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GOLD RETURNS

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ABSTRACT

From 1836 to 2011, the average real rate of price change for gold in the United States is 1.1% per year and the standard deviation is 13.1%, implying a one-standard-deviation confidence band for the mean of (0.1%, 2.1%). The covariances of gold's real rate of price change with consumption and GDP growth rates are small and statistically insignificantly different from zero. These negligible covariances suggest that gold's expected real rate of return—which includes an unobserved dividend yield—would be close to the risk-free rate, estimated to be around 1%. We study these properties within an asset-pricing model in which ordinary consumption and gold services are imperfect substitutes for the representative household. Disaster and other shocks impinge directly on consumption and GDP but not on stocks of gold. With a high elasticity of substitution between gold services and ordinary consumption, the model can generate a mean real rate of price change within the (0.1%, 2.1%) confidence band along with a small risk premium for gold. In this scenario, the bulk of gold's expected return corresponds to the unobserved dividend yield (the implicit rental income from holding gold) and only a small part comprises expected real price appreciation. Nevertheless, the uncertainty in gold returns is concentrated in the price-change component. The model can explain the time-varying volatility of real gold prices if preference shocks for gold services are small under the classical gold standard but large in other periods particularly because of shifting monetary roles for gold.

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Gold has dominated monetary systems for centuries, and it plays a prominent role in transactions among financial institutions even in modern systems that rely on fiat money. Private holdings of gold are also important, facilitated in recent years by the availability of liquid futures contracts on commodity exchanges. Gold is often viewed as a hedge against disaster scenarios, although the risk premium associated with gold is not well understood.

The present analysis begins by studying returns on gold in a Lucas-tree model that incorporates rare disasters associated with ordinary consumption. The baseline model is a two-tree version with some reasonable restrictions that deliver tractability: ordinary consumption and gold services are imperfect substitutes in an effective consumption flow, the outlay on gold services is negligible compared to that on ordinary consumption, and disaster and other shocks apply directly to ordinary consumption but not to gold. In this setting, the expected rate of return on gold ranges between the risk-free rate and the expected rate of return on consumption-tree equity if the elasticity of substitution between ordinary consumption and gold services is between infinity and one. Extensions to the model allow for a monetary role for gold and introduce shocks to preferences for ordinary consumption versus gold services. These shocks relate particularly to the shifting monetary role of gold, corresponding historically to movements off or on the gold standard.

A later section relates the model to empirical properties of real returns on gold and other assets in the United States since 1836. From 1836 to 2011, the average real rate of price change for gold is 1.1% per year, the standard deviation is 13.1%, and the covariance with consumption and GDP growth rates is small in magnitude and statistically insignificantly different from zero.

A problem is that the data reveal changes in real gold prices but not the dividend yields that correspond to service flows on gold holdings. We use the model to gauge the consequences of these omissions and find that the measured real rates of price change for gold provide substantial underestimates of expected total real rates of return. Nevertheless, the data on real rates of price change should provide good measures of the uncertainty in real gold returns, including the covariances between these returns and consumption and GDP growth rates.

The baseline model accords with the long-term data on real rates of change of gold prices if the elasticity of substitution between ordinary consumption and gold services is high. Explaining the changing volatility of real gold prices over sub-periods requires that shocks to preferences for gold services be minor under a serious gold standard, notably 1880-1913, but large in other periods, such as 1975-2011.

I. A Baseline Model of Returns on Gold with Rare Disasters

In the baseline model, the underlying demand for gold reflects a service value proportional to the stock of gold. This perspective matches up with gold used as jewelry and crafts or for electronics and medicine. As a short-hand, we refer to this array of functions as "jewelry." Gold also provides monetary services; that is, a transactions and liquid store-of-value benefit of the sort usually considered in analyses of the demand for money. This monetary role of gold is central in the operation of the world gold standard.

An important difference in the two approaches is that jewelry relates to the quantity of gold in physical units, whereas monetary services relate to the quantity of gold expressed in units of value in terms of other goods. That is, the relative price of gold and other goods enters into the monetary service flow and, hence, into household utility.

The initial model takes the view of gold as jewelry, and a later discussion considers differences resulting from the allowance for monetary services. In the initial model, "gold" can be viewed as any durable commodity that provides consumption services to households. In contrast, as stressed by Goldstein and Kestenbaum (2010), the commodities (specifically, the naturally occurring elements) that can readily provide monetary services are limited to a few precious metals, with gold emerging as the most attractive. That is, gold's prominent monetary role is not an historical accident.

The baseline model has the following key assumptions:

- Ordinary consumption and gold services are imperfect substitutes in the effective consumption flow for the representative consumer, with a constant elasticity of substitution, σ.
- The outlay on gold services is always negligible compared to that on ordinary consumption.
- Disaster and other shocks apply directly to ordinary consumption and GDP but not to gold. Specifically, even during wars and depressions, the quantity of gold never falls precipitously.

We assume that the representative household's utility depends on an effective consumption flow, c_t^*, which relates to ordinary consumption, c_t, and the flow of services from the gold stock, g_t, in a CES form:

$$(1) \qquad c_t^* = [\alpha_t \cdot c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha_t) \cdot g_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where we assume $\sigma>0$ and $0<\alpha_t<1$.[1] The variable α_t can be viewed as a preference shock for ordinary consumption compared to gold services. Given this functional form, the rental price, $\pi_t^g > 0$, for gold equals the ratio of the marginal utility of gold services to the marginal utility of ordinary consumption and is given by

[1]The limit of equation (1) as σ approaches one is the Cobb-Douglas form, ${c_t}^{\alpha_t} \cdot {g_t}^{(1-\alpha_t)}$.

(2) $$\pi_t^g = (\frac{1-\alpha_t}{\alpha_t}) \cdot (\frac{c_t}{g_t})^{1/\sigma} .$$

Hence, gold is relatively highly valued when c_t/g_t is high and α_t is low. This rental price determines the dividend flow accruing to holders of gold. We assume initially that α_t equals the constant α, where $0<\alpha<1$. In this case, π_t^g in equation (2) is proportional to c_t/g_t raised to the power $1/\sigma$:[2]

(3) $$\pi_t^g = (\frac{1-\alpha}{\alpha}) \cdot (\frac{c_t}{g_t})^{1/\sigma}.$$

The stochastic process for per capita consumption, c_t, viewed as the fruit from a Lucas tree, takes the same form as in Barro (2006, 2009):

(4) $$\log(c_{t+1}) = \log(c_t) + h + u_{t+1} + v_{t+1},$$

where $h \geq 0$ is exogenous productivity growth and u_{t+1} is an i.i.d. normal shock with mean 0 and variance σ_u^2. The number of trees is fixed, there is no possibility of loss of ownership, and the economy is closed.

The term v_{t+1} in equation (4) is a disaster shock, governed by a constant Poisson arrival probability $p \geq 0$ (expressed per unit of time) and a proportionate disaster size, $b \geq 0$, which is subject to a time-invariant probability distribution. Specifically, the disaster shock v_{t+1} equals $\log(1-b)$, where $b>0$ in a disaster state and $b=0$ in a non-disaster state. The realization of $b>0$ can be thought of as a sharp loss in productivity or as sudden depreciation or loss of trees. The expected growth rate, h^*, of c_t is given from equation (4) by

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[2]This result is reminiscent of the treatment of leverage in Campbell (1986, p. 796) and Abel (1999, p. 15). In their representations, dividends on stocks are proportional to c_t raised to the power λ, where $\lambda>1$ represents leverage. In the present model, the dividend on gold is proportional to c_t to the power $1/\sigma$, which is less than one in the cases that we emphasize (where $\sigma>1$).

(5) $$h^* = h + \left(\frac{1}{2}\right) \cdot \sigma_u^2 - p \cdot Eb .$$

Let P_t be the price of an unlevered equity claim on a tree. The gross, one-period return on tree equity is given by

(6) $$R_t = \frac{c_{t+1}+P_{t+1}}{P_t} .$$

We assume that utility is time-additive[3] and depends on c_t^* in the usual iso-elastic way with the curvature parameter (coefficient of relative risk aversion) $\gamma>0$ and time-preference rate $\rho\geq0$. A key (and reasonable) assumption that simplifies the asset-pricing analysis is that the preference parameter, α, and the per capita quantities of gold, g_t, and consumption, c_t, are always such that the outlay on gold services, $\pi_t^g g_t$, is negligible compared to c_t. This condition implies that the marginal utility of c_t can be approximated by the usual $c_t^{-\gamma}$. In this case, the first-order condition for choosing c_t over time and holding assets as equity claims on trees can be approximated using equation (6) as:

(7) $$c_t^{-\gamma} \approx e^{-\rho} \cdot E_t\left[c_{t+1}^{-\gamma} \cdot \left(\frac{c_{t+1}+P_{t+1}}{P_t}\right)\right] .$$

The consumption flow, c_t, is the dividend accruing to the owner of tree equity. Because the shocks to $\log(c_t)$ in equation (4) are i.i.d., the ratio of the consumption dividend to the equity price, c_t/P_t, will be approximately constant in equilibrium at some value denoted by $d>0$. (The approximation arises because we are neglecting effects on the first-order condition from changing ratios of c_t to g_t.) Equations (7) and (4) imply a condition for d:

(8) $$1/(1+d) \approx exp\left[(1-\gamma) \cdot h - \rho + \left(\frac{1}{2}\right)(1-\gamma)^2\sigma_u^2\right] \cdot [1 - p + p \cdot E(1-b)^{1-\gamma}] .$$

[3]With i.i.d. shocks, the main results will hold with Epstein-Zin-Weil preferences, introduced by Epstein and Zin (1989) and Weil (1990).

Define r^e to be the expectation of the rate of return on equity, R_t -1. Using equations (5), (6), and (8), this expectation (constant in this model) is given, as the period length becomes negligible, by:

$$(9) \qquad r^e \approx \rho + \gamma h^* - \frac{1}{2} \cdot \gamma \cdot (\gamma - 1) \cdot \sigma_u^2 - p \cdot [E(1-b)^{1-\gamma} - 1 - (\gamma - 1) \cdot Eb] \ .$$

We use the first-order condition for choosing c_t over time and holding assets as risk-free claims to determine the (constant) risk-free rate, denoted r^f:

$$(10) \qquad r^f \approx \rho + \gamma h^* - \frac{1}{2}\gamma(\gamma + 1)\sigma_u^2 - p \cdot [E(1-b)^{-\gamma} - 1 - \gamma \cdot Eb] \ .$$

This result also holds as an approximation as the length of the period becomes small.

