NBER WORKING PAPER SERIES

TRADE IN INTERMEDIATE INPUTS AND BUSINESS CYCLE COMOVEMENT

Robert C. Johnson

Working Paper 18240
http://www.nber.org/papers/w18240

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2012

I thank Rudolfs Bems, Andrew Bernard, Stefania Garetto, Jean Imbs, Luciana Juvenal, Esteban Rossi-Hansberg, Nina Pavcnik, and Kei-Mu Yi for helpful conversations, as well as participants in presentations at the Dallas Federal Reserve, Johns Hopkins (SAIS), Penn State, Stanford, the St. Louis Federal Reserve, UC Santa Cruz, the 2011 AEA meetings, and the 2010 EIIT Conference. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

Trade in Intermediate Inputs and Business Cycle Comovement
Robert C. Johnson
NBER Working Paper No. 18240
July 2012
JEL No. F1,F4

ABSTRACT

Do cross-border input linkages transmit shocks and synchronize business cycles across countries? I integrate input trade into a dynamic many country, multi-sector model and calibrate the model to match observed bilateral input-output linkages. With estimated productivity shocks, the model generates an aggregate trade-comovement correlation 30-40% as large as in data, and 50-75% as large for the goods producing sector. With independent shocks, the model accounts for one-quarter of the trade-comovement relationship for gross output of goods. However, shocks transmitted through input linkages do not synchronize value added. And contrary to conventional wisdom, input complementarity does not amplify value added comovement.

Robert C. Johnson
Department of Economics
Dartmouth College
6106 Rockefeller Hall
Hanover, NH 03755
and NBER
robert.c.johnson@dartmouth.edu

1 Introduction

A large empirical literature suggests that international trade transmits shocks and synchronizes economic activity across borders. For example, bilateral trade is strongly (and robustly) correlated with bilateral GDP comovement.[1] Standard international real business cycle (IRBC) models have struggled to generate strong enough propagation of shocks through trade to replicate the quantitative magnitude of this empirical correlation. Kose and Yi (2006) have dubbed this the "trade comovement puzzle."

In addressing this puzzle, recent empirical work has turned attention to the role of intermediate goods trade as a conduit for shocks. For example, Ng (2010) documents that proxies for bilateral production fragmentation predict bilateral GDP correlations, while Di Giovanni and Levchenko (2010) document that bilateral trade is more important in explaining output comovement for home and foreign sectors that use each other as intermediates. Further, Burstein, Kurz, and Tesar (2008) show that countries that intensively engage in intra-firm trade with United States multinational parents display higher manufacturing output correlations with the U.S. In a related vein, Bergin, Feenstra, and Hanson (2009) document that Mexican export assembly (maquiladora) industries are twice as volatile as their US counterparts, suggesting strong transmission of US shocks to Mexico through input linkages.

This focus on input trade is potentially important, since intermediate inputs account for roughly two-thirds of international trade. Yet, the standard IRBC model does not distinguish trade in final goods from trade in intermediate inputs, and thus is ill-suited to study propagation of shocks through input chains. To remedy this problem, I develop a many country, multi-sector extension of the IRBC model that includes sector-to-sector input-output linkages both within and across countries. This model is an open economy analog to closed economy models of sectoral linkages, pioneered by Long and Plosser (1983). I calibrate the model to data on bilateral final and intermediate goods trade flows for 22 countries and a composite rest-of-the-world region, and simulate model responses to sector-specific productivity shocks. Using simulated data, I assess the ability of the model with intermediates to explain observed bilateral output correlations, highlighting the role of input trade in driving comovement.

In the model, input trade transmits shocks across borders independent of, and in addition to, standard IRBC transmission mechanisms. In the cannonical model, idiosyncratic shocks generate output comovement by inducing comovement in factor supplies. Specifically, a positive shock in the home country raises home output and depreciates home's terms of

[1] See, for example, Frankel and Rose (1998), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderón, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), Di Giovanni and Levchenko (2010), and Ng (2010).

trade, which induces increased factor supply and hence output abroad.[2] This mechanism continues to operate in the augmented model with intermediate inputs. However, with traded intermediates, productivity shocks are passed downstream through the production chain directly.[3] One implication of this is that input linkages generate comovement in gross output even if factor supply is exogenous, which in turn implies that comovement in gross output may be delinked from comovement in real value added. Thus, the production chain puts significant additional structure to how shocks are transmitted.

To evaluate these channels quantitatively, I calibrate the model using data from a 'global bilateral input-output table.' As in Johnson and Noguera (2012), I construct this table by combining data from national input-output tables with data on bilateral trade. The table describes how individual sectors in each country source inputs from home and bilateral foreign sources, as well as how each country sources final goods. This data has several advantageous features for calibration of international macro models. First, the framework respects national accounts definitions of final and intermediate goods, and therefore is consistent with standard macro aggregates. Second, the framework explicitly accounts for the "double counting" problem in gross trade statistics, wherein the gross exports exceeds the value added content of exports. These features provide for a more realistic calibration of openness and bilateral linkages than has been previously possible in the literature.

Proceeding to the quantitative analysis, I first simulate the model using an estimated productivity process in which shocks are allowed to be correlated across countries, as in the data. This model generates an aggregate trade-comovement correlation 30-40% the size of the observed correlation. Disaggregating this result, the model generates strong cross-country correlations for goods, but not for services. For example, a trade-comovement regression for gross output of goods returns a coefficient roughly 3/4 the size of the correlation in the data, as compared to a correlation for services that is insignificantly different from zero. The aggregate trade-comovement coefficient then lies between these extremes.

These initial results represent an upper bound on the role of trade in propagating shocks, as they they confound the effects of idiosyncratic shock propagation with the correlation of shocks across countries themselves. To isolate the propagation mechanism, I simulate the model again using shocks that are uncorrelated across countries. In these simulations, the trade-comovement correlation falls substantially for real value added, both in the aggregate

[2]Several recent papers strenthen this mechanism by lowering the short run elasticity of substitution between home and foreign goods, for example by introducing durable goods (Engel and Wang (2011)) or search and matching frictions (Drozad and Nosal (2008)).

[3]Productivity shocks travel unidirectionally downstream when intermediate goods are aggregated in a Cobb-Douglas fashion, the case considered in the benchmark model below. More generally, productivity shocks travel both downstream to input users and upstream to input suppliers.

and at the sector level. This implies that the correlation of shocks across countries is primarily responsible for value added comovement.

Interestingly however, there is significant propagation of idiosyncratic shocks for gross output. For gross output, idiosyncratic shocks account for roughly one-quarter of the trade-comovement correlation in the data. This discrepancy between the comovement in real value added versus gross output points to the role of intermediates in the model. Specifically, gross output in the model is a composite of real value added and intermediate inputs. Therefore, gross output can be correlated across countries either because real value added is correlated, or because intermediate use is correlated. In the model, comovement following idiosyncratic shocks is primarily due to comovement in intermediate use. This is because intermediate trade is the primary conduit through which shocks travel in the model.

Using this framework, I explore whether complementarity among inputs amplifies comovement. I introduce complementarity in two different ways: first making intermediates complements among themselves, and second making intermediates complementary with non-produced factor inputs (i.e., capital and labor). Contrary to conventional wisdom, complementarity fails to narrow the gap between the model and data in both cases. Complementarity within the input bundle raises output comovement dramatically, but does not amplify real value added comovement. Complementarity between intermediates and factor inputs constrains fluctuations in demand for intermediates, thereby lowering comovement in gross output.

These results collectively point to a new puzzle, that could be thought of as the "input trade and comovement puzzle." In data (see references above), input trade is strongly correlated with output comovement. However, I show that a benchmark macro-model with cross-border input linkages does not generate strong real value added comovement, even with strong complementarities among inputs. The final contribution of the paper is examine this new 'puzzle' directly.

Using simulated data from the model, I argue that trade-comovement regressions that include measures of cross-border input linkages, as in Di Giovanni and Levchenko (2010) for example, are not capable of identifying the role of inputs in generating comovement. Specifically, input linkages appear to be correlated with omitted shocks driving output correlations. This suggests that the empirical puzzle is more apparent than real. At the same time, the fact that input trade does not generate stronger output comovement in the model is puzzling in its own right, and it points toward the need for richer theories of input trade and shock propagation.

In addition to the empirical work cited above, this paper is related to a number of recent attempts to incorporate production sharing into business cycle models. The closest antecedent is a two-country, two-sector IRBC model with intermediates by Ambler, Cardia, and Zimmerman (2002).[4] This paper is distinguished from Amber et al. in both scope and focus. Whereas Amber et al. focus on a stylized two country case, I calibrate and simulate a many country model to match data on bilateral final and intermediate goods linkages. Further, I hone the empirical focus toward understanding the trade-comovement puzzle, in contrast to the focus on general business cycle properties of the model in Ambler et al. Lastly, my exposition and analysis of the basic mechanisms underlying international comovement differs substantially from Ambler et al.[5]

This paper is also related in spirit to recent models by Burstein, Kurz, and Tesar (2008), Arkolakis and Ramanarayan (2009), and Bergin, Feenstra, and Hanson (2011).[6] Among these papers, the contrast to Burstein et al. is most stark. They study a two sector IRBC model in which the production sharing sector has a lower elasticity of substitution between home and foreign goods than the non-production sharing sector, which effectively lowers the aggregate elasticity of substitution and raises comovement. This low elasticity is the distinctive feature of the production sharing sector, not that it uses imported inputs per se. I discuss the relationship between the model here and the Burstein et al. model further in Section 4.3.

More broadly, the basic structure of the model in this paper has important characteristics in common with models of sectoral linkages within the domestic economy, such as those analyzed by Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Shea (2002), Carvalho (2008), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011), and Foerster, Sarte, and Watson (2011). These papers provide many insights into the role input-linkages play in transmitting shocks that are applicable to cross-border input trade. However, there is an important difference to keep in mind. Within the domestic economy, factors may be reallocated across sectors following a shock, whereas factors are comparatively immobile across countries in the international framework considered below. As we will see, the inelastic response of factor supply is a crucial impediment to comovement across countries.

Finally, in simulating a international macro model with more than two heterogeneous

[4]Both Ambler et al. and this paper are also related to Cole and Obstfeld (1991) who write down a two country model with intermediate linkages and full depreciation of capital in the spirit of Long and Plosser (1983). This seems to be an under-appreciated contribution of their paper.

[5]Ambler et al. devote attention to analyzing the role of investment frictions in their framework and explaining the differences between their empirical findings and those of Long and Plosser (1983) by appealing to different assumptions regarding capital depreciation.

[6]Arkolakis and Ramanarayan (2009) adopt a multi-stage production function, an approach that is significantly different and less tractable in a multi-country setting than the approach in this paper. Bergin, Feenstra, and Hanson (2011) work with a two sector model, in which the 'offshoring sector' involves Ricardian trade in a continuum of goods. There are not produced intermediates used in this sector, so there is no double counting in trade in the model.

countries, the paper is also related to work by Zimmerman (1997), Kose and Yi (2006), Ishise (2009, 2010), and Juvenal and Monteiro (2010). These papers emphasize that third-country effects may be important in driving bilateral correlations, effects that are picked up in my many country framework. None feature trade in inputs, however.

2 A Many Country, Multi-Sector Sector Model with Cross-Border Input Linkages

I begin by articulating a multi-sector, many country international real business cycle model that allows trade in both final and intermediate goods. The key difference between this model and the standard IRBC framework is that I specify production functions for gross output and define preferences over purchases of final goods. This has two implications.

First, I can calibrate the production structure in the model to match cross-border input-output linkages, while calibrating preferences to match shipments of final goods. As I discuss further below, this eliminates the inconsistent treatment of gross versus value added objects in standard calibrations of the IRBC framework. Second, there is a new channel for transmission of shocks through the production chain that is not operative in the standard IRBC framework. After introducing the model, I discuss both these features in greater detail.

In specifying the equilibrium in the model, I need to take a stand on financial market structure. Following Heathcote and Perri (2002) and Kose and Yi (2006), among others, I focus on the case of financial autarky on the grounds that it has been shown to generate terms of trade movements and cross-country correlations that align more closely with data. Financial autarky tends to deliver stronger comovement because it shuts down "resource-shifting" effects where in capital is reallocated toward countries with positive productivity shocks. While it is straightforward to simulate the model under alternative market structures, I examine only one here to direct attention toward the distinctive aspects of the model.

2.1 Production

Consider a multi-period world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a tradable differentiated good in sector s using capital $K_{it}(s)$, labor $L_{it}(s)$, and composite intermediate good $X_{it}(s)$, which is an aggregate of intermediate goods produced by different source countries. I assume that the aggregate production function takes a nested

CES form:

$$Q_{it}(s) = Z_{it}(s) \left(\theta_i(s)^{1-\sigma} V_{it}(s)^\sigma + (1-\theta_i(s))^{1-\sigma} X_{it}(s)^\sigma\right)^{1/\sigma} \tag{1}$$

$$\text{with} \quad X_{it}(s) = \left(\sum_{j=1}^{N}\sum_{s'=1}^{S} \omega^x_{ji}(s',s)^{1-\eta} X_{jit}(s',s)^\eta\right)^{1/\eta} \tag{2}$$

$$\text{and} \quad V_{it}(s) = K_{it}(s)^\alpha L_{it}(s)^{1-\alpha}, \tag{3}$$

where $X_{jit}(s', s)$ is the quantity of intermediate goods from sector s' in country j used by sector s in country i, $V_{it}(s)$ is a Cobb-Douglas composite domestic factor input composed of capital and labor, $Z_{it}(s)$ is exogenous sector-specific productivity, and $\{\theta_i(s), \omega^x_i(s', s), \alpha\}$ are parameters that govern shares of inputs in gross output, individual inputs in total input use, and individual factors in value added respectively.

