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AGGREGATE DEMAND, IDLE TIME, AND UNEMPLOYMENT

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ABSTRACT

This paper develops a model of unemployment fluctuations. The model keeps the architecture of the general-disequilibrium model of Barro and Grossman [1971] but takes a matching approach to the labor and product markets instead of a disequilibrium approach. On the product and labor markets, both price and tightness adjust to equalize supply and demand. Since there are two variables but only one equilibrium condition on each market, a price mechanism is needed to select an equilibrium. We focus on two polar mechanisms: fixed prices and competitive prices. When prices are fixed, aggregate demand affects unemployment: with a higher aggregate demand, firms find more customers; this reduces the idle time of their employees and thus increases their labor demand; and this reduces unemployment. We combine the predictions of the model and empirical measures of product market tightness, labor market tightness, output, and employment to assess the sources of labor market fluctuations in the US. First, we find that the product market and labor market tightnesses fluctuate a lot, which implies that the fixed-price equilibrium describes the data better than the competitive-price equilibrium. Next, we find that labor market tightness and employment are positively correlated, which suggests that labor market fluctuations are mostly due to labor demand shocks and not to labor supply or mismatch shocks. Last, we find that product market tightness and output are positively correlated, which suggests that the labor demand shocks are mostly aggregate demand shocks and not technology shocks.

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1 Introduction

Numerous hypotheses have been formulated and empirically tested to explain the extent and persistence of unemployment in the US between December 2008 and November 2013. Over that five-year period, the unemployment rate remained above 7%, peaking at 10% in October 2009. These hypotheses include high labor market mismatch, caused by major shocks to the financial and housing sectors; low search effort from unemployed workers, triggered by the long extension of unemployment insurance benefits; and low aggregate demand, caused by a sudden need to repay debts or by pessimism.¹ Low technology is another natural hypothesis since technology shocks are the main source of fluctuations in the textbook model of unemployment.²

We have learned a lot from this work. Yet, our understanding of this period of high unemployment, and of the cyclical fluctuations of the labor market in general, remains incomplete. There is a view that to make progress, we need a macroeconomic model that describes the many sources of labor market fluctuations, including aggregate demand, while permitting comparative-static analysis. The aim of this paper is to develop such a model and use it to assess the sources of labor market fluctuations in the US.

Our starting point is the general-disequilibrium model of [Barro and Grossman \[1971\]](#). The Barro-Grossman model was the first microfounded representation of the macroeconomic theory of [Keynes \[1936\]](#). The model elegantly captures the link between aggregate demand and unemployment, so it is a promising starting point.³ However, it suffers from some limitations because it relies on disequilibrium, whereby the price is fixed and demand and supply may not be equal. First, disequilibrium raises difficult theoretical questions, for instance, how to ration those who cannot buy or sell what they would like at the prevailing price. Second, disequilibrium limits tractability, because the economy can be in four different regimes, each described by a different system of equations, depending on which sides of the product and labor markets are rationed.

We keep the architecture of the Barro-Grossman model: our model is static; it has a produced

¹On mismatch, see [Sahin et al. \[2014\]](#), [Lazear and Spletzer \[2012\]](#), and [Diamond \[2013\]](#). On job-search effort, see [Elsby, Hobijn and Sahin \[2010\]](#), [Rothstein \[2011\]](#), and [Farber and Valletta \[2013\]](#). On aggregate demand, see [Farmer \[2012\]](#) and [Mian and Sufi \[forthcoming\]](#).

²See for instance [Pissarides \[2000\]](#) and [Shimer \[2005\]](#).

³General-disequilibrium theory generated a vast amount of research. For reviews of this literature, see [Grandmont \[1977\]](#), [Drazen \[1980\]](#), and [Bénassy \[1993\]](#). The macroeconomic implications of the theory are discussed in [Barro and Grossman \[1976\]](#) and [Malinvaud \[1977\]](#). For recent applications of the theory, see [Mankiw and Weinzierl \[2011\]](#), [Caballero and Farhi \[2014\]](#), and [Korinek and Simsek \[2014\]](#).

good, labor, and money; the product and labor markets are formally symmetric. But to address the limitations of the Barro-Grossman model, we take a matching approach to the product and labor markets instead of a disequilibrium approach: on each market, a matching function governs the number of trades, and buyers incur a matching cost.

The matching approach allows us to move into general-equilibrium theory. A matching market is analogous to a Walrasian market in which a seller takes as given not only the price but also the probability to sell, and a buyer takes as given not only the price but also a price wedge reflecting the cost of matching. The selling probability and price wedge are determined by the market tightness. Hence, the matching equilibrium is analogous to a Walrasian equilibrium in which not only prices but also tightnesses equalize supply and demand on all markets.

Although grounded in equilibrium theory, the matching approach allows us to introduce the price and real-wage rigidities required for aggregate demand to influence unemployment. Indeed, on each matching market, both price and tightness adjust to equalize supply and demand. Hence, many combinations of prices and tightnesses satisfy all the equilibrium conditions. This means that many price mechanisms are consistent with equilibrium; of course, once a price mechanism is specified, the equilibrium is unique. To understand the effects of aggregate demand shocks, we study an equilibrium in which the price and real wage are fixed and the product market and labor market tightnesses equalize supply and demand on all markets. We contrast the properties of this fixprice equilibrium with those of an equilibrium in which the price and real wage are competitive—they ensure that the tightnesses always maximize consumption, in the spirit of [Moen \[1997\]](#). Last, we show that the results for fixed prices hold under partially rigid prices, and the results for competitive prices hold under Nash bargained prices.

The matching approach also allows us to describe the general equilibrium of the model with one system of well-behaved equations while preserving the property of the disequilibrium approach that market conditions are favorable sometimes to buyers and sometimes to sellers. This property is essential for the propagation of aggregate demand shocks to unemployment. In the Barro-Grossman model, sellers and buyers are in a binary situation on each market—rationed or not rationed. In our model, the conditions on each matching market are captured by a tightness: a high tightness is favorable to sellers and a low tightness is favorable to buyers. Because the tightnesses are continuous and not binary variables, the equilibrium equations are well behaved and the model tractable.

Our model provides comparative statics with respect to aggregate demand shocks. Despite the different formalism, our model retains the intuition of the Barro-Grossman model that negative aggregate demand shocks propagate to the labor market by making it more difficult for firms to sell goods or services. With fixed prices, a decrease in aggregate demand lowers product market tightness, which reduces sales made by firms and increases the idle time of employed workers. Since employees are idle a larger fraction of the time, they are less profitable to firms, and the labor demand falls. Finally, the decrease in labor demand reduces the labor market tightness and raises unemployment. With competitive prices, a decrease in aggregate demand is absorbed by lower prices, so it has no effect on product market tightness and unemployment.

Besides aggregate demand shocks, our model provides comparative statics with respect to labor supply, mismatch, and technology shocks, thus representing many of the shocks cited in the context of the depressed labor market of 2008–2013. Two principles emerge from the comparative-statics analysis. First, tightnesses respond to shocks with fixed prices but not with competitive prices. Second, with fixed prices, a demand shock on a market generates a positive correlation between tightness and quantity, whereas a supply shock generates a negative correlation.

By combining the comparative-statics predictions of the model with empirical evidence, we assess the sources of labor market fluctuations in the US. Time series are available for employment, output, and labor market tightness but not for product market tightness, so we construct a time series proxying for product market tightness. The proxy is based on the capacity utilization rate measured in the Survey of Plant Capacity of the Census Bureau. Our first finding is that a fixprice equilibrium describes the data better than a competitive equilibrium. This finding is based on the observation that the product market and labor market tightnesses fluctuate a lot. We therefore use the comparative statics from the fixprice equilibrium to identify the sources of labor market fluctuations. Our second finding is that labor market fluctuations are mostly due to labor demand shocks—aggregate demand or technology shocks—and not to labor supply or mismatch shocks. This finding is based on the observation that labor market tightness is positively correlated with employment. Our third finding is that labor demand shocks mostly are aggregate demand shocks and not technology shocks. This finding is based on the observation that product market tightness is positively correlated with output.

Our findings are consonant with those obtained by other researchers. Our first finding agrees with the result from [Shimer \[2005\]](#) and [Hall \[2005\]](#) that real wage rigidity is important to explain

unemployment fluctuations over the business cycle. Our second finding is similar to the finding of [Blanchard and Diamond \[1989b\]](#) that labor market fluctuations are mostly due to aggregate activity shocks and not to reallocation or labor force participation shocks. Our third result echoes the finding of [Galí \[1999\]](#) and [Basu, Fernald and Kimball \[2006\]](#) that technology shocks do not explain most business-cycle fluctuations.

Literature Review. To explore the sources of labor market fluctuations, many models are available. We review them here. The textbook model of unemployment is the matching model of the labor market.⁴ The matching model accurately represents the mechanics of the labor market and delivers useful comparative statics for many labor market shocks. But it ignores aggregate demand, thus leaving out a potentially important source of labor market fluctuations.

To introduce aggregate demand, the matching model can be augmented with a product market combining monopolistic competition and price rigidity.⁵ If prices are fixed, the model is tractable. But because employment is solely determined by aggregate demand and technology, shocks to mismatch, job-search effort, and labor force participation have no effect on employment, so potentially important sources of employment fluctuations are ignored. If prices sluggishly adjust to shocks, for instance with [Calvo \[1983\]](#) pricing, the model can account for numerous sources of employment fluctuations.⁶ But this type of model is complex because it is inherently dynamic and relies on the Phillips curve, the Euler equation, and a monetary policy rule to describe aggregate demand. Its level of complexity goes far beyond that of a static model of the sort developed by [Barro and Grossman \[1971\]](#) or [Blanchard and Kiyotaki \[1987\]](#), making it difficult to characterize analytically the effects of shocks and thus to inspect the mechanisms behind labor market fluctuations.

To introduce aggregate demand into the matching model of the labor market, we combine it with a matching model of the product market. The literature applying the matching approach to the product market is small and scattered, so we develop a new matching model of the product market.⁷ Our model of the product market is formally symmetric to our model of the labor market. [Lehmann and Van der Linden \[2010\]](#) and [Huo and Ríos-Rull \[2013\]](#) also propose models in which

⁴See [Pissarides \[2000\]](#) for an exhaustive treatment.

⁵See [Blanchard and Kiyotaki \[1987\]](#) for the classical model of the product market with monopolistic competition.

⁶For New Keynesian model with matching frictions on the labor market and Calvo pricing, see for instance [Walsh \[2003\]](#), [Gertler, Sala and Trigari \[2008\]](#), and [Blanchard and Galí \[2010\]](#). See [Galí \[2010\]](#) for a survey of this literature. See [Rendahl \[2012\]](#) for an alternative model built around the zero lower bound on nominal interest rates.

⁷The seminal contribution to this literature is [Diamond \[1982a\]](#), and recent models include [Arseneau and Chugh \[2007\]](#), [Mathä and Pierrard \[2011\]](#), [Gourio and Rudanko \[2014\]](#), and [Bai, Rios-Rull and Storesletten \[2012\]](#).

aggregate demand influences unemployment through a product market with matching frictions. These models are quite different from ours, especially because they focus on economies with flexible prices in which dynamics play a key role.⁸

2 A Basic Model of Aggregate Demand and Idle Time

This section presents a simplified version of the complete model, which is only introduced in Section 3. In this basic model we abstract from the labor market and assume that all production directly takes place within households and not within firms. We thus simplify the presentations of the equilibrium concept and the matching frictions on the product market, which are the two most important new elements of the complete model. This section also provides empirical evidence in support of matching frictions on the product market.

2.1 Assumptions

The model is static. The assumption that the model is static will be relaxed in Section 4. The economy is composed of a measure 1 of identical households. Households produce goods or services. For concreteness, we assume that households produce services. They sell their services on a product market with matching frictions. Households also consume services, but they cannot consume their own services, so they buy services from other households on the product market. Each household also holds some money. Money is the numéraire.

The Product Market. The productive capacity of each household is k ; that is, a household is able to produce k services. Each household visits v other households to purchase their services. The number of trades y on the product market is given by a matching function with constant returns to scale. For concreteness, we assume that the matching function takes the form

$$y = (k^{-\gamma} + v^{-\gamma})^{-\frac{1}{\gamma}},$$

where k is aggregate productive capacity, v is aggregate number of visits, and the parameter γ governs the elasticity of substitution of inputs in matching. We impose $\gamma > 0$ to guarantee that y is

⁸Hall [2008], den Haan [2013], and Petrosky-Nadeau and Wasmer [2011] also take a matching approach to the product and labor markets, but they do not explicitly represent and study aggregate demand.

less than k and v .⁹ In each trade, one service is sold at price $p > 0$.

We define the product market tightness as the ratio of aggregate number of visits to aggregate productive capacity: $x = v/k$. The product market tightness is an aggregate variable taken as given by households. With constant returns to scale in matching, the tightness determines the probabilities that services are sold and that visits yield a purchase: one service is sold with probability

$$f(x) = \frac{y}{k} = (1 + x^{-\gamma})^{-\frac{1}{\gamma}},$$

and one visit yields a purchase with probability

$$q(x) = \frac{y}{v} = (1 + x^{\gamma})^{-\frac{1}{\gamma}}.$$

A useful property is that $q(x) = f(x)/x$. The function f is smooth and strictly increasing on $[0, +\infty)$, with $f(0) = 0$ and $\lim_{x \rightarrow +\infty} f(x) = 1$. The function q is smooth and strictly decreasing on $[0, +\infty)$, with $q(0) = 1$ and $\lim_{x \rightarrow +\infty} q(x) = 0$. The properties of the derivative f' will be useful later: $f'(x) = q(x)^{1+\gamma}$ so f' is strictly decreasing on $[0, +\infty)$ with $f'(0) = 1$ and $\lim_{x \rightarrow +\infty} f'(x) = 0$. An implication is that f is strictly concave on $[0, +\infty)$. The properties of f and q imply that when the product market tightness is higher, it is easier to sell services but harder to buy them.

We abstract from randomness at the household level: a household sells $f(x) \cdot k$ services and purchases $q(x) \cdot v$ services with certainty. Since a household does not sell its entire productive capacity, household members are idle part of the time. In fact, since a household only sells a fraction $f(x)$ of its productive capacity, household members are busy a share $f(x)$ of the time and idle a share $1 - f(x)$ of the time. Thus, the rate of idleness in the economy is $1 - f(x)$.

We model the matching cost as follows. Each visit requires to purchase $\rho \in (0, 1)$ services. These services for matching do not contribute to the buyer's consumption, but they are purchased like the services for consumption. A buyer doing v visits and consuming c services therefore purchases a total of $c + \rho \cdot v$ services. Since the matching process limits the purchases of a buyer doing v visits to $q(x) \cdot v$ services, the number v of visits needed to consume c services satisfies $q(x) \cdot v = c + \rho \cdot v$ or, equivalently, $v = c / (q(x) - \rho)$. This means that consuming one service

⁹The matching function is borrowed from [den Haan, Ramey and Watson \[2000\]](#). It always satisfies $y \leq \min\{k, v\}$, which is a required property for a matching function. We use this function instead of the standard Cobb-Douglas matching function, $y = k^{\gamma} \cdot v^{1-\gamma}$, because the latter must be truncated to ensure that $y \leq \min\{k, v\}$, which complicates the analysis.

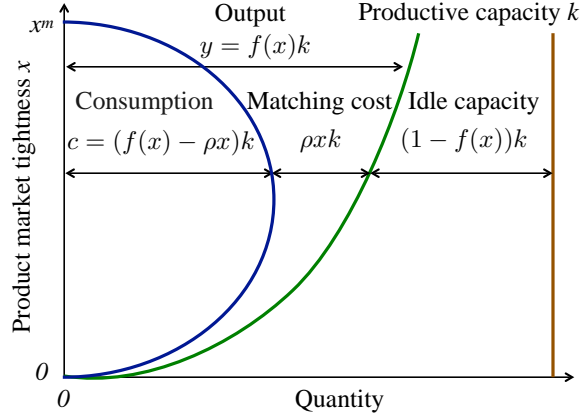


Figure 1: The Matching Frictions on the Product Market

requires to do $1/(q(x) - \rho)$ visits and thus to buy a total of $1 + \tau(x)$ services, where

$$\tau(x) \equiv \frac{\rho}{q(x) - \rho}.$$

The function τ is positive and strictly increasing for all $x \in [0, x^m)$, where $x^m > 0$ is defined by $q(x^m) = \rho$. We also have $\tau(0) = \rho/(1 - \rho)$ and $\lim_{x \rightarrow x^m} \tau(x) = +\infty$. Note that any equilibrium satisfies $x \in [0, x^m)$. Because of the matching cost, consumption is necessarily lower than output.

We define the aggregate supply as the amount of consumption traded at a given tightness:

DEFINITION 1. *The aggregate supply is a function of product market tightness defined for all $x \in [0, x^m)$ by*

$$c^s(x) = \frac{f(x) \cdot k}{1 + \tau(x)}.$$

PROPOSITION 1. *The aggregate supply satisfies*

$$c^s(x) = (f(x) - \rho \cdot x) \cdot k \tag{1}$$

for all $x \in [0, x^m)$. We define the tightness $x^* \in (0, x^m)$ by $f'(x^*) = \rho$. The aggregate supply is strictly increasing on $[0, x^*]$ and strictly decreasing on $[x^*, x^m]$. Hence, x^* maximizes the aggregate supply. Furthermore, $c^s(0) = 0$ and $c^s(x^m) = 0$.

Proof. We have $c^s(x) = f(x) \cdot k/(1 + \tau(x))$. Using the definition of τ and $f(x)/q(x) = x$, we

have $f(x)/(1 + \tau(x)) = f(x) \cdot (1 - \rho/q(x)) = f(x) - \rho \cdot x$. Hence, $c^s(x) = (f(x) - \rho \cdot x) \cdot k$. As showed above, f' is strictly decreasing on $[0, +\infty)$ with $f'(0) = 1$ and $\lim_{x \rightarrow +\infty} f'(x) = 0$. Since $dc^s/dx = (f'(x) - \rho) \cdot k$ with $\rho \in (0, 1)$, we infer that $dc^s/dx > 0$ on $[0, x^*)$, $dc^s/dx = 0$ at $x = x^*$, and $dc^s/dx < 0$ on $(x^*, +\infty)$. Thus, c^s is strictly increasing on $[0, x^*]$ and strictly decreasing on $[x^*, x^m]$. Since $f(0) = 0$ and $f(x^m)/x^m = q(x^m) = \rho$, we have $c^s(0) = 0$ and $c^s(x^m) = 0$. \square

The property that the aggregate supply is first increasing then decreasing with x is unusual, but it naturally arises from the properties of the matching function. When x is low, the matching process is congested by the available productive capacity increasing x —that is, increasing the number of visits relative to available productive capacity—leads to a large increase in the probability to sell, $f(x)$, but a small increase in the price wedge faced by buyers, $\tau(x)$. Since the aggregate supply is proportional to $f(x)/(1 + \tau(x))$, it increases. Conversely when x is high, the matching process is congested by the number of visits, and increasing x leads to a small increase in $f(x)$ but a large increase in $\tau(x)$ so an overall decrease in aggregate supply.

The aggregate supply curve is the consumption curve depicted in Figure 1. This figure illustrates the relationship between consumption, output, and productive capacity imposed by matching frictions. Output is $y = f(x) \cdot k$, an increasing and concave function of tightness. Consumption is $c = (f(x) - \rho \cdot x) \cdot k$ so it is always below output. A total of $\rho \cdot v = \rho \cdot x \cdot k$ services are used for matching; the gap between consumption and output represents this matching cost. A total of $k - f(x) \cdot k = (1 - f(x)) \cdot k$ services could have been produced during the time when workers are idle; the gap between output and productive capacity represents this idle capacity.

