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ABSTRACT

We propose a model consistent with two observations. First, the tax rates adopted by different countries
are generally uncorrelated with their growth performance. Second, countries that drastically reduce
private incentives to invest, severely hurt their growth performance. In our model, the effects of taxation
on growth are highly non-linear. Low or moderate tax rates have a very small impact on long-run growth
rates. But as tax rates rise, their negative impact on growth rises dramatically. The median voter chooses
tax rates that have a small impact on growth prospects, making the relation between tax rates and economic
growth difficult to measure empirically.

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1   Introduction

The 20th century provided two important observations on the determinants of long-run growth. The first observation is that the tax rates adopted by different countries are generally uncorrelated with their growth performance. So, are incentives to invest irrelevant for long-run growth?

The second observation is that countries that drastically reduce private incentives to innovate and invest severely hurt their growth performance. One salient example is the performance of China between 1949, when communists took over and abolished property rights, and the introduction of reforms by Deng Xiaoping in 1979. Another prominent example is the performance of India under the "permit raj" that lasted from 1947 until the reforms introduced by Rajiv Gandhi and Narasimha Rao in 1984 and 1991, respectively. Interestingly, when these countries gradually restored modest incentives to invest, growth rates increased dramatically.[1] Here, incentives to invest seem to matter for growth.

To reconcile these two observations, we propose a model where the effects of taxation on growth are highly non-linear. Low or moderate tax rates have a small impact on long-run growth rates. But, as tax rates and other disincentives to investment rise, their negative impact on growth rises dramatically.

To explain the source of this non-linearity, it is useful to describe the structure of our model. We combine the growth model proposed by Romer (1990) with the Lucas (1978) model of occupational choice. As in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), growth comes from innovation. As in Lucas (1978), the

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[1] Ahluwalia (2002) discusses the gradualist approach to reform followed by India. McMillan, Whalley and Zhu (1989) and McMillan and Naughton (1992) discuss the gradual reforms introduced in China and their impact on productivity. In China and India reforms took place in a environment of political and institutional stability. In contrast, countries from the ex-Soviet block often adopted a big-bang approach to reform that created substantial political and institutional turmoil that was generally associated with poor economic performance. See McMillan and Naughton (1992) for a comparison of the reforms in China and in the ex-Soviet block.

economy is populated by agents who differ in their ability as entrepreneurs/innovators. These agents decide optimally whether to become workers or innovators. Innovators earn profits from their patents and these profits are subject to capital-income taxation.

Motivated by the plethora of evidence on the presence of right skewness in the distribution of patents, scientific-paper citations, income, and profits, we assume that the distribution of entrepreneurial ability is skewed. Because of this skewness, most of the innovation in our economy comes from a small number of highly-productive innovators, the Bill Gates and Steve Jobs of the model.

Increasing the capital-income tax rate reduces incentives to be an entrepreneur and generates exit from the innovation sector. But, since the marginal innovator is much less productive than the average innovator, this exit has a small impact on the growth performance of the economy. In other words, the top entrepreneurs in our model are so productive that they are not deterred from innovating by low-to-moderate tax rates. As a result, there is a range of tax rates that are associated with similar growth outcomes. When taxes and other disincentives to innovate become high, high-quality entrepreneurs exit and the growth engine stalls.

We use our model economy to compute the capital-income tax rate chosen by the median voter and by a benevolent planner. We first show that, even when the median voter is a worker, he chooses a tax rate on capital income on the flat region of the function relating the tax rate to the growth rate. The reason for this choice is that workers benefit from growth in wages that results from the innovation process and understand that high taxes would severely reduce growth prospects.

Our model implies that punitive tax rates are unlikely to be adopted for extended periods of time in well-functioning democracies or, more generally, in political regimes whose decision makers value growth more than preserving the status quo or maintaining political control.[2] The optimal choices made by the median voter in our model generate

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[2]The Swedish experience in the 1970s illustrates the impact of public discourse on tax policy. Astrid Lindgren, a successful writer of children's books, published a satirical story about the adverse incentive

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a *censored* sample where the observed variation in tax rates has a small impact on growth. So, the model is consistent with the absence of a strong correlation between tax rates and growth that we observe in the data. One important implication of the model is that the lack of correlation between taxes and growth is not a global property that holds for all tax rates, but rather an artifact of the endogenous nature of taxation: tax rates that are highly detrimental to the growth process are generally not implemented.

We relate our model to the recent work of Diamond and Saez (2011) who argue that the optimal marginal income tax rate for high-income individuals is 73 percent. We argue that the Diamond-Saez calculation suffers from an important shortcoming: it considers only the static effect of taxation on current tax revenue. Implicitly, this calculation ignores dynamic effects, that is, changes in the growth rate resulting from changes in tax rates. In our model these dynamic effects are small and can be safely ignored when tax rates are low. But it is exactly when tax rates are high, in the range recommended by Diamond and Saez (2011), that these dynamic effects become important.

There are three models consistent with our observation that tax rates are uncorrelated with long-run growth rates: the neoclassical growth model, the Lucas (1988) model, and the Jones (1999) model. In all three models capital-income taxes or other disincentives to investment do not affect the steady-state growth rate. In the neoclassical model, this rate is determined by the pace of exogenous technical progress.[3] In the Lucas (1988) model, the engine of growth is the accumulation of human capital. The costs (foregone wages) and benefits (higher future wages) of this accumulation are affected by income taxes in the same proportion. As a result, the growth rate is

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effects of the punishingly high tax rates levied by Sweden. Her story sparked a public debate that is generally credited with leading to the first electoral defeat of the Social Democratic Party in 40 years and to the reform of the tax code.

[3]In the neoclassical model taxes can affect growth through transition dynamics. However, versions of the neoclassical model in which these dynamics are important tend to imply that the real interest rate takes implausibly high values. See King and Rebelo (1993) for a discussion.

independent of the income tax rate.[4] In the semi-endogenous growth model proposed by Jones (1999), the externalities from the innovation process are not strong enough to make growth sustainable. Sustained growth is feasible only when the population grows, and the steady-state growth rate is proportional to the growth rate of population. So, the only channel for taxes to influence growth is through their effect on the determinants of population growth.

These models have, in our view, two shortcomings. First, they are inconsistent with the observation that modest improvements in the incentives to invest, in economies with high disincentives to invest, can produce large growth effects. Second, they imply that long-run growth rates remain constant even when tax rates approach 100 percent.

The paper is organized as follows. In Section 2, we review and update the evidence on the empirical correlation between tax rates and growth rates. In Section 3, we present our model. In Section 4, we analyze the capital-income tax rates that the median voter and benevolent social planner would choose. We also discuss the implications of our model for the optimal income tax calculations proposed by Diamond and Saez (2011). Section 5 concludes.

2 Empirical evidence on taxation and growth

In this section, we briefly review and update the evidence on the correlation between taxation and growth. We discuss evidence both from cross-country studies and from U.S. time-series analyses.

