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A MODEL OF THE SAFE ASSET MECHANISM (SAM):
SAFETY TRAPS AND ECONOMIC POLICY

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ABSTRACT

The global economy has a chronic shortage of safe assets which lies behind many re- cent macroeconomic imbalances. This paper provides a simple model of the Safe Asset Mechanism (SAM), its recessionary safety traps, and its policy antidotes. Public debt plays a central role in SAM as long as the government has spare fiscal capacity to back safe asset production. We show that Quantitative Easing type policies have positive effects on spreads and output. In contrast, Operation Twist type policies, where the duration of public debt held by the public is reduced, can be counterproductive. Mon- etary policy commitments work if they support future bad states of nature. All these policies depend on fiscal capacity. Once the latter runs out, short term cyclical policy becomes ineffective. In contrast, credible long run fiscal consolidation relaxes the fiscal capacity constraint and enhances the effectiveness of short term policy. An economy that is near its fiscal limits is susceptible to runs on its public debt and to destabilizing feedback loops. We also show that in a SAM world, safe asset producer economies can continuously run trade deficits supported by global demand for safe assets, but they do so at the risk of exhausting their fiscal capacity.

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1 Introduction

One of the main structural features of the global economy in recent years is the apparent shortage of safe assets (see, e.g., Caballero 2010 and references therein). This deficit provided the macroeconomic force for the financial engineering behind the subprime crisis, is a paramount factor in determining the spike in funding costs when European economies switch from the core to the periphery, and has put new constraints on the effectiveness of monetary and fiscal policy. However, while much has been written about the potential importance of this scarcity, there is little formal understanding of how this shortage influences the macroeconomy and of what is the appropriate policy response. This paper's main contribution is to provide a simple model of the Safe Asset Mechanism (SAM) and its policy antidotes.

The central problem in SAM is one of "excess" demand for safe assets. This financial market problem spreads to the real economy through multiple channels. On the aggregate supply side, the actions the private sector needs to undertake to create safe assets are contrived and costly for the economy. Essentially, during a flight to safety corporations trade less potential production for safer revenue. On the aggregate demand side, SAM is characterized by the strong downward pressure it puts on safe interest rates. If there is a limit on how much these rates can drop, a *safety-trap* emerges, akin to the Keynesian liquidity trap. In this context, private wealth declines and drags alone the real economy as well as asset demand, which restores equilibrium in asset markets. However, this equilibrating mechanism is inefficient because it operates through wealth destruction rather than through safe asset creation. Overall, SAM provides a parsimonious account for symptoms also found in environments experiencing the combined effect of a credit crunch and a liquidity trap.

In SAM, as in the more conventional credit crunch mechanism, a wedge develops between the risky and riskless rate. However, in SAM, it is the safe rate drop that drives this wedge, and it is this rate that is the main sign of trouble. Although in practice both mechanisms are likely to operate in conjunction, it is important to establish their differences as they do not respond equally to different policy packages. We will show, for example, that absent a positive "signaling" effect that reduces safe-asset demand, forward guidance policies are powerless in SAM, precisely because they operate by targeting (and failing) risky assets revaluation rather than through the safe asset expansion that SAM requires.

Public debt plays a central role in SAM as long as the government has spare fiscal capacity to back safe asset production. By increasing the supply of safe assets, public debt reduces the root imbalance in the economy. We use this framework to analyze the impact of different

policy options implemented recently in the US and other developed economies. The key issue is that the government owns a disproportionate share of the capacity to create safe assets, and the private sector owns too many risky assets. We show that swapping risky private assets for safe public debt, the so called Quantitative Easing (QE) type policies, have positive effects on spreads and output. In contrast, Operation Twist (OT) type policies, where the duration of public debt held by the public is reduced, can be counterproductive. The reason for the latter result is that long term public debt, by being a "bearish" asset that can be used to hedge risky private assets, has a *safe asset multiplier effect* that short term public debt lacks. That is, long term public debt is not only a safe asset in itself, but also makes risky private assets safer through portfolio effects. We also discuss the role of monetary policy commitments in a safety trap, and show that they alleviate the shortage only if they support future *bad* states of nature; the reason is that, as we mentioned earlier, in a SAM environment the main problem is the very low equilibrium safe rate, not the large risk-spread.

Fiscal policy is the goods-markets side of public debt policy. In fact, in our analysis the proceeds of public debt sales are fully returned to the private sector. The key concept here is that of *fiscal capacity*: How much public debt can the government credibly pledge to honor, should a major macroeconomic shock take place in the future? Once fiscal capacity runs out, that is, once the government's ability to create safe assets runs out, short term fiscal policy becomes ineffective. In contrast, credible long run fiscal consolidation becomes powerful, as it relaxes the fiscal capacity constraint and enhances the effectiveness of short term fiscal policy.

Our model also captures the instabilities that have characterized the so called periphery economies of the Eurozone: Given the importance of fiscal capacity in the production of safe assets, and the centrality of the latter in determining equilibrium output in a SAM environment, an economy that is near its fiscal limits is susceptible to runs on its public debt and to destabilizing feedback loops.

The SAM environment resembles in many respects the Keynesian liquidity trap environment, but it also has important differences with it. We devote a section to highlight these differences. For example, we show that QE type policies are more powerful in SAM than in liquidity trap environments. In contrast, fiscal policy and forward guidance are more effective in the latter than in SAM.

We conclude the paper with an Open Economy section, which we use to illustrate a point and to make a conjecture. The former is that in a SAM world, it is only natural that

safe asset producer economies benefit from an "exorbitant" privilege effect, where they can continuously run trade deficits supported by global demand for safe assets. The latter, the conjecture, is that as the composition of global income gradually shifts away from safe asset producer economies to emerging market economies, SAM will worsen over time.

Related literature. Our paper is related to several strands of literature. First and most closely related is the literature that identifies the shortage of safe assets as key macroeconomic fact (see, e.g.,. Caballero 2006 and 2010, Caballero and Krishnamurthy 2009, Bernanke et al 2011, Barclay's 2012). Our paper provides a model that captures many of the key insights in that literature and that allows us to study the main macroeconomic policy implications of this environment more precisely.

Second, the literature on aggregate liquidity (see e.g. Woodford 1990 and Holmstrom and Tirole 1998) analyzes the shortage of liquidity (stores of value). It has emphasized the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Our paper shares the idea that liquidity shortages are important macroeconomic phenomena, and that the government has a special role in alleviating them. However, it shifts the focus to a very specific form of liquidity—safe assets—and works out its distinct and unique consequences.

Third, there is a literature that documents significant deviations from the predictions of standard asset-pricing models—patterns which can be thought of as reflecting money-like convenience services—in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically (Krishnamurthy and Vissing-Jorgensen 2011, 2012, Greenwood and Vayanos 2010, Duffee 1996, Gurkaynak, Sack and Wright 2006, Bansal and Coleman 1996). Our model offers a distinct interpretation of these stylized facts, where the "specialness" of public debt is its safety.

Fourth, there is an emerging literature which emphasizes how the aforementioned premium creates incentives for private agents to rely heavily on short-term debt, even when this creates systemic instabilities (Gorton and Metrick 2010, 21012, Gorton 2010, Stein 2012, Greenwood, Hanson and Stein 2012, Woodford 2012, Gennaioli, Shleifer and Vishny 2012). Our model also considers how this premium affects private agent's balance sheets, but it offers distinct mechanisms for its source, and on how it affects the economy and macroeconomic policy.

Fifth, there is the literature on liquidity trap (see e.g. Keynes 1936, Krugman 1998, Eggertsson and Woodford 2003). This literature emphasizes that the binding zero lower

bound on nominal interest rates presents a challenge for macroeconomic stabilization. Our model of a safety trap shares elements of the Keynesian liquidity trap story. However, in our model, the key interest rate is the safe interest rate, and the root cause of safety traps is an acute safe asset shortage.

Sixth, our paper relates to an extensive literature, both policy and academic, on fiscal sustainability and the consequences of current and future fiscal adjustments (see, e.g., Giavazzi and Pagano 1990, 1996, Alesina and Ardagna 1998, IMF 1996, Guihard et al 2007). Our paper revisits some of the policy questions in this literature but highlights the government's capacity to create safe assets at the margin, as the key concept to determine the potential effectiveness of further fiscal expansions as well as the benefits of future fiscal consolidations.

Finally, our paper is related to a strand of literature on global imbalances. Caballero, Farhi and Gourinchas (2008a,b) developed the idea that global imbalances originated in the superior development of financial markets in developed economies, and in particular the US. Global imbalances resulted from an asset imbalance. In our paper, it is more specifically a safe-assets imbalance that lies behind global imbalances: The latter were caused by the funding countries' demand for financial assets in excess of their ability to produce them, but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to produce them.

The paper is organized as follows. Section 2 describes our basic model and introduces the key mechanism of a safety trap. Section 3 introduces public debt and considers the effects of quantitative easing types policies as well as monetary policy commitments. Section 4 investigates the role of fiscal policy, emphasizing the effects of fiscal stimulus and fiscal consolidation as well as the importance and endogeneity of a country's fiscal capacity and safe asset status. Section 5 develops a version of a liquidity trap in the context of our model and explains the similarities and differences with a safety trap. the Section 6 introduces a model with two regions and works out the consequences of an asymmetry in the regions' capacity to produce safe assets, making the link between safe asset shortage and global imbalances. Section 7 concludes.

2   A Model of SAM

In this section we describe SAM without government intervention.

2.1 Basic Model

Setup. The basic model has exogenous output. The goal is to characterize demand and supply of safe assets, and their impact on equilibrium returns.

Output is constant, X, unless a Poisson shock takes place, in which case it drops to $\mu X < X$ forever. We focus on the period before this shock takes place. The Poisson event happens with hazard λ, and we simplify the notation by studying the limit as $\lambda \to 0$.

Population has a perpetual-youth overlapping generations structure with death and birth rates θ. Agents consume only when they die, which yields a simple aggregate consumption function $C_t = \theta W_t$, where C_t and W_t represent aggregate consumption and wealth, respectively. Note that equilibrium in goods markets pins down the equilibrium value of aggregate wealth W_t at

$$W = \frac{X}{\theta}.$$

Next, we introduce two key ingredients of the model, one about asset demand, and one about asset supply. On the asset demand side, there are two types of agents in the economy: Neutral and (locally) Knightian. Neutral agents are risk-neutral. Knightian agents are infinitely risk averse (over short time intervals): between t and $t + dt$, when forming their portfolio, they act as if the Poisson shock happens in the next infinitesimal time interval with probability one. The fraction of Neutrals in the population is $1 - \alpha$. The fraction of Knightians is α. We denote the wealth of Neutral and Knightian agents by W_t^N and W_t^K with

$$W_t^N + W_t^K = W.$$

On the asset supply side, we assume that a fraction δ of output X is pledgeable and accrues as the total dividend of Lucas trees (each tree capitalizes a stream of δ units of goods per period). The rest $(1 - \delta) X$ is distributed to newborns. The total value of assets before the Poisson shock is V and from financial market equilibrium we have:

$$V = W = \frac{X}{\theta}.$$

We assume that only a fraction ρ of these assets can be tranched to split the risky and riskless component of returns. We denote by V^μ and V^r the supply of safe and risky assets with

$$V = V^\mu + V^r.$$

A safe asset is one whose value does not change when the Poisson event takes place. Thus, we can find V^μ by solving backwards and noting that by construction a fraction ρ of the total value of assets *after* an adverse Poisson shock is safe:

$$V^\mu = \rho\mu\frac{X}{\theta}.$$

Risky assets (before the Poisson event) are worth the residual $V - V^\mu$:

$$V^r = (1 - \rho\mu)\frac{X}{\theta}.$$

Since Knightian agents only hold safe assets, their wealth holdings, W_t^K, must satisfy:

$$W_t^K \le V^\mu.$$

Let r, r^K, and δ^μ denote the (ex-ante) rate of return on risky assets, the rate of return on safe assets, and the dividend paid by safe assets, respectively. Then equilibrium is characterized by the following equations:

$$r^K V^\mu = \delta^\mu X,$$

$$rV^r = (\delta - \delta^\mu)X,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha(1-\delta)X + r^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1-\alpha)(1-\delta)X + rW_t^N,$$

$$W_t^K + W_t^N = V^\mu + V^r.$$

Two regimes. There are two regimes depending on whether the constraint $W_t^K \le V^\mu$ is slack (unconstrained regime) or binding (constrained regime).