Households. The representative household derives utility from consuming services and holding real money balances. The household's utility is given by

$$u\left(c, \frac{m}{p}\right) = \frac{\chi}{1 + \chi} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1 + \chi} \cdot \left(\frac{m}{p}\right)^{\frac{\varepsilon-1}{\varepsilon}},$$

where c is consumption of services, m are nominal money balances, m/p are real money balances, the parameter $\chi > 0$ measures the taste for consumption relative to holding money, and the parameter $\varepsilon > 1$ is the elasticity of substitution between consumption and real money balances.

The desired level of consumption determines the number of visits that the household makes. Consuming c services requires to purchase $(1 + \tau(x)) \cdot c$ services in the course of $(1 + \tau(x)) \cdot c/q(x)$

visits. For simplicity, we relegate the visits to the background and focus on consumption.¹⁰ We summarize the cost incurred by the household for the visits with a price wedge. Consuming one service requires to purchase one service for consumption plus $\tau(x)$ services to cover the cost of the visits. The total cost of consuming c therefore is $p \cdot c + p \cdot \tau(x) \cdot c = p \cdot (1 + \tau(x)) \cdot c$. From the household's perspective, it is as if it purchased c services at a unit price $p \cdot (1 + \tau(x))$. Effectively the matching frictions impose a wedge $\tau(x)$ on the price of services.

Taking as given the product market tightness and the price, the representative household chooses consumption and nominal money balances to maximize utility subject to a budget constraint. The household receives an endowment $\mu > 0$ of nominal money and income from the sale of $f(x) \cdot k$ services at price p . With these, the household purchases c services at price $(1 + \tau(x)) \cdot p$ and holds m units of nominal money balances. Hence, the household's budget constraint is

$$m + (1 + \tau(x)) \cdot p \cdot c = \mu + p \cdot f(x) \cdot k.$$

Solving the utility-maximization problem gives

$$\frac{1}{1 + \chi} \cdot \left(\frac{m}{p}\right)^{-\frac{1}{\varepsilon}} = \frac{1}{1 + \tau(x)} \cdot \frac{\chi}{1 + \chi} \cdot c^{-\frac{1}{\varepsilon}}. \quad (2)$$

This equation implies that the household is indifferent between consumption and holding money.

We define the aggregate demand as the utility-maximizing level of consumption at a given product market tightness and price, accounting for the fact that the money market clears:

DEFINITION 2. *The aggregate demand is a function of product market tightness and price*

$$c^d(x, p) = \left(\frac{\chi}{1 + \tau(x)}\right)^\varepsilon \cdot \frac{\mu}{p} \quad (3)$$

for all $(x, p) \in [0, x^m] \times (0, +\infty)$, where $x^m > 0$ satisfies $\rho = q(x^m)$.

PROPOSITION 2. *The aggregate demand is strictly decreasing in x and p . Furthermore, $c^d(0, p) = \chi^\varepsilon \cdot (1 - \rho)^\varepsilon \cdot \mu / p$ and $c^d(x^m, p) = 0$.*

Proof. Obvious from (3), since τ is strictly increasing in x . □

¹⁰This representation is slightly unconventional. The matching literature usually emphasizes the role of visits or, on the labor market, of vacancies.

The aggregate demand is the level of consumption that satisfies (2) when $m = \mu$. The properties of the aggregate demand reflect the household's indifference between consumption and holding μ/p real money balances. First, a higher p leads to lower real money balances. Households' indifference between consumption and holding money implies that they desire lower consumption when p is higher. Hence the aggregate demand decreases with p . Second, $1 + \tau(x)$ is the effective price of consumption relative to real money balances. A higher x leads to a higher relative price that reduces the attractiveness of consumption relative to that of holding real money balances, whose quantity is fixed at μ/p . Hence the aggregate demand decreases with x . The aggregate demand is plotted in Figure 3; it slopes downward in the (c, x) and (c, p) planes.

2.2 Discussion of the Assumptions

We discuss two critical assumptions of the model: matching frictions on the product market, and money in the utility function. To represent the matching frictions, we assume that the number of trades is governed by a matching function and that buyers face a matching cost. We argue that both elements realistically capture key aspects of the product market.

The Matching Function. In the same way as the production function summarizes how input are transformed into output through the production process, the matching function summarizes how available production and aggregate buying efforts are transformed into trades through the matching process. The matching function provides a tractable representation of a very complex trading process. Its main implication is that not all available productive capacity is sold and not all visits by buyers are successful. Formally, we assume that households only sell a fraction $f(x) < 1$ of their productive capacity and buyers' visits are only successful with probability $q(x) < 1$. The matching function is a useful modeling tool only if we find convincing evidence that at all times, some employed workers are idle and some visits are unsuccessful.¹¹

The prediction that not all productive capacity is sold can be examined empirically. In US data, we find that some productive capacity is idle at all time. Figure 2(a) displays the rates of idleness in non-manufacturing sectors and in the manufacturing sector. These rates indicate the share of time when employed workers are idle due to a lack of activity. These rates are constructed as one minus

¹¹Pissarides [1985] pioneered the concept of the matching function on the labor market. Pissarides [1986] and Blanchard and Diamond [1989a] first explored the empirical properties of the matching function on the labor market. See Petrongolo and Pissarides [2001] for a survey of this literature.

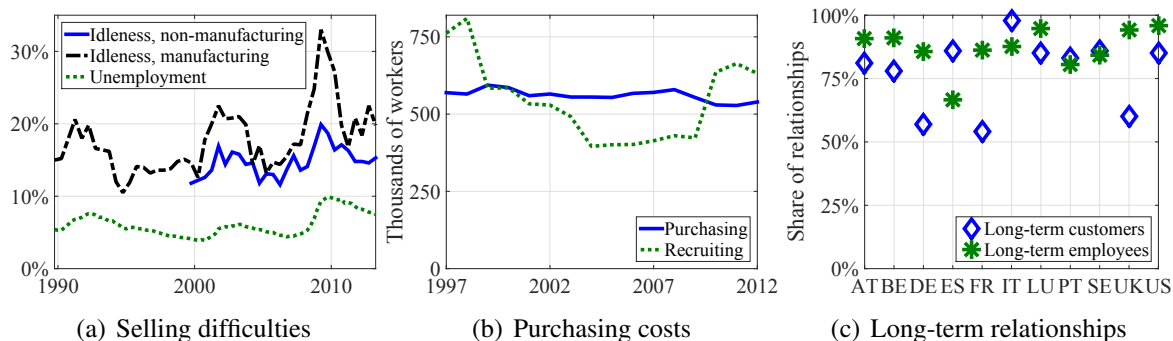


Figure 2: Evidence of Matching Frictions on the Product and Labor Markets

Notes: Panel (a): The time period is 1989:Q4–2013:Q2. The rate of idleness is one minus the operating rate measured by the ISM. For non-manufacturing sectors, the operating rate is only available after 1999:Q4. The rate of unemployment is constructed by the BLS from the CPS. Panel (b): The time period is 1997–2012. The number of workers in recruiting and purchasing occupations is from the OES database constructed by the BLS. Recruiting occupations include human resource managers, specialists, and assistants. Purchasing occupations include purchasing managers, buyers and purchasing agents, and procurement clerks. Panel (c): The share of sales to long-term customers is from the following firm surveys: Kwapil, Baumgartner and Scharler [2005] for Austria (AT); Aucremanne and Druant [2005] for Belgium (BE); Stahl [2005] for Germany (DE); Alvarez and Hernando [2005] for Spain (ES); Loupias and Ricart [2004] for France (FR); Fabiani, Gattulli and Sabbatini [2004] for Italy (IT); Lunnemann and Matha [2006] for Luxembourg (LU); Martins [2005] for Portugal (PT); Apel, Friberg and Hallsten [2005] for Sweden (SE); Hall, Walsh and Yates [2000] for the UK; and Blinder et al. [1998] for the US. All the surveys were conducted between 2000 and 2004, except in the UK and the US where they were conducted in 1995 and 1990–1992, respectively. The share of workers in long-term employment is from the OECD dataset on the incidence of permanent employment for 2005.

the operating rates measured by the Institute for Supply Management (ISM) for non-manufacturing sectors and for the manufacturing sector. The operating rate indicates the actual production level of firms as a share of their maximum production level given current capital and labor. On average the rate of idleness is 14.8% in non-manufacturing sectors and 17.3% in the manufacturing sector. For comparison, the figure also displays the rate of unemployment constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The rate of unemployment is the labor-market equivalent of the rate of idleness, but, as showed in Figure 2(a), the rate of unemployment is always much lower than the rate of idleness in the manufacturing sector or in non-manufacturing sectors. There is also microevidence that firms in the US face difficulty in selling their output. Using output and price microdata from the Census of Manufacturers, Foster, Haltiwanger and Syverson [2012] find that, despite similar or lower prices, new plants grow more slowly than similar plants with an established customer base because it is difficult for new plants to attract new customers.

Visits are the product-market equivalent of vacancies. A visit represents the process that a buyer must follow to buy an item. These visits can take different forms, depending on the buyer.

For a individual consumer, a visit may be an actual visit to a restaurant, to a hair salon, to a bakery, or to a car dealer. A visit could also be an inquiry to an intermediary, such as a travel agent, a real estate agent, or a stock broker. For a firm, a visit could be an actual visit to a potential supplier. A visit could also be the preparation and processing of a request for proposal (RFP) or request for tender (RFT) or any other sourcing process. Unlike for vacancies, however, visits are not recorded in any dataset. It is therefore difficult to provide quantitative evidence on the share of visits that are unsuccessful. The only quantitative evidence that we found is the average stockout rate provided by [Bils \[2004\]](#). Using the monthly microdata underlying the Consumer Price Index, Bils finds that temporary stockouts are quite common: the average stockout rate for consumer durables over the 1988–2004 period is 9%. A stockout is an item not available for sale, continuing to be carried by the outlet, and not seasonally unavailable; hence, a stockout indicates that a visit to the store for a buyer did not result in a purchase because the desired product was not be available. Casual observation also suggests that many visits do not generate a trade. At a restaurant, a consumer sometimes need to walk away because no tables are available or the queue is too long. The same may happen at a hair salon if no slots are available or if the salon is not open for business. At a bakery, the type of bread or cake desired by a consumer may not be available at the time of the visit, either because it was not prepared on the day or because the bakery has run out of it. At a car dealer, the specific car desired by the consumer may not be in inventory and may therefore not be available before a long time. For firms, buyers travel the world to assess production plants or examine the quality of potential suppliers, and many of these visits do not lead to a contract. When a firm issues a RFP or RFT, it considers many applications from potential suppliers, but only one supplier is eventually selected.

The Matching Cost. Empirical evidence indicates that buyers incur a broad range of matching costs on the product market. In this paper we make the assumption that the matching cost is incurred in services. This representation of the matching cost is crude but convenient: it is tractable because the cost appears as a price wedge for buyers; it is portable because we could similarly represent the matching cost incurred by a firm or a government (other representations of the matching cost, such as a utility cost, would not offer this portability); and it is isomorphic to the representation of the matching cost on the labor market in [Section 3](#). It is also conventional in the matching literature to measure matching costs in terms of output and to define consumption as output net of

matching costs, as we do here [for example, [Gertler and Trigari, 2009](#)].

Of the matching costs incurred by buyers on the product market, some are indeed service costs: for a consumer using a travel agency to book a vacation, the matching cost of purchasing hospitality services is the travel agent's fee; for a consumer who goes to a hair salon in a taxicab, the matching cost of purchasing hairdressing services is the taxicab fare; and for a firm recruiting a manager with an executive search agency, the cost of purchasing labor services is the agency's fee. Travel agent's fee, taxicab fare, and executive search agency's fee are service costs.

Besides service costs, buyers incur other types of matching costs on the product market. For consumers, the cost of a visit to a seller could be a traveling time or the time spent in a queue at a restaurant or hair salon. These time costs are not negligible: on average between 2003 and 2011 in the American Time Use Survey conducted by the BLS, people spend 47 minutes a day shopping for goods and services. For firms, a large share of the cost of sourcing goods and services is a labor cost. We quantify this cost using data from the Occupational Employment Survey (OES) database constructed by the BLS. We measure the number of workers whose occupation is buying, purchasing, and procurement.¹² Figure 2(b) displays the results. On average between 1997 and 2012, 560,600 workers were employed in such occupations. For comparison, we use the same methodology to assess the matching costs incurred by firms on the labor market. We measure the number of workers devoted to recruiting in the OES database. On average between 1997 to 2012, 543,200 workers were employed with an occupation involving recruitment, placement, screening, and interviewing. Hence, the numbers of buyers and recruiters have the same magnitude.

To confirm that our analysis is robust to alternative representations of the matching cost, we study an alternative model in which the matching cost is a time cost instead of a service cost. This alternative model is presented in Online Appendix E. In the alternative model, households share their time between supplying services and matching with other households who sell services. We find that the properties of the alternative model are almost identical to those of the original model.

Finally, sellers may also incur matching costs. Indeed, firms spend substantial resources on sales and marketing. These resources are used by firms to increase their sales for a given productive capacity. In Online Appendix F, we extend our model to include an endogenous marketing effort for sellers. We model the marketing effort as a continuous variable that increases sellers' selling probability at a cost. This extension does not alter the structure of the model or the results much.

¹²Note that the classification of occupations evolves over time so comparisons across years are not very meaningful.

Money in the Utility Function. The assumption that households derive utility from holding real money balances is borrowed from Barro and Grossman [1971]. This assumption was also used by Blanchard and Kiyotaki [1987], among many others. Introducing money in the utility function crudely but conveniently captures the fact that money provides transaction services to households. The presence of money in the utility function is necessary to obtain an interesting concept of aggregate demand in a static environment because without money, consumers would mechanically spend all their income on the produced good (Say’s law). Here households choose between buying consumption and holding money, and the aggregate demand is the desired level of consumption.

2.3 Definition of the Equilibrium

DEFINITION 3. *An equilibrium consists of a product market tightness and a price (x, p) such that aggregate supply is equal to aggregate demand:*

$$c^s(x) = c^d(x, p).$$

Since the equilibrium has two variables but only one condition, infinitely many combinations of price and tightness satisfy the equilibrium condition. To select an equilibrium, we specify a price mechanism. In Sections 2.5–2.7, we will study the equilibria selected by different mechanisms.

Figure 3 represents aggregate demand and supply, and the equilibrium. The equilibrium tightness is at the intersection of aggregate demand and supply in the (c, x) plane.¹³ The equilibrium price is at the intersection of aggregate supply and demand in the (c, p) plane.

Since many equilibrium prices are possible, we categorize equilibria into the following regimes:

DEFINITION 4. *An equilibrium is efficient if it maximizes consumption. An inefficient equilibrium can be either slack, if an increase in tightness at the equilibrium point raises consumption, or tight, if an increase in tightness at the equilibrium point lowers consumption. Equivalently, an equilibrium is efficient if $x = x^*$, slack if $x < x^*$, and tight if $x > x^*$.*

Figure 4 illustrates the three regimes in which equilibria may fall. In the efficient equilibrium, consumption is maximized. An efficient equilibrium also maximizes welfare taking real money

¹³In the equilibrium that we consider, tightness is given by the intersection with positive consumption. There is another equilibrium at the intersection with zero consumption. In that equilibrium, the tightness is x^m . We do not study that equilibrium because it has zero consumption.

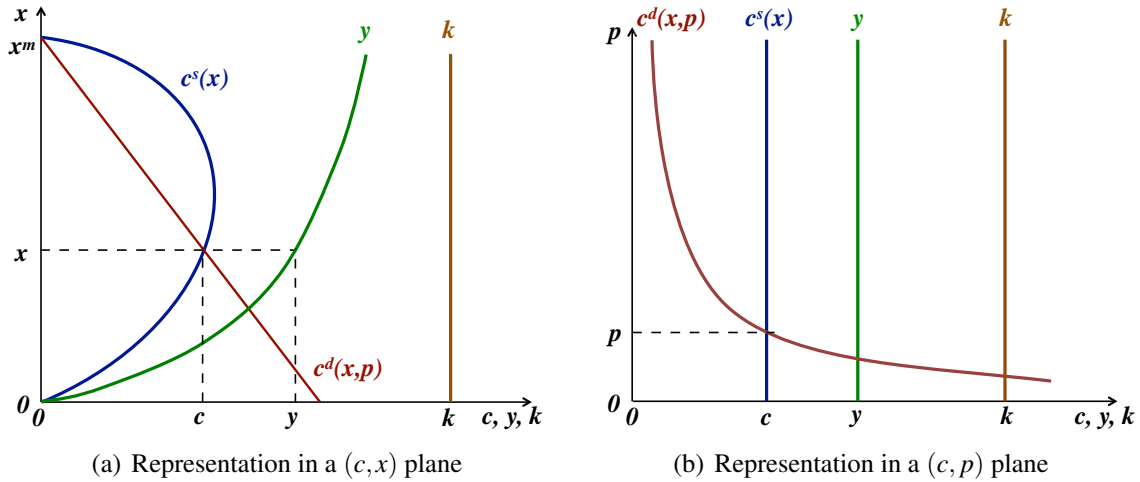


Figure 3: Aggregate Demand, Aggregate Supply, and Equilibrium in the Basic Model of Section 2

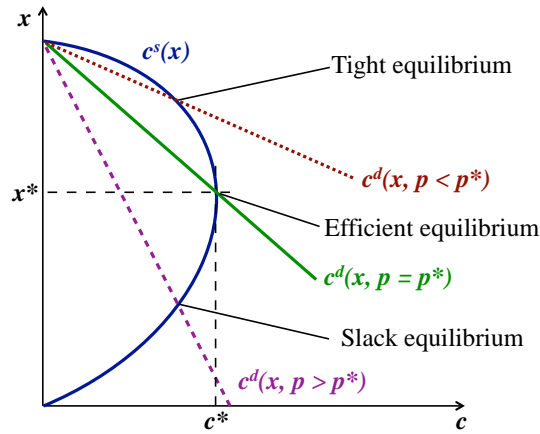


Figure 4: The Three Regimes in the Basic Model of Section 2

Notes: The figure compares the equilibria obtained for different equilibrium prices. The price p^* is given by (5).

balances as given. In a slack equilibrium, aggregate demand is too low and tightness is below its efficient level. In a tight equilibrium, aggregate demand is too high and tightness is above its efficient level. The slack and tight equilibria are inefficient because their consumption levels are below the efficient consumption level. As illustrated in Figure 1, higher output is not equivalent to higher consumption. Compared to the efficient equilibrium, a slack equilibrium has lower output and a tight equilibrium has higher output, but both have lower consumption. Given that the aggregate demand is decreasing in price, the price is too high when the equilibrium is slack and too low when the equilibrium is tight. The property that an equilibrium can be efficient, slack, or tight is true in any matching model [Pissarides, 2000, Chapter 8].

2.4 Discussion of the Equilibrium Concept

This section proposes a more detailed definition of the equilibrium concept. To make the definition more transparent, we generalize our model slightly and consider a measure 1 of households indexed by $i \in [0, 1]$. Household i has productive capacity $k(i)$ and an endowment of money $\mu(i)$. We define the equilibrium by analogy to a Walrasian equilibrium:¹⁴

DEFINITION 5. *An equilibrium is a price p , a tightness x , visits $\{v(i), i \in [0, 1]\}$, and nominal money balances $\{m(i), i \in [0, 1]\}$ such that the following conditions are satisfied:*

- (1) *Taking x and p as given, household $i \in [0, 1]$ chooses $v(i)$ and $m(i)$ to maximize its utility function subject to a budget constraint and the constraints imposed by matching frictions. The matching frictions impose that the output bought by household i is $y^b(i) = v(i) \cdot q(x)$, the output sold by household i is $y^s(i) = k(i) \cdot f(x)$, and the consumption of household i is $c(i) = y^b(i)/(1 + \tau(x))$. The budget constraint is $m(i) + p \cdot y^b(i) = \mu(i) + p \cdot y^s(i)$.*
- (2) *Quoted tightness equals actual tightness: $x = \int_0^1 v(i) di / \int_0^1 k(i) di$*

As in Walrasian theory we make the institutional assumption that a price and a tightness are quoted on the product market, and we make the behavioral assumption that households take the quoted price and tightness as given. It is natural for households to take tightness as given because the tightness is the ratio of aggregate number of visits to aggregate productive capacity, and each household is small relative to the size of the market. The issue is more complicated for the price since a buyer and a seller could bargain the transaction price once they have matched. However, the actual transaction price has no influence on households' decisions because the decisions are made before the match is realized; what matters is the price at which households expect to trade. Hence, we assume that households take the expected transaction price as given, and to ensure the consistency of the equilibrium, we require that actual and expected transaction prices are the same.

