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INFORMATION ACQUISITION IN RUMOR BASED BANK RUNS

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ABSTRACT

We study information acquisition and withdrawal decisions when a liquidity event triggers a spreading rumor and exposes a solvent bank to a run. Uncertainty about the bank's liquidity and potential failure motivates agents who hear the rumor to acquire additional signals. Depositors with unfavorable signals may wait and thus gradually run on the bank, leading to an endogenous aggregate withdrawal speed. A bank run equilibrium exists when agents aggressively acquire information. We study threshold parameters (e.g. liquidity reserve and deposit insurance) that eliminate runs. Public provision of solvency information can eliminate runs by indirectly crowding-out individual depositors' effort to acquire liquidity information. However, providing too much information that slightly differentiates competing solvent-but-illiquid banks can result in inefficient runs

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1 Introduction

Bank runs are unfortunately still with us. Runs occurred recently on major traditional banks such as Northern Rock and IndyMac, and on non-traditional "shadow" banks.¹ The largest ever bank failure resulted from the 2008 run on Washington Mutual (WaMu). Figure 1 shows that the run on WaMu was dynamic in nature: the bank experienced the gradual withdrawal of \$16 billion in the days leading up to its takeover by the FDIC. Uncertainty regarding WaMu's liquidity and hence its eventual fate remained heightened during the run.² This empirical pattern that runs have an important time dimension, and that uncertainty and learning play a key role, has been found as far back as the 1857 run on the Emigrant Industrial Savings Bank.³

In this paper, we study dynamic bank runs when depositors can acquire information about bank liquidity (reserve). We emphasize the novel role that bank liquidity reserve plays given gradual withdrawals in a dynamic bank run model with learning. With gradual withdrawals, greater liquid holdings prolong the bank's survival. Depositors who are uncertain about bank liquidity and potential failure time therefore run later, which can eliminate the run completely even for liquidity reserves below potential maximum withdrawals. This effect does not exist in a typical static Diamond and Dybvig setting where withdrawal occurs immediately.⁴

Our model incorporates uncertainty about the bank liquidity into the asynchronous awareness framework of Abreu and Brunnermeier (2003). At some unobservable random time, a liquidity event occurs for a fundamentally solvent bank, at which point it becomes illiquid (i.e., with limited amount of cash reserves and susceptible to a run) or remains liquid (with sufficient reserves and not subject to a run). The liquidity event triggers the spread of a *rumor* (i.e., the liquidity event has occurred and the bank can be either liquid or illiquid) in the population that exposes the bank to a

¹Calomiris and Mason (1997), Shin (2009), and Iyer and Puri (forthcoming) document traditional bank runs, while Gorton and Metrick (2011) and Acharya, Schnabl, and Suarez (forthcoming) document "shadow" bank runs on repo and asset-backed commercial paper conduits.

²On September 16, 2008, "*This week and next will be the moment of truth for WaMu*,' said Fred Cannon, an analyst at Keefe Bruyette & Woods." See report "WaMu faces price to keep deposits," Financial Times, by Saskia Scholtes in New York.

 $^{^{3}}$ O Grada and White (2003) find that the 1857 bank run on the Emigrant Industrial Savings Bank had an important time dimension and was driven by informational shocks in the face of asymmetric information about the true condition of bank portfolios. See Kelly and O Grada (2000) for evidence on information transmission during the run.

 $^{^{4}}$ To our knowledge, Nikitin and Smith (2008) is the only paper that studies interaction between information acquisition and bank runs. When the bank is always solvent but illiquid to different degrees, since withdrawal is immediate, the acquired information regarding bank liquidity can only play the role of a sun-spot. See footnote 7 for further discussion.



Figure 1: The Dynamic Nature of the Run on WaMu

Daily net change in deposits as reported by Washington Mutual Bank to the Office of Thrift Supervision (OTS). We take out Friday and end-of-month fixed effects, i.e. days with automatic payday deposits estimated over the preceding 52 days. The OTS appointed the FDIC as receiver of WaMu on the evening of September 25, 2008 (the vertical line in the figure) which then sold it to JPMorgan Chase.

run. Agents (depositors) learn about whether the bank is liquid from the passage of time without failure, and about the time the liquidity event started. They earn a positive convenience yield on money inside the bank, and can withdraw (redeposit) their funds from (into) the bank at any time, subject to small transaction costs.

Uncertainty motivates informed agents who hear the rumor to acquire additional information at an endogenously determined signal quality. The realized signal may be utterly uninformative, or reveals the bank liquidity state perfectly. The higher the signal's quality, the more likely is the signal to reveal the bank's liquidity status. If the bank is indeed illiquid, then in aggregate a higher quality translates to a greater fraction of agents who know the bank is illiquid.

The presence of agents with heterogeneous information allows us to derive the unique bank run equilibrium as an interior solution, given the signal quality about bank illiquidity. Conditional on the bank being illiquid, agents with informative "low" signals perceive a high bank failure hazard rate and withdraw immediately, while agents with uninformative "medium" signals wait and withdraw later. When all medium signal agents wait a bit longer, they know more agents with informative low signals have already withdrawn, and thus the bank failure hazard rate rises as more agents run earlier. This "fear-of-low-signal-agents" generates a time-varying marginal cost of waiting for agents with the medium signal, which allows us to find the unique endogenous withdrawal time that equates the marginal benefit of waiting.⁵ The endogenous waiting time of medium signal agents, combined with the exogenous rumor spreading speed, result in an endogenous withdrawal speed on the illiquid bank.

We decouple the speed by which information spreads from the length of the awareness window over which information spreads, and examine their effects on the survival of an illiquid bank.⁶ In a static model, intuitively an increase in the number of *potential* withdrawers (weakly) exacerbates runs. By contrast, we find that when the awareness window widens so everybody knows potentially more agents run on the bank, the illiquid bank survives longer. This counter-intuitive result arises due to the novel uncertainty structure we introduce, as *the agent who hears the rumor also observes that the bank is still alive.* If the awareness window is wide, the rumor could have started spreading a long time before he heard it. Thus, conditional on the bank surviving thus far, the bank is more likely to be liquid, leading to weaker running incentives.

Given the bank run equilibrium, the optimal signal quality for individual agents trades off the acquisition cost with a greater probability of receiving an informative signal. Moreover, the chosen quality of information affects the equilibrium survival time of the failing bank. This intricate feedback effect between bank runs and information acquisition generally leads to at most two equilibria: a *good equilibrium* where agents do not acquire information and do not run, and a *bad equilibrium* where agents acquire information aggressively and run on the illiquid bank. This result naturally leads us to examine how to eliminate the run equilibrium.

We find that in our dynamic setting the threshold parameters — say, the minimum liquidity reserve requirement — that eliminate the *bad* bank run equilibrium are nontrivial. This is because, with bank failure time uncertainty, depositors with medium signals may have a sufficiently large payoff from staying in the bank and delaying their withdrawal. If we increase liquidity just enough to make the *last* agent whose withdrawal exhausts the liquid reserve to wait long enough, the bank run is averted. This practically relevant mechanism depends on both the dynamic nature of the withdrawal timing and the endogenous learning about a possible bank failure. That we give depositors the option to redeposit back to a surviving bank plays an important role. Interest-

⁵This mechanism of determining unique interior equilibrium is different from Abreu and Brunnermeier (2003). See literature review at the end of introduction.

 $^{^{6}}$ By contrast, in Abreu and Brunnermeier (2003) the speed by which information spreads is one over the length of the awareness window.

ingly, the liquidity requirement is far less than the maximum potential withdrawals, which is the minimum reserve requirement from a static Diamond and Dybvig perspective. Moreover, *partial* deposit insurance suffices in our setting to prevent runs. Though beyond the scope of the current paper, a calibration of our model to data can certainly result in quantitatively meaningful policy recommendations.

We extend our baseline model along two dimensions. First, we consider fundamentally insolvent banks. When agents privately collect information about bank *solvency*, they inevitably learn about bank *liquidity*. Effort spent acquiring one reduces the effort needed to acquire the other. We show that public provision of solvency information can help curb the private information acquisition effort on bank liquidity. One such situation where information is key is at the disclosure of stress test results (Bernanke, 2010). Carefully constructed stress tests can therefore help prevent bank runs by crowding out information acquisition by individuals. Second, we find it can be beneficial to avoid providing too much information that differentiates competing solvent but potentially illiquid banks, as small differences in liquidity can result in runs on slightly weaker banks and their subsequent failure. Thus injecting noise into the system is useful, similar to the practice of the U.S. government who forced all of the "Big 9" banks to take government capital on October 13, 2008.

Related Literature

Our model is related to the vast literature on the role of information in *static* bank runs, and it is beyond the scope of our paper to have a thorough review on this topic.⁷ In the Diamond and Dybvig (1983) framework, the liquidity reserve can be interpreted as liquid short-term assets held by the bank, which affects individual depositors' payoff in both static and dynamic settings. However, we emphasize the additional effect unique to the dynamic setting with learning. In the static bank run setting, withdrawals occur immediately. By contrast, in the dynamic model considered here with

⁷To name a few, Gorton (1985), Bhattacharya and Gale (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and more recently, Ennis and Keister (2009). On the topic of information acquisition and bank runs, Nikitin and Smith (2008) study how information acquisition about bank *solvency* affects the bank run equilibrium in a static Diamond and Dybvig setting. In contrast, we are focusing on the role of information acquisition about bank *liquidity* in determining bank runs. This difference obviously matters for the first-best outcome; in addition, when Nikitin and Smith (2008) analyze banks that are always solvent (i.e., the information just tells which bank is more profitable if there is no run), information only plays the role of a sun-spot. Based on the Morris and Shin (2002) global games technique, Goldstein and Pauzner (2005) study the optimal deposit contract by deriving a unique equilibrium when depositors in a Diamond and Dybvig type setting are endowed with private noisy signals about bank fundamentals. We allow for endogenous information acquisition, and show that excessive socially wasteful learning may lead to socially inefficient runs on solvent-but-illiquid banks.

gradual withdrawals, a greater share of liquid holding also implies that it takes longer to run down the bank. Hence, through the endogenous learning channel, an increase in liquidity reduces the hazard rate of failure and cause agents to *wait* longer. This is the novel mechanism we emphasize in the paper.

Our paper contributes to a recent theoretical literature studying dynamic bank runs. As early as Wallace (1988, 1990) the dynamic nature of bank runs has been explored by studying the sequential service constraint in the Diamond and Dybvig setting. The sequential service constraint is naturally placed in period 1 when all depositors (both early and late types) are lined up to decide whether to withdraw from the bank. In two important papers that focus on the optimal mechanism design subject to sequential service constraint, Green and Lin (2003) point out that no bank runs exist if depositors know their clock times of arrival and thus infer their relative positions in the queue, while Peck and Shell (2003) show that bank runs exist if the relative queue position information is unavailable.⁸ In this regard, our paper is similar to Peck and Shell (2003) since each agent in our model assigns the same distribution to her relative position in the queue. Our paper is more closely related to Gu (2011) who takes the demand deposit contract as given (like our paper) and studies depositors' withdrawal strategy sequentially when the bank can be potentially insolvent. Because a long sequence of withdrawals indicates bank insolvency, Gu (2011) shows that herding (ignoring private information which is endowed exogenously) after a long sequence of withdrawals may lead to inefficient bank runs.

Relative to these models, we emphasize the endogenous interaction between information acquisition and bank runs, and how the government can eliminate runs when individuals acquire information about bank liquidity. Allowing for redepositing back to surviving banks is new to the literature, and affects the threshold parameters that eliminate runs. From a modeling perspective, our framework features more empirically appealing timing assumptions in that individual depositors can withdraw or redeposit their funds at any time; this represents a substantial improvement from the Diamond and Dybvig framework and the above-mentioned important extensions. In particular, if micro-level data is available, our analytically tractable model is particularly calibration-friendly

⁸There is another crucial distinction. Peck and Shell (2003) allow the utility function at period 1 (thus for early types) to be different from the utility function at period 2 (thus for late types). For other related papers, see Andolfatto, Nosal, and Wallace (2007) and Andolfatto and Nosal (2008).

thanks to its empirically-interpretable time dimension.⁹ An alternative approach to bank run modeling is He and Xiong (2012a,b) who develop dynamic debt run models of a firm with a time-varying fundamental and a staggered debt structure.

Rumors have been studied by Banerjee (1993) and van Bommel (2003). We build on the approach of Abreu and Brunnermeier (2002, 2003) who consider the asynchronous timing of awareness to study "rational" bubbles. Brunnermeier and Morgan (2010) generalize this idea to a class of "clock games" and test its main predictions in controlled experiments. We add to their model uncertainty about the capacity of the bubble (bank), allow agents to acquire additional information upon awareness, and decouple the spreading rate from the length of the awareness window. Importantly, in Abreu and Brunnermeier (2003), the interior equilibrium is generated under an ad hoc assumption that the attacking benefit (in their context, the bubble component) is exogenously decreasing over time, and arbitragers attack only when the bubble component drops to a certain level. In contrast, as shown in Section 4.2.2, the endogenous learning that we introduce gives rise to heterogeneity among agents and thus a unique bank run equilibrium without shrinking bubbles.

The paper proceeds as follows. Section 2 describes the setting and solves the agent's learning problem. Section 3 characterizes the individual agent's optimal withdrawal policy, and Section 4 analyzes the bank run equilibrium with information acquisition. Section 5 considers extensions, and we conclude in Section 6. Proofs are in the Appendix.

2 The Model

We describe the economy and the individual agent's problem, with the focus on the agent's learning.

⁹By contrast, Green and Lin (2003) and Peck and Shell (2003) only study finite-agent economy, and depositors can only either withdraw at period 1 (when he/she is called in the line) or wait until period 2. Gu (2011) considers a relaxation so that a finite number of depositors can withdraw at any time within an interim stage before period 2. A significant departure from the Diamond and Dybvig framework seems necessary for developing dynamic models that are both calibration-friendly and tractable. A sacrifice our model makes is to assume away the preference (early or late) type of depositors, an ingredient critical for Diamond and Dybvig, but inessential for our paper.

2.1 The Setting

2.1.1 Technology

Time is continuous on $t \in [0, \infty)$. A continuum of infinitely-lived risk-neutral agents (depositors) with unit mass maximize their expected utility from consumption with a zero discount rate.

Bank deposits yield a constant rate of return r > 0 when the bank is operating, while holding cash outside the bank earns zero return. Broadly, one can interpret the bank as some investment vehicle in the shadow banking system or even the entire financial system, and the positive relative wedge r > 0 reflects either a higher investment growth rate or a convenience yield for keeping funds in the institution. To avoid exploding values, we assume that the bank's growth stops at some "maturing" event modeled as a Poisson shock with intensity $\delta > r$, and afterwards the game ends in the sense that each agent gets his deposit back for consumption. The maturing event assumption plays no role in our analysis except making the value of a dollar inside the bank finite.¹⁰ Throughout, this maturing event will be independent of any other random variables that we consider.

2.1.2 Uncertainty about Bank Liquidity

There are two potential types of banks that are fundamentally solvent (i.e., bank survival is the first-best allocation), with one type of bank being "illiquid," and a second "liquid" bank impervious to runs. The uncertainty about bank liquidity is crucial to our analysis. Later we will introduce fundamentally insolvent banks as an extension.

