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# WINNERS AND LOSERS: CREATIVE DESTRUCTION AND THE STOCK MARKET

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# ABSTRACT

We develop a general equilibrium model of asset prices in which the benefits of technological innovation are distributed asymmetrically. Financial market participants do not capture all the economic rents resulting from innovative activity, even when they own shares in innovating firms. Economic gains from innovation accrue partly to the innovators, who cannot sell claims on the rents that their future ideas will generate. We show how the unequal distribution of gains from innovation can give rise to a high risk premium on the aggregate stock market, return comovement and average return differences among growth and value firms, and the failure of traditional representative-agent asset pricing models to account for cross-sectional differences in risk premia.

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Dimitris Papanikolaou Kellogg School of Management Northwestern University 2001 Sheridan Road Evanston, IL 60208 and NBER d-papanikolaou@kellogg.northwestern.edu Technological innovation is arguably the main driver of economic growth in the long run. However, the economic value generated by new ideas is usually not shared equally. The popular press is rife with rags-to-riches stories of new entrepreneurs, whose net worth rose substantially as a result of their innovative ideas during technological booms such as those experienced in the 1990s and the 2000s. In addition to the large wealth gains for successful innovators, technological progress can also create losses through creative destruction, as new technologies render old capital and processes obsolete. For instance, advances in communication technology have enabled ride-sharing companies, such as Uber, to displace traditional taxi companies.<sup>1</sup>

We show that the asymmetric sharing of gains and losses from technological innovation can give rise to well-known, prominent empirical patterns in asset price behavior, including a high risk premium on the aggregate stock market, return comovement and average return differences among growth and value firms. We build a tractable general equilibrium model in which the benefits of technological progress are distributed unevenly across investors and firms. Our model allows for two forms of technological progress. Some advances take the form of improvements in labor productivity, and are complementary to existing investments, while others are embodied in new vintages of capital. Throughout the paper we refer to the first type of technological progress as disembodied, and the second type as embodied.<sup>2</sup> The latter type of technological progress leads to more creative destruction, since old and new capital vintages are substitutes.

A prominent feature of our model is that the market for new ideas is incomplete. Specifically, shareholders cannot appropriate all the economic rents generated by new technologies, even when they own equity in the firms that develop those technologies. Our motivation for this market incompleteness is that ideas are a scarce resource, and the generation of ideas relies heavily on human capital. As a result, innovators are able to capture a fraction of the economic rents that their ideas generate. The key friction is that potential innovators cannot sell claims to these future rents. This market incompleteness implies that technological progress has an asymmetric impact on household wealth. Most of the financial benefits from innovation accrue to a small fraction of the population, while the rest bear the cost of creative destruction. This reallocative effect of technological progress is particularly strong when innovations are embodied in new capital goods. By exposing households to idiosyncratic randomness in innovation outcomes, improvements in technology can thus reduce households also care about their consumption relative to the economy-wide average,

<sup>&</sup>lt;sup>1</sup>Uber, a privately-held company, was founded in 2009. As of December 2014, Uber was valued at \$41 billion. Between December 2009 and February 2015, the value of Medallion Financial Corp. (NASDAQ: TAXI), a specialty finance company that originates, acquires, and services loans that finance taxicab medallions dropped by more than 50% relative to the value of the NASDAQ index.

<sup>&</sup>lt;sup>2</sup>Berndt (1990) gives the following definitions for these two types of technology shocks: "Embodied technical progress refers to engineering design and performance advances that can only be embodied in new plant or equipment; older equipment cannot be made to function as economically as the new, unless a costly remodelling or retrofitting of equipment occurs," and "by contrast, disembodied technical progress refers to advances in knowledge that make more effective use of all inputs, including capital of each surviving vintage (not just the most recent vintage). In its pure form, disembodied technical progress is perhaps the notion of learning curves, in which it has been found that for a wide variety of production processes and products, as cumulative experience and production increase, learning occurs which results in ever decreasing unit costs."

since households dislike being 'left behind'.

Displacement risk contributes to the equity risk premium and also leads to cross-sectional differences in asset returns. Owning shares in growth firms helps offset potential utility losses brought on by technological improvements. In our model, firms differ in their ability to acquire projects that implement new technologies. This difference in future growth opportunities implies that technological progress has a heterogeneous impact on the cross-section of asset returns. Firms with few existing projects, but many potential new investment opportunities, benefit from technological advances. By contrast, profits of firms that are heavily invested in old technologies and have few growth opportunities decline due to increased competitive pressure. In equilibrium, investors hold growth firms, despite their lower expected returns, as a hedge against the potential wealth reallocation that may result from future technological innovation. Aggregate consumption does not accurately reflect all the risks that households face as a result of technological progress, implying the failure of traditional representative-agent asset pricing models to account for cross-sectional differences in risk premia.

We estimate the parameters of the model using indirect inference. The baseline model performs well at replicating the joint properties of aggregate consumption, investment, and asset returns. The model generates an aggregate consumption process with moderate low-frequency fluctuations, volatile equity returns, a high equity premium, and a low and stable risk-free rate. The model also replicates the observed differences in average returns between value and growth stocks, and the failure of the Capital Asset Pricing Model (CAPM) to explain such cross-sectional differences. These patterns have proven challenging to reproduce in a representative-agent general equilibrium model. Our results depend on three key features of our model: technology advances that are embodied in new capital goods, incomplete market for ideas, and preferences over relative consumption. Restricted versions of the model that eliminate any one of these three features have difficulty replicating the empirical properties of asset returns, and especially the cross-sectional differences in stock returns between value and growth firms.

In addition to the features of the data that we target, we evaluate the model's other testable predictions. Most of these predictions rely on the fluctuations in the share of economic value that is generated by new innovative ideas, or blueprints – a quantity that is challenging to measure empirically. We construct an empirical estimate of this share using data on patents and stock returns collected by Kogan, Papanikolaou, Seru, and Stoffman (2012). Following Kogan et al. (2012), we infer the economic value of patents from the firms' stock market reaction following a successful patent application.<sup>3</sup> We aggregate across all patents granted in a given year and scale by market capitalization to construct an estimate for the share of economic value in the economy due to new

 $<sup>^{3}</sup>$ Relative to other measures of innovation, such as patent citations, the stock market reaction to patent issues has the unique advantage of allowing us to infer the economic – as opposed to the scientific – value of the underlying innovations. Focusing on the days around the patent is issued allows us to infer the economic value over a narrow time window. However, the stock market reaction may underestimate the value of the patent when some of the information about the innovation is already priced in by the market. Further, our analysis misses the patents issued to private firms, as well as those inventions that are not patented. Our analysis should thus be viewed as a joint test of the model and the assumption that movements in the economic value of patented innovations are representative of the fluctuations in the overall value of new inventions in the economy.

blueprints. We use this new measure to test the predictions of the model that relate consumption and asset returns to the value of new innovative activity.

Consistent with our model, we find that innovation is associated with firm displacement and higher consumption inequality. Specifically, we find that rapid technological progress within an industry is associated with lower future profitability for low Tobin's Q (value) firms relative to high-Q(growth) firms. We verify that increases in the relative value of new blueprints are associated with lower market returns, and higher returns for growth firms relative to value firms. Further, increases in the relative value of new blueprints are associated with increases in inequality – specifically, a decline in median household consumption relative to mean household consumption. We replicate these results in simulated data from the model; the empirical estimates are in most cases close to those implied by the model. We interpret these findings as providing support for the model's main mechanism.

In sum, the main contribution of our work is to develop a general equilibrium model that introduces a new mechanism that relates increases in inequality following technological innovations to the pricing of shocks to technology in financial markets.<sup>4</sup> Our model is tractable, yet it delivers rich, testable predictions regarding the cross-section of firms and households. Firm cash flows and investment decisions endogenously respond to changes in technology. Technological improvements are associated with higher inequality. Households own shares in innovating firms as a hedge against increases in inequality. Further, the model can accommodate several of the main stylized facts that have been documented about the cross-section of asset returns.<sup>5</sup> To maintain focus, we only examine a subset of the model's predictions in this paper and leave others for future work.

Our work adds to the growing literature studying asset prices in general equilibrium models.<sup>6</sup>

<sup>&</sup>lt;sup>4</sup>Kogan and Papanikolaou (2013, 2014) feature a similar structural model of the firm in a partial equilibrium setup. Partial equilibrium models are useful in connecting factor risk exposures to firm characteristics. However, they take the stochastic discount factor (SDF) as given. Reduced-form specifications of the SDF can arise in economies in which all cross-sectional variation in expected returns is due to sentiment (see, e.g. Nagel, Kozak, and Santosh, 2014). A general equilibrium model is necessary to connect the SDF to the real side of the economy.

<sup>&</sup>lt;sup>5</sup>For instance, our model allows us to address the value puzzle, which consists of two robust empirical patterns. First, firms with higher than average valuations–growth firms–experience lower than average future returns. These differences in average returns are economically large and comparable in magnitude to the equity premium. This finding has proven to be puzzling because growth firms are typically considered to be riskier and therefore should command higher average returns. Second, stock returns of firms with similar valuation ratios exhibit comovement, even across industries. These common movements are typically unrelated to the firms' exposures to fluctuations in the overall market value. See Fama and French (1992, 1993) for more details. Similar patterns to the returns of high market to book firms have been documented for firms with high past investment (Titman, Wei, and Xie, 2004), price-earnings ratios (Rosenberg, Reid, and Lanstein, 1985), labor hiring (Bazdrech, Belo, and Lin, 2009), new share issuance (Loughran and Ritter, 1995). The strong patterns of return comovement among firms with similar characteristics have motivated the use of empirical factor models (Fama and French, 1993). However, the economic origins of these empirical return factors are yet to be fully understood. In addition, Bansal, Dittmar, and Lundblad (2005) and Hansen, Heaton, and Li (2005) provide evidence that dividends of the value portfolio have higher long-run risk exposure, and grow at a faster rate, than the dividends of the growth portfolio.

<sup>&</sup>lt;sup>6</sup>Similar to this paper, Papanikolaou (2011) and Garleanu, Panageas, and Yu (2012b) are representative agent models which also feature two types of technological progress: disembodied shocks that affect the productivity of all capital, and embodied (also termed 'investment-specific' shocks) that affect the productivity only of *new* capital. Papanikolaou (2011) focuses on the pricing of embodied shocks in an environment with a representative firm and complete markets. In his model, capital-embodied shocks carry a negative risk premium due to agents' aversion to short-run consumption fluctuations. We propose a different mechanism that leads to a negative risk price for embodied shocks. Garleanu et al. (2012b) focus on understanding the joint time-series properties of consumption and excess

Closest to this paper are models with heterogeneous firms that focus on the cross-section of asset prices. Gomes, Kogan, and Zhang (2003) study the cross-section of risk premia in a model where technology shocks are complementary to all capital. Pastor and Veronesi (2009) study the pricing of technology risk in a model with a life-cycle of endogenous technology adoption. Ai, Croce, and Li (2013) analyze the value premium in a model where some technology shocks only affect the productivity of old capital. These studies consider representative-agent models. In contrast, Garleanu, Kogan, and Panageas (2012a) study the value premium puzzle in an overlapping-generations economy where technological improvements lead to inter-generational displacement risk, since existing agents cannot trade with future generations before they are born. We also study the pricing of technology risk in incomplete markets, but our model features imperfect risk sharing among the *existing* population of households, which implies a different mechanism for how the technological shocks are priced and how one may test the model empirically. Moreover, our model has a stationary distribution of firms, and firms invest and accumulate capital. These features allow for a direct comparison of the model with the data. In contrast to the papers above, we estimate the model parameters using indirect inference.

Our model contains features that connect it to several other strands of the literature. For instance, a key part of our model mechanism is that technological progress endogenously increases households' uninsurable consumption risk. The fact that time-varying cross-sectional dispersion of consumption can increase the volatility of the stochastic discount factor is well known (Constantinides and Duffie, 1996; Storesletten, Telmer, and Yaron, 2007; Constantinides and Ghosh, 2014). The existing literature uses reduced-form specifications of idiosyncratic labor income risk. In our setting, time variation in households' uninsurable risk arises as an equilibrium outcome following aggregate technological shocks. The resulting effect on asset prices is further amplified by households' preferences over relative consumption. Our work thus builds upon the extensive literature that emphasizes the role of consumption externalities and relative wealth concerns for asset prices, investment, and consumption dynamics (Duesenberry, 1949; Abel, 1990; Gali, 1994; Roussanov, 2010). Closest to our work is Roussanov (2010), who argues that households may invest in risky, zero-mean gambles whose payoff is uncorrelated with the aggregated state when they have preferences over their rank in the consumption distribution. In our setting, preferences over relative consumption induce agents to accept low risk premia (or equivalently high valuations) to hold assets that increase in value when technology prospects improve. Last, the idea that a significant fraction of the rents from innovation accrue to human capital is related to Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2013). In contrast to these papers, we explore how fluctuations in the value of these rents affect the equilibrium pricing of technology shocks.

# 1 The Model

We consider a dynamic continuous-time economy, with time indexed by t. We first introduce the productive sector of the economy – firms and the projects they own. We next introduce households,

asset returns; our main focus is on the model's cross-sectional implications.

and describe the nature of market incompleteness in our setup.

## 1.1 Firms and Technology

The basic production unit in our economy is called a project. Projects are owned and managed by firms. Each firm hires labor services to operate the projects it owns. The total output of all projects can be used to produce either consumption or investment. New production units are created using investment goods and project blueprints (ideas). Households supply labor services and blueprints to firms, and derive utility from consumption. Figure 1 summarizes the structure of our model.





### Active projects

Each firm f owns a constantly evolving portfolio of projects, which we denote by  $\mathcal{J}_{ft}$ . We assume that there is a continuum of infinitely lived firms in the economy, which we index by  $f \in [0, 1]$ .

Projects are differentiated from each other by three characteristics: a) their operating scale, determined by the amount of capital goods associated with the project, k; b) the systematic component of project productivity,  $\xi$ ; and c) the idiosyncratic, or project-specific, component of productivity, u. Project j, created at time  $\tau(j)$ , produces a flow of output equal to

$$y_{j,t} = \left(u_{j,t} e^{\xi_{\tau(j)}} k_{j,t}\right)^{\phi} \left(e^{x_t} L_{j,t}\right)^{1-\phi},$$
(1)

where  $L_{j,t}$  is labor allocated to project j. In contrast to the scale decision, the choice of labor allocated to the project  $L_{j,t}$  can be freely adjusted every period. Firms purchase labor services at the equilibrium wage  $W_t$ . We denote by

$$\pi_{j,t} = \sup_{L_{j,t}} \left[ \left( u_{j,t} e^{\xi_{\tau(j)}} k_{j,t} \right)^{\phi} (e^{x_t} L_{j,t})^{1-\phi} - w_t L_{j,t} \right]$$
(2)

the profit flow of project j under the optimal hiring policy.

We emphasize one important dimension of heterogeneity among technological innovations by modeling technological progress using two independent processes,  $\xi_t$  and  $x_t$ . First, the shock  $\xi$ reflects technological progress *embodied* in new projects. It follows an arithmetic random walk

$$d\xi_t = \mu_\xi \, dt + \sigma_\xi \, dB_{\xi,t},\tag{3}$$

where  $B_{\xi}$  is a standard Brownian motion.  $\xi_s$  denotes the level of frontier technology at time s. Growth in  $\xi$  affects only the output of new projects created using the latest frontier of technology. In this respect our model follows the standard vintage-capital model (Solow, 1960).

Second, the labor-augmenting productivity process  $x_t$  follows an arithmetic random walk

$$dx_t = \mu_x \, dt + \sigma_x \, dB_{x,t}.\tag{4}$$

Here,  $B_x$  is a standard Brownian motion independent of all other productivity shocks. In particular, the productivity process x is independent from the embodied productivity process  $\xi$ . Labor in our model is complementary to capital. Thus, in contrast to the embodied shock  $\xi$ , the technology shock x affects the output of *all* vintages of existing capital.

The level of project-specific productivity  $u_j$  is a stationary mean-reverting process that evolves according to

$$du_{j,t} = \kappa_u (1 - u_{j,t}) dt + \sigma_u u_{j,t} dB^u_{j,t},$$
(5)

where  $B_j^u$  are standard Brownian motions independent of  $B_{\xi}$ . We assume that  $dB_{j,t}^u \cdot dB_{j',t}^u = dt$  if projects j and j' belong in the same firm f, and zero otherwise. As long as  $2\kappa_u \ge \sigma_u^2$ , the ergodic distribution of u has finite first two moments (see Lemma 1 in Appendix A for details). All new projects implemented at time s start at the long-run average level of idiosyncratic productivity, i.e.,  $u_{j,\tau(j)} = 1$ . Thus, all projects created at a point in time are ex-ante identical in terms of productivity, but differ ex-post due to the project-specific shocks.

The firm chooses the initial operating scale k of a new project irreversibly at the time of its creation. Firms cannot liquidate existing projects and recover their investment costs. Over time, the scale of the project diminishes according to

$$dk_{j,t} = -\delta \, k_{j,t} \, dt, \tag{6}$$

where  $\delta$  is the economy-wide depreciation rate. At this stage, it is also helpful to define the aggregate

stock of installed capital, adjusted for quality,<sup>7</sup>

$$K_t = \int_0^1 \left( \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{s(j)}} k_{j,t} \right) df \tag{7}$$

The aggregate capital stock K also depreciates at rate  $\delta$ .

#### Creation of new projects

Creating a new project requires a blueprint and new investment goods. Firms are heterogeneous in their ability to acquire new blueprints. Inventors initially own the blueprints for creation of new projects. We assume that inventors lack the ability to implement these ideas on their own, and instead sell the blueprints for new projects to firms (we outline the details of the process for blueprint sales below).

Firms acquire projects by randomly meeting inventors who supply blueprints. The likelihood of acquiring a new project is exogenous to each firm, driven by a firm-specific doubly stochastic Poisson process  $N_{f,t}$ . The arrival rate of new projects equals  $\lambda_{f,t}$ . This arrival rate is time-varying and follows a two-state continuous-time Markov chain with high and low growth states  $\{\lambda_H, \lambda_L\}$ ,  $\lambda_H > \lambda_L$ . The transition rate matrix is given by

$$\begin{pmatrix}
-\mu_L & \mu_L \\
\mu_H & -\mu_H
\end{pmatrix}.$$
(8)

We denote the unconditional average of  $\lambda_{f,t}$  by  $\lambda$ .

