NBER WORKING PAPER SERIES

FREE LUNCH! ARBITRAGE OPPORTUNITIES IN THE FOREIGN EXCHANGE MARKETS

Takatoshi Ito
Kenta Yamada
Misako Takayasu
Hideki Takayasu

Working Paper 18541
http://www.nber.org/papers/w18541

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2012

Ito gratefully acknowledges research supports by the Japan Society for the Promotion of Science (JSPS) Grants-in-aid for Scientific Research (A) No. 20243014; and Center for Advanced Research on Finance (CARF) at University of Tokyo. Misako Takayasu acknowledges research supports by the Japan Society for the Promotion of Science (JSPS) Grants-in-aid for Scientific Research No. 22656025. The paper was presented in the Japan Economic Association Meeting in June, 2012. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Free Lunch! Arbitrage Opportunities in the Foreign Exchange Markets
Takatoshi Ito, Kenta Yamada, Misako Takayasu, and Hideki Takayasu
NBER Working Paper No. 18541
November 2012
JEL No. F31,G12,G14,G15,G23,G24

ABSTRACT

Using the "firm" quotes obtained from the tick-by-tick EBS (electronic broking system that is a major trading platform for foreign exchanges) data, it is found that risk-free arbitrage opportunities—free lunch—do occur in the foreign exchange markets, but it typically last only a few seconds. "Free lunch" is in the form of (a) negative spreads in a currency pair and (b) triangular arbitrage relationship involving three currency pairs. The latter occur much more often than the former. Such arbitrage opportunities tend to occur when the markets are active and volatile. Over the 12-year, tick-data samples, the number of free lunch opportunities has dramatically declined and the probability of the opportunities disappearing within one second has steadily increased. The size of expected profits is higher than transaction costs; trades that simultaneously take place on both sides of ask and bid (or three currency trades in case of triangular arbitrage) occur more often when free lunch appeared one second earlier than otherwise, suggesting that free lunch opportunities are actively taken. The probability of its disappearance within one second was less than 50% in 1999, but increased to about 90% by 2009. Less frequent occurrence and quicker disappearance in recent years are attributable to changes in trading microstructure: an introduction and proliferation of the Primary Customer system (weaker banks can use stronger banks' credit lines) and of direct connection of traders' programmed computers to the EBS computer.

Takatoshi Ito
Graduate School of Economics
University of Tokyo
7-3-1 Hongo, Bunkyo-ku
Tokyo 113-0033
JAPAN
and NBER
tito@e.u-tokyo.ac.jp

Kenta Yamada
Waseda Institute for Advanced Study
1-6-1 Nishi Waseda, Shinjuku-ku
Tokyo, 169-8050, Japan
yamada@aoni.waseda.jp

Misako Takayasu
Tokyo Institute of Technology
Department of Computational Intelligence and System Sciences
4259-G3-52 Nagatsuta-cho, Midori-ku
Yokohama, 226-8502, Japan
takayasu@dis.titech.ac.jp

Hideki Takayasu
Sony Computer Science Laboratories
3-14-13 Higashigotanda, Shinagawa-ku
Tokyo 141-0022, Japan
takayasu@csl.sony.co.jp

Takatoshi Ito, Kenta Yamada, Misako Takayasu, and Hideki Takayasu
"Free Lunch! Arbitrage Opportunities in the Foreign Exchange Markets"

Abstract:
Using the "firm" quotes obtained from the tick-by-tick EBS (electronic broking system that is a major trading platform for foreign exchanges) data, it is found that risk-free arbitrage opportunities—free lunch—do occur in the foreign exchange markets, but it typically last only a few seconds. "Free lunch" is in the form of (a) negative spreads in a currency pair and (b) triangular arbitrage relationship involving three currency pairs. The latter occur much more often than the former. Such arbitrage opportunities tend to occur when the markets are active and volatile. Over the 12-year, tick-data samples, the number of free lunch opportunities has dramatically declined and the probability of the opportunities disappearing within one second has steadily increased. The size of expected profits is higher than transaction costs; trades that simultaneously take place on both sides of ask and bid (or three currency trades in case of triangular arbitrage) occur more often when free lunch appeared one second earlier than otherwise, suggesting that free lunch opportunities are actively taken. The probability of its disappearance within one second was less than 50% in 1999, but increased to about 90% by 2009. Less frequent occurrence and quicker disappearance in recent years are attributable to changes in trading microstructure: an introduction and proliferation of the Primary Customer system (weaker banks can use stronger banks' credit lines) and of direct connection of traders' programmed computers to the EBS computer.

Keywords: Market microstructure, High frequency data, Arbitrage, Negative spread, Triangular arbitrage, Free lunch, Electronic Broking System, Yen, US dollar, Euro,

Acknowledgment:
Ito gratefully acknowledges research supports by the Japan Society for the Promotion of Science (JSPS) Grants-in-aid for Scientific Research (A) No. 20243014; and Center for Advanced Research on Finance (CARF) at University of Tokyo. Misako Takayasu acknowledges research supports by the Japan Society for the Promotion of Science (JSPS) Grants-in-aid for Scientific Research No. 22656025. The paper was presented in the Japan Economic Association Meeting in June, 2012.

JEL, F31, G12, G14, G15, G23, G24

1. Introduction

The foreign exchange market is one of the largest financial markets in terms of volumes of transactions and the number of market participants, who reside all over the world. The market is open 24 hours a day, (almost) seven days a week. The average daily transaction volume is estimated at 4 trillion dollars a day (BIS (2010)).[1] The market, especially for major currency pairs, is considered to be deep and liquid for virtually any hour of any business day.

As it is one of the most deep and liquid markets, the foreign exchange market is often assumed to be "efficient" in both theoretical and empirical literature. In an "efficient" market, all the risk-free arbitrage opportunities—free lunch—would not be observed.[2] "No free lunch" means that the bid (price of buyer's limit order) should always be lower than ask (price of seller's limit order). Thus, the negative spread is one form of free lunch. Among the three currency pairs, simultaneous transactions exchanging the yen (JPY) into the US dollar (USD), from the US dollar to Euro (EUR), and Euro back to the yen should not produce profits. Hence, bids and asks of the three pairs, USD/JPY, EUR/USD, and EUR/JPY, should satisfy certain inequality relationship. A theoretical model is commonly based on a "no free lunch" assumption and observations that suggest free lunch in data are most regarded as outliers and discarded.

In the real world, however, any high-frequency currency trader would know that the free lunch situation do occur rather frequently. Of course the magnitude of profits is rather small and the duration of opportunities is short. In fact, the opportunity, if it exists, is said to disappear in the matter of seconds. In fact, large financial institutions invest heavily into a real-time trading system, which would not only spot such arbitrage opportunities but also take long or short positions with a bet on the

---

[1] According to the BIS Triennial Survey, the size of the global foreign exchange market in April 2010 was the turnover of 3.98 trillion US dollars a day, of which spot transactions accounted 1.49 trillion US dollars (37%). With respect to currency pairs, the USD/EUR accounted 28%; USD/JPY 14%; and EUR/JPY 3%.

[2] There is a standard joke about the efficient market hypothesis believer: An efficient market believer and his friend are walking on the sidewalk. The friend says, "Look, there is a $100 bill on the sidewalk." The believer says, "It should be a counterfeit bill. If it is a genuine bill, someone should have picked it up already." The evidence in this paper can be interpreted as follows. Time to time, people do drop a $100 bill unknowingly. But, it does not take much time before someone finds it and picks it up, because people do check out whether the bill is genuine or counterfeit. So most of the time, you do not see $100 bill on the street.

direction of the exchange rate.[3]

In order to examine whether and how often free lunch opportunities arise and how quickly they disappear, high frequency data of an actual trading platform is needed. The major inter-bank trading platform for spot exchange rates is electronic broking services, which are dominated by the EBS (owned by ICAP) and Reuters 3000 Xtra (owned by Thomson Reuter). In the currency pairs involving USD, JPY, and EUR, the trading system by EBS has a strong market share. We are going to use the EBS data set, which record all quotes, actual deal prices and volumes at one-second time slice, so that all quotes are firm and reliable.[4]

Until the mid-1990s, human traders who have access to EBS data feed on screen, have to react to free lunch opportunities by hitting keys for buy and sell. However, in recent years, banks' computers are permitted to be connected directly to the trading network. Thus, it has become easy for banks to search for and take advantage of every profit opportunity within seconds, if and when such opportunities do arise.

The literature on negative spread is not large, as such an arbitrage is often "assumed" to be non-existent. It is known that negative spread occurs in the data with indicative quotes, such as the Olsen data, but researchers routinely "clean" them on the assumption that the data must reflect stale quotes. Negative quotes in the EBS data are real, since there is no stale quote. The triangular arbitrage has attracted more attention, since the violation to the efficient market hypothesis is not so obvious. Aiba, Hatano, Takayasu, Marumo, and Shimizu (2002), examined the data for two months in 1999 detecting triangular arbitrage among the yen, the dollar, and the euro. They claim that triangular arbitrage opportunities existed for 6.4% of total time. They constructed a time-series model in which triangular arbitrage opportunities may emerge and disappear when the three currency markets are independently moving with interaction to bring back to the arbitrage relations once the arbitrage opportunities arose. They simulated through a theoretical model the emergence and disappearance of the triangular arbitrage

[3] The latter, the bet on the direction, assumes that the exchange rate does not always follow a random walk process. See Hashimoto, et al. (2012) on this theme.

[4] The EBS data set is a proprietary data set that is commercially available. The data set from 1999 to 2010 was purchased by research grants at the University of Tokyo. The data set is proprietary information belonging to EBS. The usage of the data set purchased at the University of Tokyo is restricted to those who are affiliated at the University of Tokyo.

opportunities. Aiba, Hatano, Takayasu, Marumo, and Shimuzu (2003) and Aiba and Hatano (2004) also explored some theoretical implications of triangular arbitrage. However, the euro market was still in its infancy and the data set was different and very short. Moreover, in the data set, deal prices were not recorded and bid and ask were not necessarily firm quotes. Aiba and Hatano (2006) constructed an agent-based model to simulate the situation. Parameters were matched to replicate the observed distribution of arbitrage profits from Aiba, et al. (2002). However, these papers do not examine how observed arbitrage opportunities are related to market volatility, time of the day, and a long-run trend of technology adoption, which is the focus of this paper.

The advantage of using "firm" quotes, as opposed to "indicative" quotes, have been emphasized in Goodhart, Ito and Payne (1996), Goodhart and O'Hara (1997), and Goodhart and Payne (1996). Then, the EBS data set has been used to establish details of intra-day pattern in Ito and Hashimoto (2006); to show order flows have predictable power in Berger et al. (2006) and Ito and Hashimoto (2008). The data have been used quite successfully in the exchange rate reaction to macroeconomic statistical announcements in the US, Chaboud, Chernenko, Howorka, Iyer, Liu and Wright (2004) and Hashimoto and Ito (2010). Evidence on momentum trading is shown in Hashimoto, Ito, Ohnishi, M. Takayasu, H. Takayasu, and Watanabe (2012). Akram and Sarno (2008) investigated arbitrage opportunities in covered interest parity relationship, with Reuter tick-by-tick data from February 2004 to September 2004. Since the covered interest parity involves swap (interest rate differentials), the type of arbitrage relationship is different from our investigation, although the spirit of finding violation to risk-free arbitrage relationship is the same.

Policy makers are now concerned with a possibility that algorithmic trading—computers sending buy and sell orders directly to the EBS machine—may influence liquidity and volatility of the market. This has been examined by BIS (2011) and Chaboud, Chiquioine, Hjalmarsson, and Loretan (2007). The impact of the increasing use of algorithm trading was the focus of Chaboud, et al. (2009). They compared the computer-generated quotes and human-generated quotes, based on confidential data of EBS. They showed, among others, "despite the apparent correlation of algorithmic trades, there is no evident causal relationship between algorithmic trading and increased exchange rate volatility. If anything, the presence of more algorithmic trading is associated with lower volatility."

Marshall, Treepongkaruna, and Young (2008) investigated triangular arbitrage opportunities among the three sets of currencies, USD/CHE/EUR; GBP/EUR/USD; and USD/JPY/EUR, for the EBS data from January 1, 2005 to December 31, 2005. They found triangular arbitrage opportunities arise throughout the day. The main difference of our paper is that we cover much longer time series, with description of changes in occurrence and disappearance over time. Our paper also covers negative spread of bilateral exchange rate as well as triangular arbitrage opportunities.

The questions to be posed and answered in this paper are summarized as follows. The first question is to find out how often "free lunch" existed in the past. Contrary to conventional wisdom, we find many risk-free arbitrage opportunities in the data. Since our data are directly taken from the transaction platform, the profit opportunities are real. However, not surprisingly, the free lunch opportunities do not last long. In a matter of a few seconds, a free lunch tends to disappear.

The second question is whether the frequency of free lunch occurrence has changed over time, and if so, why. It is found that, in recent years, free lunch opportunities emerged less often, and, when they occur, they tended to disappear quicker than before. The probability of its disappearance within one second was less than 50% in 1999, but increased to about 90% by 2009. We show that these changes were due to heavier use of algorithmic (program) trading and direct connection of banks' computers to the trading platform as well as a wide-spread usage of Primary Customer system, in which less-credit-worthy banks can borrow, with fees, the name of high-credit-worthy banks that have better trading opportunities with tighter bid-ask spread.

We will show the negative correlation between the connection of banks' trading computers with the EBS computer and the occurrence and durability of arbitrage profit opportunities. The arbitrage opportunities, once they appear, may disappear in the next second, due to self-correction of bid-side or offer-side pricing by banks without transaction, or due to actual transaction, which takes out orders, making profits out of trades on the both sides of the negative spread. In the former case, the opportunities arose, but disappeared without someone taking advantage of them. Thus, the third question of the paper is whether the emergence of a profit opportunity is followed by transactions that suggest that someone took advantage of risk-free profit opportunities. It will be shown that, within one second after free lunch opportunity emerges, it is likely that trading occur on both sides of bid and ask, controlling for other factors.

