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THE OPTIMAL INFLATION RATE IN AN OVERLAPPING-GENERATIONS ECONOMY WITH LAND

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ABSTRACT

This paper is concerned with the optimal inflation rate in an overlapping-generations economy in which (i) aggregate output is constrained by a standard neoclassical production function with diminishing marginal products for both capital and labor and (ii) the transaction-facilitating services of money are represented by means of a money-in-the-utility-function specification. With monetary injections provided by lump-sum transfers, the famous Chicago Rule prescription for monetary growth is necessary for Pareto optimality but a competitive equilibrium may fail to be Pareto optimal with that rule in force because of capital overaccumulation. The latter possibility does not exist, however, if the economy includes an asset that is productive and non-reproducible--i.e., if the economy is one with land. As this conclusion is independent of the monetary aspects of the model, it is argued that the possibility of capital overaccumulation should not be regarded as a matter of theoretical concern, even in the absence of government debt, intergenerational altruism, and social security systems or other "social contrivances."

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I. Introduction

Most of the existing analyses of the optimal inflation rate[1] that have been carried out in models with finite-lived individuals have reached conclusions that seem to contradict the famous "Chicago Rule" for optimal monetary growth.[2] An exception is provided by McCallum (1983, p. 38), which suggests that analysis of overlapping-generations models is supportive of the Chicago Rule provided that these models take account of the transaction-facilitating (i.e., medium of exchange) services of money.[3] More specifically, it is shown that, in a version of the Wallace (1980) model amended to reflect the existence of monetary transaction services, Pareto optimality of a stationary competitive equilibrium requires a rate of deflation equal to the marginal product of capital.[4] That particular model is one in which the marginal product of capital does not vary with capital intensity, however, and might consequently be judged inappropriate for analysis of this issue.[5] One leading purpose of the present paper, accordingly, is to reexamine the optimal inflation issue within a specification that incorporates a standard neoclassical production function with diminishing marginal productivity for both capital and labor inputs.

The investigation confirms that, within an overlapping-generations (OG) framework of the specified type, the Chicago Rule is indeed necessary for Pareto optimality. The analysis also indicates, however, that Pareto optimality may fail to obtain for a reason not considered in my previous discussion, namely, overaccumulation of capital. In particular, the steady-state net marginal product of capital (MPK) may exceed the rate of population growth, in the manner emphasized in the famous paper by Diamond (1965).[6] Because of this possibility, then, it is not valid to conclude that competitive equilibria will be Pareto optimal (in OG models with transaction-facilitating money) provided merely that the Chicago Rule prescription for monetary growth is obeyed.

A second major purpose of the present paper, accordingly, is to show that the possibility of capital overaccumulation does not exist[7] if the economy in question is one that includes a positive quantity of an asset that is productive and non-reproducible--i.e., in an economy with land. As this conclusion is independent of the monetary aspects of the model economy, and since all actual market economies do in fact include such assets, it follows that the possibility of capital overaccumulation should not be regarded as a matter of concern, even in the absence of government debt, intergenerational altruism, and social security systems or other "social contrivances."

The paper's emphasis on this last-mentioned result should not be interpreted as a claim that it has never before been recognized by an economist. In fact, there are brief passages in papers by Samuelson (1958, p. 481) and Stiglitz (1974, p. 139) that indicate recognition of the impossibility of capital overaccumulation.[8] But the point has received very little attention in the literature of monetary and macro economics, which includes various results that are overturned by its recognition. It would seem, accordingly, that some emphasis--as well as the exposition of an elementary proof--is warranted.

There are, of course, good reasons for being interested in analysis based on different assumptions than ours concerning taxes[9] and/or the optimality criterion.[10] Both tradition and the inherent logic of economic analysis speak in favor, however, of addressing the issue initially with the Pareto criterion and in a setting that is free of distortions due to income taxes and the like.

The organization of the paper is as follows. In Section II the basic monetary model--with a neoclassical production function but no land--is specified and the conditions characterizing competitive equilibrium are derived. Next, in Section III conditions sufficient for a Pareto optimum are obtained and compared with those achieved as an automatic consequence of competitive behavior. Then in Section IV the model is extended to recognize the existence of land and the Pareto optimality analysis is conducted. Finally, in Section V some general conclusions are offered.