Output is produced under conditions of perfect competition. A representative firm in country i, sector s takes the prices for its output and inputs as given, and the firm rents capital and hires labor to solve:

$$\begin{aligned} \max \quad & p_{it}(s)Q_{it}(s) - w_{it}L_{it}(s) - r_{it}K_{it}(s) - \sum_{j=1}^{N}\sum_{s'=1}^{S} p_{jt}(s')X_{jit}(s',s) \\ \text{s.t.} \quad & L_{it}(s), K_{it}(s), X_{jit}(s',s) \geq 0 \end{aligned} \tag{4}$$

where $p_{it}(s)$ denotes the price of output, w_{it} is the wage, r_{it} is the rental rate for capital, and the production function for $Q_{it}(s)$ is given above.

This problem can be broken into two steps. In the first step, the firm chooses the amount of the composite factor $V_{it}(s)$ and intermediate $X_{it}(s)$ to use, given the prices of the composite factor $p^v_{it}(s)$ and intermediate $p^x_{it}(s)$. In the second step, the firm then chooses capital, labor, and the use of individual intermediates. This two-step formulation yields the following first order conditions:

$$V_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} \theta_i(s) \left(\frac{p^v_{it}}{p_{it}(s)}\right)^{1/(\sigma-1)} Q_{it}(s) \tag{5}$$

$$X_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} (1-\theta_i(s)) \left(\frac{p^x_{it}(s)}{p_{it}(s)}\right)^{1/(\sigma-1)} Q_{it}(s) \tag{6}$$

$$r_{it}K_{it}(s) = \alpha p^v_{it}(s) V_{it}(s) \tag{7}$$

$$w_{it}L_{it}(s) = (1-\alpha) p^v_{it}(s) V_{it}(s) \tag{8}$$

$$X_{jit}(s',s) = \omega^x_{ji}(s',s) \left(\frac{p_{jt}(s')}{p^x_{it}(s)}\right)^{1/(\eta-1)} X_{it}(s), \tag{9}$$

where $p^v_{it} = \left(\frac{r_{it}}{\alpha}\right)^{\alpha}\left(\frac{w_{it}}{1-\alpha}\right)^{1-\alpha}$ and $p^x_{it}(s) = \left(\sum_j \sum_{s'} \omega^x_{ji}(s', s) p_{jt}(s')^{\eta/(\eta-1)}\right)^{(\eta-1)/\eta}$.

Output is used both as an intermediate good in production and to produce a composite final good for consumption and investment. Within each sector, perfectly competitive firms aggregate final goods from all sources to form a sector-level composite using production function: $F_{it}(s) = \left(\sum_j \omega^f_{ji}(s)^{1-\rho} F_{jit}(s)^{\rho}\right)^{1/\rho}$. These sector composites are then aggregated to form an aggregate final good via a Cobb-Douglas technology: $F_{it} = \prod_s F_{it}(s)^{\gamma_i(s)}$, where $\gamma_i(s)$ is the expenditure share on final goods of type s in country i.[7]

A representative final goods firms maximizes:

$$\max \quad p^f_{it} F_{it} - \sum_{j=1}^{N} \sum_{s=1}^{S} p_{jt}(s) F_{jit}(s), \tag{10}$$

where p^f_{it} is the price of the composite final good. As above, this can be thought of as a two step process, where first firms choose the amount of each composite final good $F_{it}(s)$ to use given prices for those composites $p^f_{it}(s)$ and then choose final goods from individual sources to form the composites. The first order conditions can then be written as:

$$p^f_{it}(s) F_{it}(s) = \gamma_i(s) p^f_{it} F_{it} \tag{11}$$

$$F_{jit}(s) = \omega^f_{ji}(s) \left(\frac{p_{jt}(s)}{p^f_{it}(s)}\right)^{1/(\rho-1)} F_{it}(s), \tag{12}$$

where $p^f_{it}(s) = \left(\sum_j \omega^f_{ji}(s) p_{jt}(s)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$.

Aggregate final goods are used for consumption and investment: $F_{it} = C_{it} + I_{it}$.[8] Gross output equals total shipments used as intermediates and to produce final composite goods: $Q_{it}(s) = \sum_j \sum_{s'} F_{ijt}(s) + X_{ijt}(s, s')$.

2.2 Consumption and Labor Supply

Each country is populated by a representative consumer. The consumer is endowed with labor (with time endowment normalized to one) that it supplies to firms and consumes final goods. The representative consumer also owns the capital stock in her country and makes investment decisions. The capital stock evolves according to: $K_{it+1} = I_{it} + (1 - \delta) K_{it}$,

[7] Note that I assume that there is no value added at this stage to be consistent with the accounting conventions in the input-output data, which records the value of retail and distribution services as output of the services sector.

[8] This assumption implies that the aggregator is the same for consumption goods and investment goods. This assumption could be relaxed.

where $K_{it} = \sum_{s=1}^{S} K_{it}(s)$. Under financial autarky (balanced trade), expenditure on final goods must equal income in each period for the consumer: $p^f_{it} F_{it} = w_{it} L_{it} + r_{it} K_{it}$, where $L_{it} = \sum_{s=1}^{S} L_{it}(s)$.

The consumer chooses $\{C_{it}, L_{it}, K_{it+1}\}$ to solve:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{it}) - \frac{\chi \epsilon}{1+\epsilon} L_{it}^{(1+\epsilon)/\epsilon} \right] \\ & \text{s.t.} \quad p^f_{it}(C_{it} + I_{it}) = w_{it} L_{it} + r_{it} K_{it} \\ & \text{and} \quad K_{it+1} = I_{it} + (1-\delta) K_{it}, \end{aligned} \tag{13}$$

where ϵ is the Frisch elasticity of labor supply.[9] Then the Euler equation and first-order condition for labor supply are:

$$1 = \beta E_t \left[\left(\frac{C_{it}}{C_{it+1}} \right) \left(\frac{r_{it+1}}{p^f_{it+1}} + (1-\delta) \right) \right] \tag{14}$$

$$\chi L_{it}^{1/\epsilon} = \left(\frac{1}{C_{it}} \right) \frac{w_{it}}{p^f_{it}}. \tag{15}$$

2.3 Equilibrium

Given a stochastic process for productivity, an equilibrium in the model is a collection of quantities $\{C_{it}, F_{it}\}_i$ for each country, $\{Q_{it}(s), K_{it}(s), L_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p^f_{it}, \{p_{it}(s)\}_s\}_i$. These must satisfy the producers' first order conditions (5)-(9) and (11)-(12) and the consumer's Euler equation (14) and first-order condition for labor supply (15). They must also satisfy market clearing conditions $Q_{it}(s) = \sum_j \sum_{s'} F_{ijt}(s) + X_{ijt}(s, s')$ and $F_{it} = C_{it} + K_{it+1} - (1-\delta) K_{it}$, the budget constraint $p^f_{it} F_{it} = w_{it} L_{it} + r_{it} K_{it}$, and the production function in (1)-(3). The equilibrium conditions are collected explicitly in Appendix B.

2.4 Discussion

The model articulated above differs from the standard IRBC framework in that I specify a production function for gross output (Equations (1)-(3)), and therefore account directly for intermediates that are 'used up' in the production process. As mentioned above, this means that the transmission mechanisms and calibration procedure are different than the standard IRBC model. I pause here to discuss both these issues in greater detail.

[9] Quantitative results are very similar with Greenwood-Hercowitz-Huffman preferences that eliminate wealth effects on labor supply.

2.4.1 Mechanics of Comovement

In examining comovement on the production side, it is important to distinguish between real gross output and real value added. With the general CES formulation of the production function, one cannot write real value added as a closed form function of capital, labor, and productivity alone. So I will take an indirect approach and define real value added as a subfunction of gross output, and characterize how real value added changes over time. This approach is consistent with the national accounts practice of defining real GDP via double deflation.[10]

Suppose that gross output can be written as: $Q_{it}(s) = g(RVA_{it}(s), X_{it}(s); t, s)$, where $RVA_{it}(s) = h(K_{it}(s), L_{it}(s); t)$ is a function defining how real value added is produced from primary factors and $g(\cdot)$ is homogeneous of degree one. Given constant returns to scale and perfect competition, then write proportional changes in gross output as:

$$\hat{Q}_{it}(s) = s_i^v(s)\widehat{RVA}_{it}(s) + s_i^x(s)\hat{X}_{it}(s), \tag{16}$$

where $s_i^v(s) \equiv \frac{p_i^v V_i(s)}{p_i(s)Q_i(s)}$ and $s_i^x(s) \equiv \frac{p_i^x(s)X_i(s)}{p_i(s)Q_i(s)}$ are the steady-state shares of value added and intermediate inputs in gross output. Then manipulation of this expression yields:

$$\begin{aligned} \widehat{RVA}_{it}(s) &= \frac{1}{s_i^v(s)}\left[\hat{Q}_{it}(s) - s_i^x(s)\hat{X}_{it}(s)\right] \\ &= \frac{1}{s_i^v(s)}\hat{Z}_{it}(s) + \hat{V}_{it}(s), \end{aligned} \tag{17}$$

where the transition from the first to the second line uses the fact that $\hat{Q}_{it}(s) = \hat{Z}_{it}(s) + s_i^v(s)\hat{V}_{it}(s) + s_i^x(s)\hat{X}_{it}(s)$ in the model above.[11]

The need to distinguish comovement in gross output from comovement in real value added is evident on examination of these equations. Gross output is a composite of real value added and intermediate inputs, while real value added depends on productivity and factor inputs alone. Real output growth may be correlated across countries either because real value added growth is correlated, or because growth in input use is correlated across countries. Thus, traded intermediates break the link between real output and value added in the model.

---

[10] See Sims (1969) and documentation for the EU KLEMS database. Double deflation procedures use separate output price and input price deflators to define real output and input use. Real value added is then the difference between real output and real inputs.

[11] Note that we instead assume the production function is Cobb-Douglas in $V_{it}(s)$ and $X_{it}(s)$, we skip these steps and write gross output explicitly as a function of real value added: $Q_{it}(s) = RVA_{it}(s)^{\theta_i(s)} X_i^{1-\theta_i(s)}$, where $RVA_{it}(s) = Z_i^{1/\theta_i(s)} V_{it}(s)$ is real value added.

In an extreme case, gross output could be correlated across countries even if real value added is constant in all countries. I pause to discuss this special case to provide intuition regarding the role of input linkages in the model. To make the analysis tractable, I make two simplifying assumptions. First, let us assume that each country is endowed with a fixed amount of the composite factor. This both shuts down model dynamics and implies that there is no endogenous comovement in real value added. Second, assume that the production function, intermediate goods aggregator, and preferences are all Cobb-Douglas.

With these assumptions, I show in Appendix A that the proportional change output following productivity innovations is given by:

$$\hat{Q} = [I - \Omega']^{-1} \hat{Z}, \tag{18}$$

where Q and Z are vectors that stack gross output and productivity in all countries. The Ω matrix is a global bilateral input-output matrix that summarizes flows of intermediates across countries, with row i and column j elements equal to the share of expenditure on intermediates that j directly purchases from i as a fraction of the value of output in country j. The matrix $[I - \Omega']^{-1}$ provides a set of weights that indicate how production in country i responds to productivity shocks in country j. The weights can be interpreted as the total cost share of intermediates from j in production in country i, which include both direct purchases of inputs from j and indirect purchases of inputs from j embodied in purchases of inputs from third countries.

These total cost shares summarize how shocks are directly transmitted through the structure of cross-border input linkages.[12] Put simply, a positive productivity shock in country k benefits countries that use country k goods as inputs. This is true whether they use k goods directly or whether they rely on country k goods indirectly, in the sense that they source intermediates from some third country that itself relies heavily on inputs from country k. This has the implication that output will be correlated for country i and country j when they have similar overall sourcing patterns.

Broadening our focus, the general model features these input linkages alongside the standard IRBC transmission of shocks via relative prices and factor supply. If intermediates are removed from the model (setting $\theta_i(s) = 1$), then the production function is linear in the composite factor: $Q_{it}(s) = Z_{it}(s)V_{it}(s)$. When productivity shocks are uncorrelated across countries, output in country i will then be correlated with output in country j only if factor supplies V_i and V_j co-move.

Comovement in factor supplies, in turn, is driven by movements in the terms of trade

[12] I discuss this intuition for a three country case in Appendix A.

following productivity shocks. In the benchmark case below, a productivity increase in country i causes the relative price of output from country i to fall, or equivalently the relative price of output in country j to rise. This raises factor returns and hence induces increased factor supply and output in country j. The strength with which productivity shocks spill across borders then depends on: (a) how responsive prices are to the underlying shocks; (b) the elasticity of factor supply. In the extreme, when factor supply is inelastic and productivity shocks are uncorrelated across countries, there is no real GDP comovement across countries.

