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ABSTRACT

This paper compares ex-ante policy measures (such as macroprudential regulation) and ex-post policy interventions (such as bailouts) to respond to financial crises in models of financial amplification, i.e. models in which falling asset prices, declining net worth and tightening financial constraints reinforce each other. The optimal policy mix in such models involves a combination of both types of measures since they offer alternative ways of mitigating binding financial constraints. Comparing their relative merits, ex-post policy interventions are only taken once a crisis has materialized and are therefore better targeted, whereas ex-ante measures are blunter since they depend on crisis expectations. However, ex-post interventions distort incentives and create moral hazard. This introduces a time consistency problem, which can in turn be solved by ex-ante policy measures. Limiting ex-post transfers to the revenue accumulated in a bailout fund reduces welfare.

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1 Introduction

A growing literature has analyzed the role of macroprudential regulation in models of financial crises that are based on financial amplification, in which the economy experiences a feedback loop between declining asset prices and tightening financial constraints. As pointed out by Gromb and Vayanos (2002), Lorenzoni (2008) and Jeanne and Korinek (2010b), financial amplification effects involve pecuniary externalities because atomistic agents do not internalize that their individual actions lead to relative price movements that reinforce shocks in the aggregate. This argument has been used by policymakers to make the case for so-called *macroprudential regulation* (see e.g. Borio, 2003; Bank of England, 2009; Blanchard et al., 2010).

However, there has been an intense debate about the relative desirability of prudential measures that attempt to curb financial risk-taking ex ante, before crises materialize, and policy measures that are taken ex post, once a crisis has hit. In the realm of fiscal policy, such measures include bailouts, transfers and subsidies such as investment tax credits. In the realm of monetary policy, they correspond to monetary easing in response to financial crises. In this context, the so-called "Greenspan doctrine" (see Greenspan, 2002, 2011; Blinder and Reis, 2005) suggests that ex-ante interventions to prevent booms are too blunt compared to "mopping up" measures after a financial crisis has materialized.

This paper studies the desirability of these two types of policy interventions in a stylized three-period model of financial amplification and crisis that follows the spirit of Kiyotaki and Moore (1997). Entrepreneurs borrow and invest in capital in the initial period, they experience a productivity shock and reinvest in the intermediate period, and they repay their debts and consume the remainder in the final period. However, they are subject to financial constraints, which may limit how much they can reinvest in the intermediate period. If the constraint forces them to reduce their reinvestment, the value of their capital assets and therefore their collateral declines, and they have to cut back further on reinvestment, giving rise to financial amplification. We use this setup to study the desirability of ex-ante macroprudential policy interventions, which are taken in the initial period before binding financial constraints occur, as well as ex-post "mopping up" measures, which are taken once an adverse shock triggers binding financial constraints.

We first study optimal macroprudential interventions, where we define the optimal policy as the one chosen by a constrained social planner who obeys the same financial constraint as decentralized agents, but—unlike competitive agents—internalizes the effects of her actions on aggregate asset prices. As shown in the earlier literature, such a planner induces private agents to reduce borrowing in the initial period so as to mitigate financial amplification effects when the constraint is binding. This policy measure has a natural interpretation in the theory of the second-best: the planner's intervention in the initial period introduces a second-order cost, but relaxes binding constraints in the intermediate period, which results in a first-order benefit.

Next we turn our attention to ex-post policy measures: we assume a planner who has superior borrowing capacity compared to private agents and who can relax the financial constraints of private agents by providing them with a bailout transfer. The planner pays for the transfer by imposing distortionary taxes. This policy measure captures the essential characteristic of ex-post policy measures to mitigate financial amplification: it relaxes binding financial constraints but introduces a distortion into the economy.[1] The planner finds it optimal to provide such tax-financed transfers ("bailouts") when the economy experiences financial amplification, since the benefits from relaxing a binding constraint are first-order, whereas the costs from introducing a tax distortion are second-order. The transfer is financed by raising public debt that is repaid in the final period when borrowing constraints are loose again—it would be detrimental to raise taxes in the constrained period to finance a bailout.

Ex-post policy measures lead to a time consistency problem: they relax binding constraints but distort the incentives of private agents to borrow and invest more in the initial period. Ex-ante, the planner would like to commit to smaller bailouts than what is optimal under discretion so as to mitigate the overinvestment problem. However, ex-post, once the economy has entered a period with binding constraints, the planner would like to provide the optimal discretionary bailout.

We study the optimal policy mix in a setup in which the planner has access to both ex-ante macroprudential regulation and ex-post bailout tranfers. We show that a planner finds it optimal to both reduce borrowing ex-ante via macroprudential regulation and provide bailouts ex-post when there are states of nature in which borrowing constraints are binding. The optimal policy mix consists of a combination of both measures such that the marginal cost of each intervention equals its expected marginal benefit. This is consistent with the findings of the general theory of the second best (Lipsey and Lancaster, 1956): it is desirable to intervene along all available dimensions when engaging in second-best policies.

Each of the two policy instruments has specific benefits and disadvantages: bailouts are better targeted, since they are taken only once an adverse state of nature has materialized, whereas macroprudential regulation is blunter, since it is imposed in the expectation that a crisis may occur in the future. However, bailouts are subject to time consistency problems.

Macroprudential regulation, on the other hand, resolves the time consistency problem associated with bailouts since the planner can use the macroprudential policy tool to directly provide the optimal incentives for borrowing and investment and has no more reason to deviate from the optimal discretionary bailout policy in order to affect borrowing and investment. If the two policy instruments of macro-

[1] Greenspan (2002) used the term "mopping up after the crash" to refer to the use of monetary policy to support the economy after a financial crisis has occurred. Our model does not have money, but some of the ex post measures that we consider—in particular, subsidies that reduce the real interest rate—have similar economic effects on constrained borrowers as a monetary stimulus.

prudential regulation and bailouts were given to two separate agencies, the optimal policy mix could be implemented by instructing the bailout agency to provide the optimal discretionary bailouts and instructing the macroprudential agency to resolve the time consistency problem. In a way, macroprudential regulation is a substitute for commitment—the optimal policy mix could also be implemented by a planner who can credibly commit to a policy that includes carrots and sticks that are conditional on both the level of borrowing and the state of nature. Such a policy would provide bailouts for compliant borrowers and penalties for excessive borrowers.

We investigate the desirability of accumulating a bailout fund and find that welfare is generally reduced if bailouts are limited to such a fund. If the planner can supplement the bailout fund with additional tax revenue, then there are no welfare benefits to accumulating such a fund but distributions from the bailout fund distort the incentives of entrepreneurs further, which calls for an increase in macroprudential regulation. The intuition why bailout funds are not desirable in our framework is that the planner has no comparative advantage in holding savings compared to private entrepreneurs and there are no idiosyncratic risks that such a fund could insure.

Finally, we study alternative ex-post policy measures to mitigate financial amplification effects, including investment tax credits, debt forgiveness and subsidies to new borrowing, which may be interpreted as interest rate cuts or crisis lending. We find that the different policy measures are equivalent from an ex-post perspective, since what matters is only the transfer of liquidity to mitigate the constraints. However, from an ex-ante perspective, investment tax credits and borrowing subsidies provide superior incentives since they reward enterpreneurs who keep more borrowing capacity and therefore mitigate the incentives for excessive borrowing.

Literature The model setup that we study belongs to the literature on financial amplification and fire sales; see Bernanke and Gertler (1989), Shleifer and Vishny (1992, 2011) and Kiyotaki and Moore (1997). A growing literature, including Gromb and Vayanos (2002, 2010), Lorenzoni (2008), Farhi et al. (2009) and Korinek (2010) have analyzed the pecuniary externalities that arise in such models of financial constraints. Brunnermeier (2009) and Adrian and Shin (2010), among others, observe that financial amplification effects have played a crucial role in the Global Financial Crisis of 2008/09. Lorenzoni (2008) shows that there is generally excessive borrowing and investment in such a setting, and Caballero and Krishnamurthy (2003) and Korinek (2010) find that agents may not engage in sufficient insurance against adverse shocks that trigger financial amplification. Jeanne and Korinek (2010a) illustrate that total borrowing is excessive if uncontingent debt is the only financial instrument. Kato and Tsuruga (2011) study the implications of pecuniary externalities for bank leverage. Perotti and Suarez (2011) investigate whether to address such externalities via price or quantity regulations. All these papers have in common that they focus on ex-ante or macroprudential measures to reduce the risk of experiencing financial amplification effects, whereas we focus on the optimal mix and the relationship between ex-ante and ex-post policy measures.[2]

Farhi and Tirole (2012) analyze the problems that arise from policies to mitigate financial crises due to collective moral hazard, but in a setting in which there are no fire sales and pecuniary externalities. Acharya and Yorulmazer (2008) and Philippon and Schnabl (2012) compare the efficiency of different types of ex-post policy measures. Acharya and Yorulmazer use a model of liquidity constraints and cash-in-the-market pricing to show that subsidies for take-overs of failed banks provide superior incentives compared to bailouts of failed banks. Philippon and Schnabl study the optimal way of recapitalizing banks in a model of debt overhang, in which an asymmetric information problem between banks and the government is solved via a mechanism design setup. The contribution of our paper, by contrast, is to study the optimal policy mix and the interplay between both ex-ante and ex-post policy measures in a model of financial amplification and pecuniary externalities.

The remainder of this paper is structured as follows. In the ensuing section, we introduce the baseline model, characterize the first best and introduce the financial constraint that lies at the heart of our analysis. Section 3 analyzes the laissez faire equilibrium. Sections 4 and 5 introduce ex-ante macroprudential regulation and ex-post policy measures. Section 6 characterizes the optimal mix between the two policy measures, describes the interplay between the two and analyzes how macroprudential regulation can resolve the time consistency problems associated with bailouts as well as the role for a bailout fund. Section 7 discusses generalizations to a variety of alternative ex-post policy measures.

2 Model

2.1 Assumptions

We consider an economy with three time periods $t = 0, 1, 2$. Period 0 is the investment period in which the productive capital good is produced. The consumption good is produced with capital and labor in period 1 and also in period 2.

There are two classes of atomistic agents in the economy: entrepreneurs and workers. The entrepreneurs operate the productive capital and hire the workers in periods 1 and 2. The entrepreneurs do not have enough funds of their own to finance the desired level of capital in period 0 and so must borrow from the workers.

[2] In the quantitative DSGE literature, Jeanne and Korinek (2010b, 2012), Bianchi (2011) and Bianchi and Mendoza (2011) present similar findings for macroprudential regulation in models of financial feedback loops. Benigno et al. (2012) find that a planner in a small open economy who has access to lump sum transfers to relieve binding constraints does not resort to macroprudential regulation. Bianchi (2012) analyzes a quantitative model of distortionary bailouts to relax binding financial constraints. Our paper, by contrast, provides clean-cut analytical results in a stylized three period model of financial amplification.

The utility of the representative worker in period 0 is given by,

$$U^w = E_0 \left[c_0^w + c_1^w + c_2^w - \omega \ell_1 - \omega \ell_2 \right], \tag{1}$$

where $c_t^w \geq 0$ and ℓ_t are respectively the worker's level of consumption and labor supply in period t.

We assume that entrepreneurs do not supply labor in this economy to simplify the problem. Denoting consumption of entrepreneurs by $c_t \geq 0$ in period t, the utility of the representative entrepreneur is

$$U^e = E_0 \left[c_0 + c_1 + c_2 \right]$$

Output is produced by the entrepreneurs using the Cobb-Douglas production function,

$$y_t = (A_t k_t)^\alpha \ell_t^{1-\alpha},$$

where A_t is the level of capital-augmenting productivity in period $t = 1, 2$. Productivity in period 1 is taken to be stochastic and exogenous: it is the only source of uncertainty in this model.

The period-2 productivity of a given entrepreneur is increasing with an investment expenditure x that is made by the entrepreneur in period 1,

$$A_2 = A(x), A' > 0.$$

We assume that the function $A(\cdot)$ is increasing and concave, i.e., the returns on the expenditure x are decreasing. This expenditure can be interpreted for example as an investment in human capital or know-how that is complementary with the productive capital accumulated in period 0. It could also be interpreted as additional physical capital but importantly, the productivity increase brought by the expenditure x is individual-specific and inalienable. The expenditure x raises the productivity of the entrepreneur who makes the expenditure but does not raise the productivity of his capital k if it is used by other entrepreneurs.[3]

Since workers have linear disutility, the real wage must be equal to $\omega_t = \omega$ in a perfectly competitive labor market. It follows that an entrepreneur operating a quantity k_t of capital makes a profit

$$\max_{\ell_t} (A_t k_t)^\alpha \ell_t^{1-\alpha} - \omega \ell_t = \kappa A_t k_t,$$

[3] Comparing our specification to Kiyotaki and Moore (1997), investment in k in our setup corresponds to investment in land in theirs; investment in x in our setup corresponds to investment in trees in theirs. Just as they assume that trees are lost when land is transferred, we assume that the investment x is lost when an entrepreneur defaults and her capital k is seized as collateral. In both setups, the assumption that an investment that is complementary to collateral cannot be transferred ensures that collateral prices depend on aggregate variables, not individual-specific investment. This is an important ingredient to obtain the price dynamics that lead to financial amplification.

with $\kappa \equiv \alpha \left[(1-\alpha)/\omega\right]^{(1-\alpha)/\alpha}$.