As in a Walrasian equilibrium, Condition (1) imposes that households behave optimally given the quoted price and tightness. The difference with Walrasian theory is that households cannot choose the quantities that they trade. These quantities are constrained by matching frictions: as buyers, households only choose how many sellers to visit, knowing that the purchasing probability is $q(x)$ and that the purchase of one unit of output yields $1/(1 + \tau(x))$ unit of consumption; and

¹⁴For a standard definition of a Walrasian equilibrium, see [Mas-Colell, Whinston and Green \[1995\]](#).

as sellers, households only choose how much productive capacity to bring to the market, knowing that the selling probability is $f(x)$.¹⁵

Condition (2) is the equivalent of the market-clearing condition of the Walrasian equilibrium. The Walrasian market-clearing condition imposes that at the quoted price, the quantity that buyers desire to buy equals the quantity that sellers desire to sell. This condition is required to ensure the consistency of the Walrasian equilibrium because sellers and buyers make their decisions expecting to be able buy and sell any quantity at the quoted price. Similarly, Condition (3) is required to ensure the consistency of our equilibrium. Given $\{v(i), i \in [0, 1]\}$ and $\{k(i), i \in [0, 1]\}$, the number of trades is

$$\left[\left(\int v(i) di \right)^{-\gamma} + \left(\int k(i) di \right)^{-\gamma} \right]^{-\frac{1}{\gamma}} = \int k(i) di \cdot f \left(\frac{\int v(i) di}{\int k(i) di} \right) = \int v(i) di \cdot q \left(\frac{\int v(i) di}{\int k(i) di} \right).$$

These equations imply that the actual selling probability faced by households is $f(\int v(i) di / \int k(i) di)$ and the actual purchasing probability faced by households is $q(\int v(i) di / \int k(i) di)$. To ensure the consistency of the equilibrium, these probabilities must match the probabilities $f(x)$ and $q(x)$ on which households base their calculations; equivalently, the quoted tightness, x , must be equal to the actual tightness, $\int v(i) di / \int k(i) di$.

Our equilibrium has one more variable than the Walrasian equilibrium—the tightness. But the equilibrium does not have one more equation, which explains why many price-tightness pairs are consistent with the equilibrium and why a price mechanism is needed to select an equilibrium. At a microeconomic level, it is impossible to add an equilibrium condition to determine a price because each seller-buyer pair decides the price in a situation of bilateral monopoly. This situation arises because the pairing of a buyer and a seller generates a positive surplus. Since the solution to the bilateral monopoly problem is indeterminate, it cannot be used to impose a condition on the price.¹⁶ What this means is that there is no obvious economic criterion that can determine the price. For instance, when a buyer and a seller meet, there is no deviation from the quoted price that generates a Pareto improvement. Of course, a seller would be better-off with a higher price, but a buyer would be worse-off with a higher price.

¹⁵Here the productive capacity of household i is fixed to $k(i)$, but the model could be extended to have household i choose $k(i)$.

¹⁶The indeterminacy of the solution to the bilateral monopoly problem has been known since [Edgeworth \[1881\]](#). The indeterminacy is discussed by [Howitt and McAfee \[1987\]](#) and [Hall \[2005\]](#) in the context of matching models.

In a symmetric equilibrium, Definition 5 implies that $c^s(x) = c^d(x, p)$. The budget constraints of all households are satisfied, and sales equal purchases through the matching process, so Walras' law implies that the money market clears: $m = \mu$. Given the definition of the aggregate demand and the fact that $m = \mu$, Condition (1) imposes that $v(x, p) = (1 + \tau(x)) \cdot c^d(x, p)/q(x)$. Condition (2) imposes that $x = v(x, p)/k = (1 + \tau(x)) \cdot c^d(x, p)/(k \cdot q(x))$. Since $f(x) = q(x) \cdot x$, this equation implies that

$$c^d(x, p) = \frac{x \cdot q(x)}{1 + \tau(x)} \cdot k = \frac{f(x)}{1 + \tau(x)} \cdot k = c^s(x).$$

2.5 Fixprice Equilibrium

We first study a simple equilibrium in which the price is a parameter.¹⁷ In this equilibrium, only the product market tightness equilibrates the market.

DEFINITION 6. A fixprice equilibrium parameterized by $p_0 > 0$ consists of a product market tightness and a price (x, p) such that aggregate supply equals aggregate demand and the price is given by the parameter p_0 : $c^s(x) = c^d(x, p)$ and $p = p_0$.

PROPOSITION 3. For any $p_0 > 0$, there exists a unique fixprice equilibrium parameterized by p_0 with positive consumption.

Proof. In equilibrium, x satisfies $c^s(x) = c^d(x, p_0)$. We look for an equilibrium with positive consumption, so we restrict the search to $x \in (0, x^m)$. The equilibrium condition is equivalent to $(1 + \tau(x))^\varepsilon \cdot (c^s(x) - c^d(x, p_0)) = 0$ because $x \in (0, x^m)$ so $(1 + \tau(x))^\varepsilon \in (0, +\infty)$. This equation is equivalent to

$$(1 + \tau(x))^{\varepsilon-1} \cdot f(x) = \frac{\chi^\varepsilon}{k} \cdot \frac{\mu}{p_0}. \quad (4)$$

Since $\varepsilon > 1$, the function $x \mapsto (1 + \tau(x))^{\varepsilon-1} \cdot f(x)$ is strictly increasing from 0 to $+\infty$ on $[0, x^m)$. Thus, there is a unique $x \in (0, x^m)$ that solves (4). \square

We study the comparative static effects of aggregate demand and supply shocks in the fixprice equilibrium. We parameterize an increase in aggregate demand by an increase in money supply, μ ,

¹⁷In matching models of the labor market, several researchers have assumed that the wage is a parameter or a function of the parameters. See for instance Hall [2005], Blanchard and Galí [2010], and Michailat [2012, 2014].

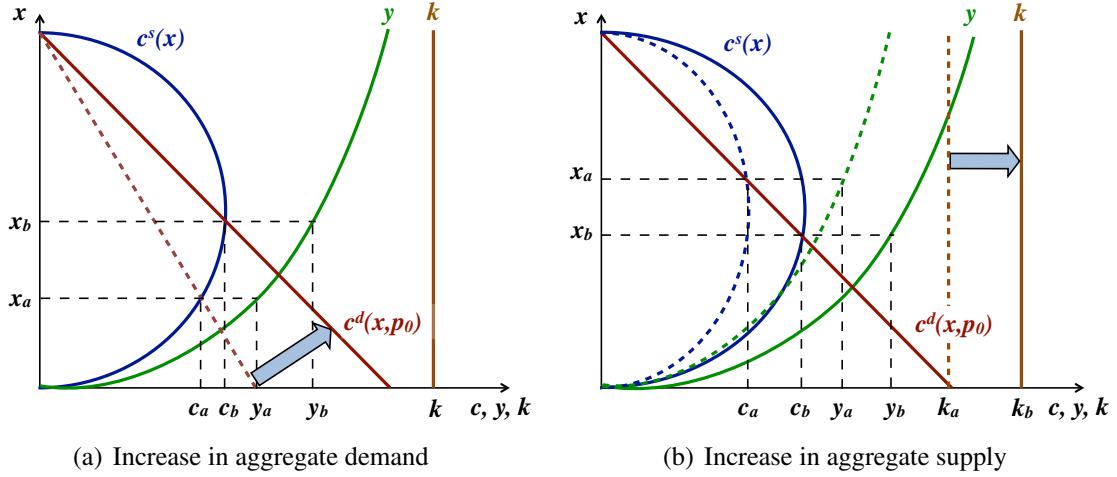


Figure 5: Shocks in the Fixprice Equilibrium of Section 2

or in the taste for consumption, χ . We parameterize an increase in aggregate supply by an increase in productive capacity, k . The following proposition summarizes the comparative statics:

PROPOSITION 4. *Consider a fixprice equilibrium with positive consumption.*

- *An increase in aggregate demand has the following effects: output and product market tightness increase; the rate of idleness decreases; consumption increases in a slack equilibrium, decreases in a tight equilibrium, and does not change in the efficient equilibrium.*
- *An increase in aggregate supply has the following effects: output increases but product market tightness decreases; the rate of idleness increases; consumption increases.*

Proof. In a fixprice equilibrium, x is the unique solution to (4). Since τ and f are strictly increasing functions of x and $\varepsilon > 1$, equation (4) implies that $dx/d\mu > 0$, $dx/d\chi > 0$, but $dx/dk < 0$. The rate of idleness is $1 - f(x)$ so its comparative statics follow from those of x . Since $y = f(x) \cdot k$, $dy/d\mu > 0$ and $dy/d\chi > 0$. We also have $y = (1 + \tau(x)) \cdot c^s(x)$ so in equilibrium, $y = (1 + \tau(x)) \cdot c^d(x, p) = (1 + \tau(x))^{1-\varepsilon} \cdot \chi^\varepsilon \cdot \mu/p$. As $1 - \varepsilon < 0$, we infer that $dy/dk > 0$. Given that $c = c^s(x)$ and the properties of c^s , we infer that $dc/d\mu > 0$ if $x < x^*$, $dc/d\mu = 0$ if $x = x^*$, $dc/d\mu < 0$ if $x > x^*$. The same is true for $dc/d\chi$. As $c = c^d(x, p)$ and c^d decreases with x , we have $dc/dk > 0$. \square

The comparative statics are summarized in panel A of Table 1. These comparative statics are illustrated in the equilibrium diagrams of Figure 5.

Figure 5(a) depicts an increase in aggregate demand. The aggregate demand curve rotates outward. Indeed, households want to consume more for a given price and tightness, either because they hold more money or because they value consumption more. Since the price is fixed, they want to consume more for a given tightness, explaining the rotation of the aggregate demand curve. To reach a new equilibrium, product market tightness necessarily increases. With a higher tightness, workers sell a larger fraction of their productive capacity, which is fixed, so output increases and the rate of idleness decreases. As the equilibrium point moves upward along the aggregate supply curve, the response of consumption clearly depends on the regime: consumption increases in the slack regime, but it decreases in the tight regime, where a higher tightness reduces consumption even though it raises output because it sharply raises the amount of output dissipated in matching.

Figure 5(b) depicts an increase in aggregate supply. The aggregate supply curve expands outward because households have a higher productive capacity. To reach a new equilibrium, product market tightness necessarily decreases. Consumption increases as the equilibrium point moves downward along the aggregate demand curve. The effect on output is not obvious on the graph because productive capacity increases whereas tightness falls so households sell a smaller fraction of a larger capacity. However, the proposition establishes that output increases. Since tightness decreases, the rate of idleness increases.

The proposition implies that aggregate demand matters with a fixed price. This result echoes the findings of a vast body of work in macroeconomics, including the contributions of [Barro and Grossman \[1971\]](#) and [Blanchard and Kiyotaki \[1987\]](#). The proposition also implies that aggregate demand shocks and aggregate supply shocks have different macroeconomic effects. Product market tightness and output are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. We will exploit this property to identify aggregate demand and supply shocks in the data in Section 5.

2.6 Competitive Equilibrium

We study an equilibrium in which the price mechanism is the polar opposite of the fixed price. In a fixprice equilibrium, the price is fixed and tightness alone equilibrates the market. In the equilibrium that we study now, the price is flexible enough to maintain the market in an efficient situation. The efficient tightness is invariant to the shocks considered, so in practice the tightness is fixed at its efficient level and the price alone equilibrates the market.

Table 1: Comparative Statics in the Basic Model of Section 2

Increase in:	Effect on:			
	Output y	Product market tightness x	Idleness $1 - f(x)$	Consumption c
<i>A. Fixprice equilibrium and equilibrium with partially rigid price</i>				
Aggregate demand	+	+	-	+ (slack) 0 (efficient) - (tight)
Aggregate supply	+	-	+	+
<i>B. Competitive equilibrium and equilibrium with Nash bargaining</i>				
Aggregate demand	0	0	0	0
Aggregate supply	+	0	0	+

Notes: An increase in aggregate demand is an increase in money supply, μ , or in the taste for consumption, χ . An increase in aggregate supply is an increase in productive capacity, k . This table summarizes the results of Propositions 4 and 6 and the results discussed in Section 2.7.

DEFINITION 7. A competitive equilibrium consists of a product market tightness and a price (x, p) such that aggregate supply equals aggregate demand and tightness is efficient: $c^s(x) = c^d(x, p)$ and $x = x^*$.

PROPOSITION 5. There exists a unique competitive equilibrium. The competitive price is

$$p^* = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{f(x^*)} \cdot \frac{\chi^\varepsilon}{k} \cdot \mu. \quad (5)$$

Proof. Obvious using equation (4). □

In Definition 7 we simply assume that the price adjusts to maintain the product market tightness at its efficient level, but market forces could achieve this through the competitive search mechanism of Moen [1997]. (We label the equilibrium as competitive in reference to the competitive search mechanism.) The mechanism lies beyond the scope of the model because it relies on directed search, whereby buyers search for the submarket offering the best price-tightness compromise, whereas our model assumes random search. Nevertheless, the mechanism is simple to understand. Starting from an equilibrium (p_a, x_a) , a subset of sellers can deviate and offer a different price, p_b . Buyers will flee or flock to the new submarket until they are indifferent between the old and new submarkets. Indifference happens when $p_b \cdot (1 + \tau(x_b)) = p_a \cdot (1 + \tau(x_a))$. By deviating,

sellers obtain a revenue $p_b \cdot f(x_b)$; thus, sellers' optimal choice is to select p_b to maximize $p_b \cdot f(x_b)$ subject to $p_b \cdot (1 + \tau(x_b)) = p_a \cdot (1 + \tau(x_a))$. This is equivalent to selecting x_b to maximize $f(x_b)/(1 + \tau(x_b)) = f(x_b) - \rho \cdot x_b$, that is, to selecting the efficient tightness. Under the competitive search mechanism, tightness is always efficient in equilibrium, and prices cannot be rigid because market forces provide an incentive for sellers to adjust their price if tightness changes.

The competitive price ensures that the aggregate demand curve is always in the position depicted in Figure 4, where the aggregate demand curve intersects the aggregate supply curve at its maximum. This price necessarily exists because by increasing the price from 0 to $+\infty$, the aggregate demand curve rotates around the point $(0, x^m)$ from a horizontal to a vertical position.

The following proposition summarizes the comparative statics in the competitive equilibrium:

PROPOSITION 6. *Consider a competitive equilibrium.*

- *An increase in aggregate demand has the following effects: output, product market tightness, the rate of idleness, and consumption remain the same; the price increases.*
- *An increase in aggregate supply has the following effects: output and consumption increase; product market tightness and the rate of idleness remain the same; the price decreases.*

Proof. The efficient tightness, x^* , satisfies $f'(x^*) = \rho$ so x^* is solely determined by f and ρ and is independent of χ , μ , and k . The comparative statics for the competitive equilibrium are therefore obvious, using $x = x^*$, $y = f(x^*) \cdot k$, and $c = (f(x^*) - \rho \cdot x^*) \cdot k$, and the expression (5) for p . \square

The comparative statics are summarized in panel B of Table 1. All the comparative statics follow from the fact that the tightness is efficient in a competitive equilibrium, and from the property that the efficient tightness does not respond to aggregate demand or aggregate supply shocks.

The proposition implies that aggregate demand shocks have no real effects in a competitive equilibrium. These results are reminiscent of those obtained by Blanchard and Galí [2010] and Shimer [2010, Chapter 2] in the context of matching models of the labor market. They find that the efficient labor market tightness does not respond to labor demand shocks taking the form of technology shocks.

2.7 Other Equilibria

We have considered a fixed price and a competitive price, but many other price mechanisms are possible. We study two of them here: Nash bargaining and a partially rigid price. Nash bargaining is the typical price mechanism in the matching literature.¹⁸ The partially rigid price is a generalization of the fixed price that partially responds to shocks. We show that the comparative statics are the same with a Nash bargained price as with a competitive price and the same with a partially rigid price as with a fixed price.

Equilibrium with Nash Bargaining. In an equilibrium with Nash bargaining, the price is the generalized Nash solution to the bargaining problem between a buyer and a seller with bargaining power $\beta \in (0, 1)$. After a match is made, the marginal surplus to the household of buying one service at price \tilde{p} is

$$\mathcal{B}(\tilde{p}) = \frac{\partial u}{\partial c} - \frac{\tilde{p}}{p} \cdot \frac{\partial u}{\partial m/p} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{1}{1 + \chi} \cdot \left[\chi \cdot c^{-\frac{1}{\varepsilon}} - \frac{\tilde{p}}{p} \cdot \left(\frac{m}{p} \right)^{-\frac{1}{\varepsilon}} \right]$$

and the marginal surplus to the household of selling one service at price \tilde{p} is

$$\mathcal{S}(\tilde{p}) = \frac{\tilde{p}}{p} \cdot \frac{\partial u}{\partial m/p} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{1}{1 + \chi} \cdot \frac{\tilde{p}}{p} \cdot \left(\frac{m}{p} \right)^{-\frac{1}{\varepsilon}},$$

where p is the price level on the product market. The Nash solution maximizes $\mathcal{B}(\tilde{p})^{1-\beta} \cdot \mathcal{S}(\tilde{p})^\beta$, so $\mathcal{S}(\tilde{p}) = \beta \cdot [\mathcal{S}(\tilde{p}) + \mathcal{B}(\tilde{p})] = \beta \cdot [\varepsilon/(\varepsilon - 1)] \cdot [\chi/(1 + \chi)] \cdot c^{-1/\varepsilon}$, and the bargained price is $\tilde{p} = p \cdot \beta \cdot \chi \cdot c^{-1/\varepsilon} / (m/p)^{-1/\varepsilon}$. In equilibrium $\tilde{p} = p$, so combining the condition on the bargained price with the aggregate demand condition, given by (2), yields

$$\beta \cdot (1 + \tau(x)) = 1. \tag{6}$$

This equation determines the product market tightness in an equilibrium with Nash bargaining.

The comparative statics are the same in the equilibrium with Nash bargaining as in the competitive equilibrium because the response of the tightness to shocks is the same. Indeed, (6) implies that tightness responds neither to aggregate demand nor to aggregate supply shocks in the equilib-

¹⁸Nash bargaining was first used in the seminal work of [Diamond \[1982b\]](#), [Mortensen \[1982\]](#), and [Pissarides \[1985\]](#).

rium with Nash bargaining, exactly as in the competitive equilibrium. These results are reminiscent of those obtained by [Blanchard and Galí \[2010\]](#), [Shimer \[2010, Chapter 2\]](#), and [Michaillat \[2012\]](#) in the context of matching models of the labor market. They assume that real wages are determined by Nash bargaining, and they study the effect of labor demand shocks taking the form of technology shocks. They find that labor market tightness does not respond to technology shocks.¹⁹

The comparative statics are the same in the competitive equilibrium and in the equilibrium with Nash bargaining despite the fact that the former is always efficient but the latter is generally inefficient. Indeed, the efficient tightness satisfies $f'(x^*) = \rho$; using $f(x)/x = q(x)$, we rewrite this condition as $x^* \cdot f'(x^*)/f(x^*) = \rho/q(x^*)$ and then as

$$\eta(x^*) = \frac{1}{1 + \tau(x^*)},$$

where $1 - \eta(x)$ is the elasticity of $f(x)$. Comparing this equation with (6) indicates that the equilibrium with Nash bargaining is only efficient if $\beta = \eta(x)$ —this is the [Hosios \[1990\]](#) condition for efficiency. Hence, the equilibrium with Nash bargaining is only efficient for a specific value of the bargaining power and not in general.

