Ireland	Italy	Portugal
EEA30 government debt	15.8%	36.2%	11.8%	10.6%	17.5%	11.7%
of which domestic government debt	14.5%	25.9%	11.2%	6.8%	15.5%	8.9%

European banks' holdings of EEA and domestic government debt as a percentage of total risk weighted assets. Source: 2011 EBA Stress Test.

The second channel arises due to the explicit (via deposit insurance) or implicit guarantees that governments provide to their banking systems. One of the contributions of the paper is to provide conditions for governments to provide guarantees.

The interactions of these two elements create an economic environment where pessimism in debt markets is transmitted to the banking system and amplified due to powerful feedback effects on the financial health of the sovereign. When the government debt market switches to a pessimistic (high interest rate, high default risk) equilibrium, government bond prices fall and the banks holding the bonds suffer losses. At this point (due to the high output costs of bank defaults), governments are forced to intervene and bail their banks out, further increasing government debt at precisely the point when high interest rates are making repayment difficult. The result is a further decline in government debt prices, leaving a deeper hole in bank balance sheets and requiring a larger bailout. This is the 'diabolic loop' between government debt and the banking system.

In this paper we study the policy options of the crisis-prone country in isolation of other members of a currency or economic union.[5] We consider two ways policy-makers and private agents (in Christine Lagarde's words) 'break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks'.

On the banking side, equity cushions can break the adverse feedbacks between banks and sovereigns. Banks that hold adequate capital against potential sovereign risks become completely insulated from developments in debt markets, severing a key channel of crisis transmission from governments to the banking system. However, we show that, when banks under-estimate the impact of the sovereign debt crisis and/or expect deposit insurance to be provided *ex post*, the incentive

[5] In future work we intend to pursue a fuller analysis of the policy options of a coalition of countries such as the European Union.

for them to self-insure by building up equity buffers against losses disappears.

On the sovereign side, we examine a key policy which affects the 'diabolic loop' - the *ex post* choice of whether to provide bailout assistance to the banking system during a crisis. We argue that in the likely case when the collapse of the financial system is very costly for the real economy, governments always provide DI *ex post*, thus removing the need for banks to self-insure *ex ante* by issuing equity.

The paper is structured as follows. Section 2 outlines the baseline model, section 3 describes the optimistic equilibrium while section 4 describes the pessimistic equilibrium and the sovereign-banking feedbacks that it triggers. Section 5 examines the government's decision on whether to provide deposit insurance *ex post*. Section 6 examines the way equity buffers can achieve the first best and discusses banks' incentives for equity issuance. Section 7 concludes.

2 Framework

Time lasts for three periods: 0, 1 and 2. The model has two principal components. The first is a banking relationship between intermediaries and depositors, following Diamond and Dybvig (1983). The second component is the pricing of government debt, following Calvo (1988) and others.[6] The debt is partially held by banks.

The intermediation process and pricing of government debt are linked in a couple of ways. First, the value of the government debt held by the banks affects their solvency. Second, the potential and realized needs to bailout the financial sector influences the value of government debt. Third, banking problems affect the real economy and impact on the size of the tax base, thus adding a further valuation effect on government debt. These interactions can be activated by either fundamental shocks or self-fulfilling expectations influencing the value of government debt.

There are four types of agents: households, banks, investors and the government. We discuss the choices and objectives of these agents and then characterize the equilibria.

2.1 Households

Households have an endowment of goods d at $t = 0$ with preferences

$$V_0^H = \pi u\left(c_1 + \beta c_2\right) + \left(1 - \pi\right) u\left(\beta c_1 + c_2\right).$$

Here β is close to 0. With probability π they are *early* consumers who prefer consuming at $t = 1$ and with probability $1 - \pi$ they are *late* consumers who prefer consuming at $t = 2$. The shares of

[6]There are now a number of papers building on Calvo (1988), including Cole and Kehoe (2000) and, more recently, Corsetti and Dedola (2012), Roch and Uhlig (2012) and Cooper (2012).

early consumers at the aggregate level is fixed at π. We assume $u(\cdot)$ is strictly increasing, strictly concave and $u(0)$ is finite.

2.2 Banks

Following Diamond and Dybvig (1983), consumers can share liquidity risk through the banking system. Banks construct a portfolio for households which provides the needed liquidity while still taking advantage of longer term investment activities. As is well understood, it is this interaction of liquidity needs and illiquid investment that can lead to fragility in the banking system.

Banks are competitive. They raise deposits d from households in period 0. Banks invest in two types of assets in period 0. They can buy government bonds b_0 at price q_0. These bonds do not pay a coupon at the middle date but can be traded in the secondary market. Second the banks can make long term investments i_0 that return $R > 1$. These investments have a liquidation value at the middle date of $0 \leq \varepsilon \leq 1$.

Ex ante banks offer contracts to consumers. The contract specifies the level of early, denoted c^E, and late consumption, denoted c^L. Decisions on holding of government securities (b_0) and long term investment (i_0) are also made *ex ante*. Banks can adjust their portfolios in the middle period. The optimal contract solves:

$$\max_{c^E, c^L, b_0, i_0, b_1, l_1, L_1} \pi u\left(c^E\right) + (1-\pi)\, u\left(c^L\right) \tag{1}$$

such that

$$i_0 + q_0 b_0 \leq d \tag{2}$$

$$\pi c^E \leq q_1 \left(b_0 - b_1\right) + \varepsilon l_1 - L_1 \tag{3}$$

$$(1-\pi)\, c^L \leq b_1 + R\left(i_0 - l_1\right)) + r^b L_1. \tag{4}$$

From (3), the funding for the payment to the early households comes from three sources. First, the bank can sell some of the government debt it acquired in period 0 to the investors to obtain goods for early consumers. Second, the bank could liquidate some of the illiquid investment, denoted l_1 in (3). The liquidation of the illiquid technology is equivalent to having access to a storage technology with a return of ε between period 0 and 1. Finally, the bank could extend loans to investors or other banks, denoted L_1 in (3), at a rate r^b. We refer to this as a loan in the interbank market.

From (4), the consumption of late households is financed by the bonds held until the last period as well as the return on the illiquid investment that was not liquidated in the middle period. Further, the bank has the returns to investor loans (L_1) made at the middle date.

2.3 Investors

Investors are risk neutral agents with endowments in periods t of A_t for $t = 0, 1, 2$. These agents consume in periods 1 and 2 with preferences given by $c_1 + \frac{c_2}{R}$. The assumption that investors discount at $\frac{1}{R}$ will determine the asset returns in equilibrium. These agents invest their endowments in government debt and lend to/borrow from banks (L_1^I).[7] They can also directly invest in the illiquid technology.

In the first period, investors allocate their endowment to the purchase of government debt, and illiquid investments, respectively:

$$A_0 = q_0 b_0^I + i_0^I. \tag{5}$$

The budget constraint in period 1 is:

$$c_1^I = A_1 + q_1[b_0^I - b_1^I] + L_1^I \tag{6}$$

as the investor can purchase government debt of b_1^I and borrow from banks. The budget constraint in period 2 is:

$$c_2^I = (1 - \tau)\, A_2 + b_1^I + R i_0^I - r^b L_1^I \tag{7}$$

where τ is the tax rate on investor's endowment. In period 2, the endowment of the investor is augmented by the returns to bond holdings and the long term investments minus the repayments on bank loans.

The investors' endowment at the final date, A_2, serves as the tax base for debt service. Its value depends on the operations of the intermediation process as well as the default choice of the government.

The dependence of A_2 on the intermediation process captures the disruptive effects of a breakdown in the financial system. For one specific model of this, see Gennaioli, Martin, and Rossi (2013). In our model, the disruption has the effect of reducing the endowment of the investors and hence the tax base. The output loss is parameterized by ψ in (8) where $\mathbb{1}\{B\} = 1$ if the intermediation process breaks down.

In addition, following Eaton and Gersowitz (1981) government default leads to output costs. This is reflected in the reduction in the $(1 - \gamma \mathbb{1}\{G\})$ term in the investors' endowment where $\mathbb{1}\{G\} = 1$ if the government defaults.

Specifically, the investor's endowment in the last period is given by:

$$A_2 = \bar{A}(1 - \psi \mathbb{1}\{B\})(1 - \gamma \mathbb{1}\{G\}). \tag{8}$$

[7] We introduce equity later in the analysis.