To implement a new blueprint as a project j at time t, a firm purchases new capital goods in quantity  $I_{j,t}$ . Investment in new projects is subject to decreasing returns to scale,

$$k_{j,t} = I_{j,t}^{\alpha}.$$
(9)

The parameter  $\alpha \in (0, 1)$  parameterizes the investment cost function and implies that costs are convex at the project level. We denote by

$$\nu_t \equiv \sup_{k_{j,t}} \left\{ \mathbf{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \, \pi_{j,s} \, ds \right] - k_{j,t}^{1/\alpha} \right\} \tag{10}$$

the net value of a new project implemented at time t under the optimal investment policy, where  $\Lambda_t$  is the equilibrium stochastic discount factor defined in Section 1.4. Since all projects created at time t are identical ex-ante,  $\nu$  is independent of j. Equation (10) is also equal to the value of a new

<sup>&</sup>lt;sup>7</sup>The definition of the aggregate capital stock, along with other aggregate quantities in the model, requires aggregating firm-level quantities or prices. Aggregation over the continuum of firms should satisfy a law of large numbers, canceling out firm-specific randomness. Several aggregation procedures with such property have been developed in the literature, and the exact choice of the aggregate as the Pettis integral. We denote the aggregate over firms by an integral over the set of firms,  $\in_0^1 \cdot df$ . For alternative constructions that deliver the law of large numbers in the cross-section, see for instance Sun (2006) and Podczeck (2010), as well as the discussion of similar issues in Constantinides and Duffie (1996).

blueprint associated at time t.

#### Aggregate output

The total output in the economy is equal to the aggregate of output of all active projects,

$$Y_t = \int_0^1 \left( \sum_{j \in \mathcal{J}_{f,t}} y_{j,t} \right) df.$$
(11)

The aggregate output of the economy can be allocated to either investment  $I_t$  or consumption  $C_t$ ,

$$Y_t = I_t + C_t. \tag{12}$$

The amount of new investment goods  $I_t$  produced is used as an input in the implementation of new projects, as given by the investment cost function defined in (9).

#### 1.2 Households

There is a continuum of households, with the total measure of households normalized to one. Households die independently of each other according to the first arrival of a Poisson process with arrival rate  $\delta^h$ . New households are born at the same rate, so the total measure of households remains constant. All households are endowed with the unit flow rate of labor services, which they supply inelastically to the firms producing the final good.

Households have access to financial markets, and optimize their life-time utility of consumption. Households are not subject to liquidity constraints; hence, they sell their future labor income streams and invest the proceeds in financial claims. We denote consumption of an individual household i by  $C_{i,t}$ .

All shareholders have the same preferences, given by

$$J_t = \lim_{\tau \to \infty} \mathcal{E}_t \left[ \int_t^\tau \phi(C_s, J_s; \bar{C}_s) \, ds \right],\tag{13}$$

where  $\phi$  is the aggregator function:

$$\phi(C, J; \bar{C}) = \frac{\rho}{1 - \theta^{-1}} \left( \frac{\left( C^{1-h} \left( C/\bar{C} \right)^h \right)^{1-\theta^{-1}}}{\left( (1-\gamma)J \right)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)J \right).$$
(14)

Households' preferences fall into the class of stochastic differential utility proposed by Duffie and Epstein (1992), which is a continuous-time analog of the preferences proposed by Epstein and Zin (1989). Relative to Duffie and Epstein (1992), our preference specification also incorporates a relative-consumption concern (otherwise termed as "keeping up with the Joneses", see, e.g., Abel, 1990). That is, households also derive utility from their consumption relative to aggregate

consumption,

$$\bar{C} = \int_0^1 C_{n,t} \, dn.$$
 (15)

The parameter h captures the strength of the relative consumption effect;  $\gamma$  is the coefficient of relative risk aversion;  $\theta$  is the elasticity of intertemporal substitution (EIS); and  $\rho$  is the effective time-preference parameter, which includes the adjustment for the likelihood of death (see Garleanu and Panageas, 2014, for a model with random life spans and non-separable preferences).

## 1.3 Household Innovation

The key feature of our model is imperfect risk sharing among investors. Households are endowed with ideas, or blueprints, for new projects. Inventors do not implement these project on their own. Instead, they sell the ideas to firms. Inventors and firms bargain over the surplus created by new projects; the inventor captures a share  $\eta$  of the net present value of a new project.

Each household receives blueprints for new projects according to an idiosyncratic Poisson process with arrival rate  $\mu_I$ . In the aggregate, households generate blueprints at the rate equal to the total measure of projects acquired by firms,  $\lambda$ . Not all innovating households receive the same measure of new blueprints. Each household *n* receives a measure of projects in proportion to her wealth  $W_{n,t}$ – that is, equal to  $\lambda W_{n,t} \left( \mu_I \int_0^1 W_{i,t} di \right)^{-1}$ . This is a technical assumption that is important for tractability of the model, as we discuss below. Thus, conditional on innovating, wealthier households receive a larger measure of blueprints.

Importantly, households cannot trade in securities contingent on future successful individual innovation. That is, they cannot sell claims against their proceeds from future innovations. This restriction on risk sharing plays a key role in our setting. In equilibrium, wealth creation from innovation leads to changes in the cross-sectional distribution of wealth and consumption, and therefore affects households' financial decisions.

### 1.4 Financial Markets

We assume that agents can trade a complete set of state-contingent claims contingent on the paths of the aggregate and idiosyncratic productivity processes, as well as paths of project arrival rates and project arrival events at the firm level. We denote the equilibrium stochastic discount factor by  $\Lambda_t$ , so the time-t market value of a time-T cash flow  $X_T$  is given by

$$\mathbf{E}_t \left[ \frac{\Lambda_T}{\Lambda_t} X_T \right]. \tag{16}$$

In addition, we follow Blanchard (1985) and assume that investors have access to competitive annuity markets that allow them to hedge their mortality risk. This assumption implies that, conditional on surviving during the interval [t, t + dt], investor *n* collects additional income proportional to her wealth,  $\delta^h W_{n,t} dt$ .

# 1.5 Discussion of the Model's Assumptions

Most existing production-economy general equilibrium models of asset returns build on the neoclassical growth framework. We depart from this literature in three significant ways.

a. Technological progress is embodied in new capital vintages. Most existing general equilibrium models that study asset prices assume that technological progress is complementary to the entire existing stock of capital – as is the case for the x shock in our model. However, many technological advances are embodied in new capital goods and thus only benefit firms which invest in the new capital vintages. Several empirical studies show substantial vintage effects in plant productivity. For instance, Jensen, McGuckin, and Stiroh (2001) finds that the 1992 cohort of new plants was 50% more productive than the 1967 cohort in their respective entry years, controlling for industry-wide factors and input differences. Further, an extensive literature documents a significant impact of embodied technological progress on economic growth and fluctuations (see, e.g. Solow, 1960; Cooley, Greenwood, and Yorukoglu, 1997; Greenwood, Hercowitz, and Krusell, 1997; Fisher, 2006). Since technological progress to obtain a broader understanding of how technological change affects asset returns.

b. Incomplete markets for innovation. In our model, inventors generate new ideas and sell them to firms. We can map this stylized process into several forms of innovation in the data. One possibility is that inventors work for existing firms, generate ideas, and receive compensation commensurate with the economic value of their ideas. Since their talent is in scarce supply, these skilled workers may be able to capture a significant fraction of the economic value of their ideas. Another possibility is that inventors implement the ideas themselves, creating startups that are partly funded by outside investors. Innovators can then sell their share of these startups to existing firms and thus capture a substantial share of the economic value of their innovations. The term 'inventors' can include highly skilled research personnel, entrepreneurs and startup employees, angel investors, or corporate executives who can generate, finance, and implement, new ideas.

An important assumption in our model is that the economic value that is generated by new ideas cannot be fully pledged to outside investors. This assumption can be motivated on theoretical grounds. New ideas are the product of human capital, which is inalienable. Hart and Moore (1994) show that the inalienability of human capital limits the amount of external finance that can be raised by new ventures. Bolton, Wang, and Yang (2015) characterize a dynamic optimal contract between a risk averse entrepreneur with risky inalienable human capital, and firm investors. The optimal contract involves a trade-off between risk sharing and incentives, and leaves the entrepreneur with a significant fraction of the upside gains.

c. Preferences over relative consumption. Our work deviates from many existing general equilibrium models by allowing households' utility to depend not only on their own consumption, but also on their consumption relative to the average per capita consumption in the economy. This assumption can be justified on empirical grounds. For instance, several studies document that, controlling for household income, the rank in the income distribution or the income of a peer group, is negatively related to self-reported measures of happiness and satisfaction (Clark and Oswald,

1996; Solnick and Hemenway, 1998; Ferrer-i Carbonell, 2005; Luttmer, 2005; Card, Mas, Moretti, and Saez, 2012). These relative income concerns are substantial; for example, the point estimates in Luttmer (2005) imply that the income of households in the same metropolitan area is more important for happiness than the households' own level of income. Frydman (2015) finds strong evidence for utility preferences over relative wealth in an experimental setting using neural data collected through fMRI.

Recent theoretical work justifies preferences over relative consumption as a reduced-form description of individual behavior. Rayo and Becker (2007) propose a theory in which peer comparisons are an integral part of the "happiness function" as a result of an evolutionary process. DeMarzo, Kaniel, and Kremer (2008) show that competition over scarce resources can make agents' utilities dependent on the wealth of their cohort.

In Section 3.3 we explore the sensitivity of our quantitative results to assumptions (a) to (c). In addition to these three main assumptions, our model deviates in some other respects from the neoclassical framework. These deviations make the model tractable but do not drive our main results.

First, we assume that projects arrive independently of the firms' own past decisions, and firms incur convex investment costs at the project level. Together, these assumptions ensure that the optimal investment decision can be formulated as a static problem, thus implying that the crosssectional distribution of firm size does not affect equilibrium aggregate quantities and prices. Second, the assumption that innovating households receive a measure of projects that is proportional to their existing wealth guarantees that the growth of household wealth is independent of its wealth level. With homothetic preferences, this assumption ensures that the households' optimal consumption and portfolio choices scale in proportion to their wealth. Thus, the cross-sectional distribution of household wealth does not affect equilibrium prices.<sup>8</sup> Third, households in our model have finite lives. This assumption ensures the existence of a stationary distribution of wealth among households. Fourth, our assumption that project productivity shocks are perfectly correlated at the firm level ensures that the firm state vector is low-dimensional. Last, there is no cross-sectional heterogeneity among the quality of different blueprints. We could easily allow for an idiosyncratic part to  $\xi$ , perhaps allowing for substantial skewness in this component, to capture the notion that the distribution of profitability of new ideas can be highly asymmetric. Our conjecture is that such an extension would strengthen our main results by raising the level of idiosyncratic risk of individual households' consumption processes.

## 1.6 Competitive Equilibrium

Here, we describe the competitive equilibrium of our model. Our equilibrium definition is standard, and is summarized below.

<sup>&</sup>lt;sup>8</sup>The assumption that the magnitude of innovation is proportional to households' wealth levels likely weakens our main results compared to the case where all households received the same measure of blueprints upon innovating. In the latter case, wealthier households would benefit less from innovation, raising their exposure to innovation shocks relative to our current specification.

**Definition 1 (Competitive Equilibrium)** The competitive equilibrium is a sequence of quantities  $\{C_t, I_t, Y_t, K_t\}$ ; prices  $\{\Lambda_t, W_t\}$ ; household consumption decisions  $\{C_{i,t}\}$ ; and firm investment and hiring decisions  $\{I_{j,t}, L_{j,t}\}$  such that given the sequence of stochastic shocks  $\{x_t, \xi_t, u_{j,t}, N_{f,t}\}$ ,  $j \in \bigcup_{f \in [0,1]} \mathcal{J}_{f,t}, f \in [0,1]$ : i) households choose consumption and savings plans to maximize their utility (13); ii) household budget constraints are satisfied; iii) firms maximize profits; iv) the labor market clears,  $\int_0^1 \left(\sum_{j \in \mathcal{J}_{f,t}} L_{j,t}\right) df = 1$ ; v) the demand for new investment equals supply,  $\int_0^1 I_{n,t} dn = I_t$ ; vi) the market for consumption clears  $\int_0^1 C_{n,t} dn = C_t$ , and vii) the aggregate resource constraint (11) is satisfied.

Because of market incompleteness, standard aggregation results do not apply. Specifically, there are two dimensions of heterogeneity in the model: on the supply side, among firms; and on the demand side, among households. Both of these sources of heterogeneity can potentially make the state space of the model infinite-dimensional. However, we can solve for aggregate-level quantities and prices in Definition 1 as functions of the low-dimensional Markov aggregate state vector  $X_t = (x_t, \xi_t, K_t)$ . Specifically, the first moment of the cross-section distribution of installed capital K summarizes all the information about the cross-section of firms relevant for the aggregate dynamics in the model. The first two assumptions discussed above in Section 1.5 enable a relatively simple characterization of equilibrium. See Lemma 3 in Appendix A for more details. We solve for equilibrium prices and quantities numerically.<sup>9</sup>

In equilibrium, aggregate output (11) is equal to

$$Y_t = e^{\chi_t} e^{-\phi \,\omega_t},\tag{17}$$

where the following two variables are a sufficient statistic for the aggregate state,

$$\omega_t = \xi_t + \alpha \,\chi_t - \log K_t,\tag{18}$$

and

$$\chi_t = \frac{1-\phi}{1-\alpha\,\phi}\,x_t + \frac{\phi}{1-\alpha\,\phi}\xi_t.$$
(19)

The variable  $\chi_t$  is a random walk, and captures the permanent effect of the two technology shocks on aggregate quantities. The variable  $\omega_t$  represents deviations of the current capital stock from its target level – and thus deviations of Y from its stochastic trend. In the model, aggregate consumption C, investment I, and labor income W are cointegrated with aggregate output Y.

# 2 Estimation

Next, we describe how we calibrate the model to the data. In Section 2.1, we describe which features of the data help identify the model's parameters. In Section 2.2, we discuss how we

<sup>&</sup>lt;sup>9</sup>We cannot rely on the standard results for uniqueness and existence of equilibrium because our model features incomplete markets and recursive preferences. In principle, there may exist multiple equilibria in our model. Our numerical solution characterizes one particular equilibrium.

choose parameters through a minimum-distance criterion. In Section 2.3, we examine the model's performance in matching the features of the data that we target, and the resulting parameter estimates.

## 2.1 Data

The model has a total of 20 parameters. Only a handful of these parameters can be calibrated using a priori evidence. We calibrate  $\phi = 1/3$  so that the labor share in the production of the final good equals two thirds. We choose the probability of household death as  $\delta^h = 1/40$  to guarantee an average working life of 40 years. We calibrate the probability of repeat innovation to equal  $\mu_I = 0.15\%$  per year, in order to generate a mean ratio of median to average consumption per capita of 0.8.<sup>10</sup> Last, we create returns to equity by levering financial wealth by a factor of 2.5.<sup>11</sup> We estimate the remaining 17 parameters using indirect inference.

The first column of Table 1 reports the 21 statistics we choose as targets. Some of these statistics are commonly used targets in the literature as, for example, the first and second moments of aggregate quantities, net payout to shareholders, returns to the market portfolio, and the risk-free rate.<sup>12</sup> We also include several other statistics that are revealing of the mechanisms in our paper. First, technological progress leads to low frequency fluctuations in consumption and output in our model (see, for instance, equation (17)). Hence, we also include as a target the estimate of the long-run standard deviation using the methodology of Dew-Becker (2014). Second, a large fraction of the parameters of the model govern the behavior of individual firms. We thus target the cross-sectional dispersion and persistence in firm investment, Tobin's Q, and profitability. Last, as we discuss in detail in Section 3, our model connects embodied technology shocks to the return differential between value and growth firms. We thus include as estimation targets the mean, volatility and CAPM alpha of a portfolio of value minus growth firms, where value and growth firms are defined according to their book-to-market ratios (following Fama and French, 1992).

We discuss the construction of these variables in detail in Appendix B. Due to data limitations, each of these statistics is available for different parts of the sample. We use the longest available

<sup>11</sup>This value lies between the estimates of the financial leverage of the corporate sector in Rauh and Sufi (2011) (which is equal to 2) and the values used in Abel (1999) and Bansal and Yaron (2004) (2.74-3). We follow the latter and use a higher leverage parameter than Rauh and Sufi (2011) to account for the effects of operating leverage.

<sup>&</sup>lt;sup>10</sup>We choose this parameter ex-post, that is, after the estimation of the other parameters. Including  $\mu_I$  in the full estimation along with the inequality moment is infeasible given the computational cost of estimating inequality in the model. The sample statistic of 0.8 is obtained using the Consumption Expenditure Survey (CEX). Our choice for  $\mu_I$ also generates a mean consumption share of the top 0.1% of households to be equal to 2.3%. Obtaining empirical estimates of consumption inequality is challenging; however, most studies agree that consumption inequality is lower than income inequality (Gordon and Dew-Becker, 2008). For comparison, the mean income share of the top 0.1% using the data of Piketty and Saez (2003) is 4.7%. We note that the exact choice of  $\mu_I$  does not seem to play a quantitative role in our results. Given the other estimates, increasing  $\mu_I$  from 0.15% to to 4.11% (the fraction of households that transition into entrepreneurship each year reported by Hurst and Lusardi (2004), a likely upper bound) has a quantitatively negligible effect on the moments of aggregate quantities and asset returns. The impact is negligible because of risk aversion: given moderate amounts of utility curvature, the certainty equivalent of a bet that pays with probability  $\mu_I$  an amount that is proportional to  $1/\mu_I$  is negligible for small  $\mu_I$ .

<sup>&</sup>lt;sup>12</sup>Dividends are not well defined in our model, hence we focus on net payout instead. Depending on the parametrization, net payout can be potentially negative. Therefore, we target the volatility of the ratio of net payout to book assets.

sample to compute them.

## 2.2 Methodology

We estimate the parameter vector p using the simulated minimum distance method (Ingram and Lee, 1991). Denote by X the vector of target statistics in the data and by  $\mathcal{X}(p)$  the corresponding statistics generated by the model given parameters p, computed as

$$\mathcal{X}(p) = \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p),$$
(20)

where  $\hat{X}_i(p)$  is the 21 × 1 vector of statistics computed in one simulation of the model. We simulate the model at a weekly frequency, and time-aggregate the data to form annual observations. Each simulation has 1,000 firms. For each simulation *i* we first simulate 100 years of data as 'burn-in' to remove the dependence on initial values. We then use the remaining part of that sample, which is chosen to match the longest sample over which the target statistics are computed. Each of these statistic is computed using the same part of the sample as its empirical counterpart. In each iteration we simulate S = 100 samples, and simulate pseudo-random variables using the same seed in each iteration.