In the unconstrained regime, Neutrals are the marginal holders of safe assets, and hence safe and risky rates must be equal. A few steps of algebra show that in this case:

$$\delta^\mu = \delta\rho\mu,$$

$$r = r^K = \delta\theta.$$

The interesting case for us is the *constrained* regime, which captures the safe asset short-

age environment. In it, Knightians gobble up all safe assets and wish they had more, so that:

$$W^K = V^\mu = \rho\mu\frac{X}{\theta}.$$

It is easy to verify that this regime holds (after possibly a transitional period) as long as $\alpha > \rho\mu$.[1] The latter is the *safe asset shortage condition*, which we shall assume holds henceforth. In this case we have:

$$\delta^\mu = \delta\rho\mu - (\alpha - \rho\mu)(1 - \delta) < \delta\rho\mu,$$

$$r^K = \delta\theta - (1 - \delta)\theta\frac{\alpha - \rho\mu}{\rho\mu} < \delta\theta,$$

$$r = \delta\theta + (1 - \delta)\theta\frac{\alpha - \rho\mu}{1 - \rho\mu} > \delta\theta.$$

It follows that in this region there is a safety premium

$$r - r^K = (1 - \delta)\theta\frac{\alpha - \rho\mu}{\rho\mu(1 - \rho\mu)} > 0.$$

The supply of safe assets is determined by the severity of the potential shock (μ) and the ability of the economy to create safe assets (ρ). In fact ρ and μ enter the equilibrium equations only through the sufficient statistic $\rho\mu$. Similarly, the demand for safe assets is summarized by the fraction of Knightians (α). Together, these sufficient statistics determine whether we are in the unconstrained regime ($\alpha < \rho\mu$) or in the constrained regime ($\alpha > \rho\mu$).

Discussion of our setup. Our model features two forms of market incompleteness. The first one is tied to our overlapping generations structure. As a result, our environment is non-Ricardian and asset supply (δ) matters. The second market incompleteness is that only a fraction of trees (ρ) can be tranched.[2] Tranching is desirable because it allows to

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[1] A delicate issue is the initial value of risky assets. It is possible for the value of risky assets (and hence total assets) to jump at date 0 and so we distinguish 0^- and 0^+. We denote by $\beta_{0_-}^\mu$ and $\beta_{0_-}^r$ the fraction of safe assets and risky assets initially owned by Knightians. The initial budget constrained of Knightians implies

$$\beta_{0_-}^r V_{0_-}^r \le (1 - \beta_{0_-}^\mu)\rho\mu\frac{X}{\theta},$$

and the absence of arbitrage for Neutrals requires that

$$V_{0_-}^r \le V_{0_+}^r = (1 - \rho\mu)\frac{X}{\theta}.$$

[2] A simple extension of our model allows us to think about pooling in addition to tranching. We refer the

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break down an asset into a safe tranche which can be sold to Knightian agents and a risky tranche that can only be sold to Neutral agents. Because agents cannot tranch assets at will, Modigliani-Miller fails in the sense that in the constrained regime the value v^t of a unit of tranched tree is higher than that of an untranched tree v^{nt}. This gap widens as the safe asset shortage $(\alpha - \rho\mu)$ worsens:

$$v^t - v^{nt} = \frac{1}{\theta} \frac{\alpha - \rho\mu}{\rho} \frac{1 - \delta}{1 - \rho\mu - (1 - \alpha)(1 - \delta)}.$$

2.2 Flight to Safety and Output Loss

Up to now we have assumed production is exogenous and focused on the spread side of SAM. But in reality production and demand decisions are also strongly influenced by risk perceptions and the corresponding spreads. In this section we focus on a supply side mechanism, while in the next one we turn to demand.

Corporations undertake a host of risk-reducing measures, such as cash preservation and productive investment postponement, despite the impact of these measures on expected revenue.[3] We capture this flight to safety aspect of SAM by adding a simple tradeoff between risk and return in production. We assume that tranching a tree into a safe part and a risky part consumes resources (which in reality could be thought of as the resources consumed by a sophisticated financial system). In particular, a tranchable tree generates a lower dividend $\hat{\delta} < \delta$.[4] Of course the advantage of tranching is that the safe part can be sold at a high price.

We denote by $\tilde{\rho} \in (0, 1)$ the fraction of trees that are tranched in equilibrium. It is easier to change variables and define

$$\rho \equiv \frac{\tilde{\rho} \frac{\hat{\delta}}{\delta}}{1 - \tilde{\rho} \left(1 - \frac{\hat{\delta}}{\delta}\right)}.$$

We exploit the monotonic (increasing) mapping between ρ and $\tilde{\rho}$ and work directly with ρ, for the analysis becomes analogous to the baseline model with this endogenous value of ρ. We now turn to its determination.

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reader to the appendix for this extension.

[3]During risk-off episodes, corporations often try to replicate bond-like payouts from their equity by implementing equity buy-back policies and predictable dividend distribution streams.

[4]For simplicity, we assume that the decision to pay the production cost does not change the fraction of the output of a tree which is capitalized. In other words, each tranched tree yields an output $\frac{\hat{\delta}}{\delta}$, a fraction δ of which is a capitalized dividend of $\hat{\delta} = \delta \frac{\hat{\delta}}{\delta}$.

In order for the equilibrium that we have characterized to be valid, it must be the case that the value of an untranched tree

$$v^{nt} = \frac{1}{\theta} \frac{\delta}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}}$$

is the same as that of a tranched tree

$$v^t = \frac{\mu}{\theta} \frac{\hat{\delta}}{\delta} \left[1 + \frac{\frac{\alpha(1-\delta)}{\rho\mu} - \left(1 - \frac{\delta}{\mu}\right)}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}} \right].$$

A necessary and sufficient condition for this is

$$\frac{\delta}{1-\delta} = \frac{\frac{\hat{\delta}}{\delta}}{1 - \frac{\hat{\delta}}{\delta}} \frac{\alpha - \rho\mu}{\rho(1 - \rho\mu)}.$$

The left hand side is constant. The right hand side is decreasing in ρ with a value of ∞ at $\rho = 0$ and of $\frac{\frac{\hat{\delta}}{\delta}}{1-\frac{\hat{\delta}}{\delta}} \frac{\alpha-\mu}{1-\mu}$ at $\rho = 1$. The solution always features $\rho > 0$, and satisfies $\rho < 1$ as long as $\frac{\delta}{1-\delta} > \frac{\frac{\hat{\delta}}{\delta}}{1-\frac{\hat{\delta}}{\delta}} \frac{\alpha-\mu}{1-\mu}$.[5]

The solution ρ is decreasing in μ and increasing in α. The effect of μ directly impacts the attractiveness of tranching, independently of its effect on the safety premium. The effect of α is entirely driven by the safety premium: the higher α, the higher the safety premium, the more agents fly to safety.

In summary, SAM affects production because in the absence of effective tranching mechanisms to satisfy demand for safe assets, production is distorted toward generating more safe assets and increase valuations. Safety rather than revenue maximization is what financial markets demand.

2.3 Safety Trap

Let us now return to the exogenous ρ model and focus on an alternative, demand type, mechanism connecting SAM to output contraction.

As the potential shock becomes more extreme (i.e., as μ drops), or the economy's ability

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[5]Note that we can have a corner solutions at $\rho = 1$ where the value of an untranched tree is less than the value of a tranched tree.

to create safe assets is more impaired (ρ drops), the supply of safe assets shrinks ($\rho\mu$ drops). Equilibrium is restored by a decline in r^K which lowers demand for safe assets. But what if some force prevents the reduction in r^K? Here we isolate the main insight that follows from that question with a disequilibrium model where we simply assume that r^K cannot be reduced below a binding value $\underline{r}^K$.

This extra restriction forces us to consider disequilibrium in some other market, which we assume to be the goods market as this connects our discussion with standard Keynesian demand arguments. We introduce a distinction between potential output X and actual output ξX. When $\xi < 1$, output is below potential. We reinterpret endowments of goods $(1 - \delta) X$ and dividends δX as endowments of a non-traded input (say labor) that can be converted into output one-to-one. When $\xi < 1$, less of this input is converted into output. This modelling strategy essentially sidesteps the labor market. In every period, there is a goods market and an asset market. Dying old agents supply assets and demand goods. Survivors and newborns demand assets and supply goods.

We postpone the discussion of the methodological issues regarding disequilibrium modelling and of the substantive issues regarding the origins of the lower bound $\underline{r}^K$. For now, we proceed to show how a recession can restore equilibrium in the safe asset market.

Let us work backwards and recall that after the negative Poisson shock hits (which never literally happens in our model since $\lambda \to 0$), uncertainty disappears and so does the safe asset shortage. This means that actual and potential output coincide after the shocks and therefore the value of safe assets (before the shock) is still given by

$$V^\mu = \rho\mu \frac{X}{\theta}.$$

Note that, mechanically, this model is identical to the basic model but with $\rho\mu$ replaced by $\frac{\rho\mu}{\xi}$ and X replaced by ξX. The requirement that $r^K = \underline{r}^K$ determines the severity of the recession ξ:

$$\underline{r}^K = \delta\theta - (1 - \delta)\,\theta \frac{\alpha - \frac{\rho\mu}{\xi}}{\frac{\rho\mu}{\xi}},$$
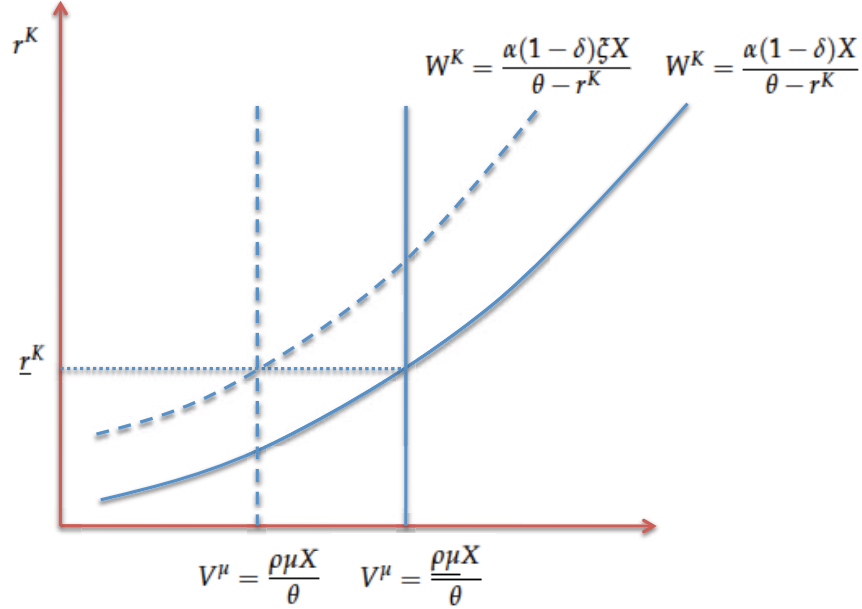
yielding

$$\xi = \frac{\rho\mu}{\underline{\rho\mu}} < 1,$$

where $\underline{\rho\mu}$ corresponds to the value of these combined parameters for which $\underline{r}^K$ is the equi-

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librium safe interest rate.[6]

Figure 1: Safety trap.

$$W^K = \frac{\alpha(1-\delta)\xi X}{\theta - r^K} \qquad W^K = \frac{\alpha(1-\delta)X}{\theta - r^K}$$

$$V^\mu = \frac{\rho\mu X}{\theta} \qquad V^\mu = \frac{\rho\mu X}{\theta}$$

Recession caused by a decrease in the supply of safe assets. The safe asset supply curve shifts left ($\rho\mu < \underline{\rho\mu}$), the endogenous recession shifts the safe asset demand curve left ($\xi < 1$), and the safe interest rate remains unchanged at $\underline{r}^K$.

Because the safe interest rate r^K cannot adjust downwards, there is a recession. This mechanism is akin to that of a liquidity trap. We call it a *"safety trap"*. A recession lowers the absolute demand for safe assets while keeping the absolute supply of safe assets fixed and restores equilibrium. Figure 1 illustrates this mechanism.

The key is to understand why when $r^K = \underline{r}^K$, a recession can restore equilibrium in the safe asset market following a shock that reduces the supply of safe assets. The supply of safe assets is given by $V^\mu = \rho\mu\frac{X}{\theta}$ and the demand for safe assets is given by $W^K = \frac{\alpha(1-\delta)\xi X}{\theta - r^K}$. Equilibrium in the safe asset market requires that $W^K = V^\mu$, i.e.

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[6]Interestingly, the risky interest rate r is increasing in ξ, so that the deeper the recession, the lower is r:

$$r = \delta\theta + (1 - \delta)\,\theta \frac{\alpha - \frac{\rho\mu}{\xi}}{1 - \frac{\rho\mu}{\xi}}.$$

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$$\frac{\alpha\left(1-\delta\right)\xi X}{\theta-r^{K}}=\rho\mu\frac{X}{\theta}.$$

Consider a shock that lowers the supply of safe assets (a reduction in $\rho\mu$). The mechanism by which equilibrium in the safe asset market is restored has two parts. The first part immediately reduces Knighitan wealth W^{K} to a lower level, consistent with the lower supply for safe assets $\rho\mu\frac{X}{\theta}$. The second part is a mechanism that maintains Knightian wealth W^{K} at this lower level.