The rest of the paper is organized as follows. Section 2 explains the EBS trading system and our data. Evidence and analysis of negative spread is presented in Section 3, and that of triangular arbitrage opportunity is presented in Section 4. Both sections depict the emergence and disappearance of such "free lunch" opportunities. Section 5 concludes the paper with thoughts on efficiency in the foreign exchange market.

2. Data

2.1. EBS

The foreign exchange market for major currencies is open and liquid around the clock, from the early morning on Monday at the Tokyo time to the late afternoon of the Friday at the New York time. There are patterns of up and down during 24 hours of the day, that is called the intra-day seasonality by Ito and Hashimoto, 2006. Although transaction volumes are relatively low in certain days, such as national holidays in one of the major markets, quotes are continuously available in the EBS system. The EBS offers a trade-platform data, whose accuracy and transactability of quotes is highly reliable.

The EBS (owned by ICAP company) is a provider of trading platforms. Traders (traditionally banks) can submit "firm" quotes (limit order), and "firm" quotes (best bid and best ask) are shown in the screen worldwide. Other traders can "hit" the bid or ask as they desire to make trade. The EBS has a strong market share in the USD/JPY (dollar/yen), EUR/USD (euro/dollar) and EUR/JPY (euro/yen) markets. It is said to cover more than 90% of the dollar/yen and euro/dollar trade.[5] Therefore, it is safe to assume that almost all electronically brokered spot deals of these two currencies are represented in the data set.

The EBS system facilitates, as part of the dealing rules, each institution to

[5] The Reuters trading system, Reuter Xtra, has significant market shares in exchanged related to sterling, Canadian dollar, and Australian dollars.

control its bilateral credit lines with counterparties. Namely, each EBS-linked institution sets credit lines (including zero) against all other potential counter-parties in the system. Therefore, an institution faces a restriction of tradable bid, offer, or deal from other institutions. When bid and offer quotes are posted for the system, they are not necessarily available to all participants of the EBS system. The EBS-registered trader's screen shows the best bid and best offer of the market as well as those for this trader. In normal times, the best bid of the market is lower than the best offer of the market.

As part of facilitating an orderly market, EBS requires any newly linked institution to secure a sufficient number of other banks that are willing to open credit lines with the new comer. A smaller or regional bank may have fewer credit-approved trading relationships. In such a case, the best bid and ask for a relatively minor institution may be much wider than those for larger institutions.

2.2. The EBS Data Set

We have purchased the EBS data set containing the price data set and the trade volume data set at a frequency of one second, from January 1, 1999 to December 31, 2010. It contains information relating to, among other things, best bid and best ask every second and deal prices done on the bid side (lowest given) and deal prices done on the ask side (highest paid) during the one-second duration.[6] The EBS price history shows whether the deal is done on the bid side (the bid was taken) or the ask side (the offer was taken). The EBS global system consists of three regional computer sites, based in Tokyo, London, and New York, and it matches orders either within the site or across different sites. Each region covers Europe, North America, and Asia, respectively.

In order to give definitions, the following notation will be used in the rest of the paper:

The currency pair is denoted by j, where j=USD/JPY, EUR/USD, or EUR/JPY,

$\mathbf{qa}_t^j$: the firm quote of the (market best of) ask (offer) rate of j at period t,

$\mathbf{qb}_t^j$: the firm quote of the (market best of) bid rate of j at period t,

---

[6] The deal (on either side) recorded at zz second includes those that took place between zz-1 second to zz second. When there are multiple trades within one second, "lowest given price" and "highest paid price" will be shown. A highest paid deal means the highest price hit (done) on the ask side within one second and the lowest given deal means the lowest price hit (done) on the bid side within one second.

$\mathbf{qm_t^j}$**:** the mid point of the best quotes for j at period t, $\mathrm{qm_t^j} = (\mathrm{qa_t^j}+\mathrm{qb_t^j})/2$

$\mathbf{spr_t^j}$ is the bid-ask spread defined as $\mathrm{spr_t^j} = \mathrm{qa_t^j} - \mathrm{qb_t^j}$

$\mathbf{da_n^j}$**:** the deal price that occurred on the ask (offer) side of j at time (tick) n;

$\mathbf{db_n^j}$**:** the deal price that occurred on the bid side of j at period n,

$\mathbf{dm_n^j}$: the mid-point of the deal prices for j at period n, $\mathrm{dm_n^j} = (\mathrm{da_n^j}+\mathrm{db_n^j})/2$

For a volatility measure, the average absolute change of the mid-deal price from one transaction to another is defined and used.

$\overline{\mathrm{dm_n^j}} = \frac{\mathrm{da}_n^j+\mathrm{db}_n^j}{2} \times 100$ (j=USD/JPY or EUR/JPY), $\overline{\mathrm{dm_n^j}} = \frac{\mathrm{da}_n^j+\mathrm{db}_n^j}{2} \times 10000$ (j=EUR/USD).

The change of the deal price at period n $\mathbf{\Delta dm_n^j}$ is defined by:

$$\mathbf{\Delta dm_n^j} = \overline{\mathrm{dm_n^j}} - \overline{\mathrm{dm_{n-1}^j}}$$

And the volatility of the j-th currency pair $\boldsymbol{V^j}$ is defined by

$$\boldsymbol{V^j} = \frac{1}{N^j - 1}\sum_{\mathrm{k}=2}^{N^j} |\Delta\, dm_k^j|$$

$\boldsymbol{N^j}$ denotes the number of deals for the j-th currency pair. Then the volatility of the three currency pairs for the month, $\boldsymbol{V^*}$**,** is defined by

$$\boldsymbol{V^*} = \frac{1}{N^1+\mathrm{N}^2 + \mathrm{N}^3 - 3}\sum_{i=1}^{3}\sum_{k=2}^{N^j} |\Delta\, dm_k^i|$$

In case of USD/JPY, the unit is the Japanese yen per US Dollar, so that the ask rate is the rate that a trader is ready to sell USD at the quoted rate and the bid rate is the rate that a trader is ready to buy USD at the quoted rate. In normal times, the seller's offer price is higher than buyer's bid price, thus, $\mathrm{spr_t^j} > 0$.

The settlement of foreign exchange transaction is globally set to be 2 business days later. However, if a national holiday takes place on the business day within or two business days within transaction, the complicated formula of interest rate differential arises. Counterparty and Herstatt risks also increase with differential in settlement days of the two currencies involved. Hence, duration of negative spread and triangular arbitrage opportunities tend to increase, unnaturally, on days that will be followed by a (non-weekend) national holiday of countries of currency pair(s). We omit observations on (i) weekend days; (ii) days with the number of transactions lower than the threshold; and (iii) days that are followed within two days by a national holiday in at least one of the three countries, the US, Japan and the UK.[7] The definition of weekend is from 6 am on Saturday to 6 am on Monday, Japan time. The threshold of low transaction is defined as one third of the average daily transaction of the year.

2.3. Negative Spread

The negative spread means that the seller's offer price is lower than the buyer's bid price, namely, the negative spread is defined to be a situation $\mathrm{spr}_t^j < 0$, for j=USD/JPY, EUR/USD, or EUR/JPY, at period t.

Since risk-free profits can be generated for someone simultaneously hitting seller's price to buy the currency at that rate as well as hitting buyer's price to sell the currency at that rate, it would be surprising to find such an opportunity observed in the market. However, there are several reasons why negative spreads may emerge. First, it could be simply a mistake. A bank quotes too high an offer price that exceeds a current bid, which will be hit by the bid side. But this is a very unlikely scenario. Second, the negative spread may occur due to an old firm quote, which was several pips behind the best offer and was left alive for some time. The market somehow moves very quickly without hitting all limit-order quotes for transactions in the process, and the old quote becomes exposed as one side of the negative spread. If this is true, negative spreads should tend to occur in volatile hours. Again, this is a possibility, but we do not think this is very likely. The most likely scenario is related to an institutional detail of the trading platform. When a bank joins the network, the bank must open credit lines with a certain number of counterparty banks. If a bank is relatively financially weak, then not

---

7 Without national holidays, settlement of interbank foreign exchange transactions takes place two business days within the transaction date. If a country of one of the currency pair has a national holiday on the following day or two days later, then the settlement day will be three business days later. An exception to this rule of delay is that if New York has a banking holiday on the day within transactions but not on two days later, then settlement will take place two days within transaction (as described in Tokyo Foreign Exchange Market Committee (2008: p.21)).

many banks would like to hold credit lines with the bank. A weak bank that maintains credit lines with relatively few banks may not be able to transact with banks that post the system-wide best bid and best offer. Suppose a situation where a small bank desperately wants to transact to cover customers' positions. It cannot hit the best bid or ask due to a lack of credit line with those banks that post the best. The weak bank may post a quote that may show a negative spread (say an offer higher than a bid) in an attempt to attract a relatively strong bank that will hit the quote immediately. The newly submitted quote (say, offer) will then constitute a part of the negative spread with the existing quote on the other side (bid). In this case, the negative spread may not be exploited by the two banks that form the bid and offer of the negative spread. However, this does not prevent another bank that has credit lines with both banks of negative spreads from taking advantage of profit opportunities. If this is the case, we predict that if financially weak banks become somehow stronger with many credit lines, negative spreads occur less often.

2.4. Market Microstructure

The EBS foreign exchange market went through several structural changes with regard to trading technology in the last ten years. In 2003, the "Ai trader" system was introduced. Ai traders are those banks that are allowed to plug their computers, which are loaded with algorithmic trading programs, directly to the EBS trading system computer. Prior to this innovation, humans have to execute orders based on bank computers buy and sell signals.[8] Computers are good at finding arbitrage opportunities and taking advantage of them—so it is expected to contribute to shortening the duration of both negative spread and triangular arbitrage opportunities.

Also in 2003, the "primary customer (PC)" system started. Under the PC system, smaller banks with manual trading (where humans are judging and hitting the button to order) can make a contract to receive the service of large banks as primary brokers. The PC banks can make orders under the name of primary broker to the EBS system, so that the primary broker collects fees, and primary customers can have better trading chances with narrower bid-ask spread. The introduction of PC is expected to

[8] Human ability to recognize opportunities and hit a button for execution takes one to two seconds, while the computer can do it in a fraction of a second.

narrow the bid-ask spread, because, under the PC agreement, small PC banks obtain access to trading with a wider set of counterparties (primary broker bank's credit-line linked counterparties), they do not have to hit the behind-the-best quote, or to post a negative spread, in order to expect trading in the subsequent second or so.[9]

As more banks face better spreads under the PC system, the credit line problem becomes less severe. This contributes to reducing the number of negative spreads occurrence, because a weak, non-primary broker bank can use and execute under a large bank's name and credit line. But it is not clear whether the PC system actually reduce occurrence and duration of both negative spreads and triangular arbitrage.

In 2004, another innovation in the market micro structure took place. The Professional Trading Community (PTC) was introduced. PTC is a category of players including hedge funds, commodity trading advisers (CTA), proprietary trading houses, and other non-bank financial institutions, almost all of which rely heavily on algorithmic trading. Starting December 2004, these non-banks were allowed to connect their computers directly to the EBS trading system.

Computer trading, both Ai and PTC, will make it possible to find arbitrage opportunities and take advantage of them. Computers can discover profit opportunities much faster than humans, and then execute orders to take advantage of the opportunities, for example, hitting with orders on both sides of negative spread. So, it is expected to contribute to shortening the duration of both negative spread and triangular arbitrage opportunities. We will sum the number of "Ai traders" and the number of "PTC traders" and treat it as a proxy for computer trading.

2.5. Triangular Arbitrage

A negative spread in one currency pair is a very obvious arbitrage opportunity

---

9 The following example explains why an introduction of PC is expected to make negative spread a rarer event. Suppose that a non-Primary broker bank become very eager to buy the currency (say, USD/JPY) quickly, either for their proprietary trading account or for the purpose of executing a customer's order, they may post the bid (say, 100.10 yen for a dollar) higher than the current market-best ask (say 100.00 yen for a dollar), where the best ask is not available to the bank due to a lack of credit line. The EBS system then shows the negative bid-ask as the best prevailing in the market. Suppose that the non-primary broker bank holds a contract with a Primary broker bank and become eligible to use the primary broker's credit-line to hit the market-best ask (100.00).

and an obvious violation to the efficiency of the market. However, even if there is no negative spread in each of the three currency pairs, USD/JPY, EUR/USD, and EUR/JPY, arbitrage opportunities may arise with transacting three currency pairs simultaneously. When one currency pair is out of line with other two currencies, that is, violating an arbitrage condition of three currency pairs, the arbitrage opportunity may exist.

The three currency pairs with each having positive bid-ask spread is denoted as follows:

For USD/JPY: $qa_t^{USD/JPY} > qb_t^{USD/JPY}$, with mid-point of $qm_t^{USD/JPY}$
For EUR/USD: $qa_t^{EUR/USD} > qb_t^{EUR/USD}$, with mid-point of $qm_t^{EUR/USD}$
For EUR/JPY: $qa_t^{EUR/JPY} > qb_t^{EUR/JPY}$, with mid-point of $qm_t^{EUR/JPY}$

Now, the three currency arbitrage condition using the mid-point is:

$$\frac{1}{qm_t^{USD/JPY}} \times \frac{1}{qm_t^{EUR/USD}} \times qm_t^{EUR/JPY} = 1 \qquad (1)$$

However, even if this condition is not met, it does not mean that there is an arbitrage profit opportunity, because buying and selling take place at particular side of the bids and quotes. Suppose a series of transactions that take the yen to the US dollar, to Euro, and back to the yen. First the buying the US dollar takes at the ask side (quoted price of USD seller); then buying the euro with the US dollar takes place at the ask side (quoted price of Euro seller); and finally exchanging the euro back to the yen should be on the bid side of the EUR/JPY. Thus, the triangular arbitrage, TA, considering the bid-ask, in the three transactions of JPY→ USD→ EUR→ JPY is defined as

$$\text{TA(JPY/USD/EUR/JPY)} = \frac{1}{qa_t^{USD/JPY}} \times \frac{1}{qa_t^{EUR/USD}} \times qb_t^{EUR/JPY}$$

Similarly, the amount of yen within the three transactions of JPY→ EUR→USD→ JPY is denoted as follows:

$$\text{TA(JPY/EUR/USD /JPY)} = \frac{1}{qa_t^{EUR/JPY}} \times qb_t^{EUR/USD} \times qb_t^{USD/JPY}$$

TA making positive profits is the value of TA(*) being more than 1, and would not make profit if the value of TA (*) is equal or less than one, where * is the order of the three sequential trading of currency pairs. It is easy to show that the mid-point arbitrage (eq. (1)) is holding the triangular arbitrage would not make profits, namely, (TA(*)) <1.