Throughout, bank liquidity is defined as the amount of depositors that it takes to run down the bank. For simplicity, we assume a binary structure for the state of bank liquidity $\tilde{\kappa}$. When the bank is illiquid, $\tilde{\kappa}$ takes a lower value $\kappa_L < 1$, and the bank is potentially subject to runs. In other words, the illiquid bank fails when more than a κ measure of depositors have fully withdrawn their funds. One can literally interpret $\tilde{\kappa}$ as the bank's cash reserves to meet withdrawals, and we broadly interpret $\tilde{\kappa}$ as the liquidity of the bank.

For the liquid bank with a higher liquidity reserve, $\tilde{\kappa} = \kappa_H > \kappa_L$. Throughout we focus on the case that there will be no run on the liquid bank; hence for now it is useful to think of $\kappa_H > 1$

¹⁰Alternatively, we could assume that each individual agent suffers liquidity shocks that require immediate consumption (therefore withdrawal). The analysis, which is available upon request, is similar but much more complicated (due to the replacement of agents).

so that the sufficiently liquid bank can survive any severe run. However, later in Section 4.5.1 we verify that our model can have $\kappa_H < 1$ but still rule out runs on the liquid bank.

In our economy, it is common knowledge that conditional on liquidity event (to be introduced shortly), $\Pr{\{\tilde{\kappa} = \kappa_L\}} = p_0 \in (0, 1)$. When the illiquid bank fails, all remaining depositors in the failed bank recover a fraction $\gamma \in (0, 1)$ of their deposits. Afterwards, all agents go to autarky consuming their remaining wealth.

2.1.3 Liquidity Event, Spreading Rumors, and Informed Agents

At t = 0 the bank is liquid, i.e., $\tilde{\kappa} = \kappa_H$. At some unobservable random time $\tilde{t}_0 > 0$ which we refer to as the liquidity event, the bank may become illiquid, and the uncertainty is as modeled in Section 2.1.2. Mapping to the 2007/08 crisis, one may think of this event as the deterioration mortgage-backed securities held by banks, which hurts banks' liquidity but keeps them solvent. Although the exact liquidity event time t_0 is publicly *unobservable*, it is common knowledge that \tilde{t}_0 is exponentially distributed on $[0, \infty)$ with cumulative distribution function $\Phi(t_0) \equiv 1 - e^{-\theta t_0}$, where θ is a positive constant.

Knowledge of this liquidity event starts spreading in the population after $\tilde{t_0}$, i.e., some agents hear "the liquidity event $\tilde{t_0}$ has occurred and thus the bank might be illiquid." Since the exact liquidity event time $\tilde{t_0}$ is unknown, this spreading information captures the essence of an unverified *rumor* of uncertain origin that spreads gradually in the depositor population. We therefore call this information about bank liquidity a "rumor." Importantly, besides knowing the bank may be illiquid, hearing the rumor also informs the agent that the liquidity event $\tilde{t_0}$ has occurred and hence other agents in the population may have heard the rumor.

We call those agents who hear the rumor "informed," and the rest "uninformed." Since at t = 0 the bank is liquid, throughout we assume that agents' beliefs are such that they expect to hold money in the bank unless they hear the rumor that the bank might be illiquid. In Section 4.5.2 we show this is indeed the case by analyzing the optimization problem of an uninformed agent.

Given a realization of $\tilde{t}_0 = t_0$, the rumor begins to spread over an interval $[t_0, t_0 + \eta]$ with a positive constant (exogenous) length η . Following Abreu and Brunnermeier (2003) we refer to η as the "awareness window." At any interval dt where $t \in (t_0, t_0 + \eta)$, uninformed agents become informed by hearing this rumor with probability βdt , where β is an exogenous positive constant. Although the rumor spreading speed is exogenous, we show later that the aggregate withdrawal speed during a bank run is endogenous. This rumor shock is i.i.d. across the population of uninformed agents. To make the problem interesting, at time $t_0 + \eta$ the fraction of informed agents is sufficient to take down the bank, if they decide to withdraw their funds. For $t \in [t_0, t_0 + \eta]$, it is easy to show that the mass of uninformed and informed agents is $e^{-\beta(t-t_0)}$ and $1 - e^{-\beta(t-t_0)}$ respectively, and the mass of newly informed agents within [t, t + dt] is $\beta e^{-\beta(t-t_0)} dt$.

This rumor spreading technology is different from that of Abreu and Brunnermeier (2003) in that we allow for separation between the spreading rate β and the awareness window η . Abreu and Brunnermeier assume a linear spreading technology with rate $\frac{1}{\eta}$ so that the entire population is informed at $t_0 + \eta$. This way, the awareness window η is artificially tied to the spreading rate $\frac{1}{\eta}$. As we show later, because of the endogenous learning, the awareness window η has interesting effects opposite to common wisdom.

2.1.4 Information Acquisition

At time $t_i > t_0$ when agent *i* hears the rumor, he can acquire an additional signal about the bank's type for some convex cost. Specifically, the agent makes an endogenous choice of information quality $q \in [0,1]$ with cost $\chi(q) = \alpha q^2/2$, where $\alpha > 0$ is a positive constant. For tractability, we assume $\chi(q)$ is the per dollar information cost, so informed agents face the same problem when they are informed at different times. Although the timing of information acquisition is by assumption immediate upon hearing the rumor, in Section 4.5.3 we show that under certain conditions it is indeed optimal to do so.

This additional signal takes three possible values $y \in \{y_L, y_M, y_H\}$ with conditional probabilities:

$$\Pr\{y = y_H | \tilde{\kappa} = \kappa_H\} = q, \Pr\{y = y_M | \tilde{\kappa} = \kappa_H\} = 1 - q, \text{ and } \Pr\{y = y_L | \tilde{\kappa} = \kappa_H\} = 0;$$
(1)
$$\Pr\{y = y_L | \tilde{\kappa} = \kappa_L\} = q, \Pr\{y = y_M | \tilde{\kappa} = \kappa_L\} = 1 - q, \text{ and } \Pr\{y = y_H | \tilde{\kappa} = \kappa_L\} = 0.$$

Figure 2 summarizes this distribution. With probability q, the bank's liquidity is perfectly revealed by the signal y_H (y_L). With probability 1 - q, the agent does not learn anything by receiving the medium signal y_M .¹¹ Conditional on the underlying bank liquidity status, The realizations of

¹¹Upon receiving y_M , the posterior probability of the bank being illiquid remains $\frac{p_0(1-q)}{p_0(1-q)+(1-p_0)(1-q)} = p_0$.





these signals are i.i.d. across agents. Unconditionally, these signals are correlated across agents, as signals tend to be low (high) in the illiquid (liquid) state.

2.1.5 Information Structure

Call the agent who is informed at $t_i \ge t_0$ simply agent t_i . Denote by $\mathcal{F}_t^{t_i}$ his information set at $t > t_i$, and $\mathbf{1}_t^{BF} \in \{1, 0\}$ the indicator of whether the bank failed or not by time t. Agent t_i 's information set is $\mathcal{F}_t^{t_i} = \{t, t_i, \tilde{y}_{t_i}, \mathbf{1}_t^{BF}\}$, i.e. the current time, the time he hears the rumor that the bank has become (potentially) illiquid, the additional signal he acquired, and whether the bank has failed or not. Recall that the liquidity event $\tilde{t_0} = t_0$ is not public information. Furthermore, consistent with electronic age banking, the agent neither observes withdrawals before him, or the potential queue in front of the bank.

2.1.6 Transaction Costs and Agent's Problem

Finally, to eliminate strategies with infinite transactions, we assume a constant (small) transaction cost k per dollar of deposits when the agent (re)deposits his cash into the bank. In sum, for an agent who hears the rumor at t_i with deposits inside the bank, he will acquire an additional signal \tilde{y} . Based on this signal, the agent can withdraw his deposits whenever he believes bank failure is imminent, and redeposit this cash in the future (and incur the proportional transaction cost k) if the bank's survival sufficiently improves his posterior belief about bank liquidity.

Risk neutrality and the bank's superior investment technology imply it is optimal for the agent to consume only at the bank's exogenous maturing event, or when the bank endogenously failed due to runs. The linearity of this problem implies a "bang-bang" strategy, i.e. keeping the entire wealth either in or out of the bank, is optimal.

2.2 Learning

2.2.1 Posterior Belief about t_0

Agent t_i updates his posterior distribution of t_0 conditional on hearing the rumor at t_i . Given t_0 , for an individual uninformed agent the probability of getting informed over $[t_i, t_i + dt]$ is determined as follows. For $t_i \in [t_0, t_0 + \eta)$, the probability of becoming informed at $[t_i, t_i + dt]$ but not before t_i is $f(t_i|t_0) = \beta e^{-\beta(t_i-t_0)} dt$. And, since the rumor stops spreading after $t_0 + \eta$, we have $f(t_i|t_0) = 0$ for $t_i \ge t_0 + \eta$. Combining with the density of $\phi(t_0) = \theta e^{-\theta t_0}$, the informed agent t_i learns about the liquidity event timing t_0 . Similar to Abreu and Brunnermeier (2003) we focus on realizations of $t_0 \ge \eta$ such that the economy is already in a stationary phase; as shown shortly agent t_i 's equilibrium strategy will be independent of the absolute timing of being informed.¹²

With $t_0 \ge \eta$, the informed agent $t_i \ge t_0 \ge \eta$ updates his posterior belief about t_0 as

$$\phi(t_0|t_i) \equiv \frac{f(t_i|t_0) \phi(t_0)}{\int_{t_i-\eta}^{t_i} f(t_i|s) \phi(s) \, ds} = \frac{\beta e^{-\beta(t_i-t_0)} \theta e^{-\theta t_0}}{\int_{t_i-\eta}^{t_i} \beta e^{-\beta(t_i-s)} \theta e^{-\theta s} ds} = \frac{\theta-\beta}{e^{(\theta-\beta)\eta}-1} e^{(\theta-\beta)(t_i-t_0)}.$$

Without loss of generality, define $\lambda \equiv \theta - \beta > 0$ so the liquidity event intensity is greater than the rumor spreading rate.¹³ Then

$$\phi(t_0|t_i) = \frac{\lambda e^{\lambda(t_i - t_0)}}{e^{\lambda \eta} - 1}.$$
(2)

¹²The finite awareness window η over which the rumor spreads makes the cases of $t_0 < \eta$ and $t_0 \ge \eta$ different. In the event of $t_0 \ge \eta$, it always holds that $t_i \ge \eta$, and rational agents know that $t_0 \in [t_i - \eta, t_i]$. In Appendix B we consider the equilibrium behavior for $t_0 < \eta$.

¹³The assumption of $\lambda > 0$ is for exposition purpose, and the analysis goes though if $\lambda < 0$. To see this, if $\lambda < 0$, then the conditional density $\phi(t_0|t_i)$ can be written as $\frac{(-\lambda)e^{\lambda(t_i-t_0)}}{1-e^{\lambda\eta}}$ which is still positive.

Integrating (2) over t_0 we get the conditional distribution function for the liquidity event's timing:

$$\Phi(t|t_i) \equiv \Pr\left(\tilde{t_0} \le t|t_i\right) = \int_{t_i-\eta}^t \phi(s|t_i) \, ds = \begin{cases} 0 & t < t_i - \eta \\ \frac{e^{\lambda\eta} - e^{\lambda(t_i-t)}}{e^{\lambda\eta} - 1} & t_i - \eta \le t \le t_i \\ 1 & t > t_i \end{cases}$$
(3)

2.2.2 Bank Failure Hazard Rate

Suppose that in a symmetric equilibrium, agents believe the illiquid bank fails at $t_0 + \zeta$ where ζ is the survival time to be determined in equilibrium (potentially infinite). Thus the event of bank failure is $\{t_0 + \zeta < t, \kappa_L\}$, and if the bank fails at t then the inferred t_0 is $t_0 = t - \zeta$. Denote by $p(t|t_i) \equiv \Pr\{\tilde{\kappa} = \kappa_L | \mathcal{F}_t^{t_i}\}$ the posterior probability at time t of the bank being illiquid. Trivially, bank failure at t reveals that $p(t|t_i) = 1$. Given y_L or y_H signals, $p(t|t_i) = 1$ or 0. For an agent t_i with a y_M signal, if the bank has not failed by t (i.e. $t < t_0 + \zeta$), then his posterior belief of the bank being illiquid is (recall that $\tilde{\kappa}$ is independent of t_0):

$$p(t|t_i) = \Pr\{\kappa_L | t < t_0 + \zeta, t_i\} = \frac{\Pr\{t < t_0 + \zeta | \kappa_L, t_i\} \Pr\{\kappa_L\}}{\Pr\{t < t_0 + \zeta | \kappa_L, t_i\} \Pr(\kappa_L) + \Pr\{\kappa_H\}}$$

=
$$\frac{[1 - \Phi(t - \zeta | t_i)] p_0}{[1 - \Phi(t - \zeta | t_i)] p_0 + 1 - p_0},$$
(4)

We can derive $p(t|t_i)$ in closed form using (3). Moreover, as shown later, in equilibrium if ζ is finite then $\zeta \leq \eta$ must hold. In this case, when agent t_i hears the rumor, his posterior probability of the bank being illiquid is

$$p(t_i|t_i) = \frac{(e^{\lambda\zeta} - 1)p_0}{(1 - p_0)(e^{\lambda\eta} - 1) + (e^{\lambda\zeta} - 1)p_0}.$$

For agent t_i and absolute time $t > t_i$, denote by $\tau \equiv t - t_i$ the time elapsed since the agent hears the rumor. The cumulative distribution of bank failure times at $t_i + \tau$, conditional on hearing the rumor at t_i and the bank has not failed by t_i , can be derived as

$$\Pi(t_i + \tau | t_i) \equiv p(t_i | t_i) \Pr\{t_i < t_0 + \zeta \le t_i + \tau | t_i, \kappa_L\} = p(t_i | t_i) \frac{\Phi(t_i + \tau - \zeta | t_i) - \Phi(t_i - \zeta | t_i)}{1 - \Phi(t_i - \zeta | t_i)}.$$

The bank failure hazard rate from the perspective of the informed agent t_i is

$$h\left(t_{i}+\tau|t_{i}\right) \equiv \frac{d\Pi\left(t_{i}+\tau|t_{i}\right)/d\tau}{1-\Pi\left(t_{i}+\tau|t_{i}\right)}.$$
(5)

We present the closed form expression for $h(t_i + \tau | t_i)$ in the following proposition. Note that bank survival after $\tau > \zeta$ reveals the bank is liquid, because the illiquid bank survives up to $t_0 + \zeta \leq t_i + \zeta$.

Proposition 1. Suppose that the illiquid bank fails at $t_0 + \zeta$ where $\zeta \leq \eta$ which holds in equilibrium. Then the bank failure hazard rate is

$$h(\tau) \equiv h(t_i + \tau | t_i) = \frac{\lambda e^{\lambda(\zeta - \tau)} p_0}{(1 - p_0) (e^{\lambda \eta} - 1) + (e^{\lambda(\zeta - \tau)} - 1) p_0} \text{ for } \tau \in [0, \zeta].$$
(6)

For $\tau > \zeta$, $h(\tau) = 0$ as the bank is revealed to be liquid.¹⁴

We have three noteworthy observations. First, the hazard rate in (6) is independent of the absolute timing of agent t_i becoming informed, therefore we can denote $h(t_i + \tau | t_i)$ by $h(\tau)$. This property guarantees the stationarity of our model.