The parameters (ψ, γ) will be important for determining the default choice as well as the government's decisions on protecting the intermediation process.

2.4 The Government

The government issues debt B_0 at price q_0 in period 0 to fund government expenditure G_0. This is two-period debt with repayment due in period 2. At the middle date, it issues additional debt at a price q_1 to fund spending of G_1.[8] The total debt stock at the start of period 2 is therefore given by:

$$B_1 = B_0 + G_1/q_1.$$

For this analysis, the level of spending is taken as given. Thus the amount of debt issued in period 1 responds to the price of debt q_1. In particular, if the price of debt is lower, the government must sell more debt in period 1 to fund its fixed expenditure of G_1.

The government taxes investors' endowments A_2 at the final date. If there is no default, the tax rate required to meet the total obligations of the government is equal to:

$$\tau = \frac{B_1}{A_2}.$$

By taxing investors' endowments, the government taxation does not directly impact the intermediation process. Any frictions that impinge on the deposit contract, such as sequential service, are irrelevant for the government's ability to collect taxes. However, the tax base does depend on the functioning of the intermediation process, as in (8).

To introduce the possibility of default into the analysis, assume the government's capacity to tax the endowment of the investors is random and drawn from a known probability distribution $F(\overline{\tau})$ with associated density $f(\overline{\tau})$. The uncertainty about tax capacity, denoted $\overline{\tau}$, is realized at the final date. This naturally leads to the possibility of default due to bad fundamentals (as opposed to strategic default): a low realization of $\overline{\tau}$ could trigger government insolvency despite a healthy tax base (A_2).

If $\overline{\tau} < \frac{B_1}{A_2}$, the government must default on its obligations where $A_2 = \bar{A}(1 - \psi \mathbb{1}\{B\})$. The probability of default is therefore equal to $F\left(\frac{B_1}{A_2}\right)$ while the probability of repayment is given by $1 - F\left(\frac{B_1}{A_2}\right)$. Once the government is forced to default, it defaults fully.

The debt is priced by risk neutral investors. Denote by r the one-period return on the outside opportunity for lenders. For now, r is arbitrary, later it is determined in equilibrium. In order for

[8] As the analysis progresses, the debt issued in period 1 will also finance transfers to the banking system.

the investors to hold the government debt in period 1, the price q_1 must satisfy

$$\frac{1-F\left(\frac{B_1}{A_2}\right)}{q_1}=r.$$

Using the government budget constraint as well as the expression for the tax base (8), this can be written

$$1-F\left(\frac{B_0+G_1/q_1}{\bar{A}\left(1-\psi\mathbb{1}\{B\}\right)}\right)=rq_1. \tag{9}$$

Looking at the government sector alone, any value of q_1 that satisfies (9) is part of a rational expectations equilibrium. Government debt is fragile in the sense that it is subject to multiple equilibria. That is, there may be multiple solutions to (9).

This reflects an underlying strategic complementarity in debt markets. If investors are pessimistic about the government repaying its debt, then q_1 will be lower. This increases the amount of period 1 debt the government must issue, thus increasing $\frac{B_1}{A_2}$. This makes default more likely.

Figure 1 illustrates the equilibria.[9] The function $[1-F(\frac{B_0+G_1/q_1}{\bar{A}(1-\psi\mathbb{1}\{B\})}]/r$ is the 'debt valuation equation' because it determines the price of government debt (as a function of itself). It is depicted as the black dashed curve. The points of intersection of this curve and the 45-degree line are solutions to (9).

There is an 'optimistic equilibrium' in Figure 1 where the default probability is zero. That is, $F\left(\frac{B_1}{A_2}\right)=0$. In addition, there is are other equilibria in which the value of debt, q_1 is lower. The resulting higher debt obligation in period 2 generates a positive probability of default.

This paper focuses on the optimistic equilibrium as well as pessimistic equilibria, with positive default probabilities. Throughout we study pessimistic equilibria which are locally stable and hence satisfy:

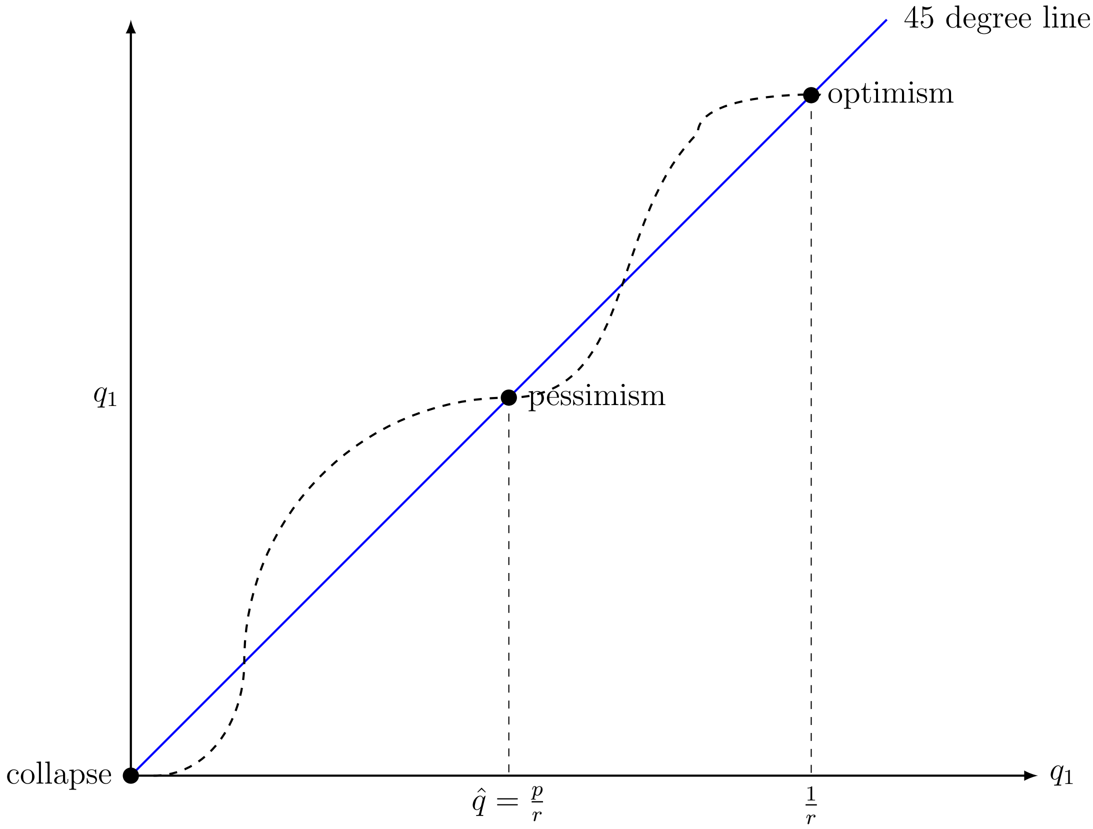
[9] We have drawn the debt valuation equation in Figure 1 with zero slope at the optimistic equilibrium and at the equilibrium with certain default, labelled 'collapse' on the graph. The slope of the debt valuation equation is given by $-\frac{G_1}{A_2}\frac{1}{q_1^2}f\left(\frac{B_0+G_1/q_1}{A_2}\right)$ where $f(\cdot)$ is the density associated with the distribution function $F(\cdot)$. This expression is zero at high levels of q_1 (the optimistic equilibrium) whenever the density of the tax capacity τ is zero at $\tau=\frac{B_0+G_1/q_1}{A_2}$ evaluated at the optimistic value of q_1. In other words, when the economy is in an optimistic equilibrium, the probability that tax capacity turns out to be insufficient is so small that small variations in bond prices make no difference to the government's probability of default.

At the collapse equilibrium with zero bond prices and certain default, the intuition for the flat debt price schedule is similar. When q_1 is close to zero, the slope of the debt valuation equation is also zero as long as the density $f(\cdot)$ goes to zero faster than q_1^{-2} goes to infinity. This property is satisfied for all distributions which have very little mass far out in the tails such as the normal distribution. Once prices go very low, τ needs to be so high to ensure repayment that this is a zero probability event. When the density of the tax capacity $f(\cdot)$ is zero at this point, small increases in debt prices make no difference to the probability that the government repays.

$$f\left(\frac{B_0 + G_1/q_1}{A_2}\right) < \frac{A_2 R q_1^2}{G_1} \tag{10}$$

for q_1 in the set of solutions to (9). This condition of local stability has implications for comparative statics. At a locally stable equilibrium an increase in the fiscal burden reduces the probability of government debt repayment. Equation (10) is a restriction that the probability of debt repayment is not falling too fast with the debt burden in the neighborhood of the locally stable pessimistic equilibrium.

