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OF EQUAL SACRIFICE AND OPTIMAL TAXATION

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Why do we Redistribute so Much but Tag so Little? The principle of equal sacrifice and optimal taxation

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### **ABSTRACT**

Tagging is a free lunch in conventional optimal tax theory because it eases the classic tradeoff between efficiency and equality. But tagging is used in only limited ways in tax policy. I propose one explanation: conventional optimal tax theory has yet to capture the diversity of normative principles with which society evaluates taxes. I generalize the conventional model to incorporate multiple normative frameworks. I then show that if the principle of equal sacrifice--a classic, comprehensive criterion of fair taxation proposed by John Stuart Mill and associated with the Libertarian normative framework--is given some weight in the social objective function, tagging generates costs that must be weighed against the benefits it generates through conventional channels. Only tags that are sufficiently predictive of ability, such as disability status, will be used. Calibrated simulations using micro data from the United States show that optimal policy may simultaneously include substantial redistribution across income-earning abilities, as in the standard model, and reject three prominently-proposed tags--gender, race, and height--as in actual policy. This explanation for limited tagging also implies that optimal marginal tax rates at high incomes are lower than in standard analysis and closer to those observed in policy.

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# Introduction

One of the most puzzling features of existing tax policies from the perspective of optimal tax theory is the simultaneous existence of substantial redistribution through distortionary income taxes and the limited role of tagging—the dependence of taxes on personal characteristics in addition to income. Tagging is strongly recommended by conventional optimal tax theory because it eases the classic tradeoff between redistribution and economic efficiency. Given the ferocity of current debates over this tradeoff, we would expect extensive use of tagging. Indeed, the stark conflict between conventional theory and the limited role of tagging in reality raises the question of whether the theory is accurately capturing the heart of the optimal tax problem.

This paper proposes a resolution to these puzzling limitations on tagging that preserves the core of the Mirrleesian approach to optimal tax, an approach many (including this author) consider fundamentally convincing as a basis for policy evaluation. Briefly, the idea is that the tradeoff between equality and efficiency is not the only one that matters for tax policy. Instead, because most individuals evaluate policy using a combination of multiple normative frameworks, society also faces a tradeoff between diverse criteria of economic justice. Utilitarianism, the framework at the heart of the Mirrleesian approach, is only one component of a multifaceted normative perspective that ought to be incorporated into optimal tax models if we want to maximize their relevance to actual policymaking.<sup>1</sup>

In particular, if a normative framework that prioritizes the principle of equal sacrifice carries some weight in society's judgments, tagging is no longer a free lunch. A tax system imposes equal sacrifice if the payment of taxes reduces each taxpayer's well-being by the same amount. Tagging distorts the allocation of resources across individuals with the same income-earning ability but different values of the tagged variable. This distortion is immaterial in conventional theory, which adopts a Utilitarian framework, but it violates equal sacrifice. As a consequence, policymaking that gives weight to equal sacrifice will reject a tag if the costs it imposes according to that principle outweigh the gains it makes possible through raising aggregate utility.

The principle of equal sacrifice is an appealing addition to the standard model of optimal taxation for at least three reasons. First, it has a long history as the normatively preferred, even dominant, criterion for fair taxation. John Stuart Mill (1871) was its most famous proponent and is worth quoting at length.

"For what reason ought equality to be the rule in matters of taxation? For the reason, that it ought to be so in all affairs of government...Equality of taxation, therefore, as a maxim of politics, means equality of sacrifice. It means apportioning the contribution of each person towards the expenses of government so that he shall feel neither more nor less inconvenience from his share of the payment than every other person experiences from his."

Mill's vision of equal sacrifice was endorsed by other influential thinkers, including Alfred Marshall and Henry Sidgwick, the latter of whom claimed it was the "obviously equitable principle—assuming that the existing distribution of wealth is accepted as just or not unjust." More recently, the late 1980s and 1990s saw a temporary resurgence of interest in equal sacrifice as a basis for policy, especially through the work of H. Peyton Young (1987, 1988, 1990, 1994) but also including Yaari (1988), Moyes (1989), Berliant and Gouveia (1993), Ok (1995), Mitra and Ok (1996), and D'Antoni (1999). That literature established conditions on

