NBER WORKING PAPER SERIES

CARRY TRADE, RESERVE ACCUMULATION, AND EXCHANGE-RATE REGIMES

Laura Alfaro
Fabio Kanczuk

Working Paper 19098
http://www.nber.org/papers/w19098

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2013

We thank Jeffrey Frankel, Márcio Garcia Ricardo, Hausmann, Lakshmi Iyer, Carmen Reinhart, Bill Russell, Eric Werker and participants of seminars at Kennedy School of Government, HBS, Fundacão Gertulio Vargas and University of São Paulo for valuable comments and suggestions. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Carry Trade, Reserve Accumulation, and Exchange-Rate Regimes
Laura Alfaro and Fabio Kanczuk
NBER Working Paper No. 19098
June 2013
JEL No. F31,F34

ABSTRACT

Carry-trade activity and foreign participation in local-currency-bond markets in emerging countries have increased dramatically over the past decade. In light of these trends, we revisit the question of the optimal exchange-rate regime when developing countries can borrow internationally with local-currency-denominated debt. We find that, as local-currency-bond markets develop, a "pseudo-flexible regime," whereby a country accumulates reserves in conjunction with debt, to be the policy that most effectively stabilizes fluctuations under real external shocks for emerging nations.

Laura Alfaro
Harvard Business School
Morgan Hall 263
Soldiers Field
Boston, MA 02163
and NBER
lalfaro@hbs.edu

Fabio Kanczuk
University of São Paulo
R. Dr Alberto Cardoso de Melo Neto 110/131A
Sao Paulo-S.P.-CEP 01455-100
BRAZIL
kanczuk@usp.br

1 Introduction

Debate over the optimal exchange-rate regime is recurrent in academic and policy circles. This paper revisits this debate under novel assumptions that reflect the new reality of capital flows to emerging markets. First and foremost, we consider the increase in foreign participation in local bond markets and the associated growth of carry-trade activity to imply that emerging countries are now borrowing internationally in domestic-currency-denominated bonds. Second, we consider active reserve accumulation coterminous with debt accumulation to constitute a policy alternative to manage the exchange rate.

Carry trade and foreign participation in local-currency-bond markets in emerging economies, our first hypothesis, became quantitatively relevant only in the present millennium.[1] Although difficult to measure with precision, carry-trade activity—that is, borrowing resources in low-interest-rate currencies and investing in high-interest-rate currencies without hedging for exchange-rate risk—has increased enormously. Data from the Bank of International Settlement (BIS) show that the share of domestic bonds has increased across different emerging markets between 2000 and 2010 (see Table 1). While debt in emerging markets increased from close to \$1 trillion in 2000 to more than \$4 trillion in 2010, the composition suggests a shift away from foreign-currency-denominated debt, from 11% in 2000 to 4% in 2010. More importantly, the participation of foreign residents has increased in the emerging markets' domestic-bond markets. The stock of Japanese households' foreign currency investments made through investment trust funds, almost nonexistent in 2002, was 35 trillion yen in 2007 (BIS, 2012). This is but one example of resources that, in search of high interest rates, often flow to emerging markets.[2] Burger,

[1] Although the literature uses different definitions, by local-currency bonds we mean the issuance of debt not indexed or linked to exchange rate independently of the legal framework adopted for its issuance. See Reinhart and Rogoff (2011) for definitions of domestic and external debt and Burger, Warnock, and Warnock (2012) for an assessment of the recent development of local-currency bonds in emerging markets.

[2] Burnside, Eichenbaum, and Rebelo (2007) show that including emerging-market currencies in the portfolio substantially increases the Sharpe ratio associated with the carry trade.

Warnock, and Warnock (2012) document a similarly dramatic post-2001 increase in U.S. investors' participation in local-currency-bond markets in emerging economies. Figure 1 shows another example: the increase in foreign participation in Brazil's domestic federal debt, which has also increasingly moved away from instruments linked to the exchange rate.[3] One might reasonably conjecture that such massive capital flows may significantly influence developing countries' exchange-rate determination and choice of exchange-rate regime.

Second, our model considers the role of reserve accumulation in the context of debt and exchange-rate management. The rapid rise in international reserves held by developing countries has spurred renewed interest in policy and academic circles about the optimal level of foreign reserves that sovereign countries should hold. Not exclusive to China or the East Asian countries, this practice has become widespread among emerging markets around the world including countries that hold a large external debt (see Figure 2). For example, the Brazilian government's net asset position has increasingly been dominated by the accumulation of close to $352 billion in reserves as of December of 2011 (against an external debt of $298 billion).

The cost of holding reserves has been estimated at close to 1% of GDP (Rodrik, 2006), which poses the question: What are the benefits? One possibility is that countries accumulate reserves as insurance against the risk of external crisis; that is, self-protection through increased liquidity.[4] Another possibility, which we consider here, is that the reserves are a mechanism for managing exchange rates and net debt positions. According to the IMF, the number of countries actively managing the exchange rate has increased over the last decade (see Habermeier, et al. 2009, IMF, 2012).[5]

---

[3] In Latin America, the share of exchange-rate-linked debt fell from close to 23% in 2005 to 13% in 2010 (BIS, 2012).

[4] There is no consensus as to whether self-insurance motivations can quantitatively account for the observed reserve accumulation in emerging markets; see Alfaro and Kanczuk (2009), Bianchi et al. (2012), Durdu, Mendoza, and Terrones (2008), and Jeanne and Rancière (2009).

[5] Over the last decade, Brazil, Chile, Colombia, Turkey, among other emerging markets with announced inflation targeting regimes, have engaged in substantial intervention of the exchange rate; see Céspesdes, Chang, and Velasco (2012b) and Aizenman, Hutchison, and Noy (2011). Calvo and Reinhart (2002) coined the term "fear of floating" for the authorities' reluctance to allow free fluctuations in the nominal (or real) exchange rate.

To address these issues, we construct and calibrate a dynamic equilibrium model of a small open economy. To smooth consumption, a benevolent government may optimally issue foreign-debt in both domestic and international currencies, denominated in the price of nontradable and tradable goods, respectively. Domestic and international interest rates may differ and we explicitly model the risks attendant on those differences. A technical consequence of these assumptions is that exchange-rate determination in our model, rather than being the price of money, is such that the portfolio allocation is satisfied.

Under this framework, we investigate the optimality conditions of different exchange-rate regimes under domestic and international shocks. We also assume that debt is noncontingent and that the government can issue reserve assets. Under these assumptions, the question of selecting an optimal exchange-rate regime looks much like a problem of optimal debt management whereby sovereigns choose a debt denomination. As such, the sustainability or viability of the exchange-rate regime becomes an explicit part of the analysis.

Our paper has four main results. First, we find, as have many studies of optimal exchange-rate regime, that the choice depends crucially on the type of shock to an economy. In our framework, a flexible-exchange-rate regime is optimal for domestic shocks and a fixed-exchange-rate regime is optimal for external shocks. This result is reminiscent of Mundell's (1968), with respect to nominal versus real shocks, obtained using a model with money and only domestic shocks. Second, we find that the traditional fixed-exchange-rate regime, albeit ideal in the presence of external shocks, is not sustainable. Third, we find that a flexible-exchange-rate regime can reduce exchange-rate volatility by issuing local-currency bonds, a policy we dub a "pseudo-flexible regime." Fourth, we find that welfare levels are better if a pseudo-flexible regime is implemented in conjunction with reserve accumulation.

