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SENTIMENTS AND AGGREGATE DEMAND FLUCTUATIONS

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ABSTRACT

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Sentiments and Aggregate Demand Fluctuations*

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Abstract

We formalize the Keynesian insight that aggregate demand driven by sentiments can generate output fluctuations under rational expectations. When production decisions must be made under imperfect information about demand, optimal decisions based on sentiments can generate stochastic self-fulfilling rational expectations equilibria in standard economies without persistent informational frictions, externalities, non-convexities or even strategic complementarities in production. The models we consider are deliberately simple, but could serve as benchmarks for more complicated equilibrium models with additional features.

Keywords: Keynesian Self-fulfilling Equilibria, Sentiments, Sunspots

1 Introduction

We construct a class of models to capture the Keynesian insight that employment and production decisions are based on consumer sentiments of aggregate demand, and that realized aggregate demand follows firms' production and employment decisions. We cast the Keynesian insight in a simple model in which (i) firms must make employment and production decisions before demand and prices are realized, and (ii) realized demand and income depend on firms' output and employment decisions. We characterize the rational expectations equilibria of this model. We find that despite the lack of any non-convexities in technologies and preferences, there can be multiple rational

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expectations equilibria. Fluctuations are driven by waves of optimism or pessimism, or as in Keynes' terminology, by "animal spirits". Sentiment-driven equilibria exist because firms must make production decisions based on signals prior to the realization of their demand and of their prices. A key feature of the class of models that we consider will be the endogeneity of the signals in rational expectations equilibria: the underlying distribution of consumers' sentiments on output that generate the signal will be consistent with the distribution of realized output.

We study models where firms produce differentiated goods, and make production and employment decisions based on signals about the demand for their goods. Trades take place in centralized markets, and at the end of each period all trading and price history is public knowledge. Consumer demand reflects fundamental preference or productivity shocks, as well as pure sentiments shocks. The firms therefore face a signal extraction problem because their optimal response to fundamental shocks is different from their optimal response to sentiment shocks. We show that under reasonable conditions the signal extraction problem of firms can give rise to sentiment-driven equilibria, in certain cases a continuum of them, in addition to equilibria solely driven by fundamentals. Such equilibria can be serially correlated over time, and are not based on randomizations over the fundamental equilibria.[1] In section 4.3 we show that the fundamental equilibrium is not stable under constant gain learning, while sentiment-driven sunspot equilibrium is stable if the gain parameter is not too large. In section 5 we provide explicit microfoundations for the signals that we consider throughout the paper.

Our models are in the spirit of the Lucas (1972) island model, as well as the models with sentiment-driven fluctuations of Angeletos and La'O (2009, 2012). In the absence of sentiments, the models that we study have unique equilibria, but sentiments and beliefs about aggregate market outcomes can affect and amplify employment, production and consumption decisions and can lead to multiple rational expectations equilibria.[2,3]

The multiplicity of rational expectations equilibria that we obtain is related to the correlated equilibria of Aumann (1974, 1987) and of Maskin and Tirole (1987).[4] They emerge naturally from

[1] For the classical work on extrinsic uncertainty and sunspot equilibria with a unique fundamental equilibrium under incomplete markets, see Cass and Shell (1983). See also Spear (1989) for an OLG model with two islands where prices in one island act as sunspots for the other and vice versa.

[2] For the possibility of multiple equilibria in the context of asymmetic information see Amador and Weill (2010), Angeletos and Werning (2006), Angeletos, Hellwig, and Pavan (2006), Angeletos, Lorenzoni and Pavan (2010), Gaballo (2012), Hellwig, Mukherji, and Tsyvinski (2006), and Hellwig and Veldkamp (2009). In particular Manzano and Vives (2010) survey the literature and study the emegerce of multiplicity when correlated private information induces strategic compementarity in the actions of agents trading in financial markets. In a number of the papers cited, prices convey noisy information about asset returns. By contrast in our model production and employment decisions are made based on expectations, but prior to the realization of demand and real prices.

[3] See also Morris and Shin (2002) and Angeletos and Pavan (2007) where agents can excessively coordinate on and overreact to public information, thereby magnifying the fluctuations caused by pure noise. By contrast, in some global games, multiple coordination equilibria may be eliminated under dispersed private signals on fundamentals, as in Morris and Shin (1998).

[4] Correlated equilibria in market economies are also discussed by Aumann, Peck and Shell (1988). See also Peck and Shell (1991), Forges and Peck (1995), Forges (2006), and more recently, Bergemann and Morris (2011) and Bergemann, Morris and Heinmann (2013).

the endogenous signals that induce imperfectly correlated employment and output decisions by firms.[5,6] In equilibrium, for every realization of sentiment shocks, the firms' expected aggregate demand is equal to the realized aggregate demand, the consumer's expected aggregate income is equal to the realized aggregate output, and the expected real wage is equal to the realized real wage.

2 The Benchmark Model

The model has a representative household, a representative final goods producer, and a continuum of monopolistic intermediate-goods producers indexed by $j \in [0, 1]$. The intermediate-goods producers each period decide on how much to produce, based on their observation of a noisy signal that they obtain from market research on their demand. The demand curve that they face depends on sentiment shocks to aggregate demand as well as on idiosyncratic demand shocks to intermediate goods from the final good sector. In section 4.1, we generalize the signal structure by allowing multiple signals. Later in section 5 we provide some explicit microfoundations for the signals that firms receive.

2.1 Households

At the beginning of the period t the representative household maximizes utility

$$\max \{\log C_t - \psi N_t\} \tag{1}$$

subject to the budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t}, \tag{2}$$

where C_t is consumption, N_t is labor supply, W_t is the wage, P_t the aggregate price level, and Π_t is aggregate profit income from firms. The first order condition of the household yields,

$$C_t = \frac{1}{\psi} \frac{W_t}{P_t}. \tag{3}$$

The households see the nominal wage and they form a conjecture on the equilibrium aggregate price level P_t and the real wage when choosing their consumption plan according to equation (3).

[5] As noted by Maskin and Tirole (1987), "Our observation that signals "matter" only if they are imperfectly correlated corresponds to the game theoretic principle that perfectly correlated equilibrium payoff vectors lie in the convex hull of the ordinary Nash equilibrium payoffs, but imperfectly correlated equilibrium payoffs need not." In Maskin and Tirole (1987) however the uninformed agents do not have a signal extraction problem as we do, so in their model in addition to the certainty Nash equilibrium, they have correlated equilibria only if there are Giffen goods.

[6] Correlated equilibria are typically defined for finite games with a finite number of agents and discrete strategy sets, but for an extention to continuous games see Hart and Schmeidler (1989) and more recently Stein, Parillo, and Ozdaglar (2008). We thank Martin Schneider for alerting us to this point.

We will see later that in equilibrium aggregate consumption C_t and the equilibrium price P_t can vary with realizations of consumer or household sentiments about aggregate output.

2.2 Final Goods Producers

The final goods firm produces output according to

$$Y_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{4}$$

where $\theta > 1$, ϵ_{jt} is a lognormal *iid* shock with unit mean. The exponential $\frac{1}{\theta}$ on shock ϵ_{jt} is just a normalization to simplify expressions later on. The final goods producer maximizes profit:

$$\max \left\{ P_t \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int P_{jt} Y_{jt} dj \right\}. \tag{5}$$

The first-order condition with respect to input Y_{jt} is $P_{jt} = P_t \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{-\frac{1}{\theta}}$, which implies

$$Y_{jt} = \left(\frac{P_t}{P_{jt}} \right)^{\theta} \epsilon_{jt} Y_t. \tag{6}$$

Substituting the last equation into the production function and rearranging gives $P_t^{1-\theta} = \int \epsilon_{jt} P_{jt}^{1-\theta} dj$.

2.3 Intermediate Goods Producers

Each intermediate goods firm produces good j to meet its demand Y_{jt} without perfect knowledge either of its idiosyncratic shock ϵ_{jt}, or the aggregate demand Y_t, which could also be random. Instead, as in the Lucas island model, firms try to infer their demand by conducting market research and market surveys and they obtain a signal s_{jt},

$$s_{jt} = \lambda \log \epsilon_{jt} + (1 - \lambda) \log Y_t + v_{jt}. \tag{7}$$

Here λ reflects the weights assigned by firms to the idiosyncratic and aggregate components of demand, and v_{jt} is a pure firm-specific *iid* noise with zero mean and variance $\sigma_v^2 \geq 0$.

In section 5 we provide explicit microfoundations to endogenize the signals s_{jt} and the weights given by λ in equation (7). In fact in certain cases the microfoundations allow a continuum of equilibrium λ values, implying the existence of a continuum of sentiment driven equilibria. For the time being we leave λ as a parameter.