Cross-country studies Easterly and Rebelo (1993) study a cross section of 125 countries for the period 1970 to 1988. Their main finding is that the correlation between various tax rate measures and growth performance is surprisingly fragile. They show

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[4]Stokey and Rebelo (1993) and Mendoza, Milesi-Ferretti, and Asea (1997) discuss variants of the Lucas (1988) model which, for certain parameter configurations, produce a small impact of taxes on long-run growth. These variants include models in which labor supply is endogenous and physical capital is an input to human capital accumulation.

that, while it is possible to select specifications for which taxes are negatively correlated with growth, this correlation is not robust to the inclusion of other controls or to changes in the sample composition.

Mendoza, Razin, and Tesar (1994) find no correlation between tax rates and growth rates in their study of panel data for 18 OECD countries. Similarly, Piketty, Saez, and Stantcheva (2011) find no correlation between growth rates and the changes in marginal income tax rates that have been implemented in OECD countries since 1975.

U.S. time-series analyses   Stokey and Rebelo (1993) argue that it is hard to detect a negative growth impact of the rise in income tax rates implemented in the U.S. after World War II. Before the Sixteenth Amendment was approved in 1913, the U.S. Constitution severely restricted the ability of the federal government to levy income taxes. Even after the approval of the Sixteenth Amendment, federal income-tax revenue remained low, representing less than 2 percent of Gross Domestic Product (GDP) between 1913 and 1940. Federal income tax revenue as a percentage of GDP rose to 15 percent by the early 1940s. Despite this increase, Stokey and Rebelo (1993) find that the average U.S. growth rate before and after World War II are not statistically different. These results were anticipated by Harberger (1964), who observed that U.S. growth rates have been invariant to changes in the tax structure.[5]

Jones (1995) makes the more general point that changes in policy variables tend to be permanent, but growth rates tend to be stationary. In related work, Easterly, Kremer, Pritchett and Summers (1993) show that persistence across decades is low for growth rates but high for policy variables. This finding suggests caution in attributing high growth rates to good policies, such as low tax rates.

In recent work, Romer and Romer (2010, 2011) use the "narrative record" on the motivation of tax policy changes in the post-war period to identify changes that are

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[5]We cannot, of course, rule out the possibility that, by coincidence, other forces offset exactly the effects of the tax hikes implemented in the post-war period, leaving the growth rate unchanged.

exogenous, in the sense that they are not a response to the growth prospects of the economy. Their paper focuses on the *short-run* effect of taxes on output. They find that, in the post-war period, a tax increase of 1 percent of GDP implies a 3 percent fall in output. Importantly, the authors assume in their empirical work that permanent changes in taxes affect output only temporarily and have no impact on the long-run growth rate of the economy.[6]

Revisiting the correlation between taxation and growth   To revisit and update the empirical work on taxation and growth, we use the method proposed by Mendoza, Razin, and Tesar (1994) to construct measures of tax rates on capital and labor income for OECD countries for the period 1965-2010.

Table 1 reports results for cross-sectional regressions. The dependent variable is the average annual growth rate of real per capita GDP. The independent variables are our labor and capital tax-rate measures. As column 1 shows, neither labor- nor capital-income taxes have a statistically significant impact on growth. Column 2 includes controls commonly used in the growth literature (see e.g. Barro (2012)).[7] Here too, the tax variables are statistically insignificant.

Table 2 shows the results for panel regressions using five-year growth rates and five-year averages of the independent variables. All regressions have time and country fixed effects. Column one shows that the coefficients on both tax variables are statistically significant. However, the sign of the labor income tax rate is positive. Once we include controls (column 2), the coefficient on labor and capital taxes become insignificant.

One concern with these regressions is that changes in tax rates might be driven by the economy's growth prospects. We explore two approaches to address this endogeneity

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[6]See Mertens and Ravn (2012) for additional evidence on the short-run effect of taxation based on Romer and Romer (2010) shocks, as well as a discussion of the related literature.

[7]These variables are: the logarithm of lagged per capita GDP, 1/life expectancy at birth, the logarithm of fertility, the investment share in GDP, male and female school years, the government consumption share in GDP, the openness ratio, and the inflation rate.

problem. The first approach is to instrument for tax rates with the lagged debt-GDP ratio. This approach is motivated by the observation that tax rate increases often result from the need to service the public debt. For example, the tax hikes implemented by many countries after World War II were not a reaction to their growth prospects. Instead, they resulted from the need to service the debt accumulated during the war years. We report the results from using this approach in columns 3-4 of Table 2.[8] In all cases, we find that the tax coefficients are insignificant.

In our second approach, we separate increases in tax rates from decreases in tax rates. The motivation for this approach is as follows. It is easy to find instances in which policy makers reduce tax rates to improve poor growth prospects. But it is much harder to find instances in which policymakers decide to raise taxes because growth prospects are too bright. So, tax increases are likely to be more exogenous to growth prospects than tax decreases. Table 3 reports results for regressions of the change in the growth rate on increases and decreases in tax rates. The first and second column show results without and with controls respectively. In both cases, the coefficients on tax variables are statistically insignificant.

We interpret the weight of the evidence gathered so far and our updated results as suggesting that there is no strong association between the tax rates adopted by different countries and their growth performance. This body of evidence is consistent with the possibility that taxes might have important *level* effects or create large deadweight losses. Higher tax rates might, for example, induce agents to work less, as emphasized by Prescott (2004), or to reallocate effort from market activities towards home production, as emphasized by Sandmo (1990). But the evidence is inconsistent with the implication, shared by many endogenous growth models, that the observed changes in capital and labor taxes have large growth effects.[9]

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[8]Since we have only one instrument, the tax variables are included in the regression one at a time.

[9]See Jones and Manuelli (1990), Barro (1990), Rebelo (1991), and Stokey and Rebelo (1995) for examples of models that share this implication.

3   Model

We consider a Romer (1990)-style model in which growth is driven by innovation that expands the variety of intermediate inputs. Agents choose whether to be workers or entrepreneurs, as in Lucas (1978). We focus our analysis on the effect of capital-income taxes on the growth rate of the economy.

Importantly, we assume that agents have different entrepreneurial ability and that this ability follows a Pareto distribution. As we discuss below, this assumption is consistent with evidence that the right tail of the U.S. income distribution is well described by the Pareto distribution (see e.g. Diamond and Saez (2011)). It is also consistent with the presence of right skewness in the cross-sectional distribution of profits from innovation, returns to entrepreneurship, and research productivity.[10] Our assumption that ability is a key driver of skewness in economic performance is motivated by the work of Huggett, Ventura and Yaron (2011) and Keane and Wolpin (1997) who find that differences in individual ability are a key source of heterogeneity in economic outcomes.[11]

3.1   Production

Final-good producers   Final-good producers operate a constant-returns-to-scale production function that combines labor (L) with a continuum of measure n of intermediate

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[10]Moskowitz and Vissing-Jorgensen (2002) document the presence of skewness in the returns to entrepreneurial activity. Scherer (1998) and Grabowski (2002) show that a small number of firms account for a disproportionate fraction of the profits from innovation. Harhoff, Scherer, and Vopel (1997), Bertran (2003), Hall, Jaffe, and Trajtenberg (2005), and Silverberg and Verspagen (2007) show that the distribution of patent values and patent citations is highly skewed. Hall, Jaffe, and Trajtenberg (2005) show that almost half of all patents receive zero or one citation and less than 0.1 percent of total patents receive more than 100 cites. Lotka (1926) and Cox and Chung (1991) show that the distribution of scientific publications per author is skewed. Redner (1998) finds similar results for the distribution of citations to scientific papers.