In our analysis, countries do not engage in high levels of reserve accumulation to deplete them in "bad" times, as usually suggested in policy circles. Instead, the issuance of domestic debt while accumulating large levels of reserve acts as a hedge against negative external shocks.[6] In this respect, this result relates to the vast literature that looks for the optimal international portfolio diversification, as in Healthcote and Perri (2013). Unlike most of this literature, rather than focusing on the relative price of exportable and importable goods, this paper examines the variation between tradeable and nontradeable prices.

In a recent paper, Céspedes, Chang, and Velasco (2012a) find that unconventional policies such as accumulation of international reserves may have a role in the presence of financial frictions. In their model, large enough reserves may signal that the government would be able to prevent the exchange rate from depreciating and thus eliminate a bad equilibrium. In contrast, in our model, reserve accumulation is justified as a policy tool in conjunction with debt management regardless of the exchange-rate misalignment. It is an optimal policy tool whether the exchange rate is appreciated or depreciated.

This paper is therefore also related to the increasing literature that examines the determinants of reserve accumulation in emerging markets. We present an exchange-rate management rationale for reserve accumulation based on the interaction of government debt and private debt flows in foreign and local currency bonds (the weakening or "redemption" of the "original sin"), to complement explanations emphasizing precautionary motives (Durdu, Mendoza, and Terrones, 2008; Jeanne and Rancière, 2011, Bianchi et al, 2012), financial stability (Obstfeld, Shambaugh, and Taylor, 2010), high pass-through coefficients that may lead to devaluation-inflation (Hausmann, Panizza, and Stein, 2001), potential externalities from the tradable sector or mercantilist view (Dooley, Folkerts-Landau, and Garber, 2003; Korinek and Servan, 2010; Benigno and Fornaro, 2011), and political economy considerations (Aizenman and Marion, 2003), among others.

[6] In Alfaro and Kanczuk (2009), we analyzed the role of reserve accumulation as a debt-management tool in a model with foreign-currency-denominated debt. In the present model, the sovereign issues domestically denominated bonds as well. See discussion in Section 8.

Conclusions reached by the academic literature about optimal exchange-rate regime vary with the hypothesis (for contrasting examples, see, among others, Helpman, 1981; Devereux and Engel, 2003; Céspedes, Chang, and Velasco, 2004; and Lahiri, Singh, and Végh, 2007). This paper's contribution to the literature derives from our hypotheses' reflection of recent trends in the international flow of funds: the development of local currency bonds in emerging markets and increased foreign participation. In the sense that our hypotheses progressively characterize the evolution of the emerging markets, our conclusions may be increasingly relevant.

Our results have normative implications. The new consensus in policy circles seems to be that textbook exchange-rate regimes are impractical (see, for example, Fischer, 2001; Calvo and Mishkin 2003; Williamson, 2010). Fixed-exchange-rate regimes, though they may have desirable features in some contexts, have been historically condemned by speculative attacks. Freely floating exchange-rates, being subject to large currency misalignments despite the absence of any shocks that might conceivably have justified them, impose substantial economic costs. We find that as emerging nation develop their local currency markets, a "pseudo-flexible regime," whereby a country accumulates reserves in conjunction with debt, to be the policy that most effectively stabilizes fluctuations under real external shocks.

The rest of the paper is organized as follows. In Section 2, we present the model. The adequacy of different exchange-rate regimes is discussed in Sections 3, 4, 5, and 6. In Section 7, we calibrate the model and evaluate welfare via quantitative simulation and Section 8 presents various robustness exercises. Section 9 concludes.

2 Model

We model the economy of a sovereign country that borrows funds from a continuum of international risk-neutral investors and faces uncertainty in output. Preferences are concave in consumption, implying that households prefer a smooth consumption profile for both tradable and nontradable goods. To smooth consumption, a benevolent government may optimally issue foreign debt in both domestic and international currencies, denominated in the price of nontradable and tradable goods, respectively.

More precisely, we assume the sovereign's preferences to be given by

$$U = E\sum_{t=0}^{\infty}\beta^t u(c_t^T, c_t^N, \frac{M_t}{E_t}), \tag{1}$$

with
$$u(c^T, c^N, \frac{M_t}{E_t}) = \frac{[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}]^{(\frac{1-\gamma}{-\eta})} - 1}{(1-\gamma)} + \phi v(\frac{M_t}{E_t}), \tag{2}$$

where ${c^T}_t$ and ${c^N}_t$ denote household consumption of tradable and nontradable goods, M_t is the stock of money, and E_t is the nominal exchange rate. The parameter $\gamma > 0$ measures the curvature of the utility, $1/(\eta + 1)$ represents the elasticity of substitution between tradable and nontradable goods, and ω indicates these goods' relative importance to household preferences. The function v is increasing, continuous, and concave and the parameter ϕ indicates the relevance of money. Finally, $\beta \in (0, 1)$ is the discount factor.

The country's budget constraint is given by

$$M_t + E_t c_t^T + p_t^N c_t^N - E_t B_t - p_t^N D_t =$$
$$M_{t-1} + E_t A^T \exp(z_t^T) + p_t^N A^N \exp(z_t^N) - E_t B_{t-1}(1+\rho_{t-1}) - p_t^N D_{t-1}(1+r_{t-1}),$$

where p_t^N is the price of nontradable goods, B_t is the foreign-denominated debt level, D_t is the domestic-denominated debt level in period t, ${z^T}_t$ and ${z^N}_i$ are the tradable and nontradable technology states in this period that determine the output level of each type of good in conjunction with the constants A^T and A^N, ρ_t is the exogenous international real interest rate, and r_t is the endogenously determined real interest rate for domestically denominated debt.

Following much of the current literature on monetary policy (Woodford, 2003), we take into consideration that, in recent decades, the monetary base became increasingly erratic and irrelevant compared to the size of the economies. Recent models designed to study optimal monetary policy deemphasize the role of money. Instead they consider that the central bank only needs to control short-term interest rates and do not explicitly consider monetary aggregates. It is thus natural to expect exchange-rate models to follow the same path.