Second, the hazard rate only depends on the remaining survival time from potential bank failure, i.e., $\zeta - \tau$. Because agents in our economy are uncertain about the exact timing of the liquidity event t_0 , the bank failure hazard rate will depend on how far the economy is away from the failure time $t_0 + \zeta$. For each agent t_i , the maximum distance is given by $t_0 + \zeta - t \leq t_i + \zeta - t = \zeta - \tau$. The equilibrium remaining survival time is important for our later analysis.

Third, the above analysis is carried out for y_M agents who remain uncertain about the bank's type. For $y_L(y_H)$ agents, we can simply replace p_0 with 1 (0) in equation (6).

With this learning process in mind, we define a bank run equilibrium and a no run equilibrium as follows:

Definition 1. A bank run equilibrium is a stationary Perfect Bayesian Nash equilibrium in which informed agents' strategies depend on the time since they heard the rumor, and the bank survival time is finite. A no run equilibrium is when the bank survival time is infinite.

¹⁴If $\lambda = 0$ the spreading speed and arrival rate of the liquidity event exactly offset. This limiting case results in a rather tractable $\lim_{\lambda \to 0} h(\tau) = \frac{p_0}{(1-p_0)\eta + (\zeta - \tau)p_0}$.

Stationarity here follows from the hazard rate's independence of the absolute timing of agent t_i becoming informed. The agent's strategy depends on $\tau = t - t_i$, not t. A bank run means the illiquid bank fails at a finite $t_0 + \zeta < \infty$, while a no run equilibrium means the bank survives forever.

2.3 Parameter Restrictions

We impose the following parametrization conditions throughout the paper. First,

$$\frac{p_0}{1-p_0} > e^{\lambda\eta} - 1; (7)$$

this implies the bank hazard rate given in Proposition 1 is increasing with τ , i.e. the time elapsed since hearing the rumor. Equivalently, this implies that the bank failure hazard rate is decreasing in the remaining survival time $\zeta - \tau$. More time without failure lowers the probability the bank is illiquid and reduces the hazard rate. But, since the illiquid bank fails a fixed amount of time after the rumor starts to spread, every minute that passes since we heard the rumor brings us closer to potential failure, which increases the hazard rate. Condition (7) guarantees the latter effect dominates. ¹⁵

Second, since the upper bound of the measure of informed agent is $1 - e^{-\beta\eta}$, for the model to be interesting we require that the illiquid bank can potentially fail if all informed agents run immediately, i.e.,

$$1 - e^{-\beta\eta} > \kappa_L. \tag{8}$$

Third, we focus on the case where both y_M agents and y_L agents are driving the bank failure. Conditional on the bank being illiquid, there could be at most $q\left(1-e^{-\beta\eta}\right)$ measure of y_L agents. To ensure that y_L agents alone are not enough to take down the bank, we further impose that the signal quality has to satisfy (instead of $q \in [0, 1]$):

$$q \in \left[0, \frac{\kappa_L}{1 - e^{-\beta\eta}}\right). \tag{9}$$

¹⁵If (7) fails, the hazard rate decreases with τ , and agents, if they ever choose to withdraw, will withdraw upon hearing the rumor $\tau = 0$ and redeposit to the bank at some endogenous time. This case with the endogenous entry could be interesting for future research on recovery after crises.

Fourth, we require that the maturity shock intensity δ is moderate:

$$\frac{\delta\left(1-p_{0}\right)\left(e^{\lambda\eta}-1\right)\frac{r\left(r-k\delta\right)}{\delta-r}}{\lambda\left(r-\lambda\left(1-\gamma\right)\right)p_{0}}\in\left(0,1\right).$$
(10)

This requirement implies $r - \lambda (1 - \gamma) > 0$. The only purpose of this condition is to guarantee the optimality of thresholds strategy.

Finally, we assume that the bank failure loss $1 - \gamma$ is significant:

$$(\lambda (1-\gamma) - r) e^{\lambda \eta} + r > 0; \tag{11}$$

as we show in Section 3.3, this implies agents with a y_L signal withdraw immediately, if the bank run equilibrium exists.

3 Optimal Withdrawal Strategies

In this section, we present the key proposition that characterizes the individual agent's optimal withdrawal policy, taking both the equilibrium bank survival time ζ and information quality q as given. The analysis involves only informed agents for reasons we explain in Section 4.5.2 which studies uninformed agents.

3.1 Value Functions

Denote by $V_I(\tau; t_i)$ ($V_O(\tau; t_i)$) the agent's value of one dollar inside (outside) the bank at time $t = t_i + \tau$ by following the optimal continuation strategy. Due to stationarity, $V_I(\tau; t_i) = V_I(\tau)$ and $V_O(\tau; t_i) = V_O(\tau)$. Because withdrawal involves no transaction cost while redepositing costs k, the optimality condition implies that $V_I(\tau) \ge V_O(\tau)$ and $V_O(\tau) \ge (1 - k) V_I(\tau)$ for all $\tau \ge 0$.

When $\tau \geq \zeta$, the surviving bank is safe for sure (recall Proposition 1). One dollar inside the bank will grow at rate r until the maturing event (occurs with Poisson intensity δ), which has a value of

$$V_I(\tau) = \int_0^\infty e^{rs} \delta e^{-\delta s} ds = \frac{\delta}{\delta - r} \text{ for } \tau \ge \zeta.$$

The value of one dollar outside the bank is $V_O(\tau) = (1-k) V_I(\tau) = \frac{(1-k)\delta}{\delta-r}$ for $\tau \ge \zeta$ which is

above 1 for sufficiently small k.

When $\tau < \zeta$, consider a dollar outside the bank. At any point in time, if the status-quo position (i.e. keeping the dollar outside the bank) is optimal, then the following ordinary differential equation (ODE), the Hamilton-Jacobi-Bellman (HJB) equation, must hold:

$$0 = h(\tau) (1 - V_O(\tau)) + \delta (1 - V_O(\tau)) + \frac{V'_O(\tau)}{\text{Bank failure}}$$
 Bank matures Time change (12)

Here, the first term is the impact of bank failure: with hazard rate $h(\tau)$ the bank fails, and the agent loses his value $V_O(\tau)$ but recovers 1, the full value for his one dollar. The second term captures the bank asset maturing event, and the third term is the change due to time elapsing. Combined with the option of redepositing immediately (with transaction cost k), the HJB equation for one-dollar outside the bank is

$$0 = max \left\{ h(\tau) (1 - V_O(\tau)) + \delta (1 - V_O(\tau)) + V'_O(\tau), (1 - k) V_I(\tau) - V_O(\tau) \right\}.$$

Similarly, for a dollar inside the bank, its value $V_{I}(\tau)$ must satisfy the following HJB equation:

$$0 = max \left\{ rV_{I}(\tau) + h(\tau)(\gamma - V_{I}(\tau)) + \delta(1 - V_{I}(\tau)) + V_{I}'(\tau), V_{O}(\tau) - V_{I}(\tau) \right\}$$
(13)
Interest growth Bank failure Bank matures Time change Withdrawal

3.2 Optimal Strategies

Define $\hat{V}_O(\tau)$ the solution to ODE (12) with boundary condition $\hat{V}_O(\zeta) = V_O(\zeta) = \frac{\delta}{\delta - r}$:

$$\widehat{V}_{O}(\tau) = \frac{e^{\lambda\eta} \left(1 - p_{0}\right) - 1 + e^{\lambda(\zeta - \tau)} p_{0} + e^{-\delta(\zeta - \tau)} \left(1 - p_{0}\right) \left(e^{\lambda\eta} - 1\right) \frac{r - k\delta}{\delta - r}}{\left(1 - p_{0}\right) \left(e^{\lambda\eta} - 1\right) + \left(e^{\lambda(\zeta - \tau)} - 1\right) p_{0}}.$$
(14)

In general $V_O(\tau) \ge \hat{V}_O(\tau)$: $\hat{V}_O(\tau)$ is the value at time τ by simply staying outside the bank until ζ (and redepositing immediately after ζ into the surviving bank), but this simple continuation strategy may not be optimal. Nonetheless, the following function captures the first-order impact of the withdrawal decision:

$$g(\tau) \equiv h(\tau) (1 - \gamma) - r \widehat{V}_O(\tau).$$
(15)

Here, $g(\tau)$ is difference between the instantaneous loss due to potential bank failure (i.e. $h(\tau)(1-\gamma)$), and $r\hat{V}_O(\tau)$ which is the instantaneous return of taking one dollar out now, keeping outside the bank until ζ , and redepositing back given survival. The underlying assumption here is that a threshold strategy is optimal (i.e. redepositing is never optimal before ζ) which is verified in Proposition 2.

At the optimal withdrawal time τ_w , we have the first-order condition $g(\tau_w) = 0$, i.e.,

$$h(\tau_w)(1-\gamma) = r\widehat{V}_O(\tau_w) = r \left[1 + (1 - p(t_i + \tau_w | t_i)) e^{-\delta(\zeta - \tau_w)} \frac{r - k\delta}{\delta - r} \right],$$
(16)
Redepositing option value

where we have used (14) and (4) in rewriting this intuitive expression. As mentioned above, the left hand side captures the marginal cost of staying, which is the hazard rate multiplying the bank failure loss. The right hand side captures the marginal benefit of staying, which is the growth rate r multiplying the agent's continuation value of one dollar by withdrawing and redepositing after ζ . For the total continuation value in the right hand side bracket of (16), the first term 1 is the principal amount which is present in Abreu and Brunnermeier (2003). The second term consists of the option value of future redepositing. Here, $1 - p(t_i + \tau_w | t_i)$ is the probability of the bank being liquid (and surviving eventually) conditional on bank survival at τ_w , $e^{-\delta(\zeta - \tau_w)}$ is the discounting, and finally

$$\frac{r-k\delta}{\delta-r} = V_O\left(\zeta\right) - 1 = \frac{(1-k)\,\delta}{\delta-r} - 1 > 0$$

is the additional payoff from redepositing. When $p_0 = 1$ so there is no uncertainty about bank liquidity, this option value term vanishes.

We now show formally that a threshold strategy is optimal based on the function $g(\tau)$.

Proposition 2. Given the equilibrium bank survival time $\zeta < \infty$, the optimal policy for the agent with y_M is as follows:

- 1. If $g(\zeta) \leq 0$, then it is optimal to stay in the bank always.
- 2. If $g(0) \ge 0$, then it is optimal to withdraw at 0 and redeposit right after ζ .
- Otherwise, we must have g(0) < 0 and g(ζ) > 0, and there exists a unique waiting time τ_w ∈ (0, ζ) so that g(τ_w) = 0, and withdrawing at τ_w and redepositing right after ζ is optimal.

The redepositing option increases the waiting benefit $r\hat{V}_O(\tau)$ in (15), towards the condition

 $g(\zeta) \leq 0$ in which agents with medium signal never run on the bank. Clearly, this option plays a role in eliminating the bank run.

3.3 Values Conditional on Signals

We have studied the optimal strategy for the agent with y_M signal. Agents with a high signal y_H (and thus knows the bank is not subject to run) keep their deposits in the bank always. For agents who receive low signals, if

$$g(0; p_0 = 1) = \frac{(\lambda (1 - \gamma) - r) e^{\lambda \zeta} + r}{e^{\lambda \zeta} - 1} > 0,$$
(17)

then it is optimal to withdraw immediately. Because $\zeta < \eta$ and $\lambda (1 - \gamma) < r$, (17) is implied by parameter condition (11). In fact, (17) turns out to be an equilibrium result (rather than a parameter restriction) in any bank run equilibrium. As shown in Section 4.2, (17) always holds with the equilibrium bank survival ζ , if a bank run equilibrium exists so that $\zeta < \infty$. More precisely, in our model if y_L agents want to wait some positive amount of time, then generically bank run equilibria do not exist. Throughout, we say that a statement holds "generically" when it fails only for some zero measure parameter set.

Proposition 3. Suppose g(0) < 0 and $g(\zeta) > 0$ so that agents with y_M signals wait τ_w given in (16). Then upon hearing the rumor, the values conditional on signals are

$$V_{I}(0|y_{L}) = 1, V_{I}(0|y_{H}) = \frac{\delta}{\delta - r},$$

$$V_{I}(0|y_{M}) = \frac{\frac{\delta(e^{\lambda\eta}(1-p_{0})-1)}{\delta - r} \left(1 - e^{-(\delta - r)\tau_{w}}\right) + \frac{\delta + \lambda\gamma}{\lambda + \delta - r} e^{\lambda\zeta} p_{0} + e^{-(\delta - r)\tau_{w}} e^{\lambda(\zeta - \tau_{w})} p_{0}\left(\frac{(\lambda + \delta)(\lambda(1-\gamma) - r)}{r(\lambda + \delta - r)}\right)}{e^{\lambda\eta} - 1 - e^{\lambda\eta} p_{0} + e^{\lambda\zeta} p_{0}},$$

where τ_w satisfies the first-order condition in (16).

4 Bank Run Equilibrium

In this section we solve for the unique bank run equilibrium and study its existence taking information acquisition effort q as given. We then endogenize this learning decision and show that a multiplicity of equilibria arises. We then study the minimum liquidity reserve to eliminate the run,



Cumulative withdrawal patterns for an illiquid bank with capacity κ_L and a liquid bank with capacity $\kappa_H > 1$. At $\tau = 0$ the rumor starts to spread. Agents with a y_M signal wait in equilibrium $\tau_w = 0.85$ before withdrawing. The illiquid bank fails while the liquid bank starts experiencing redeposits at $\zeta = 1.8$. Parameter values are r = 0.09, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

and surprising comparative statics with respect to η . Finally we consider uninformed agents and the optimal time to acquire the additional signal.

4.1 Cumulative Withdrawals Given q

In our model, the illiquid bank fails at $t_0 + \zeta$ when aggregate cumulative withdrawals by informed agents deplete the illiquid bank's capacity κ_L . Two groups of informed agents withdraw from the illiquid bank. The first group is y_L agents with q measure in aggregate per unit of time. Therefore, as the mass of newly informed agents is $\beta e^{-\beta(t_i-t_0)} dt_i$, at $t_0 + \zeta$ total withdrawals by y_L agents are

$$q \int_{t_0}^{t_0+\zeta} \beta e^{-\beta(t_i-t_0)} dt_i = q \left(1 - e^{-\beta\zeta}\right).$$
(18)

Agents with y_M signals wait for τ_w , and at $t_0 + \zeta$ their total withdrawals are

$$(1-q)\int_{t_0}^{t_0+\zeta-\tau_w}\beta e^{-\beta(t_i-t_0)}dt_i = (1-q)\left(1-e^{-\beta(\zeta-\tau_w)}\right).$$
(19)

Figure 3 depicts the cumulative withdrawal patterns for both banks. For illiquid banks, y_L agents begin to withdraw right after the liquidity event t_0 as the rumor starts spreading. At $t_0 + \tau_w$ agents with y_M signals join the force of withdrawals, until eventually cumulative withdrawals reach

 κ_L at $t_0 + \zeta$. The endogenous signal acquisition leads to an endogenous aggregate withdrawal speed.