[11]Recent papers that consider entrepreneurial ability as a major source of heterogeneity include Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014).

goods (x_i):

$$Y = L^\alpha \int_0^n x_i^{1-\alpha} di.$$

These final-good producers maximize after-tax profits, which are given by:

$$\pi_f = \left(L^\alpha \int_0^n x_i^{1-\alpha} di - \int_0^n p_i x_i di - wL \right) (1 - \tau),$$

where p_i is the price of intermediate good i, w is the wage rate, and τ is the capital-income tax rate. Both p_i and w are denominated in units of the final good. The first-order conditions for this problem are:

$$p_i = (1 - \alpha) L^\alpha x_i^{-\alpha}, \tag{1}$$

$$w = \alpha L^{\alpha-1} n x^{1-\alpha}. \tag{2}$$

The value of π_t^f is zero in equilibrium. For convenience, we normalize the number of final-goods producers to one.

Intermediate good producers/innovators Innovators own permanent patents on the production of intermediate goods. Each unit of the intermediate good, x_i, is produced with η units of the final good. The after-tax profit flow, π_i, generated by each new good is given by:

$$\pi_i = (p_i - \eta) x_i (1 - \tau). \tag{3}$$

Equations (1) and (3) imply that the optimal price and quantity produced by the innovator are:

$$p = \frac{\eta}{1 - \alpha},$$

$$x = L \left[\frac{(1 - \alpha)^2}{\eta} \right]^{1/\alpha}. \tag{4}$$

The maximal after-tax profit per patent is given by:[12]

$$\pi = \alpha(1 - \alpha)^{(2-\alpha)/\alpha} \eta^{-(1-\alpha)/\alpha} L(1 - \tau). \tag{5}$$

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[12]Since all producers choose the same price and quantity, we eliminate the subscript i in what follows.

Equations (2) and (4) imply that the equilibrium wage rate equals:

$$w_t = \alpha n_t \left[\frac{(1-\alpha)^2}{\eta} \right]^{(1-\alpha)/\alpha}. \tag{6}$$

For future reference, we note that this equation implies that the wage rate grows at the same rate as n_t.[13]

3.2   The agent's problem

Agents have identical preferences. The utility of an agent with entrepreneurial ability a, $U(a)$, is given by:

$$U(a) = \int_0^\infty e^{-\rho t} \frac{C_t(a)^{1-\sigma} - 1}{1 - \sigma} dt, \tag{7}$$

where $C_t(a)$ denotes the agent's consumption.

In each period, agents choose whether to work in the final-goods sector or become innovators. Agents who become workers receive the real wage rate w_t. An agent with entrepreneurial ability a who becomes an entrepreneur invents $\delta a n_t$ new goods and obtains a permanent patent on these inventions.

The flow budget constraint of an agent with ability a is:

$$\dot{b}_t(a) = r_t b_t(a) + w_t l_t(a) + m_t(a)\pi_t + \pi_t^f / H - C_t(a) + T_t / H, \tag{8}$$

where $l_t(a) = 1$ if the agent chooses to be a worker in period t and zero, otherwise. The variable $b_t(a)$ denotes the agent's bond holdings. The variable r_t denotes the real interest rate. The variables T_t and H denote the flow of lump-sum transfers from the government and the total size of the population, respectively. The variable $m_t(a)$ denotes the number of patents owned by an agent with entrepreneurial ability a at time

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[13]When we optimize the use of intermediate goods in the production of final goods, we obtain a reduced-form production function that is linear in labor. This result, together with the fact that p is constant, implies that the wage rate does not depend on L. This property greatly simplifies our analysis.

t. To simplify, we assume that individuals with identical ability have the same initial stock of patents. The law of motion for $m_t(a)$ is given by:

$$\dot{m}_t(a) = \delta a n_t \left[1 - l_t(a)\right]. \tag{9}$$

This equation implies that workers receive no new patents, while innovators increase in the stock of patents they hold. Finally, we assume that all agents have zero initial bond holdings:

$$b_0(a) = 0,$$

and face the non-Ponzi scheme condition,

$$\lim_{t \to \infty} e^{-\int_0^t r_s ds} b_t(a) = 0.$$

We solve the agent's problem in two steps. First, we maximize the agent's wealth. Second, we choose the optimal consumption path given the maximal level of wealth. Integrating equation (8), we obtain:

$$\int_0^\infty e^{-\int_0^s r_j dj} \left[w_s l_s(a) + m_s(a) \pi_s + \pi_s^f/H + T_t\right] ds = \int_0^\infty e^{-\int_0^s r_j dj} \left[C_s(a)\right] ds, \tag{10}$$

where the left-hand side is the wealth of the agent and the right-hand side is the present value of the agent's consumption. The wealth maximization problem can then be written as:

$$\max \int_0^\infty e^{-\int_0^s r_j dj} \left[w_s l_s(a) + m_s(a) \pi_s + \pi_s^f/H + T_t\right] ds,$$

subject to equation (9). The Hamiltonian for this problem is:

$$\mathcal{H} = w_t l_t(a) + m_t(a) \pi_t + \pi_t^f/H + T_t + V_t(a)\delta a n_t \left[1 - l_t(a)\right],$$

where $V_t(a)$, the Lagrange multiplier associated with the law of motion for $m_t(a)$, is the value of a patent. We proceed by analyzing the three first-order conditions of the problem.

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Patents accumulation   The first-order condition with respect to $m_t(a)$ is:

$$\dot{V}_t(a) = r_t V_t(a) - \pi_t.$$

Solving this differential equation, we obtain

$$V_t = \int_t^\infty \pi_j e^{-\int_t^j r_s ds} dj, \tag{11}$$

where we omit the subscript a because the value of V_t is the same for all agents. Equation (11) implies that the value of a patent for a new good is the discounted value of the profit flow associated with the patent. We show in the Appendix that this economy has no transitional dynamics, so given a tax rate, τ, the real interest rate and the growth rate are constant over time. Thus, the value of a patent is constant and equals,

$$V = \frac{\pi}{r}.$$

Choosing between being a worker and an entrepreneur   The first-order condition with respect to $l_t(a)$ implies that there is a threshold ability, a^*, that makes agents indifferent between being a worker or an entrepreneur:

$$a^* \delta n_t \frac{\pi}{r} = w_t. \tag{12}$$

Agents with ability greater (lower) than a^* choose to become entrepreneurs (workers).[14] The number of varieties in the economy, n_t, evolves according to:

$$\dot{n}_t = H \delta n_t \int_{a^*}^\infty a \Gamma(da), \tag{13}$$

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[14]There is evidence that income prospects and taxation influence career choices. For example, Philippon and Reshef (2012) show that, when finance was heavily regulated, profits and labor compensation were low. As a result, high-skill individuals did not work in the financial industry. Deregulation led to higher profits and wages, attracting high-skill individuals to finance. Cullen and Gordon (2002) use U.S. individual tax return data to show that differences in tax rates on business versus wage income have large effects on the choice to become an entrepreneur.

where $\Gamma(a)$ denotes the cumulative distribution of a. To interpret this expression, recall that H denotes the size of the population and that an agent with ability a produces δna new varieties.[15]

Consumption Finally, we note that the first-order condition for the consumer problem implies:

$$\frac{\dot{C}_t(a)}{C_t(a)} = \frac{r_t - \rho}{\sigma}. \tag{14}$$

Since all agents face the same real interest rate, their consumption grows at the same rate. We denote this growth rate by g.

