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DEBT REDEMPTION AND RESERVE ACCUMULATION

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ABSTRACT

Foreign participation in local-currency bond markets in emerging countries has increased dramatically over the past decade. In light of this trend, we revisit sovereign debt sustainability and incentives to default when the sovereign is temporarily excluded from capital markets. Differently from previous analyses, we assume that in addition to accumulating international reserves, countries can borrow internationally using their own currency. As opposed to traditional sovereign debt models (all in foreign currency), the asset valuation effects occasioned by currency depreciation (or appreciation) act to absorb global shocks and render consumption smoother. In this setting, countries do not accumulate high levels of reserves to be depleted in "bad" times. Instead, issuing domestic debt while accumulating high levels of reserves acts as a hedge against negative external shocks. A quantitative exercise, in which our model matches features of the Brazilian economic fluctuations and exchange-rate volatility, suggests this strategy to be highly effective for smoothing consumption and reducing the occurrence of default.

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1 Introduction

The past decade has seen further development of domestic government bond markets in emerging market economies (EMEs). Market depth has increased, maturities lengthened, and the investor base broadened as a consequence of active foreign participation in local-currency bond markets (Bank for International Settlements, 2012, 2015). Large accumulations of international reserves are a growing phenomenon among EMEs including those that hold large amounts of external debt.

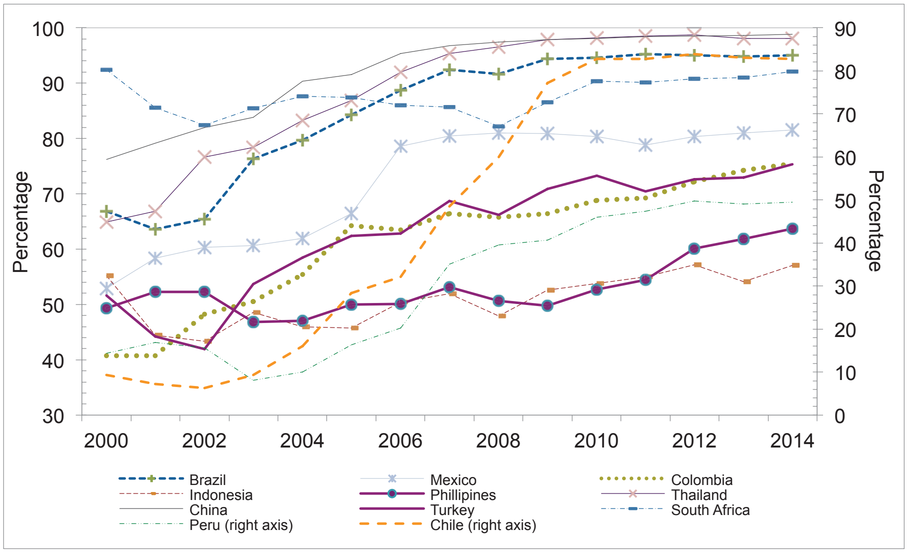
Interest earned from international reserves being much lower than that paid on EMEs' debts, this policy seems extremely costly. This behavior is all the more noteworthy when one considers that emerging markets increasingly borrow in local currency. What is the role of international reserves if these countries have the option of inflating away domestic debt and face no significant external liquidity risks? Why aren't these reserves used to repay debt or invest in productive capital? Are international reserves ultimately increasing or decreasing the sustainability of EMEs' debt?

This paper revisits these questions under novel assumptions that reflect the new reality of capital flows to emerging markets. Increased foreign participation in local-currency bond markets imply that emerging countries may now borrow internationally in domestic-currency-denominated bonds. This makes them subject to new sets of constraints regarding repayment of––or default––on their liabilities, and exposes them to new incentives to actively accumulate international reserves.

Two trends that characterize capital flows and portfolio holdings of emerging countries over the past decade are depicted in Figures 1 and 2. The first, a strong increase in foreign participation in local-currency bond markets in emerging economies, is documented in Figure 1.[1]

[1] External (and thus domestic) debt can be classified according to currency and place of issuance, and residence of the creditor. A first definition focuses on the currency in which debt is issued (external debt defined as foreign currency

Figure 1: Domestically Denominated Debt as a Fraction of Total Government Debt (%)



Source: Moody Statistical Handbook, 2014.

Although impossible to measure with precision, borrowing resources in low-interest-rate, and investing in high-interest-rate, currencies without hedging for exchange-rate risk—has increased enormously. Bank of International Settlements (2012) data show the composition of the increase in debt in emerging markets, from close to $1 trillion in 2000 to more than $4 trillion in 2010, to suggest a shift away from foreign-currency-denominated debt, from 11% in 2000 to 4% in 2010. In Latin America, the share of exchange-rate-linked debt declined from approximately 23% in 2005 to 13% in 2010.

More important is the increased participation of foreign residents in emerging markets' domestic bond markets. The stock of Japanese households' foreign currency in investment trust funds, almost nonexistent in 2002, was 35 trillion yen in 2007. This is but one example of resources in search of high interest rates flowing to emerging markets. Burger, Warnock, and

debt, and local currency as that denominated in the currency of the country of issuance). A second definition, usually adopted by the principal official compilers of statistical information, focuses on the residence of the creditor (external debt being debt owed to non-residents). Yet another definition focuses on the place of issuance and legislation that regulates the debt contract (external debt is issued in foreign countries under the jurisdiction of a foreign court). In this paper, we study foreign residents' increasing role in buying local currency debt.

Warnock (2012) document a similarly dramatic post-2001 increase in U.S. investors' participation in local-currency bond markets in EMEs.

Using a newly constructed dataset of the currency composition of sovereign and corporate external debt, Du and Schreger (2015a, b) show that over the past decade major emerging market sovereigns that borrowed as much as 85% of their external debt in foreign currency now borrow more than half in their own currency. Borrowing constraints for emerging countries are quite different from what they used to be.[2] Fifteen years ago, Eichengreen and Hausmann (1999) advanced the original-sin hypothesis on the limits of emerging markets' ability to borrow in their own currency. But the gradual redemption of these economies' debt sins over the past decade might naturally be expected to significantly affect their incentives pertaining to debt default and repayment.