For liquid banks, things are different. No agent withdraws immediately after t_0 , but y_M agents start withdrawing at $t_0 + \tau_w$. At $t_0 + \zeta$ those early informed y_M agents realize the bank is liquid and start redepositing, which makes net aggregate withdrawals decrease over time. In this range, interestingly, at the same time late informed agents are withdrawing, early informed agents who learned that the bank is liquid begin redepositing. The intriguing empirical pattern of simultaneous withdrawing-depositing is unique to our model with rumor-based bank runs, and we wait for future empirical work to test this implication.

4.2 Bank Run Equilibrium Given q

4.2.1 Two-step Procedure

Define $\tau_r \equiv \zeta - \tau_w$ as the bank's remaining survival time when y_M agents decide to stay out of the bank. The bank run equilibrium given q is determined in a straight-forward two-step procedure. We first look at the individual optimality condition for agent y_M . Define $G(\tau_r) \equiv g(\zeta - \tau_r)$, i.e., replace $\zeta - \tau$ by τ_r in (15); then (16) implies that the equilibrium redepositing time τ_r^* must satisfy

$$G(\tau_r^*) = \frac{(\lambda (1-\gamma) - r) e^{\lambda \tau_r^*} p_0 - (1-p_0) \left(e^{\lambda \eta} - 1\right) \frac{r(r-k\delta)}{\delta - r} e^{-\delta \tau_r^*} + r \left(1 - e^{\lambda \eta} \left(1 - p_0\right)\right)}{(1-p_0) \left(e^{\lambda \eta} - 1\right) + \left(e^{\lambda \tau_r^*} - 1\right) p_0} = 0.$$
(20)

One important observation emerges. In (20), the equilibrium remaining survival time τ_r^* is uniquely determined, independent of the other endogenous variables q or ζ . This is because in the agent's first-order condition in (15) regarding optimal withdrawal, both the hazard rate $h(\tau_w)$ in (6) and the continuation value $\hat{V}_O(\tau_w)$ in (14) only depend on $\tau_r = \zeta - \tau_w$.

Once we pin down τ_r^* from the individual optimality condition (20), the equilibrium survival time ζ^* follows from the aggregate withdrawal condition. Combing the withdrawal of y_L agents (18) and that of y_M agents (19), the illiquid bank fails when

$$\kappa_L = q \left(1 - e^{-\beta\zeta} \right) + (1 - q) \left(1 - e^{-\beta\tau_r} \right), \tag{21}$$

which gives the equilibrium survival time ζ^* as

$$\zeta^* = -\frac{1}{\beta} \log \left[1 - \frac{\kappa_L - (1-q)\left(1 - e^{-\beta \tau_r^*}\right)}{q} \right].$$
(22)

Before we move on, we show that $\zeta^* \leq \eta$, i.e. the illiquid bank fails before the rumor stops spreading. In fact, when we wrote down the aggregate failure condition (18), we implicitly assumed at the failure time $t_0 + \zeta^*$ there are still newly informed y_L agents withdrawing, which exactly requires that the illiquid bank fails before the rumor stops spreading, i.e., $\zeta^* \leq \eta$. We have the following lemma.

Lemma 1. Generically, for the bank run equilibrium to exist, we must have $\zeta^* \leq \eta$, and y_L agents must withdraw immediately upon hearing the rumor.

We provide intuition for why $\zeta^* \leq \eta$ holds generically. The y_M agent's individual optimality condition in (20) pins down the equilibrium remaining survival time τ_r^* based on model primitives. However, from (21), $\tau_r^* = \zeta^* - \tau_w$ also determines the cumulative withdrawals of y_M agents from the illiquid bank. Hence an extra degree of freedom is needed to ensure the consistency of both equilibrium conditions. Interestingly, the degree of freedom comes from the presence of y_L agents who withdraw immediately after hearing the rumor and are still withdrawing when the bank fails, as we in detail in the next subsection.¹⁶ A similar argument implies that generically for the bank run equilibrium to exist, y_L agents must withdraw immediately upon hearing the rumor.

4.2.2 Equilibrium Mechanism

Based on the aggregate bank failure condition, we can determine natural bounds for equilibrium remaining survival time τ_r^* . When y_M agents withdraw immediately after hearing the rumor $\tau_w = 0$, the remaining survival time $\zeta - \tau_w$ will assume its upper bound value (the second inequality is from (8)):

$$\tau_r^u = \frac{1}{\beta} \ln \frac{1}{1 - \kappa_L} < \eta.$$

¹⁶In (21), because the required y_M withdrawal is the gap between the capacity κ_L and the cumulative y_L withdrawal $q\left(1-e^{-\beta\zeta}\right)$, we can find a ζ such that y_M agents' individual incentives coincide with the required aggregate y_M withdrawal.

On the other hand, $\zeta \leq \eta$ in Lemma 1 gives the lower bound of τ_r^* :

$$\tau_r^l = \frac{1}{\beta} \ln \left(\frac{1-q}{1-\kappa_L - qe^{-\beta\eta}} \right).$$

Parameter condition (9) implies $\tau_r^l > 0$, so that y_M withdrawals contribute to the bank's failure. We have the following key proposition.

Proposition 4. Given information acquisition effort q, the equilibrium is characterized as follows:

1. If $G\left(\tau_r^l\right) \leq 0$, then there does not exist a bank run equilibrium.

2. If $G(\tau_r^u) \ge 0$, then the unique bank run equilibrium is $\tau_r^* = \zeta^* = \tau_r^u$ and $\tau_w^* = 0$, i.e. y_M agents do not wait.

3. Otherwise, we must have $G(\tau_r^u) < 0$ and $G(\tau_r^l) > 0$, and there exists a unique bank run equilibrium $\tau_r^* \in (\tau_r^l, \tau_r^u)$ so that $G(\tau_r^*) = 0$, and

$$\zeta^* = \zeta\left(\tau_r^*\right) = \frac{1}{\beta} \ln\left[\frac{q}{1 - \kappa_L - (1 - q)e^{-\beta\tau_r^*}}\right],$$

and $\tau_w^* = \zeta(\tau_r^*) - \tau_r^* > 0$ so that y_M agents wait to withdraw. The equilibrium is stable.

The economic mechanism that pins down the equilibrium is as follows. Focus on y_M agents and recall that $G(\tau_r)$ in (20) gives the sign of marginal cost minus the marginal benefit of waiting a bit longer. For illustration, start with the hypothetical equilibrium where all y_M agents withdraw immediately (i.e., $\tau_w = 0$); then the remaining survival time $\tau_r = \zeta - \tau_w$ takes its upper bound τ_r^u (one can see it graphically by setting $\tau_w = 0$ in the left panel of Figure 3). The condition $G(\tau_r^u) < 0$ implies that this conjectured equilibrium with immediate withdrawal is not incentive compatible individually: Because the marginal benefit of waiting exceeds the marginal cost, each y_M agent would like to postpone his withdrawal.

Once y_M agents decide to wait $\tau_w > 0$, there are more y_L agents withdrawing before y_M agents. Then bank failure requires less cumulative mass of withdrawing y_M agents and hence a lower remaining survival time $\tau_r = \zeta - \tau_w$ (check (21), and graphically the left panel of Figure 3). From (6), the more imminent failure gives rise to a higher failure hazard rate, pushing up the marginal cost of waiting. In the extreme, consider the equilibrium candidate where y_M agents wait sufficiently long, and the remaining survival time attains the lower bound $\tau_r = \tau_r^l$. Then the

condition $G(\tau_r^l) > 0$ implies this conjectured equilibrium is not incentive compatible either, as each individual y_M agent now wants to withdraw a bit earlier.

Combining these two results, we can find some intermediate withdrawal time τ_w^* with $G(\tau_r^* = \zeta^* - \tau_w^*) = 0$ to satisfy the individual optimality condition, which corresponds to the case 3 in Proposition 4.

4.3 Endogenous Information Acquisition

Given the bank run equilibrium characterized in Proposition 4, we first characterize the individual agent's optimal information acquisition condition. Because information quality q also affects the bank run equilibrium, this intricate feedback effect between bank runs and information acquisition may lead to multiple equilibria.

4.3.1 The First-Order Condition of Information Acquisition

Recall that by setting q, we have $\Pr\{y = y_L | t_i\} = qp(t_i | t_i)$, $\Pr\{y = y_M | t_i\} = 1-q$, and $\Pr\{y = y_H | t_i\} = q(1-p(t_i | t_i))$, while the information cost is $\chi(q) = \frac{\alpha}{2}q^2$. When agent t_i hears the rumor, he will take the equilibrium survival time ζ^* as given and spend effort to maximize:

$$v(q) = \Pr\{y = y_L | t_i\} V_I(0|y_L) + \Pr\{y = y_M | t_i\} V_I(0|y_M) + \Pr\{y = y_H | t_i\} V_I(0|y_H) - \chi(q)$$

= $qp(t_i|t_i) + q(1 - p(t_i|t_i)) \frac{\delta}{\delta - r} + (1 - q) V_I(0|y_M) - \chi(q),$

where we have used the result in Proposition 3. We are implicitly considering the bank run equilibrium here, so that y_L agents find immediate withdrawal optimal.

Taking the first order condition for v(q) with respect to q, the optimal q^* satisfies (recall parameter restriction (9))¹⁷

$$\underbrace{p\left(t_{i}|t_{i}\right)+\left(1-p\left(t_{i}|t_{i}\right)\right)\frac{\delta}{\delta-r}}_{\mathbb{E}\left[V_{I}(0)|\text{ informative signal}\right]}-\underbrace{V_{I}\left(0|y_{M}\right)}_{\mathbb{E}\left[V_{I}(0)|\text{ uninformative signal}\right]}-\alpha q^{*} \ge 0, \text{ with equality if } q^{*} < \frac{\kappa_{L}}{1-e^{-\beta\eta}}$$

$$(23)$$

This expression is intuitive. Raising q increases (lowers) the probability of (un)informative signals,

¹⁷The fact that we are focusing on bank run equilibria and that the agent with y_L finds immediate withdrawal to be optimal imply that in (23) q^* cannot bind at zero, as the Blackwell information theorem implies that $p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M) > 0$ always, i.e., information has positive value as it improves the agent's decision. On the other hand, the no run equilibrium must have $q^* = 0$ and is analyzed in Section 4.3.2.

but costs more. Combining with the dependence of ζ^* on q^* in Proposition 4 and the individual agent's optimal information acquisition condition (23), we can solve for the endogenous information acquisition q^* and survival time ζ^* simultaneously.

4.3.2 Run and No-Run Equilibria

We show that with endogenous information acquisition, in general multiple equilibria emerge. First, we check whether $q^* = 0$ is an equilibrium. Since the marginal cost of acquiring information is zero, in order for $q^* = 0$ to hold in equilibrium, it must be that there is no bank run.¹⁸ Of course, no bank run is also sufficient for not acquiring additional information $q^* = 0$. According to Proposition 4, the condition for existence of a no run equilibrium in this case is

$$G\left(\tau_r^l\left(q\right)\right)|_{q=0} \le 0.$$
(24)

When (24) fails, i.e., a bank run occurs even fixing q = 0 exogenously, then bank runs with positive information acquisition must exist. The next lemma summarizes this result.

Lemma 2. Condition (24) is a necessary and sufficient condition for the existence of an equilibrium where no bank run occurs (and therefore $q^* = 0$). If (24) fails, there exist bank run equilibria with positive information acquisition $q^* > 0$.

The above lemma only provides sufficient conditions for the existence of bank run equilibria. Specifically, bank run equilibria (with $q^* > 0$) could exist even when (24) holds. Intuitively, although no-run-no-acquisition is an equilibrium, run-acquisition might also be an equilibrium. Once other agents raise the equilibrium q^* above zero, a bank run is possible, and this makes individual information acquisition self-enforcing.

In general, even among the class of bank run equilibria with positive endogenous information acquisition, multiplicity may occur. The next lemma shows that under certain sufficient conditions provided in the Appendix A.7, we will have at most one such equilibrium. Essentially, this condition, by guaranteeing that at the equilibrium $q^* > 0$ the marginal benefit of information acquisition has to go below the marginal cost for q slightly above q^* , implies that the resulting equilibrium (if exist)

¹⁸Otherwise, with bank run, immediate withdrawal with y_L signals implies a positive value for the signal. See related argument in footnote 17.

must be unique.

Lemma 3. Under condition (30) provided in the Appendix A.7, the bank-run equilibrium with positive endogenous information acquisition, if one exists, is unique.

The next proposition follows from the two above lemmas.

Proposition 5. Suppose that condition (30) in Lemma 3 holds.

1. If the condition (24) holds, we have at most one bank run equilibrium with information acquisition so that $q^* > 0$ and $\zeta^* \leq \eta$, and there is always a no-run equilibrium with $q^* = 0$.

2. If the condition (24) fails, then we have a unique bank run equilibrium equilibrium with $q^* > 0$ and and $\zeta^* \leq \eta$.

From now on, to facilitate analysis, we focus on case 1 of Proposition 5, i.e., we have at most two equilibria: one equilibrium where agents do not acquire information and also do not run, and a second equilibrium characterized by Proposition 4 where agents acquire information aggressively and run on the illiquid bank. In particular, we will be interested in parameters that eliminate the bank run equilibrium.

4.4 Model Implications

The following analysis focuses on the run equilibrium with endogenous information acquisition. We first study the minimum reserve that eliminates rumor-based bank runs; then we show that sequential learning in our model generates intriguing comparative statics with respect to the awareness window.

4.4.1 Minimum Reserve and Deposit Insurance that Eliminate Bank Runs

In Proposition 5 we have shown that similar to the static Diamond and Dybvig setting, generally in our model there are two equilibria: one equilibrium without learning and no bank runs, and the other with active learning and bank runs. However, the dynamic run equilibrium derived in our setting has a qualitatively different nature relative to the one in the static setting. This section explores this difference by focusing on the policy implications of how to eliminate the bank run equilibrium.



Figure 4: Minimal Capacity Required to Eliminate Runs

The solid line is the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the recovery rate in case of failure γ . The dashed line is the potential mass of informed agents as in (8). Bank runs can occur only in the dark region. The dotted line shows the threshold that applies if agents ignore the redepositing option and therefore run sooner. Parameter values are r = 0.09, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\alpha = 0.7$.

Unlike the typical static setting where runs occur if bank reserves are below all potential withdrawals, our rich dynamic setting with endogenous learning gives a non-trivial reserve threshold that eliminates the run equilibrium. In other words, due to uncertainty about the bank illiquidity and other depositors' withdrawal timings, the minimum reserve requirement to fence off the bank run equilibrium in our model might be far below the level that is sufficient to cover all potential withdrawals. The same applies to deposit insurance: in our model, to eliminate runs, we only require a sufficiently high, but not full, deposit insurance.

In Figure 4, we plot the minimum reserve level κ_L and minimum recovery rate γ so that the bank run equilibrium does not exist. We plot the threshold κ_L as a function of γ . Intuitively, when depositors expect to recover more of their deposit in case of failure (larger γ) the bank is less susceptible to runs. We can interpret γ as the level of deposit insurance. The figure reveals that in our dynamic setting, bank runs can be prevented even with partial deposit insurance ($\gamma < 1$).