3.3 Government

The government rebates tax revenue back to the agents in a lump-sum manner. Profits gross of taxes are: $\left(n_t \pi_t + \pi_t^f\right)/(1-\tau)$. Tax revenue is given by: $\tau\left(n_t \pi_t + \pi_t^f\right)/(1-\tau)$. The budget constraint of the government is:

$$T_t = \tau \frac{n_t \pi_t + \pi_t^f}{1 - \tau}.$$

3.4 Consumption, borrowing and lending across agents

We show in the Appendix that the consumption of workers in period t is given by:

$$C_t(a) = w_t + \frac{\tau}{1-\tau}\frac{\pi}{H}n_t + \frac{r-g}{r}\pi m_0(a)e^{gt}, \quad a < a^*, \tag{15}$$

and the consumption of entrepreneurs in period t is given by:

$$C_t(a) = \pi n_t \left[\frac{r-g}{r}\frac{m_0(a)}{n_0} + \frac{\delta a}{r} + \frac{\tau}{1-\tau}\frac{1}{H}\right], \quad a \geq a^*. \tag{16}$$

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[15]As is common in this class of models, there is an externality in the sense that, the larger the value of n_t, the easier it is to invent new goods. This externality is essential to make sustained growth feasible.

Borrowing and lending The bond holdings in period t of an entrepreneur with ability a are given by (see Appendix for the derivation):

$$b_t(a) = \pi \left[m_0(a) - n_0 \frac{\delta a}{g} \right] \frac{e^{gt} - 1}{r}, \quad a \geq a^*.$$

This equation implies that entrepreneurs whose initial stock of patents is greater (lower) than $n_0 \delta a / g$ are lenders (borrowers). Entrepreneurs with an initial stock of patents equal to $n_0 \delta a / g$ have zero bond holdings. To understand these patterns, it is useful to integrate equation (9):

$$m_t(a) = m_0(a) + \frac{\delta a n_0}{g} (e^{gt} - 1), \quad a \geq a^*.$$

Recall that the entrepreneur's time-t income is: $m_t(a)\pi$, where π is constant (see equation (5)). When $m_0(a) = n_0 \delta a / g$, entrepreneurial income grows at the same rate as consumption, so the entrepreneur does not need to use bond markets to make his consumption stream consistent with his income stream. When $m_0 < n_0 \delta a / g$ entrepreneurial income grows faster than g, and so entrepreneurs borrow against future income. When $m_0 > n_0 \delta a / g$, entrepreneurial income grows slower than g and so entrepreneurs lend in order to consume more than their income in the future.

The bond holdings in period t of a worker are given by:

$$b_t = m_0(a)\pi \frac{(e^{gt} - 1)}{r}, \quad a < a^*.$$

When the initial stock of patents held by the workers is zero, their bond holdings are zero. When this stock is positive, workers' income grows slower than g, so workers lend in order to consume more than their labor income in the future.

The initial distribution of patents has to satisfy the following adding-up condition:

$$n_0 = H \left[\int_{a_{min}}^{a^*} m_0(a)\Gamma(da) + \int_{a^*}^{\infty} m_0(a)\Gamma(da) \right].$$

3.5 Equilibrium conditions

There are three equilibrium conditions for our economy. First, bonds are in zero net supply, so equilibrium in the credit market requires:

$$H \int_{a_{min}}^{\infty} b_t(a)\Gamma(da) = 0.$$

Second, equilibrium in the goods market requires:

$$H \int_{a_{min}}^{\infty} C_t(a)\Gamma(da) + \eta n_t x_t = Y_t.$$

Third, since agents are either workers $(l_t(a) = 1)$ or entrepreneurs $(l_t(a) = 0)$, equilibrium in the labor market requires:

$$H \int_{a_{min}}^{\infty} l_t(a)\Gamma(da) = L.$$

3.6 The evolution of the economy

The fraction of the population that works in the final-good sector is given by:

$$L = H\Gamma(a^*). \tag{17}$$

As we show in the Appendix, the threshold value a^* is implicitly defined by,

$$\delta H(1 - \alpha)a^*\Gamma(a^*)(1 - \tau) = \left[\sigma\delta H \int_{a^*}^{\infty} a\Gamma(da) + \rho\right]. \tag{18}$$

3.6.1 The effects of taxes on the growth rate

In order to study the effects of taxes on growth, we need to specify the distribution of entrepreneurial ability. Our choice is guided by the work of Diamond and Saez (2011) which shows that the right tail of the U.S. income distribution follows a Pareto distribution. We assume that ability is Pareto distributed with shape parameter k and lower bound a_{min}. Since entrepreneurial income is proportional to ability, our model is consistent with the evidence provided by Diamond and Saez (2011).

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Given this distributional assumption, the growth rate of the economy and the equation that determines a^* is given by:

$$g = \delta H \frac{k}{k-1} a_{\min}^k (a^*)^{1-k},$$ (19)

$$(1-\alpha)a^* \left[1 - \left(\frac{a_{\min}}{a^*} \right)^k \right] (1-\tau) = \sigma \left[\frac{k}{k-1} a_{\min}^k (a^*)^{1-k} \right] + \frac{\rho}{\delta H}.$$ (20)

Equation (19) implies that the effect of a tax change on the growth rate of the economy is given by,

$$\frac{dg}{d\tau} = \frac{-(\rho + \sigma \bar{g})}{(1-\tau)} \left[\frac{1}{\sigma + (1-\tau)(1-\alpha)\theta} \right],$$ (21)

where θ is:

$$\theta = \frac{1 + (k-1)(a_{\min}/a^*)^k}{k(a_{\min}/a^*)^k}.$$ (22)

The difference in the impact of taxation on growth on the two economies results from θ. When the cutoff solution is interior (i.e. $a^* > a_{\min}$) the value of θ is greater than one. This property implies that the impact of taxation on growth is always smaller in the heterogenous-ability model than in the homogeneous-ability model.

