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THE NEXT GENERATION OF THE PENN WORLD TABLE

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ABSTRACT

We describe the theory and practice of real GDP comparisons across countries and over time. Effective with version 8, the Penn World Table (PWT) will be taken over by the University of California, Davis and the University of Groningen, with continued input from Alan Heston at the University of Pennsylvania. Version 8 will expand on previous versions of PWT in three respects. First, it will distinguish real GDP on the expenditure side from real GDP on the output side, which differ by the terms of trade faced by countries. Second, it will distinguish growth rates of GDP based on national accounts data from growth rates that are benchmarked in multiple years to cross-country price data. Third, data on capital stocks will be reintroduced. Some illustrative results from PWT version 8 are discussed, including new results that show how the Penn effect is not emergent but a stable relationship over time.

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1. Introduction

For over four decades, the Penn World Table (PWT) has been a standard source of data on real GDP across countries. Making use of prices collected across countries in benchmark years by the International Comparisons Program (ICP), and using these prices to construct purchasing-power-parity exchange rates, PWT converts GDP at national prices to a common currency – U.S. dollars – making them comparable across countries. Each version of PWT is based in a newer ICP benchmark, so that PWT version 7 is based on the 2005 ICP prices. PWT version 8 will still be based on the 2005 benchmark, but its construction will pass to the University of California, Davis and the University of Groningen, while retaining the PWT initials and with continued input from Alan Heston at the University of Pennsylvania.[1]

In this paper we describe the changes to the measurement of real GDP that will be introduced in this "next generation" of PWT. We begin in section 2 with a brief overview of the theory behind real GDP comparisons, including a new theorem. That discussion is intended to indicate the challenges in making multilateral comparisons of real GDP. Diewert (1999) and Van Veelen (2002) have argued that no multilateral measure of real GDP can satisfy all the axioms we might like, so there are tradeoffs involved with any construction of this concept. Our approach is a natural extension of what is already done in PWT, but will distinguish several different concepts of real GDP.

In section 3 we describe the PWT calculation of real GDP before version 7. As argued by Feenstra, Heston, Timmer and Deng (2009), prior measurement of real GDP in PWT was closer to what is called "command-basis GDP" in the United States, or "real income" in the United Nations System of National Accounts. That is, it was a measure of real GDP that reflected the

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[1] The data are available at www.ggdc.net/pwt. After the new ICP 2011 benchmark becomes available in 2014, then PWT9 will be based on that new benchmark, again constructed at the University of California, Davis and the University of Groningen.

standard of living in an economy rather than the production possibilities. Feenstra *et al* (2009) refer to this concept as "real GDP on the expenditure side," or real GDP^e. Countries that have strong terms of trade – meaning a higher than average prices for exports or lower than average prices for imports – will have higher real GDP^e as a result. We contrast this with "real GDP on the output-side", or real GDP^o, which is intended to measure the production possibilities of an economy. Both concepts are reported in PWT8.

In section 4 we move to the measurement of real GDP over time. This is the area where there is perhaps the greatest confusion over concepts. The term "real" in multilateral comparisons of GDP refers to the use of some common "reference" prices to add up across goods and obtain real GDP in different countries. When the calculation is made in two years, the values of real GDP obtained are not necessarily comparable because reference prices can be changing. Stated differently, the purchasing-power-parity (PPP) exchange rate used to convert from national currencies to U.S. dollars changes each year. The terms "current-price real GDP" or "real GDP at current PPPs" refer to calculations across countries that are *not comparable over time*, because the comparison is based on changing PPP exchange rates each year. In contrast, the terms "constant-price real GDP" or "real GDP at chained PPPs" refer to calculations across countries that are *also comparable over time*, because an appropriate correction is made for changing reference prices and PPP exchange rates. The latter constant-price or chained concept is denoted by $RGDP^e$ and $RGDP^o$ on the expenditure and output sides, respectively. But these variables require the initial construction of current-price real GDP, denoted by $CGDP^e$ and $CGDP^o$, so both concepts are reported in PWT8. The current and constant-price variables are equal in the benchmark year 2005, but differ in other years because $RGDP^e$ and $RGDP^o$ use "real" growth rates that correct for changing reference prices. We also include a third measure of constant-price real GDP, denoted $RGDP^{NA}$, that is equal to $CGDP^{o}$ and $RGDP^{o}$ in the 2005 benchmark and uses national-accounts growth rates of real GDP to extrapolate to all other years. Earlier versions of PWT focused on $RGDP^{NA}$.[2]

As we explain in section 5, the growth rates of $RGDP^{e}$ and $RGDP^{o}$ are computed using multiple ICP benchmarks, so these growth rates can differ considerably from that in the national accounts, $RGDP^{NA}$, and therefore differ from past versions of PWT. For this reason, users who are really just interested in national accounts growth rates or who want to use a variable that is most similar to past versions of PWT should simply use $RGDP^{NA}$. In section 6 we extend our results to show how total factor productivity (TFP) across countries can be constructed in theory and in practice, using the labor and capital stock data in PWT8.

In section 7 we discuss some results from PWT version 8.0. In particular, we show how the change in $RGDP^{e}$ and $RGDP^{o}$ over time often differs substantially from the change in $RGDP^{NA}$. We also document how the new measure of factor inputs and total factor productivity constructed from them can explain more of the cross-country variation in $CGDP^{e}$ per capita than standard approaches in the literature. Finally, we show that our use of multiple ICP benchmarks has important implications for estimating the Penn effect, the positive relationship between a country's relative price level (PPP over the exchange rate) and its income level. In contrast to the finding of Bergin, Glick and Taylor (2006) that the Penn effect only gradually emerges over time, we show that this effect is continuously positive and significant as long as we rely on information from the ICP benchmarks. Section 8 concludes and the Appendix contains the proofs of our theorems.

[2] Earlier versions of PWT up to v6 constructed real GDP using a weighted average of the national accounts growth rates of the *components* of GDP, i.e. C, I and G. So the growth rate of total real GDP in PWT differed from that in the national accounts due to these weights. That approach was criticized by Johnson *et al* (2013) because the weights would change in different versions of PWT. To address this criticism, PWT7 used the national accounts growth rate of *total* GDP instead of the components to construct $RGDP^{NA}$, as we also do in v8.

2. Theory of Real Output Comparisons

Denote the final goods by i = 1,…,M, which includes all consumption goods, investment, and government expenditures. These can be thought of as the goods for which prices are collected by the ICP in benchmark years, and we treat them as non-traded in the sense that they are purchased from local retail outlets. In addition, suppose there are i = M+1,…, M+N exported and imported goods: these are treated as intermediate inputs and can be thought of as the categories in the Standard International Trade Classification (SITC), for example. Any good that is imported and then sold domestically would appear twice, once as an import and again as a non-traded final good; and a third time of that good is also exported.

The domestic price vector for the final goods is denoted by p_j in country j=1,…,C, while the free trade price vectors for exports and imports are p_j^x and p_j^m, all measured in the local currency. The domestic prices for the traded goods are $p_j^x + s_j$ and $p_j^m + t_j$, where s_j is the vector of export subsidies and t_j is the vector of import tariffs. The column vector of prices is then P_j = $(p_j, p_j^x + s_j, p_j^m + t_j)$, and we let $y_j \equiv (q_j, x_j, -m_j)$ denote the corresponding column vector of quantities (negative for imports). The revenue function for the economy is defined by:

$$r_j(P_j, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ P_j ' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}, \qquad (1)$$

where $F_j(y_j, v_j)$ is a transformation function for each country, which depends on the vector v_j representing primary factor endowments and also depends on the index of country j due to technological differences across countries.

We will distinguish the reference prices π_i for final goods, i =1,…,M, and two sets of reference prices π_i^x , π_i^m for exports and imported intermediate inputs, i =M+1,…,M+N. Denote the M+2N dimensional vector of reference prices by $\Pi = (\pi, \pi^x, \pi^m)$. We suppose that the country is engaged in *free trade* at these reference prices, and evaluate GDP on the output-side using the revenue function:

$$r_j(\Pi, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ \Pi' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}. \tag{2}$$

Then real output can be compared across countries using the ratio of revenue functions:

$$\frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)}. \tag{3}$$

In contrast to this measure of real output, the standard of living across countries can be measured by the ratio of expenditure functions as in Neary (2004). In practice, we should not expect to estimate the revenue functions across countries as Neary does for the expenditure function, because the revenue functions are indexed by the country j indicating technological differences between them. Even with a parsimonious specification of such technological differences it would be difficult to estimate revenue functions while pooling across all countries. For this reason, we must rely on indexes that can be used to approximate (3).

The simplest index that could be used to measure real output across countries is the ratio of quantities evaluated at the prices of one country or the other. Gerschenkron (1951) was the first to document that there is a systematic relationship between real output evaluated at each country's prices, which is called the "Gerschenkron effect":

$$\left(\frac{P_j' y_j}{P_j' y_k} \right) < \left(\frac{P_k' y_j}{P_k' y_k} \right). \tag{4}$$

This inequality states that real GDP is higher when measured with the prices of another country, or to put it most simply, "the grass is greener on the other side." This relationship can be interpreted by noting that the right-hand side (4) is the Laspeyres quantity index, which exceeds the Paasche quantity index on the left. That inequality is familiar from consumer theory, where goods whose prices have fallen the most will have the greatest quantity increase, and so the Laspeyres quantity index which uses the last-period prices overstates the quantity increase. The same finding holds in the cross-country comparison in (4), despite the fact that this comparison is being made using *production* data rather than consumption data. In production theory, the upward bias of the Laspeyres index is reversed (since those goods whose prices have risen the most will have the greatest quantity increase). Nevertheless, Gerschenkron and many later studies confirm that the “demand-side bias” in (4) holds. The reason for this finding is that prices are determined in general equilibrium, and with similar tastes (demand curves) across countries but different technologies (shifting supply curves), the highest-priced goods in a country will tend to have less quantity than abroad, so the upward-bias of the Laspeyres index follows.

