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ABSTRACT

Standard representative-agent models have difficulty in accounting for the weak correlation between stock returns and measurable fundamentals, such as consumption and output growth. This failing underlies virtually all modern asset-pricing puzzles. The correlation puzzle arises because these models load all uncertainty onto the supply side of the economy. We propose a simple theory of asset pricing in which demand shocks play a central role. These shocks give rise to valuation risk that allows the model to account for key asset pricing moments, such as the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals.

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1. Introduction

In standard representative-agent asset-pricing models, the expected return to an asset reflects the covariance between the asset's payoff and the agent's stochastic discount factor. An important challenge to these models is that the correlation and covariance between stock returns and measurable fundamentals, especially consumption growth, is weak at both short and long horizons. Cochrane and Hansen (1992) and Campbell and Cochrane (1999) call this phenomenon the *correlation puzzle*. More recently, Lettau and Ludvigson (2011) document this puzzle using different methods. According to their estimates, the shock that accounts for the vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.

The basic disconnect between measurable macroeconomic fundamentals and stock returns underlies virtually all modern asset-pricing puzzles, including the equity-premium puzzle, Hansen-Singleton (1982)-style rejection of asset-pricing models, violation of Hansen-Jagannathan (1991) bounds, and Shiller (1981)-style observations about excess stock-price volatility.

The key reason why classic asset-pricing models cannot account for the correlation puzzle is that all uncertainty is loaded onto the supply side of the economy. In Lucas (1978) tree models, agents are exposed to random endowment shocks, while in production economies they are exposed to random productivity shocks. Both classes of models abstract from shocks to the demand for assets.

We propose a simple theory of asset pricing in which demand shocks play a central role. In our model demand shocks arise from stochastic changes in agents' rate of time preference. An important implication of our model is that these changes are measurable because they map directly into the level of the risk free rate. So, while we are allowing for an additional shock, our analysis is disciplined by observed movements in the risk-free rate.

In our model, the representative agent has recursive preferences of the type considered by Kreps and Porteus (1978), Weil (1989), and Epstein and Zin (1991). Time-preference shocks help account for the equity premium as long as the risk-aversion coefficient and the elasticity of intertemporal substitution are either both greater than one or both are smaller than one. When the risk-aversion coefficient is equal to the inverse of the elasticity of intertemporal substitution, recursive preferences reduce to constant-relative risk aversion (CRRA) preferences. We show that, in this case, time-preference shocks have negligible

effects on key asset-pricing moments such as the equity premium.

We estimate our model using data over the sample period 1929 to 2011. The condition for time-preference shocks to help explain the equity premium puzzle is always satisfied in the different versions of the model that we estimate. Taking sampling uncertainty into account, our model accounts for the equity premium and the volatility of stock and bond returns. Critically, the model also accounts for the correlation between stock returns and fundamentals such as consumption, output, and dividend growth at short, medium and long horizons.

We define *valuation risk* as the risk associated with changes in the way that future cash flows are discounted due to time-preference shocks. According to our estimates, valuation risk is a much more important source of variation in asset prices than conventional covariance risk. The model has no difficulty in accounting for the average rate of return to stocks and bonds. But, absent preference shocks, our model implies that stocks and bonds should, on average, have very similar rates of return.

Valuation risk is an increasing function of an asset's maturity. So, a natural test of our model is whether it can account for the bond term premia and the return on stocks relative to long-term bonds. We pursue this test using stock returns as well as ex-post real returns on bonds of different maturity and argue that the model's implications are consistent with the data. We are keenly aware of the limitations of the available data on real bond returns, especially at long horizons. Still, we interpret our results as being very supportive of the hypothesis that valuation risk is a critical determinant of asset prices.

There is a literature that models shocks to the demand for assets as arising from time-preference or taste shocks. For example, Garber and King (1983) and Campbell (1986) consider these types of shocks in early work on asset pricing. Pavlova and Rigobon (2007) study the role of taste shocks in explaining asset prices in an open economy model. In the macroeconomic literature, Eggertsson and Woodford (2003) and Eggertsson (2004) model changes in savings behavior as arising from time-preference shocks that make the zero lower bound on nominal interest rates binding. A common property of these papers is that agents have CRRA preferences. Normandin and St-Amour (1998) study the impact of taste shocks in a partial-equilibrium model where the representative agent has recursive preferences. In independent work, contemporaneous with our own, Maurer (2012) explores the impact of time-preference shocks in a calibrated continuous-time representative agent model with Duffie-Epstein (1992)

preferences.

The key contribution of our paper is empirical. We consider two main variants of our model. In the first variant, time-preference shocks are uncorrelated with endowment shocks. This version of the model is very useful for highlighting the basic role of demand shocks in asset pricing. However, in a production economy, these types of shocks would generally induce changes in aggregate output and consumption. To assess the robustness of our results to this possibility, we consider a variant of the model that allows endowment and time-preference shocks to be correlated. As it turns out, the second version of the model outperforms the first version. This improved performance is reflected in the model's ability to account for the observed correlation between stock returns and fundamentals.

Our paper is organized as follows. In Section 2 we document the correlation puzzle using U.S. data for the period 1929-2011 as well as the period 1871-2006. In Section 3 we present our benchmark model where time-preference shocks are uncorrelated with the growth rate of consumption. We discuss our estimation strategy and present our benchmark empirical results in Section 4. In Section 5 we present the variant of our model in which time-preference shocks are correlated with consumption shocks and its empirical performance. In Section 6 we study the empirical implications of the model for bond term premia, as well as the return on stocks relative to long-term bonds. Section 7 concludes.

2. The correlation puzzle

In this section we examine the correlation between stock returns and fundamentals as measured by the growth rate of consumption, output, dividends, and earnings.

2.1. Data sources

We consider two sample periods: 1929 to 2011 and 1871 to 2006. For the first sample, we obtain nominal stock and bond returns from Kenneth French's website. We convert nominal returns to real returns using the rate of inflation as measured by the consumer price index. We use the measure of consumption expenditures and real per capita Gross Domestic Product constructed by Barro and Ursua (2011), which we update to 2011 using National Income and Product Accounts data. We compute per-capita variables using total population (POP).[1] We obtain data on real S&P500 earnings and dividends from Robert

[1] This series is not subject to a very important source of measurement error that affects another commonly-used population measure, civilian noninstitutional population (CNP16OV). Every ten years, the CNP16OV

Shiller's website. We use data from Ibbotson and Associates on the real return to one-month Treasury bills, intermediate-term government bonds (with approximate maturity of 5 years), and long-term government bonds (with approximate maturity of 20 years).

For the second sample, we use data on real stock and bond returns from Nakamura, Steinsson, Barro, and Ursua (2010) for the period 1870-2006. We use the same data sources for consumption, expenditures, dividends and earnings as in the first sample.

As in Mehra and Prescott (1985) and the associated literature, we measure the risk-free rate using realized real returns on nominal, one-year Treasury Bills. This measure is far from perfect because there is inflation risk, which can be substantial, particularly for long-maturity bonds.

2.2. Empirical results

Table 1, panel A presents results for the sample period 1929-2011. We report correlations at the one-, five- and ten-year horizons. The five- and ten-year horizon correlations are computed using five- and ten-year overlapping observations, respectively. We report Newey-West (1987) heteroskedasticity-consistent standard errors computed with ten lags.

There are three key features of Table 1, panel A. First, consistent with Cochrane and Hansen (1992) and Campbell and Cochrane (1999), the growth rate of consumption and output are uncorrelated with stock returns at all the horizons that we consider. Second, the correlation between stock returns and dividend growth is similar to that of consumption and output growth at the one-year horizon. However, the correlation between stock returns and dividend growth is substantially higher at the five and ten-year horizons than the analogue correlations involving consumption and output growth. Third, the pattern of correlations between stock returns and dividend growth are similar to the analogue correlations involving earnings growth.

Table 1, panel B reports results for the longer sample period (1871-2006). The one-year correlation between stock returns and the growth rates of consumption and output are very similar to those obtained for the shorter sample. There is evidence in this sample

series is adjusted using information from the decennial census. This adjustment produces large discontinuities in the CNP16OV series. The average annual growth rates implied by the two series are reasonably similar: 1.2 for POP and 1.4 for CNP16OV for the period 1952-2012. But the growth rate of CNP16OV is three times more volatile than the growth rate of POP. Part of this high volatility in the growth rate of CNP16OV is induced by large positive and negative spikes that generally occur in January. For example, in January 2000, 2004, 2008, and 2012 the annualized percentage growth rates of CNP16OV are 14.8, -1.9, -2.8, and 8.4, respectively. The corresponding annualized percentage growth rates for POP are 1.1, 0.8, 0.9, and 0.7.

of a stronger correlation between stock returns and the growth rates of consumption and output at a five-year horizon. But, at the ten-year horizon the correlations are, once again, statistically insignificant. The results for dividends and earnings is very similar across the two subsamples.

Table 2 assesses the robustness of our results for the correlation between stock returns and consumption using three different measures of consumption for the period 1929-2011, obtained from the National Product and Income Accounts. With one exception, the correlations in this table are statistically insignificant. The exception is the one-year correlation between stock returns and the growth rate of nondurables and services which is marginally significant.

In summary, there is remarkably little evidence that the growth rates of consumption or output are correlated with stock returns. There is also little evidence that dividends and earnings are correlated with stock returns at short horizons.

We have focused on correlations because we find them easy to interpret. One might be concerned that a different pictures emerges from the pattern of covariances between stock returns and fundamentals. It does not. For example, using quarterly U.S. data for the period 1959 to 2000, Parker (2001) argues that one would require a risk aversion coefficient of 379 to account for the equity premium given his estimate of the covariance between consumption growth and stock returns. Parker (2001) observes that there is a larger covariance between current stock returns and the cumulative growth rate of consumption over the next 12 quarters. However, even with this covariance measure he shows that one would require a risk aversion coefficient of 38 to rationalize the equity premium.

Viewed overall, the results in this section serve as our motivation for introducing shocks to the demand for assets. Classic asset-pricing models load all uncertainty onto the supply-side of the economy. As a result, they have difficulty in simultaneously accounting for the equity premium and the correlation puzzle. This difficulty is shared by the habit-formation model proposed by Campbell and Cochrane (1999) and the long-run risk models proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012).[2] Rare-disaster models of the type proposed by Rietz (1988) and Barro (2006) also share this difficulty because all shocks, disaster or not, are to the supply side of the model. A model with a time-varying

[2]The most recent version of the long-run risk model, proposed by Bansal, Kiku, and Yaron (2012), implies correlations between stock returns and consumption growth equal to 0.66, 0.88, and 0.92 at the one-, five- and ten-year horizon, respectively. The model implies correlations between stock returns and dividend growth equal to 0.66, 0.90, and 0.93 at the one-, five- and ten-year horizon, respectively.

disaster probability, of the type consider by Wachter (2012), might be able to rationalize the low correlation between consumption and stock returns as a small sample phenomenon. The reason is that changes in the probability of disasters induces movements in stock returns without corresponding movements in actual consumption growth. This force lowers the correlation between stock returns and consumption in a sample where rare disasters are under represented. This explanation might account for the post-war correlations. But we are more skeptical that it accounts for the results in Table 1, panel B, which are based on the longer sample period, 1871 to 2006.

Below, we focus on demand shocks as the source of the low correlation between stock returns and fundamentals, rather than the alternatives just mentioned. We model these demand shocks in the simplest possible way by introducing shocks to the time preference of the representative agent. These shocks can be thought of as capturing changes in agents' attitudes towards savings, such as those emphasized by Eggertsson and Woodford (2003). These shocks can also reflect changes in institutional factors, such as the tax treatment of retirement plans. Finally, these shocks could also capture the effects of changes in the demographics of stock market participants (see Geanakoplos, Magill, and Quinzii (2004)). In Appendix A we provide a simple example of an overlapping-generations model in which uncertainty about the growth rate of the population gives rise to shocks in the demand for assets.

3. The benchmark model

In this section, we study the properties of a simple representative-agent endowment economy modified to allow for time-preference shocks. The representative agent has the constant-elasticity version of Kreps-Porteus (1978) preferences used by Epstein and Zin (1991) and Weil (1989). The life-time utility of the representative agent is a function of current utility and the certainty equivalent of future utility, U_{t+1}^*:

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \tag{3.1}$$

where C_t denotes consumption at time t and δ is a positive scalar. The certainty equivalent of future utility is the sure value of $t+1$ lifetime utility, U_{t+1}^* such that:

$$\left(U_{t+1}^* \right)^{1-\gamma} = E_t \left(U_{t+1}^{1-\gamma} \right).$$

The parameters ψ and γ represent the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The ratio λ_{t+1}/λ_t determines how agents trade off current versus future utility. We assume that this ratio is known at time t.[3] We refer to λ_{t+1}/λ_t as the time-preference shock.

3.1. Stochastic processes

To highlight the role of time-preference shocks, we adopt a very simple stochastic process for consumption:

$$\log(C_{t+1}) = \log(C_t) + \mu + \sigma_c \varepsilon^c_{t+1}. \tag{3.2}$$

Here, μ and σ_c are non-negative scalars and ε^c_{t+1} follows an i.i.d. standard-normal distribution.