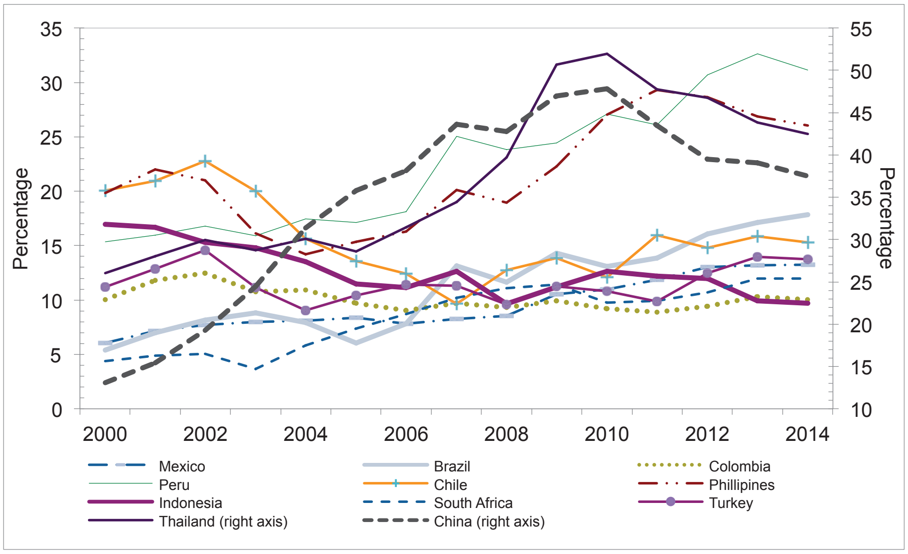
The second trend, the rapid rise in the accumulation of international reserves, is documented in Figure 2. Average reserve accumulation in 2014 was more than 25% of GDP in emerging, and only about 5% of GDP in high-income, countries. As documented by the European Central Bank (2006), the size and pace of accumulation of foreign reserves has been unprecedented. Countries have accumulated reserves greater than their IMF quota, exhibiting ratios of reserves to imports above four months' coverage, reserves to short-term external debt maturity above one year (Greenspan-Guidotti rule) and broad money. The increase in reserve assets has not been exclusive of China or the East Asian countries; its ubiquity among developing countries has raised interesting questions in the literature regarding the costs and benefits of reserve accumulation.

The accumulation of reserves has been particularly acute as countries have reduced their exposure to foreign currency debt. For example, foreign currency in countries like Thailand, Malaysia, and South Korea has shrunk to the point that EMBI+ has been forced to discontinue

[2] See also Hale, Jones, and Spiegel (2014), who document the decline in the share of international bonds denominated in major reserves, and the increase in bonds denominated in issuers' home currencies over the past two decades.

these countries' indices (Du and Schreger, 2015a, b). Yet, as of 2014, Korea held nearly $362 billion in foreign reserves (roughly 25% of GDP), Thailand $157 billion (roughly 40% of GDP), and Malaysia $116 (roughly 37% of GDP).

Figure 2: International Reserves (% GDP)



Source: Moody Statistical Handbook, 2014.

The cost of holding reserves has been estimated at close to 1% of GDP for all developing countries (Rodrik, 2006). Against the commonly advanced explanation that reserves are accumulated as insurance against the risk of an external crisis—self-protection through increased liquidity. The related literature has had difficulty quantitatively accounting for the observed reserve accumulation in emerging markets (Alfaro and Kanczuk, 2009; Bianchi et al., 2012; Durdu, Mendoza, and Terrones, 2008; and Jeanne and Rancière, 2011).

We study these recent trends by constructing a dynamic equilibrium model of a small open economy subject to international shocks. To smooth consumption, a benevolent government may issue foreign debt in domestic and international currencies denominated in the price of non-tradable and tradable goods, respectively. Domestic and international interest rates may differ, and we explicitly model the risks attendant on such differences. We calibrate our model to Brazil, a typical example of an emerging country that accumulated international reserves during debt redemption.

The basic intuition for our model goes back at least to Bohn (1990), but its empirical implementation has only recently begun to be studied (see Lane and Shambaugh, 2010 and Benetrix, Lane, and Shambaugh, 2015). Having positive net foreign currency positions (assets in foreign, and debt in domestic, currency) is optimal when a country faces international shocks (as to the endowment of tradable goods). This is because the asset valuation effects occasioned by currency depreciation (or appreciation) act to absorb global shocks and smooth consumption.

Debt and reserve accumulation also affect, and are affected by, a country's incentives to default. A large stock of domestically denominated debt could help to counterbalance an external shock, but may not be sustainable. A country may not resist the temptation to default on such debt. Very large holdings of international reserves may also not be optimal. International reserves that are not pledgeable may not increase the sustainability of debt, in fact, may reduce sustainability when debt is denominated in foreign currency (Alfaro and Kanczuk, 2009). Additionally, because holdings of international reserves shift consumption to later dates, they may be excessively costly.

We study the optimal level of international reserves and domestically denominated debt taking into account both the valuation effect and default considerations. Our quantitative results suggest that the optimal level of international reserves is fairly large as their cost is mitigated by the valuation-smoothing gains. Our model also matches some features of Brazil's economic fluctuations, being consistent, in particular, with the reduction in exchange rate volatility.

In Alfaro and Kanczuk (2009), we considered the case in which both reserves and debt were denominated in foreign currency but were not perfect substitutes (since reserves could be used even after a country has defaulted). After calibrating the model for realistic parameter values, we obtained that the reserve accumulation did not play a quantitatively important role as a way to smooth consumption. (Volatilities of 7.5 to 10 times the benchmark calibration and extremely patient sovereigns were required for the equilibrium amount of reserves to become positive). As such, for realistic parameter values reserve accumulation did not increase debt sustainability. In contrast, in this present paper debt is denominated in local currency, and reserve accumulation has a substantial impact over consumption smoothing via valuation effects. As a consequence, reserve accumulation does increase debt sustainability.