Equilibrium with Partially Rigid Price. We consider the following partially rigid price:

$$p = p_0 \cdot \left(\frac{\chi^\varepsilon}{k} \cdot \mu \right)^\xi, \quad (7)$$

where the parameter $p_0 > 0$ governs the price level and the parameter $\xi \in [0, 1)$ governs the rigidity of the price. If $\xi = 0$, the price is fixed. If $\xi = 1$, the price is proportional to and therefore as flexible as the competitive price, given by (5).²⁰ In the general case with $0 < \xi < 1$ the price is more rigid than the competitive price but less rigid than the fixed price.

In equilibrium, tightness equalizes aggregate demand and supply with the price given by (7). As in the fixprice case, there exists a unique equilibrium with positive consumption. Combining

¹⁹This result is not universal. If workers receive a fixed income when they are unemployed, labor market tightness responds to technology shocks under Nash bargaining. [Shimer \[2005\]](#) shows that if the income when unemployed is calibrated to the generosity of the unemployment insurance system in the US, the labor market tightness response is much smaller in the model than in the data. [Hagedorn and Manovskii \[2008\]](#) show that if the income when unemployed is calibrated to be very close to labor productivity, the labor market tightness response may be as large in the model as in the data. In a more complicated version of our model, households would receive a fixed income or fixed utility for each service that they do not sell, and tightness could respond to some shocks under Nash bargaining.

²⁰The competitive price is obtained by setting $\xi = 1$ and $p_0 = (1 + \tau(x^*))^{1-\varepsilon} / f(x^*)$ in (7).

$c^s(x) = c^d(x, p)$ with (7) implies that the equilibrium level of product market tightness satisfies

$$(1 + \tau(x))^{\varepsilon-1} \cdot f(x) = \left(\frac{\chi^\varepsilon}{k} \cdot \mu \right)^{1-\xi} \cdot \frac{1}{p_0}.$$

Since $\xi < 1$, the comparative statics of the tightness are the same as in the fixprice equilibrium, where the tightness satisfies (4). That is, the comparative statics of the fixprice equilibrium remain valid even if the price is partially rigid instead of being fixed.

The comparative statics of the fixprice equilibrium are therefore robust: they hold whenever the price responds less than proportionally to $\chi^\varepsilon \cdot \mu/k$, and they only break down in the knife-edge case in which the price is proportional to $\chi^\varepsilon \cdot \mu/k$. This result echoes the finding of [Blanchard and Galí \[2010\]](#) and [Michaillat \[2012\]](#) that in matching models of the labor market, the qualitative effects of technology shocks on labor market tightness are the same whether the wage is fixed or responds less than proportionally to technology.

2.8 The Case with No Matching Cost

We describe a model with no matching cost. When $\rho = 0$, we have $\tau(x) = 0$, so there is no price wedge and consumption is equal to output. Therefore, the aggregate demand is independent of tightness and the aggregate supply is increasing in tightness everywhere: $c^d(p) = \chi^\varepsilon \cdot \mu/p$ and $c^s(x) = f(x) \cdot k$. The efficient tightness, which maximizes the aggregate supply, is infinite. In [Figure 3\(a\)](#), the aggregate supply curve would take the shape of the output curve and the aggregate demand curve would become vertical at $c = \chi^\varepsilon \cdot \mu/p$.

The model with $\rho = 0$ directly relates to a Walrasian equilibrium and a rationing situation. In a Walrasian equilibrium, households are indifferent between consumption and money and the money market clears, so $c = \chi^\varepsilon \cdot \mu/p$; furthermore, the product market clears, so $c = k$ and $p = \chi^\varepsilon \cdot \mu/k$. In a rationing situation, the market does not clear because the price is at a level $p_0 > \chi^\varepsilon \cdot \mu/k$ that is too high; therefore, price and consumption are given by $p = p_0$ and $c = \chi^\varepsilon \cdot \mu/p_0 < k$.

Price and consumption are the same in the competitive equilibrium of the model with $\rho = 0$ as in a Walrasian equilibrium. Indeed, in the competitive equilibrium, the tightness is efficient so $x = x^* = +\infty$ and $c = \lim_{x \rightarrow +\infty} c^s(x) = k$ as $\lim_{x \rightarrow +\infty} f(x) = 1$; furthermore, since $c^d(p) = c = k$, the price satisfies $p = \chi^\varepsilon \cdot \mu/k$. Hence, c and p are the same in the two equilibria.

Moreover, consumption is the same in the fixprice equilibrium with price p_0 of the model

with $\rho = 0$ as in a rationing situation with price p_0 . Indeed, in the fixprice equilibrium with price $p_0 > \chi^e \cdot \mu/k$, consumption is given by $c^d(p_0) = \chi^e \cdot \mu/p_0 < k$. [Michaillat \[2012\]](#) obtains a similar result in the context of a matching model of the labor market.

The models with $\rho = 0$ and $\rho > 0$ share a number of properties since the model with $\rho = 0$ behaves like the model with $\rho > 0$ in the slack regime. In fact, the properties of all observable variables (price, output, tightness) are the same in the two models. However, assuming that $\rho = 0$ has several disadvantages. From an empirical perspective, assuming that $\rho = 0$ does not seem justified since the evidence suggest that buyers face matching costs. From a theoretical perspective, assuming that $\rho = 0$ makes the model poorer, since the model only has a slack regime so cannot capture the idea that the economy sometimes “overheats”, and less tractable, since the equilibrium only exists if the price is high enough ($p \geq \chi^e \cdot \mu/k$) and the efficient tightness is infinite.

3 A Model of Aggregate Demand, Idle Time, and Unemployment

This section develops the main model of the paper. The model keeps the architecture of the [Barro and Grossman \[1971\]](#) model but takes a matching approach to the labor and product markets instead of a disequilibrium approach.

3.1 Assumptions

The economy has a measure 1 of identical households and a measure 1 of identical firms, owned by the households. The product and labor markets are matching markets that are formally symmetric. The product market and households are the same as in [Section 2](#). The labor market and firms are described below.

The Labor Market. In each household, $h \in (0, 1)$ members are in the labor force and $1 - h$ members are out of the labor force. All the workers in the labor force are initially unemployed and search for a job. Each firm posts \hat{v} vacancies to hire workers. The number l of workers who find a job is given by the following matching function: $l = (h^{-\hat{\gamma}} + \hat{v}^{-\hat{\gamma}})^{-1/\hat{\gamma}}$, where h is the aggregate number of workers who are initially unemployed, \hat{v} is the aggregate number of vacancies, and the

parameter $\hat{\gamma} > 0$ governs the elasticity of substitution of inputs in matching.

We define the labor market tightness as the ratio of aggregate number of vacancies to aggregate number of workers who are initially unemployed: $\theta = \hat{v}/h$. The labor market tightness is an aggregate variable taken as given by the firms and households. The labor market tightness determines the probabilities that a worker finds a job and that a vacancy is filled: a worker finds a job with probability $\hat{f}(\theta) = l/h = (1 + \theta^{-\hat{\gamma}})^{-1/\hat{\gamma}}$, and a vacancy is filled with probability $\hat{q}(\theta) = l/\hat{v} = (1 + \theta^{\hat{\gamma}})^{-1/\hat{\gamma}}$. The properties of the functions \hat{f} and \hat{q} imply that when the labor market tightness is higher, it is easier to find a job but harder to fill a vacancy. We abstract from randomness at the firm and household levels: a firm hires exactly $\hat{v} \cdot \hat{q}(\theta)$ workers, and exactly $\hat{f}(\theta) \cdot h$ household members find a job.

Each firm has two types of employees: n producers and $l - n$ recruiters. The job of recruiters is to post vacancies.²¹ Posting a vacancy requires $\hat{\rho} \in (0, 1)$ recruiter, so the number of recruiters required to post \hat{v} vacancies is $l - n = \hat{\rho} \cdot \hat{v}$. Since hiring l employees requires posting $l/\hat{q}(\theta)$ vacancies, the number n of producers in a firm with l employees is limited to $n = l - \hat{\rho} \cdot l/\hat{q}(\theta)$. This relationship can be written as $l = (1 + \hat{\tau}(\theta)) \cdot n$, where $\hat{\tau}(\theta) \equiv \hat{\rho}/(\hat{q}(\theta) - \hat{\rho})$ is the number of recruiters per producer.

We define the labor supply as the number of producers employed at a given labor market tightness:

DEFINITION 8. *The labor supply is a function of labor market tightness defined by $n^s(\theta) = \hat{f}(\theta) \cdot h / (1 + \hat{\tau}(\theta))$ for all $\theta \in [0, \theta^m)$, where $\theta^m > 0$ satisfies $\hat{\rho} = \hat{q}(\theta^m)$.*

PROPOSITION 7. *The labor supply satisfies*

$$n^s(\theta) = (\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h$$

for all $\theta \in [0, \theta^m)$. We define the tightness $\theta^* \in (0, \theta^m)$ by $\hat{f}'(\theta^*) = \hat{\rho}$. The labor supply is strictly increasing on $[0, \theta^*]$ and strictly decreasing on $[\theta^*, \theta^m]$. Hence, the tightness θ^* maximizes the labor supply. Furthermore, $n^s(0) = 0$, and $n^s(\theta^m) = 0$.

Proof. Similar to the proof of Proposition 1. □

²¹In the literature, firms usually pay the cost of posting vacancies in output. Here, firms pay the cost of posting vacancies in labor as they need to employ recruiters to fill vacancies. We make this assumption because it greatly simplifies the analysis and seems more realistic. Farmer [2008] and Shimer [2010] make the same assumption.

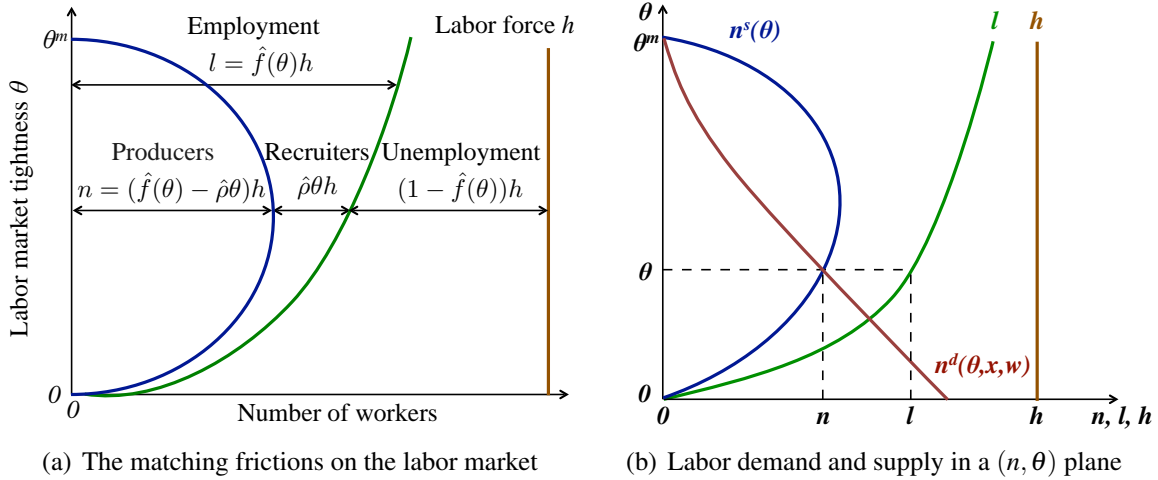


Figure 6: The Labor Market in the Model of Section 3

Figure 6 plots the labor supply. Figure 6(a) displays the numbers of producers, recruiters, and unemployed workers as a function of labor market tightness. Employment is $l = \hat{f}(\theta) \cdot h$, an increasing and concave function of tightness. The number of producers is $n = (\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h$ so it is always below the number of employed workers; the number of producers is given by the labor supply curve. The number of recruiters is $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot \theta \cdot h$; this number is represented by the gap between the labor supply and employment curves. The number of unemployed workers is $h - l = (1 - \hat{f}(\theta)) \cdot h$; this number is represented by the gap between the employment and labor force curves. The unemployment rate is $1 - l/h = 1 - \hat{f}(\theta)$. Comparing Figures 3 and 6(a) shows that the matching frictions on the product and labor markets are isomorphic.

Firms. The representative firm hires l workers. Some of the workers are engaged in production while others are engaged in recruiting. More precisely, $n < l$ producers produce a quantity k of output according to the production function $k = a \cdot n^\alpha$. The parameter $a > 0$ measures the technology of the firm and the parameter $\alpha \in (0, 1)$ captures decreasing marginal returns to labor. Because of the product market frictions, the firm only sells a fraction $f(x)$ of its productive capacity k .

The firm pays its l workers a real wage w ; the nominal wage is $p \cdot w$. The real wage bill of the firm is $w \cdot l = (1 + \hat{\tau}(\theta)) \cdot w \cdot n$. From this perspective, matching frictions in the labor market impose a wedge $\hat{\tau}(\theta)$ on the wage of producers.

Taking as given the labor market tightness, the product market tightness, the price, and the real

wage, the representative firm chooses employment to maximize profits

$$p \cdot f(x) \cdot a \cdot n^\alpha - (1 + \hat{\tau}(\theta)) \cdot p \cdot w \cdot n.$$

The profit-maximizing number of producers satisfies

$$f(x) \cdot \alpha \cdot a \cdot n^{\alpha-1} = (1 + \hat{\tau}(\theta)) \cdot w. \quad (8)$$

This equation implies that the real marginal revenue of one producer equals its real marginal cost. The real marginal revenue is the marginal product of labor, $\alpha \cdot a \cdot n^{\alpha-1}$, times the selling probability, $f(x)$. The real marginal cost is the real wage, w , plus the recruiting cost, $\hat{\tau}(\theta) \cdot w$.

We define the labor demand as the profit-maximizing number of producers at a given product market tightness, labor market tightness, and real wage:

DEFINITION 9. *The labor demand is a function of labor market tightness, product market tightness, and real wage defined by*

$$n^d(\theta, x, w) = \left[\frac{f(x) \cdot \alpha \cdot a}{(1 + \hat{\tau}(\theta)) \cdot w} \right]^{\frac{1}{1-\alpha}} \quad (9)$$

for all $(\theta, x, w) \in [0, \theta^m] \times (0, +\infty) \times (0, +\infty)$, where $\theta^m > 0$ satisfies $\hat{\rho} = \hat{q}(\theta^m)$.

PROPOSITION 8. *The labor demand is strictly increasing in x and strictly decreasing in θ and w . Furthermore, $n^d(0, x, w) = [f(x) \cdot \alpha \cdot a \cdot (1 - \hat{\rho})/w]^{1/(1-\alpha)}$ and $n^d(\theta^m, x, w) = 0$.*

Proof. Obvious from (9). □

The labor demand is the number of producers that satisfies (8). The labor demand is strictly increasing in x because when x increases, the probability $1 - f(x)$ that a producer is idle decreases, so producers become more profitable to firms. It is strictly decreasing in w because when w increases, the wage of producers increases, so producers become less profitable to firms. It is strictly decreasing in θ because when θ increases, the number $\hat{\tau}(\theta)$ of recruiters that firms must hire for each producer increases, so producers become less profitable to firms. The labor demand is depicted in Figure 6(b). The labor demand curve slopes downward in the (n, θ) plane.

Unemployment is traditionally decomposed into three components: a Keynesian component caused by deficient aggregate demand, a classical component caused by excessively high real

wages, and a frictional component caused by recruiting costs. In our model this decomposition is not meaningful because equilibrium unemployment is simultaneously determined by aggregate demand, real wage, and recruiting cost. Yet, our model of the labor demand incorporates Keynesian, classical, and frictional factors. The Keynesian factor operates through $f(x)$ in (9), because $f(x)$ describes how easy or difficult it is for firms to find customers. The classical factor operates through w in (9). The frictional factor operates through $\hat{\tau}(\theta)$ in (9), because $\hat{\tau}(\theta)$ describes how costly it is for firms to recruit workers. The cost of recruiting workers can be high either because the cost of posting a vacancy is high or because a vacancy is only filled with low probability—this happens when the labor market tightness is high.

3.2 Definition of the Equilibrium

Employed and unemployed household members pool their income before jointly deciding consumption; therefore, despite the unemployment risk, the aggregate demand is still given by (3). Firms' productive capacity is not exogenous but is endogenously determined by firms' employment level; the aggregate supply is given by

$$c^s(x, \theta) = (f(x) - \rho \cdot x) \cdot a \cdot (\hat{f}(\theta) - \hat{\rho} \cdot \theta)^\alpha \cdot h^\alpha.$$

This expression is obtained from (1) by setting the productive capacity to $a \cdot n^\alpha$ and expressing n as a function of θ using the labor supply. The equilibrium is defined as follows:

DEFINITION 10. *An equilibrium consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that aggregate supply is equal to aggregate demand and labor supply is equal to labor demand:*

$$\begin{cases} c^s(x, \theta) & = & c^d(x, p) \\ n^s(\theta) & = & n^d(\theta, x, w) \end{cases}$$

Since the equilibrium is composed of four variables that satisfy two conditions, infinitely many combinations of (x, p, θ, w) are consistent with the equilibrium conditions. To select an equilibrium, we specify a price and a wage mechanism. In Sections 3.3–3.5, we will study the equilibria selected by different mechanisms.

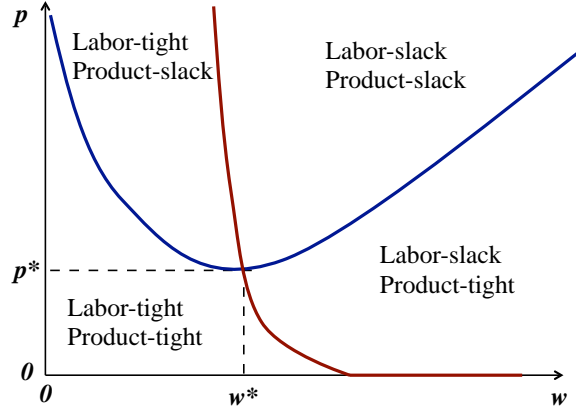


Figure 7: The Four Inefficient Regimes in the Model of Section 3

Many equilibrium prices and wages are possible, so the equilibrium may be in different regimes:

DEFINITION 11. *The equilibrium is efficient if $\theta = \theta^*$ and $x = x^*$, labor-slack and product-slack if $\theta < \theta^*$ and $x < x^*$, labor-slack and product-tight if $\theta < \theta^*$ and $x > x^*$, labor-tight and product-slack if $\theta > \theta^*$ and $x < x^*$, labor-tight and product-tight if $\theta > \theta^*$ and $x > x^*$.*

These four inefficient regimes are reminiscent of the four regimes in the Barro-Grossman model. In both models, the regimes are determined by the price and real wage being inefficiently high or low. Online Appendix B characterizes the four regions of a (w, p) plane corresponding to the four inefficient regimes. The characterization is depicted in Figure 7. The region for the labor-slack and product-slack equilibria has high price and high real wage, the region for labor-tight and product-tight equilibria has low price and low real wage, and so on. The efficient equilibrium is at the intersection of the two curves delimiting the inefficient regimes.

But there is an important difference between the Barro-Grossman model and our model. In our model the equilibrium is determined by the same system of smooth equations in any regime, whereas in the Barro-Grossman model the disequilibrium is described by a different system of equations in each regime, depending on whether the supply or the demand binds in each market. Accordingly, our representation of the four regimes is more tractable.

3.3 Fixprice Equilibrium

DEFINITION 12. *A fixprice equilibrium parameterized by $p_0 > 0$ and $w_0 > 0$ consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that supply*

equals demand on the product and labor markets and price and real wage are given by the parameter p_0 and w_0 : $c^s(x, \theta) = c^d(x, p)$, $n^s(\theta) = n^d(\theta, x, w)$, $p = p_0$, and $w = w_0$.