Figure 1: Fiscal Fragility



This is not a complete characterization of equilibria. In addition to these valuation equations and the optimal deposit contract, the markets for government bonds and for interbank loans must clear. Further, the outside option of investors will be determined in equilibrium.

3 Optimistic Equilibrium

A rational expectations equilibrium has two components. One is the optimal contract between a bank and its depositors. The second is the valuation of the government debt. These components are linked, in part, because the contractants in the banking system hold government debt and take asset prices as given.

This section characterizes a particular equilibrium of the model in which there is no default. To do so, we make two provisional assumptions.

First, we assume that there exists a solution to (9) in which there is no default. That is, if lenders believe that the default probability is zero, the debt burden of the government will always be below its minimal tax capacity. Hence we call this the “optimistic equilibrium”. In an optimistic equilibrium, there is no default so that $A_2 = \bar{A}$ and $F(\frac{B_1}{\bar{A}}) = 0$.

Second, the banking contract and lending behavior assumes that the optimistic equilibrium will occur with certainty in period 1. That is, neither lenders nor those party to the banking contract recognize the underlying strategic uncertainty.

Given these assumptions, we construct an equilibrium in which $q_0 = q_1 = \frac{1}{R}$ and $r^b = R$. We solve the banking problem given these prices, show that markets clear and that (9) holds at $r = R$.

3.1 Optimal Contract

Given debt prices q_0 and q_1, the optimal contract for the banking system solves (1) subject to the constraints as described in section 2.2. This problem generates a demand for government debt by the banking system.

Given the prices for debt and the interbank lending rate, the banks choose a contract to maximize the expected utility of depositors, given in (1). Under the assumption of optimism, neither the banks nor the depositors anticipate variations in the price of government debt.[10]

The banks hold a portfolio of government debt and long-term illiquid investment. They provide for the consumption of early households by selling government debt to investors in period 1. When the liquidation value of the illiquid investment, ε, is less than one, trading government debt strictly dominates liquidating the long-term investment. At $\varepsilon = 1$, the bank is indifferent between liquidation and the selling of government debt and we assume there is no liquidation in this case either.

Proposition 1. *In the optimal banking contract with $q_0 = q_1 = \frac{1}{R}$: (i) $c^L > c^E$ and (ii) $l_1 = 0$.*

[10] We will relax this assumption in subsequent analysis.

Proof. The first order conditions to the contracting problem in (1) are:

$$u'\left(c^E\right) - \lambda^E = 0 \tag{11}$$

$$u'\left(c^L\right) - \lambda^L = 0 \tag{12}$$

$$q_0\phi = q_1\lambda^E \tag{13}$$

$$\phi = R\lambda^L \tag{14}$$

$$(\varepsilon\lambda^E - R\lambda^L)l_1 = 0. \tag{15}$$

Combining the first four conditions and using $q_0 = q_1 = \frac{1}{R}$:

$$u'\left(c^E\right) - Ru'\left(c^L\right) = 0. \tag{16}$$

This condition implies property (i): $c^L > c^E$ for all $R > 1$ as $u(\cdot)$ is strictly concave. Using (15), l_1 is zero, and strictly so if $\varepsilon < 1$, as $\lambda^E = R\lambda^L$ from (13) and (14). □

In the subsequent discussion, let (c^{*E}, c^{*L}) denote the optimal contract characterized in Proposition 1. We will refer to this as the first best contract. The property that $c^{*L} > c^{*E}$ implies that depositors have an incentive to reveal their true taste types.[11]

From (1), there are other elements of the bank's problem to determine: (b_0, i_0, b_1, L_1). To implement the optimal contract, it is sufficient that $(b_0 = \frac{\pi c^{*E}}{q_1}, i_0 = \frac{(1-\pi)c^{*L}}{R})$ and $(b_1 = L_1 = 0)$. That is, trades in period 1 are not needed in the case of optimism, though they can be important under pessimism.

3.2 Equilibrium

Given the banking contract, the last step in constructing an equilibrium is to guarantee market clearing. There are three markets to consider: (i) the period 0 market for government debt, (ii) the period 1 market for government debt and (iii) the interbank loan market.

Proposition 2. *There exists an optimistic rational expectations equilibrium with $q_0^* = q_1^* = \frac{1}{R}$, $r^b = r = R$ and the banking contract given by (c^{*E}, c^{*L}).*

Proof. The equilibrium conditions are driven by the investors. We assume that the aggregate endowment of the investors is larger than the stock of government debt in period 0. The investors can either put their endowment directly in the illiquid technology and obtain R or purchase two

[11]As is well understood, there may also exist a bank runs equilibrium in this environment. That is not the focus of this analysis and is left aside to focus on crises emanating from uncertainty over government debt repayment.

period government debt. They are indifferent between these options if $q_0 = \frac{1}{R}$. If this condition holds, they are willing to purchase any of the government debt not held by the banking system. Since investors have linear utility of $c_1 + \frac{1}{R}c_2$, investors are indifferent between consuming their period 1 endowment and buying one period government debt if $q_1 = \frac{1}{R}$. Assuming that investors period 1 endowment is sufficiently large, if $q_1 = \frac{1}{R}$, the investors will purchase the debt sold by the banks in period 1 and the new debt issued by the government in period 1. Thus $r = R$ in (9).

Thus, at these prices, all markets clear. The excess supply of government debt in period 0 is purchased by the investors. The stock of government debt held by bank is sold to the investors along with any new debt in period 1. The market for government debt clears in both periods. Given that $q_0^* = q_1^* = \frac{1}{R}$, the probability the government will default is zero.

Further, at $r^b = R$, investors are indifferent both with respect to the timing of their consumption and the composition of their portfolio. This indifference guarantees market clearing in the interbank market at zero trades.

The fact that the first best contract is provided in equilibrium comes from Proposition 1. In equilibrium, banks will hold enough debt to finance their payment to early consumers at the anticipated period 1 price: $b_0 q_1 = \pi c^{*E}$. The debt is sold to the investors for goods and those goods are transferred to the early consumers. There are no liquidations in an optimistic equilibrium. □

4 Pessimism

The optimistic outcome is a rational expectations equilibrium. If investors are optimistic about the repayment of government debt, then the strategic uncertainty within the banking and fiscal systems is avoided. But, there is another outcome: a fiscal crises that impacts the banking system.

4.1 Fiscal Crises

This section outlines the impact of an unexpected drop in confidence in the value of government debt. The discussion highlights the adverse spillover effects from strategic uncertainty regarding government debt on the banking system. To the extent that households and banks ignored the prospect of the pessimism, the effects identified here may be excessive. We consider various remedies to the banking contract once the possibility of pessimism is recognized in the next section.

To see the impact of pessimism, assume that at $t = 0$ the equilibrium probability of repayment is considered to be unity. That is, the strategic uncertainty, i.e. the potential sunspot, influencing government debt is not considered by private agents in the pricing of government debt and the operation of the intermediaries.

Then, at the middle date, an unanticipated debt sunspot shock realizes, increasing the probability of government default. Referring to Figure 1, imagine the shift in beliefs puts the government debt market at the point labeled 'pessimism'. The price of government debt satisfies:

$$\frac{p}{\widehat{q}_1} = R \tag{17}$$

where 'hat' variables denote realizations conditional on being in a pessimistic equilibrium and $p < 1$ is the repayment probability.[12] Since $p < 1$ under pessimism, $\widehat{q}_1 < q_1^*$.

While the strategic uncertainty originates in the government debt market, it affects financial markets. The debt fragility leads to a collapse in the banking system: with the reduced value of government debt, banks are unable to meet their obligations to early and late households.