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<sup>1</sup>Examining the degree of correspondence between policy and theory is a central task of Mirrleesian optimal tax research (e.g., Saez 2001). To some, that task uncomfortably blurs the line between normative and positive analysis. In my opinion, the goal of optimal tax research is to uncover the model of policy that best matches society's true normative reasoning. Given that goal, gaps between a model and policy present opportunities to find reasonable modifications to the theory or to reject a policy as suboptimal.

the progressivity of taxes designed in accordance with equal sacrifice, and it argued for the centrality of that principle from both normative and positive perspectives.<sup>2</sup>

Second, whether or not such normative arguments for equal sacrifice are persuasive, that principle's connection to the broader, influential philosophical framework of Libertarianism implies that it plays a substantial role in real-world policy debates. As Liam Murphy and Thomas Nagel (2002) have argued: "If (and only if) [libertarianism] is the theory of distributive justice we accept, the principle of equal sacrifice does make sense." Sidgwick's statement above, with its caveat that speaks to the core of Libertarianism, suggests the same link.<sup>3</sup> Public opinion surveys estimate the proportion of individuals with traditional Libertarian views to be 10 to 20 percent in the United States (Boaz and Kirby 2007). Cappelen et al. (2011) conduct experiments in which participants' choices imply a preference among competing "fairness ideals," and in their preferred specification 18.7 percent of participants are classified as "libertarians."<sup>4</sup> The connection between equal sacrifice and Libertarianism therefore implies a sizeable portion of society may be described as using the equal sacrifice principle as its main criterion for optimal tax policy.

Third, equal sacrifice is likely to be one component of the multifaceted normative reasoning used by most individuals. For more than two decades, research<sup>5</sup> has confirmed the findings of Frohlich, Oppenheimer, and Eavey (1987) that: "...subjects preferred a compromise. This implies that *individuals treat choice between principles as involving marginal decisions. Principles are much like economic goods inasmuch as individuals are willing to trade off between them* [italics in the original]." Research has shown that normative ambivalence is widespread with regard to social welfare policies and that even those predisposed toward redistribution feel a normative pull in the opposite, Libertarian, direction. Feldman and Zaller (1992) conclude: "Most people are internally conflicted about exactly what kind of welfare system they want...Ambivalence with respect to social welfare policy is more pronounced among welfare liberals...They end up acknowledging the values of economic individualism even as they try to justify their liberal preferences."<sup>6</sup> Rhetoric over taxes also suggests that equal sacrifice is viewed by politicians, even those supporting progressivity, as a principle relevant to voters: e.g., U.S. President Barack Obama (2012) argued for a tax on large capital incomes by asking: "...should we ask middle-class Americans to pay even more at a time when their budgets are already stressed to the breaking point? Or should we ask some of the wealthiest to pay their fair share?"

Strikingly, Mill himself provides an example of exactly this form of mixed normative reasoning, writing approvingly of both equal sacrifice and minimal total sacrifice (which is similar to the Utilitarian criterion):

As a government ought to make no distinction of persons or classes in the strength of their claims on it, whatever sacrifices it requires from them should be made to bear as nearly as possible with the same pressure upon all, which, it must be observed, is the mode by which least sacrifice is occasioned on the whole.

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<sup>2</sup>Lambert and Naughton (2009) is a recent contribution that reviews much of this literature.

<sup>3</sup>Under standard versions of Libertarianism, taxes are justified to pay for public goods only. Libertarian writers are frustratingly imprecise about how to allocate required taxes and how to evaluate the harm done by deviating from that optimal allocation. Friedman (1962) does endorse a flat-rate income tax, but on intuitive rather than rigorous grounds. One natural but rigorous benchmark for dividing the resulting tax burden is to equalize the utility costs of taxation across individuals.

<sup>4</sup>Konow (2003) reports results consistent with these magnitudes.

<sup>5</sup>See, for example: Deutsch (1985); Feldman and Zaller (1992); Free and Cantril (1968); Frohlich and Oppenheimer (1992); Gainous and Martinez (2005); Hochschild (1981); Konow (2001); Miller (1976); and Mitchell, Tetlock, Mellers, and Ordonez (1993). Scott, Matland, Michelbach, and Bornstein (2001) write: "Experimental research reveals that distributive justice judgments usually involve several distinct allocation principles." Upon conducting an invaluable review of empirical findings, Konow (2003) argues that "each category [of justice theories] captures an element that is important to crafting a positive theory of justice but that no single family or theory within a family suffices to this end."

