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FRAGILE DEBT AND THE CREDIBLE SHARING OF STRATEGIC UNCERTAINTY

Russell Cooper

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I am grateful to Antoine Camous and Andrew Gimber for many comments on this paper and to Hubert Kempf and Dan Peled for discussions on related topics. Comments from conference participants at the ECB "Global Sovereign Debt" workshop in June 2012 are appreciated. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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ABSTRACT

This paper studies debt fragility. It provides conditions under which fundamentals and strategic uncertainty jointly determine the price of sovereign debt. Default arises in equilibrium both because of fundamental shocks and beliefs. The probability of default depends on borrowing rates and, in equilibrium, on the beliefs of lenders about this probability. This interaction creates a strategic complementarity and thus the basis for strategic uncertainty. The paper analyzes the role of debt guarantees as a means of sharing both fundamental and strategic uncertainty. It provides conditions for the credibility of those guarantees as well as ex post bailouts. The effects of debt purchases by a monetary authority are analyzed as well.

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September 4, 2012

Abstract

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1 Motivation

This paper provides a simple analytic framework for understanding strategic uncertainty in the valuation of sovereign debt. The analysis is motivated by the recent experience of European countries who have witnessed large movements in the price (bond spreads) of their government debt and the consequent policy discussion of debt guarantees and bailout. What are the sources of the variations in these spreads? What are the effects of credible guarantees and bailouts?

The framework addresses these questions. First, it indicates the various interactions between fundamental and strategic uncertainty underlying the volatility of spreads. The first form of uncertainty arises from the stochastic nature of underlying economic variables, such as productivity. The second source of uncertainty reflects the interaction of investors, each responding to the beliefs of others. In this model, as in Calvo (1988), the power of expectations reflects a strategic complementarity linking the beliefs of investors about repayment, interest rates and default probabilities.[1] If investors believe that default is likely, a large premium

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[1] A similar logic is present in Cole and Kehoe (2000) though that paper has a dynamic element missing in this formulation.

will arise in equilibrium. Given this large premium, a country is indeed more likely to default. Taken together, these forms of uncertainty underlie the pricing of sovereign debt and its fragility.

Second, the framework is used to evaluate policy measures, such as guarantees and bailouts, proposed as means to deal with volatile debt prices. These interventions have different effects depending on whether fundamental or strategic uncertainty is present. The analysis highlights a couple of points.

The provision of guarantees, as in the standard models of deposit insurance, provide a means of sharing strategic uncertainty.[2] But, the effects of the provision of guarantees by one country (or a group of countries in a federation) to another, depends on the economic situation in the country offering the guarantee. If, at one extreme, the country providing the guarantee is not subject to strategic uncertainty, then the guarantee removes this form of uncertainty from the determination of asset prices. In other cases though, the provision of a guarantee shifts the strategic uncertainty without extinguishing it.

Of course, guarantees have valuable only if they are credible. Even under a guarantee, some country or coalition of countries must be willing to pay the outstanding obligation in the event of default. The framework is used to evaluate credibility by studying if bailouts will be forthcoming to support the guarantee. Using the consumption smoothing motive highlighted in Cooper, Kempf, and Peled (2008), bailouts will be provided if they yield a more equitable allocation across countries within a federation than default. One leading case for bailouts is when the country on the verge of default has lower output and consumption, both with and without a bailout, than the country providing the bailout. In this case, the bailout leads to consumption smoothing across the countries in a federation.

Finally, the framework can be used to study the impact of debt purchases and changes in interest rates. Here debt purchases provide an important tool for dealing with strategic uncertainty. If a country is paying a high interest rate on its debt due to strategic uncertainty, a large buyer can coordinate on another equilibrium with lower interest rates and lower default rates through its debt purchases.

In some models, such as Tirole (1985), low interest rates support equilibria with bubbles in asset prices. Here we find that lower interest rates may reduce the prospect of default by decreasing debt service obligations. These low interest rates can also reduce strategic uncertainty.

Eaton (1987) studies capital flights in a model with multiple equilibria generated by a complementarity through the government budget constraint. More recently, Corsetti and Dedola (2012) also use a variant of the Calvo model to look at the European debt situation. While that model shares foundations with Calvo (1988), one of the distinguishing features of this paper is the construction, starting in Section 3, of a multi-country environment to study bailouts and guarantees and thus the sharing of strategic uncertainty.

[2] Bigelow, Cooper, and Ross (1993) and Cooper and Ross (1999) study guarantee funds in a variety of interactions in which complementarities are prominent.

2 Model

The analysis focuses on the financing of a given stock of debt, denoted B, in a two-period setting.[3] Given outstanding debt B which is due in the next period, the analysis initially focuses on a single question: What is the interest rate R that will be paid on the debt?

Two factors influence the return to debt holders. First, there is fundamental uncertainty over the future tax base of the borrower. This is represented through a productivity shock that determines the level of output and thus the tax base. Second, there is strategic uncertainty: the likelihood of repayment depends, in part, on the beliefs of the lenders. In the model, the fundamental and strategic uncertainty interact.

2.1 Optimization

The lenders in the model are from two groups: households in country 1 and agents from outside the country but within the federation. As we shall see, this distinction between insiders and outsiders matters for the default decision of country 1.[4]

If the debt is repaid, the representative household in country 1 has consumption and a level of utility of

$$V^r = (1-\tau)A + BR\theta = A - BR(1-\theta). \tag{1}$$

The value if repayment, V^r, is the utility from the after tax level of output, $(1-\tau)A$, where A is productivity of the unit of labor input (hence it is output) and the tax rate is τ. There are no distortions from taxes as labor supply is inelastic. The representative household owns a fraction θ of the per capita debt, B, and hence recoups $BR\theta$ when the debt is repaid. Using the government budget constraint of $\tau A = BR$, the value of repayment can be rewritten as in the second term of (1). The remainder of the debt, $(1-\theta)B$, is held by lenders outside of the country.[5]

If there is default, then the value is given by

$$V^d = (1-\gamma(\theta))A. \tag{2}$$

In this case, the country defaults on both internal and external debt. A fraction of the output, $\gamma(\theta)$ is lost. This is a conventional assumption in the debt default literature meant to capture a wide range of costs stemming from default including a reduction in trade and risk sharing opportunities, exclusion from markets in the future, etc. The details of those costs do not concern us at this stage, it is only important they are present in the analysis.