The equity premium follows (as an approximation for short periods) from equations (9) and (10):

$$(11) \qquad r^e - r^f \approx \gamma\sigma_u^2 + p \cdot [E(1-b)^{-\gamma} - E(1-b)^{1-\gamma} - Eb] \ .$$

As in previous applications of this result (such as Barro [2006] and Barro and Ursúa [2008]), we use calibrations where the disaster term involving p on the far right is the main contributor to the equity premium. The term involving σ_u^2 is negligible, as in Mehra and Prescott (1985).[4]

Consider now the pricing of gold. A unit of gold yields a dividend flow equal to the rental price, $\pi_t^g > 0$, in equation (3). We assume that gold does not depreciate in a physical sense, and there are no costs of storage or possibilities of loss of ownership. In particular, gold is

[4]Barro (2009) shows that the formula for the equity premium in equation (11) remains valid in this i.i.d. case with Epstein-Zin-Weil preferences, with γ representing the coefficient of relative risk aversion and θ (which does not enter into equation [11]) representing the reciprocal of the intertemporal elasticity of substitution. The other results require a substitution for ρ by an effective rate of time preference, ρ*, given by $\rho^* = \rho - (\gamma - \theta) \cdot \left\{h^* - \left(\frac{1}{2}\right)\gamma\sigma^2 - \left(\frac{p}{\gamma - 1}\right) \cdot [E(1-b)^{1-\gamma} - 1 - (\gamma - 1) \ \cdot Eb]\right\}$. If γ=θ, utility is time-additive, and ρ*=ρ.

not subject directly to the kinds of disturbances that beset the consumption trees—the normal shock, u_t, and the disaster shock, v_t, in equation (4). Let P_t^g be the price of gold. The gross, one-period return on holding gold is given by

$$(12) \qquad R_t^g = \frac{\pi_{t+1}^g + P_{t+1}^g}{P_t^g} .$$

The first-order condition for choosing c_t over time and holding assets as gold can be approximated, using equation (12), as

$$(13) \qquad c_t^{-\gamma} \approx e^{-\rho} \cdot E_t \left[c_{t+1}^{-\gamma} \cdot \left(\frac{\pi_{t+1}^g + P_{t+1}^g}{P_t^g} \right) \right] .$$

The assumption, as before, is that the outlay on gold is negligible compared to that on ordinary consumption, so that the marginal utility of consumption can be approximated by the usual $c_t^{-\gamma}$.[5] Recall that the rental price on gold is given by

$$(3) \qquad \pi_t^g = \left(\frac{1-\alpha}{\alpha}\right) \cdot \left(\frac{c_t}{g_t}\right)^{1/\sigma} .$$

We assume, for now, that the per capita gold stock, g_t, grows deterministically at the constant rate h_g, which could be positive due to gold discoveries or negative due to population growth and depreciation or loss of gold. In particular, we do not consider disaster or other shocks that directly affect the quantity of gold or the services provided by the gold. Unlike ordinary consumption, it is hard to see how the quantity of gold outstanding could change greatly

[5]Martin (2013) also emphasizes cases in which the shares of the dividends from some assets in total consumption are negligible. However, he assumes that the dividends from all assets are perfect substitutes in consumption, corresponding to σ being infinite in our model. Then he allows the dividends provided from the various assets to be subject to distinct shocks, whereas gold does not experience these kinds of shocks in our model. Cochrane, Longstaff, and Santa Clara (2008) also have a two-tree model in which the consumption flows are perfect substitutes, and the trees are subject to distinct shocks.

in a short period—even during a war or a depression. However, large changes in preferences for gold versus other consumption services are possible, and we explore these kinds of shocks later.

In a later section, we get a rough estimate of h_g based on the long-run growth rate of the per capita world stock of gold. The results, summarized in Figure 1, imply that this long-run growth rate (from 1875 to 2011) is between 0.4% and 0.9% per year if we neglect any loss or depreciation of the gold stock.

As with tree equity, the dividend-price ratio for gold, π_t^g/P_t^g, denoted by χ, will be constant in this i.i.d. model. The constancy of this ratio means, from equation (3), that the real gold price, P_t^g, moves along with $(\frac{c_t}{g_t})^{1/\sigma}$. Thus, although the dividend from gold is not directly subject to disasters or other shocks, its price ultimately reflects the shocks that affect consumption trees, with the sensitivity of gold prices to these shocks depending on the elasticity of substitution, σ.

Equations (3), (4), and (13) imply that the condition for determining χ is:

$$(14) \quad 1/(1+\chi) \approx exp\left[\left(\frac{1}{\sigma}-\gamma\right)h-\left(\frac{1}{\sigma}\right)h_g-\rho+\frac{1}{2}\left(\frac{1}{\sigma}-\gamma\right)^2\sigma_u^2\right]\cdot\left[1-p+p\cdot E(1-b)^{\left(\frac{1}{\sigma}-\gamma\right)}\right].$$

Note that χ does not depend on the preference parameter, α. A change in α would affect gold's dividend, π_t^g, and price, P_t^g, in the same proportion and, thereby, not affect the dividend-price ratio, χ. As α approaches one, gold has no intrinsic service value, and π_t^g approaches zero in equation (3). Since χ does not depend on α, the price of gold, P_t^g must also approach zero as α approaches one. In other words, in this model, a positive valuation of gold depends on its intrinsic usefulness—the equilibrium does not allow gold to have positive value based on a process for its real price that is unlinked to the underlying service value.

Define r^g to be the expectation of the rate of return on gold, R_t^g-1. Using equations (3), (12), and (14), this expectation is given by:

$$(15) \quad r^g \approx \rho + \gamma h^* - \frac{1}{2}\sigma_u^2 \cdot \left[\gamma \cdot (\gamma + 1 - \frac{2}{\sigma})\right] - p \cdot [E(1-b)^{\left(\frac{1}{\sigma}-\gamma\right)} - E(1-b)^{\frac{1}{\sigma}} - \gamma \cdot Eb] \, .$$

Again, this result holds as an approximation when the period length is short. Note that r^g (constant in this model) does not depend on the preference parameter, α, or the growth rate of the gold stock, h_g. (These results depend on the assumption that outlays on gold are negligible compared to outlays on ordinary consumption.)

The expected rate of return on gold, r^g from equation (15), can be compared with the expected rate of return on equity, r^e from equation (9), or the risk-free rate, r^f from equation (10). When compared to the return on equity, the result is:

$$(16) \quad r^e - r^g \approx \gamma\sigma_u^2 \cdot \left(\frac{\sigma-1}{\sigma}\right) + p \cdot [1 - Eb - E(1-b)^{1-\gamma} - E(1-b)^{\frac{1}{\sigma}} + E(1-b)^{(\frac{1}{\sigma}-\gamma)}].$$

When compared to the risk-free rate, the result is:

$$(17) \quad r^g - r^f \approx \left(\frac{\gamma}{\sigma}\right) \cdot \sigma_u^2 + p \cdot [E(1-b)^{\frac{1}{\sigma}} - E(1-b)^{\left(\frac{1}{\sigma}-\gamma\right)} - 1 + E(1-b)^{-\gamma}].$$

Equations (16) and (17) remain valid in this i.i.d. setting with a switch to Epstein-Zin-Weil preferences, with γ representing the coefficient of relative risk aversion (see n. 4).

The behavior of gold returns compared to the others depends on σ, the elasticity of substitution between gold services and ordinary consumption in the effective consumption flow in equation (1). When σ=1, returns on gold mimic the returns on tree equity, so that $r^e - r^g = 0$ in equation (16), and $r^g - r^f$ in equation (17) equals the equity premium, $r^e - r^f$, in equation (11). As σ approaches infinity, the rental price of gold, π_t^g in equation (3), becomes unresponsive to c_t/g_t, and gold becomes risk-free. Therefore, $r^g - r^f = 0$ in equation (17), and $r^e - r^g$ in equation (16) equals the equity premium, $r^e - r^f$, in equation (11). In other words, depending on the value of σ in the range where σ≥1, the expected rate of return on gold ranges between the expected rate of return on (unlevered) equity, r^e, and the risk-free rate, r^f.[6] To put it another way, given a measure of the expected rate of return on gold, r^g—and assuming that this value lies between r^f and r^e—there is a value of σ>1 that makes that observation consistent with the model.

In the baseline model—with the demand for gold derived from its role in effective consumption, c_t^*, in the form of equation (1)—there is no σ>0 that generates an expected rate of return on gold, r^g, below the risk-free rate, r^f. That is, in this model, gold never serves as enough of a disaster hedge so that its risk premium would be negative. In the next section, we show that a particular pattern of shocks to preferences, α_t, may generate $r^g<r^f$. However, since the empirically observed covariance between consumption growth and the growth rate of real gold prices turns out to be negligible, it is unclear that one wants a model that generates $r^g<r^f$.

II. Shocks to Preferences

In the baseline model, the gold price, P_t^g, is the constant multiple 1/χ of the rental price or dividend, π_t^g, given in equation (3). Therefore, P_t^g fluctuates along with $(\frac{c_t}{g_t})^{1/\sigma}$. This linkage between real gold prices and consumption (expressed relative to the stock of gold) suggests that, for reasonable values of σ, the model will not generate the high volatility of real gold prices that

[6] If σ<1, gold is riskier than tree equity, and r^g exceeds r^e.

shows up in some periods, as discussed later.[7] To remedy this shortcoming, we introduce shocks to the preference parameter, α_t, which affects the gold dividend in accordance with equation (2):

$$\pi_t^g = (\frac{1-\alpha_t}{\alpha_t}) \cdot (\frac{c_t}{g_t})^{1/\sigma}. \quad (2)$$

In a more general model (such as that considered below), a natural interpretation of a shift to α_t is that it represents a change in the monetary role of gold. In earlier periods—at least up to 1975—these shifts may reflect movements off or on aspects of the gold standard in the United States or the rest of the world.

Under some specifications of the process for α_t, the dividend-price ratio for gold, π_t^g/P_t^g, will still be a constant, χ. Under these circumstances, equation (2), (12), and (13) imply that χ must satisfy:

$$1/(1+\chi) \approx e^{-\rho-\left(\frac{1}{\sigma}\right)\cdot h_g} \cdot E_t \left\{ \left[\frac{(1-\alpha_{t+1})/\alpha_{t+1}}{(1-\alpha_t)/\alpha_t} \right] \cdot (\frac{c_{t+1}}{c_t})^{(\frac{1}{\sigma}-\gamma)} \right\}. \quad (18)$$

In the baseline model, the term involving α on the right-hand side of equation (18) equals one. More generally, if the covariance of this term with the consumption-growth term is zero and the mean of the α term equals one (implying no systematic drift in $[1-\alpha_t]/\alpha_t$), then χ will be constant. In this case, the formula for χ will still be given by equation (14), and the formula for the expected rate of return, r^g, on gold will still be given by equation (15). However, the shocks to α_t will generate fluctuations in the real gold price (matching in proportionate terms the fluctuations in the gold dividend given in equation [2], so that χ remains fixed).[8] Thus, this

[7] The baseline model also has problems replicating the volatility of dividend-price ratios for stocks. This difficulty can be alleviated by allowing for shifts to the parameters that describe uncertainty and expected growth—see Gabaix (2012), Bansal and Yaron (2004), and Barro and Ursúa (2012).

[8] For stocks, this kind of result would be unsatisfactory because observed dividend-price or earnings-price ratios for stocks are volatile (which can possibly be explained by shocks to the model's parameters that govern uncertainty and expected growth). For gold, the dividend is not directly observed, and it is unclear whether the dividend-price ratio is volatile.

extended model—which leaves intact the formula for r^g—may explain part of the observed volatility of real gold prices in some periods. Moreover, this perspective may explain the low volatility in the sub-period 1880-1913—the high point of the gold standard (as discussed below)—where the fluctuations in the preference parameter, α_t, were likely small.

The extension to allow shocks to α_t also explains how gold's expected return may fall below the risk-free rate if the covariance pattern between the preference term and the consumption-growth term is nonzero. In bad times, when c_{t+1}/c_t is low, the term involving consumption on the right-hand side of equation (18) tends to be high (if $\gamma>1/\sigma$). If the preference for gold tends to be high at these bad times—that is, if α_{t+1} tends to be low compared to α_t—the term involving α on the right-hand side of equation (18) tends also to be high. This positive covariance tends to lower the dividend-price ratio, χ, thereby lowering the expected rate of return on gold, r^g.