2.4.2 Taking the Model to Data

Before turning to calibration details, there are several broad points about matching this model to data that deserve comment. The production function and resource constraints above represent a multi-stage production process with an effectively infinite number of production stages, where value is added at each stage in a decreasing geometric sequence. Because production requires both domestic and imported intermediates, gross trade in the model will be a multiple over actual value exchanged between countries, as goods cross borders many times throughout the production process. In this sense, the model allows for 'double counting' in trade statistics associated with input trade.

The standard IRBC framework is not compatible with 'double counting' in trade data, or the use of imports to produce exports.[13] In the IRBC literature, the convention has been to write down production functions for value added, where value added is produced output of domestic factors (e.g., capital and labor). This production structure introduces several complications for calibration using conventional data.

Consistent with the value added production structure, IRBC models are typically calibrated treating gross exports and imports *as if* they are measured in value added terms. Put differently, they are calibrated under the implicit assumption that the domestic value added content of exports is equal to one. This procedure creates a model economy that is 'too open' relative to reality. Johnson and Noguera (2012) report that the ratio of value added to gross trade is about 0.7 for the median country. Therefore, treating gross exports as if they are value added implies that the economy is roughly 40% too open in the standard calibration. By calibrating a model with a production structure for gross output, I am able

[13]Some semantic confusion may arise in comparing these frameworks. Starting at least with Backus, Kehoe, and Kydland (1994), IRBC models typically talk about trade in "intermediate goods," which are aggregated to produce a "composite final good." Despite this nomenclature, trade in these models should be thought of as trade in value added or quasi-final goods, wherein output crosses an international border only once.

to circumvent this problem.[14]

On top of this problem, there are also complications in calibrating preferences in the standard IRBC framework. To be consistent with production that is measured in value added terms, the standard model must implicitly specify preferences over value added. This is problematic in the sense that substitution elasticities are always estimated using data on gross expenditure or gross trade flows. Therefore, they may not be appropriate for models with production/preferences in value added models.[15] Because I specify preferences over final goods directly, conventional expenditure-based elasticity estimates are appropriate in the context of my framework. I discuss the mapping between elasticities in the standard IRBC framework and my model further in the section on complementarity and comovement below.

3 Calibration

3.1 Technology and Preferences

To simulate the model, I need values for the following parameters: $\{\beta, \epsilon\}$ for preferences and $\{\sigma, \eta, \theta_i(s), \omega^x_{ji}(s', s), \alpha, \omega^f_{ji}(s), \rho, \gamma_i(s), \delta\}$ for the technology. Some parameters are identical across simulations, while others change. In all simulations, I set $\alpha = .33$, $\delta = .1$, $\beta = .96$, and $\epsilon = 4$ based on standard values in the literature.[16]

The elasticity parameters $\{\sigma, \eta, \rho\}$ vary across simulations to allow different degrees of complementary versus substitutability in production and preferences. In the baseline simulation below, I set $\rho = .5$, which implies the elasticity of substitution between final goods from different sources is 2. On the production side, I set $\sigma = \eta = 0$ in the baseline simulation. This implies that the production function is Cobb-Douglas in value added and the composite intermediate, and that the composite intermediate is itself Cobb-Douglas in inputs from different source countries. This parameterization implies that the elasticity of substitution within intermediates is lower than that between final goods.[17] In subsequent simulations, I

[14] The other obvious approach would be to calibrate the model using trade measured in value added terms. I am working on a project using this alternative approach with Rudolfs Bems.

[15] See Herrendorf, Rogerson, and Valentinyi (2011) for discussion of this issue in the context of models of structural transformation.

[16] On the Frisch elasticity, see King and Rebelo (1999) or Chetty, Guren, Manoli, and Weber (2011). While a Frisch elasticity of 4 is required to generate fluctuations in hours worked similar to data in the standard RBC model, it has been criticized as too high relative to micro estimates. In unreported results, I have simulated the model with a Frisch elasticity of labor supply set to 1, and the performance of the model is both qualitatively and quantitatively very similar.

[17] This echoes Burstein, Kurz, and Tesar (2008) or Jones (2011), among others, who argue that the scope for substitution across intermediate goods is lower than for final goods.

vary these elasticities and defer discussion of these alternative parameterizations until they are relevant below.

The share parameters $\{\theta_i(s), \{\omega^x_{ji}(s', s)\}, \{\omega^f_{ji}(s), \gamma_i(s)\}$ are calibrated using the GTAP 7.1 Data Base, which contains benchmark production, input-output and trade data for 2004. Due to limitations on the availability of time series data on gross production and productivity data (see below), I extract country level data for 22 countries from GTAP, covering approximately 80% of world GDP, and aggregate the remaining countries to form a composite "rest-of-the world" region.

The GTAP data allow me to match data for output and value added in each country for two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services." I set the country-sector-specific parameter $\theta_i(s)$ to match the share of intermediate goods in output for each country and sector. The median intermediate share for goods producing sectors is 0.65 for my country sample, while the corresponding share for services is 0.46.

A key part of the calibration is accurate data on bilateral intermediate and final goods flows. I construct these flows by combining input-output tables with data on bilateral trade (both from GTAP), as in Johnson and Noguera (2012). Bilateral intermediate and final goods shipments then serve as data targets for $\{\omega^x_{ji}(s', s)\}$ and $\{\omega^f_{ji}(s)\}$. See Appendix C for details on the source data, the procedure for constructing bilateral final and intermediate goods shipments, and further calibration details.

In the data, trade is unbalanced. Therefore, in calibrating the model, I allow steady state trade to be unbalanced as well to recover 'true' preference and technology parameters. I then solve for dynamics in the model by linearizing around this unbalanced steady state, assuming that trade imbalances are constant.[18] The linearized equilibrium conditions are included in Appendix B.

3.2 Productivity

To estimate stochastic processes for productivity, I use sectoral productivity data from the Groningen Growth and Development Centre's EU KLEMS and 10-Sector databases. Because data on TFP is not available for many countries over long periods of time, I follow the literature and estimate the productivity process using data on labor productivity.[19] I take

[18]An alternative approach would be to calibrate the model to the unbalanced steady state, then solve for and linearize around the corresponding balanced trade equilibrium. In practice, the differences in behavior of the model linearized around balanced steady state versus imbalanced steady states are second order.

[19]Though motivated by data constraints, using labor productivity in place of TFP implicitly assumes that capital and/or labor quality dynamics do not drive variation in labor productivity at business cycle frequencies. This assumption is common in the aggregate IRBC literature: see Backus, Kehoe, and Kydland

sectoral labor productivity growth for 19 OECD countries over the period 1970-2007 from the EU KLEMS data, where labor productivity growth is computed as the difference between real value added growth and growth in hours worked for each sector.[20] I turn to the 10-Sector data to compute productivity growth rates for three large emerging markets – Brazil, India, and Mexico – over the same period. Productivity in this data is measured as the difference between real value added growth less growth in the number of workers employed.

For each country and sector, I estimate univariate, trend stationary productivity process. Suppressing constants and time trends, the estimating equation is:

$$\log LP_{it}^{VA}(s) = \lambda_i(s) \log LP_{it-1}^{VA}(s) + \epsilon_{it}(s), \tag{19}$$

where $LP_{it}^{VA}(s)$ is the level labor productivity (measured using value added) and $\lambda_i(s)$ is the persistence parameter.[21] The correlation of productivity shocks $\epsilon_{it}(s)$ is unrestricted. To compute this correlation, I estimate equation 19 for each country and sector separately, recover regression residuals $\hat{\epsilon}_{it}(s)$, and then construct the covariance matrix of the shocks as: $\Sigma \equiv \frac{1}{T}\sum_t \hat{\epsilon}_t \hat{\epsilon}_t'$.[22] To simulate the model, I need to convert the covariance matrix Σ, constructed using residuals from estimation of the process for productivity measured using real value added, into an equivalent covariance matrix for shocks to productivity measured on a gross output basis. The adjustment multiplies each residual by the ratio of value added to output: $\hat{\hat{\epsilon}}_{it}(s) \equiv s_i^v(s)\hat{\epsilon}_{it}(s)$. See Appendix C for details.

In the simulations below, I will use this covariance matrix in two ways. One set of simulations will allow shocks to be correlated across countries, with correlations determined by the estimated covariance matrix. This is the standard approach in the literature. The shortcoming of this approach is that comovement in this set of simulations is driven both by transmission of shocks across countries via trade linkages and the direct correlation of the underlying shocks themselves.

(1992), Heathcote and Perri (2002), or Kose and Yi (2006) for example. For countries where both TFP and labor productivity growth rates are available, the correlation between the two is high.

[20]Countries include Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, Portugal, Sweden, United Kingdom, and the United States.

[21]I restrict cross-country spillovers to be zero as a matter of necessity. With N countries and 2 sectors, there are too many unrestricted spillover parameters to estimate given the relatively short length of the time series available. I have experimented with estimation of cross-sector spillovers within countries. Point estimates for cross-sector spillovers are generally unstable across countries and imprecisely estimated (often indistinguishable from zero).

[22]For three of the forty-four country-sector pairs, the estimated persistence parameters exceed one. Examination of the data indicates that this is due to breaks in the trend for these country-sector time series. For these countries, I estimate productivity processes assuming that each experiences only aggregate productivity shocks (i.e., productivity growth in goods and services is equal to aggregate productivity growth). These three countries are Italy, India, and Mexico.

To more cleanly identify the trade transmission mechanism, I will also simulate the model under the (counterfactual) assumption that shocks are uncorrelated across countries. To parameterize this counterfactual scenario, I zero out the "off-diagonal" elements of the covariance matrix.[23] Specifically, I impose $\text{cov}(Z_{it}(s), Z_{jt}(s')) = 0$ for all $i \neq j$. This allows shocks to be correlated across sectors within countries, but uncorrelated for any cross-country sector pairs. While this eliminates cross-country correlations in shocks, it should be noted that $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is an upper bound to the size of the truly independent productivity shocks.[24] This implies that simulated shocks using this method will be somewhat too large relative to the truly idiosyncratic shocks that countries face. Thus, one should interpret simulation results using these idiosyncratic shocks as an upper bound on the ability of the model to generate comovement from true idiosyncratic country shocks.

One last detail regarding the simulation is that I include a composite rest-of-the-world region in the simulations, but do not have directly measured productivity data for this composite region. Therefore, I assume that productivity shocks in the rest-of-the-world are uncorrelated with productivity shocks to countries in my sample.[25] I parameterize the persistence, variance, and cross-sector correlations of the shocks to this region based on median values in the data.

4 Results

I begin by examining the model's ability to replicate the aggregate correlation between bilateral trade and output comovement with estimated productivity shocks. In this baseline analysis, I allow productivity shocks to be correlated across countries, as in the data. To isolate the role of trade in propagation of shocks, I turn to simulations with uncorrelated productivity shocks. Here I focus on contrasting the performance of the model for gross output versus value added, and examine whether introducing stronger complementarity for intermediate goods into the production function strengthens propagation. Finally, I explore whether augmented trade-comovement regressions with vertical linkages isolate the causal influence of input linkages on comovement.

[23] This approach is adapted from Horvath (1998).

[24] For example, suppose that there are global shocks and i.i.d. country shocks. Then $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is equal to the sum of the variance of the global shock plus the variance of the idiosyncratic country shock, and hence an upper bound on the variance of the idiosyncratic shock.

[25] This assumption will likely bias downward the trade-comovement correlation in the model with correlated shocks, since in reality the rest-of-the-world productivity is likely positively correlated with most in-sample countries.

4.1 Trade-Comovement Correlations: Model vs. Data

To compare the model and data, I compute the correlation of year-on-year aggregate growth rates of gross output or real value added for each country pair. I also compute sector-level correlations across countries for three sector pairs: goods-goods, goods-services, and services-services.[26] Correlations in the model are computed as averages over 500 replications of 35 years each, roughly the same period over which correlations are computed in the data.

For aggregate output and value added, model-based correlations are positively related to data-based correlations, though the fit is imperfect. A regression line of best fit for correlations of real value added in model versus data is $\rho_{ij}(\text{data}) = .26 + .48\rho_{ij}(\text{model})$ with standard error on the slope of .08 and $R^2 = .15$. The positive intercept indicates that the model generally under predicts the average correlation in the data, which is quite reasonable given that there are other shocks not included in the model (e.g., demand shocks) that may be positively correlated across countries.[27]

To evaluate the aggregate trade-comovement relationship directly, I regress bilateral correlations in the data and model on bilateral trade intensity. Aggregate bilateral trade intensity is defined as: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, computed for the benchmark 2004 year in my data.[28] Table 1 and Table 2 contain results for gross output and real value added, respectively.[29] Panel A contains results from data, while Panel B contains results from the baseline model with correlated shocks.

Looking at the first column of Panel A in the tables, aggregate comovement is positively correlated with log bilateral trade intensity in the data. Further, comparing Tables 1 and 2, the quantitative magnitude of this relationship is similar for both gross output and real value added. Turning to model simulations in Panel B, the aggregate trade-comovement correlation is weaker, but evidently positive. Regression coefficients in the simulated data are roughly 30-40% as large as those in the actual data. Thus, while the model does not

[26]Note that for each country pair, there are two possible cross-sector (goods-services) correlations. In the analysis, I pool these correlations, so that the correlation of goods in country i with services in country j is treated the same as the correlation of services in country i with goods in country j.