The first part of the mechanism is as follows. The economy undergoes an immediate wealth adjustment (the wealth of Knighitans drops) through a round of trading between Knightians and Neutrals born in previous periods. At impact, Knightians hold assets that now carry some risk. They react by selling the risky part of their portfolio to Neutrals. This shedding of risky assets catalyzes an instantaneous fire sale whereby the price of risky assets collapses before immediately recovering once risky assets have changed hands.

The second part of the equilibrating mechanism differs depending on whether the safe interest rate r^{K} is above or at the lower bound $\underline{r}^{K}$. If $r^{K}>\underline{r}^{K}$, then a reduction in the safe interest rate r^{K} takes place. This reduction in the safe interest rate effectively limits the growth of Knightian wealth so that the safe asset market remains in equilibrium. If the safe interest rate is against the lower bound $r^{K}=\underline{r}^{K}$, then this reduction in the safe interest rate cannot take place. With full capacity utilization and $r^{K}=\underline{r}^{K}$, the growth rate of Knightian wealth is too high and an excess demand for safe assets would develop over time. Instead, a recession takes place (a reduction in ξ) which reduces the income of Knightians (newborns) and hence the growth of Knightian wealth.

Note that the recession drags down the whole economy, reducing not only the income of Knigthians, but also that of Neutrals (the dividends on risky assets and the income of Neutrals newborns) and hence the wealth of Neutrals. Of course, the flip side of this reduction in Neutral wealth is a reduction in the value of risky assets, which occurs through a reduction in dividends (and despite a decrease in the risky interest rate r). The reduction in Neutral wealth in turn reduces demand in the goods market, thereby justifying the recession.[7]

A similar logic applies if we raise the share of Knightian agents α instead of reducing $\rho\mu$,

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[7]Note that the adjustment in Knightian wealth is the same whether the safe interest rate r^{K} is or not at the lower bound $\underline{r}^{K}$. What is different is the adjustment in Neutral wealth. In response to a negative shock to the value of safe assets, Neutral wealth ends up at a lower level when the safe interest rate is against the lower bound than when it can freely adjust downwards.

in which case the recession factor is

$$\xi = \frac{\underline{\alpha}}{\alpha} < 1.$$

This interpretation resembles the paradox of thrift. Combining both, asset supply and demand factors, we have that the severity of the recession is determined by the sufficient statistic $\frac{\rho\mu}{\alpha}$ according to the simple equation:

$$\xi = \frac{\underline{\alpha}}{\underline{\rho\mu}} \frac{\rho\mu}{\alpha}.$$

Although our framework is not designed to discuss normative issues, in practice it seems evident that a recession is a costly mechanism to restore equilibrium in safe asset markets. We turn to policy options in the next section.

Discussion. Before concluding this section we address two sets of issues: (1) methodological issues regarding disequilibrium modelling, and (2) substantive issues regarding the origins of the lower bound on safe interest rates $\underline{r}^K$. We address each in turn.

(1) One possibility is to use disequilibrium theory (see e.g. Barro and Grossman 1971, Malinvaud 1977, Benassy 1986, Hall 2011a,b and Kocherlakota 2012 for more recent applications). There is excess supply in the goods market and excess demand in the asset market, and in particular an excess demand for safe assets. The gap between the notional and effective supply of goods and demand for assets can be formalized by supplementing the budget constraints of surviving old agents and newborns with quantity constraints (the demand for goods, and the supply of assets). The analysis then characterizes how a disequilibrium in the asset market spills over to a disequilibrium in the goods markets.[8]

Another possibility is to use a model closer to the New Keynesian tradition by introducing sticky prices and a monetary authority. In this setting, in every period, non-traded inputs are used to produce differentiated varieties of goods. These differentiated varieties of goods are valued by consumers according to a standard Dixit-Stiglitz aggregator. Each variety is sold by a monopolistic firm. Firms post prices. Prices are entirely fixed (an extreme form of sticky prices) and firms accommodate demand at the posted price. Finally, a monetary authority sets a safe nominal interest rate i^K for Knightians. Because prices are rigid, this determines the real interest rates $r^K = i^K$. Note that because relative prices are fixed, the demand for all varieties is the same. Output is demand-determined, and as a result, capacity

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[8]With this formalization, we need to impose the extra requirement that the utilization rate ξ of non-traded inputs is uniform. In other words, we are assuming that total demand is distributed uniformly across all production units.

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utilization rate ξ is the same for all firms (the recession is economywide).

(2) In the disequilibrium theory, the lower bound for r^K is left unexplained. In the New Keynesian there are at least two motivations for $\underline{r}^K$. First, one possibility is that there is some discontinuity in Knightians preferences: they cannot survive or suffer a very large utility loss with returns below $\underline{r}^K$ (but they still strictly prefer to hold only riskless assets over risky assets when the return on the former is $\underline{r}^K$). This could also be a stand in for a discontinuity on the supply side, stemming from institutional and cost constraints on the intermediation sector (e.g., money markets may switch in mass to gambling for resurrection once rates drop to sufficiently low levels). Depending on the exact specification, there would either be no equilibrium with $r^K < \underline{r}^K$ or only very bad equilibria (from a welfare perspective). Then either the monetary authority could be unable to set a nominal interest rate i^K below $\underline{r}^K$ or it would choose not to. A second possibility is that the lower bound on $\underline{r}^K$ is simply the zero lower bound on nominal interest rates which arises because of a no-arbitrage condition between money and bonds.[9,10,11]

3 Public Debt, Balance Sheet Policies and Monetary Policy Commitments in SAM

In this section we discuss the safe-asset aspect of several conventional macroeconomic policies used to address severe recessions stemming from financial crises.

We start by introducing public debt and discussing the role of public purchases and sales

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[9]There are several ways to introduce money into the model. An extreme version of a Money In the Utility (MIU) model is as follows. The demand for real money balances for transactional services is $\varepsilon > 0$. When $i^K > 0$, money is held only for transaction services. When $i^K = 0$ money is also held as a store of value by Knigthians. Assuming that the real money supply is constant $m = \varepsilon$, we are led to a model that becomes isomorphic to the one in the text in the cashless limit as $\varepsilon \to 0$.

[10]We have also analyzed a Cash In Advance (CIA) version of the model where agents hold a fraction κ of their wealth in the form of real money balances. This model features an additional mechanism: As Knightians begin to demand money as a store of value, the available supply of transaction money contracts for Neutrals, which in turn reduces output and restores equilibrium.

[11]Introducing money raises the question of whether printing money can be a solution to the recession. Consider for example the MIU model. If ε is small, money does not relax much the fiscal capacity of the government: issuing money while at the zero bound is equivalent to issuing short-term bonds, and both are constrained by the long-term fiscal capacity of the government If prices remain rigid after the Poisson shock, then the government must raise taxes to retire the additional money that has issued before the Poisson shock. If prices adjust after the Poisson shock, then the government can choose not to retire the extra money that it has issued, but this then leads to a jump in the price level, and money (and short term nominal bonds) cease to be safe assets and hence their issuance prior to the Poisson shock does little to alleviate the recesssion.

of such debt. To isolate the insights of this section we assume for now that private trees cannot be tranched at all ($\rho = 0$), and hence cannot produce safe assets by themselves.

We first introduce short-term public debt and study the impact of Quantitative Easing (QE) policies that swap trees for short-term public debt. These policies act to increase the supply of safe assets and therefore help reduce the safety premium. If the economy is in a safety trap and output is below potential, they have a stimulating effect on output.

We then introduce long-term public debt and study Operation Twist (OT) policies that swap long-term debt for short-term debt. In contrast to QE policies, we show that such policies can backfire. Basically, replacing long term debt by short term debt reduces the supply of safe assets. This happens despite the fact that short term debt is safe. The reason is that long term debt has a *multiplier* effect on safe asset creation which short term debt does not have: Long term debt is a "bearish" asset that can be combined with risky private assets to create safe assets. If the economy is in a safety trap so that output is below potential, OT policies have a depressing effect on output.

We then discuss the effects of forward guidance policies (commitments to low future interest rates) in a safety trap. We show that such commitments are not likely to be effective at stimulating output in a safety trap, since they are relatively powerless at increasing the value of safe assets and hence alleviate the safe asset shortage.

3.1 Short-Term Public Debt and Quantitative Easing

Introducing short-term public debt. The government taxes dividends, δX. The tax rate τ^μ after the Poisson shock occurs is fixed, while the tax rate before the Poisson shock is set to a value τ that satisfies the government flow budget constraint. The government issues a fixed amount of risk-free bonds that capitalize future tax revenues and pays a variable rate r_t^K. It is the latter feature that makes this debt "short-term," since its value remains constant over time as its coupons vary with the riskless rate. The proceeds of the sales of these bonds are rebated lump-sum to agents at date 0. Hence in this model *government debt acts exactly like tranching, with τ^μ playing the role of ρ.*

Let the value of public debt be given by D, then we have

$$D = \tau^\mu \mu \frac{X}{\theta}.$$

The equilibrium is described by the following equations:

$$r^K D = \tau \delta X,$$

$$rV = \delta (1 - \tau) X,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) X + rW^N.$$

$$W_t^K + W_t^N = D + V,$$

$$W_t^K \leq D \text{ and } r^K \leq r.$$

At a steady state of the constrained regime we have

$$W^K = D = \tau^\mu \mu \frac{X}{\theta}, \qquad W^N = V = (1 - \tau^\mu \mu) \frac{X}{\theta}$$

and

$$\delta \tau = \tau^\mu \mu - \alpha (1 - \delta),$$

$$r^K = \delta \theta - (1 - \delta) \theta \frac{\alpha - \tau^\mu \mu}{\tau^\mu \mu},$$

$$r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \tau^\mu \mu}{1 - \tau^\mu \mu}.$$

The economy is in the constrained regime if and only if $\alpha > \tau^\mu \mu$, which we assume. The safety premium is then given by

$$r - r^K = \theta (1 - \delta) \frac{\alpha - \tau^\mu \mu}{\tau^\mu \mu (1 - \tau^\mu \mu)} \geq 0.$$

In this model, the supply of safe assets comes entirely in the form of short-term public debt. The supply of the latter is determined by a notion of fiscal capacity, as measured by $\tau^\mu \mu$. *The larger fiscal capacity, the more short-term debt the government can issue, the larger the supply of safe assets and the lower the safety premium.*

Quantitative easing. We model QE as follows. The government purchases trees and issues additional short-term debt. Let $\hat{\beta}^g$ be the fraction of the trees purchased by the government. Let $\hat{D}$ be the value of government debt and let $\hat{\tau}^\mu$ be the new value of taxes after the Poisson shock (which must satisfy $\hat{\tau}^\mu \leq \tau^\mu$). We continue to assume that the stock

of short-term debt is unchanged before and after the Poisson shock. We have

$$\hat{D} = \hat{\tau}^{\mu}(1 - \hat{\beta}^g)\mu\frac{X}{\theta} + \hat{\beta}^g\mu\frac{X}{\theta}.$$

As long as

$$\hat{\tau}^{\mu}(1 - \hat{\beta}^g) + \hat{\beta}^g > \tau^{\mu},$$

the safe asset shortage is alleviated by this policy: r^K increases, r decreases, and the safety premium shrinks. QE here works not so much by removing private assets from private balance sheets, but rather by injecting public assets into private balance sheets. In other words, QE works by increasing the supply of safe assets. The government can expand this supply either because it has excess fiscal capacity or because it can implicitly tranch assets in a way the private sector cannot.[12]

If the economy is in a safety trap where the safe interest rate is fixed at $\underline{r}^K$ and output is

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[12]This mechanism has some commonality with the idea in Holmstrom and Tirole (1998) that the government has a comparative advantage in providing liquidity. In their model this result arises from the assumption that some agents (consumers in their model) lack commitment and hence cannot borrow because thay cannot issue securities that pledge their future endowments. This can result in a scarcity of stores of value. The government can alleviate this scarcity by issuing public debt and repaying this debt by taxing consumers. The proceeds of the debt issuance can actually be rebated to consumers. At the aggregate level, this essentially relaxes the borrowing constraint of consumers: they borrow indirectly through the government. The comparative advantage of the government in providing liquidity arises from its unique regalian taxation power:It is essentially better than private lenders at collecting revenues from consumers.

below potential with $\xi < 1$, then QE acts by stimulating output, increasing it from ξ to:[13,14]

$$\frac{\hat{D}}{D}\xi > \xi.$$

3.2 Long-Term Public Debt and Operation Twist

Introducing long-term public debt. The key aspect of long-term debt that we wish to capture here is its bearish nature (i.e., its ability to generate capital gains during periods of distress). In reality this feature stems from its fixed coupons and the drop in safe rates during contractions. While this drop holds in our model in the pre-Poisson phase when safety demand rises, it does not at the Poisson event since uncertainty is resolved once this shock takes place. Rather than adding shocks within the post-Poisson phase—for example a fall in interest rates induced by a deleveraging/increased in desired savings shock could be roughly captured by a decrease in θ after the Poisson shock—we introduce the bearishness of long term debt through a simple transfer mechanism.