In line with this literature, we assume M_t is negligible compared with the other terms of the budget constraint and accordingly that ϕ is small, so that money can effectively be taken out of the model. We further express all prices in terms of nontradable goods by defining $e_t = E_t / p_t^N$. The country budget constraint is now given by

$$c_t^T + \frac{c_t^N}{e_t} - B_t - \frac{D_t}{e_t} = A^T \exp(z_t^T) + \frac{A^N \exp(z_t^N)}{e_t} - B_{t-1}(1+\rho_{t-1}) - \frac{D_{t-1}(1+r_{t-1})}{e_t}. \quad (3)$$

Given the absence of money, the budget constraint is now defined in terms of the *real* exchange rate e_t. However, there are various possible assumptions that permit us to think of the real exchange rate as the nominal exchange rate. First, one may assume prices are sticky, as in any Mundell-Fleming analysis.[7] Additionally, the hypothesis that variations in the nominal exchange rate are occasioned almost entirely by variations in the real exchange rate—that is, that prices are sticky—is hardly controversial. Second, as an alternative assumption, one can assume prices are entirely flexible, and that the central bank policy, possibly through a Taylor rule, is such that the inflation of nontradable goods does not deviate from a target. In this case, the fluctuations of endowment and consumption would have to be fully accommodated by movements in the nominal exchange rate.[8]

Another assumption of the model that deserves attention is that the borrowing and lending of resources are actions of the sovereign, not of households. This surfaces two issues. First, the model analysis would be unchanged if debt or transactions between the private sector and

---

[7] In the absence of nominal rigidities, the choice of exchange-rate regime becomes irrelevant (Helpman, 1981).
[8] Calvo, Reinhart, and Végh (1995) present a theoretical and empirical discussion of policies aimed at targeting the real exchange rate. The authors conclude that the real exchange rate is a common target in developing countries. See Aizenman, Hutchison, and Noy (2011) for recent practices.

government (that is, between households and sovereign) were included, presuming no tax distortions or other imperfection. Second, in principle, households' access to international markets could affect the model results even in the case of a benevolent sovereign. However, households lack incentives to borrow or lend internationally in equilibrium because our model does not display time-inconsistency problems. Hence, the hypothesis that borrowing and lending are done by the sovereign and not by households is not crucial for this model's policy implications.

We assume that the technology states z^T_t and z^N_t can take a finite number of values and evolve over time according to a Markov transition matrix with elements $\pi^T(z^T_i, z^T_j)$ and $\pi^N(z^N_i, z^N_j)$, respectively; that is, the probability that $z^T_{t+1} = z^T_j$, given that $z^T_t = z^T_i$, is given by the matrix π element of row i and column j, and analogously for z^N. As for timing, we consider that, in each period, z^T_t and z^N_t are revealed before the sovereign chooses B_{t+1} and D_{t+t}.

We assume a net debt ceiling, or maximum sustainable debt, given by

$$B_t + \frac{D_t}{e_t} \leq S .$$

Although an extensive literature on the sustainability of debt analyzes sovereigns' incentives to repudiate debt and endogenously determine the debt ceiling, for our purposes—and to simplify—we assume the debt ceiling to be exogenously given. We discuss generalizations of this assumption in Section 8.

International investors are risk-neutral and have an opportunity cost of funds given by ρ_t, which denotes the risk-free rate denominated in the price of tradable goods.[9]

For these investors to be indifferent between the riskless asset and lending in a country's nontradable-goods denomination, it must be the case that

$$(1+r_{t-1})E_{t-1}\left[\frac{e_{t-1}}{e_t}\right] = 1+\rho_{t-1}, \tag{4}$$

[9] As discussed in the next section, in this framework, one can easily consider investors' risk aversion, shocks to the international interest rate or "contagion effects" by contemplating shocks to the parameter ρ.

which is the uncovered interest parity.[10] Given this equation and taking the prices as given, the utility maximization of the sovereign-country model implies the two usual Euler equations. One, the intertemporal decision, reflects the desire to smooth consumption while the other reflects the intratemporal choice between the two types of consumption, which is given by

$$e_t = \frac{\omega}{(1-\omega)}\left(\frac{c_t^N}{c_t^T}\right)^{1+\eta}. \quad (5)$$

A final useful equation is the market-clearing condition for nontradable goods:

$$c_t^N = A^N \exp(z_t^N) \quad (6)$$

3 Optimal Exchange Rate with Domestic and International Shocks

Before discussing the implementation of exchange-rate regimes via different types of debt, we explore which exchange-rate regime is consistent with optimal allocation. For this purpose, we examine the optimal allocation when the economy is subject to different types of shock and obtain the exchange rate in each situation.

Domestic Shocks We begin by analyzing the optimal exchange rate in an economy subject only to domestic shocks. To better understand this situation, it is useful to consider a case in which households' preferences can be separated into two types of consumption. That is, $\frac{\partial u^2(c^T, c^N)}{\partial c^T \partial c^N} = 0$. For example, when the utility function has a CES specification, as in equation (1), this happens when $\eta = 0$ and $\gamma = 1$.

Under the hypothesis that the level of nontradable consumption does not affect the marginal utility of tradable goods, households would like to smooth tradable consumption as much as possible. Given that shocks are exclusively domestic, they can achieve perfect smoothing of

[10] With some abuse of notation, E_{t-1} henceforth denotes the expectation operator with the information available at $t-1$.

tradable consumption by keeping debt constant. To see this, note that plugging (6) into (3) and making $D_t = 0$ yields

$$c_t^T - B_t = A^T \exp(z_t^T) - B_{t-1}(1+\rho_{t-1}), \tag{7}$$

where z^T and ρ are constant.

The optimal solution to the households' problem implies a constant c^T. We can use equation (5) to obtain the optimal exchange rate. Given the existence of domestic shocks, c^N—and therefore the optimal exchange rate—will both change over time. The answer in this case is therefore simple: The optimal allocation is to keep debt (or reserves) constant and let the exchange rate float freely.

In the case in which the utility function is not separable, households will prefer not to perfectly smooth the consumption of tradables. Even in this case, however, the optimal exchange rate should fluctuate inasmuch as the optimal allocation should balance intertemporal smoothing with intratemporal substitution. Thus, a fixed-exchange-rate regime again would *not* be optimal. However, it is also not optimal in this case to hold debt constant, as this would imply constant consumption of tradables.

International Shocks Now consider an economy subject only to international shocks. International shocks are variously associated with terms of trade, the price of commodities, and international interest rates. For simplicity, our model represents all international shocks by variations in z^T. But note that shocks in ρ would be qualitatively the same because they would also affect the availability of (tradable) wealth and the smoothness profile of tradable goods.[11] In this case, consumption of nontradables will be constant and, regardless of the form of the utility

[11] As in Alfaro and Kanczuk (2009), contagion can be modeled by differentiating the price of reserves, $q^R=1/(1+\rho^R)$, with ρ^R (the risk-free rate associated with reserves) taking low values, from the price of bonds $q^B=1/(1+\rho^B)$, with ρ^B oscillating between low values for tranquil times and high values for nervous times.

function, households will prefer to perfectly smooth the consumption of tradable goods. Consequently, a fixed-exchange-rate regime is optimal.

The simple conclusion to be drawn from this discussion is that the optimal exchange rate depends on the type of shock to which an economy is subject. The optimal allocation when the shock is domestic is an exchange rate that changes over time. The optimal allocation when the shock is international is a fixed exchange rate. This recalls the implication of Mundell's (1968) work. In his framework, with money and only one sector, real shocks call for a floating exchange rate and monetary shocks call for a fixed exchange rate.[12]

Another conclusion is that the optimal allocation is implemented by simply holding debt constant when shocks are domestic and the utility function is separable into tradable and nontradable goods. This is the traditional, pure, flexible-exchange-rate regime.