Example (1): In order to illustrate the point, suppose that the mid-point exchange rates are 100 yen per US dollar (JPY/USD=100); 1.1 US dollar per Euro (EUR/USD=1.1); and 110 yen per Euro (JPY/EUR=110), and the bid-ask spreads are 2 pips, each:

$qa_t^{USD/JPY} = 100.01;\ qb_t^{USD/JPY} = 99.99$

$qa_t^{EUR/USD} = 1.1001;\ qb_t^{EUR/USD} = 1.0999$

$qa_t^{EUR/JPY} = 110.01;\ qb_t^{EUR/JPY} = 109.99$

Then, the mid-point arbitrage is perfectly holding, and triangular arbitrage in either direction would not yield profit. Then suppose that the EUR/JPY jumps by 0.1 yen, namely, $qa_t^{EUR/JPY} = 110.11$; $qb_t^{EUR/JPY} = 110.09$. The exact mid-point arbitrage condition is violated:

$$\frac{1}{qm_t^{USD/JPY}} \times \frac{1}{qm_t^{EUR/USD}} \times qm_t^{EUR/JPY} = 1.0009 \qquad (2)$$

Calculating the triangular arbitrate condition of JPY→ USD→ EUR → JPY reveals that the triangular arbitrage condition is also violated:

Then TA(JPY/USD/EUR/JPY) = 1.000627 > 1

Three transactions in the other direction would not yield profits, as it can be calculated, TA(JPY/EUR/USD/JPY) = 0.998810.

Suppose that the midpoint is the same but the bid-ask spread widens to 8 pips:

$qa_t^{USD/JPY} = 100.04;\ qb_t^{USD/JPY} = 99.96$

$qa_t^{EUR/USD} = 1.1004;\ qb_t^{EUR/USD} = 1.0996$

$qa_t^{EUR/JPY} = 110.14;\ qb_t^{EUR/JPY} = 110.06$

The (not-exact) mid-point arbitrage condition (2) stays the same. But in this case, either direction of TA(*) does not produce profits.

TA(JPY/USD/EUR/JPY)=0.999782; and TA(JPY/EUR/USD/JPY)=0.997966.

This illustrates the general point that TA is more likely to be violated when the bid-ask spread is narrower, given that the degree of a deviation in mid-point arbitrage condition.

Example (2). Let us show a real world example of TA. At June 10, 1999, 06:49:02GMT, the following set of quotes are observed in the market.

$qa_t^{USD/JPY} = 118.55$; $qb_t^{USD/JPY} = 118.50$

$qa_t^{EUR/USD}$=1.0515; $qb_t^{EUR/USD}$ =1.0514;

$qa_t^{EUR/JPY} = 124.55$; $qb_t^{EUR/JPY}$ =124.45;

At this point,

TA(JPY/USD/EUR/JPY)=0.998353; and TA(JPY/EUR/USD/JPY)=1.000328

These examples show two facts with respect to triangular arbitrage. First, the violation to the mid-point arbitrage condition does not necessarily mean that there is a triangular arbitrage opportunity, because of bid-ask spreads can be wide enough to make the three transactions would not yield profits. Second, given that the mid-point triangular arbitrage condition is not holding, the wider bid-ask spreads make it *less* likely that the triangular arbitrage profit opportunity arises.

3. Negative spread, empirical results

3.1. How often did opportunities emerge?

Contrary to any prediction derived from the efficient market hypothesis, negative bid-ask spread, occurs from time to time. This is well-known among market participants and researchers handling tick-by-tick data. However, almost all researchers have "cleaned the data," discarding such observations, on an implicit assumption that they must be stale quotes or misquote of the market conditions. The assumption was realistic when researchers can obtain only the data set with indicative (not necessarily transactable) quotes, such as the Olsen data set. As the EBS data have become available, it is no longer appropriate for researchers to disregard those quotes as unreliable.

First, how often the negative spreads are observed in the data set is calculated in terms of average ratio (probability) of negative spreads. The ratio is defined as the cumulative duration (total number of seconds) of negative spreads divided by the total sample duration (seconds) in that month. The monthly probability is then shown as a time series from 1999 to 2010. **Figure 1** (Panel A for USD/JPY, Panel B for EUR/USD, and Panel C for EUR/JPY) shows the time series of such monthly ratios. For example, in the USD/JPY market, in 1999, negative spreads were observed about 0.5 percent of time. The probability declined to around 0.1 percent for the period from 2000 to 2004. The ratio then began a downward trend. In 2010, negative spreads were observed only 0.001 percent of time. The ratio was relatively higher, above the trend, from 2007 to mid-2008, most likely due to the volatile currency market in the midst of Global Financial Crisis. The magnitude of the ratio of negative spread is similar in the EUR/USD market. From 1999 to 2005, the negative spread in the EUR/USD market was observed at around 0.1 percent, but it declined to 0.001 percent by end 2010. There was a spike in late 2008, most likely reflecting the market turmoil in the wake of the Lehman Brothers failure. The EUR/JPY market shows the similar pattern to the USD/JPY market.

The total duration of a negative spread can be expressed as the number of occurrences times the average duration per occurrence (incidence). Even if a negative spread occurs often, if it disappears quickly, by dealers taking advantage of profit opportunities, then a normal condition, or efficiency, is restored very quickly. When dealers find a negative spread that is profitable even within transaction costs are paid, they have to hit both sides of bid and ask to realize profits. Hence, increased probability of observing deals on the both sides of ask and bid (double deals) one second within the negative spread (contributing to the disappearance of negative spread) is another evidence that negative spreads as free lunch are real and can be taken advantage of by dealers with quick hands or, more likely these days, by computer algorithm.

In the next subsection, we investigate when negative deals are likely to occur and in the subsection 3.3, the probability of double deals is analyzed.

3.2. Occurrence of negative spreads: Factors and Market Conditions

When are negative spreads likely to be observed? Would they occur in a thin market when not many participants are awake (for example, between the close of New York market to the open of the Tokyo market), or would they occur more likely in very active market when the prices are volatile?

The following monthly statistics are constructed for the period from 1999 to 2010 for each currency pair, j: The number of negative spread occurrence per day, N_t^j; the probability of disappearance within one second (i.e., the ratio of the disappeared within one second to total occurrence of negative spread), the number of average deals per day, and the volatility (the daily average of absolute change in terms of "pips" of the deal price (mid of deal prices on the bid and ask sides). **Figure 2** (panels A (USD/JPY), B (EUR/USD) and C (EUR/JPY)) shows these statistics.

In Figure 2, it is striking that the number of occurrence per day declined steadily in all three currency pairs. In case of EUR/USD, negative spreads were observed about twenty times a day from 1999 to 2001, followed by a declining trend. It became very close to zero by 2010. From late 2009 to 2008 there was a prominent spike in the probability of occurrences. During the period of the Global Financial Crisis, especially within the Lehman Brothers failure, the market became very volatile (volatility in the figure became record high at the time), and the number of deals were also very high. Hence, as a first step toward a rigorous analysis, we have a hypothesis that negative spreads tend to occur in an active and volatile market rather than an inactive market. Any analysis of other factors, we need to control for activity levels of the market.

Table 1 (Panels A, B, and C) is the annual average of the same statistics, the occurrence of negative spread, with breakdown by the size of negative spreads, 1-pip, 2-pip, 3-pip sizes. *N* denotes the number of negative spread occurrence of x-pip or more, per day, yearly average; *P* denotes the probability of disappearance at one second later of negative spread of x-pip or more; *D* is the number of deals (either ask side or bid side), per day, yearly average, and the volatility, *V*, is defined by the average absolute pip change in the mid-deal price (from a deal is done to the next deal), yearly average.

For USD/JPY in 1999, a negative spread with at least 1 pip magnitude, occurred 150 times a day, and they disappear within one second 46% of the time. The occurrence of negative spread with 2-pip (3-pip) magnitude is much rarer, with 45 (16, respectively) times a day, and the disappearance-within-one-second probability was 53% (57%, respectively). For USD/JPY in 2007, the negative spread with 1 pip; 2 pip; 3 pip magnitude occurred 11 times; 1.3 times; 0.3 times, respectively. The disappearance probability was 85%; 87%; 89%, respectively.

Extending these observations, with help of Figure 2 and Table 1, we can derive the following conclusion. The number of occurrences of a negative spread has been on the declining trend for all currency pairs, with an exceptional upward spike immediately after the failure of Lehman Brothers in September 2008. The timing coincided with a huge increase in volatility in all three currency pairs. The probability of disappearance within one second is on the increasing trend. However, in 2009/10, it becomes unstable. This is due to the relatively small sample days in 2009/10. For each currency, the number of deals was relatively stable until the beginning of 2007 and then rose sharply toward the end of 2008; and volatility had been on the declining trend from 1999 to mid-2007, and then it started to increase from mid-2007 to end-2008. Even in 2009/10, the level of volatility remained much higher than previous normal years (2000-2007). The raised level of volatility is more remarkable in EUR/USD and EUR/JPY. The volatility of USE/JPY has been declining since the beginning of 2009 to end-2010.

From Figure 2 and Table 1, it is not clear whether the decline of the occurrence of negative spreads and the increase of the disappearance probability within one second can be explained solely by the number of deals, which may be a proxy for the market depth, and high volatility, which may be relevant, particularly for the measurement of occurrences.

As explained earlier, the occurrence and probability of disappearance may be closely related to innovations in microstructure of the EBS system, namely the advancement of computer algorithm technology and the connection of bank computers with the EBS computer.

We have observed above that the occurrence of negative spread has declined significantly by the beginning of 2010.[10] The sample period is chosen to be from 1999 to 2009, because samples of negative spreads became too small to be reliable for a regression analysis in 2010.

The log of number of negative spread occurrence in the month t for each

[10] It is said that due to upgrading of a matching engine at the EBS, sometime in 2009, execution of matching (deal) became faster. Hence, it is possible that negative spread had occurred but been eliminated in a fraction of a second, so that our data that are on the one-second slice did not detect the existence.

currency pair, j is denoted by (N_t^j). This is regressed on the deals (D_t^j), volatility (V_t^j), number of PC (PC_t^j), and the number of the sum of Ai and PTC traders ($AiPTC_t^j$).

$$\ln\left(N_t^j\right) = c^j + \alpha_1 D_t^j + \alpha_2 V_t^j + \alpha_3 PC_t^j + \alpha_4 AiPTC_t^j + \varepsilon_t^j \qquad (3)$$

Table 2 shows the regression results for all three currency pairs. Results of estimates by both Ordinary Least Squares (OLS) and Generalized Methods of Moments (GMM) are shown.[11] Coefficients of all variables have expected signs with statistical significance, except a few in the GMM regressions that are noted below. The occurrence of negative spread increases with volatility and the number of deals, both the indication of higher market activities. (For USD/JPY and EUR/USD in GMM, the number of deals is not significant.) Controlling for those market conditions, the number of observed negative spread decreases with the widespread use of both the computer trading (Ai+PTC) and primary customers (PC) institutions. The results show that in all three currency pair, a wide-spread use of direct connection of bank computers with the EBS computer reduced the occurrence of negative spread. Since it is much easier for a computer than a human to spot the negative spread and take advantage of it in the fraction of a second, the statistically significant negative coefficient of Ai+PTC makes sense. The negative coefficient of PC implies that when not-so-strong banks started to use the name and credit lines of large banks, a chance of negative spread is reduced. [12] Without the PC contract, not-so-strong banks sometimes may be forced to post a quote that implies a negative-spread in the market due to a lack of credit lines. Widespread use of PC contracts, this possibility disappeared. (For USD/JPY, a coefficient for PC is not statistically significant in GMM, although the expected negative sign is obtained.)

Next, we investigate determinants of how quickly the negative spread will disappear. For each month, the probability of disappearance of negative spread of currency pair j is calculated as the ratio of the disappeared negative spread within one second to the total number of negative spread, p_t^{NSj}. Recall that the time series of the

[11] GMM was conducted since standard errors in the OLS regressions may not be accurate due to suspected autocorrelation and heteroscedasticity, while GMM may suffer from a (relatively) small sample data set, as all regressions are conducted in monthly frequency.

[12] The number of PC is not significant in USD/JPY with GMM estimation.

probability is shown in Figure 2. The logistic regression, estimating the probability, p_t^{NSj}, is estimated with the number of deals, volatility, and the number of the sum of Ai and PTC.

$$\ln\left(p_t^{NSj}/(1-p_t^{NSj})\right) = c^{NSj} + \beta_1^{NSj} D_t^j + \beta_2^{NSj} V_t^j + \beta_3^{NSj} AiPTC_t^j + \varepsilon_t^{NSj} \quad (4)$$

Table 3 shows the results. Almost all the variables were estimated with respectively expected signs and with statistical significance of 1 percent. The exception was the volatility variable of USD/JPY, with P-value of 0.24 in case of GMM. In all three currency pairs the within-one-second disappearance probability tends to decrease, as the level of volatility in the market increases. The number of deals, as a proxy for market activities, contributes to increase the probability of disappearance within one second. Even controlling for these market variables, the change in market microstructure may affect the probability of within-one-second disappearance. The sum of Ai and PTC traders, a good proxy for the widespread use of computer trading with computers connected to the EBS computer has the statistically significant coefficient. In all three currency pairs, it contributes to disappearance of negative spread. The introduction of machines trading with the order-matching machine significantly increased the probability of very quick disappearance of negative spread. This makes perfect sense, as one thing a machine can do best is to exploit arbitrage opportunities without any human judgment.