Let us assume that after the Poisson shock, a fraction ϕ of tax revenues $\tau^{\mu}\mu\delta$ is committed to servicing long term debt D^{long}, and the remaining fraction $(1-\phi)$ is committed to servicing short term debt D^{short}. Before the Poisson shock, we assume that tax revenues and debt policy are adjusted as follows: First, the value of outstanding short term debt is the same

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[13]In general, QE might require a transition phase where the government raises taxes before the Poisson shock in order to gradually acquire those assets. The analysis in the main text assumes that this adjustment has taken place and examines the consequences of the eventual buildup of such a portfolio. After this portfolio buildup phase, taxes are actually lower at $\hat{\tau}$ determined by:

$$\hat{\tau} = \underline{r}^K \frac{1}{\delta\theta}\frac{\tau^{\mu}\mu}{\xi} - \frac{1}{\theta}\hat{\beta}^g < \tau = \underline{r}^K \frac{1}{\delta\theta}\frac{\tau^{\mu}\mu}{\xi}.$$

[14]In certain circumstances, it is possible to design QE policies that do not require a buildup phase with increased taxes before the Poisson shock, i.e. such that the debt issuance more than covers the asset purchases. The condition is $\hat{\beta}^g\left[\frac{\hat{\xi}X}{\theta} - \hat{D}\right] \leq \hat{D} - D$, where $\hat{\xi} = \frac{\hat{D}}{D}\xi$, which assuming that $\hat{\tau}^{\mu} = \tau^{\mu}$, can be shown after some manipulation to boil down to

$$\frac{\xi - \tau^{\mu}\mu}{\mu - \tau^{\mu}\mu}\frac{\tau^{\mu} + \hat{\beta}^g(1 - \tau^{\mu})}{\tau^{\mu}} \leq 1.$$

Hence if the economy is depressed enough so that $\xi < \mu$, then it is possible to build up a QE portfolio without raising taxes, that stimulates output with $\hat{\beta}^g$ determined by the above equation. Actually if $\xi < \tau^{\mu}\mu$, then building up the QE portfolio can actually generate immediate revenues.

as after the Poisson shock (this is what makes it short term debt)

$$D^{short} = (1 - \phi)\tau^\mu \mu \frac{X}{\theta}.$$

Second, the value of long term debt is kept at a level

$$D^{long} = \psi \tau^\mu \phi \mu \frac{X}{\theta}$$

below its value $\phi\mu\frac{X}{\theta}$ after the Poisson shock, where $\psi \in (0, 1)$. This is our simple mechanism to introduce the macro insurance aspect of long term debt; the lower is ψ, the larger is the macro insurance aspect of long term debt. The required tax rate τ before the Poisson shock so that such a policy is feasible is defined implicitly by

$$r^K D^{short} + r D^{long} = \tau \delta X.$$

In this context the value of trees is

$$V = [1 - \tau^\mu \mu(1 - (1 - \psi)\phi)]\frac{X}{\theta}$$

which is increasing in ϕ, the share of long term debt in public debt. This effect is stronger, the larger is the insurance aspect of long term debt (the lower is ψ). The reason is that the presence of long term debt acts as a hedge for risky trees, and therefore allows the transformation of risky assets into safe through portfolio construction.

Let x be the fraction of the trees that uses all the stock of long term debt to generate safe asset. This fraction x is determined from the condition that the value of a safe asset can not jump at the Poisson event:

$$x[1 - \tau^\mu \mu(1 - (1 - \psi)\phi)]\frac{X}{\theta} + \psi \phi \tau^\mu \mu \frac{X}{\theta} = x(1 - \tau^\mu)\mu\frac{X}{\theta} + \phi \tau^\mu \mu \frac{X}{\theta},$$

i.e.
$$x = \frac{\phi(1 - \psi)\tau^\mu \mu}{1 - \mu + \phi(1 - \psi)\tau^\mu \mu}.$$

The total supply of safe assets is then given by

$$
\begin{aligned}
V^\mu &= D^{short} + \frac{1}{\psi} D^{long} + x(1 - \tau^\mu)\mu \frac{X}{\theta} \\
&= \tau^\mu \mu \frac{X}{\theta} + x(1 - \tau^\mu)\mu \frac{X}{\theta},
\end{aligned}
$$

which can be rewritten as

$$
V^\mu = \Gamma(\phi, \psi, \mu, \tau^\mu) \tau^\mu \mu \frac{X}{\theta},
$$

where

$$
\Gamma(\phi, \psi, \mu, \tau^\mu) = 1 + \frac{\phi(1 - \psi)(1 - \tau^\mu)\mu}{1 - \mu + \phi(1 - \psi)\tau^\mu \mu} \geq 1.
$$

The supply of risky assets is simply given by the residual

$$
V^r = \frac{X}{\theta} - V^\mu.
$$

The safe and risky interest rates can be computed to be

$$
r^K = \delta\theta - (1 - \delta)\theta \frac{\alpha - \Gamma(\phi, \psi, \mu, \tau^\mu)\tau^\mu \mu}{\Gamma(\phi, \psi, \mu, \tau^\mu)\tau^\mu \mu},
$$

$$
r = \delta\theta + (1 - \delta)\theta \frac{\alpha - \Gamma(\phi, \psi, \mu, \tau^\mu)\tau^\mu \mu}{1 - \Gamma(\phi, \psi, \mu, \tau^\mu)\tau^\mu \mu}.
$$

Operation twist. Clearly r and r^K are respectively decreasing and increasing in Γ, and Γ is increasing in ϕ. Therefore r is decreasing in ϕ and r^K is increasing in ϕ. If we interpret OT as a shortening of public debt maturity held by the private sector, i.e. a lowering of ϕ, we have that OT decreases r^K and increases r. That is, replacing long term debt by short term debt reduces the supply of safe assets. This happens despite the fact that short term debt is safe. Long term debt is a bearish asset that can be combined with risky assets to create safe assets, thus an expansion in long term debt not only creates the safe debt that is intrinsic to backed public debt, but also creates a hedge that transforms some risky private assets into safe assets. In contrast, short term debt does not have this additional hedging effect and only expands the supply of safe assets one for one.

If the economy is in a safety trap where the safe interest rate is fixed at $\underline{r}^K$ and output is

below potential with $\xi < 1$, then an OT policy that lowers ϕ to $\hat{\phi} < \phi$, further reduces ξ to

$$\frac{\Gamma(\hat{\phi}, \psi, \mu, \tau^{\mu})}{\Gamma(\phi, \psi, \mu, \tau^{\mu})} \xi < \xi.$$

A similar result can be obtained in the endogenous tranching model, since output is an increasing function of r^K.

3.3 Monetary Policy Commitments

In a safety trap, policy commitments that work support future *bad* states. This is a higher level of requirements than in the standard New-Keynesian liquidity-trap mechanism where *any* future wealth increase has the potential to unstuck the economy. We illustrate this point with an example of forward guidance policy that would work in a New-Keynesian environment but not in SAM.

Since public debt is not key to our main concern here, we revert temporarily to the model of Section 2 where there are only private assets, and focus on the New Keynesian version with rigid prices.

We introduce two extensions to that model. First, we introduce the possibility of another independent Poisson shock, which raises output to $\gamma X > X$ and removes the possibility of the bad shock. It occurs with nonzero intensity λ^G. We refer to this shock as the *good* shock, in contrast to the bad Poisson shock that concerns Knightians in our main analysis. Second we allow agents to produce $\zeta > 1$ units of output per unit of input. However, we imagine that there is a large utility loss from doing so.

This model works like that in Section 2. It features the possibility of a safety trap, indeed ξ is determined by the exact same equation. The only difference is in the (risky) interest rate r. The interest rate r is now determined by the following set of equations (and λ^G only enters the last of these equations):

$$V^r = (\xi - \rho\mu) \frac{X}{\theta},$$

$$V^{\mu} = \rho\mu \frac{X}{\theta},$$

$$\underline{r}^K V^{\mu} = \delta^{\mu} X,$$

$$rV^r = \xi\delta X - \delta^\mu X + \lambda^G (\gamma - \xi) \frac{X}{\theta}.$$

This yields

$$r = \frac{\xi\delta\theta - \underline{r}^K \rho\mu + \lambda^G (\gamma - \xi)}{\xi - \rho\mu}.$$

In New-Keynesian models of the liquidity trap (see e.g. Krugman 1998, Eggertsson and Woodford 2003 and Werning 2012), committing to keeping interest rate low in the future once the economy recovers (after the good Poisson shock) stimulates the economy—a policy often referred to as forward guidance. The latter works by creating a boom in the future, which raises current demand through a combination of a wealth effect (higher income in the future) and substitution effect (lower real interest rates because of inflation). Our model shuts down the latter mechanism, and the former is ineffective since what matters is the perceived wealth of Knightians not that of Neutrals.

In our model a commitment to low interest rates after the good Poisson shock only increases the value of risky assets. Consider the following policy. Suppose that the good Poisson shock occurs at τ. After the good Poisson shock, the central banks stimulates the economy by setting the interest rate i_t below the natural interest rate $\delta\theta$ until $\tau + T$, at which points it reverts to setting the nominal interest rate equal to the natural interest rate $i = \delta\theta$. For $t > \tau + T$, output is equal to potential so that $\zeta_t = 1$. For $\tau \le t \le t + T$, output is above potential, and capacity utilization satisfies a simple differential equation

$$\frac{\dot{\zeta_t}}{\zeta_t} = i_t - \delta\theta \le 0,$$

with terminal condition

$$\zeta_{\tau+T} = 1.$$

The solution is

$$\zeta_t = e^{\int_t^{\tau+T} (\delta\theta - i_s) ds}.$$

By lowering interest rates, the central bank creates a temporary boom after the Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from

$$\gamma \frac{X}{\theta}$$

to

$$\gamma \zeta_\tau \frac{X}{\theta} > \gamma \frac{X}{\theta}.$$

Let us now work backwards to understand the effects of this policy before the good Poisson shock, while the economy is in a safety trap. The only effect of this policy is to *increase* the interest rate r during the safety trap to

$$r = \frac{\xi \delta \theta - \underline{r}^K \rho \mu + \lambda^G (\gamma \zeta_\tau - \xi)}{\xi - \rho \mu}.$$

This increase in the interest rate is such that the value of risky assets (and hence the wealth of Neutrals) is unchanged, despite the fact that their value after a good Poisson shock has increased. Importantly, the increase in r is *orthogonal* to the safe-asset shortage problem. Since the policy leaves the supply of safe assets unchanged, it does not expand output, which remains depressed by exactly the same factor ξ.

There is one caveat to this conclusion. We have assumed that prices are entirely rigid. If prices could adjust gradually over time, then forward guidance could regain some kick: A commitment to lower interest rates after the good Poisson shock could increase inflation while the economy is in a safety trap. Under the interpretation of the model where the lower bound $\underline{r}^K$ arises because of the possibility of arbitrage between money and bonds (the zero lower bound), this would lower the safe interest rate r^K and mitigate the recession. However, under the interpretation that the lower bound $\underline{r}^K$ arises from the inability of Knightians to survive lower real interest rates (or only at a high utility cost), then our negative conclusion on the effects of standard forward guidance policy remains.[15]

SAM is addressed more directly by committing to provide support during bad rather than good times, as we now explain. What would work in a safety trap would be a commitment to lower interest i_t rates after the bad Poisson shock.[16] However it is natural to question whether monetary authorities would have the ability to lower interest rates in that state. If indeed the bad state happens to coincide with yet another safety or liquidity trap, monetary

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[15]The same comments apply to the unconventional tax policies considered by Correia, Farhi, Nicolini and Teles (2012), which here could simply take the form of an increasing path of sales taxes—say through a sales tax holiday—which would create inflation in consumer prices and hence reduce r^K.