[3]The interaction can be embedded in a dynamic equilibrium model allowing for the choice of B and intertemporal considerations for the households, for example as in Cooper, Kempf, and Peled (2008). But the essence of determining the interest rate on the outstanding debt is aptly captured by the interactions described here.

[4]The main results in Calvo (1988) only have insiders though international lending is mentioned in Section III.

[5]The country of these lenders is introduced below and the notation modified to distinguish the countries.

These costs can depend inversely on the fraction of debt held internally: $\gamma(\theta)$ would be decreasing in θ. Since there are no distortionary taxes and all households are identical, the level of consumption and welfare in the country should be the same when all debt is held within the country: hence we assume $\gamma(1) = 0$.

Using (1) and (2), the debt will be repaid iff $A \geq \hat{A}$, where

$$\hat{A} \equiv \frac{BR(1-\theta)}{\gamma(\theta)}. \tag{3}$$

The values of repayment and default and the default decision are calculated *ex post*, after the realization of the fundamental uncertainty through the productivity shock.[6] *Ex ante* A is not known. It is common knowledge that the shock is drawn comes from a distribution given by $F(A)$. We normalize the mean of A to be unity. Suppose that the domain of A is given by $A \in [A^-, A^+]$.

Thus the probability of repayment is $(1 - F(\hat{A}))$. From (3), $\hat{A}$ will depend on (B, R, θ) as well as the default costs, $\gamma(\cdot)$.

Lenders are aware of the prospect of default coming from the fundamental and strategic uncertainty. They have a safe lending opportunity which yields a real gross return of r. Letting p denote the probability of repayment of country 1 debt, the arbitrage condition (with risk neutral lenders) is simply

$$pR = r. \tag{4}$$

Of course, the probability of repayment is determined by a comparison of V^r and V^d conditional on the realized value of A. Since the loan is made *ex ante*, p is the probability that A falls outside the default range.

The equilibrium interest rate and probability of repayment are determined by two equations. The first determines the critical value of A given R, as in (3). The second uses the probability of repayment to rewrite (4) as

$$[1 - F(\hat{A})]R = r. \tag{5}$$

The strategic uncertainty comes from the dependence of the probability of repayment on the interest rate together with the dependence of the interest rate on that same probability. There is a natural strategic complementarity here: if investors think that default is likely, they demand a high interest rate and thus there is default unless a very high A is realized.[7] This is consistent with the high default probability believed by investors. This interaction between beliefs, interest rates and default decisions is brought out by the equilibrium analysis.

[6] Calvo (1988) excludes fundamental uncertainty and focuses solely on the endogenous determination of a repudiation rate by a government that lacks commitment.

[7] The complementarity takes a dynamic form in Cole and Kehoe (2000) . The prospect of a future default increases the cost of rolling over the current stock of debt and thus makes current default more likely.

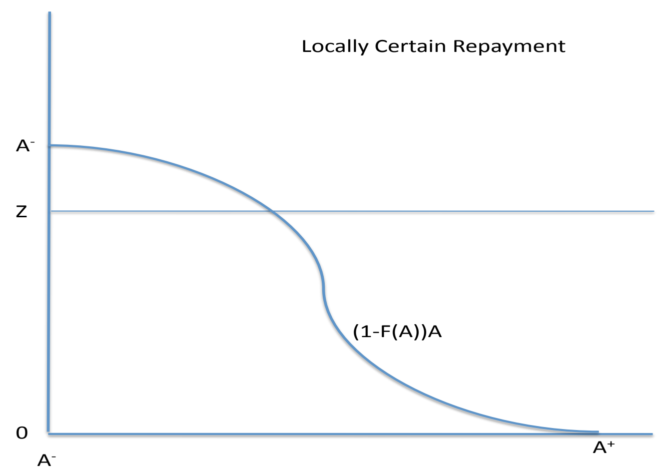

Figure 1: Locally Safe

2.2 Equilibria

An equilibrium is determined by the values of $(\hat{A}, R)$ that jointly satisfy (3) and (5). With some algebra the critical value of A determining the bound of the default region solves

$$\Gamma(A) \equiv [1 - F(A)]A = \frac{Br(1-\theta)}{\gamma(\theta)} \equiv Z. \tag{6}$$

From (3), default occurs for $A < \hat{A}$ where $\hat{A}$ is a solution to (6). The value of R is then determined from (5). In addition to the equilibria solving (6), there can be an equilibrium without any default if $\Gamma(A^-) \geq Z$.

The properties of $\Gamma(A)$ are important for the analysis. The fact that $\Gamma(A)$ may not be monotone in A is the algebraic source of the multiplicity of interior equilibria in the model. As $A \in [A^-, A^+]$, $\Gamma(A^-) = A^-$ and $\Gamma(A^+) = 0$. This implies that there is always an equilibrium with certain default and hence a zero value of debt.

Figure 1 shows the determination of an equilibrium for the model when $\Gamma(A^-) = A^- > Z$. In this case, there is an equilibrium without default, implying $R = r$. There is another equilibrium at a much higher value of $\hat{A}$ with $R > r$. The second equilibrium has default for $A \in [A^-, \hat{A})$. Clearly, strategic uncertainty is present even in the case where there is an equilibrium without default.