However, if shocks to α_t reflect particularly movements off or on the gold standard, it is unclear that the postulated covariance pattern would apply. Movements away from the gold standard tend to associate with bad economic times, notably wars and depressions. If these movements correspond to decreased demand for gold—high values of α_t—then the opposite covariance relation tends to emerge. Or, perhaps, it is satisfactory to assume that the relevant covariance is small (as turns out to be true, as discussed later, for the observed covariance between changes in real gold prices and consumption and GDP growth rates over the long run). In this case, shocks to α_t would help to explain volatility in real gold returns—perhaps differentially across sub-periods—without having major implications for the expected real rate of return on gold.

III. Shocks to the Stock of Gold

We considered how shocks to preferences, α_t, affect the rental price of gold, π_t^g, given by

(2) $$\pi_t^g = (\frac{1-\alpha_t}{\alpha_t}) \cdot (\frac{c_t}{g_t})^{1/\sigma}.$$

We can allow similarly for effects of shocks to the quantity of gold (per capita), g_t. An increase in g_t affects π_t^g and, thereby, real gold prices, in the direction opposite to that from a rise in the demand for gold (a fall in α_t).

A new consideration is that increases in π_t^g, caused by movements in c_t or α_t, tend to stimulate expanded efforts in gold mining, as discussed in Barro (1979).[9] This supply response tends to offset the effects on real gold prices already discussed for changes in c_t and α_t.

IV. Monetary Demand for Gold

We now allow for a monetary demand for gold. The total gold stock, g_t, is divided into a part used, as before, to provide jewelry-like services, g_t^c, and another for monetary services, g_t^m:

(19) $$g_t = g_t^c + g_t^m.$$

In our representative-household model, gold held for monetary purposes includes amounts held as coins, etc. by individuals. However, we assume that the analysis applies also to official reserves held by central banks and governments.

Agents can move gold costlessly between its two functions, "jewelry" and monetary gold. For individuals, monetary gold has a service or liquidity value akin to that normally considered in analyses of the demand for money. We assume that an analogous service flow—corresponding to liquidity benefits, possibly related to maintenance of exchange rates—attaches

[9]That paper also summarizes other features of classical approaches to the workings of the gold standard, as developed by Thornton (1802), Ricardo (1819), Mill (1848), Fisher (1911), and Friedman (1951).

to monetary gold held as reserves by central banks and governments. However, the service flow from monetary gold depends not on g_t^m, the physical quantity of gold held for monetary purposes, but rather on the real value of that quantity in units of consumption. Thus, the monetary service flow depends on g_t^m multiplied by the real gold price, P_t^g. As before, in the cases that we emphasize, P_t^g ends up as a constant, $1/\chi$, times the dividend flow from holding gold, π_t^g. In these cases, we can think of the monetary service flow as depending on the real rental income foregone by holding the monetary part of the gold stock, $\pi_t^g g_t^m$.

We can expand the concept of effective consumption, c_t^*, from equation (1) to incorporate monetary services from gold. An expanded CES form is:

$$(20) \qquad c_t^* = [\alpha_t \cdot c_t^{(\frac{\sigma-1}{\sigma})} + \beta_t \cdot (g_t^c)^{(\frac{\sigma-1}{\sigma})} + (1-\alpha_t-\beta_t) \cdot (\pi_t^g g_t^m)^{(\frac{\sigma-1}{\sigma})}]^{\frac{\sigma}{\sigma-1}},$$

where $0<\alpha_t<1$, $0<\beta_t<1$, and $\alpha_t+\beta_t<1$. This form assumes symmetry in the way that the three goods enter into effective consumption; that is, σ is the elasticity of substitution between any of the goods. We assume, as in the main previous analysis, that the preference parameters are constant, now at α and β.

As an analogue to equation (3), the rental price for gold used as "jewelry" satisfies:

$$(21) \qquad \pi_t^g = (\frac{\beta}{\alpha}) \cdot (\frac{c_t}{g_t^c})^{1/\sigma}.$$

Another margin of substitution is between gold used as jewelry or for monetary purposes. The associated first-order condition, which factors in the presence of π_t^g in the last term on the right-hand side of equation (20), is

$$(22) \qquad 1 = \frac{(1-\alpha-\beta)^{\sigma} \cdot \alpha^{1-\sigma}}{\beta} \cdot \left(\frac{g_t^c}{g_t^m}\right) \cdot (\frac{c_t}{g_t^c})^{(\sigma-1)/\sigma}.$$

This equation implies that, at the margin, the representative household gets the same contribution to utility from a unit of gold held as jewelry or for monetary purposes.[10] The same type of condition would hold for official gold reserves of central banks and governments.

If σ=1, equation (22) implies that variations in c_t or g_t do not affect the ratio of g_t^c to g_t^m. However, if σ>1, a rise in c_t reduces the ratio of g_t^c to g_t^m, whereas a rise in g_t raises this ratio. (These results are opposite if σ<1.) If we take σ>1 as the relevant case, the main new result is that the rental price of gold, π_t^g in equation (21), becomes more sensitive to variations in c_t because of the tendency for monetary gold, g_t^m, to move along with c_t (so that, for given g_t, gold held as jewelry, g_t^c, moves opposite to c_t).

Another result is that the equilibrium is not Pareto optimal, ultimately because of the presence of the relative price, π_t^g, in the effective consumption flow, c_t^*, in equation (20) and, thereby, in utility. The utility attained by the representative household would rise if the government taxed gold used for monetary purposes (and remitted the proceeds back to the representative household). For example, the government could require households to buy a durable certificate that has to be attached to gold to make it legitimate as "money." The price of the certificate would be in units of consumption per quantity of gold held as money. By imposing this form of tax, the government induces a decrease in the gold content of money, thereby freeing up more of the existing gold stock to be used for direct consumption purposes (as "jewelry"). The optimal tax is high enough so that the value of money approximates the value of the attached certificates, with the value of the associated monetary gold approximating zero. In other words, the solution is a form of fiat money.

[10]We can show that the equilibrium implied by equations (19), (21), and (22) exists and is unique. One feature of the results is that the real-money-balance term, $\pi_t^g g_t^m$, ends up equaling $c_t \cdot (\frac{1-\alpha-\beta}{\alpha})^\sigma$ and is, therefore, independent of g_t and positive even if β=0. As β approaches 0, g_t^c approaches 0, g_t^m approaches g_t, and π_t^g approaches $\left(\frac{c_t}{g_t}\right) \cdot (\frac{1-\alpha}{\alpha})^\sigma > 0$.

One issue about the equilibrium is that, at any finite tax rate, the government is motivated subsequently to raise the tax rate to raise revenue by imposing capital losses on existing holders of monetary gold. That is, the government has a familiar form of time-consistency problem whenever the quantity of gold held as money is positive. Since people would understand these temptations, they would be reluctant to hold fractional-gold money. That is, the only full equilibria may be either 100% gold money (where the government is somehow committed not to tax this use of gold) or 0% gold money, which amounts to paper money.

Another point is that, even with all gold driven out of monetary purposes and toward jewelry, the solution is still not Pareto optimal because of the usual issues of the "optimal quantity of fiat money," as explored by Friedman (1969). A full Pareto optimum requires the government to pay a rate of return on money either through explicit interest or through deflation of the "general price level." This setting also involves the usual time-consistency issues related to effects of inflation on the real value of the outstanding stock of paper money.

For the purposes of studying the real rate of return on gold, the most important contribution from allowing for monetary gold is likely to be the introduction of a new preference shock in the form of the term $1-\alpha_t-\beta_t$ (expressed relative to α_t and β_t) that reflects monetary services from gold. The volatility of this term and its association with movements off or on the gold standard—including changes in the demand for official gold reserves by central banks and governments—seem more compelling than fluctuations in preferences for gold jewelry versus other forms of consumption (as in the initial model).

Official gold reserves are quantitatively important when compared to the total world stock of gold. Figure 2 shows that the share of the world's gold stock held as official reserves by central banks and governments was less than 10% in 1877 but rose to a peak of 50-60% at the end of World War II in 1945. Then this share fell to around 20% in 2011.[11]

A more comprehensive measure of monetary gold would include amounts held privately as minted coins and bullion. As an example, gold in monetary circulation was roughly equal to official gold reserves in 1903 and about half of these reserves in 1913 (using data from U.S. Treasury Department, Bureau of the Mint [1904, p. 324; 1915, p. 454]). At present, we lack a long time series on a broad concept of monetary gold that includes amounts held privately as coins and bullion.

V. An Illustrative Calibration

We use a calibration of the baseline model based on an updated version of the analysis in Barro and Ursúa (2008). Thus, we assume the following:

$r^f = 0.011$ per year,
$r^e = 0.059$ per year,
$h = 0.025$ per year,
$h_g=0$,
$\sigma_u = 0.02$ per year,
$p = 0.037$ per year,
$\gamma = 3.34$,
$\rho=0.027$ per year,
$Eb = 0.208$,
$E(1\text{-}b)^{-\gamma} = 3.62$,
$E(1\text{-}b)^{1-\gamma} = 2.16$.

The values associated with disaster sizes, b, derive from the empirically observed size distribution of macroeconomic disasters (conditional on $b\geq0.095$) in the long-term history across countries based on declines in real per capita GDP (as discussed in Barro and Ursúa [2008] and

[11] At present, our measure of official gold reserves is incomplete because holdings by some countries—notably China and the Soviet Union in some periods—are excluded in standard estimates. We are working to expand the data to include broader coverage by countries and years.

Barro and Jin [2011]).[12] The value for p comes from the empirically observed probability per year of entering into these disaster states. Equation (5) and the assumed parameter values imply h*=0.0175 per year.

The value γ=3.34 was chosen to match the formula in equation (11) with the assumed (unlevered) equity premium, r^e-r^f=0.048 per year. Matching r^f=0.011 (a value discussed later) in equation (10) turns out to require ρ=0.027 per year.[13] We assume this value for ρ, although comparisons among the various rates of return do not depend on ρ (or h*). We consider below the extension of this calibration exercise to returns on gold.

As mentioned before, we estimate the long-term per capita growth rate of the gold stock, h_g, by using data on world gold production, as described in the notes to Figure 1. The data before 1875, including an estimate of cumulative world production from 1493 to 1875, are from Soetbeer (1887). The reported cumulative stock of world gold production (assuming no depreciation or loss) from 1493 to 2011 is 155,922 metric tons.

The calculations of stocks of gold require an initial stock in 1492, which is apparently subject to controversy, as noted in GoldMoney Foundation (2012). If we take this stock to be close to zero, we end up with the time series for world gold per capita since 1875 shown by the lower graph in Figure 1. To get a plausible range of possibilities, we assume as an alternative that the world gold stock in 1493 equals the reported cumulative production from 1493 to 1875 (9528 metric tons). In this case, we get the upper graph in Figure 1.[14]

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[12]Results are similar using declines in real per capita consumer expenditure, but the sample is smaller because of missing data on consumer expenditure.

[13]With Epstein-Zin-Weil preferences, the parameter ρ corresponds to an effective rate of time preference, ρ*, given in n.4.