In our analysis, countries do not accumulate high levels of reserves to be depleted in "bad" times, as is usually suggested in policy circles. Instead, issuing domestic debt while accumulating high levels of reserves acts as a hedge against negative external shocks; and the result relates to the vast literature on optimal international portfolio diversification (e.g., Cole and Obstfeld, 1991; Engel and Matsumoto, 2009; Healthcote and Perri, 2013).[3] We diverge from this literature in explicitly considering sovereigns' incentives to default, thus incorporating sustainability of portfolio choices in our analysis. In further contrast to this literature, asset yields and exchange rate are endogenously determined in the model, and are dependent on the government portfolio choice.

This paper is related to the growing literature that examines debt sustainability (see Aguiar and Amador, 2014 for a recent survey on debt sustainability) and in particular to the work analyzing the recent increasing role of local-currency debt in emerging markets (see Burger et al, 2012; Du and Schreger, 2015 a, b; Hale et al. 2014, among others). More specifically, it relates to work that examines the determinants of reserve accumulation in emerging markets. The rationale for reserve accumulation based on interaction with local currency external debt (i.e., "redemption" of the "original sin") complements explanations that emphasize precautionary motives (Durdu, Mendoza, and Terrones, 2009; Jeanne and Rancière, 2011; Bianchi et al., 2012), financial stability (Obstfeld, Shambaugh, and Taylor, 2010), potential externalities associated with the tradable sector or mercantilist view (Dooley, Folkerts-Landau, and Garber, 2003; Benigno and Fornaro, 2011) and political economy considerations (Aizenman and Marion, 2003).

[3] See Gourinhas and Rey (2014) for a comprehensive review of the role of the valuation mechanism.

The rest of the paper is organized as follows. In Section 2 we present the model, and in Section 3 we derive intuition from a two-period, stripped down version of the model. We return in Section 4 to the original model, which we calibrate to the Brazilian economy, and discuss the results of its quantitative simulation. Discussions and robustness exercises are presented in Section 5. Section 6 concludes.

2 Model

We model an economy populated by a continuum of private households, a benevolent government, and a continuum of international, risk-neutral investors. Preferences are concave, implying that households prefer a smooth consumption profile for both tradable and non-tradable goods. To smooth consumption, a benevolent government may optimally issue foreign debt in domestically denominated currency and accumulate foreign reserves. The benevolent government may further optimally choose to default on its international commitments, in which case we assume it to be temporarily excluded from borrowing in international markets.

We assume the households' preferences to be given by

$$U = E\sum_{t=0}^{\infty}\beta^{t}u(c_t^T, c_t^N) \tag{1}$$

with

$$u(c^T, c^N) = \frac{[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}]^{(\frac{1-\sigma}{-\eta})} - 1}{(1-\sigma)}, \tag{2}$$

where E is the expectation operator, ${c^T}_t$ and ${c^N}_t$ denote, respectively, household consumption of tradable and non-tradable goods, $\sigma > 0$ measures the curvature of the utility, η measures the degree of substitution between tradable and non-tradable goods, ω indicates these goods' relative importance to household preferences, and $\beta \in (0, 1)$ is the discount factor.

If the government chooses to repay its debt, the country's budget constraint is given by

$$c_t^T + \frac{c_t^N}{e_t} + q_t^R R_{t+1} - q_t^D \frac{D_{t+1}}{e_t} = y_t^T + \frac{y_t^N}{e_t} + R_t - \frac{D_t}{e_t}, \tag{3}$$

where R_t denotes the foreign reserves level , D_t denotes the domestic denominated debt level, and $y^T{}_t$ and $y^N{}_t$ are, respectively, the tradable and non-tradable goods endowments of output. The debt and reserve price functions, $q^R(s_t, R_{t+1}, D_{t+1})$ and $q^D(s_t, R_{t+1}, D_{t+1})$, and real exchange rate function, $e(s_t, R_{t+1}, D_{t+1})$, are endogenously determined in the model, and dependent on the state of the economy, s_t, as well as the government's decisions. In the benchmark version of the model, the state of the economy is completely defined by the ordered set $s_t = (R_t , D_t, y^T{}_t, y^N{}_t)$.

When the government defaults, the economy's constraint is

$$c_t^T + \frac{c_t^N}{e_t} + q_t^R R_{t+1} = y_t^D + \frac{y_t^N}{e_t} + R_t, \tag{4}$$

where y^D corresponds to the endowments of tradable goods in default periods. After defaulting, the sovereign is temporarily excluded from issuing debt. We assume θ to be the probability that the sovereign regains full access to international credit markets.

International investors are risk-neutral and have an opportunity cost of funds given by ρ, which denotes the risk-free rate denominated in the price of tradable goods. Investors' actions are to choose the debt and reserve prices q^D and q^R, which depend on the perceived likelihood of default and currency depreciation. For these investors to be indifferent between the riskless asset and lending in a country's non-tradable goods denomination, it must be the case that

$$q_t^D = \frac{(1-\psi_t)}{(1+\rho)} E_t\left[\frac{e_t}{e_{t+1}}\right] \tag{5}$$

and

$$q^R = \frac{1}{1+\rho} \tag{6}$$

where ψ_t is the probability of default, which is endogenously determined and dependent on the sovereign's incentives to repay the debt. Note (5) is a version of the uncovered interest parity condition that considers the possibility of default. The expected values are conditional on the absence of default. If default happens, international investors do not receive any payment, regardless of the exchange rate.

Because the government chooses debt and reserve levels, the problem of the households is intratemporal, and has the sole role of determining the real exchange rate. Individual household maximization equates the relative marginal utility of tradables to non-tradables to their relative prices,

$$e_t = \frac{\omega}{(1-\omega)} \left(\frac{c_t^N}{c_t^T} \right)^{1+\eta}. \tag{7}$$

The market-clearing condition for non-tradable goods is

$$c_t^N = y_t^N. \tag{8}$$

The timing of the decisions is as follows. In the beginning of each period, the government starts with debt level D_t and reserve level R_t and receives endowments $y^T{}_t$ and $y^N{}_t$. It faces the reserve price schedule $q^R(s_t, R_{t+1}, D_{t+1})$, bond price schedule $q^D(s_t, R_{t+1}, D_{t+1})$, and real exchange rate price schedule $e(s_t, R_{t+1}, D_{t+1})$. Taking these schedules as given, the government simultaneously makes three decisions. It chooses (i) the next level of reserves, R_{t+1}, (ii) whether to default on the debt, and (iii) if it decides not to default, the next level of debt, D_{t+1}.