The set of equilibria are robust to small variations in Z. As long as $\Gamma(A^-) > Z$, the equilibrium without default is robust to variations in Z. And the other equilibria remain as well.

For there to be an equilibrium without default requires, all else the same, a relatively low value of Z. From (6), a low value of Z comes from a small debt burden, BR is small relative to output. Also, if θ is large so that most of the debt is held internally, the incentive to default is reduced and Z is low as well. [8]

There are other cases to consider. Figure 2 shows a case with three equilibria. Here Z is sufficiently high so that there is no default-free equilibrium. There are two interior equilibria with different critical values of $\hat{A}$ satisfying (6) and therefore different default probabilities and associated values of R. There are multiple interior equilibria because $\Gamma(A)$ is not monotone. And, as in Figure 1, there is the equilibrium in which default always occurs.

This case is one where the debt burden is high enough relative to the lower support of A that repayment with certainty is not possible. In this case, the fundamental uncertainty is large enough that no promises of certain repayment are credible. Interestingly this creates at least two interior equilibria, in addition to the equilibrium with certain default.

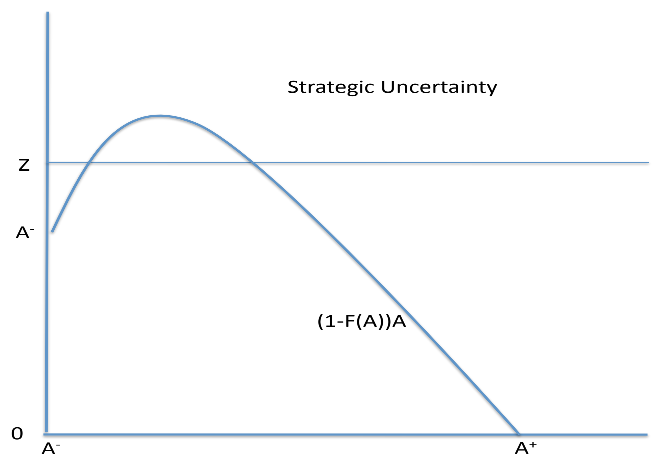

Figure 2: Strategic Uncertainty

If Z is sufficiently high and $\Gamma(A)$ does not increase rapidly enough, then there may be no equilibria with repayment. This is shown in Figure 3.

The model contains both fundamental and strategic uncertainty and they interact. Given the beliefs of investors, there is a critical A determining the default range. Without the shocks to A, if the mean is high enough relative to the debt burden, then there will be an equilibrium without default. Still, if beliefs about repayment are pessimistic enough, an equilibrium without repayment where the debt has no value will exist as well. These are extreme cases and are not as interesting as the outcomes that arise when fundamental and strategic uncertainty interacts.

[8] Assume that $\frac{(1-\theta)}{\gamma(\theta)} \to 0$ as $\theta \to 1$. So if all debt is held internally, country 1 is indifferent between default and repayment.

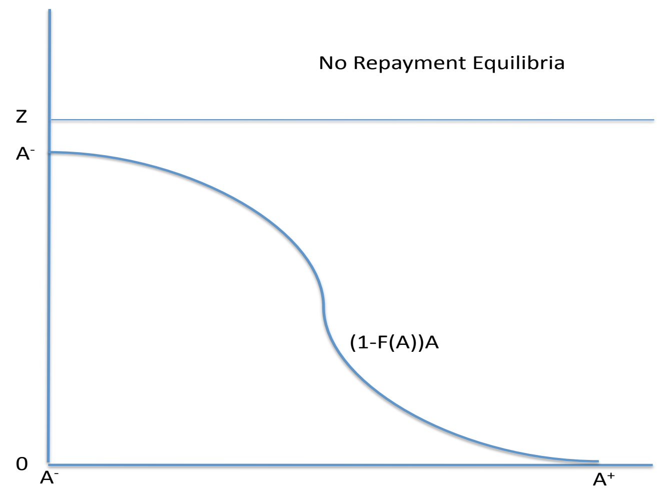

Figure 3: No equilibria with repayment

2.3 Interpreting Recent Events

Over the last few years in Europe, interest rates on country debt have experienced fairly wide variations. The return on 10 year Greek bonds has risen from around 5% in January 2010 to around 35% in January 2012. The return on 10 year Italy bonds was around 4% in January 2010 and rose to over 7% by the end of 2011. For Spain and Portugal, similar increases have been seen.

The model illustrates some of the factors that can contribute to these dramatic increases. While the goal of the paper is not to discriminate between these sources, it is nonetheless instructive to use the model to see how these changes might occur.[9].

If fundamental and strategic uncertainty interact to determine interest rates, then these same factors explain the changes. For Greece, fiscal imbalances are commonly pointed to as the source of the concerns over default. At the same time, the debt/GDP ratio was quite high for Greece during this period.

The model includes the debt/GDP ratio in (3) through the ratio $\frac{B}{A}$. The higher is B, the higher is the critical value of A to support repayment or, put differently, the higher is B the larger is the default region.[10]

At the same time, the run-up in the interest rates on Italian debt is not as easily explained by fiscal

[9] de Grauwe and Ji (2012) study these spreads and argues that at least some of the variation points to the role of beliefs not fundamentals

[10] Though not explicitly part of the model, news about future values of A and thus the future tax base, made public at the time of asset pricing, can also impact interest rates.

conditions. Over the time of the increase in interest rates, there was little change and little news about the fiscal balances of Italy. It is more natural to explain those interest rate changes as coming from beliefs of the lenders, perhaps moving from a low to a high interest rate regime.

There is another element in recent events that we will add to the model: the role of interventions in the form of bailouts and guarantees etc. As the likelihood of these interventions unfold, they too will impact on observed interest rates.