[14]The numbers on the per capita gold stock since 1875 were calculated using a time series for world population based on McEvedy and Jones (1978) and *World Development Indicators*. The WDI data were used for annual numbers since 1960, and the McEvedy-Jones data at 25-year intervals were used before 1960. The growth rate of world population from 1875 to 2011 is 1.17% per year.

The average growth rate of world gold per capita from 1875 to 2011 is 0.88% per year based on the lower graph in Figure 1 and 0.42% per year based on the upper graph. Hence, 0.4%-0.9% per year provides a reasonable range for the long-term per capita growth rate if we maintain the assumption that gold stocks had zero depreciation and loss. However, even with small rates of depreciation and loss, the long-term per capita growth rate of the world gold stock could be zero or slightly negative. We take $h_g \approx 0$ in our main calculations.

VI. Missing Data on Dividends

A problem in matching the model with data on asset returns is that dividend yields are missing for some categories of assets, notably gold and silver.[15] To assess this issue conceptually, consider first how the returns on consumption-tree equity divide up between a dividend yield and a price-appreciation term.[16] The gross return on equity is given by

(5) $$R_t = \frac{c_{t+1} + P_{t+1}}{P_t} .$$

Using the previous definition of the dividend-price ratio as $d=c_t/P_t$, the rate of return, R_t-1, on equity can be expressed as:

(23) $$R_t - 1 = d \cdot \left(\frac{c_{t+1}}{c_t}\right) + \left(\frac{c_{t+1}}{c_t} - 1\right).$$

The first term on the right-hand side is the dividend yield, and the second term is the rate of price change (because P_t always moves in the same proportion as c_t). As the length of the period becomes negligible, the first term on the right-hand side of equation (23) approaches d, the

[15]An analogous problem might apply to residential real estate, although the dividend could be approximated by the explicit or imputed rental income on housing. However, the appropriate net rental income would have to subtract maintenance costs and depreciation.

[16]Note that consumption-tree equity is an unlevered claim on the flow of consumption dividends. Therefore, a match with observed equity returns requires an adjustment for leverage. Also, in the model, dividends correspond to earnings because there is no possibility of retention of earnings by "firms."

dividend-price ratio. The uncertainty about this term is negligible for short periods;[17] that is, all of the uncertainty about the rate of return is concentrated into the second term, which reflects price changes.

We know the expectation of the rate of return in equation (23) from the formula for r^e in equation (9). The expectation of the first term on the right-hand side—the expected dividend yield (which approximates the realized yield)—comes from the formula for d in equation (8). The expectation of the second term—the expected growth rate of consumption—equals h*, given by equation (5).

In the calibration of the model, we had r^e=0.059 per year and h*=0.0175 per year. Hence, the dividend yield was 0.0415 per year (a result that can be verified from the formula for d in equation [8]). To put it another way, 30% of the overall expected rate of return on tree equity reflects expected real price appreciation and 70% represents the dividend yield. Hence, omitting the dividend yield in the data would be a major problem with respect to matching the expected rate of return in the model. On the other hand, as already noted, all of the uncertainty in the model is concentrated into the price-appreciation term, with none appearing in the dividend yield (if the length of the period is negligible). Therefore, data on asset returns that omit dividend yields should be satisfactory for gauging standard deviations of returns and covariances of returns with consumption and GDP growth rates or among assets.

We can check this theoretical reasoning against U.S. data on stock returns, as discussed more fully later. Using a total-return stock-market index and a consumer price index (provided by Global Financial Data), the average real rate of return from 1836 to 2011 was 7.4% per year,

[17] The dividend can jump in the model when a disaster occurs. However, the probability, p, of a jump occurring during a period becomes negligible as the length of the period approaches zero.

with a standard deviation of 16.1%.[18] (Note that these data refer to levered returns, rather than the unlevered ones considered in the model.) In contrast, if one uses only stock-price-index data, thereby omitting dividends, the average rate of return was 2.5% with a standard deviation of 15.4%. Therefore, the omission of dividends has a major effect on the average rate of return—with only 2.5 percentage points of the total of 7.4 percentage points or 34% captured by real price appreciation.[19] On the other hand, the standard deviation of the price-change series, 15.4%, is close to that for total returns, 16.1%. Moreover, the covariance of the growth rate of per capita consumption (personal consumer expenditure) with the total-return series is 0.00224 (correlation of 0.36),[20] compared to 0.00229 (correlation of 0.39) with the price-change series. Therefore, the covariance and correlation computed from the price-change series are close to those calculated from total returns. Hence, as suggested theoretically, the price-change series captures well the uncertainty in stock returns—and this result holds even though the dividend-price ratio is not constant in the data (unlike in the baseline model), and the data are analyzed annually, rather than at a higher frequency.

The baseline model's implications for dividend yield and price appreciation are analogous for gold. The gross return on gold, R_t^g, is defined in equation (12), the dividend on gold, π_t^g, is given by equation (3), and the dividend-price ratio, χ, is determined from equation (14). Analogous to equation (23), the rate of return on gold, $R_t^g - 1$, can be broken down into a dividend yield and a price-appreciation term:

[18]These results are based on stock-return indexes and CPI's averaged over each year, as discussed later and presented in Table 1.
[19]This kind of effect from the omission of dividends on measured mean real stock returns is well known; see, for example, Jorion and Goetzmann (1999, Table III).
[20]See Table 1. The covariance of the growth rate of real per capita GDP with the total-return series is 0.00289 (correlation of 0.39), compared to 0.00282 (correlation of 0.39) with the price-change series.

$$(24) \qquad R_t^g - 1 = \chi \cdot \left(\frac{\pi_{t+1}^g}{\pi_t^g}\right) + (\frac{\pi_{t+1}^g}{\pi_t^g} - 1).$$

As before, the first term on the right-hand side approaches the dividend-price ratio, χ, as the length of the period becomes negligible. The second term, which reflects real gold-price appreciation (because the ratio of dividend to price is constant), is more complicated than for consumption-tree equity because π_t^g depends on $(\frac{c_t}{g_t})^{1/\sigma}$.

Analogous to equity returns, when the length of the period is negligible, all of the uncertainty about gold returns is concentrated into the price-appreciation term (the second part of the right-hand side of equation [24]), and none appears in the dividend-yield term (the first term on the right-hand side). Therefore, the available data on real gold-price appreciation should provide good information about the standard deviation of gold returns and about the covariances of these returns with consumption and GDP growth rates or with other asset returns.

We know the expectation of the rate of return on gold in equation (24) from the formula for r^g in equation (15). The expectation of the first term on the right-hand side—the dividend yield—comes from the formula for χ in equation (14). The expectation of the second term—the expected growth rate of gold dividends (and, hence, of real gold prices)—can be determined as:

$$(25) \quad E\left(\frac{\pi_{t+1}^g}{\pi_t^g} - 1\right) = \left(\frac{1}{\sigma}\right) \cdot \left(h^* - h_g\right) - \left(\frac{1}{2}\right) \sigma_u^2 \cdot \frac{1}{\sigma} \cdot \left(\frac{\sigma-1}{\sigma}\right) + p \cdot [\frac{1}{\sigma} \cdot Eb - 1 + E(1-b)^{\frac{1}{\sigma}}].$$

Thus, if σ=1, the expected rate of real price change is h^*-h_g, whereas if σ is infinite, this rate is zero.

In the data discussed in the next section, the mean growth rate of real gold prices from 1836 to 2011 is 1.1% per year (based on the U.S. dollar gold price and CPI). This observation provides an estimate of the expression on the right-hand side of equation (25). If we use the parameter values assumed before, then this expected rate of real price change equals 1.1% if σ=1.5. Correspondingly, the expected overall rate of return on gold, r^g, is 4.7% (from equation [15]), and the dividend yield on gold is 3.6% (from equation [14]). That is, the unobserved dividend yield is 77% of the expected total real rate of return on gold, and the expected rate of real price change is 23% of the total.

A problem with this calculation is that the standard deviation of the observed growth rate of real gold prices is 13.1% per year, which corresponds to a standard deviation for the mean over 176 years of 1.0% per year. Therefore, a one-standard-deviation confidence interval for the mean growth rate of real gold prices is roughly (0.1%, 2.1%). This range corresponds to an interval for the estimated σ from 16 to 0.84. Correspondingly, r^g ranges from 1.6% to 6.4%, and the share of the dividend yield in this expected total return varies from 94% to 67%. In other words, this method by itself does not provide a precise way to pin down the unobserved dividend yield on gold and, thereby, the expected total real rate of return on gold. In the next section, we bring in additional information related to the observed covariance between changes in real gold prices and consumption growth.

VII. Empirical Regularities on Gold and other Asset Returns

A. Means and standard deviations

Table 1 shows means and standard deviations of U.S. real returns (computed arithmetically) from 1836 to 2011 on gold, silver, stocks, T-Bills, and 10-year U.S. government bonds, along with inflation rates and growth rates of real per capita consumption and GDP. The starting date was chosen based on the available estimates of U.S. consumption (personal consumer expenditure). The real returns are all computed from a U.S. perspective, including the deflation of U.S. dollar gold and silver prices by the U.S. CPI. An important extension of this analysis to the global economy would allow for real gold and silver returns calculated from the perspective of various countries.

As already noted, a serious problem is that we observe only the parts of returns on gold and silver based on rates of change of real gold and silver prices, computed from nominal gold and silver prices and consumer price indexes. That is, we do not observe the dividend yields, corresponding to implicit rental incomes, on these precious metals. The table covers four sub-periods, 1836-1879, 1880-1913, 1914-1974, and 1975-2011, chosen to reflect changes in the regime for gold or silver.

Figure 3 shows the U.S. dollar gold price from 1790 to 1971, the year in which the United States dropped its commitment to buy from and sell gold to foreign central banks at a fixed dollar price.[21] Figure 4 shows the real gold price since 1800 (U.S. dollar price divided by the U.S. CPI, with the real gold price in 1800 set to 1.0).

Over the full sample, 1836-2011, the mean real rates of price change on gold and silver are similar—1.1% and 1.2% per year, respectively, in columns 2 and 3 of Table 1. The mean real stock return (based on total-return indexes for the S&P 500 and analogous measures computed by Global Financial Data up to 1970) for 1836-2011 is 7.4% (column 4). Since the available proxies for returns on short-term U.S. government securities ("T-Bills") up to 1919 are unreliable,[22] it seems advisable to focus on the mean real rate of return, 1.0% per year, that applies for 1920-2011 (column 5). For longer-term U.S. government bonds (roughly 10-year maturity), the mean real rate of return for 1836-2011 is 2.9% (column 6). (The mean is similar, 2.8%, for 1920-2011, where the T-Bill data are available.)

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[21]The data on dollar gold prices are from Global Financial Data. Their main sources are Warren and Pearson (1937) and Commodity Research Bureau, *Commodity Yearbook*.
[22]Global Financial Data attempts to proxy U.S. T-Bill returns before 1919 by using nominal yields on commercial paper or coupons on U.S. government bonds.

In terms of volatilities, the standard deviation of the T-Bill return is the smallest among the assets considered—4.3% from 1920 to 2011. This standard deviation is much larger for 1920-1974, which includes the Great Depression and World War II, than for 1975-2011. The pattern in the changing volatility of real T-Bill returns reflects particularly the changing volatility of inflation, shown in column 7 of Table 1.