In the next section, we further discuss the case in which shocks are international and it is optimal for the exchange rate to be fixed. Most discussions in policy circles are about precisely the situation in which international shocks drive emerging economies' exchange rates to levels that cannot be rationalized by fundamentals.[13] Policymakers, observing that large currency misalignments incur substantial economic costs, pose the question of how to proceed in order to make exchange rates less volatile.

4 The Traditional Fixed-Exchange-Rate Regime

In this and the following sections, we focus on the case of an economy subject only to international shocks. Consequently, as discussed above, the optimal allocation involves smoothing

---

[12] Our conclusion also relates to more recent utility-maximizing work by Devereux and Engel (2003), who show how optimal exchange-rate results are sensitive to whether prices are denominated in the producer's or the consumer's currency.

[13] Notice that the case in which shocks are domestic but preferences are not separable implies in a neither flexible nor constant exchange rate. In a sense, the only international shocks example is a particular and clearer case in which the optimal exchange rate is both inflexible and constant.

the path of tradable goods as much as possible. The question is how to do so.

We consider first the traditional fixed-exchange-rate regime wherein sovereign assets must change in order to keep the exchange rate constant and the sovereign issues debt only in international currency. Plugging (5) and (6) into (3) and making $D_t = 0$ gives the path debt must follow as:

$$B_t = B_{t-1}(1+\rho) + A^N \exp(z^N)\left[\frac{\omega}{(1-\omega)e}\right]^{\frac{1}{1+\eta}} - A^T \exp(z_t^T). \quad (8)$$

Note that the only time-varying variable in this expression is ${z^T}_t$, which completely determines the evolution of debt B_t for any given and constant exchange rate e. For each z^T, this is a difference equation that determines one steady state for B; that is, an exchange rate e that makes B constant. Because (1 + ρ) > 1, this steady state is unstable. As ${z^T}_t$ is a stochastic variable, we should be looking for an invariant distribution for B_t. But because (1 + ρ) > 1, this distribution is unbounded. In other words, from (8), B_t can grow to values that contradict the sustainability constraint $B \leq S$.

This result has an important and widely recognized implication for the traditional fixed-exchange-rate regime: Although it may be optimal, as in the case of international shocks in our model, it is not sustainable. Given a sequence of bad shocks (low values for z^T), debt B_t would exceed its sustainability ceiling. Sensing this and anticipating default on the sovereign's obligations, international investors would not lend more resources. As the debt, B_t, cannot increase, the sovereign is forced to abandon the fixed-exchange-rate regime.

The unsustainable nature of the traditional fixed-exchange-rate regime in our model raises a question about the sustainability of gold standard (or "true") currency-board regimes. We are accustomed to thinking that the gold standard regime has a self-correcting mechanism that makes it sustainable. Why is it not so in our model? First, the gold standard system implicitly assumes that, since money has to be fully backed by gold, countries cannot issue debt, or at least not enough debt to make the fixed-exchange-rate regime unsustainable. Second, the gold standard system considers that prices (or wages), being flexible in the long run, would make the necessary adjustments and that the real exchange rate would therefore also adjust.[14] By assuming prices to be perpetually fixed, we miss this channel.

5 Debt Denomination and the Pseudo- Flexible-Exchange Regime

Assume again that the economy is subject only to international shocks (with the consequent optimal allocation of smoothing as much as possible the path of tradable goods and holding the exchange rate constant). But, rather than issuing internationally denominated debt, the sovereign issues domestically denominated debt. More particularly, assume that the sovereign decides to hold the (real) debt constant; that is, $\frac{D_t}{e_t} = d$. Making $B_t = 0$ and plugging (4) and (6) into (3) gives

$$c_t^T = A^T \exp(z_t^T) + d\left[1 - \frac{(1+\rho)}{e_{t-1}E_{t-1}\left[1/e_t\right]}\right], \tag{9}$$

where, plugging (6) into (5), the exchange rate is given by

$$e_t = \frac{\omega}{(1-\omega)}\left(\frac{A^N \exp z^N}{c_t^T}\right)^{1+\eta}. \tag{10}$$

The solution of the system of Equations (9) and (10) determines the consumption of tradables c^T and exchange rate e_t as functions of exogenous variables. We use d to do comparative statics.

To understand how this equilibrium looks, consider a period that experiences a good shock; that is, a high z^T. Although the first term of the right-hand side of Equation (9) implies relatively higher consumption of tradables, the second term makes consumption of tradables relatively lower

[14] Argentina's currency board problems were related to, among other things, high fiscal deficits, increased debt, and deviations of 100% backing of high-powered money. The Maastricht Treaty similarly imposed maximum deficit and debt restrictions. However, for many members, such as Greece, the rules were consistently overlooked.

because the exchange rate is smaller (from Equation (10)). Consequently, for higher values of *d*, final consumption ends up not increasing as much during a period of a good shock as for lower values of *d* (for example, when *d* is equal to zero).

The economics of this result—that, in a stochastic environment, government liability should include state-contingent securities as a hedge against macroeconomic shocks in order to achieve consumption (or tax) smoothing (Bohn, 1990; Alfaro and Kanczuk, 2010)—are well known. In the present case, debt denomination is a useful means of smoothing consumption because debt services are negatively correlated with the endowment shock.[15] Perhaps less recognized is that the potential gains of contingent services are greater than those of contingent debt. That is, as Grossman and Han (1999) show, a constant amount of debt with contingent services may engender more smoothing than would be achieved by varying the amount of outstanding debt (without contingent services).

Returning to the case at hand, we find that, to achieve consumption smoothing, the sovereign would have to increase *d*. But this strategy is limited by a sustainability ceiling expressed by the constraint $d = D/e \leq S$. Of course, there is the quantitative question of whether this constraint is binding. But, anticipating the results of Section 7, we find that it is indeed binding and that the optimal policy under this regime, in which consumption and exchange rate are smoother, is obtained by setting $d = S$.

What should we call the policy of holding debt constant and letting the exchange rate fluctuate? Traditionally, because sovereign assets are held constant, it is termed a flexible-exchange-rate regime. But, as discussed, the exchange rate under this policy is less volatile. We refer to it as a "pseudo-flexible-exchange regime."

[15] This result relates to recent explanations regarding the international diversification puzzle (home bias in portfolio allocation). Endogenous international relative price fluctuations imply that asset positions in domestic currency are a good hedge to shocks; see Cole and Obstfeld (1991), and Heathocote and Perri (2013).

One should expect that exchange rates, despite the regime's smoothing effect, will tend to fluctuate. As a consequence of the exchange-rate risk, interest rates of domestically denominated bonds will differ from the riskless rate. Although the exchange rate appreciates when the technology state z^T is good (high), it is expected to devaluate under a revert-to-mean argument. Consequently, an emerging country's "contractual" yields of local-currency bonds should be higher than the international rate. To borrow resources internationally at the low interest rate and invest in high interest rate local-bonds (without hedging for the exchange-rate risk) is to practice a carry-trade strategy. In our model, by assumption, the "peso problem" completely explains the return to carry trade. That is, exchange-rate devaluations occur sufficiently often to make ex-post returns in local bonds equal the riskless rate.