II. An OG Model with Money

As indicated above, our object is to consider the optimal money growth issue in an OG model with a specification that reflects the transaction-facilitating services of money and, consequently, that asset's distinctive role as a medium of exchange.[11] For reasons argued at length in McCallum (1983), it is my judgement that this can be accomplished more satisfactorily by means of a specification of the money-in-the-utility-function (MIUF) type than by available alternatives.[12] Actually, my argument is for a specification in which agents derive utility only from consumption and leisure, but in which an agent's shopping time necessary to obtain consumption goods is reduced by holding increased amounts of real money balances (up to some satiation level) so that larger balances enable him to consume larger quantities of goods and/or leisure. For present purposes, however, it will suffice simply to adopt a MIUF assumption,[13] keeping in mind that there is some quantity of real balances--presumably a function of planned consumption--that will result in a zero marginal utility for the services provided by those balances.

The first ingredient in our model, then, is a lifetime utility function for two-period-lived agents born in t, which we write as

(1) $u(c_t, x_{t+1}, m_{t+1})$.

Here c_t is consumption when young, x_{t+1} is consumption when old, and m_{t+1} is real money balances (after transfers) at the start of old age. It is assumed that u has first and second partial derivatives u_1, u_2, u_3, and u_{11}, u_{22}, u_{33} satisfying $u_1 > 0$, $u_2 > 0$, $u_3 \geq 0$, $u_{11} < 0$, $u_{22} < 0$, and $u_{33} \leq 0$. It is also assumed that Inada-like properties pertain to consumption when young and old, so that agents will always choose positive amounts of c_t and x_{t+1}, but that u_3 can be driven to zero for some value of m_{t+1}, denoted $\widehat{m}_{t+1}(c_{t+1})$.

These agents are endowed with one unit of labor when young, which they supply inelastically, and none when old.

The second main ingredient of the model is a production function

(2) $y_t = f(n_t, k_t)$

that is accessible to old persons. Here y_t is output by an old person in t, n_t is the number of manhours that he employs in t from young persons, and k_t is the quantity of capital that he saved when young (in period t-1). It is assumed that f is homogenous of degree one and entirely well-behaved: $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, $f_{22} < 0$, and Inada properties prevail.

The agents described by (1) and (2) live in an ongoing economy in which the rate of population growth is ν, in which there are competitive markets for labor, output, capital, and loans, and in which the only governmental activity is the injection of lump-sum monetary transfers to old agents. The real quantity of such transfers to an old person during period t is denoted v_t. Therefore, if we let P_t be the money price of output and let M_t be the nominal money stock per old person after transfers in period t, then the government budget identity can be written in per-old-person terms as

(3) $v_t P_t = M_t - (1+\nu)^{-1} M_{t-1}.$

Furthermore, if μ_t denotes the rate of growth of the aggregate money stock, so that $1+\mu_t = (1+\nu)M_t/M_{t-1}$, we also have

(4) $$v_t = \frac{(1+\mu_t)M_{t-1} - M_{t-1}}{(1+\nu)\ P_t} = \frac{\mu_t m_{t-1}}{(1+\nu)(1+\pi_{t-1})}$$

where $1+\pi_t = P_{t+1}/P_t$ defines the inflation rate, π_t. Note that the definitions of μ_t and π_t imply that in a steady state, with constant values of the growth rates of all variables, we would have $1+\pi = (1+\mu)/(1+\nu)$.

