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ABSTRACT

We study a model in which the effects of taxation on growth are highly non-linear. Marginal increases in tax rates have a small growth impact when tax rates are low or moderate. When tax rates are high, further tax hikes have a large, negative impact on growth performance. We argue that this non-linearity is consistent with the empirical evidence on the effect of taxation and other disincentives to investment and innovation on economic growth.

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1. Introduction

The 20th century provided two important observations on the determinants of long-run growth. The first observation, which we discuss in Section 2, is that tax rates are not generally correlated with long-run growth rates. So, are incentives to invest irrelevant for long-run growth?

The second observation is that countries that drastically reduce private incentives to innovate and invest severely hurt their growth performance. One salient example is the performance of China between 1949, when communists took over and abolished property rights, and the introduction of reforms by Deng Xiaoping in 1979. Another prominent example is the performance of India under the "permit raj" that lasted from 1947 until the reforms introduced by Rajiv Gandhi and Narasimha Rao in 1984 and 1991, respectively. Interestingly, when these countries gradually restored modest incentives to invest, growth rates increased dramatically.[1] Here, incentives to invest seem to matter for growth!

There are two standard models consistent with our first observation: the neoclassical growth model and the Lucas (1988) model. Income taxes or other disincentives to investment do not affect the steady-state growth rate in both models. In the neoclassical model, this rate is determined by the pace of exogenous technical progress.[2] In the Lucas (1988) model, the engine of growth is the accumulation

[1] Ahluwalia (2002) discusses the gradualist approach to reform followed by India. McMillan, Whalley and Zhu (1989) and McMillan and Naughton (1992) discuss the gradual reforms introduced in China and their impact on productivity. In China and India reforms took place in a environment of political and institutional stability. In contrast, countries from the ex-Soviet block generally adopted a big-bang approach to reform that created substantial political and institutional turmoil that was generally associated with poor economic performance. See McMillan and Naughton (1992) for a comparison of the reforms in China and in the ex-soviet block.

[2] In the neoclassical model taxes can affect growth through transition dynamics. However, versions of the neoclassical model in which these dynamics are important tend to imply that the real interest rate takes implausibly high values. See King and Rebelo (1993) for a discussion.

of human capital. The costs (foregone wages) and benefits (higher future wages) of this accumulation are affected by income taxes in the same proportion. As a consequence, the growth rate is independent of the rate of income or investment tax.[3]

These models have, in our view, two shortcomings. First, they imply that long-run growth rates remain constant even when income tax rates approach 100 percent. We can dramatize this implication by noting that, ceteris paribus, these models imply that North and South Korea should have the same long-run growth rate. Second, these models are inconsistent with the observation that modest improvements in the incentives to invest, in economies with high disincentives to investment, can produce large growth effects.

In this paper we propose a simple model that reconciles our two observations. In our model the effects of taxation on growth are highly non-linear. Taxation has a very small impact on long-run growth rates when tax rates are low or moderate. This property can create the impression that tax rates can be raised without affecting long-term economic performance. But, as tax rates and other disincentives to investment become large, their negative impact on growth rises dramatically.

To explain the source of this non-linearity, it is useful to describe the structure of our model. We combine the growth model proposed by Romer (1990) with the Lucas (1978) model of occupational choice. As in Romer (1990), Grossman and Elhanan (1991), and Aghion and Howitt (1992), growth comes from innovation. As in Lucas (1978), the economy is populated by agents with different ability as entrepreneurs/innovators. These agents decide optimally whether to become

[3]Stokey and Rebelo (1993) and Mendoza, Milesi-Ferretti, and Asea (1997) discuss variants of the Lucas (1988) model which, for certain parameter configurations, produce a small impact of taxes on long-run growth. These variants include models in which labor supply is endogenous and physical capital is an input to human capital accumulation.

workers or innovators. Motivated by the plethora of evidence on the presence of right skewness in the distribution of patents, scientific paper citations, income, and profits, we assume that the distribution of entrepreneurial ability is skewed. Because of this skewness, most of the innovation in our economy stems from a small number of highly-productive innovators. These entrepreneurs, the Bill Gates and Steve Jobs of our model economy, are unlikely to be deterred from innovating, even when tax rates are moderately high.

Increases in taxes do affect innovators who are at the margin and can lead to substantial exit from the innovation sector. But, since the marginal innovator is much less productive than the average innovator, this exit has a small impact on the growth performance of the economy. As a result, there is a range of tax rates that are associated with similar long-run growth rates. Once taxes and other disincentives to innovate are high, the entrepreneurs that drive most of the innovation in the economy no longer invest and the growth engine stalls.

Our benchmark model abstracts from the possibility that entrepreneurs might migrate to other economies. In Section 7 we introduce the possibility of "brain drain." Agents can migrate to other countries by paying a cost that is independent of their ability. So, when taxes rise, high-ability agents migrate, producing a large decline in the growth rate of the economy. The possibility of brain drain exacerbates the non-linear response of growth to taxation.

Throughout our analysis we consider models in which agents know their entrepreneurial ability. As a robustness check, we consider, in Section 8, a model in which agents have to become entrepreneurs to learn their entrepreneurial ability. In this model high taxes might have a large impact on growth by deterring agents from trying to become entrepreneurs, and learn their ability. We show that this model also exhibits a non-linear response of growth to taxation. When taxes rates are low, it is optimal for all potential entrepreneurs to try their luck and learn

their ability. So, there is a range of tax rates that is associated with the same rate of growth. The absence of a correlation between taxation and growth might lead policy makers to believe that further tax hikes have no growth impact. However, once tax rates exceed a threshold level, they have a high impact on long-run growth by reducing the number of agents who attempt to become entrepreneurs.

Our paper is organized as follows. In Section 2, we review briefly the evidence on the relation between taxation and long-run growth. In Section 3, we study the impact of taxation on growth in an endogenous growth model in which all agents have the same entrepreneurial ability. In Section 4, we consider a model in which entrepreneurial ability follows a Pareto distribution. We compare the implications of the two models for the effects of corporate income taxes (Section 5) and progressive personal income taxes (Section 6). In Section 7, we extend our model to incorporate the possibility of "brain drain," i.e. the migration of high skilled workers in response to high taxes or burdensome regulation. In Section 8, we consider the case of stochastic entrepreneurial activity. We offer some conclusions in Section 9.

2. Empirical evidence on taxation and growth

Evidence on the correlation between taxation and growth comes from a variety of sources. Easterly and Rebelo (1993a) study a cross section of 125 countries for the period 1970 to 1988. They find that the association between various tax rate measures and growth performance is surprisingly fragile. It is possible to select specifications for which taxes are negatively correlated with growth. But this correlation is not robust to the inclusion of other controls or to changes in the sample composition.

Piketty, Saez, and Stantcheva (2011) find no correlation between growth rates and the large changes in marginal income tax rates that have been implemented

in OECD countries since 1975. Similarly, Mendoza, Razin, and Tesar (1994) find no correlation between tax rates and growth rates in their study of panel data for 18 OECD countries.

Stokey and Rebelo (1993) argue that it is hard to detect a negative growth impact of the very large rise in income tax rates implemented in the U.S. after World War II. Before the Sixteenth Amendment was approved in 1913, the U.S. Constitution severely restricted the ability of the federal government to levy taxes on income. Even after approval of the amendment, income tax revenue was lower than 2 percent of output. This fraction increased dramatically in the early 1940s to 15 percent. Yet, Stokey and Rebelo (1993) cannot reject the hypothesis that the average annual U.S. per capita growth rate is the same before and after World War II. These results were anticipated by Harberger (1964), who observed that U.S. growth rates have been invariant to large changes in the tax structure. We cannot, of course, rule out the possibility that, by coincidence, other forces offset exactly the effects of the large tax increase implemented in the post-war period, leaving the growth rate unchanged.

Jones (1995) makes the more general point that changes in policy variables tend to be permanent, but growth rates tend to be stationary. Once again, it is possible that, by coincidence, all the movements in variables that can affect growth rates have been offsetting. But a more plausible interpretation of Jones's results is that permanent changes in policy have no impact on long-run growth rates.

Easterly, Kremer, Pritchett and Summers (1993) show that the persistence across decades is low for growth rates but high for policy variables. This finding suggests caution in attributing high growth rates to good policies, such as low tax rates.

Romer and Romer (2010) use the narrative record on the motivation of tax

policy changes in the post-war period to identify changes that are exogenous, in the sense that they are not a response to the growth performance of the economy. Their paper focuses on the *short-run* effect of taxes on output. They find that a tax increase of one percent of GDP implies a three percent fall in output. The authors assume in their empirical work that permanent changes in taxes affect output only temporarily and have no impact on the long-run growth rate of the economy.[4]

Romer and Romer (2011) use a similar method to study the effect of taxes on output in the U.S. during the inter-war period. They find that, despite the large changes in marginal tax rates during this period, these changes had no short-run impact on the performance of the U.S. economy.

It is always possible that better methods for measuring tax rates and taking endogeneity into account will reveal the strong connection between taxes and long-run growth that has, until now, eluded researchers. We interpret the weight of the evidence gathered so far as suggesting that there is no strong association between taxes rates and long-run growth outcomes. This body of evidence is consistent with the possibility that taxes might have important *level* effects or create large deadweight losses. High tax rates might, for example, induce agents to work less, as emphasized by Prescott (2004), or to reallocate effort from market activities towards home production, as emphasized by Sandmo (1990). But the evidence is inconsistent with the implication, shared by many endogenous growth models, that changes in income and investment taxes have large, permanent growth effects.[5]

[4] See Mertens and Ravn (2012) for additional evidence on the short-run effect of taxation based on Romer and Romer (2012) shocks, as well as a discussion of the related literature.

[5] See Jones and Manuelli (1990), Barro (1990), Rebelo (1991), and Stokey and Rebelo (1995) for examples of models that share this implication.

3. Homogeneous-ability model

Our starting point is a model where growth is driven by innovation. This innovation expands the variety of goods available as intermediate inputs, as in Romer (1990). Agents decide whether to be workers or entrepreneurs, as in Lucas (1978).

In this section we focus our analysis on the effect of the corporate income tax. We discuss the effects of a progressive personal income tax in Section 6. Throughout, we omit time subscripts whenever this omission results in no loss of clarity.