PROPOSITION 9. *For any $p_0 > 0$ and $w_0 > 0$, there exists a unique fixprice equilibrium parameterized by p_0 and w_0 with positive consumption.*

Proof. See Online Appendix A. □

We use comparative statics to describe the response of the fixprice equilibrium to aggregate demand, technology, labor supply, and mismatch shocks. We parameterize an increase in aggregate demand by an increase in money supply, μ , or in the taste for consumption, χ . We parameterize an increase in technology by an increase in the production-function parameter, a . We parameterize an increase in labor supply by an increase in the size of the labor force, h . An increase in h captures increases in labor force participation caused by demographic factors, changes to the taste for leisure and work, or changes to policies such as disability insurance. An increase in h also captures increases in job-search effort caused by changes to policies such as unemployment insurance.²² We parameterize an increase in mismatch by a decrease of the matching efficacy on the labor market along with a corresponding decrease in recruiting cost: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and $\hat{\rho}$ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$ with $\lambda < 1$.²³ Note that the function $\hat{\tau}$ and tightness θ^* remain the same after a mismatch shock. The interpretation of an increase in mismatch is that a fraction of potential workers are not suitable to employers, which reduces matching efficacy, and these unsuitable workers can be spotted at no cost, which reduces the cost of managing a vacancy.²⁴ The following proposition summarizes the comparative statics:²⁵

PROPOSITION 10. *Consider a fixprice equilibrium with positive consumption.*

²²Assume that workers receiving unemployment insurance search for a job with effort 0 or 1. A change to the generosity of unemployment insurance affects the share of workers searching with effort 1. But only workers searching with effort 1 are part of h because only these workers contribute to the matching process. Hence, changing the generosity of unemployment insurance affects h . Note that our classification of the workers receiving unemployment insurance is consistent with the definitions used in official statistics. In the statistics constructed by the Bureau of Labor Statistics from the Current Population Survey, jobseekers are counted as unemployed if they search with effort 1 and as out of the labor force if they search with effort 0, irrespective of their receipt of unemployment insurance.

²³See Shimer [2007] and Sahin et al. [2014] for microfounded models of labor market mismatch.

²⁴Another possible parameterization of mismatch shocks is a decrease in matching efficacy with no change in recruiting cost. Such a parameterization leads to less clearcut results because the mismatch shock affects both the labor demand and the labor supply.

²⁵With a linear production function ($\alpha = 1$), the comparative statics would remain the same.

- *An increase in aggregate demand has the following effects: output, product market tightness, employment, and labor market tightness increase; the rate of idleness and the rate of unemployment decrease.*
- *An increase in technology has the following effects: output increases but product market tightness decreases; employment and labor market tightness increase; the rate of idleness increases but the rate of unemployment decreases.*
- *An increase in labor supply has the following effects: output and employment increase, but product market tightness and labor market tightness decrease; the rate of idleness and the rate of unemployment increase.*
- *A decrease in mismatch has the following effects: output and employment increase, but product market tightness and labor market tightness decrease; the rate of idleness increases but the rate of unemployment decreases.*

Proof. See Online Appendix A. □

The comparative statics are summarized in panel A of Table 2. We discuss these comparative statics with the help of the equilibrium diagrams. We concentrate on the effects of shocks on tightnesses; it is simple to infer the effects on quantities from those.

First, we study an increase in aggregate demand. Since the price is fixed, the aggregate demand curve rotates upward in Figure 3(a), which raises the product market tightness. The probability to sell is higher so the rate of idleness among employees falls and hiring a worker becomes more profitable. In Figure 6(b), the labor demand curve rotates outward, which raises the labor market tightness. Since the labor market tightness rises, the aggregate supply curve shifts outward in Figure 3(a), which attenuates the increase in product market tightness.

Second, we study an increase in technology. The increase in technology simultaneously affects the labor and product markets. Since the real wage is fixed, the increase in technology raises the profitability of hiring workers. Hence, the labor demand curve rotates upward in Figure 6(b), which raises the labor market tightness. Since technology rises, the aggregate supply curve shifts outward in Figure 3(a), which lowers the product market tightness. The initial responses of the tightnesses spill over across markets: since the product market tightness falls, the labor demand rotates back inward, which attenuates the increase in labor market tightness; and since the labor

Table 2: Comparative Statics in the Model of Section 3

Increase in:	Effect on:			
	Output y	Product market tightness x	Employment l	Labor market tightness θ
<i>A. Fixprice equilibrium and equilibrium with partially rigid price and real wage</i>				
Aggregate demand	+	+	+	+
Technology	+	−	+	+
Labor supply	+	−	+	−
Mismatch	−	+	−	+
<i>B. Competitive equilibrium and equilibrium with Nash bargaining</i>				
Aggregate demand	0	0	0	0
Technology	+	0	0	0
Labor supply	+	0	+	0
Mismatch	−	0	−	0

Notes: An increase in aggregate demand is an increase in money supply, μ , or in the taste for consumption, χ . An increase in technology is an increase in the production-function parameter, a . An increase in labor supply is an increase in the size of the labor force, h . An increase in mismatch is a decrease of the matching efficacy on the labor market along with a corresponding decrease in recruiting cost. After an increase in mismatch, $\hat{f}(\theta)$, $\hat{q}(\theta)$, and $\hat{\rho}$ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$, with $\lambda < 1$. This table summarizes the results of Propositions 10 and 12 and the results discussed in Section 3.5.

market tightness increases, the aggregate supply shifts further outward, which accentuates the reduction in product market tightness.

Third, we study an increase in labor supply. In Figure 6(b), the labor force curve and the labor supply curve shift outward. Thus, the labor market tightness falls but the number of producers increases. With more producers, firms' productive capacity increases and the aggregate supply shifts outward in Figure 3(a), which reduces the product market tightness. Since the product market tightness falls, the labor demand curve rotates inward in Figure 6(b), which further reduces labor market tightness and attenuates the increase in the number of producers.

Finally, we study a reduction in mismatch. In Figure 6(b), the labor supply curve shifts outward and the labor demand curve remains the same. Like after an increase in labor supply, the labor market and product market tightnesses decrease. But unlike after an increase in labor supply, the rate of unemployment decreases. This is obvious because the reduction in mismatch leads to higher employment for the same number of labor force participants. In fact, only the mismatch shock generates a positive correlation between labor market tightness and unemployment rate.

3.4 Competitive Equilibrium

DEFINITION 13. A competitive equilibrium consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that supply equals demand on the product and labor markets and labor and product market tightnesses are efficient: $c^s(x, \theta) = c^d(x, p)$, $n^s(\theta) = n^d(\theta, x, w)$, $x = x^*$, and $\theta = \theta^*$.

PROPOSITION 11. There exists a unique competitive equilibrium. The competitive price and real wage are given by

$$p^* = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{f(x^*)} \cdot \left(\frac{1 + \hat{\tau}(\theta^*)}{\hat{f}(\theta^*)} \right)^\alpha \cdot \frac{\chi^\varepsilon}{a \cdot h^\alpha} \cdot \mu$$

$$w^* = f(x^*) \cdot \frac{\hat{f}(\theta^*)^{\alpha-1}}{(1 + \hat{\tau}(\theta^*))^\alpha} \cdot \alpha \cdot a \cdot h^{\alpha-1}.$$

Proof. See Online Appendix A. □

PROPOSITION 12. Consider a competitive equilibrium.

- An increase in aggregate demand has no effect on output, product market tightness, the rate of idleness, employment, labor market tightness, and the rate of unemployment.
- An increase in technology has the following effects: output increases; product market tightness, the rate of idleness, employment, labor market tightness, and the rate of unemployment remain the same.
- An increase in labor supply has the following effects: output and employment increase; product market tightness, the rate of idleness, labor market tightness, and the rate of unemployment remain the same.
- A decrease in mismatch has the following effects: output and employment increase; product market tightness, the rate of idleness, and labor market tightness remain the same; the rate of unemployment decreases.

Proof. Similar to the proof of Proposition 6. □

The comparative statics are summarized in panel B of Table 2. The competitive equilibrium has three notable properties. First, aggregate demand shocks have no real effects. Second, the

product market and labor market tightnesses do not respond to any of the shocks considered, not even mismatch shocks. Third, employment only responds to labor supply and mismatch shocks, and the unemployment rate only responds to mismatch shocks.

3.5 Other Equilibria

Equilibrium with Nash Bargaining. The real wage is the generalized Nash solution of the bargaining problem between a firm and a marginal worker with bargaining power $\hat{\beta} \in (0, 1)$.²⁶ After a match is made, the surplus to the firm of employing a marginal worker is $\mathcal{F}(w) = f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1} - w$, and the surplus to the worker of being employed is $\mathcal{W}(w) = w$. The Nash solution maximizes $\mathcal{F}(w)^{1-\hat{\beta}} \cdot \mathcal{W}(w)^{\hat{\beta}}$, so $\mathcal{W}(w) = \hat{\beta} \cdot [\mathcal{W}(w) + \mathcal{F}(w)] = \hat{\beta} \cdot f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1}$ and the real wage satisfies $w = \hat{\beta} \cdot f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1}$. At this wage the labor demand condition, given by (8), becomes

$$\hat{\beta} \cdot (1 + \hat{\tau}(\theta)) = 1.$$

This equation determines the equilibrium labor market tightness. Equation (6) determines the equilibrium product market tightness. In equilibrium the tightnesses solely depend on the matching costs and matching functions because they are determined by the functions τ and $\hat{\tau}$. Therefore, the comparative statics are the same as in the competitive equilibrium.

Equilibrium with Partially Rigid Price and Real Wage. We consider the following partially rigid price and real wage:

$$p = p_0 \cdot \left(\frac{\chi^\varepsilon}{a \cdot h^\alpha} \cdot \mu \right)^\xi$$

$$w = w_0 \cdot (\alpha \cdot a \cdot h^{\alpha-1})^\xi.$$

²⁶Although firm and workers are engaged in multilateral intrafirm bargaining, we abstract from possible strategic behavior. Such behavior is analyzed in [Stole and Zwiebel \[1996\]](#). Instead, we assume that the firm bargains with each worker individually, taking the worker as marginal. This would necessarily happen with a linear production function.

The parameter $\xi \in [0, 1)$ governs the rigidity of the price and the real wage.²⁷ We show in Online Appendix A that the equilibrium conditions with the partially rigid price and real wage retain the same properties as those with a fixed price and real wage; therefore, the comparative statics of the fixprice equilibrium remain valid in this equilibrium with partial rigidity.

4 A Dynamic Model with Long-Term Relationships

Long-term customer and employment relationships are prevalent. Figure 2(c) displays the fraction of sales to long-term customers reported in firm surveys in eleven countries, including the US. The average share of sales to long-term customers across countries is 77%. Figure 2(c) displays the share of workers in long-term employment relationships in the same countries. The average share of workers in long-term employment across countries is 87%. In this section we embed the static model of Section 3 into a dynamic environment to capture long-term customer and employment relationships. The dynamic model is user-friendly because at the limit without time discounting, its comparative statics are the same as those of the static model of Section 3. Since the dynamic model offers a better mapping between theoretical and empirical quantities, we will use it in the empirical analysis of Section 5.

Matching Process on the Product and Labor Markets. We work in continuous time. Firms engage in long-term relationships with customers on the product market, and they engage in long-term relationships with employees on the labor market.

At time t , there are h workers in the labor force, $l(t)$ employed workers, and $h - l(t)$ unemployed workers. Firms post $\hat{v}(t)$ vacancies. New employment relationships are formed at a rate $[(h - l(t))^{-\hat{\gamma}} + \hat{v}(t)^{-\hat{\gamma}}]^{-1/\hat{\gamma}}$. We define the labor market tightness as $\theta(t) = \hat{v}(t)/(h - l(t))$. Unemployed workers find a job at rate $\hat{f}(\theta(t))$ and vacancies are filled at rate $\hat{q}(\theta(t))$. Employment relationships are destroyed at rate $\hat{s} > 0$. The law of motion of employment is therefore given by

$$\dot{l}(t) = \hat{f}(\theta(t)) \cdot (h - l(t)) - \hat{s} \cdot l(t). \quad (10)$$

²⁷We confine our analysis to the case in which the price and the real wage have the same rigidity. This case illustrates that the comparative statics of the fixprice equilibrium are also valid with some partially rigid price and real wage. There are numerous other possible cases, depending on the rigidity of the real wage relative to that of the price.

In the law of motion, $\hat{f}(\theta(t)) \cdot (h - l(t))$ is the number of employment relationships created at t and $\hat{s} \cdot l(t)$ is the number of employment relationships destroyed at t .

The product market operates exactly like the labor market. All purchases take place through long-term customer relationships. At time t , firms have a productive capacity $k(t) = a \cdot n(t)^\alpha$ and sell output $y(t) < k(t)$. Idle capacity is $k(t) - y(t)$. Households create new customer relationships by visiting $v(t)$ firms that have $k(t) - y(t)$ productive capacity available. New customer relationships are formed at a rate $[(k(t) - y(t))^{-\gamma} + v(t)^{-\gamma}]^{-1/\gamma}$. We define the product market tightness as $x(t) = v(t)/(k(t) - y(t))$. The $k(t) - y(t)$ units of available productive capacity yield new customer relationships at rate $f(x(t))$ and the $v(t)$ visits are successful at rate $q(x(t))$. Customer relationships are destroyed at rate s . The law of motion of output is therefore given by

$$\dot{y}(t) = f(x(t)) \cdot (k(t) - y(t)) - s \cdot y(t). \quad (11)$$

In the law of motion, $f(x(t)) \cdot (k(t) - y(t))$ is the number of customer relationships created at t and $s \cdot y(t)$ is the number of customer relationships destroyed at t .

Households. The utility of the representative household is given by $\int_{t=0}^{+\infty} e^{-\delta \cdot t} \cdot u(c(t), m(t)/p(t)) dt$, where $\delta > 0$ is the time discount factor, $c(t)$ is consumption at time t , and $m(t)/p(t)$ are real money balances at time t . To consume $c(t)$, the household must make $y(t) \geq c(t)$ purchases. The $y(t)$ purchases are used for consumption, $c(t)$, and to cover the matching costs. At time t , the household adjusts its number of customer relationships by $\dot{y}(t)$, and it also replaces the $s \cdot y(t)$ relationships that have just been destroyed. Making these $\dot{y}(t) + s \cdot y(t)$ new relationships requires $(\dot{y}(t) + s \cdot y(t))/q(x(t))$ visits, each costing ρ purchases. Hence, purchases and consumption are related by

$$y(t) = c(t) + \frac{\rho}{q(x(t))} \cdot (\dot{y}(t) + s \cdot y(t)). \quad (12)$$

Nominal money balances are an asset with law of motion

$$\dot{m}(t) = p(t) \cdot w(t) \cdot l(t) - p(t) \cdot y(t) + T(t), \quad (13)$$

where $T(t)$ includes firms' nominal profits, which are rebated to the household, and transfers from the government. Given $[p(t), w(t), x(t), l(t), T(t)]_{t=0}^{+\infty}$ the household chooses $[y(t), c(t), m(t)]_{t=0}^{+\infty}$ to maximize utility subject to (12) and (13).

Firms. The representative firm employs $n(t)$ producers and $l(t) - n(t)$ recruiters. At time t , the firm adjusts its number of employees by $\dot{l}(t)$, and it also replaces the $\hat{s} \cdot l(t)$ employees that have just left the firm. Hiring these $\dot{l}(t) + \hat{s} \cdot l(t)$ new workers requires to post $(\dot{l}(t) + \hat{s} \cdot l(t)) / \hat{q}(\theta(t))$ vacancies. Each vacancy takes the time of $\hat{\rho}$ recruiters. Hence, the firm needs the following number of recruiters:

$$l(t) - n(t) = \frac{\hat{\rho}}{\hat{q}(\theta(t))} \cdot (\dot{l}(t) + \hat{s} \cdot l(t)). \quad (14)$$

The firm sells output $y(t)$ to customers. The amount of sales depend on the product market tightness and the productive capacity of the firm:

$$\dot{y}(t) = f(x(t)) \cdot (a \cdot n(t)^\alpha - y(t)) - s \cdot y(t). \quad (15)$$

Given $[w(t), x(t), \theta(t)]_{t=0}^{+\infty}$ the firm chooses $[l(t), n(t), y(t)]_{t=0}^{+\infty}$ to maximize the discounted stream of real profits, $\int_{t=0}^{+\infty} e^{-\delta \cdot t} \cdot (y(t) - w(t) \cdot l(t)) dt$, subject to (14) and (15).

Discussion of the Assumptions. We assume that in a long-term relationship, the buyer does not incur the matching cost and the seller sells one unit of good (labor or output) per unit time with certainty. These assumptions are standard in dynamic matching models of the labor market. They describe well long-term employment relationships given the nature of labor contracts.

We also think that the assumptions describe long-term customer relationships well. First, a sizable share of transactions on the product market are conducted under contract, and our assumptions describe well the terms of an explicit contract.²⁸ Second, observations from a survey of bakers that we conducted in France in the summer of 2007 suggest that even when no explicit contract is signed, long-term customer relationships are governed by implicit contracts that alleviate matching frictions in line with our assumptions.²⁹ A first observation is that customer relationships

²⁸Using BLS data on contractual arrangements between firms trading intermediate goods, [Goldberg and Hellerstein \[2011\]](#) find that one third of all transactions are conducted under contract across industries, for both goods and services.

²⁹This survey is described in [Eyster, Madarasz and Michailat \[2014\]](#).

alleviate the uncertainty associated with random demand. A baker told us that demand is difficult to predict and that having a large clientele of loyal customers who make it a habit to purchase bread in the shop was therefore important. In fact, “good” customers are expected to come everyday to the bakery. A second observation is that customer relationships alleviate the uncertainty associated with random supply. Being a customer means having the assurance that your usual bread will be available, even on days when supply runs low. Of course, this is possible because bakers know exactly what customers order every day through their long association. In fact, one baker said that it would be “unacceptable” to run out of bread for a customer, and that customers would probably “leave the bakery” if that happened.

Steady-state equilibrium. We focus on a steady-state equilibrium with no time discounting and a money supply growing at a constant rate $\dot{\mu}(t)/\mu(t) = \pi > 0$.³⁰ To maintain real money balances constant, the rate of price inflation must be $\dot{p}(t)/p(t) = \pi$; hence, the price level satisfies the differential equation $\dot{p}(t) = \pi \cdot p(t)$, where $\pi > 0$ is growth rate of the money supply and $p(0)$ is determined by the price mechanism. The real wage is constant: $w(t) = w$, where w is determined by the price mechanism. Given a price mechanism, the variables $\{l, n, y, c, \theta, x\}$ satisfy $y = [f(x)/(s + f(x))] \cdot a \cdot n^\alpha$, $l = [\hat{f}(\theta)/(\hat{s} + \hat{f}(\theta))] \cdot h$, $y = (1 + \tau(x)) \cdot c$ where $\tau(x) \equiv s \cdot \rho / (q(x) - s \cdot \rho)$, $l = (1 + \hat{\tau}(\theta)) \cdot n$ where $\hat{\tau}(\theta) \equiv \hat{s} \cdot \hat{\rho} / (\hat{q}(\theta) - \hat{s} \cdot \hat{\rho})$,

$$c = \left(\frac{\chi \cdot \pi}{1 + \tau(x)} \right)^\varepsilon \cdot \frac{\mu(0)}{p(0)}$$

$$n = \left[\frac{f(x)}{s + f(x)} \cdot \frac{\alpha \cdot a}{(1 + \hat{\tau}(\theta)) \cdot w} \right]^{\frac{1}{1-\alpha}}.$$

The first two equations are obtained by setting $\dot{l}(t) = 0$ and $\dot{y}(t) = 0$ in (10) and (11). The next two are obtained by setting $\dot{y}(t) = 0$ and $\dot{l}(t) = 0$ in (12) and (14). The last two describe the household’s optimal consumption choice combined with the market-clearing condition for money, and the firm’s optimal employment choice. These last two equations are derived in Online Appendix C.

