NBER WORKING PAPER SERIES

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Working Paper 18258
http://www.nber.org/papers/w18258

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2012

We are grateful to grant U01-AG09740 from the Behavioral and Social Research Program of the National Institute on Aging for supporting the collection of the HRS data used in this paper and to NIA grant P01AG026571 for research support. The current paper builds on ideas presented in Hill, Perry and Willis (2004); we are grateful to the late Dan Hill and to Mike Perry for their valuable insights. We are also grateful to Gábor Kézdi for many useful comments on the current paper. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w18258.ack

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NBER Working Paper No. 18258
July 2012
JEL No. C11,J1

ABSTRACT

Based on subjective survival probability questions in the Health and Retirement Study, we use an econometric model to estimate the determinants of individual-level uncertainty about personal longevity. This model is built around the Modal Response Hypothesis (MRH), a mathematical expression of the idea that survey responses of 0, 50 or 100 percent to probability questions indicate a high level of uncertainty about the relevant probability. We show that subjective survival expectations in 2002 line up very well with realized mortality of the HRS respondents between 2002 and 2010. We show that the MRH model performs better than typically used models in the literature of subjective probabilities. Our model gives more accurate estimates of low probability events and it is able to predict the unusually high fraction of focal 0, 50 and 100 answers observed in many datasets on subjective probabilities.

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Introduction

Since Frank Knight (1921) introduced the distinction, economists have recognized that risk is the special case of uncertainty in which probabilities are known. Probabilities are treated as known for outcomes of symmetric devices with known properties that are subject to random forces such as the toss of a coin or die, the shuffling of a deck of cards or the shaking of an urn containing balls of different colors. In contrast, the uncertainty that confronts economic decision makers involves probabilities that are less precisely known. [1]

In this paper, we investigate the probability beliefs held by a given individual about personal mortality risks elicited from survey questions on the Health and Retirement Study (HRS) that ask respondents about the numerical probability that he or she will survive to a given age that is 10-20 years in the future.

In forming his belief about mortality risk, a person might consult a life table based on the experience of millions of individuals. The life table provides an estimate of the mean probability of survival for persons of given age and sex with essentially no sampling error. However the individual may be ambiguous about whether this probability represents the risk he himself faces. He may have personal information that makes him think that he has a higher or lower risk than the average person of his age and sex. Moreover, he may be unsure about the influence that personal information such as the ages of death of parents and relatives, health history and current symptoms, or exercise and dietary habits will have on his likely longevity.

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[1] Paté-Cornell (1996, pp. 96-97) observes that *"… uncertainties in decision and risk analyses can be divided into two categories: those that stem from variability in known (or observable) populations and, therefore, represent randomness in samples (aleatory uncertainties), and those that come from basic lack of knowledge about fundamental phenomena (epistemic uncertainties also known in the literature as ambiguity)*."

Finally, in answering the survey question, a person must construct a probability judgment "on the fly," accessing whatever frequentist data or epistemic beliefs he may have stored in his brain and manipulating this information through reasoning or gut reaction to produce an answer to the specific question about, say, the probability he will survive to age 80, all within less than a minute.[2]

In this paper, we assume that probability beliefs about survival probabilities are ambiguous in the sense that an individual has in mind a range of possible values of the probability that can be described by a density function, $g(p)$, that can take on a variety of shapes depending on both its mean and the degree of ambiguity. The theory literature calls this density function second order probability beliefs. (See, for example, Gilboa and Maracini, 2011).

We compare the predicted survival rates of our sample members to life tables and also to the actual survival of sample members eight years later, as reported in the 2010 wave of HRS. The predictions from our models track life table and actual mortality fairly closely for sample members who are below age 80, but begin to diverge substantially at the oldest ages, with older respondents being overly optimistic. The predicted survival rates co-vary with personal characteristics, health status and cognitive status in largely the same way that they do in

[2] The attempt to determine the probabilistic beliefs of lay people using direct questions on surveys has only become commonplace in the past two decades. See Manski (2004) for a summary of the history of and rationale for eliciting numerical subjective probabilities on surveys. A special issue of *the Journal of Applied Econometrics* (2010) is devoted to analysis of subjective probability questions on a variety of topics. There is a related but somewhat separate tradition of eliciting probability beliefs of experts as part of risk assessments in engineering and operations research applications. See Paté-Cornell (1996) for a summary of this tradition.

regressions that explain actual mortality. Thus, it appears that the subjective survival probability answers are good candidates for modeling individual level heterogeneity in survival chances.[3]

The major contribution of this paper is to provide an estimate of ambiguity about survival probabilities that are embodied in the spread of $g(p)$. We find that survival expectations are very uncertain in the Health and Retirement Study (HRS) and that uncertainty varies in the population: more educated people have more certain beliefs, women have less certain beliefs, and deterioration of health, especially from previously excellent levels, leads to more uncertainty about survival chances, too.

Identifying second order probability beliefs is possible under the assumptions of our survey response model, the Modal Response Hypothesis (MRH) which is a mapping from probability beliefs $g(p)$ to survey responses. It assumes that people report the mode of $g(p)$ whenever it exists and they report 50%, whenever $g(p)$ is so ambiguous that it does not have a unique mode. The motivation for the MRH is twofold. First, imagine that people estimate the probabilities of certain events after observing some successes and failures of these events. Under some conditions a naïve estimator, which is the ratio of "good cases", is exactly equal to the mode of the Bayesian updated posterior probability distribution. Moreover, this estimator is biased in finite samples. The MRH assumes that people report this simple, naïve estimator in surveys. The second motivation is coming from the literature using numerical subjective

[3] Our findings about the external validity of the HRS subjective probability questions are consistent with those in earlier papers that have studied these questions in the HRS (Hurd and McGarry (2002), Smith, et. al. (2001). The overly optimistic expectations of people over 75 in HRS has also been noted by Hurd and Rohwedder (2008) who find the same pattern across eleven European countries in the SHARE (Survey of Health, Ageing and Retirement in Europe). There are also a few papers that use these questions in models of behavior under uncertainty such as, for example, Picone, et al. (2004) who find that people who expect to live longer are more likely to choose medical screening tests.

probabilities on surveys. When people are asked to report a numerical probability using any digit between 0 and 100, an unusually large fraction of answers are heaped on 50. The excessive use of 50 has been interpreted as occurring because many people treat “50-50” as a synonym for “I don’t know” or even for “God only knows,” a sentiment that suggests that the true probability is unknowable (Fischoff and Bruine de Bruin, 1999; Bruine de Bruin and Carman, 2012; Lillard and Willis, 2001).[4,5] Researchers have realized that these focal answers might introduce bias into estimates of subjective probabilities. As a remedy, Manski and Molinari (2010) think of focal responses as extreme versions of rounding, where some people always round to 0, 50 or 100 percent. Lumsdaine and Potter van Loon (2012) model the probability of providing a focal answer as a separate equation in their econometric model. Our approach, instead, is to model focal responses with an economic model and relate it to the precision of beliefs of individuals. In our model people answer with an epistemic 50 percent as a way of saying “I am very unsure”, when their beliefs are too ambiguous. As far as we know the MRH model is the first in the literature that makes use of the focal responses to learn something about the beliefs of individuals.

For comparison, we present an alternative model of survey response which assumes that the person reports the mean of $g(p)$. We show that the MRH model can account quite well for heaping at 0, 50 and 100 while the mean response model cannot. The overuse of focal

[4] In recent waves of the Health and Retirement Study, respondents have been asked a follow-up question if they answered 50 to survival probability question: Do you think that it is about equally likely that you will die before 75 as it is that you will live to 75 or beyond, or are you just unsure about the chances? About two-thirds say that they are “just unsure.” In this paper, we use data from the 2002 wave of HRS which did not have a follow-up question.

[5] In addition, the answers to the survival questions also exhibit some heaping at 0 and 100. Taken literally, of course, these answers cannot represent rational probability beliefs except, perhaps, in the case of zero for persons who know themselves to be terminally ill.

responses can be especially problematic when very low or very high probabilities are modeled since the high ratio of 50 percent responses can arbitrarily push the estimated mean probabilities away from their true, extreme values. We show in this paper, that our modal response model that explicitly models focal answers works better than the simple mean model typically used in research on subjective probabilities. When the modal response model is used for extreme probabilities both the estimated unconditional means and the average partial effects are closer to ones estimated from realized mortality data.[6]

Learning about the degree of uncertainty in individuals' beliefs can be important for several reasons. First, the value of information about mortality risk, for example, should be a function of this uncertainty. Uncertain people might value information more, and certain people might rationally ignore any new information since they have already established a good understanding of the risks they face. Even in a fully Bayesian SEU framework the degree of uncertainty might play an important role if the utility function is not linear in probabilities. This is the case, for example, if people can invest in learning about their own mortality risks.[7] Second, learning about the degree of uncertainty in individuals' beliefs is useful from a survey methodological point of view, too. People whose probability beliefs are more certain may answer probabilistic survey questions with greater precision. However, if beliefs are uncertain, as is the case with mortality expectations, we expect large measurement error in survey responses, that might even account for the discrepancy between subjective and objective survival probabilities at later ages. Future

[6] Alternative models of focal responses, such as Manski and Molinari (2010) and Lumsdaine and Potter van Loon (2012) might work equally well in terms of reducing this bias. The main advantage of the MRH compared to these models is that we learn about the uncertainty of individuals' beliefs, too.

[7] Another example is Bommier and Villeneuve (2012), who discuss a model of mortality risk aversion, where the utility function is not additively separable over time, and thus, mortality probabilities enter the model in a non-linear fashion as well.

research should investigate the potential role of measurement error in subjective expectation data and its link to uncertainty in individuals' beliefs. Third, learning about the degree of uncertainty in individuals' beliefs might be very important in Non-Bayesian/Non-Expected Utility models. While we don't work in these frameworks, empirical evidence on ambiguous beliefs may be useful to those who do.

The paper is organized as follows. In Section 1 we provide a quick overview of the literature on ambiguity and its role in economic decisions. Section 2 describes the subjective survival data in HRS that we use in this paper. Section 3 introduces the MRH model that can be used to model any subjective expectation data that uses the HRS framework. Section 4 describes a simple and tractable model of individual survival curves. Section 5 discusses identification and provides further details about the data used in the empirical part of the paper and Section 6 shows the result.

1. Ambiguity in economics

Knightian uncertainty, epistemic uncertainty and ambiguity are roughly synonymous terms that figure prominently in a longstanding and ongoing debate about the link between rationality and probability beliefs, on the one hand, and the relationship between beliefs and decisions, on the other hand. In an excellent and authoritative review of this debate since Pascal's famous bet on the existence of God in 1670, Gilboa and Marinacci (2011) discusses the different models of expectation formation, including Bayesian and Non-Bayesian models. They call the model we use in this paper "the smooth model of ambiguity" or "second order probability beliefs". In typical models of ambiguity agents have a set of possible probability distributions in mind but they are not able to compound this information into a single probability distribution. The smooth model, however, makes compounding possible. The real question in the smooth model is whether agents have a preference for known probabilities (in other words they are ambiguity-averse) or not (in which case they are simple Bayesians). As Gilboa and Marinacci (2011) write "*… beyond the above mentioned separation [between beliefs and utilities], the smooth preferences model enjoys an additional advantage of tractability. Especially if one specifies a simple functional form for [preferences for known probabilities], one gets a simple model in which uncertainty/ambiguity attitudes can be analyzed in a way that parallels the treatment of risk attitudes in the classical literature*.", pp. 45.

The major contribution of this paper is to provide an estimate of second order probability beliefs (a form of ambiguity or Knightian uncertainty) about survival probabilities that are embodied in the spread of $g(p)$. It is important to clarify the role of this object in alternative views of risk and uncertainty. Conventional economic theory of behavior under uncertainty, rooted in subjective expected utility (SEU) theory, is often interpreted as having erased the distinction between risk and uncertainty because expected utility is a linear function of probabilities. That is, assume that the person's ambiguous probability beliefs are described by $g(p)$. His expected utility is

$$EU = \int_0^1 [\int_0^1 pU_{live} + (1-p)U_{die}]g(p)dp = \overline{p}U_{live} + (1-\overline{p})U_{die},$$

where $\overline{p}$ is the mean of this ambiguous distribution. Clearly, expected utility is invariant to a mean preserving spread of $g(p)$; hence, decisions based on expected utility are unaffected by ambiguity.

In the famous Ellsberg experiments (Ellsberg, 1961) subjects were presented with choices of drawing balls from different urns whose compositions were either known or ambiguous. Most people revealed distaste for ambiguity which was at odds with the SEU theory. During the 50 years since Ellsberg's experiments, the implications of ambiguity and ambiguity aversion for economic theory, decision science, statistics and econometrics have generated a vast literature. It is now widely believed that ambiguity aversion is widespread, that the link between choice and probability beliefs within a Bayesian framework that was established by Savage (1954) in his development of SEU theory needs to be modified or abandoned to accommodate ambiguity, and that most of the uncertainties faced by economic decision makers in the real world involve probabilities that are not and often cannot be fully based on objective evidence. The survey by Gilboa and Marinacci (2011) provides a comprehensive and insightful discussion of these issues.