3 Enhancing Stability

The model illustrates that interest rates on the debt reflect uncertainty about repayment stemming jointly from uncertainty over the future tax base along with the beliefs of lenders. The model can be used to explore various policy measures that have been proposed to stabilize interest rates, such as debt guarantees.[11]

3.1 Guarantees

A debt guarantee is modeled as a promise by one country, or a group of countries within a federation, to pay the debt of another country in the event of default. This guarantee has effects on both the country whose debt is being guaranteed and the country providing the guarantee. The debt not held internally is held by households in this other country (or group of countries).

For now, we assume the guarantee is credible and focus on how the guarantee influences the allocation of fundamental and strategic uncertainty. The model highlights an interesting dimension of guarantees: the shifting of strategic uncertainty. The next section of the paper analyzes the credibility issue in the context of *ex post* bailouts.

If the guarantee of repayment is credible, then the interest rate paid on debt by the country receiving the guarantee obviously falls to r. If there is residual uncertainty about the credibility of the guarantee, then the interest rate will be higher than r and fluctuations in the rate can be interpreted as reflecting beliefs about the probability of the guarantee being honored, along with the probability of default. The crux of the analysis shifts to the guarantor.

To see the effects of guarantees, suppose initially that both countries are in the robust regime depicted in Figure 1. But, imagine that one of the countries, call it the safe country, is in a no default equilibrium while the other country, call it the risky country, is in the equilibrium with positive default probability. The fundamentals of the two countries are both sound: there is an equilibrium for the risky country without default. But, the risky country is affected by strategic uncertainty.

In this case, a credible guarantee implies that the rate of interest paid by the risky country will fall to r without any remaining fundamental or strategic uncertainty. This is like a selection effect: the guarantee

[11] Roch and Uhlig (2012) study the **effects** of bailouts in a model of debt dynamics along the lines of Cole and Kehoe (2000) but do not provide an analysis of the incentives for those bailouts.

eliminates the equilibrium with strategic uncertainty.

For the safe country providing the guarantee has no effects on its status as a borrower. As long as the commitment is credible, since $A^- > Z$ in the recipient country, the probability of default is zero. Thus in equilibrium the country providing the guarantee assumes no risk.

In this sense the strategic uncertainty has been shifted from one country to another. If beliefs that support the no default equilibrium in the safe country are not perturbed by its assumption of additional liabilities, then the strategic uncertainty is eliminated.

In this first case, there is no fundamental uncertainty. As an alternative, suppose that the recipient country is in the regime of multiple equilibria, as in Figure 2, so that there is fundamental uncertainty: there is no equilibrium for this country without default. Suppose the country providing the guarantee is more stable, as in the no default equilibrium of Figure 1.

As before, the provision of the guarantee will support lending at r to the recipient country. This is more than a selection effect since the country could not borrow at r. That is, borrowers are being insured here against fundamental risk.

What will happen in the safe country that is providing this contingent liability? With a positive probability it will have to make payments under the guarantee. As long as these payments are not too large, then the condition $A^- > Z$ will remain and the safe country will be able to meet its debt plus insurance obligations even when the other country has a low realization of productivity.

So, even with the shifting of this burden, the safe country can remain in the no default regime. As before, we are assuming that the assumption of the debt does not create strategic uncertainty by moving to the other equilibrium.

If the contingent obligation is large enough, then the guarantee will factor into the pricing of the debt issued by the safe country. In particular, the country providing a large enough guarantee over fundamental risk can be pushed out of the $A^- > Z$ region of parameter space. In this case, there is now fundamental uncertainty in both countries. This is perhaps reflected in the recent downgrading of Eurozone (eg. French and EFSF) debt, partly due to it implicit guarantee of the debt of other countries.

The guarantee links the two countries. The safe country absorbs both the fundamental and strategic uncertainty from the risky country. But this link creates a basis for contagion that did not exist before. Events in the risky country now have an impact of the values of debt in the safe country.

3.2 Bailouts

As noted above, this discussion assumes that the country (or federation of countries) providing the guarantee is bound to that promise. If not, we need to inspect the incentives of countries to make good on these types of promises. We do so in the context of bailouts.

A bailout is an *ex post* decision to provide a transfer to another country. As already noted, the bailout might come from one country or from a group of countries in a federation bailing out a member state. Here we view the interaction between a country and the federation to which it belongs.

The bailout could arise from the provision of a guarantee without commitment. In that case, the countries providing the guarantee has a choice to make the transfer or not. Or, as in Cooper, Kempf, and Peled (2008), the bailout can arise simply from the interaction within a federation when one country is about to default on its debt. The federation has a choice to bailout or not.

In both cases, there are two issues. First, what is the effect of the bailout on the rate at which the first country can borrow? Second, what are the incentives for this *ex post* bailout by the federation? To address these questions, the model must be enhanced to be specific about the structure and incentives of the federation.

3.2.1 Bailout Incentives

The federation has two members. One is the country whose choice of default or repay was analyzed above. This, as before, is country 1 in the model. The other member is a composite of the other countries in the federation. This is country 2 or "the federation" in the following analysis.

The interaction between country 1 and the federation is shown in Figure 4. Country 1 chooses first and can simply repay its debt. If it chooses not to repay, then the federation has an option of a bailout or allowing default. The default cost is incurred by country 1 only if the federation chooses not to bailout that country.