Productive capital is produced in period 0 with consumption good. The capital good can be produced only in period 0, implying that the aggregate stock of physical capital is constant in periods 1 and 2. The workers are endowed with a certain quantity of consumption good in period 0, y_0, whereas the entrepreneurs have no endowment. The entrepreneurs have the technology to transform consumption good into capital good, and must borrow from the workers in period 0 to produce the capital that they will use in periods 1 and 2. We assume that entrepreneurs finance their investments by issuing one-period debt in period 0.

The budget constraints of entrepreneurs and workers are collected in Table 1.

Table 1. Budget constraints

Period	Entrepreneurs	Workers
$t=0$	$c_0 + I(k) = d_0 k$	$c_0^w + b_0 = y_0$
$t=1$	$xk + c_1 + d_0 k = \kappa A_1 k + d_1 k$	$c_1^w + b_1 = \omega \ell_1 + b_0$
$t=2$	$c_2 + d_1 k = \kappa A_2 k$	$c_2^w = \omega \ell_2 + b_1$

Some parts of the budget constraints require further explanation. First, in period 0, $I(k)$ is the quantity of consumption good that the representative entrepreneur needs to produce a quantity k of productive capital. Function $I(\cdot)$ is of course increasing. We further assume that there are sufficiently decreasing returns in the production of capital, i.e., function $I(\cdot)$ is strictly convex and.satisfies $I''(k) - I'(k)/k - I(k)/k^2 > 0$. (This is for example the case for any iso-elastic function k^z with $z > 1$.)

Second, the level of borrowing by the representative entrepreneur and the level of lending by the representative worker in period t are respectively denoted by $d_t k$ and b_t. Variable d_t is the entrepreneurs' level of debt per unit of capital, or debt ratio, which is an approximate indicator for leverage. In a symmetric equilibrium, $d_t k$ must be equal to b_t times the number of workers per entrepreneur. The equilibrium interest rate on debt is equal to zero because there is no default risk and the lenders (the workers) are risk-neutral and do not discount the future. Finally, note that the productivity-enhancing expenditure x is scaled by k: a larger level of capital raises the expenditure that is required to reach a certain level of productivity.

2.2 First-Best Allocation

First, let us characterize the symmetric first-best allocation without collateral constraint. It is easy to see that the workers do not receive any surplus from working or lending (since their utility is linear in consumption and labor), so that their welfare is equal to their initial endowment, $U^w = y_0$. Using the budget constraints, it is also easy to see that the welfare of the representative entrepreneur is equal to the expected profit on the capital net of the productivity-enhancing expenditure minus the cost of producing capital,

$$\max_{k,x} E_0 \left[\kappa A_1 + \kappa A(x) - x\right] k - I(k). \tag{2}$$

The first-order conditions for welfare maximization on period-0 capital and period-1 investment are,

$$\begin{aligned} I'(k) &= E_0 \left[\kappa \left(A_1 + A_2\right) - x\right], & (3) \\ \text{and} \quad \kappa A'(x) &= 1. & (4) \end{aligned}$$

We denote the first-best levels of the variables with a superscript FB, i.e., k^{FB} and x^{FB}. The second equation determines x^{FB} independently of the levels of capital or productivity. Given x^{FB}, the first equation determines k^{FB}.

If a planner has the power to engage in lump-sum transfers, it is easy to see that she can always implement the first-best. She simply transfers $I\left(k^{FB}\right)$ and $x^{FB}k^{FB}$ from workers to entrepreneurs in periods 0 and 1 respectively, and transfer the sum of the two back in period 2. As a result, entrepreneurs invest the first-best levels and any financial market imperfections would be irrelevant. Similarly, a planner who can raise revenue via lump-sum taxes and use it to subsidize the asset price in order to fully relax binding financial constraints can restore the first-best. This is the mechanism by which e.g. the planner in Benigno et al. (2012) implements the first-best allocation.

However, we assume in the remainder of the paper that lump-sum taxes are not available. This is an important constraint on policymaking in the real world, as raising fiscal revenue generally involves distortions. It is also the starting assumption in the literature on optimal Ramsey taxation.

2.3 Financial Constraint

We assume that there is a collateral constraint coming from the fact that entrepreneurs can renegotiate their debt at the time of repayment. The constraint is the same as in Kiyotaki and Moore (1997) or Lorenzoni (2008). At the beginning of period $t = 1, 2$, the entrepreneur can make a take-it-or-leave-it offer to repay a lower amount than the debt coming due. If the creditors reject this offer, they can seize a fraction ϕ of the entrepreneur's productive capital and then sell it at price p_t. The creditors, thus, will accept the entrepreneur's offer as long as the offered repayment is not smaller than $\phi k p_t$, the amount that they would obtain by foreclosing on the capital.

We assume that debt is default-free, i.e., it is never renegotiated in equilibrium. This implies the following constraint,

$$d_t \leq \phi \min_t p_{t+1}, \tag{5}$$

where $\min_t p_{t+1}$ is the minimum possible price at which the capital of defaulting entrepreneurs can be sold in $t+1$ as anticipated in period t.

Capital, if it is seized by the creditors, is auctioned off to the non-defaulting entrepreneurs. Expressions for the equilibrium levels of p_1 and p_2 will be derived below.

3 Laissez-Faire Equilibrium

We define a symmetric equilibrium in the economy as a set of allocations $(k, b_0, b_1, d_0, d_1, x, \ell_1, \ell_2, c_0, c_1, c_2, c_0^w, c_1^w, c_2^w)$ and prices $(p_1, p_2, \omega_1, \omega_2)$ that (i) solve the optimization problems (1) and (2) of workers and entrepreneurs subject to the budget constraints in Table 1 and to the financial constraint (5) and (ii) that clear markets. We note that all variables for $t \geq 1$ are state-contingent variables that depend on the realization of the productivity shock A_1.

We solve for the equilibrium in the absence of government intervention via backward induction, starting with the last period. It will be important in some of our derivations to differentiate between variables related to an individual atomistic entrepreneur and variables related to the *representative* entrepreneur. We denote the variables related to an individual entrepreneur with a superscript i when this is necessary for clarity, whereas we denote without superscript the variables for the representative entrepreneur.

Period 2 Entrepreneur i starts period 2 with capital k^i and debt $d_1^i k^i$. If this entrepreneur came to default, his capital would be auctioned off to other entrepreneurs at a price that is equal to the return on capital for the representative entrepreneur,

$$p_2 = \kappa A_2 = \kappa A(x).$$

We write x without superscript here because the price is determined by the productivity of the *representative* entrepreneur who buys the capital rather than that of the defaulting entrepreneur.[4] In equilibrium, there is no default, and all the entrepreneurs repay their debts to workers.

Period 1 All the uncertainty is resolved in period 1. The next-period price of capital is known and the collateral constraint per unit of capital can be written as

$$d_1^i \leq \phi p_2. \tag{6}$$

Because of this constraint, entrepreneur i may not be able to finance the optimal level of productivity-enhancing expenditure, x^{FB}. Using the period-1 budget

[4] In order to ensure that the entrepreneurs have resources to buy more capital at the beginning of period 2, we can assume that they receive an exogenous endowment (which could be infinitesimally small).

constraint of entrepreneur i, the non-negativity constraint on consumption, $c_1^i \geq 0$, and $p_2 = \kappa A(x)$, the collateral constraint (6) can be written,

$$x^i + d_0^i \leq \kappa \left[A_1 + \phi A(x)\right]. \tag{7}$$

Thus, if the level of productivity, A_1, is low relative to the entrepreneur's debt ratio, d_0^i, it may be impossible to finance x^{FB}.

In a symmetric equilibrium we have $x^i = x$ and both sides of constraint (7) are increasing with x. To avoid the complications associated with multiple equilibria, we assume that the slope of the right-hand side is lower than 1.

Assumption 1 $\forall x$, $\kappa\phi A'(x) < 1$.

An important implication of equation (7) is that the impact of a negative productivity shock is amplified by the collateral constraint (financial amplification). Suppose that the level of period-1 productivity is sufficiently low that the financial constraint on entrepreneurs is binding. Assume that productivity is further reduced by a small amount $dA_1 < 0$. The first-round impact is to reduce the productivity-enhancing expenditure x by $dx = \kappa dA_1$, but the lower expenditure then reduces the price of capital p_2, which further tightens the constraint by $\phi\kappa A'(x)\, dx$. After the successive rounds of tightening have taken place (all within period 1), the net impact is given by,

$$dx = \frac{\kappa}{1 - \phi\kappa A'(x)} dA_1.$$

The denominator in this expression captures the effects of financial amplification. Individual entrepreneurs take prices as given and do not internalize the impact of financial amplification—which provides the justification for macroprudential intervention in this model.

If we denote the period-1 liquid net worth of the entrepreneur per unit of capital by $n^i = \kappa A_1 - d_0^i$, then we can express the optimization problem of the entrepreneur in period 1 as maximizing the payoff per unit of capital

$$\max_{x^i} \kappa A(x^i) - x^i + \lambda^i \left(n^i + \phi\kappa A_2 - x^i\right), \tag{8}$$

where λ_i is the shadow cost of constraint (7). Note that A_2 is taken as exogenous as it is the productivity of the average entrepreneur. This implies the first-order condition

$$\lambda^i = \kappa A'\left(x^i\right) - 1. \tag{9}$$

The period-1 price of capital, p_1, is derived in appendix B.1.

Period 0 Without loss of generality we set the entrepreneur's consumption in periods 0 and 1 to zero, $c_0 = c_1 = 0$. (If there is a possibility that the constraint is binding in period 1, then this is the optimal choice in order to minimize borrowing; otherwise it is one of a continuum of equilibrium allocations of consumption $c_0 + c_1 + c_2$ over time.) It follows that the debt ratio is a simple function of the level of capital,

$$d_0^i = d(k^i) \equiv \frac{I(k^i)}{k^i}.$$

The debt ratio function $d(k)$ is increasing with the level of capital because $I(\cdot)$ is a convex function and $I(0) = 0$. Furthermore, we assume that the period-0 borrowing constraint is loose, i.e., that $d_0 < \phi \min_0 p_1$ for the optimal d_0. The conditions on the exogenous parameters under which this is true are derived in appendix B.1.

We write the entrepreneur's period-1 welfare as the Bellman function

$$v\left(k^i\right) = \max_{x^i} \left\{ \left[\kappa A_1 + \kappa A(x^i) - x^i\right] k^i - I(k^i) + \lambda^i \left[\kappa A_1 + \phi\kappa A_2 - x^i - d(k^i)\right] k^i \right\}. \tag{10}$$

In period 0 the entrepreneur chooses the level of capital k^i that maximizes his expected welfare $E\left[v(k^i)\right]$. In the following, we denote the levels of endogenous variables in the laissez faire equilibrium by a superscript LF, and we drop the superscript i to abbreviate notation.

Proposition 1 (Level of k^{LF}) *If the period-*1 *constraint is binding with a nonzero probability under laissez-faire ($E\left(\lambda^{LF}\right) > 0$), then entrepreneurs borrow and invest less than the unconstrained first-best level in period* 0,

$$k^{LF} < k^{FB}.$$

Proof. See appendix A. ■

Intuitively, the productivity-enhancing expenditure is reduced below the first-best level because of the financial constraint. This lowers the return that the entrepreneur expects on his capital, and so his investment in period 0.

4 Ex-Ante Macroprudential Regulation

We analyze the scope for macroprudential regulation by solving the problem of a constrained social planner who makes the period-0 decisions on borrowing and investment for entrepreneurs but does not interfere otherwise with private decisions in periods 1 and 2. We assume that the social planner maximizes social welfare defined as the sum of the utilities of all agents in the economy (entrepreneurs and workers). The difference between private agents and the planner is that the latter internalizes the general equilibrium effects that are involved in financial amplification in period 1. In equilibrium, workers are always paid wages and interest rates that reflect their marginal disutility from labor and from lending. This implies that the welfare of workers is constant at y_0 so that a social planner who maximizes entrepreneurial welfare also maximizes social welfare. Any increase in social welfare is therefore a Pareto improvement.