[16]By setting the nominal interest rate i_t below the natural interest rate $\delta \theta$ after the bad Poisson shock, monetary authorities stimulate the economy and inflate the value of safe assets to $\hat{V}^\mu = \rho \mu \frac{\zeta_\tau}{\theta} X$, where $\zeta_\tau = e^{\int_\tau^{\tau+T} (\delta \theta - i_s) ds}$ (the analysis is almost identical to that of a monetary stimulus after the good Poisson shock explained in the main text). This mitigates the recession in the safety trap by raising ξ to $\xi \zeta_\tau > \xi$. Note we could just as well have used the model with public debt. The central banker's put works by increasing both the public and private sectors' ability to provide safe assets. Just like in New Keynesian models of the liquidity trap, and to the extent that they are possible at all, these forms of policy commitments are not time-consistent: Their efficacy hinges on the ability of monetary authorities to carry out credible commitments.

authorities could find themselves unable to deliver a lower interest rates. Perhaps a more realistic policy option would be a commitment by the authorities to buy up safe assets at an inflated price after the Poisson shocks—a form of government (central bank?) put. A commitment to buy up safe private assets at an inflated value $\sigma \rho \mu \frac{X}{\theta} > \rho \mu \frac{X}{\theta}$ would mitigate the recession and increase the value of ξ to

$$\sigma \xi > \xi.$$

It could be carried out by monetary authorities but it does require spare fiscal capacity (in the form of taxes or seigniorage).[17] This kind of public insurance policy has a crucial role to play in SAM.[18]

4 Fiscal Capacity and Policy Feedbacks

Fiscal capacity determines the amount of safe debt a government can issue. We start this section by showing that in a SAM environment the limits of fiscal policies are determined by fiscal capacity, in the precise sense of a government's ability to supply safe assets. By the same token, credible future fiscal consolidation enhances the government's ability to create safe assets, and therefore boosts the effectiveness of fiscal policy. We conclude with two sections highlighting the multiple feedbacks between fiscal capacity and SAM. The first one captures the perverse dynamics that emerge from a country's losing its safe asset producer status, while the second one highlights the negative feedbacks between flight to safety and fiscal capacity.

4.1 Short-Term Fiscal Stimulus and Long-Term Fiscal Consolidation

We now introduce government spending into the model. We assume that the government spends a fraction G of output X in every period until the Poisson shock hits, and a fraction

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[17]Using the model of Section 3.2,we could also intepret $(1 - \psi)$ as a government put. The lower is ψ, the higher is the government's commitment to support Knightian's portfolio should the Poisson event take place.

[18]See, e.g., Caballero and Kurlat (2010) for a proposal to increase the resilience of the financial system in a SAM environment. Also, see Brunnermeir et al (2012) for a related proposal in the context of the current Euro crisis.

G^μ of output μX after it. We index by τ^d the tax revenues allocated to paying the interest on debt and by $\hat{\tau}$ the total tax revenues, so that $\hat{\tau} - \tau^d$ is devoted to financing the flow of government expenditures. With these changes, the new equations in the system are:

$$r^K D = \tau^d \delta X,$$

$$rV = \delta (1 - \hat{\tau}) X,$$

$$GX = (\hat{\tau} - \tau^d)\delta X,$$

$$D = \tau^\mu \mu \frac{X - G^\mu}{\theta}.$$

And we still have:

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha)(1 - \delta) X + r W_t^N,$$

$$W_t^K = D,$$

$$W_t^N = V.$$

The new equilibrium rates are:

$$r^K = \delta\theta - \theta(1 - \delta)\left(\frac{\alpha}{\tau^\mu \mu}\frac{X}{X - G^\mu} - 1\right),$$

$$r = \delta\theta + \theta(1 - \delta)\left(1 - \frac{1 - \alpha}{1 - \tau^\mu \mu}\frac{X}{(X - G) + \frac{\tau^\mu \mu}{1 - \tau^\mu \mu}(G^\mu - G)}\right).$$

The system is in the constrained regime (which we assume) if and only if

$$\frac{\alpha}{1 - \alpha}\frac{1 - \tau^\mu \mu}{\tau^\mu \mu} > \frac{1}{1 + \frac{1}{1 - \tau^\mu \mu}\frac{G^\mu - G}{X - G^\mu}}.$$

Now suppose the economy is in a safety trap where the safe interest rate is at $\underline{r}^K$ and the government is using all its debt capacity. In this scenario a *fiscal expansion in G is powerless in stimulating output* because it affects neither the demand nor the supply of safe assets, and hence leaves the safe interest rate r^K unaffected. Instead, this form of government policy only affects the safety spread and interest rate r.

By contrast, a *credible decrease of G^μ to $\hat{G}^\mu < G^\mu$ stimulates output* pushing ξ to

$$\xi \frac{X - \hat{G}^\mu}{X - G^\mu} > \xi.$$

The reason is that for a given fiscal capacity, lower future government spending frees up fiscal resources and allows the government to sustain higher (safe) public debt. If the government utilizes this increased capacity in the short run, it increases the supply of safe assets, thereby alleviating their shortage. As far as government spending is concerned, *our model prescribes delayed fiscal consolidations as a remedy to safety traps.*

This conclusion has both similarities and differences with the conventional policy recommendation. Indeed proponents of fiscal stimulus in recessions typically favor a two-pronged approach combining short term fiscal stimulus for Keynesian reasons with long-term fiscal restraint to avoid generating doubts about fiscal sustainability. The short-term stimulus is the central piece of the argument, and long-run fiscal responsibility is added for robustness, almost as an afterthought. Our model identifies the same levers but reverses the pecking order by making long-run fiscal consolidation the key policy instrument since in its absence the government may not be able to create the safe debt that is in short supply.[19]

4.2   Safe Asset Status and Self-Fulfilling Debt Crises

Safe asset status is fragile. The safe asset status of a country's debt relies on the assurance that debt will be repaid should a Poisson shock occur. A country that has issued short-term debt up to its fiscal capacity constraint might find itself subject to a self-fulfilling debt crisis. This situation can lead to doubts about the ability of the government to repay its debts after the Poisson shock. These doubts jeopardize the country's safe asset status immediately, which can force it into a partial default.

We assume that the country is a small open economy facing world interest rates r and r^K

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[19]This analysis also separates SAM, which emphasizes the effect of safe asset supply, from Keynesian or New Keynesian liquidity trap economics which emphasizes the role of aggregate demand. The Keynesian view of liquidity trap argues in favor of short term fiscal stimulus, which is presumed to be especially effective at the zero lower bound because of the absence of crowding out effects through higher interest rates. The New Keynesian view emphasizes the role of inflation: Fiscal multipliers are large at the zero lower bound because government spending creates inflation and hence lowers real interest rates. (Christiano, Eichnbaum and Rebelo 2011); backloaded fiscal stimulus leads to more bang for the buck because it leads to more cumulated inflation (Farhi and Werning 2012). In contrast to the Keynesisan view, and somehwat in line with the New Keynesian view, our mechanism works through long run fiscal adjustments. Although in contrast to both the Keynesian and New Keynesian views, it works through reductions in future government spending.

with $r > r^K$, and r^μ, before and after the Poisson shock, respectively (the shock is assumed to be global). The country is up against its fiscal capacity constraint. Now imagine that when the Poisson shock hits, agents start questioning the ability of the government to repay its debt, triggering a self-fulfilling debt crisis exactly along the lines of Calvo (1988). More precisely, agents anticipate that a flow ω of debt will be repudiated per unit of time, and therefore require an interest rate $r^\mu + \omega$ on the government's debt. Moreover, we assume that a default generates extra fiscal cost α per unit of repudiated debt.[20] The only level of debt which is consistent with these expectations is

$$D = \frac{\delta \bar{\tau}^\mu \mu X}{r^\mu + \alpha \omega}.$$

The higher ω, the lower the level of debt. Note the importance of the assumption that the country is up against its fiscal capacity so that $\tau^\mu = \bar{\tau}^\mu$ cannot be increased. If instead we have

$$D = \frac{\delta \tau^\mu \mu X}{r^\mu} < \frac{\delta \bar{\tau}^\mu \mu X}{r^\mu},$$

then any ω such that

$$(r^\mu + \omega) D \leq \delta \bar{\tau}^\mu \mu X,$$

i.e.

$$\omega \leq \bar{\omega} \text{ where } \bar{\omega} = \delta (\bar{\tau}^\mu - \tau^\mu) \mu X$$

cannot be an equilibrium: The government has enough fiscal room to weather an (out of equilibrium) adverse expectational shift up to $\bar{\omega}$ by simply increasing taxes, where $\bar{\omega}$ is increasing in fiscal slack $\bar{\tau}^\mu - \tau^\mu$. In a sense, having fiscal slack restricts the possibility of self-fulfilling debt crises. As long as we are ready to impose some upper bound $\omega^{\max}$ on the adverse expectational shift ω that we consider, a country with enough fiscal slack so that $\bar{\omega} > \omega^{\max}$ will be completely immune to these crises.

We can now ask what happens before the Poisson shock, when expectations suddenly shift about the ability of the government to repay its debt should a Poisson shock occur. We start in a situation where agents anticipate $\omega = 0$ with

$$\bar{D} = \frac{\delta \bar{\tau}^\mu \mu X}{r^\mu}$$

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[20]For example, these could be legal costs. Whether the corresponding resources are dissipated or appropriated by a class of agents (lawyers) is not important in this section. In Section 6.2, we find it convenient to assume that the resources are appropriated by a class of agents, which ensures that the level of the (safe) interest rate is independent of ω.

and taxes τ are

$$\tau \delta X = r^K \bar{D}.$$

Expectations suddenly shift to a situation where $\omega > 0$. Since $\bar{D} > \frac{\delta \bar{\tau}^\mu \mu X}{r^\mu + \alpha \omega}$, debt immediately loses its safe asset status and the government must now pay an interest rate r on its debt. Indeed the government budget constraint becomes

$$\dot{D}_t = r D_t - \tau_t \delta X$$

with

$$D_0 = \bar{D}.$$

What happens next depends on the ability or willingness of the government to adjust taxes τ_t. The dynamics we have characterized are valid as long as taxes τ_t cannot be raised fast enough so that $r D_t > \tau_t \delta X$ for some time, but can eventually be increased enough to stabilize debt. However it might be impossible for the country to raise enough taxes once it faces an interest rate of r instead of r^K. For example, it could be that the country cannot raise more taxes than τ because it is against its fiscal capacity $\tau = \bar{\tau}$ even before the Poisson shock hits. In that case, an immediate partial default (haircut) must occur in order to restore fiscal balance.

Our model features multiple equilibria. Once agents start questioning the safe asset status of a country (in a way that is entirely self-fulfilling), interest rates on debt rise, triggering a gradual buildup in debt followed by a fiscal adjustment in the form of a tax increase or a partial default.

This analysis leads to a potential narrative for the recent fiscal experience of some euro-periphery countries. These countries were forced by the various fiscal costs associated with the recession—loss in fiscal revenues, government stimulus and bank bailout—against their fiscal capacity. This jeopardized their safe asset status. From safe asset producers, they suddenly became risky asset producers, facing higher interest rates. This amplification mechanism contributed to producing an unsustainable fiscal situation.

4.3   Feedback-Loop between Flight to Safety and Fiscal Capacity

We conclude our analysis of fiscal capacity by bringing back the endogenous creation of private safe assets we discussed in Section 2.2. We use this expanded model to illustrate

the feedback mechanism between (endogenous) fiscal capacity and aggregate output. This creates a feedback loop between fiscal capacity and flight to safety. The more agents fly to safety, the lower production, the lower fiscal capacity, the lower the supply of safe assets, the higher the safety premium, the more agents fly to safety, and so on.

Denoting by $\tilde{\rho}$ the fraction of tranched trees, the value of short-term debt is now given by

$$D = \left[(1 - \tilde{\rho}) \tau^{\mu} + \tilde{\rho} \tau^{\mu} \frac{\hat{\delta}}{\delta} \right] \frac{\mu X}{\theta},$$

the total value of the safe part of tranched trees is

$$\tilde{\rho}(1 - \tau^{\mu}) \frac{\hat{\delta}}{\delta} \frac{\mu X}{\theta}.$$

and hence the total value of safe assets is

$$V^{\mu} = \frac{\mu X}{\theta} \left[(1 - \tilde{\rho}) \tau^{\mu} + \tilde{\rho} \frac{\hat{\delta}}{\delta} \right].$$

By analogy to our treatment of $\tilde{\rho}$ in Section 2.2, we define $\tilde{\mu}$ as

$$\tilde{\mu}\tau^{\mu} = \mu \frac{(1 - \tilde{\rho}) \tau^{\mu} + \tilde{\rho} \frac{\hat{\delta}}{\delta}}{1 - \tilde{\rho} \left(1 - \frac{\hat{\delta}}{\delta} \right)} = \tau^{\mu} \mu \frac{1 - \tilde{\rho} \left(1 - \frac{1}{\tau^{\mu}} \frac{\hat{\delta}}{\delta} \right)}{1 - \tilde{\rho} \left(1 - \frac{\hat{\delta}}{\delta} \right)}.$$

The analysis is then identical to the baseline model with the value of μ replaced by $\tilde{\mu}$.

