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INTERNATIONAL RESERVES AND ROLLOVER RISK

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ABSTRACT

Two striking facts about international capital flows in emerging economies motivate this paper: (1) Governments hold large amounts of international reserves, for which they obtain a return lower than their borrowing cost. (2) Purchases of domestic assets by nonresidents and purchases of foreign assets by residents are both procyclical and collapse during crises. We propose a dynamic model of endogenous default that can account for these facts. The government faces a trade-off between the benefits of keeping reserves as a buffer against rollover risk and the cost of having larger gross debt positions. Long-duration bonds, the countercyclical default premium, and sudden stops are important for the quantitative success of the model.

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1 Introduction

The global financial crisis has brought cross-border capital flows to the center of policy debates. Many of these discussions have focused on the accumulation of international reserves in emerging markets. A widespread view is that international reserves are a valuable war chest against turbulence in financial markets (Feldstein, 1999; IMF, 2011).[1] Others have argued that reserves impose large financial costs and that the reserve buildup has reached excessive levels (Rodrik, 2006). Despite these extensive academic and policy debates, a quantitative theory of reserves accumulation remains elusive.

In this paper we propose a quantitative framework of optimal reserve management that accounts for two key facts of international capital flows:

Fact 1: Rate of Return Dominance. Governments hold large amounts of international reserves, for which they obtain a return lower than their borrowing cost.[2] The joint accumulation of international reserves and debt is illustrated in Figure 1. This figure plots the levels of debt and international reserves for a sample of emerging markets during the periods of 1993-2000 and 2001-2010. The figure shows that the last decade has seen a significant increase in the stock of reserves. Emerging economies' high borrowing costs are reflected in the EMBI plus sovereign spread index that averaged 4.5 percent between 2000 and 2012.

Fact 2: Gross Capital Flows Dynamics. Purchases of domestic assets by non-residents and purchases of foreign assets by residents are both procyclical and collapse during crises. These empirical regularities are documented in recent work by Broner et al. (2012), who also discuss how these facts are difficult to reconcile with the predictions of existing models (see also Forbes and Warnock, 2011).

We use a model of sovereign defaultable debt à la Eaton and Gersovitz (1981) augmented with reserves as our theoretical laboratory. We consider a benevolent government that borrows by issuing long-duration bonds (i.e., non-contingent bonds with geometrically decaying

[1] Frankel and Saravelos (2010), Dominguez et al. (2012) and Gourinchas and Obstfeld (2011) found that economies that had more reserves before the global financial crisis had milder contractions in economic activity during the crisis.

[2] IMF (2001) defines reserves as *"official public sector foreign assets that are readily available to and controlled by monetary authorities for direct financing of payments imbalances, for indirectly regulating the magnitudes of such imbalances,... and/or for other purposes."*

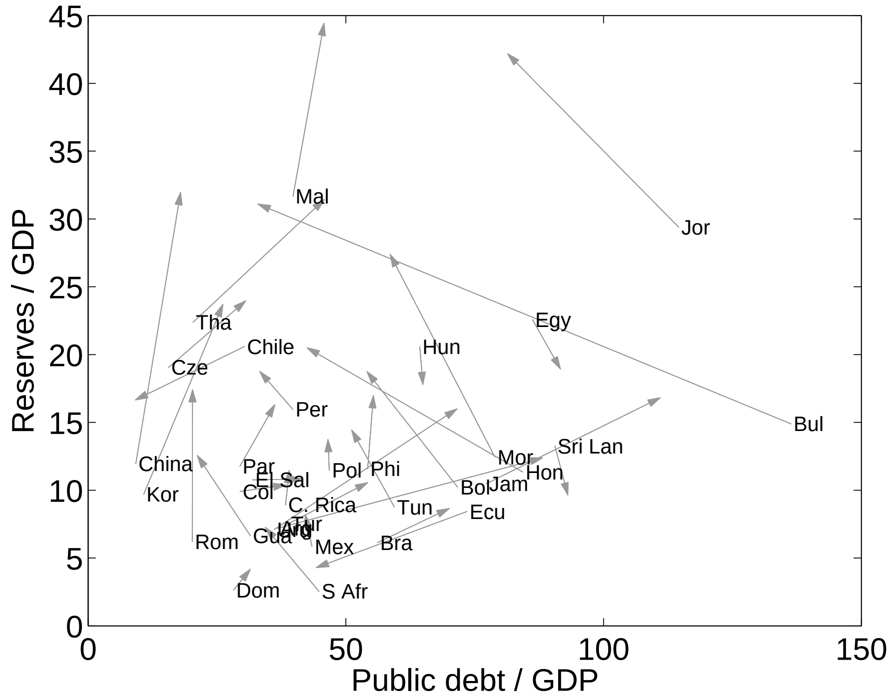

Figure 1: Evolution of international reserves (minus gold) and public debt. The beginning of each arrow corresponds to the average debt and reserves over the period of 1993-2000. The end of each arrow corresponds to the average debt and reserves over the period of 2001-2010. The sample of countries consists of middle-income countries that are not major oil exporters. Source: IMF database.

coupons) and saves by investing in a risk-free asset — reserves. A government that defaults faces an output cost and is temporarily prevented from issuing new debt but can change its reserve holdings. To capture disturbances in financial markets that are independent of the borrowing economy's fundamentals, we assume that the economy may be hit with a sudden-stop shock. During a sudden stop, the government cannot issue new debt. Sovereign bonds are priced by risk-neutral foreign investors who operate in competitive markets. Hence, in equilibrium, bond spreads reflect how both debt and reserves affect future incentives to repay.

We calibrate the model using Mexico as a reference, matching targets for the levels of debt and sovereign spread, the spread volatility, and the frequency of sudden stops. In simulations of our model, the government holds reserves with a return lower than its borrowing cost (Fact 1). The average reserve holding is equivalent to 2/3 of the average short-term debt obligations that mature within a year. This is not far from the "Greenspan-Guidotti rule" that prescribes full short-term debt coverage. Reserve holdings in the simulations are equivalent to 1/3 of the average holdings in Mexico between 1994 and 2011. Additional experiments in which reserves lower the arrival probability of a sudden stop can account for the entire reserve holdings in Mexico.

Why does the government hold international reserves with a return lower than its borrowing cost instead of allocating reserves to pay down its debt? When a government makes financial decisions, it considers not only the current borrowing cost but also future borrowing costs. In our setup, reserves provide insurance against future increases in the borrowing cost. By simultaneously issuing long-duration bonds and buying reserves the government accumulates resources that it can use in future periods with a high borrowing cost. The downside of buying reserves is that they provide a return lower than the borrowing cost. However, if there is a significant probability that the borrowing cost will increase in the future, the government may be willing to pay the financial cost of reserve accumulation.

Long-duration bonds are key for hedging rollover risk. Issuing one-period debt to finance reserves accumulation only increases the government's next-period net-asset position if the government defaults. That is, with one-period debt, issuing debt and accumulating reserves only allows the government to transfer resources to future periods in which it defaults. In contrast, with long-duration debt, issuing debt and accumulating reserves also allows the government to transfer resources to future periods in which the borrowing cost is high but there is no default. With our benchmark calibration and with one-period debt, the government does not choose significant reserve holdings (this is in line with the findings presented by Alfaro and Kanczuk, 2009).

Another key result in our model simulations is that purchases of domestic assets by nonresidents (i.e., government debt) and purchases of foreign assets by residents (i.e. international reserves) are both procyclical and collapse during crises (Fact 2).[3] The key to accounting for Fact 2 is the countercyclical nature of default risk. Consistent with the data, the model produces a lower sovereign spread in good times, reflecting the lower incentives

[3]Broner et al. (2012) show that in emerging economies changes in reserves represent about half of purchases of foreign assets by domestic agents and contract significantly during crisis episodes. In addition, they show that debt inflows play a primary role in accounting for changes in non-resident purchases of domestic assets over the business cycle and during crises. Dominguez et al. (2012) document the procyclical behavior of reserves around the global financial crisis.

to default.[4] The lower default risk provides an incentive for the government to borrow more and buy more reserves in good times. Moreover, sudden stops lead the government to cut down on borrowing and use reserves to smooth out consumption. This results in a collapse of both inflows and outflows during sudden stops.

1.1 Related Literature

We build on the quantitative sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008). With the notable exception of Alfaro and Kanczuk (2009), studies in this literature do not allow for the joint accumulation of assets and liabilities. Alfaro and Kanczuk (2009) show that while reserve accumulation is a theoretical possibility with default risk, the government's optimal policy does not feature simultaneous reserve accumulation and debt issuance for plausible parameterizations. The stark difference between our results and theirs arises because their analysis only allows for one-period debt. As we show, it is the combination of long-duration bonds and reserves that allow the government to hedge against future increases in the borrowing cost. Our modelling of long-duration bonds follows Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), and Hatchondo and Martinez (2009) who study the implications of long-duration bonds on spread dynamics.

Several other studies analyze hedging against rollover risk. Jeanne and Ranciere (2011) develop a model where the government can issue insurance contracts that pay off during a sudden stop. They analytically derive the demand for these contracts and show that this demand could be significant, depending on the probability and the size of the sudden stop. Caballero and Panageas (2008) show that there would be substantial welfare gains from having access to financial instruments that provide insurance against both the occurrence of sudden stop and changes in the sudden-stop probability. In contrast, we choose to focus on a more empirically relevant case in which reserves payoffs are not contingent on the realization of the sudden-stop shock. Durdu, Mendoza, and Terrones (2009) present a dynamic precautionary savings model where a higher net foreign asset position causes an occasionally

[4] As reported by Neumeyer and Perri (2005) and Uribe and Yue (2006), in emerging economies, government bond yields rise during economic contractions and are reduced during economic expansions (the correlation between GDP and sovereign bond spreads range between 0 and -0.8). Moreover, government bond yields are about 50 percent more volatile in emerging economies than in developed economies.

binding credit constraint to become less frequently binding. Aizenman and Lee (2007) study a Diamond-Dybvig type model where reserves serve as liquidity to reduce output costs during sudden stops. Hur and Kondo (2011) develop a model where gradual learning about the sudden stop process can account for the surge in reserves over the last decade. Overall, our paper contributes to this literature by providing a unified framework to study reserves, debt, and sovereign spreads.

Our paper is also related to Angeletos (2002) and Buera and Nicolini (2004). They show examples where issuing non-defaultable long-term debt and accumulating short-term assets can replicate complete market allocations. In their closed-economy models, changes in the interest rate arise as a result of fluctuations in the marginal rate of substitution of domestic consumers. In contrast, in our model, fluctuations in the interest rate reflect changes in the default premium that foreign investors demand in order to be compensated for the possibility of government default. Moreover, default risk introduces an additional cost of accumulating reserves, which we see as particularly relevant for emerging markets.