As in Campbell and Cochrane (1999), we allow dividends, D_t, to differ from consumption. In particular, we assume that:

$$\log(D_{t+1}) = \log(D_t) + \mu + \pi_{dc}\varepsilon^c_{t+1} + \sigma_d \varepsilon^d_{t+1}. \tag{3.3}$$

Here, ε^d_{t+1} is an i.i.d. standard-normal random variable that is uncorrelated with ε^c_{t+1}. To simplify, we assume that the average growth rate of dividends and consumption is the same (μ). The parameter $\sigma_d \geq 0$ controls the volatility of dividends. The parameter π_{dc} controls the correlation between consumption and dividend shocks.[4]

The growth rate of the time-preference shock evolves according to:

$$\log\left(\lambda_{t+1}/\lambda_t\right) = \rho \log\left(\lambda_t/\lambda_{t-1}\right) + \sigma_\lambda \varepsilon^\lambda_{t+1}. \tag{3.4}$$

Here, $\varepsilon^\lambda_{t+1}$ is an i.i.d. standard-normal random variable. In the spirit of the original Lucas (1978) model, we assume, for now, that $\varepsilon^\lambda_{t+1}$ is uncorrelated with ε^c_{t+1} and ε^d_{t+1}. We relax this assumption in Section 5. We assume that λ_{t+1}/λ_t is highly persistent but stationary (ρ very close to one). The idea is to capture, in a parsimonious way, persistent changes in agents' attitudes towards savings.

[3] We obtain similar results with a version of the model in which the utility function takes the form: $U_t = \left[C_t^{1-1/\psi} + \lambda_t \delta \left(U^*_{t+1}\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}$.

[4] The stochastic process described by equations (3.2) and (3.3) implies that $\log(D_{t+1}/C_{t+1})$ follows a random walk with no drift. This implication is consistent with our data.

The CRRA case In Appendix B we solve the model analytically for the case in which $\gamma = 1/\psi$. Here preferences reduce to the CRRA form:

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma}, \tag{3.5}$$

with $V_t = U_t^{1-\gamma}$.

The unconditional risk-free rate is affected by the persistence of volatility of time-preference shocks:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2).$$

The unconditional equity premium implied by this model is proportional to the risk-free rate:

$$E\left(R_{c,t+1} - R_{f,t+1}\right) = E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right]. \tag{3.6}$$

Both the average risk-free rate and the volatility of consumption are small in the data. Moreover, the constant of proportionality in equation (3.6), $\exp\left(\gamma\sigma_c^2\right) - 1$, is independent of σ_λ^2. So, time-preference shocks do not help to resolve the equity premium puzzle when preferences are of the CRRA form.

3.2. Solving the model

We define the return to the stock market as the return to a claim on the dividend process. The realized gross stock-market return is given by:

$$R_{d,t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_t}, \tag{3.7}$$

where $P_{d,t}$ denotes the ex-dividend stock price.

It is useful to define the realized gross return to a claim on the endowment process:

$$R_{c,t+1} = \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}, \tag{3.8}$$

where $P_{c,t}$ denotes the price of an asset that pays a dividend equal to aggregate consumption. We use the following notation to define logarithm of returns on the dividend and consumption claims, the logarithm of the price-dividend ratio, and the logarithm of the price-consumption

ratio:

$$\begin{aligned} r_{d,t+1} &= \log(R_{d,t+1}), \\ r_{c,t+1} &= \log(R_{c,t+1}), \\ z_{dt} &= \log(P_t/D_t), \\ z_{ct} &= \log(P_{c,t}/C_t). \end{aligned}$$

In Appendix C we show that the logarithm of the stochastic discount factor (SDF) implied by the utility function defined in equation (3.1) is given by:

$$m_{t+1} = \theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\, r_{c,t+1}, \tag{3.9}$$

where θ is given by:

$$\theta = \frac{1-\gamma}{1-1/\psi}. \tag{3.10}$$

When $\gamma = 1/\psi$, the case of CRRA preferences, the value of θ is equal to one and the stochastic discount factor is independent of $r_{c,t+1}$.

We solve the model using the approximation proposed by Campbell and Shiller (1988), which involves linearizing the expressions for $r_{c,t+1}$ and $r_{d,t+1}$ and exploiting the properties of the log-normal distribution.[5]

Using a log-linear Taylor expansion we obtain:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \tag{3.11}$$

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}, \tag{3.12}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ and $\Delta d_{t+1} \equiv \log(D_{t+1}/D_t)$. The constants κ_{c0}, κ_{c1}, κ_{d0}, and κ_{d1} are given by:

$$\begin{aligned} \kappa_{d0} &= \log[1 + \exp(z_d)] - \kappa_{d1} z_d, \\ \kappa_{c0} &= \log[1 + \exp(z_c)] - \kappa_{c1} z_c, \end{aligned}$$

$$\kappa_{d1} = \frac{\exp(z_d)}{1+\exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1+\exp(z_c)},$$

where z_d and z_c are the unconditional mean values of z_{dt} and z_{ct}.

[5] See Hansen, Heaton, and Li (2008) for an alternative solution procedure.

The Euler equations associated with a claim to the stock market and a consumption claim can be written as:

$$E_t\left[\exp\left(m_{t+1}+r_{d,t+1}\right)\right]=1, \tag{3.13}$$

$$E_t\left[\exp\left(m_{t+1}+r_{c,t+1}\right)\right]=1. \tag{3.14}$$

We solve the model using the method of undetermined coefficients. First, we replace m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$ in equations (3.13) and (3.14), using expressions (3.11), (3.12) and (3.9). Second, we guess and verify that the equilibrium solutions for z_{dt} and z_{ct} take the form:

$$z_{dt} = A_{d0}+A_{d1}\log\left(\lambda_{t+1}/\lambda_t\right), \tag{3.15}$$

$$z_{ct} = A_{c0}+A_{c1}\log\left(\lambda_{t+1}/\lambda_t\right). \tag{3.16}$$

This solution has the property that price-dividend ratios are constant, absent movements in λ_t. This property results from our assumption that the logarithm of consumption and dividends follow random-walk processes. We compute A_{d0}, A_{d1}, A_{c0}, and A_{c1} using the method of indeterminate coefficients.

We show in Appendix C that the conditional expected return to equity is given by:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) = & -\log\left(\delta\right)-\log\left(\lambda_{t+1}/\lambda_t\right)+\mu/\psi \\ & +\left[\frac{(1-\theta)}{\theta}\left(1-\gamma\right)^2-\gamma^2\right]\sigma_c^2/2+\pi_{dc}\left(2\gamma\sigma_c-\pi_{dc}\right)/2-\sigma_d^2/2 \\ & +\left[\left(1-\theta\right)\left(\kappa_{c1}A_{c1}\right)\left[2\left(\kappa_{d1}A_{d1}\right)-\left(\kappa_{c1}A_{c1}\right)\right]-\left(\kappa_{d1}A_{d1}\right)^2\right]\sigma_\lambda^2/2. \end{aligned} \tag{3.17}$$

Recall that κ_{c1} and κ_{d1} are non-linear functions of the parameters of the model.

We define the compensation for *valuation risk* as the part of the one-period expected return to an asset that is due to the volatility of the time preference shock, σ_λ^2. We refer to the part of the expected return that is due to the volatility of consumption and dividends as the compensation for *conventional risk*.

For stocks, the compensation for valuation risk, v_d, is given by the last term in equation (3.17):

$$v_d=\left[2\left(1-\theta\right)\left(\kappa_{c1}A_{c1}\right)\left(\kappa_{d1}A_{d1}\right)-\left(\kappa_{d1}A_{d1}\right)^2-\left(1-\theta\right)\left(\kappa_{c1}A_{c1}\right)^2\right]\sigma_\lambda^2/2.$$

To gain intuition about the determinants of v_d, it is useful to consider the simple case in which the stock market is a claim on consumption. In this case v_d is given by:

$$v_d=-\theta\left(\kappa_{c1}A_{c1}\right)^2\sigma_\lambda^2/2.$$

The compensation for valuation risk is positive as long as θ is negative. In terms of the underlying structural parameters, this condition holds as long as $\gamma > 1$ and $\psi > 1$ or $\gamma < 1$ and $\psi < 1$. Put differently, if agents have a coefficient of risk aversion higher than one, the condition requires that agents have a relatively high elasticity of intertemporal substitution. Alternatively, if agents have a coefficient of risk aversion lower than one, they must have a relatively low elasticity of intertemporal substitution. The value of θ is negative in all our estimated models, so the value of v_d is positive.

Using the Euler equation for the risk-free rate, $r_{f,t+1}$,

$$E_t\left[\exp\left(m_{t+1} + r_{f,t+1}\right)\right] = 1,$$

we obtain:

$$\begin{aligned} r_{f,t+1} &= -\log(\delta) - \log\left(\lambda_{t+1}/\lambda_t\right) + \mu/\psi - (1-\theta)\left(\kappa_{c1}A_{c1}\right)^2\sigma_\lambda^2/2 \\ &\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma_c^2/2. \end{aligned} \tag{3.18}$$

Equations (3.17) and (3.18) imply that the risk-free rate and the conditional expectation of the return to equity are decreasing functions of $\log\left(\lambda_{t+1}/\lambda_t\right)$. When $\log\left(\lambda_{t+1}/\lambda_t\right)$ rises, agents value the future more relative to the present, so they want to save more. Since risk-free bonds are in zero net supply and the number of stock shares is constant, aggregate savings cannot increase. So, in equilibrium, returns on bonds and equity must fall to induce agents to save less.

An important property of our model is that the risk-free rate is given by a constant minus $\log\left(\lambda_{t+1}/\lambda_t\right)$. This property allows us to measure movements in $\log\left(\lambda_{t+1}/\lambda_t\right)$ using the risk-free rate. This fact imposes data-based discipline on the role that we can attribute to time-preference shocks in explaining asset-price moments.

The approximate response of asset prices to shocks, emphasized by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2011), can be directly inferred from equations (3.17) and (3.18). The response of the return to stocks and the risk-free rate to a time-preference shock corresponds to that of an AR(1) with serial correlation ρ.

Using equations (3.17) and (3.18) we can write the conditional equity premium as:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) - r_{f,t+1} &= \pi_{dc}\left(2\gamma\sigma_c - \pi_{dc}\right)/2 - \sigma_d^2/2 \\ &\quad + \kappa_{d1}A_{d1}\left[2(1-\theta)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma_\lambda^2/2. \end{aligned} \tag{3.19}$$

Since the constants A_{c1}, A_{d1}, κ_{c1}, and κ_{d1} are all positive, $\theta < 1$ is a necessary condition for time-preference shocks to help explain the equity premium.

The component of the equity premium that is due to valuation risk is given by the last term in equation (3.19). It is useful to consider the case in which the stock is a claim on consumption. In this case, that term reduces to:

$$(1 - 2\theta) \left(\frac{\kappa_{c1}}{1 - \rho\kappa_{c1}} \right)^2 \sigma_\lambda^2 / 2.$$

This expression is positive as long as one of the following conditions holds:

$$\begin{array}{ll} \gamma < 0.5(1 + 1/\psi) & \text{and } \psi < 1, \\ \gamma > 0.5(1 + 1/\psi) & \text{and } \psi > 1. \end{array} \tag{3.20}$$

As it turns out, this condition is always satisfied in the estimated versions of our model.

It is interesting to highlight the differences between time-preference shocks and conventional sources of uncertainty, which pertain to the supply-side of the economy. Suppose that there is no risk associated with the physical payoff of assets such as stocks. In this case, standard asset pricing models would imply that the equity premium is zero. In our model, there is a positive equity premium that results from the different exposure of bonds and stocks to valuation risk. Agents are uncertain about how much they will value future dividend payments. Since λ_{t+1} is known at time t, this valuation risk is irrelevant for one-period bonds. But, it is not irrelevant for stocks, because they have infinite maturity. In general, the longer the maturity of an asset, the higher is its exposure to time-preference shocks and the large is the valuation risk.

Finally, we conclude by considering the case in which there are supply-side shocks to the economy but agents are risk neutral ($\gamma = 0$). In this case, the component of the equity premium that is due to valuation risk is positive as long as ψ is less than one. The intuition is as follows: stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of the representative agent, $\lambda_t C_t^{1-1/\psi}$. Even when agents are risk neutral, they must be compensated for the risk of this unwanted variation.

4. Estimating the benchmark model

We estimate the parameters of our model using the Generalized Method of Moments (GMM). Our estimator is the parameter vector $\hat{\Phi}$ that minimizes the distance between a vector of empirical moments, Ψ_D, and the corresponding model population moments, $\Psi(\hat{\Phi})$.

We proceed as follows. We estimate Ψ_D, which includes the following 16 moments: the mean and standard deviation of consumption growth, the mean and standard deviation of dividend growth, the correlation between the one-year growth rate of dividends and the one-year growth rate of consumption, the mean and standard deviation of real stock returns, the mean, standard deviation of the real risk-free rate, the correlation between stock returns and the risk-free rate, the correlation between stock returns and consumption growth at the one, five and ten-year horizon, the correlation between stock returns and dividend growth at the one, five and ten-year horizon. The parameter vector Φ includes nine parameters: γ (the coefficient of relative risk aversion), ψ (the elasticity of intertemporal substitution), δ (the rate of time preference), σ_c (the volatility of innovation to consumption growth), π_{dc} (the parameter that controls the correlation between consumption and dividend shocks), σ_d (the volatility of dividend shocks), ρ (the persistence of time-preference shocks), and σ_λ (the volatility of the innovation to time-preference shocks), and μ (the mean growth rate of dividends and consumption). We constrain the growth rate of dividends and consumption to be the same. We estimate Ψ_D using a standard two-step efficient GMM estimator with a Newey-West (1987) weighting matrix that has ten lags. The latter matrix corresponds to our estimate of the variance-covariance matrix of the empirical moments, Ω_D.

We assume that agents make decisions at a monthly frequency and derive the model's implications for population moments computed at an annual frequency, $\Psi(\Phi)$. See Appendix D for details.