In this setting, the behavior of a private agent born in t can be modelled by maximizing $u(c_t, x_{t+1}, m_{t+1})$ subject to the budget constraints faced when young and old. With w_t denoting the real wage rate in t, these constraints can be written as [14]

(5) $\quad w_t = c_t + k_{t+1} + \xi_t,$

where ξ_t denotes real money balances held at the end of t, and [15]

(6) $f(n_{t+1}, k_{t+1}) + (1-\delta)k_{t+1} - w_{t+1}n_{t+1} + v_{t+1} + \xi_t P_t/P_{t+1} = x_{t+1}.$

Also relevant, of course, is the identity

(7) $\quad m_{t+1} = v_{t+1} + \xi_t P_t/P_{t+1}.$

The first-order optimality conditions for this problem include (5)-(7) and the following:

(8) $\quad u_1(c_t, x_{t+1}, m_{t+1}) = u_2(c_t, x_{t+1}, m_{t+1}) \; [f_2(n_{t+1}, k_{t+1}) + 1-\delta]$

(9) $\quad u_3(c_t, x_{t+1}, m_{t+1})P_t/P_{t+1} = u_1(c_t, x_{t+1}, m_{t+1}) - u_2(c_t, x_{t+1}, m_{t+1})P_t/P_{t+1}$

(10) $\quad f_1(n_{t+1}, k_{t+1}) = w_{t+1}$

These determine the agent's decisions regarding c_t, k_{t+1}, ξ_t, m_{t+1}, n_{t+1}, and x_{t+1} as functions of v_{t+1} and the prices faced parametrically.

For a condition of equilibrium, we also require (3) and the following equalities:

(11) $\quad n_{t+1} = 1+\nu$

(12) $\quad f(n_t, k_t) + (1-\delta)k_t = x_t + (1+\nu)c_t + (1+\nu)k_{t+1}.$

Here (11) equates demand and supply of labor (per old person) while (12) is the overall resource constraint, also in per-old-person terms. Those three equations, in conjunction with (5)-(10), are adequate in number to govern the behavior of c_t, k_{t+1}, ξ_t, m_{t+1}, n_{t+1}, x_{t+1}, v_t, w_t, and P_t for an exogenously specified time path of the policy variable M_t (or μ_t).

For analysis of steady-state conditions, with a constant $\mu_t = \mu$, the foregoing system can be simplified to the following:

(13) $$f_1(1+\nu,k) = c + k + \xi$$

(14) $$f(1+\nu,k) + (1-\delta)k - f_1(1+\nu,k)(1+\nu) + v + \xi/(1+\pi) = x$$

(15) $$\frac{u_1(c,x,m)}{u_2(c,x,m)} = f_2(1+\nu,k) + 1-\delta$$

(16) $$\frac{u_3(c,x,m)}{u_2(c,x,m)} = [f_2(1+\nu,k) + 1-\delta]\ (1+\pi) - 1$$

(17) $$m = \nu + \xi/(1+\pi)$$

(18) $$v = \frac{\mu m}{(1+\nu)(1+\pi)}$$

(19) $$1+\pi = (1+\mu)/(1+\nu)$$

These determine c, k, x, ξ, m, v, and π as functions of the money stock and population growth rates μ and ν.

III. Conditions for Pareto Optimality

Our next step is to derive conditions relating to the attainment of Pareto optimality and to determine what inflation rate will permit these to be satisfied. Analytically, our approach will be to maximize the utility of a member of one generation subject to constrained values of the utility of members of all later generations, as well as the social feasibility requirements. Supposing that the date at which this calculation is made is t = 1, the Pareto problem is then to maximize $u(c_0,x_1,m_1)$ subject to

$$u(c_t,x_{t+1},m_{t+1}) = u_t^* \qquad t = 1,2,\ldots, \tag{20}$$

where the u_t^* are unknown solution values, and to

$$f(1+\nu,k_t) + (1-\delta)k_t = x_t + (1+\nu)c_t + (1+\nu)k_{t+1}, \tag{21}$$

for t = 1,2,.... To find conditions sufficient for optimality, we formulate the Lagrangian expression

$$L_1 = u(c_0,x_1,m_1) + \sum_{t=1}^{\infty} \theta_t[u(c_t,x_{t+1},m_{t+1})-u_t^*] + \sum_{t=1}^{\infty} \lambda_t[f(1+\nu,k_t)+(1-\delta)k_t-x_t-(1+\nu)c_t-(1+\nu)k_{t+1}] \tag{22}$$