Our work is also related to the literature on household debt. In particular, Telyukova (2011) and Telyukova and Wright (2008) address the "credit card puzzle" (i.e., the fact that households pay high interest rates on credit cards while earning low rates on bank accounts). Both the "credit card puzzle" and the "reserve accumulation puzzle" are examples of the "rate of return dominance puzzle". However, there are important differences in the environments. In their models, the demand for assets arises because credit cards cannot be used to buy some goods, whereas rollover risk and long-duration bonds are the key elements behind our theory.

Another strand of the literature focuses on the undervaluation of the exchange rate as a motive for reserve accumulation (see Dooley et al., 2003, and Benigno and Fornaro, 2012). We analyze instead the demand of international reserves for precautionary reasons, excluding other reasons for reserve accumulation (that could be relevant in accounting for the data). Our focus on the precautionary role is consistent with the formal definition of reserves that highlights the features of availability and liquidity to manage potential balance of payment crises. Moreover, building a buffer for liquidity needs is the most frequently cited reason for reserve accumulation in the IMF Survey of Reserve Managers (80 percent of respondents; IMF, 2011).[5]

The rest of the article proceeds as follows. Section 2 provides an example that highlights the key mechanism behind reserve accumulation. The model, its calibration, and results are presented in Sections 3, 4, and 5, respectively. Section 6 concludes.

2 A Three-Period Example

We first present a three-period model that allows us to illustrate the importance of rollover risk and long-duration bonds in accounting for the joint accumulation of debt and reserves. To simplify the analysis we consider only exogenous rollover risk, and abstract from endogenous rollover risk due to the possibility of default.[6]

2.1 Environment

The economy lasts for three periods $t = 0, 1, 2$. The government receives a deterministic sequence of endowments given by $y_0 = 0$, $y_1 > 0$, and $y_2 > 0$. For simplicity, the government only values consumption in period 1. The government maximizes $\mathbb{E}[u(c_1)]$, where $\mathbb{E}$ denotes the expectation operator, c_1 represents period F1 consumption, and the utility function u is increasing and concave.

The government is subject to a sudden-stop shock in period 1. When a sudden stop occurs, the government is unable to borrow. A sudden stop occurs with probability $\pi \in [0, 1]$. In the first period, the government can accumulate reserves. The interest rate the government earns on its reserves is denoted by $r_a \geq 0$ and the interest rate it pays when it borrows is denoted by $r_b \geq r_a$.

A bond issued in period 0 promises to pay one unit of the good in period 1 and $(1-\delta)$ units in period 2. Thus, the price of a bond issued in period 0 is given by $q_0 = (1 + r_b)^{-1} + (1-\delta)(1 + r_b)^{-2}$. Note that if $\delta = 1$, the government issues one-period bonds in period 0. If

[5] Empirical analysis by Aizenman and Lee (2007) and Calvo et al. (2012) also support the precautionary role in the demand of reserves.

[6] We can derive similar results with default risk, but this makes the analysis more complex. We study default risk in the model presented in the next section.

$\delta < 1$, we say that the government issues long-duration bonds in period 0. We assume that $\delta > 0$. That is, we assume that for debt issued in period 0, period 2 payments cannot be larger than period 1 payments. This assumption allows us to rule out reserve accumulation in period 1 and, thus, simplifies the exposition. A bond issued in period 1 promises to pay one unit of the good in period 2. Let b_t denote the number of bonds issued by the government in period t and a denote the amount of reserves the government accumulates in period 0. Thus, the budget constraints are:

$$a \leq y_0 + q_0 b_1,$$

$$c_1(0) \leq y_1 - b_1 + a(1 + r_a) + b_2(1 + r_b)^{-1},$$

$$c_1(1) \leq y_1 - b_1 + a(1 + r_a),$$

$$b_2 \leq y_2 - (1 - \delta) b_1,$$

where $c_1(0)$ denotes the government's period 1 consumption when it is not facing a sudden stop and $c_1(1)$ denotes period 1 consumption during a sudden stop.

2.2 Results

Without rollover risk, the government would simply consume $c_1 = y_1 + y_2/(1 + r)$. However, a sudden stop may prevent the government from borrowing in period 1. The next proposition describes how the government can use reserves and debt to smooth consumption between both period 1 states (with and without a sudden stop).

Proposition 1 (Optimal Reserve Holdings)

1. *If there is no rollover risk ($\pi = 0$) and $r_a = r_b$, gross asset positions are undetermined. In particular, the optimal allocation can be attained without reserves ($a^\star = 0$).*

2. *If there is no rollover risk ($\pi = 0$) and $r_a < r_b$, optimal reserves are zero ($a^\star = 0$).*

3. *If the government can only issue one-period debt in period 0 ($\delta = 1$) and $r_a = r_b$, gross asset positions are undetermined. In particular, the optimal allocation can be attained without reserve accumulation ($a^\star = 0$).*

4. *If the government can only issue one-period debt in period 0 ($\delta = 1$) and $r_a < r_b$, optimal reserves are zero ($a^\star = 0$).*

5. *If*

$$\pi \left[q_0(1+r_a) - 1\right] u'(y_1) > (1-\pi) \left[\frac{1-\delta}{1+r_b} + 1 - q_0(1+r_a)\right] u'\left(y_1 + y_2(1+r_b)^{-1}\right), \quad (1)$$

then the government accumulates reserves in period 0 ($a^\star > 0$). Moreover, if $r_a = r_b$, the government perfectly smooths consumption.

Proof: *See Appendix.*

Proposition 1 states that there is a fundamental role for reserves only in the presence of both rollover risk and long-duration bonds.[7] Without rollover risk, there is no need for reserve accumulation: the government can always transfer resources from period 2 to period 1 directly. If there is a sudden stop in period 1, the government cannot borrow in that period. Therefore, the government may benefit from issuing long-duration bonds to transfer resources from period 2 to period 0, and then transfer period 2 resources from period 0 to period 1 using reserves. With one-period debt, the government cannot improve its period 1 net asset position by issuing debt and accumulating reserves in period 0.

With rollover risk and long-duration bonds, the government accumulates reserves if the benefits from transferring resources from period 0 to period 1 using reserves are high enough to compensate for the financial cost of financing reserve accumulation with debt issuances. Condition (1) is sufficient for the optimality of reserve accumulation. Borrowing in period 0 and accumulating reserves instead of borrowing in period 1 allows the government to transfer resources to the period 1 sudden-stop state. The left-hand side of condition (1) represents

[7] In order to highlight the role of rollover risk and long-duration bonds in accounting for reserve accumulation, this section abstracts from the role of reserves as a way to transfer resources to default states highlighted by Alfaro and Kanczuk (2009).

the expected marginal benefit of doing so. When issuing debt and accumulating reserves in period 0, the government also transfer resources to the period 1 state without a sudden stop. But this could have been done cheaper (if $r_a < r_b$) by borrowing in period 1. The right-hand side of condition (1) represents the expected marginal cost of transferring resources to the state without a sudden stop using reserves instead of borrowing in period 1.

The financial cost of transferring resources from period 0 to period 1 by issuing debt to finance reserve accumulation appears if $r_a < r_b$. Note that, with rollover risk and long-duration bonds, condition (1) holds when $r_b = r_a$ (the left-hand side of condition (1) is positive and the right-hand side is equal to zero).

Long-duration bonds and a high enough rollover risk (high enough π) are necessary for condition (1) to hold. With one-period debt ($\delta = 1$), the left-hand side of condition (1) is equal to zero and the right-hand side is positive. Furthermore, the government is willing to pay the financial cost of issuing debt for accumulating reserves if the probability of not being able to transfer resources directly from period 2 to period 1 (π) is high enough. Recall reserves are beneficial because they increase consumption in the state in which period 1 borrowing is not possible, which occurs with probability π. In particular, note that condition (1) is not satisfied with $\pi = 0$, and is satisfied with $\pi = 1$ (and long-duration bonds).

Summing up, this section illustrates how rollover risk could play a role in accounting for reserve accumulation, and how debt duration could be a key factor for determining the importance of this role. We next study a richer model that allows us to gauge the quantitative importance of rollover risk in accounting for reserve accumulation. In this model, an endogenous sovereign default premium implies that the return on reserves is lower than the interest rate the government pays for its debt, and rollover risk arises because of both changes in the default premium (that reflect changes in the economy's fundamentals) and sudden stops (unrelated to the economy's fundamentals).

3 Model

This section presents a dynamic small-open-economy model in which the government can issue non-state contingent defaultable debt and buy risk-free assets. The economy's endowment of the single tradable good is denoted by $y \in Y \subset \mathbb{R}_{++}$. This endowment follows a Markov process.

We consider a benevolent government that maximizes:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and c_t represents consumption of private agents. The utility function is strictly increasing and concave.

The timing of events within each period is as follows. First, the income and sudden-stop shocks (to be described below) are realized. After observing these shocks, the government chooses whether to default on its debt and makes its portfolio decision subject to constraints imposed by the sudden-stop shock and its default decision. Figure 2 summarizes the timing of these events.

The sudden-stop shock follows a Markov process so that a sudden stop starts with probability $\pi \in [0,1]$ and ends with probability $\psi^s \in [0,1]$. During a sudden stop, the government cannot issue new debt and suffers an income loss of $\phi^s\left(y\right)$. However, the government can buy back debt and change its reserve holdings while in a sudden stop.

The sudden-stop shock in our model captures dislocations to international credit markets that are exogenous to local conditions. Thus, for given domestic fundamentals, a sudden stop can trigger changes in sovereign spreads and default episodes. This is important for the empirical success of the model because of a vast empirical literature showing that extreme capital flow episodes are typically driven by global factors (see, for instance, Calvo et al.,

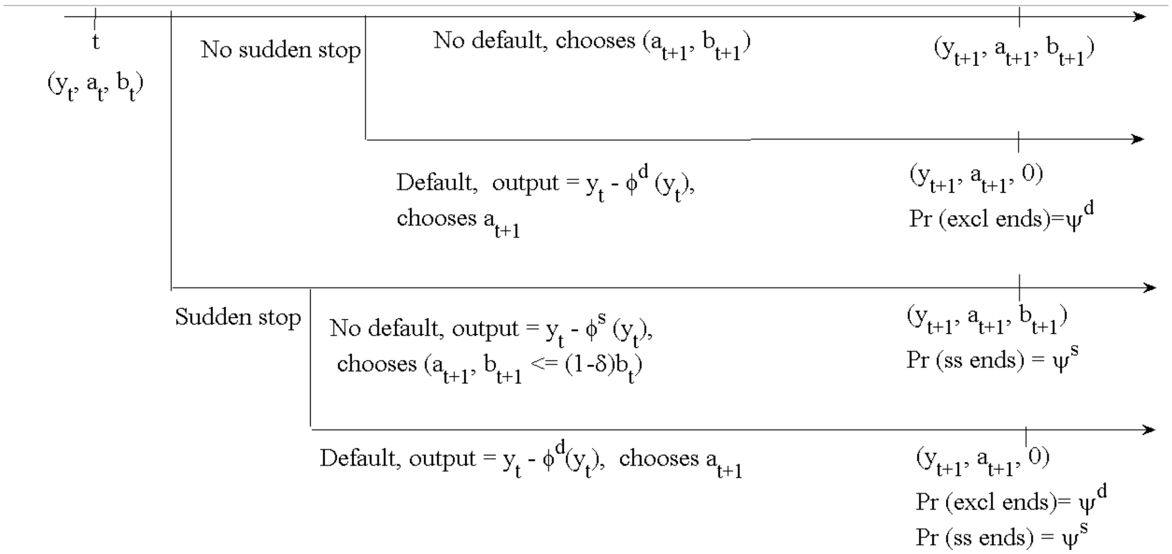

Figure 2: Sequence of events when the government is not in default. The government enters the period with debt b_t and reserves a_t. First, the income and sudden-stop shocks are realized. Second, the government chooses whether to default. Third, the government adjusts its debt and reserves positions. The government can always adjust reserve holdings and buy back debt. It can issue debt only if it did not default and is not in a sudden stop.