The figure further shows that the minimal liquidity level to eliminate the run is much lower than the potential mass of informed agents. From the view of static Diamond and Dybvig runs, the potential mass of informed agents is the relevant liquidity reserve required to eliminate runs. However, due to the asynchronous nature of our rumor-based setting, even without information acquisition, immediate withdrawal after hearing the rumor might not be an equilibrium since the bank survives for a while. Thus, eliminating dynamic rumor-based runs requires much less liquidity reserve than the one suggested by the static perspective.

Importantly, Figure 4 reveals that the redepositing option in (16) substantially reduces the minimal liquid holdings that eliminate runs. The dotted line gives the threshold that applies if agents ignore the redepositing option and therefore run sooner.¹⁹ Intuitively, the option to redeposit funds and grow them in a bank revealed to be liquid leads to an additional incentive to stay deposited for interest gains, and thus a lower minimal liquid reserve to eliminate runs.

Clearly, for the model to generate quantitatively meaningful policy recommendations on banks' capital reserve requirement, one needs reasonable parameters to begin with. As a first step toward this goal, our stylized model falls short on this dimension. However, though beyond the scope of the current paper, given the right data, a calibration (or estimation) of our model can certainly achieve this more ambitious goal.

To the best of our knowledge, no existing studies generate a result similar to the minimum liquidity reserve in Figure 4 based on static bank run models. While it may be possible to do so using the global games technique (e.g. Goldstein and Pauzner, 2005), the mechanism will be drastically different. To illustrate this point, note that the minimum liquidity reserve can be interpreted as liquid short-term assets held by the bank in the Diamond and Dybyig framework. Clearly, the portfolio share of liquid assets affects each individual depositor's payoff through some specific transformation technology between the liquid short-term asset and illiquid long-term asset. This effect shows up in both static and dynamic settings. However, there is one extra effect unique to the dynamic setting with learning. In the static bank run setting, withdrawals occur immediately. In contrast, in the dynamic model considered here with gradual withdrawal, a greater share of liquid holding also implies that it takes longer to run down the bank. Hence, through the endogenous learning channel, an increase in liquidity can reduce the hazard rate of failure and cause agents to wait longer. If we increase liquidity just enough to make the *last* agent whose withdrawal exhausts the liquid reserve to wait long enough, the bank run is averted. This practically relevant mechanism depends on both the dynamic nature of the withdrawal timing and the endogenous learning about a possible bank failure.²⁰

¹⁹The hypothetical threshold is obtained by simply setting transaction costs $k = \frac{r}{\delta}$, which only applies when redepositing, uttering the redepositing option worthless (see (16)).

²⁰In our analysis we do not consider the situation where uninformed agents run on the bank. Although Section





Solid lines show the equilibrium survival time of the illiquid bank ζ^* as a function of the rumor spreading rate β and the length of the awareness window η . Parameter values are r = 0.09, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

4.4.2 Rumor Spreading Rate β and Awareness Window η

When the rumor spreading rate β increases, all else equal, the illiquid bank will fail faster. This effect is illustrated in Figure 5. In response, each individual agent acquires more information, and the illiquid bank fails even faster. The feedback effects as discussed before are also present here, and this result is intuitive.

Relative to Abreu and Brunnermeier (2003), our model decouples the rumor spreading rate from the awareness window. When we turn to the effect of the awareness window η on the equilibrium survival time ζ^* and information precision q^* , a surprising result emerges. When η increases so that everybody knows that potentially there will be more informed agents attacking the bank, each individual agent acquires information less aggressively and the illiquid bank survives longer.

This surprising result not only comes from our decoupling of these two effects, but also relies on the novel uncertainty structure that we introduce in this framework. One can see the intuition

^{4.5.2} verifies that it is optimal for uninformed agents not to run given our rumor-based bank run equilibrium, the assumption $\tilde{\kappa} < 1$ implies that, as in Diamond and Dybvig, everybody runs on the bank (and thus the bank fails immediately) is also an equilibrium. Our paper focuses on the rumor-based bank run equilibrium because it is more appealing from a practical perspective; after all, nobody talks about runs when the banking system is healthy. To formally model this feature, one needs something similar to the *upper dominance region* in global games. For instance, we can imagine that the bank is super-liquid so that κ is above 1 before the liquidity event (e.g., even if all depositors withdraw, the bank asset is sufficiently liquid so that equity holders receive positive value in the surviving bank), and hence not running at t = 0 is the strictly dominant strategy. Then one actually can rule out one class of pure-strategy run equilibria where everybody coordinates to run on the bank at some exogenous time $t^{Run} > 0$ (suppose it exists; then since each depositor would like to preempt by running a bit earlier at $t^{Run} - \epsilon$, it can only be that $t^{Run} = 0$; but at that time everybody knows the bank being super-liquid). We leave further analysis on other classes of equilibria for future research.

by investigating the posterior probability that the bank is illiquid upon hearing the rumor and observing that the bank is still alive:

$$p(t_i|t_i) = \frac{\Pr\{\text{illiquid bank survives at } t_i|\kappa_L, t_i\} \Pr\{\kappa_L\}}{\Pr\{\text{illiquid bank survives at } t_i|\kappa_L, t_i\} \Pr(\kappa_L) + \Pr\{\kappa_H\}} = \frac{\frac{e^{\lambda\zeta} - 1}{e^{\lambda\eta} - 1}p_0}{\frac{e^{\lambda\zeta} - 1}{e^{\lambda\eta} - 1}p_0 + 1 - p_0}$$
(25)

When η is large, $t_0 \in [t_i - \eta, t_i]$ could occur a long time ago, and the probability

Pr {illiquid bank survives at
$$t_i | \kappa_L, t_i \} = \frac{e^{\lambda \zeta} - 1}{e^{\lambda \eta} - 1}$$

is lower. Consequently, conditional on the bank being alive at t_i , the bank is more likely to be liquid. Without uncertainty ($p_0 = 0$ or 1), $p(t_i|t_i)$ does not depend on the awareness window η .

This result differs from the casual intuition that runs are more severe with more prone-to-run agents, with the premise that bank failure requires a sufficient mass of running depositors. However, our result shows that when cumulative informed agents are enough to run down the bank, artificially shortening the awareness window will gives rise to a novel information effect with the exact opposite direction in a dynamic setting.²¹

A Short Awareness Window Example: Subprime Mortgage Crisis On May 17, 2007 Fed Chairman Bernanke indicated in a speech about the subprime mortgage market that looser lending standards were pervasive especially in loans originated in 2006 (Bernanke, 2007). The speech took place at a time when low teaser rates on many adjustable-rate mortgages were set to expire, suggesting that the rise in defaults was just the tip of an iceberg. Subsequently, asset-backed commercial paper (ABCP) conduits experienced the modern-day equivalent of a bank run as ABCP outstanding dropped from \$1.3 trillion in July 2007 to \$833 billion in December 2007 (Acharya, Schnabl, and Suarez, forthcoming).

One interpretation within our model of this speech is that it signaled that the awareness window was relatively short, in which case there was not much to learn from the survival of ABCP conduits

²¹Hellwig and Veldkamp (2009) show that information acquisition exhibits complementarity if and only if the actions are complementary. In our model, although running on the bank is complementary, sequential information acquisitions do not necessarily exhibit complementarity. In fact, (25) implies substitutability in information acquisition among depositors. When other depositors acquire more information, the equilibrium survival time ζ^* is shorter. Then, upon hearing the runor, the mere survival of the bank conveys more information about the bank's liquidity, and hence informed agents perceive the bank to be stronger. Substitutability naturally arises because this effect discourages each individual depositor's motivation for information acquisition.

up to that point in time. Suppose that instead, the analysis revealed that looser lending standards were pervasive in loans originated since 2003. In that case, information about the resulting weakness must have been spreading for a long time. Having observed that the ABCP market kept growing despite this fact, investors would conclude that the probability of the system being liquid enough is high and a run might not occur.

4.5 Other Theoretic Issues

In this subsection we tie some theoretic loose ends. Readers may wish to skip to the policy discussion in Section 5 and revisit this part later.

4.5.1 Minimum Threshold of κ_H to Eliminate Runs on the Liquid Bank

Thus far we have assumed that κ_H , though below 1, is sufficiently high that rumor-based bank runs do not occur for the liquid bank with $\tilde{\kappa} = \kappa_H$ (and hence staying in the bank is optimal given a y_H signal by revealing the bank being liquid). Motivated by Section 4.4.1 where a sufficiently high $\kappa_L < 1$ eliminates runs on illiquid banks, we provide a brief analysis for how to determine an upper bound of the threshold liquidity reserve κ_H that eliminates runs on a liquid bank.

Our information structure in Figure 2 implies that the agent with y_H knows the bank is liquid for sure. More importantly, the agent knows that he is among the q fraction of y_H agents, and there are 1 - q fraction of informed agents with medium signals y_M . We are interested in the equilibrium behavior of y_H agents who endogenously wait a while to withdraw, by holding the belief that 1 - q fraction of y_M agents withdraw immediately upon hearing the rumor. Thus, y_H agents face a tradeoff similar to (16). This harshest belief regarding y_M agents allows us to determine the upper bound of the κ_H threshold that rules out runs by y_H agents. Interestingly, this is in the same nature as the equilibrium for the illiquid bank studied in the main model, where y_L agents withdraw immediately while y_M agents wait for a while; and hence the same algorithm applies.²² Under our baseline parameters given in Figure 3, the upper bound on the κ_H threshold required to eliminate runs on liquid bank is 0.82.

²²More specifically, when the liquid bank reserve κ_H increases, the waiting benefit of y_H agents increases and they wish to wait longer. Once the resulting failure time ζ exceeds the awareness window η , the similar logic as in Lemma 1 and Proposition 4 implies that y_H agents will not run in equilibrium. Here, the implicit assumption is that 1 - q measure of y_M agents are not enough to take down the bank.

4.5.2 What about Uninformed Agents?

Uninformed agents play no role in our analysis of rumor-based bank runs. So far, we have assumed that (fully rational) uninformed agents remain fully deposited before they hear the rumor. We now provide justification for the optimality of this strategy.²³

Uninformed learning is more complicated because, in addition to the failure hazard rate, they must keep updating the rumor arrival rate, i.e. the probability of hearing the rumor in the next instant given that no failure and no rumor have arrived. The rumor arrival rate initially increases, and converges to a positive constant as the absolute time t grows large.²⁴ The rumor arrival rate plays a minor role in the withdrawal decision; the reason is that it does not matter much whether an agent hears the rumor inside or outside the bank, up to small transaction costs.

The failure hazard rate from the perspective of uninformed agents, on the other hand, is potentially important even when transaction costs vanish. Unlike the rumor arrival rate, the failure hazard rate converges to zero as $t \to \infty$. The usual time homogeneity property of exponentials requires one to condition on the event not happening before time t. This is not the case for our uninformed agents. As time passes and they observe no failure and no rumor, uninformed agents attach a higher and higher probability to the event that \tilde{t}_0 already happened (and they missed the rumor), but the bank is liquid so no failure occurs.

Similar to the analysis for informed agents in (15), an uninformed agent will keep their deposits inside the bank from time t going forward if the following holds:

$$h^{U}(s)(1-\gamma) \le r U_{I}(s) \text{ for } s \ge t,$$

$$(26)$$

where $h^U(s)$ and $U_I(s)$ are the conditional failure hazard rate and the value of a dollar inside the bank from the uninformed agent's perspective. Since in the limit the failure hazard rate decreases to zero, while the right hand side $U_I(s) > p_0\gamma + (1-p_0)\frac{\delta}{\delta-r}$,²⁵ condition (26) must hold for any

 $^{^{23}}$ For brevity, we provide only intuition for this behavior, though a formal analysis is available upon request.

²⁴The convergence to a positive constant when $t \to \infty$ is because becoming informed requires the liquidity event to have hit, and agents are conditioning on no failure $(\tilde{t}_0 > t - \zeta)$. Thus, the conditional distribution for \tilde{t}_0 remains fixed, which results in a positive rumor arrival rate even when t grows large.

²⁵The value $U_I(s)$ is determined under the optimal continuation strategy that always stay inside the bank, and follows optimal strategy as an informed agent once hearing the rumor. The lower bound $p_0\gamma + (1 - p_0)\frac{\delta}{\delta - r}$ is achieved under the simple strategy of never withdrawing from the bank (and the illiquid bank fails immediately). This is a lower bound because at any later time, the posterior bank illiquidity probability where agents condition on no failure is weakly lower than their prior p_0 (the formal proof is available upon request).

t such that $0 \le t^f \le t < \infty$, for some finite t^f . In fact, for the parameters chosen to illustrate our results, the maximum hazard rate is low enough so that $t^f = 0$, i.e. it is always optimal for uninformed agents to remain fully deposited.

4.5.3 Optimal Time to Acquire the Additional Signal(s)

We have so far assumed agents acquire the additional signal \tilde{y} immediately upon hearing the rumor. Although early information allows for a superior action, there is an option value derived from waiting to acquire the signal later: the agent learns from the passage of time that the bank is more likely to be liquid, and there is a probability the bank would fail (thus making the additional signal unnecessary). Proposition 6 in Appendix A.9 provides a sufficient condition for the optimality of acquiring information immediately. Intuitively, it suffices that the information acquisition cost (α) is low enough relative to the net benefit of delaying this expense. Under this condition, endogenous timing of information acquisition results in the same equilibrium behavior as we assumed before.

We could also allow agents to acquire signals more than once. But if we impose, as one should, a small fixed cost of acquiring information, we get in equilibrium a finite number of such purchases. After these, the population is fully described by the same three types (y_L, y_M, y_H) as in the main text. Thus, the bank run equilibrium structure remains qualitatively the same.

5 Extensions and Policy Analysis

We study two extensions of our base model. In Section 5.1 we introduce fundamentally insolvent banks, so that it is also *socially* efficient for individual agents to acquire information and run on insolvent banks. Section 5.2 then considers a two-bank economy where competition amplifies the individual agent's socially wasteful information production. For both extensions we discuss government policy during the recent crisis.

5.1 Insolvent Banks and Stress Tests

Thus far we have focused on runs on fundamentally solvent but potentially illiquid banks, where information is socially "bad." Of course, in practice information may be "good" because of the presence of fundamentally insolvent banks so that runs on them are socially efficient. To address this issue, we introduce insolvency into our model. We then argue that through stress tests the planner may mitigate running on fundamentally *solvent* banks by helping individual agents spot insolvent banks more easily.

5.1.1 Solvency signal versus liquidity signal

Suppose that at the liquidity event time t_0 , the bank might become insolvent, which randomly fails (rather than matures) with intensity $\xi > r$. In this event deposits recover 0 for each dollar, and therefore $\xi > r$ implies that the bank is indeed insolvent. Of course the bank can also be solvent, and if so it can be either liquid or illiquid as we modeled before.

Naturally, there will be two layers of information. The first layer of information is regarding the bank's solvency which is both socially and individually valuable. The second is information regarding the bank's liquidity condition, which is individually valuable but socially destructive when agents realize that runs on the illiquid bank become a concern. We will stress that although these two layers of information are different, they are inevitably related when individual agents are acquiring them. As shown shortly, this way we connect individuals' liquidity information choice to the active information collection about bank solvency.