Diewert (1983) refers to the measure of real output in (3) as a Samuelson-Swamy-Sato index. An alternative measure of real output, referred to as the Malmquist index, can be obtained by using the distance between the transformation functions. Specifically, Caves, Christensen and Diewert (1982a,b) define two measures of the difference in real output δ_j and δ_k as follows:

$$F_j(y_k\delta_k, v_j) = 1 \text{ and } F_k(y_j / \delta_j, v_k) = 1. \quad (5)$$

To interpret the first of these conditions, it states that if we start with the observed output vector for country *k*, and inflate it by δ_k, then we obtain an output vector that is feasible to produce with the technology and endowments of country *j*. If $\delta_k > 1$ we conclude that country *j* can produce more output than *k*. Likewise, the second condition states that by deflating the observed output vector in country *j* by δ_j, we obtain an output vector that is feasible to produce in country *k*. Again, if $\delta_j > 1$ we conclude that country *j* can produce more output than *k*. So δ_j and δ_k are both measuring (or bounding) the output of country *j* relative to *k*.

The question is as to how to measure the Malmquist distance factors without full knowledge of the transformation function. Caves, Christensen and Diewert (1982a,b) provide a powerful result by showing that if the transformation function is translog, then the geometric mean of δ_j and δ_k is measured by a Törnqvist quantity index of the outputs. We will rely on a similar result in section 6 when discussing the measurement of total factor productivity. But initially, we provide a different result that does not depend on the form of the transformation or revenue function, but just relies on the output vectors being chosen optimally at the observed prices:[3]

Theorem 1

Suppose that the outputs are revenue-maximizing and the Gerschenkron effect in (4) holds. Then there exists a reference price vector Π between P_j and P_k such that:

$$\delta_k \leq \frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)} = Q_{jk}^F \equiv \left[\left(\frac{P_j' y_j}{P_j' y_k}\right)\left(\frac{P_k' y_j}{P_k' y_k}\right)\right]^{0.5} \leq \delta_j .$$

This result says that computing a Fisher ideal quantity index Q_{jk}^F between the countries, which is a geometric mean of the Paasche and Laspeyres quantity indexes, is a valid comparison of real output between them. Remarkably, it does not depend on the functional form of the revenue function but only on optimizing behavior.

Theorem 1 gives us a compelling reason to use the Fisher quantity index when comparing real output across countries. In practice, an extension of the Fisher quantity index is used to measure real output by both by the World Bank in the *World Development Indicators* and by Eurostat and the OECD (2006), as we discuss in the next section. There is, however, a critical limitation that

[3] The proof of Theorem 1 in the Appendix uses inequalities due to Malmquist (1953), as noted by Diewert (1981).

arises with its use: the reference prices Π used in the comparison are not made explicit.[4] This limitation means that the cross-county real output comparisons made by the World Bank and Eurostat-OECD are fundamentally flawed when comparing real output over time: because the reference prices are not known, then it is impossible to correct for changes in these prices over time to obtain constant-price comparisons. Accordingly, in the next section we also discuss the leading alternative to the use of Fisher indexes, which is the Geary-Khamis (GK) method,[5] as has been used by PWT and that we shall extend. This is an attractive alternative as it involves the explicit calculation of reference prices, so that it becomes possible to correct for changes in these prices over time, as discussed in section 4.

3. Measurement of Real GDP in Practice

In place of the Fisher ideal quantity index defined in Theorem 1, we can equivalently deflate the ratio of nominal GDPs by the Fisher price index. There are two modifications that have been made to the Fisher price index, however, before it is used for multilateral comparisons. First, recall that the output vector $y_j \equiv (q_j, x_j, -m_j)$ consists of final goods, exports and (the negative of) imports. In practice, detailed data on exports and imports have never been incorporated into international comparisons of real GDP (which we shall rectify with PWT 8) and only the prices of final goods (as collected by the ICP) have been used. To mimic this approach, let the Fisher price index between country *j* and *k* computed over final goods only be denoted by,

$$P_{jk}^F = \left[\left(\frac{p_j' q_j}{p_k' q_j}\right)\left(\frac{p_j' q_k}{p_k' q_k}\right)\right]^{0.5}. \qquad (6)$$

[4] Note that the reference price vector Π referred to in Theorem 1 depends on the countries *j* and *k* being compared. When the EKS extension of the Fisher index is used to compare real GDP across countries, then it is not known whether a corresponding reference price vector like that in Theorem 1 exists.

[5] Due to Geary (1958) and Khamis (1970, 1972). A modern treatment of this method is provided by Balk (2008).

If this formula is used to measure the price of final goods in country *j* relative to *k*, and then again in country *k* relative to *l*, and we multiply these, we do not get the same numerical result as if we directly measured the price of final goods in country *j* relative to *l*. In other words, the Fisher price (or quantity) indexes are not transitive. To achieve transitivity, the second modification made to the Fisher price index is to apply the so-called EKS transformation:[6]

$$P_{jk}^{EKS} \equiv \prod_{l=1}^{C} \left(P_{jl}^{F} P_{lk}^{F} \right)^{1/C} . \tag{7}$$

It is apparent that (7) is transitive by construction. Then to obtain real GDP in country *j* relative to *k*, the ratio of nominal GDP in the two countries is deflated by this index:

$$\frac{CGDP_j}{CGDP_k} \equiv \left(\frac{GDP_j}{GDP_k} \right) / \, P_{jk}^{EKS} . \tag{8}$$

While this formula is used by the World Bank and Eurostat-OECD to measure real output, it has three limitations. First, as noted above, the EKS price index is computed using only the prices for final goods, even though it is used to deflate the ratio of nominal GDPs that include the trade balance. Second, we cannot meaningfully sum the real GDP of two countries *j* relative to *k* and *l* relative to *k*, because these two comparisons are made at different prices. Third, we likewise cannot compare the real GDP of country *j* relative to *k* at two points in time, again because the prices are changing. We shall address all these limitations in PWT8.

In contrast to the EKS system, under the Geary-Khamis (GK) system the comparison of real GDP is made by using *reference prices* π_i^e of final goods for consumption, investment and government expenditures. The reference price denoted by *e* (for expenditure) are defined by:

$$\pi_i^e = \sum_{j=1}^{C} (p_{ij} \, / \, PPP_j^e) q_{ij} \Bigg/ \sum_{j=1}^{C} q_{ij} \, , \qquad i = 1,\ldots,M. \tag{9}$$

[6] Or the GEKS system, after Gini (1931), Eltetö and Köves (1964), and Szulc (1964).

That is, the reference prices are defined as quantity-weighted average of the observed prices, after deflating by a “PPP exchange rate” for each country, which is defined by:

$$PPP_j^e = \sum_{i=1}^{M} p_{ij}q_{ij} \Big/ \sum_{i=1}^{M} \pi_i^e q_{ij} \;, \quad j = 1,\ldots,C. \tag{10}$$

The GK system (9)-(10) is $M+C$ equations that under weak conditions have a solution which is unique up to a normalization. Then *real GDP on the expenditure side* is measured by:

$$CGDP_j^e \equiv \sum_{i=1}^{M} \pi_i^e q_{ij} + (X_j - M_j) / PPP_j^e = GDP_j / PPP_j^e \,, \tag{11}$$

where the equality follows because the numerator in (10) equal nominal absorption, $C_j+I_j+G_j$, and of course, $GDP_j = C_j+I_j+G_j+ X_j - M_j$. This GK formula is the method used by PWT up to version 7 to measure real GDP in a benchmark year. We use the notation $CGDP^e$ to emphasize that (11) is actually *current-price* real GDP, because it uses reference prices π_i^e computed in the current year. Equivalently, we see from (11) that $CGDP^e$ uses current-year PPP. By definition, (11) also equals *constant-price* real GDP, or $RGDP^e$, in the benchmark year 2005, but in other years $CGDP^e$ is still computed as in (9)-(11) whereas $RGDP^e$ is computed in a different manner, as discussed in the next section.

Notice that the trade balance ($X_j - M_j$) in (11) is deflated by the PPP that is computed over *final goods* without using any prices for exports or imports, much as was done in the EKS system in (8), so the first limitation mentioned above still holds. The second limitation does not hold: we can add real GDP across countries because they are computed at common reference prices. The third limitation – not being able to compare $CGDP^e$ over time – still holds because reference prices or PPPs will change in each year. We will show in the next section how the explicit use of reference prices in the GK system allows this third limitation to be addressed, so that we can compute and compare constant-price real GDP, or $RGDP^e$, over time.

To address the first limitation, the measurement of *real GDP on the output side* under PWT8 extends the GK system by incorporating prices for N export and import goods, p_{ij}^x and p_{ij}^m, $i = M+1,\ldots,M+N$. These prices are used are used to obtain the reference prices for imports and exports, in addition to final goods, as the weighted averages across countries:

$$\pi_i^o = \sum_{j=1}^{C} (p_{ij} / PPP_j^o) q_{ij} \Big/ \sum_{j=1}^{C} q_{ij} , \qquad i = 1,\ldots,M, \tag{12}$$

$$\pi_i^x = \sum_{j=1}^{C} (p_{ij}^x / PPP_j^o) x_{ij} \Big/ \sum_{j=1}^{C} x_{ij} , \qquad i = M+1,\ldots,M+N, \tag{13}$$

$$\pi_i^m = \sum_{j=1}^{C} (p_{ij}^m / PPP_j^o) m_{ij} \Big/ \sum_{j=1}^{C} m_{ij} , \qquad i = M+1,\ldots,M+N, \tag{14}$$

from which we obtain the PPP exchange rate for each country,

$$PPP_j^o = \frac{GDP_j}{\sum_{i=1}^{M} \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij})}, \; j = 1,\ldots,C. \tag{15}$$

The system of equations in (12)-(15) has a positive solution under certain conditions (Feenstra *et al*, 2009), unique up to a normalization.[7]

The concept of real GDP^o is obtained by multiplying the reference prices for final outputs π_i^o, exports π_i^x and imports π_i^m by their respective quantities, obtaining:

$$CGDP_j^o \equiv \sum_{i=1}^{M} \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}) = \frac{GDP_j}{PPP_j^o} = \frac{C_j + I_j + G_j}{PPP_j^q} + \frac{X_j}{PPP_j^x} - \frac{M_j}{PPP_j^m}, \tag{16}$$

where the final equality follows by defining the PPPs of final goods, exports and imports:

[7] The GK system in (9)-(10) has a positive solution under very weak conditions. It is more difficult to ensure that the extended system (12)-(15) has a positive solution because real GDP^o in the denominator of (15) might become negative. This outcome can be ruled out if the trade shares are not too large (Feenstra *et al*, 2009).