We solve the problem via backward induction. The planner's expression for entrepreneurial welfare in a symmetric equilibrium of period 1 is

$$w(k) = \max_x \{[\kappa A_1 + \kappa A(x) - x]\, k - I(k) + \lambda [\kappa A_1 + \phi\kappa A(x) - x - d(k)]\, k\}\,. \quad (11)$$

This is the same optimization problem as for the entrepreneurs under laissez faire in equation (10) except that the planner internalizes that $p_2 = \kappa A(x)$ in the borrowing constraint. The associated first-order condition is

$$\tilde{\lambda} = \frac{\kappa A'(x) - 1}{1 - \phi\kappa A'(x)}, \quad (12)$$

where we use a tilde to refer to the equilibrium values of shadow prices as perceived by the planner.

The denominator of expression (12) captures that one additional dollar in period 1 leads to $1/(1 - \phi\kappa A'(x))$ additional dollars of investment in general equilibrium because of financial amplification. Comparing with equation (9), we observe that $\tilde{\lambda} > \lambda$ when the financial constraint is binding, i.e., the planner perceives the cost of binding constraints as higher than private agents. Because of this, we would expect that the social planner tries to reduce the economy's vulnerability to a credit crunch by reducing period-0 debt and investment through *macroprudential* policies. This is stated more formally in the following proposition, where we denote with a superscript MP the levels of the endogenous variables in an equilibrium in which the social planner engages in macroprudential policies.

Proposition 2 (Macroprudential Regulation) *Assume that the period-*1 *financial constraint is binding with positive probability in the laissez-faire equilibrium* $(E(\lambda^{LF}) > 0)$. *The optimal ex-ante macroprudential policy then satisfies the following properties:*

(i) the planner lowers borrowing and investment below the laissez-faire level:

$$k^{MP} < k^{LF},$$

(ii) the planner's chosen level of capital can be implemented by imposing a Pigouvian tax on borrowing or investment

$$\tau_0^{MP} > 0,$$

(iii) the planner mitigates but does not fully alleviate binding borrowing constraints,

$$E(\lambda^{LF}) > E(\lambda^{MP}) > 0.$$

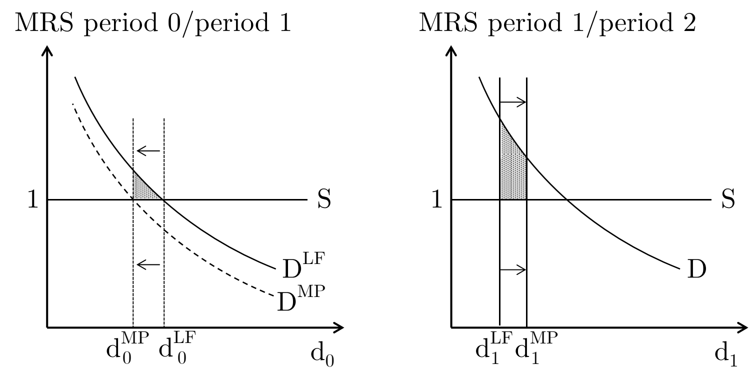

Figure 1: Macroprudential policy as a second-best intervention

Proof. See appendix A. ∎

If the financial constraints bind with a zero probability ($E\left[\lambda^{LF}\right] = 0$), period-0 investment and welfare are equal to the first-best levels and there is no justification for macroprudential intervention. If the constraints bind with a nonzero probability, the social planner recognizes that there is a trade-off between period-0 investment k and period-1 re-investment x. She invests less in period 0 than in the laissez-faire equilibrium and so increases the investment gap relative to the first best, but keeps additional borrowing capacity and raises investment in period 1.

As explained in the literature on pecuniary externalities in financial amplification, the laissez-faire equilibrium is inefficient because both the risk and severity of a credit crunch are endogenous to aggregate debt, but private entrepreneurs take aggregate debt as given (see e.g. Jeanne and Korinek, 2010ab). The planner's intervention increases welfare because reducing borrowing d_0 in period 0 below the laissez-faire level introduces a second-order distortion (i.e. a distortion that is negligible for small τ_0), but achieves a first-order benefit by relaxing the binding constraint in period 1. These welfare effects are illustrated by the shaded areas in figure 1.

In the Proposition above, there is a single policy instrument and a strictly monotonic relationship between the macroprudential policy τ_0 and the outcome k. This allows us to obtain the clear result that $k^{MP} < k^{LF}$, i.e., borrowing and investment are always lower under the macroprudential policy. As we will see below in section 6, this may no longer be the case when there are multiple policy instruments involved.

5 Ex Post Bailout Measures

In this section, we study another approach to mitigating the financial friction, in which the planner implements a transfer payment (bailout) to relax the credit constraint on entrepreneurs *ex post*. We assume that each constrained entrepreneur i

receives a transfer (subsidy) sk^i in period 1. The transfer is financed by taxes τ_1 and τ_2 on labor in periods 1 and 2.[5]

Such a generic tax-and-transfer measure captures the essential characteristic of policies to mitigate financial amplification effects ex-post: it relaxes financial constraints at the expense of introducing a distortion in the economy (here, a tax distortion). We will discuss a number of alternative common policy measures that fall into this category in section 7, including investment tax credits, subsidies to new borrowing and debt forgiveness. All these policy measures are aimed not only at alleviating financial constraints at the individual level, but also at alleviating financial amplification (systemic crises) at the aggregate level by pushing up the economy-wide level of asset prices and relaxing credit constraints across all entrepreneurs. There is thus both an "individual" and a "collective" or "systemic" element to such interventions.

If the period-1 transfer is financed with period-2 tax receipts, it requires that the planner issues public debt that is purchased by workers in period 1 and, thus, that the planner's borrowing capacity is superior to that of private agents. This is a common assumption in the literature, and it is generally justified by fact that the planner has the power to tax (Holmstrom and Tirole, 1998). The assumption is also plausible since debt-financed bailouts are commonly observed during financial crises. In such situations, we can interpret the planner's actions as lending his superior borrowing capacity to entrepreneurs at the expense of introducing a tax distortion in the economy.

The within-period optimization problem of entrepreneurs is affected by labor taxation as follows. In periods $t = 1$ and 2, the wage to workers net of taxation must still be equal to their disutility ω. After we impose a tax τ_t, entrepreneurs must therefore pay a gross wage $(1 + \tau_t)\omega$. The period profit of entrepreneurs is given by

$$\pi_t = \max_{\ell_t}(A_t k_t)^\alpha \ell_t^{1-\alpha} - (1 + \tau_t)\omega\ell_t = \kappa(\tau_t)A_t k_t,$$

where we define $\kappa(\tau) = \alpha\left[\frac{(1-\alpha)}{(1+\tau)\omega}\right]^{(1-\alpha)/\alpha}$ as the return on an effective unit of capital. We observe that labor taxation is distortionary and reduces the return $\kappa(\tau)$ per effective unit of capital Ak. However, the bailout has an a priori ambiguous impact on the period-2 price of capital $p_2 = \kappa(\tau_2)A(x)$, since it allows entrepreneurs to increase the productivity-enhancing expenditure x.

The subsidy is equal to the present value of the tax receipts per unit of capital, that is

$$sk = \tau_1\omega\ell_1 + \tau_2\omega\ell_2 = \tau_1\varepsilon(\tau_1)A_1 k + \tau_2\varepsilon(\tau_2)A_2 k, \tag{13}$$

where $\varepsilon(\tau)$ denotes the labor compensation per effective unit of capital, $\omega\ell/Ak$,

$$\varepsilon(\tau) = \omega\left[\frac{1-\alpha}{(1+\tau)\omega}\right]^{1/\alpha}. \tag{14}$$

[5] As we discussed earlier, we assume that lump-sum taxes are not available.

In a first step, we assume a time-consistent social planner who designs the bailout ex post (in period 1) to maximize domestic welfare subject to the collateral constraint. The equilibrium bailout policy can be characterized by three functions $s(n)$, $\tau_1(n)$ and $\tau_2(n)$ that map the entrepreneur's period-1 liquid net worth per unit of capital $n = \kappa A_1 - d(k)$ into the rate of subsidy and the ex-post tax rates. The properties of the equilibrium bailout policy are summarized in the following proposition, where we denote with a superscript BL the levels of the endogenous variables under the equilibrium time-consistent bailout policy.

Proposition 3 (Bailouts) *The equilibrium bailout policy under discretion satisfies the following properties:*

(i) there is a bailout if and only if the financial constraint is binding in the laissez-faire equilibrium:

$$\lambda^{LF}(n) > 0 \Longleftrightarrow s^{BL}(n) > 0.$$

(ii) the bailout is financed by issuing public debt and taxing labor in period 2*, whereas the period*-1 *tax on labor income is set to zero*

$$\tau_1^{BL}(n) = 0,$$

(iii) the bailout mitigates the constraint but does not fully alleviate it,

$$\lambda^{LF}(n) > 0 \Longrightarrow \lambda^{LF}(n) > \lambda^{BL}(n) > 0,$$

(iv) if the financial constraints bind with a nonzero probability ($E\left[\lambda^{LF}\right] > 0$), the expectation of bailouts increases period-0 *investment above the laissez-faire level,*

$$k^{BL} > k^{LF}.$$

Proof. See appendix A. ■

The intuition behind points (i) and (ii) is the following. A bailout raises welfare to the extent that it relaxes the credit constraint. There is no benefit for the planner to impose a tax in period 1 and transfer the receipts to entrepreneurs, since such a policy would both introduce a distortion into the resource allocation of the economy and tighten the financial constraint. On the other hand, by borrowing to make a transfer in period 1, the planner lends her own superior borrowing capacity to entrepreneurs. This yields a first-order welfare benefit since it relaxes a binding borrowing constraint, but comes at a second-order welfare cost by reducing the ratio ℓ_2/Ak in period 2 because of the future tax burden. The welfare effects are illustrated by the shaded areas in figure 2. According to the theory of the second best, it is always desirable to engage in some bailout when the financial constraint is binding, but not to fully undo the constraint, as noted in point (iii). The reason is that if the constraint were fully alleviated, the last bit of such a policy would have

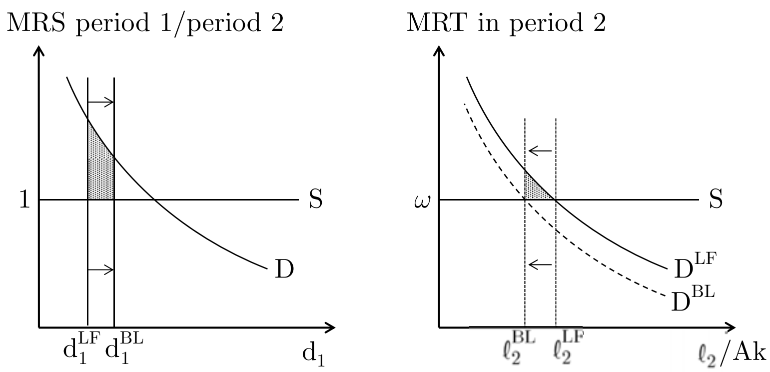

Figure 2: Bailouts as second-best interventions

only second-order welfare benefits but would come at a first-order welfare cost. As for point (iv), the intuition is that the bailouts raise the return on capital ex post and so enhance the incentives to invest in capital k ex ante.

In the extreme, the incentive effects of bailouts on capital investment k may lead to multiple equilibria. The period-0 optimality condition of private entrepreneurs on capital investment defines k as an increasing function of expected bailouts τ_2, and the period-1 optimality condition of the planner on the optimal bailout measure defines τ_2 as an increasing function of capital investment k, since a greater capital stock implies more debt and tighter financial constraints.[6] If the two functions intersect more than once, there are multiple equilibria with smaller or larger bailouts: if entrepreneurs expect small bailouts, they will be prudent and invest less, which in turn makes it optimal for the planner to provide only small bailouts; if entrepreneurs expect large bailouts, they will invest more, experience tighter constraints, and the planner will find it ex-post optimal to provide large bailouts.[7] For the remainder of our paper, we assume that the equilibrium is unique or that private agents always manage to coordinate on the better equilibrium.

Bailouts increase welfare ex post (in period 1), but their impact on ex-ante (period-0) welfare is in general ambiguous since they increase investment in k, which magnifies the overinvestment problem identified in Proposition 2. If the planner can commit to a less generous bailout policy $s^{BLc}(n)$, she would like to do so.[8]

[6] The period-0 optimality condition for capital investment is $Ev_k^{BL} = 0$ as defined in (27). The optimality condition for bailouts is (24).

[7] We refer to Farhi and Tirole (2012), who term this phenomenon "collective moral hazard," for a rigorous discussion of multiple equilibria under bailouts.

[8] Our result is reminiscent of (but not quite the same as) many similar results in the literature on financial safety nets in which discretionary bailouts induce excessive risk-taking ex ante. The difference is that the excessive risk-taking, in our model, involves a systemic component and exists even in the absence of bailouts because of pecuniary externalities. See also Davila (2011) for an analysis of time consistency problems in models of fire-sale externalities.