We compute our estimator $\hat{\Phi}$ as:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\Psi(\Phi) - \Psi_D\right]' \Omega_D^{-1} \left[\Psi(\Phi) - \Psi_D\right].$$

Table 3 reports our parameter estimates along with GMM standard errors. Several features are worth noting. First, both the estimates of the coefficient of risk aversion and the intertemporal elasticity of substitution are close to one. The point estimates satisfy the condition $\theta < 1$ which is necessary for time-preference shocks to help explain the equity premium. The estimates also satisfy the more stringent condition (3.20), required for a positive equity premium in the absence of consumption and dividend shocks. Second, the growth rate of λ_t is estimated to be highly persistent, with a first-order serial correlation close to one (0.9936). Third, the volatility of the innovation to the growth rate of dividends is much higher than that of the innovation to the growth rate of consumption. Finally, the estimate of δ is close to one.

Table 4 compares the moments implied by the benchmark model with the estimated data moments. Recall that in estimating the model parameters we impose the restriction that the unconditional average growth rate of consumption and dividends coincide. To assess the robustness of our results to this restriction, we present two versions of the estimated data moments, one that imposes this restriction and one that does not. With one exception, the constrained and unconstrained moment estimates are similar, taking sampling uncertainty into account. The exception is the average growth rate of consumption, where the constrained and unconstrained estimates are statistically different.

Table 4 shows that the model generates a high average equity premium (5.47) and a low average risk-free rate (0.80). Neither of these model moments is statistically different from our estimates of the corresponding data moments. Even though the coefficient of relative risk aversion is close to one, the model is consistent with the observed equity premium. This result might seem surprising because our estimates of γ and $1/\psi$ are close to each other. However, the implied value of θ, the key determinant of the equity premium, is -2.56.

The basic intuition for why our model generates a high equity premium despite a low coefficient of relative risk aversion is as follows. From the perspective of the model, stocks and bonds are different in two ways. First, the model embodies the conventional source of an equity premium, namely bonds have a certain payoff that does not covary with the SDF while the payoff to stocks covaries negatively with the SDF (as long as $\pi_{dc} > 0$). Since γ is close to one, this traditional covariance effect is very small. Second, the model embodies a compensation for valuation risk that is particularly pronounced for stocks given their long-lived nature relative to bonds. Recall that, given our timing assumptions, when an agent buys a bond at t, the agent knows the value of λ_{t+1}, so the only source of risk are movements in the marginal utility of consumption at time $t+1$. In contrast, the time-t stock price depends on the value of λ_{t+j}, for all $j > 1$. So, agents are exposed to valuation risk, a risk that is particularly important because time-preference shocks are very persistent.

In Table 5 we decompose the equity premium into the valuation risk premium and the conventional risk premium. We calculate these premia at the benchmark parameter estimates using various values of ρ. Two key results emerge from this table. First, the conventional risk premium is always roughly zero. This result is consistent with Kocherlakota's (1996) discussion of why the equity premium is not explained by endowment models in which the representative agent has recursive preferences and consumption follows a martingale.

Second, consistent with the intuition discussed above, the valuation risk premium and the equity premium are increasing in ρ. The larger is ρ, the more exposed agents are to large movements in stock prices induced by time-preference shocks.

Implications for the correlation puzzle Table 6 reports the model's implications for the correlation of stock returns with consumption and dividend growth. Recall that consumption and dividends follow a random walk. In addition, the estimated process for the growth rate of the time-preference shock is close to a random walk. So, the correlation between stock returns and consumption growth implied by the model is essentially the same across different horizons. A similar property holds for the correlation between stock returns and dividend growth.

The model does well at matching the correlation between stock returns and consumption growth in the data, because this correlation is similar at all horizons. In contrast, the empirical correlation between stock returns and dividend growth increases with the time horizon. The estimation procedure chooses to match the long-horizon correlations and does less well at matching the yearly correlation. This choice is dictated by the fact that it is harder for the model to produce a low correlation between stock returns and dividend growth than it is to produce a low correlation between stock returns and consumption growth. This property reflects the fact that the dividend growth rate enters directly into the equation for stock returns (see equation (3.11)).

Implications for the risk-free rate A problem with some explanations of the equity premium is that they imply counterfactually high levels of volatility for the risk-free rate (see e.g. Boldrin, Christiano and Fisher (2001)). Table 4 shows that the volatility of the risk-free rate and stock market returns implied by our model are similar to the estimated volatilities in the data. Notice also that, taking sampling uncertainty into account, the model accounts for the correlation between the risk-free rate and stock returns.

An empirical shortcoming of the benchmark model is its implication for the persistence of the risk-free rate. Recall that, according to equation (3.18), the risk-free rate has the same persistence as the growth rate of the time-preference shock. Table 4 shows that the AR(1) coefficient of the risk free rate, as measured by the ex-post realized real returns to one-year treasury bills, is only 0.61, with a standard error of 0.11, which is substantially smaller that our estimate of ρ (0.95). We address this issue in the next section.

5. Extensions of the benchmark model

In this section we present two extensions of the benchmark model. In the first extension we present a simple perturbation of the benchmark model that renders it consistent with the observed persistence of the risk free rate. We refer to this extension as the *augmented model*. Second, we modify this extension to allow for correlation between time preference shocks and the growth rate of consumption and dividends. We refer to this version as the *quasi-production model*.

An important advantage of our benchmark model is its simplicity and its ability to account for both the equity premium and the correlation puzzle. However, this model suffers from an important shortcoming: it overstates the persistence of the risk-free rate. It is straightforward to resolve this issue by assuming that the time-preference shock is the sum of a persistent shock and an i.i.d. shock:

$$\begin{aligned} \log(\lambda_{t+1}/\lambda_t) &= x_{t+1} + \sigma_\eta \eta_{t+1}, \\ x_{t+1} &= \rho x_t + \sigma_\lambda \varepsilon^\lambda_{t+1}, \end{aligned} \tag{5.1}$$

where $\varepsilon^\lambda_{t+1}$ and η_{t+1} are uncorrelated, i.i.d. standard normal shocks. If $\sigma_\eta = 0$ and $x_1 = \log(\lambda_1/\lambda_0)$ we obtain the specification of the time-preference shock used in the benchmark model. Other things equal, the larger is σ_η, the lower the persistence of the time-preference shock.

We define the augmented model as a version of the benchmark model in which we replace equation (3.4) with (5.1). We estimate the augmented model by adding σ_η to the vector Φ and the AR(1) coefficient of the risk-free rate, τ, to our specification of Ψ_D. Tables 3, 4, and 6 report our results. With the exception of σ_η, the estimated structural parameters are very similar across the two models. With one important exception, the models' implications for the data moments are also very similar, taking sampling uncertainty into account. The exception pertains to the serial correlation of the risk-free rate, which falls from 0.95 in the benchmark model to 0.62 in the augmented model. The latter value is very close to our estimate of the analogue object in the data (0.61). According to our point estimates, the i.i.d. component of the time-preference shock accounts for 83 percent of the variance of the shock.

We now turn to a more interesting shortcoming of the benchmark and augmented models: they do not allow the growth rate of consumption and/or dividends to be correlated with

the time-preference shocks. In a production economy, time-preference shocks would generally induce changes in aggregate consumption. For example, in a simple real-business-cycle model, a persistent increase in λ_{t+1}/λ_t would lead agents to reduce current consumption and invest more in order to consume more in the future. Taken literally, an endowment economy specification does not allow for such a correlation. We can, however, modify the augmented model to mimic a production economy along this dimension by allowing the growth rate of dividends, consumption and the time-preference shock to be correlated. We refer to this extension as the quasi-production model.

We assume that the stochastic process for consumption and dividend growth is given by:

$$\begin{aligned} \log(C_{t+1}) &= \log(C_t) + \mu + \sigma_c \varepsilon^c_{t+1} + \pi_{c\lambda} \varepsilon^\lambda_{t+2}, \\ \varepsilon^c_{t+1} &\sim N(0,1), \end{aligned} \tag{5.2}$$

$$\begin{aligned} \log(D_{t+1}) &= \log(D_t) + \mu + \pi_{dc} \varepsilon^c_{t+1} + \sigma_d \varepsilon^d_{t+1} + \pi_{d\lambda} \varepsilon^\lambda_{t+2}, \\ \varepsilon^d_{t+1} &\sim N(0,1), \end{aligned}$$

where ε^c_{t+1}, ε^d_{t+1}, $\varepsilon^\lambda_{t+1}$, and η_{t+1} are mutually uncorrelated. As long as the two new parameters, $\pi_{c\lambda}$ and $\pi_{d\lambda}$ are different from zero, $\log(\lambda_{t+1}/\lambda_t)$ is correlated with $\log(C_{t+1}/C_t)$ and $\log(D_{t+1}/D_t)$. Only the innovation to time-preference shocks enters the law of motion for $\log(C_{t+1}/C_t)$ and $\log(D_{t+1}/D_t)$. So, we are not introducing any element of long-run risk into consumption or dividend growth. As in the benchmark and augmented models, both consumption and dividends are martingales.

In estimating the model we add $\pi_{c\lambda}$, $\pi_{d\lambda}$, and σ_η to the vector Φ and the AR(1) coefficient of the risk-free rate, τ, to our specification of Ψ_D. Tables 3 reports our point estimates. As in the benchmark model, γ and ψ are still close to one, although they are estimated with more imprecision. The point estimates continue to satisfy the condition $\theta < 1$, required for time-preference shocks to generate an equity premium. The absolute value of θ is smaller than in the benchmark model, which is consistent with equity premium also being smaller. Second, even though ρ continues to be close to one, the growth of λ_t is less persistent than in the benchmark model because of the i.i.d. shock in equation (5.1). The value of $\pi_{c\lambda}$ is insignificantly different from zero. In contrast, the value of $\pi_{d\lambda}$ is negative and statistically significant. Below we argue that this value allows the model to match the yearly correlation between stock returns and dividend growth.

Tables 4 and 6 reports the implications of the quasi-production model for various data moments. A number of features are worth noting. First, this version of the model generates a lower equity premium than the benchmark model (3.73 percent versus 5.47 percent). However, the equity premium is still quite large and within two standard deviations of our point estimate. Second, the average risk-free rate implied by the model is negative (-0.92), but it is still within two standard deviations of our point estimate. Third, the volatility of stock returns and the risk-free rate implied by the model are close to the point estimates. Fourth, taking sampling uncertainty into account, the model accounts for the correlation between the risk-free rate and stock returns. Fifth, the persistence of the risk-free rate implied by the model is similar to that in the data (0.55 in the model versus 0.61 in the data).

Recall that the benchmark model produces correlations between stock returns and consumption growth that are similar to those in the data. The quasi-production model continues to succeed on this dimension by setting the $\pi_{c\lambda}$ to a value that is close to zero. The coefficient $\pi_{d\lambda}$ allows the model to fit the low level of the one-year correlation between stock returns and dividend growth. The cost is that the model does less well than the benchmark model at matching the five and ten-year correlation. The reason the estimation procedure chooses to match the one-year correlation is that this correlation is estimated with more precision than the ten-year correlation.

To document the relative importance of the correlation puzzle and the equity premium puzzle, we re-estimate the model subject to the constraint that it matches the average equity premium and the average risk-free rate. We report our results in Tables 3, 4 and 6. Even though the estimates of γ and ψ are similar to those reported before, the implied value of θ goes from -0.79 to -6.65, which is why the equity premium implied by the model rises. This version of the model continues to produce low correlations between stock returns and consumption growth. However, the one-year correlation between stock returns and dividend growth implied by the model is much higher than that in the data (0.56 versus 0.08). The one-year correlation between stock returns and dividend growth is estimated much more precisely than the equity premium. So, the estimation algorithm chooses parameters for the quasi-production model that imply a lower equity premium in return for matching the one-year correlation between stock returns and dividend growth.

6. Bond term premia

As we emphasize above, the equity premium in our estimated models results primarily from the valuation risk premium. Since this valuation premium increases with the maturity of an asset, a natural way to assess the plausibility of our model is to evaluate its implications for the slope of the real yield curve.

Table 7 reports the mean and standard deviation of ex-post real returns to short-term (one-month) Treasury Bills, intermediate-term government bonds (with approximate maturity of five years), and long-term government bonds (with approximate maturity of twenty years). A number of features are worth noting. First, consistent with Alvarez and Jermann (2005), the term structure of real returns is upward sloping. Second, the real yield on long-term bonds is positive. This result is consistent with Campbell, Shiller and Viceira (2009) who report that the real yield on long-term TIPS has always been positive and is usually above two percent.

Our model implies that long-term bonds command a positive risk premium that increases with the maturity of the bond. The latter property reflects the fact that longer maturity assets are more exposed to valuation risk. Table 7 shows that, taking sampling uncertainty into account, both the augmented and the quasi-production model are consistent with the observed one-year holding returns for short-, intermediate- and long-term bonds. The table also shows that the estimated models account for the volatility of the returns on short-, intermediate-, and long-term bonds. So, our model can account for key features of the intermediate and long-term bond returns, even though these models were not used to estimate the model.

According to Table 7, the augmented and quasi-production models imply that the difference between stock and long-term bond returns is roughly 3 percent. This value is well within two standard errors of our point estimate. From the perspective of our model, the positive premium that equity commands over long-term bonds reflects the difference between an asset of infinite and twenty-year maturity. Consistent with this perspective, Binsbergen, Hueskes, Koijen, and Vrugt (2011) estimate that 90 (80) percent of the value of the S&P 500 index corresponds to dividends that accrue after the first 5 (10) years.