$$PPP_j^q = \frac{\sum_{i=1}^{M} p_{ij} q_{ij}}{\sum_{i=1}^{M} \pi_i^o q_{ij}}, \quad PPP_j^x = \frac{\sum_{i=M+1}^{M+N} p_{ij}^x x_{ij}}{\sum_{i=M+1}^{M+N} \pi_i^x x_{ij}}, \quad PPP_j^m = \frac{\sum_{i=M+1}^{M+N} p_{ij}^m m_{ij}}{\sum_{i=M+1}^{M+N} \pi_i^m m_{ij}}, \quad j = 1,\ldots,C. \quad (17)$$

It is apparent that nominal exports and imports in (16) are *not* deflated by a PPP computed over final goods, but rather, are deflated by PPP exchange rates that are specific to exports and imports. Feenstra *et al* (2009) argue that this feature makes real GDP^o an appropriate measure of the real output of countries, whereas real GDP^e is a measure of the standard of living across countries. As these two concepts are defined the in systems (9)-(10) and (12)-(16), however, they require separate normalizations. We resolve this issue in PWT8 by re-defining real GDP^e from (10) by instead using the same reference prices for final goods π_j^o found in (12), obtaining:

$$CGDP_j^e \equiv \sum_{i=1}^{M} \pi_i^o q_{ij} + (X_j - M_j)/PPP_j^q = GDP_j/PPP_j^q. \quad (18)$$

The system (12)-(18) defines both real GDP^e and real GDP^o, and is unique up to a single normalization. It is apparent from our notation that both $CGDP^e$ and $CGDP^o$ are current-price versions of real GDP, because they are using reference prices and PPPs computed in the current year. By definition, they equal constant-price real GDP, $RGDP^e$ and $RGDP^o$, in the benchmark year 2005, but will differ in other years as discussed in the next section.

Real GDP on the expenditure side and output side will differ due to the terms of trade faced by countries. This is apparent by taking the difference between (18) and (17):

$$CGDP_j^e - CGDP_j^o = = \left(\frac{PPP_j^x}{PPP_j^q} - 1 \right) \frac{X_j}{PPP_j^x} - \left(\frac{PPP_j^m}{PPP_j^q} - 1 \right) \frac{M_j}{PPP_j^m}. \quad (19)$$

To simplify this expression, we can divide by $CGDP_j^o$ and re-arrange terms to obtain:

$$\underbrace{\frac{CGDP_j^e - CGDP_j^o}{CGDP_j^o}}_{\text{Gap}} = \underbrace{\frac{1}{2}\left(\frac{PPP_j^x}{PPP_j^q} - \frac{PPP_j^m}{PPP_j^q}\right)}_{\text{Terms of trade}} \underbrace{\left(\frac{X_j / PPP_j^x}{CGDP_j^o} + \frac{M_j / PPP_j^m}{CGDP_j^o}\right)}_{\text{Real Openess}} + \underbrace{\left[\frac{1}{2}\left(\frac{PPP_j^x + PPP_j^m}{PPP_j^q}\right) - 1\right]}_{\text{Traded/Nontraded Price}} \underbrace{\left(\frac{X_j / PPP_j^x}{CGDP_j^o} - \frac{M_j / PPP_j^m}{CGDP_j^o}\right)}_{\text{Real Balance of Trade share}} . \quad (20)$$

We see that the gap between real GDP^e and real GDP^o can be expressed as the sum of two terms: the first is the terms of trade (expressed as a difference rather than a ratio) times real openness; and the second is the relative prices of traded goods (again expressed as a difference) times the real balance of trade. The influence of both these terms on the gap between real expenditure and real output has also been shown by Kohli (2004, 2006) and Reinsdorf (2010), and we will illustrate this relation with some examples from PWT8.0 in section 7.

4. Real GDP over Time

To summarize our results so far, Theorem 1 gave us a compelling reason to use Fisher index quantity indexes to compare real output across countries, or the EKS extension of that method defined in (7)-(8). But as we noted earlier, there is a fundamental difficulty in applying this approach to obtain a comparison of real GDP over time: because there are no explicit reference prices, we cannot control for changes in such prices over time. So instead of the EKS method, PWT8 will use the extended GK system defined in (12)–(18). To relate this extended GK system back to Theorem 1, notice that the ratio of revenue functions in (6) is:

$$\frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)} = \frac{\sum_{i=1}^{M} \pi_i q_{ij}^* + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij}^* - \pi_i^m m_{ij}^*)}{\sum_{i=1}^{M} \pi_i q_{ik}^* + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ik}^* - \pi_i^m m_{ik}^*)} . \quad (21)$$

We use the superscript * to emphasize that the final goods, exports and imports are all evaluated at the *optimal* quantities given the reference prices. In contrast, the ratio of real GDP^o for countries *j* and *k* measured by the extended GK system in (16) is:

$$\frac{CGDP_j^o}{CGDP_k^o} = \frac{\sum_{i=1}^{M} \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij})}{\sum_{i=1}^{M} \pi_i^o q_{ik} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ik} - \pi_i^m m_{ik})}. \tag{22}$$

This measured ratio is evaluated at the *observed* rather than the optimal quantities, which is the principal difference between the theoretical ratio in (21) and its measured counterpart in (22).[8]

In the context of a similar consumer problem, Neary (2004) shows how an *estimated expenditure* rather than revenue function can be used to obtain the theoretical ratio in (21). We believe that it will be much harder to estimate a revenue function over many countries while allowing for technological differences, and for this reason, are willing to accept (22) as an approximation to (21). While this approximation is the cost of using the extended GK system, the great benefit is that we will be able to correct for changing reference prices over time, thereby obtain constant-price real GDP, or real GDP at chained PPPs. In this section we show how such a constant-price comparison can be made in theory and in practice.

We now introduce a subscript *t* on all variables to indicate time. It turns out that we can readily apply Theorem 1 to obtain a consistent comparison of real GDP over space and time. Specifically, suppose that we start in a situation where we have two reference price vectors at two points in time, $\Pi_\tau = \left(\pi_t, \pi_\tau^x, \pi_\tau^m\right)$, $\tau = t\text{-}1, t$, using the reference prices from the GK system extended to include exports and imports. In order to also compare real output over time, it would be desirable to use a single vector Π and compute the ratios:

$$\frac{r_{jt}(\Pi, v_{jt})}{r_{jt-1}(\Pi, v_{jt-1})}, \qquad j = 1,\ldots,C, \tag{23}$$

---

[8] The other difference between (21) and (22) is that the reference prices used in each case can be different.

for each country. Notice that the endowments in (23) can change over time, as well as the revenue function itself due to technological change.

We can apply Theorem 1 by treating the bilateral comparison as between country *j* using reference prices Π_{t-1} and Π_t in the two periods. The optimal outputs at these prices are denoted by $y^*_{j\tau} \equiv \partial r_{j\tau}(\Pi_\tau, v_{j\tau}) / \partial \Pi_\tau, \tau = t-1, t.$ We assume that the time-series analogue of the Gerschenkron effect holds, which states that for country *j*:

$$\left(\frac{\Pi_t' y^*_{jt}}{\Pi_t' y^*_{jt-1}} \right) < \left(\frac{\Pi_{t-1}' y^*_{jt}}{\Pi_{t-1}' y^*_{jt-1}} \right). \tag{24}$$

Again, we interpret (24) as stating that the Laspeyres quantity index (on the right) exceeds the Paasche quantity index (on the left). This inequality cannot hold for the optimal quantities obtained from a revenue function *in the absence of* changes in endowments or technology, since in that case the goods with rising relative prices will also have rising quantities, and the left side of (24) will exceed the right. But as Gerschenkron (1951) found when comparing countries, the time-series evidence within a country also supports the idea that goods with the greatest increase in quantity (due to changing endowments or technology) have falling relative prices, so that (17) holds. Then an immediate corollary of the earlier theorem is obtained by changing the notation to compare time periods rather than countries, as follows:

Corollary 1

Suppose that the outputs are revenue-maximizing and the Gerschenkron effect in (24) holds. Then there will exist reference prices Π between Π_{t-1} and Π_t such that:

$$\frac{r_{jt}(\Pi, v_{jt})}{r_{jt-1}(\Pi, v_{jt-1})} = \left[\left(\frac{\Pi_{t-1}' y^*_{jt}}{\Pi_{t-1}' y^*_{jt-1}} \right) \left(\frac{\Pi_t' y^*_{jt}}{\Pi_t' y^*_{jt-1}} \right) \right]^{0.5}. \tag{25}$$

In words, this result states that we can take the geometric mean of the growth rates obtained at the reference prices of each period to obtain a constant-reference-price growth rate. To see the usefulness of this result, suppose that instead of using the *optimal* quantities in (25) we apply this formula using *observed* quantities, obtaining the growth rate of constant-price real GDP:

$$\left(\frac{RGDP^o_{jt}}{RGDP^o_{jt-1}}\right) \equiv \left[\left(\frac{\Pi'_{t-1}y_{jt}}{\Pi'_{t-1}y_{jt-1}}\right)\left(\frac{\Pi'_t y_{jt}}{\Pi'_t y_{jt-1}}\right)\right]^{0.5}$$
$$= \left[\left(\frac{\sum_i \pi^o_{it-1}q_{ijt} + \pi^x_{it-1}x_{ijt} - \pi^m_{it-1}m_{ijt}}{\sum_i \pi^o_{it-1}q_{ijt-1} + \pi^x_{it-1}x_{ijt-1} - \pi^m_{it-1}m_{ijt-1}}\right)\left(\frac{\sum_i \pi^o_{it}q_{ijt} + \pi^x_{it}x_{ijt} - \pi^m_{it}m_{ijt}}{\sum_i \pi^o_{it}q_{ijt-1} + \pi^x_{it}x_{ijt-1} - \pi^m_{it}m_{ijt-1}}\right)\right]^{0.5}. \qquad (26)$$

Thus, the quantities of final goods, exports and imports change from t–1 to t in both ratios, and are evaluated using the reference prices from one period or the other, and then taking the geometric mean. PWT8 uses the growth rates from this formula to compute constant-price real GDP^o in all years other than the 2005 benchmark, where in that benchmark year the extended GK system (12)-(18) is used with $RGDP^o = CGDP^o$.