The model described is a stochastic dynamic game. We focus exclusively on the Markov perfect equilibria, whereby the government does not have commitment and players act sequentially and rationally.

Note that international investors and households are passive, and their actions can be completely described by equations (5), (6), and (7). To write the government problem recursively, let ν^G denote the value function if the sovereign decides to maintain a good credit history this period (G stands for good credit history), and ν^B the value function if the sovereign decides to default (B stands for bad credit history). The value of being in good credit standing at the start of a period can then be defined as

$$v = Max\{v^G, v^B\}. \tag{9}$$

This indicates that the sovereign defaults if $v^G < v^B$. The value function v^G can be written as

$$v^G(s_t) = Max\{u(c_t^T, c_t^N) + \beta E v(s_{t+1})\}, \tag{10}$$

subject to (3), and the value function v^B as

$$v^B(s_t) = Max\{u(c_t^T, c_t^N) + \beta[\theta E v^G(s_{t+1}) + (1-\theta) E v^B(s_{t+1})]\}, \tag{11}$$

subject to (4).

The recursive equilibrium is defined by the set of policy functions for government asset holdings and default choice and the price functions for domestic bonds, reserves, and real exchange rate such that, (i) taking the price functions as given, the government policy functions satisfy the government optimization problem, and (ii) prices of domestic bonds, reserves, and exchange rate are consistent with the government decisions.

This definition of equilibrium, identical to that of Arellano (2008) and Alfaro and Kanczuk (2009), among many others, reflects a game played by a large agent (the government) against many small agents (the continua of investors and households). It implies that the government internalizes the effects of its actions over the prices. In our model, the government internalizes the effect of its asset holdings over the real exchange rate.

3 A Two-period Economy

In this section, we develop a two-period, stripped down version of the model to provide some intuition for the joint determination of international reserves and domestically denominated debt. Although it cannot shed light on the sovereign's "willingness to pay" incentives, which hinge on the cost of exclusion from the market, this simpler version underscores how the combination of reserves and debt both provide insurance against international shocks and allow for intertemporal consumption smoothing.

In this simple economy, the sovereign's preferences are given by

$$u(c^N{}_1, c^T{}_1, c^N{}_{2,} c^T{}_2) = c^N{}_1 + c^T{}_1 + \beta\, E\, [c^N{}_2 + \log(c^T{}_2)\,],$$

where households' consumption of non-tradable and tradable goods are respectively denoted, in the first period by $c^N{}_1$ and $c^T{}_1$, and in the second, by $c^N{}_2$ and $c^T{}_2$. The parameter $\beta \in (0, 1)$ is the discount factor. In the first period, households receive an endowment of a unit of consumption in both the non-tradable and tradable sectors, denoted $y^N{}_1 = 1$ and $y^T{}_1 = 1$, respectively. In the second period, the endowment of non-tradable goods is again equal to one unit, $y^N{}_2 = 1$, but the endowment of the tradable good follows a stochastic process:

$y^T{}_2 = (1 + \sigma)$, with probability equal to ½ (good state of nature), and

$y^T{}_2 = (1 - \sigma)$, with probability equal to ½ (bad state of nature).

Note that in the second period, we retain, for tradables, only the curvature of the utility function, which is sufficient to create the gains from smoothing consumption across the different states of nature. The parameter β indicates the benefit of consuming earlier rather than later. The parameter σ equals the standard deviation of the second period tradable goods endowment process.

The sovereign has two instruments with which to transfer resources across periods and states: domestically denominated bonds and international reserves. Reserves correspond to riskless bonds that bear interest rates given by ρ; defaultable bonds are contingent claims. We assume the sovereign to repay debt in both the good and bad states of nature. That is, default cannot be used to smooth consumption. Households' budget constraint can be written as

$$c_1^T + \frac{c_1^N}{e_1} = 1 + \frac{1}{e_t} - R + \frac{D}{e_1} \text{ in the first period,}$$

$$c_{2G}^T + \frac{c_{2G}^N}{e_{2G}} = (1+\sigma) + \frac{1}{e_{2G}} + R(1+\rho) - \frac{D(1+r)}{e_{2G}} \text{ in the good state of second period, and}$$

$$c_{2B}^T + \frac{c_{2B}^N}{e_{2B}} = (1+\sigma) + \frac{1}{e_{2B}} + R(1+\rho) - \frac{D(1+r)}{e_{2B}}$$ in the bad state of second period,

where D is the debt level, R the amount of reserves, and ρ the international interest rate, and r denotes the interest rate on domestically denominated debt. International investors are risk neutral, and must be indifferent between the international asset and domestic bonds. This implies

$$(1+\rho) = (1+r)\left[½\frac{e_1}{e_{2G}} + ½\frac{e_1}{e_{2B}} \right].$$

In each period and each state of nature, household optimization determines the real exchange rate according to the relative marginal utility of tradable and non-tradable goods. In the first period, utility functions being linear, $e_1 = 1$. In the second period, the utility of the tradable good being logarithmic, the exchange rates are given by $e_{2G} = c^T_{2G}$ and $e_{2B} = c^T_{2B}$.

Because market clearing forces consumption of nontradables to equal the endowment of nontradables, and plugging the exchange rate into the constraints, the government problem can be written as

maximize $u(c^T_{1,}, c^T_2) = c^T_1 + \beta\,[½ \log(c^T_{2G}) + ½ \log(c^T_{2B})\,]$

subject to

$$c^T_1 = 1 - R + D$$

$$c^T_{2G} = 1 + \sigma + R(1+\rho) - D.\ c^T_{2G}\,(1+r)$$

$$c^T_{2B} = 1 - \sigma + R(1+\rho) - D.\ c^T_{2B}\,(1+r)$$

$$(1+r)(\,c^T_{2G} + c^T_{2B}) = 2(1+\rho).$$

Before numerically finding the D and R that solve this problem, we can use another approximation to derive some intuition. Because second period output fluctuates around one, suppose that $c^T_{2G} + c^T_{2B}$ equals approximately two such that the last constraint can be approximately written as $r = \rho$. In this case, it becomes straightforward to plug the constraints into the maximization as follows,

$$u = (1 + D - R) + \frac{\beta}{2}\log\left(\frac{1 + \sigma + R(1+\rho)}{1 + D(1+\rho)}\right) + \frac{\beta}{2}\log\left(\frac{1 - \sigma + R(1+\rho)}{1 + D(1+\rho)}\right),$$

where the amount of debt D and reserves R should be set to maximize utility u, and the three terms on the right-hand side correspond to the first period, the second period in the good state, and the second period in the bad state.