The standard deviation of real stock returns from 1836 to 2011 is high, 16.1% per year. The means and standard deviations of real stock returns are reasonably stable over the sub-periods, and the standard deviation is well above those on T-Bills and government bonds.

For 1836-2011, the standard deviations of the real rates of price change for gold and silver, 13.1% and 17.9%, respectively, are similar to those on stocks.[23] As noted before, these measures should be reasonable proxies for standard deviations of total returns even though the dividend yields are not observed. However, the standard deviations depend a good deal on the sub-period considered. For example, the standard deviation of real rates of price change for gold for 1880-1913 (the peak of the world gold standard; see Figure 3) is only 2.6% per year, whereas that for 1975-2011 is 20.7%, even higher than that for stocks. (The standard deviations of real rates of price change for silver are 6.9% for 1880-1913 and 29.4% for 1975-2011.)

B. **Covariances with consumption and GDP growth rates**

Table 2 shows sample covariances and correlations of the various real asset returns and the inflation rates with growth rates of real per capita consumption (personal consumer expenditure) and GDP. Also included are 95% confidence intervals for these statistics, based on bootstrap methods.

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[23]According to data provided on Bob Shiller's website (www.econ.yale.edu/~shiller/data.htm), the mean of the real rate of price change for U.S. residential housing from 1891 to 2011 was 0.4% per year, with a standard deviation of 7.3%.

For 1920-2011, the sample covariance of the real T-Bill return with the growth rate of real per capita consumption (based on personal consumer expenditure) is small in magnitude, -0.0003 (with a correlation of -0.21), and is statistically insignificantly different from zero (column 5). In a standard model with iso-elastic utility and i.i.d. normal shocks, the return premium for T-Bills over a hypothetical risk-free claim equals the covariance of the real T-Bill return with consumption growth multiplied by the coefficient of relative risk aversion, γ. Hence, if we gauge the covariance between real bill returns and consumption growth by the sample covariance of -0.0003, then a γ around 4[24] implies that the mean real bill return of 1.0% (for 1920-2011) would be about one-tenth of a percentage point below the risk-free rate; that is, the (average) risk-free rate would be around 1.1%.

The real stock return is clearly procyclical, gauged by the covariances with consumption or GDP growth rates shown over the various sub-periods in Table 2 (column 4). For 1836-2011, the sample covariance with consumption growth is 0.0022 (correlation of 0.36) and is statistically significantly different from zero. The sample correlations over the sub-periods range from 0.22 for 1836-1879 to 0.47 for 1914-1974 and all except for 1836-1879 are statistically significantly different from zero.

One reason that the annual correlations of stock returns with consumption or GDP growth rates are strongly positive is that the data were assembled to achieve reasonably comparable timing in the variables. The growth rates of consumption or GDP involve first differences of annual flows observed each year. The comparable stock returns (and other returns considered in Tables 1 and 2) involve averages of nominal total returns and CPI values for each year, based on

[24]A γ around 3-4 emerges in the analysis of stock and bill returns in Barro and Ursúa (2008) and Barro and Jin (2011).

underlying daily or monthly values.[25] A more common procedure computes annual real stock returns (or other asset returns) based on year-end values of total-return indexes and price levels. This approach yields the average real stock return experienced during each year but does not line up with the available measures of annual consumption and GDP growth rates (because of the lack of data for much of the sample on the macroeconomic aggregates applying close to the end of each year). If the year-end procedure had been applied to calculating real stock returns, the correlation with consumption growth for 1836-2011 turns out to be 0.17, rather than 0.36, and the corresponding covariance turns out to be 0.00125, rather than 0.00224.[26] That is, the sample covariance would have been lower by a factor of two. (For GDP growth rates, the sample covariance would have been lower by a factor of three.)

In the standard model with iso-elastic utility and i.i.d. normal shocks, the equity premium (excess of the expected real stock return over the risk-free rate) equals the covariance of the real stock return with consumption growth multiplied by the coefficient of relative risk aversion, γ. Hence, if we gauge the covariance between stock returns and consumption growth by the sample value of 0.0022, a γ of 4 means that the model explains about 0.009 of the observed equity premium of 0.063 (based on a mean stock return of 0.074 and a risk-free rate of 0.011). To put it another way, the model requires γ=28 to explain the equity premium of 0.063. In contrast, if we use the sample covariance of 0.0012 computed from year-end-return values, the model with γ=4 would explain only 0.005 of the observed equity premium, and γ=50 would be required to

[25]The available price-level data before 1875 are annual, but these data are probably satisfactory because they approximate annual averages for each year. The data for 1875-1912 are rough estimates of monthly values from Snyder (1924). The data since 1913 are the standard monthly CPI numbers for urban consumers from the Bureau of Labor Statistics.

[26]The main effect comes from the timing of the stock-return data, not the price-level data. If we compute real stock returns from averages of daily or monthly total-return indexes for stocks along with year-end data for price levels (while retaining the available annual data before 1875), the covariance of the real stock return with consumption growth is 0.00208, with a correlation of 0.35, not much different from the values in Table 2, which are based on averages of monthly data on the price level since 1875.

generate the equity premium of 0.063. This last finding is reminiscent of the results stressed by Mehra and Prescott (1985).

The rare-disasters literature makes a major adjustment by noting that the assumed normality of the shocks greatly understates the fatness of the tails for consumption growth and stock returns. In this view, the observed covariance between stock returns and consumption growth of 0.00224 substantially understates the risks associated with holding stocks. However, the fat-tail or disaster effects cannot be satisfactorily estimated solely from the U.S. data considered in Tables 1 and 2. Rather, the approach originated by Rietz (1988) and applied in Barro (2006) and Barro and Ursúa (2008) relies on the broad history of macroeconomic disasters observed for many countries over a century or more.

As already noted, the mean real rate of price change for gold for 1836-2011 (column 2 of Table 1) is 1.1%, with a standard deviation of 13.1%. In contrast, during the high point of the world gold standard from 1880 to1913 (following the U.S. resumption in 1879), the average real rate of price change was slightly negative, -1.0%, with a small standard deviation, 2.6%. Since the nominal gold price was essentially constant (Figure 3), these patterns reflect the behavior of inflation (Table 1, column 6).

In the first sub-period, 1836-1879, the U.S. monetary system was linked to silver or gold for much of the sample, but a suspension of monetary convertibility occurred near the beginning of the U.S. Civil War in 1861, with full convertibility restored only in 1879 (see Figure 3). For 1914-1974, the United States maintained aspects of the gold standard, but a rise in the nominal gold price and the prohibition of private holdings of monetary gold occurred in 1933 (Figure 3). In 1971, the United States formally dropped its commitment to foreign central banks to convert U.S. dollars into gold at a fixed dollar price. Then at the beginning of 1975, the prohibition on private holdings of monetary gold in the United States was lifted. In the most recent sub-period, 1975-2011, gold retained a commodity-reserve role for central banks but one that was largely divorced from domestic monetary systems.

Table 1, column 2, shows that, for gold's real rate of price change, the mean and standard deviation vary substantially across the sub-periods. It is also clear, however, that focusing on a period such as 1880-1913—the high point of the international gold standard—involves a strong element of ex post selection. From an ex ante perspective, the 1880-1913 period likely shares with the preceding and subsequent periods the possibility of moving off the gold standard in certain circumstances, particularly associated with war. Examples are the U.S. movement off of a gold/silver commodity standard in 1861 during the Civil War, movements of many countries off gold with the start of World War I in 1914, and, much earlier, the British movement off gold in 1797 during the long period of wars with France. At times of suspension, an important issue was the prospect of eventual return to the gold or gold/silver standard, likely during peacetime, and at what parity. The British resumption of the gold standard in 1821 was at the old parity (as urged by David Ricardo), as was the U.S. resumption in 1879 (Figure 3). However, the British return to gold in 1925 at the previous parity proved unsuccessful in the wake of the Great Depression.

When compared with earlier times, the most recent sub-period, 1975-2011, shows a substantially higher mean and standard deviation of the real rate of price change for gold—4.0% and 20.7%, respectively. For silver, the mean and standard deviation are even higher: 5.1% and 29.4%, respectively. Although the increase in volatility of these real rates of price change for the most recent sub-period is clear, the change in the mean is less sure because it is hard to pin down the expected value when the volatility is this high. For example, if the annual standard deviation were known to be 20.7% (the sample standard deviation for the real rate of price change for gold for 1975-2011), the standard error of the mean over 37 years would be 3.4%. That is, the observed mean real rate of price change of 4.0% is not statistically significantly different from zero (or from the mean rates of change in earlier periods) at typical significance levels.[27]

Given this last perspective and the sample-selection issues, it seems best to focus on statistics for gold over the full sample, 1836-2011. In this context, the key points from Tables 1 and 2 are that the mean real rate of price change is 1.1% per year, the standard deviation is 13.1%, and the covariance with consumption growth is small, -0.0002 (with a correlation of -0.05), and not statistically significantly different from zero. With a coefficient of relative risk aversion, γ, around 4 (and with i.i.d. normal shocks), the observed covariance implies that the expected real rate of return on gold should be close to the risk-free rate; as a point estimate, about 1.0% if the risk-free rate is 1.1%.

In the baseline model, it is impossible to get the expected real rate of return on gold, r^g, to fall below the risk-free rate, r^f, although a high value of σ gets r^g close to r^f. Therefore, we need a high value of σ to be consistent with the negligible covariance between gold's real rate of price change and consumption growth.

We can combine the last observation with the finding from before that the long-term sample mean of gold's real rate of price change—1.1% per year, with a one-standard-deviation confidence interval of roughly (0.1%, 2.1%)—corresponded in the baseline model to a range for σ from 16 to 0.84. Low values of σ within this range are inconsistent with r^g being close to r^f. For example, using equation (15), σ=1 implies r^g = 5.9% (the same as the expected return on unlevered equity), and σ=2 implies r^g =4.0%. Therefore, with a risk-free rate around 1.1%, the

[27]We reach the same conclusion by noting that, with the standard deviation having to be estimated, the ratio of the sample mean, 4.0%, to the sample standard deviation for the mean of 3.4% follows a t-distribution with 36 degrees of freedom.

risk premia on gold implied by these values of σ are too high, given the negligible observed covariance of gold's real rate of price change with consumption growth.

The high end of the range for σ produces more satisfactory results. For example, σ=10 generates r^g=1.8%, with an expected rate of change of real gold prices of 0.2%, and σ=16 generates r^g=1.6%, with an expected rate of change of real gold prices of 0.1%. Hence, these specifications with high values of σ generate values of r^g that are only small amounts above the estimated risk-free rate of 1.1%, while also producing expected rates of change of real gold prices that fall within the one-standard-error band for the long-run mean, (0.1%, 2.1%). A notable feature of these results is that the bulk of gold's expected rate of return comes from the unobserved dividend yield, with only a small part reflecting the expected real rate of price change.

C. **Covariances between gold returns and other asset returns**

In the baseline model, with only i.i.d. shocks to GDP and consumption, the covariance between real gold returns and the growth rates of GDP and consumption is positive but small if the elasticity of substitution, σ, is high. The same prediction applies to the covariance between real gold returns and real stock returns.

Table 3 shows covariances and correlations of real gold returns with other real asset returns and inflation rates for 1836-2011 and the various sub-periods. For stock returns (column 3), the covariance is significantly positive for 1836-1879 but otherwise small in magnitude and statistically insignificantly different from zero. The results for the periods since 1880 are similar to those for the covariance between real gold returns and consumption or GDP growth rates shown in Table 2 (column 1). As discussed before, these results accord reasonably well with the model if σ is high.