Our estimate of the parameter vector is given by

$$\hat{p} = \arg\min_{p \in \mathcal{P}} (X - \mathcal{X}(p))' W(X - \mathcal{X}(p)),$$
(21)

where  $W = diag(XX')^{-1}$  is our choice of weighting matrix that ensures that the estimation method penalizes proportional deviations of the model statistics from their empirical counterparts.

We compute standard errors for the vector of parameter estimates  $\hat{p}$  as

$$V(\hat{p}) = \left(1 + \frac{1}{S}\right) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p)\right)^{-1} \frac{\partial}{\partial p} \mathcal{X}(p)' W' V_X(\hat{p}) W \frac{\partial}{\partial p} \mathcal{X}(p) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p)\right)^{-1},$$
(22)

where

$$V_X(\hat{p}) = \frac{1}{S} \sum_{i=1}^{S} (\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p})) (\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p}))'$$
(23)

is the estimate of the sampling variation of the statistics in X computed across simulations. The standard errors in (22) are computed using the sampling variation of the target statistics across simulations (23). We use (23), rather than the sample covariance matrix, because the statistics that we target are obtained from different datasets (e.g. cross-sectional versus time-series), and we often do not have access to the underlying data. Since not all of these statistics are moments, computing the covariance matrix of these estimates would be challenging even with access to the underlying data. Under the null of the model, the estimate in (23) would coincide with the empirical estimate. If the model is misspecified, (23) does not need to be a good estimate of the true covariance matrix

# of $X.^{13}$

Solving each iteration of the model is computationally costly, and thus computing the minimum (21) using standard methods is infeasible. We therefore use the Radial Basis Function (RBF) algorithm in Björkman and Holmström (2000).<sup>14</sup>

Statistic	Data		Model			
	Data	Mean	RDev2	(5%)	(95%)	
Aggregate quantities						
Consumption growth, mean	0.015	0.014	0.001	0.004	0.024	
Consumption growth, volatility	0.036	0.042	0.028	0.038	0.047	
Consumption growth, long-run volatility	0.041	0.056	0.128	0.040	0.072	
Investment, mean share of output	0.089	0.055	0.148	0.028	0.083	
Investment, volatility	0.130	0.116	0.012	0.094	0.137	
Investment and consumption, correlation	0.472	0.383	0.036	0.167	0.554	
Net payout to assets, coefficient of variation	0.575	0.520	0.009	0.291	0.850	
Asset Prices						
Market Portfolio, excess returns, mean	0.063	0.068	0.005	0.049	0.083	
Market Portfolio, excess returns, volatility	0.185	0.152	0.033	0.138	0.166	
Risk-free rate, mean	0.020	0.025	0.065	0.019	0.033	
Risk-free rate, volatility	0.007	0.007	0.000	0.003	0.012	
Value factor, mean	0.065	0.054	0.031	0.031	0.075	
Value factor, volatility	0.243	0.191	0.045	0.134	0.278	
Value factor, CAPM alpha	0.040	0.034	0.026	0.007	0.059	
Cross-sectional (firm) moments						
Firm investment rate, IQR	0.175	0.200	0.021	0.153	0.249	
Firm investment rate, serial correlation	0.223	0.191	0.020	0.014	0.429	
Firm investment rate, correlation with Tobin's Q	0.237	0.205	0.019	0.143	0.328	
Firm Tobin's Q, IQR	1.139	0.750	0.117	0.534	1.045	
Firm Tobin's Q, serial correlation	0.889	0.924	0.002	0.897	0.947	
Firm profitability, IQR	0.902	0.963	0.005	0.855	1.123	
Firm profitability, serial correlation	0.818	0.755	0.006	0.733	0.773	
Distance criterion (mean of Rdev2)			0.034			

This table reports the fit of the model to the statistics of the data that we target. Growth rates and rates of return are reported at annual frequencies. See main text for details on the estimation method and Appendix B for details on the data construction. We report the mean statistic, along with the 5% and 95% percentiles across simulations. We also report the squared relative deviation of the mean statistic to their empirical counterparts,  $Rdev2_i = (X_i - \mathcal{X}_i(p))^2/X_i^2$ .

<sup>&</sup>lt;sup>13</sup>Partly for these reasons, we specify the weighting matrix as  $W = diag(XX')^{-1}$ , rather than scaling by the inverse of the sample covariance matrix of X. In principle we could weigh moments by the inverse of (23). However, doing so forces the model to match moments that are precisely estimated but economically less interesting, such as the dispersion in firm profitability or Tobin's Q.

<sup>&</sup>lt;sup>14</sup>The Björkman and Holmström (2000) algorithm first fits a response surface to data by evaluating the objective function at a few points. Then, it searches for a minimum by balancing between local and global search in an iterative fashion. We use a commercial implementation of the RBF algorithm that is available through the TOMLAB optimization package.

## 2.3 Estimation Results

Examining columns two to five of Table 1 we see that the baseline model fits the data reasonably well. The model generates realistic patterns for aggregate consumption, investment and corporate payout. In terms of the cross-section of firms, the model largely replicates the empirical cross-sectional dispersion in investment and profitability, the low persistence in firm investment rates, and the weak empirical relation between firm investment and average Q. At the same time, the model replicates key features of asset returns. The model generates a high equity premium, low and stable risk free rate, the value premium, the value factor, and the failure of the Capital Asset Pricing Model (CAPM). Consistent with the data, value firms in the model have higher average returns than growth firms, yet this difference in risk premia is not accounted by their differential exposure to the market portfolio.

Parameter	Symbol	Estimate	SE
Preferences			
Risk aversion	$\gamma$	105.856	37.987
Elasticity of intertemporal substitution	$\theta$	2.207	5.613
Effective discount rate	ho	0.040	0.043
Preference weight on relative consumption	h	0.947	0.121
Technology and Production			
Decreasing returns to investment	$\alpha$	0.362	0.071
Depreciation rate	$\delta$	0.033	0.094
Disembodied technology growth, mean	$\mu_x$	0.016	0.113
Disembodied technology growth, volatility	$\sigma_x$	0.077	0.008
Embodied technology growth, mean	$\mu_{\xi}$	0.004	0.221
Embodied technology growth, volatility	$\sigma_{\xi}$	0.137	0.013
Transition rate to low-growth state	$\mu_L$	0.283	0.237
Transition rate to high-growth state	$\mu_H$	0.015	0.017
Project mean arrival rate, low growth state	$\lambda_L$	0.122	0.043
Project mean arrival rate, high growth state	$\lambda_H$	8.588	2.106
Project-specific productivity, volatility	$\sigma_{u}$	0.636	0.188
Project-specific productivity, persistence	$\kappa_u$	0.303	0.055
Incomplete Markets			
Fraction of project NPV that goes to inventors	$\eta$	0.785	0.371

 Table 2: Benchmark model: parameter estimates

This table reports the estimated parameters of the model. When constructing standard errors, we approximate the gradient  $\partial \mathcal{X}(p)/\partial p$  using a five-point stencil centered at the parameter vector  $\hat{p}$ .

Although most of the statistics in simulated data are close to their empirical counterparts, there are a few exceptions. The model's equilibrium consumption process has somewhat higher long-run consumption volatility than its empirical counterpart (0.056 vs 0.041), although the empirical value still falls within the model's 90% confidence interval. More importantly, the model generates stock returns that are smoother than their empirical counterparts by approximately 30%. As is the case

with almost all general equilibrium models, the need to match the relatively smooth dynamics of aggregate quantities imposes tight constraints on the shock volatilities  $\sigma_x$  and  $\sigma_{\xi}$ . Mechanisms that lead to time-variation in risk premia – for example, time-variation in the volatility of the shocks  $\sigma_x$  and  $\sigma_{\xi}$  – may help increase the realized variation in asset returns. Due to the associated increased computational complexity, we leave such extensions to future research. Last, the model has difficulty replicating the cross-sectional dispersion in Tobin's Q observed in the data; in the model, the inter-quartile range (IQR) in Q is 34% smaller than the data. Given the fact that part of this dispersion may be measurement error due to imperfections in the empirical measures of the replacement cost of capital, this under-performance is not a major concern.

We report the estimated parameters in Table 2, along with their standard errors. The estimated utility curvature parameter is high, ( $\hat{\gamma} = 106$ ). The parameter controlling the elasticity of intertemporal substitution ( $\hat{\theta} = 2.21$ ), and the estimated preference parameter ( $\hat{h} = 0.947$ ) implies that agent place high weight on relative consumption.<sup>15</sup> The estimated share of project surplus that goes to innovating households is  $\hat{\eta} = 0.785$ ; thus, incomplete risk sharing is an important feature of the estimated model. The volatility of the two technology shocks is  $\hat{\sigma}_x = 7.7\%$  and  $\hat{\sigma}_{\xi} = 13.7\%$ . Last, the estimate for the parameter governing decreasing returns to scale in investment is  $\hat{\alpha} = 0.36$ , implying an investment cost function that is not far from quadratic at the project level. The estimated parameters governing the evolution of  $\lambda_{f,t}$  imply that the high-growth and low-growth states have very different project acquisition rates ( $\hat{\lambda}_H = 8.6$ ,  $\hat{\lambda}_L = 0.12$ ), with the high-growth state being highly transitory ( $\hat{\mu}_H = 0.015$ ,  $\hat{\mu}_L = 0.283$ ).

Not all of the parameters are precisely estimated. Their precision reflects the degree to which the output of the model is sensitive to the individual parameter values. For instance, the rate of capital depreciation, the mean values of the two technology shocks, and the preference parameter  $\theta$  are estimated with large standard errors. As it is typically the case, shock volatilities are fairly precisely estimated.

Relative to existing general equilibrium production-based models of asset prices, the main success of our model is in replicating the cross-sectional patterns in asset returns.<sup>16</sup> These patterns arise primarily from value and growth firms having differential exposures to the two technology shocks x and  $\xi$ , imperfect risk sharing, and preferences over relative consumption. The next Section of the paper details the specific mechanisms that lead to these results.

<sup>&</sup>lt;sup>15</sup>The preference parameters  $\theta$  and  $\gamma$  are high relative to the range of values typically considered in the literature. However, relative consumption preferences alter the meaning of these parameters. For instance, estimating the EIS via a Hall-style regression with aggregate consumption in simulated data from the model yields a mean estimate of 1.08 across simulations. Regarding the other two preference parameters, in Section 3.3.2 we show that their values are significantly lower when extending the model, for instance, by introducing limited asset market participation.

<sup>&</sup>lt;sup>16</sup>Recent contributions include Boldrin, Christiano, and Fisher (2001); Guvenen (2009); Campanale, Castro, and Clementi (2010); Kaltenbrunner and Lochstoer (2010); Garleanu et al. (2012b); Croce (2014). Some of these models succeed in generating smooth consumption paths with low-frequency fluctuations and volatile asset returns, sometimes at the cost of a volatile risk-free rate. The models are often hampered by the fact that consumption rises while dividends fall after a positive technology shock, leading to a negative correlation between aggregate payouts of the corporate sector and consumption (see e.g., Rouwenhorst, 1995). In our setup, consumption and dividends are positively correlated, which helps the model deliver a sizeable equity premium.

# 3 Examining the Model's Mechanism

To obtain some intuition about the asset pricing predictions of the model, it is helpful to analyze the relation between technological progress, the stochastic discount factor, and asset returns. We begin our analysis in Section 3.1, where we examine how technology shocks enter into the investors' stochastic discount factor (SDF). In Section 3.2 we examine how these two shocks impact asset returns in the cross-section. In Section 3.3 we examine which of the model's non-standard features are important for the model's quantitative performance. In addition, we introduce an extension of the model that allows for limited participation in financial markets.

## 3.1 The pricing of technology risk

To understand how the two technology shocks x and  $\xi$  affect the equilibrium SDF, we need to establish their impact on the consumption of individual agents. Because of imperfect risk sharing, there is a distinction in how these shocks affect aggregate quantities and how they affect individual households. To emphasize this distinction, we first analyze the impact of technology on aggregate economic output and consumption, and then examine its impact on the distribution of consumption for individual households.

### 3.1.1 Aggregate quantities and asset prices

We compute impulse responses for aggregate output  $Y_t$ , consumption  $C_t$ , investment  $I_t$ , labor income  $W_t$  and aggregate payout to shareholders  $D_t$  to the two technology shocks x and  $\xi$ . The latter is equal to total firm profits minus investment expenditures and payout to new inventors,

$$D_t = \phi Y_t - I_t - \eta \lambda \nu_t. \tag{24}$$

In the model, the aggregate payout D is not restricted to be positive. However, using the baseline parameter estimates, D becomes negative only in the extreme ranges of the state space that are rarely reached in model simulations. We compute impulse responses taking into account the nonlinear dynamics of the economy. The shape of these impulse responses depends on the current state vector X. In our model, the scalar state variable  $\omega$  summarizes all relevant information for the model's non-linear dynamics. We compute impulse responses at the mean of the stationary distribution of  $\omega$ .

We plot the impulse responses to the technology shocks in Figure 2. Panel A shows that a positive disembodied technology shock x leads to an increase in output, consumption, investment, payout, and labor income. The increase in investment leads to higher capital accumulation, so the increase in output is persistent. However, since x is complementary to existing capital, most of its benefits are immediately realized. In panel B, we plot the response of these equilibrium quantities to a technology shock  $\xi$  that is embodied in new capital. In contrast to the disembodied shock x, the technology shock  $\xi$  affects output only through the formation of new capital stock. Consequently, it has no immediate effect on output, and only leads to a reallocation of resources from consumption to investment on impact. Further, shareholder payout declines immediately after the shock, as firms

cut dividends to fund new investments. In the medium run, the increase in investment leads to a gradual increase in output, consumption, payout, and the equilibrium wage (or labor income).

#### Figure 2: Technology and Aggregate Quantities

This figure plots the impulse response of aggregate output, investment and consumption expenditures to the two technology shocks in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time t = 0 without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of  $\omega$ . We report the log difference between the mean response of the perturbed and unperturbed series (multiplied by 100).



Next, we examine the impact of technology on the prices of financial assets and human capital (i.e., the households tradeable wealth). The total wealth of all existing households,

$$W_t \equiv \int_0^1 W_{n,t} \, dn = V_t + G_t + H_t, \tag{25}$$

equals the sum of three components. The first part is the value of a claim on the profits of all existing projects  $\mathcal{J}_t$ ,

$$V_t \equiv \int_0^1 \mathcal{E}_t \left[ \sum_{j \in \mathcal{J}_{f,t}} \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \, \pi_{j,s} \right] \, df.$$
(26)

The second component is the value of new growth opportunities that accrues to shareholders,

$$G_t \equiv (1 - \eta) \int_0^1 \mathcal{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \lambda_{f,s} \nu_s \, ds \right] \, df \tag{27}$$

where  $\nu_t$  is the net present value of a new project implemented at time t – defined in equation (10).

The last part denotes the present value of labor services to households,

$$H_t \equiv \mathcal{E}_t \left[ \int_t^\infty e^{-\delta^h(s-t)} \,\frac{\Lambda_s}{\Lambda_t} \, \mathcal{W}_s \, ds \right].$$
(28)

In Figure 3 we see how technology shocks affect the risk-free rate, the value of installed assets  $V_t$ , growth opportunities  $G_t$ , and human capital. In our subsequent analysis, the ratio of the value of new blueprints  $\nu_t$  to total wealth  $W_t$  plays a key role. Thus, we also plot the response of  $\nu_t/W_t$  to a positive technology shock. A positive technology shock increases expected consumption growth; hence, as we see in the first column, the risk-free rate rises on impact. The next three columns plot the response of aggregate assets in place  $(V_t)$ , the present value of growth opportunities  $(G_t)$ , and human capital  $(H_t)$  to a technology shock. A positive disembodied shock x is complementary to installed capital, hence the value of assets in place and growth opportunities rises on impact. By contrast, the technology shock  $\xi$  that is embodied in new capital lowers the value of existing assets V but it increases the value of growth opportunities G.

#### Figure 3: Technology and Asset Prices

This figure plots the impulse response of aggregate output, investment and consumption expenditures to the two technology shocks in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time t = 0 without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of  $\omega$ . In the first row, we report the difference between the mean response of the perturbed and unperturbed risk-free rate. For all other series we report the log of the ratio of the perturbed to the unperturbed series. The vertical axis is in percentage points.



The value of new blueprints  $\nu$  relative to total wealth W rises in response to either technology shock, as we see in the last column of Figure 3. Importantly, the effect is quantitatively much larger for the technological shock that is embodied in *new* projects compared to advances in technology that affect both existing and new projects. This difference is important because the responses of aggregate consumption in Figure 2 to technology shocks mask substantial heterogeneity in the consumption paths of individual households. As we show next, larger changes in  $\nu/W$  lead to greater reallocation of wealth among households.

### 3.1.2 Technology and individual consumption

The consumption of an individual household differs from aggregate consumption  $C_t$  due to to imperfect risk sharing. The current state of a household can be summarized by its current share of total wealth,  $w_{n,t} \equiv W_{n,t}/W_t$ . The functional form of preferences (13-14) together with our assumption that the scale of the household-level innovation process is proportional to individual wealth imply that optimal individual consumption and portfolio plans are proportional to individual wealth. Then, a household's consumption share is the same as its wealth share,  $c_{n,t} = w_{n,t}$ . Therefore, individual consumption satisfies

$$C_{n,t} = C_t \, w_{n,t}.\tag{29}$$

The dynamic evolution of households' share of aggregate wealth is

$$\frac{dw_{n,t}}{w_{n,t}} = \delta^h dt + \frac{\lambda}{\mu_I} \frac{\eta \nu_t}{W_t} \left( dN_{it}^I - \mu_I dt \right), \tag{30}$$

where  $N_{n,t}^{I}$  is a Poisson process that counts the number of times that household n has acquired a new blueprint.