In addition, the constant-price growth rates of real GDP^e are obtained by using only the reference prices π^o_{t-1} and π^o_t of the M final consumption goods. $RGDP^e = CGDP^e$ is defined by (18) in the benchmark year 2005, and its growth rate to other years is obtained as:

$$\left(\frac{RGDP^e_{jt}}{RGDP^e_{jt-1}}\right) \equiv \left[\left(\frac{\sum_i \pi^o_{it-1}q_{ijt} + \frac{X_{jt}}{PPP^q_{jt-1}} - \frac{M_{jt}}{PPP^q_{jt-1}}}{\sum_i \pi^o_{it-1}q_{ijt-1} + \frac{X_{jt-1}}{PPP^q_{jt-1}} - \frac{M_{jt-1}}{PPP^q_{jt-1}}}\right)\left(\frac{\sum_i \pi^o_{it}q_{ijt} + \frac{X_{jt}}{PPP^q_{jt}} - \frac{M_{jt}}{PPP^q_{jt}}}{\sum_i \pi^o_{it}q_{ijt-1} + \frac{X_{jt-1}}{PPP^q_{jt}} - \frac{M_{jt-1}}{PPP^q_{jt}}}\right)\right]^{0.5}. \qquad (27)$$

Notice that in (27) we deflate nominal exports and imports by PPP^q_{jt-1} and PPP^q_{jt}, computed from the reference prices for final goods as in (17). This is in contrast to (26) where the actual reference prices of exports and imports are used.

5. Data, Interpolation and Normalization

While the above sections give an accurate theoretical description of the calculations in PWT8, we should clarify how these calculations are implemented and especially how the growth rates of real GDP^e and real GDP^o make use of *multiple years* of ICP data. Like past versions of PWT, we begin with aggregation of ICP data using the EKS technique. The Fisher price indexes in (6) are computed over the prices of final goods obtained from the ICP, within the categories of consumption, investment, and government expenditures. Then the EKS transformation in (7) is applied to obtain the price indexes P_{jt}^C, P_{jt}^I, P_{jt}^G for each country relative to the U.S., so that M =3 is the number of final goods used in the extended GK system.

Notice that this aggregation procedure for final goods can be done for the set of countries in each year that ICP prices are available, i.e. 1970, 1975, 1980, 1985, 1996, 2005. There is an expanded set of countries available from the ICP in each benchmark, and in total, 167 countries are used in one benchmark or another. That becomes the set of countries included in PWT8 (this set will expand as more countries are included in future benchmarks). For each country, we keep track of which benchmarks were used: those *benchmark* years are denoted by B; years in-between benchmarks will have the prices for final goods *interpolated* using the corresponding price trends from national accounts data, and are denoted by I; and for years before the first or after the last benchmark for each country the prices of final goods are *extrapolated* using national account data, and are denoted by E.

We should explain exactly how the interpolation procedure is done, since in it we are reconciling cross-country benchmarks in two years with (possibly inconsistent) price trends from national accounts data. Consider the case of India, which was included in all ICP benchmarks except for 1996, and for illustration purposes, focus on household consumption. We apply an

EKS aggregation procedure to compute the relative price of household consumption in India (and every other country included in the ICP) relative to the U.S. in each benchmark year, obtaining $P^{C}_{IND,t}$. In the years after 2005, we have no further benchmarks, so we extrapolate forward using the trend in consumption prices from the national accounts:

$$P^{C}_{IND,2006} = P^{C}_{IND,2005} \times \frac{P^{CPI}_{IND,2006}}{P^{CPI}_{IND,2005}},$$

where the last ratio in this expression is taken from the Indian CPI data. For years in-between the 1985 and 2005 benchmarks, we extrapolate forward from 1985 and backward from 2005, using appropriate weights to make this interpolation for $1985 \le t \le 2005$:

$$P^{C}_{IND,t} = P^{C}_{IND,1985} \times \frac{P^{CPI}_{IND,t}}{P^{CPI}_{IND,1985}}\left(\frac{2005-t}{2005-1985}\right) + P^{C}_{IND,2005} \times \frac{P^{CPI}_{IND,t}}{P^{CPI}_{IND,2005}}\left(\frac{t-1985}{2005-1985}\right).$$

This approach to interpolation and extrapolation is similar in spirit to the approach of Rao et al (2010), who also discuss a method for estimating PPPs for a full set of years and countries using benchmark PPPs and National Accounts deflators. The key distinction is that we always force the price series to be equal to the benchmark estimates, while this is a special case of Rao et al (2010); see also Hill (2004).

For traded goods, the prices used are actually quality-adjusted unit values obtained from Feenstra and Romalis (2012). These prices are available in every year from 1984 to 2007, and then EKS aggregation is used to obtain the prices of exports and imports for every country relative to the U.S. in the six Broad Economic Categories, i.e. food and beverages, other consumer goods, capital, fuels, intermediate inputs, and transport equipment (while excluding miscellaneous goods). So N=6 in the extended GK system, and there is no need to interpolate in-between any years. Before 1984 and after 2007, however, we extrapolate these export and import prices using the price movement in aggregate exports and imports from national accounts data.

The prices for final goods, exports and imports can then used in the extended GK system (12)-(18) for each year, to compute reference prices. A normalization is needed, and denoting the national accounts GDP deflator for the United States by $P_{US,t}^{GDP}$, we normalize each year by:

$$PPP_{US,t}^{o} = P_{US,t}^{GDP}, \quad t = 1950, \ldots, 2011, \tag{28}$$

which states that the PPP for GDP in the U.S. equals GDP deflator from national accounts, relative to 2005. Since the PPP for the United States is used to deflate nominal U.S. GDP in (16), this normalization implies that all reference prices are in 2005 U.S. dollars and therefore correct for U.S. inflation. By using the resulting reference prices for each year in the calculation of $CGDP^o$ and $CGDP^e$ in (16) and (18), we obtain current-price real GDP. As just noted, these values correct for U.S. inflation but are still using different reference prices each year. They allow for a consistent comparison of real GDP across countries, but as we have stressed, they do not allow for a consistent comparison across time because of changing reference prices.

To achieve a consistent comparison across countries and time, we apply the growth rates in (28) and (29) to compute $RGDP^o$ and $RGDP^e$, respectively, in all years other than the 2005 benchmark. In addition, PWT8 provides a third measure of real GDP that equals $CGDP^o$ and $RGDP^o$ in the 2005 ICP benchmark and then uses national-accounts growth rates for real GDP to extrapolate to all other years, obtaining $RGDP^{NA}$. PWT8 also includes the nominal national accounts data on which our calculations are based. There can be substantial changes to these national accounts data over time, which will be the principal source of changes to any of our *interpolated* values for the growth of $RGDP^o$ and $RGDP^e$ as new versions of PWT become available. That is, our methods guarantee in principal that the growth of $RGDP^o$ and $RGDP^e$ will not change in-between existing benchmark years even as new benchmarks become available, unless the underlying national accounts data or the quality-adjusted trade prices are revised. This "invariance of growth rates between benchmarks" was not previously the case in PWT, as discussed by Johnson *et al* (2013), where new ICP benchmarks often led to a change in real GDP growth rates for all prior years. This invariance has been achieved here by virtue of our interpolation procedures. That invariance comes at a cost, however: because we interpolate between multiple ICP benchmarks, there is no guarantee that the growth rates of $RGDP^{o}$ and $RGDP^{e}$ will necessarily be close the national accounts growth rate, $RGDP^{NA}$. So except for this latter series, we violate a long-standing tenet of PWT whereby the growth rates should be "similar" to those in the national accounts.[9] Our growth rates of $RGDP^{o}$ and $RGDP^{e}$ can be quite different from that in national accounts if that is what is indicated by ICP data, as we shall illustrate in section 7.

6. Total Factor Productivity

PWT version 8 will re-introduce a capital stock series for countries up to 2011, which was last available for 1990. The calculation of the capital stock as well as the measurement of labor input, human capital and the share of labor income in GDP is given in Inklaar and Timmer (2013). In this section, we extend our earlier theoretical discussion to show how total factor productivity can be computed, across countries and over time. We rely heavily on the results of Caves, Christensen and Diewert (CCD, 1982a,b) and Diewert and Morrison (DM, 1986).

We drop the time subscript and return to the ratio of revenue functions given in (3), $r_j(\Pi, v_j)/r_k(\Pi, v_k)$, which measures real output in country *j* relative to *k*. Real output can vary due to differing factor endowments, as indicated by v_{lj} and v_{lk} for factors $l = 1,\ldots,L$, or due to differing technologies, as indicated by the country subscript *j* and *k* on the revenue function. We

[9] As discussed in note 2, versions of PWT up to v6 constructed real GDP using a weighted average of the national accounts growth rates of the *components* of GDP. That is why the growth rate of real GDP did not exactly match the growth rate from the national accounts.

can isolate the effect of technological differences alone by considering two alternative ratios:

$$A_j \equiv \frac{r_j(\Pi, v_j)}{r_k(\Pi, v_j)}, \quad \text{and} \quad A_k \equiv \frac{r_j(\Pi, v_k)}{r_k(\Pi, v_k)}.$$

Both of these ratios measure the overall productivity of country *j* to country *k*, holding fixed the level of factor endowments. Neither ratio can be measured directly from the data, however, because the numerator or the denominator involves a revenue function that is evaluated with the technology of one country but the endowments of the other. But the results of CCD and DM tell us that if the revenue function has a translog functional form, then we can precisely measure the geometric mean of these two ratios:

<u>Theorem 2</u>

Assume that the revenue functions $r_j(\Pi, v_j)$ and $r_k(\Pi, v_k)$ are both translog functions that are homogeneous of degree one in Π and in v_j and have the same coefficients on factor endowments, but can have different coefficients on prices and on interaction terms due to technological differences across countries. Then the overall productivity of country *j* relative to *k* can be measured by:

$$\left(A_j A_k\right)^{1/2} = \frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)} / Q_T(v_j, v_k, w_j^*, w_k^*), \tag{29}$$

where $Q_T(v_j, v_k, w_j^*, w_k^*)$ is the Törnqvist quantity index of factor endowments, defined by:

$$\ln Q_T(v_j, v_k, w_j^*, w_k^*) = \sum_{l=1}^{L} \frac{1}{2}\left(\frac{w_{lj}^* v_{lj}}{\sum_m w_{mj}^* v_{mj}} + \frac{w_{lk}^* v_{lk}}{\sum_m w_{mk}^* v_{mk}}\right) \ln\left(\frac{v_{lj}}{v_{lk}}\right), \tag{30}$$

and where $w_{lj}^* = \dfrac{\partial r_j(\Pi, v_j)}{\partial v_{lj}}$, $w_{lk}^* = \dfrac{\partial r_k(\Pi, v_k)}{\partial v_{lk}}$ are the factor prices using goods prices Π.