<sup>6</sup>Though the connection to problems of taxation is imperfect, Frohlich, Oppenheimer, and Kurki (2004) show that "just deserts" or "entitlements" exert an influence on allocations for most dictators in allocation games with production.

Mill is incorrect, as many others have noted, in the assertion that equal sacrifice implies minimized total sacrifice. But this mistake reveals that, for Mill, both equal and minimized total sacrifice were principles he believed appealing and likely to be accepted by his readers. Mill's split normative intuition is more the rule than the exception, and I explore the implications of it in this paper.

The payoff from recognizing and incorporating this form of normative diversity<sup>7</sup> in the optimal tax model goes well beyond simply providing a reason to avoid tagging. A role for equal sacrifice can improve the match between the recommendations of optimal tax theory and the reality of tax policy in several ways.

A role for equal sacrifice can explain not only why tagging is limited in practice but also the circumstances under which tagging is likely to be optimal. Tags that are strongly correlated with underlying income-earning ability, such as blindness, disability status, and old age, will generate smaller losses according to the principle of equal sacrifice and therefore be more acceptable to a society that balances Utilitarianism and the equal sacrifice principle when evaluating policy.

Moreover, incorporating equal sacrifice can explain why tagging may be rejected while substantial redistribution across income-earning abilities is sustained. The equal sacrifice principle places no value on redistribution, but it is consistent with progressive taxation to pay for public goods if the utility sacrifice caused by any given rate of taxation is smaller for a higher-income individual than a lower-income one.<sup>8</sup> Thus, by adding a role for the principle of equal sacrifice, we can in principle reject tagging without jettisoning modern tax theory, a possibility raised in Mankiw and Weinzierl (2010).

In this paper, I show that the potential for the equal sacrifice principle to explain these features of tax policy is not merely theoretical. Using microeconomic data on earnings and personal characteristics, I find that calibrations of the optimal policy model exist in which society rejects the use of three prominently-proposed but as yet unused tags—height, gender, and race—but accepts substantial redistribution through progressive income taxes, as in the current U.S. tax code.

Finally, if correct, this paper's explanation for limited tagging has additional implications; in fact it may contribute to resolving a second high-profile puzzle in optimal tax research. Diamond and Saez (2011) show that the top marginal tax rate for the United States, according to standard theory, is substantially (30 percentage points) higher than in current U.S. policy. Using a calibration of the model with a concern for equal sacrifice that rejects tagging on height, gender, and race but retains redistribution, I show that the optimal top marginal tax rate falls by seven percentage points relative to a conventional Utilitarian model. While the magnitude of this effect would vary across calibrations, the lesson is clear: if we care about equal sacrifice and avoid most tags because of it, we are likely to moderate our use of high marginal income tax rates at the top of the income distribution.

This paper proceeds as follows. Section 1 briefly reviews prior results and discussions of tagging in optimal tax research, including a discussion of the relationship between this paper and the concept of horizontal equity. Section 2 generalizes the conventional optimal tax model to include multiple normative frameworks. Section 3 applies this generalized model in the case of two normative frameworks, Utilitarianism and Libertarianism, whose priorities I assume to be maximal aggregate utility and equal sacrifice respectively. Section 4 derives conditions on optimal tagging and marginal distortions to labor supply in that model, formally establishing

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<sup>7</sup>Using the approach described by this paper to include other normative frameworks, such as the Rawlsian priority on the least fortunate, would likely lead to further insights on existing policies.

<sup>8</sup>Also, to the extent that Utilitarian reasoning causes society to want to support low-ability individuals with redistribution, the optimal sharing of that burden according to the principle of equal sacrifice, much less to Utilitarians, is likely to be progressive.

that a role for the principle of equal sacrifice reduces optimal tagging. Section 5 performs calibrated numerical simulations of optimal policy, and section 6 concludes.

## 1 Prior work on Tagging

Tagging has an illustrious theoretical pedigree. James Mirrlees (1971) noted the potential of tagging in only the fifth sentence of his Nobel Prize-winning analysis of optimal taxation. George Akerlof (1978), also a recipient of the Nobel Prize, worked out the basic theory of tagging in a seminal paper just seven years later. Forty years into the modern optimal tax literature, recent analyses have shown the substantial potential gains from tagging according to three specific personal characteristics: height, gender, and race (see Mankiw and Weinzierl 2010; Alesina, Ichino, and Karabarbounis 2011; and Blumkin, Margalioth, and Sadka 2009).