1993, Uribe and Yue, 2006 and Forbes and Warnock, 2011).[8] The loss of income triggered by a sudden stop is also consistent with empirical studies (e.g. Calvo et al., 1993) and can be rationalized by the adverse effects of these episodes on the economy, which are often associated with currency and banking crises with deep recessions.[9]

As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period t promises an infinite stream of coupons that decrease at a constant rate δ. In particular, a bond issued in period t promises to pay $(1-\delta)^{j-1}$ units of the tradable good in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented as follows:

$$b_{t+1} = (1-\delta)b_t + i_t,$$

[8]Changes in credit conditions triggered by "global factors" could also be modeled by shocks that affect the risk compensation demanded by international investors (see Borri and Verdelhan, 2009; Arellano and Bai, 2012; Lizarazo, 2011). Alternatively, one could model an increase in the probability of a self-fulfilling rollover crises à la Cole-Kehoe. In both cases the global factor amounts to an increase in the cost of issuing debt and resemble our sudden-stop shock. However, the analysis Chatterjee and Eyigungor (2012) suggests that the role of self-fulfilling crises may be limited once debt duration is assumed to display the levels observed in the data.

[9]Modelling currency or banking crises are beyond the scope of this paper.

where b_t is the number of coupons due at the beginning of period t, and i_t is the number of bonds issued in period t.

Let $a_t \geq 0$ denote the government's reserve holdings at the beginning of period t. The budget constraint conditional on having access to credit markets is represented as follows:

$$c_t = y_t - b_t + a_t + i_t q_t - \frac{a_{t+1}}{1+r},$$

where q_t is the price of the bond issued by the government, which in equilibrium will depend on exogenous shocks and the policy pair (b_{t+1}, a_{t+1}), and $1+r$ is the per period return on reserves.[10]

When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments and it is a standard assumption in the literature.[11] As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan lenders recover after a default) is zero.[12]

A default event triggers exclusion from credit markets for a stochastic number of periods. Income is given by $y - \phi^d(y)$ in every period in which the government is excluded from credit markets because of a default. Thus, the income level of an economy in default is independent of whether the economy is facing a sudden stop. This implies that the income loss triggered by a default is effectively lower for an economy facing a sudden stop (since the sudden-stop income would be $y - \phi^s(y)$ in case the government repays). This assumption is justified because the income losses during both defaults and sudden stops intend to capture local disturbances caused by the loss of access to international credit markets. This assumption also allows the model to capture that some but not all sudden stops trigger defaults. The

[10]Because the return per period is fixed, modelling long-duration reserves would deliver identical results. We do not allow a_t to take negative values. Because markets are incomplete, it is possible that the government may want to issue one-period bonds and buy reserves, but computational reasons prevent us from introducing one-period debt as a third endogenous state variable.

[11]Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a debt payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

[12]Yue (2010) and Benjamin and Wright (2008) present models with endogenous recovery rates.

government does not have access to debt markets in the default period and then regains access to debt markets with constant probability $\psi^d \in [0, 1]$.

Foreign investors are risk-neutral and discount future payoffs at the rate r (which is also the rate of return on reserves). Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero-expected-profit condition.

The government cannot commit to future (default, borrowing, and saving) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (default, borrowing, and saving) strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period the government's equilibrium default, borrowing, and saving strategies depend only on payoff-relevant state variables.

3.1 Recursive Formulation

We now describe the recursive formulation of the government's optimization problem. The sudden-stop shock is denoted by s, with $s = 1$ ($s = 0$) indicating that the economy is (is not) in a sudden-stop.

Let V denote the value function of a government that is not currently in default. For any bond price function q, the function V satisfies the following functional equation:

$$V(b, a, y, s) = \max \left\{ V^R(b, a, y, s), V^D(a, y, s) \right\}, \tag{2}$$

where the government's value of repaying is given by

$$V^R(b, a, y, s) = \max_{a' \geq 0, b', c} \left\{ u\left(c\right) + \beta \mathbb{E}_{(y', s')|(y,s)} V(b', a', y', s') \right\}, \tag{3}$$

subject to

$$c = y - s\phi^s\left(y\right) - b + a + q(b', a', y, s)\left[b' - (1-\delta)b\right] - \frac{a'}{1+r},$$

and if $s = 1$, $b' - (1-\delta)b \leq 0$.

The value of defaulting is given by:

$$V^D(a,y,s) = \max_{a'\geq 0,c} u\left(c\right) + \beta \mathbb{E}_{(y',s')|(y,s)}\left[(1-\psi^d)V^D(a',y',s') + \psi^d V(0,a',y',s')\right], \quad (4)$$

subject to

$$c = y - \phi^d(y) + a - \frac{a'}{1+r}.$$

The solution to the government's problem yields decision rules for default $\hat{d}(b,a,y,s)$, debt $\hat{b}(b,a,y,s)$, reserves $\hat{a}^i(b,a,y,s)$, and consumption $\hat{c}^i(b,a,y,s)$ for $i = R, D$. The superindex R (D) indicates that the government is (is not) in default. The default rule $\hat{d}(\cdot)$ is equal to 1 if the government defaults, and is equal to 0 otherwise.

In a rational expectations equilibrium (defined below), investors use these decision rules to price debt contracts. Because investors are risk neutral, the bond-price function solves the following functional equation:

$$q(b',a',y,s)(1+r) = \mathbb{E}_{(y',s')|(y,s)}(1-\hat{d}(b',a',y',s'))(1+(1-\delta)q(b'',a'',y',s')), \qquad (5)$$

where

$$\begin{aligned} b'' &= \hat{b}(b',a',y',s') \\ a'' &= \hat{a}^R(b',a',y',s'). \end{aligned}$$

Condition (5) indicates that in equilibrium, an investor has to be indifferent between selling a government bond today and investing in a risk-free asset, and keeping the bond and selling it next period. If the investor keeps the bond and the government does not default in the next period, he first receives a one unit coupon payment and then sells the bonds at market price, which is equal to $(1-\delta)$ times the price of a bond issued next period.

Notice that while investors receive on expectation the risk free rate, the cost of borrowing is higher than the risk free rate for the government since it suffers output costs and exclusion after defaulting. Therefore, the costs of defaulting leads the government to avoid paying high spreads on borrowing.

3.2 Recursive Equilibrium

A *Markov Perfect Equilibrium* is characterized by

1. a set of value functions V, V^R and V^D,
2. rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\left\{\hat{a}^R, \hat{a}^D\right\}$, and consumption $\left\{\hat{c}^R, \hat{c}^D\right\}$,
3. and a bond price function q,

such that:

i. given a bond price function q; the policy functions $\hat{d}$, $\hat{b}$, $\hat{a}^R$, $\hat{c}^R$, $\hat{a}^D$, $\hat{c}^D$, and the value functions V, V^R, V^D solve the Bellman equations (2), (3), and (4).

ii. given policy rules $\left\{\hat{d}, \hat{b}, \hat{a}^R\right\}$, the bond price function q satisfies condition (5).

4 Calibration

The utility function displays a constant coefficient of relative risk aversion, i.e.,

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \text{ with} \gamma \neq 1.$$

The endowment process follows:

$$\log(y_t) = \left(1-\rho\right)\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

Following Arellano (2008), we assume an asymmetric cost of default $\phi^d\left(y\right)$, so that it is proportionally more costly to default in good times. This is a property of the endogenous default cost in Mendoza and Yue (2012) and, as shown by Chatterjee and Eyigungor (2012), allows the equilibrium default model to match the behavior of the spread in the data. In particular, we assume a quadratic loss function for income during a default episode $\phi^d\left(y\right) = d_0 y + d_1 y^2$, as in Chatterjee and Eyigungor (2012).

Table 1: Parameter Values.

Risk aversion	γ	2
Risk-free rate	r	1%
Income autocorrelation coefficient	ρ	0.94
Standard deviation of innovations	σ_ϵ	1.5%
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$
Debt duration	δ	0.033
Probability of reentry after default	ψ^d	0.083
Probability of entering a SS	π	0.025
Probability of reentry after SS	ψ^s	0.25
Discount factor	β	0.9745
Income cost of defaulting	d_0	-1.01683
Income cost of defaulting	d_1	1.18961
Income cost of sudden stops	λ	0.5

We also assume that the income loss during a sudden stop is a fraction of the income loss after a default: $\phi^s(y) = \lambda\phi^d(y)$. With this assumption, we have to pin down only one more parameter value in order to determine the cost of sudden stops. Since both sovereign defaults and sudden stops are associated with disruptions in the availability of private credit, it is natural to assume that the cost of these events is higher in good times when investment financed by credit is more productive.

Table 1 presents the benchmark values given to all parameters in the model. A period in the model refers to a quarter. The coefficient of relative risk aversion is set equal to 2, and the risk-free interest rate is set equal to 1 percent. These are standard values in quantitative business cycle and sovereign default studies.

We use Mexico as a reference for choosing the parameters that governs the endowment process, the level and duration of debt, and the mean and standard deviation of spread. This choice is guided by the fact that business cycles in Mexico display the same properties that are observed in small open developing economies (see Aguiar and Gopinath, 2007; Neumeyer and

Perri, 2005; and Uribe and Yue, 2006). Unless we explain otherwise, we compare simulation results with data from Mexico from the first quarter of 1980 to the fourth quarter of 2011. Therefore, the parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Mexico during that period.

We set $\delta = 3.3\%$. With this value, bonds have an average duration of 5 years in the simulations, which is roughly the average debt duration observed in Mexico according to Cruces et al. (2002).[13] As in Mendoza and Yue (2012), we assume an average duration of sovereign default events of three years ($\psi^d = 0.083$), in line with the duration estimated in Dias and Richmond (2007).