To think about the government problem, consider first consumption smoothing across the two states of the second period. To make second period consumption the same in the good and bad states, the government could set either a very high D or very high R, or both.

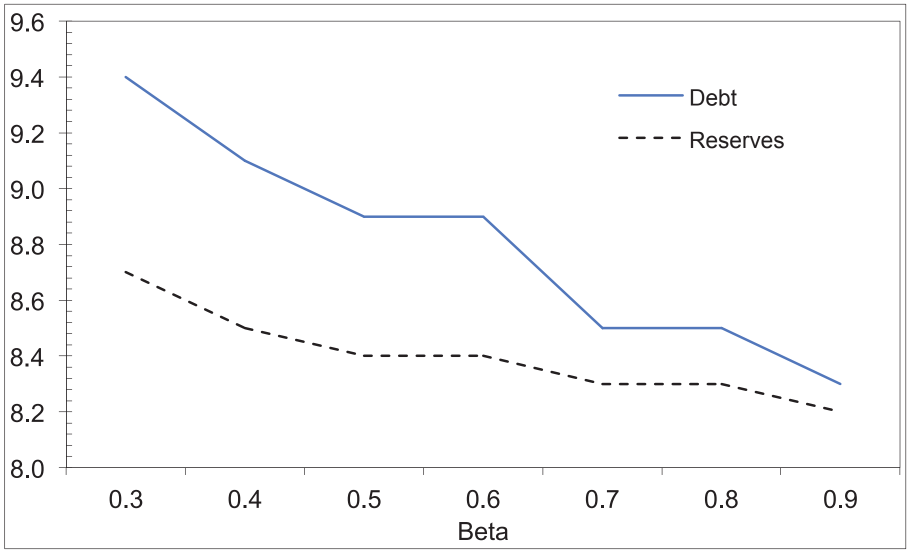
Looking at first period consumption reveals the difference between debt and reserves to determine consumption smoothing over time. If debt is much higher than reserves, first period consumption becomes much greater than second period consumption, regardless of the latter's state.

The solution thus involves setting debt and reserve levels sufficiently high to smooth across states of nature, while balancing the difference between these large quantities so as to achieve smoothing over time.

Returning to the two-period model without interest approximation, we can numerically solve for the optimal D and R by setting $\sigma = 0.05$ and $\rho = 0.01$. Figure 3 shows the optimal levels of debt and reserves for various values of β.

Note that optimal levels of debt and reserves are quite high compared to output, which is normalized to one. Also note that the difference between debt and reserves decreases as beta increases. That is, if households are more patient, the government will shift their consumption to the second period.

Figure 3: Two-period Economy Solution



4 Quantitative Analysis

Our two-period model abstracts from the possibility of default and results in extremely large debt and reserve levels. To sharpen these results, we return to our infinite period model, calibrated to the Brazilian economy. We solve the model numerically to evaluate its quantitative predictions regarding the occurrence of default events and the business cycle properties of the exchange rate.

4.1 Calibration

A relevant difficulty in performing the quantitative analysis is that emerging countries only began to issue relevant amounts of domestically-denominated bonds in the middle of the past decade. The data time span of the current regime, especially as concerned with episodes of default, is consequently relatively small. In Brazil, for example, the last default episode was between 1983 and 1990 (Reinhart, 2010).

We address this problem by calibrating some of the parameters using a much longer time horizon, during which international debt was denominated mainly in foreign currency. In transforming our economy to consider the case in which bonds were denominated in foreign currency, we assume the country budget constraint to be given by

$$c_t^T + \frac{c_t^N}{e_t} + q_t^R R_{t+1} - q_t^B B_{t+1} = y_t^T + \frac{y_t^N}{e_t} + R_t - B_t \qquad (3')$$

rather than equation (3).

This is effectively the case considered by Alfaro and Kanczuk (2009), in which B denotes holdings of foreign bonds denominated in foreign currency. As above, in the case of a debt default, reserves R continue to be held and can be used to smooth consumption. We proceed with calibration by considering annual data since 1965.

We set the international interest rate $\rho = 0.04$ and inter-temporal substitution parameter $\sigma = 2$, as is usual in real-business-cycle research in which each period corresponds to one year (see Kanczuk, 2004). There being considerable disagreement about the intratemporal elasticity of substitution between tradable and non-tradable goods (Akinci, 2011), we make elasticity equal to one (the middle of the many possible estimations), and, for that, set $\eta = 0$. Our results are robust to many other parameter values. For the weight of tradables, we use the share of output that corresponds to industry and agriculture, and set $\omega = 0.35$.

Because non-tradable consumption goods cannot be smoothed, we focus on the case in which shocks are exclusively external, that is, on the tradable endowment. We thus make $y^N = 1$ for all periods. We then set $y^T{}_t = \exp(z^T{}_t)$, and assume that $z^T{}_t$ can take a finite number of values and evolves over time according to a Markov transition matrix with elements $\pi^T (z^T{}_i , z^T{}_j)$; that is, the probability that $z^T{}_{t+1} = z^T{}_j$, given that $z^T{}_t = z^T{}_i$, is given by the matrix π element of row i and column j.