For simplicity, let us introduce the notation $u_{jt} = u_j(c_t,x_{t+1},m_{t+1})$ and $f_{jt} = f_j(n_t,k_t)$. The implied first-order conditions are then as follows, for t = 1,2,...:

(23a) $\theta_t u_{1t} - \lambda_t(1+\nu) = 0$

(23b) $\theta_t u_{2t} - \lambda_{t+1} = 0$

(23c) $\theta_t u_{3t} = 0$

(23d) $\lambda_{t+1}[f_{2t+1} + 1-\delta] - (1+\nu)\lambda_t = 0.$

In addition, we have (20) and (21) plus

(24a) $u_2(c_0,x_1,m_1) - \lambda_1 = 0$

(24b) $u_3(c_0,x_1,m_1) = 0$

and the transversality condition[16]

$$(25) \quad \lim_{t\to\infty} \lambda_{t+1} k_{t+2} = 0$$

All of these would more appropriately be expressed as two-part Kuhn-Tucker conditions, reflecting the non-negativity of most of the model's variables, but our assumptions on u and f are adequate to ensure that positive values will be relevant and that the simpler equalities can be used.

To determine whether the foregoing conditions will be satisfied by a competitive equilibrium, we refer back to equations (5)-(12) in the previous section. Doing so, we immediately see that the only possibilities for failure involve (23c), (24b), and (25). For the first of these to be satisfied, it must be the case--as we see from equations (8) and (9)--that P_t/P_{t+1} equals $f_{2t+1} + 1-\delta$. That requirement can be reexpresssed as

$$(26) \quad \frac{P_t - P_{t+1}}{P_{t+1}} = f_{2t+1} - \delta$$

which is, of course, precisely the Chicago Rule prescription that the rate of deflation be equated to the (net) marginal product of capital.[17] Condition (24b) will obtain, moreover, if the Chicago Rule held in the past--and otherwise dictates the value of M_1.

Thus we see that inflation at the Chicago-Rule rate is necessary for Pareto optimality. Since the discussion of Weiss (1980) might appear to deny this, a brief word of explanation may be useful. The basic point is that Weiss's assumptions concerning the utility function imply that u_{3t}, the marginal service yield of real money balances, is strictly positive for all values of m_{t+1}. Thus the possibility of monetary satiation is precluded by assumption, with the consequence that Weiss's model is one in which no Pareto-optimal equilibrium can exist. That this aspect of his specification makes Weiss's model inapplicable to issues regarding Pareto optimality has been recognized by Abel (1984)--whose own optimality criterion is more demanding--and by Park (1986).

It cannot be concluded, however, that a policy of creating money and inflation in accordance with the Chicago Rule is <u>sufficient</u> for Pareto optimality, for that rule does not guarantee satisfaction of the transversality condition (25). A simple way of seeing that point is to rearrange (23d) as follows:

$$\lambda_{t+1} = \frac{(1+\nu)\lambda_t}{f_{2t+1}+1-\delta}. \tag{27}$$

Consider, then, the limiting behavior of λ_t as the system approaches a steady state with a constant value of $k_{t+1} = k$. Clearly, if the steady-state k is such that $1+\nu > f_2 + 1-\delta$, the transversality condition (25) will not be satisfied and the possibility of Pareto non-optimality will be introduced. In fact, in this case the economy's parameters are such that the competitive equilibrium leads to a steady state with capital overaccumulation, so the equilibrium will not be Pareto optimal despite adherence by the monetary authority to the Chicago Rule prescription. This "market failure" is, of course, the same as that featured in the analyses of Diamond (1965), Cass and Yaari (1967), and Phelps (1966).

IV. Extension to an Economy with Land

Reflection upon the nature of the capital overaccumulation phenomenon suggests, however, that a crucial feature of reality has been omitted from the model at hand. In particular, the reason for the phenomena's possible occurrence is simply that, as expressed by Cass and Yaari (1967, p. 251), "at efficient rates of interest consumers may want to hold more real assets than are available in the existing capital stock" (plus, in the present case, the real money stock). But in an economy with land--a non-reproducible, non-depreciating, and productive asset--this possibility can not obtain, for the real exchange value of land can and will be as large as is needed to accommodate desired private saving at an efficient rate of interest.