Proposition 4 (Bailouts Under Commitment) *If the planner can commit to a bailout policy $s^{BLc}(n)$, she would choose a lower level of bailouts than under discretion. This implies that the planner faces a time consistency problem in designing her optimal bailout policy.*

Proof. See appendix A. ■

The intuition is that committing to smaller bailouts reduces investment incentives. In a time-consistent equilibrium, the last unit of the bailout has only second-order ex-post benefits, but imposes a first-order cost by increasing ex-ante investment incentives, which are already excessive. If the overall ex ante welfare impact of bailouts is negative, it may even be optimal for the social planner to commit to do no bailouts whatsoever than to allow for discretionary bailouts. In our numerical illustration below, we will discuss the conditions under which this case may arise.

6 Optimal Policy Mix

We have now laid the groundwork to address the paper's core question: to integrate ex-ante macroprudential regulation and ex-post bailouts in a common framework and compare the benefits and costs as well as the interplay of the two policies. In this section we assume a social planner who can use the full set of instruments considered in the previous two sections: the macroprudential tax on period-0 borrowing, τ_0, as well as a period-1 bailout s that is financed by a tax τ_2 on labor in period 2. As before, taxation in period 1 will not be used and we accordingly omit the tax τ_1 from the problem. We start by describing the optimal policy mix under discretion; then we will show that our solution coincides with the optimal policy mix under commitment.

Observe that the ex-ante policy in the optimal policy mix can be described in terms of setting the instrument τ_0 or in terms of setting the outcome k. Depending on the results that we analyze, it is useful to focus on one or the other. For example, we obtain sharp results on the sign of the optimal policy instrument τ_0 in the following proposition, but the implications for the direction of change in the outcome k is ambiguous. By contrast, we obtain a clean characterization of the complementarity of capital investment k and bailouts s, but we show that the complementarity or substitutability of the ex-ante policy instrument τ_0 and the bailout policy s is generally ambiguous.

Proposition 5 (Optimal Policy Mix) *Assume that the period-1 financial constraint is binding with positive probability in the laissez-faire equilibrium ($E\left[\lambda^{LF}\right] > 0$). The optimal policy mix under discretion then satisfies the following properties:*

(i) the planner imposes a positive Pigouvian tax on borrowing or investment,

$$\tau_0^{MIX} > 0,$$

(ii) the planner provides the optimal discretionary bailout whenever the financial constraint is binding

$$\lambda^{LF}(n) > 0 \Longleftrightarrow s^{MIX}(n) > 0.$$

This mitigates the binding constraint but does not fully relax it, i.e. $\lambda^{MIX}(n) > 0$*.*

Proof. See appendix A. ■

The optimal policy mix gives a role to both macroprudential policy and bailouts. Since the probability of binding financial constraints remains positive under the optimal policy mix, our earlier intuition from the theory of the second-best is still valid: both macroprudential regulation and the bailout introduce a second-order distortion into the economy but achieve a first-order benefit from mitigating binding constraints through two alternative channels.

Comparative Merits Each of the two policy instruments has specific benefits and disadvantages:

Macroprudential regulation is somewhat blunt but can correct the distortions in investment incentives introduced by bailouts. Specifically, macroprudential policy measures are taken in the *expectation* of a systemic crisis in the following period. If the economy enters a good state of nature in the following period, then macroprudential measures have introduced a distortion without any corresponding ex-post benefit. (This is in the nature of all prudential interventions – their costs are incurred with certainty whereas their benefits materialize only in certain states of nature.) In this sense, macroprudential regulation is a somewhat "blunter" policy instrument than ex-post interventions.

However, an important second role for macroprudential policy is to correct the increased investment incentives created by bailouts. Since bailouts are contingent on the scale k of entrepreneurs, they provide them with additional incentives to invest in capital. Macroprudential regulation counteracts this distortion by taxing borrowing/investment.

The comparative advantage of bailouts is that they are more state-contingent. They are implemented conditional on the *realization* of a systemic crisis to alleviate financial constraints ex post. Their magnitude can be precisely targeted at the tighteness of binding constraints $\tilde{\lambda}$ in a given state of nature. When the economy enters a good state, no bailout is given and no cost is incurred. When the economy enters a bad state, a large bailout is given.

An analytic example that illustrates this distinction between macroprudential regulation and state-contingent bailouts is given by the following limit case:

Example 6 *If the probability of binding constraints as captured by* $E[\lambda]$ *goes to zero, the planner ceases to use macroprudential regulation* $\tau_0 \to 0$*. However, if a state with a strictly binding constraint* $\lambda > 0$ *occurs, the planner will engage in a strictly positive bailout* $s > 0$*.*

Macroprudential regulation is a function of both the probability of experiencing binding constraints and the tightness of such constraints. It is only useful if the probability of experiencing a financial crisis is bounded away from zero.

Effects on Capital Investment k Let us next translate the implications of the optimal policy mix for capital investment k.

Proposition 7 (Effects of Optimal Policy Mix) *Under the optimal policy mix, the level of capital investment k is in between the levels under macroprudential regulation only and under bailouts only,*

$$k^{MP} < k^{MIX} < k^{BL}.$$

Proof. See appendix A. ∎

The results of the Proposition are intuitive: macroprudential policy reduces investment compared to laissez-faire; bailouts increase investment compared to laissez faire. If a planner employs both instruments, the resulting level of capital investment is in between the ones that results from just one of the two measures.

However, capital investment in the optimal policy mix compared to the laissez faire equilibrium can go either up or down, i.e. $k^{MIX} \gtrless k^{LF}$ is ambiguous. Introducing the optimal macroprudential measure τ_0^{MIX} reduces capital investment k compared to the laissez-faire equilibrium; introducing the optimal level of bailouts $s^{MIX}(n)$ for given τ_0^{MIX} raises capital investment k. Taking the two policy changes together, borrowing and investment can either go up or down compared to the laissez faire equilibrium. The effects on the optimal macroprudential tax τ_0 are also ambiguous, $\tau_0^{MIX} \gtrless \tau_0^{MP}$: introducing bailouts in an economy with macroprudential policies raises the incentives for the planner to invest in capital k, which allows for a relaxation of macroprudential restrictions, but also raises the private incentives to invest, which calls for a tightening of macroprudential restrictions. The overall effect is indeterminate. We illustrate these ambiguities in the numerical illustration below.

The literature on macroprudential policy has sometimes described the finding that entrepreneurs borrow and invest more than a social planner ($k^{MP} < k^{LF}$) as "overborrowing." In models that focus exclusively on ex-ante policy measures (section 4), this description of outcomes mirrors the optimal policy prescription that $\tau_0^{MP} > 0$. Once we introduce multiple policy measures, as we do in our optimal policy mix here, a simple comparison between k^{MIX} and k^{LF} no longer reflects the direction of the optimal policy τ_0. In our framework, it is always desirable to set $\tau_0 > 0$, but, as shown in the numerical illustration below, situations arise in which $k^{MIX} > k^{LF}$ since the bailout policy has an independent effect on k.[9]

[9] Benigno et al. (2010) also illustrate such situations.

6.1 Time Consistency of the Optimal Policy Mix

As we noted in section 5, the bailout policy suffers from a time-consistency problem. The time-consistent bailout policy is excessively generous because it does not take into account its impact on the ex-ante accumulation of capital. One possible advantage of macroprudential policy is that it may help resolve the time consistency problem in the bailout policy by restricting investment in period 1. In fact, as we show below, the time consistency problem in the bailout policy is perfectly resolved by macroprudential policy in the optimal policy mix. Respectively denoting by $s^{MIXc}(n)$ and $s^{MIXd}(n)$ the optimal bailout policy under commitment (i.e., when it is chosen in period 0) and under discretion (when it is chosen in period 1), this result is stated formally in the following proposition.

Proposition 8 (Resolving Time Inconsistency) *The optimal policy mix resolves the time consistency problem introduced by bailouts, i.e., the optimal policy mix under commitment is identical to the optimal policy mix under discretion,*

$$s^{MIXc}(\cdot) = s^{MIXd}(\cdot).$$

Proof. Assume that a planner has chosen the optimal policy mix under discretion described by k^{MIX} and $s^{MIX}(n)$ as characterized in Proposition 5. If this policy is time inconsistent, then a planner under commitment would choose a different bailout policy $s^{MIX,c}(n) \neq s^{MIX}(n)$. Since $s^{MIX}(n)$ maximizes the period 1 payoff per unit of capital for a given n, the only reason could be to affect k^{MIX}. But if it is welfare-improving to deviate from k^{MIX}, then the discretionary planner would also choose a different k, contradicting our assumption that k^{MIX} was optimal. ■

At the optimal policy mix, there is no conflict between using macroprudential policy to solve the time consistency problems of bailouts and correcting the pecuniary externalities from financial amplification. The only reason for the time consistency problem in the absence of macroprudential regulation was that lower bailouts would reduce the incentives for excessive period-0 investment, leading to a conflict between what is optimal ex ante and ex post. If we add the macroprudential tax τ_0, then the planner has an independent instrument to set the correct ex-ante incentives for period-0 investment and the conflict is resolved. Macroprudential regulation therefore kills two birds with one stone.

The Proposition mirrors a more general insight about time consistency in optimal policy problems: time consistency problems generally reflect a lack of policy instruments and can be solved if a planner has sufficient instruments available. The reason is that time inconsistency arises when the expectation of a planner's optimal actions affects the behavior of private agents in earlier periods in an undesirable way. In our setup, for example, time inconsistency under bailouts only arises because a planner's optimally chosen bailouts in period 1, given the capital investment chosen by private agents in period 0, distorts the level of capital investment in period 0.

If the planner has no other instruments at hand, he would like to commit to being tough and reduce the size of bailouts in order to improve the incentives for period 0 investment of private agents. However, if the planner can control capital investment in period 0 directly via a macroprudential policy instrument τ_0, then there is no more reason to deviate from the ex-post optimal level of capital investment, and the time consistency problem disappears.

In short, time consistency arises when the planner has a lack of instruments and attempts to use one instrument, the bailout, to affect two targets, the incentive to invest in period 0 and the tightness of constraints in period 1. The time consistency problem is resolved if the planner can target these two objectives independently.

Corollary 9 (Allocation of Policy Objectives) *Assume that the bailout policy $s(n)$ and macroprudential regulation τ_0 are granted to two different agencies. Then the constrained optimal allocation can be achieved by giving the mandate of maximizing welfare ex post to the bailout agency and the mandate of removing the time-inconsistency in bailouts to the macroprudential agency.*

Proof. Let us analyze the optimal bailout policy $s(n)$ for a given τ_0. Then any change in the bailout policy affects k. The optimal bailout policy, given τ_0, is no longer necessarily the same under discretion and under commitment. For $\tau_0 = \tau_0^{MIX}$, however, the optimal bailout policy remains $s^{MIX}(n)$ under both commitment and discretion. Assuming otherwise leads to a contradiction with the fact that k^{MIX}, $s^{MIX}(n)$ maximizes welfare under commitment. By implication, one can characterize the optimal level of macroprudential taxation as the level of τ_0 such that the optimal bailout policy is time-consistent. ■

Our analysis above indicates that macroprudential regulation obviates the benefits of commitment and allows us to achieve the optimal policy mix while implementing the optimal discretionary bailout policy. An interesting question is whether bailout policy under commitment to a more refined set of state variables can obviate macroprudential policy and replicate the optimal policy mix. We find that this is indeed the case, but only under a restrictive set of assumptions that may be difficult to replicate in reality:

Corollary 10 (Commitment as a Substitute to Macroprudential Policy) *If the planner can commit to a bailout policy $s(k, A_1)$ that is conditional on both k and A_1 and unrestricted in sign, then she can replicate the optimal policy mix described in Proposition 5 without macroprudential regulation.*

Proof. Assume that the planner commits to a bailout policy

$$s(k, A_1) = \begin{cases} s^{BL}(\kappa A_1 - d(k)) & \text{for} \quad k \leq k^{MIX} \\ -\underline{s} & \text{for} \quad k > k^{MIX} \end{cases}$$

Under full commitment and for a sufficiently large penalty $\underline{s}$, this replicates the optimal policy mix since it ensures that entrepreneurs will find it optimal to invest at most k^{MIX} and since the policy provides the optimal bailout $s^{MIX}(n)$ where $n = \kappa A_1 - d(k)$. ■

The corollary captures that if a planner can commit to "carrots and sticks," i.e. to rewarding prudent entrepreneurs with a bailout and to punishing reckless entrepreneurs with a sufficient penalty, then she can implement the optimal policy mix. The crucial feature of this policy is that the planner can condition the payoff of entrepreneurs on the variable k that is the target of macroprudential policy. Bailout policy under commitment is a substitute to macroprudential policy if (i) it is contingent on the outcome targeted by prudential policy and (ii) it has not only carrots but also sticks.[10]

If we make the assumption that $\underline{s} = 0$, i.e. that the planner cannot impose an outright penalty on entrepreneurs who over-invest, then we remove the "sticks" and the planner has to rely exclusively on the "carrot" of bailouts. In that case, entrepreneurs can choose whether to operate under the bailout umbrella or not, and commitment works only under a limited set of cirumstances. In general, entrepreneurs will accept the conditions of the bailout umbrella if restricting their investment k is relatively cheap because the implied macroprudential tax τ_0 is low—this is the case if crises are rare—and if the benefit of receiving a bailout is sufficiently high—this will be the case if those rare crises are sufficiently deep. Otherwise entrepreneurs will opt out from the bailout umbrella and implement the laissez-faire equilibrium.