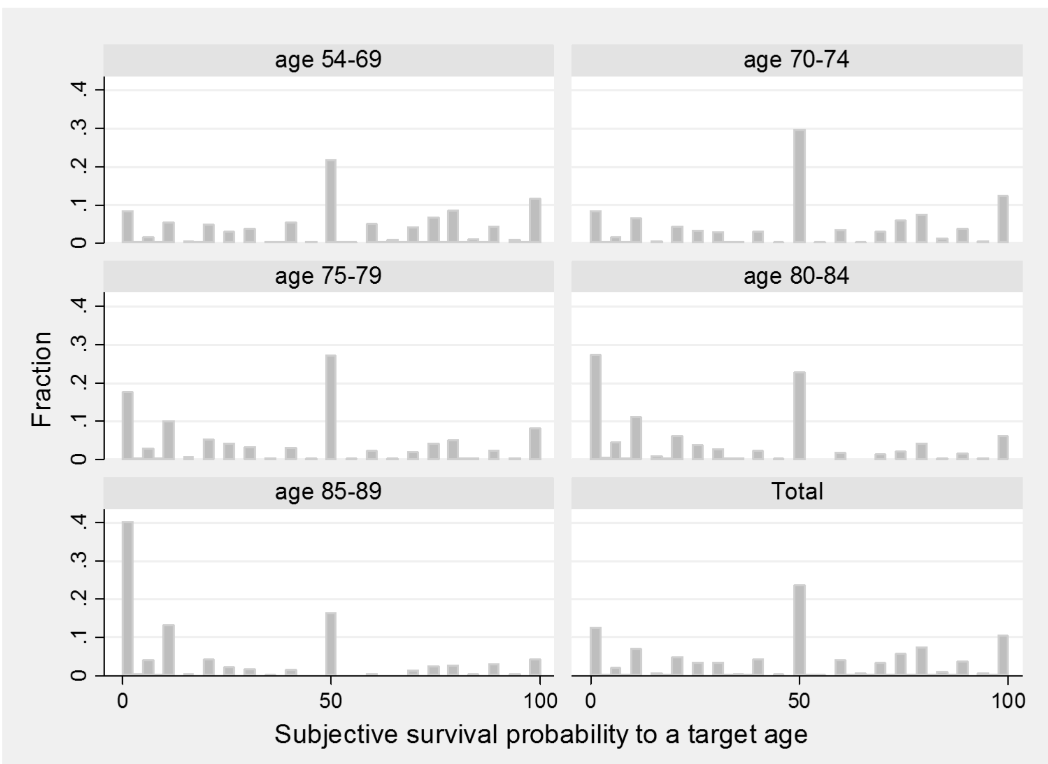
We separate the issues concerning ambiguity of probability beliefs from those concerning the effects of epistemic uncertainty and ambiguity aversion on decisions. We do so by utilizing survey data that asks directly about people's probability beliefs. As Manski (2004) emphasizes, this approach differs from the practice in much of applied economics of assuming that individuals have exogenously given probabilities. It also differs from Savage's theory which infers probability beliefs from choice situations. This means that we can explore empirically how beliefs about risk and uncertainty vary in the population without being required to take a stand on how decisions are affected by probability beliefs.

2. Subjective Survival Probability Questions on the HRS

The Health and Retirement Study has asked probabilistic expectation questions in various topics since its beginning in 1992. The survival question that we use in this paper comes from the 2002 wave and it reads as follows: "What is the percent chance that you will live to be [TARGET AGE] or more?" The target age exceeds the individual's age by at least 10 years: it is 80 years for people below 70, and 85, 90, 95 and 100 for individuals in successive five year age intervals.

Although the subjective probability responses in HRS seem reasonable when averaged across respondents, individual responses appear to contain considerable noise and are often heaped on values of "0", "50" and "100" (See for example Manski, 2004, for a discussion). Considering the whole group of probability questions in HRS-1998, for example, while only 5% of respondents refused to answer the probability questions, 52% of questions were heaped on either "0" or "100" and an additional 15% were heaped on "50".

Figure 1: Distribution of Survival Probabilities to Target Age, by Age of Respondent, HRS-2002



Note: Target age is 80 years for people below 70, and it is 85, 90, 95 and 100 for individuals in successive five year age intervals

These patterns are illustrated in Figure 1 by histograms of responses to the HRS-2002 survival probability question. We have included separate histograms by the target age used in the survival question and a total histogram in the six panels of Figure 1. Each histogram shows a high frequency of focal answers, especially at 50. The ratio of 50 responses is somewhat smaller at old ages where the actual survival probabilities are low, but it still accounts for 16 percent of the responses for people above 84.

Some psychologists, especially Fischhoff, Bruine de Bruin and their colleagues (Fischhoff and Bruine de Bruin, 1999; Bruine de Bruin et al., 2000) have argued that answers of "50" may reflect "epistemic uncertainty;" that is, a failure to have any probability belief at all about the event in question or, at least, to have no clear idea of what the probability could be. Alternatively, of course, an answer of "50" might reflect a very precise belief about the probability that a fair coin will come up heads or perhaps a somewhat less precise belief that a given event is about equally likely to occur or not occur. Indeed, while HRS probability questions offer participants a scale of integers from 0 to 100, the large majority of "non-focal" answers are integers ending in "5" or "0", suggesting that responses from most people involve rounding or approximation. See Manski and Molinari (2010) for a discussion of the different rounding practices of survey respondents in the HRS and of potential remedies.

There has been much less emphasis in the psychological literature on focal answers at "0" or "100". When a probability question concerns an event such as the chance of being alive ten or fifteen years from now, it does not seem credible to assume that a respondent who gives such an answer of "100" is completely certain he will be alive then and, apart from a person diagnosed with a terminal illness, an answer of "0" should not be taken at face value, either.

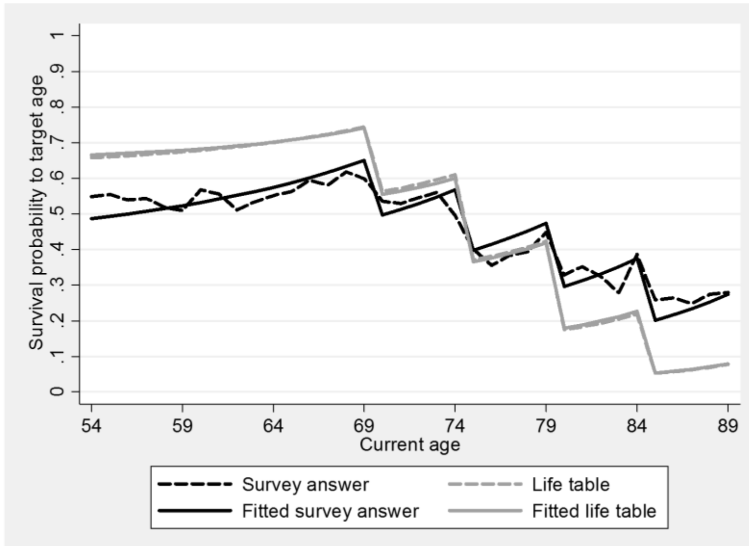
It is possible to regard answers of “0” or “100” as approximations which are no different in kind than rounded answers of “5”, “40” or “95”. However, in a discussion of Gan, Hurd and McFadden (2005), Willis (2005) provides evidence against this interpretation in a simple regression of actual mortality by 1995 on individual answers to the subjective survival probability question in 1993 using a sample of persons over 70 from the AHEAD cohort of HRS. He finds that a person who responds “0” has an actual survival probability that is 13 percentage points higher than a person who gives very low non-focal answer. Similarly, a person who responds “100” has a survival chance that is 3.8 percentage points lower than a person who gives a non-focal answer near 100. This suggests that focal answers at the extreme may, like answers at “50”, reflect more imprecise, ambiguous or uncertain probability beliefs than those of persons who give non-focal answers.

Previous researchers have found that the tendency to give focal answers is associated with lower cognitive ability (Lillard and Willis, 2001; Hurd and McGarry, 1995). In particular, age and education, both strong correlates of mortality risk, are also related to the tendency to give focal answers to probability questions on a wide variety of topics.

Focal responses, thus, might bias estimated average survival probabilities if we take them at face value and the bias might be stronger in situations where the underlying probability is far from 50 percent. Figure 2 and 3 compare the average responses to the survival questions by age and gender to the corresponding numbers from life tables. The target age in the survival question changes with the age of the respondents, hence the apparent sawtooth shape of the curves in Figure 2 and 3.

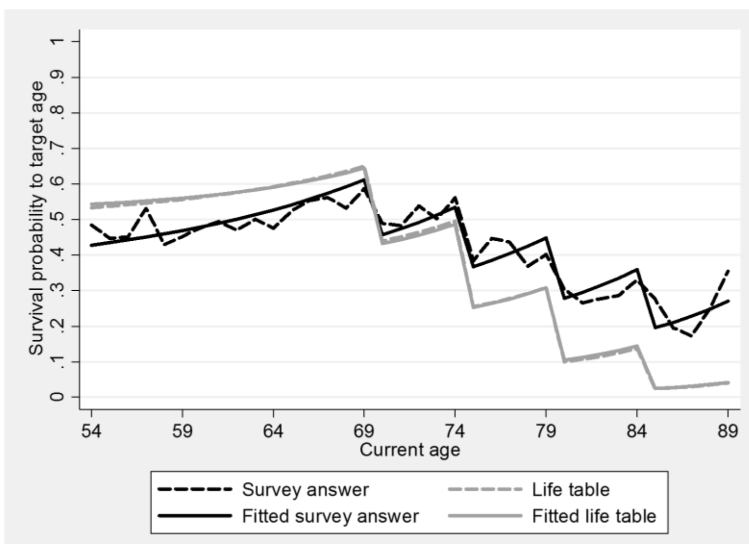
Comparing the HRS responses to life tables might be problematic for reasons we will shortly discuss, but we can see a quite strong bias in the survey responses toward 50 percent. We shall investigate in this paper whether the discrepancy between subjective and objective survival probabilities is a consequence of epistemic responses.

Figure 2: Subjective and life table survival probabilities to a target age, HRS-2002, females



Note: Target age is 80 years for people below 70, and it is 85, 90, 95 and 100 for individuals in successive five year age intervals; The model fits Gompertz survival functions on life table and age-aggregated subjective probabilities; Weighted numbers

Figure 3: Subjective and life table survival probabilities to a target age, HRS-2002, males



Note: Target age is 80 years for people below 70, and it is 85, 90, 95 and 100 for individuals in successive five year age intervals; The model fits Gompertz survival functions on life table and age-aggregated subjective probabilities; Weighted numbers

It is problematic to directly compare the objective and the subjective survival curves. Life table probabilities are estimated from current mortality data and it is known that mortality is changing over time. An important advantage of the subjective survival data is that it is measured directly for each cohort and we do not need to extrapolate information from previous cohorts (Perozek, 2008; Gan et al., 2005). In the empirical part of the paper, thus, we will compare estimated individual survival curves to actual survival of the respondents eight years later in 2010, which is the last available wave of the HRS.

3. Probability Beliefs and the Modal Response Hypothesis

In this section, we describe a theoretical model which attempts to relate answers that an individual gives to a survey question about the subjective probability of a given event and his underlying probability beliefs. In our model we distinguish between *ambiguity*, which we define as a second order probability distribution represented by the density function, $g(p)$, and *epistemic uncertainty*, which we define as an inability or unwillingness to report a probabilistic belief. Although the model is designed to apply to any subjective probability question using the HRS format, in this paper we discuss it in the context of survival probabilities.

Let us assume that person i is faced with the problem of estimating the probability p_i of an event A. Initially he has no information about the probability of this event, he has an uninformed prior $p_i^{prior} \sim U(0,1)$, where U denotes the uniform distribution. The person observes event A happening $\alpha_i - 1$ times and event A not happening $\beta_i - 1$ times. In the survival context, for example, this means that a person is aware of $\alpha_i - 1$ people similar to himself who survived to the given target age and $\beta_i - 1$ similar people who died before reaching that age. It is well known that if this new information is used to Bayesian update one's beliefs about p_i, the posterior distribution has a Beta distribution[8] with parameters α_i and β_i,

$p_i \sim Beta(\alpha_i, \beta_i)$.

When faced with a survey question about the probability of event A the person might respond with the mean or the mode of this distribution. When α_i and β_i are larger than one[9], the mean and the mode of the Beta distribution are

$$p_i^{mean} = \frac{\alpha_i}{\alpha_i + \beta_i} \equiv \mu_i \quad (1)$$

$$p_i^{\text{mode}} = \frac{\alpha_i - 1}{\alpha_i + \beta_i - 2} \quad (2)$$

A Bayesian agent would report p_i^{mean} which is the expected value of the Bayesian updated posterior distribution. Note that p_i^{mode} is exactly equal to the naïve estimator of the probability that can be computed by the number of "good cases" which is $\alpha_i - 1$ over the number of all cases which is $\alpha_i + \beta_i - 2$. A frequentist agent, thus, would not report the mean but, rather, the mode of the distribution $g_i(p)$. The modal response hypothesis assumes that people report p_i^{mode} rather than p_i^{mean} to probabilistic survey questions for at least two reasons. First, as Lillard and Willis (2001) argue, it is cognitively less burdensome for a respondent to answer a

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[8] Hill, Perry and Willis (2004) used a model that was similar in spirit and its qualitative properties to the model used in this paper, but it had a number of disadvantages. First, the expected value of the subjective probability distribution they used did not have a closed form. Second, their distribution was not based on Bayesian updating and its parameters did not have a clear economic meaning. Third, as we discuss in the text, in our model the mode of the Beta distribution is exactly equal to a rule-of-thumb estimator for the probability in question.

[9] Other cases will be discussed below.

survey probability question quickly by reporting the most likely value of *p*, given by the mode of $g(p)$, than it is to report the expected value given by $p_i^{mean} = \int p g_i(p) dp$. Second, p_i^{mode} is equal to a very simple rule-of-thumb estimator for the probability in question: the frequentist response. It seems a reasonable assumption that in a survey situation where people have to answer many questions in a very short timeframe give frequentist approximations to probability questions instead of Bayesian updating their priors. Moreover, in this model the mode is often a good approximation of the mean[10]. The formula in (2) does not give the mode of the distribution when either α_i or β_i is smaller than 1. Whenever $\alpha_i < 1, \beta \geq 1$ the distribution is always decreasing and has a unique mode at zero. Whenever $\alpha_i \geq 1, \beta < 1$ the distribution is always increasing and its unique mode is at one. Finally, if $\alpha_i < 1, \beta < 1$ the distribution has a U-shape and it has two maxima at zero and one. In this case one finds it more probable that the probability of event A is zero or one than that it is 50 percent.

We have motivated the use of the Beta distribution with a Bayesian framework where agents observe certain numbers of successes and failures. As we shall show, however, the distributions that occur when either α_i or β_i is smaller than 1 cannot be derived from Bayesian updating based on evidence. Indeed, these distributions correspond to situations in which, in effect, the agent has very little objective evidence on which to base his beliefs and, lacking evidence, tends to give conventional "epistemic " responses to survey questions about his probabilistic beliefs . In particular, we hypothesize that the person will respond with an extreme value of either zero or one when $g(p)$ is montonically decreasing or increasing. When the distribution is U-shaped,

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[10] Engelberg et al. (2009) find that the large majority of respondents in the Survey of Professional Forecasters have tight subjective probability distributions in their head about future GDP and inflation, and thus, the means, the modes and medians of these distributions are very close to each other.

we hypothesize that the person will answer "50" as a synonym for "God only knows" rather than as necessarily a belief that the outcome in question is equally likely to occur or not.[11]

To show how the shape of $g(p)$ is related to the amount of evidence on which an individual bases his beliefs, it is convenient to introduce two more parameters that are functions of α and β:

$$\mu_i = \frac{\alpha_i}{\alpha_i + \beta_i} \tag{3}$$

$$n_i = \alpha_i + \beta_i \tag{4}$$

μ_i is the expected value of the distribution of the probability in question, and n_i is a measure of the precision of beliefs. Higher n_i means more precise beliefs, that is, a tighter $g_i(p)$ density function. This density function has the form

$$g_i(p) = g(p \mid \mu_i, n_i) = \frac{p^{\mu_i n_i - 1}(1-p)^{(1-\mu_i)n_i - 1}}{B(\mu_i n_i, (1-\mu_i)n_i)} \tag{5}$$

where $B(\cdot)$ is the Beta function.