Heterogeneous-ability versus homogenous-ability economy   In order to isolate the role of heterogeneity in ability in our model, it is useful to compare an economy with heterogeneous ability with one where ability is homogeneous ($a = 1$ for all agents). In the homogeneous-ability economy, the free entry condition into the entrepreneurial sector is,

$$\frac{\delta n_t \pi}{r} = w_t,$$

so all agents are indifferent in equilibrium between being workers and entrepreneurs.

To make the heterogenous- and homogeneous-ability comparable, we assume that the two economies grow at the same rate, $\bar{g}$, have the same structural parameters α

and ρ, and the same tax rate, τ. The effect of τ on g in the economy with homogenous ability is given by:

$$\frac{dg}{d\tau} = \frac{-(\rho + \sigma \bar{g})}{(1 - \tau)} \left[\frac{1}{\sigma + (1 - \tau)(1 - \alpha)} \right]. \tag{23}$$

In the heterogenous-ability economy, this effect is given by:

$$\frac{dg}{d\tau} = \frac{-(\rho + \sigma \bar{g})}{(1 - \tau)} \left[\frac{1}{\sigma + (1 - \tau)(1 - \alpha)\theta} \right], \tag{24}$$

where θ is:

$$\theta = \frac{1 + (k - 1)(a_{\min}/a^*)^k}{k (a_{\min}/a^*)^k}.$$

The difference in the impact of taxation on growth on the two economies results from θ. When the cutoff solution is interior (i.e. $a^* > a_{\min}$) the value of θ is greater than one. This property implies that the impact of taxation on growth is always smaller in the heterogenous-ability model than in the homogeneous-ability model.

As we discuss in the introduction, the intuition for this result is that in the heterogenous-ability economy, the marginal innovator is much less productive than the average innovator. So, the exit of the marginal entrepreneurs in response to a tax rise has a smaller growth effect than in the homogenous-ability economy, where the marginal innovator is as productive as the average innovator.

Equations (23), and (24) show that, in the heterogenous-ability economy, the impact of taxation on growth rises with the tax rate. Since $da^*/d\tau > 0$, θ falls as τ rises, increasing the value of $dg^*/d\tau$. The intuition for this result is when τ is high, a marginal increase in τ leads to the exit of high-ability entrepreneurs which has a large growth effect.

Numerical example We use a numerical example to illustrate the effects of changes in the capital-income tax rate in economies with homogeneous and heterogenous ability. The following parameterization is shared by both economies. We set the labor share in the production of final goods to 60 percent ($\alpha = 0.60$). We assume that $\sigma = 2$

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and we choose $\rho = 0.01$, so that the annual real interest rate in an economy with no growth is one percent. Without loss of generality, we normalize δ and η to one. Finally, in both the homogeneous and heterogenous case, we choose the value of H so that, when $\tau = 0.3$, the growth rate of the economy is 2 percent per year. This value of τ corresponds to the average capital-income tax rate in the U.S., computed as the tax revenue divided by the base. We normalize the value of $a_{\min}$ to one and choose $k = 1.5$. This choice of k implies that the right tail of the income distribution implied by the model is the same as that estimated by Diamond and Saez (2011) for the U.S. economy.

The first panel of Figure 1 shows the effect of changes in τ on the growth rate of the two economies. In the homogenous-ability model, the growth rate of the economy is approximately linear in τ. The growth rate ranges from 2.9 percent, when $\tau = 0$, to zero when $\tau = 0.87$. Doubling the capital-income tax rate from 30 to 60 percent, halves the growth rate from 2 percent to 1 percent. Higher taxes reduce the incentives to innovation, reducing the number of entrepreneurs. Since all agents in the economy are equally good at being entrepreneurs, this reduction has a large impact on the rate of growth.

In the heterogeneous-ability model, the growth rate is a non-linear function of the tax rate. The growth rate ranges from 2.2 percent, when $\tau = 0$, to zero when $\tau = 1$. Doubling the tax rate from 30 to 60 percent reduces the growth rate from 2 to 1.65 percent. This reduction is much smaller than that implied by the homogeneous-ability model (from 2 to 1 percent). The impact of taxation is highly non-linear in the heterogenous-ability economy: increasing τ from 60 to 79 percent reduces the growth rate by as much as doubling τ from 30 to 60 percent.

The second panel of Figure 1 depicts the fraction of entrepreneurs in the population for different values of τ. In the homogeneous-ability model, this fraction ranges from 15 percent, when $\tau = 0$ to zero when $\tau = 0.87$. The strong, negative effect of taxes on the number of entrepreneurs is at the core of the homogenous agents model's large impact of taxation on growth. In contrast, in the heterogeneous ability model the fraction of

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agents who choose to be entrepreneurs ranges from 5.2 percent, when $\tau = 0$, to zero when $\tau = 1$. As τ rises, the number of entrepreneurs declines roughly linearly. But the impact on growth is highly nonlinear, because the ability of the entrepreneurs who exit rises with τ.

4   The Median Voter and the Social Planner

In this section, we analyze the capital-income tax rates that the median voter and a benevolent social planer would choose for our economy. We conclude this section by relating our findings to the recent work of Diamond and Saez (2011) on the optimal income tax rate for high-income individuals.

4.1   Median voter

We analyze an economy in which the median voter is a worker and workers own no patents at time zero. This case provides an upper bound on the tax rate chosen by the median voter for two reasons. First, if workers held some patents, they would have less of an incentives to tax capital income. Second, if the median worker was an entrepreneur, he would choose a lower tax rate than a worker, since he receives capital income.[16]

A worker who is the median voter faces the following tradeoff. On the one hand, higher capital-income taxes result in higher tax revenue and higher lump-sum transfers, which benefit workers. On the other hand, higher taxes lead to lower growth in wages, which hurts workers.

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[16]A median voter who is an entrepreneur might choose a positive capital-income tax rate in order to redistribute income from high-ability to low-ability entrepreneurs.

Computing life-time utility   Since the consumption of both workers and entrepreneurs grow at a constant rate, we can re-write lifetime utility, defined in equation (7), as:

$$U = \frac{C_0^{1-\sigma}}{1-\sigma} \int_0^{\infty} \exp\left\{[-\rho + (1-\sigma)g]\,t\right\} dt.$$

Assuming that $(1-\sigma)g < \rho$, so that lifetime utility is finite, the value of U is given by:

$$U = \frac{C_0^{1-\sigma}}{\rho(1-\sigma) - (1-\sigma)^2 g}. \tag{25}$$

Equations (15) and (16) imply that the time-zero consumption of workers is given by:

$$C_0 = w_0 + \frac{\tau}{1-\tau}\frac{\pi}{H}n_0 + \frac{r-g}{r}\pi m_0(a), \quad a < a^*, \tag{26}$$

The tradeoff faced by the median voter is clear from equations (25) and (26). A higher tax rate benefits workers since it increases the lump-sum transfer they receive from the government (the term $[\tau/(1-\tau)]\,(\pi/H)\,n_0$ in equation (26)). At the same time, a higher tax rate reduces the growth rate and lowers utility by reducing the denominator in equation (25).