Final-good producers The final-good producers operate a constant-returns-to-scale production function that combines labor (L) with a continuum of measure n of intermediate goods (x_i):

$$Y = L^{\alpha} \int_0^n x_i^{1-\alpha} di.$$

The objective of the final-good producer is to maximize after-tax profits, which are given by:

$$\pi_f = \left(L^{\alpha} \int_0^n x_i^{1-\alpha} di - \int_0^n p_i x_i di - wL \right) (1-\tau),$$

where p_i is the price of intermediate good i, w is the wage rate, and τ is the corporate income tax rate. Both p_i and w are denominated in units of the final good. The first-order conditions for this problem are:

$$p_i = (1-\alpha) L^{\alpha} x_i^{-\alpha}, \tag{3.1}$$

$$w = \alpha L^{\alpha-1} n x^{1-\alpha}. \tag{3.2}$$

The value of π_t^f is equal to zero in equilibrium. For convenience, we normalize the number of final-goods producers to one.

Intermediate good producers/innovators Each agent in the economy chooses whether to work in the final-goods sectors or become an innovator. Agents who choose the former, receive the wage rate w. Agents who choose the latter, invent δn new goods and obtain a permanent patent on these inventions.[6] The number of varieties in the economy, n, evolves according to:

$$\frac{\dot{n}}{n} = \delta \left(H - L \right), \tag{3.3}$$

where H is the size of the population.

Each unit of the intermediate good, x_i, requires an input of η units of the final good. The after-tax profit flow generated by each new good, π_i, is given by:

$$\pi_i = \left(p_i - \eta \right) x_i (1 - \tau). \tag{3.4}$$

Equations (3.1) and (3.4) imply that the optimal price and quantity produced by the innovator are:

$$\begin{aligned} p &= \frac{\eta}{1-\alpha}, \\ x &= L \left[\frac{(1-\alpha)^2}{\eta} \right]^{1/\alpha}. \end{aligned} \tag{3.5}$$

Since all producers make the same price and quantity decision, we eliminate the subscript i. The maximal after-tax profit per patent is given by:

$$\pi = \alpha (1-\alpha)^{(2-\alpha)/\alpha} \eta^{-(1-\alpha)/\alpha} L (1-\tau). \tag{3.6}$$

Equations (3.2) and (3.5) imply that the equilibrium wage rate is equal to:

$$w = \alpha n \left[\frac{(1-\alpha)^2}{\eta} \right]^{(1-\alpha)/\alpha}. \tag{3.7}$$

[6] As is common in this class of models, there is an externality in the sense that, the larger the value of n, the easier it is to invent new goods. This externality is essential to be feasible for the economy to grow at a constant rate.

This equation implies that the wage rate grows at the same rate as n.[7] For future reference, we note that the ratio of tax revenue to GDP is:

$$\frac{\tau n\widetilde{\pi}}{wL + n\widetilde{\pi}} = \frac{\tau}{1 + (1-\alpha)^{-1}}. \tag{3.8}$$

where $\widetilde{\pi}$ denotes the pre-tax profits, $\widetilde{\pi} = \pi/(1-\tau)$.

The agent's problem Agents have identical preferences. The utility of agent i, U_i, is given by:

$$U_i = \int_0^\infty e^{-\rho t} \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} dt,$$

where C_t^i denotes the consumption of agent i. We assume, without loss of generality, that agents own an equal share of the final-goods firm. The budget constraint of agent i is:

$$\dot{b}_t^i = r_t b_t^i + w_t l_t^i + m_t^i \pi_t + \pi_t^f / H - C_t^i + T_t, \tag{3.9}$$

where $l_t^i = 1$ if agent i chooses to be a worker in period t and zero, otherwise. The variable b_t^i denotes the agent's bond holdings. The variable r_t and T_t denote the real interest rate and the flow of lump-sum transfers from the government, respectively.

The variable m_t^i denotes the number of patents owned by agent i at time t. The law of motion for m_t^i is given by:

$$\dot{m}_t^i = \delta n_t (1 - l_t^i).$$

This equation implies that agents who choose to be workers have a constant number of patents in their portfolio. Agents who become innovators see an instantaneous increase in the number of patents they hold.

[7] When we optimize the use of intermediate goods in the production of final goods, we obtain a reduced-form production function that is linear in labor. This result, together with the fact that p is constant, implies that the wage rate does not depend on L. This property greatly simplifies our analysis.

The non-Ponzi game condition,

$$\lim_{t\to\infty} \int_0^t e^{-\int_0^s r_j dj} b_s^i ds = 0,$$

completes the description of the problem.

The first-order condition for the consumer problem implies that:

$$\frac{\dot{C}_t^i}{C_t^i} = \frac{r_t - \rho}{\sigma}. \tag{3.10}$$

Since all agents face the same real interest rate, they choose the same growth rate of consumption. We denote this growth rate by g.

We assume that, at time zero, each of the H agents in the economy has an identical share of the existing patents and zero bond holdings:

$$\begin{aligned} m_0^i &= n_0/H, \\ b_0^i &= 0, \end{aligned}$$

for all i. As we discuss below, this assumption ensures that the path of consumption is the same for all agents.

Solving the agent's problem We solve the agent's problem in two steps. The first step is to maximize the agent's wealth. The second step is to choose the optimal consumption path given the maximal level of wealth.

Integrating equation (3.9), we obtain:

$$\int_0^\infty e^{-\int_0^s r_j dj} \left(w_s l_s^i + m_s^i \pi_s + \pi_s^f / H \right) ds = \int_0^\infty e^{-\int_0^s r_j dj} \left(C_s^i \right) ds, \tag{3.11}$$

where the left-hand side is the wealth of agent i and the right-hand side is the present value of this agent's consumption.

The wealth maximization problem can be written as:

$$\max \int_0^\infty e^{-\int_0^s r_j dj} \left(w_s l_s^i + m_s^i \pi_s + \pi_s^f / H \right) ds \,,$$

subject to:

$$\dot{m}_t^i = \delta n_t (1 - l_t^i).$$

The Hamiltonian for this problem is:

$$\mathcal{H} = \left(w_t l_t^i + m_t^i \pi_t + \pi_t^f / H \right) + V_t^i \delta n_t (1 - l_t^i),$$

where V_t^i denotes the Lagrange multiplier associated with the law of motion for m_t^i. The first-order condition with respect to m_t^i is:

$$\dot{V}_t^i = r_t V_t^i - \pi_t.$$

Solving this differential equation, we obtain:

$$V_t = \int_0^\infty \pi_t e^{-\int_0^s r_j dj} dt, \tag{3.12}$$

where we omit the subscript i because the value of V_t is identical across agents. Equation (3.12) implies that the value of a patent for a new good is the discounted value of the profit flow.

The first-order condition with respect to l_t^i is:

$$\begin{array}{lll} n\delta V_t < w_t, & & l_t^i = 1, \\ n\delta V_t = w_t, & l_t^i \in \{0, 1\}, & \\ n\delta V_t > w_t, & & l_t^i = 0 \end{array} \tag{3.13}$$

The maximal value of wealth is identical across agents. Since the growth rate of consumption is also identical, all agents have the same consumption path.

Government The government rebates taxes back to the agents in a lump-sum manner. Since profits are expressed net of taxes, the budget constraint of the government is:

$$\tau \frac{n_t \pi_t + \pi_t^f}{1 - \tau} = T_t.$$

Equilibrium conditions Bonds are in zero net supply, so equilibrium in the credit market requires:

$$\int_0^H a_t^i di = 0.$$

Recall that the path for consumption is the same for all agents. Since workers and entrepreneurs have different income paths, there can be borrowing and lending across agents in equilibrium.

Equilibrium in the goods market implies:

$$\int_0^H C_t^i di + \eta n_t x_t = Y_t. \tag{3.14}$$

Equilibrium in the labor market implies that agents are either workers ($l_t^i = 1$) or entrepreneurs ($l_t^i = 0$):

$$\int_0^H l_t^i di + \int_0^H (1 - l_t^i) di = H.$$

3.1. The fraction of entrepreneurs in the economy

Using the first-order condition from the household problem, we obtain the following condition for the value of L:

$$\begin{array}{ll} n\delta V_t < w_t, & L = H, \\ n\delta V_t = w_t, & L < H. \end{array} \tag{3.15}$$

When $n\delta V_t < w_t$, the rewards to innovating are lower than the opportunity cost, so there is no innovation. In this case, all agents work in the production sector, $L = H$ and the number of goods in the economy remains constant.

When $n\delta V_t = w_t$, there is an interior solution for the number of agents who decide to innovate ($H - L$). The value of L is always strictly positive, otherwise there is no production of final goods and the value of innovating is zero.

To derive the value of L when the solution is interior, we first note that this economy has no transition dynamics, so the real interest rate and the rate of

growth are constant over time (see proof in Appendix). Using this result, we can write equation (3.15) as:

$$\delta n \frac{\pi}{r} = w. \tag{3.16}$$

Replacing π and w using equations (3.6) and (3.7), we obtain:

$$L = \frac{r}{\delta(1-\alpha)(1-\tau)}. \tag{3.17}$$

Since the real interest rate and the growth rate of the economy are constant, equation (3.10) implies that,

$$r = \sigma g + \rho. \tag{3.18}$$

Using this result, we can rewrite equation (3.17) as:

$$L = \frac{\sigma g + \rho}{\delta(1-\alpha)(1-\tau)}. \tag{3.19}$$

In a symmetric equilibrium, output is given by:

$$Y = L^{\alpha} n x^{1-\alpha}. \tag{3.20}$$

Equation (3.19) implies that L is constant. Equation (3.5) implies that x is also constant. These two properties, together with equation (3.20), imply that output grows at the same rate as the number of varieties. Equation (3.14) implies that consumption also grows at the same rate as n.

Deriving the growth rate of n Equation (3.3) and the fact that consumption, output and the number of varieties grow at the same rate imply:

$$g = \delta \left(H - L \right). \tag{3.21}$$

Combining this result with equation (3.19), we obtain the following expression for the growth rate of the economy:

$$g = \frac{\delta H(1-\alpha)(1-\tau) - \rho}{(1-\alpha)(1-\tau) + \sigma}. \tag{3.22}$$

Equation (3.22) implies that the measure of agents who work in the final production sector is given by:

$$L = \frac{\sigma H \delta + \rho}{\delta \left[(1-\alpha)(1-\tau) + \sigma \right]}.$$

The impact of taxes Equation (3.22) implies that, when the solution for L is interior, the marginal impact of taxes on growth is negative and given by:

$$\frac{dg}{d\tau} = -\frac{(1-\alpha)\left(\rho + \delta H \sigma\right)}{\left[(1-\alpha)(1-\tau) + \sigma\right]^2} < 0. \tag{3.23}$$

There is a corner solution for L ($L = H$) whenever:

$$\tau \geq 1 - \frac{\rho}{\delta H (1-\alpha)}. \tag{3.24}$$

The growth rate of the economy is zero for values of τ that satisfy equation (3.24). We return to this result in Section 5 when we compare these predictions for the effects of taxation on growth with those of a model where entrepreneurs have an heterogeneous ability.