6.2 Bailout Fund

Since it is optimal to tax borrowing or investment in period 0 and to implement bailouts in period 1, one might be tempted to combine the two policy measures and use the proceeds of the period-1 prudential tax to finance the bailouts. This can be done by accumulating the prudential tax proceeds in a "bailout fund" that is distributed in the future if entrepreneurs experience binding financial constraints.[11] It would seem much preferable to finance the bailouts with a tax that tends to correct the distortions induced by the expectation of bailouts than by a tax that introduces new distortions in the economy.

We analyze this policy proposal by considering a planner who saves the tax revenue $T_0 = \tau_0 I(k)$ raised via macroprudential taxation in period 0 and uses it to bail out entrepreneurs in period 1 in order to relax their financial constraints. We continue to assume that this bailout is made in proportion to the capital holdings k of entrepreneurs so that each unit of asset receives an additional payment from the bailout fund of T_0/k. In order to keep our analysis as general as possible, we

[10] One interpretation of our result is that committing to a penalty on entrepreneurs with $k > k^{MIX}$ is effectively a form of macroprudential regulation.

[11] This is for example common practice for most deposit insurance systems (see Garcia, 1999).

allow for an additional potential bailout that is financed by a period 2 tax and that is described by the function $\tau_2(n)$. The total bailout received by an individual entrepreneur is then described by

$$s^{BF}(n) = T_0/k + \tau_2 \varepsilon(\tau_2) A(x)$$

If $s^{BF}(n) = s^{BL}(n)$, then entrepreneurs obtain the optimal discretionary bailout described in section 5.

The period-1 problem of entrepreneurs under a bailout fund, denoted by the superscript BF, is[12]

$$v^{BF}(k, d_0; T_0) = \max_x \left[\kappa A_1 + \kappa(\tau_2(n)) A(x) - x - d_0 + s^{BF}(n)\right] k + \\ + \lambda \left\{\kappa A_1 - x - d_0 + s^{BF}(n) + \phi p_2\right\},$$

and similarly for $w^{BF}(k, d_0; T_0)$ under the planner. The welfare properties of a bailout fund are described in the following.

Proposition 11 (Bailout Fund) *(i) Limiting bailouts to the resources available from a bailout fund reduces welfare compared to the optimal policy mix.*

(ii) Introducing a bailout fund in addition to the optimal policy mix described above does not affect welfare, but increases the required level of macroprudential taxation $\tau_0^{BF} > \tau_0^{MIX}$.

Proof. See appendix A. ■

The intuition for our results is that introducing a bailout fund does not yield any efficiency benefits — the planner has no comparative advantage in holding precautionary savings against systemic risk compared to entrepreneurs, as long as she can determine the correct level of private savings via macroprudential regulation. As we observe in point (i), limiting bailouts to the resources available from the fund therefore replicates the macroprudential equilibrium, which exhibits lower welfare than the equilibrium under the optimal policy mix.

In addition, observe that the bailout fund distorts incentives, since it implies greater transfers in period 1 than if entrepreneurs had held their savings privately. This calls for an even higher level of the macroprudential tax τ_0^{BF} in order to undo the distortion. As stated in point (ii), the equilibrium with a bailout fund and additional discretionary bailouts therefore implements the same equilibrium as the optimal policy mix, but with a higher macroprudential tax rate.

Our result that a bailout fund is undesirable as a precautionary instrument against aggregate risk contrasts with the desirability of funds that are used to share

[12] For ease of notation, we assume that the transfer T_0/k is made to entrepreneurs no matter if their financial constraint is binding or not. Similar results are obtained if the planner rebates the tax revenue T_0 in other ways when the financial constraint on entrepreneurs is loose, e.g. in lump-sum fashion.

idiosyncratic risk: if a planner can pool the idiosyncratic risks of *heterogeneous* entrepreneurs in a common fund, then she can reduce the total amount of savings held and thereby improve efficiency. We conclude that accumulating bailout funds only helps with idiosyncratic risk, not aggregate or systemic risk.

6.3 Numerical Illustration

We now present a numerical illustration of the results that we have obtained to provide additional intuition. For the sake of simplicity, we assume that the investment cost function $I(k)$ and the productivity $A(x)$ are given by

$$\begin{aligned} I(k) &= k^2 \\ \text{and} \quad A(x) &= \min(x, \bar{x}). \end{aligned}$$

The numerical values of the parameters chosen are given below in Table 1. Furthermore, we assume that A_1 follows a symmetrically truncated normal distribution with mean 1 and standard deviation σ.[13]

α	ω	ϕ	$\bar{x}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	1

Table 1: Parameter values for numerical illustration

In Figure 3 we vary the standard deviation σ and illustrate the effects on the equilibrium across the different policy regimes: laissez faire (LF), macro-prudential regulation (MP), bailouts (BL) and the optimal policy mix (MIX). As the standard deviation σ increases, the probability of binding constraints in the economy rises.

Panel 1 illustrates the effects on welfare under the different regimes. Under all four regimes, welfare is a strictly declining function of σ. The welfare losses are minimized under the optimal policy mix. For a low σ, using discretionary bailouts is superior to using macroprudential regulation – this is because the probability of binding constraints is low and bailouts allow for greater state contingency. For $\sigma > 0.47$, macroprudential regulation is superior to bailouts. In the Figure, welfare is always lowest under laissez faire. This is not necessarily always the case, since discretionary bailouts create moral hazard.[14]

In panel 2, we depict the initial capital investment k. Under macroprudential policy, capital k is always lower than under laissez faire. With bailouts, k is always higher than under laissez faire. Under the optimal policy mix, capital k is higher than laissez faire as long as $\sigma < 0.07$ and lower than laissez faire if $\sigma > 0.08$. This is

[13] An in-depth description of how we implemented the numerical simulation is available from the authors upon request.

[14] In our simulations, we found that $W^{BL} < W^{LF}$ may occur if the probability of being constrained is close to 1 and if amplification effects are strong, i.e. $\phi\kappa$ is close to 1.

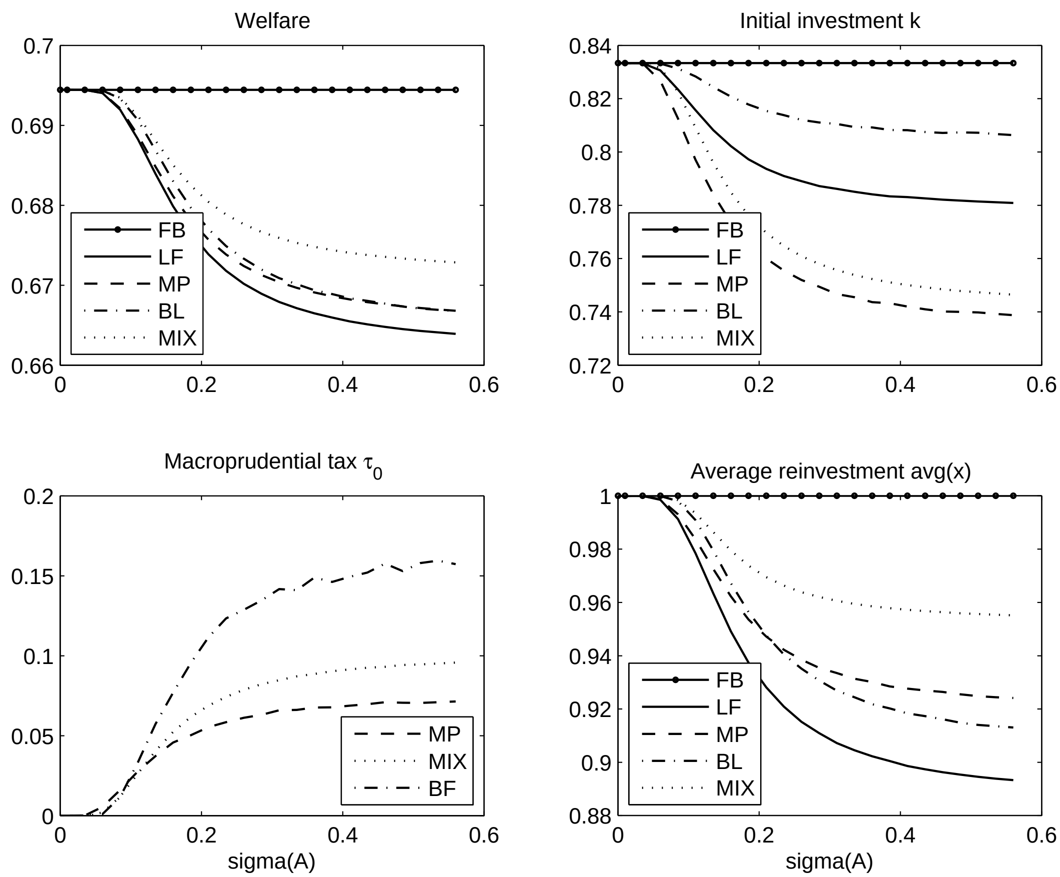

Figure 3: Numerical illustration of policy regimes

because for a low probability of being constrained, the planner relies more on bailouts and less on macroprudential regulation. Finally, observe that investment under the optimal policy mix is always greater than under macroprudential regulation alone.

Panel 3 describes the optimal macroprudential instrument τ_0. For low values of $\sigma < 0.12$, the planner imposes a lower τ_0 in the optimal policy mix than if only macroprudential regulation is available. This is because she relies mostly on bailouts to address binding financial constraints, and bailouts occur relatively rarely. For higher levels of σ, binding constraints and bailouts are increasingly common, and both factors induce the planner to raise τ_0 more heavily under the optimal policy mix. The regime under a bailout fund BL always requires greater macroprudential taxation than the optimal policy mix, but delivers the same real allocation.

Finally, panel 4 illustrates the average re-investment x across the different policy regimes. Reinvestment is greatest under the optimal policy mix and lowest under laissez faire. For low values of $\sigma < 0.21$, average reinvestment x is greater under bailouts; however, bailouts provide increasingly stronger incentives for additional investment in k, which increases the tightness of constraints. If σ is above 0.21, then reinvestment is actually greater under macro-prudential regulation.

More broadly speaking, the described economy faces a trade-off between how much to invest ex-ante in k and how much to reinvest ex-post in x when financial constraints are binding. Macroprudential regulation allows the planner to target the former and is most useful when constraints bind frequently. Bailouts allow her to target the latter, but distort investment in k: they are most useful when constraints bind rarely or when the distortion can be offset by macroprudential regulation.

7 Alternative Ex-Post Policy Measures

This section shows that our main results extend to alternative ex-post policy measures, including investment tax credits, debt forgiveness and subsidies to new borrowing. All of these measures have different distortive effects on the ex-ante incentives of entrepreneurs, but ultimately implement the same optimal policy mix when combined with the appropriate level of macroprudential regulation. Furthermore, the optimal policy mix is time consistent for each of these ex-post policy measures and coincides with the constrained efficient equilibrium of the economy described in Proposition 5.

From the perspective of period 1, the effects of alternative ex-post policy measures during financial amplification do not depend on the type of policy instrument. The behavior of entrepreneurs is determined by binding financial constraints not their optimality conditions. What matters for constrained entrepreneurs in period 1 is the total amount of liquid resources that is transferred to them to relax the financial constraint, not what this transfer is contingent on. We may call this the "liquidity effect" of an ex-post policy measure. Viewed from period 1, direct lump-sum transfers, investment tax credits, debt forgiveness or subsidies to new borrowing are all equivalent as long as they transfer the same amount of liquidity.

From the perspective of period 0, however, alternative ways of providing this transfer have different effects on ex-ante incentives, i.e. different "moral hazard" effects. Each of the discussed policy measures is contingent on different variables and therefore affects initial capital investment k through different channels. For example, in our benchmark ex-post policy measure, a transfer s is given per unit of capital, which rewards period 0 capital investment and therefore encourages more of it.[15] In addition, all the discussed ex-post policy measures, including lump-sum transfers, indirectly encourage more period 0 capital investment (which is already excessive) because they increase the return on investment by relaxing the financial constraint. However, the period 2 tax to raise the fiscal revenue required for the transfer discourages capital investment because it lowers the return on it.

In the following we will describe each of the alternative ex-post policy measures that we consider in some more detail. Then we will formally describe that they all implement the same optimal policy mix when combined with the correct level of macroprudential regulation.