Median voter's preferred tax rate   The tax rate that maximizes the utility of the median voter for the parameters considered in Section 3 is 35 percent. We find this result interesting for two reasons. First, this tax rate is very close to the average capital-income tax rate, computed as the tax revenue divided by the base, which is roughly 30 percent. Second, the tax rate chosen by the median voter suggests that democracies are unlikely to choose tax rates that are very detrimental to growth prospects. When τ is equal to 35 percent, the economy grows at an annual rate of 1.95 percent. Reducing the capital-income tax rate to zero would increase the growth rate only to 2.25 percent.

The fact that the median voter avoids implementing tax rates that lead to poor growth prospects has important implications for the samples used to study the empirical relation between taxation and growth. These samples are censored in the sense that they generally don't contain the full range of values of τ. Instead, they contain values

of τ on the flat region of the function relating the tax rate to the growth rate, making the relation between taxation and growth hard to detect.

4.2 Social Planner

We now consider a social planner who chooses the tax rate to maximize a weighted average of the utility of workers and entrepreneurs:

$$\max_\tau \bar{U} = \int_{a_{\min}}^\infty \int_0^\infty \left\{ e^{-\rho t} \frac{[C_t(a)]^{1-\sigma} - 1}{1 - \sigma} dt \right\} \Gamma(da).$$

Recall that consumption of both workers and entrepreneurs grows at rate g. The time-zero consumption of workers is given by equation (26). The time-zero consumption of entrepreneurs is given by:

$$C_0 = \pi n_0 \left(\frac{r - g}{r} \frac{m_0(a)}{n_0} + \frac{\delta a}{r} + \frac{\tau}{1 - \tau} \frac{1}{H} \right), \quad a \geq a^*. \tag{27}$$

For the parameters considered in Section 3, the value of τ that maximizes $\bar{U}$ is 31 percent.[17] Since the planner places a positive weight on the welfare of the entrepreneurs, he always chooses a lower tax rate than the worker. The tax rate selected by the social planner is remarkably close to the U.S. capital-income tax rate.

4.3 Relation to the modern public finance literature

The tax rates chosen by the median voter and the social planner in our model are much lower than the optimal tax rates advocated in the modern public finance literature. For example, Diamond and Saez (2011) argue that the optimal income tax rate for high-income individuals is 73 percent. This high optimal tax rate results from the fact

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[17]We choose the initial distribution of patents so that the social planner has a strong redistribution motive. As in the case of the median voter, we assume that workers do not own any initial patents. In addition, we distribute initial patents equally among the top 1 percent of entrepreneurs ranked by ability. The tax rate chosen by the social planner is basically identical if the initial patents are distributed equally among the top 2 or 3 percent of entrepreneurs ranked by ability.

that high-income individuals have a low elasticity of taxable income with respect to the tax rate.

The Diamond-Saez calculation suffers from an important shortcoming. It considers only the static effect of taxation on current tax revenue. It ignores dynamic effects by implicitly assuming that the growth rate of the economy is invariant with respect to the tax rate. In our model, these dynamic effects are indeed small and can be safely ignored when tax rates are low. But it is exactly when tax rates become high, in the range recommended by Diamond and Saez (2011), that these dynamic effects become significant.[18]

To see this problem, suppose the government adopts very high tax rates. The static effects of this action is positive for the workers: their wage remains the same and they see a rise in the lump-sum transfers they receive from the government. But the dynamic effects can be disastrous for worker's welfare, since higher taxes lead to a fall in the growth rate of wages. We can use our model to illustrate how misleading the focus on the static effects of taxation can be. Total pre-tax income in the economy is given by:

$$w_t H \Gamma(a^*) + n_t \frac{\pi_t}{1-\tau}.$$

An increase in τ leads to a rise in pre-tax income. To see this result, note that the derivative of pre-tax income with respect to τ is given by:

$$w_t H \Gamma'(a^*) \frac{da^*}{d\tau} + n_t \frac{d\left[\pi_t/(1-\tau)\right]}{d\tau} > 0. \qquad (28)$$

This expression reflects the fact that the wage rate is solely a function of n_t (see equation (2)). Since n_t is fixed in the short run, w_t is not affected by an increase in τ.

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[18]Scheuer (2014) discusses another shortcoming of the Diamond-Saez calculation: it implicitly assumes that tax-driven changes in the labor supply of high-income individuals do not affect the productivity of other agents in the economy. Scheuer (2014) illustrates this issue in a model in which the labor supply of entrepreneurs and workers are complements, so reductions in the entrepreneurial labor supply lead to a decline in the productivity of workers. Our model embodies a natural source of complementarity between entrepreneurs and workers.

As the tax rate rises, the ability threshold also rises ($da^*/d\tau > 0$), and some entrepreneurs become workers. So, the first term in equation (28) is positive. The second term is also positive because a rise in the number of workers leads to a rise in pre-tax profits (see equation (5)).

Since the short-run response of pre-tax income to the tax rate is positive, the tax rate that maximizes short-run tax revenue is 100 percent! A planner who considers only the static effects of taxation would choose a high tax rate in order to redistribute income from the entrepreneurs to the workers in order to equalize their marginal utility. But this choice would be misguided since it ignores the impact of taxation on growth which is a key driver of welfare.

5   Conclusion

In this paper, we propose a model in which the effects of taxation on growth are highly non-linear. Taxes have a small impact on long-run growth when taxes rates and other disincentives to investment are low or moderate. But, as tax rates rise, the marginal impact of taxation on growth also rises. This non-linearity is generated by heterogeneity in entrepreneurial ability. In a low-tax economy, the ability of the marginal entrepreneur is relatively low. So, increasing the tax rate leads to the exit of low-ability entrepreneurs and to a small decline in the growth rate. In a high-tax economy the ability of the marginal entrepreneur is relatively high. So, increasing the tax rate leads to the exit of high-ability entrepreneurs and in a large decline in the growth rate.

Table 1: Cross-section regressions

Dependent variable: growth rate of real, per capita GDP

Regression	1	2
Labor income tax	0.80 (1.55)	0.44 (2.30)
Capital income tax	-1.90 (2.93)	3.44 (3.74)
Controls	no	yes
R^2	0.04	0.56
Number of observations	17	17