In the modern theory of optimal taxation, tagging is a free lunch. That theory starts with the assumption that individuals differ in their unobservable abilities to earn income but are equally able to enjoy consumption. If social welfare is a weakly concave function of all individual utilities, income ought to be redistributed from those with high ability to those with low ability. But, there is a tradeoff. Taxing endogenous income rather than exogenous ability discourages effort, reducing economic activity overall. Tags carry information about ability but are hard to modify, so taxing them allows for redistributive gains without efficiency losses.

According to this theory, a wide variety of candidate tags exist. Any observable and largely inelastic characteristic across which the distribution of abilities differs ought to affect tax schedules. For example, groups with higher mean ability ought to be taxed to support other groups, while groups with a higher variance of ability ought to face a more progressive within-group tax policy. As Mirrlees wrote: "One might obtain information about a man's income-earning potential from his apparent I.Q., the number of his degrees, his address, age or colour..."<sup>9</sup> There are many other potential tags—height, gender, facial symmetry, place in birth order, native language, parental traits, macroeconomic conditions at age 18, and so on—all of which relate systematically to income-earning ability and are largely exogenous to the individual. Genetic information may someday provide particularly powerful tags.<sup>10</sup>

In comparison, the role for tagging in modern tax policy is highly constrained. Some sizeable tagging does occur, but only for tags that are virtually guaranteed to indicate that a taxpayer has low income-earning ability. For example, disability benefits are common among developed countries, as are programs aimed at alleviating poverty among the elderly. Indeed, nearly two-thirds of U.S. federal entitlement spending goes to programs generally limited to the elderly and disabled (Viard, 2001). These groups are the prototypical examples of those with systematically low income-earning ability.<sup>11</sup> The other large example of tagging is payments to families with young children, where the *per capita* ability to earn income is mechanically low when compared to childless households. Other, isolated programs such as benefits for the blind follow a similar pattern, so that existing tagging bears little resemblance to the broad and nuanced application recommended by modern optimal tax theory.

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<sup>9</sup>Despite this quotation, age should not be considered a tag. Unlike these other characteristics, age is shared by all individuals (abstracting from mortality variation), so that age-dependent taxes do not achieve support for a disadvantaged group by taxing another. In particular, age-dependent taxes do not violate equal sacrifice once the full lifecycle of each taxpayer is considered. See Weinzierl (2011) for a study of this and other aspects of age-dependent taxes.

<sup>10</sup>Note that privacy concerns may be relevant for some potential tags, such as genetic information. A concern for privacy is one example of a value that could be incorporated into the optimal tax model using the approach of this paper, provided that it can be translated convincingly into a preference over final allocations.

<sup>11</sup>The economic prospects for people over the age of 65 have improved in the decades since the programs designed to support the elderly were created. The current debate over raising the retirement age in these programs may reflect, in part, skepticism that age 65 is still a reliable indicator of lower income-earning ability.

The main reasons why tagging may be unappealing in practice have been discussed from the beginning. Akerlof (1978) himself writes: "the disadvantages of tagging... are the perverse incentives to people to be identified as needy (to be tagged), the inequity of such a system, and its cost of administration."

Akerlof's first and third disadvantages of tagging are straightforward but of limited effect. Tags are undoubtedly less appealing if they are easily mimicked—as they would then distort behavior while failing to redistribute—or costly to monitor and administer. Most of the candidate tags mentioned above and considered in modern tax theory, however, are inelastic and cheap to enforce. Even a statistic such as "apparent I.Q.", which may seem both elastic and costly to monitor, has such large implications outside the tax system for individuals that we might argue it would be largely immune to these concerns.<sup>12</sup> Certainly a characteristic such as gender is highly inelastic and could be cheaply incorporated into the tax system.

Akerlof's remaining disadvantage of tagging is that it could violate horizontal equity: the notoriously difficult-to-define principle that "equals ought to be treated equally". This is a prominent concern: Boadway and Pestieau (2006) write: "Of course, such a system may be resisted because, if the tagging characteristic has no direct utility consequences, a differentiated tax system violates the principle of horizontal equity". Similar statements are made by, e.g., Atkinson and Stiglitz (1980) and Auerbach and Hassett (1999).