Now when agent t_i hears the rumor, the possibility of insolvency motivates him to spend a fixed amount of effort $\underline{e} > 0$ to obtain a signal $\mathbf{1}_z \in \{0, 1\}$; for simplicity, this signal $\mathbf{1}_z$ perfectly reveals whether the bank is solvent or not.²⁶ We focus on situations where, in equilibrium, the agent always finds it optimal to acquire this solvency signal (which is guaranteed by a sufficiently high default intensity ξ). Therefore, conditional on the bank being insolvent, all agents who hear the rumor know that the bank is insolvent and therefore run on the bank immediately, and the acquisition of solvency information is socially optimal.

A by-product of the agent's private learning about bank solvency is that he also learns something about the liquidity of the bank. We assume that given the effort \underline{e} of figuring out whether the bank is insolvent, if the bank turns out to be solvent, then the baseline quality of the bank's liquidity signal y—which is the signal we modeled in Section 2.1.4—is just \underline{e} . As a result, the agent's additional liquidity information precision choice is $q \geq \underline{e}$ with acquisition cost $\frac{\alpha}{2} (q - \underline{e})^2$. The interpretation is that the process of collecting insolvency information inevitably teaches the agent something about

 $^{^{26}}$ This assumption is innocuous and clarifies the economic channel. See detailed discussion in footnote 27.

bank liquidity. The more the agent collects insolvency information, the more he knows about the bank's liquidity. Our modeling that the effort of collecting insolvency information (i.e., \underline{e}) becomes the baseline quality of liquidity information captures this idea in the simplest way.

5.1.2 Policy Implication: Stress Tests

The above setting has important implications for stress tests in revealing the fundamentally problematic banks. By *providing insolvency information alone*, the government can use stress tests to reduce \underline{e} to eliminate runs on solvent-but-illiquid banks. According to our model, if a great effort is required to learn about insolvent banks (say Lehman), i.e., a higher \underline{e} , then each agent will be automatically endowed with significant information about the liquidity of solvent banks (say Citibank). As a result, runs on solvent banks may start, which pushes each agent to acquire even more liquidity information.

Stress tests provide transparency on potentially insolvent banks (Bernanke, 2009), and therefore reduce \underline{e} . Government, by providing higher quality information about banks' insolvency, can crowd out private acquisition of insolvency information. Because public information can be better targeted at insolvency alone, while the process of private acquisition of solvency information inevitably reveals liquidity information, public provision of solvency information helps all agents know that other agents do not have superior information regarding banks liquidity situation. Therefore, our model suggests that the public provision of insolvency information indirectly reduces the socially wasteful information acquisition regarding liquidity, and therefore make runs on illiquid banks less likely.

The perfect revelation of insolvent bank clarifies the channel that we are emphasizing. Releasing better solvency information helps illiquid banks, but it is not through a higher average bank quality once the stress test isolates those insolvent ones.²⁷ Rather, the channel is through the strategic interaction of individual agents as now everybody knows that everybody will wait to see the stress test and therefore not scramble to search the insolvent banks. As a result everybody will have less

²⁷When the planner varies \underline{e} , agents can always perfectly spot insolvent banks, therefore the channel of insolvency information is shut down. In fact, our model can handle the case that insolvent banks are imperfectly revealed. As shown in Lemma 1, in our model generically a bank run equilibrium exists only when y_L agents withdraw immediately, and the equilibrium analysis is identical if we instead assume that agents cannot perfectly tell insolvent banks from illiquid ones (so they will withdraw immediately once receiving y_L signal or $\mathbf{1}_z = 1$). However, under this alternative assumption, it is quite obvious that better solvency information helps illiquid banks since it allows individual agents to tell them apart from insolvent ones.

Figure 6: Minimal Capacity and Information Collection Effort Required to Eliminate Runs



We plot the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the solvency information collection effort \underline{e} . Parameter values are r = 0.09, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\alpha = 0.7$, $\gamma = 0.75$.

precise information on which solvent bank is less liquid and susceptible to a run.

This view is consistent with the Federal Reserve Board's recent break from the traditional supervisory view of opaqueness in favor of more public disclosure to restore the confidence of investors. However, our rationale is different than the one described by Bernanke (2010) who argues that transparency allows for scrutiny by outside analysts, which enhances the credibility of the tests. Instead, we argue that by providing more information, the government crowds out the information collection effort by individuals about the solvency of the banks. Since liquidity and solvency are tightly related, this government policy has the useful by-product of reducing information collection about bank liquidity and therefore reducing the incidence of runs on illiquid banks.

Figure 6 plots the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information collection effort \underline{e} . We see from Figure 6 that as the government provides more information about the solvency of the bank (lower \underline{e}), less liquid banks (lower κ_L) can avoid runs.

5.2 Multiple Solvent Banks and Policy Implications

We now investigate the model with competing solvent banks. Instead of holding cash, a bank run in this setting involves the transfer of deposited funds from one illiquid bank to other more liquid one.²⁸ Here, the two banks' difference might be minuscule, and in fact transfers between the two institutions only involve social losses (transaction fees and bank failure). The planner could inject noise (about liquidity) into the system, so that individual agents with less informative signals are more likely to stay in their original bank without knowing which one is the (more) liquid one.

Suppose that there are two banks, ex-ante identical except that half the population deposits in bank A and half deposits in bank B. Both banks promise the same rate of return r. However, transferring funds between banks requires a transaction cost k. A liquidity event occurs at a random time t_0 , and a rumor starts that exactly one of the banks is illiquid (with κ_L) while the other is liquid (with κ_H). The prior probability that each bank is illiquid is $p_0 = 0.5$ since they are ex-ante identical. The learning process in the two bank set-up is simpler, because the passage of time without a failure teaches agents nothing about the relative viability of their bank.

As in the setup above, agents are allowed to acquire a costly signal $\tilde{y} \in \{y_L, y_M, y_H\}$ about the status of their bank with probability distribution as in (1). Agents who receive the y_H signal know that their bank is the liquid one and therefore never withdraw; agents who receive the y_M signal gain no useful information and staying in their original bank is optimal; and agents with y_L signals run on their bank immediately. ²⁹ Proposition 7 in the Appendix A.10 characterizes the equilibrium in this setting with two banks.

In this extension, higher information quality q about the liquidity of two solvent banks is socially undesirable, as it shortens the survival time of the illiquid bank by inducing more agents to realize one bank strictly dominates the other. Injecting noise can alleviate the problem, and the simplest interpretation of injecting noise is to raise the information acquisition cost α . Figure 7 plots the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information cost α for the two bank setup (see Appendix A.10 for details). The injection of noise into the economy (higher α) blurs the differences between competing solvent banks. The noise reduces the equilibrium information quality q, and as a result, a run is easier to eliminate and

²⁸Some anecdotal evidence that differences in depositor perception about bank liquidity motivates them to withdraw from illiquid banks is provided by Sidel, Enrich, and Fitzpatrick (2008): "Melody Williams, 50 years old, said in the past 30 days she has moved about \$25,000 out of Washington Mutual, spreading it to other financial institutions she thought were stronger, including Wells Fargo & Co. Ms. Williams, the controller for an architecture firm, said she thought that Washington Mutual had gotten 'too big for their britches' with too many deals over the years."

²⁹From the perspective of y_L agents, the value of a dollar in their bank falls and the value of a dollar in the competing bank increases to a riskless $\frac{\delta}{\delta-r}$; as long as the transaction cost k is small enough so that $V_I(0|y_M) < (1-k) V_{-I}(0|y_H) = (1-k) \frac{\delta}{\delta-r}$, immediate withdraw is optimal. Here, information is (privately) more valuable in this setup since the outside option is a nearly identical bank rather then holding cash.

Figure 7: Minimal Capacity and Information Cost Required to Eliminate Runs



We plot the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information cost α for the two bank setup. Parameter values are r = 0.09, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $\delta = 0.12$, $k = 10^{-6}$, $\gamma = 0.75$.

requires less reserves by the illiquid bank.

Consider the recent financial crisis in 2008. A fear that some banks were insolvent prompted the Capital Purchase Program commonly known as the bailout of the nine largest U.S. financial institutions on October 13, 2008. When presenting the program to the CEOs of the 9 banks, Secretary of Treasury Henry M. Paulson was concerned that the strongest banks (e.g. JPMorgan) would not participate.³⁰ To make sure they do, government officials suggested that if a bank refused the funds, its regulator would later force it to raise capital anyway and under worse terms.³¹ The government was in fact injecting noise about the liquidity of competing solvent banks into the economy. By pooling banks together the incentive to transfer funds between them was kept low enough so that none of the nine banks suffered a run.

6 Conclusion

We study above new dimensions of bank runs, previously unexplored, by focusing on information acquisition about bank liquidity in rumor-based runs. These dimensions include the spreading rate of information and the awareness window over which it spreads. We show that individuals acquire

 $^{^{30}}$ "I was concerned about Jamie Dimon, because JPM organ appeared to be in the best shape of the group, and I wanted to be sure he would accept the capital." - Paulson (2010)

 $^{^{31}}$ "Look, we're making you an offer," I said, jumping in. "If you don't take it and sometime later your regulator tells you that you are undercapitalized and you have to raise private-sector capital but you are unable to do so, you may not like the terms if you have to come back to me." - Paulson (2010)

information excessively about the liquidity of banks subject to runs, study the minimum liquidity reserve to eliminate bank runs which is far below that of traditional static bank run models, and analyze the role government information policy can play in bank run prevention.

Empirically, our model can shed new light on both traditional bank runs and debt runs more broadly. Our model makes new predictions linking the likelihood of a run to the cost of acquiring information, the spreading rate of the rumor and its awareness window. It also predicts that for banks that survive a run, we should observe agents withdraw at the same time earlier informed agents redeposit. Our model is rich enough for structural estimation or calibration of its parameters given detailed withdrawals data, which can result in meaningful policy implications.

Finally, the dynamic bank run model we provide above can shed new light on other economic settings such as arbitrageur behavior, currency attacks, and R&D investment games.

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A Appendix A

A.1 Proof of Proposition 1

We have two cases to consider for the hazard rate. First, suppose $\zeta > \eta$. Then an agent informed at t_i learns nothing from the fact that the bank has not failed by t_i . His distribution of failure dates $t_i + \tau$ is

$$\Pi\left(t_{i}+\tau|t_{i}\right) = \begin{cases} 0 & \tau < \zeta - \eta\\ p_{0} \frac{e^{\lambda \eta} - e^{\lambda(\zeta - \tau)}}{e^{\lambda \eta} - 1} & \zeta - \eta \le \tau < \zeta\\ p_{0} & \zeta \le \tau. \end{cases}$$

with non-zero density $\pi(t_i + \tau | t_i) = p_0 \frac{\lambda e^{\lambda(\zeta - \tau)}}{e^{\lambda \eta} - 1}$ for $\zeta - \eta < \tau < \zeta$. On the other hand, if $\zeta \leq \eta$, then agent t_i 's distribution of failure dates $t_i + \tau$ is

$$\Pi\left(t_{i}+\tau|t_{i}\right) = \begin{cases} 0 & \tau < 0\\ p\left(t_{i}|t_{i}\right) \frac{1-e^{-\lambda\tau}}{1-e^{-\lambda\zeta}} & 0 \le \tau < \zeta\\ p\left(t_{i}|t_{i}\right) & \zeta \le \tau. \end{cases}$$

non-zero density $\pi(t_i + \tau | t_i) = p(t_i | t_i) \frac{\lambda e^{-\lambda \tau}}{1 - e^{-\lambda \zeta}}$ for $0 < \tau < \zeta$. Plugging either of the pairs into the definition of the hazard rate (5) yields after some algebraic manipulation the same expression (6) for any $\tau \ge \max\{0, \zeta - \eta\}$ and zero elsewhere.

A.2 Proof of Proposition 2

We first establish the following lemma.

Lemma 4. The function $g(\tau)$ crosses zero from below at most once in the interval $[0, \zeta]$.

Proof: Since it is the change in the numerator that dominates around $g(\tau) = 0$, it suffices to show that the numerator of $g(\tau)$ (ignoring the constant)

$$\left(\lambda\left(1-\gamma\right)-r\right)e^{\lambda\left(\zeta-\tau\right)}p_{0}-\left(1-p_{0}\right)\left(e^{\lambda\eta}-1\right)\frac{r\left(r-k\delta\right)}{\delta-r}e^{-\delta\left(\zeta-\tau\right)}$$
(27)

is increasing over the interval $[0, \zeta]$. Furthermore, since $\lambda (1 - \gamma) - r < \lambda (1 - \gamma) - r (1 - k) < 0$ from (10), it follows that (27) is concave in τ . Let $\overline{\tau}$ be the unique maximizer. At the maximum

$$\zeta - \overline{\tau} = \frac{1}{\lambda + \delta} \ln \frac{\delta \left(1 - p_0\right) \left(e^{\lambda \eta} - 1\right) \frac{r(r - k\delta)}{\delta - r}}{\lambda \left(r - \lambda \left(1 - \gamma\right)\right) p_0} < 0$$

due to (10). Thus, the function in (27) attains its maximum to the right of ζ and is therefore increasing over $[0, \zeta]$.

Lemma 4 implies that if $g(\zeta) \leq 0$ $(g(0) \geq 0)$ then $g(\tau) < 0$ $(g(\tau) \geq 0)$ always for $\tau \in [0, \zeta]$. We next consider the optimal strategy for the three cases of the proposition:

Case 1. If $g(\zeta) \leq 0$, then it is optimal to stay in the bank always. To prove our claim, it suffices to show that $V_I(\tau) > V_O(\tau)$ for $\tau \in [0, \zeta]$. Suppose not, then there must exist some τ_w so that $V_I(\tau_w) = V_O(\tau_w)$ and $V'_I(\tau_w) > V'_O(\tau_w)$ because $V_I(\zeta) > V_O(\zeta)$. From HJB equations, we have

$$h(\tau_w)(1-\gamma) - rV_I(\tau_w) = V'_I(\tau_w) - V'_O(\tau_w) > 0$$

However, since $V_I(\tau_w) \ge V_O(\tau_w) \ge \hat{V}_O(\tau_w)$ by definition (the first inequality is because there is no transaction cost to take one dollar out, and the second inequality is because $\hat{V}_O(\tau_w)$ may be derived under suboptimal policy), we have

$$h(\tau_w)(1-\gamma) - rV_I(\tau_w) < h(\tau_w)(1-\gamma) - r\hat{V}_O(\tau_w) = g(\tau_w) \le 0,$$

a contradiction.

Case 2. If $g(0) \ge 0$, then it is optimal to withdraw at 0 and redeposit right after ζ . Well, if $g(0) \ge 0$, then $g(\tau) \ge 0$ always for $\tau \in [0, \zeta]$, and $g(\zeta) > 0$. Using $g(\zeta) > 0$, we first show that since k is arbitrarily small, there exists some $\hat{\tau}$ close to ζ so that $V_O(\hat{\tau}) = V_I(\hat{\tau})$. To show this, we show that $V'_O(\zeta) - V'_I(\zeta)$ is strictly below zero when k is arbitrarily small. To see this, from the HJB equations we know that

$$V_O'(\zeta) - V_I'(\zeta) = h(\zeta) (V_O(\zeta) - V_I(\zeta) + \gamma - 1) + \delta (V_I(\zeta) - V_O(\zeta)) + rV_I(\zeta)$$

= $-g(\zeta) + (h(\zeta) + \delta - r) (V_O(\zeta) - V_I(\zeta))$

The first is strictly negative while the second term converges to zero as $k \to 0$. Therefore, when k is arbitrary small there exists some ε so that $V_I(\zeta - \varepsilon) < V_O(\zeta - \varepsilon)$. Due to continuity and the fact that $V_I(\zeta) = \frac{V_O(\zeta)}{1-k} > V_O(\zeta)$, there exists some $\hat{\tau}$ close to ζ so that $V_I(\hat{\tau}) = V_O(\hat{\tau})$. Note that $V_O(\hat{\tau}) = \hat{V}_O(\hat{\tau})$.