CCD establish a result like Theorem 2 using the distance function, whereas DM establish an analogous result using a time-series rather than cross-country comparison. For completeness, we include a proof in the Appendix which clarifies how the coefficients of the factor endowments must be the same for countries *j* and *k*, but other coefficients can differ.

Theorem 2 tells us that by deflating the observed difference in real GDP^o by the Törnqvist quantity index of factor endowments, we obtain a meaningful measure of the productivity difference between the countries. The Törnqvist quantity index is constructed using the factor prices that are implied by the reference prices for goods Π . In practice we do not observe these factor prices, and so we replace the theoretical expressions in (29)-(30) with versions that we can measure from the data:

$$CTFP_{jk} \equiv \frac{CGDP_j^o}{CGDP_k^o} / Q_T(v_j, v_k, w_j, w_k), \tag{31}$$

where we use $CTFP_{jk}$ to denote the (current-price) productivity of country *j* relative to *k*, and the Törnqvist quantity index of factor endowments is evaluated with observed factor prices and shares. PWT8 includes $CTFP_{jk}$ computed with current reference prices for each country *j* relative to the United States. Combining (31) with (20), we obtain a decomposition of current-price real GDP on the expenditure side:

$$\frac{CGDP_j^e}{CGDP_k^e} = CTFP_{jk} \times Q_T(v_j, v_k, w_j, w_k) \times \left(\frac{1+Gap_j}{1+Gap_k} \right), \tag{32}$$

where the *Gap* between real GDP^e and real GDP^0 is defined by the various terms-of-trade and balance-of-payments expressions on the right of (20). We report summary statistics from this decomposition in the next section.

An analogous expression can be obtain for the *productivity growth* in each country, which is defined by re-introducing time subscripts and using constant-price real GDP:

$$RTFP_{j,t,t-1}^{NA} \equiv \frac{RGDP_{jt}^{NA}}{RGDP_{jt-1}^{NA}} / Q_T(v_{jt}, v_{jt-1}, w_{jt}, w_{jt-1}). \tag{33}$$

PWT8 also reports $RTFP_{j,t,t-1}^{NA}$ computed with constant-price real GDP growth rates from the national accounts.

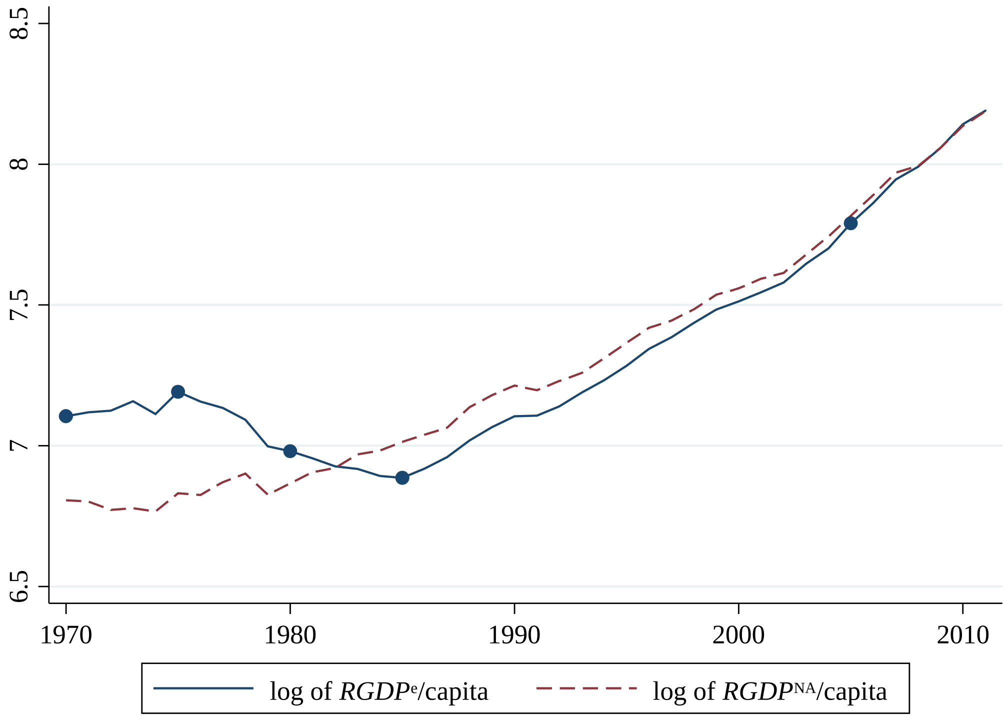
7. Results from PWT version 8.0

One of the main consequences of using computing real GDP based on multiple benchmark comparisons is that the change in $RGDP^{o}$ or $RGDP^{e}$ can be very different from the change in $RGDP^{NA}$. To illustrate this point, Figure shows (the log of) $RGDP^{e}$ per capita and $RGDP^{NA}$ per capita for India since 1970, the year of the first ICP benchmark, in which India participated. This benchmark year and the other four in which India participated are indicated by dots. The figure shows that while $RGDP^{NA}$ has grown almost continuously over the 1970-2005 period, comparative living standards as measured by $RGDP^{e}$ has not. Between 1975 and 1985, comparative living standards declined at an average pace of 0.7% per year, while $RGDP^{NA}$ increased by an average annual rate of 4.2%.[10]

This result indicates that between 1975 and 1985, the overall PPP for India computed from the ICP benchmark prices increased at a much more rapid pace than indicated by the relative inflation rates (from national accounts) of India and the U.S. That finding could reflect measurement error in the ICP benchmark prices and therefore in the PPPs, and/or in the inflation rates. However, these differences may well be informative. Deaton (2012) discusses this problem

[10] Though not shown, Figure 1 would look very similar if $RGDP^{o}$ had been graphed instead of $RGDP^{e}$.

Figure 1: Real GDP per capita in India since 1970, *RGDP^e* vs. *RGDP^NA*

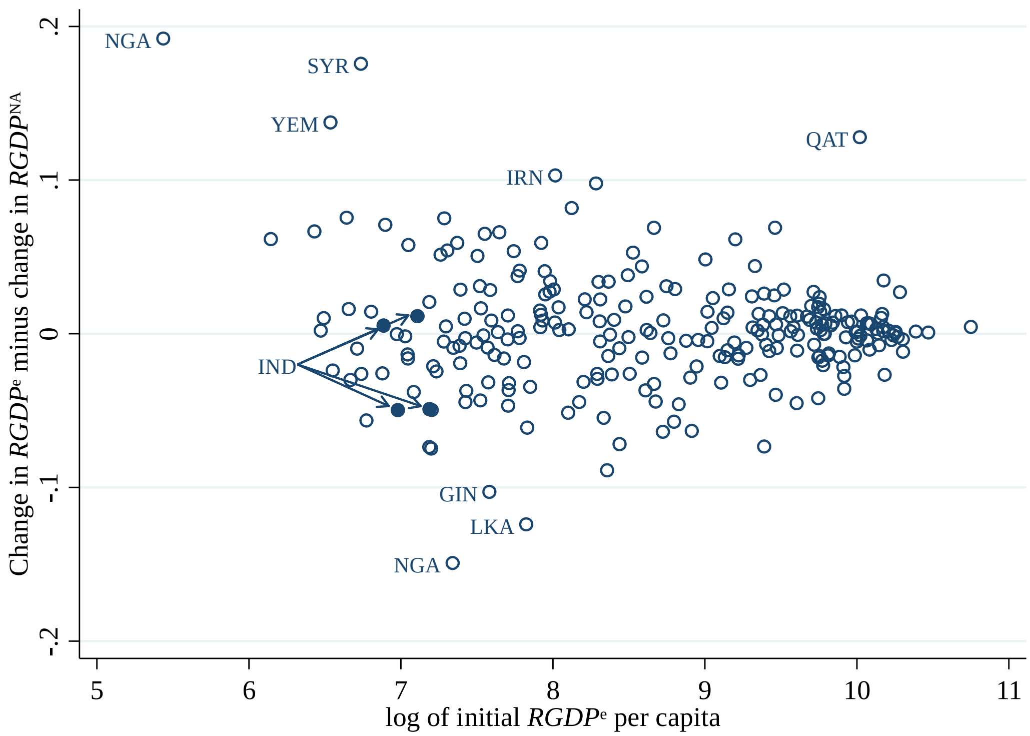


Note: the dots on the 'log of $RGDP^e$/capita' line indicate the ICP benchmark years in which India participated.

in general and emphasizes how inflation and changes in PPPs can be very different because the aim in measurement is different. Inflation only tracks domestic price changes and should thus only take domestic budget shares into account. In contrast, PPPs aim to compare prices across countries, which requires taking budget shares from multiple countries into account. Deaton (2012) shows that if budget shares and inflation rates for different products differ across countries and over time, the type of discrepancy shown in Figure can easily arise. It is thus important to conceptually distinguish the change in $RGDP^{NA}$, measuring economic growth, from the change in $RGDP^e$, which measures the change in comparative living standards, or $RGDP^o$, which measures the change in comparative productive capacity.

The difference shown in Figure is not unique to India and depending on the country and time period, $RGDP^e$ and $RGDP^o$ can change at a faster or slower pace than $RGDP^{NA}$. In Figure 2, we compute the average annual change in real GDP between each set of ICP benchmark observations for $RGDP^e$ and $RGDP^{NA}$ and plot the difference between these for different levels of $RGDP^e$ per capita. The first observation is that differences are often substantial and the differences seen for India in Figure are not atypical. Furthermore, the differences are smaller at higher levels of $RGDP^e$ per capita. This finding could occur because higher-income countries are more similar in terms of expenditure shares and product-level inflation rates, thus reducing the conceptual gap between relative inflation rates and changes in PPPs. Alternatively, measurement errors in the original ICP price surveys could be less for higher-income countries.