We now turn to the determination of $\tilde{\rho}$. In order for $\tilde{\rho} \in (0, 1)$ to be an equilibrium, the value of an untranched tree:

$$v^{nt} = \frac{\delta}{\theta} \frac{(1 - \tau^{\mu})}{1 - \frac{(1-\alpha)(1-\delta)}{1 - \tau^{\mu} \tilde{\mu}}}.$$

must be the same as that of a tranched tree:

$$v^{t} = \mu \frac{\hat{\delta}}{\delta} (1 - \tau^{\mu}) \frac{1}{\theta} \left[1 + \frac{\frac{\alpha(1-\delta)}{\tau^{\mu} \tilde{\mu}} - \left(1 - \frac{\delta}{\mu} \right)}{1 - \frac{(1-\alpha)(1-\delta)}{1 - \tau^{\mu} \tilde{\mu}}} \right].$$

Hence we have $v^{nt} = v^{t}$ if and only if

$$\frac{\delta}{1 - \delta} = \frac{\frac{\hat{\delta}}{\delta}}{1 - \frac{\hat{\delta}}{\delta}} \frac{\mu \left(\alpha - \tau^{\mu} \tilde{\mu} \right)}{\tau^{\mu} \tilde{\mu} \left(1 - \tau^{\mu} \tilde{\mu} \right)}.$$

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The right hand side is decreasing in $\tilde{\rho}$. It is also increasing in α and decreasing in μ. Hence $\tilde{\rho}$ is increasing in α and decreasing in μ.

The important point is that there is a feedback effect from flight to safety to fiscal capacity. Keeping $\tilde{\rho}$ constant, when α increases or μ decreases, the safety premium

$$r - r^K = \theta \left(1 - \delta\right) \frac{\alpha - \tau^\mu \tilde{\mu}}{\tau^\mu \tilde{\mu} \left(1 - \tau^\mu \tilde{\mu}\right)}$$

rises, which triggers a flight to safety in production (an increase in $\tilde{\rho}$) that increases the supply of safe assets and reduces the safety premium. But the flight to safety also reduces fiscal capacity, which offsets the reduction in the safety premium, reinforcing the flight to safety. At the end of this feedback loop, there are fewer public safe assets and more private safe assets. The total supply of safe assets rises just enough to stabilize the safety premium. Taking into account the endogenous response of $\tilde{\rho}$, the safety premium is unchanged in response to a shock to α, and increases in response to a shock to μ.

Formally, for $\zeta \in \{\alpha, \mu\}$ we can decompose the total effect $\frac{d\left(r - r^K\right)}{d\zeta}$ on the safety premium into a direct effect $\frac{\partial\left(r - r^K\right)}{\partial\zeta}\big|_{\tilde{\rho}}$ keeping $\tilde{\rho}$ constant and an indirect effect through the flight to safety $\frac{\partial\left(r - r^K\right)}{\partial\tau^\mu\tilde{\mu}} \frac{d\tau^\mu\tilde{\mu}}{d\tilde{\rho}} \frac{d\tilde{\rho}}{d\zeta}$:

$$\frac{d\left(r - r^K\right)}{d\zeta} = \frac{\partial\left(r - r^K\right)}{\partial\zeta}\big|_{\tilde{\rho}} + \frac{\partial\left(r - r^K\right)}{\partial\tau^\mu\tilde{\mu}} \frac{d\tau^\mu\tilde{\mu}}{d\tilde{\rho}} \frac{d\tilde{\rho}}{d\zeta}.$$

We can prove that $\frac{d\tilde{\rho}}{d\zeta}$ is larger, the higher is τ^μ.[21] In other words, the *flight to safety is more severe, the more potent is the endogenous fiscal capacity feedback loop.*

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[21] We have

$$\frac{d\tilde{\rho}}{d\zeta} = \frac{\frac{d\tau^\mu\tilde{\mu}}{d\zeta}}{\frac{d\tau^\mu\tilde{\mu}}{d\tilde{\rho}}},$$

where

$$\frac{d\tau^\mu\tilde{\mu}}{d\tilde{\rho}} = \tau^\mu \mu \frac{\frac{\hat{\delta}}{\delta}\left(\frac{1}{\tau^\mu} - 1\right)}{\left[1 - \left(1 - \frac{\hat{\delta}}{\delta}\right)\tilde{\rho}\right]^2}$$

is lower, the higher τ^μ, and $\frac{d\tau^\mu\tilde{\mu}}{d\zeta}$ is independent of τ^μ, so that $\frac{d\tilde{\rho}}{d\zeta}$ is larger, the higher is τ^μ.

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5   Safety Traps vs. Liquidity Traps

The SAM environment and its safety trap shares many features in common with the Keynesian liquidity trap, however there are also important differences between these two. In this section we highlight some of these differences by comparing the impact of several macroeconomic policy on them.

We consider the following simple model of a liquidity trap. It is a version of our model where the economy is in the unconstrained regime so that $r = r^K$ (which means the distinction between risky and riskless assets is irrelevant). We add one ingredient: the possibility of the bad shock is $\lambda > 0$ rather than $\lambda = 0$. This is necessary for interest rates r to reach zero. Interest rates r cannot go below zero because of the zero lower bound on nominal interest rates.

As long as the zero bound is not binding we have

$$r = \delta\theta - \lambda\left(1 - \mu\right) > 0.$$

When the zero bound $r = 0$ binds, the economy enters a recession ($\xi < 1$) where ξ is determined by the requirement that $r = 0$:

$$0 = \delta\theta - \lambda\left(1 - \frac{\mu}{\xi}\right),$$

i.e.

$$\xi = \frac{\mu}{1 - \frac{\delta\theta}{\lambda}}.$$

The recession originates from a scarcity of assets (stores of value). It is more severe, the worse is the expected bad shock (the lower is μ), the more likely is the expected bad shock (the higher λ), the higher the propensity to save (the lower θ), and the lower is the ability of the economy to create assets that capitalize future income (the lower is δ).

We can use this model to examine the effects of the same policies that we have considered in the context of the safety trap: QE, OT, monetary policy commitments, and fiscal policy.

5.1 QE and OT

We start with QE and OT. We introduce public debt in the model exactly as in Sections 3.1 and 3.2. The key point is that QE and OT have *no* effect at all on the recession ξ in the liquidity trap model. This precise irrelevance result relies on our assumption that dividends are taxed while the endowment of newborns (wages) is not. As a result, QE and OT simply reshuffle the fraction of dividends that accrues to private asset holders and the fraction of dividends that is absorbed by taxes to pay interest on debt of various maturities. This assumption essentially renders our framework Ricardian, despite the fact that we have overlapping generations of agents.[22] These conclusions about the irrelevance of QE and OT in liquidity traps must be contrasted with those reached in Sections 3.1 and 3.2 in safety traps. The effects of QE and OT in safety traps rely entirely on the (assumed) superior ability of the government to address a form of market incompleteness—the difficulty to isolate safe from risky assets.

5.2 Monetary Policy Commitments

We then turn to monetary policy commitments. To do so, we introduce the possibility of a good shock as in Section 3.3. The Poisson shock now has intensity $\hat{\lambda}$. When the Poisson shock hits, output increases to γX with $\gamma > 1$ with probability $\lambda^G = \pi \hat{\lambda}$ and decreases to μX with $\mu < 1$ with probability $\lambda = (1 - \pi) \hat{\lambda}$. In a liquidity trap $r = 0$, the recession is now determined by

$$0 = \delta\theta - \lambda \left(1 - \frac{\mu}{\xi} \right) - \lambda^G \left(1 - \frac{\gamma}{\xi} \right),$$

i.e.

$$\xi = \frac{\frac{\lambda}{\lambda + \lambda^G} \mu + \frac{\lambda^G}{\lambda + \lambda^G} \gamma}{1 - \frac{\delta\theta}{\lambda + \lambda^G}}.$$

Consider the following policy. Suppose that the good Poisson shock occurs at τ. After the good Poisson shock, the central banks stimulates the economy by setting the interest rate i_t below the natural interest rate $\delta\theta$ until $\tau + T$, at which points it reverts to setting the nominal interest rate equal to the natural interest rate $i = \delta\theta$. For $t > \tau + T$, output is equal to potential so that $\zeta_t = 1$. For $\tau \leq t \leq t + T$, output is above potential, and capacity

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[22]If we allowed the endowments of newborns to be taxed, then QE and OT could have some non-Ricardian effects, depending on how taxes are levied.

utilization satisfies a simple differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta\theta \leq 0,$$

with terminal condition

$$\zeta_{\tau+T} = 1.$$

The solution is

$$\zeta_t = e^{\int_t^{\tau+T}(\delta\theta - i_s)ds}.$$

By lowering interest rates, the central creates a temporary boom after the good Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from

$$\gamma\frac{X}{\theta}$$

to

$$\gamma\zeta_\tau\frac{X}{\theta} > \gamma\frac{X}{\theta}.$$

This policy alleviates the recession while the economy is in a liquidity trap to

$$\xi\frac{\frac{\lambda}{\lambda+\lambda^G}\mu + \frac{\lambda^G}{\lambda+\lambda^G}\zeta_\tau\gamma}{\frac{\lambda}{\lambda+\lambda^G}\mu + \frac{\lambda^G}{\lambda+\lambda^G}\gamma} > \xi.$$

Basically, committing to low interest rates after the good Poisson shock increases the value of assets while the economy is in the liquidity trap. This wealth effect increases demand and mitigates the recession. Forward guidance works by alleviating the asset shortage that is at the root of the recession.[23]

Forward guidance is not time consistent. It trades off a future boom against a mitigation of the current recession. Because of the utility loss that comes with the boom, monetary authorities are tempted to renege on their commitment to keep interest rates low when time comes to deliver on this promise. Nevertheless, our main point here is that the effectiveness of forward guidance in liquidity traps is to be contrasted with its relative ineffectiveness in safety traps.

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[23]We should emphasize that one channel through with forwad guidance works in traditional New Keynesian models of the liquidity trap—reducing real interest rates by creating inflation—is absent from this model because we have assumed that prices are rigid. Instead, forward guidance works entirely through a wealth effect by boosting asset values, wealth and hence spending.

5.3 Fiscal Stimulus

Finally we consider the effects of fiscal stimulus (government spending paid for by taxes on dividends) in a liquidity trap as in Section 4.1. The value of assets must be

$$\theta V = (1 - G)\, \xi X,$$

which can be combined with the asset pricing equation to yield

$$rV = (\delta - G)\, \xi X - \lambda\, (\xi\, (1 - G) - \mu\, (1 - G^\mu))\, \frac{X}{\theta},$$

implying that the interest rate is

$$r = \frac{(\delta - G)\, \xi\theta - \lambda\, (\xi\, (1 - G) - \mu\, (1 - G^\mu))}{(1 - G)\, \xi}.$$

If the zero bound is binding so that $r = 0$ and $\xi < 1$, we have

$$\xi = \frac{\mu\, (1 - G^\mu)}{1 - \frac{\delta\theta}{\lambda} - G\left(1 - \frac{\theta}{\lambda}\right)}.$$

As long as $\frac{\theta}{\lambda} < 1$, an increase in G increases ξ (output). This can be understood through the effects of this policy on the natural interest rate (in the flexible price economy). On the one hand, an increase in G reduces private wealth and asset values, and hence moderates the drop in asset values that occurs if the bad shock occurs. This increases the natural interest rate. On the other hand, an increase in G is financed by taxes on dividends, which reduces the natural interest rate. The parameter $\frac{\theta}{\lambda}$ indexes the relative strength of the second effect vs. the first effect. When $\frac{\theta}{\lambda} < 1$, the first effect dominates. Because the interest rate is stuck at zero, the increase in G reduces the gap between the actual and natural interest rates and mitigates the recession.[24]

The important point for us here is that the potential expansionary effect of short-term fiscal stimulus is in stark contrast with our safety trap result in Section 4.1, where we found that such policy was powerless at stimulating output once the government is unable to produce additional safe assets.

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[24]Once again, one channel (the inflation channel) through which government spending stimulates output in traditional New Keynesian models of the liquidity trap is absent in our model. Fiscal multipliers are large at the zero lower bound because government spending creates inflation and hence lowers real interest rates.

Conversely, just as in Section 4.1, long-run fiscal consolidation (a reduction in G^μ) also stimulates output. The long-term reduction in government spending increases asset values and increases private current spending through a wealth effect. It works to reduce the asset shortage that is at the root of the liquidity trap.

6 Two Regions

In Caballero-Farhi-Gourinchas (2008) we showed the importance of heterogeneity in countries' ability to produce financial assets for capital flows. Here we highlight a related but distinct source of heterogeneity: their ability to produce *safe* assets. In this context we show the "exorbitant privilege" of safe-asset producers, as well as the dangers of a world where the share of output of these asset-producers is shrinking over time.

6.1 The Exorbitant Privilege

We study a two-region version (U and R) of our model. The difference between U and R is that public debt in U is safe, while public debt in R is not. We also assume that private assets (trees) cannot be tranched ($\rho = 0$), and that both regions have similar compositions of Neutrals and Knightians.