Earlier, we argued that an uninformed agent with a uniform prior over the unit interval would update his prior after observing $\alpha - 1$ successes and $\beta - 1$ failures in a sample of

[11] In the survival context, a U-shaped distribution could represent the beliefs of someone who is unsure whether he had inherited a genetically transmitted disease. In case he did, he might face a low survival probability to the target age, but if he did not he has a high probability of surviving. The posterior distribution of the survival probability in this case can have a U shape, where the extreme probabilities are more likely than any middle values. However, it is not plausible that such situations are common enough to account for the large number of "50" responses that we see in survey responses.

$N=\alpha+\beta-2$. Note that $B(1,1)$ is a uniform distribution so that $n_i=\alpha+\beta=2$ for an uninformed agent. Equivalently, such an agent observes no data since $N=0$. Thus, a necessary condition for Bayesian updating is that the agent observe a positive amount of data, which implies that $\alpha+\beta>2$. As we have seen, any Beta function satisfying this condition is unimodal where the mode falls in the interval, $0<p<1$. Conversely, when $1\leq\alpha+\beta<2$, $g(p)$ may be monotonically increasing, decreasing or U-shaped depending on the value of μ and if $\alpha+\beta<1$, $g(p)$ is always U-shaped. Obviously, Bayesian updating cannot be the source of such beliefs since the implied sample size is negative! That is why we label such beliefs as "epistemic" and distinguish them from "ambiguous" beliefs that can be represented by a second order probability.

Up to now, we have assumed that agents observe a certain number of successes and failures in a sample of a given size, implying that α and β are integers. The Beta distribution, however, is well defined for any positive real values of α and β. The intuition for observing non-integer numbers of successes/failures could be that the observed success/failure is only partially relevant to the question. For example, assume that a person's father died at the age of 78. When this person faces a survey question about the probability of surviving to at least 80 he might consider his father's early death as a negative signal about his own survival chances. However, his father lived in a time when the health care system was not as good as today; his father might have lived an unhealthier life; etc. The person, thus, might put a less than unitary weight on his father's death as it is only partially relevant to his own survival chances.

In our development of the Beta model, the precision parameter, $n_i=\alpha_i+\beta_i$, is assumed to be equal to the size of the sample less two that is observed by agent *i*. A broader and more useful interpretation of precision is that it measures the confidence that an individual has in his judgment of the risk of a given event. The person's confidence may be either warranted or unwarranted when tested against objective evidence. For example, in this paper we find that, on average, people aged 50-70 have quite accurate estimates of their survival chances 10-15 years in the future while people older than 70 are significantly overoptimistic. However, confidence in one's probabilistic beliefs is often based on evidence. For instance, educated individuals can utilize, in addition to their personal experience, a broader knowledge of evidence about mortality and its causes from past coursework, wider reading and better informed family and social networks. Thus, we may interpret precision as a measure of a person's capacity to assess his survival risks based on his knowledge of mortality risks and his ability to translate personal information about his own health, health behavior and family history into its implications for survival chances.[12] By considering n_i and μ_i together and comparing the accuracy of subjective beliefs against objective evidence, we can assess whether increases in confidence tend to be warranted.

The relationship between n_i, μ_i and $g(p)$ is depicted in Figure 4. The figure presents a matrix of 81 probability density functions— $g(p \mid \mu_i, n_i)$ in equation (5)—corresponding to nine different values of the mean of $g(p)$, given by μ_i on the horizontal axis, and nine different degrees of precision, measured by n_i, on the vertical axis. In our model, these densities

[12] Of course, it is possible that a person may have a very unambiguous view about the "true" survival probability that is based on little or no evidence. While our model cannot identify whether and how people process evidence, we will compare the effects of covariates on subjective probabilities and the actual mortality experience of the same people eight years later. With a few exceptions, subjective and objective covariates have the same signs, but the coefficients of covariates estimated from the subjective data are smaller than those estimated from actual mortality.

represent different possible beliefs about the probability of survival to a target age. For example, consider the densities for which $\mu_i = 0.5$ so that the mean survival probability is one-half. These are represented by the column of density graphs in the center of the figure. The density in the lowest graph in the column, corresponding to a very high level of precision of $n_i = \alpha_i + \beta_i = 5000$, has almost all its probability mass concentrated at 0.5. As n_i decreases, the densities spread. For example, when $n_i = 100$, $g(p)$ has a single mode at 0.5 and probability mass near zero for $p < 0.4$ and $p > 0.6$. In this case, an answer of "50" to the survey question means that the respondent believes that the probability is between 40 and 60 percent, so that 50 is a reasonable approximation. When $n_i = 2$, the density is uniform, corresponding to the case when the posterior belief is the same as the prior because no new information was acquired ($\alpha_i - 1 = 0$ and $\beta_i - 1 = 0$). In this case all probabilities between 0 and 1 are equally likely. For values of $n_i < 2$, the density is U-shaped with two maxima at 0 and 1. According to the *MRH*, a survey respondent would report "50" in any of these cases. Thus, an answer of "50" by a survey respondent might reflect a very precise view that the chance of an event is one half; a view that the probability is approximately one half; or a view that the probability could be anything so that an answer of "50" indicates epistemic uncertainty.

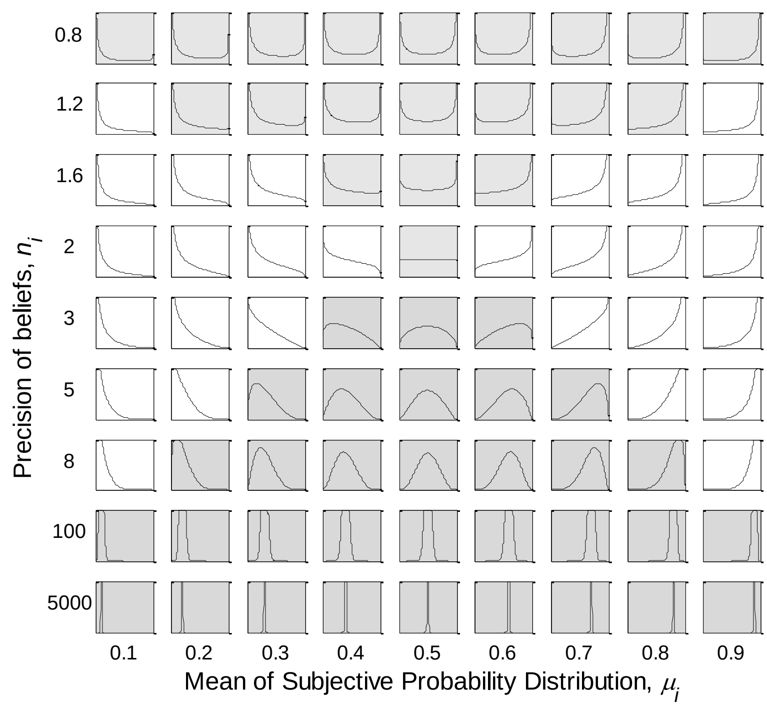
Figure 4 also illustrates the boundary between ambiguous beliefs that can be represented by a second order probability distribution based on Bayesian principles and epistemic uncertainty in which the individual has too little knowledge about the risk in question to be able to form an evidence-based probability judgment. Possible ambiguous densities appear in the darkly shaded, inverted U-shaped region in Figure 4 for which $n_i > 2$. Note that throughout this region, reports of p^{mean} and p^{mode} tend to be very close to one another. The reason is twofold.

First, as we have remarked earlier, p^{mode} provides a good estimate value of p^{mean} when the mean is in an intermediate range of about 30-70 percent even with levels of precision as low as $n_i = 5$. Second, when the individual's subjective value of μ is more extreme, he will provide a non-epistemic response (i.e., not 0, 50 or 100) only if he has a relatively high level of precision. For example, the MRH implies that a person who believes that the mean is 10 percent would report (a rounded) 10 percent if $n_i = 100$, but would report zero if $n_i = 8$. The combinations of μ_i and n_i for which the MRH predicts epistemic responses are shown in Figure 4 by the two unshaded triangular regions corresponding to answers of 0 and 100; finally, in the lightly shaded, inverted U-shaped region with a minimum at $\mu_i = 0.5$ and $n_i = 2$, a respondent always reports 50 regardless of their belief about risk.

To summarize, the modal response hypothesis claims that survey respondents report a potentially rounded version of the mode of $g_i(p)$ whenever it exists and they report 50 percent whenever $g_i(p)$ has a U shape.

$$p_i^{mrh} = \begin{cases} round\left(\dfrac{\alpha_i - 1}{\alpha_i + \beta_i - 2}\right) & \text{if} \quad \alpha_i > 1, \beta_i > 1 \\ 1 & \text{if} \quad \alpha_i > 1, \beta_i \le 1 \\ 0 & \text{if} \quad \alpha_i \le 1, \beta_i > 1 \\ 0.5 & \text{if} \quad \alpha_i \le 1, \beta_i \le 1 \end{cases} \tag{6}$$

Figure 4: Density of probability beliefs ($g_i(p)$) for different mean (μ_i) and precision (n_i) values



The *round* function can be rounding to the closest 1 percent, 5 percent, 10 percent or anything that seems appropriate in the context of the survey. Manski and Molinari (2010), for example, use a framework where there are individual differences in rounding practices. Their approach can also be modeled in our framework by letting the *round* function vary across individuals.

The hypothesis of this paper is that people answer subjective probabilistic expectation questions according to the MRH. It would be desirable, however, to test the MRH against other survey response models. A natural candidate for comparison is the mean model where people respond a potentially rounded version of the mean of $g_i(p)$.

$$p_i^{mm} = round\left(\mu_i\right) \tag{7}$$

In the mean model the precision of beliefs (n_i) is not identified, only the mean (μ_i) is. The mean and the mode models however converge to each other as n_i goes to infinity. One natural test of the MRH, thus, can be carried out by estimating the MRH model and testing whether the estimated precision is finite. As we will show, this is indeed the case. We will also provide other tests. We will simulate probabilistic survey responses from the estimated mean and MRH models and compare their histogram to the histogram of responses from the HRS. As we will show while the MRH model tracks the true histogram very closely, the mean model performs poorly, mostly because it cannot explain the large fraction of focal answers. Lastly, we will compare subjective and actual survival probabilities. We will show that while the MRH model still has problems predicting low probability events precisely, such as survival probabilities at old ages, it still performs better than the mean model.

4. Individual subjective survival curves

In the previous section, we introduced two survey response models that transform second order probability distributions $g\left(p \mid \mu_i, n_i\right)$ into the survey response p_i^{mrh} or p_i^{mm}. These models can be applied to any subjective probabilities and not just to survival data. To close the model, however, we need to specify the mean (μ_i) and the precision (n_i) of beliefs. There is no unique way of modeling these two variables; it is the task of the researcher to find the appropriate model in the context of the particular project. In this section, we show how μ_i and n_i can be modeled in the context of survival probabilities.

The so-called Gompertz model of longevity has been widely used in both demography and biology because its increasing mortality hazard assumption lines up with mortality data of humans and other species very well (Vaupel, 1997). The Gompertz model assumes that the hazard of death is exponentially increasing with age:

$$h(a) = \gamma_0 \gamma_1 \exp(\gamma_1 a) \tag{8}$$

where γ_0 is a positive scale and γ_1 is a positive shape parameter. By simple calculation, (8) leads to the following survival probability from age *a* to age *t*:

$$S(a,t) = \exp\left(-\gamma_0 \left(\exp(\gamma_1 t) - \exp(\gamma_0 a)\right)\right). \tag{9}$$

We have already discussed Figure 2 and 3 in Section 2 when we compared subjective survival probabilities to life tables. The two figures also show fitted Gompertz values. For estimation, we collapsed the probabilities into age-gender cells and used non-linear least squares on the aggregated data to recover the unknown scale (γ_0) and shape (γ_1) parameters. As we can see, the Gompertz model gives a very good fit for both genders and for both subjective and objective life table probabilities. Thus, we shall maintain the assumption of exponentially increasing mortality hazards in this paper.

The main advantage of subjective survival data is that we can estimate individual heterogeneity in survival chances. With objective survival data we can only identify group-specific survival probabilities by computing the ratio of survivors in a particular group. Unobserved heterogeneity within groups, however, is not identified. In contrast, subjective survival data enables us to estimate individual heterogeneity in survival chances as we collect probability data on the individual level. Following Vaupel (1979) we allow the scale parameter (γ_0) to have a distribution but fix the shape parameter (γ_1) in the population. We are going to use the assumption that the scale parameter has a gamma distribution with shape parameter k and scale parameter θ

$$\gamma_{0i} \sim \Upsilon(k,\theta). \tag{10}$$

The expected value of the gamma distribution is $k\theta$ and thus both parameters increase mortality chances and decrease the probability of survival (see equation (9)). Using the gamma distribution has several advantages. The first is that under this assumption the average survival probabilities can be derived analytically. As we show in Appendix B, in the gamma-gompertz model the average survival probabilities from age *a* to age *t* is

$$E\left(S_i\left(a,t\,|\,k,\theta,\gamma_1\right)\right)=\left(1+\theta\left(\exp\left(\gamma_1 t\right)-\exp\left(\gamma_1 a\right)\right)\right)^{-k}. \tag{11}$$

The second advantage of the gamma-gompertz framework is that we can analytically derive the effect of individual heterogeneity on sample selection. Different survival chances are modeled by letting γ_0 have a distribution in the population. In this paper we refer to γ_0 as "frailty" (Vaupel, 1979), which includes genetic, environmental and behavioral factors that affect the underlying mortality of individuals other than age. As long as survival chances are heterogeneous in a population, fit individuals will be overrepresented in the sample over time as frail individuals are more likely to die and not participate in the HRS. By applying the formula from Vaupel (1979) we can analytically characterize this sample selection. Let k^a and θ^a denote the shape and scale parameters in (10) in cohort a. The following is true for any cohorts

$$k^a = k^r \tag{12}$$

$$\theta^a = \frac{1}{\frac{1}{\theta^r} + \left(\exp(\gamma_1 a) - \exp(\gamma_1 r)\right)} \quad (13)$$

where r is a reference cohort. As we can see, older and younger cohorts share the same shape parameter k^a, but older cohorts have lower scale parameter θ^a (lower frailty) than younger cohorts.