On its own, the principle of horizontal equity offers an unsatisfying explanation for the limits to tagging. First, it is a tautological solution: it literally assumes that tagging is costly. It is also an unreliable solution, as the choice of which characteristics are to be treated as "horizontal" is, at heart, arbitrary. For example, if "equals" are defined by income, they cannot also be defined by income-earning ability if preferences over consumption and leisure are heterogeneous. In that case, the principle of horizontal equity gives no guidance as to how to resolve this contradiction.

The core of the problem for horizontal equity is that it is not based on a comprehensive criterion of optimal taxation; as Kaplow (2008) writes, it "lacks affirmative justification." For example, horizontal equity offers no guidance on how taxes required for public goods ought to be assigned.

In contrast, the principle of equal sacrifice is just such a comprehensive criterion that, as one of its outcomes, discourages tagging. The distaste for tagging under equal sacrifice comes from such personal characteristics being irrelevant to the sacrifice an individual bears to pay a given tax. In other words, rather than a requirement of horizontal equity acting an ad hoc explanation for limited tagging, in this paper a concern for horizontal equity arises endogenously out of the classic principle of equal sacrifice. One interpretation of this paper, therefore, is as providing a normatively rigorous foundation for the concern over horizontal equity long intuited as the obstacle to greater tagging and, then, examining the broader consequences of that foundation for income taxation.

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<sup>12</sup>Mirrlees (1971) makes the same point on I.Q. See page 208.

## 2 Generalizing the Optimal Tax Model for Normative Diversity

The first step in explaining the limited role of tagging is to generalize the conventional optimal tax model so that the criterion of equal sacrifice can play a role. That step requires, in particular, that the normative perspective of the model be broadened.

Starting with Mirrlees (1971), conventional optimal tax analysis has assumed a straightforward normative framework: generalized Utilitarianism. According to Utilitarianism, a centralized planner ought to maximize the sum of the utilities of a population of individuals, in some cases applying a concave transformation to the utilities before summing. Combined with the assumptions that individuals differ only in their innate ability to earn income and that preferences over consumption and leisure are common, this Utilitarian framework powerfully recommends income redistribution.<sup>13</sup>

Economists, and especially optimal tax theorists, have been largely united around this Utilitarian perspective.<sup>14</sup> The canonical justification for it is due to John Harsanyi (1953, 1955), who argued that "value judgments concerning social welfare and the cardinal utility maximized in choices involving risk may be regarded as being fundamentally based upon the same principle." In other words, expected utility maximization, when the expectation is taken over all individuals in society, is pure-sum Utilitarianism.

As even a casual observer of policy debates can attest, discussions of taxes by members of the public, policymakers, and scholars do not reflect such a pure normative perspective. While some individuals find the Utilitarian criterion appealing, others are drawn to sharply opposing frameworks. For example, Milton Friedman wrote in 1962: "I find it hard, as a liberal, to see any justification for graduated taxation solely to redistribute income. This seems to me a clear case of using coercion to take from some in order to give to others and thus to conflict head-on with individual freedom." The statements by Mill and Sidgwick above similarly show that the current scholarly convergence on a Utilitarian perspective is at odds with important and long-lived strains of thinking on the topic.

More important, as the discussion in the Introduction and Mill's ambivalence over equal sacrifice and Utilitarianism suggests, and as decades of research in psychology, political science, and economics has shown, most individuals are not normative purists. Those who are fully convinced by single frameworks are best seen as outliers occupying the extremes of a continuum, the interior of which is populated by those for whom multiple normative frameworks have appeal. As Feldman and Zaller (1992) state: "Our results offer strong support to studies, especially that of Hochschild, that have identified ambivalence as a fundamental feature of political belief systems...Even those who take consistently pro- or consistently antiwelfare positions often cite reasons for the opposite point of view." The Hochschild (1981) study to which they refer consisted of long-term, in-depth interviews of a group of individuals across a wide range of socioeconomic status. It concluded: "Some people...hold beliefs that are predominately clear and sharp—but even they express some ambivalence. Others...hold beliefs that are predominately ambivalent and blurred—but even they express

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<sup>13</sup>See Lockwood and Weinzierl (2012) for a treatment of optimal taxation with preference heterogeneity.