The evolution of a household's relative wealth in (30) is conditional on the household survival; thus, the first term captures the flow payoff of the annuity, as it is standard in perpetual youth OLG models (Blanchard, 1985). The second term captures changes in the households' wealth resulting from innovation. Both the drift and the return to successful innovation depend on the fraction of shareholder wealth that accrues to all successful inventors,  $\eta \nu_t/W_t$ . Each period, a household yields a fraction  $\lambda \eta \nu_t/W_t$  of its wealth share to successful innovators. This wealth reallocation occurs because households own shares in all firms, and these firms make payments to new inventors in return for their blueprints. During each infinitesimal time period, with probability  $\mu_I dt$ , the household is itself one of the innovators, in which case it receives a payoff proportional to  $\eta \nu_t W_{n,t}$ . The magnitude of wealth reallocation depends on the contribution of new investments to total wealth  $\nu_t/W_t$ . Recall that, as described in Section 3.1.1, an increase in either x or  $\xi$  implies that the equilibrium value of new blueprints to total wealth  $\nu/W$  increases. This increase in  $\nu_t/W_t$  leads to an increase in the households' idiosyncratic risk – similar to the model of Constantinides and Duffie (1996).

The process of wealth reallocation following positive technology shocks is highly skewed. Equation (30) shows that the rise in gains from successful innovation, i.e., the rise in  $\nu_t/W_t$ , implies that most households experience higher rates of relative wealth decline, as captured by a reduction in the drift of  $dw_t$ . By contrast, the few households that innovate increase their wealth shares greatly, as captured by the jump term  $\nu_t/W_t dN_t^I$ . From the perspective of a household at time t, the distribution of future consumption becomes more variable and more skewed following a positive technological shock, even though on average the effect is zero – a positive technological shocks magnifies both the extremely high realizations of w and the paths along which w declines persistently.

Figure 4 illustrates the impact of technology shocks on the consumption path of an individual household. Our objects of interest are the households' relative wealth share  $w_i$ , the households' consumption  $C_i$ , and household consumption adjusted for relative preferences  $C_i^{1-h}w_i^h$ . In the first three columns, we plot the impulse response of these variables to the two technology shocks. In addition to the response of the mean, we also plot how the median of the future distribution of these variables changes in response to the two technology shocks. Unlike the mean, the median of w is not influenced by the rare but extremely positive outcomes, and instead reflects the higher likelihood of large gradual relative wealth declines in response to technology shocks.

The first column of Figure 4 summarizes the role of incomplete risk sharing in our model. A technology shock – either x or  $\xi$  – has no impact on the expected future wealth share w at any horizon, because in our model technology shocks have ex-ante a symmetric effect on all households. However, the lack of an effect on the average wealth share masks substantial heterogeneity in individual outcomes. Specifically, the response of the median of the distribution is significantly negative at all horizons. The very different responses of the mean and the median wealth share suggest a highly skewed effect of technological shocks on individual households, which is key to understanding the effects of technology shocks on the stochastic discount factor in our model.

The next two columns of Figure 4 examine the response of household consumption. In the second column, we see that the responses of the mean and median future consumption to a disembodied shock x are not substantially different. The difference in responses between the mean and the median consumption is most stark when technology is embodied in new vintages. A positive embodied shock  $\xi$  leads to an increase in the share of value due to new blueprints  $\nu/W$ , and thus to greater wealth reallocation among households. The next column shows the role of relative consumption preferences. If households care about their relative consumption  $w_i$  in addition to their own consumption  $C_i$ , then the impact of technology shocks on their adjusted consumption flow is a weighted average of the first two columns.

The difference between the response of the mean and the median of the distribution of future consumption highlights the asymmetric benefits of technology shocks. Since households are risk averse, the mean response is insufficient to characterize the impact of technology on their indirect utility. When evaluating their future utility, households place little weight on the extremely high paths of w. Hence, the median response is also informative. In other words, in addition to their effect on the mean consumption growth, technology shocks also affect the variability of consumption because they affect the magnitude of the jump term in (30). Even though the conditional risk of individual wealth shares is idiosyncratic, this risk depends on the aggregate state of the economy. Therefore, innovation risk affects the stochastic discount factor, similarly to the model of Constantinides and Duffie (1996). To illustrate this connection, the last column of Figure 4 plots the increase in the variance of instantaneous consumption growth – adjusted for relative preferences. We see that both technology shocks lead to higher consumption volatility. The effect is substantially higher for  $\xi$  than for x, again due to its higher impact on the returns to innovation  $\nu/W$ .

#### Figure 4: Technology and Household Consumption

The first three columns of this figure plot the impulse response of the household wealth share  $(w_i)$ , household consumption  $(C_i)$ , and consumption adjusted for relative consumption preferences  $C_i^{1-h} w_i^h$ . We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time t = 0 without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of  $\omega$ . We report the log difference between the mean (median) response of the perturbed and unperturbed series in the solid (dashed) line. The last column plots the impulse response of the conditional variance of instantaneous log consumption growth, adjusted for relative preferences.



In sum, Figure 4 shows that the economic growth that results from technological improvements is not shared equally across households. Specifically, innovation reallocates wealth shares from most households to a select few. Although an increase in  $\nu_t/W_t$  does not affect the *expected* wealth share of any household, it raises the magnitude of unexpected changes in households' wealth shares. Since households are risk averse, they dislike the resulting variability of changes in their wealth. Preferences over relative consumption (h > 0) magnify the negative effect of relative wealth shocks on indirect utility. This effect on households' indirect utility has important implications about the pricing of these shocks, as we discuss next.

#### 3.1.3 The stochastic discount factor

In this section, we examine the stochastic discount factor. Financial markets in our model are incomplete, since some of the shocks (specifically, the acquisition of blueprints by individual households) are not spanned by the set of traded financial assets. As a result, there does not exist a unique stochastic discount factor (SDF) in our model. Similar to Constantinides and Duffie (1996), the utility gradients of various agents are not identical and each can serve as a valid SDF. To facilitate the discussion of the aggregate prices, we construct an SDF that is adapted to the market filtration  $\mathcal{F}$  generated by the aggregate productivity shocks  $(B_x, B_\xi)$ . This SDF is a projection of agent-specific SDFs (utility gradients) on  $\mathcal{F}$ . The following proposition illustrates how to construct a valid SDF in our economy.

**Proposition 1 (Stochastic Discount Factor)** The process  $\Lambda_t$ , given by the following equation, is adapted to the market filtration  $\mathcal{F}$  and is a valid SDF:

$$\log \Lambda_t = \int_0^t b(\omega_s) \, ds - \gamma_1 \chi_t - \frac{1}{\theta_1} \left( \log C_t - \chi_t \right) - \frac{1 - \kappa}{\kappa} \log f(\omega_t). \tag{31}$$

In the above equation,  $\kappa \equiv \frac{1-\gamma}{1-\theta^{-1}}$ ,  $\gamma_1 \equiv 1 - (1-\gamma)(1-h)$ , and  $\theta_1 \equiv (1 - (1-\theta^{-1})(1-h))^{-1}$ . In the first term, the function  $b(\omega)$  is defined in the proof of the proposition in the Appendix. In the last term, the function  $f(\omega)$  is related to the value function J of an investor with relative wealth  $w_{it}$ ,

$$f(\omega_t) = (1 - \gamma) J(w_{i,t}, \chi_t, \omega_t) \left( w_{i,t}^{1 - \gamma} e^{(1 - \gamma_1)\chi_t} \right)^{-1}.$$
 (32)

**Proof:** See Appendix A.

Equilibrium prices of the aggregate technological shocks stem from four stochastic terms in the expression for the SDF in (31). We construct impulse responses for the log SDF – and its components – taking into account the nonlinear nature of equilibrium dynamics; we introduce an additional one-standard deviation shock at time t without altering the realizations of all future shocks. We plot these responses in Figure 5.

The first two components in (31) capture the permanent component of the SDF. In the second term,  $\chi_t$  captures the permanent impact of technology shocks on the aggregate consumption process. The third term in (31) reflects the pricing of transitory shocks to aggregate consumption. This transitory deviation of log consumption from its stochastic trend  $\chi_t$  is a function of the stationary process  $\omega_t$ . The coefficients  $\gamma_1$  and  $\theta_1$  are essentially the risk aversion and EIS coefficients modified towards one to account for relative consumption considerations. These coefficients determine the price of risk for the permanent and transitory components of aggregate consumption, respectively, while holding constant the effect of these shocks on the future distribution of the households' relative wealth share  $w_i$ . As we can see from the fourth column of Figure 5, the contribution of transitory shocks in aggregate consumption to the SDF is quantitatively small.

Most importantly, the last term in (31) captures fluctuations in an investor's indirect utility that result from technology shocks, holding fixed the investor's current wealth share w and stochastic

#### Figure 5: Technology and the Stochastic Discount Factor

This figure plots the impulse response of the log stochastic discount factor, as well as its four components, to the two technology shocks in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time t = 0 without altering the realizations of all future shocks. The impulse responses are computed at the mean of the stationary distribution of  $\omega$ . We report the difference between the mean response of the perturbed and unperturbed series.



trend  $\chi$ . This term summarizes the effect of technology shocks on the distribution of future consumption growth for individual households. Importantly, this term also captures the effect of household expectations about the fluctuations of their future wealth shares  $w_i$  due to incomplete markets. The impact of technology on the indirect utility function is further exacerbated by the fact that individual innovation shocks have a highly positively skewed distribution. A positive technology shock implies a higher payoff to individual households from successful innovation and a larger decline in relative consumption absent thereof. An investor with concave preferences does not place as much weight on the low-probability, high-payoff outcomes as he does on the highly likely persistently low consumption growth. Thus, a positive technology shock can have a large negative effect on the indirect utility of the investors. As we can see from the impulse responses shown in Figure 5, this is a quantitatively important aspect of how technology shocks are priced in our model, particularly for the embodied shock  $\xi$ . The difference in the magnitude in the response of indirect utility arises because the reallocative effects of an embodied shock, given by its impact on the returns to innovation  $\nu_t/W_t$ , is an order of magnitude higher than the reallocative effects of the disembodied shock, as we saw in the last column of Figure 3.

Comparing panels A and B of Figure 5, we see that the two technology shocks carry opposite prices of risk in our model. A positive disembodied shock x negatively affects the SDF on impact,

implying a positive risk premium. By contrast, a positive embodied shock  $\xi$  leads to a rise in the SDF on impact, implying a negative risk premium. As a result, households value securities that provide a hedge against states of the world when  $\xi$  is high and x is low. This difference in how the SDF responds to the two technology shocks stems primarily from the response of the indirect utility term  $f(\omega_t)$  in the SDF. Both technology shocks x and  $\xi$  lead to an increase in the permanent component  $\chi_t$  of consumption, which by itself causes the SDF to fall. However, in the case of the embodied shock, the fall in indirect utility due to the unequal sharing of benefits from technological progress is sufficiently large to offset the benefits of higher aggregate consumption. The resulting demand for insurance against high realizations of  $\xi$  is driven by the endogenous increase in the consumption uncertainty of individual investors.

### 3.2 Technology Shocks and the Cross-section of Firms

Next, we examine the impact of technology shocks on individual firms. The firm's current state is fully characterized by the aggregate state  $X_t$ , its probability of acquiring new projects  $\lambda_{f,t}$ , its relative size,

$$k_{f,t} \equiv \frac{1}{K_t} \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{s(j)}} k_{j,t}.$$
(33)

and its current average productivity across projects

$$\bar{u}_{f,t} \equiv \left( \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{s(j)}} u_{j,t} k_{j,t} \right) \bigg/ \left( \sum_{j \in \mathcal{J}_{f,t}} e^{\xi_{s(j)}} k_{j,t} \right).$$
(34)

A major challenge in examining the predictions of the model for the cross-section of firms is that a firm's current state  $(\lambda_{ft}, k_{ft}, \bar{u}_{ft})$  is unobservable. Hence, we have to rely on observable proxies. The most commonly used measure of a value or growth firm is its ratio of market value of the firm to its book value of assets. This ratio is also referred to as the firm's (average) Tobin's Q. In our model, a firm's log market to book ratio can be written as

$$\log Q_{ft} - \log Q_t = \log \left[ \frac{V_t}{V_t + G_t} \left( 1 + \widetilde{p}(\omega_t) \left( \overline{u}_{ft} - 1 \right) \right) + \frac{G_t}{P_t + G_t} \frac{1}{k_{ft}} \left( 1 + \widetilde{g}(\omega_t) \left( \frac{\lambda_{ft}}{\lambda} - 1 \right) \right) \right],$$
(35)

where  $Q_t$  is the market-to-book ratio of the market portfolio;  $V_t$  and  $G_t$  are defined in equations (26) and (27) respectively; and  $\tilde{p}(\omega)$  and  $\tilde{g}(\omega)$  are defined in the Appendix.

Examining (35), we note that a firm's market-to-book ratio is increasing in the likelihood of future growth  $\lambda_f$ , decreasing in the firm's relative size  $k_f$ , and increasing in the firm's current productivity  $\bar{u}_f$ . As we see below, the main determinant of firms' cross-sectional differences in cashflow risk is the ratio of  $\lambda_f$  to  $k_f$ . Differences in  $\bar{u}_f$  play some role due to cashflow duration effects; in our current calibration, however, this effect is quantitatively minor. Firms' technology risk exposures are increasing in Tobin's Q, as we describe in detail in the Online Appendix.

#### 3.2.1 Technology and creative destruction

We next examine how the impact of technology shocks on firm outcomes varies with the current state of the firm. In our model, technological progress indirectly leads to displacement of installed capital due to general equilibrium effects. To see this, consider the effect of technological progress on firm profitability. The profit flow of a firm f can be expressed as

$$\pi_{f,t} \equiv \sum_{j \in \mathcal{J}_{f,t}} \pi_{j,t} = \phi Y_t \, \bar{u}_{f,t} \, k_{f,t}. \tag{36}$$

Cross-sectional differences in firms' systematic cashflow risk depend on the response of the firm's relative size  $k_f$  to technology shocks. The dynamics of  $k_f$  are given by

$$\frac{dk_{f,t}}{k_{f,t}} = a_0 \frac{\nu_t}{V_t} \left(\frac{\lambda_{f,t}}{\lambda k_{f,t}} - 1\right) dt + a_0 \frac{\nu_t}{V_t} \frac{1}{\lambda k_{f,t}} \left(dN_{f,t} - \lambda_{f,t} dt\right),\tag{37}$$

where  $a_0$  is a constant and  $N_{f,t}$  is a Poisson process that counts the number of times that firm f has acquired a new project.

The first term in (37) captures fluctuations in the firm's expected cashflow growth due to changes in technology. The conditional growth rate of the aggregate capital stock K depends on the value of new blueprints  $\nu$  relative to the value of installed capital V. Each firm's' cashflow risk exposure to shocks to  $\nu/V$  depends on its current investment opportunities  $(\lambda_f)$  and its current scale of operations  $(k_f)$ . Relative to the average firm in the economy, firms with high levels of  $\lambda_f/k_f$  derive more of their value from their future growth opportunities rather than their existing operations. We refer to such firms as growth firms. Conversely, firms with low levels of  $\lambda_f/k_f$  derive most of their market value from their existing operations. We refer to these firms as value firms. The last term in (37) captures idiosyncratic shocks due to the acquisition of new projects.

To illustrate the heterogeneous impact of technology shocks on firm profitability and investment, we examine separately two firms with high and low levels of  $\lambda_f/k_f$  in Figure 6. For both firms, we set productivity at its long run mean,  $\bar{u}_f = 1$ . As we see in panel A, improvements in technology that are complementary to all capital lead to an immediate increase in profitability for both types of firms. The growth firms are more likely to have higher investment opportunities than the value firms; hence, on average, they increase investment. While growth firms pay lower dividends in the short run, their payouts rise over the long run. As a result, the market value of a growth firm appreciates more than the market value of a value firm. In panel B, we see that value and growth firms have very different responses to technology improvements embodied in new vintages. The technology shock  $\xi$  leaves the output of existing projects unaffected, since it only increases the productivity of new investments. Due to the equilibrium response of the price of labor services, the profit flow from existing operations falls. Growth firms increase investment, and experience an increase in profits and market valuations. In contrast, value firms have few new projects to invest in, and therefore experience a decline in their profits and valuations.

In sum, growth firms have higher cashflow and stock return exposure to either technology shock

#### Figure 6: Technology Shocks and Firms

This figure plots the dynamic response of firm profits, investment, dividends and stock prices to the two technology shocks x and  $\xi$  in the model. We construct the impulse responses taking into account the nonlinear nature of equilibrium dynamics: we introduce an additional one-standard deviation shock at time t = 0 without altering the realizations of all future shocks. We report separate results for two types of firms. The solid line represents the responses for a Growth firm, defined as a firm with  $\lambda_{f,t} = \lambda_H$  and  $k_{f,t} = 0.5$ . The dotted line indicates the responses for a value firm, defined as a firm with low investment opportunities  $\lambda_{f,t} = \lambda_L$  and large size  $k_{f,t} = 2$ . For both firms, the level of average profitability is equal to its long-run mean,  $u_{f,t} = 1$ . The initial value of the state variable  $\omega$  is set to its unconditional mean,  $\omega_0 = E[\omega_t]$ . Columns 1 and 4 plot percentage changes, columns 2 and 3 plot changes in the level (since both dividends and investment need not be positive) normalized by the aggregate dividend and investment at time t = 0.



than value firms, and the difference is quantitatively larger for technological improvements embodied in new capital vintages  $\xi$  versus shocks to labor productivity x. These differential responses of growth and value firms to technology shocks translate into cross-sectional differences in risk and risk premia.

## 3.3 Sensitivity Analysis and Extensions

Here, we explore the impact of alternative formulations of the model on our main findings. Our model has three relatively non-standard features. First, our model features technology shocks that are embodied in new capital. Second, markets are incomplete in that households cannot sell claims on their proceeds from innovation. Third, household preferences are affected by their consumption relative to the aggregate economy. Here, we examine how important these three features are for the quantitative performance of the model. In addition, we estimate an extended version of the model that allows for limited stock market participation.

#### 3.3.1 Sensitivity to modeling assumptions

We estimate three restricted versions of the model. The first version sets  $\eta = 0$ , which effectively completes the markets for innovation outcomes. In this case, all proceeds from new projects accrue to financial market participants. The second restricted model constrains h = 0 so that households have no preferences over relative consumption. The third restricted model features no embodied technological change – we restrict  $\mu_{\xi} = 0$  and  $\sigma_{\xi} = 0$ . To estimate these models, we repeat the procedure detailed in Section 2. We report the performance in matching the target set of empirical statistics of the restricted versions in columns (R1) through (R3) of Table 3. We report the corresponding parameter estimates in column (R1)-(R3) of Table 4.