In the baseline model, which abstracts from inflation or default on government debt, the real T-bill return is constant and equals the short-term risk-free rate, and the real term structure is flat. Hence, the model predicts zero covariance between real gold returns and real returns on T-Bills or 10-year government bonds. This prediction might change if we bring in effects of inflation on the real returns on government securities or if we allow for stochastic shifts in parameters, such as the disaster probability, p, or the preference parameter, α, that governs demand for gold services versus ordinary consumption.

In Table 3, columns 4 and 5, the covariance between real gold returns and real returns on the two forms of government securities is positive for sub-periods between 1836 and 1974 and negative for 1975-2011. These covariances are statistically significantly different from zero at the 5% level for 10-year bonds for 1836-1879 and 1880-1913 and for T-Bills for 1920-1974 and 1975-2011. The results are nearly statistically significant at the 5% level for 10-year bonds for 1914-1974 and 1975-2011.

The relation between real gold returns and real returns on government securities likely derives from effects of inflation on the real returns on assets that are denominated in nominal terms. The covariances between the real returns on the two forms of government securities and inflation are strongly negative for the overall sample, 1836-2011, and the various sub-periods. The covariances between real gold returns and inflation are also negative in periods where the nominal gold price is virtually constant (1880-1913) or has some element of nominal pegging (1836-1879 and 1914-1974)—as shown in Table 3, column 6. Therefore, variations in inflation associate with movements in the same direction for real returns on government securities and gold over the sub-periods from 1836 to 1974 (columns 4 and 5). From 1975 to 2011, there is no longer any nominal pegging of the gold price, and the covariance between real gold returns and inflation becomes positive, though not statistically significantly different from zero (column 6). This changed pattern between real gold returns and inflation likely explains why the covariances between real returns on gold and real returns on the two forms of government securities are negative in this period.

The covariance between real gold returns and real silver returns (Table 3, column 1) is significantly positive except during the high point of the classical gold standard from 1880 to 1913. In this period, the nominal price of gold, but not silver, was essentially constant. For 1836-1879, the correlation between real gold and silver returns is close to one, because the ratio of gold to silver prices changes little, reflecting the bimetallic standard that was partially maintained in the United Kingdom and the United States. The ratio of gold to silver prices from 1790 to 2011 is shown in Figure 5. For 1836-1879, this ratio varies relatively little around the median for that sub-period of 16.0.[28]

Figure 5 also shows the ratio of world stocks of silver to world stocks of gold for 1790-2011. This ratio falls from 34 in 1840 to 19 in 1875 (partly due to gold discoveries), then trends downward further to 9 in 2011.[29] This pattern in the quantity ratio does not align in an obvious way with the fluctuations in the price ratio. An interesting extension would relate the price and quantity ratios to exogenous changes in the relative supplies of and demands for these two precious metals.

VIII. Summary Observations

Our main objective was to match empirical regularities for gold returns with the predictions from a simple asset-pricing model. As to regularities, we observe first that, from

[28]Nevertheless, the changes in the U.S. mint ratio in 1834 and 1853 and the full demonetization of silver in 1873 (the "crime of 1873") received a lot of attention—see Laughlin (1894, Chs. IV, V, and VII).
[29]The corresponding stock of silver in 2011 is 1,371,239 metric tons.

1836 to 2011, the average real rate of price change for gold in the United States is 1.1% per year with a standard deviation of 13.1%, implying a one-standard-deviation confidence band for the mean of (0.1%, 2.1%). Second, over the same period, the covariances of gold's real rate of price change with consumption and GDP growth rates are small and statistically insignificantly different from zero. These negligible covariances imply that gold should carry a small risk premium; that is, gold's expected real rate of return—which includes an unobserved dividend yield—should be close to the risk-free rate, estimated from real returns on Treasury Bills to be around 1.1%. Third, the volatility of the growth rate of real gold prices is small under the classical gold standard from 1880 to 1913 but high—comparable to that on stocks—in other periods, including 1975 to 2011.

Key features of our baseline model are, first, ordinary consumption and gold services are imperfect substitutes for the representative household; second, outlays on gold services (for jewelry, crafts, electronics, medicine, monetary purposes, and so on) are always minor compared to ordinary consumption; and third, disaster and other shocks impinge directly on consumption and GDP but not on stocks of gold.

With a high elasticity of substitution between gold services and ordinary consumption, the model can generate a mean real rate of price change within the observed one-standard-deviation confidence band, (0.1%, 2.1%), along with a small risk premium for gold. In this scenario, the bulk of gold's expected rate of return reflects the unobserved dividend yield and only a small part comprises expected real price appreciation. Nevertheless, the uncertainty in gold returns is concentrated in the price-change component. The model can explain the time-varying volatility of real gold prices if preference shocks for gold services are small under the classical gold standard but large in other periods particularly because of shifting monetary roles for gold.

One useful extension would consider gold as a tradable good in a world of open economies. This framework would include tradable consumer goods and non-tradable goods consumed within each country. The risk characteristics of gold from the perspective of each country then depend on the quantity of world tradables and on country variables that determine the real exchange rate, in the sense of the relative price of non-tradable home goods and tradable goods, including gold.

Other extensions would study empirically the shifting demands for gold, related especially to official holdings by central banks and governments (Figure 2). From a data standpoint, we may be able to estimate the quantity of the world's gold held as coins and jewelry and for other "non-monetary" purposes. The time series of world gold production can also be analyzed, including incentive effects on gold mining from shifts in the real price of gold. This analysis can be carried out jointly with silver (Figure 5).

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Table 1
U.S. Real Asset Returns: Means and Standard Deviations

(1) Period	(2) Gold	(3) Silver	(4) Stocks	(5) T-Bills	(6) 10-Year Bonds	(7) Inflation Rate	(8) C growth	(9) GDP growth
1836-2011	0.0112 (0.1306)	0.0123 (0.1788)	0.0740 (0.1607)	0.0097† (0.0431)	0.0287 (0.0766)	0.0225 (0.0542)	0.0159 (0.0385)	0.0191 (0.0467)
1836-1879	-0.0008 (0.0819)	-0.0041 (0.0808)	0.0760 (0.1553)	-- --	0.0446 (0.0887)	0.0067 (0.0727)	0.0118 (0.0484)	0.0143 (0.0345)
1880-1913	-0.0102 (0.0257)	-0.0261 (0.0686)	0.0626 (0.1251)	-- --	0.0262 (0.0412)	0.0109 (0.0263)	0.0129 (0.0393)	0.0186 (0.0453)
1914-1974	0.0145 (0.1338)	0.0219 (0.1792)	0.0779 (0.1933)	0.0072†† (0.0523)	0.0087 (0.0723)	0.0283 (0.0580)	0.0180 (0.0399)	0.0239 (0.0641)
1975-2011	0.0398 (0.2074)	0.0511 (0.2937)	0.0753 (0.1410)	0.0134 (0.0238)	0.0450 (0.0871)	0.0421 (0.0291)	0.0201 (0.0163)	0.0175 (0.0201)

†1920-2011
††1920-1974

Notes to Table 1

All asset returns are in real terms. Data on nominal asset returns, nominal gold and silver prices, and CPI's are from Global Financial Data (GFD).

The return on gold refers to the growth rate of real gold prices, calculated arithmetically for each year as: -1 + (gold price/CPI)/[gold price (-1)/CPI(-1)], where (-1) indicates an annual lag. The U.S. dollar gold price reported by GFD up to 1933 is the official price set by the U.S government, except for 1861-1878, when the data come from *Commercial and Financial Chronicle*. Data after 1933 are from Commodity Research Bureau, *Commodity Yearbook*. The CPI values reported by GFD derive from the BLS consumer price index for urban consumers since 1913. Data before 1913 are based on information from the Federal Reserve Bank of New York, including monthly data since 1875 described in Snyder (1924). The gold price used for each year is the average of daily or monthly values during the year. The CPI value used for each year since 1875 is the average of monthly values during the year. Only annual data on consumer prices are available before 1875.

The real silver return is computed analogously, based on averages of daily or monthly U.S. dollar silver prices. The silver prices reported by GFD come from Officer (2008), Warren and Pearson (1937), and Commodity Research Bureau, *Commodity Yearbook*. Recent New York quotes are from Handy and Harman.

The real stock return is computed analogously, based on averages of daily or monthly nominal total-return indexes computed by GFD for the S&P 500. Values before 1971 are based on GFD estimates of total-return indexes comparable to the S&P 500.

The real T-Bill return is computed analogously, based on averages of monthly nominal total-return indexes for 90-day U.S. Treasury Bills. The estimates of total returns from GFD since 1929 derive from yields on 90-day bills. Estimates from 1919 to 1928 are based on yields on short-term U.S. Treasury bonds.

The real return on 10-year U.S. government bonds is computed analogously, based on averages of monthly nominal total-return indexes for U.S. government bonds with roughly 10-year remaining maturity. Values from 1919-1940 are based on the Federal Reserve's 10-15 year Treasury bond index. Values before 1919 are based on various long-term U.S. government bonds. (Data for 1836 to 1841 are from Boston city bonds.)

The inflation rate is calculated as -1 + CPI/CPI(-1), using the CPI data described above.

Consumption (C) and GDP growth are real per capita growth rates calculated arithmetically from the Barro-Ursúa annual data on real per capita personal consumer expenditure and GDP, available at www.rbarro.com/data-sets.