Lump-Sum Transfer The first alternative ex-post policy measure that we consider is a lump-sum transfer T^{LS} that is only contingent on aggregate variables. For a given set of state variables (k, A_1) the planner chooses the optimal transfer such that

$$T^{LS}(k, A_1) = T^*(k, A_1) = \tau_2 \varepsilon(\tau_2) A(x) k$$

where τ_2 is the optimal time consistent tax rate chosen to finance the bailout as described in Proposition 3 and k and x are the aggregate levels of period 0 capital investment and period 1 reinvestment implied by (k, A_1).

Lump sum transfers are difficult to implement in practice since entrepreneurs are heterogeneous, but the measure is nonetheless a useful benchmark to consider: Lump sum transfers do not have direct effects on capital investment, but they still indirectly increase such investment because they relax the financial constraints of entrepreneurs, which allows for higher reinvestment x^i and therefore makes initial capital investment more profitable. This is undesirable because initial capital investment is excessive in the laissez faire equilibrium.

Debt Forgiveness A policy of partial debt forgiveness in period 1 corresponds to a transfer to each entrepreneur i of a fraction of her period 0 debt level. Since the planner finds it ex-post optimal to transfer $T^*(k, A_1)$ to entrepreneurs as described in Proposition 3, the optimal debt forgiveness policy defines the transfer contingent on the debt level as $T^{DF} = d_0^i \cdot T^*/d_0$ where $d_0^i = d(k^i)$ and $d_0 = d(k)$ are the

[15] Subsidies to period-1 production or period-1 employment have similar effects on ex-ante incentives since production and employment increase in k, but they introduce an additional distortion in the choice of labor ℓ_1, which reduces the period 1 productivity of capital κA_1.

individual and aggregate debt level of entrepreneurs. This policy acts as a state-contingent subsidy to indebtedness and therefore exacerbates the overborrowing problem that is present in the laissez faire equilibrium.

Investment Tax Credit Investment tax credits or, more generally, subsidies to new investment are transfers that are contingent on the level of new investment $x^i k^i$ of individual entrepreneurs, $T^{ITC} = x^i k^i \cdot T^* / (xk)$. Such a subsidy makes period 1 investment more desirable, which induces entrepreneurs to invest less in period 0 so that they have more precautionary savings that can be used to increase period 1 investment, thereby mitigating the overborrowing problem somewhat.

Subsidy to New Borrowing A subsidy to new borrowing is a transfer that depends on an an entrepreneur's level of new borrowing d_1^i and can be captured by $T^{BS} = d_1^i k^i \cdot T^* (k, A_1) / (d_1 k)$. We can think of subsidies to borrowing e.g. as being implemented through interest rate cuts. Furthermore, crises lending programs often also include an implicit subsidy to new borrowing, as governments provide loans below market interest rates. Subsidies to borrowing reward entrepreneurs who have access to borrowing and therefore encourages them to borrow less in period 0 so as to keep spare borrowing capacity for period 1, which also mitigates the overborrowing problem.

Proposition 12 (Alternative Ex-Post Measures) *(i) For any of the discussed alternative ex-post policy measures $P \in \{LS, DF, ITC, BS\}$ there exists an optimal macroprudential tax τ_0^P such that the policy mix $\left(\tau_0^P, T^P\right)$ implements the optimal policy mix described in Proposition 5.*

(ii) The optimal policy mix $\left(\tau_0^P, T^P\right)$ is time consistent.

Proof. See appendix A. ■

The intuition for these results is analogous to the intuition of Proposition 5 on the optimal policy mix and Proposition 8 on resolving the time consistency problem of ex-post policy measures: from the perspective of period 1, the planner finds it optimal to transfer an identical amount T^* to entrepreneurs, no matter through what instrument this transfer occurs. The amount of this transfer is determined such that the marginal benefit of relaxing the financial constraint of entrepreneurs equals the marginal tax distortion introduced in period 2. No matter what the incentive effects of the ex-post policy measure, the planner's desired level of ex-ante capital investment is given by k^{MIX}. For each of the ex-post policy measures, there is an optimal period 0 tax τ_0^P that precisely implements this level of capital investment, i.e. that simultaneously internalizes the pecuniary externality and corrects the distortions in ex-ante incentives created by the ex-post intervention. The reason why the policy mix is time consistent for any such pair is that the period 0 policy measure can perfectly target the optimal level of capital investment k^{MIX}; therefore the planner has no incentives to commit to a different action in period 1 than what is ex-post optimal.

In conclusion, we note that alternative forms of providing bailouts have different incentive effects for capital investment k in period 0, but (except in knife-edge cases) do not implement the constrained efficient equilibrium. For any ex-post policy P, we can find an optimal policy mix $\left\{\tau_0^P, T^P\right\}$ such that the same constrained efficient equilibrium is implemented and such that the time consistency problem is solved.

8 Conclusions

This paper develops a simple framework of optimal policies in an environment where collateral-dependent borrowing constraints lead to financial amplification. If policy-makers have access to lump-sum transfers, they can restore the first-best equilibrium in which borrowing constraints are irrelevant. Otherwise, all policies fall into the category of second-best interventions, i.e. they achieve first-order welfare gains by mitigating binding borrowing constraints in the economy, but at the expense of introducing second-order distortions, i.e. distortions that are initially negligible but grow with the square of the policy intervention.

In accordance with the theory of the second-best (see Lipsey and Lancaster, 1956), it is optimal to use all second-best instruments available in such a setting. In particular, we show that it is optimal to both restrict borrowing ex-ante via macroprudential regulation and to relax borrowing constraints ex-post by providing bailouts or other transfers. This implies that in our model, policymakers should both "lean against the wind" and "mop up after the crash."

In comparing the relative benefits and disadvantages, we find that ex-post policy measures are better targeted because they are conditional on an adverse state of nature having materialized, but they lead to problems of time consistency as they distort the ex-ante incentives of entrepreneurs to invest and lead to moral hazard. Macroprudential regulation is blunter since it is imposed in the anticipation that crises may occur in the future, but it can resolve the time-inconsistency problem. These distinctions between the two policy measures reinforce the message that it is generally desirable in models of financial amplification to use both of them.

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A Proofs

Proof of Proposition 1 (Level of k^{LF}) We show that $I'(k^{LF}) < I'(k^{FB})$, which will prove the proposition since $I'(\cdot)$ is convex. The first-order condition for the entrepreneur's problem is

$$E\left[v'(k)\right] = 0. \tag{15}$$

We use equation (10) and the envelope theorem; furthermore we observing that in equilibrium, $\lambda\left[\kappa A_1 + \phi\kappa A_2 - x - d(k)\right] = 0$. This implies

$$I'(k^{LF}) = E\left[\kappa A_1 + \kappa A(x^{LF}) - x^{LF}\right] - E\left(\lambda^{LF}\right) k^{LF} d'(k^{LF}). \tag{16}$$

In the special case where there is no collateral constraint this equation becomes the first-order condition for the first-best level of capital

$$I'\left(k^{FB}\right) = E_0\left[\kappa A_1 + \kappa A(x^{FB}) - x^{FB}\right]. \tag{17}$$

Comparing equations (16) and (17) shows that $I'(k^{LF}) < I'\left(k^{FB}\right)$ for two reasons. First, the fact that x^{LF} sometimes falls below x^{FB} reduces the first term on the r.h.s. of (16) below the r.h.s. of (17). The constraint reduces the average level of productivity-enhancing expenditure and so the return on capital. Second, the second term on the r.h.s. of (16) is negative because $E\left(\lambda^{LF}\right) > 0$ and $d'(k^{LF}) > 0$. This reflects another cost of increasing capital: it raises the debt ratio d_0 and so tightens the constraint on the productivity-enhancing expenditure. ■

Proof of Proposition 2 (Macroprudential Regulation) Using equations (10), (11), the envelope theorem and equations (9) and (12), the optimality conditions of entrepreneurs and the planner are respectively

$$\begin{aligned} Ev'(k) &= E\left[\kappa A_1 + \kappa A(x) - x\right] - I'(k) - kE\left[\kappa A'(x) - 1\right] d'(k) = 0, \quad (18) \\ Ew'(k) &= E\left[\kappa A_1 + \kappa A(x) - x\right] - I'(k) - kE\left[\frac{\kappa A'(x) - 1}{1 - \phi\kappa A'(x)}\right] d'(k) = 0. \quad (19) \end{aligned}$$

Both $Ev'(k)$ and $Ew'(k)$ are decreasing in k, since our assumption $I''(k) - I'(k)/k - I(k)/k^2 > 0$ ensures that $kd'(k)$ is increasing in k. The equilibrium levels of capital under laissez-faire and the social planner respectively satisfy $Ev'(k^{LF}) = 0$ and $Ew'(k^{MP}) = 0$. Observe that when the economy is constrained, the level of x is determined by the constraint and thus is the same whether or not there is a social planner (given k). Hence, for any given k, whenever $E[\lambda] = E[\kappa A'(x) - 1] > 0$, comparing (18) and (19) shows that $Ew'(k) < Ev'(k)$, i.e., the social planner has a strictly lower marginal valuation of capital than individual entrepreneurs. If the laissez-faire equilibrium satisfies $E\left[\lambda^{LF}\right] = E[\kappa A'(x) - 1] > 0$, then $Ev'(k^{LF}) = 0 > Ew'(k^{LF})$ and the planner finds it optimal to reduce capital investment and borrowing to a lower level $k^{MP} < k^{LF}$. This proves point (i) of the Proposition.

To see how the planner's equilibrium can be implemented via Pigouvian taxation, consider a tax $\tau_0 > 0$ on period-0 investment that is rebated to entrepreneurs in lump-sum fashion so as to be wealth-neutral. This modifies the period-0 budget constraint of entrepreneurs to

$$c_0 + (1+\tau_0)I(k) = d_0 k + T,$$

where the rebate satisfies $T = \tau_0 I(k)$.[16] The tax modifies the optimality condition of entrepreneurs (18) by pre-multiplying the marginal cost of investment and adding a term to the perceived cost of binding constraints,

$$Ev'(k) = E\left[\kappa A_1 + \kappa A(x) - x\right] - (1+\tau_0)\, I'(k) - E[\lambda]\left[kd'(k) + \tau_0 I'(k)\right] = 0. \quad (20)$$

(A tax on borrowing would introduce an equivalent wedge.) The optimal tax rate τ_0 is then chosen such that $Ev'\left(k^{MP}\right) = 0$ so that the decentralized equilibrium

[16] One interpretation of the rebate is that the policy is introduced not literally as a tax, but instead as a quantity regulation, which implies that entrepreneurs keep the surplus that results from restricting borrowing and investment.

replicates the social planner's equilibrium. Substituting equations (19) and (20) and using the expressions (12) and (9) for $\tilde{\lambda}^{MP}$ and λ^{MP} we obtain

$$\tau_0^{MP} = \frac{E\left[\tilde{\lambda}^{MP} - \lambda^{MP}\right]}{1 + E\left[\lambda^{MP}\right]} \cdot \frac{d'\left(k^{MP}\right) k^{MP}}{I'\left(k^{MP}\right)}. \tag{21}$$

The level of the optimal tax τ_0 is strictly positive if there is a positive probability that the financial constraint is binding and that $\tilde{\lambda}^{MP} > \lambda^{MP}$. This proves point (ii) of the Proposition.

If $E\left[\lambda^{LF}\right] > 0$, then there will still be a strictly positive expected cost of binding constraints $E\left[\lambda^{MP}\right] > 0$ even after the planner's intervention. Otherwise equation (19) would imply $k^{MP} > k^{LF}$, a contradiction with point (i) of the Proposition. The lower level of capital investment $k^{MP} < k^{LF}$ implies a lower debt ratio $d_0 = d\,(k)$. If the constraint is binding for a given pair (k^{LF}, A_1), then it is therefore looser for the pair (k^{MP}, A_1) and the expenditure x satisfies $x^{MP} > x^{LF}$. Thus, $\lambda^{MP} < \lambda^{LF}$ for the realizations of A_1 such that the financial constraint is binding under laissez-faire, and since these realizations have a non-zero probability, one has $E\left[\lambda^{MP}\right] < E\left[\lambda^{LF}\right]$. This proves (iii). ■

Proof of Proposition 3 (Bailouts) The period-1 welfare of an entrepreneur who takes the subsidy rate, the tax rates and the collateral price as given, is

$$\begin{aligned} v^{BL}(k^i) &= \max_{x^i} \left[\kappa(\tau_1)A_1 + \kappa(\tau_2)A(x^i) + s - x^i\right] k^i - I(k^i) \\ &\quad + \lambda^i \left[\kappa(\tau_1)A_1 + \phi p_2 + s - x^i - d(k^i)\right] k^i. \end{aligned} \tag{22}$$

Substituting for s and p_2, a planner who enters period 1 facing a set of state variables (k, A_1) solves

$$\begin{aligned} w^{BL}\,(k) &= \max_{x,\tau_1,\tau_2} \left[\eta\,(\tau_1)\,A_1 + \eta\,(\tau_2)\,A\,(x) - x\right] k - I\,(k) \\ &\quad + \tilde{\lambda}\left\{\eta\,(\tau_1)\,A_1 + \left[\phi\kappa\,(\tau_2) + \tau_2\varepsilon\,(\tau_2)\right] A\,(x) - x - d(k)\right\} k, \end{aligned} \tag{23}$$

where we denote by $\eta\,(\tau) = \kappa\,(\tau) + \tau\varepsilon\,(\tau)$ the social net return on capital, i.e., the entrepreneur's return plus tax revenue per unit of capital. We observe that $\eta'\,(\tau) = \tau\varepsilon'\,(\tau) < 0$ for $\tau > 0$, i.e., the social net return on capital is decreasing with the level of taxation.