Another advantage of the gamma-gompertz framework is that it is relatively easy to add covariates to the model and recover interesting structural parameters using the delta method. Without a closed analytical formula for the average survival chances in (11), the use of the delta method would be complicated. We recommend adding covariates to θ^r, which is the scale parameter of the gamma distribution in the reference cohort. To sum up, we use the following structural equations.

$$\mu_i \equiv S_i(a,t) = \exp\left(-\gamma_{0i}\left(\exp(\gamma_1 t) - \exp(\gamma_0 a)\right)\right) \quad (14)$$

$$\gamma_{0i} \sim \Upsilon(k, \theta_i) \quad (15)$$

$$\theta_i = \frac{1}{\frac{1}{x_i^{'}\beta_\theta} + \left(\exp(\gamma_1 a) - \exp(\gamma_1 0.5)\right)} \quad (16)$$

In equation (16) we have set the reference age to 50 ($r = 0.5$) and we have added covariates to the scale parameter of the 50 year old cohort ($\theta_i^{0.5} = x_i^{'}\beta_\theta$). After fitting this model one can recover the average partial effect of any covariate x_j on the survival probability from age a_1 to age a_2 by substituting the estimated coefficients into the following equation.

$$APE_j(a_1, a_2) = E_x\left[\frac{\partial S_i(a_1, a_2)}{\partial x_j}\right] \quad (17)$$

Details about estimating the partial effects in (17) can be found in Appendix B.

So far we have only talked about how to model the mean survival probability, μ_i. For modeling the precision of beliefs (n_i), we use a very simple log-normal framework.[13]

$$\ln\left(n_i\right) = z_i^{'}\beta_n + u_{ni} \tag{18}$$

$$u_{ni} \sim N\left(0, \sigma_n^2\right) \tag{19}$$

Equation (14)-(19) together with the survey response models of the previous section fully specify our model and we are ready to estimate it with maximum likelihood.

5. Estimation and identification

Our structural model has two unobservables, λ_{0i} which is a function of the mean survival probability and u_{ni} which is the unobserved heterogeneity in the precision of beliefs. Based on the distributional assumptions from the previous section, the model is fully specified and it can be estimated with maximum likelihood.

We only observe one survival probability answer in HRS. In Section 3, we proposed two survey response models. The mean model assumed that people report a rounded version of their true survival chances, while the MRH model assumed that people report a rounded version of the

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[13] This specification is a very simple way of modeling uncertainty. One important issue is that when $n<1$, all responses will be epistemic 50%. The empirical model, thus, can identify the ratio of responses where $n<1$, but whether the log-normality assumption is appropriate for the conditional density of n in this region cannot be determined from the data.

mode of the distribution of probability beliefs $g\left(p \mid \mu_i, n_i\right)$ or 50 percent when the mode does not exist.

The joint distribution of the two random variables λ_{0i} and u_{ni} is complicated because one is gamma, the other is normal and they enter the model in a non-linear fashion. The estimation of the MRH, thus, can be carried out by maximum simulated likelihood. The estimation of the mean model is more straightforward as the precision of beliefs plays no role in the model.

5.1. Estimating the mean model

The likelihood function can be written as

$$l_i = \Pr\left(\underline{p}_i \leq S_i\left(a,t\right) \leq \overline{p}_i\right) \tag{20}$$

where $\underline{p}_i$ and $\overline{p}_i$ denote the lower and upper bound probabilities that would be rounded to the survey response. For example, if the rounding function rounds to the closest 1 percent and the survey response is 27 percent, then $\underline{p}_i = 0.265$ and $\overline{p}_i = 0.275$. If the rounding function rounds to the closest 5 percent, then the corresponding probabilities would be $\underline{p}_i = 0.225$ and $\overline{p}_i = 0.275$.[14] The likelihood function, thus, is

[14] As we can see, a response that is not a multiple of 5 percent cannot be a rounded version of a true latent probability when we round to the closest 5 percent. The rounding model we have in mind is one where agents only identify bins (e.g. 0%-2.5%, 2.5%-7.5%, …, 97.5%-100%) and any response in a particular bin only tells the econometrician that the true latent probability is also in the same bin. An alternative model would be that of Manski and Molinari (2010) where they identify individuals' rounding practices across many probability questions and use different rounding functions for each individual.

$$l_i = \Pr\left(\frac{-\ln\left(\underline{p}_i\right)}{\left(\exp\left(\gamma_1 t\right) - \exp\left(\gamma_0 a\right)\right)} \geq \lambda_{0i} \geq \frac{-\ln\left(\overline{p}_i\right)}{\left(\exp\left(\gamma_1 t\right) - \exp\left(\gamma_0 a\right)\right)}\right) \tag{21}$$

which can be easily computed from the c.d.f. of the gamma distribution in (10) with parameters k and θ_i.

5.2. Estimating the modal response model

The estimation of the MRH can be carried out by maximum simulated likelihood (MSL). MSL computes the likelihood function by drawing many values from the distribution of one (or several) random variables and computing the average conditional likelihood, conditioning on those values. In our case, it is worth simulating values from the distribution of u_{ni}. A standard version of the simulated likelihood would look like the following.[15]

$$l_i = \frac{1}{S}\sum_{s=1}^{S} l_i\left(u_{ni}^{s}\right) \tag{22}$$

where S is the number of simulation draws. The problem with this approach is that our model is full of discontinuities that make this approach infeasible. Let us take a look at the MRH formula by rewriting (6) with our new variables, μ_i and n_i. Let us denote $\underline{n}_i \equiv \frac{1}{n_i}$ and $\overline{n}_i \equiv 1 - \frac{1}{n_i}$. The MRH model assumes that the survey response is

[15] Note that in case the simulated values are drawn from the distribution of u_{ni}, we do not need to weight the terms in (22) by the density of the draw as the simulation itself already weights the data.

$$p_i^{mrh} = \begin{cases} round\left(\dfrac{\mu_i n_i - 1}{n_i - 2}\right) & \text{if} \quad \overline{n}_i > \mu_i > \underline{n}_i \\ 1 & \text{if} \quad \mu_i > \underline{n}_i, \overline{n}_i \\ 0 & \text{if} \quad \mu_i < \underline{n}_i, \overline{n}_i \\ 0.5 & \text{if} \quad \overline{n}_i < \mu_i < \underline{n}_i \end{cases} . \tag{23}$$

As we can see, whenever $n_i \leq 1$ an answer can only be 50 percent. Whenever $n_i \leq 2$, an answer can only be 0, 50 or 100 percent. These discontinuities make the simulation model in (22) hard for the following reason. Imagine that during the maximization of the likelihood function we get into a region where the precision of beliefs n_i is always below 2 for each simulation draw. This could happen if $z_i^{'}\beta^k{}_n \ll 2$ and $\sigma_n^{2,k}$ is small, where k indexes the actual guesses for the parameters. In this case, the likelihood function would be undefined and the numerical maximization would fail. As a remedy, we recommend drawing separate simulation values from each region of n_i; this assures that the likelihood is well-defined in each iteration of the maximization.[16] The likelihood function, thus, is written

$$\begin{aligned} l_i = & \frac{1}{S_1}\sum_{s=1}^{S_1} l_i\left(u_{ni}^s, n_i \leq 1\right)\Pr\left(n_i \leq 1\right) + \frac{1}{S_2}\sum_{s=1}^{S_2} l_i\left(u_{ni}^s, 1 < n_i \leq 2\right)\Pr\left(1 < n_i \leq 2\right) \\ & + \frac{1}{S_3}\sum_{s=1}^{S_3} l_i\left(u_{ni}^s, 2 < n_i\right)\Pr\left(2 < n_i\right). \end{aligned} \tag{24}$$

Now, whenever $z_i^{'}\beta^k{}_n \ll 2$ and $\sigma_n^{2,k}$ is small in a particular iteration for a non-focal answer, the likelihood is a well-defined, extremely small number.

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[16] Hill, Perry and Willis (2004) did not use this trick when they estimated a very similar model. The consequence was that they had to make restrictions on their model to be able to carry out the numerical estimation. It turned out that our model is identified and estimable in practice under mild conditions once these discontinuities are properly taken care of.

The probabilities of the different regions of n_i are trivial since n_i is assumed to have a log-normal distribution. Whenever $n_i \leq 1$ an answer can only be fifty and thus the conditional likelihood is

$$l_i\left(u_{ni}^s, n_i \leq 1\right) = \begin{cases} 1 & \text{if } p_i^{hrs} = 50 \\ 0 & \text{otherwise} \end{cases}. \tag{25}$$

When $1 < n_i \leq 2$, it is assured that $\underline{n}_i \geq \overline{n}_i$ and thus

$$l_i\left(u_{ni}^s, 1 < n_i \leq 2\right) = \begin{cases} \Pr\left(\overline{n}_i < S_i\left(a,t\right) < \underline{n}_i\right) & \text{if } p_i^{hrs} = 50 \\ \Pr\left(S_i\left(a,t\right) \leq \overline{n}_i\right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(S_i\left(a,t\right) \geq \underline{n}_i\right) & \text{if } p_i^{hrs} = 100 \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

These probabilities can be computed analogously to the mean model derived in Section 5.1.

The most complicated, although still very straightforward, case is the conditional likelihood in the region where $n_i > 2$. In this region $\underline{n}_i < \overline{n}_i$. The only complication is that a 0 and a 100 answer can now be either a focal answer or an exact rounded answer. 50 answers in this region cannot be focal answers as $n_i > 2$ and thus either α_i or β_i is larger than one (See equation(4)). The conditional likelihood is

$$l_i\left(u_{ni}^s, n_i > 2\right) = \begin{cases} \Pr\left(S_i\left(a,t\right) \le \underline{n}_i \text{ OR } \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \le \overline{p}_i \right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(S_i\left(a,t\right) \ge \overline{n}_i \text{ OR } \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \ge \underline{p}_i \right) & \text{if } p_i^{hrs} = 100. \\ \Pr\left(\underline{p}_i \le \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \le \overline{p}_i \right) & \text{otherwise} \end{cases} \tag{27}$$

After straightforward algebra the conditional likelihood becomes

$$l_i\left(u_{ni}^s, n_i > 2\right) = \begin{cases} \Pr\left(S_i\left(a,t\right) \le \max\left\{ \dfrac{\overline{p}_i\left(n_i - 2\right)+1}{n_i}, \underline{n}_i \right\} \right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(\min\left\{ \dfrac{\underline{p}_i\left(n_i - 2\right)+1}{n_i}, \overline{n}_i \right\} \le S_i\left(a,t\right) \right) & \text{if } p_i^{hrs} = 100. \\ \Pr\left(\dfrac{\underline{p}_i\left(n_i - 2\right)+1}{n_i} \le S_i\left(a,t\right) \le \dfrac{\overline{p}_i\left(n_i - 2\right)+1}{n_i} \right) & \text{otherwise} \end{cases} \tag{28}$$

and again these probabilities can be computed analogously to the mean model derived in Section 5.1.

5.3. Identification

We seek to estimate the following set of parameters: $\lambda_1, k, \beta_\theta, \beta_n, \sigma_n$. In the case of the mean model, parameters of the belief precision, β_n and σ_n are not identified. It is worth thinking about where the identification of the different parameters is coming from. Parameter λ_1 is the shape parameter of the individual survival function and it is identified from how fast the probability responses change with age ($\partial E\left(p_i^{hrs} \mid a\right)/\partial a$). Parameters k and β_θ determine the scale parameter of the individual subjective survival curves (λ_{0i}) and they are identified from the location and dispersion of the individual responses ($E\left(p_i^{hrs} \mid x_i\right)$ and $V\left(p_i^{hrs} \mid x_i\right)$).

The identification of the belief precision parameters β_n and σ_n is primarily coming from the fraction of different focal answers in different demographic groups. If we have many focal answers, we expect n_i to be small. If we have many different types of focal answers (0, 50 and 100), we expect a high σ_n, indicating a high dispersion of belief precision in the population.

Finally it is worth noting that both the mean and the MRH models are identified even unconditionally. There is no need for instruments, exclusion restrictions, or any other variables beyond a single subjective survival probability question of the form used in HRS.

6. Empirical Analysis

Beliefs about subjective survival probabilities presumably depend on an individual's knowledge of his situation, his ability to translate this information into a probability and on his level of optimism or pessimism. In the empirical model estimated in this section, we use the information available in the HRS to try to capture several of the major determinants of beliefs in a parsimonious fashion.

6.1. Sample and Measures Employed

The sample used in the empirical analysis consists of 13,038 respondents to the 2002 Health and Retirement Study, over age 54 in 2002,[17] who provided responses to the subjective survival

[17] The HRS is a representative panel sample of the 50+ population and their spouses. The sample is refreshed in every 6 years and the last time it was refreshed before 2002 was in 1998. The sample still contains some people younger than 54 in 2002 (mostly spouses of older people), but given that their

probability questions. Excluded from the sample are proxy respondents and non-respondents in the 2000 wave of data collection. Also excluded are persons over 90 who were not asked the survival probability questions and people for whom we did not have realized survival information in 2010.[18] Table A1 in Appendix A presents the statistics for the variables used in our analyses. The average age of our sample members is just over 68 years (ranging from 54 to 89 years) and on average the target age was 16 years from their current age. The modal sample member is a white female with a high school education although there is substantial variance in each of these dimensions.

The age of death of the parents might be a primary source of information for individuals about their own survival chances. As Table A1 shows, 16 percent of our sample still has a living mother but only 5 percent has a living father. The average age of death of the mothers is 76 years while the corresponding number for the fathers is 72 years. For each parent, we construct a variable equal to the current age of the parent if he or she is alive and the age of death otherwise.

We also included in our analysis three sets of variables on health related behavior. As Table A1 shows, 43 percent of our sample reports regular exercise, at least three times a week. While only 14 percent of the sample smoked in 2002, almost 60 percent reported having smoked in the past. There is a big variation in the sample in drinking behavior. Roughly half of our sample (48 percent) reports that they drink alcohol sometimes, but the majority are not regular alcohol consumers. Among those who are, the average number of days when they drink is 3.4 days a week, and the average number of alcoholic beverages consumed is 1.9.

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number is small and age plays a crucial role in our survival framework we decided to drop these observations.

[18] Actual mortality of HRS respondent is very precisely measured from administrative data (the National Health Index) and it is even available for people who dropped out of the survey in a later wave.

Self-rated health in the HRS is measured on a five-point scale--1) Excellent, 2) Very Good, 3) Good, 4) Fair and 5) Poor. We translated these into three categories: 1) excellent/very good; 2) good; 3) fair/poor. We then constructed dummy variables representing the combination of self-rated health in 2000 and 2002 for each respondent with excellent/very good in both years as the baseline case. In Table A1 we only show the marginals of this joint distribution. As we can see the fraction of people in "excellent/very good" health decreased from 47 percent in 2000 to 43 percent in 2002. The fraction of people in "good" health did not change much (31 and 32 percent) and the fraction in "fair/poor" health increased from 22 to 25 percent.