Table 2: Covariances [Correlations] of Real Asset Returns with Consumption and GDP Growth Rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Period	**Gold**	**Silver**	**Stock**	**T-Bill**	**10-Year Bond**	**Inflation rate**
			Results for C			
1836-2011	-0.0002 (-.0086, .0036) [-0.05] (-.20, .10)	0.0008 (-.0001, .0017) [0.11] (-.03, .26)	0.0022* (.0012, .0034) [0.36*] (.23, .49)	-0.0003† (-.0009, .0002) [-0.21]† (-.52, .20)	0.0004 (-.0002, .0010) [0.12] (-.07, .30)	-0.0001 (-.0005, .0003) [-0.03] (-.22, .17)
1836-1879	0.0002 (-.0009, .0014) [0.05] (-.26, .32)	0.0002 (-.0008, .0014) [0.04] (-.27, .32)	0.0016 (-.0007, .0041) [0.22] (-.10, .53)	-- -- 	0.0014 (.0000, .0034) [0.34] (-.01, .59)	-0.0009 (-.0019, .0002) [-0.26] (-.50, .00)
1880-1913	-0.0004* (-.0009, -.0002) [-0.42*] (-.67, -.10)	0.0004 (-.0005, .0013) [0.14] (-.21, .44)	0.0020* (.0004, .0036) [0.42*] (.10, .64)	-- -- 	0.0001 (-.0005, .0007) [0.05] (-.30, .41)	0.0004* (.0000, .0010) [0.43*] (.02, .70)
1914-1974	-0.0001 (-.0013, .0011) [-0.02] (-.26, .23)	0.0019 (-.0003, .0043) [0.27] (-.05, .59)	0.0036* (.0013, .0062) [0.47*] (.20, .67)	-0.0006†† (-.0015, .0003) [-0.28]†† (-.62, .19)	-0.0002 (-.0012, .0008) [-0.07] (-.39, .31)	0.0002 (-.0006, .0010) [0.07] (-.33, .42)
1975-2011	-0.0011 (-.0027, .0002) [-0.33] (-.62, .07)	-0.0004 (-.0020, .0011) [-0.08] (-.41, .27)	0.0010* (.0002, .0019) [0.44*] (.13, .69)	0.0001 (.0000, .0002) [0.31] (-.03, .59)	0.0003 (-.0002, .0008) [0.21] (-.18, .51)	-0.0001 (-.0003, .0001) [-0.23] (-.57, .20)
			Results for GDP			
1836-2011	-0.0005 (-.0012, .0003) [-0.07] (-.19, .04)	0.0004 (-.0008, .0016) [0.05] (-.10, .21)	0.0029* (.0016, .0043) [0.39*] (.22, .54)	-0.0007† (-.0015, .0000) [-0.32]† (-.59, .03)	-0.0002 (-.0007, .0003) [-0.06] (-.20, .09)	0.0003 (-.0002, .0007) [0.11] (-.07, .28)
1836-1879	-0.0001 (-.0011, .0011) [-0.03] (-.41, .33)	-0.0001 (-.0011, .0011) [-0.02] (-.41, .35)	0.0025* (.0010, .0041) [0.47*] (.25, .67)	-- -- 	0.0005 (-.0003, .0012) [0.16] (-.11, .43)	-0.0003 (-.0011, .0005) [-0.14] (-.45, .20)
1880-1913	-0.0004 (-.0009, .0000) [-0.34] (-.61, .01)	0.0008 (-.0004, .0020) [0.25] (-.15, .56)	0.0036* (.0017, .0055) [0.65*] (.43, .80)	-- -- 	0.0004 (-.0003, .0010) [0.20] (-.16, .52)	0.0004 (.0000, .0009) [0.34] (-.02, .61)
1914-1974	-0.0006 (-.0022, .0010) [-0.07] (-.27, .13)	0.0008 (-.0025, .0041) [0.07] (-.21, .39)	0.0038* (.0004, .0079) [0.32*] (.03, .56)	-0.0012†† (-.0025, .0001) [-0.36]†† (-.67, .02)	-0.0009 (-.0022, .0003) [-0.20] (-.46, .08)	0.0008 (-.0003, .0020) [0.21] (-.10, .50)
1975-2011	-0.0008 (-.0025, .0007) [-0.19] (-.53, .19)	-0.0001 (-.0019, .0016) [-0.01] (-.34, .30)	0.0012* (.0002, .0022) [0.43*] (.10, .68)	0.0001 (-.0001, .0003) [0.16] (-.26, .55)	-0.0001 (-.0006, .0005) [-0.05] (-.39, .28)	-0.0001 (-.0003, .0002) [-0.10] (-.46, .32)

*Statistically significant at 5% level.
†1920-2011
††1920-1974

Table 3: Covariances [Correlations] of Real Gold Returns with other Real Asset Returns

(1) Period	(2) Silver	(3) Stock	(4) T-Bill	(5) 10-Year Bond	(6) Inflation rate
1836-2011	0.0167* (.0089, .0260) [0.71*] (.58, .81)	0.0001 (-.0028, .0039) [0.03] (-.13, .20)	-0.0001† (-.0013, .0011) [-0.01]† (-.19, .16)	0.0007 (-.0016, .0027) [0.07] (-.14, .31)	-0.0009 (-.0023, .0008) [-0.13] (-.38, .09)
1836-1879	0.0065* (.0031, .0116) [0.98*] (.95, .995)	0.0056* (.0023, .0108) [0.44*] (.21, .63)	-- --	0.0037* (.0011, .0063) [0.51*] (.21, .75)	-0.0024 (-.0052, .0004) [-0.41] (-.83, .05)
1880-1913	-0.0001 (-.0006, .0004) [-0.05] (-.36, .28)	-0.0003 (-.0013, .0008) [-0.09] (-.41, .26)	-- --	0.0006* (.0003, .0009) [0.60*] (.35, .80)	-0.0007* (-.0010, -.0004) [-0.999*] (-.9997, -.9992)
1914-1974	0.0124* (.0012, .0259) [0.52*] (.09, .74)	-0.0003 (-.0089, .0078) [-0.01] (-.30, .35)	0.0016*†† (.0002, .0031) [0.23*]†† (.02, .53)	0.0026 (-.0001, .0052) [0.27] (.00, .66)	-0.0020 (-.0042, .0004) [-0.26] (-.66, .05)
1975-2011	0.0505* (.0227, .0810) [0.83*] (.72, .91)	-0.0033 (-.0096, .0029) [-0.11] (-.36, .12)	-0.0027* (-.0045, -.0011) [-0.55*] (-.76, -.30)	-0.0061 (-.0153, .0017) [-0.34] (-.68, .12)	0.0014 (-.0021, .0058) [0.24] (-.54, .66)

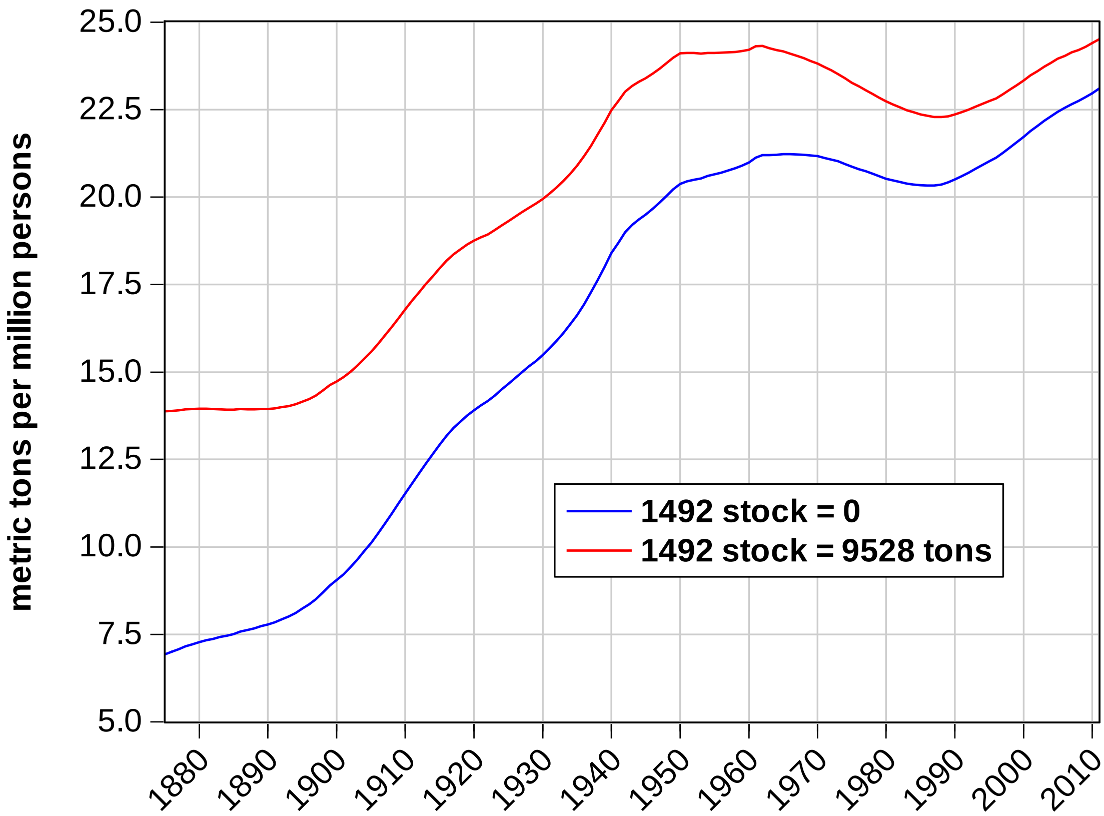
*Statistically significant at 5% level.
†1920-2011
††1920-1974

Notes to Tables 2 and 3

The data on real asset returns, inflation rates, and growth rates of consumption and GDP are described in the notes to Table 1. The upper part of Table 2 applies to covariances and correlations (shown in brackets) with the growth rate of consumption. The lower part applies to the growth rate of GDP. 95% confidence intervals are in parentheses below each sample value for covariance or correlation. These intervals were generated from percentile-method bootstraps with 100,000 iterations**.** Table 3 shows the covariances and correlations (shown in brackets) of real gold returns with other real asset returns and the inflation rate. 95% confidence intervals were constructed as described above.

Figure 1

World Gold Stock per Person, 1875-2011

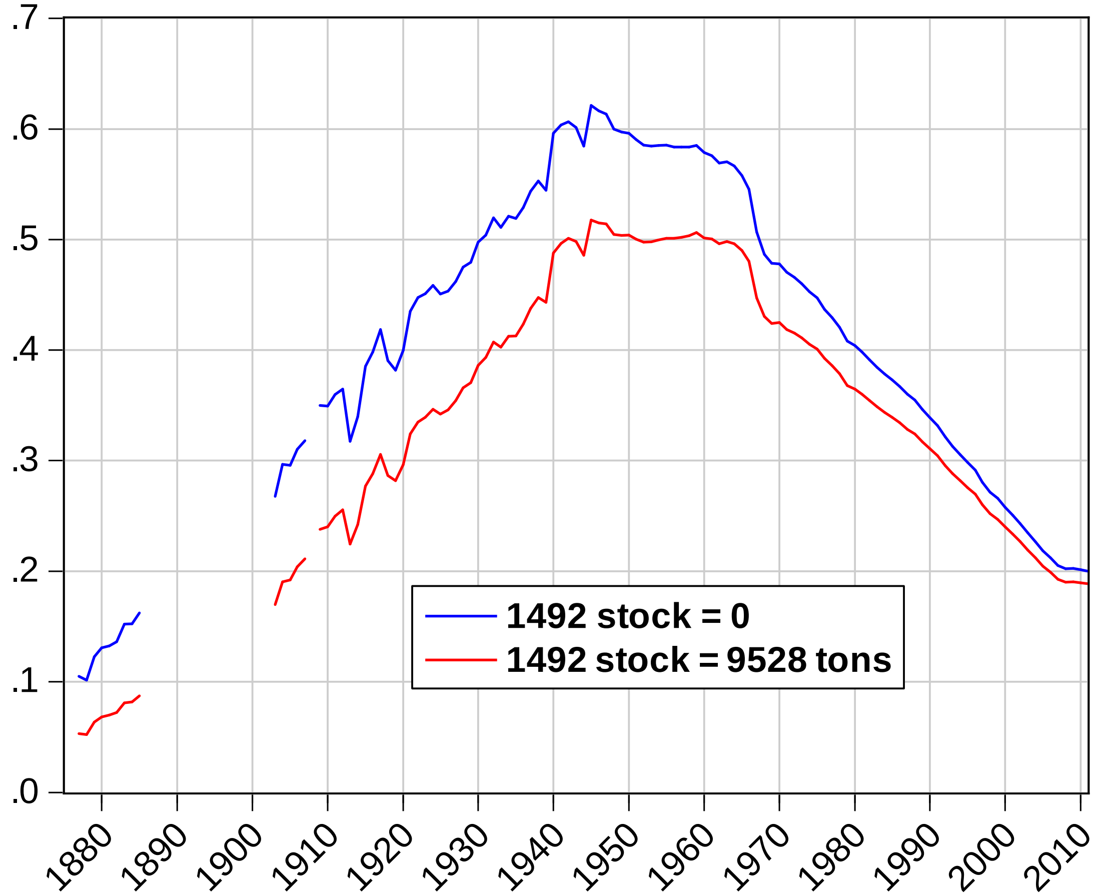


Note: The blue graph assumes that the gold stock in 1492 equals zero. The red graph assumes that this stock equals 9528 tons, the reported cumulative production from 1493 to 1875. The calculations for the stock after 1492 use the available gold production data (annual since 1876 and at longer intervals before 1876), assuming zero depreciation and loss on the existing stock.