<sup>14</sup>Some important exceptions to this statement exist. Stiglitz (1987) and Werning (2007) describe Pareto-optimal taxation. Their efforts are similar in spirit to mine, in that they widen the model's normative perspective. They differ, however, in that my approach provides a way to include a specific combination of normative perspectives held by society, while these authors remain agnostic and, therefore, are able to provide less specific guidance to or explanation of policy. Related, recent research by Saez and Stantcheva (2012) focuses on marginal social welfare weights through which tax reforms may be evaluated. They allow these weights to take any positive values, including values based on principles or priorities at odds with Utilitarianism. Their approach is complementary to mine, in that they focus on the welfare weights that arise from, at least in part, the normative reasoning I model directly. Finally, specific normative limitations of the conventional model have been addressed directly. Fleurbaey and Maniquet (2006) allow for considerations of fairness and responsibility with respect to preference heterogeneity. Besley and Coate (1992) allow for society to place particular emphasis on poverty alleviation. Auerbach and Hassett (1992) model a concern for horizontal equity. This paper's general framework for enriching the conventional model could accommodate these concerns.



the dominant pattern much of the time."<sup>15</sup> Policy driven by individuals (i.e., voters) in that interior will therefore balance these competing normative frameworks.

Appealing as it may be to generalize the optimal tax model's normative perspective to capture the diversity of moralities that drive public and scholarly debate over taxes, there is a methodological obstacle. Many of the most prominent normative frameworks evaluate outcomes in ways that are not directly commensurable. For example, Utilitarianism is a consequentialist framework, basing its evaluations solely on ends, not means. It is the most prominent example of a consequentialist framework "with *additional* demands, particularly 'welfarism,' which insists that states of affairs must be judged exclusively by the utility information related to the respective states," in the words of Sen (2000). In contrast, some normative frameworks, such as Libertarianism, stress the moral relevance of concerns such as freedom, rights, and rules, rather than the ends emphasized by Utilitarianism. These frameworks are often referred to as deontological. A long-standing concern in moral philosophy is whether the judgments of such frameworks can be compared—for example, can we meaningfully measure Utilitarian gains against Libertarian losses?

This paper ensures commensurability by representing the priorities of each normative framework with a consequentialist loss function that depends on deviations of the actual allocation of resources from each framework's optimal allocation. Of course, some may object to consequentialist representations of deontological approaches. In the end, the appeal of my analysis will depend on how closely the optimal allocations and loss functions I use align with the priorities of the normative frameworks. These loss functions can be specified in a way that respects Pareto efficiency, as the examples in Section 3 below illustrate, avoiding the problem with non-welfarist criteria noted by Kaplow and Shavell (2001).

In this paper, a social planner minimizes a "social loss function" that is the weighted sum of these framework-specific losses. The weight on a given framework's loss represents the force that framework exerts on society's moral evaluations. The social planner can be interpreted as either a normatively-optimal authority or, from a positive perspective, an authority that uses the median voter's preferred objective.

This approach to combining disparate normative frameworks appears to be consistent with the moral reasoning of Amartya Sen. In Sen (1982) he writes: "...both welfarist consequentialism (such as utilitarianism) and constraint-based deontology are fundamentally inadequate because of their failure to deal with certain important types of interdependences present in moral problems. This leads to an alternative approach... which incorporates, among other things, some types of rights in the evaluation of states of affairs, and which gives these rights influence on the choice of actions through the evaluation of consequent states of affairs." In Sen (2000), he names this approach "consequential evaluation". Sen does not specify how these frameworks ought to be combined, but a suggestive passage indicates that my approach of social loss minimization may not be far off the mark: "...rights-inclusive objectives in a system of consequential evaluation can accommodate certain rights the fulfillment of which would be excellent but not guaranteed, and we can still try to minimize the shortfall." Now I develop this generalized optimal tax model formally.

## 2.1 The model

Individuals differ in their innate ability to earn income, denoted  $w^i$  for types  $i \in \{1, 2, \dots, I\}$ , with the proportion of the population with ability  $i$  denoted  $p^i$  such that  $\sum_{i=1}^I p^i = 1$ . An individual of type  $i$  derives utility from consumption  $c$  and disutility from exerting labor effort  $y/w$  to earn income  $y$ . Denote the utility function  $U(c, y/w)$ .

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<sup>15</sup>Similarly, Gainous and Martinez (2005) conclude that "a sizable chunk of the American public is, in fact, ambivalent to some degree about social welfare."