We see that all three of these assumptions play an important role. The model with complete markets (R1) generates essentially zero risk premia, both on average (the market portfolio) and also in the cross-section (the value factor). The model without relative preferences (R2) generates realistic aggregate dynamics and moments of the market portfolio, and in contrast to models (R1) and (R3) uses only moderate levels of utility curvature ( $\hat{\gamma} = 17.8$ ). However, (R2) generates virtually no cross-sectional dispersion in risk premia. Last, the model with only disembodied technology shocks (R3) has difficulty jointly matching the dynamics of aggregate consumption and investment. It does generate a sizable equity premium, however it generates almost no cross-sectional dispersion in risk premia.

#### 3.3.2 Limited participation

Our baseline model is quite successful in matching empirical facts about economic quantities and asset returns. Based on introspection, however, some of the parameter estimates may seem implausibly high – for instance, the degree of utility curvature ( $\hat{\gamma} = 106$ ), the preference weight on relative consumption ( $\hat{h} = 0.95$ ) and the share of the surplus that accrues to innovators ( $\hat{\eta} = 0.79$ ) may seem relatively large.

One plausible reason why these estimated parameters are high may be that the model underestimates the risks that financial markets participants face from innovation. As we saw in Figure 2, labor income rises in response to improvements in technology. This increase in labor income acts as a natural hedge for the displacement of households that occurs through financial markets. However, the hedging benefit of labor income is an artifact of the stylized nature of our model. Specifically, we assume that technology has no displacive effect on labor, and that labor income is tradable without any frictions. A more realistic model that allows for endogenous displacement of human capital and possibly frictions, such as credit constraints, is outside the scope of this paper.

	Data	Baseline		testricted vers	ions	Extension
			(R1)	(R2)	(R3)	(X1)
			Complete	No Relative	No Embod.	LimPart
			markets $[\eta = 0]$	cons prei $[h=0]$	tech shock $[(\mu_{\xi}, \sigma_{\xi}) = 0]$	$[q_S < 1]$
Aggregate quantities						
Consumption growth, mean	0.015	0.014	0.013	0.017	0.012	0.015
Consumption growth, volatility	0.036	0.042	0.042	0.036	0.040	0.038
Consumption growth, long-run risk	0.041	0.056	0.055	0.048	0.055	0.051
Shareholder Consumption, mean share of total	0.429	ı	ı	ı		0.362
Shareholder Consumption, volatility	0.037	ı	ı	ı	·	0.040
Investment, mean share of output	0.089	0.055	0.087	0.078	0.146	0.078
Investment, volatility	0.130	0.116	0.099	0.088	0.067	0.118
Investment and consumption, correlation	0.472	0.383	0.433	0.372	0.989	0.345
Net payout to assets, coefficient of variation	0.575	0.520	0.378	0.509	0.261	0.486
Asset Prices						
Market Portfolio, excess returns, mean	0.063	0.068	0.012	0.061	0.066	0.077
Market Portfolio, excess returns, std	0.185	0.152	0.133	0.122	0.095	0.134
Risk-free rate, mean	0.020	0.025	0.024	0.020	0.023	0.021
Risk-free rate, volatility	0.007	0.007	0.007	0.006	0.003	0.007
Value factor, mean	0.065	0.054	-0.004	0.004	-0.007	0.058
Value factor, volatility	0.243	0.191	0.100	0.063	0.236	0.168
Value factor, CAPM alpha	0.040	0.034	-0.004	0.001	0.018	0.037
Cross-sectional (firm) moments						
Investment rate, IQR	0.175	0.200	0.187	0.153	0.077	0.167
Investment rate, serial correlation	0.223	0.191	0.186	0.238	0.181	0.203
Investment rate, correlation with Tobin's Q	0.237	0.205	0.204	0.281	0.155	0.171
Tobin's Q, IQR	1.139	0.750	0.699	0.452	0.347	0.908
Tobin's Q, serial correlation	0.889	0.924	0.933	0.880	0.814	0.946
Profitability, IQR	0.902	0.963	0.930	0.827	0.808	0.927
Profitability, serial correlation	0.818	0.755	0.930	0.880	0.728	0.848
Distance criterion (mean of Rdev2)		0.034	0.192	0.151	0.255	0.026

Table 3: Comparison across restricted models: goodness of fit

Table compares the fit of the baseline model (column Bench) to three restricted versions of the model: a model where shareholders capture full rents from innovation, or  $\eta = 0$  (R1); a model where households have no preferences over relative consumption, or h = 0 (R2), and a model without embodied technology shocks,  $(\mu_{\xi}, \sigma_{\xi}) = 0$  (R3); and an extended version with limited asset market participation. See the main text for details on the estimation method.

Parameter	$\operatorname{Symbol}$	Baseline		Restricted ver	sions	Extension
			(R1)	(R2)	(R3)	(X1)
			Complete	No Relative	No Embod.	LimPart
			markets	cons pref	tech shock	$[q_S < 1]$
			$[\eta=0]$	$[0= \eta]$	$[(\mu_{\xi},\sigma_{\xi})=0]$	
Preferences						
Risk aversion	7	105.856	94.682	17.826	89.860	58.250
Elasticity of intertemporal substitution	θ	2.207	2.088	1.305	1.947	1.851
Effective discount rate	θ	0.040	0.028	0.045	0.059	0.046
Preference weight on relative consumption	h	0.947	0.990	I	0.849	0.804
Technology and Production						
Decreasing returns to investment	σ	0.362	0.372	0.313	0.598	0.532
Depreciation rate	δ	0.033	0.029	0.056	0.087	0.029
Disembodied technology growth, mean	$\mu_x$	0.016	0.016	0.021	0.015	0.015
Disembodied technology growth, volatility	$\sigma_x$	0.077	0.078	0.064	0.080	0.070
Embodied technology growth, mean	$\mu_{\xi}$	0.004	0.000	0.002	ı	0.006
Embodied technology growth, volatility	σξ	0.137	0.127	0.106	ı	0.113
Transition rate to low-growth state	$\mu_L$	0.283	0.452	0.277	0.351	0.353
Transition rate to high-growth state	HH	0.015	0.009	0.059	0.056	0.025
Project mean arrival rate, low growth state	$\lambda_L$	0.122	0.152	1.821	0.235	0.111
Project mean arrival rate, high growth state	$\lambda_{H}$	8.588	6.708	3.642	17.744	11.983
Project-specific productivity, volatility	$\sigma_u$	0.636	0.258	0.349	0.647	0.466
Project-specific productivity, persistence	$\kappa_u$	0.303	0.050	0.100	0.318	0.160
Incomplete Markets						
Fraction of project NPV that goes to inventors	μ	0.785	0.000	0.727	0.900	0.591
Probability of born a stockholder	dS	ı	'	I	ı	0.073

 Table 4: Comparison across restricted models: parameter estimates

Table compares estimated parameters in the baseline model (column Bench) to three restricted versions of the model: a model where shareholders capture full rents from innovation, or  $\eta = 0$  (R1); a model where households have no preferences over relative consumption, or h = 0 (R2), and a model without embodied technology shocks,  $(\mu_{\xi}, \sigma_{\xi}) = 0$  (R3); and an extended version with limited asset market participation. See the main text for details on the estimation method.

However, recognizing that only a subset of households participates in financial markets serves to reduce the usefulness of labor income as a hedge against innovation shocks. For instance, Poterba and Samwick (1995) report that the households in the top 20% in terms of asset ownership consistently own more than 98% of all stocks. If most workers do not participate in financial markets, the share of labor income accruing to market participants can actually be rather small. We thus estimate an extension of our baseline model that allows for limited participation in the stock market.

We model limited participation by assuming that newly born households are randomly assigned to one of two types, shareholders (with probability  $q_S$ ) and workers (with probability  $1 - q_S$ ). Shareholders have access to financial markets, and optimize their life-time utility of consumption, just like the households in our baseline model. Workers are instead hand-to-mouth consumers. They do not participate in financial markets, supply labor inelastically, and consume their labor income as it arrives. Workers can also successfully innovate (just like shareholders); those that do so become shareholders. To estimate the extended model, we include two additional target statistics. First, we target a mean consumption share of 42.9% based on the estimates of Guvenen (2006). Second, we target a volatility of shareholder consumption growth of 3.7% based on the unpublished working paper version of Malloy, Moskowitz, and Vissing-Jorgensen (2009), which includes an adjustment for measurement error.

We report the performance of the extended model in matching the target set of empirical statistics of the restricted versions in column (X1) of Table 3 and the corresponding parameter estimates in column (X1) of Table 4. We see that the extended model indeed does better in matching the empirical facts with lower levels of risk aversion ( $\hat{\gamma} = 58 \text{ vs } 106$ ), share of surplus to innovators ( $\hat{\eta} = 0.59 \text{ vs } 0.79$ ) and preference share over relative consumption ( $\hat{h} = 0.80 \text{ vs } 0.95$ ). The estimate  $\hat{q}_s = 0.073$  implies that the steady-state fraction of households that participate in financial markets is approximately 12%, consistent with the evidence in Guvenen (2006), who reports that the top 10% of households own approximately 84% of all financial assets.

# 4 Additional Predictions

Here, we examine the performance of the model in replicating some features of the data that we do not use as an explicit estimation target in Section 2. In Section 4.1, we consider the correlation between consumption growth, dividends and asset returns. In Section 4.2, we examine some direct predictions of the model mechanism.

## 4.1 Consumption, Dividends and Asset Returns

We begin by examining the implications of the model for the joint distribution of consumption, dividends, and asset returns. In addition to the results for the baseline model, we also report the results for the model with limited market participation in Section 3.3.2. Motivated by that extension, we examine the correlation of asset returns and dividends with the consumption of households that participate in the stock market. For this purpose, we use the data of Malloy et al. (2009), which is available over the 1982-2002 period. To facilitate comparison with the existing literature, we

relate the properties of aggregate payout  $D_t$  in the model to the empirical dynamics of dividends per share for the market portfolio.<sup>17</sup> We compute correlations of asset returns and dividends from the market portfolio with the consumption of stockholders in absolute terms, but also relative to the consumption of non-participants. The behavior of this variable plays an important role in the extended model with limited participation in Section 3.3.2. We follow standard practice and aggregate consumption and dividend growth over multiple horizons. We report results using 2-year growth rates, but the results are qualitatively similar using longer horizons.

Table 5 shows that the baseline model generates empirically plausible levels of correlation between asset returns, market dividends, and aggregate consumption. Consumption and aggregate stock market returns are more highly correlated in the model, but the difference between the correlations in the model and in the data is not statistically significant. The model also reproduces the low empirical correlation between aggregate consumption growth and the value factor, which contributes to the failure of the Consumption CAPM to capture the value premium in simulated data (see the Online Appendix). Further, we see that the extended model with limited market participation replicates one of the main findings of Malloy et al. (2009): returns of value firms covary more with shareholder consumption than returns of growth firms. However, the extended model does not capture the relatively low correlation of shareholder consumption and the market portfolio (equal to 21% in the 1982-2002 sample versus 72% in the model).

#### 4.2 Implications of the Model Mechanism

Our analysis thus far follows closely the existing literature and evaluates the success of the model based on the model-implied correlations between macroeconomic quantities and prices. Constructing direct tests of the model is challenging: a part of the model mechanism relies on fluctuations in the value of new blueprints  $\nu_t$ , a quantity that is difficult to observe empirically. Here we construct an empirical measure for the value of new blueprints based on prior research on the economic value of innovative activity. We then use this empirical proxy to examine the predictions of the model's main mechanism.

#### 4.2.1 Estimating the value of new blueprints

The market value of new blueprints  $\nu_t$  plays a key role in the model's predictions, both for the dynamics of firm cashflows (37) and for the evolution of investors' wealth (30). To test the model's mechanism, we use data on patents and stock returns to construct an empirical proxy for  $\nu_t$ . Specifically, we view patents as empirical equivalents to the blueprints in our model. Kogan et al. (2012) show that the time series of their patent-based measure is informative about the overall pace of technological innovation in the economy, and that it captures meaningful differences in outcomes across firms and industries.

<sup>&</sup>lt;sup>17</sup>Given the estimated parameters, aggregate net payout  $D_t$  is almost always positive – it becomes negative only in extreme ranges of the state space that are unlikely to be reached across 1,000 simulations. However, the same is not true for the dividends arising from the value and growth portfolios; the growth portfolio often has a negative net payout. In the Online Appendix we investigate the long-run cash flow risk properties of the value and growth portfolios.

Variable	Consumption growth	Data	]	Model
	eonsumption growth		Baseline	LimitedPart
Market	Aggregate	0.395	0.627	0.639
returns		[0.147]	[0.069]	[0.073]
	Shareholders	0.212		0.717
		[0.117]		[0.118]
	Shareholders,	0.111		0.296
	relative to non-shareholders	[0.154]		[0.21]
Value Factor	Aggregate	0.162	-0.015	-0.009
returns		[0.13]	[0.209]	[0.19]
	Shareholders	0.351		0.178
		[0.184]		[0.187]
	Shareholders,	0.410		0.457
	relative to non-shareholders	[0.182]		[0.112]
Aggregate	Aggregate	0.351	0.368	0.434
DPS growth		[0.250]	[0.176]	[0.171]
	Shareholders	0.282		0.616
		[0.133]		[0.188]
	Shareholders,	0.359		0.531
	relative to non-shareholders	[0.126]		[0.235]

Table compares the empirical correlations between three consumption measures (aggregate consumption, consumption of stock holders, and relative consumption of stockholders) with the corresponding correlations in the model. Market returns are in excess of the risk-free rate. The empirical correlations with shareholder consumption are based on the consumption data in (Malloy et al., 2009), covering the 1982-2002 period.

Following Kogan et al. (2012), we estimate the net present value of a patent as the change in the dollar value of the firm around a three-day window after the market learns that the firm's patent application has been successful.<sup>18</sup> To replicate this construction in simulated data, we employ an approximation that does not require the estimation of new parameters.<sup>19</sup> Kogan et al. (2012) document a strong positive relation between estimated patent values and future citations the patent receives–a commonly used indicator of the scientific value of new patents. To evaluate the quality of our approximation, we replicate their result using our approximation  $\hat{\nu}_j$  and plot the results in panel

<sup>&</sup>lt;sup>18</sup>The dollar reaction around the issue date is an understatement of the dollar value of a patent. The market value of the firm is expected to change by an amount equal to the NPV of the patent times the probability that the patent application is unsuccessful. This probability is not small; in the data less than half of the patent applications are successful. See Kogan et al. (2012) for a detailed discussion of this empirical procedure.

<sup>&</sup>lt;sup>19</sup>Kogan et al. (2012) allow for movements in stock returns around the announcement window that is unrelated to the value of the patent. They construct a filter of the estimated patent value using specific distributional assumptions, and propose a methodology to empirically estimate those parameters using high-frequency data. This is quite difficult to replicate in simulated data because it requires re-estimating parameters in each sample. Instead, we recognize that their optimal filter can be approximated by the function f(x) = max(x, 0). Using this approximate construction, we can replicate their main empirical results. The details are available upon request, and also appear in earlier working paper versions of Kogan et al. (2012).

A of Figure 7. Comparing panel A to the corresponding figure in their paper, we find a qualitatively similar relation between  $\hat{\nu}_i$  and future citations.

#### Figure 7: Estimated Value of new Patents

Panel A of this figure plots the relation between forward patent citations and the estimated market value of patents. We group the patent data into percentiles. The vertical axis plots the average number of patent citations in each percentile–minus the average number of citations to patents in the same technology class that were granted in the same year. The horizontal axis plots the logarithm of the average estimated patent value in each percentile (deflated by the CPI). Panel B plots the time series of the ratio of the estimated value of new blueprints  $\hat{\nu}_t$  to the value of the stock market. See the main text, the appendix and Kogan et al. (2012) for more details. Both panels cover data over the 1927-2008 period.



### 4.2.2 Innovation and aggregate dynamics

We begun by studying the relation between our estimate of technological progress, aggregate consumption, and stock returns. We construct an estimate of the aggregate value of new blueprints at time t as

$$\hat{\omega}_t = \log\left(\frac{\sum_{j \in P_t} \hat{\nu}_j}{M_t}\right),\tag{38}$$

where  $P_t$  denotes the set of patents granted to firms in our sample in year t. Since the value of assets is not observable, we scale by total stock market capitalization M = V + G instead. In the model,  $\nu_t/V_t$  and  $\nu_t/M_t$  (as well as  $\nu_t/W_t$ ) are monotonically increasing functions of the state variable  $\omega_t$ , hence, we can interpret (38) as an empirical proxy for  $\omega$ .

We plot the constructed  $\hat{\omega}_t$  in Panel B of Figure 7. Similar to Kogan et al. (2012), this time-series series lines up well with the three major waves of technological innovation in the U.S. – the 1930s, 1960s and early 1970s, and 1990s and 2000s. In Table 6, we compare the empirical properties of the innovation series  $\hat{\omega}_t$  to the properties of  $\hat{\omega}$  in simulated data. In the data,  $\hat{\omega}_t$  is more volatile and less persistent than in the model, consistent with the presence of substantial measurement error. We compute correlations of  $\Delta \hat{\omega}$  with the aggregate consumption growth, with the consumption growth of stock-holders (as in Section 4.1 above), and with stock market returns and with the value factor.

Statistic	Data	Model	
	Data	Bench	LimitedPart
$\hat{\omega}$ , serial correlation	0.765	0.906	0.907
	(0.068)	(0.055)	(0.053)
$\Delta \hat{\omega}$ , volatility	0.303	0.156	0.152
	(0.047)	(0.012)	(0.015)
$\Delta \hat{\omega}$ , correlation with value factor	-0.284	-0.350	-0.353
	(0.078)	(0.102)	(0.115)
$\Delta \hat{\omega}$ , correlation with market portfolio	-0.499	-0.419	-0.379
	(0.088)	(0.067)	(0.081)
$\Delta \hat{\omega}$ , correlation with agg. cons. growth	-0.216	-0.001	0.003
	(0.113)	(0.148)	(0.153)
$\Delta \hat{\omega}$ , correlation with shareholder cons. growth	-0.303		-0.229
	(0.137)		(0.235)
$\Delta \hat{\omega}$ , correlation with shareholder relative cons. growth	-0.354		-0.643
	(0.126)		(0.108)

 Table 6: Properties of estimated value of blueprints to total wealth

This table reports the moments of the measure of the value of new blueprints to the value of the market portfolio,  $\hat{\omega}$ , in the data and in the model. We report correlations of  $\Delta \hat{\omega}$  with aggregate consumption growth (NIPA), the consumption of shareholders, and the consumption of shareholders relative to non-stockholders; also, correlations with the excess returns on the market portfolio and the value factor. See Section 4.1 and Appendix B for more details on the consumption of stockholders. Numbers in brackets are standard errors. In panel A, the standard errors are Newey-West with 5 lags. In panel B, the standard errors are computed based on the standard deviation of the estimated coefficients across 1,000 simulations.