On the basis of its signal, the firm chooses its production to maximize profits. An intermediate goods producer j has the production function

$$Y_{jt} = AN_{jt}. \tag{8}$$

So the firm maximizes expected nominal profits $\Pi_{jt} = P_{jt}Y_{jt} - \frac{W_t}{A}Y_{jt}$ by solving

$$\max_{Y_{jt}} E_t \left[\left(P_t Y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{jt} \right) \middle| \, s_{jt} \right]. \tag{9}$$

The first order condition for Y_{jt} is given by

$$\left(1 - \frac{1}{\theta}\right) Y_{jt}^{-\frac{1}{\theta}} E_t \left[P_t (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} \middle| \, s_{jt} \right] = \frac{1}{A} E_t \left[W_t | \, s_{jt} \right]. \tag{10}$$

Using equation (3) and the equilibrium condition $C_t = Y_t$ we have $P_t = \frac{1}{\psi}\frac{W_t}{Y_t}$. We can normalize either by price level P_t or by the nominal wage W_t, for simplicity we set $W_t = 1$. Equation (10) then becomes

$$Y_{jt} = \left\{ \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} E_t \left[(\epsilon_{jt})^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} \middle| \, s_{jt} \right] \right\}^{\theta}. \tag{11}$$

The final aggregate output is given in equation (4). Note from (11) that in equilibrium the optimal firm output declines with aggregate output since $\frac{1}{\theta} - 1 < 0$, which implies that we have strategic substitutability. Despite this, we will show that the rational expectations equilibrium is not unique.[7]

2.4 Equilibrium

We denote consumers' sentiments about aggregate output in period t as Z_t. Aggregate quantities and prices are given by $C_t = C(Z_t), Y_t = Y(Z_t)$ and $P_t = P(Z_t)$. An equilibrium consists of an endogenous distribution of sentiment shocks Z_t such that $Z_t = C_t = Y_t$ and equations (3), (4), (6) and (11) are satisfied. The time profile for the realization of the equilibrium is as follows:

1. The household draws a sentiment Z_t about aggregate output from an equilibrium distribution $F(Z)$ and conjectures that the aggregate price is $P_t = P(Z_t)$ and the real wage is $1/P_t\,(Z_t)$.

2. Based on the conjectured aggregate price and real wage, the household decides its consumption plan $C_t = C(Z_t)$.

[7] We can also replace the firm's problem with $\max_{Y_{jt}} E_t \left[\left(\Lambda_t P_t Y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} \Lambda_t Y_{jt} \right) \middle| \, s_{jt} \right]$, where Λ_t is the marginal utility of the households. Note the optimal response of the firm changes to $Y_{jt} = \left\{ \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi E_t(\Lambda_t | s_{jt})} E_t \left[\Lambda_t \, (\epsilon_{jt})^{\frac{1}{\theta}} \, Y_t^{\frac{1}{\theta}-1} \middle| \, s_{jt} \right] \right\}^{\theta}$. It is easy to show after taking logs, terms associated with Λ_t disappear through cancellation, and our equilibrium conditions do not change.

3. The final good producer has the same information set as the household, and chooses demands $Y_{jt} = \left(\frac{P_t(Z_t)}{P_{jt}}\right)^{\theta} \epsilon_{jt} Y_t\left(Z_t\right)$ based on the sentiment shock Z_t, the idiosyncratic shock ϵ_{jt}, and their conjectured intermediate-good prices $P_{jt} = Q(Z_t, \epsilon_{jt})$ for $j \in [0, 1]$.

4. Each intermediate-good firm j uses F as the prior distribution of Y_t. It receives a signal s_{jt} about its demand according to the signal (7).

5. Based on the signal, each intermediate-good firm j produces according to (11).

6. Given the production decision Y_{jt}, the aggregate production Y_t is realized according to (4), and all prices are realized according to (6).

7. The equilibrium is reached if the conjectured prices equal the actual realized prices and the conjectured quantities equal the realized quantities, that is the consumer's consumption plan C_t equals the actual final good production Y_t. Also, as $Y_t = C_t = Z_t$, the actual aggregate output Y_t follows a distribution consistent with F, namely $\Pr(Y_t \leq X_t) = F(X_t)$.

3 The Certainty Equilibrium

There exists a fundamental certainty equilibrium, defined as the allocation with $Y_t = Y^*$ and $P_t = P^*$. The certainty equilibrium is obtained under perfect information. When information is perfect, the signals obtained by firms fully reveal their own demand in each period. Equation (11) becomes

$$Y_{jt}^{\frac{1}{\theta}} = \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi} \epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1-\theta}{\theta}}, \tag{12}$$

or if we use $P_{jt} = P_t Y_t^{\frac{1}{\theta}} \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{-\frac{1}{\theta}}$ and $P_t = \frac{1}{\psi}\frac{1}{Y_t}$, equation (12) becomes

$$P_{jt} = \left(\frac{\theta}{\theta - 1}\right) \frac{1}{A} = \bar{P}. \tag{13}$$

Then if their demand curve shifts by ϵ_{jt} units, their optimal output will change in such a way as to leave their prices invariant, so all firms will charge the same price. Substituting P_{jt} into $P_t^{1-\theta} = \int \epsilon_{jt} P_{jt}^{1-\theta} dj$ gives

$$P_t = \left(\frac{\theta}{\theta - 1}\right) \frac{1}{A} \left[\int \epsilon_{jt} dj\right]^{\frac{1}{1-\theta}}. \tag{14}$$

Hence, equation (3) implies

$$C^* = Y^* = \frac{A}{\psi}\left(1 - \frac{1}{\theta}\right)\left[\int \epsilon_{jt} dj\right]^{\frac{1}{\theta-1}}. \tag{15}$$

If without loss of generality we normalize $\left(1-\frac{1}{\theta}\right)\frac{A}{\psi}=1$, we then have $\log Y_t = \frac{1}{\theta-1}\log E\exp(\varepsilon_{jt})$, where $\varepsilon_{jt} \equiv \log \epsilon_{jt}$ has zero mean and variance σ_ε^2. Therefore, under the assumption of a normal distribution for ε_{jt},

$$\log Y_t = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2 = \bar{\phi}_0, \tag{16}$$

which is an alternative way of expressing equation (15).

4 Sentiment-Driven Equilibrium

We conjecture that there exists another equilibrium, such that aggregate output may not be a constant. In particular we assume that

$$y_t \equiv \log Y_t = \phi_0 + z_t, \tag{17}$$

where z_t is a normally distributed random variable with zero mean and variance σ_z^2.[8] The noisy signal received by each firm is given by (7). Note that with fluctuations in aggregate output, the firm's signal is no longer fully revealing even with $\sigma_v^2 = 0$.[9]

We may view z_t as a sentiment held by consumers about aggregate demand. We will show that in our sentiment-driven equilibrium the distribution of the sentiments $\{z_t\}$ assumed by the firms will be consistent with the realized distribution of aggregate output $\{y_t\}$ given by equation (17). In a rational expectations equilibrium, realizations of the sentiment variable z_t will in fact be equal to aggregate output $y_t \equiv \log Y_t$.

Let $\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2+\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2}$. We first state the result for the certainty equilibrium when $\sigma_v^2 \geq 0$:

Proposition 1 *Under the signal given by (7) there is a constant certainty equilibrium,* $y_t = \tilde{\phi}_0$*, given by*

$$\tilde{\phi}_0 = \frac{1}{2}\left[\left(\frac{\theta+\theta\mu\lambda(\theta-1)+(\theta\mu\lambda(\theta-1))^2}{\theta^2(\theta-1)}\right)\sigma_\varepsilon^2 + (\theta-1)(\theta\mu)^2\sigma_v^2\right]. \tag{18}$$

Proof. See Appendix. ■

Proposition 2 *Let* $\lambda \in (0, 1/2)$*, and* $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$*. In addition to the certainty equilibrium given in Equation (18), there also exists a sentiment-driven rational expectations equilibrium with*

[8] For convenience, in the rest of the paper we denote the logarithm of the sentiment variable by $z_t \equiv \log Z_t - \phi_0$.

[9] Note that here we define the signal as the weighted sum of the idiosyncratic shock and the innovation to aggregate demand. The mean of the log of aggregate demand will be absorbed by the constant ϕ_0 in equation (17) and incorporated into output decisions of firms. So $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t + v_{jt}$ is equivalent to $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t + v_{jt}$ as ϕ_0 is common knowledge.

stochastic aggregate output, $\log Y_t$, *that has a mean*

$$\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\,\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\,\sigma_v^2}{2\theta^2(1-\lambda)^2} \tag{19}$$

and a variance $\sigma_y^2 = \sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$.

Proof. See the Appendix. ■

If firms believe that their signals contain information about changes in aggregate demand in addition to the firm-level demand shocks, then these beliefs will partially coordinate their output responses, up or down, and sustain sentiment-driven fluctuations consistent with their beliefs about the distribution of output. Both the variance of the sentiment shock σ_z^2 and λ affect the firms' optimal output responses through their signal extraction problems. Given λ, the variance of the idiosyncratic shock σ_ε^2, and the variance of the noise σ_v^2, for markets to clear for all possible realizations of the aggregate demand sentiment z_t, the variance σ_z^2 has to be precisely pinned down, as indicated in Proposition 2.[10] As we show in section 5 where we consider the microfoundations of the signals and the permissible values of λ, there may well be a continuum of equilibrium λ values, and therefore a continuum of sentiment-driven equilibria parametrized by λ or σ_z^2. Sentiments, realized under their equilibrium distributions parametrized by σ_z^2, serve to correlate firm decisions and give rise to a continuum of correlated equilibria.

Notice that if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda\,(1-2\lambda)\,\sigma_\varepsilon^2$, then equilibrium would require $\sigma_z^2 < 0$, suggesting that the only equilibrium is $z_t = 0$. Hence, to have a sentiment-driven equilibrium, we require $\lambda \in \left(0, \frac{1}{2}\right)$ and $\sigma_v^2 < \lambda\,(1-2\lambda)\,\sigma_\varepsilon^2$. The value of λ pins down the equilibrium value of $\sigma_z^2 > 0$ as a function of σ_ε^2 and σ_v^2. Note that the extra noise v_{jt} in the signal makes output in the sentiment-driven equilibrium less volatile. The reason for the smaller volatility of output when $\sigma_v^2 > 0$ is that the signal is more noisy, and firms attribute a smaller fraction of the signal to demand fluctuations. Note however that this requires the additional restriction that the variance of the extra noise cannot be too big, $\sigma_v^2 < \lambda\,(1-2\lambda)\,\sigma_\varepsilon^2$, to ensure that $\sigma_z^2 > 0$.