As in Jeanne and Ranciere (2011), we define a sudden stop in the data as an annual fall in net capital inflows of more than 5 percent of GDP. Using this definition, the same sample of countries considered by Jeanne and Ranciere (2011), and the IMF's International Financial Statistics annual data from 1970 to 2011, we find one sudden stop every 10 years (as they do). Thus, we set $\pi = 0.025$.

We set ψ^s to match the duration of sudden stops in the data. We estimate the duration of sudden stops using quarterly data from 1970 to 2011. We define ca_t as the ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters.[14] We identify quarters in which $ca_{t+4} < ca_t - 0.05$. For such quarters, a sudden-stop episode begins the first quarter between t and $t+4$ in which ca_t falls. This sudden stop ends the first period in which ca_t increases. Following this methodology, we find a mean duration of a sudden stop of 1.12 years, and set accordingly $\psi^s = 0.25$.[15] These parameter values are similar to the ones we would have obtained using only data for Mexico, which has experienced three sudden stops since 1979 with an average duration of 1.4 years. The appendix presents the list of sudden stops we identify and the evolution of net capital inflows for each country in our sample.

[13] We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+r^*}{\delta+r^*}$, where r^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of foreign sovereign debt in emerging economies—in 2000—of 4.77 years, with a standard deviation of 1.52.

[14] Net capital inflows are measured as the deficit in the current account minus the accumulation of reserves and related items.

[15] This estimation is close to the results obtained by Forbes and Warnock (2011) who use gross capital inflows. Jeanne and Ranciere (2011) do not report the duration of sudden stops.

We need to calibrate the value of four other parameters: the discount factor β, the parameters of the income cost of defaulting d_0 and d_1, and the parameter determining the relative income cost of a sudden stop compared with a default λ. Chatterjee and Eyigungor (2012) calibrate the first three parameter values to target the mean and standard deviation of the sovereign spread, and the mean debt level. We follow their approach but incorporate as a fourth target the average accumulated income cost of a sudden stop.[16] We target an average accumulated income cost of a sudden stop of 14 percent of annual income, which is at the lower end of the range of estimated values (see Becker and Mauro, 2006; Hutchison and Noy, 2006; and Jeanne and Ranciere, 2011). Section 5.5 present results for different values of λ.

In order to compute the sovereign spread implicit in a bond price, we first compute the yield i an investor would earn if it holds the bond to maturity (forever) and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}}{(1+i)^j}.$$

The sovereign spread is the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r}\right)^4 - 1.$$

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b_t(1+r)(\delta+r)^{-1}$.

4.1 Computation

We solve for the equilibrium of the finite-horizon version of our economy as in Hatchondo et al. (2010). That is, the approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough that the maximum deviation between the value and bond price functions in the first and second period is no larger than 10^{-6}. The recursive problem is solved using value function iteration. We solve the optimal portfolio allocation in each state by searching over a grid

[16] The time series for the spread is taken from Neumeyer and Perri (2005) for the period 1994-2001 and from the EMBI+ index for the period 2002-2011. The data for public debt is taken from Cowan et al. (2006).

of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions V^D and V^R and the function that indicates the equilibrium bond price function conditional on repayment $q\left(\hat{b}(\cdot), \hat{a}^R(\cdot), \cdot, \cdot\right)$ are approximated using linear interpolation over y and cubic spline interpolation over debt and reserves positions. We use 20 grid points for reserves, 20 grid points for debt, and 25 grid points for income realizations. Expectations are calculated using 50 quadrature points for the income shocks.

5 Quantitative Results

We start the quantitative analysis by showing that the model simulations match the calibration targets and other non-targeted moments in the data. We also show that the model generates joint debt and reserve accumulation together with a significant default premium (Fact 1), and gross capital flows dynamics consistent with the ones in the data (Fact 2). We then show that long-duration bonds, sudden stops, and the endogenous and countercyclical default risk are important ingredients for the quantitative success of the model. Before concluding, we show that the model generates more reserve accumulation when we allow reserves to lower the probability of sudden stops.

5.1 Model Simulations

Table 2 reports moments in the data and in the model simulations. The table shows that the simulations match the calibration targets reasonably well. The model also does a good job in mimicking other non-targeted moments such as the ratio of the volatilities of consumption and income. Overall, Table 2 shows that the model can account for distinctive features of business cycles in Mexico and other emerging economies, as documented by Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006). Previous studies show that the sovereign default model without reserve accumulation can account for these features of the data. We show that this is still the case when we extend the baseline model to allow for the empirically relevant case in which indebted governments can hold reserves and choose to do so.

Table 2: Business Cycle Statistics

Targeted moments	Model	Data
Mean Debt-to-GDP	42	43
Mean r_s	3.4	3.4
$\sigma(r_s)$	1.3	1.5
Mean sudden stop income cost (% annualized)	14	14
Non-Targeted moments		
$\sigma(c)/\sigma(y)$	1.3	1.2
$\sigma(tb)$	1.3	1.4
$\rho(tb, y)$	-0.5	-0.7
$\rho(c, y)$	0.96	0.93
$\rho(r_s, y)$	-0.4	-0.5
$\rho(r_s, tb)$	0.3	0.6
Mean Reserves-to-GDP	2.5	7.0
$\rho(\Delta a, y)$	0.4	0.4
$\rho(\Delta b, y)$	0.5	0.95
$\rho(a, r_s)$	-0.4	-0.2

Note: The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Changes in debt and reserves levels are denoted by Δa and Δb, respectively. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of $1,600$. Moments for the simulations correspond to the mean value of each moment in 250 simulation samples, with each sample including 120 periods (30 years) without a default episode. Default episodes are excluded to improve comparability with the data; our samples start at least five years after a default. Consumption and income are expressed in logs. Due to data availability, debt statistics are at annual frequency.

5.2 Reserve Accumulation

As indicated in Table 2, in the model simulations an indebted government paying a significant spread chooses to hold a significant amount of international reserves (Fact 1). Average reserve holdings in the simulations represent 66 percent of short-term debt (i.e., debt maturing within a year). Interestingly, this is quite close to the "Greenspan-Guidotti rule" often targeted by policymakers, which prescribes full short-term debt coverage.

Section 2 shows that the accumulation of reserves financed by debt issuances to hedge rollover risk is a theoretical possibility with long-duration bonds. The simulations of our model indicate that this is not only a theoretical possibility but it is also quantitatively important.

Average reserve holdings in the simulation are about 1/3 of average holdings in Mexico between 1994 and 2011. Reserve holdings in the simulations are close to holdings in Mexico in the second half of the 1990s. There has been a fast growth of reserve accumulation in Mexico since the early 2000s. Section 5.7 extends our baseline model by allowing reserves to lower the sudden-stop probability and shows that simulations of the extended model can account for up to 100 percent of the average reserve holdings in Mexico. Of course, as mentioned in the introduction, motives for reserve accumulation other than rollover risk that are not discussed in this paper could also be important to account for a fraction of reserve holdings.

5.3 Capital Flows Over the Cycle and during Sudden Stops

Table 2 shows that purchases of domestic assets by non-residents (changes in debt levels) and purchases of foreign assets by residents (changes in reserve levels) are both procyclical in the simulations. This is consistent with recent evidence presented in Broner et al. (2012) (Fact 2).

The procyclicality of reserve accumulation and debt issuances is a consequence of the effect of income on the availability of credit. Figure 3 illustrates how borrowing conditions deteriorate when income falls. As illustrated in the three-period model presented in Section 2, when borrowing conditions are good, a fraction of debt issuances are allocated to accumulate reserves. When borrowing conditions deteriorate, the government borrows less and sells reserves. Thus, given the positive effect of income on borrowing conditions, changes in debt and reserves levels are both procyclical.

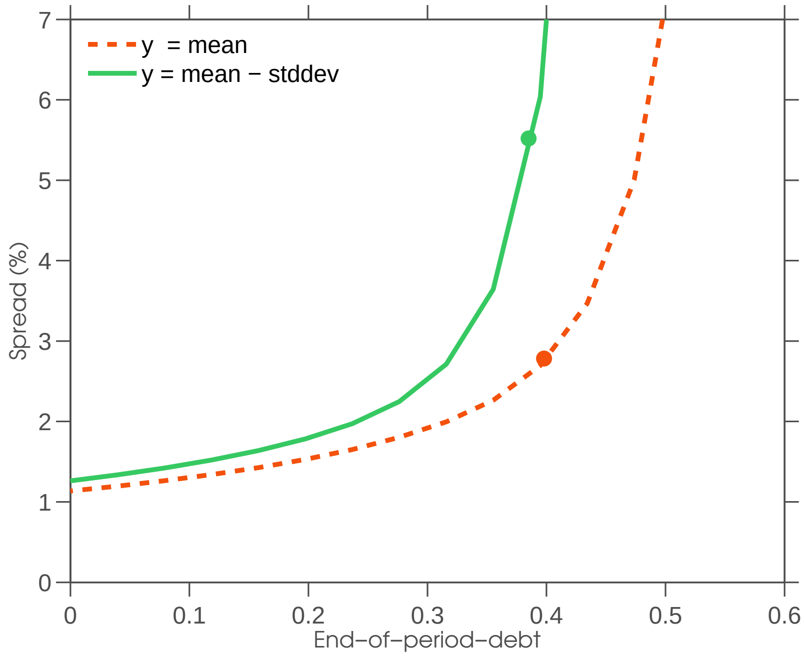

Figure 3: Menus of spread and end-of-period debt levels available to a government that is not facing a sudden stop and chooses a level of reserves equal to the mean in the simulations, i.e., $r^s(b', \bar{a}, y, 0)$, where $\bar{x}$ denotes the sample mean value of variable x. The solid dots present the spread and debt levels chosen by the government when it starts the period with debt and reserves levels equal to the mean levels observed in the simulations (for which it does not default).

To illustrate the mechanism, Figure 4 presents the policy functions for the changes in debt (left panel) and reserves (right panel) as a function of current income. The policies correspond to the case in which, at the beginning of the period, the government is not in default and holds an initial level of debt and reserves equal to the mean levels observed in the simulations. The straight (broken) line indicates the demand for reserves when the economy is (is not) in a sudden stop. In all the figures, we express debt and reserves normalized by annualized income so that all expressions can be understood as fractions of GDP.

The vertical dotted lines correspond to the default threshold that separate repayment region and default region. When the government is not hit by a sudden-stop shock, the government repays the debt for shocks to income higher than -5.2 percent and defaults otherwise. The fact that the default region is decreasing in the level of income is standard in the literature and reflects the fact that repayment is more costly for low income levels and that the punishment is also lower. Moreover, the default threshold when the economy is in a sudden stop is strictly higher, i.e., the government is more likely to default if it faces a sudden stop. This reflects the fact that default entails less of a punishment during a sudden stop as the government already faces restrictions to credit market and income losses due to the sudden stop.