3.3. Double Deals

We have established that negative spreads do appear time to time, but they disappear very quickly within a few seconds. If the disappearance occurs due to correction of a bid or an offer without actual transactions, we expect that the number of deals one second within the emergence of negative spread should not necessarily increase. On the other hand, if disappearance is due to some bank taking advantage of negative spreads, then we expect an increase in deals on both the bid side and offer side at one second within the appearance of negative spread. So, comparing to other times, if the probability of simultaneous deals on the both sides (double deals) is higher at one second within the appearance of negative spread, then we take it as evidence that some dealers with quick hands detect the profit opportunities and take actions to exploit the opportunities.

Figure Panel 3-j for currency pair j shows two lines. The dotted (lower) line shows the probability of double deals for all seconds, q_t^{DDj}, that is the ratio of the number of seconds with double deals to the number of total sample seconds. The solid (higher) line shows the probability of double deals conditional on negative spread at one second earlier, that is the ratio of the number of seconds with double deals that took place one second within negative spreads to the number of seconds with negative spreads, q_t^{NSDDj}. The probabilities (ratios) are calculated monthly. The tight range of error bar (confidence interval of 1 sigma) at each month on either line shows the standard error of Bernoulli process for that month.

Several observations are obtained. First, in each figure, both lines show an increasing trend. What is remarkable is the large difference between conditional probability and unconditional probability lines. This implies that negative spreads do prompt double deals one second within such an appearance. When there is a negative spread, it is very likely to stimulate deals on both sides of ask and bid one second later. This is consistent with a scenario that when traders see negative spread, they actively trade to exploit the opportunity. This also is an empirical proof that the expected profits, conditional on both trades hitting, exceeds the transaction costs.

Second, the probability of double deals one second within negative spread occurrence has steadily increased faster than unconditional double deals throughout sample years. The probability became almost unity at around 2006 for EUR/USD (panel 3-B). Big dips in the probability in 2008 and 2009 are due to the extreme market volatility (and failure to exploit the profit opportunity) and the small number occurrence (denominator).

Similarly, All panels 3-A, 3-B and 3-C show the same trend and characteristics. But, conditional probability of double deals is slightly lower in USD/JPY and EUR/JPY than EUR/USD. It suggests that the free lunch opportunities are detected and taken advantage of by execution of double deals.

Table 4 shows the regression results of the probability of double deals for all samples (that corresponds to lower line in Figure 3), q_t^{DDj}, and that for negative spread samples (that corresponds to higher line in Figure 3), q_t^{NSDDj}.

$$\ln\left(q_t^{DDj}/(1-q_t^{DDj})\right) = c^{DDj} + \beta_1^{DDj}D_t + \beta_2^{DDj}V_t + \beta_3^{DDj}AiPTC_t + \varepsilon_t^{DDj} \quad (5)$$

$$\ln\left(q_t^{NSDDj}/(1-q_t^{NSDDj})\right) = c^{NSDDj} + \beta_1^{NSDDj}D_t + \beta_2^{NSDDj}V_t + \beta_3^{NSDDj}AiPTC_t + \varepsilon_t^{NSDDj} \quad (6)$$

The coefficients of the constant term are significantly different in each currency pair. The difference of the constant terms roughly corresponds to the vertical difference of two lines in Figure 3. In addition, the introduction of the machines connected to the EBS system (AiPTC) has made a measurable difference in the probability of double deals. Without being conditional on the appearance of a negative spread, the more computers connected to the system implies more double deals. However, the increase in probability of double deals conditional on the negative spreads once second earlier is much more pronounced when the number of machines in the system becomes higher.

4. Triangular Arbitrage

In this section, we describe and analyze how often the triangular arbitrage opportunities have emerged in our sample; how quickly they have disappeared; and whether disappearance is correlated with simultaneous triangular transactions one second later. Investigation methods in this section parallel with the preceding section on negative spread.

As explained in Section 2, the triangular arbitrage opportunities may occur in the direction of either JPY→ USD → EUR → JPY; or JPY→ EUR→ USD → JPY. When an arbitrage opportunity arises in either direction, we count it as one event of triangular arbitrage (TA) opportunity. As explained in Section 2.5., TA opportunity cannot simultaneously emerge in both directions.

First, **Table 5** shows how often TA opportunities emerged, by counting the number of occurrence, which is defined by one continuous occurrence of the TA opportunities, however many seconds it continued. The unit is the yearly average of TA (measured in the number of occurrence) per day.[13] In the table, arbitrage opportunities are counted by the size of potential profits: "N_0", "N_10" and "N_30" corresponds to "more than break-even," "more than $10," and "more than $30" per one million dollar contract (minimum unit of transaction)," respectively.

The probability of disappearance is calculated for each of the profit size: "P_0", "P_10" and "P_30" correspond to probability of "N_0", "N_10" and "N_30", respectively. The $6 is approximate transaction costs for three currency pair trades (JPY/USD, EUR/USD, EUR/JPY) for large-volume banks; and the $22.5 is approximate transaction costs for three currency trades for small-volume banks.[14] Hence, N_10 and N_30 show the numbers of potential profits opportunities that emerged for large-volume financial institutions and small-volume financial institutions, respectively, net of transaction costs.

Table 5 shows that on average, N_10 of TA (more than $10 gain) opportunities emerged about 480 times a (sample) day in 1999 and 2000, but spiked up to 524 in 2001. The number gradually decreased to about 350 in 2008. The number sharply declined to 123 in 2009, and then to less than 50 times in 2010. The spike up in 2001 may be related to the introduction of EURO cash (paper currency) in January 2001, and increased transaction and volatility in the EURO rates. The larger profit opportunity, namely the $30 or more profit opportunities (N_30) was much rarer an event. In 2001, the number was 390 times per day, but by 2008, it declined to 184. It sharply declined to 68 in 2009, and to 24 in 2010. The number for N_30 in each of 2008-10 was about one half of the number for N_10.

The probability of disappearance one second within emergence of such a TA opportunity (P_10) rose from 0.36 in 1999 to 0.87 in both 2009 and 2010. The same statistics for larger profit opportunities (P_30) has been slightly higher than that of P_10, rising from 0.38 in 1999 to 0.90 in 2009 and slightly declined to 0.88 in 2010. In the first half of the 2000s, there were significant differences between the probability of

---

[13] As explained in Section 2.1., data on days with very low number of deals are not included, and days that are two-day prior to a national holiday are not used. If they are included, the probability increases slightly, since there used to be occasions of prolonged TA opportunities during a holiday season.

[14] This estimate is based on our interpretation of and interview on the transaction fees charged by EBS on financial institutions.

disappearance between small and large profit opportunities, that is, between P_0 and P_10; and between P_10 and P_30. This is consistent with our presumption that market participants reacted much faster to larger potential profit opportunities. But, the differences narrowed to a few percentage points by 2008. It should be noted that the occurrence itself became much fewer and the probability may have reached its practical ceiling for all profit sizes.

It is also remarkable that TA opportunities have emerged much more often than negative spread. By comparing 1-pip negative spread in Table 1 (panels A, B, and C) and small-profit TA opportunity (N_10) in Table 5, we observe that the number of TA opportunities are typically more than ten times of the number of negative spread in any of the three currency pairs (N_1pip). For example, the number of negative spread for the three currency pairs ranged from 19 to 150 in 1999, but the number for TA opportunity was 477. In 2010, the number of negative spread for the three currency pairs was down to the range of 0.9 to 1.1, while the number for TA opportunity declined to 47.

The reason that TA opportunities emerge more often than negative spread may be due to its difficulty in being detected and taken advantage of, at least for humans, relative to negative spread. Similarly, the increase in the probability of disappearance within one second was much faster for negative spread than TA opportunities. The disappearance probability of negative spread reached 85% by 2006, while that of TA opportunities reached 85% for all profit sizes only in 2009. But, by 2010, the disappearance probabilities of negative spreads and of TA became much similar.

The last two columns of Table 5 show the number of deals and volatility in the three currency markets. The sum of the numbers of USD/JPY deals, EUR/USD deals, and EUR/JPY deals per day is shown as “deals”. The number of the deals rose from about 29,000 in 2001 to 42,000 in 2010. This is a reflection of the expansion of the global foreign exchange market transactions. The volatility measure is the average absolute value of pip changes of three currencies, one second after deals were recorded.

The unconditional probability of triangular arbitrage for a month is defined as “the number of seconds when triangular arbitrage opportunities existed” divided by “the number of total seconds in sample.”[15] If triangular arbitrage opportunities existed 2

[15] How certain days are omitted from the sample was explained in Section 2.2.

seconds, then both seconds are counted toward the calculation, that is, the numerator, in the calculation of probability. This definition is thus duration-sensitive, unlike "occurrence" in Table 5.

Figure 4 shows the probability of triangular opportunities (the number of seconds with TA divided by the total number of seconds in sample) calculated monthly, from 1999 to 2010. Three lines show the different profit sizes, $0, $10, and $30. The probability was at around 2% in 1999, gradually decreasing to 1% by 2007.[16] The probability shows a sharp decline from 2008 to 2009 and, then again from 2009 to 2010. By 2010, the probability was at around 0.1%.

Figure 5 shows the monthly movements of the number of TA opportunities; the probability of disappearance one second within its emergence, the number of simultaneous three currency deals; and the volatility measure. For the first two pieces of information, we draw three lines. The lowest line (>1.00000) is a TA opportunity with any positive arbitrage value; the middle line (>1.00001) is a TA opportunity with more than 0.001% profit ($10 per 1 million USD contract); and the top line is a TA (>1.00003) opportunity with more than 0.003% ($30 per 1 million USD contract) profit. The larger the potential TA profits, the quicker it disappears within one second. Volatility had gradually declined from 1999 to 2007, but then rose sharply in mid-2007, and even more sharply in late-2008, with a much smaller spike in the spring of 2010, which reflect the US sub-prime crisis, the Lehman Brothers collapse and its impact, and the Euro sovereign debt crisis, respectively. These periods correspond to spikes in the number of TA opportunities. TA opportunities seem to increase, ceteris paribus, with volatility and the number of deals being transacted.

The top panel of Figure 5 shows the daily average of the number of triangular arbitrage opportunities for that month. There are three lines for positive arbitrage opportunity (more than break even); for the size of opportunities that exceeds $10 per $1 million contract; and for the size of opportunities that exceeds $30 per $1 contract.

The second panel of Figure 5 shows the probability of disappearance within

[16] Aiba, Hatano, Takayasu, Marumo, and Shimizu (2002) claimed that they observed triangular arbitrage for 6.4% of total time in January – March 1999. However, our results show it was around 2 to 2.5% at in months. The difference is due to the data set (ours is the actual transaction platform and theirs is not) and our screening of the data.

one second for each of the three sizes of profit opportunities. Between 2004 and 2008, there are significant differences among the three lines, the bigger the expected profits, the higher the probability of differences.

In terms of daily average number, TA opportunities (Table 5; N_0) occur typically 50 times more often than negative spread of 3 currency pairs (measured in Table 1-A, B, and C) in 2010. The negative spreads (of more than 1 pip) hardly occur any longer, that is, on average, less than once in a day for any of the three currency pairs, while the TA opportunities (of 0.001%, or more) do occur. This reflects the technical difficulty, even for computers, to avoid or take advantage of profit opportunities. It is more difficult to detect TA opportunity than negative spread, and it takes three transactions, rather than two, to take profit from such a position. However, with advancement of the transmission speed of electronic order execution and computer calculation speed, the number of such profit opportunities will continue to decline.

In sum, over the years, the TA opportunities have become less frequent events, and once they appear, they disappear with increasing speed. Fluctuations in the number of TA opportunities seem to be correlated with the volatility of the market.

Below we concentrate in the case of potential arbitrage opportunities of at least 0.001% profits (>1.00001). We have observed that the number of TA profit opportunities has declined significantly by the beginning of 2010. Hence, in the regression analysis below, we terminate samples at the end of 2009.

We investigate the impact of the microstructure innovation on TA opportunities, controlling for the number of deals and volatility, using a regression analysis.

$$\ln\left(\mathrm{N}_{\mathrm{t}}^{\mathrm{TA}}\right) = c^{TA} + \alpha_1^{TA} D_t + \alpha_2^{TA} \mathrm{V}_t + \alpha_3^{TA} PC_t + \alpha_4^{TA} AiPTC_t + \varepsilon_t^{TA} \qquad (7)$$

Table 6 shows the results. Estimates are all statistically significant (except N_PC in GMM), and imply that more deals (market activities) lead to an environment where it is more likely that a TA opportunity emerges.[17] It is not true that TA

[17] Marshall, Treepongkaruna, and Young (2008) found that triangular arbitrage opportunities emerge constantly and more often when quote activities are high. The finding in our paper that volatility is positively correlated with occurrence of triangular arbitrage opportunities is consistent with their explanation, since volatility in the mid-point revisions occurs when new quotes come in to the market.

opportunity occurs in times where the market is thin, but it happens in times with more active trading. Second, higher volatility tends to be associated with more TA opportunities. The very high volatility of prices is likely to produce not-exact mid-point triangular arbitrage condition, since one currency pair may move first, while there may be a delay in other currency pairs to catch up to recover the exact triangular arbitrage condition.

The institutions do matter. The introduction and proliferation of the Primary Customer system may mean narrower bid-ask spread (Sec 2.3). The narrower spread meant that it is more likely to observe a TA opportunity, given the jump in the mid-point of one currency pair, holding other currency pairs constant (Sec. 2.4.). Hence, the coefficient of the number of primary customers (PC) is expected to be positive, and this is indeed the case in Panel of OLS, but not in GMM. The more computer trading institutions there are, the less likely are the opportunities of TA, but it is not statistically significant (in GMM).

It is clear that the introduction of computer trading contributes to declining occurrence of TA opportunities, because computers can revise bid and ask prices to restore the mid-point arbitrage condition (eq. (1)), by adjusting the other two currency pairs at the time of changing one currency pair. Alternatively, if a computer quickly discovers a particular TA opportunity and carries out trades to take profits within one second (say, 250 millisecond), then it does not show up in our sample which is based on one-second slice. The increase in computer trading is expected to strongly contribute to the decline in the number of TA opportunity.