4. Heterogenous-ability model

A large literature shows that firm size and executive compensation are skewed to the right and follow, approximately, a Pareto distribution.[8] This skewness is also present in variables related to innovation and entrepreneurship. Moskowitz and Vissing-Jørgensen (2002) document the presence of skewness in the returns to entrepreneurial activity. Scherer (1998) and Grabowski (2002) show that a small number of firms account for a disproportionate fraction of the profits from

[8] Simon and Bonini (1958), Ijiri and Simon (1964, 1971, 1974), Axtel (2001), Luttmer (2007) and Gabaix and Landier (2008) provide evidence on the distribution of firm size. Roberts (1956), Baker, Jensen, and Murphy (1988), Barro and Barro (1990), and Rosen (1992) provide evidence on the distribution of executive compensation.

innovation. Harhoff, Scherer, and Vopel (1997), Bertran (2003), Hall, Jaffe, and Trajtenberg (2005), and Silverberg and Verspagen (2007) show that the distribution of patent values and patent citations is highly skewed. Hall, Jaffe, and Trajtenberg (2005) show that almost half of all patents receive zero or one citation and less than 0.1 percent of total patents receive more than 100 cites.[9]

There is also evidence of skewness in the productivity of scientists. Lotka (1926) and Cox and Chung (1991) show that the distribution of scientific publications per author is skewed. Redner (1998) finds similar results for the distribution of citations to scientific papers. Azoulay, Zivin and Wang (2010) show that the premature death of an academic "superstar" has a sizable, permanent negative impact on the productivity of the superstar's co-authors.

One important question is: what is the source of skewness in economic performance? Huggett, Ventura and Yaron (2011) and Keane and Wolpin (1997) find that differences in individual ability are a key driver of heterogeneity in economic outcomes. Graham, Li and Qui (2012) find that ability is a key driver of executive compensation. This body of evidence suggests that there is substantial heterogeneity in ability or productivity. In this sections we incorporate this type of heterogeneity in the model of Section 3.[10]

We assume that ability, a, follows a continuous distribution with cumulative distribution function, $\Gamma(a)$. To simplify, we suppose that all agents are equally productive as workers but differ in their entrepreneurial ability. An agent with ability a can produce $\delta n a$ new goods per period.

As in section 3, this economy has no transitional dynamics, so the real interest

[9] These authors also show that citations are a good proxy for the value of a patent. The citation-weighted stocks of patents have a higher correlation with the market value of the patents than the unweighted stocks of patents.

[10] Recent papers that consider entrepreneurial ability as a major source of heterogeneity include Buera, Kaboski, and Shin (2012), and Midrigan and Xu (2010). Kortum (2007) and Jones (2007) consider models in which new ideas are productivity levels that follow a Pareto distribution.

rate is constant (see Appendix). The flow profit per patent, π, that accrues to the innovator and the value of an additional patent are the same as in the previous section. Following the same steps used in Section 3, we obtain:

$$\delta n a^* \frac{\pi}{r} = w, \tag{4.1}$$

where a^* is the ability of the marginal innovator who is indifferent between being an innovator and a worker.

The fraction of the population that works in the final-production sector is then:

$$L = H\Gamma(a^*). \tag{4.2}$$

Using equations (3.6), (3.7), (4.1), and (4.2), we obtain:

$$\delta a^*(1-\alpha)H\Gamma(a^*)(1-\tau) = r.$$

Substituting r from equation (3.10), we obtain:

$$\delta a^*(1-\alpha)H\Gamma(a^*)(1-\tau) = \sigma g + \rho. \tag{4.3}$$

To solve for a^* we first note that, as in Section 3, the growth rate of the economy is equal to the growth rate of the number of varieties,

$$g = \delta H \int_{a^*}^{a_{\max}} a\Gamma(da). \tag{4.4}$$

To interpret this expression, recall that all agents with ability greater than a^* become entrepreneurs. There is a mass of agents with ability a which is equal to $\Gamma(da)$. Each of these agents produces $\delta n a$ varieties.

Using equation (4.4) to replace g in equation (4.3), we obtain the following implicit equation for a^*:

$$(1-\alpha)a^*\Gamma(a^*)(1-\tau) = \left[\sigma \int_{a^*}^{a_{\max}} a\Gamma(da) + \rho/(\delta H)\right]. \tag{4.5}$$

We assume that a follows a truncated Pareto distribution with shape parameter equal to one, lower bound $a_{\min}$, and upper bound $a_{\max}$. We choose this distribution for three reasons. First, it allows us to obtain some additional analytical results. Second, this distribution is a good description of the empirical distribution of variables such as firm size (Luttmer (2010)). Third, as we discuss below, it allows us to choose $a_{\max}$ so that our numerical example is consistent with the distribution of employment across firms in the U.S.[11].

Under these assumptions, we can rewrite equation (4.5), which determines the threshold value, a^*, as:

$$a^*(1-\tau) + \frac{\sigma}{(1-\alpha)} a_{\min} \log(a^*/a_{\max}) = (1-\tau)a_{\min} + \frac{\rho\left(1 - a_{\min}/a_{\max}\right)}{(1-\alpha)\delta H}. \quad (4.6)$$

The growth rate of the economy, implied by equation (4.4), is given by:

$$g = \frac{\delta H}{1 - (a_{\min}/a_{\max})} \left\{(a_{\min}) \log(a_{\max}/a^*)\right\}. \quad (4.7)$$

Using equation (4.6) we can write $a_{\min} \log(a_{\max}/a^*)$ as:

$$\frac{1}{\sigma}\left\{(a^* - a_{\min})(1-\tau)(1-\alpha) - \frac{\rho\left(1 - a_{\min}/a_{\max}\right)}{\delta H}\right\} = a_{\min} \log(a_{\max}/a^*).$$

Using this expression and equation (4.7) we obtain:

$$g = \frac{1}{\sigma}\left[\frac{\delta H\left(a^* - a_{\min}\right)(1-\tau)(1-\alpha)}{1 - (a_{\min}/a_{\max})} - \rho\right]. \quad (4.8)$$

Differentiating equation (4.8) with respect to τ we obtain:

$$\frac{dg}{d\tau} = \frac{1}{\sigma} \frac{(1-\alpha)\,\delta H}{1 - (a_{\min}/a_{\max})} \left[\frac{da^*}{d\tau}(1-\tau) - (a^* - a_{\min})\right].$$

[11] We studied numerically the case in which the shape parameter of the Pareto distribution is bigger than one. There is no discontinuity in the effects of taxes on growth as the parameter approaches one.

The effect of a change in τ on the ability of the marginal innovator, $da^*/d\tau$, is given by:

$$\frac{da^*}{d\tau} = \frac{a^* - a_{\min}}{(1-\tau) + \sigma a_{\min}/[(1-\alpha)a^*]}.$$

The effect of a marginal increase in τ on the growth rate of the economy is negative and given by:

$$\frac{dg}{d\tau} = -\frac{\delta H(1-\alpha)\left(a^* - a_{\min}\right)}{1-(a_{\min}/a_{\max})}\left[\frac{a_{\min}}{(1-\alpha)\left(1-\tau\right)a^* + \sigma a_{\min}}\right] < 0. \tag{4.9}$$

5. Homogeneous versus heterogenous ability

In what follows we compare the effects of changes in τ in economies with homogenous and heterogenous entrepreneurial ability. It is useful to consider two economies that are growing at the same rate, g^*, have the same structural parameters α and ρ, and the same corporate tax rate, τ.

We begin by deriving the effects of τ on g in the economy of homogenous entrepreneurial ability. It is useful to rewrite equation (3.22) as:

$$\left[g^* - \delta H\right](1-\alpha)(1-\tau) = -\left(\rho + \sigma g^*\right). \tag{5.1}$$

Totally differentiating this equation we obtain:

$$\frac{dg^*}{d\tau} = \frac{\left[g^* - \delta H\right](1-\alpha)}{\{(1-\alpha)(1-\tau)+\sigma\}}.$$

Using equation (5.1) we obtain:

$$\frac{dg^*}{d\tau} = \frac{-\left(\rho + \sigma g^*\right)}{(1-\tau)}\frac{1}{\{(1-\alpha)(1-\tau)+\sigma\}}.$$

Consider the economy with heterogenous entrepreneurial ability. We can rewrite equation (4.8) as:

$$\frac{\sigma g^* + \rho}{(1-\tau)} = \left[\frac{\delta H\left(a^* - a_{\min}\right)(1-\alpha)}{1-(a_{\min}/a_{\max})}\right].$$

Using equation (4.9):

$$\frac{dg^*}{d\tau} = \frac{-\left(\rho + \sigma g^*\right)}{(1-\tau)} \left[\frac{1}{(1-\alpha)\,(1-\tau)\,(a^*/a_{\min}) + \sigma} \right].$$

Since, in the absence of changes in τ, the two economies grow at the same rate and share the same parameters, the difference in the slope comes from the term in square brackets, which is given by

$$\begin{aligned} \text{Heterogenous model} \quad &: \quad \frac{1}{(1-\tau)(1-\alpha)a^*/a_{\min} + 1} \\ \text{Homogenous model} \quad &: \quad \frac{1}{(1-\alpha)(1-\tau) + 1} \end{aligned}$$

Note that $a^*/a_{\min} > 1$ as long as $a_{\max} > a_{\min}$. So, the slope in the heterogenous agents model is smaller in absolute value implying that the effect of an increase in τ on g is always smaller in the economy with heterogeneous ability.

5.1. Numerical example

We use a numerical example to compare the effects of changes in the corporate income tax rate in economies with homogeneous and heterogenous agents. The following parameterization is shared by both economies. We set the labor share in the production of final goods to 60 percent ($\alpha = 0.60$). We assume that $\sigma = 1$ (log preferences). We choose $\rho = 0.01$, so that the annual real interest rate in an economy with no growth is one percent. Without loss of generality, we normalize δ and η to one. Finally, in both the homogeneous and heterogenous case, we choose the value of H so that, when $\tau = 0.35$, the growth rate of the economy is 2 percent per year. This value of τ corresponds to the U.S. Federal corporate income tax rate.