[27]One possible candidate for these omitted shocks would be monetary shocks. Indeed, examining the model's fit for EU-pairs versus non-EU pairs (or Eurozone versus non-Eurozone), the model does a better job explaining variation in bilateral correlations for non-EU pairs than among EU-pairs. While the model does not fit EU-pairs in the aggregate, I show below that it does fit EU-pairs well for the goods sector. This is indirect evidence that demand shocks could be an important driver of services correlations observed in the data that cannot be explained by the model.

[28]Because trade shares are stable over time, results are not sensitive as to whether one computes bilateral trade intensity using trade data single year or averages bilateral trade over time prior to computing the metric. The basic results also hold if the level, rather than log, of bilateral trade intensity is used.

[29]Gross output correlations are computed using the Groningen EU KLEMS database, which implies that I cannot calculate correlations for pairs involving Brazil, India, and Mexico. Therefore, I also omit them in calculating gross output correlations in the model.

explain the aggregate trade-comovement correlation entirely, it accounts for a significant share of it.

To break down this result, I turn to sector-level correlations for output and value added. Figure 1 plots bilateral sector-level correlations in the data and model with correlated shocks for gross output of goods and services separately. The upper panel contains the data for each country's goods sector paired with a bilateral foreign goods sector, and the lower panel contains the same for services. The results are striking: the model with correlated shocks does a good job predicting gross output correlations for goods, but does a weak job for services. The correlation of model and data-based correlations is .46 for goods, and only .15 for services. Cross-sector pairs are in between with a correlation of .25.[30] This basic dichotomy – the model fits relatively well for goods and poorly for services – is borne out no matter whether one looks at gross output or real value added.

Not surprisingly, the good model fit for goods and poor fit for services manifests itself in trade-comovement regressions. Tables 1 and 2 reports regression coefficients for each sector pairing – goods-goods, services-services, and goods-services (cross) – separately. In these sector level regressions, log bilateral trade intensity between sector s in country i and sector s' in country j is defined as: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.[31]

Looking at results for both gross output and real value added, trade predicts comovement for all sector pairs in the data. While trade somewhat better job predicting comovement for goods-goods sector pairs, regression coefficients are positive, significant, and large for all sector pairs. Turning to the model with correlated shocks, the model generates significant trade-comovement correlations only for goods-goods pairs. For goods-goods pairs, the coefficient on trade is roughly 3/4 the size of the correlation in the data for gross output and 1/2 for real value added.[32] In contrast, the model generates markedly weaker correlations for other sector pairings. For services-services pairs, trade is only weakly and insignificantly correlated with comovement for both gross output and real value added.

These results help us understand the origins of the aggregate trade-comovement correlation. The model with correlated shocks yields a strong relationship between trade and comovement for goods, but not for services. The aggregate trade-comovement coefficient then lies between these extremes, pulled toward zero by the model's inability to explain

[30] A regression line for goods is $\rho^g_{ij}(\text{data}) = .29 + .48\rho^g_{ij}(\text{model})$ with standard error on the slope of .07 and $R^2 = .21$, while for services the line of best fit is $\rho^s_{ij}(\text{data}) = .22 + .15\rho^s_{ij}(\text{model})$ with standard error on the slope of .08 and $R^2 = .02$. For cross sector pairs, $\rho^c_{ij}(\text{data}) = .24 + .27\rho^c_{ij}(\text{model})$ with standard error on the slope of .06 and $R^2 = .06$.

[31] This definition follows di Giovanni and Levchenko (2010). One could alternatively define bilateral trade intensity using sector-to-sector shipments $EX_{ij}(s, s')$.

[32] If all 231 country pairs are included in the simulated data regression, the coefficient rises to .07 for gross output.

services sector correlations. To raise the model implied trade-comovement correlation would require introducing elements that raise the correlation of services sectors across countries. Put differently, neither the measured correlation of services productivity shocks across countries, nor the transmission of idiosyncratic shocks through trade, is strong enough to generate a large aggregate correlation of trade with aggregate output comovement in this model.

4.2 Propagation of Idiosyncratic Shocks via Trade

The model-based trade-comovement correlations reported above represent an upper bound on the role of trade in generating comovement. Specifically, the trade-comovement regressions confound two possible reasons why trade predicts comovement. Bilateral trade can predict comovement either because it propagates shocks across border, or because it is a proxy for another force that generates comovement. Of principal concern, countries that trade more may have more correlated underlying productivity shocks.

To focus on pure propagation of idiosyncratic shocks, I turn to simulated data from the model with uncorrelated shocks. Panel C of Tables 1 and 2 report trade-comovement regressions for these simulations. In the first column, the aggregate trade-comovement correlation declines substantially once one removes common shocks from the productivity process. This decline is particularly pronounced for real value added in Table 2, where the coefficient is roughly one-fifth the size of the coefficient in the model with correlated shocks and only one-twentieth the size of the coefficient in the data.

The inability of the model here to generate a sizable correlation between trade and comovement when shocks are uncorrelated is the analog to the Kose and Yi (2006) trade-comovement puzzle in my framework. In a three-country IRBC model, Kose and Yi vary bilateral trade-intensity exogenously by manipulating trade costs, holding the correlation of shocks across countries constant. Then comparing value added correlations across equilibria with different trade costs, they compute a trade-comovement quasi-regression coefficient that is at most 1/10th the size of the coefficient in the data, a similar order of magnitude to the coefficients here. Given that Kose-Yi examine a model without intermediate goods trade, this leads to the conclusion that input trade does not "solve" the trade-comovement puzzle, at least in this standard class of models. Despite the introduction of input trade into the IRBC model, trade does not propagate shocks strongly enough to generate much comovement in aggregate GDP. This implies that the positive coefficient on bilateral trade in data and the model with correlated shocks arises because bilateral trade intensity proxies for the correlation of shocks themselves.

An important caveat, however, is that there is significant propagation of idiosyncratic shocks for gross output. The trade-comovement correlation in the model with uncorrelated shocks is roughly 60% of the correlation in the model with correlated shocks. Thus, there is an important discrepancy between the model's ability to generate comovement in gross output versus comovement in real value added. This result deserves separate attention, as it highlights the role that intermediates play in this framework.

The discrepancy between the propagation mechanism for gross output versus real value added is mostly clearly illustrated by examining cross-country correlations for goods production, so I focus on this sub-set of the data. For gross output of goods, propagation of independent shocks explains roughly one-third of the observed comovement in the data. Figure 2 plots actual gross output correlations for goods against those predicted by the model with uncorrelated shocks. There is a clear positive relationship, particularly among EU country pairs. The U.S.-Canada outlier is particularly instructive. The predicted correlation is near .25, while the actual correlation in the data is near .75, roughly a ratio of three to one. More generally, this magnitude is consistent with the overall spread in the data. Focusing on EU-pairs, predicted correlations vary in the range $(0, .15)$ while actual correlations lie in the range $(.25, .75)$, so the ratio of the ranges is roughly $.5/.15$ or three to one.[33]

These relationships are borne out in looking at the trade-comovement regressions for goods trade in Panel C of Table 1, where the coefficient generated by the model with uncorrelated shocks is one-third the size of the coefficient in the model with correlated shocks and one-quarter of that in the data. Thus, while two-thirds of the goods trade-comovement relationship for gross output is due to correlated shocks in the model, one-third is explained by the propagation of uncorrelated shocks across countries. At the same time, the model generates much weaker comovement in real value added, even for goods-goods sector pairs. One can see this by comparing the trade-comovement regression for goods in Panel C of Table 2 to those in Table 1, where the coefficient for value added is near zero.

To tie these results together, one needs to look at how correlations in gross output are related to real value added in the model. I plot the correlation of gross output against the correlation for real value added for goods sector-pairs in Figure 3. The left figure depicts the relationship in the data (which is matched by the model with correlated shocks), while the right figure depicts this relationship in the model with uncorrelated shocks. Whereas correlations for value added and gross output track each other closely in the data, there are large differences between the two in the model. First, dispersion in correlations of real value added across country is much smaller than the variance of correlations in gross output.

[33] In this comparison, I relate changes in comovement across pairs to changes in predicted model correlations for EU pairs. This obviously ignores the fact that the model grossly underestimates the median correlation. The median ratio of the model correlation with uncorrelated shocks to the actual correlation is $\approx 10\%$ for EU pairs.

Second, the correlation of gross output is typically larger (sometimes much larger) than the correlation of real value added for individual country pairs.

These discrepancies shed light on the role of intermediate goods in the model. Recall from the discussion in previous sections that gross output is a composite of real value added and intermediate inputs, as in Equation (16). The correlation of gross output can then be decomposed into a weighted sum of the correlation of real value added across countries, the correlation of input use across countries, and the cross-correlation of real value added and input use:

$$\rho_{ij}(Q) = w_{ij}^{vv}\rho_{ij}(V) + w_{ij}^{xx}\rho_{ij}(X) + w_{ij}^{vx}\rho_{ij}(V,X) + w_{ij}^{xv}\rho_{ij}(V,X), \tag{20}$$

where $w_{ij}^{vv}, w_{ij}^{xx}, w_{ij}^{vx}, w_{ij}^{xv}$ are the appropriate weighting terms for each correlation, themselves functions of the shares and standard deviations of gross output, real value added, and input use. To provide a visual sense of how these correlations aggregate, I plot the correlations $\rho_{ij}(V)$ and $\rho_{ij}(X)$ for select country pairs in Figure 4. As is evident, the correlation in input use across countries dwarfs the correlation in real value added. Further, the correlation of output lies somewhere in between, near the simple average of these two correlations.[34] Thus, the correlation of gross output is high because intermediate use is highly correlated, not because value added is highly correlated.

The fact that intermediate use is highly correlated is direct evidence that productivity shocks are being forcefully transmitted through cross-border production chains in the model. Because the share of intermediates in gross output for goods is roughly 2/3, this translates into significant output comovement. On the other hand, value added comovement is not high. Recall that one reason value added comoves in the model is that factor supply responds to relative prices. The low comovement of real value added indicates this channel is relatively weak in the model. To raise comovement in value added, one would need to strengthen this channel. In particular, the model would need to be adapted to translate the relatively strong comovement in intermediate use into stronger comovement in value added. With this motivation, I turn to analyzing whether input complementarity amplifies comovement.

4.3 Complementarity and Comovement

A recent strain of thought holds that disruptions in input-sourcing produce large output losses because inputs are complements in production. This argument surfaces in Burstein,

[34] In the simulated data, the weights on each term are approximately equal (roughly 1/4) and the typical cross-correlation ($\rho_{ij}(V,X)$ or $\rho_{ij}(V,X)$) is relatively close to $\rho_{ij}(Q)$, lying between the extremes of $\rho_{ij}(V)$ and $\rho_{ij}(X)$. Hence, the simple average of $\rho_{ij}(V)$ and $\rho_{ij}(X)$ approximates $\rho_{ij}(Q)$ quite well.

Kurz, and Tesar (2008), Jones (2011), Di Giovanni and Levchenko (2010), or news coverage of the economic repercussions of the 2011 earthquake and tsunami in Japan for global supply chains. This is intuitively plausible, as negative supply shocks in a particular country or sector should be particular painful to upstream input users who have limited ability to substitute toward using inputs from alternative suppliers, or toward using non-produced factors of production (i.e., capital and labor) more intensively.

Building on these ideas, there are two distinct ways to introduce limited substitution into the production function used in previous sections. First, inputs may complements to each other. In the model above, the elasticity parameter η in Equation (2) governs this form of complementarity.[35] Second, inputs may be complementary to other factors of production. Put differently, inputs may be complementary to value added. The strength of the this form of complementarity is governed by the parameter σ in Equation (1).

There is scant evidence as to which form of complementarity is more important empirically, particularly in contexts with imported intermediates. Perhaps this is unsurprising, as nearly all IRBC models ignore input trade. To illustrate the consequences of complementarity, I simulate the model for several extreme cases. In the first case, I assume the production function is Cobb-Douglas ($\sigma = 0$) and the intermediate goods aggregator is near-Leontief (setting $\eta = -19$, corresponding to an elasticity of substitution equal to .05). In the second case, I assume the intermediate goods aggregator is Cobb-Douglas ($\eta = 0$) and the production function is near-Leontief in $V_{it}(s)$ and $X_{it}(s)$ ($\sigma = -19$).[36] Third, I examine an combination of the two cases, setting $\sigma = -1$ and $\eta = -19$. Finally, I also consider a variant of the model in which final goods, rather than inputs, are complements. In this fourth simulation, I re-set the production structure to the benchmark case ($\sigma = \eta = 0$), and make preferences near-Leontief ($\rho = -19$).

Using these alternative parameterizations, I re-simulate the model with uncorrelated productivity shocks as in previous sections and run trade-comovement regressions in this new simulated data. I present the results for sector-level correlations for the goods-goods sector pairing in Table 3. The columns labeled "Benchmark" repeat results from previous tables for reference, while columns labeled "Simulations with Complementarity" contain the new results.