A planner chooses allocations  $\{c_*^i, y_*^i\}_{i=1}^I$  to minimize social loss subject to feasibility and incentive compatibility constraints. Formally, the planner's problem is:

**Problem 1** *Social planner's problem (general case)*

$$\min_{\{c_*^i, y_*^i\}_{i=1}^I \in \mathbb{F} \cap \mathbb{IC}} \mathcal{L} = \sum_{\phi \in \Phi} \alpha_\phi \mathcal{L}_\phi \left( \{c_\phi^i, y_\phi^i\}_{i=1}^I, \{c_*^i, y_*^i\}_{i=1}^I \right), \quad (1)$$

where the framework-specific loss functions  $\mathcal{L}_\phi$  are defined below;

$\mathbb{F}$  denotes the set of feasible allocations for the economy:

$$\mathbb{F} = \left\{ \{c^i, y^i\}_{i=1}^I : \sum_{i=1}^I p^i (y^i - c^i) \geq G \right\}, \quad (2)$$

where  $G$  is exogenous, required government spending on public goods;

$\mathbb{IC}$  denotes the set of incentive compatible allocations:

$$\mathbb{IC} = \left\{ \{c^i, y^i\}_{i=1}^I : U(c^i, y^i/w^i) \geq U(c^j, y^j/w^i) \text{ for all } i, j \in \{1, 2, \dots, I\} \right\}. \quad (3)$$

The novel component of this planner's problem is its objective, captured in expression (1), to minimize the weighted sum of losses across a set of normative frameworks. These framework-specific losses are calculated through loss functions that are designed to capture the priorities of the normative frameworks.

Formally, a set  $\Phi$  of normative frameworks enters the planner's objective. Each normative framework  $\phi \in \Phi$  implies a (possibly incomplete) preference relation  $\succeq_\phi$  on the set  $\mathbb{F}$ , so that we say allocation  $\{c_1^i, y_1^i\}_{i=1}^I \in \mathbb{F}$  is weakly preferred under the framework  $\phi$  to allocation  $\{c_2^i, y_2^i\}_{i=1}^I \in \mathbb{F}$  if

$$\{c_1^i, y_1^i\}_{i=1}^I \succeq_\phi \{c_2^i, y_2^i\}_{i=1}^I.$$

Given  $\succeq_\phi$ , the strict preference relation  $\succ_\phi$  is defined as usual: for any  $\{c_1^i, y_1^i\}_{i=1}^I, \{c_2^i, y_2^i\}_{i=1}^I \in \mathbb{F}$ ,

$$\{c_1^i, y_1^i\}_{i=1}^I \succ_\phi \{c_2^i, y_2^i\}_{i=1}^I \Leftrightarrow \{c_1^i, y_1^i\}_{i=1}^I \succeq_\phi \{c_2^i, y_2^i\}_{i=1}^I \text{ but not } \{c_2^i, y_2^i\}_{i=1}^I \succeq_\phi \{c_1^i, y_1^i\}_{i=1}^I.$$

These preference relations allow the identification of a preferred, economically-feasible allocation of consumption and income across types for each normative framework.<sup>16</sup> I label such an allocation an " $\phi$ -optimal feasible allocation", denote it  $\{c_\phi^i, y_\phi^i\}_{i=1}^I$ , and formally define it as follows.<sup>17</sup>

**Definition 1** *An  $\phi$ -optimal feasible allocation  $\{c_\phi^i, y_\phi^i\}_{i=1}^I$  is any allocation in the set  $\mathbb{F}$  for which there is no other allocation  $\{c^i, y^i\}_{i=1}^I$  in the set  $\mathbb{F}$  such that:  $\{c^i, y^i\}_{i=1}^I \succ_\phi \{c_\phi^i, y_\phi^i\}_{i=1}^I$ .*

These  $\phi$ -optimal feasible allocations provide a key link between normative frameworks. With them, the priorities of any framework can be represented by a loss function that measures the costs of deviations from the framework's most preferred allocation. I denote these loss functions  $\mathcal{L}_\phi \left( \{c_\phi^i, y_\phi^i\}_{i=1}^I, \{c_*^i, y_*^i\}_{i=1}^I \right)$

<sup>16</sup>Some frameworks may not be able to rank all possible outcomes, i.e., exhibit incompleteness. In that case, the social planner picks one of the non-dominated outcomes as the framework's optimum.