6 Reserve Accumulation

In the previous sections, we considered only sovereign borrowing (denominated either in domestic or foreign currencies). An additional possibility would be sovereign assets in an international denomination. For reasons as yet unclear, rapid accumulation of reserves has recently become common practice in emerging countries. Given that the interest earned from these reserves is much lower than the interest paid on these countries' debts, this policy would seem extremely costly to these countries.

By way of example, the interest on Brazil's total government debt, approximately 60% of GDP in 2011[16], was about 12% per year, whereas the interest rate on its reserves, 15% of GDP in 2011, was approximately 2% per year. It is not clear why Brazil did not use its reserves to reduce the outstanding debt, given that the difference between the two interest rates multiplied by the amount of reserves implied a cost of about 1.5% of GDP.

[16] The number includes all public debt (domestic and external). As mentioned in the introduction, external debt in in Brazil was close to $298 billion in 2011.

Our model suggests a potential rationale for this type of policy. As discussed in the previous section, although it is optimal for a sovereign to issue as much debt as possible to smooth consumption, there is a sustainability limit to the outstanding debt. Having reserves or internationally denominated assets that bear the (risk-free) international rate enables a sovereign to maintain the same net debt and increase the stabilizing effect of its domestically denominated debt. Suppose, for example, that instead of $D/e = d = S$ domestically denominated debt, the sovereign asset position is $d' - R = S$, where R is the amount of reserves or internationally denominated assets. This would result in an increase in the domestically denominated debt by the amount of the reserves ($d' = S + R$) without increasing the net debt ceiling.

In this case, Equation (9) becomes

$$c_t^T = A^T \exp(z_t^T) + (S+R)\left[1 - \frac{(1+\rho)}{e_{t-1}E_{t-1}[1/e_t]}\right] + \rho R, \qquad (11)$$

where e_t is given by (10). Keeping net debt equal to its ceiling (S) when the sovereign accumulates reserves concurrently with debt, increases the importance of the second term of the right-hand side of (11) thereby smoothing consumption of tradables.

The logic of accumulating both reserves and debt is thus precisely that it is costly during good periods. When an international shock is favorable (z^T is high), debt services become higher and consumption is reduced. When an international shock is unfavorable, debt services become lower and consumption increases. When the whole invariant distribution of shocks is taken into account, a country will have a more stable consumption level and higher welfare. (The quantitative application presented in Section 7 makes this logic clearer.)

Note that in the proposed construction, the level of reserves remains high during unfavorable periods. Thus, contrary to the usual argument in policy circles, reserves are not an insurance that can be "used" in bad times.[17] The idea is not to buy consumption goods that deplete the stock of reserves, but rather to maintain a constant reserve stock that serves as insurance by increasing the stabilizing effect of domestically denominated debt.

7 Quantitative Experiment

To sharpen our results and quantify the welfare costs of the alternative exchange-rate regimes, we simulate our model under alternative hypotheses, calibrating it according to the following parameters.

We set the international interest rate $\rho = 0.04$ and the intertemporal substitution parameter $\sigma = 2$ as is usual in real-business-cycle research, in which each period corresponds to one year (see Kanczuk, 2004). To make the elasticity of substitution between tradable and nontradable goods equal to 0.5, in line with the evidence for Latin American countries, we make $\eta = 1$ (see, for example, Alves et al., 2003). The same evidence points to equal weights of tradables and nontradables, that is, $\omega = 0.5$. We set $\rho = 0.05$ and $A^T = A^N = 1$. The debt ceiling is $(B + D/e) \leq M = 0.6$, which corresponds to 60% of the tradable GDP in a neutral state ($z^T = z^N = 0$). To obtain reasonable levels of debt in equilibrium, we set the intertemporal factor at the relatively low value of $\beta = 0.90$, which is common practice in debt models (Alfaro and Kanczuk, 2009).

For the interesting case in which shocks are external, we make $z^N{}_t = 0$ for any t. To calibrate the technology state z^T, we consider the (logarithm) of the tradable GDP to follow an AR(1) process; that is, $z_{t+1}^T = \alpha z_t^T + \varepsilon_{t+1}$, where $\varepsilon_t \approx N(0,\sigma^2)$. We assume in our experiments that $\alpha = 0.5$ and use various standard deviations σ. We discretize this technology state into five possible values, spaced so that the extreme values are 2.5 standard deviations away from the mean, and use

[17] Aizenman and Hutchison (2010), for example, document what the authors labeled "fear of reserve loss": emerging markets reluctance to deplete their foreign reserves during the Subprime crisis.

the Quadrature Method (Tauchen, 1986) to calculate transition probabilities. We also discretize the space state of debt sufficiently to avoid affecting the decision rules.

Our first experiment considers a country that issues only bonds denominated in international currency. In this situation, the sovereign changes the level of outstanding debt in order to smooth tradable goods consumption as much as possible. Figure 3 shows, for the case σ = 20%, the policy function—, the level of debt B_t— to be a function of the state variables B_{t-1} (horizontal axis) and z^T_t (various curves). As a direct consequence of our hypothesis, the sovereign can hold, even for unfavorable states, a maximum level of debt that is up to 60% of GDP. When this restriction is not binding, the sovereign adjusts the level of debt in the expected way: in favorable states of nature (high z^T), the sovereign prefers to decrease the level of outstanding debt.

Table 2 reports the results for different volatilities of the endowment process *(σ)*. As expected, both the volatility of the consumption of tradables and the exchange rate increase with the volatility of the endowment. Less obvious is that average debt level decreases when the volatility of the endowment increases. When volatility is high, the sovereign opts to increase the volatility of debt, changing the outstanding debt to smooth consumption. A constraint in the maximum debt, however, forces the sovereign to operate with a smaller average debt.[18] When the volatility of the endowment is low, debt is used mainly to front-load consumption and the sovereign chooses to hold the maximum allowed debt.

Our second economy issues local-currency debt and potentially holds reserves. Its results, for the case σ = 20%, are reported in Table 3. We choose this volatility level as a benchmark because it implies volatilities of exchange rate close to 40% (Table 2), which is what we observe in emerging countries.

Starting with the case in which the sovereign has no reserves, it opts to hold the maximum outstanding debt. In fact, the policy function (which we do not graph) is to hold the maximum debt

[18] As discussed in the next section, the result is robust to endogenous debt limits.

next period regardless of the period state. This is because the stabilization effect of issuing local-currency debt affords sufficient consumption smoothing so that the debt can only be used to front-load consumption. When we increase the reserves, the *gross* debt also increases so that *net* debt remains at the maximum level. As discussed above, this amplifies the consumption-smoothing effect of local-currency debt.

We report the welfare of our second economy relative to the welfare of the first economy (only international-currency bonds) with the same endowment volatility (σ = 20%). Note that even without reserves the local-currency-bonds economy achieves substantially higher welfare (1.40% in consumption terms). In the case with a lot of reserves (400% of GDP), this economy performs even better (2.20% gain in consumption). Perhaps due to the numerical imprecision of our computational experiments, higher holdings of reserves do not further increase welfare.