The planner's optimality condition on τ_1 is

$$\tau_1\varepsilon'\,(\tau_1)\,A_1(k + \tilde{\lambda}) = 0,$$

which implies that $\tau_1 = 0$. This proves point (ii) of the Proposition.

Using $\varepsilon'(\tau) = -\frac{1}{\alpha}\varepsilon(\tau)/(1+\tau)$ and $\kappa'(\tau) = -\varepsilon(\tau)$, the optimality condition for τ_2 can be written

$$\frac{\tau_2}{1+\tau_2} = \alpha\left(1-\phi\right)\frac{\tilde{\lambda}}{1+\tilde{\lambda}}. \tag{24}$$

The shadow costs λ and $\tilde{\lambda}$ are respectively given by

$$\lambda = \kappa(\tau_2)A'(x) - 1, \tag{25}$$

$$\tilde{\lambda} = \frac{\kappa(\tau_2)A'(x) - 1}{1 - \left[\phi\kappa\left(\tau_2\right) + \tau_2\varepsilon\left(\tau_2\right)\right]A'\left(x\right)}. \tag{26}$$

Observe that as in the laissez faire equilibrium above, the period-1 liquid net worth n, which determines the tightness of the constraint in (23), is a sufficient statistic for the optimal tax rate $\tau_2\left(n\right)$ and the bailout $s\left(n\right) = \tau_2\left(n\right)\varepsilon\left(\tau_2\left(n\right)\right)A\left(x\right)$.

The social planner still values liquidity more than entrepreneurs in a constrained equilibrium, i.e., $\tilde{\lambda} > \lambda$ if $\lambda > 0$. It follows that if $\lambda^{LF}\left(n\right) > 0$, one must have a strictly positive tax rate τ_2 to finance a bailout in the amount of $s = \tau_2\varepsilon\left(\tau_2\right)A\left(x\right)$. If not, (i.e., if τ_2 were equal to zero), then there would be no bailout, implying $\lambda = \lambda^{LF}\left(n\right) > 0$ and $\tilde{\lambda}$, being larger than λ, would be strictly positive, which would contradict equation (24). Conversely, if $\lambda^{LF}\left(n\right) = 0$, the laissez-faire equilibrium is unconstrained and the social planner does not increase welfare by implementing a bailout. This proves points (i) of the Proposition.

Furthermore, equation (24) also implies that the constraint is still binding under the optimal bailout measure; otherwise $\tilde{\lambda}$ would be equal to zero and so would τ_2. This shows $\lambda^{LF}(n) > 0 \implies \lambda^{BL}(n) > 0$. To show that $\lambda^{BL}(n)$ is smaller than $\lambda^{LF}(n)$, observe that the planner chooses (τ_2, x) in optimization problem (23) so as to maximize the period-2 net return $\eta\left(\tau_2\right)A\left(x\right) - x$ per unit of capital subject to the borrowing constraint. This implies that the return at the planner's optimum, is greater than the return in the absence of intervention, $\eta\left(\tau_2^{BL}\right)A\left(x^{BL}\right) - x^{BL} > \kappa A\left(x^{LF}\right) - x^{LF}$, which in turn is possible only if the bailout raises the expenditure, $x^{BL} > x^{LF}$. Then for $\tau_2 > 0$ we have $\kappa(\tau_2)A'(x^{BL}) < \kappa(0)A'(x^{LF})$, which with (25) implies $\lambda^{BL} < \lambda^{LF}$. This proves point (iii) of the Proposition.

Finally, we show that the expectation of bailouts raises investment. Taking the derivative of (22) and using the envelope theorem and $\tau_1 = 0$, an individual entrepreneur perceives the marginal benefit of investing in capital as

$$v_k^{BL} = \kappa A_1 + \eta\left(\tau_2\right)A\left(x^i\right) - x^i - I'\left(k^i\right) - \left[\kappa(\tau_2)A'(x^i) - 1\right]d'\left(k^i\right). \tag{27}$$

Increasing τ_2 from 0 to the optimal level set by the social planner (given by (24)) raises v_k^{BL} for two reasons. Firstly, as noted above, the return at the planner's optimum is greater than the return in the absence of intervention, $\eta\left(\tau_2^{BL}\right)A\left(x^{BL}\right) - x^{BL} > \eta(0)A\left(x^{LF}\right) - x^{LF}$; secondly, $x^{BL} > x^{LF}$ implies that $\kappa\left(0\right)A'\left(x^{LF}\right) > \kappa\left(\tau_2^{BL}\right)A'\left(x^{BL}\right)$. Combining these two observations with equation (27), we observe that for any realization of A_1 with binding constraints, v_k^{BL} is increased above the laissez faire level by the bailout. This implies that entrepreneurs choose a higher level of period 0 capital investment $k^{BL} > k^{LF}$, proving point (iv) of the Proposition.

Lastly, note that we have not taken into account the period-1 implementability constraint that private entrepreneurs are willing to invest the chosen level of x, i.e., $\kappa(\tau_2) A'(x) \geq 1$. But taking this constraint into account does not change our results. In general, the planner raises the tax either until either her optimal tax rate τ_2 determined by equation (24) is reached or the implementability constraint becomes binding so that $\kappa(\tau_2) A'(x) = 1$. In both cases, the chosen tax rate is strictly positive $\tau_2 > 0$. ■

Proof of Proposition 4 (Bailouts under Commitment) Assume a planner who can commit to a bailout policy $s^{BLc}(n)$ financed by a tax $\tau_2^{BLc}(n)$, which are both functions of the aggregate period 1 liquid net worth per unit of capital $n = \kappa A_1 - d(k)$. Such a planner will solve the following optimization problem, denoted by superscript BLc to capture bailouts under commitment:

$$\max_{k,x(n),\tau_2(n)} E\left\{w^{BLc}(k, x(n), \tau_2(n))\right\} \quad \text{s.t.} \quad Ev_k^{BLc}(k; \tau_2(n)) = 0 \quad [\xi]$$

where

$$\begin{aligned} w^{BLc}(k, x(n), \tau_2(n)) &= \{\kappa A_1 + \eta(\tau_2) A(x(n)) - x(n)\} k - I(k) + \\ &\quad +\lambda\{[\kappa A_1 + [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)] A(x(n)) - x(n)] k - I(k)\} \end{aligned}$$

and $\quad v_k^{BLc} = \kappa A_1 + \eta(\tau_2(n)) A(x(n)) - x(n) - I'(k) - \left[\kappa(\tau_2(n))A'(x) - 1\right] d'(k)k$

Observe that individual entrepreneurs take aggregate net worth and therefore the tax rate and bailout as given.

The difference between this optimization problem and the time-consistent problem is that the planner sets $x(n)$, $\tau_2(n)$ and by implication $s(n)$ already in period 0 and internalizes how her decisions affect the ex-ante incentives of entrepreneurs to invest, as reflected by the implementability constraint $Ev_k^{BLc} = 0$. Unlike in our formulation of the discretionary planning problem, the function $w^{BLc}(\cdot)$ is not a Bellman equation that is maximized – it is just short-hand notation for the payoff of entrepreneurs, given the optimal policies chosen in period 0. To save on notation we will omit the argument n on x and τ_2 in the following. The Lagrangian associated with the planner's problem is

$$L = E\left\{w^{BLc}(k, x, \tau_2)\right\} - \xi E v_k^{BLc}(k, \tau_2)$$

The planner's optimal capital investment k is determined by the condition

$$Ew_k^{BLc} = \xi E v_{kk}^{BLc}$$

However, the implementability constraint captures that private agents determine k according to their first-order condition $Ev_k^{BLc} = 0$. As we observed earlier, $Ew_k < 0$ at the privately optimal level of k. The planner's optimality condition on k therefore pins down the shadow price

$$\xi = \frac{Ew_k}{Ev_{kk}} > 0.$$

The optimality conditions on the optimal tax τ_2 and period 1 investment x are

$$\left[(1+\lambda)\,\tau_2\varepsilon'(\tau_2) + \lambda(1-\phi)\,\varepsilon(\tau_2)\right] A(x)\,k = \xi v_{k\tau_2} \tag{28}$$

$$\left\{\eta(\tau_2)\,A'(x) - 1 - \lambda\left[1 - (\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2))\,A'(x)\right]\right\} k = \xi v_{kx} \tag{29}$$

For loose borrowing constraints, the two conditions can be simplified to yield $\tau_2\varepsilon(\tau_2)\,A(x) = 0$ and $\kappa A'(x) = 1$, implying that the planner does not intervene and the first-best level of investment x^{FB} is implemented.

If the borrowing constraint is binding, we observe that $v_{kx} > 0$ and $v_{k\tau_2} > 0$ for $\tau_2 < \tau_2^{BLd}$. The optimality condition on τ_2 implies

$$\lambda^{BLc} = \frac{\eta(\tau_2)\,A'(x) - 1 - \xi/kv_{kx}}{1 - [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)]\,A'(x)} \tag{30}$$

The planner under commitment perceives the shadow price of relaxing the constraint as lower than the shadow price under discretion because of the term $-\xi/kv_{kx} < 0$. This term captures that relaxing the constraint induces entrepreneurs to engage in more period 0 capital investment, which is already excessive.

The optimality condition on τ_2 becomes

$$\frac{1+\lambda}{\alpha}\frac{\tau_2}{1+\tau_2} = \lambda(1-\phi) - \frac{\xi v_{k\tau_2}}{\varepsilon(\tau_2)\,A(x)\,k}$$

Comparing this expression to equation (24), the optimal tax rate $\tau_2^{BLc}(n)$ in a bailout regime under commitment is below the optimal tax rate under discretion $\tau_2^{BLd}(n)$ for two reasons: first, the planner perceives a lower cost of binding constraints λ^{BLc}; second, the planner lowers the transfer in order to reduce the incentives for excessive borrowing and investment, as captured by the term on the right-hand side.

In short, the planner reduces the magnitude of the bailout measures that are ex-post efficient in order to provide better ex-ante incentives. ■

We observe that if the planner committed to a bailout policy that is a function $s(A_1)$ of the period 1 productivity shock A_1 rather than a function $s(n)$, the outcome would be identical since entrepreneurs take both A_1 and aggregate n as given and since both are sufficient statistics for the state of nature. By contrast, if the planner can commit to a bailout transfer policy that is conditional on k and that includes penalties on entrepreneurs who borrow excessively, then commitment may solve the time consistency problem. We discuss this in detail in section 6.

Proof of Proposition 5 (Optimal Policy Mix) We proceed by backward induction. The proof of point (ii) of the Proposition is identical to the proof of point (i) of Proposition 3, and we find that the magnitude of the optimal bailout policy is identical $s^{MIX}(n) = s^{BL}(n)$.

To show point (i), we follow similar steps as in the proof of Proposition 2. Observe that the ex-ante optimization problem of the planner under discretion (superscript d) is

$$\max_k Ew^{MIX,d}(k),$$

where $w^{MIX,d}(k) = w^{BL}(k)$ as given in equation (23), and similarly $v^{MIX,d}(k) = v^{BL}(k)$. The optimality conditions of entrepreneurs and the planner are, respectively

$$Ev_k^{MIX,d} = E[\kappa A_1 + \eta(\tau_2)A(x) - x] - I'(k) - E[\lambda]\, d'(k) = 0, \tag{31}$$

$$Ew_k^{MIX,d} = E[\kappa A_1 + \eta(\tau_2)A(x) - x] - I'(k) - E\left[\tilde{\lambda}\right] d'(k) = 0, \tag{32}$$

where λ and $\tilde{\lambda}$ are given by equations (25) and (26).