Next, we investigate the probability of TA opportunity disappearance one second within it emerges ($\mathrm{p}_\mathrm{t}^{\mathrm{TA}}$), aggregated monthly, as a logistic regression.

$$\ln\left(\mathrm{p}_\mathrm{t}^{\mathrm{TA}}/(1-\mathrm{p}_\mathrm{t}^{\mathrm{TA}})\right) = \mathrm{c} + \beta_1^{TA}\mathrm{D}_\mathrm{t} + \beta_2^{TA}\mathrm{V}_\mathrm{t} + \beta_3^{TA}\mathrm{AiPTC}_\mathrm{t} + \varepsilon_t^{TA} \qquad (8)$$

The regression is to explain the probability of TA opportunity to emerge by the number of deals, the volatility measure and the number of computer trading connected to the EBS system (AiPTC). The estimation results are shown in **Table 7**.

Table 7 shows that the probability of TA opportunity disappears one second within it emerges increases as the volatility is lower and the number of deals is higher. Controlling for those factors, an increase in bank computers being connected to the EBS computer increases the probability of disappearance within one second. This is expected that TA opportunities would be more difficult to spot by human eyes than by computer algorithm.[18]

Next, we investigate whether the disappearance is associated with actual deals, in this case the simultaneous triple deals in the three currency pairs. **Figure 6** shows two sets of line. The three top lines show the probability of triple deals conditional on the existence of triangular arbitrage opportunity one second earlier. The three probability lines correspond to potential profit sizes of \$30 or more, \$10 or more, and \$0 or more. The lower line shows the probability of simultaneous triple deals for all seconds. As was the case for the negative spread, a big difference between the two lines is consistent with the proposition that the TA opportunities prompt triple deals in order to take profits out of the opportunities.

The unconditional probability of triple deals, $\mathrm{q}_{\mathrm{t}}^{\mathrm{TA}}$, is first calculated monthly, and then estimated as a logistic regression with Deals, Volatility and the number of computers connected to EBS. Similarly, the probability of triple deals conditional of the TA opportunity one second earlier ($\mathrm{q}_{\mathrm{t}}^{\mathrm{TATD}}$) is calculated monthly and tested with a logistic regression.

$$\ln(\mathrm{q}_{\mathrm{t}}^{\mathrm{TD}}/(1-\mathrm{q}_{\mathrm{t}}^{\mathrm{TD}})) = \mathrm{c} + \beta_1^{TD}\mathrm{D}_{\mathrm{t}} + \beta_2^{TD}\mathrm{V}_{\mathrm{t}} + \beta_3^{TD}\mathrm{AiPTC}_{\mathrm{t}} + \varepsilon_t^{TD} \qquad (9)$$

$$\ln\left(\mathrm{q}_{\mathrm{t}}^{\mathrm{TATD}}/(1-\mathrm{p}_{\mathrm{t}}^{\mathrm{TATD}})\right) = \mathrm{c} + \beta_1^{TATD}\mathrm{D}_{\mathrm{t}} + \beta_2^{TATD}\mathrm{V}_{\mathrm{t}} + \beta_3^{TATD}\mathrm{AiPTC}_{\mathrm{t}} + \varepsilon_t^{TATD} \qquad (10)$$

The results are shown in **Table 8**. The difference in the constant terms corresponds with the vertical difference of the two lines of Figure 4. The constant term

[18] The same point is stated without evidence in Chaboud, et al. (2009): "Interestingly, the difference in price impact in the cross-rate, the euro-yen exchange rate, is very small. In this market, computers have a clear advantage over humans in detecting and reacting more quickly to triangular arbitrage opportunities so that a large proportion of algorithmic trading contributes to more efficient price discovery. It is then not so surprising that in this market, computers and humans, on average, appear to be equally informed."

is higher in the conditional (TA opportunity) equation (lower panel), compared to the unconditional equation (upper panel). The sensitivity to the number of trading computers connected to the EBS system (AiPTC) is also different between the two regressions. In sum, the findings are consistent with a view that the TA opportunities increase the probability of triple deals conducted one second later, controlling for the volatility and the number of deals. Hence, the connection of banks' computers to the EBS system made it much faster to take advantage of TA profit opportunities, resulting in the disappearance of the opportunities within one second.

5. Concluding Remarks

This paper shows that free lunch—risk-free arbitrage opportunities—does exist, but only for a very brief period, that is, in the matter of a few seconds. The probability of occurrence has declined and the probability of disappearance within one second has increased in the past ten years.[19] Changes in market microstructure have contributed to disappearing free lunch. Since the mid-2000s, an increasing number of banks' computers are allowed to be directly connected to the EBS computer. The machines undoubtedly are faster than humans in detecting and taking advantage of free lunch opportunities. The power of computer (algorithmic) trading is shown in this paper as a primary cause for this change. Evidence shows that machines have made the market more efficient by almost eliminating negative spreads and triangular arbitrage opportunities. The probability of negative spread disappearing within one second was less than 50% in 1999, but increased to about 90% by 2009.

Details of major innovation and finding this paper can be summarized as follows. First, in order to recreate the real trading situation, the EBS data set that includes "firm" (transactable) quotes and transactions is used. The electronic broking system is a major trading platform for the yen/dollar, dollar/euro and yen/euro foreign exchange markets. Second, risk-free profitable arbitrage opportunities—free lunch—are defined as a negative spread in any currency pair and triangular arbitrage conditions involving three currency pairs, net of transaction cost. Contrary to almost all theoretical

[19] In the analogy of mistakenly dropping a genuine $100 bill on the street, it has become less frequent that someone drops the bill onto the street, and it has become much faster that the money is picked up by someone else, once it is dropped.

predictions, risk-free arbitrage opportunities are found to emerge time to time. Arbitrage opportunities tend to occur when the markets are active with large transaction volumes and high volatility. Third, over the 12-year sample period, the frequency of free lunch has declined and the probability of its disappearance within one second has increased. Fourth, The number of trades simultaneously on both ask and bid sides increases in one second within an arbitrage opportunity emerges (controlling for time of the day, and volatilities in preceding seconds), suggesting the free lunch opportunities, when occur, prompt actual trades in an attempt of taking advantage of free lunch. Fifth, during the period, market microstructure has changed. Starting 2003, major banks were allowed to connect their computers with algorithm trading programs directly to the EBS system. The direct connections have quickly spread to other banks. Starting December 2004, non-bank financial institutions, all of them being algorithmic traders, were also allowed to have a direct connection. Sixth, econometric analysis suggests that an increase in algorithmic trading contributed to declining occurrence of free lunch and increasing probability of its disappearance within one second. Seventh, the Primary Customer system introduced in 2003 allowed weaker banks with limited credit line counterparts to trade under the name of the primary broker banks. This led to a decrease the number of negative spread events, but increased the number of triangular arbitrage opportunities. The latter may be counterintuitive, but this can be understood as follows. The Primary Customer system contributed to narrowing the bid-ask spread, one of the efficient market criteria. But narrower spreads made it easier for the triangular arbitrage relationship to be violated in a volatile market.

Broader implications of the study extend from theory to business implications. First, assumptions in finance theory, with regard to no free lunch, are much more realistic now than ten years ago, thanks to algorithmic trading. Second, cross rates –exchange rates that do not involve US dollar—are deeper but more volatile now since computers are adjusting the rates to maintain arbitrage conditions. Third, algorithmic trading seems to have reached almost its maximum potential in the area of taking advantage of free lunch—negative spread and triangular arbitrage—by now. Unless the machine beats the competitor machines in its calculation and communication speed somehow, in the order of nano-seconds, a new entrant may not expect easy profits.

Fourth, once the algorithm to detect and take advantage of free lunch is written, it can be applied to any currency. Although it is not verified in this paper, we conjecture that other non-major currencies do satisfy arbitrage conditions, without experiencing the period of big opportunities.

References:

Aiba, Yukihiro and Naomichi Hatano, (2004), "Triangular Arbitrage in the Foreign Exchange Market," *Physica A* 344 (2004) 174–177.

Aiba, Yukihiro and Naomichi Hatano, (2006), "A microscopic model of triangular arbitrage" *Physica A* 371 (2006) 572–584.

Aiba, Yukihiro; Naomichi Hatano; Hideki Takayasu; Kouhei Marumo; and Tokiko Shimizu, (2002), "Triangular arbitrage as an interaction among Foreign exchange rates," *Physica A* 310 (2002) 467 – 479.

Aiba, Yukihiro; Naomichi Hatano; Hideki Takayasu; Kouhei Marumo; and Tokiko Shimizu, (2003) "Triangular arbitrage and Negative Auto-correlation of Foreign Exchange Rates," *Physica A* 314 (2003) 253 – 257.

Akram, Farooq and Lucio Sarno, (2008), "Arbitrage in the Foreign Exchange Market: Turning on the Microscope", *Journal of International Economics,* vol. 76, 2008, pp. 237-253.

Bank of International Settlements, (2010), *Triennial Central Bank Survey: Report on global foreign exchange market Activity in 2010*, December.

Bank of International Settlements, (2011), "High-frequency trading in the foreign exchange market," Study Group established by the Markets Committee, September.

Berger, David W.; Alain P. Chaboud; Sergey V. Chernenko; Edward Howorka; and Jonathan H. Wright, (2006). "Order Flow and Exchange Rate Dynamics in Electronic Brokerage System Data," Board of Governors of the Federal Reserve System International Finance Discussion Papers, Number 830r, April 2005, Revised September.

Chaboud, Alain P.; Sergey V. Chernenko; Edward Howorka; Raj S. Krishnasami Iyer; David Liu; and Jonathan H. Wright, (2004), "The High-Frequency Effects of U.S. Macroeconomic Data Releases on Prices and Trading Activity in the Global Interdealer Foreign Exchange Market," Board of Governors of the Federal Reserve System International Finance Discussion Papers, Number 823, November 2004.

Chaboud, A., B. Chiquoine, E. Hjalmarsson, and M. Loretan, 2007. Frequency of Observation and the Estimation of Integrated Volatility in Deep and Liquid Markets, *Journal of Empirical Finance*, Forthcoming.

Chaboud, Alain; Benjamin Chiquoine; Erik Hjalmarsson; and Clara Vega, (2009). "Rise of the Machines: Algorithmic Trading in the Foreign Exchange Market," Board of Governors of the Federal Reserve System, International Finance Discussion Papers, Number 980, October 2009.

Goodhart, Charles, Ito, Takatoshi, and Payne, Richard, 1996. One day in June 1993: a study of the working of the reuters 2000-2 electronic foreign exchange trading system. In: Frankel, J.A., Galli, G., Giovannini, A. (Eds.), The Microstructure of Foreign Exchange Markets. The University of Chicago Press, Chicago, pp. 107–179.

Goodhart, C.A.E. and O'Hara, M., (1997), "High frequency data in financial markets: Issues and applications." *Journal of Empirical Finance*, 1997, 4, 73–114.

Goodhart, Charles, Payne, Richard, 1996. Microstructural dynamics in a foreign exchange electronic broking system. *Journal of International Money and Finance* 15 (6), 829–85.

Hashimoto, Yuko and Takatoshi Ito**,** (2010), **"**Effects of Japanese Macroeconomic Announcements on the Dollar/Yen Exchange Rate: High-Resolution Picture," *Journal of the Japanese and International Economies* vol. 24, 2010: 334-354.

Hashimoto, Yuko; Takatoshi Ito; Takaaki Ohnishi; Misako Takayasu; Hideki Takayasu, and Tsutomu Watanabe, (2012), "Random Walk or a Run—Market Microstructure Analysis of the Foreign Exchange Rate Movements based on Conditional Probability—", *Quantitative Finance*, vol. 12, no. 6, June: 893-905.

Ito, Takatoshi and Yuko Hashimoto, (2006), "Intra-day Seasonality in Activities of the Foreign Exchange Markets: Evidence from the Electronic Broking System" *Journal of the Japanese and International Economies*, vol. 20, no. 4, December 2006: 637-664.

Ito, Takatoshi and Yuko Hashimoto (2008). "Price Impacts of Deals and Predictability of the Exchange Rate Movements," in T. Ito and A. Rose (eds.) *International Financial Issues in the Pacific Rim***,** NBER East Asia Seminar on Economics, Volume 17, Chicago: University of Chicago Press, 2008: 177-215.

Marshall, Ben R., Treepongkaruna, Sirimon and Young, Martin R., (2008). "What Level of Compensation is Available to Arbitrageurs in the Foreign Exchange Market?". AFA 2008 New Orleans Meetings Paper. Available at SSRN: http://ssrn.com/abstract=970410

Tokyo Foreign Exchange Market Committee, (2008). "Code of Conduct," 2008 Edition, http://www.fxcomtky.com/index_e.html.