When the economy is not in a sudden stop and the economy is in the repayment region, both borrowing and reserves are increasing with respect to income. In particular, notice that the government increases its reserve holdings when income is above trend in line with the permanent income hypothesis. The permanent income hypothesis would also imply that borrowing should be *decreasing* with respect to income. However, because income is persistent, a high current income improves borrowing opportunities (Figure 3) and leads to more borrowing. Moreover, once the government is allowed to accumulate reserves, there is an extra motive for borrowing more when income is higher: financing reserve accumulation to hedge rollover risk. In the default region, the government sells reserves (and debt levels are equal to zero).

Figure 4 also shows that a sudden stop causes a reduction in borrowing and reserve accumulation. During a sudden stop, the government sells reserves and makes coupon payments. As illustrated by the flat policy function for borrowing, the government is constrained and does not repurchase debt. Notice that changes in reserves are slightly decreasing in the level of income, reflecting the fact that the government expects a low future interest rate when it regains access to credit markets.

Figure 5 presents an event analysis of capital flows around sudden stops for the model and the data. To construct the event analysis in the model, we run a long time-series simulation and identify all the periods that are hit by a sudden stop. Then, we construct windows of five years around those episodes. The simulations show that the model predicts a collapse in both inflows and outflows during sudden stops. This is consistent with the behavior of flows around crises documented by Broner et al. (2012) (Fact 2) and reproduced in Figure 5.

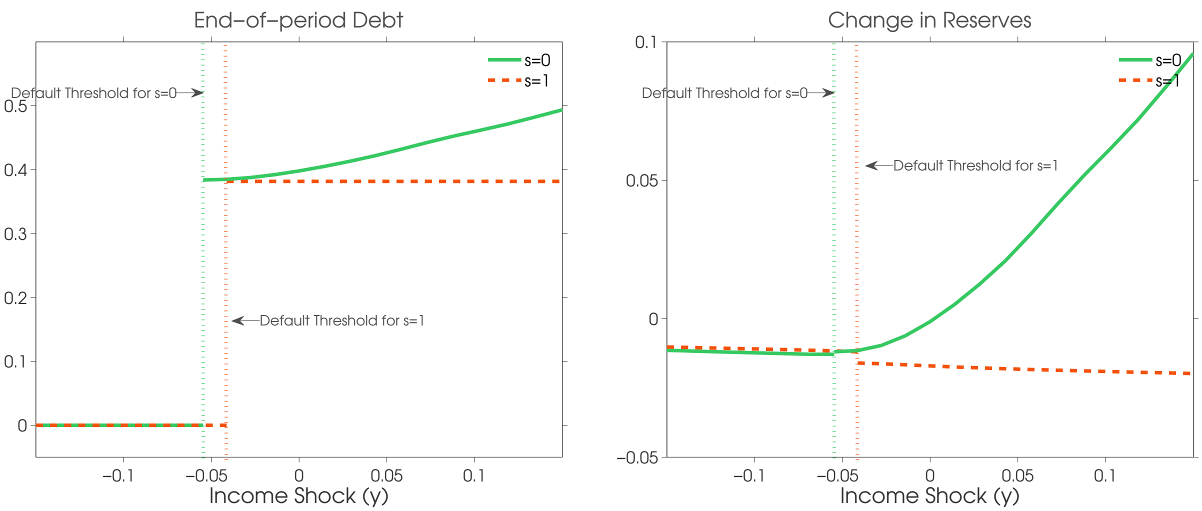

Figure 4: Equilibrium borrowing and reserve accumulation policies for a government that starts the period with levels of reserves and debt equal to the mean levels in the simulations. Debt levels and variations in reserves are presented as a percentage of the mean annualized income (4). That is, the left panel plots $\hat{b}\left(\bar{b}, \bar{a}, y, s\right)/4$ and the right panel plots $\left(\hat{a}\left(\bar{b}, \bar{a}, y, s\right) - \bar{a}\right)/4$.

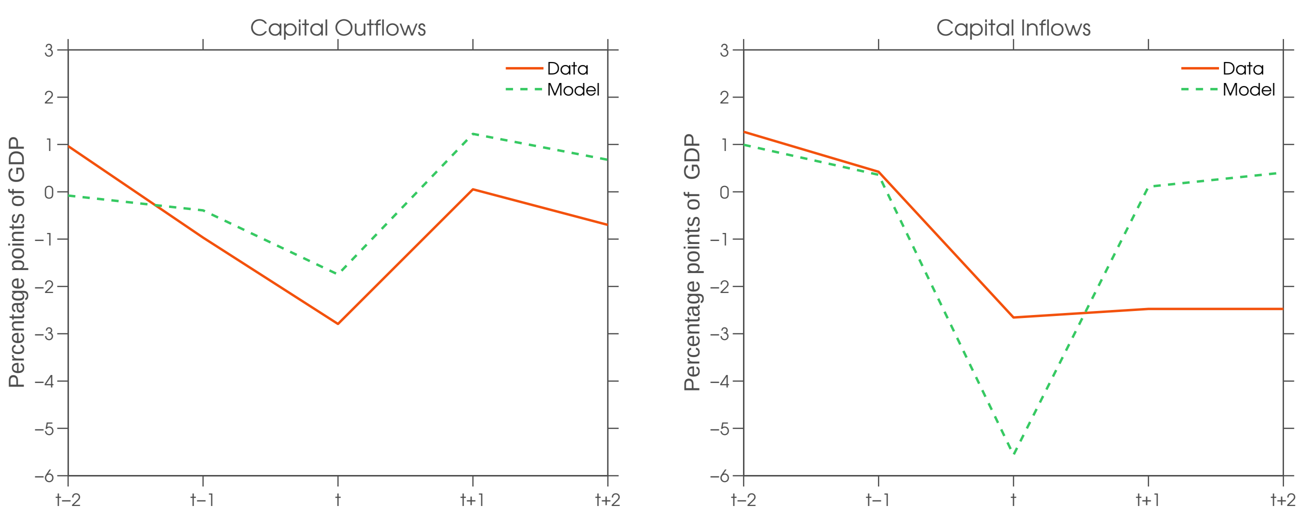

Figure 5: Average gross capital flows as a percentage of trend GDP in the simulations and in the data. The crisis year is denoted by t. In the simulations, we consider only sudden-stop episodes that do not trigger a default (in default episodes changes in the debt level do not correspond to changes in capital inflows). The behavior of flows in the data is the one presented by Broner et al (2012).

5.4 Role of Long-Duration Bonds

We next show how assuming that the government can issue long-duration bonds plays a critical role in allowing the model to simultaneously generate significant levels of debt and reserves. Table 3 presents simulation results for our benchmark calibration, but assuming one-period bonds ($\delta = 1$) instead of long-duration bonds.[17] The table shows that the mean debt-to-income ratio in the simulations drops to 3 percent of annual income, compared with 42 percent in the simulations with long-duration bonds, and reserves drop to 0.01 percent compared with 2.5 percent in the benchmark.

There are three fundamental reasons that explain why the presence of long-duration bonds influences incentives for reserve accumulation in our model. First, long-duration bonds are essential for reserves to play a role in hedging against rollover risk. As shown in Section 2, when the government only issues one-period bonds, reserves play no role in insuring against future increases in the borrowing costs.

Second, with one-period debt, the government chooses low debt levels for which default risk is negligible. Therefore, the expansion in the consumption space spanned by portfolios

Table 3: Business Cycle Statistics with One-Period Bonds.

	Benchmark	One-period bonds
Mean debt-to-GDP	42	3
Mean reserves-to-GDP	2.5	0.01
Mean r_s	3.4	0.0
$\sigma(r_s)$	1.3	0.0
$\sigma(tb)$	1.3	2.0

[17] Among combinations of reserves and debt levels that command a spread equal to zero, gross asset positions are undetermined: the government only cares about its net position. This is not a problem when solving the model for our benchmark calibration because such combinations of reserves and debt levels are never optimal. However, this becomes a problem when we assume one-period bonds. In order to sidestep this problem, we solve the model with one-period bonds by allowing the government to choose only its net asset position. As indicated by the negligible mean sovereign spread in Table 3, the government chooses net asset positions that command a spread equal to zero in almost all simulation periods.

with positive reserves is unlikely to be significant.[18] With one-period bonds, the government has to roll over (or pay back) 100 percent of its debt each quarter. Hence, a government facing a sudden stop or a sharp increase in spreads would have to use a large fraction of its income (160 percent of its quarterly income to repay the average value of debt in the data). Because investors will charge a high spread anticipating a government default, the government chooses instead low debt levels that imply a negligible default probability. In contrast, with long-duration bonds reserve holdings of 2.5 percent of income represent 66 percent of the average short-term debt and, thus, provide meaningful insurance against increases in the borrowing cost.

Third, allowing for long-duration bonds also changes the link between reserve accumulation and the current cost of borrowing. This can be illustrated by considering the Euler equation with respect to reserves:[19]

$$u'(t)(1 + (\partial q_t/\partial a_{t+1})(b_{t+1} - b_t(1 - \delta))) = R\beta E_t u'(t + 1) \tag{6}$$

The term $(\partial q_t/\partial a_{t+1})$ reflects how reserve accumulation affects the price at which the government issues new debt in equilibrium and is key in determining whether the government accumulates reserves. Figure 6 shows that in our benchmark model larger reserve holdings tend to improve the government's current borrowing opportunities, providing larger incentives to accumulate reserves. This does not happen in a model with one-period debt (see Alfaro and Kanczuk, 2009) where a Bulow-Rogoff type argument causes spreads to be increasing in the level of reserves. In particular, higher reserves reduce the cost of defaulting because autarchy becomes relatively more attractive. The reason is that a higher reserve level enables the government to smooth out the fall in consumption implied by the income loss triggered by a default. Thus, as illustrated in the left panel of Figure 7, the next-period default probability increases when the government accumulates more reserves in the current period. In a model with one-period debt, the spread increases with the next-period default probability.

[18]In a model with defaults, reserves allow the government to transfer resources to the states in which it will choose to default (see Alfaro and Kanczuk, 2009).

[19]For illustration purposes, we assume differentiability and that the constraint on reserves is not binding.