Piazzesi and Schneider (2007) and Beeler and Campbell (2012) argue that the bond term premium and the yield on long-term bonds are useful for discriminating between competing asset pricing models. For example, they stress that long-run risk models, of the type

pioneered by Bansal and Yaron (2004), imply negative long-term bond yields and a negative bond term premium. The intuition is as follows: in a long-run risk model agents are concerned that consumption growth may be dramatically lower in some future state of the world. Since bonds promise a certain payout in all states of the world, they offer insurance against this possibility. The longer the maturity of the bond, the more insurance it offers and the higher is its price. So, the term premium is downward sloping. Indeed, the return on long-term bonds may be negative. Beeler and Campbell (2012) show that the return on a 20-year real bond in the Bansal, Kiku and Yaron (2012) model is -0.88.

Standard rare-disaster models also imply a downward sloping term structure for real bonds and a negative real yield on long-term bonds. See, for example the benchmark model in Nakamura, Steinsson, Barro, and Ursúa (2010). According to these authors, these implications can be reversed by introducing the possibility of default on bonds and to assume that probability of partial default is increasing in the maturity of the bond.[6] So, we cannot rule out the possibility that other asset-pricing models can account for bond term premia and the rate of return on long-term bonds. Still, it seems clear that valuation risk is a natural explanation of these features of the data.

We conclude with an interesting challenge posed to a large class of asset pricing models by Binsbergen, Brandt, and Koijen (2012). Using data over the period 1996 to 2009, these authors decompose the S&P500 index into portfolios of short-term and long-term dividend strips. The first portfolio entitles the holder to the realized dividends of the index for a period of up to three years. The second portfolio is a claim on the remaining dividends. Binsbergen et al (2012) find that the short-term dividend portfolio has a higher risk premium than the long-term dividend portfolio, i.e. there is a negative stock term premium. They argue that this observation is inconsistent with habit-formation, long-run risk models and standard of rare-disaster models.[7] Our model, too, has difficulty in accounting for the Binsbergen et al (2012) negative stock term premium. Of course, our sample is very different from theirs and their negative stock term premium result is heavily influenced by the recent financial crisis.[8] An open, important question is whether the Binsbergen et al (2012) results hold over

[6] Nakamura et al (2010) consider a version of their model in which the probability of partial default on a perpetuity is 84 percent, while the probability of partial default on short-term bonds is 40 percent. This model generates a positive term premium and a positive return on long-term bonds.

[7] Recently, Nakamura et al (2012) show that a time-vaying rare disaster model in which the component of consumption growth due to a rare disaster follows an AR(1) process, is consistent with the Binsbergen et al (2012) results.

[8] See Figure 6 of Binsbergen et al (2011).

a longer sample period.

7. Conclusion

In this paper we argue that allowing for demand shocks in an otherwise standard asset pricing model substantially improves the performance of the model. Specifically, it allows the model to account for the equity premium, bond term premia, and the correlation puzzle. According to our estimates, valuation risk is by far the most important determinant of the equity premium and the bond term premia.

We introduced these demand shocks by allowing for shocks to a representative agent's rate of time preference. These shocks can be measured as movements in the risk-free rate. Estimated versions of our model are consistent with the key empirical properties of the risk-free rate.

The recent literature has incorporated many interesting features into standard asset-pricing models to improve their performance. Prominent examples, include habit formation, long-run risk, time-varying endowment volatility, and model ambiguity. We abstract from these features to isolate the empirical role of valuation risk. But they are, in principle, complementary to valuation risk and could be incorporated into our analysis. We leave this task for future research.

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8. Appendix

8.1. Appendix A

This appendix provides a simple overlapping-generations model in which uncertainty about the growth rate of the population gives rise to shocks in the demand for assets. In period t there are x_t young agents and x_{t-1} old agents. Young agents have an endowment (labor income) of w and can buy S_t stock shares. These shares yield $P_{t+1} + D_{t+1}$ at time $t+1$, where P_{t+1} is the price at which the generation that is young at time $t+1$ is willing to buy the stock. We normalize the total number of stock shares to one. The economy's time t output is: $x_t w + D_t$. We will show that x_t and D_t represent two sources of aggregate risk.

Consider the optimization problem faced by a young agent at time t. Assume for simplicity that agents have logarithmic preferences. Each young agent solves the following problem:

$$\max_{c_t^y, c_{t+1}^o, S_t} \left[\log\left(c_t^y\right) + \delta E_t\left(\log\left(c_{t+1}^o\right)\right)\right],$$

subject to the resource constraint as young

$$c_t^y = w - P_t S_t,$$

and the resource constraint as old

$$c_{t+1}^o = S_t\left(P_{t+1} + D_{t+1}\right).$$

The first-order condition for S_t is:

$$P_t\left(w - P_t S_t\right)^{-1} = \delta E_t\left[\left(S_t\left(P_{t+1} + D_{t+1}\right)\right)^{-1}\left(P_{t+1} + D_{t+1}\right)\right],$$

or

$$\frac{P_t S_t}{w - P_t S_t} = \delta. \tag{8.1}$$

In period t, the equilibrium in the stock market requires that the young buy all the shares from the old:

$$x_t S_t = 1.$$

Substituting the equilibrium condition in equation (8.1), we obtain the solution for the stock price:

$$P_t = \frac{\delta}{1+\delta} x_t w.$$

We can compute the risk-free rate using the condition:

$$(c_t^y)^{-1} = R_{f,t+1}\delta E_t\left(\left(c_{t+1}^o\right)^{-1}\right).$$

Substituting in the equilibrium values of c_t^y and c_{t+1}^o obtain:

$$R_{f,t+1} = E_t\left[\left(\frac{x_{t+1}}{x_t} + \frac{D_{t+1}}{\frac{\delta}{1+\delta}x_t w}\right)^{-1}\right]^{-1}.$$

The equity premium is given by:

$$E_t\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) - R_{f,t+1} = E_t\left(\frac{x_{t+1}}{x_t} + \frac{D_{t+1}}{\frac{\delta}{1+\delta}x_t w}\right) - E_t\left[\left(\frac{x_{t+1}}{x_t} + \frac{D_{t+1}}{\frac{\delta}{1+\delta}x_t w}\right)^{-1}\right]^{-1}.$$

The risk premium thus depends on the volatility of x_{t+1}/x_t, the volatility of dividends and the covariance between x_{t+1}/x_t and D_{t+1}.

8.2. Appendix B

In this appendix, we solve the model in Section 3 analytically for the case of CRRA utility. Let $C_{a,t}$ denote the consumption of the representative agent at time t. The representative agent solves the following problem:

$$U_t = \max E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} \frac{C_{a,t+i}^{1-\gamma}}{1-\gamma},$$

subject to the flow budget constraints

$$W_{a,i+1} = R_{c,i+1}\left(W_{a,i} - C_{a,i}\right),$$

for all $i \geq t$. The variable $R_{c,i+1}$ denotes the gross return to a claim that pays the aggregate consumption as in equation (3.8), financial wealth is $W_{a,i} = \left(P_{c,i} + C_i\right) S_{a,i}$, and $S_{a,i}$ is the number of shares on the claim to aggregate consumption held by the representative agent. The first-order condition for $S_{a,t+i+1}$ is:

$$\delta^i \lambda_{t+i} C_{a,t+i}^{-\gamma} = E_t\left(\delta^{i+1}\lambda_{t+i+1} C_{a,t+i+1}^{-\gamma} R_{c,i+1}\right).$$

In equilibrium, $C_{a,t} = C_t$, $S_{a,t} = 1$. The equilibrium value of the intertemporal marginal rate of substitution is:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{8.2}$$

The Euler equation for stock returns is the familiar,

$$E_t\left[M_{t+1}R_{c,t+1}\right]=1.$$

We now solve for $P_{c,t}$. It is useful to write $R_{c,t+1}$ as

$$R_{c,t+1}=\frac{\left(P_{c,t+1}/C_{t+1}+1\right)}{P_{c,t}/C_t}\left(\frac{C_{t+1}}{C_t}\right).$$

In equilibrium:

$$E_t\left[M_{t+1}\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)\left(\frac{C_{t+1}}{C_t}\right)\right]=\frac{P_{c,t}}{C_t}. \tag{8.3}$$

Replacing the value of M_{t+1} in equation (8.3):

$$E_t\left[\delta\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)\left(\frac{C_{t+1}}{C_t}\right)\right]=\frac{P_{c,t}}{C_t}.$$

Using the fact that λ_{t+1}/λ_t is known as of time t we obtain:

$$\delta\frac{\lambda_{t+1}}{\lambda_t}E_t\left[\exp\left(\mu+\sigma_c\varepsilon^c_{t+1}\right)^{1-\gamma}\left(\frac{P_{ct+1}}{C_{t+1}}+1\right)\right]=\frac{P_{ct}}{C_t}.$$

We guess and verify that P_{ct+1}/C_{t+1} is independent of ε^c_{t+1}. This guess is based on the fact that the model's price-consumption ratio is constant absent time-preference shocks. Therefore,

$$\delta\frac{\lambda_{t+1}}{\lambda_t}\exp\left[(1-\gamma)\mu+(1-\gamma)^2\sigma_c^2/2\right]E_t\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)=\frac{P_{c,t}}{C_t}. \tag{8.4}$$

We now guess that there are constants k_0, k_1,..., such that

$$\frac{P_{c,t}}{C_t}=k_0+k_1\left(\lambda_{t+1}/\lambda_t\right)+k_2\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho}+k_3\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho+\rho^2}+... \tag{8.5}$$

Using this guess,

$$\begin{aligned}
&E_t\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)\\
=\;&E_t\left(k_0+k_1\left(\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda\varepsilon^\lambda_{t+2}\right)\right)+k_2\left(\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda\varepsilon^\lambda_{t+2}\right)\right)^{1+\rho}+...+1\right)\\
=\;&k_0+k_1\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda^2/2\right)+k_2\left(\lambda_{t+1}/\lambda_t\right)^{\rho(1+\rho)}\exp\left((1+\rho)^2\sigma_\lambda^2/2\right)+...+1.
\end{aligned} \tag{8.6}$$

Substituting equations (8.5) and (8.6) into equation (8.4) and equating coefficients leads to the following solution for the constants k_i:

$$\begin{aligned}
k_0&=0,\\
k_1&=\delta\exp\left[(1-\gamma)\mu+(1-\gamma)^2\sigma_c^2/2\right],
\end{aligned}$$

and for $n \geq 2$

$$k_n = k_1^n \exp\left\{\left[1 + (1+\rho)^2 + \left(1+\rho+\rho^2\right)^2 + ... + \left(1 + ... + \rho^{n-2}\right)^2\right]\sigma_\lambda^2/2\right\}.$$

We assume that the series $\{k_n\}$ converges, so that the equilibrium price-consumption ratio is given by equation (8.5). Hence, the realized return on the consumption claim is

$$R_{c,t+1} = \frac{C_{t+1}}{C_t}\frac{k_1\left(\lambda_{t+2}/\lambda_{t+1}\right) + k_2\left(\lambda_{t+2}/\lambda_{t+1}\right)^{1+\rho} + ... + 1}{k_1\left(\lambda_{t+1}/\lambda_t\right) + k_2\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho} + ...}. \tag{8.7}$$

The equation that prices the one-period risk-free asset is:

$$E_t\left[M_{t+1}R_{f,t+1}\right] = 1.$$

Taking logarithms on both sides of this equation and noting that $R_{f,t+1}$ is known at time t, we obtain:

$$r_{f,t+1} = -\log E_t\left(M_{t+1}\right).$$

Using equation (8.2),

$$\begin{aligned} E_t\left(M_{t+1}\right) &= \delta\frac{\lambda_{t+1}}{\lambda_t}E_t\left[\exp\left(-\gamma\left(\mu + \sigma_c\varepsilon_{t+1}^c\right)\right)\right] \\ &= \delta\frac{\lambda_{t+1}}{\lambda_t}\exp\left(-\gamma\mu + \gamma^2\sigma_c^2/2\right). \end{aligned}$$

Therefore,

$$r_{f,t+1} = -\log\left(\delta\right) - \log\left(\lambda_{t+1}/\lambda_t\right) + \gamma\mu - \gamma^2\sigma_c^2/2.$$

Using equation (3.4), we obtain

$$E\left[\left(\lambda_{t+1}/\lambda_t\right)^{-1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right).$$

We can then write the unconditional risk-free rate as:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2).$$

Thus, the equity premium is given by:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right],$$

which can be written as:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right].$$

8.3. Appendix C

This appendix provides a detailed derivation of the equilibrium of the model economy where the representative agent has Epstein-Zin preferences and faces time-preference shocks. The agent solves the following problem:

$$U\left(W_t\right)=\max_{C_t}\left[\lambda_t C_t^{1-1/\psi}+\delta\left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}, \tag{8.8}$$

where $U_{t+1}^*=\left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}$. The optimization is subject to the following budget constraint:

$$W_{t+1}=R_{c,t+1}\left(W_t-C_t\right).$$

The agent takes as given the stochastic processes for the return on the consumption claim $R_{c,t+1}$ and the preference shock λ_{t+1}. For simplicity, we omit the dependence of life-time utility on the processes for λ_{t+1} and $R_{c,t+1}$.