Table 1. Negative spread: Annual summary of per-day occurrence and disappearance probability

Panel 1-A. USD/JPY

	The number of occurrence by the size of negative spread			Probability of disappearance within 1 second by the size of negative spread			# of deals	Volatility
Year	N_1PIP	N_2PIP	N_3PIP	P_1PIP	P_2PIP	P_3PIP		
1999	149.52	45.02	16.43	0.46	0.53	0.57	11184.18	0.46
2000	68.35	15.26	4.86	0.44	0.51	0.53	8725.24	0.36
2001	67.81	14.54	4.62	0.47	0.53	0.55	9506.25	0.38
2002	61.71	11.63	3.80	0.52	0.56	0.54	10592.55	0.36
2003	50.38	8.95	3.07	0.52	0.59	0.60	9854.52	0.32
2004	42.86	7.32	2.11	0.54	0.61	0.65	10597.00	0.32
2005	21.82	2.18	0.63	0.71	0.76	0.72	11609.93	0.28
2006	13.94	1.30	0.32	0.85	0.89	0.90	12889.79	0.27
2007	10.64	1.37	0.34	0.85	0.87	0.89	16315.07	0.29
2008	13.88	2.90	1.02	0.88	0.88	0.89	20075.77	0.43
2009	4.50	0.54	0.14	0.94	0.96	0.90	12563.88	0.43
2010	1.08	0.19	0.06	0.85	0.85	0.93	11651.96	0.34

(Panel 1-B, 1-C, to continue next page)

Panel 1-B. EUR/USD

	The number of occurrence by the size of negative spread			Probability of disappearance within 1 second by the size of negative spread			# of deals	Volatility
Year	N_1PIP	N_2PIP	N_3PIP	P_1PIP	P_2PIP	P_3PIP		
1999	71.15	11.35	3.54	0.47	0.56	0.59	13532.38	0.27
2000	89.07	14.80	4.33	0.50	0.56	0.61	15774.50	0.28
2001	64.16	9.54	2.44	0.54	0.60	0.64	14977.65	0.25
2002	42.75	5.03	1.22	0.55	0.61	0.57	14648.50	0.21
2003	62.95	7.88	2.14	0.59	0.63	0.65	18940.71	0.25
2004	53.16	6.81	1.93	0.62	0.68	0.70	20065.75	0.27
2005	24.29	2.27	0.62	0.73	0.76	0.73	19563.42	0.23
2006	9.16	0.88	0.26	0.81	0.80	0.77	18320.78	0.21
2007	4.59	0.43	0.12	0.85	0.85	0.89	17582.78	0.20
2008	17.16	2.57	0.81	0.91	0.89	0.89	27073.84	0.41
2009	8.43	0.91	0.25	0.94	0.89	0.84	23453.63	0.44
2010	2.31	0.37	0.14	0.91	0.81	0.77	24080.97	0.38

Panel 1-C. EUR/JPY

	The number of occurrence by the size of negative spread			Probability of disappearance within 1 second by the size of negative spread			# of deals	Volatility
Year	N_1PIP	N_2PIP	N_3PIP	P_1PIP	P_2PIP	P_3PIP		
1999	18.52	8.52	4.61	0.42	0.48	0.51	2147.72	1.05
2000	18.68	6.81	2.89	0.44	0.49	0.50	2705.94	0.86
2001	20.41	6.67	2.69	0.46	0.50	0.50	3228.39	0.74
2002	13.65	3.64	1.36	0.45	0.49	0.46	2840.32	0.61
2003	12.64	3.16	1.40	0.47	0.54	0.55	3176.54	0.62
2004	9.86	2.78	1.30	0.52	0.56	0.53	3474.76	0.68
2005	5.28	0.89	0.38	0.71	0.76	0.74	4138.71	0.51
2006	3.24	0.50	0.19	0.83	0.76	0.79	4718.20	0.46
2007	6.31	1.20	0.50	0.86	0.86	0.84	8749.50	0.56
2008	7.36	2.27	1.08	0.90	0.90	0.91	9138.94	0.97
2009	2.53	0.74	0.28	0.94	0.94	0.95	5990.26	1.04
2010	0.92	0.21	0.09	0.91	0.84	0.75	5452.19	0.92

Source: ICAP, EBS data mine. Level 2 (one-second slice), 1999-2010
Note: Authors' calculation

Table 2. Factors of occurrence of negative spread, (equation (3))

2-OLS

Equation (3)	USD/JPY			EUR/USD			EUR/JPY		
	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	1.41***	0.12	0.000	1.60***	0.113157	0.000	1.01***	0.12	0.000
Volatility	5.74***	0.40	0.000	7.73***	0.528806	0.000	1.42***	0.15	0.000
N_Deal	7.03 E-05***	9.06 E-06	0.000	3.48 E-05***	7.96 E-06	0.000	2.23 E-04***	1.58 E-05	0.000
N_PC	-4.97 E-03***	7.12 E-04	0.000	-5.99 E-03***	0.000632	0.000	-7.57 E-03***	7.73 E-04	0.000
N_AI+N_PTC	-4.11 E-03***	4.12 E-04	0.000	-4.22 E-03***	0.000453	0.000	-1.99 E-03***	4.72 E-05	0.000
Adj R-squared	0.95			0.94			0.89		
DW	1.41			0.78			1.52		
NOB	132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)		

2-GMM

Equation (3)	USD/JPY			EUR/USD			EUR/JPY		
	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	1.55***	0.43	0.000	1.713963***	0.26249	0.000	0.87764***	0.210283	0.000
Volatility	7.12***	1.06	0.000	7.622008***	1.277365	0.000	1.703904***	0.242737	0.000
N_Deal	4.01 E-06	1.96 E-05	0.838	2.95 E-05	1.87 E-05	0.116	0.000194***	2.68 E-05	0.000
N_PC	-1.70 E-03	1.18 E-03	0.155	-0.005991***	0.001261	0.000	-0.006136***	0.001136	0.000
N_AI+N_PTC	-5.43 E-03***	8.06 E-04	0.000	-0.00411***	0.001029	0.000	-0.002738***	0.000902	0.003
Adj R-squared	0.93			0.94			0.89		
DW	1.64			0.76			1.59		
NOB	131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)		

Note: (1) Panel OLS is estimated with Ordinary Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Table 3. Probability of disappearance of negative spread within one second (equation (4))

3-OLS

Equation (4)	USD/JPY			EUR/USD			EUR/JPY		
	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	0.72***	0.19	0.000	0.31**	2.09	0.038	0.79***	0.173850	0.000
Volatility	-2.96***	0.47	0.000	-3.22***	-5.93	0.000	-1.40***	0.196145	0.000
N_Deal	4.06 E-05***	1.11 E-05	0.000	4.54 E-05***	5.01	0.000	7.58 E-05***	2.40 E-05	0.002
N_AI+N_PTC	5.47 E-03***	2.53 E-04	0.000	5.82 E-03***	23.5	0.000	5.71 E-03***	0.000424	0.000
Adj R-squared	0.86			0.88			0.81		
DW	0.99			1.21			0.97		
NOB	132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)			129 (Jan 1999 – Dec 2009)		

3-GMM

Equation (4)	USD/JPY			EUR/USD			EUR/JPY		
	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	-0.04	0.37	0.917	-0.26	0.205853	0.203	0.42	0.321049	0.189
Volatility	-2.38**	1.05	0.024	-3.93***	1.177295	0.001	-1.23***	0.290292	0.000
N_Deal	9.18 E-05***	2.32 E-05	0.000	8.83 E-05***	1.83E-05	0.000	1.47 E-04***	5.56 E-05	0.009
N_AI+N_PTC	4.77 E-03***	4.72 E-04	0.000	5.33 E-03***	0.000366	0.000	4.75 E-03***	0.000629	0.000
Adj R-squared	0.83			0.85			0.80		
DW	1.14			1.39			1.04		
NOB	131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)			128 (Feb 1999 – Dec 2009)		

Note: (1) Panel OLS is estimated with Ordinary Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Table 4. Regression of Double Deals probabilities on Microstructure (Equations (5) and (6))

4-OLS

	USD/JPY (unconditional)			EUR/USD (unconditional)			EUR/JPY (unconditional)		
Equation (5)	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	-5.89***	0.08	0.000	-5.18***	0.08	0.000	-7.22***	0.09	0.000
Volatility	-0.61***	0.19	0.002	-0.50*	0.28	0.079	-1.33***	0.11	0.000
N_Deal	9.71 E-05***	4.59 E-06	0.000	6.92 E-05***	4.70 E-06	0.000	2.48 E-04***	1.24 E-05	0.000
N_AI+N_PTC	1.59 E-03***	1.05 E-04	0.000	1.00 E-03***	1.28 E-04	0.000	3.78 E-03***	2.06 E-04	0.000
Adj R-squared	0.92			0.85			0.95		
DW	1.32			1.25			1.21		
NOB	132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)		
	USD/JPY (conditional on negative spread)			EUR/USD (Conditional on negative spread)			EUR/JPY (conditional on negative spread)		
Equation (6)	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	-0.78***	0.24	0.001	-1.21***	0.21	0.000	-0.77***	0.252746	0.003
Volatility	-5.96***	0.60	0.000	-7.21***	0.78	0.000	-4.20***	0.287384	0.000
N_Deal	8.97 E-05***	1.42 E-05	0.000	1.04 E-04***	1.30 E-05	0.000	1.57 E-04***	3.33 E-05	0.000
N_AI+N_PTC	6.13 E-03***	3.25 E-04	0.000	6.30 E-03***	3.55 E-04	0.000	9.12 E-03***	0.000556	0.000
Adj R-squared	0.85			0.82			0.86		
DW	0.77			1.00			0.71		
NOB	132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)			132 (Jan 1999 – Dec 2009)		

(Table 4 continued)

4-GMM

Equation (5)	USD/JPY (unconditional)			EUR/USD (unconditional)			EUR/JPY (unconditional)		
	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	-6.15***	0.14	0.000	-5.63***	0.14523	0.000	-7.44***	0.21	0.000
Volatility	-0.67***	0.24	0.007	-1.44***	0.378881	0.000	-1.26***	0.21	0.000
N_Deal	1.22 E-04***	1.00 E-05	0.000	1.08 E-04***	1.12 E-05	0.000	2.99 E-04***	3.11 E-05	0.000
N_AI+N_PTC	1.29 E-03***	1.62 E-04	0.000	6.34 E-04***	2.12 E-04	0.003	3.21 E-03***	3.33 E-04	0.000
Adj R-squared	0.90			0.79			0.94		
DW	1.57			1.78			1.33		
NOB	131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)		
	USD/JPY (conditional on negative spread)			EUR/USD (conditional on negative spread)			EUR/JPY (conditional on negative spread)		
Equation (6)	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat	Coeff	s.e.	P-stat
C	-1.27*	0.74	0.089	-2.40***	0.37	0.000	-1.04	0.88	0.240
Volatility	-6.13***	1.61	0.000	-9.15***	1.81	0.000	-4.14***	0.88	0.000
N_Deal	1.39 E-04***	4.01 E-05	0.001	2.00 E-04***	2.90 E-05	0.000	2.25 E-04*	1.18 E-04	0.058
N_AI+N_PTC	5.55 E-03***	6.74 E-04	0.000	5.32 E-03***	6.58 E-04	0.000	8.37 E-03***	1.19 E-03	0.000
Adj R-squared	0.83			0.73			0.85		
DW	0.88			1.46			0.74		
NOB	131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)			131 (Feb 1999 – Dec 2009)		

Note: (1) Panel OLS is estimated with Ordinary Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Table 5. Triangular arbitrage opportunities: Occurrence and disappearance

	The number of occurrence by the size of potential profits			Probability of disappearance within 1 second by the size of potential profits			deals	Volatility
Year	N_0	N_10	N_30	P_0	P_10	P_30		
1999	538.90	477.14	370.75	0.35	0.36	0.38	28625.80	0.41
2000	546.56	481.71	371.27	0.38	0.39	0.41	27936.83	0.37
2001	605.69	524.42	390.36	0.39	0.41	0.44	28528.60	0.35
2002	512.67	427.97	294.83	0.42	0.43	0.47	28977.71	0.31
2003	624.71	518.33	351.40	0.43	0.45	0.49	32634.79	0.31
2004	649.81	533.80	362.48	0.51	0.55	0.59	34875.77	0.33
2005	574.88	445.06	264.23	0.62	0.67	0.75	35867.23	0.28
2006	521.88	372.09	187.02	0.68	0.75	0.86	36641.93	0.27
2007	552.49	373.58	163.13	0.73	0.79	0.88	42446.70	0.29
2008	486.24	357.17	184.20	0.81	0.84	0.89	56424.41	0.50
2009	167.59	122.94	67.96	0.85	0.87	0.90	42694.76	0.52
2010	67.17	46.66	24.38	0.84	0.87	0.88	41629.64	0.44

Source: ICAP, EBS data mine. Level 2 (one-second slice), 1999-2010.
Note: Authors' calculation.

Table 6. Triangular arbitrage opportunity: Factors for Occurrence

Equation (7)

6-OLS

Equation (7)	Coeff	s.e.	P-stat
C	5.00***	0.07	0.000
Volatility	1.19***	0.22	0.000
N_Deal	2.56 E-05***	2.17 E-06	0.000
N_PC	1.22 E-03***	3.92 E-04	0.002
N_AI+N_PTC	-4.41 E-03***	2.53 E-04	0.000
Adj R-squared	0.91		
DW	1.16		
NOB	132 (Jan 1999 – Feb 2009)		

6-GMM

Equation (7)	Coeff	s.e.	P-stat
C	4.93***	0.14	0.000
Volatility	0.65	0.44	0.140
N_Deal	3.43 E-05***	4.59 E-06	0.000
N_PC	3.83 E-04	7.99 E-04	0.632
N_AI+N_PTC	-4.11 E-03***	5.92 E-04	0.000
Adj R-squared	0.90		
DW	1.39		
NOB	131 (Jan 1999 – Dec 2009)		

Note: (1) Panel OLS is estimated with Ordinary Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Table 7. Triangular Arbitrage Opportunities: Probability of Disappearance within 1 second, Equation (8)

7-OLS

Equation (8)	Coeff	s.e.	P-stat
C	-0.15	-1.03	0.306
Volatility	-2.62***	-7.27	0.000
N_Deal	2.92 E-05***	7.73	0.000
N_AI+N_PTC	4.29 E-03***	18.0	0.000
Adj R-squared	0.86		
DW	0.57		
NOB	132 (Jan 1999 – Dec 2009)		

7-GMM

Equation (8)	Coeff	s.e.	P-stat
C	-0.99***	0.40	0.001
Volatility	-2.42***	0.79	0.002
N_Deal	5.41 E-05***	8.86 E-06	0.000
N_AI+N_PTC	3.23 E-03***	1.29 E-03	0.000
Adj R-squared	0.81		
DW	0.99		
NOB	131 (Feb 1999 – Dec 2009)		

Note: (1) Panel OLS is estimated with Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Table 8. Regression, Probability of triple deals, Equations (9) and (10)

8-OLS

Eq. (9): Unconditional	Coeff	s.e.	P-stat
C	-7.87***	0.19	0.000
Volatility	-5.80***	0.49	0.000
N_Deal	7.85 E-05***	5.09 E-06	0.000
N_AI+N_PTC	3.71 E-03***	3.2 E-04	0.000
Adj R-squared	0.86		
DW	0.97		
NOB	132 (Jan 1999 – Dec 2009)		
Eq. (10): Conditional on TA	Coeff	s.e.	P-stat
C	-3.24***	0.39	0.000
Volatility	-11.7***	0.97	0.000
N_Deal	1.04 E-04***	1.02 E-05	0.000
N_AI+N_PTC	7.44 E-03***	6.39 E-04	0.000
Adj R-squared	0.79		
DW	0.60		
NOB	132 (Jan 1999 – Dec 2009)		