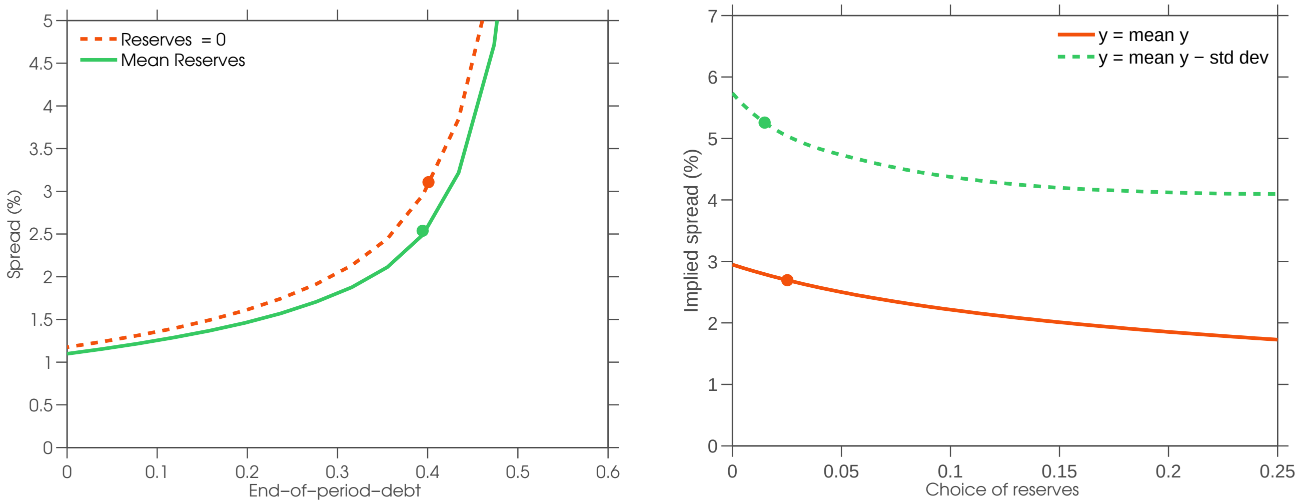

Figure 6: Effect of reserves on credit availability. The left panel presents menus of spread $(r^s(b', a', \bar{y}, 0))$ and end-of-period debt levels $(b'(1+r)[4(\delta+r)]^{-1})$ available to a government that starts the period with the mean income and that does not face a sudden stop in the current period. Solid dots indicate optimal choices conditional on the assumed value of a'. The right panel presents the spread the government would pay if it chooses the optimal borrowing level and different levels of reserves, $r^s(\hat{b}(\bar{b}, \bar{a}, y, 0), a', y, 0)$. Solid dots indicate optimal choices $(\hat{a}(\bar{b}, \bar{a}, y, 0), r^s(\hat{b}(\bar{b}, \bar{a}, y, 0), \hat{a}(\bar{b}, \bar{a}, y, 0)y, 0))$.

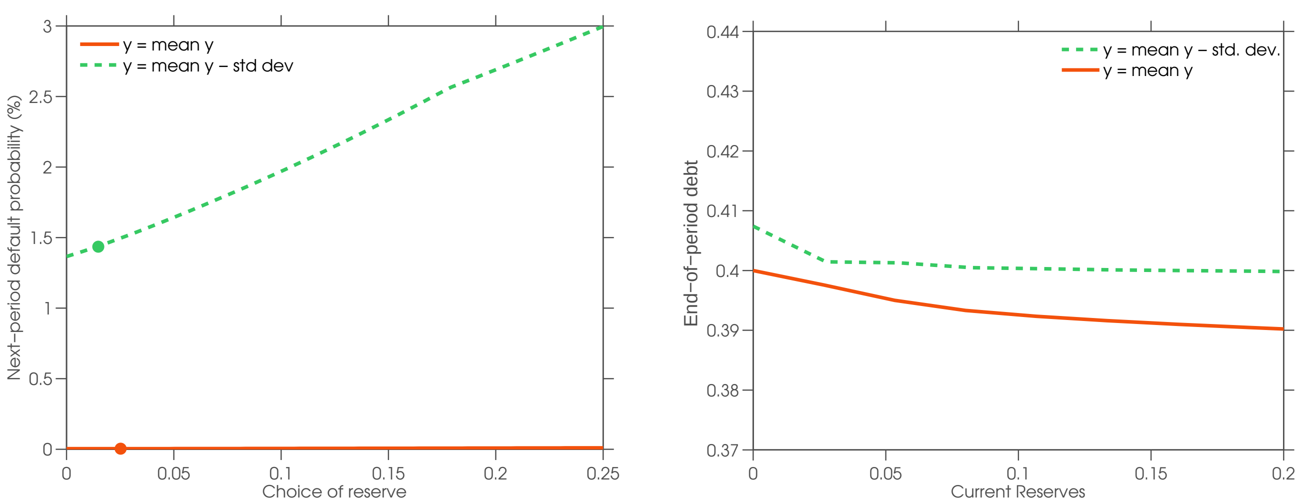

Figure 7: Effect of reserves on next-period default probability and borrowing. The left panel presents the next-period default probability $\left(Pr\left(V^D\left(b', a', y', s'\right) > V^R\left(b', a', y', s'\right) \mid y, s\right)\right)$ as a function of a' when $b' = \hat{b}\left(\bar{b}, \bar{a}, y, 0\right)$. Solid dots mark the optimal choice of reserves when initial debt and reserves levels are equal to the mean levels in the simulations $\left(\hat{a}\left(\bar{b}, \bar{a}, y, 0\right)\right)$. The right panel presents the optimal debt choice $\hat{b}\left(\bar{b}, a, y, 0\right)$ as a function of initial reserve holdings (a), assuming that the initial debt stock equals the mean debt stock in the simulations.

In a model with long-duration debt, current spread reflects not only next-period default probability but also default probabilities in other future periods. The right panel of Figure 7 shows that accumulating reserves in the current period tends to lower borrowing in the next period. Thus, higher reserve holdings at the end of the period lead creditors to expect lower future debt levels, which in turn leads them to expect lower default probabilities in other future periods. Figure 6 shows that the effect of reserves on the next-period default probability may be dominated by their effects on the default probability in other future periods, in which case the spread decreases with respect to reserve holdings.

5.5 Role of Sudden Stops

We now present sensitivity analysis with respect to the frequency and cost of sudden stops. All remaining parameters take the same values of our benchmark calibration.

Figure 8 presents simulation results obtained for different sudden stop processes. The left panel shows that higher frequency of sudden stops generate higher reserve holdings and lower debt levels. In particular, the figure shows that sudden stops play an important role in accounting for reserve accumulations in our benchmark: without sudden stops, reserve holdings decline from 2.5 percent of income in the benchmark to 0.4 percent. The right panel of the figure presents simulation results for different magnitudes of income losses while in sudden stop. The figure shows that for a higher sudden-stop cost, the government chooses higher reserve holdings and lower debt levels.

It has been argued that the surge in reserve holdings during the past decade could be related to the crises observed in many emerging economies in the late 1990s (see, for example, Ghosh et al., 2012). If the number or severity of sudden-stop episodes observed in those years increased their perceived frequency or cost, this would be captured in the model by a higher arrival rate or cost of sudden stops. Results in Table 4 suggest that the quantitative contribution of those channels is significant.

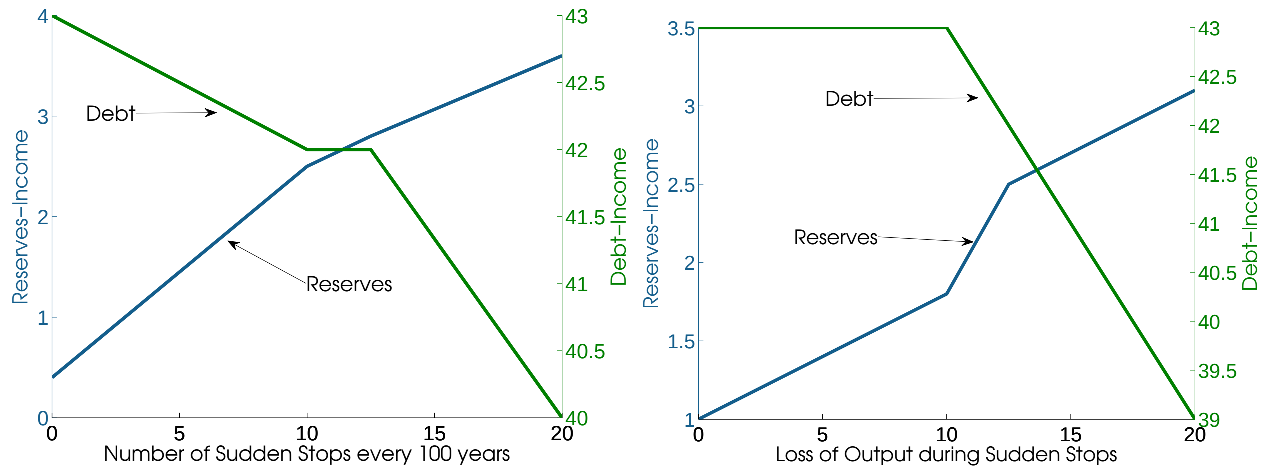

Figure 8: Mean Debt and Reserves for different sudden stop processes.

5.6 Role of the Endogenous and Countercyclical Spread

We now show that the endogenous and countercyclical sovereign spread plays a key role in generating demand for reserves in our model. To gauge the importance of allowing for an endogenous and countercyclical sovereign spread, we solve a version of the model without the default option. In this case income shocks do not affect the government's borrowing opportunities, which implies that there is no time-varying endogenous rollover risk associated with the possibility of default. The government continues to face sudden stops and pays a constant and exogenous spread for its debt issuances. Because of sudden stops and the presence of long-duration bonds, gross asset positions are relevant despite the lack of default risk. Formally, we solve the following recursive problem:

$$W(b, a, y, s) = \max_{a' \geq 0, b', c} \left\{ u(c) + \beta \mathbb{E}_{(y', s')|(y,s)} W(b', a', y', s') \right\},$$

subject to

$$c = y - s\phi^s(y) - b + a + q^*(b' - (1-\delta)b) - \frac{a'}{1+r},$$

$$b' \leq \bar{B},$$

$$b' - (1-\delta)b \leq 0 \text{ if } s = 1,$$

where $q^* = \frac{1}{r^*+\delta}$, r^* represents the interest rate demanded by investors to buy sovereign bonds, and $\bar{B}$ is an exogenous debt limit. The values of r^* and $\bar{B}$ are chosen to replicate the mean spread and debt levels in Mexico (also targeted in our benchmark calibration). Remaining parameter values are identical to the ones used in our benchmark calibration.

Table 4 presents simulation results obtained with the no-default model. The table indicates that the endogenous source of rollover risk is important in accounting for reserve accumulation. Simulated reserve holdings decline from 2.5 percent of income in the benchmark to 0.1 percent with an exogenous and constant sovereign spread. Two factors are important for this result. First, rollover risk is lower in the no-default model because borrowing opportunities are independent from the income shock. Second, a model with the spread level observed in the data but without default overstates the financial cost of accumulating reserves financed by borrowing. In a default model, since the government always receives the return from reserve holdings but does not always pay back its debt, the financial cost of accumulating reserves financed by borrowing is lower than in a no-default model with the same spread.

Table 4: Debt and Reserve Levels in a Model without Default and a Constant Spread.