The distribution of ability in the economy with heterogeneity is governed by the two parameters of the Pareto distribution: $a_{\min}$ and $a_{\max}$. Without loss of generality we set $a_{\min} = 1$. To choose $a_{\max}$ we build on Luttmer's (2010) finding

that the largest 1,000 U.S. firms in terms of employment account for roughly 25 percent of total employment. Since there are roughly 6 million employer firms in the U.S., these firms represent a mere 0.017 percent of U.S. firms.[12] In what follows we show how we can map this statistic into our model, using the fact that firm size is proportional to the ability of the entrepreneur. This property enables us to calibrate $a_{\max}$ to match the Luttmer's firm size statistic.

We proceed as follows. Suppose that firms are vertically integrated, so that research firms hire workers to produce the final output goods. This assumption generates a non-trivial distribution of employment. We also assume that the ownership of the initial stock of patents is distributed among entrepreneurs in proportion to their ability:

$$s(a) = \frac{a}{H \int_{a^*}^{a_{\max}} a\Gamma(da)}, \tag{5.2}$$

where $s(a)$ is the initial share of patents attributed to an entrepreneur of ability a. We assume that entrepreneurs take the initial distribution of patents as given, so this distribution does not affect the choice of being a worker or an entrepreneur. The number of patents held by each entrepreneur grows at rate g and the share of patents held by this agent remains constant over time.[13] Recall that all innovators produce the same quantity of intermediate goods, x, per patent and that the amount of labor employed in producing a given good is proportional to x. Under these conditions, firm size is proportional to the ability of the entrepreneur.

We choose $a_{\max}$ so that the top 0.017 percent entrepreneurs account for 25 percent of employment. We use an iterative process to find this value of $a_{\max}$. For

[12] Luttmer's (2010) figure 3 shows that this statistic is stable over time.

[13] To see this property, suppose that the agent enters period t with $s(a)$ shares. The instantaneous growth rate in the number of patents held by this agent, m_t, is given by: $\dot{m}_t/m_t = n_t \delta a/m_t$. Using equation (5.2), together with the fact that $m_t = s(a)n_t$, we obtain $\dot{m}_t/m_t = \delta H \int_{a^*}^{a_{\max}} a\Gamma(da) = g$. Since m_t grows at the same as n_t, the share of patents remains constant over time.

a given $a_{\max}$ we compute a^* and find $\bar{a}$, which denotes the the lower bound of the interval that contains the top 0.017 entrepreneurs.

$$\frac{\int_{\bar{a}}^{a_{\max}} \Gamma(da)}{\int_{a^*}^{a_{\max}} \Gamma(da)} = 0.017. \tag{5.3}$$

Using the Pareto distribution, this equation can be written as:

$$\bar{a} = \frac{a_{\min}}{0.017\left[(a_{\min}/a^*) - (a_{\min}/a_{\max})\right] + (a_{\min}/a_{\max})}$$

Since employment is proportional to ability, the requirement that the top 0.017 percent of entrepreneurs account for 25 percent of employment can be written as:

$$\frac{\int_{\bar{a}}^{a_{\max}} a\Gamma(da)}{\int_{a^*}^{a_{\max}} a\Gamma(da)} = 0.25. \tag{5.5}$$

Using the Pareto distribution, we can write this equation as:

$$\frac{\log(a_{\max}/\bar{a})}{\log(a_{\max}/a^*)} = 0.25.$$

We iterate on $a_{\max}$ until both equations (5.3) and (5.5) hold; this convergence occurs for a value of $a_{\max} = 5000$.

The first panel of Figure 1 shows the effect of changes in τ on the growth rate of the two economies. In the homogenous ability model the growth rate of the economy is roughly linear in τ.[14] The growth rate ranges from 3.15 percent, when $\tau = 0$, to zero when $\tau = 0.816$. Doubling the corporate income tax rate from 35 to 70 percent, reduces the growth rate from 2 percent to 0.56 percent. Higher taxes reduce the incentives to innovation, reducing the number of entrepreneurs. Since all agents in the economy are equally good at being entrepreneurs, this reduction has a large impact on the rate of innovation and growth. As we discuss in Section 2, it is difficult to empirically find this large impact of taxation on growth.

[14]It is possible to generate a non-linear response of the growth rate to τ in the homogeneous ability model. But it requires using values of ρ and σ that are close to zero.

The heterogeneous ability model exhibits a non-linear response of growth to taxation. The growth rates ranges from 2.15 percent, when $\tau = 0$, to zero when $\tau = 1$. Doubling the tax rate from 35 to 70 percent reduces the growth rate from 2 percent to 1.73 percent. This reduction is much smaller than that implied by the model with homogeneous agents. This result might suggest to policy makers that taxes have no impact on the tax rate. But tax effects are highly non-linear: increasing taxes from 70 to 85 percent reduces the growth rate by as much as doubling the tax rate from 35 to 70 percent.

The second panel of Figure 1 depicts the fraction of entrepreneurs in the population for different values of τ. In the homogeneous ability model, this fraction ranges from 23.3 percent, when $\tau = 0$ to zero when $\tau = 0.816$. The strong, negative effect of taxes on the number of entrepreneurs is at the core of the model's large impact of taxation on growth. In contrast, in the heterogeneous ability model the fraction of agents who choose to be entrepreneurs ranges from 4.8 percent, when $\tau = 0$, to zero when $\tau = 1$. As τ rises the number of entrepreneurs declines roughly linearly. But the impact on growth is highly nonlinear, because the ability of the entrepreneurs that exit rises with τ.

Finally, the third panel of Figure 1 displays tax revenues as a percentage of total output. This variable is given by equation (3.8) for both the homogeneous ability and the heterogeneous ability model. Suppose that the government wants to obtain a ratio of taxes to GDP of 25 percent. In both economies this objective would require a tax rate of 87.5 percent. This tax rate would reduce growth to zero in the homogeneous ability economy. In contrast, the heterogenous ability economy still grows at 1.43 percent.

To summarize, in this section we considered numerical versions of the homogeneous and heterogeneous ability models. In the model with heterogenous ability the effects of corporate income taxes on growth are highly non-linear. These ef-

fects are small when tax rates are low or moderate and are high once tax rates are high.

This non-linearity would be even stronger if we introduced the production complementarities emphasized by Kremer (1993) and Gabaix and Landier (2008). In models with production complementarities, it is optimal to implement assortative matching. In Kremer's (1993) model it is optimal to form groups of agents with similar abilities. In Gabaix and Landier (2008) it is optimal to match the best managers with the most productive firms. In both cases, the skewness of the distribution of productivity of profits is a magnified version of the skewness in the distribution of ability.

6. Progressive personal income taxes

In this section we compare the effects of progressive personal income taxes in our homogeneous and heterogeneous-agent models. Our motivation is three fold. First, income tax systems are generally progressive, both in developed and developing countries (see Easterly and Rebelo (1993b)). Second, changes in income taxes often take the form of changes in the degree of progressivity (see Piketty, Saez, and Stantcheva (2011)). Third, a progressive income tax can introduce important distortions in the choice to be an entrepreneur, since entrepreneurs generally forego income in the present in return for higher future income.

To simplify, we assume that agents choose at time zero whether to be workers or entrepreneurs. Moreover, we assume that the initial number of patents in the economy is distributed equally across all agents:

$$m_0 = n_0/H.$$

This assumption implies that workers and entrepreneurs have different income paths, even though the present value of their incomes is the same. Absent differ-

ences in income paths, progressive taxation would have no effects.

The gross income of the worker has two components. The first component is the wage rate, which grows at rate g. The second component is constant, and corresponds to the profit from the initial stock of patents, $m_0\pi$. The total gross income of the worker grows at a rate that is lower than g. The present value of the after tax income of a worker is: $w_0/(r-g) + m_0\pi/r$.

We consider a simple income tax system where agents pay a rate ω_1 for income levels lower or equal than the total income received by a worker at time t ($w_t + m_0\pi$) and a marginal tax rate $\omega_1 + \omega_2$ for income exceeding that amount. The total income tax paid by an agent with income y_t is given by:

$$\Omega(y_t; w_t) = \begin{cases} \omega_1 y_t, & \text{for } y_t \leq w_t + m_0\pi, \\ \omega_1 y_t + \omega_2 \left[y_t - (w_t + m_0\pi) \right], & \text{for } y_t > w_t + m_0\pi \,. \end{cases}$$

To isolate the effects of personal income taxes we assume that corporate income taxes are zero ($\tau = 0$).

The gross income of the entrepreneur is given by $m_t\pi$. At time zero, the income of the entrepreneur is lower than that of the worker, since the worker receives $w_t + m_0\pi$, while the entrepreneur receives only $m_0\pi$.

Homogeneous agent model Recall that the law of motion for m_t is given by:

$$\dot{m}_t = \delta n_t = \delta n_0 e^{gt}.$$

Integrating this equation we obtain:

$$m_t = m_0 + \frac{\delta n_0}{g} \left(e^{gt} - 1 \right). \tag{6.1}$$

The entrepreneur's income, $m_t\pi$, grows at rate:

$$\frac{\dot{m}_t}{m_t} = \frac{g}{1 + e^{-gt} \left[g/\left(\delta H \right) - 1 \right]}$$

Let's assume that H is large enough that $g/(\delta H) < 1$. In this case, the income of the entrepreneurs grows faster than g.

We denote by t^* the moment at which the income of the worker and the entrepreneur are equalized:

$$w_0 e^{gt^*} + m_0\pi = m_{t^*}\pi. \tag{6.2}$$

Until t^* both agents pay the same marginal income tax rate. Using equation (6.1) to replace m_{t^*} in equation (6.2) and solving for t^*, we obtain:

$$t^* = \frac{1}{g}\ln\left[\frac{\pi\delta}{\pi\delta - gw_0/n_0}\right]. \tag{6.3}$$

Since there is free-entry entry into the entrepreneurial activity, agents must be indifferent at time zero between being workers and entrepreneurs:

$$\begin{aligned}(1-\omega_1)\left(\frac{w_0}{r-g}+\frac{m_0\pi}{r}\right) &= (1-\omega_1)\int_0^\infty \left(\pi m_t e^{-rt}\right)dt \\ &\quad -\omega_2\int_{t^*}^\infty \left(\pi m_t - w_t - \pi m_0\right)e^{-rt}dt.\end{aligned} \tag{6.4}$$

The left-hand side of this expression is the present value of a worker's after-tax income. The right-hand side is the present value of an entrepreneurs's after-tax income.