The correlations in the model are largely consistent with their empirical counterparts. In the data, there is a negative (-50%) correlation between innovation shocks  $\Delta \hat{\omega}$  and stock market returns. The value factor and  $\Delta \hat{\omega}$  have correlation of -28%, implying that value stocks covary more negatively with innovation shocks than growth stocks. The model counterparts to the above correlations are -42% and -35% respectively. The correlation between  $\Delta \hat{\omega}_t$  and aggregate consumption in the data is also negative and equal to -20%, while in the model this correlation is approximately zero. In the data, the negative relation between innovation shocks and consumption is stronger for the consumption series of shareholders (-30%) or the consumption of stockholders relative to non-stock holders (-35%). These correlations are broadly consistent with both the baseline model and the extension with limited participation.

### 4.2.3 Technological innovation and firm cashflows

We next study the model's predictions about the cross-section of firms. Kogan et al. (2012) document substantial heterogeneity in innovation across industries. Here, we exploit this heterogeneity to get sharper estimates of the impact of technological progress on firm outcomes. Specifically, we construct a direct analogue of  $\nu/V$  at the industry level as

$$\hat{\omega}_{I\setminus f} = \frac{\sum_{f'\in I\setminus f} \sum_{j\in P_{f',t}} \hat{\nu}_j}{\sum_{f'\in I\setminus f} M_{f',t}},\tag{39}$$

where industry I is defined by the 3-digit SIC code and  $P_{ft}$  is the set of patents issued to firm f in year t.  $I \setminus f$  denotes the set of all firms in industry I excluding firm f.

We are mainly interested in how the relation between technology shocks and future firm profitability depends on the current state of the firm, that is, the interaction of  $\nu_t/V_t$  with  $\lambda_f/k_f$  in (37). We estimate the impact of technology on log firm profitability using the following approximation to (36),

$$\log \pi_{f,t+T} - \log \pi_{f,t} = (a_0 + a_1 q_{f,t}) \ \hat{\omega}_{I \setminus f,t} + a_t + a_I + c_1 G_{f,t} + c_2 \ \hat{N}_{f,t} + c_l s_{f,t} + c_k k_{f,t} + \varepsilon_{t+T}.$$
(40)

Based on the discussion in Section 3.2, we allow this relation to vary as a function of the firms' Tobin's Q. We classify firms as either value  $(q_{f,t} = 0)$  or growth  $(q_{f,t} = 1)$  depending on whether their Tobin's Q falls below or above the industry median at time t. We control for innovation outcomes by firm f – the analogue of the  $dN_{f,t}$  term in equation (37) – through  $\hat{N}_{f,t} = \sum_{j \in P_{f,t}} \hat{\nu}_j / M_{f,t}$ , where  $\hat{\nu}$  is our estimate of patent value constructed in Section 4.2.1. Consistent with (36)-(37) we also include controls for lagged log profits  $y_{f,t}$  and log size  $k_{f,t}$ . In the empirical specification, we include time and industry dummies, and cluster the standard errors at the firm level. We also estimate equation (40) in simulated data from the model. Since the model contains no industries, we drop the time and industry dummies. We scale  $\hat{\omega}_{I\setminus f}$  to unit standard deviation to facilitate comparison between the model and the data.

Table 7 compares the estimated coefficients  $a_0$  and  $a_1$  across horizons of 1 to 6 years in the data and the model. In sum, our empirical estimates support the notion that improvements in technology benefit firms that have the ability to implement these technologies at the expense of firms that do not. The estimated coefficient  $a_0$  in the data (panel A) is negative and statistically different from zero, implying that the impact of technological progress on the expected profitability of low-Qfirms is negative. Further, the estimated coefficient  $a_1$  is positive and statistically significant across horizons, revealing substantial heterogeneity in how the profits of low-Q and high-Q firms respond to an increase in innovation at the industry level. The estimates of  $a_0$  in the model (panel B) are roughly half the magnitude of the empirical values. However, the point estimates of  $a_0$  vary significantly across simulations. The model implies somewhat larger heterogeneity between value and growth firms; specifically, the estimated coefficient  $a_1$  is higher in simulated data. However, there is significant variation across simulated samples, and the empirical estimates lie inside the 90% confidence intervals implied by model simulations. We conclude that the empirical tests are qualitatively consistent with the model, but a quantitative comparison of the point estimates between the data and the model is challenging.

Table 7: Technology shocks and firm profitability

			A. I	Data		
Horizon	1	2	3	4	5	6
$\hat{\omega}_{I\setminus ft}$	-0.039	-0.040	-0.049	-0.049	-0.055	-0.060
	(0.003)	(0.005)	(0.006)	(0.008)	(0.009)	(0.010)
$\hat{\omega}_{I\setminus ft} \times G_{ft}$	0.022	0.016	0.018	0.017	0.017	0.018
	(0.003)	(0.005)	(0.006)	(0.007)	(0.008)	(0.008)
	B. Model					
Horizon	1	2	3	4	5	6
$\hat{\omega}_{I\setminus ft}$	-0.015	-0.019	-0.022	-0.025	-0.027	-0.028
	(0.047)	(0.061)	(0.091)	(0.108)	(0.127)	(0.132)
$\hat{\omega}_{I\setminus ft} \times G_{ft}$	0.006	0.014	0.024	0.031	0.038	0.046
	(0.004)	(0.008)	(0.012)	(0.017)	(0.020)	(0.024)

This table summarizes the estimated coefficients  $a_0$  and  $a_1$  from equation (40) in the data (panel A) and in simulated data from the model (panel B). We report standard errors in parenthesis. In panel A, the standard errors are clustered at the firm level. In panel B, the standard errors are computed based on the standard deviation of the estimated coefficients across 1,000 simulations.

### 4.2.4 Innovation and inequality

A key feature of our model is that the benefits of innovation are asymmetrically distributed. This mechanism is summarized by Figure 4, which shows that improvements in technology have very different effects on the *average* versus the *median* future consumption path of an individual household. Here, we provide supporting evidence that demonstrates that technological improvements are indeed associated with a decline in the median consumption relative to the average.

We do so using the following specification,

$$(c_T^{md} - c_t^{md}) - (c_T - c_t) = b_0 + b_1 \left(\hat{\omega}_T - \hat{\omega}_t\right) + b_2 \left(c_t^{md} - c_t\right) + b_3 \,\hat{\omega}_t + u_t.$$
(41)

Here,  $c_t$  refers to log aggregate per capita NIPA personal consumption expenditures. We estimate the log median per capita total consumption expenditures  $c_t^{md}$  in year t using the Consumer Expenditure Survey (CEX). The sample covers the 1982-2010 period. We also estimate (41) using simulated data from the model. Each simulation contains 10,000 households over 29 years.

We report the estimated coefficients  $b_1$  in Table 8 for horizons of one to five years. Consistent with the model, the empirical estimates of  $\hat{b}_1$  are negative. However, given the short length of the sample, the coefficients are fairly imprecisely estimated and not statistically different from zero for some horizons. The same is true in simulated data. The empirical estimates are fairly close in magnitude to those implied by the model.

We conclude that estimates of (41) are qualitatively consistent with the model, but due to the wide standard errors comparing magnitudes between the model and the data is difficult. Further,

one concern with using CEX data is that it tends to undersample rich households. Even though using the full consumption distribution of the CEX is problematic for our purposes, the effect of undersampling on the median should be minor. However, we emphasize that, since the full extent of the sample selection biases in the CEX data is unknown, our results should be interpreted with caution.

		Horiz	zon (year	(s, T)	
	(1)	(2)	(3)	(4)	(5)
Data					
$\hat{\omega}_T - \hat{\omega}_t$	-0.209	-0.480	-0.413	-0.379	-0.318
	(0.26)	(0.21)	(0.17)	(0.18)	(0.25)
Model					
$\hat{\omega}_T - \hat{\omega}_t$	-0.168	-0.223	-0.266	-0.302	-0.332
	(0.12)	(0.16)	(0.18)	(0.20)	(0.21)

 Table 8: Technology shocks and consumption growth

This table reports the estimated coefficient  $b_1$  from equation (41). We construct the median using total household expenditures scaled by number of its members. Data period is 1982-2010. Simulated samples have the same length. All variables are standardized to mean zero and unit standard deviation. Standard errors are in parenthesis. In panel A, the standard errors are Newey-West, with maximum lag length equal to T + 1. In panel B, the standard errors are computed based on the standard deviation of the estimated coefficients across 1,000 simulations.

# 5 Conclusion

We develop a general equilibrium model to study the effects of innovation on asset returns. The main feature of our model is that the benefits from technological progress are not shared symmetrically across all agents in the economy. Specifically, technological improvements partly benefit agents that are key in the creation and implementation of new ideas. As a result, technology shocks also lead to substantial reallocation of wealth among households. Embodied shocks have a large reallocative effects, whereas disembodied shocks have mostly a level effect on household consumption. In equilibrium, shareholders invest in growth firms despite their low average returns, as they provide insurance against increases in the probability of future wealth reallocation. Our model delivers rich cross-sectional implications about the effect of innovation on firms and households that are supported by the data.

Our work suggests several promising avenues for future research. First, labor income in our model is homogenous, and therefore workers benefit from both types of technological progress. In practice, however, technological advances are often complementary to only a subset of workers' skills. Recent evidence, for example, shows that the job market has become increasingly polarized (Autor, Katz, and Kearney, 2006). Thus, quantifying the role of technological progress as a determinant of the risk of human capital may be particularly important. Second, technological progress tends to disrupt traditional methods of production, leading to periods of increased uncertainty. If some

agents have preferences for robust control, higher levels of uncertainty will likely increase the agents' demand for insurance against improvements in technology embodied in new vintages. Last, our model implies that claims on any factor of production that can be used across different technology vintages (as, for instance, land) can have an insurance role similar to the one played by growth firms in our current framework. Our model would therefore imply that claims on such factors should have lower equilibrium expected returns.

# **Appendix A: Proofs and Derivations**

Here, we provide proofs and detailed derivations. To conserve space, we provide the solution for the extended model with limited participation. We begin by describing the differences between the extended model and our baseline model. The solution to the baseline model is a special case in which  $q_S = 1$ .

## Extended Model

There are two types of households in our economy, *workers* and *shareholders*. There is continuum of each type, with the total measure of households normalized to one. We denote the set of workers by  $\mathcal{W}_t$  and the set of shareholders by  $\mathcal{S}_t$ 

#### Workers

Workers in this economy are hand-to-mouth consumers. They do not participate in financial markets, supply labor inelastically and consume their labor income as it arrives. When a new household is born, it becomes a worker, independently of all other households and all other sources of randomness in the economy, with probability  $1 - q_s$ . Each worker receives a measure  $\lambda (1 - \psi)/\mu_I$  of blueprints upon innovating.

#### Shareholders

A newly born household becomes a shareholder with probability  $q_S$ . Workers that successfully innovate (see below) also become shareholders. Shareholders have access to financial markets, and optimize their life-time utility of consumption. Shareholders are not subject to liquidity constraints. In particular, shareholders sell their future labor income streams and invest the proceeds in financial claims. All shareholders have the same preferences, given by (13)-(14). Each shareholder *i* receives a measure of projects in proportion to her wealth  $W_{i,t}$  relative to shareholders as a group, specifically  $\lambda \psi \mu_I^{-1} W_{i,t} \left( \int_{j \in S_t} W_{j,t} dj \right)^{-1}$ . Here,

$$\psi = \frac{\mu_I + q_S \,\delta^h}{\mu_I + \delta^h} \tag{A.1}$$

is the steady-state fraction of households that participate in financial markets. The case  $q_S = 1$  (or equivalently,  $\psi = 1$ ) corresponds to our baseline model.

### **Proofs and Derivations**

Lemma 1 (Stationary distribution for u) The process u, defined as

$$du_t = \kappa_u (1 - u_t) \, dt + \sigma_u \, u_t \, dB_t^u \tag{A.2}$$

has a stationary distribution given by

$$f(u) = cu^{-2 - \frac{2\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right),\tag{A.3}$$

where c is a constant that solves  $\int_0^\infty f(u) du = 1$ . Further, as long as  $2 \kappa_u \ge \sigma_u^2$ , the cross-sectional variance of u is finite.

**Proof.** We follow (Karlin and Taylor, 1981, p. 221). The forward Kolmogorov equation for the stationary transition density f(u) yields the ODE

$$0 = -\kappa_u \frac{\partial}{\partial u} \left[ (1-u)f(u) \right] + \frac{1}{2}\sigma_u^2 \frac{\partial^2}{\partial u^2} \left[ u^2 f(u) \right]$$
(A.4)

Integrating the above with respect to u yields

$$k = -\kappa_u \left[ (1-u)f(u) \right] + \frac{1}{2}\sigma_u^2 \frac{\partial}{\partial u} \left[ u^2 f(u) \right]$$
(A.5)

where k is a constant of integration. We set k = 0 and find

$$f(u) = cu^{-2 - \frac{2\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right),\tag{A.6}$$

where c is an unknown constant. By construction, the function f is positive. Further, setting the constant c to

$$\left(\int_0^\infty u^{-2-\frac{2\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right) \, du\right)^{-1} \tag{A.7}$$

(the above integral is finite as long as  $\kappa_u > 0$ ) guarantees that  $\int_0^\infty f(u) du = 1$ , and therefore f(u) is the stationary density of the diffusion process u.

The last part of the proof is to show that the variance of u is finite and positive as long as  $2 \kappa_u - \sigma_u^2 > 0$ . Given the solution for c,

$$\int_0^\infty (u-1)^2 c \, u^{-2-\frac{2\kappa_u}{\sigma_u^2}} \exp\left(-\frac{2\theta}{u\sigma_u^2}\right) du = \frac{\sigma_u^2}{2\kappa_u - \sigma_u^2},\tag{A.8}$$

which is finite as long as  $2 \kappa_u - \sigma_u^2 > 0$ .

Before proving the Propositions in the main text, we establish some preliminary results. First, we show how to relate the stochastic discount factor (SDF) to the value function of an investor. This is a straightforward application of the results in Duffie and Skiadas (1994) on the relation between the utility gradient and the equilibrium SDFs.

We focus on a single household an omit the household index. To simplify exposition, we present the result in a slightly more general form, not limiting it to the exact structure of our economy. As in our model, the household is solving a consumption-portfolio choice problem with one non-standard element: it receives a stochastic stream of gains from innovation in proportion to its financial wealth. Let  $W_t$  denote the household's wealth.

The market consists of I financial assets that pay no dividends. Let  $S_t$  denote the vector of prices of the financial assets.  $S_t$  is an Ito process

$$dS_t = \mu_t \, dt + \sigma_t \, dB_t. \tag{A.9}$$

The first asset is risk-free, its price growth at the equilibrium rate of interest  $r_t$ . Let  $\mathcal{F}$  denote the natural filtration generated by the Brownian motion vector  $B_t$ .

The investor receives a flow of income from innovation projects according to an exogenous Poisson process N with the arrival rate  $\lambda$ . The process N is independent of the Brownian motion B. We assume that conditional on innovating, household's wealth increases by a factor of  $\exp(\rho_t)$ , where the process  $\rho$  is adapted to the filtration  $\mathcal{F}$ . The consumption process of the household, C, and its portfolio vector  $\theta$ , are adapted to the filtration generated jointly by the exogenous processes N and B.

As in our model, the investor maximizes the stochastic differential utility function given by equations (13-14) in the main text, where we take the process  $\overline{C}_t$  to be a general Ito process adapted to the filtration  $\mathcal{F}$ , subject to the dynamic budget constraint

$$dW_t = \delta W_t \, dt - C_t \, dt + (e^{\varrho_t} - 1) \, W_t \, dN_t + \theta_t dS_t, \quad W_t = \theta_t S_t, \tag{A.10}$$

and a credit constraint, which rules out doubling strategies and asymptotic Ponzi schemes:

$$W_t \ge 0. \tag{A.11}$$

Note that the first term in (A.10) captures the flow of income from annuities that the household collects conditional on its continued survival. The death process is a Poisson process with arrival rate  $\delta$ , which is independent of N and B. We are now ready to define an SDF in relation to the value function of the household. In particular, we construct an SDF process that is adapted to the filtration  $\mathcal{F}$ , and hence does not depend on the household-specific innovation arrival process N.

**Lemma 2 (SDF)** Let  $C_t^*$ ,  $\theta_t^*$ , and  $W_t^*$  denote the optimal consumption strategy, portfolio policy, and the wealth process of the household respectively. Let  $J_t^*$  denote the value function under the optimal policy. Define the process  $\Lambda_t$  as

$$\Lambda_t = \exp\left(\int_0^t \delta + \frac{\partial \phi(C_s^\star, J_s^\star; \overline{C}_s)}{\partial J_s^\star} + \lambda \left(e^{(1-\gamma)\varrho_s} - 1\right) ds\right) A_t, \tag{A.12}$$

where

$$A_t = \frac{\partial \phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star} \exp\left(\int_0^t \gamma \varrho_s \, dN_s\right). \tag{A.13}$$

Then  $\Lambda_t$  is a stochastic discount factor consistent with the price process S and adapted to filtration  $\mathcal{F}$ .