Let $X = X^U + X^R$ and denote the relative sizes of the two regions by $x^U = \frac{X^U}{X}$ and $x^R = 1 - x^U$. In the constrained regime (which remains our focus), we have

$$W^K = D = \tau^\mu \mu x^U \frac{X}{\theta},$$

$$W^N = V = \left(1 - \tau^\mu \mu x^U\right) \frac{X}{\theta}.$$

where D denotes the public debt in U, and V the value of private assets. Global equilibrium is characterized by:

$$r^K D = \tau x^U \delta X,$$

$$rV = \delta X - \tau x^U \delta X,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) X + r W_t^N.$$

The solution for taxes in the U region before the Poisson shock τ, the safe interest rate r^K and the interest rate r is

$$\delta x^U \tau = \tau^\mu \mu x^U - \alpha (1 - \delta),$$

$$r^K = \delta\theta - \theta (1 - \delta) \frac{\alpha - \tau^\mu \mu x^U}{\tau^\mu \mu x^U},$$

$$r = \delta\theta + \theta (1 - \delta) \frac{\alpha - \tau^\mu \mu x^U}{1 - \tau^\mu \mu x^U}.$$

and the condition for the constrained regime is

$$\alpha \geq \tau^\mu \mu x^U.$$

Let us now turn to the regions' net foreign asset positions and external accounts. We denote by TV^U the total value of U assets (public and private) and by TV^R the total value of R assets. We have that

$$TV^U = \frac{X}{\theta} \left[x^U + (1 - x^U) (1 - \delta) \frac{\alpha - \tau^\mu \mu x^U}{\alpha (1 - \delta) + \delta - \tau^\mu \mu x^U} \right] > x^U \frac{X}{\theta}.$$

That is, the total share of U assets in the world portfolio exceeds U's GDP share in world GDP (and it follows that the opposite holds for region R).

What about U's residents share of global wealth?[25] It is straightforward to show (see the appendix) that in the long-run steady state $W^U = \frac{x^U X}{\theta}$.

It follows that in the long run trade is balanced:

$$TB^U = X^U - \theta W^U = 0.$$

while U has a *negative* net foreign asset position:

$$NFA^U = W^U - TV^U = x^U \frac{X}{\theta} - TV^U < 0.$$

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[25]One issue is that the portfolios of Neutrals is not determinate, but this is only relevant after the Poisson shocks hits. Hence before the Poisson shock, we can make unambiguous predictions about net foreign asset positions and external accounts.

The remarkable implication of this analysis is that U has a negative net foreign asset position but runs a balanced trade. In other words, the negative net foreign asset position does not require U to ever run trade surpluses. How can that be? The key is that there is a favorable return differential between U's assets and liabilities. This is because the share of safe assets in U's liabilities is higher than the share of assets that U's agents hold. Interpreting U as the U.S. and R as the rest of the world, this observation is very much consistent with the findings of Gourinchas and Rey (2007) who used the expression *exorbitant privilege* to describe this situation.[26],[27] The safety premium is larger, and U's net foreign asset position (normalized by its GDP) is more negative, the larger is the relative size x^R of the R region.

Naturally, there is a transition phase behind the steady state result. Suppose that the initial wealth of U agents is equal to the total value of their assets $W_0^U = TV^U$ so that $NFA_0^U = 0$. (We do not need take a stand on how the initial wealth of U is split between Knightians and Neutrals in U). Then we have

$$W_t^{U,K} = W_0^{U,K} e^{(r^K - \theta)t} + \alpha x^U X \int_0^t (1 - \delta) e^{(r^K - \theta)(t-s)} ds,$$

$$W_t^{U,N} = W_0^{U,N} e^{(r - \theta)t} + (1 - \alpha) x^U X \int_0^t (1 - \delta) e^{(r - \theta)(t-s)} ds.$$

which converge asymptotically to

$$W_\infty^{U,K} = x^U \frac{\tau^\mu \mu x^U X}{\theta},$$

$$W_\infty^{U,N} = \left(1 - \tau^\mu \mu x^U\right) \frac{x^U X}{\theta},$$

with

$$W_\infty^{U,K} + W_\infty^{U,N} = \frac{x^U X}{\theta} < TV^U = W_0^U.$$

That is, the wealth of U agents decreases over time and asymptotes towards $W_\infty^U = W^U = \frac{x^U X}{\theta} < TV^U = W_0^U$ so that $NFA_\infty^U = NFA^U < 0$. During the transition U runs a trade deficit $TB_t^U < 0$ and a current account deficit $CA_t^U < 0$. These deficits decrease over time and asymptote to $TB_\infty^U = TB^U = 0$ and $CA_\infty^U = CA^U = 0$. How can U start with a zero net foreign asset position and run trade deficits in all periods? Because U essentially perceives an

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[26]The expressions "exorbitant privilege" is usually attributed to Valery Giscard D'Estaing in 1960, when he was finance minister under the De Gaulle government.

[27]The analysis with a strictly positive rate of growth, $g > 0$, is analogous to that in the main text, with one important difference: Now U runs a permanent current account deficit $CA^U = g NFA^U < 0$.

intermediation rent, a compensation for the *safety transformation* services that it provides. In our model, these services are the counterpart of the exorbitant privilege.

6.2 Fiscal Capacity and Global Instability

We now consider the consequences of a faster rate of growth in R than in U. That is a scenario where the relative size x^U declines over time. The direct impact of this differential growth is that the scarcity of safe assets rises over time. The model predicts that the global economy converges toward a new steady state with a lower r^K, a higher r, and a larger negative net foreign asset position in U.

As long as U has some fiscal slack so that $\tau^\mu < \bar{\tau}^\mu$, this trend increases the incentives for U to issue more debt because it now needs lower taxes τ to sustain its debt before the Poisson shock. U might respond to these incentives and run up its debt, mitigating the increase in the safety premium. However, it is easy to imagine how this virtuous response might run into perverse consequences as U overstretches itself. Indeed, as U runs up its debt, it reduces its fiscal slack $\bar{\tau}^\mu - \tau^\mu$, and might become susceptible to self-fulfilling default crises. It is then exposed to the possibility of suddenly losing its safe asset status, exactly along the lines of the mechanism described in Section 4.2.[28]

For simplicity, we focus on the case where U is against its fiscal capacity before and after the Poisson shock, so that $\tau = \bar{\tau}$, $\tau^\mu = \bar{\tau}^\mu$ and its debt is $D = \frac{\bar{\tau}\delta x^U X}{r^K} = \frac{\bar{\tau}^\mu \mu x^U X}{\theta}$. Then a self-fulfilling adverse shift in ω can take place. If agents suddenly anticipate that this might happen (once the Poisson shock hits), then U finds itself with too high a debt to retain its safe asset status and is forced into an immediate default.

Discussion. The idea that fiscal capacity is a key determinant of the supply of safe assets echoes some influential ideas about the functioning of international of the international monetary system. Indeed, under Bretton Woods, gross foreign exchange reserves constituted the most important source of international liquidity. In a rapidly growing world economy, the demand for reserves rapidly outgrew the demand for monetary gold. The U.S. dollar filled the void. Triffin famously argued that this system was unstable, and formulated his well-

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[28]As long as we consider U to be composed of a (continuum) set of small open economies, the analysis of Section 4.2 is literally correct for an individual economy within this set. Otherwise general equilibrium implications of this behavior, which affects the interest rates r and r^K, have to be explicitly dealt with. However, the post-Poisson shock analysis is unchanged since the level of the world (riskless) interest rate $r^\mu = \delta\theta$ after the Poisson shocks is independent of the expected value of ω. The only difference is what happens before the Poisson shock.

known dilemma: Either the world supply of international reserves would become insufficient, or foreign dollar holdings would expand beyond the amount the U.S. could redeem in gold at the statutory peg, triggering the collapse of Bretton Woods (see Obstfeld 2011).

Today's world features some elements that are reminiscent of the Triffin logic (see Farhi, Gourinchas, and Rey 2011). Rapidly growing emerging and developing countries (R in our model) have large demands for reserve assets. They hold their reserves in relatively low-risk assets, mostly government or government-guaranteed debt instruments from creditworthy developed countries (U in our model). The supply of these safe assets is limited by the fiscal capacity of a few rich and creditworthy reserve issuers whose economies are shrinking as a fraction of world GDP. Triffin's pronouncements foretelling the demise of the Bretton Woods system then offer a cautionary tale. The fiscal capacity of reserve issuers are limited. A government increasingly willing to issue safe liabilities and invest in risky assets eventually becomes more likely to encounter fiscal problems in a systemic crisis – precisely the moment its creditor will wish to liquidate its supposedly safe claims.

7   Final Remarks

Our model illustrates the essence of SAM and its implications for standard macroeconomic policy tools, but it leaves untouched critical considerations for the dynamics of the world economy in the medium term.

Given the faster growth of safe-asset-consumer economies than that of safe-asset-producer economies, SAM is only likely to get worse over time, absent major financial innovations. Aside from temporary cyclical recoveries in spreads and output, it is our conjecture that SAM will remain as a structural drag, lowering safe rates, increasing safety spreads, straining the financial system, and weakening the effectiveness of conventional monetary policy.

There is significant incentive to produce major financial innovations to address the safety gap, which can be a source of concern in itself, as the financial system capacity to extract safe tranches from risky assets is likely to get stretched along lines similar to those behind the subprime crisis. However, it is important to recognize that this is a chronic structural and macroeconomic problem that does not get remedied, but only reallocated, with standard regulatory pressure (which may have merits on its own). Instead, what is needed is public-private cooperation in increasing the efficiency of production of safe assets, as well as policies that reduce the demand for such assets.

What are these policies? What are the political limits on pre-paid "bailout" agreements and cross-country insurance arrangements that may be needed to reduce the SAM problem? What are the intergenerational transfers that SAM requires and, are these acceptable, especially for current retirees? Will SAM lead to widespread "gambling for resurrection" type mechanisms from those institutions and corporations whose implicit liabilities are inconsistent with safe rates of return? How much of the adjustment will take place on the supply side of the economy via reduced entrepreneurial risk taking? We are just beginning to comprehend the implications of SAM for macroeconomic policy and politics, but it is already apparent that these are broad and significant.

References

[1] Acharya, Viral V. and Philipp Schnabl. 2009. "Do Global Banks Spread Global Imbalances? The Case of Asset-Backed Commercial Paper During the Financial Crisis of 2007–09," Paper presented at the 10th Jacques Polak Annual Research Conference, International Monetary Fund. November.

[2] Bansal, Ravi and John W. Coleman, 1996, "A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles," Journal of Political Economy, University of Chicago Press, vol. 104(6), pp. 1135-71, December.

[3] Barro, Robert .J and Herschel I. Grossman, 1971, "A General Disequilibrium Model of Income and Employment," American Economic Review, 61(1), pp. 82-93, March.

[4] Bénassy, Jean-Pascal, 1986, "Macroeconomics: An Introduction to the Non-Walrasian Approach," Academic Press, Orlando.

[5] Bernanke, Ben, Carol Bertaut, Laurie Ponder DeMarco, and Steven Kamin, 2011. "International Capital Flows and the Returns to Safe Assets in the United States, 2003-2007," International Finance Discussion Paper 1014, Federal Reserve.

[6] Caballero, Ricardo J., 2006. "On the Macroeconomics of Asset Shortages." In The Role of Money: Money and Monetary Policy in the Twenty-First Century The Fourth European Central Banking Conference 9-10 November, Andreas Beyer and Lucrezia Reichlin, editors, pp. 272-283.

[7] Caballero, Ricardo J., 2009. "The 'Other' Imbalance and the Financial Crisis," Paolo Baffi Lecture, Bank of Italy.

[8] Caballero, Ricardo J., Emmanuel Farhi and Pierre-Olivier Gourinchas, 2008a, "Financial Crash, Commodity Prices, and Global Imbalances." Brookings Papers on Economic Activity, Fall, pp. 1-55.

[9] Caballero, Ricardo J., Emmanuel Farhi and Pierre-Olivier Gourinchas, 2008b, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates." American Economic Review, 98(1), pp. 358-393.

[10] Caballero, Ricardo J. and Arvind Krishnamurthy, 2009, "Global Imbalances and Financial Fragility," American Economic Review, May.

[11] Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, 2011, "When Is the Government Spending Multiplier Large?," Journal of Political Economy, 119(1), pp. 78-121.

[12] Correia, Isabel, Emmanuel Farhi, Juan-Pablo Nicolini and Pedro Teles, "Unconventional Fiscal Policy at the Zero Bound," forthcoming in the American Economic Review.

[13] Coval, Joshua D., Jakub W. Jurek and Erik Stafford. 2009, "The Economics of Structured Finance," Journal of Economic Perspectives, Vol. 23, No. 1, Winter.