At the certainty or fundamental equilibrium we have $Y_{jt} = \epsilon_{jt} Y_t^{1-\theta}$, so firm-level outputs depend negatively on aggregate output . This strategic substitutability implies that the certainty equilibrium is unique. When aggregate demand is sentiment driven, if we increase σ_z^2 the firm attributes more of the signal to an aggregate sentiment shock. The optimal supply of the firm's output however depends positively on firm-level demand shocks. Consequently, if firms cannot distinguish firm-level shocks from aggregate demand shocks, informational strategic complementarities can arise so that higher realizations of z result in higher optimal firm outputs, giving rise to

[10] To see this look at equations (A.9), (A.10), (A.11) and (A.13) in the proof of the Proposition in the Appendix.

sentiment-driven equilibria. In the sentiment-driven equilibrium, σ_z^2 is determined at a value that will clear markets for all realizations of the sentiment z. Note also that the mean output ϕ_0 in the sentiment-driven equilibrium will be lower than the output $\bar{\phi}_0$ under the certainty equilibrium, and the mean markup will be higher.

4.1 Multiple Sources of Signals

The government and public forecasting agencies as well as news media often release their own forecasts of the aggregate economy. Such public information may influence and coordinate output decisions of firms and affect the equilibria. Suppose firms receive two independent signals, s_{jt} and s_{pt}. The firm-specific signal s_{jt} is based on a firm's own preliminary information about its demand and is identical to that in equation (7). The public signal in the case of the sentiment-driven equilibrium is

$$s_{pt} = z_t + e_t \tag{20}$$

where we can interpret e_t as common noise in the public forecast of aggregate demand with mean 0 and variance σ_e^2. For example, if consumers sentiments were heterogeneous and differed by *iid* shocks, then a survey of a subset of consumer sentiments would have sampling noise e_t.[11]

We also assume that $\sigma_e^2 = \tilde{\gamma}\sigma_z^2$, where $\tilde{\gamma} > 0$. This assumption states that the variance of the forecast error of the public signal for aggregate demand is proportional to the variance of z, or equilibrium output. Then in the certainty equilibrium where output is constant over time, the public forecast of output is correct and constant as well.

Proposition 3 *If $\lambda \in (0, 1/2)$, and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$, then there exists a sentiment-driven rational expectations equilibrium with stochastic aggregate output* $\log Y_t = y_t = z_t + \eta e_t + \phi_0 \equiv \hat{z}_t + \phi_0$, *which has mean* $\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$ *and variance* $\sigma_y^2 = \sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2 > 0$ *with* $\eta = -\frac{\sigma_z^2}{\sigma_e^2} = -\frac{1}{\tilde{\gamma}}$. *In addition, there is a certainty equilibrium with constant output identical to that given in Proposition 1 with* $\sigma_z^2 = \tilde{\gamma}\sigma_e^2 = 0$.

Proof. See the Appendix. ■

As shown in the proof of Proposition 3, firms choose their optimal outputs based both on z_t and e_t. When $\sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$, the optimal weight that they place on the public signal becomes zero. Nevertheless aggregate output is stochastic, and driven by the volatility of $\hat{z}_t \equiv z_t + \eta e_t$. It is easy to see that the certainty equilibrium of Proposition 1 with $\sigma_z^2 = 0$ also applies

[11] In the working paper, Benhabib, Wang and Wen (2013), we explicitly consider the case where sentiment signals observed by consumers are heterogenous but correlated. Consumers in this case also have a signal extraction problem in forming an expectation of the real wage.

to Proposition 3 since we have $\sigma_e^2 = \tilde{\gamma}\sigma_z^2 = 0$, i.e. the public signal also becomes a constant. We can then directly apply Proposition 1 to find the equilibrium output (see the proof in Appendix).

4.2 A Simple Abstract Version of the Model

To illustrate the forces at work that produce the sentiment-driven stochastic equilibrium, we can abstract from the household and production side of our model. Let us assume for simplicity that the economy is log-linear, so optimal log output of firms is given by the rule

$$y_{jt} = E_t\{[\beta_0\varepsilon_{jt} + \beta y_t]\,|s_{jt}\}. \tag{21}$$

This log-linear specification allows us to avoid any constant term in the equilibrium output, so we can maintain a zero mean for y_t. The coefficient β can be either negative or positive, so we can have either strategic substitutability or strategic complementarity in firms' actions. The signal s_{jt} is given by

$$s_{jt} = v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)\,y_t, \tag{22}$$

where both the exogenous noise v_{jt} and the idiosyncratic demand shock ε_{jt} are *iid* and normally distributed with a zero mean. Market clearing then requires

$$y_t = \int y_{jt}dj. \tag{23}$$

In the certainty equilibrium y_t is constant, so equation (21) yields $y_{jt} = \beta y_t + \frac{\lambda\beta_0\sigma_\varepsilon^2}{\sigma_v^2+\lambda^2\sigma_\varepsilon^2}(v_{jt}+\lambda\varepsilon_{jt})$. Substituting this solution into equation (23) and integrating give

$$y_t = \int y_{jt}dj = \beta y_t. \tag{24}$$

So unless $\beta = 1$, in which case there is a continuum of certainty equilibria, the unique certainty equilibrium is given by $y_t = 0$.

In the sentiment-driven stochastic equilibrium, assume that y_t is normally distributed with zero mean and variance σ_z^2. Based on the simple response function given by equation (21), signal extraction implies

$$y_{jt} = \frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}\left[v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)\,y_t\right]. \tag{25}$$

Then market clearing requires

$$y_t = \int y_{jt}dj = \frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda)y_t. \tag{26}$$

Since this relationship has to hold for every realization of y_t, we need

$$\frac{\lambda\beta_0\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda) = 1 \tag{27}$$

or, for $\beta \neq 1$,

$$\sigma_z^2 = \frac{\lambda(\beta_0 - (1+\beta_0)\lambda)\sigma_\varepsilon^2 - \sigma_v^2}{(1-\lambda)^2(1-\beta)}. \tag{28}$$

Thus, σ_z^2 is pinned down uniquely and it defines the sentiment-driven equilibrium. Note that if $\beta < 1$, then a necessary condition for σ_z^2 to be positive is $\lambda \in \left(0, \frac{\beta_0}{1+\beta_0}\right)$. If $\beta_0 = 1$, this restriction becomes $\lambda \in \left(0, \frac{1}{2}\right)$, as in Propositions 2 or 3. In particular, under the usual Dixit-Stiglitz specification with strategic substitutability across intermediate goods, we have $\beta = (1-\theta) < 0$. Note however that if $\beta > 1$, which may correspond to a special model with externalities, σ_z^2 will be positive if $\lambda \in \left(\frac{\beta_0}{1+\beta_0}, 1\right)$. If $\beta > 1$, firm output will respond more than proportionately to aggregate demand, a situation that may in some sense be unstable. However, in the sentiment-driven equilibrium where firms respond to the imperfect signal of aggregate demand, this more than proportionate response is moderated if the signal is weakly related to aggregate demand, that is if $\lambda \in \left(\frac{\beta_0}{1+\beta_0}, 1\right)$.[12]

4.3 Stability Under Learning

Our model is essentially static, but we can investigate whether the equilibria of the model are stable under adaptive learning. For simplicity we will confine our attention to the simplified abstract model of section (4.2), and also set $\sigma_v^2 = 0$. The signal is $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)\log Y_t$. Together with this signal, equilibrium is defined by equations (21) and (23), where without loss of generality we set $\beta_0 = 1$. This model has two equilibrium solutions , the certainty equilibrium with $\sigma_z^2 = 0$, and, from equation (28), the sunspot equilibrium with $\sigma_z^2 = \frac{\lambda(\beta_0-(1+\beta_0)\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)} = \frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}$. We can renormalize our model so that the sentiment or sunspot shock z_t has unit variance by redefining output as $y_t = \log Y_t = \sigma_z z_t$. The variance of output y_t then is still σ_z^2. Solving for equilibria and rewriting equation (26) with $y_t = \sigma_z z_t$ we have:

$$\sigma_z z_t = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda)\sigma_z z_t. \tag{29}$$

[12] In the knife-edge case where $\beta = 1$ we have a continuum of certainty equilibria since any y_t satisfies (24). Similarly if $\beta = 1$, $\lambda = \frac{\beta_0}{1+\beta_0}$ and $\sigma_v^2 = 0$, there is a continuum of self-fulfilling stochastic equilibria since any σ_z^2 satisfies (27).

We obtain our previous two rational expectations equilibria of section (4.2): (i) $\sigma_z^2 = 0$ and $y_t = 0$; and (ii) $\sigma_z^2 = \frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}$ and $y_t = \left(\frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}\right)^{\frac{1}{2}} z_t$.

We now turn to learning. Suppose that agents understand that equilibrium y_t is proportional to z_t and they try to learn σ_z. If agents conjecture at the beginning of the period t that the constant of proportionality is $\sigma_{zt} = \frac{y_t}{z_t}$, then the realized output is

$$y_t = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_{z_t}^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_{z_t}^2}(1-\lambda)\sigma_{zt}z_t. \tag{30}$$

Under adaptive learning with constant gains $g = 1-\alpha$, agents update σ_{zt}:

$$\begin{aligned} \sigma_{zt+1} &= \alpha\sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t}\right) \\ &= \sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t} - \sigma_{zt}\right) \end{aligned} \tag{31}$$

For any initial $\sigma_{zt} > 0$, we will show that σ_{zt} does not converge to 0, the certainty equilibrium. By contrast the sentiment-driven sunspot equilibrium is stable under learning provided the gain $g = 1-\alpha$ is not too large.