If financial constraints are binding with positive probability in the laissez-faire equilibrium $E\left[\lambda^{LF}\right] > 0$ then, by Proposition 3 $E\left[\lambda^{MIX}\right] > 0$. Furthermore, since $\tilde{\lambda}^{MIX} > \lambda^{MIX}$ in that case, the marginal return on capital is strictly smaller for the social planner than for entrepreneurs, $Ew_k^{MIX,d} < Ev_k^{MIX,d}$. The social planner, as a result, chooses a strictly lower level of capital investment $k^{MIX} < k^{BL}$. Following the steps of Proposition 2 in deriving equation (21), this capital level can be implemented by setting the macroprudential tax τ_0 to

$$\tau_0^{MIX} = \frac{E\left[\tilde{\lambda}^{MIX} - \lambda^{MIX}\right]}{1 + E\left[\lambda^{MIX}\right]} \cdot \frac{d'\left(k^{MIX}\right) k^{MIX}}{I'\left(k^{MIX}\right)}. \tag{33}$$

If there is a nonzero probability that the financial constraint is binding, then $E\left[\tilde{\lambda}^{MIX}\right] > E\left[\lambda^{MIX}\right] > 0$ and this expression is strictly positive. ■

Proof of Proposition 7 (Effects of Optimal Policy Mix) The first inequality in the Proposition holds because capital investment and bailouts are complements for the planner. Looking at expression (32), increasing the bailout s for given k reduces the tightness of financial constraints and lowers $\tilde{\lambda}$. For any level of the bailout $s \leq s^{MIX}$, it also raises the expected period 2 return on capital $[\eta(\tau_2) A(x) - x]$. This implies that the cross derivative satisfies $Ew_{ks}^{MIX} > 0$ and the planner finds it optimal to increase capital investment. The second inequality is analogous to point (i) of Proposition 2 and can be proven in the same manner. ■

Proof of Proposition 11 (Bailout Fund) Using the period 0 budget constraint of entrepreneurs $d_0 k = (1+\tau_0) I(k)$ together with the planner's budget constraint $T = \tau_0 I(k)$, we find that the revenue accumulated in the bailout cancels out from the period 1 budget constraint, i.e. $d_0 - T/k = I(k)/k = d(k)$ and therefore $w^{BF}(k, d_0; T) = w^{BF}(k, d(k); 0)$. This implies that for the planner, the optimal level of capital investment remains unchanged from what it was in the absence of a bailout fund.

The private optimality condition for period 0 investment of entrepreneurs is

$$Ev_k^{BF} = E[\kappa A_1 + \kappa A(x) - x] + T/k - (1+\tau_0) I'(k) - kE[\lambda]\left[d'(k) + \tau_0 I'(k)/k\right] = 0,$$

and is increased because $v_k^{BF} = v_k^{MIX}(k) + T/k$. This captures that the bailout fund increases moral hazard because entrepreneurs expect to receive greater transfers.

Equating $Ev_k^{BF} = Ew_k$ therefore requires that we set the macroprudential tax to

$$\tau_0^{BF} = \frac{E\left[\tilde{\lambda} - \lambda\right] d'(k)\, k}{(1 + E[\lambda])\, I'(k) - d(k)},$$

in order to implement the same equilibrium as in the absence of the bailout fund. This expression differs from the optimal tax τ_0 in (21) and (33) by the term $-d(k)$ in the denominator. This term clearly increases the optimal macroprudential tax rate above the level in the respective equilibrium without bailout fund.

To prove statement (i) in the Proposition, observe that setting $\tau_2 = 0$ implies that the equilibrium under the bailout fund implements precisely the equilibrium under macroprudential regulation MP. Welfare in this equilibrium is below welfare under the optimal policy mix, since we have restricted the magnitude of bailouts.

Statement (ii) in the Proposition follows from the observation that the transfer from the bailout fund cancels out and the equilibrium under the optimal policy mix is implemented. ■

Proof of Proposition 12 (Alternative Ex-Post Measures) Each of the described ex-post policy measures $P \in \{MIX, LS, DF, ITC, BS\}$ can be captured by a transfer function $T^P\left(A_1, k^i, k\right)$ that is conditional on the productivity shock in period 1 as well as on the individual and aggregate levels of capital investment. We defined the transfer functions above in a way that they each result in the ex-post optimal transfer $T^*(k, A_1)$ in a symmetric equilibrium. This implies that the transfer coincides with the solution of the optimization problem (23) in each of these cases and implements the optimum in period 1 for $k = k^{MIX}$.

For a given ex-post policy measure $T^P\left(A_1, k^{MIX}, k^{MIX}\right) = T^*\left(A_1, k^{MIX}\right)$, the period 1 optimization problem of an individual entrepreneur is

$$\begin{aligned} v^T(k^i) \;\; &= \;\; \max_{x^i} \left[\kappa A_1 + \kappa(\tau_2) A(x^i) - x^i\right] k^i + T^{P,i} - I(k^i) + \\ &\quad + \lambda^i \left\{\left[\kappa A_1 + \phi p_2 - x^i - d(k^i)\right] k^i + T^{P,i}\right\} \end{aligned}$$

The associated optimality condition on x is

$$\begin{aligned}\left[\kappa(\tau_2)A'(x^i)-1\right]k^i+\partial T^{P,i}/\partial x^i &= \lambda^i\left[k^i+\partial T^{P,i}/\partial x^i\right]\\ \text{so}\quad \lambda^{P,i} &= \frac{\left[\kappa(\tau_2)A'(x^i)-1\right]+T_x^{P,i}/k^i}{1+T_x^{P,i}/k^i}\end{aligned}$$

Using the envelope theorem, the private marginal incentive to invest in period 0, given a macroprudential tax τ_0^P that is rebated lump-sum, is

$$\begin{aligned}Ev_k^P(k^i) &= E\Big\{\kappa A_1+\kappa(\tau_2)A(x^i)-x^i+T_k^{P,i}-\left(1+\tau_0^P\right)I'(k^i)-\\ &\quad -\lambda^{P,i}\left[\left(1+\tau_0^P\right)d'(k)\,k+\tau_0^P d(k)+T^{P,i}/k^i-T_k^{P,i}\right]\Big\}\end{aligned}$$

This expression depends on the ex-post policy measure T^P via the partial derivatives $T_k^{P,i}$ and $T_x^{P,i}$ as well as T^P/k^i. We summarize these terms for the discussed ex-post policy measures in the table below:

Ex-post policy P	$T^P\left(A_1,k,k^i\right)$	T_x^P	T_k^P	T^P/k^i
Lump-sum	$T^*\left(A_1,k\right)$	0	0	T^*/k^i
Benchmark	$\frac{T^*(A_1,k)}{k}k^i$	0	T^*/k^i	T^*/k
Debt forgiveness	$\frac{T^*}{d(k)}d\left(k^i\right)$	0	$\frac{T^*}{d(k)}d'\left(k^i\right)$	$\frac{T^*}{d(k)}\frac{d(k^i)}{k^i}$
Inv. tax credit	$\frac{T^*(A_1,k)}{xk}x^ik^i$	$\frac{T^*}{xk}k^i$	$\frac{T^*}{xk}x^i$	$\frac{T^*}{xk}x^i$
Borrowing subsidy	$\frac{T^*(A_1,k)}{d_1k}d_1^ik^i$	$\frac{T^*}{d_1k}k^i$	$\frac{T^*}{d_1k}d_1^i$	$\frac{T^*}{d_1k}d_1^i$

A planner who will use the optimal ex-post policy $T^P\left(A_1,k^i,k\right)$ can implement the constrained optimum by setting the tax rate τ_0^P to ensure

$$Ev_k^P\left(k^{MIX}\right)=Ew_k\left(k^{MIX}\right)=0$$

where the planner's period 0 optimality condition Ew_k is given by

$$\begin{aligned}w_k(k) &= \kappa A_1+\eta(\tau_2)A(x)-x-I'(k^i)-\tilde{\lambda}d'(k)\,k\\ &\text{with}\quad \tilde{\lambda}=\frac{\eta(\tau_2)\,A'(x)-1}{1-\left[\phi\kappa(\tau_2)+\tau_2\varepsilon(\tau_2)\right]A'(x)}\end{aligned}$$

Substituting the expressions for v_k^P and w_k from above, this requires

$$E\left\{T_k^{P,i}-\tau_0^P I'(k^i)-\lambda^i\left[(1+\tau_0)\,d'(k)\,k+\tau_0 d(k)+T^{P,i}/k^i-T_k^{P,i}\right]\right\}=E\left[s^{MIX}-\tilde{\lambda}d'(k)\,k\right]$$

$$\text{or}\quad \tau_0^P=\frac{E\left[\left(\tilde{\lambda}-\lambda^i\right)d'(k)\,k\right]+E\left\{\left(1+\lambda^i\right)\left[T_k^{P,i}-T^P/k\right]\right\}}{\left(1+E\left[\lambda^i\right]\right)I'(k^i)} \tag{34}$$

at $k = k^{MIX}$. This tax rate is well-defined for any policy measure P. In general, the optimal tax τ_0^P is different from zero, except in knife-edge cases. We present an affine example of such a knife-edge case in appendix B.2.

The proof for why the policy mix $\left(\tau_0^P, T^P\right)$ is time-consistent follows the same arguments as Proposition 8. ■

B Further Results

B.1 Binding constraint in period 0

This appendix first derives the price of capital that an individual entrepreneur is willing to pay in period 1; then we express the condition under which we can omit the period-0 borrowing constraint from the problem.

If an entrepreneur defaults in period 0, his capital is auctioned to a random set of entrepreneurs at the beginning of period 1, who will solve the following optimization problem, where we denote their additional purchases of capital by Δk^i,

$$\max_{x^i, \Delta k^i} \left[\kappa A_1 + \kappa A(x^i) - x^i\right] \left(k^i + \Delta k^i\right) - d_0^i k^i - p_1 \Delta k^i +$$
$$+ \lambda^i \left\{\left[\kappa A_1 + \phi \kappa A_2 - x^i\right] \left(k^i + \Delta k^i\right) - d_0^i k^i - p_1 \Delta k^i\right\}.$$

From the optimality condition with respect to Δk^i we obtain

$$p_1 = \kappa A_1 - x + \phi \kappa A_2 + \frac{(1-\phi)\,\kappa A_2}{1+\lambda_1} = \kappa A_1 - x + \frac{1+\phi\lambda_1}{1+\lambda_1}\kappa A_2.$$

Entrepreneurs are willing to buy the additional capital if the market price corresponds to the return that they can obtain on it over periods 1 and 2. The net return of the additional capital in period 1 is $\kappa A_1 - x + \phi \kappa A_2$ (including additional borrowing capacity) and the net return in period 2 is $(1-\phi)\,\kappa A_2$ (accounting for the repayment on additional borrowing).

The lowest possible realization of this price is given for $A_1 = A^{\min}$, which, under a binding period 1 borrowing constraint, implies a reinvestment $x^{\min}$ that is the solution to the implicit equation $x^{\min} = \kappa\left[A^{\min} + \phi A(x^{\min})\right] - d_0(k)$ and a shadow price $\lambda^{\max} = \kappa A\left(x^{\min}\right) - 1$. Under our assumption that $\kappa\phi A'(x) < 1$, the resulting $x^{\min}$ is a strictly increasing function of $A^{\min}$ and $\lambda^{\max}$ is a strictly decreasing function of $A^{\min}$. The corresponding minimum asset price is

$$p_1^{\min} = \kappa A^{\min} - x^{\min} + \frac{1+\phi\lambda^{\max}}{1+\lambda^{\max}}\kappa A\left(x^{\min}\right)$$

and is strictly increasing in $A^{\min}$. For a sufficiently large minimum return $A^{\min}$ in period 1, we conclude that the period-0 borrowing constraint is always loose, i.e., $d(k) < \phi p_1^{\min}$ for a given level of capital k. We assume that this is the case for the first-best level of capital investment k^{FB}. By implication, it will also be true for the other equilibria that we investigate. Our numerical illustration in section 6.3 illustrates that it is indeed easy to satisfy this restriction.

B.2 Time-consistent optimal ex-post policies

Assume an affine function

$$T^A\left(A_1, k^i, k\right) = T^* + \alpha\left(k^i - k\right)$$

and observe

$$T_k^A = \alpha, \quad T_x^A = 0, \quad T^A/k^i = T^*/k^i$$

Assume $\lambda^i \neq 1$ and set

$$\alpha = \frac{\lambda^i T^*/k^i - \left(\tilde{\lambda} - \lambda^i\right) d'\left(k^i\right) k^i - \tau_2 \varepsilon(\tau_2) A(x^i)}{1 - \lambda^i}.$$

Then the described ex-post policy $T^A\left(A_1, k^i, k\right)$ implements the optimum and is time-consistent, i.e. it does not require ex-ante macroprudential regulation.

We note that there is no natural intuitive interpretation to this policy measure. Therefore it would be difficult to commit a policymaker to implementing such a measure.

We can also interpret the "carrot-and-stick" measure from section 6 as satisfying the requirements of a time-consistent optimal policy mix as captured by equation (34), since it has a discontinuity at $k^i = k^{MIX}$. This admits any derivative $T_k^i \in (0, \infty)$, including the value that satisfies the equation.