Using equation (6.1) to replace m_t in equation (6.4) we obtain:

$$\begin{aligned}(1-\omega_1)\frac{w_0/n_0}{r-g} &= (1-\omega_1)\frac{\delta\pi}{g}\left(\frac{1}{r-g}-\frac{1}{r}\right) \\ &\quad -\omega_2\left[\pi\frac{\delta}{g}\frac{e^{(g-r)t^*}}{r-g} - \pi\frac{\delta}{g}\frac{e^{-rt^*}}{r} - \frac{w_0/n_0}{r-g}e^{(g-r)t^*}\right]\end{aligned} \tag{6.5}$$

Recall that w_t/n_t is a constant (see equation (3.7)) and that π is a function of L (see equation (3.6)).

We can characterize the equilibrium of this economy by solving a system of six equations to six unknowns. The equations are: (3.7), (3.6), (6.3), (6.5), (3.21),

and (3.18). Equation (3.21) expresses g as a function of L, while equation (3.18) expresses r as a function of g. The unknowns are g, r, L, t^*, w_0/n_0, and π.

Introducing heterogeneity The key difference between the homogeneous-agent model and the heterogeneous agent-model is that in the latter the law of motion for the number of patents held by an entrepreneur depends on his or her ability. The number of patents held by an entrepreneur with ability a, m_t^a, evolve according to:

$$\dot{m}_t^a = a\delta n_t = a\delta n_0 e^{gt}.$$

Integrating this equation we obtain:

$$m_t = m_0 + a\frac{\delta n_0}{g}\left(e^{gt} - 1\right). \tag{6.6}$$

In order to determine the ability of the marginal entrepreneur (a^*) we need to compute the time t^* at which the incomes of the marginal entrepreneur and the worker are equalized:

$$w_0 e^{gt^*} + m_0\pi = m_{t^*}^{a^*}\pi.$$

Replacing $m_{t^*}^{a^*}$ using equation (6.6) and solving for t^* we obtain:

$$t^* = \frac{1}{g}\ln\left[\frac{\pi a\delta}{\pi a\delta - g w_0/n_0}\right]. \tag{6.7}$$

The marginal entrepreneur is indifferent at time zero between being a worker or an entrepreneur:

$$\begin{aligned}(1-\omega_1)\left(\frac{w_0}{r-g} + \frac{m_0\pi}{r}\right) &= (1-\omega_1)\int_0^\infty \left(\pi m_t^{a^*} e^{-rt}\right) dt \\ &\quad -\omega_2 \int_{t^*}^\infty \left(\pi m_t^{a^*} - w_t - \pi m_0\right) e^{-rt} dt.\end{aligned}$$

Replacing $m_{t^*}^{a^*}$ using equation (6.6) we obtain:

$$(1-\omega_1)\frac{w_0/n_0}{r-g} = (1-\omega_1)\,a^*\frac{\delta\pi}{g}\left(\frac{1}{r-g}-\frac{1}{r}\right) - \omega_2\left[\pi a^*\frac{\delta}{g}\frac{e^{(g-r)t^*}}{r-g} - \pi a^*\frac{\delta}{g}\frac{e^{-rt^*}}{r} - \frac{w_0/n_0}{r-g}e^{(g-r)t^*}\right] \tag{6.8}$$

We can characterize the equilibrium by solving a system of seven equations and seven unknowns. The equations are: (3.7), (3.6), (4.2), (6.7), (6.8), (4.4), and (3.18). The unknowns are g, r, L, t^*, a^*, w_0/n_0, and π.

A numerical example We use a simple numerical example to illustrate the effects of changes in the degree of progressivity in heterogenous-ability economies. We use the same parameters as in the previous section, except for the corporate income tax rate, which we set to zero. We choose $\omega_1 = 0.15$ and $\omega_2 = 0.20$. These parameter values imply that entrepreneurs represent 5 percent of the population. Our choice of ω_1 is consistent with the estimates reported in Piketty and Saez (2007). These authors find that the average federal income tax paid by the 95 percent of the population with the lowest income was approximately 15 percent in 2004. Our choice of ω_2 was motivated by the fact that the highest marginal income tax rate in the U.S. in 2012 is 35 percent, which corresponds to $\omega_1 + \omega_2$.

Figure 2 depicts the response of the growth rate to ω_2 in the homogenous and heterogenous-ability models. This response shares the two features of the effects of corporate income taxes that we emphasize in section 5. First, the impact of ω_2 on growth is more nonlinear in the heterogenous-ability model than in the homogeneous-ability model. Second, the heterogenous-ability model features a range of ω_2 for which growth is relatively constant. Increases in ω_2 beyond that range have a very large impact on the growth rate of the economy.

7. Brain drain

A potentially important effect of high taxes rates or burdensome regulation is the migration of high-skill individuals, a phenomenon often referred to as "brain drain."[15] We explore this phenomenon in this section.

Consider a small open economy that can borrow and lend at a constant real interest rate, r. The rest of the world has a stock of patents $\hat{n}_t$ that grows at a constant rate, $\hat{g}$. To simplify, we assume that there is no trade between the small open economy and the rest of the world.

An agent in the small open economy can migrate to the rest of the world and work or innovate there. For simplicity, we assume that this outside option can be summarized as follows. An agent with entrepreneurial ability a who migrates, receives a flow income in the rest of the world equal to $\hat{x}_t a$, where $\hat{x}_t$ grows at rate $\hat{g}$. To migrate, the agent pays a cost of $\theta \hat{x}_t$ per period. Since this cost is proportional to $\hat{x}_t$, it grows at rate $\hat{g}$.

The problem of an entrepreneur in the home country is to maximize:

$$\max_{l_t^i} \int_t^\infty e^{-r(s-t)} \left[\hat{x}_t a l_t^i - \theta x_t l_t^i + m_s^i \pi_s + \pi_s^f / H \right] ds,$$

subject to:

$$\dot{m}_t^i = \delta n_t a (1 - l_t^i),$$

where l_t^i is an indicator function that takes the value one when the agent migrates and zero otherwise. The stock of domestic patents owned by an agent who migrates remains constant over time. The Hamiltonian for the entrepreneur's problem is:

$$\mathcal{H} = \hat{x}_t a l_t^i - \theta x_t l_t^i + m_t^i \pi_t + \pi_t^f / H + V_t^i \delta n_t a (1 - l_t^i).$$

[15] See Commander, Kangasniemi, and Winters (2004) and Beine, Docquier, and Rapoport (2008) for a summary of the literature on the brain drain phenomenon. Kleven, Landais, and Saez (2010) provide evidence on the impact of migration incentives for European soccer players following the 1995 liberalization of the soccer labor market.

The first-order condition for l_t^i is:

$$\hat{x}_t a - \theta \hat{x}_t = \frac{\pi}{r} a \delta n_t.$$

This condition implies that the cutoff for ability above which an entrepreneur migrates is:

$$\hat{a}_t = \frac{\theta}{1 - \pi\delta \left(n_t / \hat{x}_t\right) / r}. \tag{7.1}$$

The cutoff level of ability above which it is optimal to be an entrepreneur, a^*, is given by:

$$\delta n a^* \frac{\pi}{r} = w.$$

Replacing π and w using equations (3.6) and (3.7), and using equation (**??**) we obtain:

$$a^* = a_{\min} + \frac{r\left[1 - (a_{\min}/a_{\max})\right]}{\delta(1-\alpha)H(1-\tau)}. \tag{7.2}$$

The growth rate is given by:

$$g = \delta H \frac{a_{\min} \log\left(a_{\max}/a^*\right)}{1 - (a_{\min}/a_{\max})}. \tag{7.3}$$

We assume that $a^* < \theta$. This condition ensures that $\hat{a}_t > a^*$ for all t (see equation (7.1)). In this case, the ability of the marginal migrant is higher than that of the marginal entrepreneur, so only entrepreneurs migrate.

Suppose that the rate of corporate tax in the domestic economy, τ, is such that this economy grows at the same rate as the rest of the world and that the cost of moving, θ, is high enough that $\hat{a} > a^{\max}$, so no one migrates. In what follows we analyze the effects of a tax rate increase.

We denote by τ^* the threshold value for the corporate tax rate below which there is no migration. Replacing the value of π in equation (7.1), we obtain the following equation for τ^*:

$$\frac{\theta}{1 - \alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}(1-\tau_t^*)L\delta\left(n_t/\hat{x}_t\right)/r} = a.$$

The effect of an increase in taxes to a level $\tau' < \tau_t^*$ In this experiment we analyze the effect of a permanent increase in the tax rate to a new level $\tau' < \tau_t^*$. Since $\tau' < \tau_t^*$ there is no immediate flow of migration. However, the growth rate of the economy falls below $\hat{g}$ in response to the tax increase, according to the mechanism discussed in Section 4. This fall implies that over time, $n_t/\hat{x}_t$ declines and thus, eventually $\hat{a}_t$ falls below $\mathrm{a_{max}}$. At this point, migration begins (see equation (7.1)), as foreign opportunities improve faster than domestic opportunities ($g_t < \hat{g}$). This divergence between domestic and foreign growth rates lead to a smooth flow of migration that generates a slow decline in growth. The growth rate is given by:

$$g_t = \begin{cases} \delta H \int_{a^*}^{a_{\max}} a\Gamma(da) = \delta H \frac{a_{\min}\log(a_{\max}/a^*)}{1-a_{\min}/a_{\max}} & \hat{a}_t \geq a_{\max} \\ \delta H \int_{a^*}^{\hat{a}_t} a\Gamma(da) = \delta H \frac{a_{\min}\log(\hat{a}_t/a^*)}{1-a_{\min}/a_{\max}} & \hat{a}_t < a_{\max} \end{cases} \tag{7.4}$$

Equation (7.1) implies that the migration threshold, $\hat{a}_t$, keeps falling, converging to θ. This behavior of $\hat{a}_t$ implies that, asymptotically, the growth rate converges to a new lower value given by:

$$g_t = \delta H \frac{a_{\min}\log(\theta/a^*)}{1-a_{\min}/a_{\max}}$$

So, the elasticity of the growth rate with respect to taxation is low in the short run and high in the long run. This pattern is illustrated in the top panel of Figure 3.