These equations describe the output, employment, aggregate supply, labor supply, aggregate demand, and labor demand curves. These curves are exactly those of the static model of Section 3 once χ , $f(x)$, and $\hat{f}(\theta)$ are replaced by $\chi \cdot \pi$, $f(x)/(s + f(x))$, and $\hat{f}(\theta)/(\hat{s} + \hat{f}(\theta))$ and the param-

³⁰Introducing positive inflation ensures that households consume some produced good even when it becomes infinitely patient at the limit without time discounting. Without inflation, at the limit without time discounting, households would use all their income to increase their money balances and aggregate demand would be zero.

eters ρ and $\hat{\rho}$ are replaced by the parameters $s \cdot \rho$ and $\hat{s} \cdot \hat{\rho}$ in τ and $\hat{\tau}$. All the relevant properties of the functions f and \hat{f} are preserved by the transformation to $f/(s+f)$ and $\hat{f}/(\hat{s}+\hat{f})$. Hence, the comparative statics are the same as here as in the static model of Section 3, both in a fixprice equilibrium in which $p(0)$ and w are fixed, and in a competitive equilibrium in which $p(0)$ and w ensure that the product market and labor market tightnesses are always efficient.

5 Exploration of the Sources of Labor Market Fluctuations

In this section we combine the comparative-statics predictions of the dynamic model of Section 4 with empirical evidence to assess the sources of labor market fluctuations in the US.³¹ The analysis suggests that labor market fluctuations in the US are mainly driven by aggregate demand shocks.

5.1 A Proxy for Product Market Tightness

The empirical analysis relies on the cyclical behavior of product market tightness, x_t . We are not aware of any measure of product market tightness, so we construct a proxy for the cyclical component of product market tightness in the US.³² Our proxy is the cyclical component of the labor utilization rate, $f(x_t)/(s+f(x_t))$. The labor utilization rate is one minus the rate of idleness of employed workers.

We construct our proxy from the capacity utilization rate cu_t measured by the Census Bureau in the Survey of Plant Capacity (SPC) from 1973:Q4 to 2013:Q2. We choose the measure of capacity utilization from the SPC because, compared to other measures of utilization, it is available for the longest period and uses the broadest sample of firms. The measure applies to the manufacturing sector. The SPC measures fourth-quarter capacity utilization rates until 2007 and quarterly capacity utilization rates after that. To obtain a quarterly series for the entire period, we use a linear interpolation of the annual series into a quarterly series for 1973:Q4–2007:Q4 and combine it with the quarterly series for 2008:Q1–2013:Q2.

³¹The assumption underlying our strategy is that the comparative statics provide a good approximation to the actual dynamic effects of shocks. This assumption is justified if the labor and product markets quickly reach their steady states. Shimer [2005] and Pissarides [2009] argue that this assumption is justified for the labor market because the rates of inflow to and outflow from unemployment are large. Online Appendix A5 in Michailat [2012] validates this approximation with numerical simulations. There is little evidence on the size of customer flows on the product market, making it difficult to validate the approximation for the product market.

³²For a measure of tightness on the capital market, see Ottonello [2014].

We need to correct cu_t to obtain $f(x_t)/(s + f(x_t))$ because $f(x_t)/(s + f(x_t))$ is the share of the productive capacity at current employment that is actually sold, whereas cu_t is the share of the productive capacity at full employment that is actually sold [Morin and Stevens, 2004]. Let $g(a, n, k) = a \cdot n^\alpha \cdot k^{1-\alpha}$ be a firm's productive capacity with technology a , employment n , and capital k . Let K be the current stock of capital, which is also the stock of capital that respondents take into account when they report cu . Let N be the full employment level that respondents take into account when they report cu . Let n be the current employment level. We assume that N and K move slowly over time so that their cyclical components are zero. The firm's capacity is $g(a, n, K)$ under current employment and $g(a, N, K)$ under full employment. We can write the firm's output in two different ways:

$$y_t = cu_t \cdot g(a_t, N, K) = \frac{f(x_t)}{s + f(x_t)} \cdot g(a_t, n_t, K).$$

Taking log and recombining implies that

$$\ln\left(\frac{f(x_t)}{s + f(x_t)}\right) = \ln(cu_t) - \alpha \cdot \ln(n_t) + \alpha \cdot \ln(N). \quad (16)$$

We use (16) to construct the cyclical component of $f(x_t)/(s + f(x_t))$, which is our proxy for the cyclical component of product market tightness. We denote this proxy by x_t^c . First, we measure n_t as the quarterly average of the seasonally adjusted monthly employment level in the manufacturing sector constructed by the BLS Current Employment Statistics program, and, following conventions, we set $\alpha = 2/3$. Then, we remove from $\ln(cu_t)$ and $\ln(n_t)$ a low frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter 1600—to obtain the cyclical components of cu_t and n_t , which we denote by cu_t^c and n_t^c . Finally, using (16), we construct our proxy as

$$x_t^c = cu_t^c - \alpha \cdot n_t^c.$$

Figure 8(a) plots the proxy for 1973:Q4–2013:Q2.

Our proxy for product market tightness is not ideal. First, it is constructed from a measure of capacity utilization instead of a direct measure of labor utilization. Second, the measure of capacity utilization applies to the manufacturing sector, and it may therefore be influenced by some logistical issues, such as peak load and inventory management. We address these problems

in Online Appendix D. In that appendix we show that all our empirical results are robust to using an alternative proxy for product market tightness. This alternative proxy is constructed from the operating rate in non-manufacturing sectors measured by the Institute for Supply Management (ISM) and published in their Semiannual Reports. The operating rate is the actual production level of firms as a share of their maximum production level given current capital and labor, so it exactly corresponds to our concept of labor utilization. Unfortunately, the time series for the operating rate only starts in 1999:Q4, so it is too brief to permit a thorough empirical analysis.

The empirical analysis also requires measures of output, employment, and labor market tightness for the US from 1973:Q4 to 2013:Q2. We measure output and employment using seasonally adjusted quarterly indices for real output and employment for the nonfarm business sector constructed by the MSPC program of the BLS. We construct labor market tightness as the ratio of vacancy to unemployment. We measure vacancy with the quarterly average of the monthly vacancy index constructed by [Barnichon \[2010\]](#). This index combines the online and print help-wanted indices of the Conference Board. We measure unemployment with the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. We construct the cyclical components of these series by taking their log and removing a low frequency trend—a HP filter with smoothing parameter 1600.

5.2 Evidence of Price and Real-Wage Rigidity

The equilibria that we have studied can be sorted in two groups based on their comparative statics. The first group includes the fixprice equilibrium and the equilibrium with partially rigid price and real wage. Their comparative statics are reported in panel A of Table 2. Since shocks are not entirely absorbed by the price and real wage and transmit to tightnesses, we say that these equilibria exhibit *price and real-wage rigidity*. The second group includes the competitive equilibrium and the equilibrium with Nash bargaining. Their comparative statics are reported in panel B of Table 2. Since shocks are entirely absorbed by the price and real wage and do not transmit to tightnesses, we say that these equilibria exhibit *price and real-wage flexibility*.

The two groups of equilibria have starkly different comparative statics, so the first step of the empirical analysis is to determine which group better describes the data. To do so, we observe the cyclical behavior of the product market and labor market tightnesses, and we exploit the property that the tightnesses respond to shocks only in equilibria with price and real-wage rigidity.

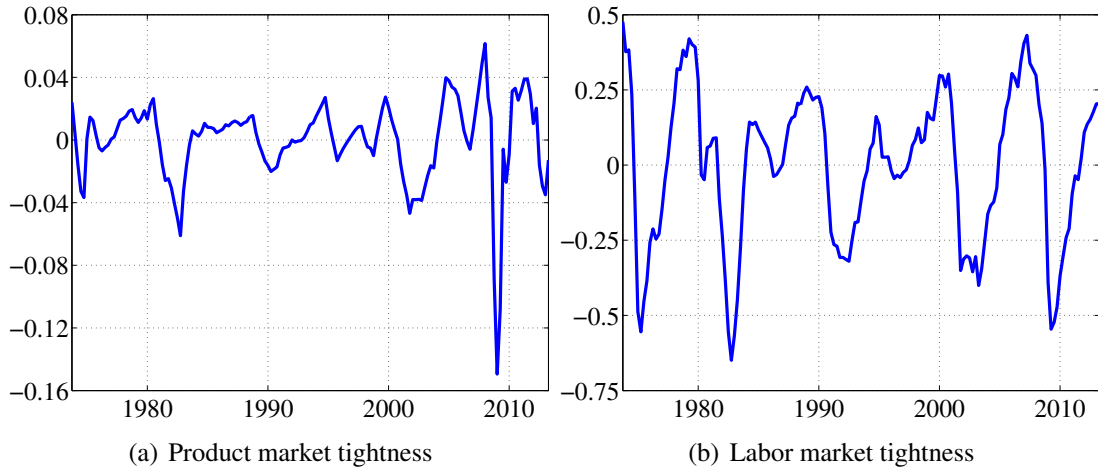


Figure 8: The Cyclical Components of Product Market Tightness and Labor Market Tightness

Notes: The time period is 1973:Q4–2013:Q2. Panel (a) displays the proxy for the cyclical component of product market tightness, x_t^c . The construction of x_t^c is explained in the text. Panel (b) displays the cyclical component of labor market tightness, θ_t^c . Labor market tightness is constructed as $\theta_t = v_t/u_t$, where v_t is the quarterly average of the monthly vacancy index constructed by [Barnichon \[2010\]](#), and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. We construct θ_t^c by removing from $\ln(\theta_t)$ the trend produced by a HP filter with smoothing parameter 1600.

Figure 8 displays the cyclical components of the product market and labor market tightnesses. Figure 8(a) shows that the cyclical component of the product market tightness is subject to fluctuations.³³ Figure 8(b) confirms the well-known fact that the labor market tightness is subject to large fluctuations over the business cycle.³⁴ For instance, the cyclical component of the labor market tightness fell to -0.5 in 2009, which indicates that the labor market tightness was broadly 50% below trend in 2009. While the drop in labor market tightness in 2008–2009 was commensurate to the drops in previous recessions, the drop in product market tightness in 2008–2009 was unprecedented—it was three times as large as the drops in 1981–1982 and 2001.

The cyclical fluctuations of the product market and labor market tightnesses suggest that the equilibria with price and real-wage rigidity are more appropriate to describe the business cycle than the equilibria with price and real-wage flexibility. In the rest of the analysis, we therefore use the predictions from the equilibria with price and real-wage rigidity. Relatedly, [Shimer \[2005\]](#) and [Hall \[2005\]](#) observe that labor market tightness is subject to large fluctuations in the US and

³³The cyclical fluctuations of the product market tightness have not been analyzed before. Yet, the observation that the product market tightness is subject to large cyclical fluctuations is not very surprising: everybody knows that queues at restaurants systematically vary depending on the time of the day or the day of the week, which indicates that prices do not adjust sufficiently to absorb variations in demand.

³⁴See for instance the empirical work of [Blanchard and Diamond \[1989b\]](#) and [Shimer \[2005\]](#).

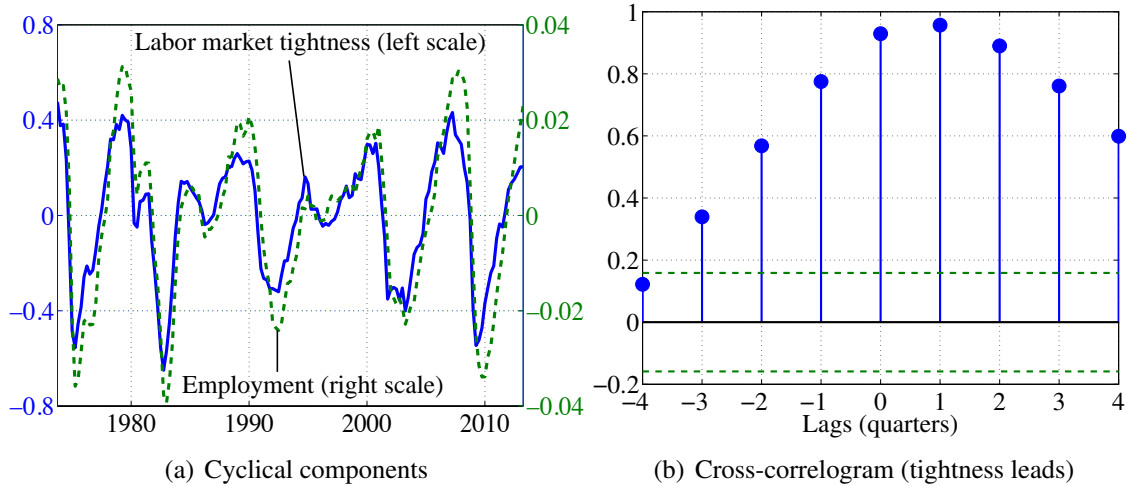


Figure 9: Correlation Between Labor Market Tightness and Employment

Notes: The time period is 1973:Q4–2013:Q2. Panel (a) displays the cyclical component of labor market tightness, θ_t^c , and the cyclical component of employment, l_t^c . Labor market tightness is constructed as $\theta_t = v_t/u_t$, where v_t is the quarterly average of the monthly vacancy index constructed by Barnichon [2010], and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. Employment is the seasonally adjusted quarterly index for employment in the nonfarm business sector constructed by the BLS MSPC program. We construct θ_t^c and l_t^c by removing from $\ln(\theta_t)$ and $\ln(l_t)$ the trends produced by a HP filter with smoothing parameter 1600. Panel (b) displays the cross-correlogram between θ_t^c and l_t^c . The cross-correlation at lag i is the correlation between θ_{t-i}^c and l_t^c . The green dashed lines are the two-standard-deviation confidence bounds.

conclude that real wages must be somewhat rigid.

5.3 Evidence of Labor Demand Shocks

We evaluate whether labor market fluctuations are caused by labor demand, labor supply, or mismatch shocks. Labor demand shocks encompass aggregate demand and technology shocks.

The comparative statics in equilibria with price and real-wage rigidity, reported in panel A of Table 2, indicate that labor demand shocks have different effects from labor supply and mismatch shocks. Labor demand shocks produce a positive correlation between labor market tightness and employment. In contrast, labor supply and mismatch shocks produce a negative correlation between labor market tightness and employment.

To assess the prevalence of labor demand shocks, we therefore measure the correlation between the cyclical components of labor market tightness and employment. In Figure 9(a) the cyclical components of labor market tightness and employment appear strongly positively correlated. Figure 9(b) formalizes this observation by reporting the cross-correlogram of labor market tightness

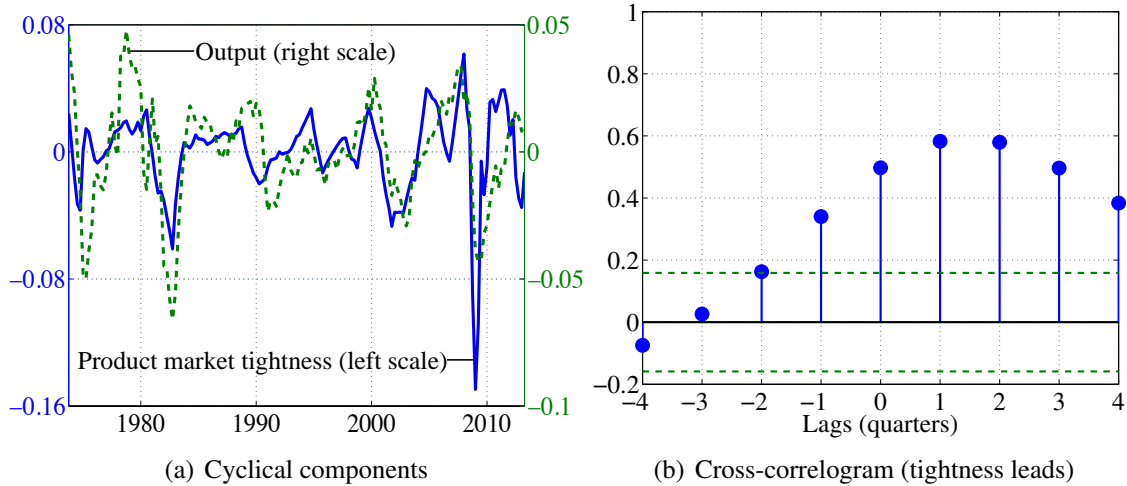


Figure 10: Correlation Between Product Market Tightness and Output

Notes: The time period is 1973:Q4–2013:Q2. Panel (a) displays the proxy for the cyclical component of product market tightness, x_t^c , and the cyclical component of output, y_t^c . The construction of x_t^c is explained in the text. Output, y_t , is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. We construct y_t^c by removing from $\ln(y_t)$ the trend produced by a HP filter with smoothing parameter 1600. Panel (b) displays the cross-correlogram between x_t^c and y_t^c . The cross-correlation at lag i is the correlation between x_{t-i}^c and y_t^c . The green dashed lines are the two-standard-deviation confidence bounds.

and employment: labor market tightness leads employment by one lag; at one lag, the correlation is large, 0.95; the contemporaneous correlation is broadly the same, 0.93; and all the correlations are statistically different from 0.

In the context of our model, these positive correlations imply that it is labor demand shocks and not labor supply or mismatch shocks that generate labor market fluctuations. Relatedly, [Blanchard and Diamond \[1989b\]](#) observe that vacancy rate and unemployment rate are negatively correlated in US data, and they conclude that the source of labor market fluctuations must be aggregate activity shocks and not labor force participation or reallocation shocks.

5.4 Evidence of Aggregate Demand Shocks

Having found that labor demand shocks are the prevalent source of labor market fluctuations, we evaluate whether these labor demand shocks are aggregate demand shocks or technology shocks.

The comparative statics in equilibria with price and real-wage rigidity, reported in panel A of Table 2, indicate that aggregate demand shocks have different effects from technology shocks. Aggregate demand shocks produce a positive correlation between product market tightness and output. In contrast, technology shocks produce a negative correlation between product market

tightness and output.

To assess the prevalence of aggregate demand shocks, we therefore measure the correlation between the cyclical components of product market tightness and output. In Figure 10(a) the cyclical components of product market tightness and output appear positively correlated. Particularly, large drops in product market tightness followed the output drops of 1981–1982, 2001, and 2008–2009, suggesting that these recessions were caused by a negative aggregate demand shock. There are some exceptions, however. From 2004 to 2006, output was increasing while product market tightness was falling. This observation suggests a positive technology shock in the 2004–2006 period. Figure 10(b) reports the cross-correlogram of product market tightness and output: product market tightness leads output by one lag; at one lag, the correlation is quite large, 0.59; the contemporaneous correlation is 0.49; and all the correlations are statistically different from 0.

In the context of our model, these positive correlations imply that it is aggregate demand shocks and not technology shocks that are the main source of labor market fluctuations. Even though it is obtained very differently, our result echoes the finding by Galí [1999] and Basu, Fernald and Kimball [2006] that technology shocks are not the main source of business-cycle fluctuations.

6 Conclusion

We use a simple model and direct evidence to explore the sources of unemployment fluctuations in the US. The model makes predictions about the comovements between tightnesses and quantities for a broad set of shocks that could potentially explain these fluctuations. We compare these predictions with the comovements between product market tightness, labor market tightness, output, and employment observed in US data. The comparison suggests that unemployment fluctuations are driven by aggregate demand shocks that shift the labor demand in the presence of price and real wage rigidities. The comparison also suggests that technology, labor supply, and mismatch shocks are not an important source of unemployment fluctuations.

In the model, aggregate demand arises from a choice between consumption and holding money in a static environment. Usually, we think that aggregate demand arises from a choice between consumption, holding money, and saving with interest-bearing assets in a dynamic environment. In Michailat and Saez [2014], we extend the model in that direction and show that the properties of the aggregate demand and equilibrium are robust. We also use the extended model to study

the role and limits of conventional and unconventional monetary and fiscal policies in stabilizing unemployment fluctuations.

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Online Appendix A: Long Proofs

Proof of Proposition 9. In a fixprice equilibrium parameterized by $p_0 > 0$ and $w_0 > 0$, (x, θ) satisfies $n^s(\theta) = n^d(\theta, x, w_0)$ and $c^s(x, \theta) = c^d(x, p_0)$. We look for equilibria with positive consumption. These equilibria necessarily have $\theta \in (0, \theta^m)$ and $x \in (0, x^m)$.