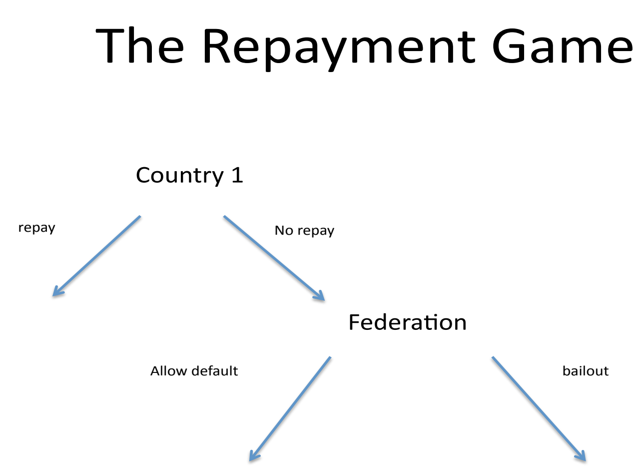

Figure 4: Game between Country 1 and the Federation

We first make explicit the consumption allocations under the options of debt repayment, debt default and bailout. We use these to construct an equilibrium.

To fix notation, let N_i be the fraction of the population in country i. Recall that B is the total debt outstanding of country 1 per capita of that country. Let b_i be the holding of country 1 debt by a representative agent in country i. Hence $N_1 \times b_1 + N_2 \times b_2 = N_1 B$ in equilibrium. Further, A_i is the realized productivity in country i, $Y_i = N_i A_i$ is total country i output and $\bar{Y} = \sum_i Y_i$ is total output of the federation.

Looking first at the representative household in country 1, using (1), the consumption allocation under repayment is $c_1^r = A_1 - BR(1-\theta)$. Here $\theta = \frac{b_1}{B}$ is the fraction of country 1 debt held by country 1 households. When $\theta = 1$, the repayment of debt is just a reshuffling within a household's budget constraint without any real implications. Using (2), the consumption allocation for country 1 agents under default is $c_1^d = (1-\gamma(\theta))A_1$.

Similar expressions hold for the consumption of agents in country 2 under repayment and default on country 1 debt. If there is repayment, $c_2^r = A_2 + b_2 R$. If country 1 defaults, $c_2^d = A_2$.

The final outcome is a bailout by the federation. To start, we study a complete bailout of country 1 debt and then introduce partial bailouts as the analysis develops. In the event of a complete bailout, a common tax is levied, denoted $\bar{\tau}$, to cover the debt obligations of country 1: $\bar{\tau}\bar{Y} = N_1 BR$.

In the case of a bailout, the consumption of country i agents is $c_i^b = A_i(1-\bar{\tau}) + b_i R$. With a little algebra, the consumption of country 1 agents becomes $c_1^b = A_1 + BR(\theta - \frac{Y_1}{\bar{Y}})$.

The last ingredient is the objective of the federation. Assume that the federation has an objective of the weighted average of utility levels from the households in the two countries: $W(c_1, c_2) = \sum_i \Delta_i v(c_i)$ where Δ_i is a welfare weight and $v(\cdot)$ is strictly increasing and strictly concave. The weights Δ_i might coincide with population weights or could reflect broader objectives of the federation.

The bailout incentive comes from the curvature in the objective function of the federation. In the model, households are assumed (for simplicity) to be risk neutral.[12] Thus the assumed preferences of the federation have more curvature in consumption that the representative household. This could reflect the views of the policymakers that take into account the joint welfare of all households in the federation or a political process that puts more value on equality of consumption across members of the federation.

It is difficult to rationalize bailout activity without some type of objective which takes the consumption of households across federation members into account. The curvature ensures that the gains to the joint activities are split cross the federation members.

This maximization of a weighted sum of the strictly concave utilities obtained from the member countries, creates, as in Cooper, Kempf, and Peled (2008), a consumption smoothing motive for bailout. In the leading case of $\Delta_i = N_i$, the federation prefers to equalize consumption across agents, given a fixed amount of resources.

[12] Allowing risk averse households would not influence the conditions for default and repayment. If lenders are risk neutral, then the asset pricing equation remains, as does the analysis of the multiplicity.

3.2.2 Bailout Equilibria

Given these incentives, we study the conditions under which a bailout occurs in equilibrium. We first look at the choices of the federation and then return to the decisions of country 1.

Incentives for the Federation A sufficient condition for bailout is that the consumption allocation after bailout is more equitable than the one with default. Formally,

Proposition 1 *If* $\Delta_1 \geq N_1$,$\theta > \frac{Y_1}{\bar{Y}}$ *and* $c_1^b < c_2^b$, *then the federation prefers bailout to default.*

Proof. $\Delta_1 = N_1$, so that $\Delta_2 = N_2$, implies that the preferred allocation of total output, $\bar{Y}$, is equal consumption. Thus if the allocation under bailout is more equitable than the allocation of consumption under default, a bailout will be provided.

Using $c_1^b = A_1 + BR(\theta - \frac{Y_1}{\bar{Y}})$, if $\theta > \frac{Y_1}{\bar{Y}}$, as hypothesized, then the bailout redistributes consumption from country 2 to country 1. Hence $c_1^b < c_2^b$ requires that $A_1 < A_2$. From this, we know that $c_1^d < c_2^d$.

Putting this together, in both the default and bailout allocations the consumption per agent in country 2 exceeds that in country 1. The bailout allocation redistributes toward country 1 agents. Hence it is more equitable and thus preferred by the federation.

If $\Delta_1 > N_1$, then the redistribution from a bailout to country 1 is more desirable than when $\Delta_1 = N_1$. Hence a bailout that is preferred to default when $\Delta_1 = N_1$ is also preferred when $\Delta_1 > N_1$. ■

The proposition is illustrated in Figure 5. The consumption allocation under a default is indicated by point "D" and that under bailout is point "B". As shown, the bailout reallocates consumption towards country 1, which has lower consumption than country 2 both before and after the bailout.