Sources:
Gold production data. 1493-1884: Soetbeer (1887, Tables 1 and 2). 1885-1899: U.S. Treasury Department, Bureau of the Mint (various years). 1900-2011: U.S. Geological Survey (2010; 2012, p. 67).

Population data. World population since 1960 is based on annual data from *World Development Indicators*. Earlier numbers are based on data at 25-year intervals from McEvedy and Jones (1978).

Figure 2
Share of Official Gold Reserves in World Stock

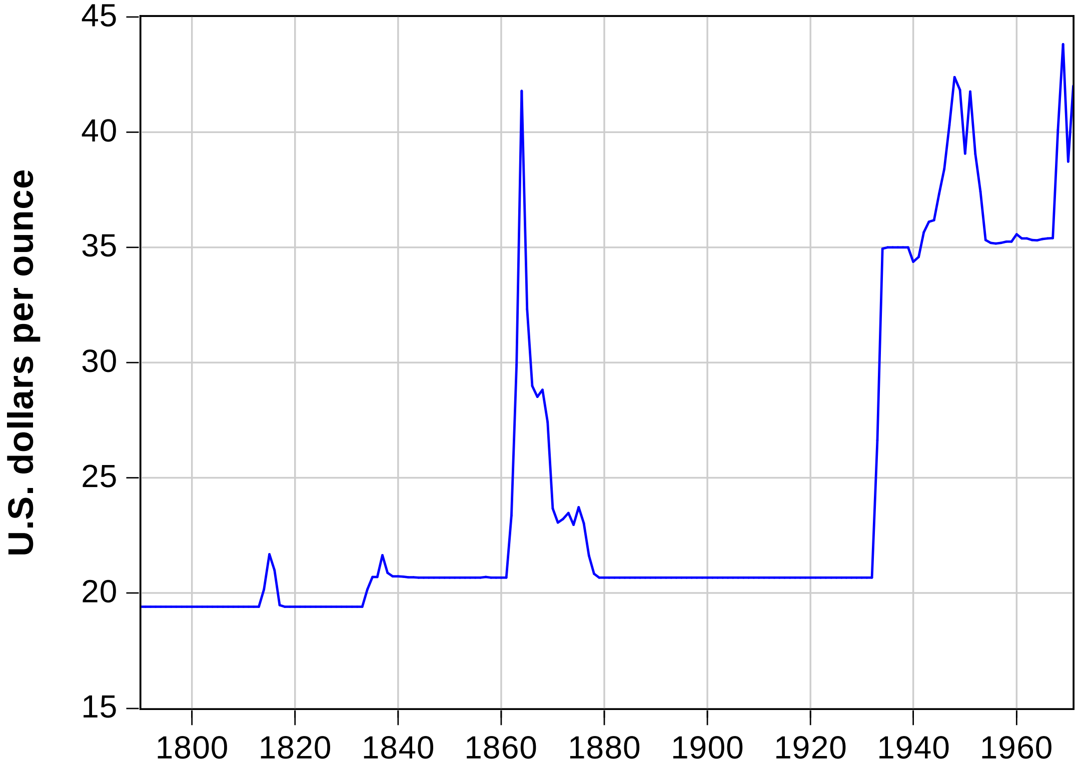


Note: The graphs show the ratio of gold held by central banks and governments to the total world stock. The blue graph assumes that the world gold stock in 1492 equals zero. The red graph assumes that this stock equals 9528 tons, the reported cumulative production from 1493 to 1875 (see Figure 1).

Sources: The world gold stock comes from the sources detailed in the notes to Figure 1. Data on official gold reserves of central banks and governments are from Soetbeer (1887); U.S. Treasury Department, Bureau of the Mint (various years); Board of Governors of the Federal Reserve System (1943, 1976); League of Nations (various years); and *International Financial Statistics*. These sources exclude holdings over some periods at least by China and the Soviet Union. We are working to fill these gaps and also to expand the coverage to years prior to 1877 and for missing years between 1886 and 1908. In some of these years, the available data combine official holdings with gold coins in circulation.

Figure 3

U.S. Dollar Gold Price, annual average, 1790-1971

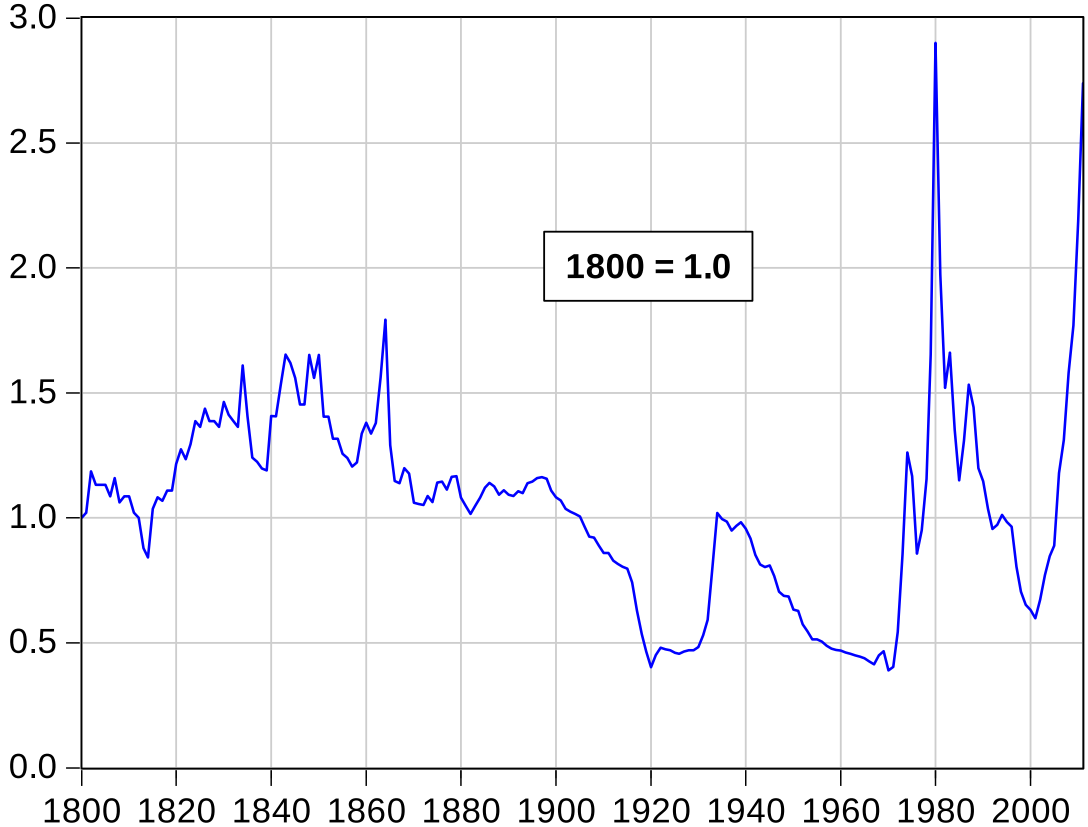


Note: See the notes to Table 1 for the sources of data on annual-average U.S. dollar gold prices. After 1971, the U.S. dollar gold price became highly variable, reaching an annual average of $613 for 1980 and $1573 for 2011.

Figure 4

Real Gold Price, annual average, 1800-2011

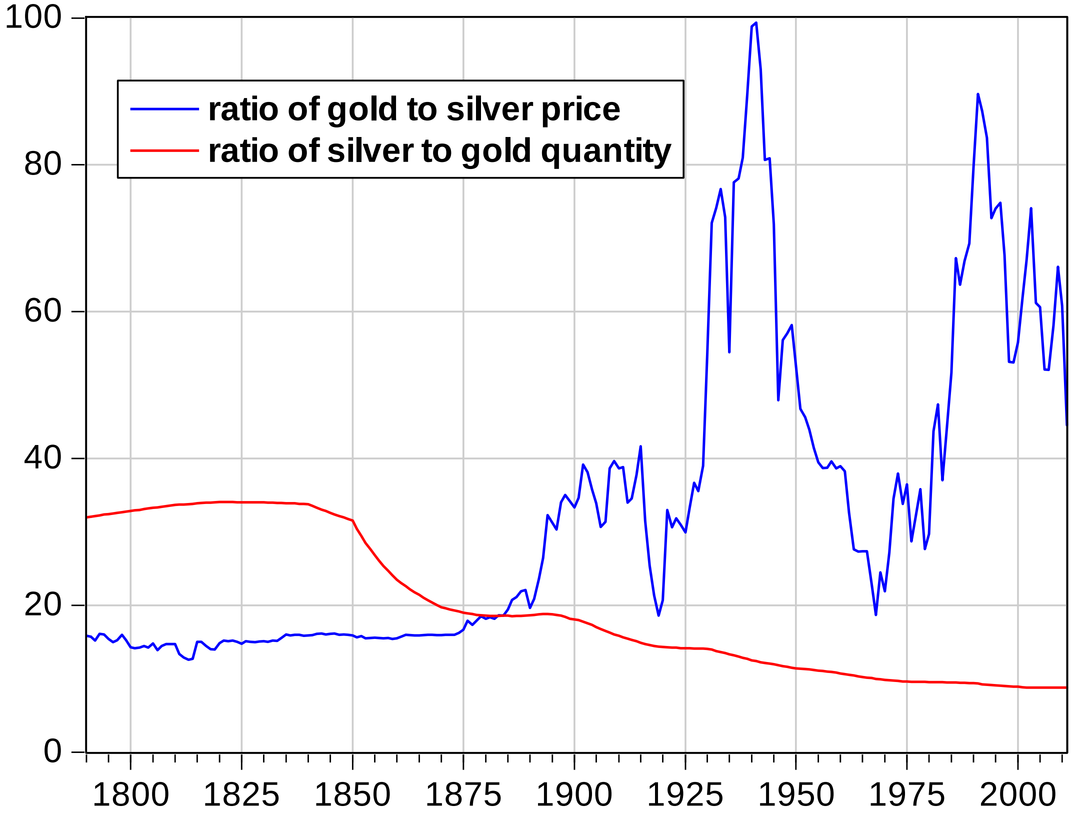
U.S. dollar price, divided by U.S. CPI



Note: See the notes to Table 1 for the sources of data on annual average U.S. dollar gold prices and CPI.

Figure 5

Ratios of Gold to Silver Prices and Quantities



Note: See the notes to Table 1 for the sources of data on annual average U.S. dollar gold and silver prices for 1790-2011. The accumulated stock of world gold comes from the sources detailed in the notes to Figure 1. Figure 5 uses the series corresponding to a world stock of zero in 1492 (see Figure 1). The sources for the data on world production of silver since 1493 are the same as those for gold. Figure 5 uses a series corresponding to a world stock of zero in 1492 and assumes zero depreciation and loss on the existing stock. The underlying data on world gold and silver production are annual since 1876 and at 5-, 10-, or 20-year intervals from 1790 to 1875.