The dynamics of σ_{zt} are given by:

$$\begin{aligned} \sigma_{zt+1} &= \alpha\sigma_{zt} + (1-\alpha)\left(\frac{y_t}{z_t}\right) \\ &= \alpha\sigma_{zt} + (1-\alpha)\frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)\beta\sigma_{zt}^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_{zt}^2}(1-\lambda)\sigma_{zt}, \end{aligned} \tag{32}$$

Let $h(\sigma_z) = \alpha\sigma_z + (1-\alpha)\frac{\lambda\sigma_\varepsilon^2+(1-\lambda)\beta\sigma_z^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\sigma_z^2}(1-\lambda)\sigma_z$. So we then have

$$\sigma_{zt+1} = h(\sigma_{zt}). \tag{33}$$

There are two solutions to the above fixed point problem, $\sigma_z = \sqrt{\frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}}$ and $\sigma_z = 0$. We have

$$h'(0) = \alpha + (1-\alpha)\frac{(1-\lambda)}{\lambda} > 1 \tag{34}$$

as $\lambda < 1/2$. It follows that the certainty equilibrium $\sigma_z = 0$ is not stable. Any initial belief of $\sigma_{zt} > 0$ will lead the economy away from the fundamental certainty equilibrium.

To check the stability of the sentiment-driven equilibrium we evaluate $h'(\sigma_z)$ at $\sigma_z = \sqrt{\frac{\lambda(1-2\lambda)\sigma_\varepsilon^2}{(1-\lambda)^2(1-\beta)}}$. This yields

$$h'(\sigma_z) = 1 - (1-\alpha)2\sigma_z \frac{(1-\lambda)^2(1-\beta)^2}{\left[(1-\beta)\lambda^2\sigma_\varepsilon^2 + \lambda(1-2\lambda)\sigma_\varepsilon^2\right]}. \tag{35}$$

So the sentiment-driven or sunspot equilibrium is stable under learning if $|h'(\sigma_z)| < 1$. This will be true if the gain $1-\alpha$ is sufficiently small.

5 Microfoundations for the signals

Since intermediate goods firms make employment and production decisions prior to the realization of market clearing prices, they have to form expectations about their demand and about the real wage. So far we simply assumed that firms receive signals of the type given in equation (7) based on their market research, market surveys, early orders, initial inquiries and advanced sales, to form such expectations. In particular, we assumed that the signals obtained by intermediate goods firms consist of a weighted sum of the fundamental and the sentiment shocks, and that the relative weights λ and $(1-\lambda)$ attached to these shocks are exogenous. It is therefore desirable to spell out in more detail the microfoundations of how firms can obtain these signals.

We begin with a special signal that reveals, except for *iid* noise, the precise weighted sum of the fundamental and the sentiment shock that the intermediate goods firms need to know to make its optimal production and employment decisions. Such a signal, under the simple informational frictions of our model, eliminates the signal extraction problem of the firm and excludes the possibility of sentiment-driven equilibria.[13]

To see this note that the demand curve of firm j is given by $Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} Y_t \epsilon_{jt}$. Since from the labor market first order conditions we have $P_t = \frac{1}{\psi Y_t}$, the logarithm of the demand curve, ignoring constants that can be filtered, becomes $y_{jt} = \varepsilon_{jt} + (1-\theta)\, y_t - \theta p_{jt}$. Suppose firm j can post a hypothetical price $\tilde{p}_{jt}$ and ask a subset of consumers about their intended demand given this hypothetical price. The firms can then obtain a signal about the intercept of their demand curve, possibly with some noise v_{jt} if consumers have heterogenous sentiments, $s_{jt} = \varepsilon_{jt} + (1-\theta)\, y_t + v_{jt}$.[14] The optimal output decision of firms is given by

$$y_{jt} = E\left[\varepsilon_{jt} + (1-\theta)y_t\right]|\, s_{jt} = \frac{\sigma_s^2 - \sigma_v^2}{\sigma_s^2}\left[\varepsilon_{jt} + (1-\theta)y_t + v_{jt}\right] \tag{36}$$

where σ_s^2 is the variance of the signal.[15] Integrating across firms, $y_t = \int y_{jt}dj$, and equating coefficients of y_t yields $1 = \frac{\sigma_s^2 - \sigma_v^2}{\sigma_s^2}(1-\theta)$. This equality is impossible (even if $\sigma_v^2 = 0$) since by

[13] We thank an anonymous referee for pointing this out to us.

[14] See Benhabib, Wang and Wen (2013) for the case where consumer sentiments are heterogenous but correlated.

[15] Note that $\sigma_s^2 - \sigma_v^2 > 0$ is the variance of $\varepsilon_{jt} + (1-\theta)y_t$, or $Cov\left(\varepsilon_{jt} + (1-\theta)y, s_{jt}\right)$.

construction $\sigma_s^2 > \sigma_v^2$ and $\theta > 1$.[16] In other words, a constant output with $y_t = 0$ is the only equilibrium.

To restore the possibility of sentiment-driven equilibria, we can either slightly complicate the signal extraction problem of the firm by adding an extra source of uncertainty, or we can modify the signal so that it does not eliminate the signal extraction problem faced by the firm. We provide microfoundations for both of these approaches below.

First we study a model with an additional source of uncertainty. We still allow a firm to post a hypothetical price $\tilde{p}_{jt}$ and to ask a subset of consumers about their intended demand at that hypothetical price. However at the time of the survey the preference shock is not yet realized with certainty: each consumer i receives a signal for his/her preference shock ϵ_{jt}: $s_{ht}^{j} = \varepsilon_{jt} + h_{jt}^{i}$ which forms the basis of their response to the posted hypothetical price. This extra source of uncertainty now enters the demand signal received by the intermediate goods firms, and re-establishes the sentiment-driven equilibria.

In our second approach the firms, instead of learning the demand for their good for a particular hypothetical posted price, receive a signal from consumers about the quantity of the demand for their good. Consumers, responding to demand surveys do so on the basis of their expectations of equilibrium prices. Firms therefore still face downward sloping demand curves, but the signal transmitted to them is now only a quantity signal. Therefore they still have to optimally extract from their signal the magnitude of the fundamental and sentiment shocks because the realization of real prices and real wages depend on the relative magnitude of these shocks. We show that (i) a continuum of endogenous sentiment-driven equilibria arise in this setup even if firms can observe the quantity of their demand perfectly (i.e., even if their signal is $s_{jt} = y_{jt}$);[17] and (ii) the signal $s_{jt} = y_{jt}$ is isomorphic to that specified in equation (7) with $\lambda \in \left(0, \frac{1}{2}\right)$.

Finally we construct another case where we introduce extra uncertainty on the cost side, where a cost shock is correlated with the preference or intermediate-good demand shock, possibly because a high demand may affect marketing or sales costs for the firm.

5.1 Consumer Uncertainty

To re-establish sentiment-driven equilibria when firms can extract information about the intercept of their demand curves by posting hypothetical prices, we introduce an additional informational friction into the benchmark model. Consider the aggregate utility function, $\frac{C_t^{1-\gamma}-1}{1-\gamma} - \psi N_t$, or al-

[16] In general, a model with $y_{jt} = E\left[\beta_0\varepsilon_{jt} + \beta y_t\right] | \left[\beta_0\varepsilon_{jt} + \beta y_t + v_{jt}\right]$ and $y_t = \int y_{jt} dj$ cannot have sentiment-driven equilibria. To see that we have $y_{jt} = \frac{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2}{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2 + \sigma_v^2}\left[\beta_0\varepsilon_{jt} + \beta y_t + v_{jt}\right]$. A sunspot equilibrium requires $1 = \frac{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2}{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2 + \sigma_v^2}\beta$, which is impossible since $\beta < 1$ and $0 < \frac{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2}{\beta_0^2\sigma_\varepsilon^2 + \beta^2\sigma_z^2 + \sigma_v^2} < 1$.

[17] We keep the notation that lower-case letters denote the demeaned logarithm of the capital-letter variables.

ternatively the utility function $\log(C_t - \psi\frac{N_t^{1+\gamma}}{1+\gamma})$, where $C_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} C_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$. Note that here without loss of generality we bypass the final good sector and use a utility aggregator for consumption goods C_{jt}. For notational convenience and consistency we continue to use $Y_t \equiv C_t$ and $Y_{jt} = C_{jt}$ in this and subsequent sections. Both utility functions yield, using the first order conditions in the labor market, the same first order conditions in the labor market, $p_t \equiv -\gamma y_t$, where $\gamma \neq 1$ ($\gamma = 1$ corresponds to our benchmark model). There are a continuum of identical consumers indexed by $i \in [0,1]$, as before. Now suppose each firm j conducts early market surveys by posting a hypothetical price p_{jt} to consumer i. As before, consumers' demand for good j can be affected both by the aggregate sunspot $Z_t = Y_t$ and the variety-specific preference shock ϵ_{jt}. However, we assume that at the moment of the survey, the preference shock ϵ_{jt} is not yet established with certainty, but that each consumer i receives a signal for his/her preference shock ϵ_{jt}: $s_{ht}^{j} = \varepsilon_{jt} + h_{jt}^{i}$. So if firm j posts a hypothetical price $\tilde{P}_{jt}$ to consumer i, the demand for variety j at the posted price $\tilde{P}_{jt}$ will be

$$\tilde{Y}_{jt}^{i} = \left(\frac{\tilde{P}_{jt}}{P_t}\right)^{-\theta} Y_t \left(E\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)\Big| \left(\varepsilon_{jt} + h_{jt}^{i}\right)\right]\right)^{\theta}. \tag{37}$$

Notice that all consumers $i \in [0,1]$ are identical up to their idiosyncratic signal h_{jt}^{i}. Aggregating across the consumers yields

$$\tilde{Y}_{jt} = \left(\frac{\tilde{P}_{jt}}{P_t}\right)^{-\theta} Y_t \int_0^1 \left(E\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)\Big| \left(\varepsilon_{jt} + h_{jt}^{i}\right)\right]\right)^{\theta} di. \tag{38}$$