Table 2: Panel regressions
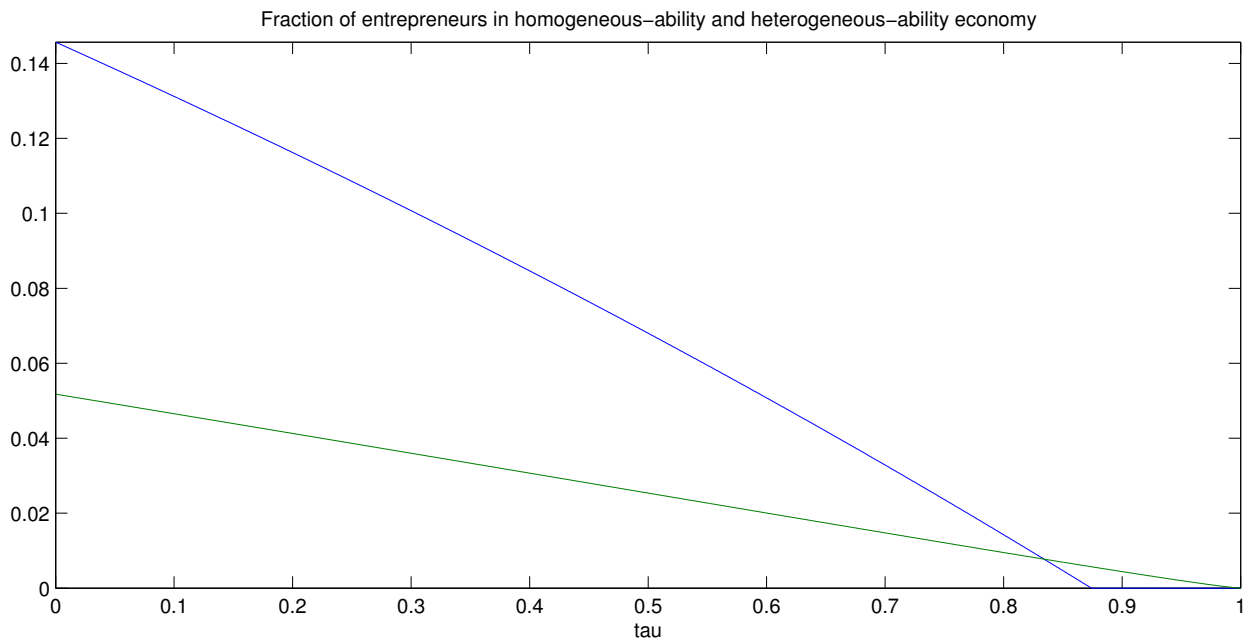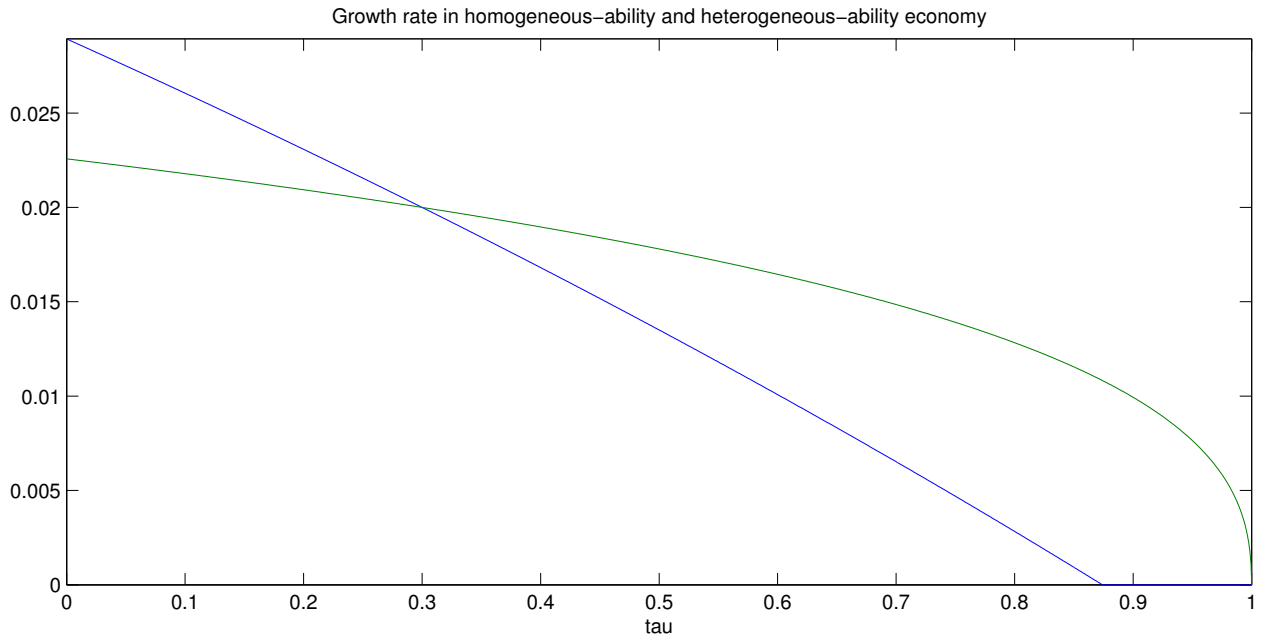
Dependent variable: growth rate of real, per capita GDP

Regression	1	2	3	4
Labor income tax	0.15 (0.06)	−0.05 (0.09)	−0.26 (0.45)	
Capital income tax	−0.09 (0.03)	−0.06 (0.03)		−0.14 (0.25)
Controls	no	yes	yes	yes
R^2	0.72	0.82	0.04	0.13
Fixed effects	yes	yes	yes	yes
Time effects	yes	yes	yes	yes
Instrument	no	no	lagged $\frac{Debt}{GDP}$	lagged $\frac{Debt}{GDP}$
Number of observations	78	71	70	70

Table 3: Panel regressions

Dependent variable: change in the growth rate of real, per capita GDP

Regression	1	2
Increase in labor income tax	0.12 (0.24)	0.18 (0.27)
Increase in capital income tax	−0.08 (0.08)	−0.12 (0.09)
Decrease in labor income tax	0.08 (0.26)	0.23 (0.23)
Decrease in capital income tax	−0.25 (0.20)	−0.19 (0.24)
Controls	no	yes
R^2	0.69	0.78
Fixed effects	yes	yes
Time effects	yes	yes
Number of observations	63	59

Figure 1

Growth rate in homogeneous−ability and heterogeneous−ability economy

Fraction of entrepreneurs in homogeneous−ability and heterogeneous−ability economy

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7 Appendix: Not for Publication

7.1 Transitional dynamics

7.1.1 Homogeneous-ability model

We first show that the homogenous-ability model has no transitional dynamics.

$$\pi = \alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L(1-\tau). \tag{29}$$

Equations (2) and (4) imply that the equilibrium wage rate is given by:

$$w_t = \alpha n_t \left[\frac{(1-\alpha)^2}{\eta}\right]^{(1-\alpha)/\alpha}. \tag{30}$$

Recall that the value of a patent for a new good is :

$$V_t = \int_t^\infty \pi_j e^{-\int_t^j r_s ds} dj. \tag{31}$$

When there is an interior solution for the number of entrepreneurs, we have:

$$\delta n_t V_t = w_t. \tag{32}$$

Using equation (31) to compute $\dot{V}_t$, we obtain:

$$\dot{V}_t = \frac{d}{dt}\int_t^\infty \pi_j e^{-\int_t^j r_s ds} dj.$$

Define:

$$f(t,j) = e^{-\int_t^j r_s ds}\pi_j.$$

Using Leibnitz's rule we obtain:

$$\dot{V}_t = \frac{d}{dt}\int_t^\infty f(t,j)dj = -\pi_t + r_t V_t. \tag{33}$$

Equation (30) implies that w_t/n_t is constant. This property, together with equation (32), implies that the value of the firm, V_t is also constant. Given that $\dot{V}_t = 0$, equation (33) implies:

$$V_t = \frac{\pi_t}{r_t}. \tag{34}$$

Replacing π_t and r_t in equation (34) for V_t,

$$V_t = \frac{\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L_t(1-\tau)}{\sigma\delta(H-L_t)+\rho}.$$

Rearranging,

$$\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L_t(1-\tau) = V\left[\sigma\delta(H-L_t)+\rho\right].$$

Differentiating with respect to time, using the fact that $\dot{V}_t = 0$, we have

$$\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}\dot{L}_t(1-\tau) = -\dot{L}_t V\sigma\delta.$$

This equation implies that $\dot{L}_t = 0$, so L_t is constant. This property implies that π_t is constant (equation (29)). Equation (34) implies that r_t is constant. In sum, the model has no transition dynamics.