8 Robustness and Discussion

Volatility of Exchange Rate In our model, as in other standard models of exchange rate, the volatility of the exchange rate is directly linked to the volatility of the ratio of tradable and nontradable goods consumption. As such, it only generates reasonable levels of exchange-rate volatility if we assume the volatility of consumption is much higher than that observed in the data. Adding to this puzzle, Engel (1999) shows that movements in the U.S. exchange rate are driven almost exclusively by changes in prices of tradable goods, which are usually assumed to be equal across countries.

To address these issues in the context of exchange-rate stabilizations, Burstein, Neves, and Rebelo (2003) introduce a distribution sector in an otherwise standard model and show that it can dramatically improve the model's performance. Rather than fully adding a new sector into our model, we follow their claim that taking a standard model and modifying preferences can mimic the introduction of distribution costs. These modifications are a larger share of nontradables in the utility and a lower elasticity of substitution between tradables and nontradables.

In particular, we modify the utility function, making the share of tradable goods $\omega = 0.25$ and choosing η such that the exchange-rate volatility is comparable to the observed value even when the volatility of consumption is only $\sigma = 2\%$. We obtained that when $\eta = 20$, the volatility of the exchange rate is 43.4% in the economy with international-currency debt. Next, we simulated the economy with local-currency debt. As before, we found that the exchange rate is less volatile and that its volatility decreases with the accumulation of reserves (Table 4). The only difference between the results of this experiment and the one in Section 6 is quantitative. In this alternative economy, the welfare gains reach only 0.13% (in consumption terms).

Endogenous Debt Another robustness consideration is how debt sustainability could change the results. For simplicity, our model assumes that the debt ceiling is exogenous. In order to make it endogenous, one has to study the sovereign's willingness-to-pay, which is a comparison of the gains and costs of defaulting. This, in turn, involves specifying the benefits of not servicing the debt and the costs related to not being able to smooth consumption using the international market and to the reduction in output associated with defaulting (Alfaro and Kanczuk, 2005). After this specification, our model could be written with the parameters related to output costs, the curvature of utility, and the volatility of the shocks, which determine the maximum sustainable debt. Rather than change our results, they would be expressed in terms of these parameters.

More interestingly, there is a question of how reserve accumulation in conjunction with debt affects sustainability. In Alfaro and Kanczuk (2009), we considered a contingent debt-service model in which both reserves and debt were denominated in international currency but were not perfect substitutes (since reserves can be used even after a country has defaulted). As Grossman and Han (1999) show, smoothing consumption through increasing debt is less effective than smoothing consumption through defaulting. Or, using their typology, "contingent service" generates more consumption smoothing than "contingent debt" (see also Grossman and Van Huyck, 1988). And since reserves are useful even after defaulting, they can be even more useful when the sovereign opts to pay service contingently.

After calibrating the model for realistic parameter values and abstracting from transaction costs, we determined that the reserve accumulation did not play a quantitatively important role. For reasonable parameter calibrations, higher welfare is achieved using reserves to pay down debt. Reserve accumulation did not increase debt sustainability and the optimal policy was not to hold reserves at all. (It took an extremely patient sovereign and volatilities of 7.5 to 10 times the benchmark calibration for the equilibrium amount of reserves and debt to become positive.)

In contrast, in the present paper, debt is denominated in local currency and reserve accumulation has a substantial impact on consumption smoothing via the management of the exchange rate. That is, reserve accumulation amplifies the insurance benefits of issuing domestically denominated bonds. As a consequence, reserve accumulation increases debt sustainability.

Net and Gross Debt Our results were obtained under the hypothesis of an exogenous ceiling for net debt. Normatively speaking, this could be justifiable, for example, by the option of pledging reserves as collateral.[19] Since reserves do smooth consumption in the model, in this scenario, their accumulation would increase the sustainability ceiling if it were endogenously modeled. As a consequence, in this case, the benefits of reserve accumulation could be even stronger. Our results also underscore the importance of valuation effects accruing to different gross and net positions.

More generally, assuming debt ceilings on gross positions, independently of net positions,

[19] There are different views the role of reserves as collateral. Under the Foreign Sovereign Immunities Act of 1976 in the United States and similar laws in other countries, Central Bank assets, including international reserves, are usually protected against attachment by creditors. The Bank for International Settlements, in Switzerland, is also protected against attachment proceedings. This, however, is not the case in Germany, where, under German law, reserves are open for attachment; see Scott (2005) and Sturzenegger and Zettelmeyer (2006).

does not seem to be borne by the data. This, in turn, may justify the view that *net* debt—the amount of debt minus the amount of reserves—may be an appropriate yardstick to measure a sovereign's sustainability. More importantly, the focus of this paper is to analyze the optimality of different exchange rate regimes against the fact that emerging markets increasingly have been able to issue debt in local currency while engaging in active reserve accumulation and active management of the exchange rate. These facts suggest potential hedging benefits of asset positions in domestic currency associated to endogenous relative price fluctuations of traded and nontraded goods (the exchange rate).

9 Conclusion

According to the Mundell-Fleming model, in an economy hit by foreign real shocks, flexible exchange rates dominate fixed rates. The intuition, stressed by Milton Friedman, is that nominal rigidities make it both faster and less costly to adjust the nominal exchange rate in response to a shock that requires a fall of the real exchange rate. But what if it were possible to offset the foreign shock so that the real exchange rate did not have to change?

We revisit the exchange-rate-regime choice under the assumption that emerging markets can borrow internationally in local currency. This hypothesis reflects a new trend in international capital flows: carry trade and relevant foreign participation in local-currency-bond markets. Moreover, emerging markets countries are increasingly deviating from inflation targeting regimes, managing their exchange rate and engaging in exchange-rate accumulation. Our main result is that, by means of international borrowing in domestic currency, emerging countries can partially offset foreign shocks. In conjunction with reserve accumulation, they can consequently implement a less flexible exchange regime, which we term pseudo-flexible, that is sustainable and yields higher welfare than alternative regimes.

References

Aizenman, Joshua, and Michael M. Hutchison (2010). “Exchange Market Pressure and Absorption by International Reserves: Emerging Markets and Fear of Reserve Loss During the 2008-09 Crisis,” NBER Working Paper No. 16260.

Aizenman, Joshua, Michael M. Hutchison, and Ilan Noy (2011). “Inflation Targeting and Real Exchange Rates in Emerging Markets,” *World Development* 39, 712-724.

Aizenman, Joshua, and Nancy Marion (2003). “International Reserve Holdings with Sovereign Risk and Costly Tax Collection,” *The Economic Journal* 114, 569-591.

Alfaro, Laura, and Fabio Kanczuk (2005). “Sovereign Debt as a Contingent Claim: A Quantitative Approach,” *Journal of International Economics* 65, 297-314.

Alfaro, Laura, and Fabio Kanczuk (2009). "Optimal Reserve Management and Sovereign Debt," *Journal of International Economics* 77, 23-36.

Alfaro, Laura, and Fabio Kanczuk (2010). "Nominal versus Indexed Debt: A Quantitative Horse Race," *Journal of International Money and Finance* 29, 1706-1726.

Alves, Denisar, Eustaquio Reis, Eduardo Fiúza, and Rodrigo Bueno (2003). “From Cross-Section to Time-Series Analysis: What Can Be Learned about the Elasticity of Tradable and Non-Tradable Goods in Brazil,” Inter-American Development Bank.