[14] Eggertsson, Gauti B. and Michael Woodford, 2003, "The Zero Bound on Interest Rates and Optimal Monetary Policy," Brookings Papers on Economic Activity, pp. 139-211.

[15] Farhi, Emmanuel, Pierre-Olivier Gourinchas, and Helene Rey, "Reforming the International Monetary System," CEPR ereport, Setpember 2011.

[16] Farhi, Emmanuel, and Ivan Werning, 2012, "Fiscal Multipliers: Liquidity Traps and Currency Unions," NBER working paper 18381.

[17] Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2011. "Neglected Risks, Financial Innovation, and Financial Fragility," forthcoming in the Journal of Financial Economics.

[18] Gorton, Gary B., 2010, "Slapped by the Invisible Hand: The Panic of 2007," Oxford University Press.

[19] Gorton, Gary B. and Andrew Metrick, 2012, "Securitized Banking and the Run on Repo," Journal of Financial Economics 104, 425-451.

[20] Gorton, Gary B. and Andrew Metrick, 2010, "Haircuts," Federal Reserve Bank of St. Louis, Review 92(6).

[21] Gorton, Gary B., and Nicholas Souleles. 2006. "Special Purpose Vehicles and Securitization," in The Risks of Financial Institutions, edited by Rene Stulz and Mark Carey. University of Chicago Press.

[22] Gourinchas, Pierre-Olivier and Hélène Rey. 2007. "From World Banker to World Venture Capitalist: US External Adjustment and The Exorbitant Privilege," in Richard Clarida, editor, G7 Current Account Imbalances: Sustainability and Adjustment, The University of Chicago Press, pp. 11-55.

[23] Greenwood Robin, and Dimitri Vayanos, 2010, "Price Pressure in the Government Bond Market," American Economic Review, 100(2), pp. 585-90, May.

[24] Greenwood Robin, Sam Hanson, and Jeremy Stein, 2012, "A Comparative-Advantage Approach to Government Debt Maturity", mimeo, Harvard.

[25] Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, "The U.S. Treasury yield curve: 1961 to the present," Journal of Monetary Economics, Elsevier, 54(8), pp. 2291-2304, November.

[26] Hall, Robert E., 2011a, "The Long Slump," American Economic Review, 101, pp. 431-469.

[27] Hall, Robert E., 2011b, "Clashing Theories of Unemployment," mimeo, Hoover Institution and Stanford University.

[28] Holmstrom, Bengt and Jean Tirole, 1998, "Private and Public Supply of Liquidity," Journal of Political Economy, 106(1), pp. 1-40.

[29] Keynes, John M, 1936, "The General Theory of Employment, Interest, and Money," Macmilan.

[30] Kocherlakota, Narayana, 2012, "Incomplete Labor Markets," mimeo, Federal Reserve Bank of Minneapolis.

[31] Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2011, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," Brookings Papers on Economic Activity.

[32] Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2012, "The Aggregate Demand for Treasury Debt," Journal of Political Economy, 120:2, pp. 233-267

[33] Krugman, Paul R., 1998, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," Brookings Papers on Economic Activity, 29(2), pp. 137-206.

[34] Malinvaud, Edmond. 1977, "Theory of Unemployment Reconsidered," Oxford: Blackwell Publishers.

[35] Obstfeld, Maury, 2011, "International Liquidity: The Fiscal Dimension," Monetary and Economic Studies, pp. 33-48, November.

[36] Stein, Jeremy C., 2012, "Monetary Policy as Financial-Stability Regulation," Quarterly Journal of Economics, 127(1), pp. 57-95.

[37] Werning, Ivan, 2012, "Managing a Liquidity Trap: Monetary and Fiscal Policy," MIT mimeo.

[38] Woodford, Michael, 1990, "Public Debt as Private Liquidity," American Economic Review, Papers and Proceedings 80, pp. 382-88, May.

8 Appendix

8.1 Proofs for Section 2.1

The goods market clearing condition is

$$W = \frac{X}{\theta}.$$

The financial market clearing condition is

$$W = V.$$

If we are in the unconstrained regime, then we have $r_t = r_t^K$ and summing up the wealth evolution equations

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r_t^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) X + r_t W_t^N,$$

we find

$$r_t = \delta \theta.$$

If we are in the constrained regime, then

$$W_t^K = V^\mu = \rho \mu \frac{X}{\theta},$$

which can be combined with the wealth evolution equation for Knightians

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r_t^K W_t^K,$$

to get

$$r_t^K = \delta \theta - (1 - \delta) \theta \left(\frac{\alpha - \rho \mu}{\rho \mu} \right).$$

We can then use the asset pricing equation for safe assets

$$r_t^K V^\mu = \delta_t^\mu X,$$

to get

$$\delta_t^\mu = \delta \rho \mu - (\alpha - \rho \mu)(1 - \delta).$$

Similarly we can use the fact that

$$W_t^N = V - V^\mu = (1 - \rho\mu) \frac{X}{\theta},$$

combined with the wealth accumulation equation for Neutrals

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha)(1 - \delta) X + r_t W_t^N,$$

to get

$$r_t = \delta\theta + (1 - \delta)\theta \frac{\alpha - \rho\mu}{1 - \rho\mu}.$$

We can verify that we have $r > r^K$ if and only if $\alpha > \rho\mu$.

If this condition is verified, then the economy always converges to the constrained region in the long run. If it is violated, then the economy is always in the unconstrained regime and the wealth of Knightians and Neutrals converges to

$$W_\infty^K = \alpha \frac{X}{\theta} < \rho\mu \frac{X}{\theta} = V^\mu,$$

$$W_\infty^N = (1 - \alpha) \frac{X}{\theta}.$$

8.2   Extension of the Model with Pooling

Assume that agents cannot by themselves diversify their portfolios. Otherwise the issue of pooling becomes irrelevant. The reason could be that they do not have the expertise to do that. Alternatively, there could be some indivisibilities in assets (in other words vertical—as opposed to horizontal—tranching is not perfect, vertical tranches cannot be too thin), that could arise from a desire to minimize structuration/issuance/transaction costs.

To introduce a role for pooling, we imagine that trees have idiosyncratic risk. The dividend of each tree follows the process: on top of the macro Poisson shock, there is a sectoral shock ε that hits the dividend of the tree if the macro Poisson shock occurs. Hence the dividend is 1 if there is no Poisson shock and $\varepsilon\mu$ if a Poisson shock has occurred, where ε is idiosyncratic to a sector. The shock ε has mean 1 and the lower bound of its support is $\underline{\varepsilon} < 1$. There is a continuum of sectors. Assets from different sectors can be pooled. The financial sector has the ability to pool assets from a mass γ of different sectors. The rest $1 - \gamma$ of the sectors cannot be pooled. Only a fraction $\hat{\rho}$ of the assets that can be

pooled can be tranched, and only a fraction $\tilde{\rho}$ of the assets that cannot be pooled can be tranched. The model is then isomorphic to the model with an imperfect ability to tranch with $\rho = \hat{\rho}\gamma + \tilde{\rho}(1 - \gamma)\varepsilon$.

8.3 Proofs for Section 2.2

Total output after a Poisson shock is

$$(1 - \tilde{\rho})\mu X + \tilde{\rho}\mu \frac{\hat{\delta}}{\delta}X$$

and hence the total value of assets is

$$\frac{\mu X}{\theta}\left[1 - \tilde{\rho}\left(1 - \frac{\hat{\delta}}{\delta}\right)\right].$$

The total dividend of the untranched unfertilized trees is $\hat{\delta}\tilde{\rho}\mu X$. The total dividend of the untranched fertilized trees is $\delta(1 - \tilde{\rho})\mu X$. Hence the value of safe assets (safe tranches of tranched trees and untranched unfertilized trees) after the Poisson shock is

$$V^{\mu} = \frac{\frac{\hat{\delta}}{\delta}\tilde{\rho}\mu X}{\theta}.$$

Let

$$\rho = \frac{\tilde{\rho}\frac{\hat{\delta}}{\delta}}{1 - \tilde{\rho}\left(1 - \frac{\hat{\delta}}{\delta}\right)}.$$

The analysis is then completely similar to the baseline model with this value of ρ (the only difference is that and X must be replaced by $\left[1 - (1 - \chi)\left(1 - \frac{\hat{\delta}}{\delta}\right)\right]X$).

We now turn to the determination of ρ. In order for the equilibrium that we have characterized to be valid, it must be the case that the value of tranched and untranched trees are equal. We have

$$r = \theta - \theta\frac{(1 - \alpha)(1 - \delta)}{1 - \rho\mu},$$

so that the value of an untranched tree is

$$v^{nt} = \frac{\delta}{r} = \frac{1}{\theta}\frac{\delta}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}}.$$

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The value of a the safe part of a tranched tree is

$$\frac{\mu}{\theta}\frac{\hat{\delta}}{\delta}.$$

The associated dividend of the safe part is

$$r^K\frac{\mu}{\theta}\frac{\hat{\delta}}{\delta} = \delta^t,$$

where

$$r^K = \theta - \theta\frac{\alpha\left(1-\delta\right)}{\rho\mu},$$

so that

$$\delta^t = \mu\frac{\hat{\delta}}{\delta}\left[1 - \frac{\alpha\left(1-\delta\right)}{\rho\mu}\right],$$

and hence the value of the risky part is

$$\frac{1}{\theta}\frac{\frac{\hat{\delta}}{\delta}\delta - \delta^t}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}} = \frac{\mu}{\theta}\frac{\frac{\hat{\delta}}{\delta}\left[\frac{\alpha(1-\delta)}{\rho\tilde{\mu}} - \left(1 - \frac{\delta}{\mu}\right)\right]}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}}.$$

Hence

$$v^t = \frac{\mu}{\theta}\frac{\hat{\delta}}{\delta}\left[1 + \frac{\frac{\alpha(1-\delta)}{\rho\tilde{\mu}} - \left(1 - \frac{\delta}{\mu}\right)}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}}\right]$$

Hence we have $v^{nt} = v^t$ if and only if

$$\frac{\delta}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}} = \mu\frac{\hat{\delta}}{\delta}\left[1 + \frac{\frac{\alpha(1-\delta)}{\rho\tilde{\mu}} - \left(1 - \frac{\delta}{\mu}\right)}{1 - \frac{(1-\alpha)(1-\delta)}{1-\rho\mu}}\right],$$

$$\delta = \mu\frac{\hat{\delta}}{\delta}\left[1 - \frac{\left(1-\alpha\right)\left(1-\delta\right)}{1 - \rho\mu} + \frac{\alpha\left(1-\delta\right)}{\rho\mu} - \left(1 - \frac{\delta}{\mu}\right)\right],$$

$$\delta = \mu\frac{\hat{\delta}}{\delta}\left[\frac{\delta}{\mu} + \frac{\left(1-\delta\right)\left(\alpha - \rho\mu\right)}{\rho\mu\left(1 - \rho\mu\right)}\right].$$

8.4   Proofs for Section 4.1

We have

$$r^K D = \tau \delta X,$$

$$rV = \delta (1 - \hat{\tau}) X,$$

$$\dot{W}_t^K = -\theta W_t^K + \alpha (1 - \delta) X + r^K W_t^K,$$

$$\dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) X + r W_t^N,$$

$$GX = (\hat{\tau} - \tau) \delta X,$$

$$D = \tau^\mu \mu \frac{X - G^\mu}{\theta},$$

$$W_t^K = D,$$

$$W_t^N = V.$$

This implies

$$D + V = \frac{X - G}{\theta},$$

$$V = \frac{X - G}{\theta} - \tau^\mu \mu \frac{X - G^\mu}{\theta} = (1 - \tau^\mu \mu) \frac{X - G}{\theta} + \tau^\mu \mu \frac{G^\mu - G}{\theta},$$

$$\left(\theta - r^K\right) D = \alpha (1 - \delta) X,$$

$$(\theta - r) V = (1 - \alpha) (1 - \delta) X,$$

and hence

$$r^K = \theta - \theta (1 - \delta) \frac{\alpha}{\tau^\mu \mu} \frac{X}{X - G^\mu},$$

$$r = \theta - \theta (1 - \delta) \frac{1 - \alpha}{1 - \tau^\mu \mu} \frac{X}{(X - G) + \frac{\tau^\mu \mu}{1 - \tau^\mu \mu} (G^\mu - G)}.$$

We are in the constrained regime if and only if

$$\frac{\alpha}{1 - \alpha} \frac{1 - \tau^\mu \mu}{\tau^\mu \mu} > \frac{X - G^\mu}{(X - G) + \frac{\tau^\mu \mu}{1 - \tau^\mu \mu} (G^\mu - G)},$$

i.e.

$$\frac{\alpha}{1-\alpha} \frac{1-\tau^\mu \mu}{\tau^\mu \mu} > \frac{1}{1 + \frac{1}{1-\tau^\mu \mu} \frac{G^\mu - G}{X - G^\mu}}.$$