7.1.2 Heterogenous-ability model

The free entry condition in the heterogenous-ability model is:

$$n\delta a_t^* V_t = w_t.$$

Since w_t/n_t is constant we have:

$$\frac{\dot{a}_t^*}{a_t^*} + \frac{\dot{V}_t}{V_t} = 0.$$

Conjecture that $\dot{a}^* = 0$, so $a_t^* = a^*$. The free entry condition is:

$$V_t = \frac{w_t}{n\delta a^*}.$$

We can now proceed as in the homogenous agent model and show that the real interest rate and the growth rate are constant and that all equations are satisfied, so that a constant value of a_t^* is indeed a solution.

7.2 Consumption and bond holdings

Worker's consumption Consider an agent who is a worker, so $l_t(a) = 1$. The worker's lifetime budget constraint is given by:

$$\int_0^\infty e^{-rt} \left(w_0 e^{gt} + m_0(a)\pi + T_t/H \right) dt = \int_0^\infty e^{-rt} \left[C_0(a) \right) e^{gt} \right] dt. \tag{35}$$

Recall that $\pi_t^f = 0$ and π_t is constant. The lump-sum transfers received by the worker are:

$$\frac{T_t}{H} = \frac{\tau}{1-\tau}\frac{\pi}{H}n_0 e^{gt}.$$

Integrating equation (35), we obtain the following equation for consumption at time zero:

$$C_0(a) = w_0 + \frac{\tau}{1-\tau}\frac{\pi}{H}n_0 + \frac{r-g}{r}\pi m_0(a), \, a < a^*.$$

Since consumption grows at rate g, consumption at time t is given by:

$$C_t(a) = w_t + \frac{\tau}{1-\tau}\frac{\pi}{H}n_t + \frac{r-g}{r}\pi m_0(a) e^{gt}, \, a < a^*.$$

Entrepreneur's consumption Recall that the law of motion for the patents of an entrepreneur with ability a is:

$$\dot{m}_t(a) = \delta a n_t = \delta a n_0 e^{gt}, \, a \geqslant a^*.$$

Integrating this equation, we obtain:

$$m_t(a) = m_0(a) + \frac{\delta a n_0}{g} \left(e^{gt} - 1 \right), \, a \geqslant a^*. \tag{36}$$

The growth rate of $m_t(a)$ is given by:

$$\frac{\dot{m}_t(a)}{m_t(a)} = \frac{g}{1 + e^{-gt} \left[g m_0(a)/(\delta a n_0) - 1 \right]}, \, a \geqslant a^*.$$

This entrepreneur's life-time budget constraint is given by

$$\int_0^\infty e^{-rt} \left(m_t(a)\pi + T_t/H \right) dt = \int_0^\infty e^{-rt} \left(C_0^e(a) \right) e^{gt} \right) dt, \, a \geqslant a^*. \tag{37}$$

Using equations (36) and (37), we obtain:

$$C_0(a) = \pi \left[\left(\frac{r-g}{r} \right) m_0(a) + \frac{\delta a n_0}{r} + \frac{1}{H} \frac{\tau n_0}{1-\tau} \right], \, a \geqslant a^*.$$

Since consumption grows at rate g, consumption at time t is given by:

$$C_t(a) = \pi n_t \left[\left(\frac{r-g}{r} \right) \frac{m_0(a)}{n_0} + \frac{\delta a}{r} + \frac{1}{H} \frac{\tau}{1-\tau} \right], \, a \geqslant a^* \tag{38}$$

$$\dot{b}_t(a) = \pi \left[m_0(a) + \frac{\delta a}{g}(n_t - n_0) \right] - \pi \left[m_0(a)e^{gt} + \frac{\delta a n_t}{r} \right], \, a \geqslant a^*.$$

Borrowing and lending between agents   The evolution of bond holdings for an entrepreneur of ability a given by:

$$\dot{b}_t^e(a) = r b_t^e(a) + m_t^e(a)\pi - C_t^e(a) + T_t/H.$$

Using equation (36) and the expression for consumption we can rewrite the evolution of the bond holding as:

$$\dot{b}_t^e(a) = r b_t^e(a) + \pi \left[m_0^e(a) - n_0 \frac{\delta a}{g} \right] + \pi \left[n_0 \delta a \left(\frac{1}{g} - \frac{1}{r} \right) - m_0^e(a) \frac{r-g}{r} \right] e^{gt}.$$

Integrating this equation, we obtain:

$$b_t^e(a) = \pi \left[m_0^e(a) - n_0 \frac{\delta a}{g} \right] \frac{e^{gt} - 1}{r}.$$

With the appropriate modification of the problem, we can also obtain the bond holding of a worker (recall that the stock of patents held by workers is constant over time).

$$b_t(a) = m_0(a)\pi \frac{(e^{gt} - 1)}{r}, \, a < a^*.$$

7.3   Derivation of the threshold value for entrepreneurs

Equation (18) implicitly defines the threshold value a^*. To derive it recall that this threshold value is given by:

$$a^* \delta n_t \frac{\pi}{r} = w_t.$$

Replacing π using equation (5) and w using equation (6) and using the fact that the measure of workers is given by

$$L = H\Gamma(a^*),$$

we can then rewrite the threshold value equation as:

$$\delta a^*(1 - \alpha)H\Gamma(a^*)(1 - \tau) = r.$$

Substituting r from equation (14), we obtain:

$$\delta a^*(1 - \alpha)H\Gamma(a^*)(1 - \tau) = \sigma g + \rho. \tag{39}$$

To solve for a^* we note that the growth rate of the economy is equal to the growth rate of the number of varieties,

$$g = \delta H \int_{a^*}^{\infty} a\Gamma(da). \tag{40}$$

Using equation (40) to replace g in equation (39), we obtain the implicit equation (18) for a^*:

$$\delta H(1 - \alpha)a^*\Gamma(a^*)(1 - \tau) = \left[\sigma\delta H \int_{a^*}^{\infty} a\Gamma(da) + \rho\right].$$