Equation $n^d(\theta, x, w_0) = n^s(\theta)$ is equivalent to $[n^d(\theta, x, w_0)]^{1-\alpha} = [n^s(\theta)]^{1-\alpha}$. Following the logic of the proof of Proposition 3, we can show that the latter equation is equivalent to

$$F(\theta, x, a, h, w) \equiv f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^\alpha \cdot h^{1-\alpha} \cdot \frac{w}{\alpha \cdot a} = 0. \quad (\text{A1})$$

Since $\alpha < 1$, the function $\theta \mapsto \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^\alpha$ is strictly increasing from 0 to $+\infty$ on $[0, \theta^m)$. Hence, (A1) implicitly defines θ as a function Θ^F of $x \in [0, +\infty)$. Since f is strictly increasing from 0 to 1 on $[0, +\infty)$, Θ^F is strictly increasing on $[0, +\infty)$, $\Theta^F(0) = 0$, and $\lim_{x \rightarrow +\infty} \Theta^F(x) = \theta^F$ where $\theta^F \in (0, \theta^m)$ is defined by $\hat{f}(\theta^F)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta^F))^\alpha \cdot h^{1-\alpha} \cdot w / (\alpha \cdot a) = 1$.

The proof of Proposition 3 shows that the equation $c^s(x, \theta) = c^d(x, p_0)$, combined with $k = a \cdot n^\alpha$ is equivalent to

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1 + \hat{\tau}(\theta)} \cdot h \right)^\alpha \cdot (1 + \tau(x))^{\varepsilon-1} = \chi^\varepsilon \cdot \frac{\mu}{p}.$$

Using (A1), we transform this equation to

$$G(\theta, x, h, \chi, \mu, w, p) \equiv \hat{f}(\theta) \cdot (1 + \tau(x))^{\varepsilon-1} - \frac{\alpha}{w \cdot h} \cdot \chi^\varepsilon \cdot \frac{\mu}{p} = 0. \quad (\text{A2})$$

If $\alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h) \geq 1$, we define $x^G(p, w)$ by $(1 + \tau(x^G))^{\varepsilon-1} = \alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h)$. If $\alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h) < 1$, we set $x^G(p, w) = 0$. Since $\varepsilon > 1$, the function $x \mapsto (1 + \tau(x))^{\varepsilon-1}$ is strictly increasing from 1 to $+\infty$ on $[0, x^m)$; therefore, x^G is well defined and $x^G(p, w) \in (0, x^m)$. Since \hat{f} is strictly increasing from 0 to 1 on $(0, +\infty)$, (A2) implicitly defines θ as a function Θ^G of $x \in (x^G(p, w), x^m)$. Moreover, Θ^G is strictly decreasing on $(x^G(p, w), x^m)$, $\lim_{x \rightarrow x^G(p, w)} \Theta^G(x) = +\infty$, and $\lim_{x \rightarrow x^m} \Theta^G(x) = 0$.

The system of (A1) and (A2) is equivalent to the system of $\Theta^F(x) = \Theta^G(x)$ and $\theta = \Theta^F(x)$. The properties of Θ^F and Θ^G imply that this system admits a unique solution (x, θ) with $x \in (x^G(p, w), x^m)$ and $\theta \in (0, \theta^F)$.

Proof of Proposition 10.

Aggregate Demand Shocks. We parameterize an increase in aggregate demand by an increase in χ or μ . The function F in (A1) satisfies $\partial F/\partial \theta < 0$, $\partial F/\partial x > 0$, $\partial F/\partial a > 0$, and $\partial F/\partial h < 0$. Using the implicit function theorem, we write the solution θ to $F(\theta, x, a, h, w) = 0$ as a function $\Theta^F(x, a, h)$ with $\partial \Theta^F/\partial x > 0$, $\partial \Theta^F/\partial a > 0$, and $\partial \Theta^F/\partial h < 0$.

The function G in (A2) satisfies $\partial G/\partial \theta > 0$, $\partial G/\partial x > 0$, $\partial G/\partial h > 0$, $\partial G/\partial \chi < 0$, and $\partial G/\partial \mu < 0$. Using the implicit function theorem, we write the solution θ to $G(\theta, x, h, \chi, \mu, w, p) = 0$ as a function $\Theta^G(x, h, \chi, \mu)$ with $\partial \Theta^G/\partial x < 0$, $\partial \Theta^G/\partial h < 0$, $\partial \Theta^G/\partial \chi > 0$, and $\partial \Theta^G/\partial \mu > 0$.

In equilibrium, x satisfies $G(\Theta^F(x, a, h), x, h, \chi, \mu) = 0$. Given that $\partial\Theta^F/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial\chi < 0$, the implicit function theorem implies that $\partial x/\partial\chi > 0$. We can show similarly that $\partial x/\partial\mu > 0$. Since $\theta = \Theta^F(x, a, h)$ with $\partial\Theta^F/\partial x > 0$, we also have $\partial\theta/\partial\chi > 0$ and $\partial\theta/\partial\mu > 0$. Equation (8) yields $y = \hat{f}(\theta) \cdot h \cdot w/\alpha$; therefore, the comparative statics of θ imply that $\partial y/\partial\chi > 0$ and $\partial y/\partial\mu > 0$. Since $l = \hat{f}(\theta) \cdot h$, the comparative statics of θ also imply that $\partial l/\partial\chi > 0$ and $\partial l/\partial\mu > 0$.

Technology Shocks. We parameterize an increase in technology by an increase in a . In equilibrium, x satisfies $F(\Theta^G(x, h, \chi, \mu), x, a, h) = 0$. Given that $\partial\Theta^G/\partial x < 0$, $\partial F/\partial\theta < 0$, $\partial F/\partial x > 0$, and $\partial F/\partial a > 0$, the implicit function theorem implies that $\partial x/\partial a < 0$. Since $\theta = \Theta^G(x, h, \chi, \mu)$ with $\partial\Theta^G/\partial x < 0$, we obtain $\partial\theta/\partial a > 0$. The logic presented for aggregate demand shocks implies that since $\partial\theta/\partial a > 0$, then $\partial y/\partial a > 0$ and $\partial l/\partial a > 0$.

Labor Supply Shocks. We parameterize an increase in labor supply by an increase in h . The functions F and G both depend on h , so it is impossible to obtain comparative statics for x and θ from them. To obtain the comparative statics, we manipulate and combine (A1) and (A2), and we obtain

$$H(\theta, x) \equiv (1 + \hat{\tau}(\theta))^\alpha - f(x) \cdot (1 + \tau(x))^{(1-\alpha) \cdot (\varepsilon-1)} \cdot a \cdot \left(\frac{\alpha}{w}\right)^\alpha \cdot \left(\chi^\varepsilon \cdot \frac{\mu}{p}\right)^{\alpha-1} = 0. \quad (\text{A3})$$

The function H satisfies $\partial H/\partial\theta > 0$ and $\partial H/\partial x < 0$. The function H does not depend on h , which resolves the earlier problem. Using the implicit function theorem, we write the solution θ to $H(\theta, x) = 0$ as a function $\Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$.

In equilibrium, x satisfies $G(\Theta^H(x), x, h) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial h > 0$, the implicit function theorem implies that $\partial x/\partial h < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we obtain $\partial\theta/\partial h < 0$. We find that $\partial y/\partial h > 0$ because $y = (1 + \tau(x)) \cdot c^d(x, p) = (1 + \tau(x))^{1-\varepsilon} \cdot \chi^\varepsilon \cdot \mu/p$ and $1 - \varepsilon < 0$ and $\partial x/\partial h < 0$. We also find that $\partial l/\partial h > 0$ because $l = \alpha \cdot y/w$ (from equation (8)) and $\partial y/\partial h > 0$.

Mismatch Shocks. We parameterize an increase in mismatch by a decrease in matching efficacy on the labor market along with a corresponding decrease in recruiting cost: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$ with $\lambda < 1$. Consequently, the function $\hat{\tau}$ remains the same. With the parameter λ for mismatch, the functions H and Θ^H are the same, but the function G depends on λ with $\partial G/\partial\lambda > 0$. In equilibrium, x satisfies $G(\Theta^H(x), x, \lambda) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial\lambda > 0$, the implicit function theorem implies that $\partial x/\partial\lambda < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we have $\partial\theta/\partial\lambda < 0$. The logic presented for labor supply shocks implies that since $\partial x/\partial\lambda < 0$, then $\partial y/\partial\lambda > 0$ and $\partial l/\partial\lambda > 0$.

Comparative Statics with Partially Rigid Price and Real Wage. Using the expressions of the partially rigid price and real wage, we rewrite (A1), (A2), and (A3) as

$$\begin{aligned} f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^\alpha \cdot \frac{w_0 \cdot h^{(1-\alpha) \cdot (1-\xi)}}{(\alpha \cdot a)^{1-\xi}} &= 0 \\ \hat{f}(\theta) \cdot (1 + \tau(x))^{\varepsilon-1} - \frac{\alpha^{1-\xi}}{w_0 \cdot h^{1-\xi}} \cdot \frac{(\chi^\varepsilon \cdot \mu)^{1-\xi}}{p_0} &= 0 \\ (1 + \hat{\tau}(\theta))^\alpha - f(x) \cdot (1 + \tau(x))^{(1-\alpha) \cdot (\varepsilon-1)} \cdot a^{1-\xi} \cdot \frac{\alpha^{\alpha(1-\xi)}}{w_0^\alpha} \cdot \left[\frac{(\chi^\varepsilon \cdot \mu)^{1-\xi}}{p_0} \right]^{\alpha-1} &= 0. \end{aligned}$$

The implicit functions defined by these equations have exactly the same properties as the functions F , G , and H . Hence, the comparative statics for x and θ are the same in the fixprice equilibrium as in the equilibrium with partially rigid prices. Finally, the arguments used above for the fixprice equilibrium imply that the comparative statics for y and l are the same in the fixprice equilibrium as in the equilibrium with partially rigid prices.

Proof of Proposition 11. In a competitive equilibrium, the pair (p^*, w^*) satisfies $n^s(\theta^*) = n^d(\theta^*, x^*, w^*)$ and $c^s(x^*, \theta^*) = c^d(x^*, p^*)$. Following the same steps as in the proof of Proposition 9, we can show that (p^*, w^*) satisfies

$$\begin{aligned} w^* &= f(x^*) \cdot \hat{f}(\theta^*)^{\alpha-1} \cdot (1 + \hat{\tau}(\theta^*))^{-\alpha} \cdot h^{\alpha-1} \cdot \alpha \cdot a \\ p^* &= \frac{(1 + \tau(x^*))^{1-\varepsilon}}{\hat{f}(\theta^*)} \cdot \frac{\alpha}{w^* \cdot h} \cdot \chi^\varepsilon \cdot \mu. \end{aligned}$$

This system admits a unique solution. Thus, there exists a unique competitive equilibrium. The wage w^* clearly satisfies the expression in the proposition. Combining these two equations, we find that the price p^* satisfies the expression in the proposition.

Online Appendix B: The Four Inefficient Regimes

This appendix establishes the boundaries in a (w, p) plane of the four inefficient regimes of the model of Section 3. These boundaries are depicted in Figure 7.

PROPOSITION A1. *There exists a function $w \mapsto p^x(w)$ such that for any $w > 0$, the product market is slack if $p > p^x(w)$ and tight if $p < p^x(w)$. There exists a function $w \mapsto p^\theta(w)$ such that for any $w > 0$, the labor market is slack if $p > p^\theta(w)$ and tight if $p < p^\theta(w)$. The function p^x is strictly decreasing for $w \in (0, w^*]$, strictly increasing for $w \in [w^*, +\infty)$, $\lim_{w \rightarrow 0} p^x(w) = +\infty$, and $\lim_{w \rightarrow +\infty} p^x(w) = +\infty$. The function p^θ is strictly decreasing for $w \in (0, w^F]$, $\lim_{w \rightarrow 0} p^\theta(w) = +\infty$, and $p^\theta(w) = 0$ for $w \in [w^F, +\infty)$, where $w^F \in (w^*, +\infty)$. Furthermore, $p^x(w^*) = p^\theta(w^*) = p^*$.*

Proof. We build on the proof of Proposition 9. We define the function $\Theta^F : [0, +\infty) \times (0, +\infty) \rightarrow (0, \theta^m)$ such that $F(\Theta^F(x, w), x, a, h, w) = 0$. The function Θ^F is strictly increasing in x and strictly decreasing in w . We define the function $\Theta^G : \{(x, w, p) | p > 0, w > 0, x^G(p, w) < x < x^m\} \rightarrow$

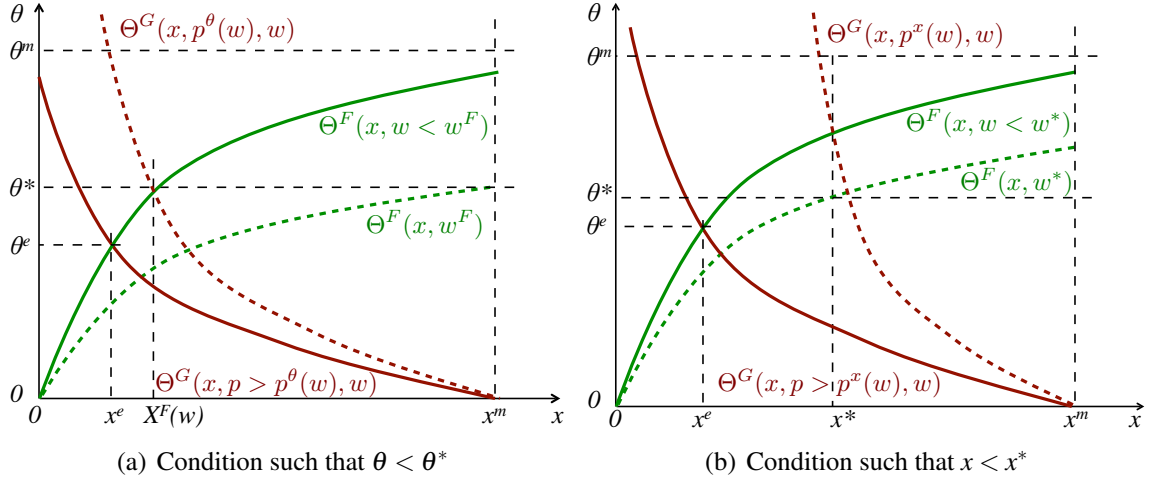


Figure A1: Illustration of the proof of Proposition A1

$(0, +\infty)$ such that $G(\Theta^G(x, w, p), x, h, \chi, \mu, w, p) = 0$. The function Θ^G is strictly decreasing in x , p , and w . The proof is illustrated in Figure A1.

First part: condition such that $\theta < \theta^*$. Let w^F be defined by $\Theta^F(x^m, w^F) = \theta^*$. For all $w > w^F$ and for all $x \in [0, x^m]$, $\Theta^F(x, w) < \theta^*$. For all $w \leq w^F$, there exists a unique $x \in [0, x^m]$ such that $\Theta^F(x, w) = \theta^*$. We define the function $X^F : (0, w^F] \rightarrow [0, x^m]$ by $\Theta^F(X^F(w), w) = \theta^*$. The function X^F is strictly increasing, $\lim_{w \rightarrow 0} X^F(w) = 0$, $X^F(w^*) = x^*$, and $X^F(w^F) = x^m$.

Next, for all $w \leq w^F$, there exists a unique $p \in (0, +\infty)$ such that $\Theta^G(X^F(w), p, w) = \theta^*$. We define the function $p^\theta : (0, w^F] \rightarrow (0, +\infty)$ by $\Theta^G(X^F(w), p^\theta(w), w) = \theta^*$. The function p^θ is strictly decreasing, $\lim_{w \rightarrow 0} p^\theta(w) = +\infty$, $p^\theta(w^*) = p^*$, and $p^\theta(w^F) = 0$. We extend the definition of p^θ by setting $p^\theta(w) = 0$ for all $w \in (w^F, +\infty)$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e . For any $w > w^F$, $\theta^e = \Theta^F(x^e, w) < \theta^*$ by definition of w^F . Consider $w \leq w^F$ and $p > p^\theta(w)$. Then $\Theta^G(X^F(w), p, w) < \Theta^G(X^F(w), p^\theta(w), w) = \theta^* = \Theta^F(X^F(w), w)$ because Θ^G is strictly decreasing in p . Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < X^F(w)$. Thus, $\theta^e = \Theta^F(x^e, w) < \Theta^F(X^F(w), w) = \theta^*$ because Θ^F is strictly increasing in x . In sum, for any $w > 0$ and $p > p^\theta(w)$, we have $\theta^e < \theta^*$. Following the same logic, we find that for any $w > 0$ and $p < p^\theta(w)$, we have $\theta^e > \theta^*$.

Second part: condition such that $x < x^*$. We define the function $p^x : (0, +\infty) \rightarrow (0, +\infty)$ by

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{h \cdot \hat{f}(\Theta^F(x^*, w))} \cdot \chi^\varepsilon \cdot \frac{\alpha}{w} \cdot \mu.$$

The function p^x has the property that $\Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$. Hence, $p^x(w^*) = p^*$.

Next, we define the auxiliary function $Z : (0, +\infty) \rightarrow (0, +\infty)$ by

$$Z(w) = f(x^*) \cdot a \cdot \alpha \cdot n^s(\Theta^F(x^*, w)).$$

Given that $\Theta^F(x^*, w^*) = \theta^*$ and Θ^F is strictly decreasing in w , $\Theta^F(x^*, w) \in [\theta^*, \theta^m]$ if $w \in (0, w^*]$ and $\Theta^F(x^*, w) \in (0, \theta^*]$ if $w \in [w^*, +\infty)$. Since n^s is strictly increasing on $[0, \theta^*]$ and strictly decreasing on $[\theta^*, \theta^m]$ and Θ^F is strictly decreasing in w , we infer that Z is strictly increasing for $w \in (0, w^*]$ and strictly decreasing for $w \in [w^*, +\infty)$. Since $n^s(0) = n^s(\theta^m) = 0$, $\lim_{w \rightarrow 0} \Theta^F(x^*, w) = \theta^m$, and $\lim_{w \rightarrow +\infty} \Theta^F(x^*, w) = 0$, we infer that $\lim_{w \rightarrow 0} Z(w) = 0$ and $\lim_{w \rightarrow +\infty} Z(w) = 0$.

The definition of Θ^F implies that $Z(w) = h \cdot w \cdot \hat{f}(\Theta^F(x^*, w))$. Thus,

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{Z(w)} \cdot \chi^\varepsilon \cdot \alpha \cdot \mu.$$

The properties of Z imply that the function p^x is strictly decreasing for $w \in (0, w^*]$ and strictly increasing for $w \in [w^*, +\infty)$, $\lim_{w \rightarrow 0} p^x(w) = +\infty$, and $\lim_{w \rightarrow +\infty} p^x(w) = +\infty$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e . Consider $w \in (0, +\infty)$ and $p > p^x(w)$. Then $\Theta^G(x^*, p, w) < \Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$ because Θ^G is strictly decreasing in p . Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < x^*$. In sum, for any $w > 0$ and $p > p^x(w)$, we have $x^e < x^*$. Similarly, for any $w > 0$ and $p < p^x(w)$, we have $x^e > x^*$. \square

In Figure 7, the function p^θ is represented by the downward-sloping red line and the function p^x is represented by the u-shaped blue line. The two curves intersect at (w^*, p^*) .

Online Appendix C: Optimal Control Problems

This appendix solves the optimal control problems of the household and firm in the dynamic model of Section 4.

The Optimal Control Problem of the Household. Let $b(t) \equiv m(t)/p(t)$ denote real money balances. The law of motion of $b(t)$ is obtained from (13):

$$\dot{b}(t) = w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)}$$

To solve the problem, we set up the current-value Hamiltonian:

$$\begin{aligned} \mathcal{H}(t, c(t), y(t), b(t)) = & \frac{\chi}{1 + \chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1 + \chi} \cdot b(t)^{\frac{\varepsilon-1}{\varepsilon}} + Y(t) \cdot \left[\frac{q(x(t))}{\rho} \cdot (y(t) - c(t)) - s \cdot y(t) \right] \\ & + B(t) \cdot \left[w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)} \right] \end{aligned}$$

with control variable $c(t)$, state variables $y(t)$ and $b(t)$, and costate variables $Y(t)$ and $B(t)$.