The first-order condition with respect to consumption is,

$$\lambda_t C_t^{-1/\psi}=\delta\left(U_{t+1}^*\right)^{-1/\psi}\left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1}E_t\left(U\left(W_{t+1}\right)^{-\gamma}U'\left(W_{t+1}\right)R_{c,t+1}\right),$$

and the envelope condition is

$$U'\left(W_t\right)=U\left(W_t\right)^{1/\psi}\delta\left(U_{t+1}^*\right)^{-1/\psi}\left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1}E_t\left(U\left(W_{t+1}\right)^{-\gamma}U'\left(W_{t+1}\right)R_{c,t+1}\right).$$

Combining the first-order condition and the envelope condition we obtain:

$$U'\left(W_t\right)=U\left(W_t\right)^{1/\psi}\lambda_t C_t^{-1/\psi}. \tag{8.9}$$

This equation can be used to replace the value of $U'\left(W_{t+1}\right)$ in the first order condition:

$$\lambda_t C_t^{-1/\psi}=\delta\left(U_{t+1}^*\right)^{-1/\psi}\left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1}E_t\left(U\left(W_{t+1}\right)^{1/\psi-\gamma}\lambda_{t+1}C_{t+1}^{-1/\psi}R_{c,t+1}\right).$$

Using the expression for U_{t+1}^* this last equation can be written compactly after some algebra as,

$$1=E_t\left(M_{t+1}R_{c,t+1}\right). \tag{8.10}$$

Here, M_{t+1} is the stochastic discount factor, or intertemporal marginal rate of substitution, which is given by:

$$M_{t+1}=\delta\frac{\lambda_{t+1}}{\lambda_t}\frac{U\left(W_{t+1}\right)^{1/\psi-\gamma}}{\left(U_{t+1}^*\right)^{1/\psi-\gamma}}\frac{C_{t+1}^{-1/\psi}}{C_t^{-1/\psi}}.$$

We guess and verify the policy function for consumption and the form of the utility function. As in Weil (1989) and Epstein and Zin (1991), we guess that:

$$\begin{aligned} U(W_t) &= a_t W_t, \\ C_t &= b_t W_t. \end{aligned}$$

Replacing these guesses in equation (8.9) and simplifying yields:

$$a_t^{1-1/\psi} = \lambda_t b_t^{-1/\psi}. \tag{8.11}$$

Substitute the guess also in the Hamilton-Jacobi-Bellman equation (8.8) and simplifying we obtain:

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta \left(\left[E_t \left(\left(a_{t+1} \frac{W_{t+1}}{W_t} \right)^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}.$$

Finally, use the budget constraint to replace W_{t+1}/W_t and get

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta \left(\left[E_t \left(\left(a_{t+1} \left(1 - b_t \right) R_{c,t+1} \right)^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}. \tag{8.12}$$

Equations (8.11) and (8.12) give a solution to a_t and b_t.

Combining equations (8.11) and (8.12) gives:

$$\lambda_t b_t^{-1/\psi} \left(1 - b_t \right) = \delta \left(\left[E_t \left(\left(a_{t+1} \left(1 - b_t \right) R_{c,t+1} \right)^{1-\gamma} \right) \right]^{1/(1-\gamma)} \right)^{1-1/\psi},$$

which we can replace in the expression for the stochastic discount factor together with (8.11) to obtain:

$$M_{t+1} = \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right)^{(1-\gamma)/(1-1/\psi)} \left(\frac{b_{t+1}}{b_t} \left(1 - b_t \right) \right)^{-((1/\psi-\gamma)/\psi)/(1-1/\psi)} \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left(R_{c,t+1} \right)^{1/\psi-\gamma}.$$

Now note that $\theta = \left(1 - \gamma \right) / \left(1 - 1/\psi \right)$, and that

$$\frac{\frac{C_{t+1}}{C_t}}{R_{c,t+1}} = \frac{b_{t+1} R_{c,t+1} \left(W_t - C_t \right) / \left(b_t W_t \right)}{R_{c,t+1}} = \frac{b_{t+1} \left(1 - b_t \right)}{b_t},$$

to finally get,

$$M_{t+1} = \left(\delta \frac{\lambda_{t+1}}{\lambda_t} \right)^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(R_{c,t+1} \right)^{\theta-1}.$$

Taking logarithms on both sides and equating the consumption of the representative agent to aggregate consumption yields equation (3.9).

The rest of the equilibrium derivation solves for $r_{c,t+1}$ and $r_{d,t+1}$ as well as for the risk free rate $r_{f,t+1}$. Up to now, we did not need to specify the process for the time-preference shock, the process for consumption growth or the process for dividend growth. We solve the rest of the model assuming the general processes of Section 5 given in equations (5.1) through (**??**). Recovering the equilibrium values for the benchmark model is easily done by setting $\pi_{c\lambda} = \pi_{d\lambda} = \sigma_\eta = 0$.

To price the consumption claim, we must solve the pricing condition:

$$E_t\left[\exp\left(m_{t+1} + r_{c,t+1}\right)\right] = 1.$$

Guess that the log of the price consumption ratio, $z_{ct} \equiv \log\left(P_{c,t}/C_t\right)$, is

$$z_{ct} = A_{c0} + A_{c1}x_{t+1} + A_{c2}\eta_{t+1},$$

and approximate

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta c_{t+1}. \tag{8.13}$$

Replacing the approximation and the guessed solution for z_{ct} on the pricing condition gives

$$E_t\left[\exp\left(\theta\log\left(\delta\right) + \theta\log\left(\lambda_{t+1}/\lambda_t\right) + \left(1-\gamma\right)\Delta c_{t+1} + \theta\kappa_{c0} + \theta\kappa_{c1}z_{ct+1} - \theta z_{ct}\right)\right] = 1.$$

Calculation of the expectation requires some algebra and yields the equation

$$\begin{aligned} 0 &= \theta\log\left(\delta\right) + \theta\kappa_{c0} + \theta\kappa_{c1}A_{c0} - \theta A_{c0} + \left(1-\gamma\right)\mu + \left(1-\gamma\right)^2\sigma_c^2/2 + \left(\left(1-\gamma\right)\pi_{c\lambda} + \theta\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2 \\ &\quad + \left(\theta\kappa_{c1}A_{c2}\right)^2/2 + \theta\left(\kappa_{c1}A_{c1}\rho - A_{c1} + 1\right)x_{t+1} + \theta\left(\sigma_\eta - A_{c2}\right)\eta_{t+1}. \end{aligned}$$

In equilibrium, this equation must hold in all possible states resulting in the restrictions:

$$A_{c1} = \frac{1}{1-\kappa_{c1}\rho},$$

$$A_{c2} = \sigma_\eta,$$

and

$$A_{c0} = \frac{\theta\log\left(\delta\right) + \theta\kappa_{c0} + \left(1-\gamma\right)\mu + \left(1-\gamma\right)^2\sigma_c^2/2 + \left(\left(1-\gamma\right)\pi_{c\lambda} + \theta\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2 + \left(\theta\kappa_{c1}A_{c2}\right)^2/2}{\theta\left(1-\kappa_{c1}\right)}.$$

To solve for the risk free rate, we again use the stochastic discount factor to price the risk free asset. In logs, the Euler equation is

$$\begin{aligned} r_{f,t+1} &= -\log\left(E_t\left(\exp\left(m_{t+1}\right)\right)\right) \\ &= -\log\left(E_t\left(\exp\left(\theta\log\left(\delta\right) + \theta\log\left(\lambda_{t+1}/\lambda_t\right) - \frac{\theta}{\psi}\Delta c_{t+1} + \left(\theta-1\right)r_{c,t+1}\right)\right)\right). \end{aligned}$$

Using equation (8.13), we get:

$$r_{f,t+1} = -\log\left(E_t\left(\exp\left(\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)-\gamma\Delta c_{t+1}+\left(\theta-1\right)\left(\kappa_{c0}+\kappa_{c1}z_{ct+1}-z_{ct}\right)\right)\right)\right).$$

Substituting in the consumption process and the solution for the price consumption ratio, and after much algebra, we obtain,

$$\begin{aligned} r_{f,t+1} &= -\log\left(\delta\right)-\log\left(\lambda_{t+1}/\lambda_t\right)+\frac{\mu}{\psi}-\frac{1-\theta}{\theta}\left(\left(1-\gamma\right)\pi_{c\lambda}+\theta\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2 \\ &\quad+\left[\frac{\theta-1}{\theta}\left(1-\gamma\right)^2-\gamma^2\right]\sigma_c^2/2-\left(\left(\theta-1\right)\kappa_{c1}A_{c1}\sigma_\lambda-\gamma\pi_{c\lambda}\right)^2/2-\left(1-\theta\right)\left(\kappa_{c1}A_{c2}\right)^2/2. \end{aligned}$$

Setting $\pi_{c\lambda}=\sigma_\eta=0$ we get the benchmark-model value of the risk free rate (3.18).

Finally, we price a claim to dividends. Again, we assume the price dividend ratio is given by

$$z_{dt} = A_{d0}+A_{d1}x_{t+1}+A_{d2}\eta_{t+1},$$

and approximate the log linearized return to the claim to the dividend:

$$r_{d,t+1} = \kappa_{d0}+\kappa_{d1}z_{dt+1}-z_{dt}+\Delta d_{t+1}. \tag{8.14}$$

The pricing condition is

$$E_t\left[\exp\left(m_{t+1}+r_{d,t+1}\right)\right]=1.$$

Substituting in for m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$,

$$\begin{aligned} 1 &= E_t\left(\exp\left(\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)-\frac{\theta}{\psi}\Delta c_{t+1}+\left(\theta-1\right)\left(\kappa_{c0}+\kappa_{c1}z_{ct+1}-z_{ct}+\Delta c_{t+1}\right)\right.\right. \\ &\quad\left.\left.+\kappa_{d0}+\kappa_{d1}z_{dt+1}-z_{dt}+\Delta d_{t+1}\right)\right). \end{aligned}$$

Further substitution of the consumption growth and dividend processes and of the price consumption and price dividend ratios, and after significant algebra, we get that in equilibrium

$$A_{d1} = \frac{1}{1-\kappa_{d1}\rho},$$

$$A_{d2} = \sigma_\eta,$$

and

$$\begin{aligned} &A_{d0}\left(1-\kappa_{d1}\right) \\ &= \theta\log\left(\delta\right)+\left(1-\gamma\right)\mu+\left(\left(\left(\theta-1\right)\kappa_{c1}A_{c1}+\kappa_{d1}A_{d1}\right)\sigma_\lambda-\gamma\pi_{c\lambda}+\pi_{d\lambda}\right)^2/2+\left(\pi_{dc}-\gamma\sigma_c\right)^2/2 \\ &\quad+\kappa_{d0}+\left(\theta-1\right)\left(\kappa_{c0}+\kappa_{c1}A_{c0}-A_{c0}\right)+\left(\kappa_{d1}A_{d2}+\left(\theta-1\right)\kappa_{c1}A_{c2}\right)^2/2+\sigma_d^2/2. \end{aligned}$$

Having solved for these constants, we can compute the expected return on the dividend claim $E_t(r_{d,t+1})$. When $\pi_{c\lambda} = \pi_{d\lambda} = \sigma_\eta = 0$ we obtain the benchmark-model value of $E_t(r_{d,t+1})$ given in equation (3.17).

We are now solve for the conditional risk premium:

$$E_t(r_{d,t+1}) - r_{f,t+1} = E_t(\kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}) - r_{f,t+1}.$$

Substituting the values of z_{dt} and Δd_{t+1},

$$\begin{aligned} E_t(r_{d,t+1}) - r_{f,t+1} &= E_t\left(\kappa_{d0} + \kappa_{d1}\left(A_{d0} + A_{d1}x_{t+2} + A_{d2}\eta_{t+2}\right) - A_{d0} - A_{d1}x_{t+1} - A_{d2}\eta_{t+1}\right) \\ &\quad + E_t\left(\mu + \sigma_d \varepsilon^d_{t+1} + \pi_{d\lambda}\varepsilon^\lambda_{t+2} + \pi_{dc}\varepsilon^c_{t+1}\right) - r_{f,t+1}. \end{aligned}$$

Computing expectations:

$$E_t(r_{d,t+1}) - r_{f,t+1} = \kappa_{d0} + \kappa_{d1}(A_{d0} + A_{d1}\rho x_{t+1}) - A_{d0} - A_{d1}x_{t+1} - A_{d2}\eta_{t+1} + \mu - r_{f,t+1}.$$

Substituting the values of A_{d0} and $r_{f,t+1}$ and simplifying, we obtain:

$$\begin{aligned} E_t(r_{d,t+1}) - r_{f,t+1} &= \pi_{dc}(2\gamma\sigma_c - \pi_{dc})/2 - \sigma_d^2/2 + \kappa_{d1}(2(1-\theta)\kappa_{c1} - \kappa_{d1})\sigma_\eta^2/2 \\ &\quad + (\kappa_{d1}A_{d1}\sigma_\lambda + \pi_{d\lambda})(2((1-\theta)\kappa_{c1}A_{c1}\sigma_\lambda + \gamma\pi_{c\lambda}) - (\kappa_{d1}A_{d1}\sigma_\lambda + \pi_{d\lambda}))/2. \end{aligned}$$

8.4. Appendix D

This appendix provides details on the model's implications for population moments computed at an annual frequency.