Using $p_t = -\gamma y_t$, the intercept of the demand curve for variety j (in logarithm) is given by

$$(1-\theta\gamma)\, y_t + \int_0^1 E\left[\varepsilon_{jt} | \left(\varepsilon_{jt} + h_{jt}^{i}\right)\right] dj = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_h^2}\varepsilon_{jt} + (1-\theta\gamma)\, y_t. \tag{39}$$

Hence, the signal firm j can obtain through its market surveys is $s_{jt} = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2}\varepsilon_{jt} + (1-\theta\gamma)y_t$, which is isomorphic to

$$s_{jt} = \frac{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2}}{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2} + (1-\theta\gamma)}\varepsilon_{jt} + \left[1 - \frac{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2}}{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2} + (1-\theta\gamma)}\right] y_t. \tag{40}$$

Clearly, for sentiment-driven equilibria to exist as required by the Propositions of section 4 we need

$$\lambda \equiv \frac{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2}}{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2} + (1-\theta\gamma)} \in \left(0, \frac{1}{2}\right) \tag{41}$$

which will hold if $1-\theta\gamma > \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_h^2}$.[18]

5.2 Quantity Signal

Instead of introducing additional sources of uncertainty to recapture sentiment-driven equilibria, we instead modify the signals so that they do not eliminate the signal extraction problem faced by firms. As discussed earlier in this section suppose the intermediate-good firms, instead of learning the demand for their good for a particular hypothetical posted price, receive a signal from consumers about the quantity of the demand Y_{jt} for their good. As in section 5.1, we drop the final-good sector and assume instead that the representative household purchases a variety of goods C_{jt} to maximize utility $\log C_t - \psi N_t$, where $C_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} C_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$. The households choose their demand C_{jt} and labor supply N_{jt} based on the sentiment shock Z_t and the preference shocks ϵ_{jt}. Note that in equilibrium $C_{jt} = Y_{jt}$. The household demand function for each variety is:

$$C_{jt} = \left(\frac{P_t}{P_{jt}}\right)^{\theta} \epsilon_{jt} C_t = \left(\frac{P_t}{P_{jt}}\right)^{\theta} \epsilon_{jt} Y_t. \tag{42}$$

Since demand depends on the consumers' conjectures about their price $P_{jt} = P_t(\epsilon_{jt}, Z_t)$, the demand C_{jt} is a function of preference shocks and the sentiment, $C_{jt} = C(\epsilon_{jt}, Z_t)$. The first order condition for labor, as in the benchmark model, is $C_t = \frac{1}{\psi}\frac{1}{P_t}$, with the nominal wage normalization $W_t = 1$.

The intermediate-good firms, based on the signal $s_{jt} = c_{jt}$ choose their production according to the first order condition given by

$$C_{jt} = Y_{jt} = \left\{\left(1-\frac{1}{\theta}\right)\frac{A}{\psi} E_t\left[\epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} \middle| S_{jt}\right]\right\}^{\theta}, \tag{43}$$

We conjecture that in equilibrium,

$$c_{jt} = \phi\varepsilon_{jt} + \phi_z z_t \tag{44}$$

where ϕ and ϕ_z are undetermined coefficients. Under the assumption of lognormal distributions, our model is log-linear. So log-linearizing equation (43) yields

$$\begin{aligned} c_{jt} &= E_t\left[\left(\varepsilon_{jt} + \beta y_t\right)|c_{jt}\right] \\ &= E_t\left[\left(\varepsilon_{jt} + \beta y_t\right)|\left(\phi\varepsilon_{jt} + \phi_z z_t\right)\right], \end{aligned} \tag{45}$$

[18] This restriction on the value of γ can be further relaxed if we extend our model to allow for heterogenous labor supply so that each intermediate goods firm faces its own wage rate.

where $\beta \equiv 1-\theta$. By definition aggregate output is $y_t = \frac{1}{\left(1-\frac{1}{\theta}\right)} \int_0^1 \left(\frac{1}{\theta}\varepsilon_{jt} + \left(1-\frac{1}{\theta}\right) y_{jt}\right) dj = \int y_{jt} dj$. Finally, we require consumers to have correct endogenous sentiments: for each realization of z_t,

$$y_t = z_t. \tag{46}$$

Equations (44) and (45) imply

$$\begin{aligned} \phi\varepsilon_{jt} + \phi_z z_t &= E\left[(\varepsilon_{jt} + \beta z_t) \mid (\phi\varepsilon_{jt} + \phi_z z_t)\right] \qquad (47) \\ &= \frac{\phi\sigma_\varepsilon^2 + \beta\phi_z\sigma_z^2}{\phi^2\sigma_\varepsilon^2 + \phi_z^2\sigma_z^2}\left(\phi\varepsilon_{jt} + \phi_z z_t\right). \end{aligned}$$

Equating coefficients we have $\frac{\phi\sigma_\varepsilon^2 + \beta\phi_z\sigma_z^2}{\phi^2\sigma_\varepsilon^2 + \phi_z^2\sigma_z^2} = 1$. Note that integrating equation (44) and using (46) we have $\phi_z = 1$. Hence we can solve

$$\sigma_z^2 = \frac{\phi(1-\phi)}{1-\beta}\sigma_\varepsilon^2, \tag{48}$$

where $\beta \equiv 1-\theta < 1$. So for sentiment-driven equilibria to exist with $\sigma_z^2 > 0$, ϕ can take any value in the interval $[0,1]$. However, the value of σ_z^2 is determined in the interval $\phi \in \left[0, \frac{1}{2}\right]$ because $\arg\max \phi(1-\phi) = \frac{1}{2}$. Therefore, since ϕ is indeterminate in the interval $\left[0, \frac{1}{2}\right]$, we have the following Proposition:

Proposition 4 *There is a continuum of sentiment-driven equilibria indexed by* $\sigma_z^2 \in \left[0, \frac{1}{4(1-\beta)}\sigma_\varepsilon^2\right]$.

This establishes that given the other parameters of the model, the existence of sentiment-driven equilibria is robust to perturbations of σ_z^2 within the range $\sigma_z^2 \in \left[0, \frac{1}{4(1-\beta)}\sigma_\varepsilon^2\right]$.[19] To solve for the prices, note that c_{jt} satisfies the household's first-order conditions (42) and (3), which (taking logs) can be written as

$$\begin{aligned} c_{jt} &= \theta(p_t - p_{jt}) + \varepsilon_{jt} + c_t \qquad (49) \\ &= -\theta p_{jt} + \varepsilon_{jt} + (1-\theta) y_t, \end{aligned}$$

which can be used to solve for p_{jt} and p_t.

Finally in the next proposition we show that the equilibria of this model can be mapped into the equilibria of our benchmark model parametrized by $\lambda = \frac{\phi}{\phi+1}$:

[19] The results on the continuum of equilbria also hold if the signal is not on the firm specific demand Y_{jt}, but on aggregate demand Y_t. See Benhabib, Wang and Wen (2013).

Proposition 5 *The sentiment-driven equilibria of this model with signal $s_{jt} = c_{jt}$ can be mapped one-to-one to the sentiment-driven equilibria of our benchmark model with the signal $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t$.*

Proof. *First, we scale the signal by a constant, namely*

$$\phi\varepsilon_{jt} + z_t \Leftrightarrow \frac{\phi}{\phi+1}\varepsilon_{jt} + \frac{1}{\phi+1}y_t. \tag{50}$$

Second, define $\frac{\phi}{\phi+1} = \lambda$, so that $\phi = \frac{\lambda}{1-\lambda}$. It follows that

$$\sigma_z^2 = \frac{1}{(1-\beta)}(\phi - \phi^2)\sigma_\varepsilon^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2, \tag{51}$$

which is exactly the result of Proposition 2. Notice that $\lambda \in \left(0, \frac{1}{2}\right)$ is equivalent to $\phi \in (0,1)$. ■

The sentiment-driven equilibria are driven by fundamental shocks ε_{jt} and sentiments z_t, but in addition, they may also depend of firm-specific *iid* noise, v_{jt}, as shown in the following Proposition:

Proposition 6 *Under the signal $s_{jt} = c_{jt}$, there also exists another type of sentiment-driven equilibria with firm-level output driven not only by the fundamental shock ε_{jt} and aggregate sentiment shock z_t, but also by a firm-specific iid shock v_{jt} with zero mean and variance σ_v^2:*

$$c_{jt} = \phi\varepsilon_{jt} + z_t + (1+\phi)v_{jt}. \tag{52}$$

Furthermore the signal $s_{jt} = c_{jt}$ is isomorphic to the signal $s_{jt} = \lambda \log\epsilon_{jt} + (1-\lambda)y_t + v_{jt}$ in equation (7).