The effect of an increase in taxes to a level $\tau' > \tau_t^*$ Suppose now that the tax rate is increased to a value greater than τ_t^*. In this case, there is an immediate flow of migration at time t with all agents with abilities greater than $\hat{a}_t$ leaving the economy. This brain drain leads to a discrete decline in the rate of growth of the economy (see the lower branch in equation (7.4)). The new value of g_t is lower than that implied by equation (7.3) for two reasons. First, $\hat{a}_t < a_{\max}$,

which reflects the fact that agents with skill above $\hat{a}_t$ migrate, reducing the flow of innovation. Second, as discussed in Section 4, higher tax rates induce more agents to become workers, so there a rise in a^* which reduces the flow of innovation. The initial fall in the growth rate generates an immediate second wave of migration. This second wave is similar to the one that eventually occurs when $\tau < \tau_t^*$. This pattern is illustrated in the bottom panel of Figure 3.

In sum, this model generates non-linear effects of taxation on growth that are similar to those of the previous models. Raising the tax rate to a value below τ_t^* results in no immediate migration and in a relatively small decline in the rate of growth, similar to the one discussed in Section 4. The growth rate remains stable for a while but, eventually, migration starts, causing further reductions in the rate of growth. Raising the tax rate to a value above τ^* results in an immediate flow of migration and a discrete decline in the growth rate, followed by additional reductions in growth.

8. Stochastic ability

So far we have assumed that agents know their entrepreneurial ability. In this section we consider the case where entrepreneurs do not know their true ability before they try to become entrepreneurs, as in Jovanovic (1982). High tax rates might deter agents from discovering their entrepreneurial ability. To isolate the effect of this informational friction, we assume that all successful entrepreneurs have the same ability. In the model that we analyze, the impact of taxes on growth is non-linear as in the previous sections.

We consider a very simple scenario in which only a fraction μ of the population, H, can be an entrepreneurs, but they do not know their entrepreneurial ability. A candidate entrepreneur has high ability with probability ϕ. High-ability entrepreneurs discover δn new varieties. Low-ability entrepreneurs produce no new goods

and end up operating a backyard technology that has productivity λw, $\lambda < 1$. At the beginning of time, agents have to commit to being workers or entrepreneurs.

In equilibrium, the measure of workers in the economy, L, has to such that:

$$\begin{cases} \phi U(\delta; L) + (1-\phi)U(\lambda w) < U(w) & L = H \\ \phi U(\delta; L) + (1-\phi)U(\lambda w) = U(w), & L \geq (1-\mu)H \\ \phi U(\delta; L) + (1-\phi)U(\lambda w) > U(w), & L = (1-\mu)H \end{cases}$$

where $U(\delta; L)$ is the utility of a successful entrepreneur when the number of workers in the economy is equal to L.

When the expected utility of an entrepreneur is equal to that of a worker, the solution for the number of entrepreneurs is interior and the number of entrepreneurs is lower or equal to μH. There are also two corner solutions. The first corresponds to the case in which the expected utility of becoming an entrepreneur is higher than the utility of a being a worker. In this case, all potential μH entrepreneurs decide to become entrepreneurs. The second corresponds to the case in which the expected utility of becoming an entrepreneur is lower than the utility of a being a worker. In this case no one tries to become an entrepreneur.

The successful entrepreneur's utility To be consistent with balanced growth, we assume that the initial number of patents, n_t, is equally distributed among successful entrepreneurs. As a result, the consumption at time zero, or at any time t, of a successful entrepreneur is:

$$C_t = n_t \pi / \left[\phi(H-L)\right] + \tau n_t \pi / \left[H(1-\tau)\right].$$

Their utility is:

$$U^e = \frac{(n_0 \pi)^{1-\sigma} \left\{1/\left[\phi(H-L)\right] + \tau/\left[H(1-\tau)\right]\right\}^{1-\sigma}}{\rho(1-\sigma) - (1-\sigma)^2 g}$$

The number of varieties in the economy continues to evolve according to

$$\dot{n} = n\delta\phi(H-L)$$

The worker's utility A worker's consumption is given by:

$$C_t = w_t + \tau n_t \pi / \left[H(1-\tau) \right];$$

implying

$$U^w = \frac{(n_0)^{1-\sigma} \left\{ w_0/n_0 + \tau\pi / \left[H(1-\tau) \right] \right\}^{1-\sigma}}{\rho(1-\sigma) - (1-\sigma)^2 g}$$

The failed entrepreneur's utility The consumption of failed entrepreneurs is given by:

$$C_t = \lambda w_t + \tau n_t \pi / \left[H(1-\tau) \right];$$

implying that

$$U^f = \frac{(n_0)^{1-\sigma} \left\{ \lambda w_0/n_0 + \tau\pi / \left[H(1-\tau) \right] \right\}^{1-\sigma}}{\rho(1-\sigma) - (1-\sigma)^2 g}$$

Equilibrium When the solution for L is interior, the value of L is given by:

$$\begin{aligned} & \phi \left(n_0 \pi \right)^{1-\sigma} \left\{ 1/\left[\phi(H-L) \right] + \tau / \left[H(1-\tau) \right] \right\}^{1-\sigma} \\ & + (1-\phi) \left(n_0 \right)^{1-\sigma} \left\{ \lambda w_0/n_0 + \tau\pi / \left[H(1-\tau) \right] \right\}^{1-\sigma} \\ = \; & \left(n_0 \right)^{1-\sigma} \left\{ w_0/n_0 + \tau\pi / \left[H(1-\tau) \right] \right\}^{1-\sigma}. \end{aligned}$$

Rearranging terms,

$$L = H - \frac{1}{\phi} \left\{ \left\{ \frac{\left[\frac{w_0}{n_0 \pi} + \frac{\tau}{H(1-\tau)} \right]^{1-\sigma} - (1-\phi) \left[\frac{\lambda w_0}{\pi n_0} + \frac{\tau}{H(1-\tau)} \right]^{1-\sigma}}{\phi} \right\}^{1/(1-\sigma)} - \frac{\tau}{H(1-\tau)} \right\}^{-1}.$$

Figure 4 shows results for a numerical example.[16] Taxes have no impact on growth for values of τ between zero and 0.75. For tax rates in this range, the

[16] The parameters used in this example are: $\alpha = 0.6$, $\sigma = 2$, $\rho = 0.01$, $H = 1$, $\delta = 2$, $\tau = 0.35, \eta = 1$, $\lambda = 0.91$, $\phi = 0.1$, $\mu = 0.1$.

expected utility of being an entrepreneur is higher than the expected utility of being a worker. As a result, all potential entrepreneurs choose to be entrepreneurs. When tax rates are higher than 0.75, there is a larger effect of taxes on growth because the solution for L is interior. As tax rates increase, the number of entrepreneurs declines. This decline leads to a reduction in the number of successful entrepreneurs and in the growth rate of the economy.

9. Conclusion

In this paper we discuss several models in which the effects of taxation on growth are highly non-linear. Taxes have a small impact on long-run growth when taxes rates and other disincentives to investment are low or moderate. But, as tax rates rise, the marginal effect of taxation also increases. In our benchmark model this non-linearity is generated by heterogeneity in entrepreneurial ability. In a low-tax economy the ability of the marginal entrepreneur is much lower than that of the average entrepreneur. Increases in taxes results in the exit of low-ability entrepreneurs and in a small decline in the rate of growth of the economy. In a high-tax economy the ability of the marginal entrepreneur is similar to that of the average entrepreneur. Increases in taxes result in the exit of high-ability entrepreneurs and in a large decline in the rate of growth of the economy.

We show that these non-linear effects of taxation on growth emerge naturally in two extensions of our model. In the first extension agents can migrate by paying a flow cost. Since this cost is assumed to be independent of ability, it creates an incentive for high-ability workers to migrate. There is a range of tax rates for which there is no immediate migration, so the effects of taxation on growth are small. But, as tax rates exceed a certain threshold, high-ability agents migrate, reducing the rate of innovation and producing a large decline in growth rates.

The second extension is a model in which potential entrepreneurs do not know

their ability. This ability is learned only when agents become entrepreneurs. In this economy there is a range of tax rates for which all potential entrepreneurs try to become entrepreneurs. Changes in tax rates in this range have no impact on growth rates. But, once taxes exceed a certain threshold, the number of potential entrepreneurs decline reducing the growth rate of the economy.

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11. Appendix

11.1. Homogeneous agent model

In this appendix we show that the model in Section 3 has no transitional dynamics. The same property applies to the model in Section 4.

$$\pi = \alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L(1-\tau). \tag{11.1}$$

Equations (3.2) and (3.5) imply that the equilibrium wage rate is given by:

$$w = \alpha n\left[\frac{(1-\alpha)^2}{\eta}\right]^{(1-\alpha)/\alpha}. \tag{11.2}$$

Recall that the value of a patent for a new good is :

$$V_t = \int_t^\infty e^{-\int_t^s r_s ds}\pi_s ds.$$

When there is an interior solution for the number of entrepreneurs, we have:

$$\delta n_t V_t = w_t.$$

Differentiating with respect to time:

$$\dot{n}\delta V_t + \delta n_t \dot{V}_t = \dot{w}_t,$$

$$\dot{V}_t = \frac{d}{dt}\int_t^\infty e^{-\int_t^s r_s ds}\pi_s ds.$$

Define:

$$f(t,s) = e^{-\int_t^s r_s ds}\pi_s.$$

Using Leibnitz's rule:

$$\frac{d}{dt}\int_t^\infty f(t,s)ds = -f(t,t) + \int_t^\infty f_1(t,s)ds,$$

$$\frac{d}{dt}\int_t^\infty f(t,s)ds = -\pi_s + \int_t^\infty f_1(t,s)ds,$$

$$\frac{d}{dt}\left[-\int_t^s r_s ds\right] = r_t,$$

$$f_1(t,s) = \pi_s \frac{d}{dt} e^{-\int_t^s r_s ds} = \pi_s r_t e^{-\int_t^s r_s ds},$$

$$\dot{V}_t = \frac{d}{dt}\int_t^\infty f(t,s)ds = -\pi_t + r_t \int_t^\infty \left(\pi_s e^{-\int_t^s r_s ds}\right) ds,$$

$$\dot{V} = \frac{d}{dt}\int_t^\infty f(t,s)ds = -\pi_t + r_t V_t.$$

Taking time derivatives of the free-entry condition, we have:

$$\dot{n}_t \delta V_t + \delta n_t \left(r V_t - \pi_t\right) = \dot{w}_t.$$

Using the free-entry condition:

$$\delta n_t V_t = w_t. \tag{11.3}$$

Equation (11.2) implies that w_t/n_t is constant. Equation (11.3) implies that the value of the firm, V_t is also constant.