To demonstrate the validity of this claim, we now modify the model of previous sections by changing the per-capita production function to

(2') $y_t = f(n_t, k_t, \ell_t)$

where ℓ_t is land employed by a producer—an old agent—in t and where f is again homogeneous of degree one and well-behaved. In addition, it is assumed that the economy (i.e., f and u) is capable of attaining a unique steady state and that its dynamics are such that this steady state will be approached as time passes.[18]

With this modification, the budget constraints for an agent born in t become

(5') $w_t = c_t + k_{t+1} + \xi_t + q_t \ell_{t+1}$

and

(6') $f(n_{t+1}, k_{t+1}, \ell_{t+1}) + (1-\delta)k_{t+1} - w_{t+1}n_{t+1}$

$+ v_{t+1} + \xi_t P_t/P_{t+1} + q_{t+1}\ell_{t+1} = x_{t+1}$

where q_t is the real price in period t of a unit of land. The private optimality conditions then become (5'), (6'), and (7')[19] plus

(8') $u_{1t} = u_{2t}(f_{2t+1} + 1-\delta)$

(9') $u_{3t}\, P_t/P_{t+1} = u_{1t} - u_{2t}P_t/P_{t+1}$

(10') $f_{1t+1} = w_{t+1}$

and also

(28) $u_{1t} = u_{2t}(f_{3t+1} + q_{t+1})/q_t$.

For competitive equilibrium, we require satisfaction of (5')-(10'), (28), the government budget constraint (3), and the following three supply-equal-demand conditions:

(11') $n_{t+1} = 1+\nu$

(12') $f(n_t,k_t,\ell_t) + (1-\delta)k_t = x_t + (1+\nu)c_t + (1+\nu)k_{t+1}$

(29) $\ell_{t+1} = \ell_0/(1+\nu)^{t+1}$.

The last of these expresses an equality between the quantity of land demanded and supplied per old person with ℓ_0 the land to old person ratio in period 0. The 11 mentioned equations determine (for a given path of M_t or μ_t) the values of c_t, k_{t+1}, ℓ_{t+1}, n_{t+1}, ξ_t, m_{t+1}, x_{t+1}, v_t, w_t, P_t, and q_t.

Next we turn to the Pareto problem. In the present case it should be clear that the relevant Lagrangian expression is

(30) $$L_1 = u(c_0,x_1,m_1) + \sum_{t=1}^{\infty} \theta_t[u(c_t,x_{t+1},m_{t+1})-u_t^*]$$
$$+ \sum_{t=1}^{\infty} \lambda_t[f(1+\nu,k_t,\ell_0/(1+\nu)^t) + (1-\delta)k_t-x_t-(1+\nu)c_t-(1+\nu)k_{t+1}]$$

and that the first-order and transversality conditions are as follows:

(31a) $\theta_t u_{1t} - \lambda_t(1+\nu) = 0$

(31b) $\theta_t u_{2t} - \lambda_{t+1} = 0$

(31c) $\theta_t u_{3t} = 0$

(31d) $\lambda_{t+1}[f_{2t+1} + 1-\delta] - (1+\nu)\lambda_t = 0$

(32a) $u_2(c_0,x_1,m_1) - \lambda_1 = 0$

(32b) $u_3(c_0,x_1,m_1) = 0$

(33) $\lim_{t\to\infty} \lambda_{t+1}k_{t+2} = 0.$

As in the previous section, it is easy to see that the Chicago Rule condition

(34) $\frac{P_t-P_{t+1}}{P_{t+1}} = f_{2t+1} - \delta$

must hold, and also that the behavior of λ_t can be expressed as

(35) $\lambda_{t+1} = \frac{(1+\nu)\ \lambda_t}{f_{2t+1} + 1-\delta}.$

But the latter can now, in view of (28), alternatively be written as

(36) $\frac{\lambda_{t+1}}{\lambda_t} = \frac{(1+\nu)q_t}{f_{3t+1}+q_{t+1}}.$

Since the economy approaches a steady state, the limiting behavior of λ_{t+1}/λ_t can be deduced from the limiting behavior of the right-hand side of (36). To the evaluation of that expression we accordingly turn our attention.