Two cognitive measures are used in the analyses: "Vocabulary" and the "27-point cognitive capacity scale"[19]. The first score "Vocabulary" aims at measuring established knowledge or crystallized intelligence of respondents by asking them to define 5 randomly selected words, such as "fabric", "domestic", "remorse" or "plagiarize".

The second measure, the "27-point cognitive capacity scale" is a composite measure aiming to classify HRS respondents into three cognitive function categories: Normal (12 – 27); Borderline (7 – 11); and Impaired (0 – 6). The questions used are 1) an immediate and delayed 10-noun free recall test to measure memory; 2) a serial seven subtraction test to measure working memory; and 3) a counting backwards test to measure speed of mental processing (Fisher et al., 2012).

We have standardized both scores on the total 54-90 year old population in HRS. The means of these variables in Table A1, thus, also gives us a sense of how representative our final sample is. As we can see, the average respondent in our sample is 0.08 standard deviation above the

[19] The 27-point scale Langa-Weir method is discussed in Crimmins, et al. (2011). HRS cognitive measures are described in Fisher, et al. (2012).

average cognitive capacity of the HRS respondents, and he is also above average in his vocabulary (0.11 standard deviation).

Since mood can affect the level of optimism we also include in our model a count of the number of depressive symptoms the respondent has exhibited over the recent past. These range from disturbed sleep patterns, through feelings of hopelessness all the way up to thoughts of suicide. In all there are eight such symptoms measured and they are used to construct the CESD depression scale (Ofstedal, et. al., 2002). On average respondents have less than one and one-half depressive symptoms. Table A1 shows that our sample is less depressed (by 0.04 standard deviation) than the average HRS respondents in the same cohort.

6.2 Maximum Likelihood Model Estimates

The objective of this section is to test the modal response hypothesis (MRH) through a series of performance tests. All the results we present in this section are based on the six estimated models shown in Table A2 and A3. Table A2 reports models without covariates and Table A3 shows estimated models with covariates appearing in the equations of θ^{50} (or γ_0) and n. Both tables present three models. The first columns show actual eight year survival of the HRS respondents between 2002 and 2010 estimated with nonlinear least squares. The second columns called "Mean model" show results based on the assumption that HRS respondents reported a rounded version of the mean of the subjective distribution $g_i(p)$ when they answered the probabilistic survival questions in HRS in 2002. The last columns present the results of the MRH model where the assumption is that respondents reported a rounded version of the mode of $g_i(p)$ if it existed and 50 percent when the mode did not exist. In this paper we assume that respondents round to the closest 1 percent, but the results are not sensitive to this assumption.

All the parameters of our model ($\gamma_0, \theta^{50}, n, \gamma_1, k, sd(n)$) are assumed to be positive and thus their logarithms enter the likelihood function. Covariates potentially enter two equations. The first is the equation of θ^{50} which is the scale parameter of the gamma distribution of the mortality hazard at the age of 50. Covariates with positive coefficients are estimated to increase the mortality hazard and decrease the survival chances. The magnitudes of these coefficients will be analyzed later when we derive average partial effects of them on various survival probabilities. The second equation where covariates appear is the equation of the precision of beliefs (n). Positive coefficients mean tighter, more precise probability beliefs.

Figure 6: 8 year actual and expected survival probabilities by age in 2002

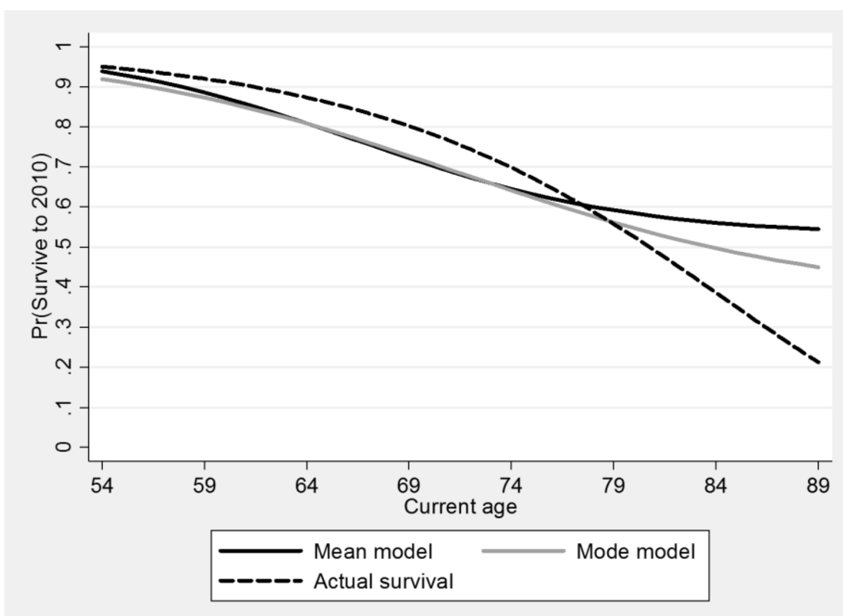


Figure 6 compares estimated actual 8 year survival probabilities of HRS respondents to subjective survival beliefs in 2002 computed from the models in Table 2. The horizontal axis shows the current age of respondents in 2002 and the vertical axis shows the fitted average 8 year survival chances from the three models. As we discussed in Section 3, heterogeneity in survival chances leads to sample selection as people with better fitness are more likely to survive and become respondents of the HRS survey at older ages. In the case of realized survival, only the interpretation of the estimates changes, but we do not need to make any further adjustments of the parameters. In the demography literature they call survival tables of this sort "Survival probabilities of the survivors". In the case of subjective survival chances, however, we do have to properly adjust for the unmeasured genetic and environmental differences of cohorts with the formula in (13). As we can see, both the mean and the MRH model track the actual survival chances very well up until about age 84, when the subjective probabilities become too optimistic. While the 8 year actual survival chance of the 90 year old is roughly 20 percent, the corresponding numbers in the MRH and mean models are ~ 45 and 55 percent, respectively. Thus, although the MRH model provides numbers that are closer to the true survival chances at old ages, these numbers are still too large on average. It is not obvious, however, whether these overly optimistic numbers are biases in people's heads or biases due to measurement error in the survey. In this paper we do not try to separate these two types of bias and we simply compare the mean, the mode and the actual survival models using the raw data.

Figure 7: Heterogeneity in the subjective survival curves, MRH model

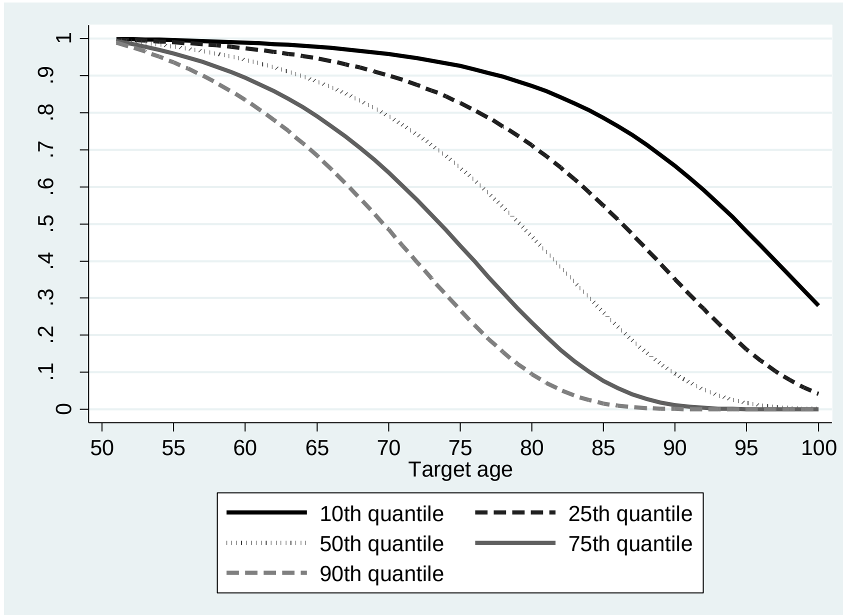
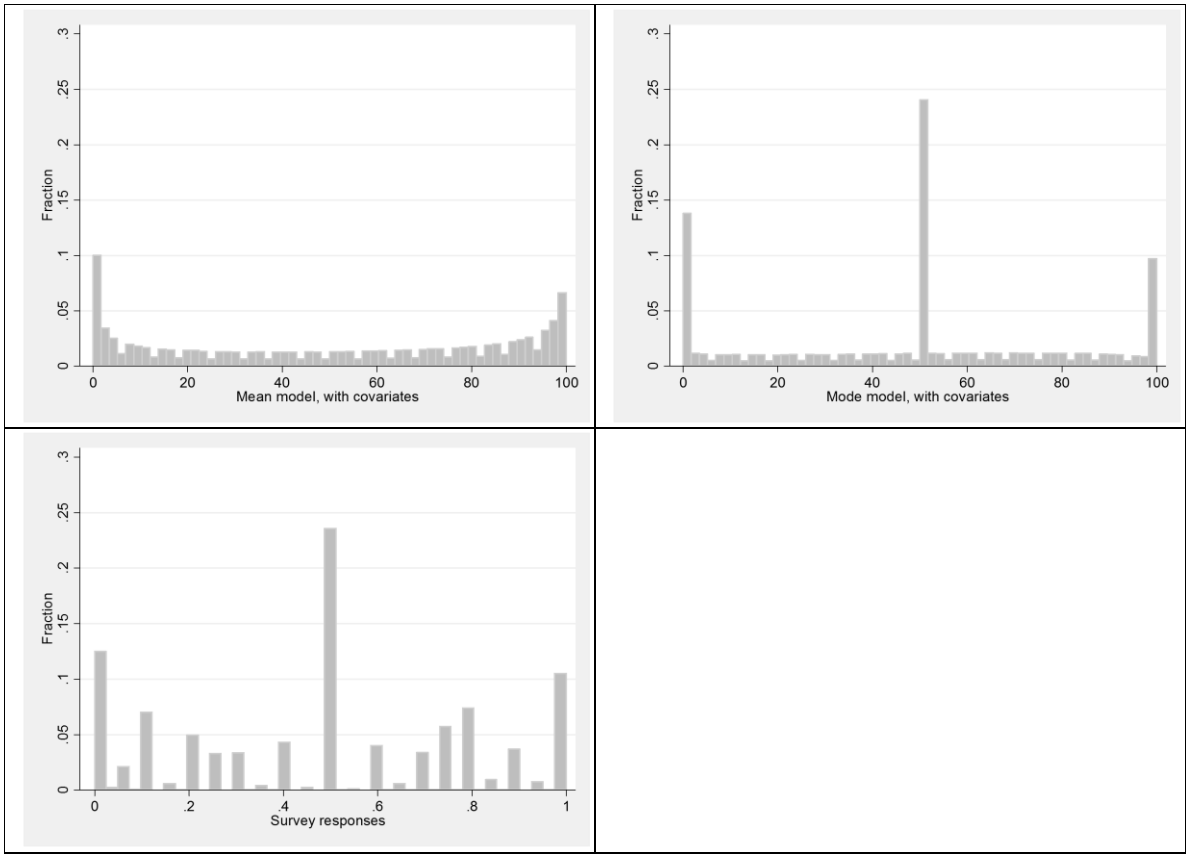


Figure 7 shows the estimated heterogeneity of survival chances in our sample. The different curves correspond to different values of γ_0 with lower values meaning better fitness. As we can see, there is notable variability in survival chances. For example, the difference in median survival (i.e., half-life) between those in the 10th and 90th percentiles of the estimated frailty distribution is about 25 years. That is, comparing two groups of 50 year olds, half of those in the 90th percentile are expected to survive to age 70 while among those in the 10th percentile half are expected to survive to age 95. Using only mortality data, one cannot identify the unobserved heterogeneity in survival chances.[20]

[20] There is a long history of discussion about the difficulty in separately identifying duration effects and unobserved heterogeneity. See, for example, Vaupel (1979) and Heckman and Singer (1984). Using subjective survival data, however, identifying unobserved heterogeneity in frailty is easier, because we observe probabilities of survival on the individual level. This contrasts with the use of mortality data, where it is hard to know which survivor is fit and which is simply luckier than other non-survivors. Even though we use a particular functional form for how unobserved heterogeneity enters the model (the

Figure 8: Simulated survey responses based on mean and mode models with covariates and the empirical distribution of survey responses



The reason the MRH model is somewhat better than the mean model in predicting low probability events is that the high fraction of 50 percent answers are allowed to be focal answers that do not arbitrarily push the mean survival chances up. To visualize this effect we simulated survey responses based on the estimated models of subjective survival chances in Table A3. In order to get precise numbers, we used 651.900 observations for simulation which is 50 times the size of our dataset ($651,900 = 50 \times 13,038$). As we can see in Figure 8, the MRH model is able to predict histograms of responses that are very similar to the histogram of actual responses in the bottom panel. The ratio of 50 percent answers is around 25 percent, while the