Proposition 3. *If $\varepsilon < 1$, then the pessimistic debt sunspot triggers bank insolvency and hence the collapse of the banking system.*

Proof. Consider a bank at the middle date. In the optimistic equilibrium, it was selling debt b_0 at price q_1^* to meet the needs to early consumers: $b_0 q_1^* = \pi c^{*E}$. The consumption of late consumers was financed by the return on the illiquid technology: $i_0 R = (1-\pi)c^{*L}$. With $\widehat{q}_1 < q_1^*$, the bank is unable to meet needs of early households out of its debt holdings. Either it must default on its obligations to the early households or it must liquidate enough of the illiquid investment so that $b_0\widehat{q}_1 + \varepsilon l_1 = \pi c^{*E}$. As $\varepsilon < 1$, the bank will be unable to meet the demands of both early and late households regardless of whether the government defaults or not. Consequently, the bank is insolvent.

Depositors at the bank realize that the fall in debt prices has made the bank insolvent. This leads to a run on the bank. The bank liquidates its illiquid investment and uses the resources to meet a fraction of the withdrawals of c^{*E}. □

If $\varepsilon = 1$ and *ex post* the government does not default on its debt, then the bank will have enough resources to pay late households. That is, if the bank does not sell any of its debt in period 1, then $l_1 = \pi c^{*E}$ implying that the late households will get $R(i_0 - l_1) + b_0 = R(i_0 - \pi c^{*E}) + b_0$ which does equal Ri_0 and thus $(d - \pi c^{*E})$ if the government does not default.

The collapse of the banking system in the pessimistic equilibrium is in stark contrast to its smooth functioning in the optimistic equilibrium. The unanticipated wealth shock which hits banks due to the switch to the pessimistic equilibrium renders the banking system insolvent.

[12] At this point, which locally stable pessimistic equilibrium is selected is not important.

4.2 Interactions from Government Intervention

So far we have studied the sovereign-banking linkages without considering any forms of *ex post* government intervention. In practice, governments provide a financial safety net to their banks in the form of Deposit Insurance (DI) as well as various other forms of government assistance. Government intervention to stabilize the banking system enriches the interaction between these sectors of the economy.

Pessimism can be magnified and propagated through the economy by government reactions through two programs: (i) the provision of deposit insurance and (ii) debt buy-backs. The ability of DI to stop bank runs and achieve the first best has been studied by Diamond and Dybvig (1983). In this section we assume that these *ex post* interventions do take place and study their implications for the nature of the sovereign-banking interactions. We subsequently derive conditions under which it is optimal for the government to provide DI *ex post*.

Suppose that the government provides deposit insurance so that banks are able to meet the liquidity needs of all households claiming to be early consumers without liquidating any of the illiquid investment. The government finances deposit insurance by selling more debt to the investors, just as it did to finance government spending in period 1. Will this be enough to avoid a shutdown of the banking system?

If the banking contract offers each early consumer c^{*E}, then the maximal amount of deposit insurance the government provides is

$$DI = \pi c^{*E} - \hat{q}_1 b_0. \tag{18}$$

Here $\hat{q}_1$ is the value of government debt in period 1.

Under local stability, the added debt burden from DI provision implies that q_1 will be even lower than initially.[13] At this point the strategic complementarity outlined above comes into play. The additional reduction in the value of the government debt implies that the bank again cannot meet the demands of the early consumers without liquidating some of the illiquid investment. Either the banking system is insolvent or the government must provide additional resources to the banking system in order to sustain it. We detail how this is determined in equilibrium below.

An alternative way to support the banking system is through debt buybacks. In the event of pessimism, the government purchases its own bonds from banks at the optimistic price q_1^*. This insulates the assets of banks from the fiscal crisis, leaving them with adequate resources to meet the demands of early households.

[13] The additional debt burden shifts down the debt valuation equation in Figure 1. Under local stability at the pessimistic equilibrium (guaranteed by (10)), this reduces the price of government debt.

To do so, the government must sell additional debt equal to $DB_1 = (q_1^* - \hat{q}_1)b_0$. As with deposit insurance, this additional debt burden will have an influence on the period 1 price of debt and thus the amount of debt buybacks required. This is determined, as characterized below, through the equilibrium price of debt.

In our environment, the provision of DI and a debt buy-back program are equivalent ways to support depositors. That is, either depositors are supported by direct transfers supported by new debt through the DI system, or through support to the banks via debt financed purchases of bank held debt, with the proceeds transferred to depositors. In environments where there are other claimants on the banks, such as equity holders, these programs may not be equivalent. Or, if the provision of DI required costly liquidations, then the policies would also differ. Since our model has neither of these features, as the discussion progresses, we use the two policies interchangeably.

4.3 A Generalized Model of Pessimism

To determine an equilibrium with DI and/or debt buy-back, it is necessary to study asset pricing with these government responses. With the inclusion of these policies, (9) becomes:

$$1 - F\left(\frac{B_0 + G_1/q_1 + T_1(q_1)/q_1}{A_2}\right) = Rq_1 \tag{19}$$

where T_1 are transfers to the banking system. In the event of DI and/or Debt Buybacks, $T_1 = \pi c^{*E} - q_1 b_0 = (q_1^* - q_1)b_0$. Note that $T(q_1)$ is decreasing in q_1.

The support of the banking system adds another source of complementarity to the pricing equation. Inspecting (19), the argument of $F(\cdot)$ is decreasing in q_1. This was the case without the government support of the banking system and becomes even more strongly the case with that support since $\frac{T_1(q_1)}{q_1}$ is decreasing in q_1.

Even if $G_1 = 0$, so that the original source of strategic uncertainty in the debt market was not present, the government support of the banking system would induce strategic uncertainty in the debt market. In this sense, a government with implicit liabilities to the banking system may be subject to strategic uncertainty even though it has no debt outstanding.[14]

4.4 Is Pessimism Endemic?

The analysis supports the presence of a 'diabolic loop' between the financial and fiscal sectors. Key elements are the presence of standard deposit contracts and bank holdings of government debt.

[14]So if there was an independent source of either fundamental or strategic (bank runs) uncertainty in the banking system, it would be propagated and magnified through this link with the fiscal system.

These features are clearly consistent with evidence. As documented earlier, banks hold a substantial amount of government debt. Further, government actions are consistent with the assumed *ex post* public guarantee of the banking system.

Using our model, we can explore whether these key features are robust in a number of directions. First, does the government actually have an incentive to provide Deposit Insurance to banks? Section 5 studies this question and argues that this support will generally be forthcoming.

Second, the banking contract is simplistic as it ignores the possibility of equity issuance by the bank to potentially buffer against fluctuations in the value of government debt. This makes the banks particularly susceptible to shocks. Section 6 enhances the bank contract to include these elements. We then find conditions under which the diabolic loop remains.

5 Public Response: Provision of Deposit Insurance

In this section we examine the problem of a government which is faced with a pessimism shock to government finances and decides whether to bail out its banks through the provision of DI. The bail-out decision entails elements of redistribution between investors and depositors, as in Cooper and Kempf (2013). Moreover, a bail-out avoids costly liquidations, as in Acharya and Yorulmazer (2008) and thus disruptions of the intermediation process possibly at the cost of an increased probability of default. Our analysis will focus primarily on the second trade-off.

5.1 Welfare when DI is provided

The equilibrium when DI is provided starts with a shock to the beliefs of investors about debt repayment. This leads, as in the analysis of pessimism, to a lower price of government debt. This would be a solution to (19) with $q_1 < q_1^*$.[15]

In this equilibrium, at the middle date investors consume the difference between their endowment, the bonds they buy and the amount they borrow from banks. The amount of bonds they buy is equal to the bond sales by banks $(b_0^B - b_1^B)$ plus new issuance $((G_1 + T(q_1))/q_1)$ needed to finance spending as well as to bail out the banking system.

$$c_1^{I,DI} = A_1 - q_1 \left(b_0^B - b_1^B + (G_1 + T(q_1))/q_1\right) + L_1.$$

At the final date, investors consume all their net worth. The level of consumption depends on the default decision of the government. Throughout this discussion, let $(\mathbb{1}\{G\} = 0)$ denote the states in which the government repays its debt and let $(\mathbb{1}\{G\} = 1)$ denote default states.

[15] Exactly which equilibrium obtains is not important as long as there is movement away from the optimistic equilibrium.

If the government repays its debt, consumption is given by:

$$c_2^{I,DI}(\mathbb{1}\{G=0\}) = \bar{A} + B_1 + Ri_0^I - RL_1 - T_2. \tag{20}$$
$$= \bar{A} + Ri_0^I - RL_1$$

where the second equality follows from the fact that the banking system sells its entire holdings of government debt in the pessimistic equilibrium and therefore investors receive debt repayments exactly equal to their tax obligations.