In a steady state y_t/k_t must be constant, with the values of the denominator and numerator each proportional to $(1+\nu)^{(\alpha-1)t}$ where α is a positive fraction such that aggregate (not per capita) output grows according to $(1+\nu)^{\alpha t}$.[20] The steady state condition also requires that factor shares be constant,[21] so from the capital share expresion $f_{2t}k_t/y_t$ we see that f_{2t}--i.e., the marginal product of capital--must be constant. The share of land, by contrast, is $f_{3t}\ell_t/y_t$ from which we deduce that f_{3t} grows in the steady state according to $f_{3t} = f_{30}\ (1+\nu)^{\alpha t}$. Finally, with n_t constant, the labor-share expression $f_{1t}n_t/y_t$ implies that $w_t = f_{1t}$ grows at the same (negative) rate as y_t--i.e., that $w_t = f_{1t} = f_{10}(1+\nu)^{(\alpha-1)t}$.

Continuing with the implications of steady state growth, we refer to equation (12'), the economy's overall resource constraint. Since the left-hand side grows like $(1+\nu)^{(\alpha-1)t}$, so must each term on the right-hand side--which implies that c_t/x_{t+1} is constant with numerator and denominator each growing like $(1+\nu)^{(\alpha-1)t}$. Inspection of (5') or (6') then indicates that the product $q_t \ell_t$ must grow like $(1+\nu)^{(\alpha-1)t}$ which in turn implies that q_t behaves, in the steady state, according to $q_t = q_0(1+\nu)^{\alpha t}$.

From the foregoing, then, we can write the steady state value of the right-hand side of (36) as

$$(37) \quad \frac{(1+\nu)q_0(1+\nu)^{\alpha t}}{f_{30}(1+\nu)^{\alpha(t+1)}+q_0(1+\nu)^{\alpha(t+1)}} = \frac{(1+\nu)^{1-\alpha} q_0}{f_{30} + q_0},$$

where $(1+\nu)^{\alpha t}$ has been cancelled from the latter expression. But we also know that $k_{t+1}/k_t = (1+\nu)^{\alpha-1}$ in the steady state. Consequently, the steady state behavior of $\lambda_{t+1}k_{t+2}$ is given by

$$(38) \quad \frac{\lambda_{t+1}k_{t+2}}{\lambda_t k_{t+1}} = \frac{q_0}{f_{30} + q_0}.$$

But with $f_{30} > 0$, as assumed, the right-hand side of (38) is a positive fraction which implies that the limiting behavior of $\lambda_{t+1}k_{t+2}$ is to approach 0 as $t \to \infty$. This guarantees that the transversality condition (33) is satisfied. Thus all of the sufficient conditions for Pareto optimality are satisfied by the competitive equilibrium in the economy under discussion provided only that the money growth rate is such as to produce the Chicago Rule rate of inflation that induces satiation in real money balances.[22,23]

V. Conclusions

The foregoing line of argument is straightforward enough that a summary should be redundant. Instead, we conclude with a few observations on the assumptions utilized and then on the significance of the main result.

Throughout the foregoing discussion, it has been implicitly assumed that the economy under consideration does not benefit from technical progress. It would appear, however, that a simple modification of the proof employed would remain applicable in the presence of technical progress so long as the latter is of a type that will accommodate a steady state. What then of the assumption that steady state growth is feasible? It is my guess that that condition too is unnecessary for the result, but this guess is at present only that--I do not have a line of attack to propose for the more general case.

Mention should perhaps be made of the possibility of multiple solution paths,[24] which may obtain even under conditions implying a unique steady state. In a very useful and somewhat neglected paper, Calvo (1978) has shown that multiple solutions--bubble paths converging to the steady state--are possible in a model with land but no capital or money, and it seems clear that similar solution paths would also be possible in the model of Section IV. As long as these paths approach a steady state, however, their existence will not invalidate our argument.[25]

As for the significance of our results, the main point regarding monetary theory is simply that the insights expressed in the Chicago Rule are applicable even to economies without infinite-lived agents. It could be added--at the risk of belaboring the obvious--that the Rule would remain applicable if the economy's money paid interest; the optimum would then require not a zero nominal rate of interest on non-monetary assets, however, but a nominal rate equal to that paid on money.