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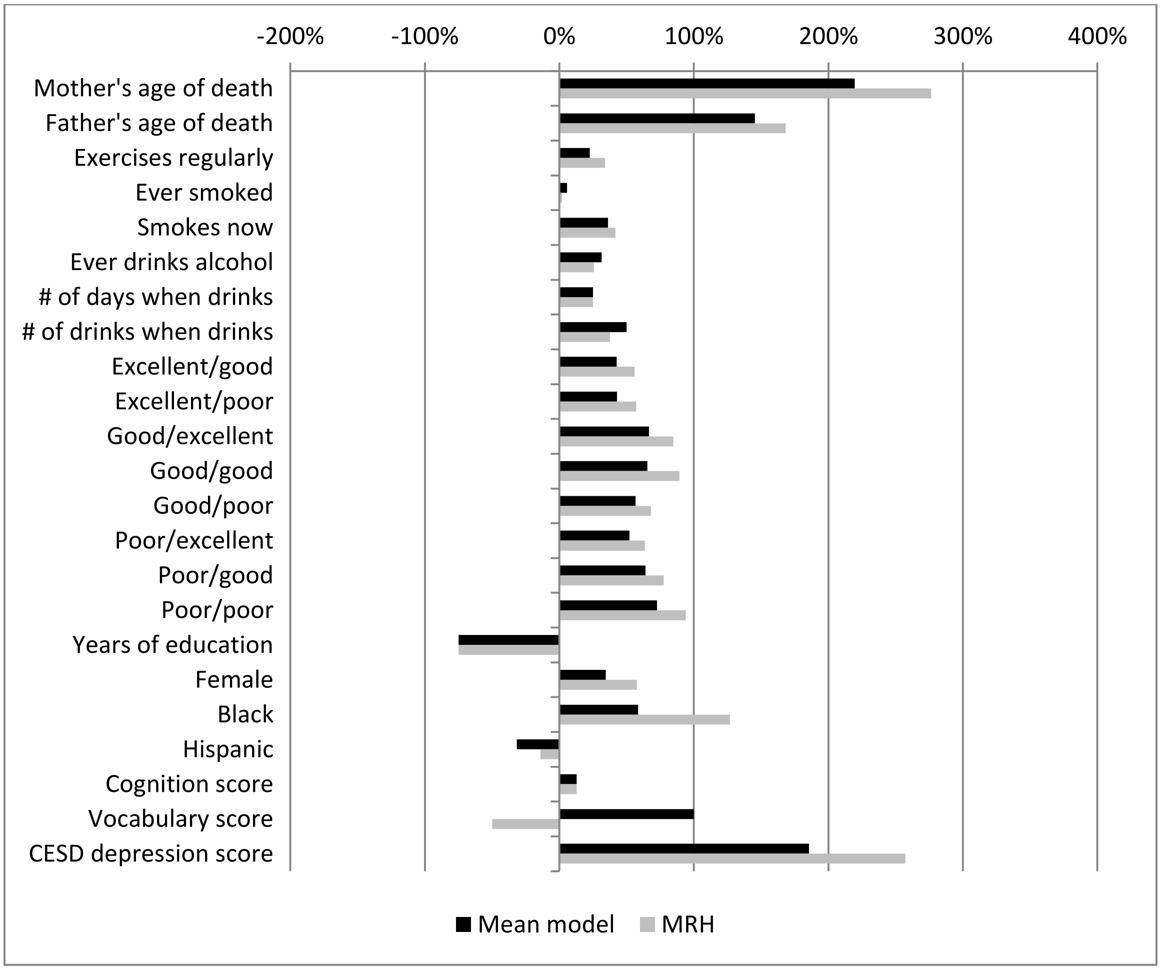
gamma-gompertz framework) it is important to note that these functional form assumptions are not needed for identifying unobserved heterogeneity in subjective frailty.

ratio of 0 and 100 percent answers are both around 10 percent. What is more important, the MRH model recognizes that the high fraction of focal answers should not be taken at face value as a large fraction of them only reflect imprecise knowledge. The mean model, however, takes all the focal answers at face value. Consequently, the mean model is not able to predict a histogram similar to actual responses, and it seriously biases the estimation of low or high probability events.

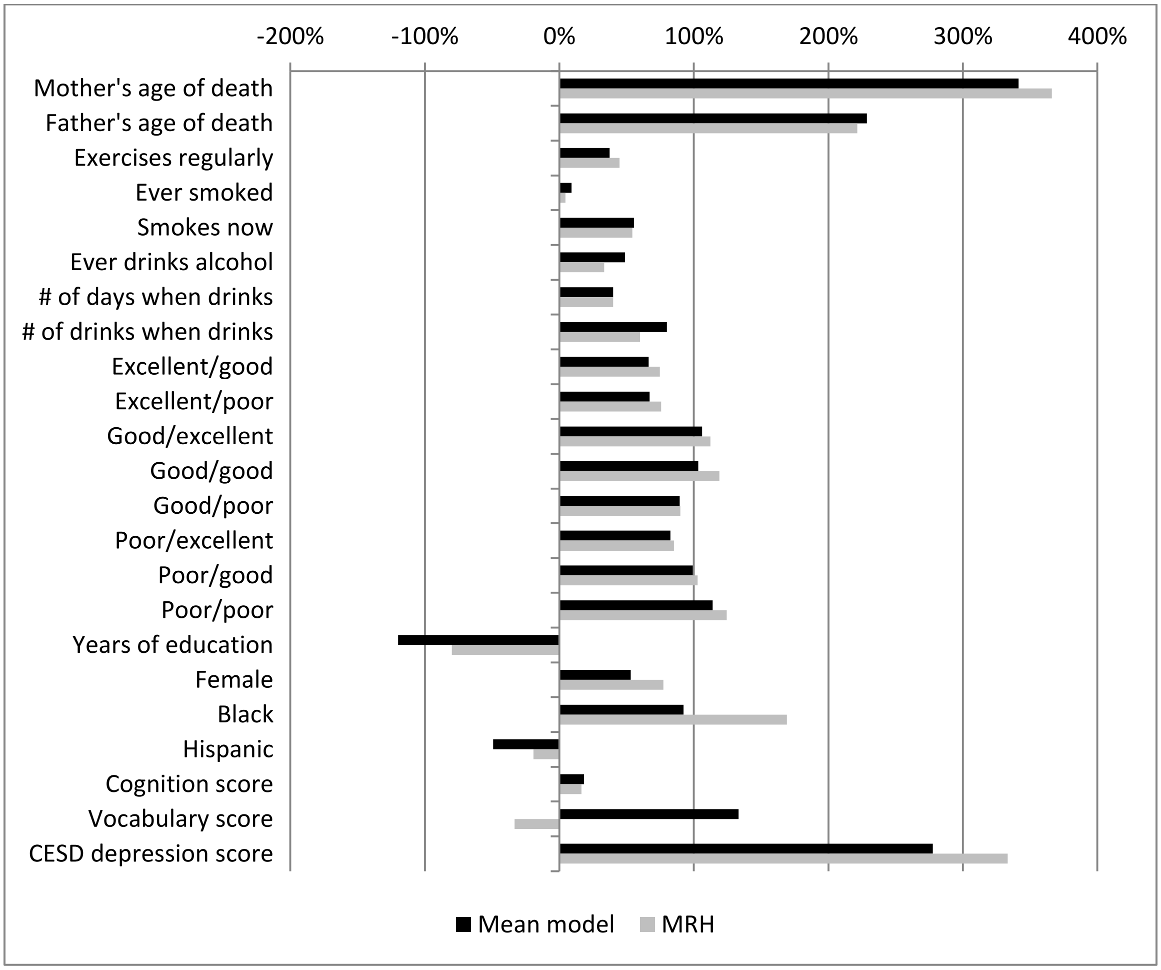
Tables A4-A6 show estimated average partial effects of surviving between three different age intervals: current age to eight years later, 55-75 and 75-95. For all the three tables, we use results from Table A3 and the formula in (17). Figures 9-11 show the estimated average partial effects from the subjective survival models as a percentage of partial effects from mortality data. A 100% on these figures means that the estimated average partial effect is the same in the subjective and the actual mortality models, and a number that is higher than 100% means that the given covariate has a stronger effect on subjective than on objective mortality. The general pattern is the following. The partial effects of different covariates are in general smaller in absolute value on subjective than on objective survival probabilities. This can be seen on Figure 9-11 where the majority of the bars lie below 100%. The only important exceptions are parental longevity and the depression score which seem to have stronger effects on subjective than on objective survival chances. This result is consistent with a model in which agents base their expectations on easily observable determinants of mortality (parents' survival) because they have limited information on demographic differences in the society and on the role of different behavioral factors, such as smoking and exercising, on survival. Although this result is not shown in this paper, it is important to note that covariates other than parents' mortality and the depression score are smaller in absolute value even if we do not control for these covariates.

Figure 9: Average partial effects of surviving eight more years in the subjective survival models as a percentage of partial effects using mortality data



The second general finding in Tables A4-A6 and Figure 9-11 is that the discrepancy between objective and subjective survival probabilities is higher for low probability events than for moderate probability events. While the different columns contain similar numbers for the probability of surviving from age 55 to age 75, the numbers are wildly different when we look at survival chances from age 75 to 95. Whether this is evidence for biased expectations or measurement error in the HRS cannot be determined from our tables.

Figure 10: Average partial effects of survival from age 55 to 75 in the subjective survival models as a percentage of partial effects using mortality data



The third general finding is that the MRH model seems to outperform the mean model as the partial effects derived from the MRH model are closer to the ones derived from actual mortality. The difference between the mean and the MRH models are more obvious for low probability events (surviving from age 75 to 95), when the estimated partial effects are roughly two times as big in the MRH model than in the mean model. We take it as evidence that by directly modeling focal responses we are able to get rid of an important bias in subjective survival data.

Figure 11: Average partial effects of survival from age 75 to 95 in the subjective survival models as a percentage of partial effects using mortality data

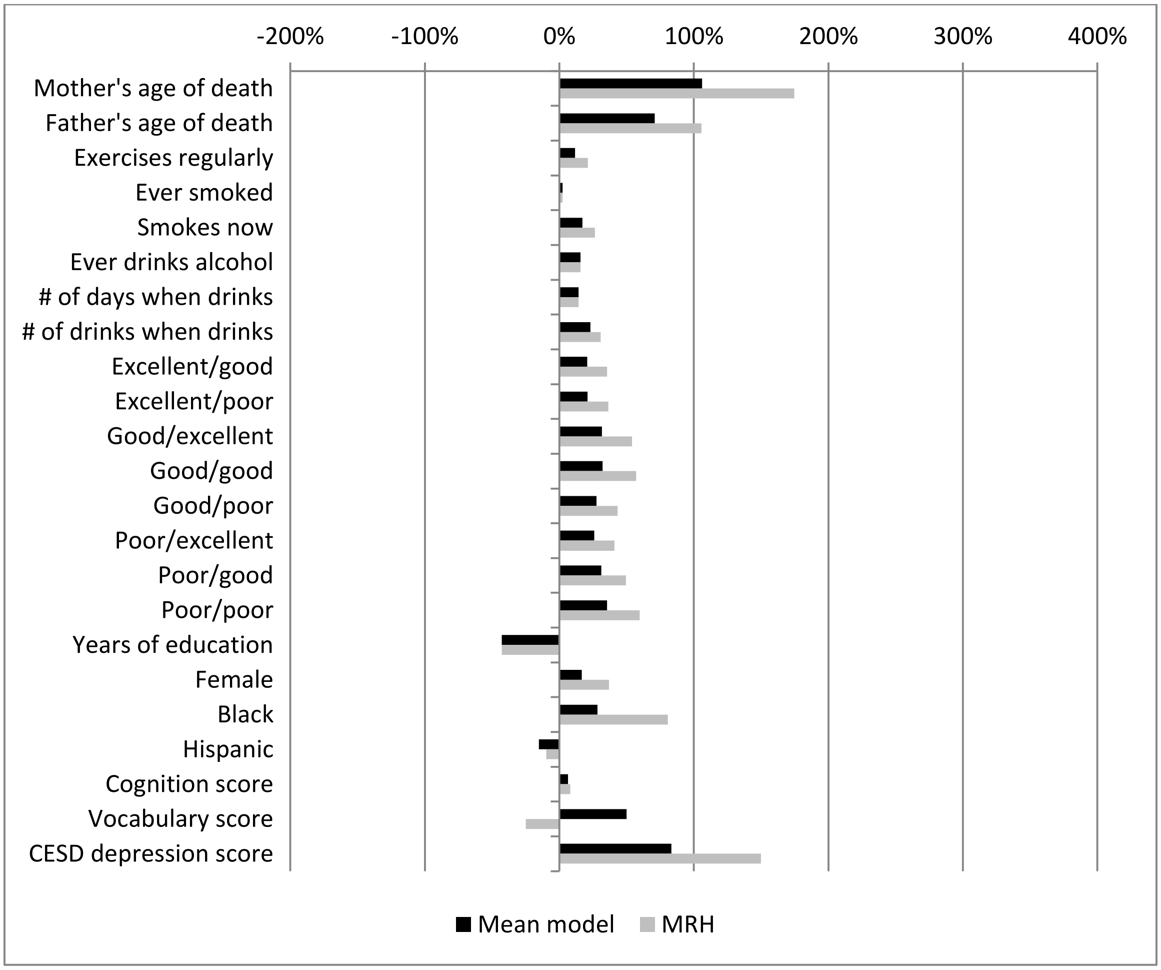


Table A4 shows average partial effects of surviving 8 more years until 2010. As we can see almost all coefficients are closer to zero in the subjective survival models than in the realized survival model. The main exception is parental mortality, where the effects are higher for subjective than for objective survival in both the mean and the MRH models. Furthermore, almost all coefficients in the MRH model are closer to the ones in the actual survival model than the coefficients of the mean model. For example, people who were in poor health both in 2000 and in 2002 had a 16 percent lower 8 year survival probability than people who were in excellent health, while the corresponding numbers in the MRH and mean models are 15 percent and 12 percent respectively; females had a 5.6 percent higher survival chances than males, but the subjective probabilities were 3 percent and 1.8 percent in the MRH and in the mean models; other things equal blacks had a 4.1 percent higher[21] survival chance and the subjective estimated probabilities were 5.2 and 2.4 percent. Thus, not only is the MRH model better in predicting low probability events compared to the mean model, the estimated partial effects of different covariates are also closer to the realized partial effects estimated from mortality data. A surprising finding in Table A4 is that while education has a positive partial effect on subjective survival chances, it has negative partial effect on realized mortality. This is entirely driven by the cognitive capacity control variable that positively affects survival and is also strongly correlated with the years of education variable. Another interesting finding in Table A4 is about the role of smoking. The actual mortality data shows us that while, other things equal, smokers have a 12.4 percent lower chance of surviving 8 more years ($5.2+7.2=12.4$), those who quit smoking are still 5.2 percent more likely to die than those who never smoked. The effect of smoking on subjective probabilities is different. Smokers seem to understand the health risks of their addiction, although they strongly underestimate it, but quitters falsely believe that their survival chances are the same as those who never smoked. These findings are in line with Sloan et al. (2003). Such beliefs might help convince smokers to quit, but nevertheless these beliefs are not based on strong evidence.