If the government defaults, investors' final period consumption is given by:

$$c_2^{I,DI}(\mathbb{1}\{G=1\}) = \bar{A}\,(1-\gamma) + Ri_0^I - RL_1. \tag{21}$$

The only difference between (20) and (21) lies in the investors' endowment. When the government defaults, this endowment is reduced by a factor of γ which represents the real costs of government default. Because investors are holding the entire the debt stock and paying all the tax at the final date, the government's decision to default or repay only affects investors' net worth and consumption via its impact on A_2.

The utility of depositors when the government provides DI is independent of its decision to default and is given by the utility delivered by the standard banking contract

$$W^{C,DI} = \pi u\left(c^{*E}\right) + (1-\pi)\,u\left(c^{*L}\right).$$

The utility of depositors under DI is independent of government default since banks sell their entire bond holding to investors as soon as the economy switches to the pessimistic equilibrium. The government's provision of DI insulates depositors from risks to the bank at the middle date. The bank's sale of bonds to investors insulates depositors from any risks coming from defaults at the final date. As a result, depositors obtain the first best contract promised to them by banks at the initial date.

Investors' welfare is affected by the default decision due to the output costs of default assumed in our specification of the final date endowment. Investors' welfare when the government repays ($\mathbb{1}\{G\}=0$) and defaults ($\mathbb{1}\{G\}=1$) is:

$$W^{I,DI}\left(\mathbb{1}\{G\}\right) = c_1^{I,DI} + \frac{1}{R}c_2^{I,DI}\left(\mathbb{1}\{G\}\right).$$

The government will make a decision on whether to provide DI or not based on expected social welfare in the middle date when the pessimism shock hits. The social welfare function weights depositors and investors together, where the latter's Pareto weight is ω. Aggregate social welfare when DI is provided is therefore given by:

$$\begin{aligned} W^{AGG,DI} &= \pi u\left(c^{*E}\right) + (1-\pi)\, u\left(c^{*L}\right) \\ &+ \omega\left[A_1 - q_1^{DI}\left(B_0 + \left(G_1 + T\left(q_1^{DI}\right)\right)/q_1^{DI}\right)\right] \\ &+ \frac{\omega}{R}\left(\bar{A} + Ri_0^I\right) - \frac{\omega}{R}\left(1-p^{DI}\right)\gamma\bar{A}. \end{aligned} \tag{22}$$

Here p^{DI} is the probability that the government repays at the final date conditional upon DI being provided:

$$p^{DI} = 1 - F\left(\frac{B_0 + G_1/q_1^{DI} + T_1(q_1^{DI})/q_1^{DI}}{\bar{A}}\right). \tag{23}$$

In (22), the notation for the price of government debt in the middle period if there is pessimism and deposit insurance is provided is q_1^{DI}. This is determined from the investor's arbitrage condition of $\frac{p^{DI}}{q_1^{DI}} = R$ and (23). Finding the (p^{DI}, q_1^{DI}) that solves these two conditions is the same as finding a solution to (19), other than the optimistic equilibrium.

5.2 Welfare when DI is not provided

In section 4 we analyzed a situation in which a pessimism shock hits and the banking system is exposed to government debt and is entirely deposit funded. We saw that the financial system collapsed and experienced a fundamentals-based run unless DI was provided. This is the situation we analyze in this subsection.

The consumption of investors under repayment and default respectively are given by:

$$c_2^{I,NI}\left(\mathbb{1}\{G=0\}\right) = \bar{A}\left(1-\psi\right) + Ri_0^I$$

and

$$c_2^{I,NI}\left(\mathbb{1}\{G=1\}\right) = \bar{A}\left(1-\psi\right)\left(1-\gamma\right) + Ri_0^I.$$

As the government does not provide a financial safety net, the sovereign crisis leads to a banking crisis that reduces investors' endowments by a factor of $(1-\psi)$ due to the disruption of the intermediation process. If, in addition, the government ends up defaulting at the final date ($\mathbb{1}\{G\} = 1$) and investors' endowments are reduced further, by the factor $(1-\gamma)$ which represents the output costs of default in our model.

Consumers run on the bank when no deposit insurance is provided. Hence, their welfare depends on their place in the banking queue and on whether they are early or late consumers. A fraction

$$k_1 = \frac{q_1^{NI} b_0^B + \varepsilon i_0^B}{c^{*E}}$$

get served in equilibrium and they obtain the consumption allocation of the early consumers. Of those who get served, π fraction are early consumers who actually like consuming at the middle date. The rest, $1-\pi$, are late consumers who get reduced utility from consuming at the early date. Those who do not get served, a fraction $(1-k_1)$ of the total, receive zero consumption which delivers a low but finite utility $u(0)$. The price of government debt in the middle period if there is pessimism and deposit insurance is not provided is q_1^{NI}.

The total utility of depositors in the regime without DI is given by:

$$W^{C,NI} = k_1 \left(\pi u \left(c^{*E}\right) + (1-\pi) u \left(\beta c^{*E}\right)\right) + (1-k_1) u(0).$$

Investors' welfare when the government does (does not) repay is:

$$W^{I,NI}(\mathbb{1}\{G\}) = c_1^{I,NI} + \frac{1}{R} c_2^{I,NI}(\mathbb{1}\{G\}).$$

Aggregate welfare is a weighted sum of the welfare of depositors and investors:

$$\begin{aligned} W^{AGG,NI} &= k_1 \left(\pi u \left(c^{*E}\right) + (1-\pi) u \left(\beta c^{*E}\right)\right) + (1-k_1) u(0) \\ &+\omega \left[A_1 - q_1^{NI}\left(B_0 + G_1/q_1^{NI}\right)\right] \\ &+\frac{\omega}{R}\left(\bar{A} + R i_0^I\right) - \frac{\omega}{R}\left(1-p^{NI}\right)\gamma \bar{A} - \frac{\omega}{R}\psi \bar{A}. \end{aligned} \tag{24}$$

Here p^{NI} is the probability that the government repays conditional upon no DI being provided, given by:

$$p^{NI} = 1 - F\left(\frac{B_0 + G_1/q_1^{NI}}{\bar{A}(1-\psi)}\right). \tag{25}$$

Note the presence of ψ in the denominator of $F(\cdot)$ since there are bank defaults when deposit insurance is not provided.

This is determined from the investor's arbitrage condition of $\frac{p^{NI}}{q_1^{NI}} = R$ where p^{NI} is given in (25). Finding the (p^{NI}, q_1^{NI}) that solves these two conditions is the same as finding a solution to (19) with $T(q_1) \equiv 0$, other than the optimistic equilibrium.

5.3 The DI Provision Decision

The difference in the value of the social welfare function between providing DI and not is:

$$\begin{aligned} &W^{AGG,DI} - W^{AGG,NI} \\ &= \pi(1-k_1)\left[u\left(c^{*E}\right) - u(0)\right] + (1-\pi)\left[u\left(c^{*L}\right) - \left(k_1 u\left(\beta c^{*E}\right) + (1-k_1) u(0)\right)\right] \\ &- \omega T\left(q_1^{DI}\right) + \frac{\omega}{R}\left[\left(p^{DI} - p^{NI}\right)\gamma + \psi\right]\bar{A}. \end{aligned} \tag{26}$$

There are three terms in (26). The first is the gain by depositors from DI. This is unambiguously positive because $u\left(c^{*E}\right) > u(0)$ and $u\left(c^{*L}\right) > \left(k_1 u\left(\beta c^E\right) + (1-k_1)\, u(0)\right)$. The second term, $-\omega T\left(q_1^{DI}\right)$, is the loss for investors due to higher taxation under DI. The third term is the gain for investors as there are no bank default costs if DI is provided and a (possible) loss from higher expected default costs.[16]

To separate these influences, let ω^* satisfy:

$$\pi\,(1-k_1)\left[u\left(c^{*E}\right) - u(0)\right] + (1-\pi)\left[u\left(c^{*L}\right) - \left(k_1 u\left(\beta c^{*E}\right) + (1-k_1)\, u(0)\right)\right] = \omega^* T\left(q_1^{DI}\right). \quad (27)$$

In this case, the welfare effects of the insurance gains and tax costs just balance.