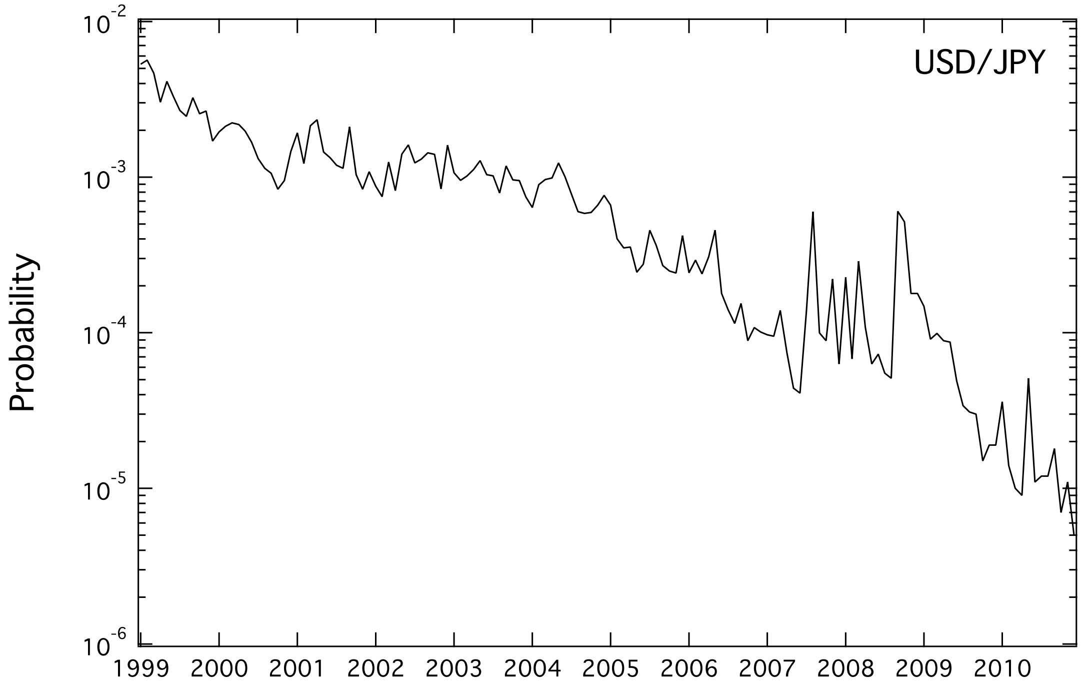
8-GMM

Eq. (9): Unconditional	Coeff	s.e.	P-stat
C	-8.96***	0.34	0.000
Volatility	-6.20***	0.82	0.000
N_Deal	1.17 E-04***	1.12 E-05	0.000
N_AI+N_PTC	2.32 E-03***	5.84 E-03	0.000
Adj R-squared	0.79		
DW	1.45		
NOB	131 (Feb 1999 – Dec 2009)		
Eq. (10): Conditional of TA	Coeff	s.e.	P-stat
C	-5.30***	0.78	0.000
Volatility	-11.8***	2.01	0.000
N_Deal	1.71 E-04***	2.64 E-05	0.000
N_AI+N_PTC	4.84 E-03***	1.15 E-03	0.000
Adj R-squared	0.71		
DW	1.05		
NOB	131 (Feb 1999 – Dec 2009)		

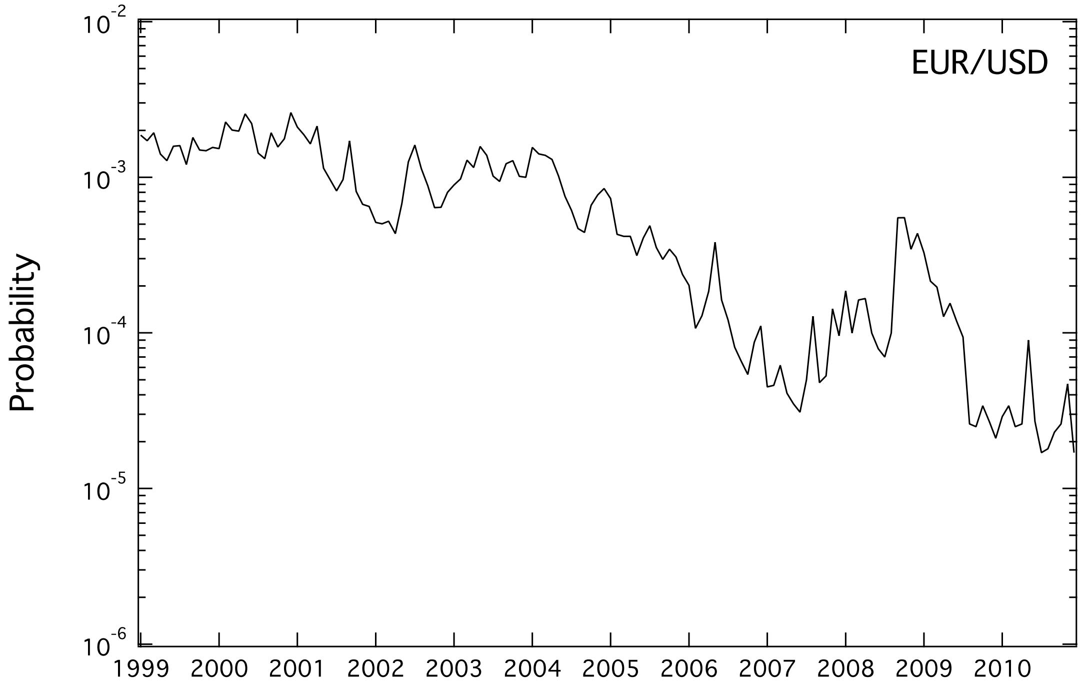
Note: (1) Panel OLS is estimated with Ordinary Least-Squares methods; Panel GMM is estimated with Generalized Methods of Moments.
(2) Significance level is indicated with *** for 1 percent; ** for 5 percent; and * for 10 percent.

Figure 1. Probability of negative spreads duration

Panel 1-A. USD/JPY



Panel 1-B. EUR/USD



Panel 1-C. EUR/JPY

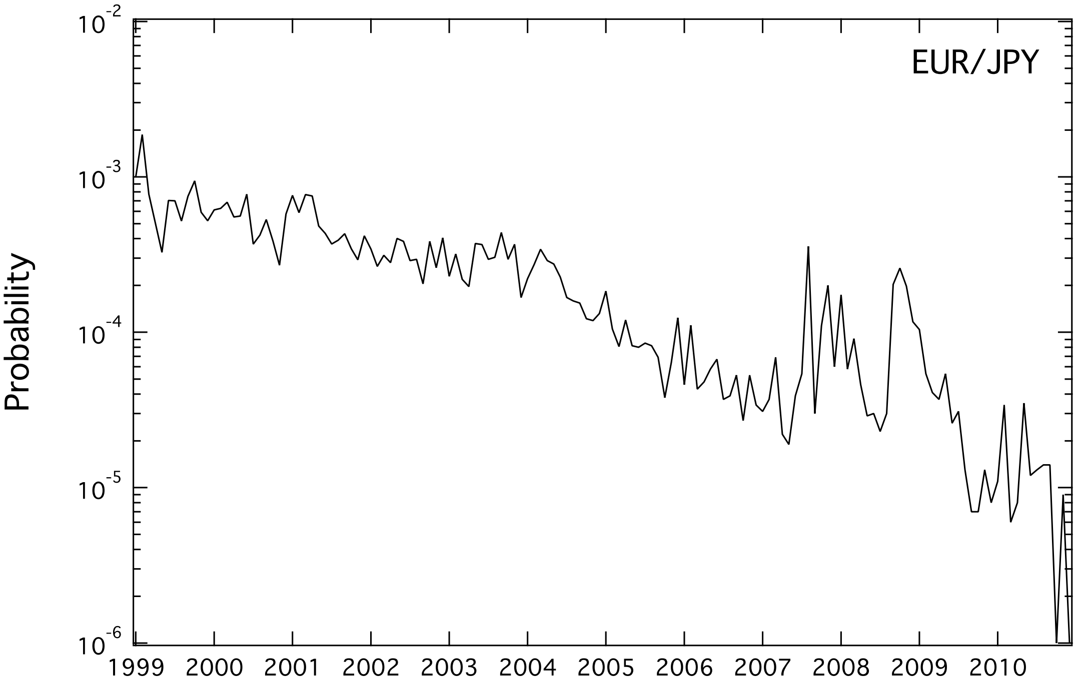
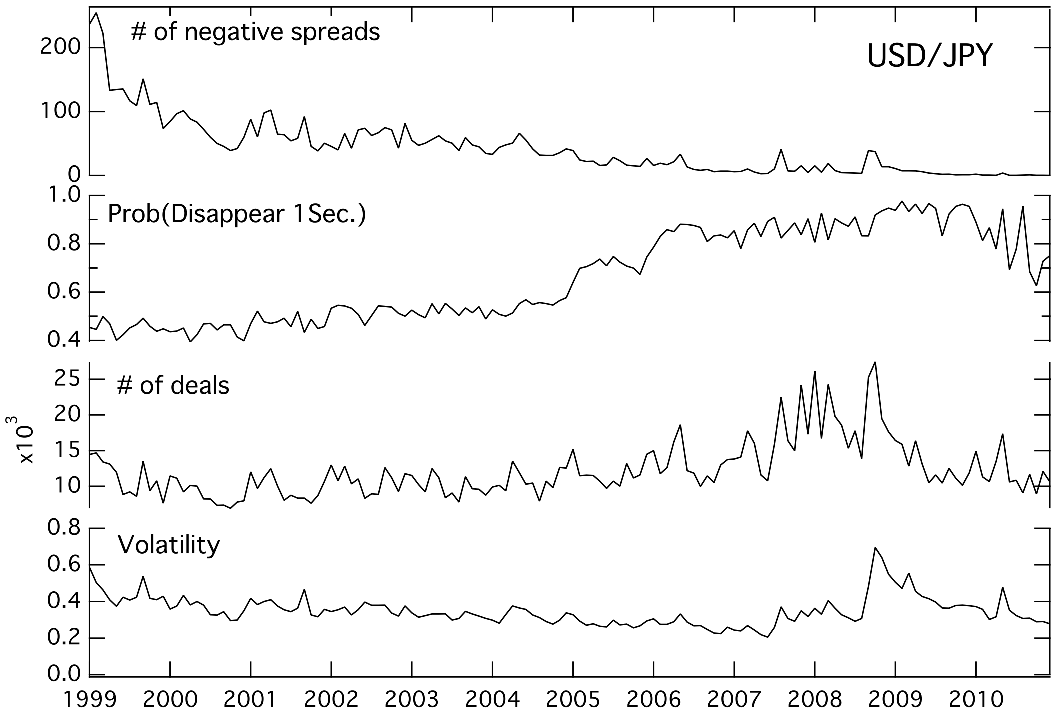