Equation (11.3) implies:

$$\frac{\dot{n}_t}{n_t} + \frac{r_t V_t - \pi_t}{V_t} = \frac{\dot{w}_t}{w_t}.$$

Recall from equation (11.2) that $\dot{w}_t/w_t = \dot{n}_t/n_t$, so we have:

$$V_t = \frac{\pi_t}{r_t}. \tag{11.4}$$

Replacing π_t and r_t in equation (11.4) for V_t,

$$\frac{\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L_t(1-\tau)}{\sigma\delta(H-L_t)+\rho} = V.$$

Rearranging,

$$\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}L_t(1-\tau) = V\left[\sigma\delta(H-L_t)+\rho\right].$$

Differentiating with respect to time, using the fact that $\dot{V}_t = 0$, we have

$$\alpha(1-\alpha)^{(2-\alpha)/\alpha}\eta^{-(1-\alpha)/\alpha}\dot{L}_t(1-\tau) = -\dot{L}_t V\sigma\delta.$$

This equation implies that $\dot{L}_t = 0$, so L_t is constant. This property implies that π_t is constant (equation (11.1)). Equation (11.4) implies that r_t is constant. In sum, the model has no transition dynamics.

11.2. Heterogenous agent model

The equation for the identity of the marginal innovator is given by (3.15)

$$n\delta a_t^* V_t = w_t.$$

where

$$V_t = \int_t^\infty e^{-\int_t^s r_s ds}\pi_s ds.$$

The wage rate is given by

$$w = \alpha n\left[\frac{(1-\alpha)^2}{\eta}\right]^{(1-\alpha)/\alpha},$$

implying that:

$$a_t^*\delta V_t = \alpha\left[\frac{(1-\alpha)^2}{\eta}\right]^{(1-\alpha)/\alpha}.$$

Differentiating with respect to time,

$$\frac{\dot{a}_t^*}{a_t^*} + \frac{\dot{V}_t}{V_t} = 0.$$

The term $\dot{V}$ equals:

$$\dot{V}_t = r_t V_t - \pi_t,$$

implying,

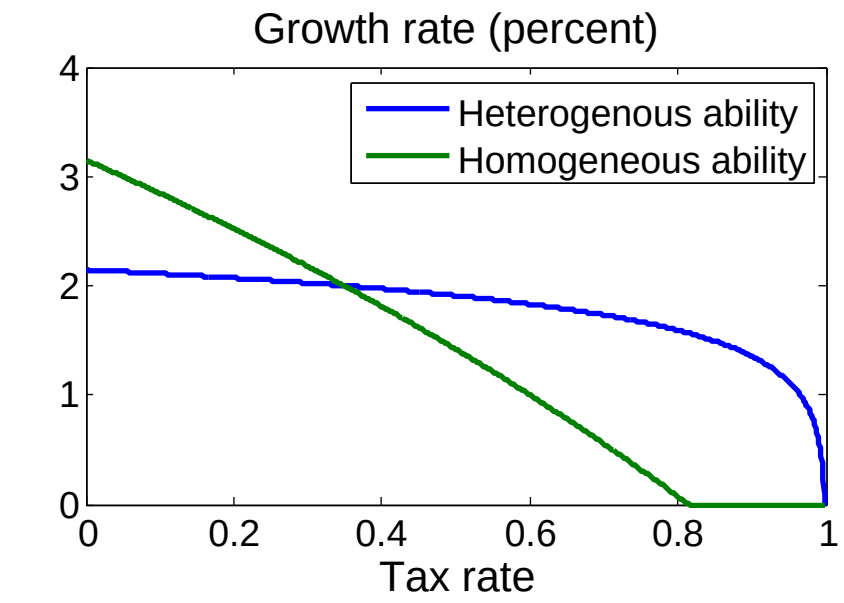
$$\begin{aligned} \frac{\dot{a}_t^*}{a_t^*} + \frac{r_t V_t - \pi_t}{V_t} &= 0, \\ \frac{\dot{a}_t^*}{a_t^*} + r_t - \frac{\pi_t}{V_t} &= 0. \end{aligned}$$

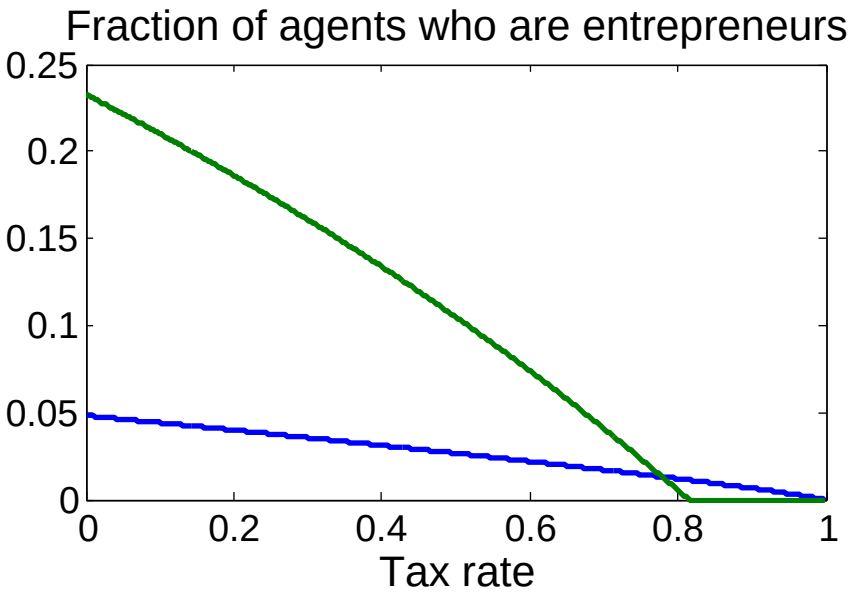
Conjecture that $\dot{a}^* = 0$, so $a_t^* = a^*$. The free entry condition,

$$V_t = \frac{w_t}{n\delta a^*}$$

implies that V_t is constant, given that w_t/n_t is constant. We can now proceed as in the homogenous agent model and show that the real interest rate and the growth rate is constant and that all equations are satisfied, so that a constant value of a_t^* is a solution.

Figure 1


Growth rate (percent)
Heterogenous ability
Homogeneous ability
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
Tax rate


Fraction of agents who are entrepreneurs
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
Tax rate