The results point to several problems with the conventional view that complementarity

[35] In this model, complementaries among inputs are symmetric across inputs from different countries and sectors. Complementaries could also be allowed to vary among subsets of inputs (e.g., home and foreign inputs could be complements, while foreign inputs are substitutable among themselves).

[36] Strong complementary between factors and intermediates is common within the static computable general equilibrium trade literature, where Leontief production functions have been commonly employed. See Kehoe and Kehoe (1994), for example.

(either among intermediate or final goods) is important in explaining comovement. Introducing complementarity among intermediates (column 2) does substantially strengthen the propagation of shocks for gross output. In fact, the model here generates a trade-comovement coefficient that exceeds the coefficient in data. However, even with this extreme comovement in output, the model does not generate any comovement in real value added. This implies that even extremely strong transmission of shocks through input linkages fails to generate enough comovement in factor supplies across countries to replicate real value added correlations.

In contrast, complementarity between inputs and factors fails on both counts: it neither generates comovement in gross output, nor real value added. In particular, the trade-comovement correlation for gross output is even lower than in the benchmark model. What is going on here? When agents are unable to substitute between factor inputs (V) and intermediate inputs (X), the less responsive input effectively constrains fluctuations in demand for the other input. In the model, factor input supply is fairly inelastic. This dampens fluctuations in input use, which weakens the transmission of shocks through intermediate linkages and lowers comovement in gross output.

Two further exercises serve to deepen the complementarity puzzle further. Combining complementarity within intermediates with complementarity between inputs and factors, with results presented in the column labeled "w/in X + b/n V&X", does not change this result. As before, complementarity within the intermediates bundle amplifies gross output comovement alone. And finally, moving the complementarity from production to consumption also does not change the results. In the final simulation, presented in the column labeled "w/in X," I reset the the production elasticities to the benchmark case and then drive down the final goods elasticity to make preferences near Leontief. Here too the model fails, and in fact performs worse than in the benchmark scenario.

These results run counter to the received wisdom regarding the role of intermediates in propagation of shocks. Many recent papers have improved the performance of standard international business cycle models by lowering the elasticity of substitution between home and foreign goods, including Kose and Yi (2006), Corsetti, Dedola, and Leduc (2008), Burstein, Kurz, and Tesar (2008), and Drozad and Nosal (2008). For example, Burstein, Kurz, and Tesar (2008) suggest that complementary intermediates are important for understanding the trade-comovement relationship.

The relationship between results in this paper and this related work can be understood by noting that these models follow the IRBC tradition and write down production functions for value added. These papers then introduce complementarity across value added coming from different sources. In the Burstein, Kurz, Tesar model, for example, the "production-sharing" (vertically integrated) sector features a low elasticity of substitution between home and foreign value added. As in the standard IRBC model, this low elasticity amplifies comovement, because low elasticities imply volatile relative prices and strong transmission of shocks. I do not directly assume home and foreign value added are complementary, but rather embed complementarity into the production function for gross output. One way of reading my results is that complementarity of this form is not sufficient to induce the complementarity between home and foreign value added needed to replicate observed comovement.

4.4 Vertical Linkages in Trade-Comovement Regressions

In previous sections, I have used simple bivariate trade-comovement regressions to compare model and data. Several recent papers have attempted to isolate the role of intermediates in explaining comovement using more sophisticated specifications. Specifically, Di Giovanni and Levchenko (2010) and Ng (2010) both construct proxies for bilateral vertical linkages by combining trade and input-output data, and look at the partial effect of these linkages controlling for overall bilateral trade intensity. Further, Di Giovanni and Levchenko also estimate sector-level regressions with sector-pair and/or country-pair fixed effects to control for common shocks across countries. It is an open question whether these augmented trade-comovement regressions with vertical linkages can be interpreted as evidence of a causal relationship between vertical linkages and output comovement. I therefore explore this question using my simulated data.

Because Di Giovanni and Levchenko (2010) examine sector-level data, it is straightforward to map their empirical exercise to my framework and therefore I focus on their work. Di Giovanni and Levchenko attack the identification problem by estimating trade-comovement regressions at the sector level, pooling across sectors, and adding fixed effects to absorb particular unobservable shocks. Specifically, they construct a metric of bilateral vertical linkages at the sector level to capture the intensity with which exports from sector s in country i are used as intermediates by sector s' in country j (and vice versa). This takes the form: $\left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s')\right]$, where $\text{IO}(s,s')$ is a measure of input-output linkages between sectors s and s' taken from a single country's input-output table and $\text{Exports}_{ij}(s) = \log\left(\frac{EX_{ij}(s)}{GDP_i + GDP_j}\right)$ is the log of exports from i to j in sector s normalized by the sum of value added in the source and destination countries.[37]

[37] I use the direct input-requirements $\text{IO}(s,s')$ the U.S. to proxy for cross-sector input links. Di Giovanni and Levchenko also use input links for a single country.

Then, Di Giovanni and Levchenko estimate the following regression:

$$\begin{aligned}\rho_{ij}(s,s') =& \alpha + \beta \text{Trade}_{ij}(s,s') \\ &+ \gamma \left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s')\right] \\ &+ \text{FE} + \epsilon_{ij}(s,s'),\end{aligned} \tag{21}$$

where $\text{Trade}_{ij}(s,s') \equiv \log\left(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j}\right)$ and FE denotes fixed effects that vary by specification. One specification includes sector-pair fixed effects, while a second specification includes sector-pair fixed effects and country-pair fixed effects. These fixed effects are introduced to address concerns about omitted common shocks. The sector pair effects control for worldwide sector-specific shocks (possibly correlated across sectors) that hit all countries simultaneously. The country pair fixed effects control for aggregate shocks that may be correlated across countries, but hit all sectors symmetrically within each country.[38]

I report the results of running these regressions in my data in Table 4. Focusing on results for gross output, the regression results in the actual data are generally consistent with those reported in Di-Giovanni and Levchenko. Both bilateral trade and vertical linkages (Trade $\times$ IO) are positively correlated with bilateral sector-level comovement. Vertical linkages are significant in both specifications, while trade intensity is not significant when country fixed are included (though the t-stat of 1.47 is sizable).[39]

Examining results in the model with correlated shocks, vertical linkages remain significant and the coefficient magnitudes are the same or larger than those found in the data. Turning to the model with uncorrelated shocks, however, the magnitude of the coefficient on vertical linkages drops significantly, explaining at most 1/5 of the magnitude of the coefficients in the data. Further, looking at real value added, the model with correlated shocks continues to generate coefficients on vertical linkages similar to those in the data, though smaller in magnitude. However, the sign on vertical linkages actually flips sign in regressions in the simulated data with uncorrelated shocks.

Recall that the fixed effects are intended to control for correlated shocks driving correlations in the data. If these fixed effects adequately control for these shocks, one should expect that regression results in the model with uncorrelated shocks to be similar to those in the data (alternatively, the model with correlated shocks). Given that they are not, this suggests that vertical linkages proxies in the data may themselves be picking up shocks that

[38] Ng (2010) embeds a vertical linkages metric into an aggregate trade-comovement regression, and therefore cannot use pair fixed effects to absorb common shocks. Instead, he includes other possible determinants of correlations (e.g., financial openness, output composition, etc.) directly as control variables in the regression.

[39] One point to note is that my country sample is much smaller than Di Giovanni and Levchenko (2010), so lower significance levels may be expected.

vary by country-pair and sector-pair that the fixed effects cannot absorb. As such, these regressions are of dubious value as evidence that vertical linkages play an important role in propagation of shocks.

5 Conclusion

This paper uses a multi-sector, many country extension of the IRBC model with trade in both final and intermediate goods to dissect the trade-comovement puzzle. Using the model, I attempt to refine our understanding of the trade comovement puzzle along several dimensions.

First, input trade does not resolve the aggregate trade-comovement puzzle in a straightforward manner. That said, input trade does appear to play a role in explaining the relatively good fit of the model for the gross output of goods. Surprisingly, however, transmission of shocks through intermediate input channels does not generate strong comovement in value added. Moreover, complementarity for intermediates within the production function does not resolve this problem, or strengthen the trade transmission mechanism. This result points to a deeply puzzling mismatch between preference elasticities used in standard value added macro models and elasticities in the gross framework laid out here. Exploring this mapping is a fruitful project for future work.

Second, and more generally, the aggregate trade-comovement correlation in the model is induced by correlation of shocks across countries. In this, the low aggregate correlation of trade with comovement in the model is due to the low correlation of shocks to services productivity across countries. This suggests that closing the gap between model and data will require expanding the set of shocks considered beyond productivity to include shocks that synchronize services more forcefully.

Third, trade-comovement regressions are difficult to interpret because it is not generally possible to control unobservable common shocks. This is true for plain-vanilla specifications, as well as augmented specifications that include proxies for vertical linkages and/or employ sector-level data with fixed effects. Model simulations with uncorrelated shocks suggest that the "causal" role of bilateral trade and/or vertical linkages is much smaller than suggested by raw regression estimates.

Despite these results, there is promising evidence here that the introduction of intermediates into macro models alters the role of trade as a conduit of shocks. While intermediates in this benchmark IRBC framework do not replicate observed value added comovement, this may speak to the shortcomings of the IRBC framework rather than to the role of inputs per se. That is, there may be an important role for intermediates in models that capture the micro-structure of trading relationships more accurately. For example, the bulk of intermediates are traded within multinational firms, and this concentration of input trade among largest firms in the economy may mean shocks to intermediate suppliers are passed to aggregates. It is also true that many traded intermediates are tailored to a specific input purchaser – e.g., screens for the iPad. This specificity at the firm-product level is difficult to capture in the type of aggregate model developed in this paper. More careful consideration of these microeconomic features of input trade would be useful in future work.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2011. "The Network Origins of Aggregate Fluctuations." Unpublished Manuscript, MIT.

Ambler, Steve, Emanuela Cardia, and Christian Zimmerman. 2002. "International transmission of the business cycle in a multi-sector model." *European Economic Review*, 46: 273–300.

Arkolakis, Costas, and Anath Ramanarayanan. 2009. "Vertical Specialization and International Business Cycle Synchronization." *Scandinavian Journal of Economics*, 111(4): 655–680.

Backus, David, Patrick Kehoe, and Finn Kydland. 1994. "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?" *The American Economic Review*, 84(1): 84–103.

Baxter, Marianne. 1995. "International Trade and Business Cycles." In *Handbook of International Economics*. Vol. 3, , ed. Gene Grossman and Kenneth Rogoff, Chapter 35, 1801–1864. Elsevier.

Baxter, Marianne, and Michael Kouparitsas. 2005. "Determinants of business cycle comovement: a robust analysis." *Journal of Monetary Economics*, 52: 113–157.

Bems, Rudolfs, Robert Johnson, and Kei-Mu Yi. 2010. "Demand Spillovers and the Collapse of Trade in the Global Recession." *IMF Economic Review*, 58(2): 295–326.

Bergin, Paul, Robert Feenstra, and Gordon Hanson. 2009. "Offshoring and Volatility: Evidence from Mexico's Maquiladora Industry." *The American Economic Review*, 99(4): 4.

Burstein, Ariel, Christopher Kurz, and Linda Tesar. 2008. "Trade, production sharing, and the international transmission of business cycles." *Journal of Monetary Economics*, 55: 775–795.

Calderon, Cesar, Alberto Chong, and Ernesto Stein. 2007. "Trade intensity and business cycle sychronization: Are developing countries any different?" *Journal of International Economics*, 71: 2–21.

Carvalho, Vasco M. 2008. "Aggregate Fluctuations and the Network Structure of Intersectoral Trade." Unpublished, CREi and Universitat Pompeu Fabra.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Does Indivisible Labor Explain the Difference between Mico and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities." *American Economic Review Papers and Proccedings*, 101(2).

Cole, Hal, and Maurice Obstfeld. 1991. "Commodity trade and international risk sharing." *Journal of Monetary Economics*, 28: 3–24.

di Giovanni, Julian, and Andrei Levchenko. 2010. "Putting the Parts Together: Trade, Vertical Linkages and Business Cycle Comovement." *American Economic Journal: Macroeconomics*, 2(2): 95–124.

Drozad, Lukasz, and Jaromir Nosal. 2008. "Long-Run Price Elasticity of Trade and the Trade-Comovement Puzzle." Unpublished, University of Wisconsin.

Dupor, Bill. 1999. "Aggregation and irrelevance in multi-sector models." *Journal of Monetary Economics*, 43: 391–409.

Engel, Charles, and Jian Wang. 2011. "International Trade in Durable Goods: Understanding Volatility, Cyclicality, and Elasticities." *Journal of International Economics*, 83(1): 37–52.

Foerster, Andrew, Pierre-Danieal Sarte, and Mark Watson. 2011. "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *The Journal of Political Economy*, 119(1): 1–38.

Frankel, Jeffrey, and Andrew Rose. 1998. "The Endogeneity of the Optimum Currency Area Criteria." *The Economic Journal*, 108: 1009–1025.

Heathcote, Jonathan, and Fabrizio Perri. 2002. "Financial autarky and international business cycles." *Journal of Monetary Economics*, 49: 601–627.

Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2011. "Two Perspectives on Preferences and Structural Transformation." Unpublished Manuscript, Arizona State University.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks." *Review of Economic Dynamics*, 1: 781–808.