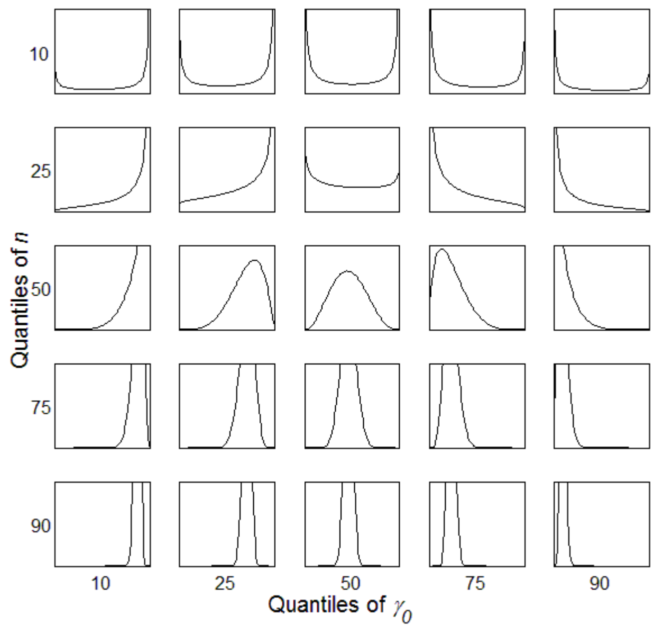
In Table A5 and A6 and Figure 10-11 we estimated partial effects of surviving from age 55 to age 75 and from age 75 to age 95. This exercise is useful to see whether the estimated partial effects behave differently for moderate and relatively low probability events. In Table A5 we can see that for moderate probability events (surviving from 55 to 75) the mean and the MRH models give very similar average partial effects of almost all coefficients, and even the actual survival

[21] Unconditionally, though, blacks have lower survival probabilities than whites.

chances are very close. For example, the partial effect of being in poor as opposed to excellent health in 2000 and 2002 decreases survival chances by 21 percent, while the mean model predicts 24 and the MRH model predicts 26 percent differentials. The corresponding numbers for gender are 6.6 percent, 3.5 percent and 5.1 percent. We can also see that parental mortality has a stronger effect on subjective as opposed to objective survival probabilities. Thus, both the mean and the MRH subjective survival models are performing comparably well for predicting partial effects on moderate probabilities. In Table A6, though, we can see that the subjective survival models are performing poorly at lower probabilities when we estimate the average partial effects of surviving from age 75 to 95. Almost all coefficients are closer to zero in the subjective models compared to the objective actual survival model and the differences are quite substantial. Nevertheless, the MRH model seems to be closer to actual survival data than the mean model. For example, while the partial effect of being in poor health in 2000 and 2002 decreases survival chances by 27 percent, the mean model predicts only 9 and the MRH model predicts only 16 percent differentials. The corresponding numbers for gender are 8.4 percent, 1.4 percent and 3.1 percent; for blacks 6.7 percent, 1.9 percent and 5.4 percent. The effect of parental mortality is estimated to be stronger in the MRH model than in the realized mortality model, but now the mean model gives similar numbers to the objective survival model. The effect of smoking is similar to the models when we looked at 8 year survival chances. Present smokers seem to understand the health risks of smoking, although they underestimate its magnitude, but the average quitter holds the false belief that his survival chance is as good as that of someone who has never smoked before.

To sum up, we found evidence that the subjective survival data gives very reasonable estimates of unconditional and conditional probabilities for moderate probability events but they both perform poorly for low probability events, with the MRH model being better than the mean model.

Figure 12: Estimated distribution of probability beliefs $g_i(p)$ of surviving from age 50 to age 80



Finally let us take a look at the estimated distribution of probability beliefs (second order probability distribution, $g_i(p)$) of HRS respondents. Based on the MRH model without covariates (Table A2) we computed the 10th, 25th, 50th, 75th and 90th quantiles of belief precision (n) and the scale parameter of the survival function (γ_0) for the cohort of age 50. These numbers can be found in Table A7. Figure 12 shows the corresponding probability belief distributions of the probability of surviving from age 50 to age 80. As we can see there is an enormous heterogeneity in probability beliefs. The median responder in HRS (3rd row and 3rd column of Figure 12) has a belief distribution that is single peaked but wide, having significant probability mass for any possible probability values between zero and one. It means that although the median responder's best guess for the probability of surviving from age 50 to age 80 is roughly 50 percent, she is quite unsure about this probability. People with even less precise beliefs are very unsure. For example, already at the 25th percentile of belief precision (where $n=1.58$) everyone provides a focal response of either 0, 50 or 100. At the 10th percentile (where $n=0.4$) everyone has U-shaped beliefs and, thus, responds with an epistemic 50%..[22] As we increase belief precision to the 75th percentile, the second order probability belief distribution becomes quite tight, having most of its mass in the neighborhood of the mean probability. It means that at least 25 percent of the respondents have very precise beliefs about their own survival chances.

Determinants of belief precision appear in the last column of Table A3. Positive coefficients mean tighter, more precise beliefs. As we can see more educated people have more certain beliefs. This is consistent with our hypothesis, discussed in Section 3, that more educated people may have a broader knowledge of evidence about mortality and its causes. We can also see that the deterioration of health, especially from previously excellent levels, leads to more uncertainty about survival chances, perhaps because of uncertainty about the future course of of a new disease. Those who were in poor health both in 2000 and in 2002, however, hold the most certain and pessimistic beliefs about their survival chances. The effect of age and the time horizon of the survival question in HRS have complicated relations to uncertainty. For a fixed time horizon, the net effect age on uncertainty is positive. It seems older people are less sure about their survival chances. Whether it is an age or a cohort effect cannot be determined from this table. For a fixed age, the net effect of the time horizon on uncertainty is negative, because

[22] Note, however, that the particular shape of the distribution of beliefs is only identified from the lognormal functional form in the region where *n* is below 1. Because there are many focal responses in the HRS data, the model estimates many uncertain responses where $n<1$. It is hard to know, however, what the distribution of n looks like, conditional on being below 1. The log-normality assumption might or might not describe this conditional distribution well. It is possible, for example, that no-one has U-shaped beliefs, but all epistemic focal responses come from a uniform distribution. If that is the case, then the log-normality assumption of n is inappropriate.

the interaction term dominates within the age interval used in this project. It seems people hold more precise views about their long run than their short run survival chances. Table 3 also shows that women and those who quit smoking have less precise expectations (the latter not significant); infrequent alcohol consumers and those whose father lived longer have more precise beliefs.

Conclusion

The modal response hypothesis is used in this paper as the foundation for an econometric model that is intended to provide a mapping between survey responses to probability questions and the underlying subjective probability beliefs of individuals about their chances of surviving to a target age. In this paper, we have presented the MRH as a hypothesis designed to capture the kinds of "gut response" to such questions that would be made after about 15 seconds of consideration by persons who vary in the amount of information they have about actuarial risks to health, about their own health-related circumstances and in their capacity to process such information into subjective beliefs. We argued in Section 3 that reporting the mode is relatively easier from a cognitive point of view than the mean or the median; the mode is equal to a very simple rule-of-thumb estimator for the probability in question; and that the mode often provides a good approximation to the expected probability that is called for in *SEU* theory.

Our empirical findings suggest that there is considerable heterogeneity in subjective survival risks, some of it associated with age, sex, race, education, health related behavioral factors, parental mortality and cognitive capacity. We have shown that subjective survival expectations line up with actual mortality very well when the objective probabilities are moderate. The subjective survival probabilities, however, become overly optimistic at old ages when the true survival probabilities are relatively low. We have shown that the MRH model does a better job compared to a standard mean model in reducing this bias as the MRH models focal answers in an explicit way. It remains for future research to learn whether the overly optimistic subjective expectations are biases in individuals' head, potentially having behavioral consequences, or they are a result of survey measurement error, potentially being related to uncertain beliefs.

In the empirical section of this paper we have also found substantial uncertainty about mortality risks which is manifested by considerable spread in the estimated distribution of subjective survival probabilities for a typical respondent. In addition, we found significant variation in uncertainty, holding expected survival risk constant. It remains for future work to explore the explanation of these findings more deeply and to see whether survival risk and uncertainty about this risk play a role in decisions made by HRS respondents.

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Appendix A: Tables and figures

Table A1: Descriptive statistics, HRS-2002

	mean	sd
Alive in 2010	0.77	0.42
Subjective survival probability to target age	48.40	32.13
Age	68.15	8.69
Target age less actual age	15.97	4.17
Female	0.59	0.49
Black	0.12	0.33
Hispanic	0.06	0.24
Years of education	12.54	3.00
Mother is alive	0.16	0.37
Mother's age of death/100 or current age	0.76	0.15
Father is alive	0.05	0.23
Father's age of death/100 or current age	0.72	0.14
Exercises at least 3 times a week	0.43	0.49
Ever smoked	0.59	0.49
Smokes now	0.14	0.34
Ever drinks alcohol	0.48	0.50
# of days a week when drinks alcohol	1.10	2.08
# of days a week when drinks alcohol if positive	3.42	2.34
# of drinks when drinks alcohol	0.61	1.18
# of drinks when drinks alcohol if positive	1.92	1.36
Health excellent / very good, 2002	0.43	0.49
Health good, 2002	0.32	0.47
Health fair / poor, 2002	0.25	0.43
Health excellent / very good, 2000	0.47	0.50
Health good, 2000	0.31	0.46
Health fair / poor, 2000	0.22	0.42
Cognition score, std.	0.08	1.01
Vocabulary score, std.	0.11	0.97
CESD depression score, std.	-0.04	1.04
N	13038	

Table A2: Actual survival until 2010 and the mean and MRH models of subjective survival expectations, models without covariates

	Actual survival	Mean model	MRH
$\ln(\gamma_0)$	-8.404		
	[0.21]***		
$\ln(\theta^{50})$		-11.174	-8.827
		[0.150]***	[0.169]***
ln(n)			1.978
			[0.040]***
$\ln(\gamma_1)$	2.277	2.73	2.397
	[0.024]***	[0.012]***	[0.020]***
ln(k)		-0.656	0.121
		[0.011]***	[0.025]***
ln(sd(n))			0.814
			[0.025]***
N	13038	13038	13038
Log-likelihood		-57961.459	-47058.606

*, ** and *** denote significance at 10, 5 and 1 percent

Table A3: Actual survival until 2010 and the mean and MRH models of subjective survival expectations, models with covariates

	Actual survival	Mean model	MRH	
	$\ln(\gamma_0)$	$\ln(\theta^{50})$	$\ln(\theta^{50})$	ln(n)
Mother's age of death/100 or current age	-0.305	-1.292	-1.044	-0.194
	[0.101]**	[0.137]**	[0.095]**	[0.169]
Father's age of death/100 or current age	-0.265	-0.744	-0.544	0.358
	[0.113]*	[0.128]**	[0.089]**	[0.173]*
Exercises at least 3 times a week	-0.313	-0.144	-0.13	0.023
	[0.038]**	[0.034]**	[0.024]**	[0.053]
Ever smoked	0.311	0.032	0.011	-0.096
	[0.038]**	[0.036]	[0.026]	[0.055]
Smokes now	0.431	0.298	0.221	0.008
	[0.049]**	[0.051]**	[0.037]**	[0.078]
Ever drinks alcohol	-0.21	-0.127	-0.068	0.184
	[0.043]**	[0.041]**	[0.030]*	[0.064]**
# of days a week when drinks alcohol	-0.024	-0.009	-0.008	-0.015
	[0.011]*	[0.011]	[0.008]	[0.017]
# of drinks when drinks alcohol	0.047	0.048	0.026	-0.033
	[0.019]*	[0.019]*	[0.014]	[0.029]
Health in 2000/2002				

Excellent/excellent	*ref.*	*ref.*	*ref.*	*ref.*
Excellent/good	0.445	0.366	0.311	-0.182
	[0.065]**	[0.055]**	[0.040]**	[0.087]*
Excellent/poor	0.831	0.69	0.588	0.122
	[0.091]**	[0.120]**	[0.085]**	[0.163]
Good/excellent	0.231	0.3	0.243	-0.102
	[0.079]**	[0.061]**	[0.044]**	[0.098]
Good/good	0.445	0.571	0.498	-0.057
	[0.059]**	[0.049]**	[0.035]**	[0.076]
Good/poor	0.891	0.985	0.751	-0.062
	[0.066]**	[0.086]**	[0.061]**	[0.108]
Poor/excellent	0.569	0.582	0.452	-0.109
	[0.119]**	[0.155]**	[0.106]**	[0.204]
Poor/good	0.663	0.814	0.637	-0.054
	[0.076]**	[0.087]**	[0.063]**	[0.120]
Poor/poor	0.978	1.374	1.135	0.183
	[0.057]**	[0.069]**	[0.051]**	[0.088]*
Years of education	0.026	-0.037	-0.019	0.055
	[0.006]**	[0.007]**	[0.005]**	[0.010]**
Female	-0.311	-0.206	-0.222	-0.177
	[0.035]**	[0.035]**	[0.026]**	[0.054]**
Black	-0.245	-0.279	-0.385	0.014
	[0.053]**	[0.051]**	[0.037]**	[0.078]
Hispanic	-0.341	0.209	0.061	0.141
	[0.077]**	[0.078]**	[0.056]	[0.109]
Cognition score, std.	-0.23	-0.053	-0.036	0.026
	[0.019]**	[0.020]**	[0.015]*	[0.029]
Vocabulary score, std.	-0.013	-0.022	0.004	0.011
	[0.018]	[0.020]	[0.014]	[0.029]
CESD depression score, std.	0.043	0.148	0.13	0.049
	[0.016]**	[0.020]**	[0.014]**	[0.027]
Age / 100				-2.895
				[1.475]*
Horizon (Target age - age)/100				-8.636
				[6.251]
Age X Horizon				26.986
				[10.572]*
Constant	-7.611	-9.285	-6.646	1.771
	[0.261]**	[0.228]**	[0.195]**	[0.969]
Other parameters				
$\ln(\gamma_1)$	2.152	2.694	2.264	
	[0.027]**	[0.012]**	[0.021]**	

ln(k)		-0.541	0.329
		[0.011]**	[0.025]**
ln(sd(n))			0.794
			[0.026]**
N	13038	13038	13038
Log-likelihood		-57001.593	-45889.598

*, ** and *** denote significance at 10, 5 and 1 percent

Table A4: Average partial effects of surviving 8 more years

	Actual survival	Mean model	MRH
Mother's age of death/100 or current age	0.051	0.112	0.141
	[0.017]**	[0.019]**	[0.027]**
Father's age of death/100 or current age	0.044	0.064	0.074
	[0.019]*	[0.014]**	[0.017]**
Exercises at least 3 times a week	0.053	0.012	0.018
	[0.006]**	[0.003]**	[0.005]**
Ever smoked	-0.052	-0.003	-0.001
	[0.007]**	[0.003]	[0.004]
Smokes now	-0.072	-0.026	-0.03
	[0.008]**	[0.006]**	[0.007]**
Ever drinks alcohol	0.035	0.011	0.009
	[0.007]**	[0.004]**	[0.004]*
# of days a week when drinks alcohol	0.004	0.001	0.001
	[0.002]*	[0.001]	[0.001]
# of drinks when drinks alcohol	-0.008	-0.004	-0.003
	[0.003]*	[0.002]*	[0.002]
Health in 2000/2002			
Excellent/excellent	*ref.*	*ref.*	*ref.*
Excellent/good	-0.075	-0.032	-0.042
	[0.011]**	[0.007]**	[0.009]**
Excellent/poor	-0.14	-0.06	-0.08
	[0.016]**	[0.013]**	[0.018]**
Good/excellent	-0.039	-0.026	-0.033
	[0.013]**	[0.006]**	[0.008]**
Good/good	-0.075	-0.049	-0.067
	[0.010]**	[0.008]**	[0.013]**
Good/poor	-0.15	-0.085	-0.102
	[0.012]**	[0.014]**	[0.019]**
Poor/excellent	-0.096	-0.05	-0.061
	[0.020]**	[0.015]**	[0.018]**
Poor/good	-0.111	-0.071	-0.086
	[0.013]**	[0.012]**	[0.017]**