Figure 2. Negative Spread, Emergence and disappearance

Panel 2-A. USD/JPY



Panel 2-B. EUR/USD

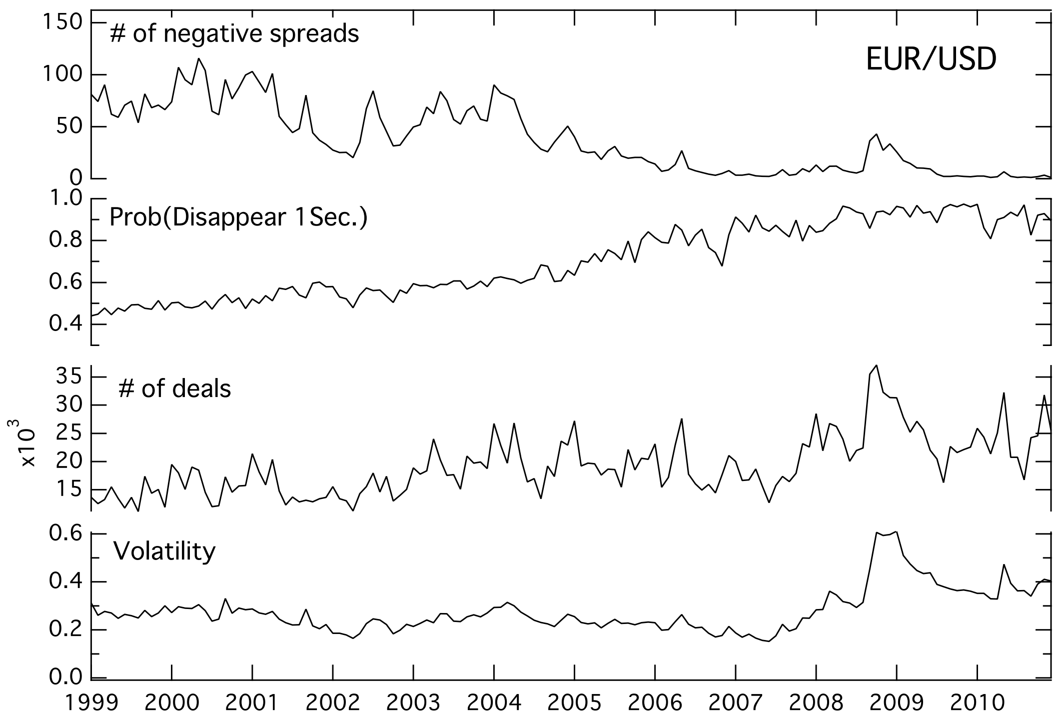


Figure 2-C. EUR/JPY

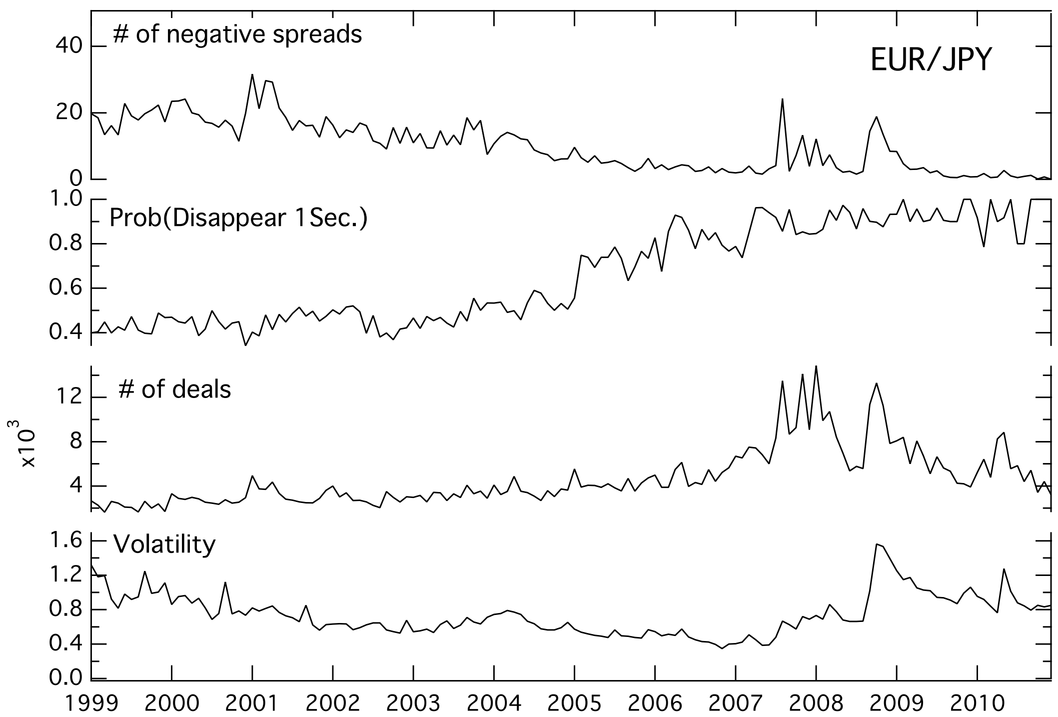
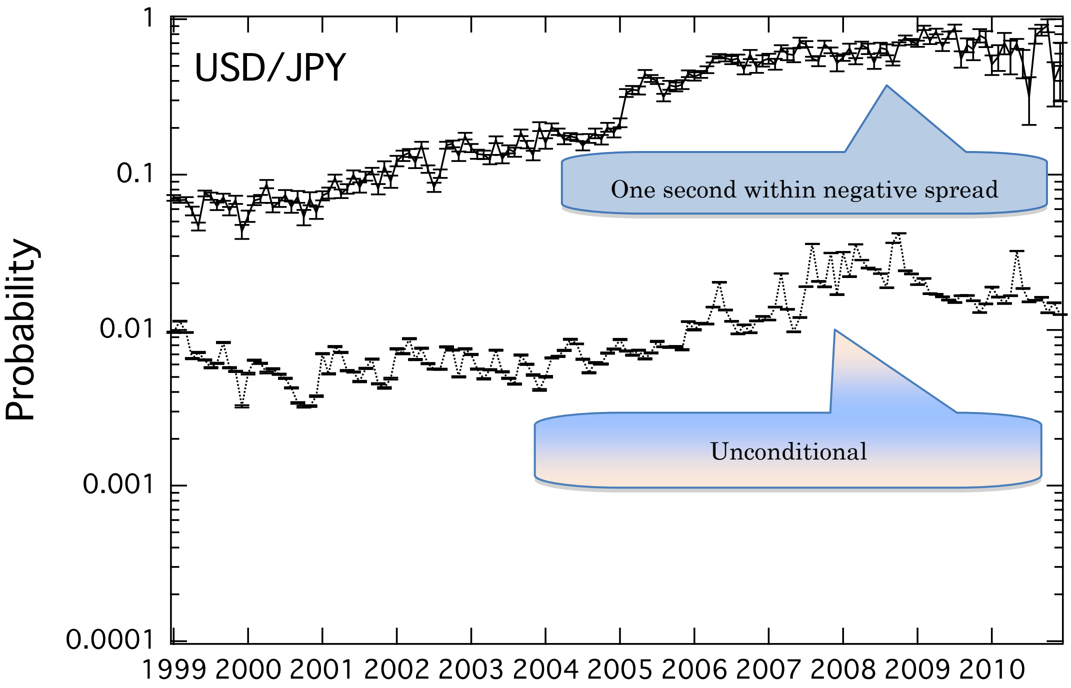
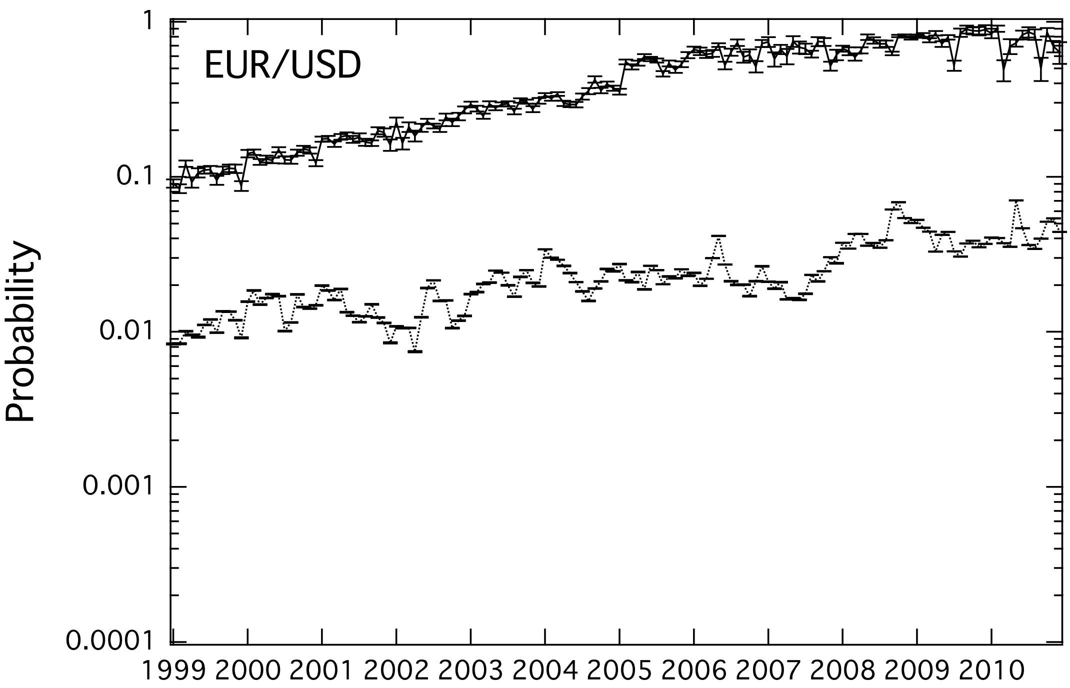


Figure 3. Probability of double deals

Panel 3-A. USD/JPY



Panel 3-B. EUR/USD



Panel 3-C. EUR/JPY

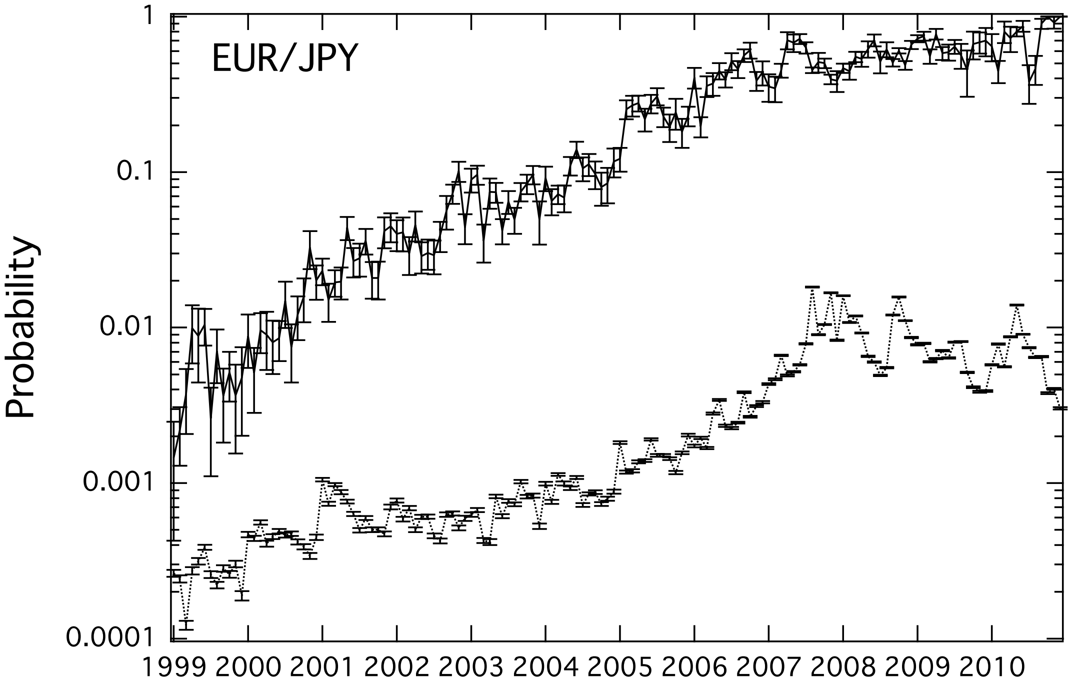


Figure 4. Probability of Triangular Arbitrage Opportunities: defined by duration

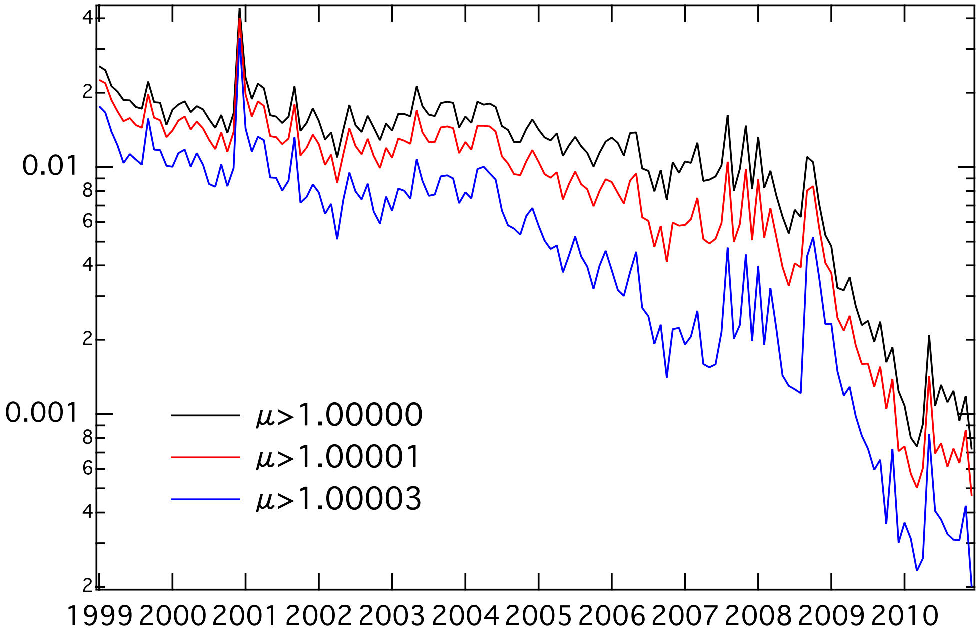


Figure 5. Triangular Arbitrage Opportunities

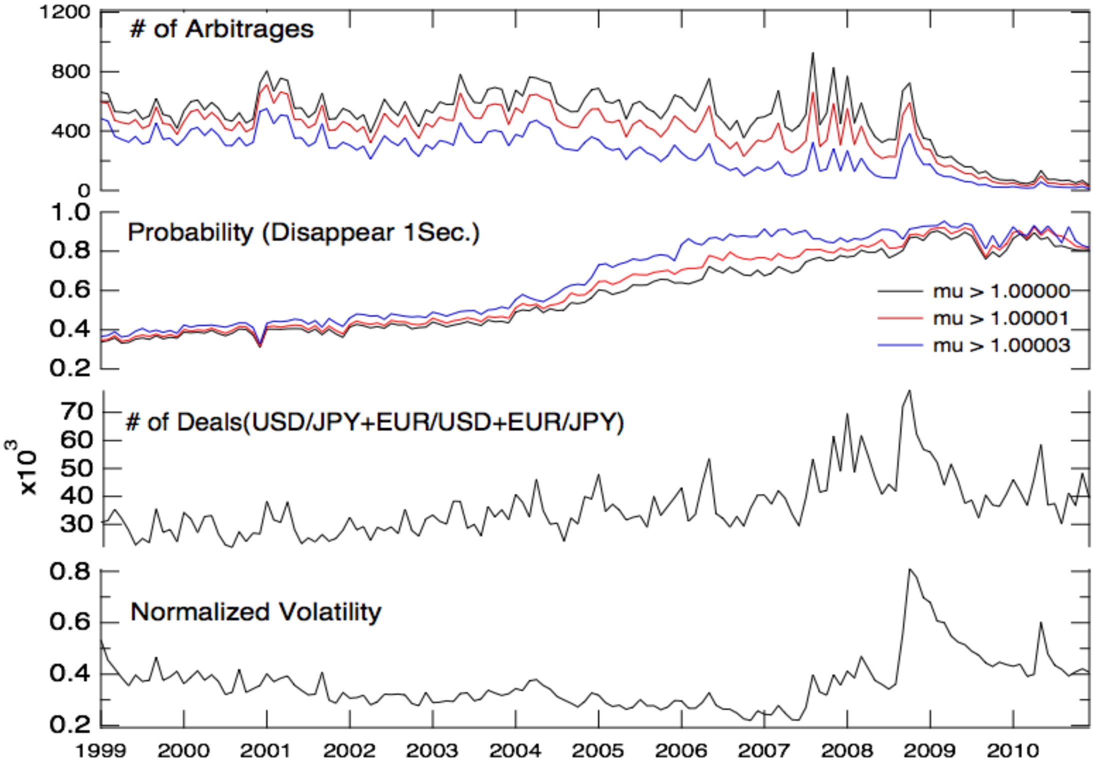


Figure 6. Probability of Triple deals