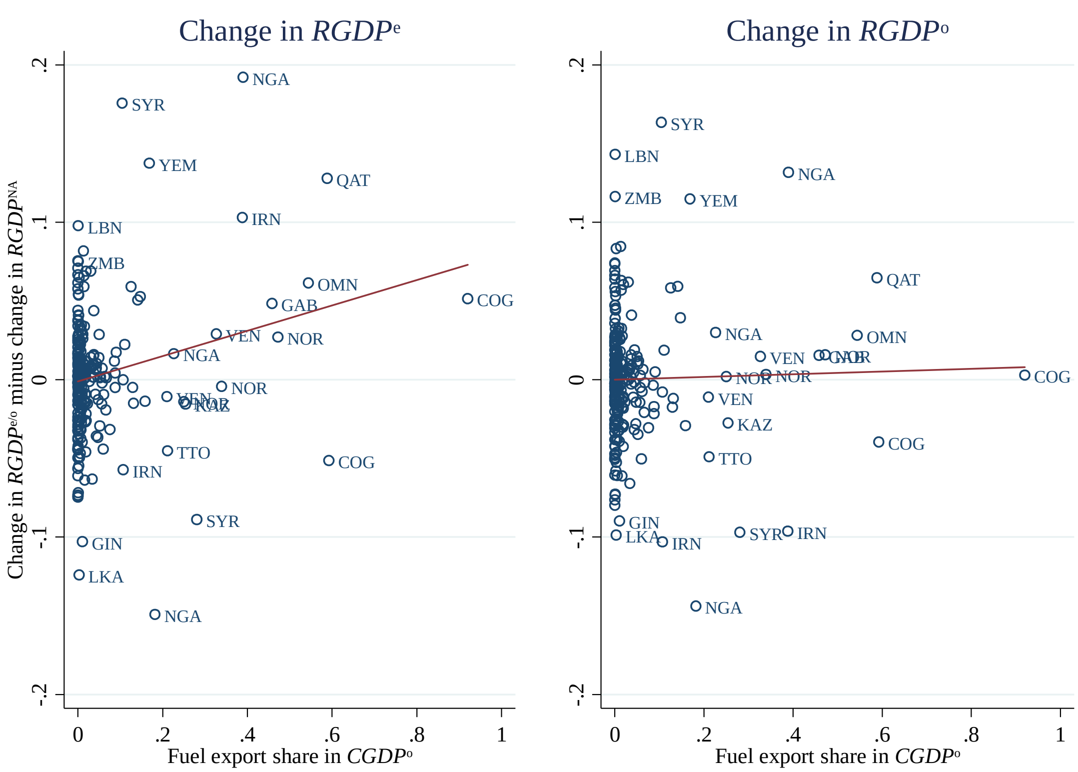
Figure 2: Difference in average annual real GDP changes between ICP benchmarks and levels of *RGDPe* per capita



Notes: Every observation denotes on the y-axis the difference between average annual change in $RGDP^e$ and $RGDP^{NA}$ between two consecutive ICP benchmarks. So for India, the four included observations are for growth between 1970 and 1975, 1975 and 1980, 1980 and 1985, and 1985 and 2005. On the x-axis, the initial level of $RGDP^e$ per capita is shown, so India's 1975 level with the difference in *RGDP* change between 1975 and 1980.

Although there is no systematic relationship from Figure 2 between the difference in *RGDP*^{e} and *RGDP*^{NA} growth and the initial *RGDP*^{e} per capita level, Figure 3 shows that there are signs of a relationship depending on the importance of oil in the economy. The left-hand panel plots the same differences between the change in *RGDP*^{e} and *RGDP*^{NA} between benchmarks, but now against the share of fuel exports in *CGDP*^{o}. The left panel shows that oil-exporting countries tend to have faster increases in *RGDP*^{e} than in *RGDP*^{NA}. The right-hand panel shows no such positive relationship when comparing the change in *RGDP*^{o} to *RGDP*^{NA}, which indicates that those oil-exporting countries benefited from improvements in their terms of trade, rather than from improvements in productive capacity.

Figure 3: Difference in average annual real GDP changes between ICP benchmarks and fuel export shares



Notes: See the notes to Figure 2 for a discussion of the variable on the y-axis. On the x-axis is the share of fuel exports in *CGDP*^{o}, variable csh_x3 in the detailed trade data file available on the PWT website.

We now turn to the decomposition of differences across countries in $CGDP^e$ per capita from equation (32). That decomposition can be performed for each country and year and, in line with earlier work by Hall and Jones (1999), Klenow and Rodriguez-Clare (1997) and Caselli (2005), the variation in $CGDP^e$ per capita can then be decomposed into the variance of each of the components in equation (32) and their covariances.

Table 1: Decomposing the cross-country variation in $CGDP^e$ per capita

	2005			1996	1988
	Baseline	α=.3	α=.3 & simple k	α=.3 & simple k	α=.3 & simple k
var(log($CGDP^e$))	1.542	1.542	1.542	1.243	1.106
var(log(*Q*))	0.485	0.332	0.362	0.334	0.357
var(log(*CTFP*))	0.398	0.512	0.474	0.381	0.304
var(log(1+*Gap*))	0.005	0.005	0.005	0.003	0.002
covar(log(*Q*),log(*CTFP*))	0.323	0.343	0.347	0.248	0.210
covar(log(*Gap*),log(*CTFP*))	0.000	0.000	-0.001	0.005	0.006
covar(log(*Gap*),log(*Q*))	0.004	0.004	0.005	0.008	0.006
Variance explained by factor inputs	0.314	0.215	0.235	0.269	0.323
Number of countries	111	111	111	110	94

Notes: α is the share of capital income in GDP, which varies by country and year in the baseline. 'simple k' indicates that the capital stock is computed using information on total investment and assuming a constant depreciation rate of 6%. In the baseline, investment in up to 6 assets is distinguished and depreciation rates vary by asset. 'Variance explained by factor inputs' is computed as var(log(*Q*))/var(log($CGDP^e$)).

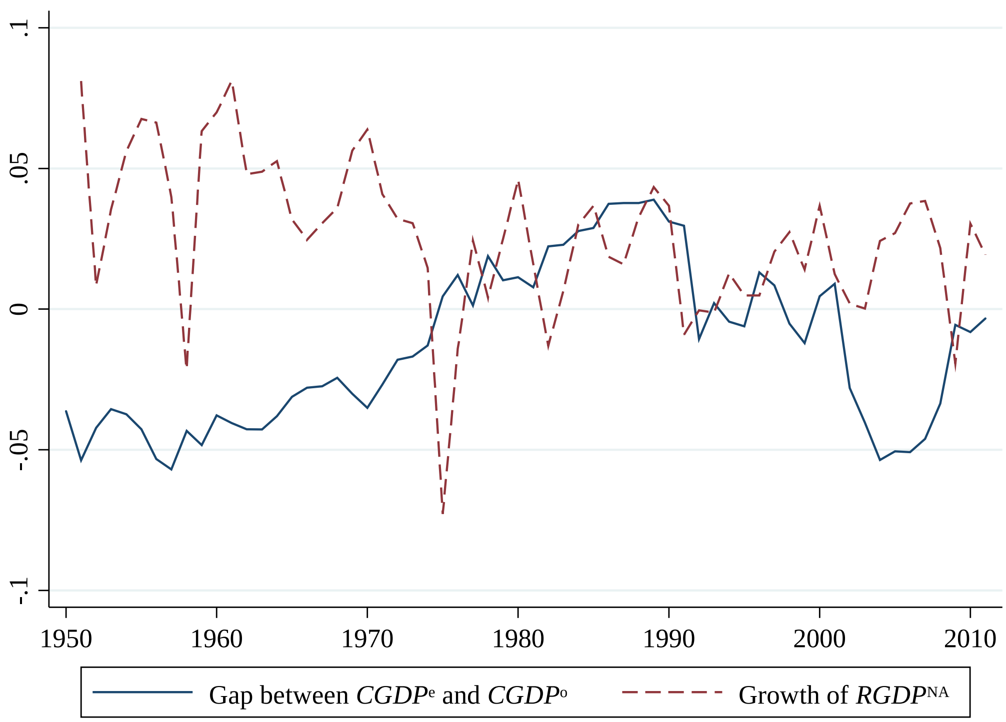
Table 1 shows the results of this decomposition for a number of cases. The first column uses the data on factor inputs as given in PWT8 for 2005, i.e. using estimates of the capital and labor share and accounting for differences in the asset composition of the capital stocks (see Inklaar and Timmer, 2013). The cross-country variance of $CGDP^e$ is 1.542 and nearly all of this is explained by the variance of factor inputs, the variance of *CTFP* and the covariance between these two. The "success" measure of Caselli (2005) is the share of variance in $CGDP^e$ that is explained by the variation of factor inputs, and in the baseline case this share is 0.314, reported near the bottom of the first column.

The subsequent two columns in Table 1 simplify the capital measure, by first assuming a common capital share of 0.3 in column 2 and then by assuming that the capital stock consists of a single asset that depreciates at 6 percent a year in column 3. The move from country-specific to a common capital share greatly reduces the cross-country variation that can be explained by factor inputs. The final two columns show the same decomposition as in column 3 but in earlier years, to match earlier decomposition analyses for 1996 (Caselli, 2005) and 1988 (Hall and Jones, 1999). Factor inputs explain less of the cross-country variation in $CGDP^e$ per capita in 2005 than in previous years, and the variance explained in those earlier years is smaller than in the earlier studies: Caselli (2005) found that factor inputs explained almost 40 percent of variation in GDP per worker in both 1988 and 1996. Since the method for computing factor inputs in column 3-5 matches the earlier studies, this difference in outcome could reflect a different country coverage or data revisions, such as in the educational attainment data of Barro and Lee (2010). The finding that factor inputs as included in PWT8.0 explain more of the cross-country variation in $CGDP^e$ per capita than simpler capital measures is found across all years, though. The fact that this is driven by the use of country-specific capital shares confirms Caselli's (2005) argument that such results are sensitive to this measure. Furthermore, the finding of Inklaar and Timmer (2013) that, in contrast to Gollin (2002), labor shares vary considerably across countries and over time is further reason to use these country-specific labor and capital shares in PWT8.0.

The other main finding in Table is that the *Gap* between $CGDP^e$ and $CGDP^o$, reflecting differences in terms of trade and the trade balance, explains almost nothing of the cross-country variation in $CGDP^e$ per capita: the variance is relatively small and the covariances are also close to zero. The finding that the covariance between the *Gap* and *CTFP* is almost zero is consistent with the argument by Kehoe and Ruhl (2008) that terms of trade shocks are very different from productivity shocks: the data in PWT8.0 show that there is little to no systematic correlation between these measures across countries.