Yearly average of consumption growth:

$$E\left(\sum_{j=0}^{11}\Delta c_{t-j}\right) = 12\mu.$$

Yearly standard deviation of consumption growth:

$$\sqrt{V\left(\sum_{j=0}^{11}\Delta c_{t-j}\right)} = \sqrt{12(\sigma_c^2 + \pi_{c\lambda}^2)}.$$

Yearly mean growth rate of dividends:

$$E\left(\sum_{j=0}^{11}\Delta d_{t-j}\right) = 12\mu.$$

Yearly standard deviation of the mean growth rate of dividends:

$$\sqrt{V\left(\sum_{j=0}^{11}\Delta d_{t-j}\right)}=\sqrt{12(\pi_{dc}^2+\sigma_d^2+\pi_{d\lambda}^2)}.$$

Yearly risk-free rate:

$$r_f\equiv E\left(\sum_{j=0}^{11}r_{f,t+1-j}\right)=12\left\{\begin{array}{c}-\log(\delta)+\frac{\mu}{\psi}-\left(\gamma^2-\frac{\theta-1}{\theta}(1-\gamma)^2\right)\sigma_c^2/2+(\theta-1)\left(\kappa_{c1}A_{c2}\right)^2/2\\+\frac{\theta-1}{\theta}\left((1-\gamma)\pi_{c\lambda}+\theta\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2-\left((\theta-1)\kappa_{c1}A_{c1}\sigma_\lambda-\gamma\pi_{c\lambda}\right)^2/2\end{array}\right\}.$$

The n-period risk-free rate:

$$\begin{aligned}\sum_{j=0}^{n}r_{f,t+1-j}=&\text{constant}-\sum_{j=0}^{n}\left(\sigma_\eta\eta_{t-j}+x_{t-j}\right)\\=&\text{constant}-\sigma_\eta\eta_t-\sigma_\lambda\varepsilon_t^\lambda-\rho\sigma_\lambda\varepsilon_{t-1}^\lambda-\rho^2\varepsilon_{t-2}^\lambda+\cdots\\&-\sigma_\eta\eta_{t-1}-\sigma_\lambda\varepsilon_{t-1}^\lambda-\rho\sigma_\lambda\varepsilon_{t-2}^\lambda-\rho^2\varepsilon_{t-3}^\lambda+\cdots\\&+\cdots\\&-\sigma_\eta\eta_{t-n}-\sigma_\lambda\varepsilon_{t-n}^\lambda-\rho\sigma_\lambda\varepsilon_{t-n-1}^\lambda-\rho^2\sigma_\lambda\varepsilon_{t-n-2}^\lambda+\cdots\end{aligned}$$

The variance of the n-period risk-free rate is given by:

$$V\left(\sum_{j=0}^{n}r_{f,t+1-j}\right)=(n+1)\sigma_\eta^2+\sum_{m=0}^{n-1}\left(\sigma_\lambda\sum_{j=0}^{m}\rho^j\right)^2+\frac{\left(\sigma_\lambda\sum_{j=0}^{n}\rho^j\right)^2}{1-\rho^2}.$$

The covariance between the year t and the year $t-1$ interest rate is given by:

$$
\begin{aligned}
&E\left\{[\sum_{j=0}^{11}(r_{f,t+1-j}-r_f)][\sum_{j=0}^{11}(r_{f,t+1-12-j}-r_f)]\right\}\\
=&E[(\sum_{j=0}^{11}[x_{t-j}+\sigma_\eta\eta_{t-j}])(\sum_{j=0}^{11}[x_{t-12-j}+\sigma_\eta\eta_{t-12-j}])\\
=&E(\sum_{j=0}^{11}x_{t-j})(\sum_{j=0}^{11}x_{t-12-j})\\
=&E\Big[\sigma_\lambda\varepsilon_t^\lambda+\rho\sigma_\lambda\varepsilon_{t-1}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-2}^\lambda+\cdots+\rho^{22}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{23}x_{t-23}\\
&+\sigma_\lambda\varepsilon_{t-1}^\lambda+\rho\sigma_\lambda\varepsilon_{t-2}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-3}^\lambda+\cdots+\rho^{21}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{22}x_{t-23}\\
&+\sigma_\lambda\varepsilon_{t-2}^\lambda+\rho\sigma_\lambda\varepsilon_{t-3}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-4}^\lambda+\cdots+\rho^{20}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{21}x_{t-23}\\
&+\cdots+\\
&+\sigma_\lambda\varepsilon_{t-11}^\lambda+\rho\sigma_\lambda\varepsilon_{t-12}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-13}^\lambda+\cdots+\rho^{11}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{12}x_{t-23}\Big]\\
&\times\Big[\sigma_\lambda\varepsilon_{t-12}^\lambda+\rho\sigma_\lambda\varepsilon_{t-13}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-14}^\lambda+\cdots+\rho^{10}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{11}x_{t-23}\\
&+\sigma_\lambda\varepsilon_{t-13}^\lambda+\rho\sigma_\lambda\varepsilon_{t-14}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-15}^\lambda+\cdots+\rho^{9}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{10}x_{t-23}\\
&+\sigma_\lambda\varepsilon_{t-14}^\lambda+\rho\sigma_\lambda\varepsilon_{t-15}^\lambda+\rho^2\sigma_\lambda\varepsilon_{t-16}^\lambda+\cdots+\rho^{8}\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho^{9}x_{t-23}\\
&+\cdots+\\
&\sigma_\lambda\varepsilon_{t-22}^\lambda+\rho x_{t-23}\\
&+x_{t-23}\Big].
\end{aligned}
$$

Grouping terms we obtain:

$$\sigma_\lambda^2\sum_{m=0}^{10}E(\varepsilon_{t-12-m}^\lambda)^2\rho^{m+1}\sum_{n=0}^{m}\rho^n\sum_{j=0}^{11}\rho^j+E(x_{t-23}^2)\rho^{12}\sum_{n=0}^{11}\rho^n\sum_{j=0}^{11}\rho^j,$$

which simplifies to

$$\sigma_\lambda^2\sum_{m=0}^{10}\rho^{m+1}\sum_{n=0}^{m}\rho^n\sum_{j=0}^{11}\rho^j+\frac{\sigma_\lambda^2}{1-\rho^2}\rho^{12}\sum_{n=0}^{11}\rho^n\sum_{j=0}^{11}\rho^j.$$

The AR(1) statistic is given by:

$$\frac{\sigma_\lambda^2\sum_{m=0}^{10}\rho^{m+1}\sum_{n=0}^{m}\rho^n\sum_{j=0}^{11}\rho^j+\frac{\sigma_\lambda^2}{1-\rho^2}\rho^{12}\sum_{n=0}^{11}\rho^n\sum_{j=0}^{11}\rho^j}{12\sigma_\eta^2+\sum_{m=0}^{11}\left(\sigma_\lambda\sum_{j=0}^{m}\rho^j\right)^2+\frac{\left(\sigma_\lambda\sum_{j=0}^{11}\rho^j\right)^2}{1-\rho^2}}.$$

The one-year average equity return is:

$$r_d \equiv E\left(\sum_{j=0}^{11} r_{d,t-j}\right) = 12\left\{\kappa_{d0} + (\kappa_{d1}-1)A_{d0} + \mu\right\}.$$

The n-period standard deviation of equity returns is given by the expression below. Notice that

$$\begin{aligned}
r_{d,t} =& \text{constant} + \kappa_{d1} z_{d,t} - z_{d,t-1} + \sigma_d \varepsilon_t^d + \pi_{d\lambda}\varepsilon_{t+1}^{\lambda} + \pi_{dc}\varepsilon_t^c \\
=& \text{constant} + \kappa_{d1}\sigma_\eta \eta_{t+1} - \sigma_\eta \eta_t + A_{d1}\kappa_{d1} x_{t+1} - A_{d1} x_t + \sigma_d \varepsilon_t^d + \pi_{d\lambda}\varepsilon_{t+1}^{\lambda} + \pi_{dc}\varepsilon_t^c \\
=& \text{constant} + \kappa_{d1}\sigma_\eta \eta_{t+1} - \sigma_\eta \eta_t \\
&+ A_{d1}\kappa_{d1}\left(\sum_{m=0}^{\infty} \rho^m \sigma_\lambda \varepsilon_{t+1-m}^{\lambda}\right) - A_{d1}\left(\sum_{m=0}^{\infty} \rho^m \sigma_\lambda \varepsilon_{t-m}^{\lambda}\right) + \sigma_d \varepsilon_t^d + \pi_{d\lambda}\varepsilon_{t+1}^{\lambda} + \pi_{dc}\varepsilon_t^c \\
=& \text{constant} + \kappa_{d1}\sigma_\eta \eta_{t+1} - \sigma_\eta \eta_t \\
&+ (A_{d1}\kappa_{d1}\sigma_\lambda + \pi_{d\lambda})\varepsilon_{t+1}^{\lambda} + A_{d1}(\kappa_{d1}\rho - 1)\left(\sum_{m=0}^{\infty} \rho^m \sigma_\lambda \varepsilon_{t-m}^{\lambda}\right) + \sigma_d \varepsilon_t^d + \pi_{dc}\varepsilon_t^c.
\end{aligned}$$

Note that $(\kappa_{d,1}\rho - 1)A_{d,1} = -1$, so

$$r_{d,t} = \text{constant} + \kappa_{d1}\sigma_\eta \eta_{t+1} - \sigma_\eta \eta_t + (A_{d1}\kappa_{d1}\sigma_\lambda + \pi_{d\lambda})\varepsilon_{t+1}^{\lambda} - \sigma_\lambda\left(\sum_{m=0}^{\infty} \rho^m \varepsilon_{t-m}^{\lambda}\right) + \sigma_d \varepsilon_t^d + \pi_{dc}\varepsilon_t^c$$

Then

$$\begin{aligned}
\sum_{j=0}^{n} r_{d,t-j} =& \text{constant} + \sum_{j=0}^{n}\left\{\kappa_{d1}\sigma_\eta \eta_{t+1-j} - \sigma_\eta \eta_{t-j} + (A_{d1}\kappa_{d1}\sigma_\lambda + \pi_{d\lambda})\varepsilon_{t+1-j}^{\lambda} - \sigma_\lambda\left(\sum_{m=0}^{\infty} \rho^m \varepsilon_{t-m-j}^{\lambda}\right)\right. \\
&\left. + \sigma_d \varepsilon_{t-j}^d + \pi_{dc}\varepsilon_{t-j}^c\right\}
\end{aligned}$$

So

$$\begin{aligned}
V\left(\sum_{j=0}^{n} r_{d,t-j}\right) =& [\kappa_{d1}^2 + n(\kappa_{d1}-1)^2 + 1]\sigma_\eta^2 \\
&+ (A_{d1}\kappa_{d1}\sigma_\lambda + \pi_{d\lambda})^2 + \sum_{j=0}^{n-1}\left(A_{d1}\kappa_{d1}\sigma_\lambda + \pi_{d\lambda} - \sum_{m=0}^{j} \rho^j \sigma_\lambda\right)^2 \\
&+ \frac{\sigma_\lambda^2}{1-\rho^2}\left(\sum_{j=0}^{n} \rho^j\right)^2 + (n+1)(\sigma_d^2 + \pi_{dc}^2)
\end{aligned}$$

The one-year covariance between consumption and dividend growth is given by:

$$
\begin{aligned}
&E\left[\sum_{j=0}^{11}(\Delta c_{t-j}-\mu)\right]\left[\sum_{j=0}^{11}(\Delta d_{t-j}-\mu)\right] \\
&= E\left[\sum_{j=0}^{11}\left(\pi_{c\lambda}\varepsilon^{\lambda}_{t+2-j}+\sigma_c\varepsilon^{c}_{t+1-j}\right)\right]\left[\sum_{j=0}^{11}\left(\sigma_d\varepsilon^{d}_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}+\pi_{dc}\varepsilon^{c}_{t+1-j}\right)\right] \\
&= 12(\pi_{c\lambda}\pi_{d\lambda}+\pi_{dc}\sigma_c).
\end{aligned}
$$

The n-year covariance between consumption growth and equity returns is given by:

$$
\begin{aligned}
&E\left[\sum_{j=0}^{12N-1}(\Delta c_{t-j}-\mu)/N\right]\left[\sum_{j=0}^{12N-1}(r_{d,t-j}-\mu_r)/N\right] \\
&= E\left[\sum_{j=0}^{12N-1}(\pi_{c\lambda}\varepsilon^{\lambda}_{t+2-j}+\sigma_c\varepsilon^{c}_{t+1-j})/N\right]\times \\
&\left[\sum_{j=0}^{12N-1}\left(\kappa_{d1}\left(z_{d,t+1-j}-z_d\right)-\left(z_{d,t-j}-z_d\right)+\sigma_d\varepsilon^{d}_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}+\pi_{dc}\varepsilon^{c}_{t+1-j}\right)/N\right] \\
&= \frac{12}{N}\pi_{dc}\sigma_c + E\left[\sum_{j=0}^{12N-1}\pi_{c\lambda}\varepsilon^{\lambda}_{t+2-j}/N\right]\times \\
&\left[\sum_{j=0}^{12N-1}\left(\kappa_{d1}\left(z_{d,t+1-j}-z_d\right)-\left(z_{d,t-j}-z_d\right)+\sigma_d\varepsilon^{d}_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}\right)/N\right].
\end{aligned}
$$

Now consider the second term:

$$
\begin{aligned}
&\frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}\pi_{c\lambda}\varepsilon^{\lambda}_{t+2-j}\right]\times \\
&\left[\kappa_{d,1}A_{d,1}x_{t+2}-A_{d,1}x_{t-12N+2}+\sum_{j=0}^{12N-2}(\kappa_{d1}-1)A_{d,1}x_{t+1-j}+\sum_{j=0}^{12N-1}\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}\right].
\end{aligned}
$$