So leave aside this part, the focus of Cooper and Kempf (2013), to study the effects of DI on economic activity and thus the tax base, i.e. the last term of (26) which is proportional to:

$$\left(p^{DI} - p^{NI}\right)\gamma + \psi. \quad (28)$$

The first term, $\left(p^{DI} - p^{NI}\right)\gamma$, is the difference in the expected output costs of default due to the provision of DI relative to the situation where DI is not provided. Note that the sign of this term is not determined. That is, the provision of deposit insurance may increase the probability of sovereign default or it may reduce it.

The probabilities that the government repays the debt were given in (23) and (25). The two opposing effects of DI provision on the probability that the government repays the debt are apparent from these expressions. On the one hand, DI provision protects the intermediation process thus maintaining the tax base which is equal to $\bar{A}$ in (23) ('the tax base effect'). On the other hand, DI provision involves fiscal outlays $T_1(q_1^{DI})/q_1^{DI}$ which add to the debt burden ('the debt burden effect'). Which of the two dominates depends on the size of bank government debt holdings, which determine $T_1(q_1^{DI})/q_1^{DI}$, relative to the output costs of banking collapse ψ.

Proposition 4. *There is a critical value ψ^* such that DI is provided when $\psi \geqslant \psi^*$ and $\omega \leq \omega^*$.*

Proof. Let $\Delta(\psi) \equiv \left(p^{DI} - p^{NI}\right)\gamma + \psi$ with p^{NI} depending on ψ from (25). Set $\omega = \omega^*$ so the incentive to provide DI is determined by the sign of $\Delta(\psi)$. The proof shows that $\Delta(\psi)$ crosses zero exactly once as ψ varies between 0 and 1 using: $\Delta(0) < 0$, $\Delta(1) > 0$ and $\Delta'(\psi) > 0$.

$\Delta(0) < 0$ since at $\psi = 0$, $p^{NI} > p^{DI}$ from (25) and (23). Too see this, note that at $\psi = 0$, government debt is more valuable if DI is not provided since then default probabilities are lower: $q_1^{NI} > q_1^{DI}$. $\Delta(1) > 0$, since at $\psi = 1$, $p^{NI} = 0$ so that $p^{NI} < p^{DI}$ from (25) and (23).

[16] The sign of $(p^{DI} - p^{NI})$ is not determined *a priori*, as discussed further below.

To prove that $\Delta(\psi)$ is increasing in ψ, the derivative of (28) with respect to ψ implies:

$$\Delta'(\psi) = 1 - \frac{\partial p^{NI}}{\partial \psi}\gamma.$$

The first effect is direct as ψ enters $\Delta(\psi)$. The second is indirect, through the effect of ψ on p^{NI}. p^{NI} and q^{NI} are jointly determined with the equilibrium value of q^{NI} given by (19) with $T(q_1) \equiv 0$ and $p^{NI} = q^{NI}R$. By assumption, the pessimistic equilibrium is a locally stable solution to (19) and $\partial q^{NI}/\partial\psi < 0$. Hence $\partial p^{NI}/\partial\psi < 0$.

From this structure of $\Delta(\psi)$, there is a critical value of ψ such that the positive effect from protecting intermediation is exactly equal to the increased expected sovereign default costs as the result of DI provision. For $\psi \geqslant \psi^*$ DI is provided.

If DI is provided for a given ψ with $\omega = \omega^*$, then it will be provided if $\omega \leq \omega^*$. From (26), $\omega < \omega^*$ implies that the gains from insurance outweigh the costs so that the first term is positive. □

The case of costly disruption of intermediation as a basis for bailouts seems powerful. The Lehman Brothers collapse in September 2008 demonstrated clearly how costly the collapse of the financial system can be. Lehman Brothers was allowed to fail, partly at least due to the strong public outcry following the bailouts of Bear Stearns in March 2008 and of Fannie Mae and Freddie Mac during the summer of 2008. Yet, once the real effects of the crisis became apparent, the bailouts were extended more widely than ever before, covering insurance companies such as AIG and even the entire money market mutual fund industry. For this reason, assume DI is provided *ex post* by the government when the banking system is threatened with insolvency without it.

If bank or government defaults are not costly in terms of output (ψ and γ are close to zero) the DI provision decision is entirely determined by the balance of the insurance benefits (to depositors) versus the tax costs (to investors) of providing DI. This balance depends critically on the relative Pareto weight of investors (ω) as demonstrated in equation (27). Hence, as in Cooper and Kempf (2013), DI will still be provided even if $\psi = \gamma = 0$ if $\omega \leq \omega^*$.

6 Private Response: Equity Buffer

This section looks at private remedies for the spillovers between banks and governments. Subsection 6.1 shows that equity issuance, in particular, can, if done in sufficient quantities, restore the first best **without** the need for DI. The equity holders absorb the losses which occur when sunspot shocks hit the debt market.

From Proposition 4 the government will provide DI to banks threatened by insolvency. Subsection 6.2 shows how, when banks anticipate such government bailouts, this destroys their incentive to self-insure through building up an equity buffer against sovereign risk. Sovereign-banking loops reappear despite the presence of private as well as public remedies for them.

6.1 Self Insurance Through Equity Buffers

Suppose the bank can raise x_0 goods from investors at the initial date by selling shares in the bank. The equity entitles the owner to a state contingent dividend δ_2 at the final date, which is equal to the residual value of the bank after all depositors have received what was promised to them. Hence the equity will trade at a price e_0 which equals the expected value of dividends at the final date discounted back to time 0 by investors' required rate of return R. Investors will participate in this share offering if $e_0 \geqslant x_0$. Put differently, the equity investment in the bank will yield the same return as the outside option of the investors to put resources into the two-period, illiquid technology.

With an appropriate equity investment, the bank can offer households the first best contract, i.e. the allocation to early and late consumers, from Proposition 1 even in the presence of variations in debt prices. The resulting allocation insulates the banking system from debt fragility and the 'diabolic loop' breaks down.

6.1.1 Optimal Contract

To show how equity issuance can deal with the source of strategic uncertainty, we focus on a 'bankruptcy-proof' contract in which the bank issues sufficient equity to cover losses on sovereign debt holdings. The contract is further restricted so that allocations to early and late consumers are state noncontingent despite the presence of strategic uncertainty. As we demonstrate, this 'bankruptcy-proof' contract implements the first-best contract.

Even though the bank's promises to early and late consumers are noncontingent, its investments as well as the dividend payments $\delta_2(s)$ to shareholders are contingent upon the realization of sunspot shocks. Let s index the sunspot shock in the sovereign debt market: $s = 1$ when pessimism occurs and $s = 0$ when it does not. Let $\nu(s)$ be the probability of sunspot state s.

We will construct an equilibrium in which sunspots do not affect the consumption of early and late consumers. The value of government debt is state contingent. In the optimistic state, the value of government debt is, $q_1(0) = \frac{1}{R}$, characterized in Proposition 2. In the pessimistic state, the value of government debt, $q_1(1) < q_1(0)$ is given by an interior equilibrium from Figure 1.

The optimal contract with equity solves:

$$\max_{\delta_2(s), l_1(s), b_0, b_1(s), L_1(s), i_0, c^E, c^L} \pi u\left(c^E\right) + (1-\pi)\, u\left(c^L\right) \tag{29}$$

such that

$$i_0 + q_0 b_0 \leq d + x_0 \tag{30}$$

$$\pi c^E \leq q_1(s)(b_0 - b_1(s)) + \varepsilon l_1(s) - L_1(s) \tag{31}$$

$$(1-\pi) c^L \leq b_1(s) + R(i_0 - l_1(s)) - \delta_2(s) + r^b L_1(s) \tag{32}$$

$$e_0 = \frac{\sum_s \nu(s)\, \delta_2(s)}{R} \geqslant x_0. \tag{33}$$

The contract is modified in several ways compared to the contract in section 3. First of all, the sale of equity to investors adds further resources x_0 at the initial date. Secondly, the constraint (31) is modified because the choice of how many bonds to roll over $b_1(s)$ and how much to lend to investors $L_1(s)$ are now contingent upon the realization of the government debt sunspot shock. (33) is the participation constraint of the equity investors. It states that the amount investors inject into the bank must be greater than or equal to the expected value of dividends at the final date $(\sum_s \nu(s)\, \delta_2(s))$ discounted back to period 0 by the investors' opportunity cost of funds R.[17]

What remains unchanged is the fact that the promises to early and late consumers are non-contingent on the realization of the sunspot. In what follows below we do not solve (29) directly but instead show that it achieves the first best and fully insulates the bank from sunspot shocks in sovereign debt markets.