Horvath, Michael. 2000. "Sectoral shocks and aggregate fluctuations." *Journal of Monetary Economics*, 45: 69–106.

Imbs, Jean. 2004. "Trade, Finance, Specialization, and Synchronization." *The Review of Economics and Statistics*, 86: 723–734.

Inklaar, Robert, Richard Jong-A-Pin, and Jakob de Haan. 2008. "Trade and business cycle synchronization in OECD countries – A re-examination." *European Economic Review*, 52(4): 646–666.

Ishise, Hirokazu. 2009. "Trade Costs and Business Cycle Transmission in a Multi-Country, Multi-Sector Model." Unpublished Manuscript, Boston University.

Ishise, Hirokazu. 2010. "The World Has More Than Two Countries: Implications of Multi-Country International Real Business Cycle Models." Unpublished Manuscript, Boston University.

Johnson, Robert C., and Guillermo Noguera. 2012. "Accounting for Intermediates: Production Sharing and Trade in Value Added." *Journal of International Economics*, 86(2): 224–236.

Jones, Charles. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics*, 3(2): 1–28.

Juvenal, Luciana, and Paulo Santos Monteiro. 2010. "Trade and Synchronization in a Multi-Country Economy." Federal Reserve Bank of St. Louis Working Paper 2010-038C.

Kehoe, Patrick, and Timothy Kehoe. 1994. "A Primer on Static Applied General Equilibrium Models." *Federal Reserve Bank of Minneapolis Quarterly Review*, 18(1): 2–16.

King, Robert, and Sergio Rebelo. 1999. "Resuscitating Real Business Cycles." In *Handbook of Macroeconomics*. Vol. 1, , ed. J. B. Taylor and M. Woodford, Chapter 14, 927–1007. Elsevier.

Kose, M Ayhan, and Kei Mu Yi. 2006. "Can the standard international business cycle model explain the relation between trade and comovement?" *Journal of International Economics*, 68: 267–295.

Long, John, and Charles Plosser. 1983. "Real Business Cycles." *Journal of Political Economy*, 91(1): 39–69.

Ng, Eric C.Y. 2010. "Production fragmentation and business-cycle comovement." *Journal of International Economics*, 82(1): 1–14.

O'Mahoney, Mary, and Marcel Timmer. 2009. "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database." *The Economic Journal*, 119: F374–F403.

Shea, John. 2002. "Complementarities and Comovements." *Journal of Money, Credit, and Banking*, 34: 412–433.

Sims, Christopher. 1969. "Theoretical Basis for A Double Deflated Index of Real Value Added." *The Review of Economics and Statistics*, 51(4): 470–471.

Uhlig, Harald. 1999. "A toolkit for analysing nonlinear dynamic stochastic models easily." In *Computational Methods for the Study of Dynamic Economies.* , ed. Ramon Marimon and Andrew Scott, Chapter 3, 30–61. Oxford University Press.

Zimmerman, Christian. 1997. "International Real Business Cycles among Heterogeneous Countries." *European Economic Review*, 41: 319–355.

Table 1: Trade-Comovement Regressions for Real Gross Output: Data and Model

Panel A: Data

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.093***	0.090***	0.071***	0.071***
	(0.012)	(0.013)	(0.013)	(0.009)
N	171	171	171	342
R-sq	0.25	0.23	0.15	0.16

Panel B: Model with Correlated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.036**	0.063***	0.015	0.030***
	(0.016)	(0.013)	(0.014)	(0.009)
N	171	171	171	342
R-sq	0.04	0.12	0.01	0.03

Panel C: Model with Uncorrelated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.023***	0.023***	0.011***	0.018***
	(0.002)	(0.002)	(0.001)	(0.001)
N	171	171	171	342
R-sq	0.50	0.54	0.32	0.47

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log bilateral trade for aggregate regression: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$. Log bilateral trade for sector-level regressions: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.

Table 2: Trade-Comovement Regressions for Real Value Added: Data and Model

Panel A: Data

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.104***	0.098***	0.054***	0.055***
	(0.014)	(0.012)	(0.013)	(0.009)
N	231	231	231	462
R-sq	0.18	0.22	0.07	0.08

Panel B: Model with Correlated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.031**	0.052***	0.013	0.022***
	(0.013)	(0.012)	(0.012)	(0.008)
N	231	231	231	462
R-sq	0.02	0.08	0.01	0.02

Panel C: Model with Uncorrelated Shocks

	Aggregate	Goods	Services	Cross
Log Bilateral Trade	0.006***	0.003***	0.003***	0.005***
	(0.001)	(0.000)	(0.001)	(0.000)
N	231	231	231	462
R-sq	0.33	0.16	0.14	0.25

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log bilateral trade for aggregate regression: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$. Log bilateral trade for sector-level regressions: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$.

Table 3: Trade-Comovement Regressions with Complementarity: Results for Goods-Goods Sector Pair in Model with Uncorrelated Shocks

Panel A: Gross Output

	Benchmark		Simulations with Complementarity			
	Data	Model	w/in X	b/n V & X	w/in X + b/n V & X	w/in F
Log Bilateral Trade	0.090***	0.023***	0.092***	0.010***	0.114***	0.012***
	(0.012)	(0.002)	(0.011)	(0.001)	(0.011)	(0.002)
N	171	171	171	171	171	171
R-sq	0.23	0.54	0.22	0.45	0.29	0.37

Panel A: Real Value Added

	Benchmark		Simulations with Complementarity			
	Data	Model	w/in X	b/n V & X	w/in X & b/n V & X	w/in F
Log Bilateral Trade	0.098***	0.003***	-0.001	0.003***	-0.001	0.003***
	(0.012)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	231	231	231	231	231	231
R-sq	0.22	0.16	0.00	0.16	0.00	0.10

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log Bilateral Trade: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$. The column labeled "w/in X" presents results with $\sigma = 0$ and $\eta = -19$. The column labeled "b/n V & X" presents results with $\sigma = -19$ and $\eta = 0$. The column labeled "w/in X + b/n V & X" presents results with $\sigma = -1$ and $\eta = -19$. The column labeled w/inF presents results for $\sigma = 0$, $\eta = 0$, and $\rho = -19$. See the text for details.

Table 4: Disaggregate Trade-Comovement Regressions with "Vertical Linkages"

Panel A: Gross Output

	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.046***	0.007	0.014***	0.025	0.019	0.005***
	(0.013)	(0.015)	(0.002)	(0.017)	(0.017)	(0.001)
Trade x IO	0.059**	0.054**	0.006	0.036**	0.053***	0.007***
	(0.024)	(0.028)	(0.005)	(0.018)	(0.018)	(0.002)
N	684	684	684	684	684	684
R-sq	0.23	0.08	0.48	0.68	0.69	0.92

Panel B: Real Value Added

	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.021	0.006	0.005***	-0.031**	0.002	0.002***
	(0.014)	(0.013)	(0.001)	(0.013)	(0.016)	(0.001)
Trade x IO	0.088***	0.042*	-0.002*	0.075***	0.046**	-0.002**
	(0.024)	(0.024)	(0.001)	(0.016)	(0.019)	(0.001)
N	924	924	924	924	924	924
R-sq	0.13	0.04	0.21	0.70	0.57	0.68

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Log Bilateral Trade: $\text{Trade}_{ij}(s,s') \equiv \log\left(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j}\right)$. Trade x IO: $\left[\text{IO}(s,s') \times \text{Exports}_{ij}(s) + \text{IO}(s',s) \times \text{Exports}_{ji}(s')\right]$, where $\text{IO}(s,s')$ is a measure of input-output linkages between sectors s and s' and $\text{Exports}_{ij}(s) = \log\left(\frac{EX_{ij}(s)}{GDP_i+GDP_j}\right)$. See the text for details.

Figure 1: Correlations in Data vs. Model with Correlated Shocks

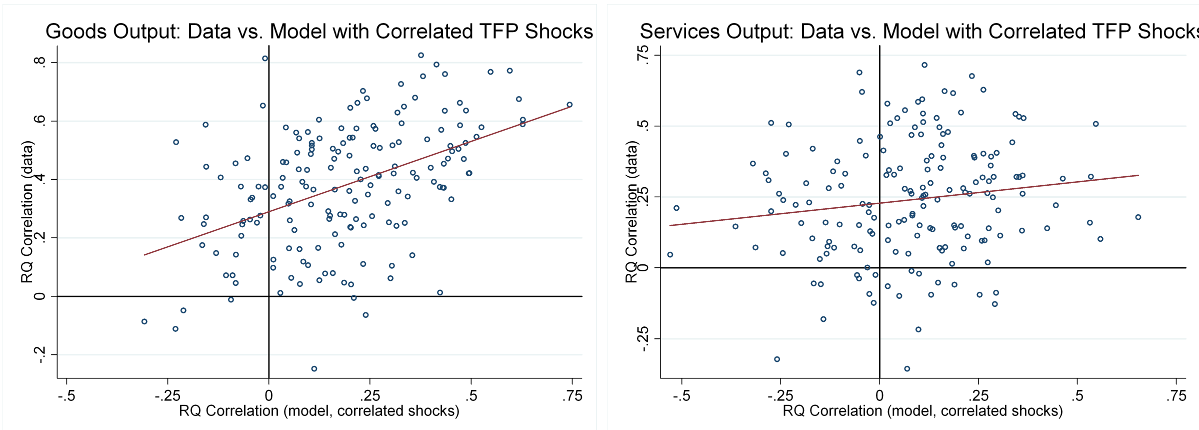


Figure 2: Goods Gross Output Correlations in Data vs. Model with Uncorrelated Shocks