Benigno, Gianluca, and Luca Fornaro (2012). “Reserve Accumulation, Growth, and Financial Crises,” CEP Discussion Paper No 1161.

Bank for International Settlements (2012). “Developments of Domestic Government Bond Markets in EMEs and their Implications.” BIS Papers, no 67.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez (2012). “International Reserves and Rollover Risk,” working paper.

Bohn, Henning (1990). “Tax Smoothing with Financial Instruments,” *American Economic Review* 80, 1217-1230.

Burger, John D., Francis E. Warnock, and Veronica Cacdac Warnock (2012). “Investing in Local-currency-bonds Markets,” *Financial Analysts Journal* 68, 73-93.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (2007). “The Returns to Currency Speculation in Emerging Markets," *American Economic Review* 97, 333-338.

Burstein, Ariel T., Joao C. Neves, and Sergio Rebelo (2003). “Distribution Costs and Real Exchange-rate Dynamics during Exchange-Rate-Based Stabilizations,” *Journal of Monetary Economics* 50, 1189-1214.

Calvo, Guillermo, and Frederic Mishkin (2003). “The Mirage of Exchange-rate Regimes for Emerging Market Countries,” *The Journal of Economic Perspectives* 17, 99-118.

Calvo, Guillermo, and Carmen Reinhart (2002). “Fear of Floating,” *Quarterly Journal of Economics* 117 (2), 379-408.

Calvo, Guillermo, Carmen Reinhart and Carlos Végh (1995). “Targeting the Real Exchange Rate: Theory and Evidence,” *Journal of Development Economics* 47, 97-133.

Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco (2004). “Balance Sheets and Exchange-rate Policy,” *American Economic Review* 94, 1183-1193.

Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco (2012a). “Financial Intermediation, Exchange-rates, and Unconventional Policy in an Open Economy," NBER Working Paper 18431.

Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco (2012b). “Is Inflation Target Still on Target? Working paper.

Cole, Harald L. and Maurice Obstfeld (1991). “Commodity Trade and International Risk Sharing,” *Journal of Monetary Economics,* 28(1): 3-24.

Devereux, Michael B., and Charles Engel (2003). “Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility,” *Review of Economic Studies* 70, 765-783.

Dooley, M., D. Folkerts-Landau, and P. Garber (2003). “An Essay on the Revived Bretton Woods System,” NBER Working Paper 9971.

Fischer, Stanley (2001). “Exchange-rate Regimes: Is the Bipolar View Correct?” *Journal of Economic Perspectives* 15, 3-24.

Grossman, H. I., and J. B. Van Huyck (1988). “Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation,” *American Economic Review* 78, 1088-1097.

Grossman, Herschel I., and Taejoon Han (1999). “Sovereign Debt and Consumption Smoothing,” *Journal of Monetary Economics* 44, 149-158.

Habermeier, Karl Friedrich, Kokenyne, Annamaria, Veyrune, Romain and Anderson, Harald (2009), “Revised System for the Classification of Exchange Rate Arrangements,” *IMF Working Paper* 09/211.

Hausmann, Ricardo, Ugo Panizza, and Ernesto Stein (2001). “Why Do Countries Float the Way They Float?” *Journal of Development Economics* 66, 387-414.

Heathcote, Jonathan and Perri, Fabrizio (2013) “The International Diversification Puzzle is not as Bad as You Think” *Journal of Political Economy*, forthcoming.

Helpman, Elhanan (1981). “An Exploration in the Theory of Exchange-rate Regimes,” *Journal of Political Economy* 89, 865-890.

International Monetary Fund (2012). *Annual Report on Exchange Arrangements and Exchange Restrictions 2012*. Washington D.C.

Jeanne, Olivier, and Romain Rancière (2011). “The Optimal Level of Reserves for Emerging Market Countries: A New Formula and Some Applications,” *Economic Journal* 121, 905-930.

Kanczuk, Fabio (2004). “Real Interest Rates and Brazilian Business Cycles,” *Review of Economic Dynamics* 7, 436-455.

Lahiri, Amartya, Rajesh Singh, and Carlos A. Végh, (2007). “Segmented Asset Markets and Optimal Exchange-rate Regimes,” *Journal of International Economics*, 1-21.

Mundell, Robert A. (1968). *International Economics*. New York: MacMillan.

Obstfeld, Maurice, Jay C. Shambaugh, and Alan M. Taylor (2010). "Financial Stability, the Trilemma, and International Reserves," *American Economic Journal: Macroeconomics* 2, 57-94.

Reinhart, Carmen, and Kenneth Rogoff (2011). “The Forgotten History of Domestic Debt,” *The Economic Journal* 121, 319-350.

Rodrik, Dani. (2006). “The Social Cost of Foreign Exchange Reserves,” NBER Working Paper 11952.

Scott, Hal (2005). *International Finance: Transactions, Policy, and Regulation*, 12^{th} Edition. New York: Foundation Press.

Sturzenegger, Federico, and Jeremy Zettelmeyer (2007). *Debt Defaults and Lessons from a Decade of Crisis*. Cambridge, MA: MIT Press.

Tauchen, G. (1986). “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions,” *Economics Letters* 20, 177-181.

Williamson, John (2010). “Exchange-rate Policy in Brazil,” Peterson Institute for International Economics Working Paper 10-16.

Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Table 1: Outstanding Stocks of Domestic Government Debt Securities (Billions of U.S. Dollars)

	2000		2005		2010	
	Billions US$	% GDP	Billions US$	% GDP	Billions US$	% GDP
Asia	434	14.8%	1068	21.6%	2418	23.0%
China	111	9.3%	335	14.8%	1006	17.0%
Hong Kong	15	8.9%	18	10.1%	30	13.4%
India	112	23.6%	268	32.1%	608	36.1%
Indonesia	45	27.3%	40	14.0%	68	9.6%
Korea	62	11.6%	231	27.3%	331	32.6%
Malaysia	28	29.9%	51	37.0%	125	52.6%
Philippines	20	24.7%	40	38.8%	62	31.1%
Singapore	25	26.1%	47	38.1%	102	47.9%
Thailand	16	13.0%	38	21.5%	86	27.0%
Latin America	356	19.2%	639	26.5%	1346	29.3%
Argentina	33	11.6%	48	26.2%	31	8.4%
Brazil	219	34.0%	416	47.2%	949	44.3%
Chile	1	1.3%	3	2.4%	17	7.9%
Colombia	16	15.9%	40	27.3%	70	24.2%
Mexico	75	12.9%	115	13.5%	247	23.8%
Peru	4	7.5%	6	7.6%	14	9.1%
Venezuela	8	6.8%	11	7.6%	18	4.6%
Central Europe	56	20.3%	164	30.1%	270	33.9%
Czech Rep.	8	13.6%	26	20.0%	52	26.3%
Hungary	16	34.5%	43	39.0%	50	38.9%
Poland	32	18.7%	95	31.3%	168	35.8%
Other EMEs	110	16.7%	282	18.9%	421	16.3%
Russia	8	3.1%	25	3.3%	67	4.5%
S. Africa	47	35.4%	74	30.0%	125	34.4%
Turkey	55	20.6%	183	37.9%	229	31.3%
Total	**956**	**16.7%**	**2153**	**22.9%**	**4455**	**24.1%**

Notes: Central bank issues are excluded. In the BIS securities statistics, domestic debt securities are defined as issues by residents in the local market in local currency; some foreign currency issues are included in these data, but they are small; Central Bank issues are excluded. In Mexico, the numbers include debt resulting from the rescue of the banking sector, originally issued off-balance-sheet but now included in the government balance sheet. In Brazil, part of the increase in debt represents conversion of former central bank issues into government debt. For Chile, figures were taken from the Ministry of Finance.
Source: Taken from BIS, 2012 (BIS securities statistics; JPMorgan Chase; national data.)