Proposition 5. *Optimal equity issuance implements the first-best contract.*

Proof. The proof argues that there exists e_0 such that (i) the contract with equity supports the first best allocation despite stochastic government debt prices and (ii) investors receive their required rate of return from the equity investment. We first determine the level of equity investment needed to support the first best contract, (c^{*E}, c^{*L}). We then argue that the return on this equity equals the outside option of the investors.

Let x_0 denote the period 0 investment of equity holders into the bank. The bank's resource constraint becomes:

$$q_0 b_0 + i_0 = d + x_0. \tag{34}$$

In the first best contract, the expected net present value of promises to depositors equals the amount they deposit at the initial date:

$$\pi c^{*E} + \frac{(1-\pi) c^{*L}}{R} = d. \tag{35}$$

[17] As we discussed in section 2, investors can place unlimited time deposits with banks and these time deposits pay a return of R. Hence risk-neutral investors will require the same expected return from any other asset they invest in.

To support the first best allocation, the bank must have sufficient resources to meet the contractual commitment to early consumers regardless of the realized government debt price:

$$\pi c^{*E} = q_1(1)\, b_0 \tag{36}$$

where $q_1(1)$ is the period 1 price of government debt under pessimism. In this state, the bank sells its entire bond holding in order to pay off early consumers. Promises to late consumers are met through the illiquid investment:

$$(1-\pi)\, c^{*L} = Ri_0. \tag{37}$$

The cash flow for dividend payments to shareholders is only generated in the optimistic state. The bank rolls over its bond not needed to fund the early consumers:

$$\pi c^{*E} = q_1(0)\,(b_0 - b_1(0)). \tag{38}$$

The rolled over bond holding is then used to pay dividends to shareholders at the final date:

$$\delta_2(0) = b_1(0).$$

The value of the equity to the shareholder is the discounted expected value of this dividend:

$$e_0 = \frac{\nu b_1(0)}{R}. \tag{39}$$

For the equity investment to be undertaken in equilibrium it must be the case that this expected value equals the equity put into the bank by the investor, i.e. $x_0 = e_0$. Substituting (35), (36) and (37) into (34) yields $x_0 = q_0 b_0 - \pi c^{*E}$. From this the equity investment needed is:

$$x_0 = \nu \frac{(1-p)}{p} \pi c^{*E}. \tag{40}$$

Combining (36) with (38) we get:

$$b_1(0) = R\frac{(1-p)}{p} \pi c^{*E}.$$

Hence

$$e_0 = \nu \frac{(1-p)}{p} \pi c^{*E} = x_0. \tag{41}$$

□

The effect of the equity issuance is to remove all risks from the bank's bond purchases. This is very intuitive. The deep pocket risk-neutral equity investors absorb all the risk and leave the bank and its depositors with a safe consumption profile.

It is useful to define

$$\kappa^* \equiv \frac{x_0}{b_0} = \nu \frac{(1-p)}{R} \tag{42}$$

as the amount of equity issued for given purchases of government debt. This is chosen by the bank in its own prudential capital choice (under the belief that no *ex post* assistance will be provided). The form of the capital charge consists of two key parts. It is increasing in the difference between the value of government debt in the optimistic state and the pessimistic state

$$q_1(0) - q_1(1) = \frac{(1-p)}{R}$$

because this difference determines the volatility of bank asset values and the size of dividends in the good state.

When p is small and the government is expected to default with a high probability in the pessimistic state of the world, the bank needs to buy a lot of bonds at the initial date in order to ensure that their value under pessimism is sufficient to meet depositor withdrawals. When the economy remains in the optimistic equilibrium (this happens with probability ν which is also part of the κ^* expression), the difference between bank assets and bank liabilities is very big and shareholders receive a large dividend. So κ^* has the interpretation of a 'risk weight' - it is growing in the volatility of government debt prices between the pessimistic and optimistic states of the world.

6.1.2 Crisis Insulation

Returning to our generalized model of pessimism in the section 4.3, we can see how equity issuance affects the government transfers which drive the fiscal-banking interactions. Since banks are stabilized, there are no government transfers due to the banking sector. Government debt losses in the pessimistic sovereign debt market state are entirely absorbed by setting share dividend payments to zero and therefore when (31) holds with equality. Transfers become zero in all states of the world and (19) reverts back to its formulation without sovereign-banking linkages and without liquidations of the long technology:

$$1 - F\left(\frac{B_0 + G_1/q_1}{\bar{A}}\right) = Rq_1. \tag{43}$$

When banks issue enough equity, multiple equilibria in the debt market still survive but carry no implications for the solvency of the banking system. In addition, the feedbacks from banking to sovereign fragility also disappear because no bank failures and long asset liquidations occur in the pessimistic equilibrium of our economy.

6.2 The Limits to Self-Insurance

The previous subsection painted an optimistic picture: sovereign debt crises generate no spillovers as long as the banking system raises sufficient equity. But, as we know, this did not happen in practice and a number of European economies did experience severe positive feedback loops between the financial health of governments and their banks. In this section, we explain the limited equity issuance as a response of banks to anticipated bailouts.

Of course, there are other arguments for the fact that banks were exposed to variations in government debt prices. An immediate explanation is some element of irrationality. Either banks did not anticipate the possibility of pessimism at all or did not appreciate the severity of the debt crisis through the complicated linkages from the equilibrium interactions between debt prices, bailouts and bank balance sheets.

But even if banks correctly anticipate the likelihood and severity of sovereign and banking crises, a moral hazard problem exists from the government's strong incentives to provide DI (see Proposition 4 in section 5). Banks fail to shield their balance sheets from the impact of the crises anticipating government debt buybacks at above market prices if banks experience losses.

6.2.1 Optimal Contract

From section 5, *ex post* the government will purchase bonds from banks at a price q_1^* which is chosen such that the bank can meet payments to early as well as late consumers for any realization of the government debt sunspot shock s. This government intervention makes the consumption allocation received by depositors riskless. It also obviates the need for self-insurance through equity buffers.

Note that we do not generate this effect through a high cost of equity. The return on equity is R, exactly the same as on all the other assets held by investors. The unwillingness to issue equity comes purely from the desire by the banks to exploit the financial safety net to the maximum extent possible.[18]

But the effects go further. The implicit bailout assistance which banks rationally anticipate the government to provide *ex post* creates incentives for excessive risk-taking through holdings of fragile government debt. This implication of the model is consistent with the large exposures to sovereign debt documented in Table 1.

The banking contract with expected bailouts but without equity issuance solves

$$\max_{b_0, i_0, L_1, b_1, c^E, c^L} \pi u\left(c^E\right) + (1-\pi)\, u\left(c^L\right)$$

[18]To rule out permanent transfers to the banking sector we assume that the government can commit to limit transfers to zero in the optimistic equilibrium. In other words $q_1^* \leq q_1(0)$ so transfers only occur during a fiscal crisis.

such that

$$i_0 + q_0 b_0 \leq d$$

$$\pi c^E = q_1^* (b_0 - b_1) - L_1 \tag{44}$$

$$(1 - \pi) c^L = R i_0 + b_1 + r^b L_1. \tag{45}$$

Here the government buys government bonds from the banks at q_1^*, the optimistic equilibrium price. The allocation is independent of the sunspot.