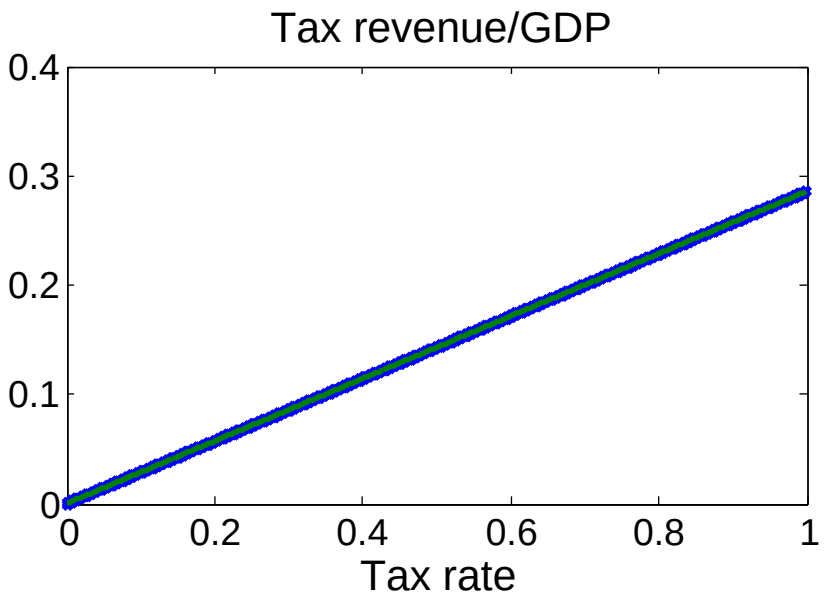

Tax revenue/GDP
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
Tax rate

Figure 2

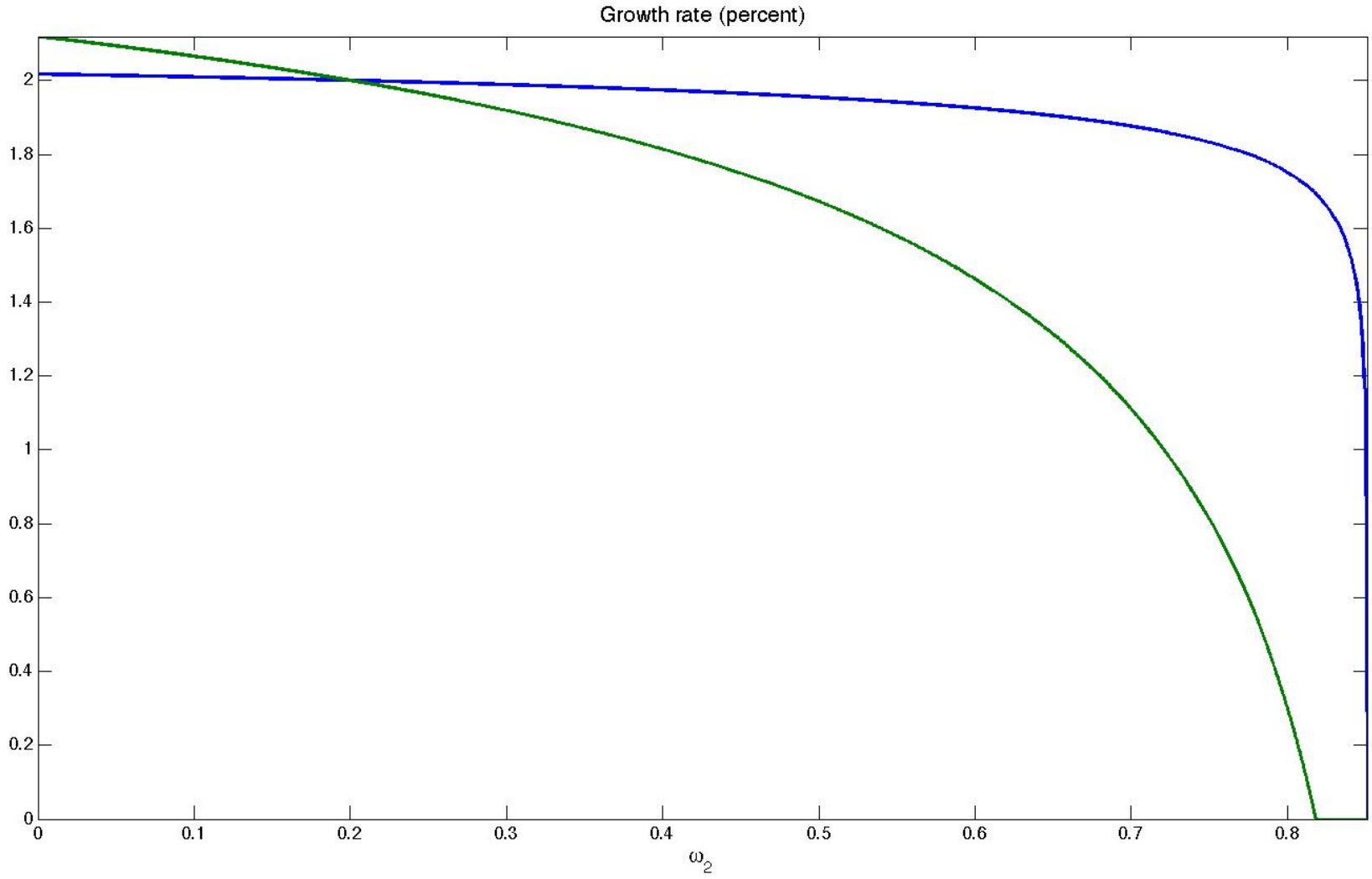
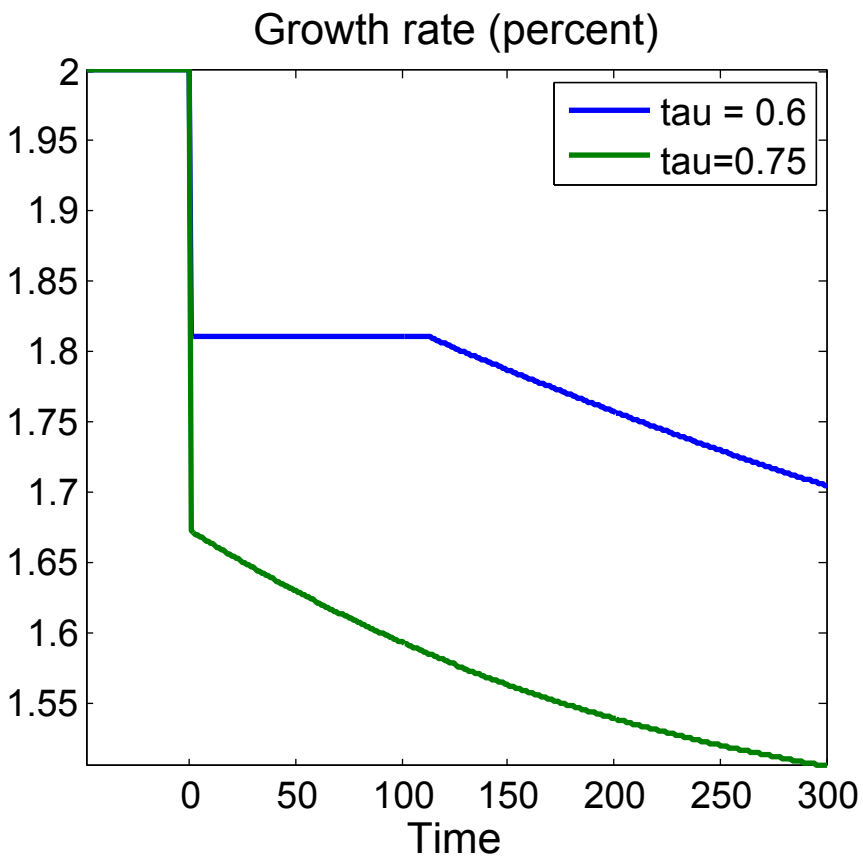

Growth rate (percent)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
ω2

Figure 3


Growth rate (percent)
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
1.6
1.55
0
50
100
150
200
250
300
Time
tau = 0.6
tau=0.75

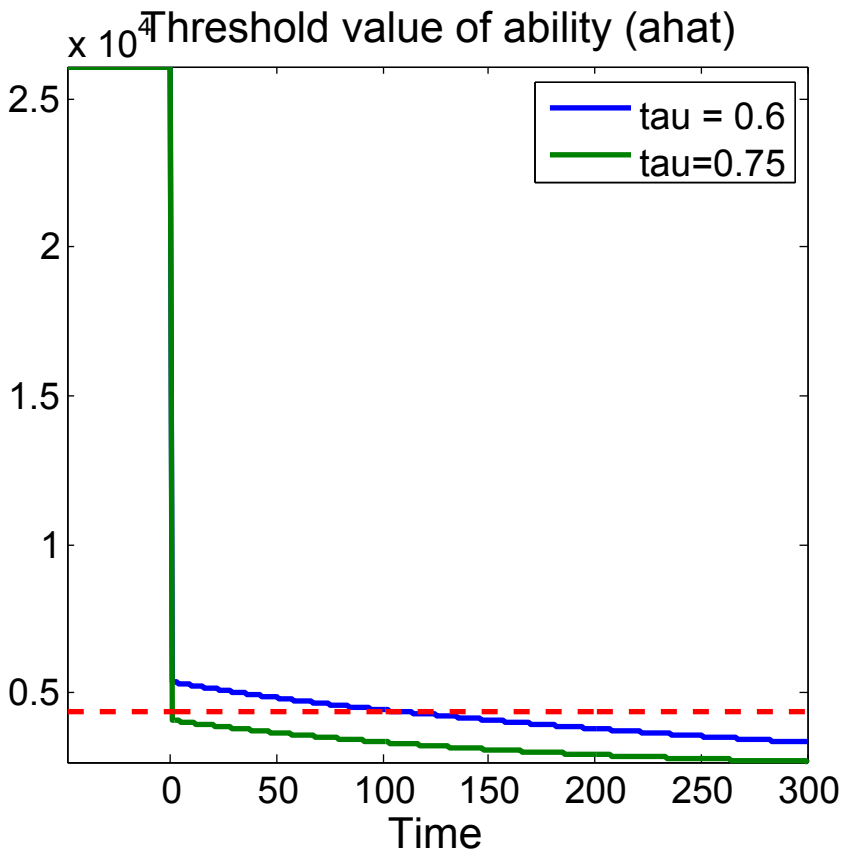
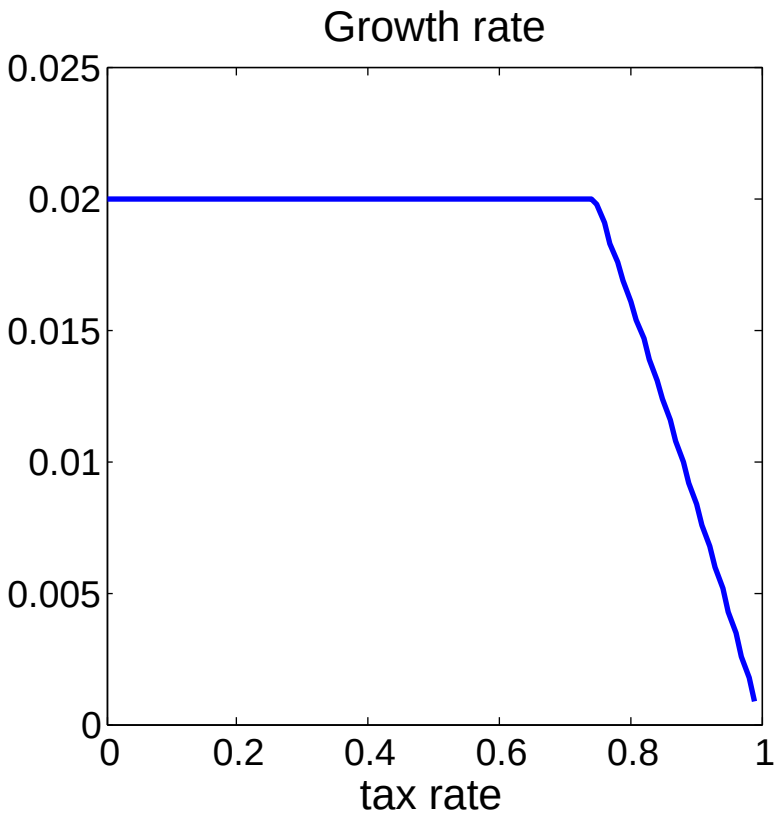
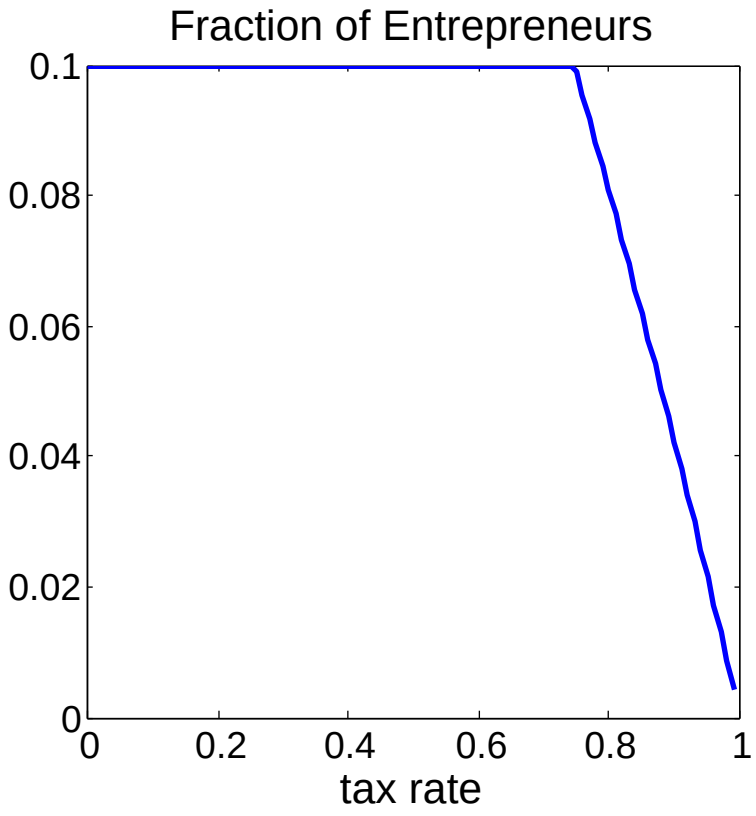

Threshold value of ability (ahat)
x 10^4
2.5
2
1.5
1
0.5
0
50
100
150
200
250
300
Time
tau = 0.6
tau=0.75

Figure 4


Growth rate
0.025
0.02
0.015
0.01
0.005
0
0
0.2
0.4
0.6
0.8
1
tax rate


Fraction of Entrepreneurs
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
tax rate