The gap is between $CGDP^e$ and $CGDP^o$ can still be of interest for some countries and years, however. To illustrate, Figure 4 shows the *Gap* for Switzerland and the growth of $RGDP^{NA}$ between 1950 and 2011. While $RGDP^{NA}$ declined substantially beginning in 1975, living standards held up considerably better as indicated by the increase in the gap. This example illustrates the differing opinions of Kehoe and Prescott (2002) and Kehoe and Ruhl (2005), who characterize of the Swiss economy as being in a great depression over the period 1974-2000, versus Kohli (2004), who focuses on the improving terms of trade between 1980 and 1996. Although this terms of trade gain was fortuitous for the Swiss, a relationship between strong terms of trade (measured by the *Gap*) and low growth of $RGDP^{NA}$ is not found for all years either

Figure 4: Gap between $CGDP^e$ and $CGDP^o$ and $RGDP^{NA}$ growth in Switzerland, 1950-2011



in Switzerland or systematically for the dataset as a whole. Still, this figure helps to illustrate that there can be substantial deviations between $CGDP^e$ and $CGDP^o$ that imply meaningful differences between output and the standard of living for some countries and periods.[11]

As discussed earlier, PWT8.0 makes a distinction between observations based on *benchmark or interpolated* PPP estimates and PPP estimates based on *extrapolations* using information on national accounts price trends. This distinction turns out to be important for estimates of the "Penn effect", i.e. the positive relationship between the log of a country's price level (its PPP over its exchange rate) and the log level of *CGDP* per capita. Named as such by Samuelson (1994), the effect inspired the work by Balassa (1964) and Samuelson (1964) who hypothesized that productivity increases in the traded sector and stagnant productivity in the non-traded sector would lead to increases in prices. More recently, Bergin, Glick and Taylor (2006) found that there was no evidence of a Penn effect in the early 1950s, but that effect gradually became significant and strengthened over time. Their analysis was based on PWT version 6 and we revisit it using version 8.0. Consider the following model:

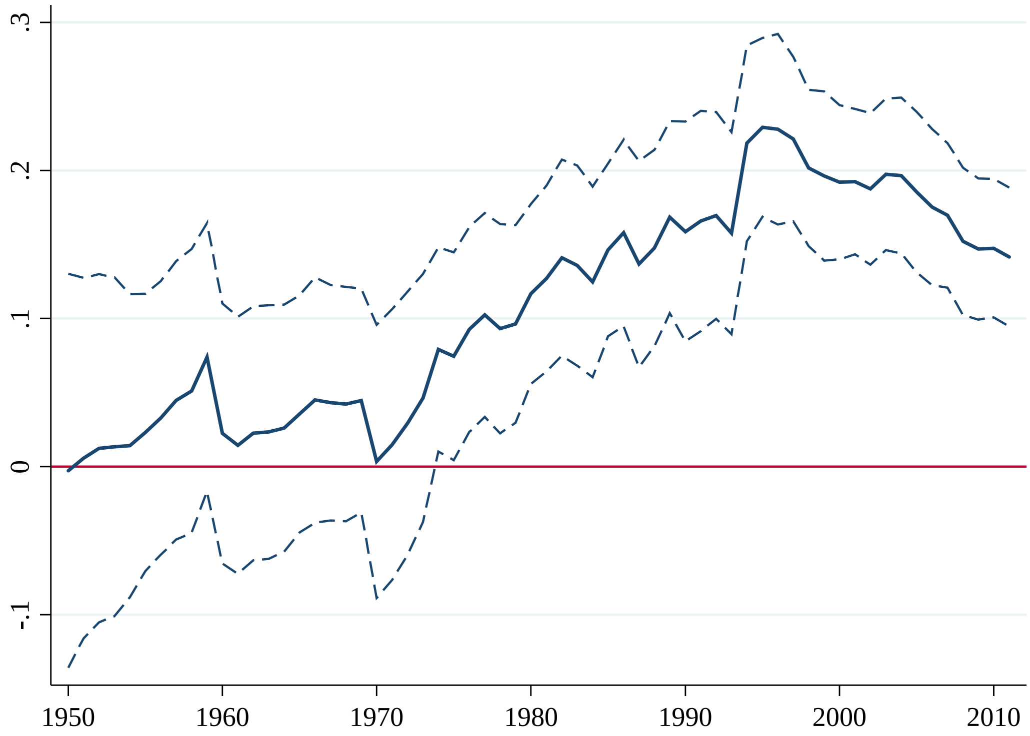
$$\log\left(\frac{PPP_{it}^e}{XR_{it}}\right) = \beta_0 + \beta_{1t}\log\left(\frac{CGDP_{it}^e}{POP_{it}}\right) + \varepsilon_{it}\,, \tag{34}$$

where *XR* is the exchange rate and *POP* is the population of country *i* at time *t*.[12] For each year, a separate cross-sectional regression is run and coefficient β_{1t} and its 95% confidence interval is reported in Figure 5. The figure shows a qualitatively similar pattern as in Bergin et al. (2006), with a low and insignificant coefficient in the early years, which rises steadily and becomes

[11] Australia has benefited from an increase in mineral prices since 2003, as discussed by Gregory (2012). When Figure 4 is constructed for Australia, it indeed shows a rising *Gap* after 2003.

[12] Bergin et al. (2006) divide the country's GDP per capita level by the U.S. level in every year, but this only affects the estimate of β_0. Also note that sometimes the exchange-rate-converted GDP per capita level is used as the explanatory variable instead of the PPP-converted GDP per capita level. We follow the approach of Bergin et al. (2006), which was also advocated by Officer (1982). Officer also argued that a productivity measure would be preferable to a GDP per capita level. Results using $CGDP^o$ per capita or *CTFP* are very similar.

Figure 5: The Penn effect in PWT8.0, 1950-2011



Notes: The figure plots β_{1t} from equation (34) and its 95% confidence interval for all observations in PWT8.0. Each β_{1t} is from a cross-sectional regression for all observations in year *t*. Excluded are those observations for which, due to extreme swings, the market exchange rate is replaced by an exchange rate based on a relative PPP assumption.

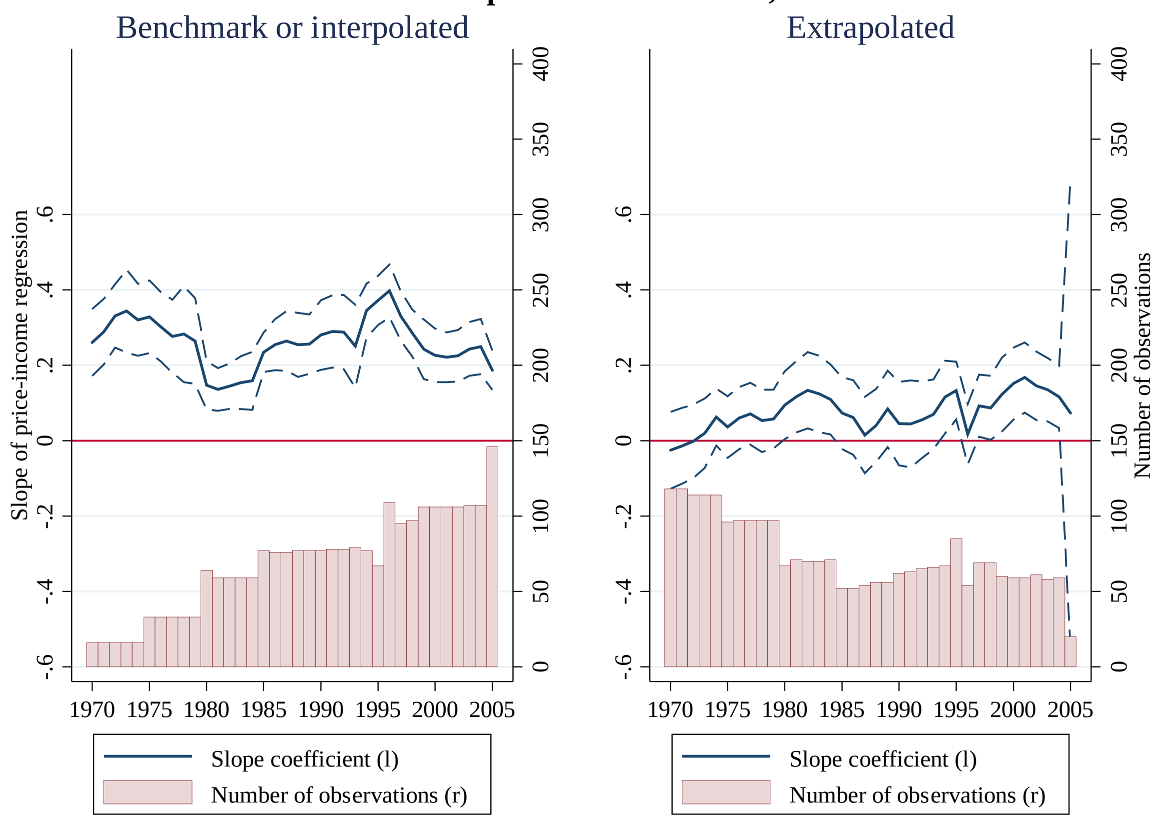
highly significant in later years. The main substantive difference is that it takes until 1974 before the Penn effect coefficient turns significantly positive, versus the mid-1950s in Bergin et al. (2006).[13] This is a challenging finding, as the Penn effect was already identified in data for the 1950s and 1960s, in e.g. Balassa (1964), therefore raising the question why the effect shows up so much later in PWT8.0.

To determine what is driving the result in Bergin et al. (2006) and in Figure 5, we distinguish between observations from PPP benchmarks or interpolated between benchmarks on the one hand, and observations that are extrapolated on the other hand. Figure 6 shows the result

[13] The same analysis based on PWT7.1 shows a qualitatively similar pattern, but the coefficient does not turn significantly positive until 1988.

from the same cross-sectional regressions as in Figure 5, but run separately on the two subsamples. Note that there are no benchmark observations before 1970 and that the latest global comparison was in 2005. Figure 6 shows that in the *benchmark/interpolated* sample there is a consistently positive and significant Penn effect of 0.3 on average, while in the *extrapolated* sample, the estimated Penn effect is rarely significantly different from zero. As the bars at the bottom of each panel indicate, the number of observations in the benchmark/interpolated sample increases, from 16 countries in 1970 to 146 countries in 2005, while the reverse is the case for the 'extrapolated' sample.