The proposition requires $c_1^b < c_2^b$, defined as the consumption levels under a full bailout, as a sufficient condition for bailout. As noted above, $c_i^b = A_i(1 - \bar{\tau}) + b_i R$ so that the ordering of the bail-out consumption levels is a restriction on exogenous variables.

The fact that $c_1^b < c_2^b$ is sufficient but not necessary is shown in Figure 6. Here there are clearly consumption smoothing gains to bailout though $c_1^b > c_2^b$. The bailout is so excessive that the consumption of country 1 exceeds that of country 2, though the resulting allocation is still socially preferred to default.

Of course, it can be that the consumption reallocation through bail-out is so excessive that welfare is lower under default. In that case, as well as in the case illustrated in Figure 6, a partial bailout is socially preferred. A partial bailout occurs when revenue from common taxation is used to pay a fraction, denoted ν, of country 1 debt. In this case, the tax rate is given by: $\bar{\tau}\bar{Y} = \nu N_1 BR$.

Proposition 2 *If* $\Delta_1 = N_1$,$\theta > \frac{Y_1}{\bar{Y}}$, $A_1 < A_2$ *and* $c_1^b > c_2^b$, *then the federation prefers a partial bailout to default.*

Proof. The argument builds from Proposition 1 by showing that partial bailout, parameterized by ν ,can generate an outcome with equal consumption which is preferred to default. The conditions $A_1 < A_2$ and

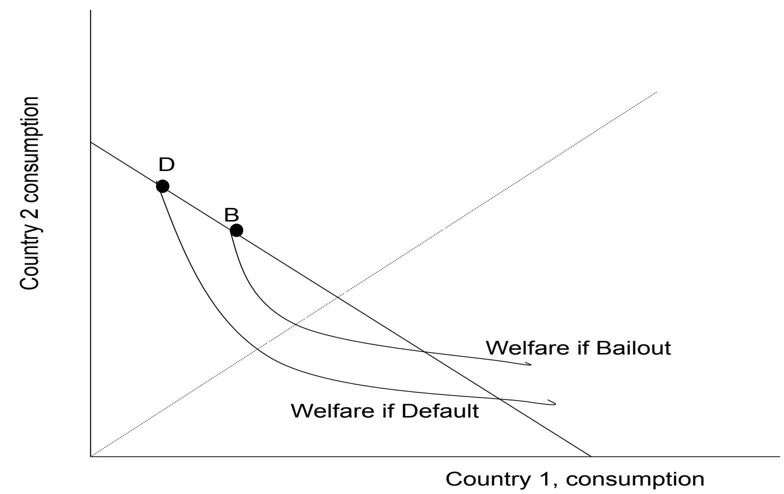

Figure 5: Gains from Bailout

$c_1^b > c_2^b$ implies that consumption of country 1 is less than that of country 2 if $\nu = 0$ and consumption of country 1 exceeds that of country 2 when $\nu = 1$. Since the choice of ν redistributes existing resources, by continuity there exists a partial bailout such that the consumption levels in the two countries are equal. This allocation with partial bailout is preferred to default. ■

In these cases, country 1 has relative low output and a lot of the debt is held internally. A bailout then redistributes from rich to poor and increases social welfare. A bailout can also arise if country 1 is relatively rich and holds very little of its debt. Then a bailout redistributes from country 1 to country 2.

Proposition 3 *If $\Delta_1 \leq N_1, \theta < \frac{Y_1}{Y}$ and $A_1 > A_2$, then the federation prefers bailout to default.*

Proof. Suppose default costs are near zero: $\gamma(\theta) = 0$ for all θ. In this case, default levels of consumption are given by A_i so that $c_1^d > c_2^d$. If $\theta < \frac{Y_1}{Y}$, as hypothesized, then the bailout redistributes consumption from country 1 to country 2 relative to default. This is socially desirable as $c_1^d > c_2^d$ and $\Delta_1 \leq N_1$. If there are default costs, then the allocation under default is made worse and the argument for bailout is stronger. ■

Of course, bailout may also be undesirable if the redistribution is from poor to rich. The following proposition summarizes that case:

Proposition 4 *If $\Delta_1 \leq N_1, \theta > \frac{Y_1}{Y}$, $A_1 > A_2$ and small default costs, then the federation will prefer default to bailout.*

Proof. The argument parallels that of Proposition 1. Assume $\Delta_1 = N_1$. Using $c_1^b = A_1 + BR(\theta - \frac{Y_1}{Y})$, if $\theta > \frac{Y_1}{Y}$, as hypothesized, then the bailout redistributes consumption from country 2 to country 1. Hence

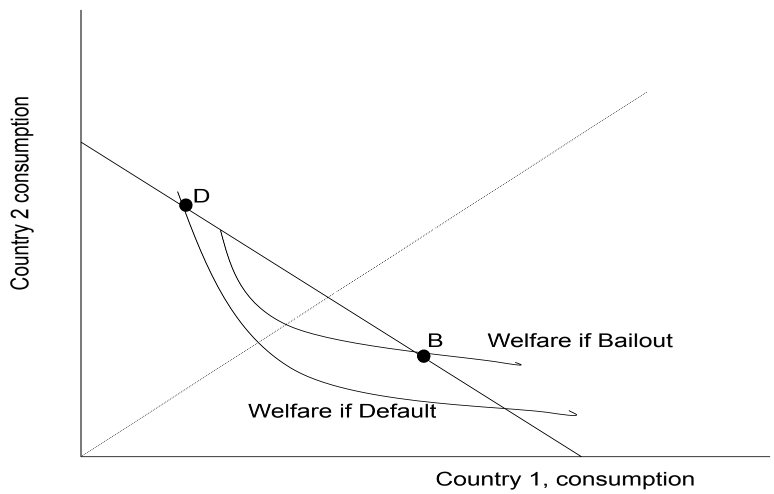

Figure 6: Gains from Excessive Bailout

$c_1^b > c_2^b$ as $A_1 > A_2$. If default costs, $\gamma(\theta)$, are zero then $c_1^d = A_1 > c_2^d = A_2$. In this case, a bailout would move the consumption allocation away from equality and hence social welfare would fall. The argument holds by continuity for small enough default costs.