#### Proof.

Let  $\mathcal{M}$  denote the market under consideration, and define a fictitious market  $\mathcal{\hat{M}}$  as follows.  $\mathcal{\hat{M}}$  has the same information structure as  $\mathcal{M}$ , with modified price processes for financial assets. Specifically, let

$$R_t = \exp\left(\int_0^t \delta \, ds + \varrho_s \, dN_s\right) \tag{A.14}$$

and define price processes in the market  $\hat{\mathcal{M}}$  as

$$\hat{S}_t = R_t S_t. \tag{A.15}$$

The budget constraint in the market  $\hat{\mathcal{M}}$  is standard,

$$d\hat{W}_t = -C_t \, dt + \hat{\theta}_t d\hat{S}_t, \quad \hat{W}_t = \hat{\theta}_t \hat{S}_t. \tag{A.16}$$

If a consumption process  $\{C\}$  can be financed by a portfolio policy  $\theta$  in the original market  $\mathcal{M}$ , it can be financed by the policy  $R^{-1}\theta$  in the fictitious market  $\hat{\theta} = R^{-1}\theta$ , and vice versa. Thus, the set of feasible consumption processes is the same in the two markets, and therefore the optimal consumption processes are also the same. Since the consumption-portfolio choice problem in the fictitious market is standard, according to (Duffie and Skiadas, 1994, Theorem 2), the utility gradient of the agent at the optimal consumption policy defines a valid SDF process  $\hat{\Lambda}_t$ ,

$$\hat{\Lambda}_t = \exp\left(\int_0^t \frac{\partial \phi(C_s^\star, J_s^\star; \overline{C}_s)}{\partial J_s^\star} \, ds\right) \, \frac{\partial \phi(C_t^\star, J_t^\star; \overline{C}_t)}{\partial C_t^\star}.\tag{A.17}$$

Thus, for all t < T,

$$\hat{\Lambda}_t R_t S_t = \hat{\Lambda}_t \hat{S}_t = \mathcal{E}_t \left[ \hat{\Lambda}_T \hat{S}_T \right] = \mathcal{E}_t \left[ \hat{\Lambda}_T R_T S_T \right]$$
(A.18)

and therefore  $\Lambda'_t = \hat{\Lambda}_t R_t$  is a valid SDF in the original market  $\mathcal{M}$ . Note that  $\Lambda'_t$  is not adapted to the filtration  $\mathcal{F}$ , since it depends on the agent's innovation process N. In other words,  $\Lambda'_t$  is an agent-specific SDF process.

The last remaining step is to show that the process  $\Lambda_t$  is adapted to the filtration  $\mathcal{F}$  and a valid SDF. First, we show that the process  $\exp\left(\int_0^t \partial \phi(C_s^\star, J_s^\star; \overline{C}_s)/\partial J_s^\star\right)$  is adapted to  $\mathcal{F}$ , and the process  $\partial \phi(C_t^\star, J_t^\star; \overline{C}_t)/\partial C_t^\star$ can be decomposed as  $A_t R_t^{-\gamma}$ , where  $A_t$  is also adapted to  $\mathcal{F}$ . Given the homotheticity of the stochastic differential utility function and the budget constraint (A.10), standard arguments show that the agent's value function and the optimal consumption policy can be expressed as

$$J_t^{\star} = \left(W_t^{\star}\right)^{1-\gamma} \varpi_{J,t},\tag{A.19}$$

where  $\varpi_{J,t}$  is a stochastic process adapted to  $\mathcal{F}$ . The optimal wealth and consumption processes then take form

$$W_t^{\star} = R_t \varpi_{W,t}, \quad C_t^{\star} = R_t \varpi_{C,t}, \tag{A.20}$$

where  $\varpi_{W,t}$  and  $\varpi_{C,t}$  are adapted to  $\mathcal{F}$ , and therefore

$$J_t^{\star} = R_t^{1-\gamma} \varpi_{W,t}^{1-\gamma} \varpi_{J,t}. \tag{A.21}$$

We next use these expressions to evaluate the partial derivatives of the aggregator  $\phi$ :

$$\frac{\partial \phi(C_t^{\star}, J_t^{\star}; \overline{C}_t)}{\partial J_t^{\star}} = -\frac{\rho(1-\gamma)}{1-\theta^{-1}} - \frac{\rho(1-\gamma)^{\frac{1-\theta^{-1}}{\gamma-1}}(\gamma-\theta^{-1})}{1-\theta^{-1}} \left(\varpi_{C,t}^{1-h} \left(\varpi_{C,t}/\overline{C}_t\right)^h\right)^{1-\theta^{-1}} \left(\varpi_{W,t}^{1-\gamma} \varpi_{J,t}\right)^{\frac{1-\theta^{-1}}{\gamma-1}},\tag{A.22}$$

which is adapted to  $\mathcal{F}$ ; and

$$\frac{\partial \phi(C_t^{\star}, J_t^{\star}; \overline{C}_t)}{\partial C_t^{\star}} = \rho(1-\gamma)^{\frac{1-\theta^{-1}}{\gamma-1}} \overline{C}_t^{h(1-\theta^{-1})} \overline{\omega}_{C,t}^{-\theta^{-1}} (\overline{\omega}_{W,t}^{1-\gamma} \overline{\omega}_{J,t})^{\frac{1-\theta^{-1}}{\gamma-1}} R_t^{-\gamma}.$$
(A.23)

Thus, the process

$$A_t = \frac{\partial \phi(C_t^{\star}, J_t^{\star}; \overline{C}_t)}{\partial C_t^{\star}} e^{-\gamma \delta t} R_t^{\gamma}$$
(A.24)

is adapted to  $\mathcal{F}$ . Based on the above results, we express  $\Lambda'_t$  as

$$\Lambda'_{t} = \exp\left(\int_{0}^{t} \frac{\partial\phi(C_{s}, J_{s}; \overline{C}_{s})}{\partial J_{s}} + \gamma\delta \, ds\right) \, A_{t} R_{t}^{1-\gamma} \tag{A.25}$$

Define  $\Lambda_t = E[\Lambda'_t | \mathcal{F}_t]$ . Since all asset price processes in the original market are adapted to  $\mathcal{F}$ ,  $\Lambda_t$  is also a valid SDF process.

Using the equality (see below)

$$\mathbf{E}\left[R_t^{1-\gamma}|\mathcal{F}_t\right] = \exp\left(\int_0^t \delta(1-\gamma) + \lambda\left(e^{(1-\gamma)\varrho_s} - 1\right)\,ds\right),\tag{A.26}$$

we find

$$\Lambda_t = \exp\left(\int_0^t \frac{\partial \phi(C^*{}_s, J^*{}_s; \overline{C}{}_s)}{\partial J^*{}_s} + \delta + \lambda \left(e^{(1-\gamma)\varrho_s} - 1\right) ds\right) A_t.$$
(A.27)

To complete the proof, we show that

$$\mathbf{E}\left[\left(R_{t}\right)^{1-\gamma}|\mathcal{F}_{t}\right] = \exp\left(\int_{0}^{t}\delta(1-\gamma) + \lambda\left(e^{(1-\gamma)\varrho_{s}} - 1\right)\,ds\right).\tag{A.28}$$

Fix the path of  $\rho_s$  and consider only the uncertainty associated with Poisson process N. Define

$$M_t = \exp\left(\int_0^t \varrho_s(1-\gamma) \, dN_s - \int_0^t \lambda\left(e^{(1-\gamma)\varrho_s} - 1\right) \, ds\right). \tag{A.29}$$

Then

$$dM_t = -M_t \lambda \left( e^{(1-\gamma)\varrho_s} - 1 \right) dt + \left( e^{(1-\gamma)\varrho_s} - 1 \right) M_t dN_t, \tag{A.30}$$

and therefore

$$\mathbf{E}\left[dM_t|N_s,\,s\le t\right] = 0\tag{A.31}$$

and  $M_t$  is a martingale. So,  $E[M_t|\mathcal{F}_t] = 1$ .

Next, we consider some of the equilibrium relations in order to gain intuition for the overall structure of the solution. Define

$$\zeta_{j,t} = u_{j,t} \, e^{\xi_{\tau(j)}} \, k_{j,t}. \tag{A.32}$$

and

$$Z_t = \int_{J_t} e^{\xi_{s(j)}} u_{j,t} \, k_{j,t} \, dj.$$
 (A.33)

The labor hiring decision is static. The firm managing project j chooses  $L_{jt}$  as the solution to

$$\pi_{jt} = \sup_{L_{jt}} \left[ \zeta_{jt}^{\phi} \left( e^{x_t} L_{jt} \right)^{1-\phi} - W_t L_{j,t} \right]$$
(A.34)

The firm's choice

$$L_{j,t}^{\star} = \zeta_{j,t} \left( \frac{(1-\phi) e^{(1-\phi)x_t}}{W_t} \right)^{\frac{1}{\phi}}.$$
 (A.35)

After clearing the labor market,  $\int_{\mathcal{J}_t} L_{j,t} dj = 1$ , the equilibrium wage is given by

$$W_t = (1 - \phi) e^{(1 - \phi) x_t} Z_t^{\phi}, \qquad (A.36)$$

and the choice of labor allocated to project j is

$$L_{j,t}^{\star} = \zeta_{j,t} \, Z_t^{-1}. \tag{A.37}$$

Aggregate output of all projects equals

$$Y_t = \int_{J_t} \zeta_{j,t} \, e^{(1-\phi) \, x_t} \, Z_t^{\phi-1} \, dj = e^{(1-\phi) \, x_t} \, Z_t^{\phi}. \tag{A.38}$$

The project's flow profits are

$$\pi_{j,t} = \sup_{L_{j,t}} \left[ \zeta_{jt}^{\phi} \left( e^{x_t} L_{j,t} \right)^{1-\phi} - W_t \, l_{j,t} \right] = p_t \, \zeta_{jt} \tag{A.39}$$

where

$$p_t = \phi Y_t Z_t^{-1} \tag{A.40}$$

Because firms' investment decisions do not affect its own future investment opportunities, each investment maximizes the net present value of cash flows from the new project. Thus, the optimal investment in a new project j at time t is the solution to

$$\sup_{k_{j,t}} \mathcal{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} \, \pi_{j,s} \, ds \right] - k_{j,t}^{1/\alpha} = \sup_{k_{j,t}} \left[ P_t \, k_{j,t} e^{\xi_t} - k_{j,t}^{1/\alpha} \right], \tag{A.41}$$

where  $P_t$  is the time-t price of the asset with the cash flow stream  $\exp(-\delta(s-t))p_s$ :

$$P_t = \mathcal{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} e^{-\delta(s-t)} p_s \, ds \right]. \tag{A.42}$$

The optimal scale of each new project is then given by

$$k_t^{\star} = \left(\alpha \, e^{\xi_t} \, P_t\right)^{\frac{\alpha}{1-\alpha}}.\tag{A.43}$$

Note that the solution does not depend on the identity of the firm, i.e., all firms, faced with an investment decision at time t, choose the same scale for the new projects. The optimal investment scale depends on the current market conditions, specifically, on the current level of the embodied productivity process  $\xi_t$ , and the

current price level  $P_t$ .

We thus find that the aggregate stock of quality-adjusted installed capital in the intermediate good sector, defined by (7), evolves according to

$$dK_t = \left(-\delta K_t + \lambda e^{\xi_t} k_t^{\star}\right) dt = \left(-\delta K_t + \lambda e^{\xi_t} \left(\alpha e^{\xi_t} P_t\right)^{\frac{\alpha}{1-\alpha}}\right) dt.$$
(A.44)

An important aspect of (A.44) is that the growth rate of the capital stock  $K_t$  depends only on its current level, the productivity level  $\xi_t$ , and the price process  $P_t$ . Furthermore, as we show below, we can clear markets with the price process  $P_t$  expressed as a function of the state vector  $X_t = (x_t, \xi_t, K_t)$ . Thus,  $X_t$  follows a Markov process in equilibrium.

We express equilibrium processes for aggregate quantities and prices as functions of  $X_t$ . For instance, the fact that investment decisions are independent of u implies that  $Z_t = K_t$ . Aggregate investment  $I_t$  is given by

$$I_t = \lambda \left(k_t^\star\right)^{1/\alpha},\tag{A.45}$$

The aggregate consumption process satisfies

$$C_t = Y_t - I_t = K_t^{\phi} e^{(1-\phi)x_t} - \lambda \left(k_t^{\star}\right)^{1/\alpha}.$$
(A.46)

Prices of long-lived financial assets, such as the aggregate stock market, depend on the behavior of the stochastic discount factor. In equilibrium, the SDF is determined jointly with the value function of the households, as shown in Lemma 2. Below we fully characterize the equilibrium dynamics and express  $\Lambda_t$  as a function of  $X_t$ .

Define the two variables

$$\chi_t = \frac{1-\phi}{1-\alpha\phi} x_t + \frac{\phi}{1-\alpha\phi} \xi_t.$$
(A.47)

and

$$\omega_t = \left(\xi_t + \alpha \,\chi_t - \log K_t\right) \tag{A.48}$$

 $\omega_t$  and  $\chi_t$  are linear functions of the state vector  $X_t$ . In Lemma 3 below, we characterize the SDF and aggregate equilibrium quantities as functions of  $\omega_t$  and  $\chi_t$ .

In the formulation of the lemma, we characterize the value function of a household, as well as prices of financial assets, such as  $P_t$  in (A.42), using differential equations. Verification results, such as (Duffie and Lions, 1992, Sec. 4), show that a classical solution to the corresponding differential equation, subject to the suitable growth and integrability constraints, characterizes the value function. Similarly, the Feynman-Kac Theorem (Karatzas and Shreve, 1991, e.g., Theorem 7.6) provides an analogous result for the prices of various financial assets. Because we solve for equilibrium numerically, we cannot show that the classical solutions to our differential equations exist and satisfy the sufficient regularity conditions. With this caveat in mind, in the following lemma we characterize the equilibrium processes using the requisite differential equations.

**Lemma 3 (Equilibrium)** Let the seven functions,  $f(\omega)$ ,  $s(\omega)$ ,  $\kappa(\omega)$ ,  $i(\omega)$ ,  $v(\omega)$ ,  $g(\omega)$ ,  $h(\omega)$  solve the following system of four ordinary differential equations,

$$0 = A_1(\omega)f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + f(\omega)\left\{c_0^f - (1-\gamma)s(\omega) + \left[\left(1 + \frac{\psi}{\mu_I}s(\omega)\right)^{1-\gamma} - 1\right]\mu_I\right\} + f'(\omega)\left\{c_1^f - (1-\alpha\phi)\kappa(\omega)\right\} + f''(\omega)c_2^f,$$
(A.49)

$$0 = \phi e^{-\phi \omega} B(\omega) + v'(\omega) \left\{ c_1' - (1 - \alpha \phi) \kappa(\omega) \right\} + v''(\omega) c_2',$$
  
+  $v(\omega) \left\{ c_0^f - \frac{\gamma - \theta^{-1}}{1 - \gamma} A_1(\omega) f(\omega)^{\frac{1 - \theta^{-1}}{\gamma - 1}} + \mu_I \left( \left( 1 + \frac{\psi}{\mu_I} s(\omega) \right)^{1 - \gamma} - 1 \right) + \gamma s(\omega) - \kappa(\omega) \right\}, \quad (A.50)$ 

$$0 = (1 - \eta) (1 - \alpha) v(\omega) \kappa(\omega) + g'(\omega) \left\{ c_1^f - (1 - \alpha \phi) \kappa(\omega) \right\} + g''(\omega) c_2^f + g(\omega) \left\{ c_0^f - \frac{\gamma - \theta^{-1}}{1 - \gamma} A_1(\omega) f(\omega)^{\frac{1 - \theta^{-1}}{\gamma - 1}} + \mu_I \left( \left( 1 + \frac{\psi}{\mu_I} s(\omega) \right)^{1 - \gamma} - 1 \right) + \gamma s(\omega) \right\},$$
(A.51)

$$0 = (1 - \phi) e^{-\phi \omega} B(\omega) + h'(\omega) \left\{ c_1^f - (1 - \alpha \phi) \kappa(\omega) \right\} + h''(\omega) c_2^f + h(\omega) \left\{ c_0^f - \delta^h - \frac{\gamma - \theta^{-1}}{1 - \gamma} A_1(\omega) f(\omega_t)^{\frac{1 - \theta^{-1}}{\gamma - 1}} + \mu_I \left( \left( 1 + \frac{\psi}{\mu_I} s(\omega_t) \right)^{1 - \gamma} - 1 \right) + \gamma s(\omega) \right\}, \quad (A.52)$$

and three algebraic equations,

$$s(\omega) = \frac{\eta \left(1 - \alpha\right) v(\omega) \kappa(\omega)}{v(\omega) + g(\omega) + \psi h(\omega)},\tag{A.53}$$

$$\kappa(\omega) = \lambda^{1-\alpha} e^{(1-\alpha\phi)\omega} [i(\omega)]^{\alpha}, \tag{A.54}$$

$$\left(\frac{i(\omega_t)}{\lambda}\right)^{1-\alpha} = \alpha e^{(1-\alpha\phi + \phi(1-\theta_1^{-1}))\omega} v(\omega) f(\omega)^{\frac{\gamma-\theta^{-1}}{1-\gamma}} \left[(1-i(\omega))\right]^{\theta_1^{-1}} \left(1 - \frac{(1-\psi)(1-\phi)}{1-i(\omega)}\right)^{1/\theta}.$$
 (A.55)

The constants  $c_0^f$ ,  $c_1^f$ ,  $c_2^f$  and  $\phi^d$  are

$$c_{0}^{f} = \left\{ \delta^{h}(1-\gamma) - \frac{\rho(1-\gamma)}{1-\theta^{-1}} + (1-\gamma_{1})(1-\phi)\mu_{x} + \frac{1}{2}(1-\phi)^{2}\sigma_{x}^{2}(1-\gamma_{1})^{2} + \frac{1}{2}\left(\frac{\phi(1-\gamma_{1})}{1-\alpha\phi}\right)^{2}\left(\sigma_{\xi}^{2} + \alpha^{2}(1-\phi)^{2}\sigma_{x}^{2}\right) + \frac{\phi(1-\gamma_{1})}{1-\alpha\phi}\left(\mu_{\xi} + \alpha(1-\phi)\mu_{x} + (1-\gamma_{1})\alpha(1-\phi)^{2}\sigma_{x}^{2}\right) \right\},$$
(A.56)

$$c_{1}^{f} = \left\{ \mu_{\xi} + \alpha \left(1 - \phi\right) \mu_{x} + \left(1 - \alpha \phi\right) \delta + \left(1 - \gamma_{1}\right) \alpha \left(1 - \phi\right)^{2} \sigma_{x}^{2} + \frac{\phi(1 - \gamma_{1})}{1 - \alpha \phi} \left(\sigma_{\xi}^{2} + \alpha^{2} (1 - \phi)^{2} \sigma_{x}^{2}\right) \right\}, \quad (A.57)$$

$$c_2^f = \frac{1}{2} \left( \sigma_{\xi}^2 + \alpha^2 (1 - \phi)^2 \sigma_x^2 \right)$$
(A.58)

and the functions  $A_1(\omega)$  and  $B(\omega)$  are defined as

$$A_1(\omega) = \frac{\rho(1-\gamma)}{1-\theta^{-1}} \left[ (1-i(\omega)) \right]^{1-\theta_1^{-1}} \left( 1 - \frac{(1-\psi)(1-\phi)}{1-i(\omega)} \right)^{1-\theta^{-1}} e^{-\phi(1-\theta_1^{-1})\omega},$$
(A.59)

$$B(\omega) = \left[ (1 - i(\omega)) \right]^{-\theta_1^{-1}} \left( 1 - \frac{(1 - \psi)(1 - \phi)}{1 - i(\omega)} \right)^{-1/\theta} f(\omega)^{\frac{\gamma - \theta^{-1}}{\gamma - 1}} e^{\phi \, \theta_1^{-1} \, \omega}.$$
(A.60)

Then we can construct price processes and individual policies that satisfy the definition 1, so that the value function of a shareholder household n with relative wealth  $W_n/W = w_n$  is given by

$$J(w_n, \chi, \omega) = \frac{1}{1 - \gamma} w_n^{(1 - \gamma)} e^{(1 - \gamma_1)\chi} f(\omega),$$
(A.61)

where  $\gamma_1 = 1 - (1 - \gamma)(1 - h)$ , and  $K_t$  follows

$$\frac{dK_t}{K_t} = -\delta \, dt + \kappa(\omega_t) \, dt. \tag{A.62}$$

)

**Proof.** We start with a conjecture, which we confirm below, that the equilibrium price process  $P_t$  satisfies

$$P_t = K_t^{-1} e^{\chi_t} v(\omega_t) B(\omega_t)^{-1},$$
(A.63)

the equilibrium aggregate value of assets in place is

$$V_t = e^{\chi_t} v(\omega_t) B(\omega_t)^{-1}, \tag{A.64}$$

the value of growth opportunities for the average firm  $(\lambda_f = \lambda)$  is

$$G_t = e^{\chi_t} g(\omega_t) \left( B(\omega_t) \right)^{-1}, \tag{A.65}$$

and the aggregate value of human capital is

$$H_t = e^{\chi_t} h(\omega_t) \left( B(\omega_t) \right)^{-1}.$$
(A.66)

We then characterize the equilibrium SDF and the optimal policies of the firms and households, and show that all markets clear and the above conjectures are consistent with the equilibrium processes for cash flows and the SDF.