Proof. *Given the signal $s_{jt} = c_{jt}$, conjecture that $c_{jt} = \phi\varepsilon_{jt} + z_t + (1+\phi)v_{jt}$. The firm's first-order condition in equation (45) becomes*

$$\begin{aligned}
\phi\varepsilon_{jt} + z_t + (1+\phi)v_{jt} &= E\left[(\varepsilon_{jt} + \beta z_t)\right| \left(\phi\varepsilon_{jt} + z_t + (1+\phi)\,v_{jt}\right)\right], \qquad (53)\\
&= \frac{\phi\sigma_\varepsilon^2 + \beta\sigma_z^2}{\phi^2\sigma_\varepsilon^2 + \sigma_z^2 + (1+\phi)\sigma_v^2}\left(\phi\varepsilon_{jt} + z_t + (1+\phi)\,v_{jt}\right).
\end{aligned}$$

Comparing coefficients gives $\frac{\phi\sigma_\varepsilon^2 + \beta\sigma_z^2}{\phi^2\sigma_\varepsilon^2 + \sigma_z^2 + (1+\phi)^2\sigma_v^2} = 1$, or

$$\sigma_z^2 = \frac{\phi(1-\phi)\sigma_\varepsilon^2 - (1+\phi)^2\sigma_v^2}{(1-\beta)}. \tag{54}$$

Hence, there exists an continuum of sentiment-driven equilibria with $\sigma_z^2 = \frac{\phi(1-\phi)-(1+\phi)^2\sigma_v^2}{\theta}$ *and* $\sigma_v^2 \leq \frac{\phi(1-\phi)}{(1+\phi)^2}\sigma_\varepsilon^2$*. Let* $\lambda = \frac{\phi}{\phi+1}$ *or* $\phi = \frac{\lambda}{1-\lambda}$*. It follows that*

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)\sigma_\varepsilon^2 - \sigma_v^2}{\theta(1-\lambda)^2}, \quad \sigma_v^2 \leq \lambda(1-2\lambda)\sigma_\varepsilon^2; \tag{55}$$

which exactly correspond to the results of Proposition 3. Since sunspots can exist only for $\phi \in (0,1)$*, we then require* $\lambda \in \left(0, \frac{1}{2}\right)$*. Hence, the signal* $s_{jt} = c_{jt}$ *is isomorphic to the signal* $s_{jt} = \lambda \log \epsilon_{jt} + (1-\lambda)y_t + v_{jt}$ *in equation (7).* ■

5.3 Cost Shocks

As in the case of consumer uncertainty of section 5.1, we assume that the household utility function is $\frac{C_t^{1-\gamma}-1}{1-\gamma} - \psi N_t$. Each firm j's total production cost (or labor productivity) is affected by an idiosyncratic shock that is correlated with the demand shock ϵ_{jt}. For example marketing costs may be lower under favorable demand conditions: for a higher amount of sales, labor becomes more productive so that labor demand $N_{jt} = Y_{jt}\epsilon_{jt}^{-\tau}$ is lower, where $\tau > 0$ is a parameter. Alternatively, if marketing costs increase with sales and labor demand is higher, we may have $\tau < 0$.

Firm j's problem is:

$$\max_{Y_{jt}} E_t \left[\left(P_t Y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} \epsilon_{jt}^{-\tau} Y_{jt} \right) \middle| S_{jt} \right]. \tag{56}$$

The first-order condition is

$$Y_{jt} = \left\{ \left(1 - \frac{1}{\theta}\right) \frac{A}{\psi E_t \left[\epsilon_{jt}^{-\tau} | s_{jt}\right]} E_t \left[\epsilon_{jt}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-\gamma} \middle| S_{jt} \right] \right\}^{\theta}, \tag{57}$$

which (after taking logs and filtering out constants) may be written as

$$y_{jt} = E_t \left[((1+\theta\tau)\varepsilon_{jt} + (1-\theta\gamma)y_t) | \, s_{jt} \right]. \tag{58}$$

Now assume that firms can then obtain a signal on the intercept of their demand curve with a noise η_{jt}:

$$s_{jt} = \varepsilon_{jt} + (1-\theta\gamma)y_t + \eta_{jt}. \tag{59}$$

Using this signal, even assuming firms can gain perfect information with $\eta_{jt} \equiv 0$, the first-order condition becomes

$$y_{jt} = E_t \left[((1+\theta\tau)\varepsilon_{jt} + (1-\theta\gamma)y_t) | \, (\varepsilon_{jt} + (1-\theta\gamma)y_t) \right]. \tag{60}$$

Hence, we obtain

$$y_{jt} = \frac{(1+\theta\tau)\sigma_\varepsilon^2 + (1-\theta\gamma)^2\sigma_z^2}{\sigma_\varepsilon^2 + (1-\theta\gamma)^2\sigma_z^2} \left(\varepsilon_{jt} + (1-\theta\gamma)y_t\right). \tag{61}$$

Integration yields $y_t = \frac{(1-\theta\gamma)^2\sigma_z^2+(1+\theta\tau)\sigma_\varepsilon^2}{\sigma_\varepsilon^2+(1-\theta\gamma)^2\sigma_z^2}(1-\theta\gamma)y_t$, or $\frac{(1-\theta\gamma)^2\sigma_z^2+(1+\theta\tau)\sigma_\varepsilon^2}{\sigma_\varepsilon^2+(1-\theta\gamma)^2\sigma_z^2}(1-\theta\gamma) = 1$. Define $\beta = (1-\theta\gamma) < 1$, we have

$$\sigma_z^2 = \frac{[(1+\theta\tau)\beta - 1]}{\beta^2(1-\beta)}\sigma_\varepsilon^2. \tag{62}$$

For sentiment-driven equilibria to exist we need

$$(1+\theta\tau)\beta > 1. \tag{63}$$

Notice that if $\tau = 0$ (as in our benchmark model), sentiment-driven equilibria will not be possible, but they can exist if $\tau \neq 0$. In that case, the sign of τ depends the sign of β: If $\beta > 0$ (the case where firm output and aggregate output are complements) , we need $\tau > \frac{1-\beta}{\beta\theta}$. If $\beta < 0$ (the case where firm output and aggregate output are substitutes) we need $\tau < \frac{1-\beta}{\beta\theta}$ and τ can be negative.

6 Extensions

6.1 Price-Setting

So far we considered cases where firms decide how much to produce before they know their demand. We now briefly consider the case where intermediate goods firms must set prices first and commit to meeting demand at the announced prices.[20] The Dixit-Stiglitz structure of our model implies that the optimal price for an intermediate goods firm under perfect information is

$$P_{jt} = \frac{\theta}{\theta - 1}W_t. \tag{64}$$

Note that whether we normalize the final goods price or nominal wages to be unity, the optimal price does not depend on idiosyncratic preference shocks. Sentiment-driven equilibria cannot exist as firms do not face signal extraction problems. Therefore we assume, as in section (5.3), that the firm's costs are positively correlated with firm's demand. We use the final good price as the numeraire.[21] The firm's problem is

$$\max_{Y_{jt}} E_t\left[\left(P_{jt}^{1-\theta}\epsilon_{jt}Y_t - \frac{W_t}{A}\epsilon_{jt}^{-\tau}\left(P_{jt}^{-\theta}\epsilon_{jt}Y_t\right)\right)\middle| s_{jt}\right], \tag{65}$$

[20] In models where money plays a role and agents choose to hold money, rigidities in price-seeting can be adressed via monetary policy to alleviate or eliminate inefficient equilibria. We do not have money in our simple model of price setting, so we cannot explore the role of monetary policy.

[21] Our result also holds if we use the nominal wage as the numeraire.

where we substitute $N_{jt} = Y_{jt}/\epsilon_{jt}^{\tau}$ and $Y_{jt} = P_{jt}^{-\theta}\epsilon_{jt}Y_t$. The optimal price is then

$$(\theta-1)P_{jt}^{-\theta}E_t\left[\epsilon_{jt}Y_t|S_{jt}\right] = \theta P_{jt}^{-\theta-1}E_t\left[W_t\epsilon_{jt}^{1-\tau}Y_t|S_{jt}\right]. \tag{66}$$

That is, $P_{jt} = \frac{\theta}{\theta-1}\frac{E_t\left[W_t\epsilon_{jt}^{1-\tau}Y_t|S_{jt}\right]}{E_t\left[\epsilon_{jt}Y_t|S_{jt}\right]}$. Since from the first order condition for labor supply we have $W_t = \psi Y_t^{\gamma}$, taking logs leads to

$$p_{jt} = E\left[\left(\gamma y_t - \tau\varepsilon_{jt}\right)|s_{jt}\right], \tag{67}$$

where $\varepsilon_{jt} \equiv \log\epsilon_{jt}$. The aggregate price index is normalized to unity $P_t = \left[\int \epsilon_{jt}P_{jt}^{1-\theta}dj\right]^{\frac{1}{1-\theta}} = 1$, which implies that $\int_0^1 p_{jt}dj = 0$. Notice that since $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t$, we have

$$p_{jt} = \frac{\gamma(1-\lambda)\sigma_z^2 - \tau\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{jt} + (1-\lambda)y_t). \tag{68}$$

Then sentiment-driven equilibria require $\gamma(1-\lambda)\sigma_z^2 - \tau\sigma_\varepsilon^2 = 0$, or

$$\sigma_z^2 = \frac{\tau\sigma_\varepsilon^2}{\gamma(1-\lambda)} > 0, \tag{69}$$

which holds for any γ and λ. Note that here even if firms can post a price $\tilde{P}_{jt}$ and obtain the intercept term in the demand curve $\tilde{Y}_{jt} = \tilde{P}_{jt}^{-\theta}\epsilon_{jt}Y_t$ which reveals the sum $\varepsilon_{jt} + y_t$, sentiment-driven equilibria will still exist.

6.2 Persistence

Persistence in output can be introduced in a variety of ways. The simplest way is to note that the productivity parameter A in the benchmark model of section 2 can be a stochastic process that is observed at the beginning of each period. Since periods are independent of time the equilibrium over time would be driven by the stochastic process for A.[22]

Finally, in a model with aggregate fundamental preference shocks (or final good productivity shocks), we may assume that the fundamental shock is a stochastic process but that intermediate goods firms only observe aggregate demand C_t and its history. They do not separately observe the past or present values of the preference shocks or sentiments Z_t. Then equilibrium output will also be persistent, as shown in Benhabib, Wang and Wen (2013).

[22] Another approach to obtain persistence, making use of the multiplicity of equilibria, is to introduce a simple Markov sunspot process that selects the equilibria in each period.