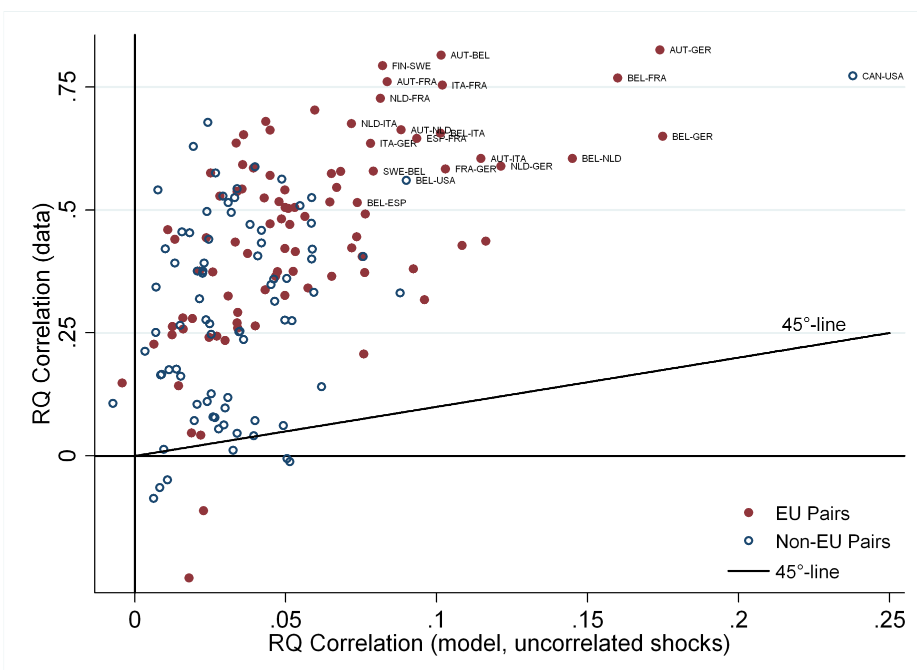


Figure 3: Goods Gross Output vs. Value Added Correlations in Data and Model

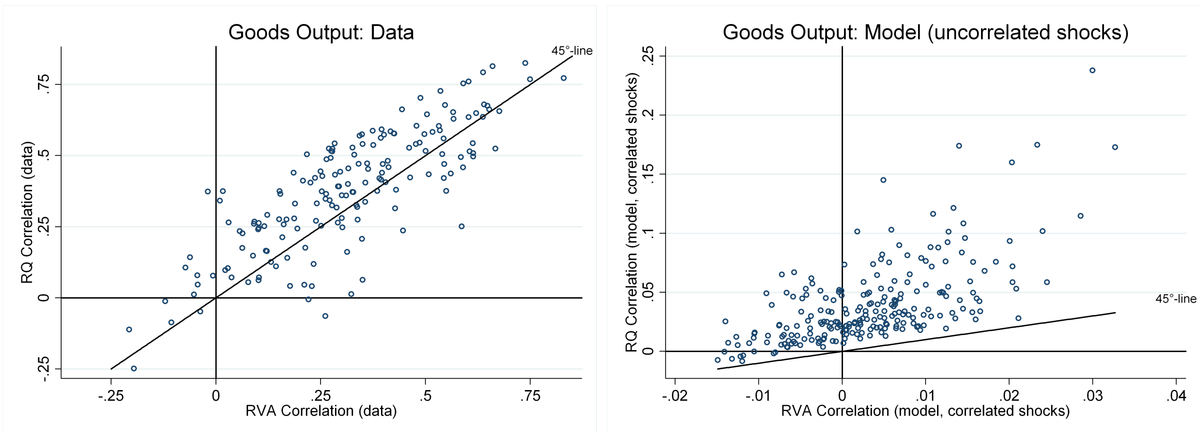
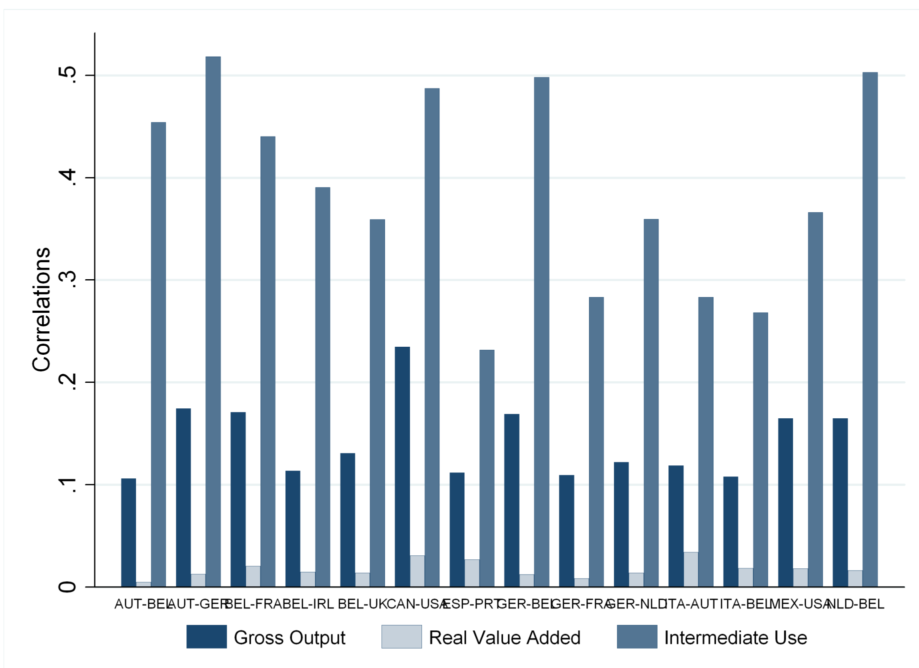


Figure 4: Correlations of Goods Output and Components in Model with Uncorrelated Shocks



Appendix A

This appendix lays out the algebra behind the derivation of Equation (18) for the special case discussed in Section 2.4.1. I also work out a three country case to provide intuition regarding how shocks are transmitted through input trade linkages.

A.1 Cobb-Douglas Model with Fixed Factor Inputs

As stated in the text, there are two major assumptions underlying that special case. First, I assume that each country is endowed with a fixed amount of the composite factor, denoted $\bar{V}_i$. Second, I assume that the production function takes a nested Cobb-Douglas form, and that preferences are also Cobb Douglas. Further, in this appendix, I focus on the solution here for a one sector version of the model to lighten the notation. With these assumptions, preferences and production functions can be written as:

$$U_i = \log\left(\prod_j F_{ji}^{\omega_{ji}^f}\right) \tag{A1}$$

$$Q_{it} = Z_{it}\bar{V}_i^{\theta_i}\left(\prod_j X_{ji}^{\omega_{ji}^x}\right)^{1-\theta_i}, \tag{A2}$$

where ω_{ji}^f and ω_{ji}^x are shares of goods from j in preference and technologies for country i.

Given this Cobb-Douglas structure, relative price changes in a decentralized, balanced trade equilibrium provide complete insurance.[40] Therefore, I can characterize the solution to the problem via the social planner's problem. The social planner maximizes $\sum_i \mu_i U_i$ by choosing $\{F_{ji}, X_{ji}\}_{\forall j,i}$, given the production function above and resource constraint $Q_{it} = \sum_j F_{ij} + X_{ij}$. The first order conditions are:

$$\mu_i \omega_{ji}^f = \lambda_j F_{ji} \tag{A3}$$

$$(1-\theta_i)\omega_{ji}^x \lambda_i Q_i = \lambda_j X_{ji}, \tag{A4}$$

where λ_j denotes the shadow price of output from country j.

These first order conditions along with the technology and resource constraints can be linearized around the static equilibrium to analyze the effects of a productivity shock. It is convenient to stack the equilibrium conditions to generate the following system:

$$\hat{\mathbb{F}} = -M_s\hat{\lambda} \tag{A5}$$

$$\hat{\mathbb{X}} = M_\lambda\hat{\lambda} + M_Q\hat{Q} \tag{A6}$$

$$\hat{Q} = S_X\hat{\mathbb{X}} + S_F\hat{\mathbb{F}} \tag{A7}$$

$$\hat{Q} = \hat{Z} + diag(1-\theta_i)W\hat{\mathbb{X}}, \tag{A8}$$

where $\hat{\mathbb{Z}} = [\hat{Z}_{11}, \hat{Z}_{12}, \ldots, \hat{Z}_{1N}, \hat{Z}_{21}, \hat{Z}_{22}, \ldots]'$ denotes an $(N^2 \times 1)$ vector of bilateral shipments

[40] The argument is the same as in Cole and Obstfeld (1991).

for $Z \in \{F, X\}$, and $\{\hat{\lambda}, \hat{Q}, \hat{Z}\}$ are $(N \times 1)$ vectors of the underlying variables for each country.

The matrices are defined as follows:

$$\begin{gathered} M_s \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}, \\ M_\lambda \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N} - I_{N\times N} \otimes \mathbf{1}_{N\times 1}, \\ M_Q \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}, \\ W \equiv [diag(\omega_1), diag(\omega_2), \ldots] \quad \text{with} \quad \omega_i = [\omega_{i1}, \cdots, \omega_{iN}] \\ S_Z \equiv \begin{pmatrix} s_1^z & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^z & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{with} \quad s_i^z = [s_{i1}^z, \cdots, s_{iN}^z] \quad \text{and} \quad s_{ij}^z = \frac{Z_{ij}}{Q_i}, \end{gathered}$$

where again $Z \in \{F, X\}$.

Then derivation of equation (18) proceeds in two steps. First, the production function in equation (A8) can be re-written as:

$$\hat{Q} = \hat{Z} + diag(1-\theta_i) W M_\lambda \hat{\lambda} + diag(1-\theta_i) W M_Q \hat{Q}, \tag{A9}$$

using (A6). Second, the market clearing and first order conditions can be used to solve for shadow prices as a function of quantities. Specifically, using (A6) and (A5), the market clearing constraint in (A7) can be rewritten as:

$$\hat{Q} = \left(S_X M_\lambda - S_F M_s\right) \hat{\lambda} + S_X M_Q \hat{Q}. \tag{A10}$$

This equation reduces due to the fact that $\left(I - S_x M_Q\right)^{-1} \left(S_X M_\lambda - S_F M_s\right) = -I$. So then, $\hat{Q} = -\hat{\lambda}$. This result that relative prices are proportional to relative quantities is a familiar result from Cobb-Douglas models. Substituting this into (A9) then yields:

$$\hat{Q} = \Omega' \hat{Q} + \hat{Z}, \tag{A11}$$

because $\Omega' = diag(1-\theta_i) W \left(M_Q - M_\lambda\right)$. Manipulation of this equation then completes the derivation of equation (18).

A.2 A Three Country Example

To fix ideas regarding what the total input requirements matrix $(I - \Omega')^{-1}$ captures, it is helpful to examine a three country version of the model above in which there are no domestic intermediates ($\omega_{ii}^x = 0$). Then the solution for the vector of output growth takes the form:

$$\hat{Q} = [I - \Omega']^{-1} \hat{Z} \quad \text{with} \quad \Omega = \begin{pmatrix} 0 & \omega_{12}^x & \omega_{13}^x \\ \omega_{21}^x & 0 & \omega_{23}^x \\ \omega_{31}^x & \omega_{32}^x & 0 \end{pmatrix}. \tag{A12}$$

This solution can be rewritten as:

$$\hat{Q} = M \begin{pmatrix} 1-\omega^x_{32}\omega^x_{23} & \omega^x_{21}+\omega^x_{23}\omega^x_{31} & \omega^x_{31}+\omega^x_{32}\omega^x_{21} \\ \omega^x_{12}+\omega^x_{13}\omega^x_{32} & 1-\omega^x_{31}\omega^x_{13} & \omega^x_{32}+\omega^x_{31}\omega^x_{12} \\ \omega^x_{13}+\omega^x_{12}\omega^x_{23} & \omega^x_{23}+\omega^x_{21}\omega^x_{13} & 1-\omega^x_{21}\omega^x_{12} \end{pmatrix} \hat{Z}, \tag{A13}$$

where $M = \frac{1}{\det[I-\Omega']^{-1}}$. There are two points to note regarding this solution.

First, for each country, the loadings on foreign country shocks are a function both of parameters associated with bilateral trade as well as trade with third countries. For example, the impact of a productivity innovation in country 2 on country 1's output is: $M(\omega^x_{21}+\omega^x_{23}\omega^x_{31})\hat{z}_2$. This effect is a function of both the intensity with which country 1 sources intermediates from country 2 (ω^x_{21}) and the compound term $\omega^x_{23}\omega^x_{32}$. This compound term picks up the indirect effect of country 2 productivity shocks operating via country 1's sourcing intermediates from country 3. Specifically, a shock in country 2 raises the supply of the country 2 intermediate good. This benefits country 1 directly because it uses this intermediate in production, but also benefits it indirectly because it uses intermediates from country 3 and country 3 intermediates are in turn produced using country 2 goods. Thus, the structure of the entire production chain matters, not just bilateral input sourcing patterns.[41]

Second, there is multiplier effect that controls the magnitude of the effect of shocks on each country. To see this clearly, I re-write output growth for country 1 as:

$$\hat{Q}_1 = M_1 \left[\hat{Z}_1 + \left(\frac{\omega^x_{21}+\omega^x_{23}\omega^x_{31}}{1-\omega^x_{32}\omega^x_{23}} \right) \hat{Z}_2 + \left(\frac{\omega^x_{31}+\omega^x_{32}\omega^x_{21}}{1-\omega^x_{32}\omega^x_{23}} \right) \hat{Z}_3 \right], \tag{A14}$$

where I define $M_1 = \frac{1-\omega^x_{32}\omega^x_{23}}{\det[I-\Omega']^{-1}}$ to be country 1's multiplier. M_1 summarizes how much country 1 output increases with shocks to its own productivity and is generally greater than one. Thus, the sensitivity of output to shocks for different countries can be decomposed into a country specific effect M_i and a vector of weights on different shocks that varies across countries.

Appendix B

The equilibrium conditions for the model in Section 2 are collected here. The first order conditions for the consumer and production problems are given by:

[41] In a concrete example, the U.S. benefits from productivity increases in China not only because it imports from China, but also because the U.S. sources intermediates from Japan and Japan uses Chinese goods as inputs in production.

$$1 = \beta E_t \left[\left(\frac{C_{it}}{C_{it+1}} \right) \left(\frac{r_{it+1}}{p^f_{it+1}} + (1-\delta) \right) \right] \tag{B1}$$

$$\chi L_{it}^{1/\epsilon} = \left(\frac{1}{C_{it}} \right) \frac{w_{it}}{p^f_{it}} \tag{B2}$$

$$p^f_{it}(s) F_{it}(s) = \gamma_i(s) p^f_{it} F_{it} \tag{B3}$$

$$F_{jit}(s) = \omega^f_{ji}(s) \left(\frac{p_{jt}(s)}{p^f_{it}(s)} \right)^{1/(\rho-1)} F_{it}(s) \tag{B4}$$

$$V_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} \theta_i(s) \left(\frac{p^v_{it}}{p_{it}(s)} \right)^{1/(\sigma-1)} Q_{it}(s) \tag{B5}$$

$$X_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} (1-\theta_i(s)) \left(\frac{p^x_{it}(s)}{p_{it}(s)} \right)^{1/(\sigma-1)} Q_{it}(s) \tag{B6}$$

$$r_{it} K_{it}(s) = \alpha p^v_{it}(s) V_{it}(s) \tag{B7}$$

$$w_{it} L_{it}(s) = (1-\alpha) p^v_{it}(s) V_{it}(s) \tag{B8}$$

$$X_{jit}(s', s) = \omega^x_{ji}(s', s) \left(\frac{p_{jt}(s')}{p^x_{it}(s)} \right)^{1/(\eta-1)} X_{it}(s) \tag{B9}$$

The market clearing conditions are given by:

$$Q_{it}(s) = \sum_{j=1}^{N} \sum_{s'=1}^{S} F_{ijt}(s) + X_{ijt}(s, s') \tag{B10}$$

$$F_{it} = C_{it} + K_{it+1} - (1-\delta) K_{it} \tag{B11}$$

$$p^f_{it} F_{it} = w_{it} L_{it} + r_{it} K_{it} + T_i \tag{B12}$$

$$K_{it} = \sum_{s=1}^{S} K_{it}(s) \tag{B13}$$

$$L_{it} = \sum_{s=1}^{S} L_{it}(s). \tag{B14}$$

And remaining production functions and composite aggregators are given by:

$$Q_{it}(s) = Z_{it}(s)\left(\theta_i(s)^{1-\sigma}V_{it}(s)^\sigma + (1-\theta_i(s))^{1-\sigma}X_{it}(s)^\sigma\right)^{1/\sigma} \tag{B15}$$

$$X_{it}(s) = \left(\sum_j \sum_{s'} \omega^x_{ji}(s',s)^{1-\eta} X_{jit}(s',s)^\eta\right)^{1/\eta} \tag{B16}$$

$$V_{it}(s) = K_{it}(s)^\alpha L_{it}(s)^{1-\alpha} \tag{B17}$$

$$F_{it}(s) = \left(\sum_j \omega^f_{ji}(s)^{1-\rho} F_{jit}(s)^\rho\right)^{1/\rho} \tag{B18}$$

$$F_{it} = \prod_s F_{it}(s)^{\gamma_i(s)}. \tag{B19}$$

These equations represent $7N + 10(S\times N) + 6N^2$ equations (minus one after choosing a numeraire) in the same number of unknowns. The unknowns include $\{C_{it}, F_{it}, K_{it}, L_{it}\}$ for each country, $\{Q_{it}(s), V_{it}(s), X_{it}(s), K_{it}(s), L_{it}(s), F_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s',s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p^f_{it}, \{p^f_{it}(s), p^v_{it}(s), p^x_{it}(s), p_{it}(s)\}_s\}_i$.[42] The $\{T_i\}$ terms in the budget constraint represent time-invariant transfers across countries, which allow me to fit the model to a steady state with unbalanced trade. As discussed in the calibration section, I linearize around this unbalanced trade equilibrium assuming that trade imbalances are constant.