We denote the time-t net present value of the new projects (the maximum value in (A.41)) by  $\nu_t$ . According to equations (A.43, A.45) above,

$$\nu_t = \left(\alpha^{\alpha/(1-\alpha)} - \alpha\right) \left(P_t e^{\xi_t}\right)^{1/(1-\alpha)} \tag{A.67}$$

The aggregate investment process, according to (A.43, A.45), is given by

$$I_t = \lambda \left( \alpha \, e^{\xi_t} \, P_t \right)^{\frac{1}{1-\alpha}} \tag{A.68}$$

Using (A.68) and market clearing (A.45),  $K_t$  follows

$$\frac{dK_t}{K_t} = \left(-\delta K_t + \lambda e^{\xi_t} \left(\alpha e^{\xi_t} P_t\right)^{\frac{\alpha}{1-\alpha}}\right) dt = -\delta dt + \kappa(\omega_t) dt,$$

where we have used (A.63), (A.55), and (A.60) for the last equality. The equilibrium dynamics of the aggregate quality-adjusted capital stock thus agrees with (A.62).

Next, we establish the dynamics of the SDF  $\Lambda$ . Consider the evolution of household's wealth share. All shareholders solve the same consumption-portfolio choice problem, different only in the level of household wealth, and households have homothetic preferences. Thus, the evolution of a shareholder household's wealth share (defined as the ratio of household wealth to the total wealth of all shareholders) is given by an equation similar to (30) but taking into account the presence of households that do not participate in financial markets:

$$\frac{dw_{n,t}}{w_{n,t}} = \delta^h \, dt - \frac{\lambda \eta \nu_t}{V_t + G_t + \psi \, H_t} \, dt + \psi \, \frac{\lambda \eta \nu_t}{V_t + G_t + \psi \, H_t} \, \mu_I^{-1} dN_{it}^I. \tag{A.69}$$

Equation (A.69) takes into account that the total measure of new blueprints that accrue to shareholders is equal to  $\psi$ . The benchmark model in the paper corresponds to the case where  $\psi = 1$ . Based on the asset prices in (A.64–A.66) and (A.53), we find that

$$\frac{\lambda \eta \nu_t}{V_t + G_t + \psi H_t} = s(\omega_t),\tag{A.70}$$

and therefore wealth shares follow

$$\frac{dw_{n,t}}{w_{n,t}} = (\delta^h - s(\omega_t)) dt + \psi \mu_I^{-1} s(\omega_t) dN_{it}^I.$$
(A.71)

Next, we derive the the consumption process of households from the market clearing conditions. Then, optimality of this process follows from asset prices being consistent the SDF implied by this process. Based on the aggregate consumption process (A.46) and equilibrium wage process (A.36), along with the definition

of  $\omega$  and  $\chi$ , the consumption of shareholders as a group is

$$C_t^S \equiv \int_{i \in \mathcal{S}_t} C_{i,t} \, di = C_t - (1 - \psi) \, \mathbf{w}_t = e^{\chi_t} e^{-\phi \, \omega_t} \left( 1 - i(\omega_t) - (1 - \psi)(1 - \phi) \right). \tag{A.72}$$

Preference homotheticity implies that the consumption of a each shareholder household is proportional to its wealth share, so

$$C_{n,t} = w_{n,t} C_t^S.$$
 (A.73)

Optimality of household consumption and portfolio choices implies that the SDF in Lemma 2 above, defined using a shareholder households' consumption process, is a valid equilibrium SDF in this economy. In particular, we obtain

$$\Lambda_t = \Lambda_0 \, \exp\left(\int_0^t \delta^h + \frac{\partial \phi(C_{n,s}, J_{n,s}; \overline{C}_s)}{\partial J_{n,s}} + \mu_I \left(\left(1 + \frac{\psi}{\mu_I} s(\omega_s)\right)^{1-\gamma} - 1\right) \, ds\right) \, A_t, \tag{A.74}$$

where  $\Lambda_0$  is a constant and

$$A_t \equiv \frac{\partial \phi(C_{n,t}, J_{n,t}; \overline{C}_t)}{\partial C_{n,t}} \exp\left(\int_0^t \gamma \log\left(1 + \frac{\psi}{\mu_I} s(\omega_s)\right) \, dN_{n,s}\right). \tag{A.75}$$

In the formulation of Lemma 2, we set the gain from innovation to its equilibrium value,

$$\varrho_t \equiv \log\left(1 + \frac{\psi}{\mu_I}s(\omega_t)\right). \tag{A.76}$$

Note also that the SDF is defined only up to a multiplicative positive constant.

Equation (A.49) is the Hamilton-Jacobi-Bellman equation for the value function, if the latter is expressed in the form of (A.61). Using this expression for the value function and the process for shareholder consumption (A.73), we find that

$$A_t = A_0 \, e^{-\gamma_1 \, \chi_t} \, \hat{w}_t^{-\gamma} \, B(\omega_t), \tag{A.77}$$

where  $A_0$  is a constant and  $B(\omega)$  satisfies (A.60) and

$$d\hat{w}_t = \left(\delta^h - s(\omega_t)\right) dt. \tag{A.78}$$

The process  $\hat{w}_t$  is the same as the process for the household wealth shares, conditional on no innovation shocks, i.e., setting  $N_{n,t}$  to be constant. Further,

$$\frac{\partial \phi(C_t^{\star}, J_t^{\star}; \overline{C}_t)}{\partial J_t^{\star}} = -\frac{\rho}{1 - \theta^{-1}} \left( (\gamma - \theta^{-1}) \, l(\omega)^{1 - \theta^{-1}} \left[ f(\omega) \right]^{\frac{1 - \theta^{-1}}{\gamma - 1}} + (1 - \gamma) \right) \tag{A.79}$$

where

$$l(\omega) \equiv (1 - i(\omega) - (1 - \psi)(1 - \phi)) ((1 - i(\omega)))^{-h}.$$
 (A.80)

We are now in a position to complete the proof by verify that the conjectured price processes in (A.63–A.66) are consistent with the equilibrium SDF above. Note that equations (A.50–A.52) are the valuation equations for  $V_t$ ,  $G_t$ , and  $H_t$  respectively, based on the Feynman-Kac Theorem (Karatzas and Shreve, 1991, e.g, Theorem 7.6), given the equilibrium SDF above and the conjectured expressions in (A.64–A.66). By definition of  $V_t$  and  $K_t$ ,  $P_t = K_t^{-1}V_t$ , which establishes the consistency of A.63.

**Proof of Proposition 1.** Proposition follows directly from (A.74) and (A.77) in the proof of the Lemma 3 above. The process  $b(\omega_t)$  is given by

$$b(\omega) = (1 - \gamma)\delta^{h} - \rho \kappa - \rho (1 - \kappa) (1 - i(\omega_{t}))^{(1 - \theta_{1}^{-1})} (f(\omega))^{-\kappa^{-1}} + \gamma s(\omega) + \mu_{I} \left( \left( 1 + \frac{\psi}{\mu_{I}} s(\omega) \right)^{1 - \gamma} - 1 \right).$$
(A.81)

where the functions  $i(\omega)$ ,  $f(\omega)$ , and  $s(\omega)$  are defined in Lemma 3 above.

Lemma 4 (Market value of a firm) The market value of a firm equals

$$S_{f,t} = e^{\chi_t} \left[ v(\omega_t) \frac{z_{f,t}}{\bar{u}_{ft}} \left( 1 + \frac{v_1(\omega_t)}{v(\omega_t)} \left( \bar{u}_{f,t} - 1 \right) \right) + g(\omega_t) + \left( \frac{\lambda_{f,t}}{\lambda} - 1 \right) g_1(\omega_t) \right] \left( B(\omega_t) \right)^{-1}$$
(A.82)

where  $v(\omega)$  and  $g(\omega)$  are defined above and the functions  $v_1$  and  $g_1$  solve the ODEs

$$0 = \phi e^{-\phi \omega} B(\omega) + v_1'(\omega) \left\{ c_1^f - (1 - \alpha \phi) \kappa(\omega) \right\} + v_1''(\omega) c_2^f + v_1(\omega) \left\{ c_0^f - \kappa_u - \frac{\gamma - \theta^{-1}}{1 - \gamma} A_1(\omega) f(\omega)^{\frac{1 - \theta^{-1}}{\gamma - 1}} + \mu_I \left( \left( 1 + \frac{\psi}{\mu_I} s(\omega) \right)^{1 - \gamma} - 1 \right) + \gamma s(\omega) - \kappa(\omega) \right\}$$
(A.83)  
$$0 = (1 - \eta) (1 - \alpha) v(\omega) \kappa(\omega) + g_1'(\omega) \left\{ c_1^f - (1 - \alpha \phi) \kappa(\omega) \right\} + g_1''(\omega) c_2^f + g_1(\omega) \left\{ c_0^f - \mu_L - \mu_H - \frac{\gamma - \theta^{-1}}{1 - \gamma} A_1(\omega) f(\omega)^{\frac{1 - \theta^{-1}}{\gamma - 1}} + \mu_I \left( \left( 1 + \frac{\psi}{\mu_I} s(\omega) \right)^{1 - \gamma} - 1 \right) + \gamma s(\omega) \right\}$$
(A.84)

**Proof.** The proof follows closely the derivations of equations (A.63) and (A.65) above. We have that the value of assets in place for an existing firm with capital stock  $K_{f,t}$  and profitability  $Z_{f,t}$  are given by

$$VAP_{f,t} = P(X_t)K_{f,t} + P_1(X_t) \left( Z_{f,t} - K_{f,t} \right), \tag{A.85}$$

where

$$P_1(X_t) \equiv \mathcal{E}_t \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} e^{-(\delta + \kappa_u)(s-t)} p_s \, ds \right] = K_t^{-1} e^{\chi_t} \, v_1(\omega_t) B(\omega_t)^{-1} \tag{A.86}$$

where  $v_1(\omega)$  satisfies the ODE (A.83). As above, we have used the SDF (A.74), equation (A.40), the definition of  $\chi$  and  $\omega$  and the Feynman-Kac theorem. Similarly, the present value of growth opportunities for a firm equals

$$PVGO_{f,t} \equiv (1-\eta) \operatorname{E}_{t} \left[ \int_{t}^{\infty} \frac{\Lambda_{s}}{\Lambda_{t}} \lambda_{f,s} \nu_{s} \, ds \right]$$
  
=  $PVGO_{t} + \lambda \left(1-\eta\right) \left(\frac{\lambda_{f,t}}{\lambda} - 1\right) \operatorname{E}_{t} \int_{t}^{\infty} \frac{\Lambda_{s}}{\Lambda_{t}} e^{(\mu_{L}+\mu_{H})(s-t)} \nu_{s} \, ds$  (A.87)

$$= e^{\chi_t} g(\omega_t) \left( B(\omega_t) \right)^{-1} + e^{\chi_t} g_1(\omega_t) \left( B(\omega_t) \right)^{-1}$$
(A.88)

where  $g_1(\omega)$  satisfies the ODE (A.83). As above, we have used the SDF (A.74), the definition of z and  $\omega$  and the Feynman-Kac theorem, and the fact that

$$\mathbf{E}[\lambda_{fs}|\lambda_{f,t}] = \lambda + \lambda \left(\frac{\lambda_{f,t}}{\lambda} - 1\right) e^{(\mu_L + \mu_H)(s-t)}.$$
(A.89)

# Appendix B: Data

## **Construction of Estimation Targets**

Aggregate consumption: We use the Barro and Ursua (2008) consumption data for the United States, which covers the 1834-2008 period. We compute the estimate of long-run risk using the estimator in Dew-Becker (2014). We thank Ian Dew-Becker for sharing his code.

Volatility of shareholder consumption growth: The volatility of shareholder consumption growth is from the unpublished working paper version of Malloy et al. (2009) and includes their adjustment for measurement error. Data period is 1980-2002. We are grateful to Annette Vissing-Jorgensen for suggesting this.

Aggregate investment and output: Investment is non-residential private domestic investment. Output is gross domestic product. Both series are deflated by population and the CPI. Data on the CPI are from the BLS. Population is from the Census Bureau. Data range is 1927-2010.

**Dividends and payout**: Moments of net payout to assets are from Larrain and Yogo (2008) who use flow of funds data. Data range is 1929-2004. We compute the growth in dividends per share based on the differences between the return with dividends and without dividends of the CRSP value-weighted portfolio, see Hansen et al. (2005), among others, for more details. Data range for dividends per share is 1927-2010.

Firm Investment rate, Tobin's Q and profitability: Firm investment is defined as the change in log gross PPE. Tobin's Q equals the market value of equity (CRSP December market cap) plus book value of preferred shares plus long term debt minus inventories and deferred taxes over book assets. Firm profitability equals gross profitability (sales minus costs of goods sold) scaled by book capital (PPE). When computing correlation coefficients, we winsorize the data by year at the 1% level to minimize the effect of outliers. We simulate the model at a weekly frequency, dt = 1/50 and time aggregate the data at the annual level. In the model, we construct Tobin's Q as the ratio of the market value of the firm divided by the replacement cost of capital using end of year values. Replacement cost is defined as the current capital stock adjusted for quality,  $\hat{K}_{ft} = e^{-\xi_t} K_{ft}$ . The investment rate is computed as  $I_{ft}/\hat{K}_{ft-1}$ , where  $I_{ft}$  is the sum of firm investment expenditures in year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t - 1. Similarly, we compute firm profitability as  $p_{Z,t}Z_{f,t}/\hat{K}_{f,t-1}$ , where  $p_{Z,t}Z_{f,t}$  is the accumulated profits in year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$  is capital at the end of year t and  $\hat{K}_{f,t-1}$ 

Market portfolio and risk-free rate moments: We use the reported estimate from the long sample of Barro and Ursua (2008) for the United States and cover the 1870-2008 sample (see Table 5 in their paper). In the data, the risk-free rate is the return on treasury bills of maturity of three months or less. The reported volatility of the interest rate in Barro and Ursua (2008), which equals 4.8%, is the volatility of the realized rate. Hence it is contaminated with unexpected inflation. We therefore target a risk-free rate volatility of 0.7% based on the standard deviation of the annualized yield of a 5-year Treasury Inflation Protected Security (the shortest maturity available) in the 2003-2010 sample. In the model,  $r_f$  is the instantaneous short rate; and  $R_M$  is the return on the value-weighted market portfolio. We simulate the model at a weekly frequency, dt = 1/50 and time aggregate the data at the annual level.

Value factor moments: We use the 10 value-weighted portfolios sorted on book-to-market from Kenneth French's Data Library. The value factor is the 10 minus 1 portfolio of firms sorted on book-to-market. Data range is 1927-2010.

**Consumption share of stockholders:** Consumption share of stock holders is from Table 2 of Guvenen (2006). This number is also consistent with Heaton and Lucas (2000): using their data on Table AII we obtain an income share for stockholders of approximately 43%.

**Consumption growth of shareholders:** We use the series constructed in Malloy et al. (2009), which covers the 1980-2002 period. We follow Jagannathan and Wang (2007) and construct annual consumption growth rates by using end-of-period consumption. In particular, we focus on the sample of households that are interviewed in December of every year, and use the average 8 quarter consumption growth rate of non-stockholders and stockholders, defined as in Malloy et al. (2009). We focus on 2-yr horizon. Results using longer horizons are available upon request.

Value of new blueprints: We create  $\hat{\omega}$  using a non-parametric variant of the Kogan et al. (2012) procedure. First, we create idiosyncratic stock returns for firm f around the day that patent j is granted to equal the 3-day return of the firm minus the return on the CRSP value-weighted index around the same window,

$$r_{fj}^e = r_{fj} - r_{mj}.$$
 (B.1)

Patents are issued every Tuesday. Hence,  $r_{fj}$  are the accumulated return over Tuesday, Wednesday and Thursday following the patent issue. Second, we compute an estimate of the value of patent j as the firm's market capitalization on the day prior the patent announcement  $V_{fj}$  times the idiosyncratic return to the firm truncated at zero,

$$\hat{\nu}_j = \frac{1}{N_j} \max(r_{fj}^e, 0) \, V_{fj}. \tag{B.2}$$

If multiple patents were granted in the same day to the same firm, we divide by the number of patents N. Relative to Kogan et al. (2012), we replace the filtered value of the patent  $E[x_{fd}|r_{fd}]$  with  $\max(r_{fj}^e, 0)$ . Our construction is an approximation to the measure in Kogan et al. (2012) that can be easily implemented in simulated data without the additional estimation of parameters. In particular, we follow a similar approach when constructing  $\hat{\nu}_j$  in simulated data. We compute the excess return of the firm as in equation (B.1) around the times that the firm acquires a new project, and then construct  $\hat{\nu}_j$  as in equation (B.2). Data range is 1927-2010.

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