	Benchmark	Constant exogenous spread
Mean debt-to-GDP	42	43
Mean reserves-GDP	2.5	0.1

5.7 Reserve Accumulation for Crisis Prevention

In this subsection we show how the optimal level of reserves increases when we assume reserves are useful for preventing sudden stops. This assumption is consistent with recent evidence (see e.g., Calvo et al., 2012) showing that international reserves reduce the likelihood of a sudden stop.[20] Following Jeanne and Ranciere (2011), we assume that the probability of a sudden stop is given by

$$\hat{\pi}\left(\frac{a}{\varrho(b)}\right) = G\left(m - w\frac{a}{\varrho(b)}\right), \tag{7}$$

where $\varrho(b) = b\sum_{t=1}^{t=4}\frac{(1-\delta)^{t-1}}{(1+r)^t}$ denotes the level of short-term debt, i.e., debt obligations maturing within the next year, and G denotes the standard normal cumulative distribution function. Note that our benchmark calibration is a special case of equation (7) with $w = 0$. We assume that m is such that the probability of a sudden stop is 10 percent (our benchmark target) when $w = 0$.

Table 5 presents simulation results for $w \in [0, 0.15]$, which lies within the lower half of values considered by Jeanne and Ranciere (2011). As in the previous sensitivity analysis, all other parameters take the values used in our benchmark calibration. Table 5 shows that as we allow reserves to be more effective in reducing the probability of a sudden stop, optimal reserve holdings increase. In particular, when $w = 0.15$, the model replicates the average reserve level in Mexico. At that value of reserves, the government reduces the frequency of sudden stops from 10 episodes every 100 years to 6 episodes every 100 years.

Table 5: Simulation Results when Reserves Reduce the Probability of a Sudden Stop.

	$\boldsymbol{w} = \mathbf{0}$	$w = 0.05$	$w = 0.10$	$w = 0.15$
Mean debt-to-GDP	**42**	42	42	42
Mean reserves-to-GDP	**2.5**	4.0	5.7	7.2
Sudden stops per 100 years	**10**	8	7	6

Note: cells in boldface correspond to the benchmark parameterization.

[20]In contrast, several previous empirical studies do not find evidence of reserves significantly reducing the probability of a sudden stop (e.g., Jeanne, 2007). The relationship between reserves and the probability of a sudden stop is difficult to estimate: sudden stops are relatively rare events and the relationship between sudden stops and economic fundamentals may differ across countries. Several studies using a broader definition of crises do find that higher levels of reserves are associated with a lower crisis probability (see Berg et al., 2005; Frankel and Saravelos, 2010; and Gourinchas and Obstfeld, 2011).

6 Conclusions

This paper proposed a model of optimal reserve management that is consistent with two salient features of international capital flows: (1) indebted governments hold large amounts of international reserves and the yield of the debt they issue is significantly higher than the return they obtain from holding reserves; (2) non-resident purchases of domestic assets and purchases of foreign assets by domestic agents are both procyclical and collapse during crises.

In the model, the government faces a trade-off between the insurance benefits of reserves and the cost of having larger gross debt positions. Because default risk is countercyclical, the government accumulates both reserves and debt in good times. On the other hand, when income is low and borrowing costs rise, the government uses reserves to make debt repayments and smooth consumption. We show that long-duration bonds, sudden stops, and countercyclical spreads are key ingredients for the quantitative success of the model.

Looking forward, our analysis suggests several avenues for further research. For instance, it would be interesting to study the interaction of the debt maturity structure and reserve holdings. In addition, the mechanisms studied in this paper could be relevant for understanding the financial decisions of corporate borrowers facing rollover risk.

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A Appendix

A.1 Proof of Proposition 1

Without rollover risk, the optimal allocation is such that $c_1 = y_1 + y_2(1+r_b)^{-1}$. If $r_a = r_b$ (point 1 of Proposition 1), any combination of debt issuances and reserve holdings such that $b_0 q_0 = a$ and $b_1 = y_2 - (1-\delta)b_0$ attain the optimal allocation. In particular, the optimal allocation can be attained without reserve accumulation ($a = b_0 = 0$, and $b_1 = y_2$).

If $r_a < r_b$ and there is no rollover risk (point 2 of Proposition 1), the government can only attain the optimal allocation if it chooses to not accumulate reserves. Let us consider any levels of period-0 savings and borrowing $\hat{a} = \hat{b}_0 q_0 > 0$. It is easy to show that the government can do better choosing $a = b_0 = 0$. Since $r_a < r_b$, $\hat{a}(1+r_a) < \hat{b}_0[1+(1-\delta)(1+r_b)]^{-1}$. Therefore, the level of period-2 consumption is higher with $b_0 = a = 0$ than with $\hat{a} = \hat{b}_0 q_0 > 0$, and $\hat{a} = \hat{b}_0 q_0 > 0$ cannot be part of an equilibrium.

Suppose the government can only issue one-period debt and $r_a = r_b$ (point 3 of Proposition 1). Since $q_0 = (1+r_a)^{-1}$, $c_1 = y + b_1(1+r_b)^{-1}$ for all possible equilibrium borrowing and saving choices satisfying $b_0 q_0 = a$. Then, gross asset positions are undetermined and the optimal allocation can be attained without reserve accumulation ($a = b_0 = 0$).

Suppose now the government can only issue one-period debt and $r_a < r_b$ (point 4 of Proposition 1). Let us consider any levels of period-0 savings and borrowing $\hat{a} = \hat{b}_0 q_0 > 0$. Then, period-1 consumption is given by $c_1 = y_1 + b_1(1+r_b)^{-1} + \hat{b}(1+r_b)^{-1}(1+r_a) - \hat{b} < y + b_1(1+r_b)^{-1}$. Therefore, the level of period-1 consumption would be higher if the government chooses $a = b_0 = 0$, and $\hat{a} = \hat{b}_0 q_0 > 0$ cannot be part of an equilibrium.

Next, we show that condition (1) is sufficient for reserve accumulation (point 5 of Proposition 1). Since $b_0 q_0 = a$, the government's well defined maximization problem can be written as:

$$\max_{b_0} \left\{ \pi u(y_1 + b_0 q_0(1+r_a) - b_0) + (1-\pi) u\left(y_1 + b_0 q_0 (1+r_a) - b_0 + \frac{y_2 - (1-\delta)b_0}{1+r_b} \right) \right\}.$$

The first-order condition of the government's problem is given by:

$$\begin{aligned}
&\pi \left[q_0(1+r_a)-1\right] u'(y_1+b_0q_0(1+r_a)-b_0) \leq \\
&(1-\pi)\left[\frac{1-\delta}{1+r_b}+1-q_0(1+r_a)\right] u'\left(y_1+b_0q_0(1+r_a)-b_0+\frac{y_2-(1-\delta)b_0}{1+r_b}\right).
\end{aligned} \tag{8}$$

Condition (1) states that the left-hand side of condition (8) is higher than the right-hand side of condition (8) when evaluated at $b_0 = 0$. Therefore, if condition (1) holds, $a = b_0 = 0$ cannot be part of an equilibrium.

A.2 Sudden Stops

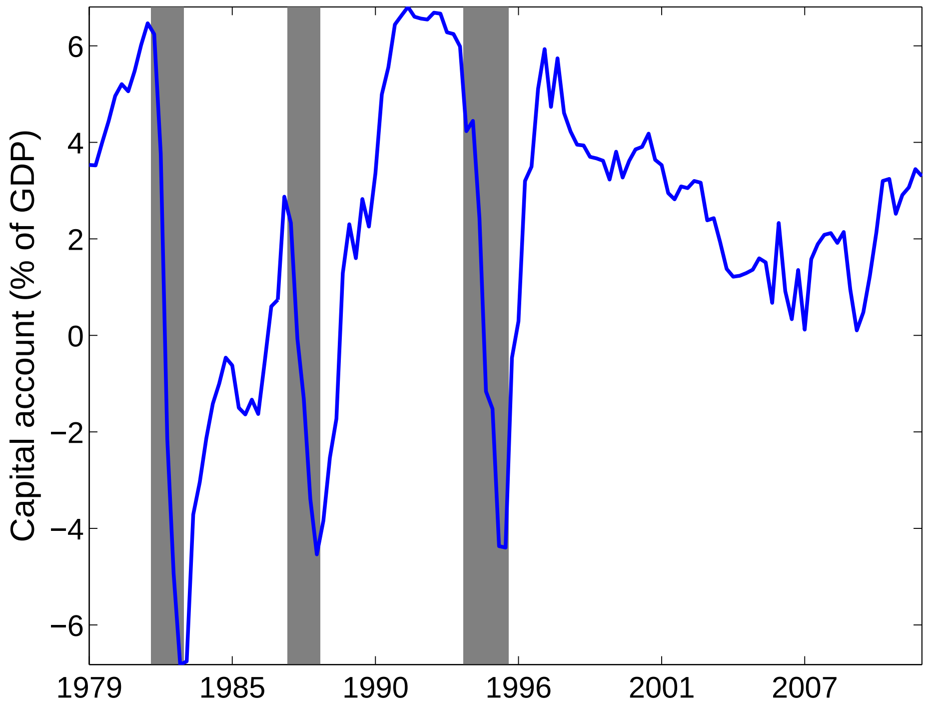
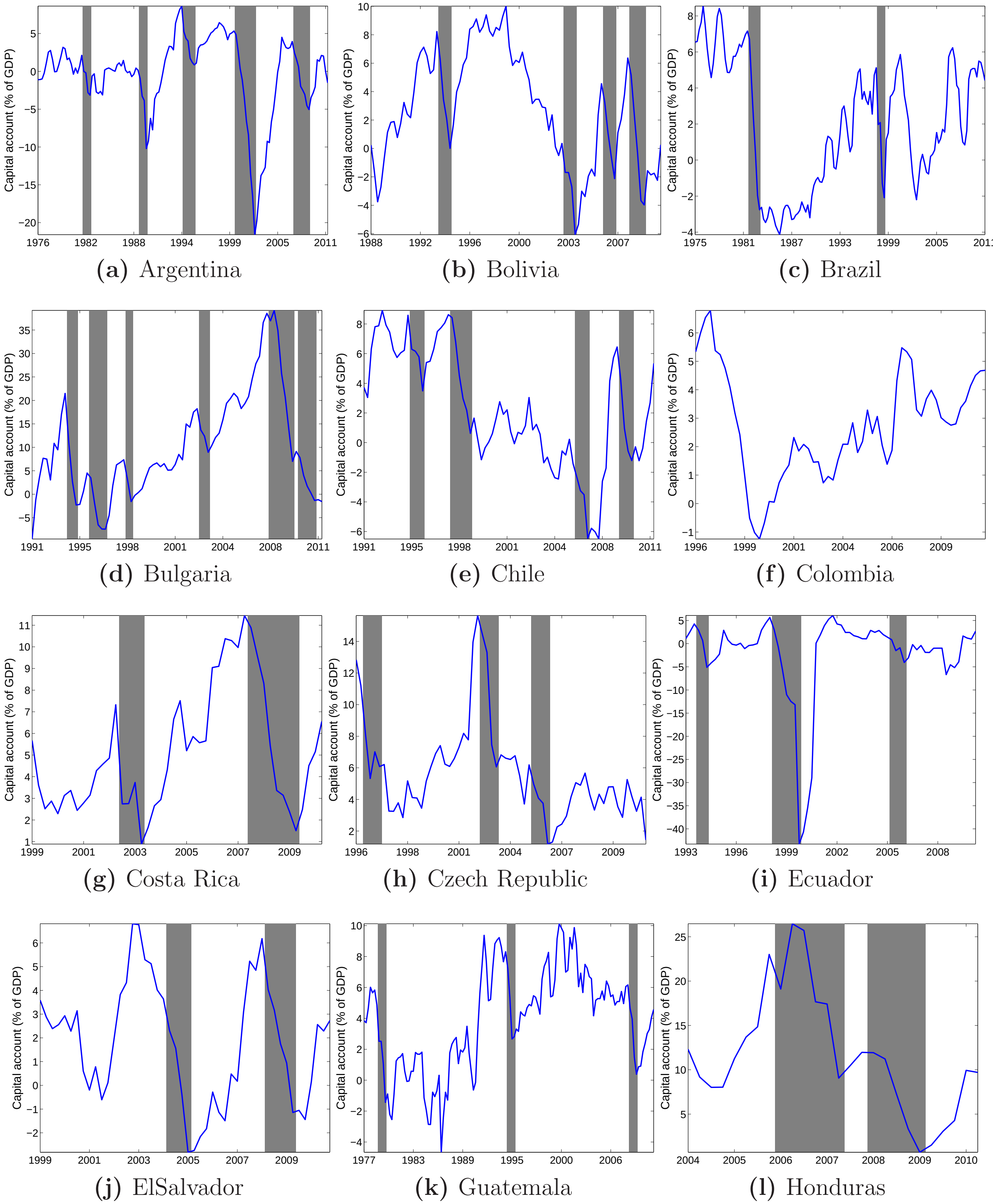