Figure 6: The Penn effect in PWT8.0 for benchmark or interpolated and for extrapolated observations, 1970-2005



Note: the figure plots β_{1t} from equation (34) and its 95% confidence interval for all observations in PWT8.0. Each β_{1t} is from a cross-sectional regression in year *t*. The left-hand panel only includes observations from PPP benchmarks or interpolated between PPP benchmarks. The right-hand panel includes observations extrapolated from PPP benchmarks using inflation rates, i.e. all other observations. Excluded are those observations for which, due to extreme swings, the market exchange rate is replaced in PWT8.0 by an exchange rate based on a relative PPP assumption.

The pattern in Figure 5 can thus be best understood as a weighted average between a positive and significant coefficient in the benchmark/interpolated sample and a zero coefficient in the extrapolated sample. As the weight of the benchmark/interpolated sample increases over time, the overall coefficient increases and turns significant. The Bergin et al. (2006) result is thus the result of the extrapolation methodology used exclusively in earlier versions of PWT and partially in PWT8.0, rather than an economically relevant phenomenon. By distinguishing between benchmark/interpolated/extrapolated observations in PWT8, we are thus able to show how the Penn effect is always valid in in-between benchmarks. This finding reinforces the importance of incorporating historical benchmark material, since an economically relevant phenomenon like the Penn effect would otherwise disappear from the data when going back further in time.

8. Conclusions

From its inception, the ICP only collected the prices of final products – for consumption, investment and the government – across countries. It was prohibitively expensive to further collect comparable prices for the whole range of industrial and intermediate inputs used in economies, many of which are also traded. This limitation means that the PWT calculations based on ICP prices are best thought of as representing the standard of living of countries rather than their production possibilities. Feenstra *et al* (2009) argued that a measure of the productive capacity of countries could be obtained by combining the ICP data with prices for exports and imports. These two approaches lead to real GDP on the expenditure-side and real GDP on the output-side, respectively, both of which will be included in PWT version 8.0.

The second contribution of PWT8 will be to improve upon the growth of real GDP previously reported in PWT, which was based on national accounts data. Johnson *et al* (2013) criticized that growth rate as being dependent on the benchmark year of ICP data, and thereby dependent on the version of PWT being used. That problem is resolved in two ways: (i) by including the series $RGDP^{NA}$ that uses the 2005 benchmark and then the growth rates of real GDP from the national accounts to extrapolate to all other years; (ii) by using *multiple* ICP benchmarks to construct $RGDP^{e}$ and $RGDP^{o}$. Under this second approach, the relative living standards or productive capacity of a country can change at a different pace than implied by national accounts GDP growth. India, for example, is found to have a higher standard of living in its 1975 ICP benchmark than predicted from the 1985 benchmark and back-casting using the growth of national accounts prices. It follows that the change in real GDP from 1975 onwards is correspondingly reduced. We have shown that incorporating multiple ICP benchmarks also ensures that relationships such as the Penn effect remain apparent in the dataset, rather than disappearing when going back further in time.

The final contribution of PWT8 is to reintroduce a measure of the capital stock and, for the first time, include a measure of relative TFP across countries. We have shown that, compared to standard approaches in the literature, cross-country variation in factor inputs can account for more of the cross-country variation in $CGDP^{e}$ per capita. This is mostly because PWT8.0 incorporates new estimates of the labor share in GDP that vary in a meaningful fashion across countries and over time.

Taken together, these contributions show that PWT version 8 breaks new ground in providing a cross-country dataset that is broader, more consistent over time and more transparent in its methods. To facilitate this, we have also made available a User’s Guide in Feenstra, Inklaar and Timmer (2013). We hope that this new version provides a fresh impetus for research aiming to explain cross-country differences in economic performance and living standards.

Appendix A

Proof of Theorem 1:

Because the outputs y_k are feasible for country *k*, but not optimal at the prices P_j, it follows that $r_k(P_j, v_k) \geq P_j' y_k$. This establishes the first inequality below and the second is established similarly:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j' y_j}{P_j' y_k}\right) \quad \text{and} \quad \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)} \geq \left(\frac{P_k' y_j}{P_k' y_k}\right).$$

Using the Gerschenkron effect it follows that:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j' y_j}{P_j' y_k}\right) < \left[\left(\frac{P_j' y_j}{P_j' y_k}\right)\left(\frac{P_k' y_j}{P_k' y_k}\right)\right]^{0.5} < \left(\frac{P_k' y_j}{P_k' y_k}\right) \leq \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)}. \tag{A1}$$

Now consider the first condition in (5), which states that $y_k \delta_k$ is feasible using the technology of county *j*. Because these outputs are not optimally chosen for the prices P_j it is immediate that:

$$P_j'(y_k \delta_k) \leq P_j' y_j.$$

It follows that $\delta_k \leq P_j' y_j / P_j' y_k$. Then consider the second condition in (5), which states that (y_j / δ_j) is feasible using the technology of county *k*. Because these outputs are not optimally chosen for the prices P_k it is immediate that:

$$P_k'(y_j / \delta_j) \leq P_k' y_k.$$

It follows that $\delta_j \geq P_k' y_j / P_k' y_k$. Using the Gerschenkron effect again we have therefore shown:

$$\delta_k \leq \left(\frac{P_j' y_j}{P_j' y_k}\right) < \left[\left(\frac{P_j' y_j}{P_j' y_k}\right)\left(\frac{P_k' y_j}{P_k' y_k}\right)\right]^{0.5} < \left(\frac{P_k' y_j}{P_k' y_k}\right) \leq \delta_j. \tag{A2}$$

Using (A1) and (A2), and by continuity of the function $r_j(\Pi, v_j)/r_k(\Pi, v_k)$, there exists a value for Π between P_j and P_k such that Theorem 1 holds. QED

Proof of Corollary:

This follows immediately from Theorem 1 by indexing country *j* with the time subscript *t*, and then treating country *k* as identical to country *j* in year *t-1*.

Proof of Theorem 2:

Since the reference prices Π are equal in the revenue functions $r_j(\Pi, v_j)$ and $r_k(\Pi, v_k)$ we can be quite flexible about how they appear. In particular, suppose that $r_j(\Pi, v_j)$ is of the form:

$$\ln r_j(\Pi, v_j) = \ln h_j(\Pi) + \sum_{l=1}^{L} \alpha_{lj} \ln v_{lj} + \sum_{l=1}^{L} \sum_{i=1}^{M+N} \beta_{lij} \ln v_{lj} \ln \pi_i + \tfrac{1}{2} \sum_{l=1}^{L} \sum_{m=1}^{L} \gamma_{lm} \ln v_{lj} \ln v_{mj}, \qquad \text{(A3)}$$

where $h_j(\Pi)$ is homogenous of degree one. The function $\ln r_k(\Pi, v_k)$ is specified similarly but with *k* replacing *j*. Without loss of generality we can assume that $\gamma_{lm} = \gamma_{ml}$, and notice that only these coefficients do not depend on countries *j* or *k*. In order for the translog function to be homogeneous of degree one in Π and in v_j, the coefficients must satisfy the restrictions:

$$\sum_{l=1}^{L} \alpha_{lj} = 1 \text{ and } \sum_{l=1}^{L} \gamma_{lm} = \sum_{m=1}^{L} \gamma_{lm} = \sum_{l=1}^{L} \beta_{lij} = \sum_{i=1}^{M+N} \beta_{lij} = 0.$$

Using the definition of the factor prices $w_{jl}^* = \partial r_j(\Pi, v_j)/\partial v_{jl}$ and the fact that $r_j(\Pi, v_j)$ is homogeneous of degree one in endowments, we have that $\frac{w_{lj}^* v_{lj}}{\sum_m w_{mj}^* v_{mj}} = \frac{\partial \ln r_j(\Pi, v_j)}{\partial \ln v_{jl}}$. It follows from (A3) that:

$$\frac{w_{lj}^* v_{lj}}{\sum_m w_{mj}^* v_{mj}} = \alpha_{lj} + \sum_{i=1}^{M+N} \beta_{lij} \ln \pi_i + \sum_{m=1}^{L} \gamma_{lm} \ln v_{mj}. \qquad \text{(A4)}$$

Then using the definition of A_j and A_k along with (A3) and (A4), we can compute:

$$\ln\left(A_j A_k\right)^{1/2} = \frac{1}{2}\left[r_j(\Pi, v_j) - r_k(\Pi, v_j) + r_j(\Pi, v_k) - r_k(\Pi, v_k)\right]$$

$$= \left[r_j(\Pi, v_j) - r_k(\Pi, v_k)\right] - \frac{1}{2}\left[r_j(\Pi, v_j) + r_k(\Pi, v_j) - r_j(\Pi, v_k) - r_k(\Pi, v_k)\right]$$

$$= \left[r_j(\Pi, v_j) - r_k(\Pi, v_k)\right]$$
$$-\frac{1}{2}\left[\sum_{l=1}^{L}\left(\alpha_{lj} + \alpha_{lk} + \sum_{i=1}^{M+N}(\beta_{lij} + \beta_{lik})\ln \pi_i\right)\ln\left(\frac{v_{lj}}{v_{lk}}\right) + \sum_{l=1}^{L}\sum_{m=1}^{L}\gamma_{lm}\ln v_{lj}\ln v_{mj} - \sum_{l=1}^{L}\sum_{m=1}^{L}\gamma_{lm}\ln v_{lk}\ln v_{mk}\right]$$

$$= \left[r_j(\Pi, v_j) - r_k(\Pi, v_k)\right]$$
$$-\frac{1}{2}\left[\sum_{l=1}^{L}\left(\alpha_{lj} + \alpha_{lk} + \sum_{i=1}^{M+N}(\beta_{lij} + \beta_{lik})\ln \pi_i\right)\ln\left(\frac{v_{lj}}{v_{lk}}\right) + \sum_{l=1}^{L}\sum_{m=1}^{L}\gamma_{lm}(\ln v_{mj} + \ln v_{mk})(\ln v_{lj} - \ln v_{lk})\right]$$

$$= \left[r_j(\Pi, v_j) - r_k(\Pi, v_k)\right] - \frac{1}{2}\left[\sum_{l=1}^{L}\left(\frac{w_{lj}^* v_{lj}}{\sum_m w_{mj}^* v_{mj}} + \frac{w_{lk}^* v_{lk}}{\sum_m w_{mk}^* v_{mk}}\right)\ln\left(\frac{v_{lj}}{v_{lk}}\right)\right],$$

where the first equality follows directly from the definition of A_j and A_k; the second equality follows from simple algebra; the third equality follows from the translog formula in (A3); the fourth equality follows from algebra on the double-summations; and the final equality follows from the share formula in (A4). QED

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