In terms of the capital accumulation issue, our result--that overaccumulation is precluded by the presence of land--can readily be seen to obtain whether or not the economy is one with a medium of exchange. Consequently, it suggests that conclusions of a non-monetary type that require the possibility of overaccumulation should be reconsidered (at best).[26] As a prominent example, consider the argument developed in the final sections of Samuelson's original OG paper (1958, pp. 476-482). Evidently, these sections are intended to suggest that Samuelson's OG model provides analytical support for the notion that social "compacts" or "contrivances"--over and above the existence of markets--are apt to be necessary to avoid Pareto suboptimality in laissez faire economies that go on indefinitely.[27] That this suggestion is not overturned by relaxation of the assumption that all goods are highly perishable is implied by Diamond's (1965) demonstration that inefficient steady-state equilibria may exist in his model, a result that is in this respect a generalization of Samuelson's. But the analysis of the present paper indicates that this type of inefficiency--capital overaccumulation in a competitive economy free from tax distortions--requires the assumed absence of assets like land, an assumption that seems decidedly counterfactual and analytically inappropriate.

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Footnotes

1. This issue, also referred to under the heading of the "optimum quantity of money," is of course concerned with the optimal average rate of inflation over an extended period of time. Consequently, it abstracts entirely from matters concerned with cyclical fluctuations.

2. An incomplete but representative list of examples includes Stein (1971), Helpman and Sadka (1979), Wallace (1980), Weiss (1980), Drazen (1981), and Woodford (1985).

3. The main purpose of McCallum (1983) is to argue that those special overlapping-generations models (or other models!) that fail to take account of the transaction-facilitating services of money are highly inappropriate vehicles for monetary analysis, i.e., for analysis in which it is necessary to distinguish between assets that serve as a medium of exchange and those that do not. Further discussion of the issue is presented in McCallum (1986).

4. This rule, which received its most famous exposition in Friedman (1969), was mentioned earlier by Friedman (1960, p. 73) and given a very clear statement by Marty (1961, p. 57). It has been termed the Chicago Rule by Niehans (1978, p. 93) and Weiss (1980). The discussion here and throughout the present paper assumes that monetary injections are made by way of lump-sum transfers to old agents.

5. That a variable marginal product of capital is crucial in this context has apparently been suggested by Weiss (1980, p. 970).

6. It has, of course, been pointed out by Barro (1974) that Diamond's result presumes the absence of operative intergenerational transfers. Throughout the present paper such an absence is taken for granted, not because I believe that to be a particularly realistic assumption, but in order to consider its implications for the optimality of the Chicago Rule. For the same reason, the

possibility that population growth might be endogenous--as considered by Meltzer and Richard (1985)--is ignored. For an introduction to the concept of capital overaccumulation, see Burmeister (1980, pp. 57-74).

7. Even in the absence of intergenerational altruism, social security systems, and government debt.

8. An ambitious recent paper by Tirole (1985), which is devoted primarily to an investigation of the possibility of asset-price bubbles, considers a version of the Diamond model extended to include rents. Much of the analysis assumes that the aggregate quantity of rent is exogenously fixed, an assumption that leads to some conclusions that do not pertain to an economy with a fixed stock of land. (Tirole's Proposition 2, for example, seems to be inapplicable to the economy described in the present paper.) In one place, Tirole mentions the possibility that rents could grow at the rate of population growth in which case "a perfect foresight equilibrium must be efficient" (1985, p.1079), a conclusion that is similar in spirit to mine. He does not, however, consider a specification in which land, an asset fixed in total quantity, appears as a useful input to the productive process for aggregate output. Thus, his analysis provides no reason for believing that rents will tend to grow at the same rate as output--and in that sense does not include my result as a special case. A similar statement also applies to a much earlier but unpublished paper by Scheinkman (1980), which emphasizes the implausibility of capital overaccumulation. I am indebted to Olivier Blanchard for calling my attention to Tirole's paper.