Figure 9: Mexico: Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops.



(a) Argentina (b) Bolivia (c) Brazil

(d) Bulgaria (e) Chile (f) Colombia

(g) Costa Rica (h) Czech Republic (i) Ecuador

(j) ElSalvador (k) Guatemala (l) Honduras

Figure 10: Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops.

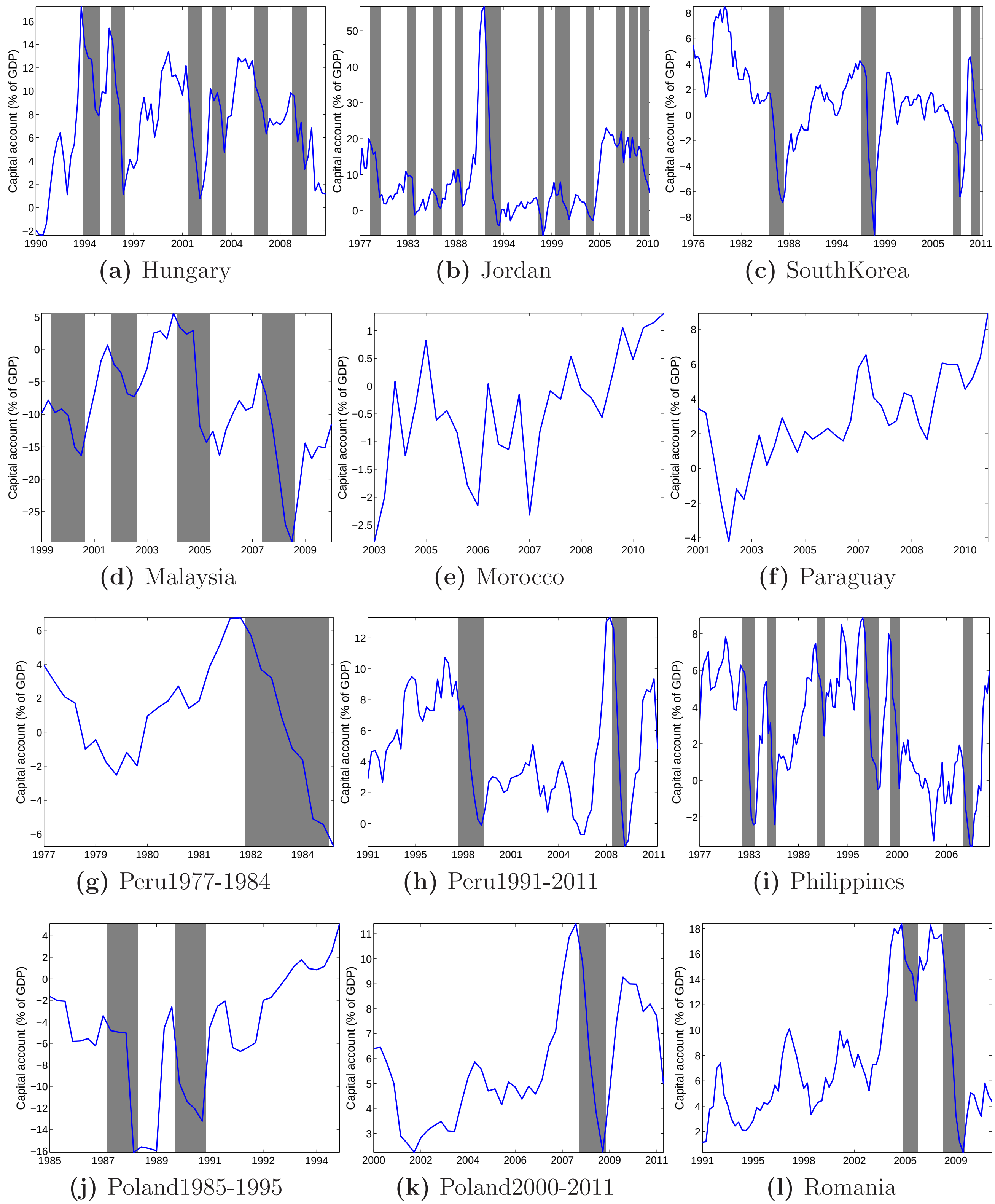
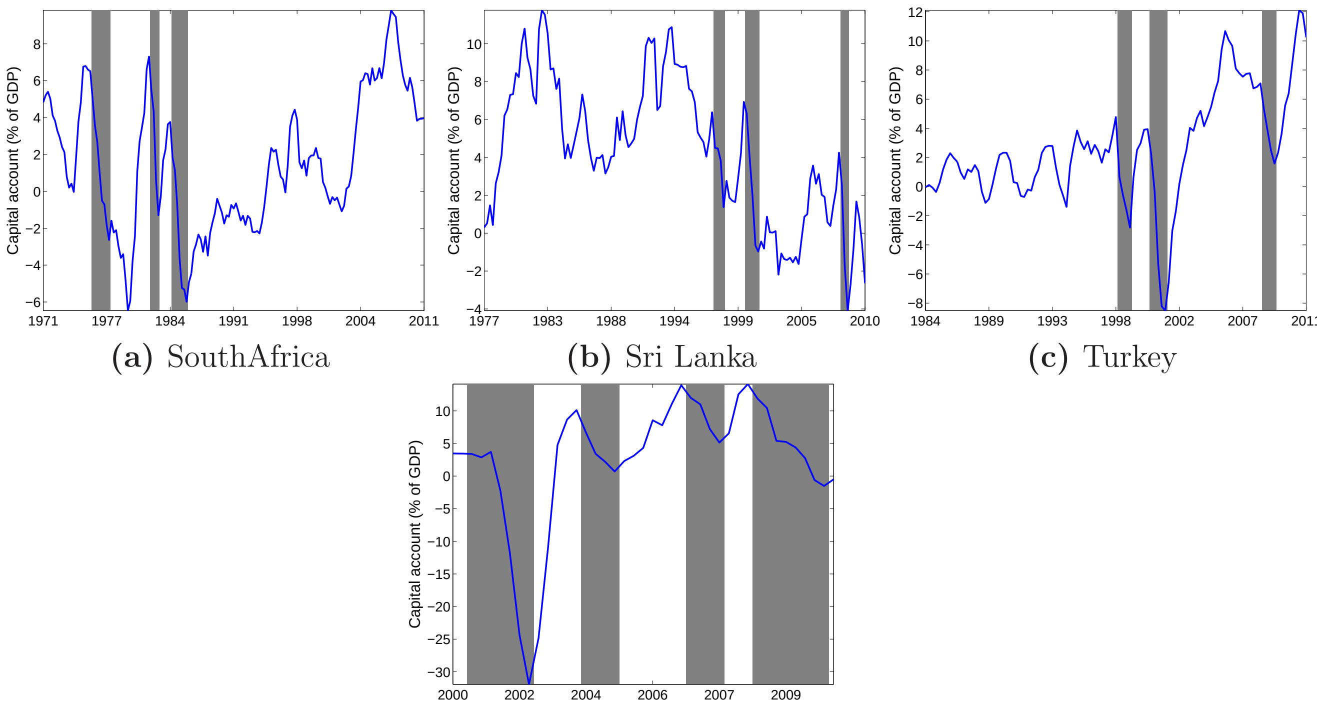


Figure 11: Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops.



(a) SouthAfrica **(b)** Sri Lanka **(c)** Turkey

(d) Uruguay

Figure 12: Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of sudden stops.

Table 6: Sudden-Stop Episodes.

Argentina	1989, 2001
Bolivia	1980, 1982, 1994
Botswana	1977, 1987, 1991, 1993, 2001, 2003, 2010
Brazil	1983
Bulgaria	1990, 1994, 1996, 1998, 2003, 2008
Chile	1982, 1985, 1991, 1995, 1998, 2009
China, P.R.	
Colombia	
Costa Rica	2009
Czech Republic	1996, 2003
Dominican Republic	1993, 2003
Ecuador	1979, 1986, 1988, 1992, 1999, 2006
Egypt	1990, 1993
El Salvador	1979, 1986, 2005, 2009
Guatemala	
Honduras	2008
Hungary	1994, 1996, 2009
Jamaica	1983, 1985, 1988, 2002, 2009
Jordan	1976, 1979, 1984, 1989, 1992, 1998, 2001, 2008, 2010
Korea, Republic of	1986, 1997, 2008
Malaysia	1984, 1987, 1994, 1999, 2005, 2008
Mexico	1982, 1988, 1995
Morocco	1978, 1995
Paraguay	1988, 1995, 2002
Peru	1983, 1998, 2009
Philippines	1983, 1997, 2000
Poland	1981, 1988, 1990
Romania	1981, 1988, 2008
South Africa	1985
Sri Lanka	
Thailand	1982, 1997, 2009
Tunisia	
Turkey	2001
Uruguay	1982, 2002, 2004, 2007, 2009

Note: Sudden-stop episodes correspond to years in which the ratio of net capital inflows to GDP falls by more than 5 percentage points. Source: IMF's International Financial Statistics annual data from 1970 to 2011