Expanding x_j, this second term becomes

$$
\begin{aligned}
&\frac{E\left(\sum_{j=0}^{12N-1}\pi_{c\lambda}\varepsilon^{\lambda}_{t+2-j}\right)}{N^2}\left(\kappa_{d,1}A_{d,1}\sum_{j=0}^{\infty}\rho^j\sigma_\lambda\varepsilon^{\lambda}_{t+2-j}+\sum_{j=0}^{12N-2}(\kappa_{d1}-1)A_{d,1}\sum_{m=0}^{\infty}\rho^m\sigma_\lambda\varepsilon^{\lambda}_{t+1-j-m}+\sum_{j=0}^{12N-1}\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}\right) \\
&= \frac{1}{N^2}\left[\kappa_{d,1}A_{d,1}\pi_{c\lambda}\sigma_\lambda\sum_{j=0}^{12N-1}\rho^j+12N\pi_{c\lambda}\pi_{d\lambda}+\pi_{c\lambda}(\kappa_{d,1}-1)A_{d,1}\sigma_\lambda\sum_{m=0}^{12N-2}\sum_{j=0}^{m}\rho^j\right].
\end{aligned}
$$

So the covariance is given by:

$$
\frac{12}{N}\pi_{dc}\sigma_c+\frac{\pi_{c\lambda}}{N^2}\left[\sigma_\lambda\kappa_{d,1}A_{d,1}\sum_{j=0}^{12N-1}\rho^j+12N\pi_{d\lambda}+\sigma_\lambda(\kappa_{d,1}-1)A_{d,1}\sum_{m=0}^{12N-2}\sum_{j=0}^{m}\rho^j\right].
$$

The n-year covariance between dividend growth and equity returns:

$$
\begin{aligned}
& E\left[\sum_{j=0}^{12N-1}(\Delta d_{t-j}-\mu)/N\right]\left[\sum_{j=0}^{12N-1}(r_{d,t-j}-\mu_r)/N\right] \\
&= E\left[\sum_{j=0}^{12N-1}(\sigma_d\varepsilon^d_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}+\pi_{dc}\varepsilon^c_{t+1-j})/N\right] \\
&\quad\times\left[\sum_{j=0}^{12N-1}\left(\kappa_{d1}\left(z_{d,t+1-j}-z_d\right)-\left(z_{d,t-j}-z_d\right)+\sigma_d\varepsilon^d_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}+\pi_{dc}\varepsilon^c_{t+1-j}\right)/N\right].
\end{aligned}
$$

We can exploit the similarities with the analogue expression for consumption growth. The first term on the left hand side is new. However, it reduces to $12N\sigma_d^2$. The third term yields $12N\pi_{dc}^2$. The middle term is the same as in the previous analysis with $\pi_{d\lambda}$ replacing $\pi_{c\lambda}$. Thus, the covariance is:

$$
\frac{12}{N}\sigma_d^2+\frac{12}{N}\pi_{dc}^2+\frac{\pi_{d\lambda}}{N^2}\left[\sigma_\lambda\kappa_{d,1}A_{d,1}\sum_{j=0}^{12N-1}\rho^j+12N\pi_{d\lambda}+\sigma_\lambda(\kappa_{d,1}-1)A_{d,1}\sum_{m=0}^{12N-2}\sum_{j=0}^{m}\rho^j\right].
$$

We now compute the covariance between the risk-free rate and the equity return. In terms of timing, note that $r_{f,t+1}$ and $r_{d,t+1}$ are both returns from t to $t+1$. We compute the covariance over $12N$ months and annualize:

$$
\begin{aligned}
& \frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}\left(r_{f,t+1-j}-r_f\right)\sum_{j=0}^{12N-1}\left(r_{d,t+1-j}-r_d\right)\right] \\
= \;& \frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}\left(-\sigma_\eta\eta_{t+1-j}-x_{t+1-j}\right)\sum_{j=0}^{12N-1}\left(\kappa_{d1}\left(z_{d,t+1-j}-z_d\right)-\left(z_{d,t-j}-z_d\right)+\left(\Delta d_{t+1-j}-\mu\right)\right)\right].
\end{aligned}
$$

Using

$$
z_{d,t}=A_{d,0}+A_{d,1}x_{t+1}+\sigma_\eta\eta_{t+1},
$$

the covariance can be written as,

$$
\begin{aligned}
& \frac{1}{N^2}E\Bigg[\sum_{j=0}^{12N-1}\left(-\sigma_\eta\eta_{t+1-j}-x_{t+1-j}\right)\times \\
& \sum_{j=0}^{12N-1}\Bigg[\kappa_{d1}\left(A_{d,1}x_{t+2-j}+\sigma_\eta\eta_{t+2-j}\right)-\left(A_{d,1}x_{t+1-j}+\sigma_\eta\eta_{t+1-j}\right) \\
& +\sigma_d\varepsilon^d_{t+1-j}+\pi_{d\lambda}\varepsilon^{\lambda}_{t+2-j}+\pi_{dc}\varepsilon^c_{t+1-j}\Bigg]\Bigg].
\end{aligned}
$$

Simplifying this expression gives,

$$\frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}\left(-\sigma_\eta\eta_{t+1-j}-x_{t+1-j}\right)\times\right.$$
$$\left.\sum_{j=0}^{12N-1}\left[\kappa_{d1}\left(A_{d,1}x_{t+2-j}+\sigma_\eta\eta_{t+2-j}\right)-\left(A_{d,1}x_{t+1-j}+\sigma_\eta\eta_{t+1-j}\right)+\pi_{d\lambda}\varepsilon^\lambda_{t+2-j}\right]\right],$$

or,

$$-\frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}\sigma_\eta\eta_{t+1-j}\sum_{j=0}^{12N-1}\left[\kappa_{d1}\sigma_\eta\eta_{t+2-j}-\sigma_\eta\eta_{t+1-j}\right]\right]$$
$$-\frac{1}{N^2}E\left[\sum_{j=0}^{12N-1}x_{t+1-j}\sum_{j=0}^{12N-1}\left[\kappa_{d1}A_{d,1}x_{t+2-j}-A_{d,1}x_{t+1-j}+\pi_{d\lambda}\varepsilon^\lambda_{t+2-j}\right]\right].$$

Consider the first term:

$$-\frac{\sigma_\eta^2}{N^2}E\left[\kappa_{d1}\sum_{j=0}^{12N-1}\eta_{t+1-j}\sum_{j=0}^{12N-1}\eta_{t+2-j}-\sum_{j=0}^{12N-1}\eta_{t+1-j}\sum_{j=0}^{12N-1}\eta_{t+1-j}\right]$$
$$=-\frac{\sigma_\eta^2}{N^2}E\left[\kappa_{d1}\sum_{j=0}^{12N-2}\eta_{t+1-j}\sum_{j=1}^{12N-1}\eta_{t+2-j}-12N\right]$$
$$=-\frac{\sigma_\eta^2}{N^2}\left[\kappa_{d1}\left(12N-1\right)-12N\right].$$

Consider the second term:

$$-\frac{1}{N^2}E\left[\kappa_{d1}A_{d,1}\sum_{j=0}^{12N-1}x_{t+1-j}\sum_{j=0}^{12N-1}x_{t+2-j}-A_{d,1}\left(\sum_{j=0}^{12N-1}x_{t+1-j}\right)^2+\pi_{d\lambda}\sum_{j=0}^{12N-1}x_{t+1-j}\sum_{j=0}^{12N-1}\varepsilon^\lambda_{t+2-j}\right].$$

Note that,

$$x_{t+1-j}=\sum_{m=0}^{\infty}\rho^m\sigma_\lambda\varepsilon^\lambda_{t+1-j-m},$$

so that,

$$\begin{aligned}
\sum_{j=0}^{12N-1}x_{t+1-j} &= \sigma_\lambda\sum_{m=0}^{\infty}\sum_{j=0}^{12N-1}\rho^m\varepsilon^\lambda_{t+1-j-m}\\
&= \sigma_\lambda\left(\varepsilon^\lambda_{t+1}+\varepsilon^\lambda_t+...+\varepsilon^\lambda_{t+1-(12N-1)}\right)\\
&\quad+\sigma_\lambda\rho_x\left(\varepsilon^\lambda_t+\varepsilon^\lambda_{t-1}+...+\varepsilon^\lambda_{t-(12N-1)}\right)\\
&\quad+...\\
&= \sigma_\lambda\left[\varepsilon^\lambda_{t+1}+\left(1+\rho\right)\varepsilon^\lambda_t+...+\left(1+...+\rho^{12N-1}\right)\varepsilon^\lambda_{t+1-(12N-1)}\right]\\
&\quad+\sigma_\lambda\rho\left(1+...+\rho^{12N-1}\right)\sum_{n=0}^{\infty}\rho^n\varepsilon^\lambda_{t-(12N-1)-n}.
\end{aligned}$$

We may then compute,

$$\begin{aligned} E\left[\left(\sum_{j=0}^{12N-1} x_{t+1-j}\right)^2\right] &= \sigma_\lambda^2 E\left[\sum_{j=0}^{12N-1}\left(\sum_{m=0}^{\infty}\rho^m\varepsilon_{t+1-j-m}^{\lambda}\right)\sum_{j=0}^{12N-1}\left(\sum_{n=0}^{\infty}\rho^n\varepsilon_{t+1-j-n}^{\lambda}\right)\right] \\ &= \sigma_\lambda^2\left[1+(1+\rho)^2+...+\left(1+...+\rho^{12N-1}\right)^2\right]+\sigma_\lambda^2\rho^2\left(1+...+\rho^{12N-1}\right)^2\sum_{n=0}^{\infty}\rho^{2n}. \end{aligned}$$

Also,

$$\begin{aligned} & E\left[\sum_{j=0}^{12N-1} x_{t+1-j}\sum_{j=0}^{12N-1} x_{t+2-j}\right] \\ = \; & \sigma_\lambda^2 E\left[\sum_{j=0}^{12N-1}\left(\sum_{m=0}^{\infty}\rho^m\varepsilon_{t+1-j-m}^{\lambda}\right)\sum_{j=0}^{12N-1}\left(\sum_{n=0}^{\infty}\rho^n\varepsilon_{t+2-j-n}^{\lambda}\right)\right] \\ = \; & E\left[\begin{array}{c}\sigma_\lambda\left[\varepsilon_{t+1}^{\lambda}+(1+\rho)\varepsilon_t^{\lambda}+...+\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+1-(12N-1)}^{\lambda}\right] \\ +\sigma_\lambda\rho\left(1+...+\rho^{12N-1}\right)\sum_{n=0}^{\infty}\rho^n\varepsilon_{t-(12N-1)-n}^{\lambda}\end{array}\right] \\ & \times\left[\begin{array}{c}\sigma_\lambda\left[\varepsilon_{t+2}^{\lambda}+(1+\rho)\varepsilon_{t+1}^{\lambda}+...+\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+2-(12N-1)}^{\lambda}\right] \\ +\sigma_\lambda\rho\left(1+...+\rho^{12N-1}\right)\sum_{n=0}^{\infty}\rho^n\varepsilon_{t+1-(12N-1)-n}^{\lambda}\end{array}\right] \\ = \; & E\left[\begin{array}{c}\sigma_\lambda\left[\varepsilon_{t+1}^{\lambda}+(1+\rho)\varepsilon_t^{\lambda}+...+\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+1-(12N-1)}^{\lambda}\right] \\ +\sigma_\lambda\rho\left(1+...+\rho^{12N-1}\right)\sum_{n=0}^{\infty}\rho^n\varepsilon_{t-(12N-1)-n}^{\lambda}\end{array}\right] \\ & \times\Bigg[\sigma_\lambda\left[(1+\rho)\varepsilon_{t+1}^{\lambda}+...+\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+2-(12N-1)}^{\lambda}\right] \\ & +\sigma_\lambda\rho\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+1-(12N-1)}^{\lambda}+\sigma_\lambda\rho^2\left(1+...+\rho^{12N-1}\right)\sum_{n=0}^{\infty}\rho^n\varepsilon_{t-(12N-1)-n}^{\lambda}\Bigg] \\ = \; & \sigma_\lambda^2\left[(1+\rho)+(1+\rho)\left(1+\rho+\rho^2\right)+...+\left(1+...+\rho^{12N-2}\right)\left(1+...+\rho^{12N-1}\right)\right] \\ & +\sigma_\lambda^2\left(1+...+\rho^{12N-1}\right)\rho\left(1+...+\rho^{12N-1}\right)+\sigma_\lambda^2\rho^3\left(1+...+\rho^{12N-1}\right)^2\sum_{n=0}^{\infty}\rho^{2n}, \end{aligned}$$

and lastly,

$$\begin{aligned}
E\left[\sum_{j=0}^{12N-1} x_{t+1-j} \sum_{j=0}^{12N-1} \varepsilon_{t+2-j}^{\lambda}\right] &= E\left[\sum_{j=0}^{12N-1}\left(\sum_{m=0}^{\infty}\rho^{m}\sigma_{\lambda}\varepsilon_{t+1-j-m}^{\lambda}\right)\sum_{j=0}^{12N-1}\varepsilon_{t+2-j}^{\lambda}\right] \\
&= E\Bigg[\Bigg(\sigma_{\lambda}\left[\varepsilon_{t+1}^{\lambda}+(1+\rho)\,\varepsilon_{t}^{\lambda}+...+\left(1+...+\rho^{12N-1}\right)\varepsilon_{t+1-(12N-1)}^{\lambda}\right] \\
&\quad +\sigma_{\lambda}\rho\left(1+...+\rho^{12N-1}\right)\times \\
&\quad \sum_{n=0}^{\infty}\rho^{n}\varepsilon_{t-(12N-1)-n}^{\lambda}\Bigg)\left[\varepsilon_{t+2}^{\lambda}+\varepsilon_{t+1}^{\lambda}+...+\varepsilon_{t+2-(12N-1)}^{\lambda}\right]\Bigg] \\
&= \sigma_{\lambda}E\left[\varepsilon_{t+1}^{\lambda}+(1+\rho)\,\varepsilon_{t}^{\lambda}+...+\left(1+...+\rho^{12N-2}\right)\varepsilon_{t+2-(12N-1)}^{\lambda}\right] \\
&\quad \times\left[\varepsilon_{t+1}^{\lambda}+...+\varepsilon_{t+2-(12N-1)}^{\lambda}\right] \\
&= \sigma_{\lambda}\left[1+(1+\rho)+...+\left(1+...+\rho^{12N-2}\right)\right].
\end{aligned}$$