7 Conclusion

In their discussion of correlated and sunspot equilibria, Aumann, Peck and Shell (1988) note: "Even if economic fundamentals were certain, economic outcomes would still be random... Each economic actor is uncertain about the strategies of the others. Business people, for example, are uncertain about the plans of their customers... This type of economic randomness is generated by the market economy: it is thus endogenous to the economy, but extrinsic to the economic fundamentals." Along similar lines, we explore the Keynesian idea that sentiments or animal spirits can influence the level of aggregate income and give rise to recurrent boom-bust cycles. In particular, we show that when consumption and production decisions must be made separately by consumers and firms who are uncertain of each other's plans, the equilibrium outcome can indeed be influenced by animal spirits or sentiments, even though all agents are fully rational. The key to generating our results is a natural friction in information: Even if firms can perfectly observe or forecast demand, they cannot separately identify the components of demand stemming from consumer sentiments as opposed to preference or fundamental shocks. Sentiments matter because they are correlated across households, and they affect aggregate demand and real wages differently than shocks to productivity or preferences. Faced with a signal extraction problem, firms make optimal production decisions that depend on the degree of sentiment uncertainty, or the variance of sentiment shocks. Such sentiment shocks can give rise to sentiment-driven rational expectations equilibria in addition to equilibria driven solely by fundamentals. We show that in a production economy, pure sentiments, completely unrelated to fundamentals, can indeed affect economic performance and the business cycle even though (i) expectations are fully rational and (ii) there are no externalities, non-convexities or even strategic complementarities in production. Furthermore, in our model with microfounded signals there can exist a continuum of (normally distributed) sentiment shocks, indexed by their variances, that give rise to sentiment-driven rational expectations equilibria. Such self-fulfilling sunspot equilibria are not based on randomizations over fundamental equilibria, and they are stable under constant gain learning if the gain parameter is not too large.

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A Appendix

This Appendix provides brief proofs for Propositions 1-3 in the paper. More detailed proofs can be found in our NBER working paper (Benhabib, Wang and Wen, 2012).

A. 1 Proofs of Proposition 1 and Proposition 2

Proof. We start with the proof of Proposition 2, and give the proof of Proposition 1 further below.

1. **The Sentiment-Driven Equilibrium**. Let $s_{jt} = v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t$. Firms conjecture that output is equal to

$$\log Y_t = y_t = \phi_0 + z_t, \tag{A.1}$$

where ϕ_0, and σ_z^2 are constants to be determined. The optimal output of a firm can be written as

$$y_{jt} = (1-\theta)\phi_0 + \theta \log E_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) \middle| s_{jt} \right]$$

Note that

$$E_t \left[\exp(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t) \middle| s_{jt} \right] = \exp\left(E\left[\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) \middle| s_{jt} \right] + \frac{1}{2} var\left(\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) \middle| s_{jt} \right) \right) \tag{A.2}$$

where

$$\begin{aligned} E\left[\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) \middle| s_{jt} \right] &= \frac{cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt} \right)}{var\left(s_{jt} \right)} s_{jt} \\ &= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t). \end{aligned} \tag{A.3}$$

Denote the conditional variance by

$$\Omega_s = var\left(\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) \middle| s_{jt} \right). \tag{A.4}$$

Since $\frac{1}{\theta}\varepsilon_{jt}$ and $\frac{1-\theta}{\theta} z_t$ are Gaussian, the conditional variance Ω_s will not depend on the observed s_{jt} and will be given by

$$\Omega_s = var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t \right) - \frac{\left(cov\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta} z_t, s_{jt} \right) \right)^2}{var\left(v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t \right)}. \tag{A.5}$$

We then have

$$
\begin{aligned}
y_{jt} &= (1-\theta)\phi_0 + \theta \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t) + \frac{\theta}{2}\Omega_s \qquad \text{(A.6)} \\
&\equiv \varphi_0 + \theta\mu(v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t)
\end{aligned}
$$

where

$$
\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} \qquad \text{(A.7)}
$$

$$
\varphi_0 = (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s \qquad \text{(A.8)}
$$

Now for equilibrium to hold we need aggregate demand to equal the output of the final good. From equation (4), markets will clear if for each z_t we have

$$
\begin{aligned}
\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) &= \log \int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}} dj \qquad \text{(A.9)} \\
&= \log E \exp\left[\frac{1}{\theta}\varepsilon_t + \left(1-\frac{1}{\theta}\right)[\varphi_0 + \theta\mu(v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda) z_t)]\right] \\
&= \left(1-\frac{1}{\theta}\right)\varphi_0 + \left[\left(1-\frac{1}{\theta}\right)\theta\mu(1-\lambda)\right] z_t \\
&\quad + \frac{1}{2}\left[\frac{1}{\theta} + \left(1-\frac{1}{\theta}\right)\theta\mu\lambda\right]^2 \sigma_\varepsilon^2 + \frac{1}{2}\left[\left(1-\frac{1}{\theta}\right)\theta\mu\right]^2 \sigma_v^2
\end{aligned}
$$

Matching the coefficients yields two constraints: If $\mu \neq 0$, then

$$
\theta\mu = \frac{1}{1-\lambda}, \qquad \text{(A.10)}
$$

and

$$
\phi_0 = \varphi_0 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left(\frac{1}{\theta-1} + \theta\mu\lambda\right)^2 \sigma_\varepsilon^2 + (\theta\mu)^2\sigma_v^2\right) \qquad \text{(A.11)}
$$

Notice $\theta\mu = \frac{1}{1-\lambda}$ (when $\mu \neq 0$) implies

$$
\theta\mu = \theta \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} = \frac{1}{1-\lambda} \qquad \text{(A.12)}
$$

or we have

$$
\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2. \qquad \text{(A.13)}
$$

Notice that if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda(1-2\lambda)\sigma_\varepsilon^2$, then $\sigma_z^2 < 0$, suggesting that the only equilibrium is $z = 0$. Hence, to have a self-fulfilling expectations equilibrium, we require $\lambda \in \left(0, \frac{1}{2}\right)$ and $\sigma_v^2 < \lambda(1-2\lambda)\sigma_\varepsilon^2$. This pins down σ_z^2, the variance of z or of output as a function of σ_ε^2 and σ_v^2. Note that introducing the noise v_{jt} into the signal makes output in the self-fulfilling equilibrium less noisy: If the signal was $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda) z_t$, then we would have $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$. The reason is that the signal is now more noisy, and firms attribute a smaller fraction of the signal to demand fluctuations.

Now we consider the two constants ϕ_0 and φ_0. First, using (A.12), we have

$$\begin{aligned} \Omega_s &= var\left(\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}z_t\right)\Big| s_{jt}\right) \\ &= \left[\left(\frac{1}{\theta}\right)^2 - \frac{1}{1-\lambda}\frac{1}{\theta^2}\lambda\right]\sigma_\varepsilon^2 + \left[\left(\frac{1-\theta}{\theta}\right)^2 + \frac{\theta-1}{\theta^2}\right]\sigma_z^2 \end{aligned} \tag{A.14}$$

Since $\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2\theta}\sigma_v^2$ from (A.13), we have

$$\Omega_s = \frac{(1-\lambda+(\theta-1)\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\sigma_v^2}{\theta^2(1-\lambda)^2} \tag{A.15}$$

Then, from equation (A.8),

$$\varphi_0 = (1-\theta)\phi_0 + \frac{1}{2\theta}\frac{(1-\lambda+(\theta-1)\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\sigma_v^2}{(1-\lambda)^2}.$$

From equation (A.11) we have,

$$\phi_0 = \varphi_0 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left(\frac{1}{\theta-1} + \theta\mu\lambda\right)^2\sigma_\varepsilon^2 + (\theta\mu)^2\sigma_v^2\right). \tag{A.16}$$

Combining these implies

$$\begin{aligned} \phi_0 &= \frac{1}{2}\frac{(1-\lambda+(\theta-1)\lambda)(1-2\lambda)\sigma_\varepsilon^2 - (\theta-1)\sigma_v^2}{\theta^2(1-\lambda)^2} \\ &\quad + \frac{1}{2}\frac{1}{\theta-1}[\frac{1}{\theta} + (1-\frac{1}{\theta})\theta\mu\lambda]^2\sigma_\varepsilon^2. \end{aligned} \tag{A.17}$$

Simplifying further gives,

$$\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)}\frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}. \tag{A.18}$$

Therefore the outputs of intermediate goods firms, conditioned on signals $s_{jt} = v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)z_t$, are given by

$$y_{jt} \equiv \varphi_0 + \theta\mu(v_{jt} + \lambda\varepsilon_{jt} + (1-\lambda)z_t). \tag{A.19}$$

They constitute a market clearing stochastic rational expectations equilibrium. We now turn to the proof of Proposition 1.