Proposition 6. *Equilibrium under the debt buyback scheme features maximum bank exposure to strategic uncertainty from the government debt market: $i_0 = 0$ and $d = q_0 b_0$. Equity is not issued voluntarily.*

Proof. The debt price reflects investors' valuation which takes the risk of sunspot shocks into account.

$$q_0 = \frac{\nu + (1 - \nu) p}{R}. \tag{46}$$

The first order conditions for the optimal contract are:

$$u' \left(c^E\right) = \lambda^E = 0$$

$$u' \left(c^L\right) = \lambda^L = 0$$

$$\left(q_0 \phi + q_1^* \lambda^E\right) b_0 = 0$$

Using (46) and $q_1^* = 1/R$, the first order condition with respect to bond purchases becomes

$$\left(-\phi + \frac{\lambda^E}{(\nu + (1 - \nu) p)}\right) b_0 = 0. \tag{47}$$

The rest of the first order conditions are:

$$\left(-\phi + R\lambda^L\right) i_0 = 0, \tag{48}$$

$$\left(-\lambda^E + r^b \lambda^L\right) L_1 = 0, \tag{49}$$

$$\left(-q_1^* \lambda^E + \lambda^L\right) b_1 = 0. \tag{50}$$

It is clear from the above first order conditions that $i_0 = 0$ and b_0 is greater than zero. To see why this is, remember that $r^b = R$ and substitute (49) into (47) to get:

$$-\phi + \frac{R}{\nu + (1 - \nu) p} \lambda^L = 0.$$

Since $\nu + (1-\nu) p < 1$, $-\lambda^E + R\lambda^L < 0$. Hence $i_0 = 0$ and $q_0 b_0 = d$.

Choosing $i_0 = 0$ and $q_0 b_0 = d$ maximises the expected payments from the deposit insurance fund. It implies that the net present value of promises to the two types of consumers is:

$$\pi c^E + \frac{(1-\pi) c^L}{R} = \frac{q_1^*}{q_0} d \geqslant d. \tag{51}$$

Using the first order conditions for c^E, c^L, and L_1 we can see that the promises to the early and late consumers continue to be governed by

$$u'\left(c^E\right) = Ru'\left(c^L\right).$$

which is identical to the first best. Since the ratio c^E/c^L is equal to the first best but the net present value of consumption promises is higher by a factor of $\frac{q_1^*}{q_0}$ than that in the first best, this implies that the consumption of early and late consumers is each higher by the same factor.

Since the consumption of all consumers in the banking contract is higher under no equity issuance and maximum sovereign debt exposure, this implies that competitive banks optimally issue no equity if it is allowed to choose its capital structure freely. □

Proposition 6 demonstrated two important consequences of the anticipation by banks of government bailouts.[19] First, the bank invests heavily in fragile government debt. Secondly, the bank does not issue equity despite the absence of any costs of doing so.

The lack of equity issuance is actually the more serious consequence of deposit insurance. Large holdings of risky sovereign debt are not necessarily a problem for bank solvency as long as they are backed by sufficiently large equity buffers. But we saw in Proposition 6 that this is not the case. The key reason for this is that when the bank issues equity, its losses are borne by investors who demand compensation through a competitive return on the funds they provide. In contrast, the

[19] The results in Proposition 6 crucially depend on the fact that DI will in fact be provided *ex post*. A key determinant of the government's DI provision is the output cost of bank default as measured by ψ in equation (8). We assumed for simplicity that ψ is a parameter which is independent of the nature of bank portfolios. Another possible assumption is that bank default costs only arise when bankrupt banks are forced to liquidate long term projects early. In other words, when banks hold only government bonds (as shown in Proposition 6), ψ should be very low and possibly zero. Even when $\psi = 0$ we know from section 5 that DI will still be provided if the government cares more about depositors than about investors ($\omega \leq \omega^*$).

Another possibility is that banks behave strategically and hold just enough long term projects to make sure that they are important enough to get bailed out by the government. Banks then place the rest of their deposits in government debt in order to benefit from bailouts.

Such strategic considerations behind banks' portfolio choices are not crucial for the main message of the paper and we do not pursue them here for the sake of brevity. Instead we assume that ψ is sufficiently large and ω is sufficiently small so that DI is provided even when banks hold only government bonds.

government does not require compensation for the bailout assistance it provides *ex post* when the banking system gets into difficulty. Banks and their depositors rationally prefer to resort to free government insurance rather than relying on costly private alternatives.[20]

6.2.2 Crisis Amplification

In this environment, as shown above, banks invest all deposits at $t = 0$ in government bonds. The transfer from the government to banks in the pessimistic state of the world therefore becomes:

$$T(q_1) = (q_1^* - q_1)\frac{d}{q_0}.$$

Compared to a situation in which banks did not anticipate the debt buyback policy, here banks maximize their investment in sovereign debt thus making the pessimistic equilibrium even more damaging to bank and sovereign solvency. This is because banks' exposure to sovereign debt losses is greater, requiring larger bailouts and larger debt issuance at the middle date. The 'diabolic loop' is again alive and well.

7 Conclusions

This paper builds a model of the feedback loops between banks and sovereigns in Europe. From Diamond and Dybvig (1983) and Calvo (1988), banks as well as sovereign debt markets are individually subject to powerful sources of strategic uncertainty, which can lead to multiple Pareto-ranked equilibria. Our paper characterizes a 'diabolic loop' that links these markets and thus propagates and amplifies the impact of strategic uncertainty.

The interactions work from sovereigns to banks as well as in the opposite direction. Bank solvency is affected by sovereign bond market turmoil because the financial system holds a large amount of (largely domestic) government debt. In turn, government solvency is affected due to the implicit or explicit guarantees extended by governments to their banking systems. These interactions amplify the impact of pessimism in the government debt market. The initial decline in government debt prices reduces bank solvency and causes the implicit government promises to its banks to turn into explicit debt issuance at precisely the time when the government is least able to issue debt on favorable conditions. The higher debt issuance then pushes government debt prices even lower, completing the diabolic loop which has been rocking a number of European economies

[20] Note that we are implicitly assuming here that investors must hold government debt directly. The option of holding bonds via the banking system in order to take advantage of the deposit guarantee is ruled out.

since 2010. The impact of these feedbacks is to make sovereign-banking crises much more severe than they otherwise would have been.

While we study the effects of strategic uncertainty in debt markets as the initial shock, the model is general enough to accommodate other sources and types of uncertainty. As is well understood, the Diamond and Dybvig (1983) model often has a bank run equilibrium which itself could influence debt valuation through the cost of a government fulfilling obligations to banks. In fact, even if the bank run could be avoided through government intervention, the costs of those actions through debt valuation would be an important part of the calculus concerning the *ex post* provision of deposit insurance. Moreover, fundamental shocks to either the banking system or the government fiscal situation would be magnified and propagated through the mechanisms identified in our model.

Having built a model of the crisis, we study a number of simple remedies for cutting the diabolic loop. One often suggested policy is just to let the banking system fail, imposing losses on depositors. Such a policy, if credible, would have multiple benefits. First, it would reduce the need for bailout assistance to add to government debt during a sovereign crisis. This, taken in isolation, would diminish the crisis amplification mechanisms we study in our paper. Second, when banks know they will not be bailed out, they will issue equity which will absorb losses from sovereign bond holdings without needing government assistance. Hence, bank solvency becomes completely decoupled from government solvency, severing a key linkage that has amplified the financial crisis in a number of EU countries.

The problem with such a commitment to let the banking system fail is that it is not credible. Governments, acting with discretion after a crisis, prefer (with some restrictions) to bailout the banking system rather than incur the output losses associated with a breakdown of the intermediation process.

In turn, banks, anticipating that government assistance will be provided, have little incentive to issue equity. This is because they would like to take advantage of the 'heads-I-win, tails-you-lose' nature of the financial safety net. If the economy finds itself in an optimistic equilibrium, banks profit from high *ex post* bond returns. When the economy finds itself in the pessimistic equilibrium, the bank expects the government to bail it out in order to protect household deposits. This strategy extracts a transfer from taxpayers to bank depositors which makes the latter better off. As a result, banks rationally prefer to remain exposed to a sovereign debt crisis.

This moral hazard by banks might be corrected by regulatory interventions which impose capital requirements on banks' sovereign debt holdings until they become insulated from shocks in the debt market. In the light of this finding, it is puzzling that the new Basel III regime continues to favor domestic government debt over other assets by assigning it a zero capital weight. Moreover, domestic government debt continues to be exempt from large exposure limits creating exactly the kinds of incentives for banks to become overexposed to it described in our paper. While beyond the scope of this paper, adding an explicit analysis of government capital regulation seems to us like a interesting avenue for future research.

Future work will analyze the international dimension of the European twin crisis. Financial stability policy in Europe is undergoing major reform with the establishment of the Single Supervisory Mechanism (SSM) and with the use of Outright Monetary Transactions (OMT) by the European Central Bank. We intend to embed our single country model into a multi-country setting and consider the union-wide policies which can limit the economic damage done by the 'diabolic loop'.

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