We calibrate the technology state z^T by considering the (logarithm) of the GDP to follow an $AR(1)$ process; that is, $z_{t+1}^T = \alpha z_t^T + \varepsilon_{t+1}$ where $\varepsilon_t \approx N(0, \sigma_\varepsilon{}^2)$. We obtain $\alpha = 0.85$ and $\sigma = 0.12$. The apparently high value of the standard deviation reflects the fact that the tradable sector corresponds to roughly one-third of total output. To make the model consistent with the data, the volatility of the tradable sector must thus be about three times that of total output. We discretize this technology state into three possible values, spaced such that the extreme values are three standard deviations away from the mean, and use the Quadrature Method (Tauchen, 1986) to calculate transition probabilities. We also discretize the space state of debt sufficiently to avoid affecting the decision rules.

Setting the probability of redemption at $\theta = 0.5$ implies an average stay in autarky of two years, in line with estimates by Gelos et al. (2011). Direct output costs are modeled from default and assumed to be asymmetric: $y^D = y^{DEF}$ in case $y^D > y^{DEF}$. Setting $y^{DEF} = 0.85y^T$ implies that tradable output costs of defaulting equal 15%, the relatively large number again reflecting the fact that the tradable sector corresponds to one-third of the economy.

To obtain reasonable levels of debt in equilibrium, we set the intertemporal factor at the relatively low value of $\beta = 0.80$, which is common practice in debt models (Alfaro and Kanczuk, 2009).

Table 1 summarizes the parameter values.

Table 1: Calibration

Technology autocorrelation	$\alpha = 0.85$
Technology standard deviation	$\sigma_{\varepsilon} = 0.12$
Probability of redemption	$\theta = 0.50$
Output costs	$y^{DEF} = 0.85y^T$
Risk aversion	$\sigma = 2$
Risk free interest rate	$\rho = 0.04$
Discount factor	$\beta = 0.80$

4.2 Simulation results

We first simulate our economy under the assumption that debt is denominated in foreign currency. For the chosen parameters, the invariant distribution displays 48% of GDP of debt and a

6.1% frequency of default (case (i) in Table 2). These numbers are very much in line with the historical data for Brazil and other emerging countries presented in many other papers. The equilibrium level of reserves is zero, a reincarnation of Alfaro and Kanczuk's (2009) result in a model with two sectors (but shocks in only one).

As discussed extensively in that paper, there is a potential role in this setup for reserves to be used to smooth consumption when the country is excluded from international markets. Because reserve holdings reduce the sustainability of debt, however, quantitatively their optimal holding is zero. Optimal government policy is to hold (foreign denominated) debt and take recourse to default in extremely bad times.

As an intermediate step, assuming the government cannot hold reserves (case (ii)), we simulate the economy with locally denominated bonds. We obtain, in this case, that the government holds a fairly small amount of debt (9.8% of GDP) and virtually does not default. Note that the volatility (standard deviation) of the exchange rate drops from 26.2% in the case of foreign-denominated debt to 7.5% in the case of domestically-denominated debt with no reserves. Thus, even without resorting to default, domestic denomination results in more consumption smoothing.

Simulating the economy with locally denominated bonds, but allowing the government to hold positive amounts of reserves (case (iii)), we obtain that, in the invariant distribution, the economy displays 28.6% of GDP in (locally denominated) debt, with 24% of GDP in reserves. The possibility of holding reserves increases debt holding; as in case (ii), the government virtually does not resort to default as a means to smooth consumption. Note also that the volatility of consumption drops even more, the standard deviation of the exchange rate falling to 4.2.

The intuition for holding both (domestically-denominated) debt and reserves, developed in Section 3, allows for consumption smoothing across both states and time. But the experiment with the full-fledged model yields some novel results. First, debt and reserve levels look more reasonable. That they may, in fact, be smaller than anticipated suggests that this scheme for smoothing consumption is fairly powerful.

Second, the experiment shows the proposed scheme to be, in fact, sustainable in the sense that the government (almost) never defaults. Related to this observation, we learned that accumulation of reserves does not seem to be a problem in terms of reducing the sustainability of debt when debt is in local currency, the reason being that consumption is already so smooth that the government has no incentive to default. Put differently, in both the foreign denominated and locally denominated debt experiments, international reserves play a role when a country is excluded from capital markets. Although quantitatively unimportant in conjunction with foreign denominated bonds when a country has access to international markets, reserves are, owing to their valuation effect, which helps to smooth consumption, nevertheless highly useful in conjunction with domestically-denominated bonds. This, in turn, reduces the incentive to default.

Table 2: Invariant Distribution Properties

Case	Foreign Den. Debt (% GDP)	Locally Den. Debt (% GDP)	Internat. Reserves (% GDP)	Probab. Default (%)	Exchange Rate S.D. (%)	Output S.D. (%)	Ratio
(i)	48.0	-	0.0	6.1	26.2	2.8	9.4
(ii)	-	9.8	-	0.4	7.5	2.8	2.7
(iii)	-	28.6	24.0	0.4	4.2	2.8	1.5

Given that the interest earned from their reserves is much lower than that paid on their debt, emerging economies' reserve accumulation policy seems extremely costly. For example, Brazil's total government debt (domestic and international) in 2011 was approximately 60% of GDP and paid annual interest of about 12%. Its holdings of international reserves, at 15% of GDP, earning interest of approximately 2% per year, it is unclear why Brazil did not use its reserves to reduce its outstanding debt, the difference between the two interest rates multiplied by the amount of reserves implying a cost of about 1.5% of GDP.

According to our model, the logic of accumulating both reserves and domestically denominated debt is precisely that it is costly during good periods. When an international shock is favorable, debt service is higher and consumption is reduced, when unfavorable, debt service is reduced and consumption increases. When the whole invariant distribution of shocks is taken into account, a country will enjoy more stable consumption and higher welfare.