Table 2: Invariant Distributions for International Currency Debt Economy

Endowment Standard Deviation (σ) (%)	*Debt (% GDP)*	*Consumption Standard Deviation (%)*	*Exchange-rate Standard Deviation (%)*
2	60	2.5	5.0
4	59.9	5.0	9.9
5	59.8	6.2	12.3
10	55.0	10.9	21.8
20	29.7	18.6	37.2
30	-15.2	25.0	49.9
40	-61.4	31.5	63.0

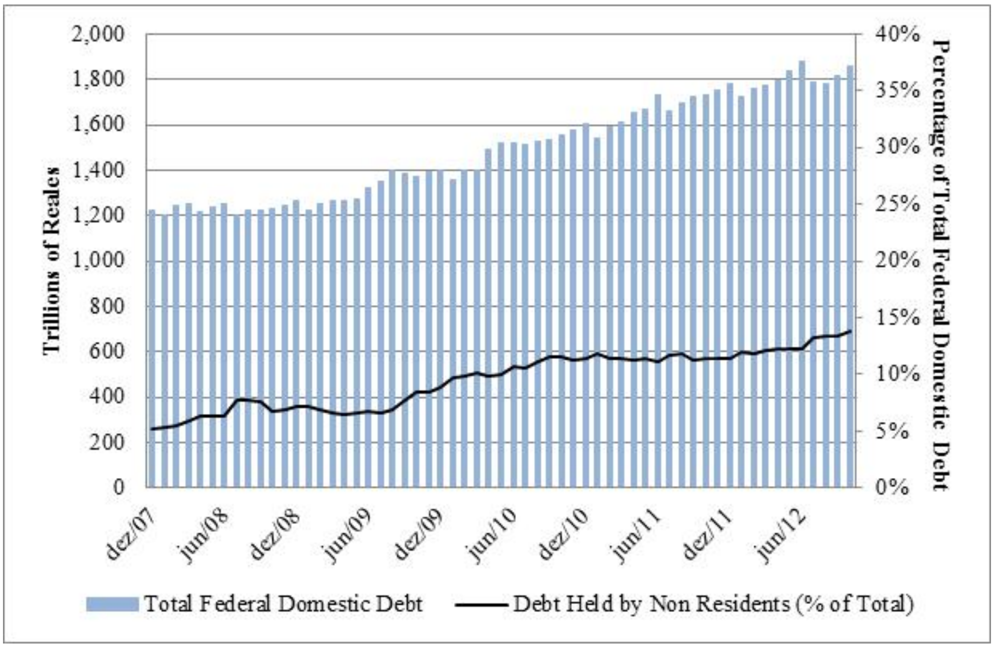
Table 3: Invariant Distributions for Local Currency Debt Economy with σ = 20%

Reserves (% GDP)	*Debt (% GDP)*	*Consumption Standard Deviation (%)*	*Exchange-rate Standard Deviation (%)*	*Welfare (% GDP)*
0	60	15.4	30.7	1.40
10	70	14.8	29.6	1.49
30	90	13.8	27.6	1.65
60	120	12.5	25.0	1.83
100	160	11.1	22.2	1.98
200	260	8.7	17.4	2.15
300	360	7.2	14.3	2.20
400	460	6.1	12.2	2.21
500	560	5.3	10.6	2.20

Table 4: Invariant Distributions for Alternative Economy (γ = 20, σ = 2%)

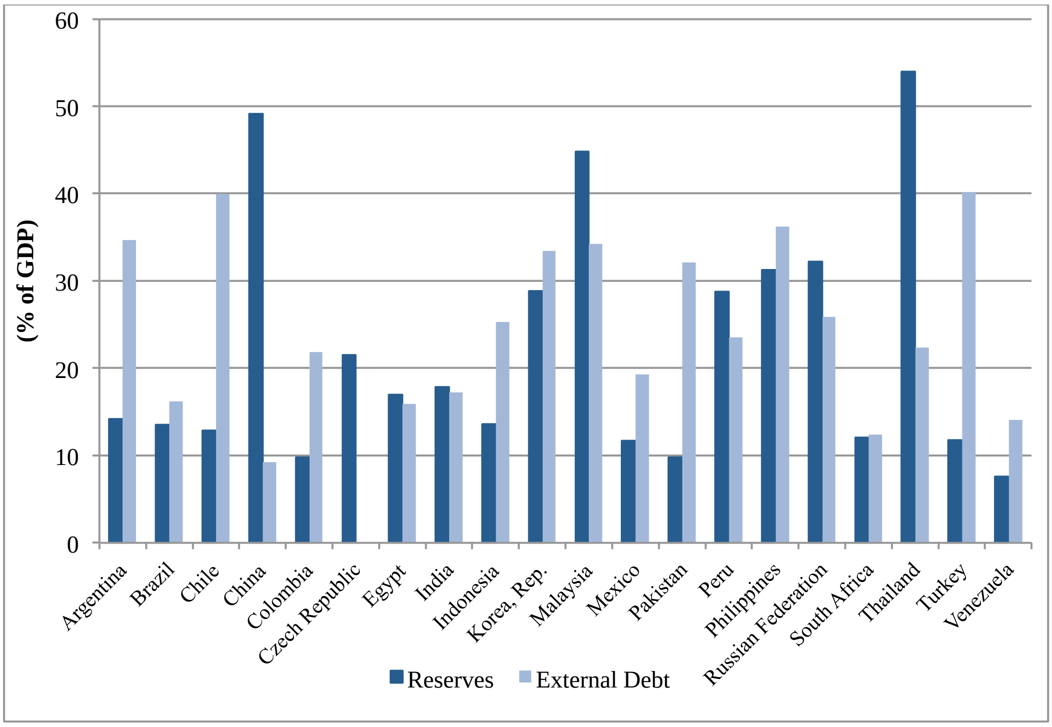
	Reserves (% GDP)	***Exchange-rate Standard Deviation (%)***
International Currency	0	43.4
Local Currency	0	6.8
Local Currency	50	4.7
Local Currency	100	3.6
Local Currency	500	1.3

Figure 1: Brazil's Total Federal Domestic Debt and Non-resident Participation



Source: Brazil, National Treasury (https://www.tesouro.fazenda.gov.br)

Figure 2: External Debt and Reserve Holdings of Emerging Countries in 2010 (% GDP)



Source: World Bank, World Development Indicators. For Czech Republic and Korea: gross debt; for others: external debt.

Figure 3: Debt Decision in the International Currency Debt Economy