So, the covariance is:

$$\begin{aligned}
&-\frac{\sigma_{\eta}^{2}}{N^{2}}\left[\kappa_{d1}\left(12N-1\right)-12N\right] \\
&-\frac{\kappa_{d1}A_{d1}}{N^{2}}\Bigg(\sigma_{\lambda}^{2}\left[(1+\rho)+(1+\rho)\left(1+\rho+\rho^{2}\right)+...+\left(1+...+\rho^{12N-2}\right)\left(1+...+\rho^{12N-1}\right)\right] \\
&+\sigma_{\lambda}^{2}\left(1+...+\rho^{12N-1}\right)\rho\left(1+...+\rho^{12N-1}\right)+\sigma_{\lambda}^{2}\rho^{3}\left(1+...+\rho^{12N-1}\right)^{2}\sum_{n=0}^{\infty}\rho^{2n}\Bigg) \\
&+\frac{A_{d,1}}{N^{2}}\left(\sigma_{\lambda}^{2}\left[1+(1+\rho)^{2}+...+\left(1+...+\rho^{12N-1}\right)^{2}\right]+\sigma_{\lambda}^{2}\rho^{2}\left(1+...+\rho^{12N-1}\right)^{2}\sum_{n=0}^{\infty}\rho^{2n}\right) \\
&-\frac{\pi_{d\lambda}}{N^{2}}\sigma_{\lambda}\left[1+(1+\rho)+...+\left(1+...+\rho^{12N-2}\right)\right].
\end{aligned}$$

or

$$\begin{aligned}
&-\frac{\sigma_{\eta}^{2}}{N^{2}}\left[\kappa_{d1}\left(12N-1\right)-12N\right] \\
&-\frac{\sigma_{\lambda}^{2}\kappa_{d1}A_{d1}}{N^{2}}\left(\sum_{j=1}^{12N-1}\left(\sum_{m=0}^{j}\rho^{m}\right)\left(\sum_{m=0}^{j-1}\rho^{m}\right)+\left(\sum_{j=0}^{12N-1}\rho^{j}\right)^{2}\left(\rho+\rho^{3}\sum_{n=0}^{\infty}\rho^{2n}\right)\right) \\
&+\sigma_{\lambda}^{2}\frac{A_{d,1}}{N^{2}}\left(\sum_{j=0}^{12N-1}\left(\sum_{m=0}^{j}\rho^{m}\right)^{2}+\frac{\rho^{2}}{1-\rho^{2}}\left(\sum_{j=0}^{12N-1}\rho^{j}\right)^{2}\right) \\
&-\frac{\pi_{d\lambda}}{N^{2}}\sigma_{\lambda}\sum_{j=0}^{12N-2}\left(\sum_{m=0}^{j}\rho^{m}\right).
\end{aligned}$$

Table 1

Correlation between stock returns and per capita growth rates of fundamentals

Panel A, 1929-2011

Horizon	Consumption	Output	Dividends	Earnings
1 year	-0.05	0.05	0.05	0.10
	(0.12)	(0.10)	(0.11)	(0.10)
5 years	0.01	0.00	0.30	0.20
	(0.19)	(0.12)	(0.13)	(0.13)
10 years	-0.01	-0.09	0.59	0.30
	(0.49)	(0.14)	(0.14)	(0.11)

Panel B, 1871-2006

Horizon	Consumption	Output	Dividends	Earnings
1 year	0.09	0.14	-0.04	0.13
	(0.09)	(0.10)	(0.10)	(0.10)
5 years	0.40	0.25	0.38	0.44
	(0.18)	(0.14)	(0.15)	(0.18)
10 years	0.25	0.00	0.64	0.41
	(0.18)	(0.11)	(0.17)	(0.13)

Table 2

Correlation between stock returns and per capita growth rates of fundamentals

NIPA measures of consumption, 1929-2011

Horizon	Durables	Non-durables	Non-durables and services
1 year	-0.04	0.05	0.27
	(0.13)	(0.14)	(0.12)
5 years	0.07	-0.08	0.18
	(0.11)	(0.09)	(0.10)
10 years	0.21	-0.2	0.06
	(0.15)	(0.13)	(0.13)

Table 3

	Benchmark model	Augmented model	Quasi-production	Quasi-production match equity premium
γ	1.0684 (5.555)	1.0398 (0.95939)	0.9302 (7.6689)	1.0951
ψ	1.0275 (2.2738)	1.0311 (0.84281)	0.91839 (8.4925)	1.0145
δ	0.99802 (0.00069517)	0.99925 (0.013551)	0.99951 (0.00587697)	0.99813
σ_c	0.0066807 (0.00020834)	0.0075147 (0.00026327)	0.0081094 (0.00025137)	0.0066978
μ	0.0014517 (6.0378e-05)	0.0015846 (5.6632e-05)	0.0010629 (6.703e-05)	0.0017229
$\pi_{c\lambda}$	0.00	0.00	3.2471e-05 (0.00014922)	-5.1591e-05
σ_d	0.015811 (0.00048413)	0.017987 (0.00071243)	0.022763 (0.00082418)	0.019945
π_{dc}	3.0489e-06 (0.00054383)	5.2506e-06 (0.00064363)	2.2163e-05 (0.00077632)	5.7025e-06
$\pi_{d\lambda}$	0.00	0.00	-0.0089244 (0.0012766)	3.4262e-06
σ_η	0.00	0.0078202 (0.00062626)	0.0090905 (0.00052927)	0.0058789
ρ	0.9936 (0.0026001)	0.9936 (0.0026932)	0.99875 (0.0017456)	0.99892
σ_λ	0.00040109 (8.0343e-05)	0.00040109 (0.00012958)	0.00014709 (0.00010173)	0.0001648
Implied value of θ	-2.56	-1.32	-0.79	-6.65

Table 4

Selected moments	Data (Constrained)	Data (Unconstrained)	Benchmark model	Augmented model	Quasi-production	Quasi-production match equity premium
Average growth rate of consumption	1.44	2.24	1.74	1.90	1.28	2.07
	(0.32)	(0.23)				
Average growth rate of dividends	1.44	-0.12	1.74	1.90	1.28	2.07
	(0.32)	(0.75)				
Standard deviation of the growth rate of consumption	2.08	2.15	2.31	2.60	2.81	2.32
	(0.38)	(0.31)				
Standard deviation of the growth rate of dividends	6.82	7.02	5.48	6.23	8.47	6.91
	(1.35)	(1.31)				
Contemporaneous correlation between consumption and dividend growth	0.17	0.16	0.00	0.00	0.00	0.00
	(0.12)	(0.09)				
Average return to equities	7.55	6.20	6.27	4.30	2.81	7.55
	(1.74)	(1.87)				
Standard deviation of return to equities	17.22	17.49	14.10	16.85	18.24	12.40
	(1.31)	(1.39)				
Average risk-free rate	0.36	0.06	0.80	-0.27	-0.92	0.36
	(0.81)	(0.83)				
Standard deviation of the risk-free rate	3.20	3.47	4.21	4.47	4.72	4.72
	(0.80)	(0.80)				
First-order serial correlation of the risk-free rate	0.61	0.60	0.95	0.62	0.55	0.81
	(0.11)	(0.08)				
Correlation between equity returns and risk-free rate	0.20	0.26	0.13	0.10	0.10	0.26
	(0.10)	(0.09)				
Equity premium	7.19	6.13	5.47	4.57	3.73	7.19
	(1.77)	(1.84)				

Table 5: Equity and Valuation Risk Premium

Benchmark Model

ρ	Equity premium	Conventional Risk Premium	Valuation Risk Premium
0.000	-0.0015	-0.0015	0.0000
0.100	-0.0015	-0.0015	0.0000
0.200	-0.0015	-0.0015	0.0000
0.300	-0.0015	-0.0015	0.0000
0.400	-0.0015	-0.0015	0.0000
0.500	-0.0015	-0.0015	0.0000
0.600	-0.0015	-0.0015	0.0000
0.700	-0.0014	-0.0015	0.0001
0.800	-0.0014	-0.0015	0.0001
0.900	-0.0009	-0.0015	0.0006
0.950	0.0007	-0.0015	0.0022
0.960	0.0018	-0.0015	0.0033
0.970	0.0042	-0.0015	0.0057
0.980	0.0104	-0.0015	0.0119
0.990	0.0330	-0.0015	0.0345
0.991	0.0378	-0.0015	0.0393
0.992	0.0435	-0.0015	0.0450
0.993	0.0502	-0.0014	0.0516
0.994	0.0580	-0.0015	0.0595
0.995	0.0670	-0.0015	0.0685
0.996	0.0775	-0.0015	0.0790
0.997	0.0893	-0.0015	0.0908
0.998	0.1027	-0.0015	0.1042
0.999	0.1174	-0.0015	0.1189

Table 5 (continuation): Equity and Valuation Risk Premium

Augmented Model

ρ	Equity premium	Conventional Risk Premium	Valuation Risk Premium
0.000	-0.0006	-0.0006	0.0000
0.100	-0.0006	-0.0006	0.0000
0.200	-0.0006	-0.0006	0.0000
0.300	-0.0006	-0.0006	0.0000
0.400	-0.0006	-0.0006	0.0000
0.500	-0.0006	-0.0006	0.0000
0.600	-0.0006	-0.0006	0.0000
0.700	-0.0006	-0.0006	0.0000
0.800	-0.0006	-0.0006	0.0000
0.900	-0.0005	-0.0006	0.0001
0.950	-0.0003	-0.0006	0.0003
0.960	-0.0002	-0.0006	0.0004
0.970	0.0002	-0.0006	0.0008
0.980	0.0011	-0.0006	0.0017
0.990	0.0055	-0.0006	0.0061
0.991	0.0068	-0.0006	0.0074
0.992	0.0084	-0.0006	0.0090
0.993	0.0106	-0.0006	0.0112
0.994	0.0137	-0.0006	0.0143
0.995	0.0179	-0.0006	0.0185
0.996	0.0240	-0.0006	0.0246
0.997	0.0324	-0.0006	0.0330
0.998	0.0439	-0.0007	0.0446
0.999	0.0590	-0.0006	0.0596

Table 5 (continuation): Equity and Valuation Risk Premium

Quasi-production model

ρ	Equity premium	Conventional Risk Premium	Valuation Risk Premium
0.000	-0.0023	-0.0023	0.0000
0.100	-0.0023	-0.0023	0.0000
0.200	-0.0023	-0.0023	0.0000
0.300	-0.0023	-0.0023	0.0000
0.400	-0.0023	-0.0023	0.0000
0.500	-0.0023	-0.0023	0.0000
0.600	-0.0023	-0.0023	0.0000
0.700	-0.0024	-0.0024	0.0000
0.800	-0.0024	-0.0024	0.0000
0.900	-0.0024	-0.0024	0.0000
0.950	-0.0024	-0.0025	0.0001
0.960	-0.0024	-0.0026	0.0002
0.970	-0.0024	-0.0028	0.0004
0.980	-0.0021	-0.0029	0.0008
0.990	-0.0004	-0.0034	0.003
0.991	0.0001	-0.0035	0.0036
0.992	0.0009	-0.0036	0.0045
0.993	0.0020	-0.0037	0.0057
0.994	0.0035	-0.004	0.0075
0.995	0.0060	-0.0042	0.0102
0.996	0.0098	-0.0045	0.0143
0.997	0.0161	-0.0049	0.021
0.998	0.0263	-0.0053	0.0316
0.999	0.0418	-0.006	0.0478

Table 6

Moments	Data (Constrained)	Data (Unconstrained)	Benchmark model	Augmented model	Quasi-production	Quasi-production match equity premium
1-year correlation between equity returns and consumption growth	-0.03	-0.05	0.00	0.00	0.00	-0.01
5-year correlation between equity returns and consumption growth	0.07	0.00	0.00	0.00	0.00	0.00
	(0.17)	(0.14)				
10-year correlation between equity returns and consumption growth	-0.02	-0.11	0.00	0.00	0.00	0.00
	(0.30)	(0.20)				
1-year correlation between equity returns and dividend growth	0.08	0.05	0.39	0.37	0.07	0.56
	(0.12)	(0.11)				
5-year correlation between equity returns and dividend growth	0.27	0.3	0.39	0.36	0.09	0.48
	(0.30)	(0.20)				
10-year correlation between equity returns and dividend growth	0.51	0.59	0.39	0.35	0.11	0.43
	(0.22)	(0.14)				

Table 7

Moments	Data (Unconstrained)	Benchmark model	Augmented model	Quasi-production model
Average return				
Long-term bond	1.66	5.21	1.98	0.74
	(0.85)			
Intermediate-term bond	1.06	3.67	1.06	0.05
	(0.90)			
Short-term bond	0.36	0.8	-0.27	-0.92
	(0.81)			
Return to equity minus long-term bond yield	4.54	1.23	3.16	3.00
	(1.84)			
Standard deviation				
Long-term bond	3.54	2.22	2.96	2.72
	(0.76)			
Intermediate-term bond	1.06	3.04	3.29	2.93
	(0.82)			
Short-term bond	3.20	4.21	4.47	4.72
	(0.80)			
Return to equity minus long-term bond yield	16.7	13.8	16.9	18.9
	(1.48)			