If $\Delta_1 < N_1$, then the redistribution from a bailout to country 1 is less desirable than when $\Delta_1 = N_1$. Hence if a bailout is not preferred to default when $\Delta_1 = N_1$ it is also not preferred when $\Delta_1 < N_1$. ■

We can summarize this discussion by defining a set Λ as the combinations of (A, θ) such that the federation prefers to bailout the debt of country 1 over default. Propositions 1 and 3 characterize some of the elements of this set indicating, among other things, that Λ is non-empty.

Building on Proposition 1, if $(\hat{A}, \hat{\theta}) \in \Lambda$ then a full bailout occurs for any $A \leq \hat{A}$. A lower value of A enhances the incentives for a bailout, all else the same. Likewise, if $(\hat{A}, \hat{\theta}) \in \Lambda$ then a full bailout occurs for any $\theta \geq \hat{\theta}$ as long as $c_1^b < c_2^b$. As we shall see, the conditions for Proposition 1 will underlie bailout equilibria.

Before proceeding to the incentives of country 1, it is instructive to compare the conditions for a bailout here with those in Cooper, Kempf, and Peled (2008). While the consumption smoothing motive for bailout from that paper is present here, it takes a different form. In their argument, the country potentially in default (country 1) has higher consumption than the other country (country 2). The bailout redistributes consumption away from the defaulting country towards households in country 2 since the country 1 agents hold relatively small levels of country 1 debt. In equilibrium, the country 1 agents prefer the bailout allocation to repayment of the debt. Default does not occur since the central government prefers the bailout to allowing default.

The analysis in this model, summarized in Figure 5, is a bit different. Here country 1 has lower output and thus, all else the same, lower consumption than country 2. The bailout of country 1 actually favors that country. This is apparent from the restriction in the propositions that the share of debt held internally is high enough: $\theta > \frac{Y_1}{Y}$.

Incentives for Country 1 The analysis thus far focuses on the bailout incentives of the federation by providing sufficient conditions for a bailout. Recall from Figure 4 that country 1 chooses to repay or not prior to the choice of the federation. Anticipating a bailout, what is the response of country 1?

If the bailout leads to higher consumption for country 1 than default, as in Proposition 1 and summarized by Figure 5, country 1 obviously prefers the bailout outcome.

Proposition 5 *If $(A, \theta) \in \Gamma$ and $\theta > \frac{Y_1}{Y}$, then in equilibrium country 1 chooses not to repay its debt and the federation chooses a full bailout.*

Proof. By definition if $(A, \theta) \in \Gamma$ implies the federation will choose a full bailout. Since $\theta > \frac{Y_1}{Y}$, the bailout redistributes in favor of country 1. Hence, it prefers bailout to default.

Under a bailout, country 1 residents have consumption of $c_1^b = A_1 + BR(\theta - \frac{Y_1}{Y})$ while under repayment, consumption is $c_1^r = A_1 + BR(\theta - 1)$. From this, $c^b \geq c^r$ iff $\theta - \frac{Y_1}{Y} \geq (\theta - 1)$. The left side of this inequality is positive from the hypothesis of the proposition and the right side is negative as $\theta \leq 1$ since θ is a share of debt held by country 1 agents. Hence $c_1^b \geq c_1^r$ and country 1 prefers bailout to repayment of debt. ■

In addition, country 1 will also choose not to repay its debt if the conditions for a partial bailout hold, as in Proposition 2.

Proposition 6 *If $\Delta_1 \geq N_1$,$\theta > \frac{Y_1}{Y}$, $A_1 < A_2$ and $c_1^b > c_2^b$, then in equilibrium country 1 chooses not to repay its debt and the federation chooses a partial bailout.*

Proof. By the conditions of the proposition, the federation prefers a partial bailout to default. This partial bailout redistributes in favor of country 1 relative to a costless default and thus relative to a costly default. Since repayment has lower consumption than a costless default, country 1 prefers the partial bailout to repayment. ■

These propositions characterize the outcome for low values of A_1 compared to A_2. In this case, country 1 is relatively poor and, when θ is large enough, redistribution is in its favor.

In some cases though, as in Proposition 3, the bailout redistributes from a rich country 1 to the rest of the federation. For small default costs, country 1 would prefer default to redistribution through a bailout. This redistribution can be avoided if country 1 repays its debt. At the extreme of $\theta = 1$, then the repayment of debt is purely a redistribution within country 1. This is no worse than default. Hence for θ near 1, when redistribution from a bailout is against country 1, it will prefer to repay its debt.

For (A_1, θ) outside of Γ, the analysis of section 2.1 applies. Country 1 will choose between repayment and default using the critical value of $\hat{A}$ characterized in (3).

3.2.3 Effects on Interest Rate

The effect of the bailout on the borrowing rate of the country is easy to determine. If lenders believe that the bailout will be provided, then the country can borrow at r. If the bailout is not going to be provided, then the analysis of the country is not altered.

If there is a credible partial bailout, then this will be taken into account in the pricing of sovereign debt. In particular, (4) is modified to reflect a bailout of a fraction ν of the debt so that the interest rate on the debt is given by:

$$R[p + (1 - p)\nu] = r. \tag{7}$$

Clearly R moves inversely with ν.

In this case, there is no uncertainty over the provision of a bailout. Either θ is in the region of a bailout or not.

While outside of the model, if θ was not public information, then it would not be clear to lenders whether a bailout would be provided or not. This would create additional uncertainty about the return on the debt of an individual country. Fluctuations in the interest rates of individual countries might then reflect bailout uncertainty.