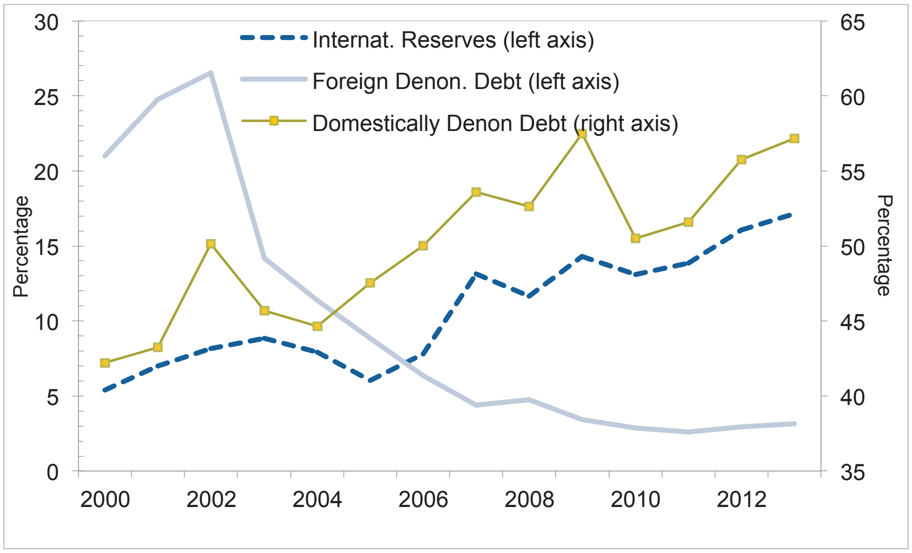
Note that in the proposed construction, the level of reserves remains high during unfavorable periods. Contrary to the usual argument in policy circles, reserves are thus not insurance that can be "used" in bad times. The idea is not to buy consumption goods that deplete the stock of reserves, but rather to maintain a constant reserve stock that serves as insurance by increasing the stabilizing effect of domestically-denominated debt.

In fact, the optimal policy function is to hold the amount of debt and reserves next period constant regardless of the period state. For this reason, we opted not to depict the debt and reserve policy functions, since they are just simple horizontal lines. The essential intuition is that the stabilization effect of issuing local-currency debt results in sufficient consumption smoothing that there is no need to change the levels of debt and reserves.

4.3 Comparison with the Brazilian data

We now compare the model's outcomes with recent data from Brazil. Figure 4 plots the evolution of government holdings of international reserves and foreign and domestically denominated debt. These assets (or liabilities) could potentially be held against both the Brazilian private sector and the rest of the world.

Figure 4: Government Holdings (% GDP)



Source: Brazilian Central Bank.

In our model, only the government is assumed to be able to hold international assets. However, the position of the full country (government and private sector) against the rest of the world is, in fact, the closest real concept to be contrasted with the model, the challenge in doing so being the absence of comprehensive data about the denomination of private sector holdings.

Although some information about private sector debt holdings is available, it is common practice for firms to change the denomination of debt using market derivatives. It is similarly common for foreigners to use derivatives to gain exposure to the Brazilian currency and invest in carry trade strategies. That these derivatives to swap currencies are often traded over the counter makes it impossible to assess their dimension.[4] Anecdotal information indicates that even large firms that issue bonds denominated in dollars hedged most of their currency exposure after the substantial depreciation in 1999 (Central Bank of Brazil (2014)). Given the lack of comprehensive

[4] Benetrix, Lane, and Shambaugh (2015) document dramatic improvements in net foreign asset values for emerging market and developing economies in the last two decades.

data, we assume international reserves and foreign denominated government debt to roughly correspond to the country position against the rest of the world.

The case of domestically-denominated bonds is more complicated. We know that before 2003, foreign exposure to Brazilian currency debt was quite small. In 2002, the Brazilian five-year CDS, which measures risk of dollar denominated debt, was as high as 4,000 basis points. In periods with such risk of default, it is highly unlikely that foreigners would hold local currency. Post 2002, however, we do not know how much of the increase in government domestically denominated debt was held by foreigners. Because the increase in debt was concomitant with the accumulation of reserves, as depicted in Figure 4, it is natural to assume foreigners to be responsible for a large fraction of it.

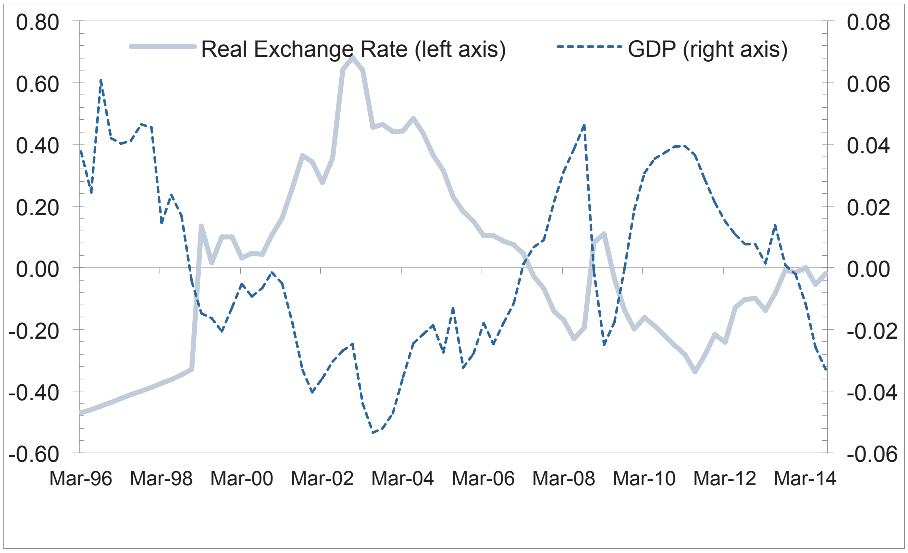
Table 3 summarizes the data. Rather than guess the holdings of assets and liabilities, we indicate the net position in each denomination. Before 2006, there was virtually no locally denominated external debt. After 2006, holdings of reserves were higher than holdings of foreign denominated debt. Thus, the net position of foreign denominated securities switched from debt (liabilities) to assets.

Table 3: Brazilian Data

Period	Foreign Denominated	Locally Denominated	Exch. Rate S.D. (%)	Output S.D. (%)	Ratio	Correl.
1996 to 2005	Debt	0	35.9	3.0	11.8	-0.90
2006 to 2014	Assets	Debt	12.3	2.2	5.6	-0.87

Information about the cyclical behavior of output and real exchange rate is presented in Figure 5. There being no available data on the consumption of tradables and non-tradables, the exchange rate is contrasted with the model. The sample being short, a single crisis could imply differing output volatility. We thus determine the ratio between the exchange rate standard deviation and output deviation to be the best way to compare the model with the data.