Now to prove that "it is optimal to withdraw at 0 and redeposit right after ζ ," we only need to show that $V_I(\tau) = V_O(\tau)$ holds for all $\tau \in [0, \hat{\tau}]$ (intuitively, at any point of time a dollar inside the bank has the value of taking outside, it is always optimal to keep the money outside). Suppose that this does not hold; since $V_I(\tau) \ge V_O(\tau)$ in general, there must exits some point $\tau_w \in [0, \hat{\tau}]$ so that $V_I(\tau_w) = V_O(\tau_w)$ and $V'_I(\tau_w) < V'_O(\tau_w)$. Choosing the largest value τ_w , so that $V_O(\tau_w) = \hat{V}_O(\tau_w)$ holds (i.e., the optimal continuation strategy is wait outside the bank until ζ). Similar to the argument before, we have

$$h(\tau_w)(1-\gamma) - rV_O(\tau_w) = V'_I(\tau_w) - V'_O(\tau_w) < 0,$$

but this contradicts the fact that $h(\tau_w)(1-\gamma) - rV_O(\tau_w) = h(\tau_w)(1-\gamma) - r\hat{V}_O(\tau_w) \ge 0$ since $g(\tau) \ge 0$ always.

Case 3. It follows from the Lemma 4 that $g(\zeta) > 0$ and g(0) < 0 imply that there exists a unique $\tau_w \in (0, \zeta)$ so that $g(\tau_w) = 0$, $g(\tau) > 0$ for $\tau \in (\tau_w, \zeta)$ and $g(\tau) < 0$ for $\tau \in (0, \tau_w)$. Following the same argument in the second part by replacing 0 with τ_w , we know that it is optimal to withdraw at τ_w and redeposit at $\zeta +$, and $V_I(\tau_w) = V_O(\tau_w) = \hat{V}_O(\tau_w)$. Then to prove our claim we only need to show that $V_I(\tau) > V_O(\tau)$ for $\tau \in (0, \tau_w)$.

Let $H(\tau) \equiv V_I(\tau) - V_O(\tau)$ with $H(\tau_w) = 0$, we need to show that $H(\tau) > 0$ for $\tau \in (0, \tau_w)$, as $H(\tau) = V_I(\tau) - V_O(\tau) \ge 0$ in general. First, we show that it is impossible to have $H(\tau) = 0$ uniformly on any interval $(\tau_w - \Delta, \tau_w)$ where $\Delta > 0$; if it is true then it must be that $V_I(\tau) = 0$

 $V_{O}\left(\tau\right)=\widehat{V}_{O}\left(\tau\right)$ on that interval so that

$$0 = rV_{I}(\tau) + h(\tau)(\gamma - V_{I}(\tau)) + \delta(1 - V_{I}(\tau)) + V'_{I}(\tau)$$

= $r\hat{V}_{O}(\tau) + h(\tau)(\gamma - \hat{V}_{O}(\tau)) + \delta(1 - \hat{V}_{O}(\tau)) + \hat{V}'_{O}(\tau)$
= $r\hat{V}_{O}(\tau) - h(\tau)(1 - \gamma) = -g(\tau) > 0$

where the first equality is (13) and third equality is using the ODE for $\hat{V}_O(\tau)$ with $0 = h(\tau) \left(1 - \hat{V}_O(\tau)\right) + \delta\left(1 - \hat{V}_O(\tau)\right) + \hat{V}'_O(\tau)$. This contradiction implies that we must have $V_I(\tau) > V_O(\tau)$ for some τ close to τ_w . Now suppose that there exists another point $\tau^1_w < \tau_w$ so that $V_I(\tau^1_w) = V_O(\tau^1_w)$. Take τ^1_w that is closet to τ_w so that $V'_I(\tau^1_w) \ge V'_O(\tau^1_w)$. At τ^1_w , $V_O(\tau^1_w)$ must satisfy the HJB in (18) (because τ^1_w is in the inaction region around the neighborhood, i.e., $(1 - k) V_I(\tau) < V_O(\tau)$ for τ close to τ_w). Then

$$0 = rV_{I}\left(\tau_{w}^{1}\right) + h\left(\tau_{w}^{1}\right)\left(\gamma - V_{I}\left(\tau_{w}^{1}\right)\right) + \delta\left(1 - V_{I}\left(\tau_{w}^{1}\right)\right) + V_{I}'\left(\tau_{w}^{1}\right)$$

$$\geq rV_{O}\left(\tau_{w}^{1}\right) + h\left(\tau_{w}^{1}\right)\left(\gamma - V_{O}\left(\tau_{w}^{1}\right)\right) + \delta\left(1 - V_{O}\left(\tau_{w}^{1}\right)\right) + V_{O}'\left(\tau_{w}^{1}\right)$$

$$= rV_{O}\left(\tau_{w}^{1}\right) - h\left(\tau_{w}^{1}\right)\left(1 - \gamma\right) \geq -g\left(\tau_{w}^{1}\right) > 0$$

where we have used the HJB for $V_O(\tau_w^1)$, and the fact that $V_O(\tau_w^1) \ge \hat{V}_O(\tau_w^1)$ in general. Again we get a contradiction with (13).

A.3 Proof of Proposition 3

Given τ_w simple integration yields

$$V_{I}\left(0|y_{M}\right) = \frac{\left[\begin{array}{c}\frac{\delta\left(e^{\lambda\eta}(1-p_{0})-1\right)}{\delta-r}\left(1-e^{-(\delta-r)\tau_{w}}\right) + \frac{\delta+\lambda\gamma}{\lambda+\delta-r}e^{\lambda\zeta}p_{0}\left(1-e^{-(\lambda+\delta-r)\tau_{w}}\right)\right]}{+e^{-(\delta-r)\tau_{w}}\left(e^{\lambda\eta}-1-\left(e^{\lambda\eta}-e^{\lambda(\zeta-\tau_{w})}\right)p_{0}\right)V_{O}\left(\tau_{w}\right)}\right]}{e^{\lambda\eta}-1-e^{\lambda\eta}p_{0}+e^{\lambda\zeta}p_{0}}$$

Note that $h(\tau_w)(1-\gamma) = rV_O(\tau_w)$ implies that

$$\left(e^{\lambda\eta} - 1 - \left(e^{\lambda\eta} - e^{\lambda(\zeta - \tau_w)}\right)p_0\right)V_O\left(\tau_w\right) = \frac{\lambda\left(1 - \gamma\right)e^{\lambda(\zeta - \tau_w)}p_0}{r}$$

where we used the definition of h in (5). Then note that

$$\frac{\lambda\left(1-\gamma\right)}{r} - \frac{\delta + \lambda\gamma}{\lambda + \delta - r} = \frac{\left(\lambda + \delta\right)\left(\lambda\left(1-\gamma\right) - r\right)}{r\left(\lambda + \delta - r\right)}$$

which gives our expression.

A.4 Proof of Lemma 1

Suppose that $\zeta > \eta$, so that at ζ the cumulative withdrawal from y_L agents is $q\left(1 - e^{-\beta\eta}\right)$.

Then using the aggregate condition (21), we can back out the equilibrium τ_r for y_M agents as

$$\tau_r = -\frac{1}{\beta} \ln \left(1 - \frac{\kappa_L - q \left(1 - e^{-\beta \eta} \right)}{1 - q} \right).$$

However, unless parameters are such that the above τ_r happens to satisfy $G(\tau_r) = 0$ which is the y_M agents' optimal waiting decision, generically this cannot occur.

Now we rule out the case that y_L agents wait a positive time $\tau_w^L > 0$. Clearly, y_M agents will set $\tau_w^M > \tau_w^L > 0$. Importantly, their respective redepositing times $(\tau_r^M \text{ and } \tau_r^L)$ have to satisfy the individual optimality conditions in the nature of (20), which determine τ_r^M and τ_r^L fully. However, generically these τ_r^M and τ_r^L will not satisfy the aggregate withdrawal condition: either both agents are withdrawing at ζ so that the aggregate withdrawal condition is $\kappa_L = (1-q) \left(1-e^{-\beta\tau_r^M}\right) + q \left(1-e^{-\beta\tau_r^M}\right) + q \left(1-e^{-\beta\tau_r^M}\right) + q \left(1-e^{-\beta\tau_r^M}\right) + q \left(1-e^{-\beta\eta}\right).$

A.5 Proof of Proposition 4

First note that the G function is the mirror image of individual FOC condition function, i.e., $G(\tau_r) = g(\zeta - \tau_r)$, and it shares the same (but opposite) property of $g(\cdot)$ shown in Lemma 4:

Corollary 1. $G(\tau_r)$ crosses zero from above at most once on $\tau_r \in [0, \zeta]$.

This result implies that the following holds for the three cases of the proposition:

Case 1. If $G(\tau_r^l) \leq 0$, then $G(\tau_r) \leq 0$ for all $\tau_r \geq \tau_r^l$. Thus if all other agents strategy is to redeposit after any $\tau_r \geq \tau_r^l$, it is optimal for the individual agent to deviate and wait a bit longer. Therefore, $\zeta^* \to \infty$ and no run equilibrium exists.

Case 2. If $G(\tau_r^u) \ge 0$, then $G(\tau_r) \ge 0$ for all $\tau_r \le \tau_r^u$. Thus, if all other agents' strategy is to withdraw at some interior $\tau_r \le \tau_r^u$, it is optimal for the individual agent to deviate and withdraw earlier. Therefore, agents withdraw immediately in the only symmetric equilibrium.

Case 3. Finally, if $G(\tau_r^u) < 0$ and $G(\tau_r^l) > 0$ then by continuity of G and Corollary 1, there exists a unique bank run equilibrium $\tau_r^* \in (\tau_r^l, \tau_r^u)$ so that $G(\tau_r^*) = 0$. Plugging into (21) we get the equilibrium survival time ζ^* and waiting time τ_w^* . A second implication of Corollary 1 is that $G'(\tau_r^*) < 0$. Therefore the equilibrium is stable.

A.6 Proof of Lemma 2

The proof of the first statement is given in the text. To show the second statement, we need to show that the agent's FOC in information acquisition

$$A(q) \equiv p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M) - \alpha q \ge 0 \text{ with equality if } q < \frac{\kappa_L}{1 - e^{-\beta\eta}},$$

combined with bank-run equilibrium condition, has a solution. Note that both $p(t_i|t_i)$ and $V_I(0|y_M)$ (given in (25) and Proposition 3) depend on ζ that is determined in Proposition 4 about bank run equilibrium given q. Now because (24) fails, the bank run equilibrium exists even with exogenously

given q = 0. From (17) it is optimal to withdraw given y_L . The agent's strategy depends on his signal, and therefore information has a positive value, i.e. $A(0) = p(t_i|t_i) + (1 - p(t_i|t_i))\frac{\delta}{\delta - r} - V_I(0|y_M) > 0$. Now suppose that q takes its upper bound $\frac{\kappa_L}{1 - e^{-\beta\eta}}$; if $A\left(q = \frac{\kappa_L}{1 - e^{-\beta\eta}}\right) > 0$ then the upper bound information quality and associated bank run equilibrium is just the equilibrium that we are after. If instead $A\left(q = \frac{\kappa_L}{1 - e^{-\beta\eta}}\right) < 0$, then an interior equilibrium exists because of the continuity of A(q).

A.7 Proof of Lemma 3

First, when the bank run equilibrium occurs with corner solution $\zeta = \tau_r^u = \frac{1}{\beta} \ln \frac{1}{1-\kappa_L}$, then the marginal benefit of information $MB = p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M)$ is independent of q, and the equilibrium q^* equates $MC = \alpha q^* = MB$. Therefore the equilibrium is unique and stable (*MB* is constant while *MC* increases with q). Also if q^* takes the upper bound corner value, the associated run equilibrium is unique as well. So the rest of proof focus on the case where both the information quality of equilibrium survival time take interior solutions.

From now on we focus on interior bank run equilibrium. Importantly, this implies that τ_r is determined in (20) which only depends on primitives. Therefore we treat τ_r as a primitive parameter. The FOC (23) when the agent sets q^* is

$$\begin{bmatrix}
\left(e^{\lambda\zeta}-1\right)p_{0}+\left(1-p_{0}\right)\left(e^{\lambda\eta}-1\right)\frac{\delta}{\delta-r}-\frac{\delta\left(e^{\lambda\eta}\left(1-p_{0}\right)-1\right)}{\delta-r}\left(1-e^{-\left(\delta-r\right)\tau_{w}}\right)\\-\frac{\delta+\lambda\gamma}{\lambda+\delta-r}e^{\lambda\zeta}p_{0}-e^{-\left(\delta-r\right)\tau_{w}}e^{\lambda\tau_{r}}p_{0}\left(\frac{\left(\lambda+\delta\right)\left(\lambda\left(1-\gamma\right)-r\right)}{r\left(\lambda+\delta-r\right)}\right)-\alpha q\left(\left(1-p_{0}\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_{0}\right)\\\left(1-p_{0}\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_{0}=0$$
(28)

where ζ and $\tau_w = \zeta - \tau_r$ depend on q through (22). We have

$$\zeta'\left(q\right) = \tau'_{w}\left(q\right) = \frac{e^{-\beta\zeta} - e^{-\beta\tau_{r}}}{q\beta e^{-\beta\zeta}} = \frac{1 - e^{\beta\tau_{w}}}{q\beta} < 0$$

The derivative at the point where (28) takes zero value yields:

$$\begin{split} \lambda\zeta' e^{\lambda\zeta} p_0 &- \delta \left(e^{\lambda\eta} \left(1 - p_0 \right) - 1 \right) e^{-(\delta - r)\tau_w} \tau'_w - \frac{\delta + \lambda\gamma}{\lambda + \delta - r} p_0 \lambda e^{\lambda\zeta} \zeta' + (\delta - r) e^{-(\delta - r)\tau_w} e^{\lambda\tau_r} p_0 \left(\frac{(\lambda + \delta) \left(\lambda \left(1 - \gamma \right) - r \right)}{r \left(\lambda + \delta - r \right)} \right) \tau'_w \\ &- \alpha \left(\left(1 - p_0 \right) \left(e^{\lambda\eta} - 1 \right) + \left(e^{\lambda\zeta} - 1 \right) p_0 \right) - \alpha q e^{\lambda\zeta} \lambda\zeta' p_0 \\ &= \lambda\zeta' e^{\lambda\zeta} p_0 \left[\frac{\lambda \left(1 - \gamma \right) - r}{\lambda + \delta - r} - \alpha q \right] + e^{-(\delta - r)(\zeta - \tau_r)} \tau'_w \left[e^{\lambda\tau_r} p_0 \left(\delta - r \right) \left(\frac{(\lambda + \delta) \left(\lambda \left(1 - \gamma \right) - r \right)}{r \left(\lambda + \delta - r \right)} \right) - \delta \left(e^{\lambda\eta} \left(1 - p_0 \right) - 1 \right) \right] \\ &- \alpha \left(\left(1 - p_0 \right) \left(e^{\lambda\eta} - 1 \right) + \left(e^{\lambda\zeta} - 1 \right) p_0 \right) \\ &= \zeta' \left(q \right) e^{-(\delta - r)\zeta} \left[e^{(\delta - r)\tau_r} \left[e^{\lambda\tau_r} p_0 \left(\delta - r \right) \left(\frac{(\lambda + \delta) \left(\lambda \left(1 - \gamma \right) - r \right)}{r \left(\lambda + \delta - r \right)} \right) - \delta \left(e^{\lambda\eta} \left(1 - p_0 \right) - 1 \right) \right] + \lambda e^{(\lambda + \delta - r)\zeta} p_0 \left[\frac{\lambda \left(1 - \gamma \right) - r}{\lambda + \delta - r} - \alpha q \right] \\ &- \alpha \left(\left(1 - p_0 \right) \left(e^{\lambda\eta} - 1 \right) + \left(e^{\lambda\zeta} - 1 \right) p_0 \right) \end{split}$$