These equilibrium conditions can be linearized around the steady state as follows. The linearized and stacked first order conditions are given by:

$$0 = E_t\left[\hat{C}_t - \hat{C}_{t+1} + (1-\beta(1-\delta))\left(\hat{r}_{t+1} - \hat{p}^f_t\right)\right] \tag{B20}$$

$$0 = \frac{1}{\epsilon}\hat{L}_t + \hat{C}_t - \hat{w}_t + \hat{p}^f_t \tag{B21}$$

$$0 = \hat{p}^f_t + \hat{F}_t - \hat{p}^f_t(s) - \hat{F}_t(s) \tag{B22}$$

$$0 = \hat{\mathbb{F}}_t(s) + \frac{1}{1-\rho}M_i\hat{p}_t(s) - \frac{1}{1-\rho}M_j\hat{p}^f_t(s) - M_j\hat{F}_t(s) \tag{B23}$$

$$0 = \hat{V}_t(s) + \frac{\sigma}{1-\sigma}\hat{Z}_t(s) + \frac{1}{1-\sigma}\hat{p}^v_t(s) - \frac{1}{1-\sigma}\hat{p}_t(s) - \hat{Q}_t(s) \tag{B24}$$

$$0 = \hat{X}_t(s) + \frac{\sigma}{1-\sigma}\hat{Z}_t(s) + \frac{1}{1-\sigma}\hat{p}^x_t(s) - \frac{1}{1-\sigma}\hat{p}_t(s) - \hat{Q}_t(s) \tag{B25}$$

$$0 = \hat{\mathbb{X}}_t(s',s) + \frac{1}{1-\eta}M_i\hat{p}_t(s) - \frac{1}{1-\eta}M_j\hat{p}_t(s') - M_j\hat{X}_t(s) \tag{B26}$$

$$0 = \hat{p}^v_t(s) + \hat{V}_t(s) - \hat{r}_t - \hat{K}_t(s) \tag{B27}$$

$$0 = \hat{p}^v_t(s) + \hat{V}_t(s) - \hat{w}_t - \hat{L}_t(s), \tag{B28}$$

where $M_i \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}$ and $M_j \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}$. Here $\{r_t, w_t, p^f_t, p^f_t(s), p^x_t(s), p_t(s)\}$ and $\{C_t, L_t, F_t, Q_t(s), X_t(s), K_t(s), L_t(s), F_t(s)\}$ are vectors of prices and quantities, with element

[42] The equilibrium here can be easily reduced to the equilibrium defined in the main text via substitution.

i equal to the relevant variable for country i. The vector $\mathbb{F}_t(s)$ is a N^2 dimensional vector that records final goods shipments for sector s, while $\mathbb{X}_t(s', s)$ is a N^2 dimensional vector that records intermediates goods flows from sector s' to sector s:

$$\hat{\mathbb{F}}_t(s) = [\hat{F}_{11t}(s), \hat{F}_{12t}(s), \ldots, \hat{F}_{1Nt}(s), \hat{F}_{21t}(s), \hat{F}_{22t}(s), \ldots]'$$
$$\hat{\mathbb{X}}_t(s', s) = [\hat{X}_{11t}(s', s), \hat{X}_{12t}(s', s), \ldots, \hat{X}_{1Nt}(s', s), \hat{X}_{21t}(s', s), \hat{X}_{22t}(s', s), \ldots]'$$

The stacked and linearized market clearing conditions are given by:

$$0 = \hat{Q}_t(s) - S_F(s)\hat{\mathbb{F}}_t(s) - \sum_{s'} S_X(s, s')\hat{\mathbb{X}}(s, s') \tag{B29}$$

$$0 = \hat{F}_t - diag\left(\frac{\bar{C}_i}{\bar{F}_i}\right)\hat{C}_t - diag\left(\frac{\bar{K}_i}{\bar{F}_i}\right)\hat{K}_{t+1} + diag\left(\frac{\bar{K}_i(1-\delta)}{\bar{F}_i}\right)\hat{K}_t \tag{B30}$$

$$0 = \hat{p}_t^f + \hat{F}_t - diag\left(\frac{\bar{w}_i \bar{L}_i}{\bar{p}_i^f \bar{F}_i}\right)(\hat{w}_t + \hat{L}_t) - diag\left(\frac{\bar{r}_i \bar{K}_i}{\bar{p}_i^f \bar{F}_i}\right)(\hat{r}_t + \hat{K}_t) \tag{B31}$$

$$0 = \hat{K}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{K}_i(s)}{\bar{K}_i}\right)\hat{K}_t(s) \tag{B32}$$

$$0 = \hat{L}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{L}_i(s)}{\bar{L}_i}\right)\hat{L}_t(s). \tag{B33}$$

The bar notation denotes steady state values. The matrices $S_F(s)$ and $S_X(s, s')$ collect the share of output allocated to final and intermediate use in destinations as follows:

$$S_F(s) \equiv \begin{pmatrix} s_1^f(s) & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^f(s) & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X(s, s') \equiv \begin{pmatrix} s_1^x(s, s') & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x(s, s') & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix}$$
$$\text{with} \quad s_i^f(s) = [s_{i1}^f(s), \cdots, s_{iN}^f(s)], \quad s_{ij}^f(s) = \frac{F_{ij}(s)}{Q_i(s)},$$
$$s_i^x(s, s') = [s_{i1}^x(s, s'), \cdots, s_{iN}^x(s, s')], \quad s_{ij}^x(s, s') = \frac{X_{ij}(s, s')}{Q_i(s)}.$$

Finally, the stacked and linearized production functions and aggregators are given by:

$$0 = \hat{Q}_t(s) - \hat{Z}_t(s) - diag\left(\frac{p_i^v V_i(s)}{p_i(s)Q_i(s)}\right)\hat{V}_t(s) - diag\left(\frac{p_i^x X_i(s)}{p_i(s)Q_i(s)}\right)\hat{X}_t(s) \tag{B34}$$

$$0 = \hat{V}_t(s) - \alpha\hat{K}_t(s) - (1-\alpha)\hat{L}_t(s) \tag{B35}$$

$$0 = \hat{X}_t(s) - \sum_{s'} W_X(s', s)\hat{\mathbb{X}}_t(s', s) \tag{B36}$$

$$0 = \hat{F}_t(s) - W_F(s)\hat{\mathbb{F}}_t(s) \tag{B37}$$

$$0 = \hat{F}_t - \sum_{s} diag(\gamma_i(s))\hat{F}_t(s). \tag{B38}$$

The matrices $W_F(s)$ and $W_X(s', s)$ are sourcing shares for final and intermediate goods:

$$W_F(s) \equiv [diag(w_1^f(s)), diag(w_2^f(s)), \ldots]$$
$$\text{with} \quad w_i^f(s) = [w_{i1}^f(s), \cdots, w_{iN}^f(s)], \quad w_{ij}^f(s) \equiv \frac{p_i(s)F_{ij}(s)}{p_j^f(s)F_j(s)},$$
$$\text{and} \quad W_X(s', s) \equiv [diag(w_1^x(s', s)), diag(w_2^x(s', s)), \ldots]$$
$$\text{and} \quad w_i^x(s', s) = [w_{i1}^x(s', s)), \cdots, w_{iN}^x(s', s)], \quad w_{ij}^x(s', s) \equiv \frac{p_i(s')X_{ij}(s', s)}{p_j^x(s)X_j(s)}.$$

To compute the dynamics, one needs to modify these conditions to reflect the choice of numeraire. Further, to reduce the computational burden, I manually substitute out for final and intermediate goods shipments $\{\{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}\}$, thereby reducing the size of the system by $6N^2$ (3174 with 23 countries) elements. Obviously other manual substitutions further reduce the dimensionality, but eliminating unknowns that increase in the square of the number of countries is most important. I use Harald Uhlig's "Toolkit for Analyzing Nonlinear Dynamic Stochastic Models" in MATLAB to compute solutions to this system.[43].

Appendix C

C.1 Final and Intermediate Goods Data and Calibration

The GTAP 7.1 Database is assembled by the Global Trade Analysis Project at Purdue University based on three main sources: (1) World Bank and IMF macroeconomic and Balance of Payments statistics; (2) United Nations Commodity Trade Statistics (Comtrade) Database; and (3) input-output tables from national statistical sources. To reconcile data from these different sources, GTAP researchers adjust the input-output tables to be consistent with international data sources. The data set includes internally consistent bilateral trade statistics combined with domestic and import input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004.

From the GTAP database, I extract disaggregate input use tables for domestic input purchases and imported inputs. I then use bilateral trade data to split imported input use across bilateral partners, assuming that input purchases from each source are proportional to bilateral trade shares within a given sector. I split final goods imports across source countries using trade shares in a similar way. This yields bilateral final and intermediate goods shipments for 57 sectors. I then aggregate data on sectoral production, trade, final and intermediate shipments across sectors to form two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services."

This data is the main input to calibrating $\{\omega_{ji}^x(s', s), \omega_{ji}^f(s), \theta_i(s)\}$. On the production

[43]See Uhlig (1999) or http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

side, the relevant first order conditions are Equations (5) and (9):

$$\frac{p_i^v(s)V_i(s)}{p_i(s)Q_i(s)} = Z_i(s)^{\sigma/(\sigma-1)}\theta_i(s)\left(\frac{p_i^v(s)}{p_i(s)}\right)^{\sigma/(\sigma-1)} \tag{C1}$$

$$\frac{p_i(s')X_{ji}(s',s)}{p_i^x(s)X_i(s)} = \omega_{ji}^x(s',s)\left(\frac{p_j(s')}{p_i^x(s)}\right)^{\eta/(\eta-1)}. \tag{C2}$$

Then choosing quantity units so that the prices and productivity are equal to one in the steady state, then the ratio of value added to gross output pins down $\theta_i(s)$ and data on the share of inputs from a particular source country and sector in total input use in the destination pins down $\omega_{ji}^x(s',s)$.

To calibrate $\{\omega_{ji}^f(s), \gamma_i(s)\}$, note that the final goods producer's first order conditions in Equations (11)-(12) can be rewritten in share form as:

$$\frac{p_i(s)F_{ij}(s)}{p_j^f F_j} = \gamma_j(s)\omega_{ij}^f(s)\left(\frac{p_i(s)}{p_j^f(s)}\right)^{-\rho/(1-\rho)}, \tag{C3}$$

where $\frac{p_i(s)F_{ij}(s)}{p_j^f F_j}$ is the share of final goods of sector s sourced from country i in total final goods expenditure in j. The share of final expenditure on goods of sector s – $\gamma_j(s)$ – can be computed directly in the data. Then, choosing quantity units so that the price of gross output and the final goods are equal to one in the steady state, $\{\omega_{ij}^f\}$ can computed by combining these expenditure shares.

C.2 Productivity Adjustment

To understand the productivity adjustment, recall the discussion in Section 2.4.1 about distinguishing gross output from real value added. TFP measured using gross output is $\widehat{TFP}_{it}^Q(s) = \hat{Z}_{it}(s)$, while TFP measured using real value added is $\widehat{TFP}_{it}^V(s) = \frac{1}{s_i^v(s)}\hat{Z}_{it}(s)$, as in Equation (17). The two TFP measures are related by $\widehat{TFP}_{it}^Q(s) = s_i^v(s)\widehat{TFP}_{it}^V(s)$, so shocks to productivity measured using value added will be larger than the corresponding shocks measured using gross output. This explains the need to adjust Σ and means that the correct covariance matrix for simulation is $\widetilde{\Sigma} = \frac{1}{T}\sum_t \hat{\tilde{\epsilon}}_t \hat{\tilde{\epsilon}}_t'$, where $\hat{\tilde{\epsilon}}_{it}(s) \equiv (1-\theta_i(s))\hat{\epsilon}_{it}(s)$ as in the main text. The persistence parameter $\lambda_i(s)$ obtained in estimation of (19) can be directly used in simulations, as it does not depend on which definition of productivity is used in the estimation.