The necessary conditions for an interior solution to this maximization problem are $\partial \mathcal{H} / \partial c(t) = 0$, $\partial \mathcal{H} / \partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial \mathcal{H} / \partial b(t) = \delta \cdot B(t) - \dot{b}(t)$, together with the transversality conditions $\lim_{t \rightarrow +\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \rightarrow +\infty} e^{-\delta \cdot t} \cdot B(t) \cdot b(t) = 0$. Given that \mathcal{H} is concave in (c, y, b) , these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned} \frac{\chi}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot c(t)^{\frac{-1}{\varepsilon}} &= Y(t) \cdot \frac{q(x(t))}{\rho} \\ Y(t) \cdot \left(\frac{q(x(t))}{\rho} - s \right) - B(t) &= \delta \cdot Y(t) - \dot{y}(t) \\ \frac{1}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot b(t)^{\frac{-1}{\varepsilon}} - B(t) \cdot \pi &= \delta \cdot B(t) - \dot{b}(t) \end{aligned}$$

In steady state, $\dot{y}(t) = \dot{b}(t) = 0$. Hence, after eliminating the costate variables B and Y , we find that the optimal consumption decision of the household is

$$c = \chi^\varepsilon \cdot (\delta + \pi)^\varepsilon \cdot \left[1 - (\delta + s) \cdot \frac{\rho}{q(x)} \right]^\varepsilon \cdot b.$$

We obtain the equation in the text by setting $\delta = 0$ and $b = \mu(0)/p(0)$ in the above equation.

The Optimal Control Problem of the Firm. To solve this problem, we set up the current-value Hamiltonian:

$$\begin{aligned} \mathcal{H}(t, n(t), y(t), l(t)) &= y(t) - w(t) \cdot l(t) + Y(t) \cdot [f(x(t)) \cdot (a \cdot n(t)^\alpha - y(t)) - s \cdot y(t)] \\ &\quad + L(t) \cdot \left[\frac{\hat{q}(\theta(t))}{\hat{\rho}} \cdot (l(t) - n(t)) - \hat{s} \cdot l(t) \right] \end{aligned}$$

with control variable $n(t)$, state variables $y(t)$ and $l(t)$, and current-value costate variables $Y(t)$ and $L(t)$. The necessary conditions for an interior solution to this maximization problem are $\partial \mathcal{H} / \partial n(t) = 0$, $\partial \mathcal{H} / \partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial \mathcal{H} / \partial l(t) = \delta \cdot L(t) - \dot{l}(t)$, together with the transversality conditions $\lim_{t \rightarrow +\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \rightarrow +\infty} e^{-\delta \cdot t} \cdot L(t) \cdot l(t) = 0$. Given that \mathcal{H} is concave in (n, y, l) , these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned} Y(t) \cdot f(x(t)) \cdot \alpha \cdot a \cdot n(t)^{\alpha-1} &= L(t) \cdot \frac{\hat{q}(\theta(t))}{\hat{\rho}} \\ 1 &= Y(t) \cdot (f(x(t)) + s) + \delta \cdot Y(t) - \dot{y}(t) \\ L(t) \cdot \left(\frac{\hat{q}(\theta(t))}{\hat{\rho}} - \hat{s} \right) &= w(t) + \delta \cdot L(t) - \dot{l}(t) \end{aligned}$$

In steady state, $\dot{l}(t) = \dot{y}(t) = 0$. Hence, after eliminating the costate variables L and Y , we find

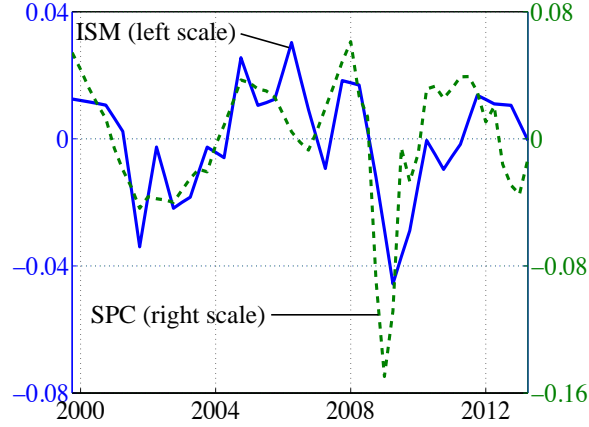


Figure A2: The Two Proxies for the Cyclical Component of Product Market Tightness

Notes: The time period is 1999:Q4–2013:Q2. The ISM proxy is constructed in Online Appendix D from the operating rate in non-manufacturing sectors constructed by the ISM. The SPC proxy is constructed in Section 5 from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the SPC.

that the optimal employment decision of the firm satisfies

$$n = \left\{ \frac{\alpha \cdot a}{w} \cdot \frac{f(x)}{\delta + s + f(x)} \cdot \left[1 - (\delta + \hat{s}) \cdot \frac{\hat{\rho}}{\hat{q}(\theta)} \right] \right\}^{\frac{1}{1-\alpha}}.$$

We obtain equation in the text by setting $\delta = 0$ in the above equation.

Online Appendix D: Another Proxy for Product Market Tightness

This appendix proposes another proxy for the cyclical component of product market tightness, and it shows that all the empirical results are robust to using this alternative proxy. The proxy is constructed from the operating rate in non-manufacturing sectors measured by the Institute for Supply Management (ISM) and published in their Semiannual Reports. This operating rate is available for the 1999:Q4–2013:Q2 period. In the text, the proxy is constructed from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the Survey of Plant Capacity (SPC).

Using the operating rate from the ISM is conceptually better than using the capacity utilization rate from the SPC. First, the operating rate is a direct measure of labor utilization; therefore, it is directly linked to product market tightness. Second, the operating rate applies to non-manufacturing sectors, where logistical issues such as peak load and inventory management do not influence labor utilization. We do not use this alternative proxy in the text, however, because it is only available for a brief period (1999:Q4–2013:Q2) that does not cover sufficiently many business cycles to permit a thorough empirical analysis.

We construct our alternative proxy for the cyclical component of product market tightness as follows. The operating rate or_t measured by the ISM is the actual production level of firms as a

share of their maximum production level given their current capital stock and workforce. Since the operating rate takes labor as a fixed factor, it exactly corresponds to our concept of labor utilization: $or_t = f(x_t)/(s + f(x_t))$.³⁵ The ISM measures or_t in the second and fourth quarter; we use a linear interpolation of the biannual series to transform it into a quarterly series for the 1999:Q4–2013:Q2 period. Then, we remove from $\ln(or_t)$ the trend produced by a HP filter with smoothing parameter 1600. The resulting detrended series is our proxy for the cyclical component of product market tightness. This proxy is plotted in Figure A2, together with the proxy used in the text. We refer to the proxy in the text as the *SPC proxy* and to this alternative proxy as the *ISM proxy*.

Over the 1999:Q4–2013:Q2 period, the correlation between the two proxies is 0.67. As showed in Figure A2, the two proxies behaved similarly over the period: both fell after 2001, picked up in the 2004–2008 period, and collapsed in 2009, before recovering. The main difference is that the SPC proxy is subject to larger fluctuations than the ISM proxy.

In the text, we use the SPC proxy to identify aggregate demand and technology shocks. The finding reported in Figure 10 is that the SPC proxy and output are positively correlated, which implies that aggregate demand shocks are the main source of labor market fluctuations. Figures 3(a) and 3(b) confirm that this result remains valid if we focus on the correlation between SPC proxy and output over the subperiod 1999:Q4–2013:Q2. The correlations between SPC proxy and output are slightly higher: at one lag, the correlation is 0.68; the contemporaneous correlation is 0.60. These correlations are statistically significant.

Figures 3(c) and 3(d) shows that we obtain the same result by using the ISM proxy instead of the SPC proxy. The correlations between ISM proxy and output are even slightly higher than those of SPC proxy and output: the contemporaneous correlation is 0.82; at one lag, the correlation is 0.77. These correlations are statistically significant.

Online Appendix E: Another Type of Matching Cost

This appendix proposes an alternative to the basic model of Section 2 in which the cost of matching is a time cost instead of an output cost. In this alternative model, households share their time between supplying services and matching with other households who sell services. In the original model, households spend all their time supplying services and they purchase services to match with other households. Yet, all the results of Section 2 remain valid in this alternative model.

Households employ their time to purchase services: the more time they spend on purchasing services, the less time they have to supply services. A visit takes away an amount $\rho > 0$ of the household's productive capacity; therefore, the actual productive capacity of a household making v visits is $k - \rho \cdot v$. The number of matches on the product market is given by

$$y = [(k - \rho \cdot v)^{-\gamma} + v^{-\gamma}]^{-\frac{1}{\gamma}},$$

³⁵This is a major difference with the capacity utilization rate from the SPC, which takes labor as a variable factor and thus requires a correction to be converted into a labor utilization rate. [Morin and Stevens \[2005\]](#) discuss the difference between the capacity utilization rate collected in the SPC and the operating rate collected in the ISM survey.

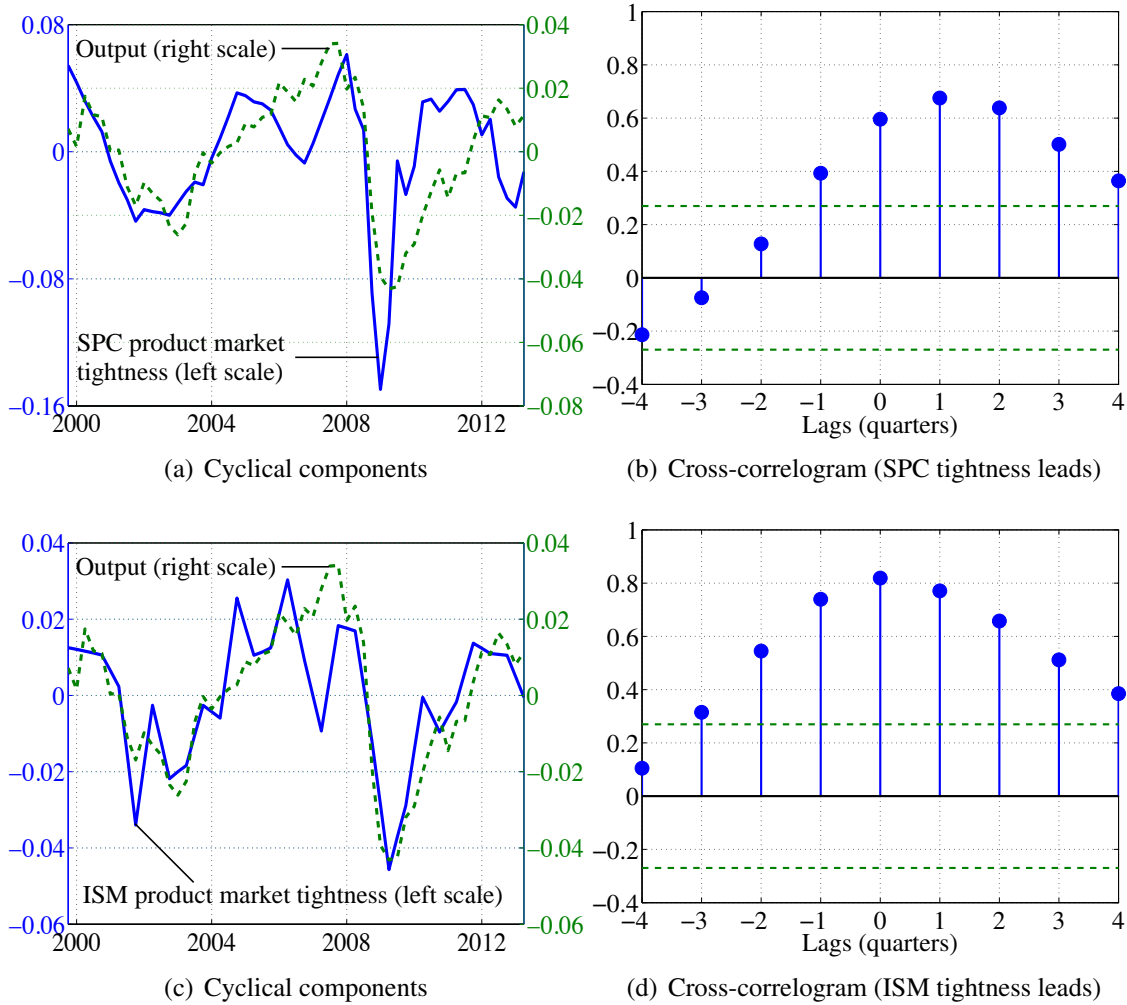


Figure A3: Correlation Between Product Market Tightness and Output

Notes: The time period is 1999:Q4–2013:Q2. Panel (a) displays the SPC proxy for the cyclical component of product market tightness, x_t , and the cyclical component of output, y_t . The SPC proxy is constructed in Section 5 from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the SPC. Output is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. The cyclical component of y_t is constructed by removing the trend produced by a HP filter with smoothing parameter 1600 from $\ln(y_t)$. Panel (b) displays the cross-correlogram between the SPC proxy and the cyclical component of y_t . The cross-correlation at lag i is the correlation between x_{t-i} and y_t . Panel (c) displays the ISM proxy for the cyclical component of product market tightness and the cyclical component of output. The ISM proxy is constructed in Online Appendix D from the operating rate in non-manufacturing sectors constructed by the ISM. Panel (d) displays the cross-correlogram between the ISM proxy and the cyclical component of y_t . The cross-correlation at lag i is the correlation between x_{t-i} and y_t . The green dashed lines are the two-standard-deviation confidence bounds.

and the product market tightness is defined by

$$x = \frac{v}{k - \rho \cdot v}.$$

The probability to sell one of the $k - \rho \cdot v$ services for sale is $f(x)$. The probability that a visit is successful is $q(x)$.

To purchase c services, household need to make $c/q(x)$ visits that take away an amount $\rho \cdot c/q(x)$ of their productive capacity. Households are left with a capacity $k - \rho \cdot c/q(x)$, and they sell a fraction $f(x)$ of it. Furthermore, output is equal to consumption—and welfare—because no part of output is used for matching. Therefore, $c = y = f(x) \cdot (k - \rho \cdot c/q(x))$. The aggregate supply is the amount of consumption that solves this equation. Since $f(x)/q(x) = x$, the aggregate supply admits the following expression:

$$c^s(x) = \frac{f(x)}{1 + \rho \cdot x} \cdot k.$$

The aggregate supply is strictly increasing for $x \in [0, x^*]$ and strictly decreasing for $x \in [x^*, +\infty)$ where x^* is the unique solution to

$$q(x)^\gamma = \frac{\rho \cdot x}{1 + \rho \cdot x}.$$

This equation is obtained by rearranging $dc^s/dx = 0$ and using the fact that $f'(x) = q(x)^{1+\gamma}$. It admits a unique solution because q is strictly decreasing from 1 to 0 on $[0, +\infty)$ while $x \mapsto (\rho \cdot x)/(1 + \rho \cdot x)$ is strictly increasing on 0 to 1 on $[0, +\infty)$. The tightness x^* is the efficient tightness: it maximizes welfare for a given level of real money balances. As in the original model, x^* depends only on the matching function and on the matching cost.

Since all output is used for consumption, there is no price wedge due to matching, and the price of consumption is p . However, the selling capacity of the household, and thus its income, is reduced because of the time spent buying consumption. The household's budget constraint is therefore modified to

$$m + p \cdot c = \mu + p \cdot f(x) \cdot \left(k - \rho \cdot \frac{c}{q(x)} \right).$$

To see the parallel between this budget constraint and the budget constraint in Section 2, it is convenient to rewrite this constraint as

$$m + p \cdot (1 + \rho \cdot x) \cdot c = \mu + p \cdot f(x) \cdot k.$$

From the household's perspective, the time required to buy consumption imposes a wedge $\rho \cdot x$ on the price of consumption. Accordingly, the optimal level of consumption for the household is the same as in the model of Section 2 after replacing the old price wedge, $\tau(x)$, by the new price

wedge, $\rho \cdot x$. The aggregate demand therefore becomes

$$c^d(x, p) = \left(\frac{\chi}{1 + \rho \cdot x} \right)^\varepsilon \cdot \frac{\mu}{p}.$$

The aggregate demand is strictly decreasing in x and p for all $x > 0$ and $p > 0$.

Given that the aggregate supply and demand are isomorphic to those in the original model once $\tau(x)$ is replaced by $\rho \cdot x$, we can analyze this alternative model by following the same steps. We can show that all the properties of the original model carry over, with one exception. In a fixprice equilibrium, the comparative statics of output and consumption are the same in the alternative model, and they are the same as the comparative statics of consumption in the original model. But they are different from the comparative statics of output in the original model. In the slack regime this difference is mute because consumption and output move together in the original model. But this difference is visible in the tight regime where, after an increase in aggregate demand, output increases in the original model but decreases in the alternative model.

We think that the original model of Section 2 is more realistic than this alternative model because the result that output is higher when the economy becomes tighter seems more realistic, at least for Western economies. The main difference between the original and the alternative model is that the resources devoted to matching are marketed in the original model but not in the alternative model. The alternative model perhaps describes better the centralized economies of the Soviet Union where fewer services were marketed. It is possible that in those economies, people spent so much time queuing to buy goods and services that they had to reduce their supply of labor, and output was lower than if aggregate demand had been lower.

Online Appendix F: Endogenous Marketing Effort

This appendix proposes an extension of the basic model of Section 2 in which households devote marketing effort to increase their sales. In this alternative model, households share their productive capacity between supplying services and marketing these services. In the original model, households spend all their productive capacity on supplying services. All the results of Section 2 remain valid in this extension.

Households spend an amount $a \leq k$ of their productive capacity on marketing. The amount of productive capacity left for supplying services is $k - a$. Marketing increases the visibility of services for sale and thus their probability of being sold. The function $e : [0, k] \rightarrow [0, 1]$ describes the effectiveness of marketing. We assume that e is strictly increasing and concave. To ensure an interior solution with positive marketing effort, we assume $e(0) = 0$. To simplify, we assume that the function e has a constant elasticity ε . As in Pissarides [2000, Chapter 5], the number of matches on the product market is given by

$$y = \{ [e(a) \cdot (k - a)]^{-\gamma} + v^{-\gamma} \}^{-\frac{1}{\gamma}},$$

and the product market tightness is defined by

$$x = \frac{v}{e(a) \cdot (k - a)}.$$

The probability to sell one service is $e(a) \cdot f(x)$. Hence, a higher amount of marketing generates more sales. The probability that a visit is successful is $q(x)$.

Households choose their marketing effort to maximize their income. Given x , they choose a to maximize $e(a) \cdot f(x) \cdot (k - a)$. The optimal a satisfies $\varepsilon = a/(k - a)$, which can be rewritten as

$$a = \frac{\varepsilon}{1 + \varepsilon} \cdot k.$$

It is optimal for households to devote a fraction $\varepsilon/(1 + \varepsilon)$ of their productive capacity to marketing.

The aggregate supply describes the amount of consumption sold given the matching process and the optimal marketing decision of households. The aggregate supply admits the following expression:

$$c^s(x) = (f(x) - \rho \cdot x) \cdot e\left(\frac{\varepsilon}{1 + \varepsilon} \cdot k\right) \cdot \frac{1}{1 + \varepsilon} \cdot k.$$

Although the aggregate supply admits a different expression from in the model without marketing, it keeps the same properties. Furthermore, the aggregate demand remains the same because the trade-off between consumption and holding money is not affected by the marketing effort.

We can analyze this extension with endogenous marketing effort following the same steps as in the original model. Since the aggregate demand and supply retain the same properties as in the original model, we can show that all the properties of the original model carry over.