9. Helpman and Sadka (1980) assume that labor and capital returns are taxed at flat rates in a setting otherwise similar to mine.

10. Notable discussions of the appropriate criterion have been provided by Samuelson (1967)(1968), Abel (1984), Calvo and Obstfeld (1985), and others.

11. An extensive discussion of the issue in an OG model in which "money" does not provide transaction-facilitating services to its holders is given by Wallace (1980).

12. Cash-in-advance models amount to a special case of shopping-time or MIUF models. For a useful recent discussion of some ways of recognizing the transaction services of money see Feenstra (1985).

13. Related analysis focussing explicitly on the shopping time specification has been conducted by Park (1986).

14. Strictly speaking, (5) and (6) should be written as inequalities. Throughout the paper, however, we shall simplify by using equalities when the conditions of the problem imply that they will hold as such in equilibrium.

15. The possibility of making loans to (or borrowing from) other individuals of the same generation is not made explicit in (6) because the equilibrium quantity of such loans will be zero for each individual, as they are all alike. The existence of an (inactive) loan market is assumed, however, and justifies the form of condition (8), which implies that the (common) real rate of return on capital and loans is taken exogenously by each individual.

16. That the transversality condition and first-order conditions are jointly sufficient for optimality in a setting such as this one is well-known from the work of Weitzman (1973) and others. The first-order conditions are also necessary.

17. That the deflation rate is here measured as $(P_t-P_{t+1})/P_{t+1}$ rather than $(P_t-P_{t+1})/P_t$ is an unimportant manifestation of our discrete-time framework.

18. These assumptions make the situation with respect to existence, uniqueness, and stability similar to that presumed by Diamond (1965, p. 1134). Some comments on these assumptions will be provided below in Section V. With a fixed total stock of land, the assumed possibility of a steady state

comes close to a requirement that f is Cobb-Douglas, a fact mentioned in a different but related context by Solow (1974).

19. Here (7') is simply a new label for equation (7).

20. If the production function is Cobb-Douglas with factor exponents of α_1, α_2, and α_3 (for n_t, k_t, and ℓ_t, respectively), then α would be equal to $\alpha_1/(\alpha_1+\alpha_3)$)

21. This requirement stems from the same arithmetic fact that necessitates the constancy of y_t / k_t, namely, that for two terms and their sum all to grow at constant rates, those rates must be equal.

22. This statement presumes that the Chicago Rule inflation rate held in the most recent period so that the <u>initial</u> stock of real money balances induces satiation.

23. That the steady-state value of the net marginal product of capital must exceed the rate of growth can be seen, incidentally, by noting from (8') and (28) that $f_{2t} + 1-\delta = (f_{3t}+q_{t+1})/q_t$ and then deducing that the steady-state value of the right-hand side of the latter is $[f_{30} + q_0(1+\nu)^\alpha]/q_0 > (1+\nu)^\alpha$.

24. Some analysts would probably contend that bubble solutions should be accorded more emphasis then this statement implies. I would suggest, however, that bubble phenomena and the possibility of capital overaccumulation are distinct subjects that can best be understood in isolation. In particular, it would seem appropriate to discuss the overaccumulation possibility first under the assumption that bubble paths are excluded from consideration. (They are, of course, absent from the analysis of Diamond (1965).)

25. For a much more complete discussion of multiple solutions in OG models the reader is referred to Tirole (1985). The reader should note, however, that Tirole's definition of bubbles (and market fundamentals) differs, in the case of

Wallace (1980)-style OG models, from terminology previously employed by many authors--including McCallum (1983, p.15). Consequently, some of Tirole's conclusions must be interpreted carefully.

26. Here I am taking it for granted that no one would wish to argue that actual economies possess no assets with the properties of land. For many issues, of course, it is convenient and not misleading to ignore such assets in the analysis. But for issues relating to overaccumulation, recognition is apparently essential.

27. In economies in which, in Samuelson's (1958, p. 482) words, "every today is followed by a tomorrow."