Poor/poor	-0.164	-0.119	-0.154
	[0.011]**	[0.018]**	[0.027]**
Years of education	-0.004	0.003	0.003
	[0.001]**	[0.001]**	[0.001]**
Female	0.052	0.018	0.03
	[0.006]**	[0.004]**	[0.006]**
Black	0.041	0.024	0.052
	[0.009]**	[0.006]**	[0.011]**
Hispanic	0.057	-0.018	-0.008
	[0.013]**	[0.007]*	[0.008]
Cognition score, std.	0.039	0.005	0.005
	[0.003]**	[0.002]*	[0.002]*
Vocabulary score, std.	0.002	0.002	-0.001
	[0.003]	[0.002]	[0.002]
CESD depression score, std.	-0.007	-0.013	-0.018
	[0.003]**	[0.002]**	[0.004]**

*, ** and *** denote significance at 10, 5 and 1 percent

Table A5: Average partial effects of survival from age 55 to age 75

	Actual survival	Mean model	MRH
Mother's age of death/100 or current age	0.065	0.222	0.238
	[0.022]**	[0.038]**	[0.043]**
Father's age of death/100 or current age	0.056	0.128	0.124
	[0.024]*	[0.028]**	[0.028]**
Exercises at least 3 times a week	0.067	0.025	0.03
	[0.008]**	[0.007]**	[0.007]**
Ever smoked	-0.066	-0.006	-0.003
	[0.009]**	[0.006]	[0.006]
Smokes now	-0.092	-0.051	-0.05
	[0.011]**	[0.011]**	[0.011]**
Ever drinks alcohol	0.045	0.022	0.015
	[0.009]**	[0.008]**	[0.007]*
# of days a week when drinks alcohol	0.005	0.002	0.002
	[0.002]*	[0.002]	[0.002]
# of drinks when drinks alcohol	-0.01	-0.008	-0.006
	[0.004]*	[0.003]*	[0.003]
Health in 2000/2002			
Excellent/excellent	*ref.*	*ref.*	*ref.*
Excellent/good	-0.095	-0.063	-0.071
	[0.014]**	[0.013]**	[0.015]**
Excellent/poor	-0.177	-0.119	-0.134
	[0.021]**	[0.026]**	[0.029]**

Good/excellent	-0.049	-0.052	-0.055
	[0.017]**	[0.013]**	[0.013]**
Good/good	-0.095	-0.098	-0.113
	[0.013]**	[0.016]**	[0.020]**
Good/poor	-0.19	-0.17	-0.171
	[0.016]**	[0.028]**	[0.031]**
Poor/excellent	-0.121	-0.1	-0.103
	[0.026]**	[0.030]**	[0.029]**
Poor/good	-0.141	-0.14	-0.145
	[0.017]**	[0.024]**	[0.027]**
Poor/poor	-0.208	-0.237	-0.259
	[0.015]**	[0.035]**	[0.043]**
Years of education	-0.005	0.006	0.004
	[0.001]**	[0.002]**	[0.001]**
Female	0.066	0.035	0.051
	[0.008]**	[0.008]**	[0.010]**
Black	0.052	0.048	0.088
	[0.011]**	[0.011]**	[0.017]**
Hispanic	0.073	-0.036	-0.014
	[0.017]**	[0.014]*	[0.013]
Cognition score, std.	0.049	0.009	0.008
	[0.004]**	[0.004]*	[0.004]*
Vocabulary score, std.	0.003	0.004	-0.001
	[0.004]	[0.003]	[0.003]
CESD depression score, std.	-0.009	-0.025	-0.03
	[0.003]**	[0.005]**	[0.006]**

*, ** and *** denote significance at 10, 5 and 1 percent

Table A6: Average partial effects of survival from age 75 to age 95

	Actual survival	Mean model	MRH
Mother's age of death/100 or current age	0.083	0.088	0.145
	[0.027]**	[0.018]**	[0.032]**
Father's age of death/100 or current age	0.072	0.051	0.076
	[0.031]*	[0.012]**	[0.020]**
Exercises at least 3 times a week	0.085	0.01	0.018
	[0.011]**	[0.003]**	[0.005]**
Ever smoked	-0.084	-0.002	-0.002
	[0.011]**	[0.003]	[0.004]
Smokes now	-0.117	-0.02	-0.031
	[0.013]**	[0.005]**	[0.008]**
Ever drinks alcohol	0.057	0.009	0.009

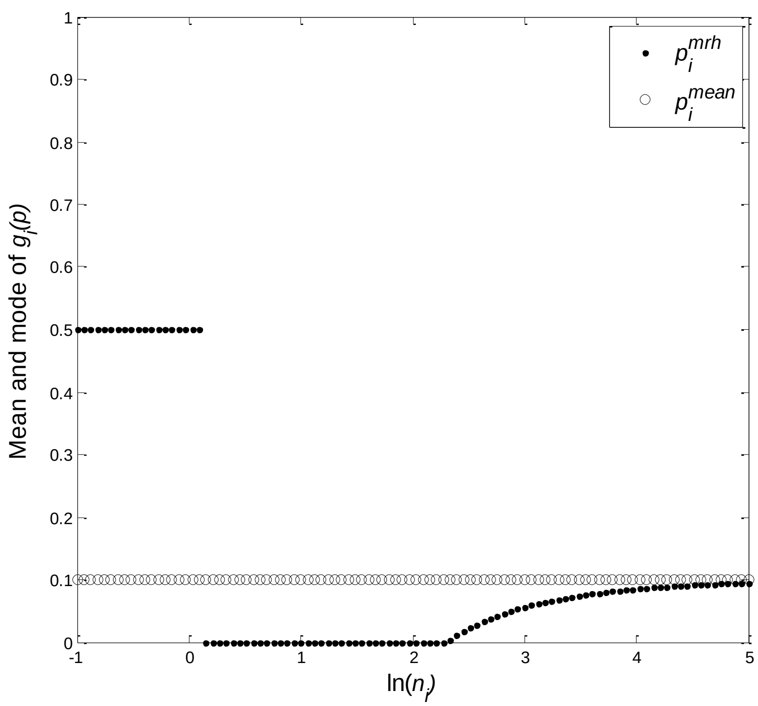
	[0.012]**	[0.003]**	[0.005]*
# of days a week when drinks alcohol	0.007	0.001	0.001
	[0.003]*	[0.001]	[0.001]
# of drinks when drinks alcohol	-0.013	-0.003	-0.004
	[0.005]*	[0.001]*	[0.002]
Health in 2000/2002			
Excellent/excellent	*ref.*	*ref.*	*ref.*
Excellent/good	-0.121	-0.025	-0.043
	[0.019]**	[0.006]**	[0.011]**
Excellent/poor	-0.225	-0.047	-0.082
	[0.027]**	[0.012]**	[0.021]**
Good/excellent	-0.063	-0.02	-0.034
	[0.022]**	[0.005]**	[0.009]**
Good/good	-0.121	-0.039	-0.069
	[0.017]**	[0.008]**	[0.015]**
Good/poor	-0.242	-0.067	-0.105
	[0.021]**	[0.013]**	[0.023]**
Poor/excellent	-0.154	-0.04	-0.063
	[0.033]**	[0.013]**	[0.020]**
Poor/good	-0.18	-0.056	-0.089
	[0.022]**	[0.011]**	[0.021]**
Poor/poor	-0.265	-0.094	-0.158
	[0.020]**	[0.017]**	[0.033]**
Years of education	-0.007	0.003	0.003
	[0.002]**	[0.001]**	[0.001]**
Female	0.084	0.014	0.031
	[0.010]**	[0.003]**	[0.007]**
Black	0.067	0.019	0.054
	[0.015]**	[0.005]**	[0.013]**
Hispanic	0.092	-0.014	-0.009
	[0.022]**	[0.006]*	[0.008]
Cognition score, std.	0.062	0.004	0.005
	[0.006]**	[0.002]*	[0.002]*
Vocabulary score, std.	0.004	0.002	-0.001
	[0.005]	[0.001]	[0.002]
CESD depression score, std.	-0.012	-0.01	-0.018
	[0.004]**	[0.002]**	[0.004]**

*, ** and *** denote significance at 10, 5 and 1 percent

Table A7: Quantiles of belief precision (n) and probabilities of surviving from age 50 to age 80

quantiles	n	γ_0	S(50,80)
10	0.40	0.000022	0.87
25	1.58	0.000054	0.71
50	7.23	0.000120	0.47
75	33.12	0.000229	0.23
90	130.31	0.000370	0.10

Figure A1: The hypothetical modal response probability answer by the log of precision (n_i), when the mean is set to 0.1



Appendix B: Derivations

Proof of equation (11)

The density function of the gamma distribution is

$$f(x)=\frac{1}{\Upsilon(k)\theta^k}x^{k-1}exp\left(-\frac{x}{\theta}\right)=c(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}\right)$$

It is well known that expected value of the gamma function is

$$E(x)=\int_0^\infty xc(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}\right)dx=\int_0^\infty c(k,\theta)x^k exp\left(-\frac{x}{\theta}\right)dx=k\theta$$

The expected value of a scaled gamma function is also gamma and its expected value is

$$E(cx)=ck\theta$$

The expected value of the negative exponentiated gamma function is

$$E\left(exp(-x)\right)=\int_0^\infty exp(-x)c(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}\right)dx$$

$$=\int_0^\infty c(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}-x\right)dx=\int_0^\infty c(k,\theta)x^{k-1}exp\left(-\frac{x}{\frac{\theta}{\theta+1}}\right)dx=*$$

Note that this is very similar to the expected value formula of the gamma function, thus

$$*=\frac{(k-1)\theta}{\theta+1}\frac{c(k,\theta)}{c\left(k+1,\frac{\theta}{\theta+1}\right)}=\frac{(k-1)\theta}{\theta+1}\frac{\Upsilon(k-1)\left(\frac{\theta}{\theta+1}\right)^k}{\Upsilon(k)\theta^k}=*$$

Note that $\Upsilon(k)=(k-1)\Upsilon(k-1)$ and thus

$$*=\frac{k-1}{k-1}\frac{\theta^k}{\theta^k}(1+\theta)^{-k}=(1+\theta)^{-k}$$

Thus if $\lambda_{0i}\sim\Upsilon(k,\theta)$ then

$$E\left[S_i(a,T)\right]=E\left[exp\left(-\lambda_{0i}\left[exp(\lambda_1 T)-exp(\lambda_1 a)\right]\right)\right]=\left(1+\theta\left[exp(\lambda_1 T)-exp(\lambda_1 a)\right]\right)^{-k}$$

Details about the use of the Delta method to derive average partial effects

The goal is to derive point estimates and standard errors of the partial effects of the form in (17)

. The point estimates can be computed by expanding (17).

$$APE_j(a,T) = E_x\left[\frac{\partial E(S_i(a,T))}{\partial x_j}\right] = E_x\left[\frac{\partial\left(1+\theta^a\left[exp(\lambda_1 T) - exp(\lambda_1 a)\right]\right)^{-k}}{\partial x_j}\right] = *$$

Let us denote $e(a',a) = exp(\lambda_1 a') - exp(\lambda_1 a)$. Then

$$* = E_x\left[\frac{\partial\left(1+\dfrac{e(T,a)}{exp(-\beta' x_i)+e(a,r)}\right)^{-k}}{\partial x_j}\right]$$

$$= E_x\left[-k\left(1+\frac{e(T,a)}{exp(-\beta' x_i)+e(a,r)}\right)^{-k-1}\frac{e(T,a)exp(-\beta' x_i)\beta_j}{\left(exp(-\beta' x_i)+e(a,r)\right)^2}\right]$$

$$= E_x\left[-k\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-1}e(T,a)exp(-\beta' x_i)\beta_j\right]$$

By substituting the estimated coefficients into this formula we have a point estimate for the average partial effect of x_j on the survival probability from age a to age T. The standard errors can be computed with the delta method. For any differentiable transformation $g(\beta)$ and variance-covariance matrix Σ, the variance covariance matrix of $g(\beta)$ is $(\nabla g)^T \Sigma(\nabla g)$.

Thus, we only need to compute the first derivatives of g. Let us see them one-by-one.

$$\frac{\partial APE_j(a,T)}{\partial \beta_l} = -I(j=l)\times$$

$$\times E_x\left[k\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-1}e(T,a)exp(-\beta' x_i)\right]$$

$$-E_x\left[k(k+1)\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-2}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-1}e(T,a)exp(-2\beta' x_i)\beta_j x_{il}\right]$$

$$+E_x\left[k(k-1)\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-2}e(T,a)exp(-2\beta' x_i)\beta_j x_{il}\right]$$

$$+E_x\left[k\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-1}e(T,a)exp(-\beta' x_i)\beta_j x_{il}\right]$$

$$\frac{\partial APE_j(a,T)}{\partial k} = -E_x\left[\left(exp(-\beta' x_i)+e(T,50)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,50)\right)^{k-1}e(T,a)exp(-\beta' x_i)\beta_j\right]$$

$$-E_x\left[k\left(exp(-\beta' x_i)+e(T,r)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,r)\right)^{k-1}e(T,a)exp(-\beta' x_i)\times\right.$$

$$\left.\times\beta_j\left(-ln\left(exp(-\beta' x_i)+e(T,50)\right)\right)\right]+E_x\left[ln\left(exp(-\beta' x_i)+e(a,50)\right)\right]$$

$$\frac{\partial APE_j(a,T)}{\partial \lambda_1} = \left[Texp(\lambda_1 T)-50exp(\lambda_1 50)\right]\times$$

$$\times\left\{E_x\left[k(k+1)\left(exp(-\beta' x_i)+e(T,50)\right)^{-k-2}\left(exp(-\beta' x_i)+e(a,50)\right)^{k-1}e(T,a)exp(-\beta' x_i)\beta_j\right]\right.$$

$$\left.-E_x\left[k(k-1)\left(exp(-\beta' x_i)+e(T,50)\right)^{-k-1}\left(exp(-\beta' x_i)+e(a,50)\right)^{k-2}e(T,a)exp(-\beta' x_i)\beta_j\right]\right\}$$

By substituting the estimated coefficients into these formulas we have an estimator for the variance covariance matrix of the average partial effects.