The second line is clearly negative. If we can ensure that

$$e^{(\lambda+\delta-r)\tau_r}p_0\left(\delta-r\right)\frac{(\lambda+\delta)\left(\lambda\left(1-\gamma\right)-r\right)}{r\left(\lambda+\delta-r\right)} + e^{(\delta-r)\tau_r}\delta\left(1-e^{\lambda\eta}\left(1-p_0\right)\right) + \lambda e^{(\lambda+\delta-r)\zeta}p_0\left[\frac{\lambda\left(1-\gamma\right)-r}{\lambda+\delta-r} - \alpha q\right] > 0, \quad (29)$$

then since $\zeta'(q) < 0$, the first line is negative, and as a result the derivative of (28) is always negative. Therefore, when (28) equals zero, it must go down. Combined with differentiability of (28), this result rules out multiple solutions, because if there exist, then there must have one solution with the local slope being nonnegative. Therefore, (28) crosses zero at most once from above, which implies that the bank run equilibrium, if exists, is unique and stable. To finish the argument, we show that the sufficient condition for condition (29) is (note that $e^{\lambda \eta} (1 - p_0) < 1$)

$$e^{(\lambda+\delta-r)\tau_r^u} p_0\left(\delta-r\right) \frac{(\lambda+\delta)\left(\lambda\left(1-\gamma\right)-r\right)}{r\left(\lambda+\delta-r\right)} + e^{(\delta-r)\tau_r^d}\delta\left(1-e^{\lambda\eta}\left(1-p_0\right)\right)$$
(30)
+ $\lambda e^{(\lambda+\delta-r)\eta} p_0\left[\frac{\lambda\left(1-\gamma\right)-r}{\lambda+\delta-r} - \alpha\frac{\kappa_L}{1-e^{-\beta\eta}}\right] > 0.$

We only need to verify that (29) dominates (30) term by term. The first term is because $\lambda (1 - \gamma) - r < 0$ (recall (10)) and $\tau_r \leq \tau_r^u$; the second term is because $\tau_r \geq \tau_r^d$, and the third term is because $\lambda (1 - \gamma) - r < 0$, $\zeta < \eta$, and $q^* < \frac{\kappa_L}{1 - e^{-\beta\eta}}$.

A.8 Proof of Proposition 5

Directly follows from Lemma 2 and Lemma 3.

A.9 Optimal Time to Acquire the Additional Signal(s)

Proposition 6. The optimal time to acquire the additional signal \tilde{y} is immediately upon hearing the rumor if

$$\left[\delta + h\left(\zeta\right)\right] \frac{\alpha}{2} \frac{\kappa_L}{1 - e^{-\beta\eta}} - \frac{r - k\delta}{\delta - r} \delta\left[1 - p\left(0\right)\right] < 0, \tag{31}$$

and

$$\left[\delta - r + h\left(\zeta\right)\right] \frac{\alpha}{2} \frac{\kappa_L}{1 - e^{-\beta\eta}} - \left(\left(1 - \gamma\right) h\left(0\right) - rp\left(0\right)\right) < 0.$$
(32)

Proof. Let $\tau_y \equiv t_y - t_i$ denote the time an agent waits between hearing the rumor and acquiring the additional signal y. We first show that under condition (31), if $0 \leq \tau_y \leq \tau_w$ then the optimal $\tau_y^* = 0$. We then show that under condition (32), if $\tau_w \leq \tau_y \leq \zeta$ then $\tau_y^* = \tau_w$. Finally, we show that $\tau_y = \tau_w$ is dominated by $\tau_y = \tau_w - \epsilon$, which implies setting $\tau_y^* = 0$ is everywhere optimal.

First, suppose $0 \le \tau_y \le \tau_w$. Informed agents maximize the value of a deposited dollar at t_i :

$$\max_{q,\tau_y} v(0;q,\tau_y) = \int_0^{\tau_y} \delta e^{-(\delta-r)s} (1 - \Pi(s)) \, ds + \int_0^{\tau_y} \gamma e^{-(\delta-r)s} \pi(s) \, ds + e^{-(\delta-r)\tau_y} (1 - \Pi(\tau_y)) v(\tau_y;q,\tau_y) \, .$$

The marginal benefit of earlier information acquisition time τ_y dominates the marginal cost if

$$\left[\delta - r + h\left(\tau_y\right)\right] \chi\left(q\right) < q\left(\left(1 - \gamma\right)h\left(\tau_y\right) - rp\left(\tau_y\right)\right) \text{ for all } \tau_y \in [0, \tau_w].$$

Condition (31) guarantees this is the case.

Second, suppose $\tau_w < \tau_y \leq \zeta$. The agent now maximizes:

$$\max_{q,\tau_y} v\left(\tau_w; q, \tau_y\right) = \int_{\tau_w}^{\tau_y} \delta e^{-\delta s} \left(1 - \Pi\left(s\right)\right) ds + \int_{\tau_w}^{\tau_y} e^{-\delta s} \pi\left(s\right) ds + e^{-\delta \tau_y} \left(1 - \Pi\left(\tau_y\right)\right) v\left(\tau_y; q, \tau_y\right).$$

For this case, the marginal benefit of earlier information acquisition time τ_y dominates if

$$\left[\delta+h\left(\tau_{y}\right)\right]\chi\left(q\right) < q\frac{r-k\delta}{\delta-r}\delta\left[1-p\left(\tau_{y}\right)\right] \text{ for all } \tau_{y}\in\left[\tau_{w},\zeta\right],$$

which is guaranteed by condition (32).

The earlier region dominates if $v(\tau_w^-;q) \ge v(\tau_w^+;q)$, i.e. if acquiring the signal just before withdrawing dominates acquiring it just after. Using the fact that $V_I(\tau_w|y_M) = V_O(\tau_w|y_M)$, the earlier region dominates if and only if $k\left[q(1-p(\tau_w))\frac{\delta}{\delta-r}\right] \ge 0$ which is holds under our assumptions. The intuition is that since the money is inside the bank just before τ_w , an agent who withdraws, acquires information, and then redeposits would waste the transaction cost.

A.10 Equilibrium with Multiple Solvent Banks

At the time of hearing the rumor, the value of a dollar in the bank for agents with y_L and y_H signals are respectively $V_I(0|y_L) = \frac{(1-k)\delta}{\delta-r}$ and $V_I(0|y_H) = \frac{\delta}{\delta-r}$. In order to calculate the value for agents with the y_M signal, note that with probability 1/2 the original bank is the illiquid one, but the agent can deposit his funds (after the liquidation cost $1 - \gamma$) to the liquid one. As a result, the value with y_M signal is

$$\begin{split} V_{I}\left(0|y_{M}\right) &= \frac{1}{2} \frac{1}{1 - \Pi\left(0|\kappa_{L}\right)} \int_{0}^{\zeta} \left[\delta e^{-(\delta - r)s} \left(1 - \Pi\left(s|\kappa_{L}\right)\right) + e^{-(\delta - r)s} \pi\left(s|\kappa_{L}\right) \gamma\left(1 - k\right) \frac{\delta}{\delta - r} \right] ds + \frac{1}{2} \frac{\delta}{\delta - r} \\ &= \frac{1}{2} \int_{0}^{\zeta} \left[\delta e^{-(\delta - r)s} \left(1 - \frac{1 - e^{-\lambda s}}{1 - e^{-\lambda \zeta}}\right) + e^{-(\delta - r)s} \frac{\lambda e^{-\lambda s}}{1 - e^{-\lambda \zeta}} \gamma\left(1 - k\right) \frac{\delta}{\delta - r} \right] ds + \frac{1}{2} \frac{\delta}{\delta - r} \\ &= \frac{1}{2} \frac{1}{1 - e^{-\lambda \zeta}} \int_{0}^{\zeta} \left[-e^{-\lambda \zeta} \delta e^{-(\delta - r)s} + \delta e^{-(\delta - r + \lambda)s} + \lambda \gamma\left(1 - k\right) \frac{\delta}{\delta - r} e^{-(\delta - r + \lambda)s} \right] ds + \frac{1}{2} \frac{\delta}{\delta - r} \\ &= \frac{1}{2} \frac{1}{1 - e^{-\lambda \zeta}} \left[-e^{-\lambda \zeta} \frac{\delta}{\delta - r} \left(1 - e^{-(\delta - r)\zeta}\right) + \frac{\delta}{\delta - r + \lambda} \left(1 - e^{-(\delta - r + \lambda)\zeta}\right) + \frac{\lambda \gamma\left(1 - k\right)}{\delta - r + \lambda} \frac{\delta}{\delta - r} \left(1 - e^{-(\delta - r + \lambda)\zeta}\right) \right] + \frac{1}{2} \frac{\delta}{\delta - r} \\ &= \frac{1}{2} \frac{1}{1 - e^{-\lambda \zeta}} \left[-e^{-\lambda \zeta} \frac{\delta}{\delta - r} \left(1 - e^{-(\delta - r)\zeta}\right) + \frac{\delta}{\delta - r + \lambda} \left(1 + \frac{\lambda \gamma\left(1 - k\right)}{\delta - r}\right) \left(1 - e^{-(\delta - r + \lambda)\zeta}\right) \right] + \frac{1}{2} \frac{\delta}{\delta - r}. \end{split}$$

Proposition 7. Under the two banks setup, the bank run equilibrium $\{\zeta^*, q^*\}$ is determined by the following two equations:

$$\zeta^* = -\frac{1}{\beta} \ln\left(1 - \frac{\kappa_L}{q^*}\right), \text{ and } \frac{1}{2} (1-k) \frac{\delta}{\delta - r} + \frac{1}{2} \frac{\delta}{\delta - r} - V_I(0|y_M; \zeta^*, q^*) = \alpha q^*.$$

A bank run equilibrium requires that withdrawals by the y_L agents alone can destroy a bank, i.e. $\kappa_L < q \left(1 - e^{-\beta\eta}\right)$.³² The threshold \overline{q} so that no run would occur is $\overline{q} \equiv \frac{\kappa_L}{1 - e^{-\beta\eta}}$. If the planner

³²In this two-bank setting we no longer impose the restriction of $q \leq \frac{\kappa_L}{1 - e^{-\beta \eta}}$ as in condition (9), because now only

raises α so that the marginal benefit of acquiring information is below its marginal cost, i.e.,

$$\frac{1}{2}(1-k)\frac{\delta}{\delta-r} + \frac{1}{2}\frac{\delta}{\delta-r} - V_I(0|y_M) \le \alpha \overline{q},$$

then the illiquid bank is always sufficiently liquid to sustain a run.

B Appendix B

We consider the non-stationary part of the model here. If $t_0 < \eta$, then some early informed agents with $t_i < \eta$ knows that $t_0 \in [0, t_i]$, and this truncation implies strategy may be t_i -dependent. However, as shown in Abreu and Brunnermeier (2003), those early agents will be bunching together to eliminate the non-stationarity. We modify their results to our setting.

Focus on the bank being illiquid. To be precise, follow Abreu and Brunnermeier (2003) in our model the agents who hears the rumor before $\zeta - \tau_w = \tau_r$ will behave as if the agent who hears the rumor exactly at τ_r . The strategy of the agent who hears rumor at τ_r is that, independent of signal $(y_L \text{ or } y_M)$ he will withdraw at $\tau_r + \tau_w = \zeta$. Moreover, for agents who hear rumor at $t_i \in [\tau_r, \zeta]$, they take the following strategy. If they receive y_L signal then he will withdraw at ζ , while if they get y_M signal then they withdraw at $t_i + \tau_w$. This additional modification is because relative to Abreu and Brunnermeier (2003) agents may have different signals in our model.

For illustration, suppose that $t_0 = 0$ so the bank should fail at ζ . Recall that there are q measure of y_L signals and 1-q measure of y_M signals. Since the information keeps spreading at ζ (recall that $\eta < \zeta$) and all agents hears the rumor before ζ will withdraw at ζ , there are $q\left(1-e^{-\beta\zeta}\right)$ measure of y_L agents withdrawing. On the other hand, y_M agents who hear the rumor in the interval $[0, \tau_r]$ are withdrawing at ζ , with a total mass of $(1-q)\left(1-e^{-\beta\tau_r}\right)$. Therefore, we have

$$(1-q)\left(1-e^{-\beta\tau_r}\right)+q\left(1-e^{-\beta\zeta}\right)=\kappa_L,$$

which is exactly (21). A similar argument can be applied to the case of $t_0 > 0$ so that the bank failure time is $t_0 + \zeta$: this is because endogenously there are less agents bunching at the physical time ζ , so the bank failure time is postponed to $t_0 + \zeta > \zeta$.

There is one issue that our richer (than Abreu and Brunnermeier (2003)) setting leads to potential non-stationarity. Although withdraw behavior can be stationary, the endogenous learning about bank liquidity is non-stationary when $\eta < \zeta$. In fact, initially when $t_0 = 0$, agents have no other information so $p(t_i|t_i) = p_0$ must holds. In stationary state, $p(t_i|t_i) = \hat{p}_0 \equiv \frac{(e^{\lambda\zeta}-1)p_0}{(1-p_0)(e^{\lambda\eta}-1)+(e^{\lambda\zeta}-1)p_0} < p_0$. This difference potentially alters the optimal withdraw strategies for agents with different timings. To resolve this issue, we simply assume that for $t_i < \eta$, the prior is time-varying

$$p_0(t_i) = \frac{\left(e^{\lambda t_i} - 1\right) p_0}{\left[\left(1 - p_0\right) \left(e^{\lambda \eta} - 1\right) + p_0\left(e^{\lambda t_i} - 1\right)\right]}$$

and one can show that with this specification, the resulting posterior upon hearing the rumor,

 y_L agents are withdrawing to potentially take down the illiquid bank.

 $p(t_i|t_i)$, is always \hat{p}_0 . One can presumably achieve this by a more structural way; for instance, introduce other shocks so that, conditional on survival and hearing the rumor, the posterior of the bank being illiquid is always \hat{p}_0 . Also, we have to fix the signal quality structure the q in (1) is the same as in the stationary phase. We deem these technical issue non-essential for the economic questions that we are after in this paper.