2. The Certainty Equilibrium. Firms take aggregate output as constant, so $z_t = 0$ and $\log Y_t = y_t = \phi_0$, but the signal $s_{jt} = \lambda\varepsilon_{jt} + v_{jt}$ gives them imperfect information on their idiosyncratic shock. We can compute the certainty equilibrium by setting $z_t = \sigma_z^2 = 0$, and we have

$$\mu = \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2}{\sigma_v^2 + \lambda^2\sigma_\varepsilon^2} \tag{A.20}$$

$$\Omega_s = var(\frac{1}{\theta}\varepsilon_{jt}|s_{jt}] = \left(\frac{1}{\theta}\right)^2 (1-\mu\theta\lambda)\,\sigma_\varepsilon^2 \tag{A.21}$$

$$\varphi_0 = (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s = (1-\theta)\phi_0 + \frac{\theta}{2}\left(\frac{1}{\theta}\right)^2 (1-\mu\theta\lambda)\,\sigma_\varepsilon^2 \tag{A.22}$$

$$\phi_0 = \varphi_0 + \frac{\theta-1}{\theta}\frac{1}{2}\left(\left[\frac{1}{\theta-1} + \theta\mu\lambda\right]^2 \sigma_\varepsilon^2 + (\theta\mu)^2\,\sigma_v^2\right) \tag{A.23}$$

so that

$$\phi_0 = \frac{1}{2}\left[\left(\frac{\theta + \theta\mu\lambda\,(\theta-1) + (\theta\mu\lambda\,(\theta-1))^2}{\theta^2\,(\theta-1)}\right)\sigma_\varepsilon^2 + (\theta-1)\,(\theta\mu)^2\,\sigma_v^2\right]. \tag{A.24}$$

■

A. 2 Proof of Proposition 3

Proof. In our previous case, output was equal to $y_t = z_t + \phi_0$. Now the agent receives two signals. The first is $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)y_t + v_{jt}$, which is equivalent to $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)z_t + v_{jt}$ as ϕ_0 is common knowledge. The second signal is $s_{pt} = z_t + e_t$, where we can interpret e_t as common noise in the public forecast of aggregate demand. Conjecture that output is equal to

$$\log Y_t = y_t = \phi_0 + z_t + \eta e_t, \tag{A.25}$$

where ϕ_0, σ_z^2 and η are constants to be determined. In that case,

$$cov(s_{pt}, y_t) = \sigma_z^2 + \eta\sigma_e^2. \tag{A.26}$$

(Note that if $\eta = -\frac{\sigma_z^2}{\sigma_e^2}$, then this covariance term becomes zero.) The agent has two signals. The private signal is

$$s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[z_t + \eta e_t] + v_{jt} \tag{A.27}$$

and the public signal is

$$s_{pt} = z_t + e_t \tag{A.28}$$

so we have

$$y_{jt} \equiv (1-\theta)\,\phi_0 + \theta \log E_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t) \right) \middle| \{s_{jt}, s_{pt}\} \right] \tag{A.29}$$

Since the random variables are assumed normal, we can write

$$y_{jt} \equiv (1-\theta)\,\phi_0 + \frac{\theta}{2}\Omega_s + \theta[\beta_0 s_{jt} + \beta_1 s_{pt}] \tag{A.30}$$

where Ω_s is the conditional variance of $x_{jt} = \frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t)$ based on s_{jt} and s_{pt}. Market clearing implies

$$Y_t^{1-\frac{1}{\theta}} = \int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{1-\frac{1}{\theta}} dj, \tag{A.31}$$

so taking logs and equating the stochastic elements on the left and right, we must have

$$\begin{aligned} \frac{z_t + \eta e_t}{\theta} &= \beta_0 \int s_{jt} dj + \beta_1 s_{pt} \\ &= \beta_0(1-\lambda)(z_t + \eta e_t) + \beta_1(z_t + e_t) \end{aligned} \tag{A.32}$$

which requires

$$\frac{1}{\theta} = \beta_0(1-\lambda) + \beta_1 \tag{A.33}$$

$$\frac{\eta}{\theta} = \beta_0(1-\lambda)\eta + \beta_1 \tag{A.34}$$

If $\beta_1 = 0$ these two equations collapse to $\frac{1}{\theta} = \beta_0(1-\lambda)$.

We first explore the self-fulfilling equilibrium with stochastic output where $\beta_1 = 0$. Note that the optimal solutions for β_0 and β_1 must satisfy

$$Ex_{jt}s_{jt} - \beta_0\sigma_{s_{jt}}^2 - \beta_1 cov(s_{jt}, s_{pt}) = 0, \tag{A.35}$$

$$Ex_{jt}s_{pt} - \beta_0 cov(s_{jt}, s_{pt}) - \beta_1\sigma_{s_{pt}}^2 = 0. \tag{A.36}$$

From (A.35),

$$\beta_0 = \frac{\lambda \frac{1}{\theta}\sigma_\varepsilon^2 + (1-\lambda)\frac{1-\theta}{\theta}(\sigma_z^2 + \eta^2\sigma_e^2)}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2(\sigma_z^2 + \eta^2\sigma_e^2) + \sigma_v^2} = \frac{1}{\theta}\frac{1}{1-\lambda}, \tag{A.37}$$

which yields

$$\sigma_z^2 + \eta^2\sigma_e^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2 \tag{A.38}$$

Again, as in Proposition 3, if either $\lambda \geq \frac{1}{2}$, or $\sigma_v^2 > \lambda(1-2\lambda)\sigma_\varepsilon^2$, then $\sigma_z^2 + \eta^2\sigma_e^2 \leq 0$ and there is only the certainty equilibrium.

Now we need to determine η. Notice that

$$E(x_{jt}s_{pt}) = E\left[(\frac{1}{\theta}\varepsilon_{jt} + \frac{1-\theta}{\theta}(z_t + \eta e_t)) \times (z_t + e_t)\right] = \frac{1-\theta}{\theta}(\sigma_z^2 + \eta\sigma_e^2). \tag{A.39}$$

and

$$\begin{aligned} cov(s_{jt}, s_{pt}) &= E(\lambda\varepsilon_{jt} + (1-\lambda)(z_t + \eta e_t)) \times (z_t + e_t) \\ &= (1-\lambda)(\sigma_z^2 + \eta\sigma_e^2) \end{aligned} \tag{A.40}$$

If $\beta_0 \neq 0$ in this case we have

$$\sigma_z^2 + \eta\sigma_e^2 = 0, \tag{A.41}$$

or

$$\eta = -\frac{\sigma_z^2}{\sigma_e^2} \tag{A.42}$$

and (A.36) is satisfied. By our assumption $\sigma_e^2 = \tilde{\gamma}\sigma_z^2$ we have $\eta = -\frac{1}{\tilde{\gamma}}$. Suppose that $\lambda < \frac{1}{2}$. We have to find out whether it is possible to have a rational expectation equilibrium satisfying $\sigma_z^2 > 0$. Note from (A.38) that

$$\sigma_z^2 + \eta^2\sigma_e^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2 \tag{A.43}$$

Substituting η into the expression we then have

$$\left(\sigma_e^2\right)^{-2}\left(\sigma_z^2\right)^2 + \sigma_z^2 = \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right) \tag{A.44}$$

Using the relationship between σ_e^2 and σ_z^2 we have

$$\frac{1+\tilde{\gamma}}{\tilde{\gamma}}\sigma_z^2 = \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2}\sigma_v^2\right) \tag{A.45}$$

Notice that the above equation has an unique solution for $\sigma_z^2 > 0$:

$$\sigma_z^2 = \frac{1+\tilde{\gamma}}{\tilde{\gamma}} \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2} \sigma_v^2 \right) \tag{A.46}$$

If $\tilde{\gamma}$ approaches zero, σ_z^2 also approaches to zero. However, since $\sigma_e^2 = \tilde{\gamma}\sigma_z^2$ and $\eta = -\frac{1}{\tilde{\gamma}}$, the variance of output is given by

$$\sigma_y^2 = \frac{1+\tilde{\gamma}}{\tilde{\gamma}} \sigma_z^2 = \left(\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2} \sigma_v^2 \right), \tag{A.47}$$

which is not affected and the uncertainty equilibrium will continue to exist.

Finally, since the public signal is not informative at all, the firm's effective signal is only the private one. We can redefine

$$\hat{z}_t = z_t + \eta e_t = z_t - \frac{1}{\tilde{\gamma}} e_t \tag{A.48}$$

which then has variance

$$\sigma_{\hat{z}}^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 - \frac{1}{\theta(1-\lambda)^2} \sigma_v^2 \tag{A.49}$$

where we again use $\sigma_e^2 = \tilde{\gamma}\sigma_z^2$ and $\eta = -\frac{1}{\tilde{\gamma}}$ to derive (A.49). So output will be as in Proposition 3,

$$y_t = z_t + \eta e_t + \phi_0 = \hat{z}_t + \phi_0, \tag{A.50}$$

where the constant term is $\phi_0 = \frac{1}{2}\left(\frac{(1-\lambda+(\theta-1)\lambda)}{\theta(1-\lambda)} \frac{1}{(\theta-1)}\right)\sigma_\varepsilon^2 - \frac{(\theta-1)\sigma_v^2}{2\theta^2(1-\lambda)^2}$. With z_t redefined as $\hat{z}_t$, the property of output fluctuations is not affected.

We now turn to the certainty equilibrium. From (A.33) and (A.34), if $\beta_1 \neq 0$, we must have $\eta = 1$. Namely aggregate output will be

$$y_t = \phi_0 + z_t + e_t, \tag{A.51}$$

If the public signal is still as $s_{pt} = z_t + e_t$ it fully reveals aggregate demand y_t. The private signal would now be $s_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[z_t + e_t] + v_{jt} = \lambda\varepsilon_{jt} + (1-\lambda)[(y_t - \phi_0)] + v_{jt}$ where by construction $y_t - \phi_0$ will be known. If we define $\hat{z}_t = z_t + e_t$, and attempt to define an equilibrium analogous to the certainty equilibrium of Proposition 1, with the difference that the aggregate demand shock $\hat{z}_t = z_t + e_t$ is not taken as zero but is perfectly observed each period prior to the production decision, we reach a contradiction. Setting $z_t = 0$, the "constant" term ϕ_0 can be defined to include e_t and solved as in Proposition 1 as a function of time-invariant parameters of the model. However this will contradict the randomness of e_t unless $e_t = 0$ for all t. The certainty equilibrium of Proposition

1 with constant output is not compatible with a time-varying public forecast of aggregate demand since firms would forecast the constant output. The public signal $s_{pt} = z_t + e_t$ would be observed in the self-fulfilling equilibrium, but in the certainty equilibrium the public forecast of aggregate output would be a constant, and identical to the equilibrium in Proposition 1. If on the other hand we use our assumption that the variance of the forecast error of the public signal is proportional to the variance of z, that is if $\sigma_e^2 = \tilde{\gamma}\sigma_z^2$, then we can recover the certainty equilibrium of Proposition 1 where output is constant: for this equilibrium we would have $z_t = e_t = 0$ for all t. ■