Figure 5: Cyclical Properties of the Brazilian Economy



Source: Brazilian Institute of Statistics and Geography and Brazilian Central Bank.

The comparison between Table 2 and Table 3 requires some discussion. Our argument is that with debt redemption, the Brazilian economy should move from case (i) in Table 2 (foreign-denominated debt) to case (iii) in the same table (domestically-denominated debt and international reserves). As is seen in Figure 5, however, this change occurs only gradually. The country seems to be in the process of changing from one steady state to the other, whereas Table 2 shows the economy to already be in the steady state invariant distribution.

We propose that one think of the 2006-2014 time horizon of Table 3 data as an intermediate step in the transition from case (i) to case (iii) in Table 2. Interpreted thus, the model accounts for the exchange rate's cyclical behavior reasonably well. The ratio of volatility of the real exchange rate to that of output dropped by half, from 11.8 to 5.6 (Table 3). According to our model, this ratio should drop from 9.4 to 1.5 if the country moves from one steady state to the other (Table 2). Brazil, however, is still far from converging on the steady state. During the 2006-2014 period, Brazil's holdings of international reserves were 13.4% of GDP. In the proposed steady state, these holdings will reach 24% of GDP.

For completeness, Table 3 also reports the correlation of the exchange rate and output. For the two periods considered, the correlation was -0.90 and -0.87. In our model, there being shocks only to tradable goods, this correlation is equal to -1. A simple way to reduce this correlation would be to add (uncorrelated) shocks to the non-tradable sector. We nevertheless construe this comparison to support our hypothesis that non-tradable sector shocks are not a quantitatively important factor in our analysis.

5 Robustness and Discussion

5.1 Volatility of exchange rate

In our model, as in other standard exchange rate models, the volatility of the exchange rate is directly linked to the volatility of the ratio of tradable and non-tradable goods consumption. If the volatility of consumption is much higher than that observed in the data, our model generates only reasonable levels of exchange-rate volatility. Adding to this puzzle, Engel (1999) shows movements in the U.S. exchange rate to be driven almost exclusively by changes in prices of tradable goods, which are usually assumed to be equal across countries.

Burstein, Neves, and Rebelo (2003), addressing these issues in the context of exchange-rate stabilization, introduce in an otherwise standard model a distribution sector that can dramatically improve the model's performance. Rather than adding a new sector to our model, we invoke their claim that modifying preferences in a standard model can mimic the introduction of distribution costs.

We modify, in particular, the utility function, making the share of tradable goods $\omega = 0.10$. As above, we find the optimal policy to be to accumulate reserves in conjunction with locally denominated debt, and this strategy to be effective at smoothing consumption across both different states of nature and time. The only difference between the results of this experiment and the one in

Section 4 is quantitative; the level of locally denominated debt and reserves as a percentage of GDP being, respectively, 10.4% and 3.2% in this alternative economy, the decrease in the importance of the tradable sector implies, as expected, a reduction in debt and reserve accumulation.

5.2 Reserve accumulation and exchange rate management

A branch of the literature on reserve accumulation argues that government motivation is a form of exchange rate management (Dooley et al., 2003). That is, reserve accumulation is a means of keeping the exchange rate depreciated and thereby helping to protect a country's industry and stimulate exports.

Although we did not use the foregoing motivation in this paper, our results can be connected to this literature. In our model, the rationale for reserve accumulation is to smooth consumption of tradable goods. But as a direct consequence the exchange rate is also smoothed. In particular, the exchange rate does not appreciate as much in good times due to the accumulation of reserves.

There are, however, two observational differences between the motivation referenced in the literature and our results. First, in contrast to the literature, in which reserves are to be accumulated during good times and depleted in bad times, in our model reserves are kept constant over different states of nature. Second, depreciation is viewed in our model as something to be avoided.

5.3 Rationale for Debt Redemption

In our model, debt redemption, that is, the possibility of a country issuing external debt denominated in local currency, because it implies fewer occurrences of default, does not explain why emerging countries were unable to issue domestically-denominated external debt previously, in particular, during the 1980s and into the 1990s. Although a complete investigation is beyond the scope of this paper, we conjecture that there were two reasons for debt redemption.

One possibility is that investors could not identify the type of government issuing the bonds. As Alfaro and Kanczuk (2005) argue, sovereign default data seems consistent with reputation building. This, in turn, is consistent with the existence of different types of governments including those that would default independently of the state of nature ("inexcusable defaults"). It is possible that this additional factor could increase risk sufficiently to shut down the market. As the type of government in control became clearer, and the risk was reduced, international investors became more disposed to buy debt issued in local markets.

A second, related issue is inflation. Inflation and inflation volatility were extremely high in Latin America during the 1980s, making returns on domestically-denominated bonds extremely risky for international investors, possibly so high that investor appetite for this type of asset was insufficient for the existence of the market.

5.4 Private-Sector Debt

In our model, debt is issued exclusively by the benevolent government; households (i.e., the private sector) cannot issue debt and choose their inter-temporal consumption. This assumption raises two issues. First, the analysis would be unchanged were private sector debt to be included, assuming no distortions or other imperfections (taxes, externalities, time inconsistency issues) that could drive a wedge between the objectives of the benevolent government and those of the households.

Second, in the event that the objectives of the government and the households do conflict, the government would attempt to offset, perhaps even prohibit, household debt and reserve accumulation by creating rules and changing the law. Thus, unless political economy issues are considered, the assumption that households cannot issue debt is not crucial to the analysis.

6 Conclusion

The past decade was characterized by two new trends in international capital flows to emerging markets, (1) carry trade activity and associated foreign participation in local-currency bond markets, and (2) large accumulations of international reserves. We believe that both can be rationalized as optimal debt management strategy. Borrowing in domestic currency can insure emerging countries against international shocks because the valuation effect that results from currency appreciation has a negative correlation with the shock, an intuition that dates to Bohn (1990).

Our quantitative study of how reserve accumulation affects governments' decisions to default finds that optimal holdings turn out to be as large as those presently observed. Our results match some of the characteristics of the Brazilian business cycle.

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