3.2.4 Value of Guarantees

Having established the conditions for a bailout, we return to the discussion of guarantees. These points are related as long as the guarantee is made without a credible commitment to meeting the promises of the guarantee.

If there is no commitment to meeting the guarantee, then the incentives for following through on that promise matter. If the conditions for a bailout are met, then the guarantee of the debt is credible. In this case, the federation has an *ex post* incentive for bailout and thus to meet the terms of a debt guarantee.

If the conditions for a bailout are not met the guarantee could still be credible if there is a cost, different from default, that the federation incurs if it does not meet the terms of a guarantee. This cost could be seen as lost prestige or more severe measures taken in the event of a default on a guarantee contract.

Finally, a guarantee might be more than a promise to act. It could entail a type of bond such that the resources needed to meet the guarantee are committed at the time the promise is made. In this case, there is no question of *ex post* credibility. But most guarantees work as promises without the immediate commitment of resources.

3.3 Other Forms of Intervention

We study other forms of intervention which entail active monetary policy. While the analysis stretches the model somewhat since it does not any nominal assets, it is useful to understand these other types of policies.

3.3.1 Interest Rates and Monetary Policy

Here we think of monetary policy as being able to influence the risk free rate, r. The effects of interest rates on debt prices can be seen from (6). If monetary policy leads to lower interest rates, then Z, on the right-side of (6), will be lower. Using this, we can trace out the effects of a reduction in r on the probability of default and on the equilibrium interest rate.

The change in the critical value, $\hat{A}$ and thus on the interest rate R will depend on the nature of the equilibrium. Inspecting Figure 1, if the economy was at the locally risk free equilibrium, then a reduction in the interest rate has no effect on the probability of default. In this case, R would fall with r as $R = r$ in equilibrium.

But when there is a positive probability of default, $\hat{A}$ will change with r and this will have an additional effect on the equilibrium interest rate. The response of $\hat{A}$ to changes in r depends on the local properties of the equilibrium. From (6), Z is increasing in r but, as discussed earlier, $\Gamma(A)$ is not a monotone function of A. Thus, $\hat{A}$ will fall as r falls **if** the equilibrium is along an upward sloping portion of the function $\Gamma(A)$. The comparative static is reversed if $\Gamma(A)$ is downward sloping at the initial equilibrium point.

To see this, consider Figure 2. In this case, there is an equilibrium where the function $\Gamma(A)$ is upward sloping. The comparative static in this case is that an reduction in r leads to a decrease $\hat{A}$ and thus a reduction in the probability of default. From (4), the fall in R would be larger than the reduction in r due to the effects of r on the default probability. The magnification effect of changes in r on R comes from the endogenous default decision.

If that economy was at the interior equilibrium where $\Gamma(A)$ was downward sloping, then the reduction in r would lead to an increase in $\hat{A}$ as Z falls. This would lead to an increase in the default probability. Thus, from (4), the fall in R would be less than the reduction in r. The offsetting influence of monetary policy comes from the effects of r on the default probability.

Finally, as in the case of Figure 3, it is possible that a reduction in the interest rate made it credible for a country to borrow which was otherwise excluded from the market. Of course, the risk premium on this loan would be quite high.

3.3.2 Debt Purchases

Another form of intervention is a debt purchase by the federation directly or through a central bank. To highlight the potential gains from such an intervention, suppose that both countries have fundamentals to support default free equilibria but country 1 is in an equilibrium in which strategic uncertainty has driven up its cost of debt. This is the same situation studied earlier to see the gains from the sharing of strategic uncertainty through a debt guarantee.

Suppose that the federation purchases the debt of country 1 at the fundamental price, ignoring the pessimism that has driven prices down and yields up on country 1 debt. A large enough intervention could coordinate the market on the fundamental price and hence $R = r$. A that lower return, country 1 would be back to the default free equilibrium.

4 Conclusion

This paper studies the conditions for debt runs. It does so in a model where default is possible due to fundamental shocks. Prices reflect this risk. In addition, the beliefs of investors have an independent influence on debt prices. Thus debt prices reflect both fundamental and strategic uncertainty.

The model is a vehicle to explore policy measures intended to stabilize debt prices. A key issue is the incentive for a federation to bailout the debt of a member country, particularly in the presence of strategic uncertainty. The analysis highlights the sharing of fundamental and strategic uncertainty as well as the conditions for a bailout.

There are a number of interesting extensions to consider. First, the model ignores an active monetary authority. As discussed, if we assume a monetary authority controls the real interest rate, then a channel exists between monetary policy and the volatility of debt prices. Integrating the model into a monetary framework would allow a more thorough investigation of this channel as well as the consideration of bailouts through monetary policy.

Second, there is no banking sector in this economy. Informally, there seems to be another important complementarity at work between country debt and the banking system. If investors are pessimistic about debt repayment, the value of country debt falls and the balance sheets of banks worsen. This implies a higher liability for countries under deposit insurance schemes and/or guarantees (either implicit or explicit) of the banking system. But the assumption of these additional liabilities supports the initial pessimistic beliefs about debt repayment. Formalizing this interaction and using it to explore additional policy measures through active monetary interventions could prove insightful.

Third, moral hazard effects are ignored. These can arise as a consequence of a bailout in a couple of forms. As studied by Cooper, Kempf, and Peled (2010), for example, the prospect of bailout can induce countries to issue more debt. Moreover, for countries under pressure to reform, a bailout can relax that pressure and be counterproductive.

Finally, the only debt is issued by a particular country. Another remedy for stabilizing debt prices could be the introduction of federation bonds, so-called euro bonds.[13] Understanding the impact of euro-bonds in this framework would be of interest as well.

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