<sup>17</sup>No incentive compatibility constraints are imposed on the set of feasible allocations because we want to compare allocations to a constant ideal for each framework.

The loss functions  $\{\mathcal{L}_\phi\}_{\phi \in \Phi}$  ought to satisfy some natural properties for all frameworks  $\phi \in \Phi$ . Specifically, the loss functions I use in this paper satisfy the following two conditions.

**Remark 1** For all  $\phi \in \Phi$ , the loss function  $\mathcal{L}_\phi(x, y)$  satisfies:

1. *Ordinality:* For any  $\{c_1^i, y_1^i\}_{i=1}^I, \{c_2^i, y_2^i\}_{i=1}^I \in \mathbb{F}$ ,

$$\mathcal{L}_\phi \left( \{c_\phi^i, y_\phi^i\}_{i=1}^I, \{c_1^i, y_1^i\}_{i=1}^I \right) \leq \mathcal{L}_\phi \left( \{c_\phi^i, y_\phi^i\}_{i=1}^I, \{c_2^i, y_2^i\}_{i=1}^I \right) \Leftrightarrow \{c_1^i, y_1^i\}_{i=1}^I \succeq_\phi \{c_2^i, y_2^i\}_{i=1}^I,$$

so that the loss from one allocation is no greater than that from another to which it is weakly preferred under framework  $\phi$ ;

2. *Normalization:*  $\mathcal{L}_\phi \left( \{c_\phi^i, y_\phi^i\}_{i=1}^I, \{c_\phi^i, y_\phi^i\}_{i=1}^I \right) = 0$ , so that the loss is zero<sup>18</sup> when the equilibrium allocation equals the  $\phi$ -optimal feasible allocation.

The weights applied to each loss function represent the importance each normative framework plays in society's evaluations of policy. A number of models of the policymaking process could be used to generate such weights. The most straightforward is that the median (pivotal) voter has his or her own weights on each normative framework, adopted by policymakers as a result of electoral competition.<sup>19</sup> One implication of this paper's analysis is that future research identifying the values of these weights would be valuable.

Next, I apply this general approach to a specific case with two prominent normative frameworks.

### 3 A two-framework case: Utilitarianism and Libertarianism

Feldman and Zaller (1992) summarize a prominent specific instance of normative ambivalence documented in the United States as between "two major elements of the political culture: achievement and equality..., capitalism and democracy..., or freedom and equality..." As shorthand for this division in the context of optimal taxation, I will refer to this as the tension between Libertarianism and Utilitarianism.

The first step in applying the previous section's approach is to define the preference relations for the Utilitarian and Libertarian frameworks that determine their  $\phi$ -optimal feasible allocations. The preference relation for Utilitarianism is familiar from the conventional optimal tax literature: allocations are preferred that generate a greater sum of individual utilities. Formally,  $\succeq_{Util}$  is defined by:

$$\{c_1^i, y_1^i\}_{i=1}^I \succeq_{Util} \{c_2^i, y_2^i\}_{i=1}^I \Leftrightarrow \sum_{i=1}^I p^i U(c_1^i, y_1^i/w^i) \geq \sum_{i=1}^I p^i U(c_2^i, y_2^i/w^i). \quad (4)$$

The Utilitarian-optimal feasible allocation is therefore, for all possible  $\{c^i, y^i\}_{i=1}^I \in \mathbb{F}$ :

$$\{c_{Util}^i, y_{Util}^i\}_{i=1}^I \in \mathbb{F} : \sum_{i=1}^I p^i U(c_{Util}^i, y_{Util}^i/w^i) \geq \sum_{i=1}^I p^i U(c^i, y^i/w^i).$$

<sup>18</sup> Any constant would accomplish the same normalization, though zero is the natural choice.

<sup>19</sup> If one wished to consider, instead, different groups engaged in a policy-setting game, alternative approaches could be used. For example, the Nash bargaining solution would optimize a weighted combination of their interests. "Veto" models such as that in Moulin (1981) would allow a coalition of voters to block some alternatives. Such formulations are conceptually similar to this paper's, as the key to this paper's results is not the specific formalization of the tradeoff between normative frameworks but rather that the tradeoff is included at all.

The preference relation for Libertarianism requires more discussion. In their critique of Libertarianism, Murphy and Nagel (2002) write: "The implication for tax policy of rights-based libertarianism in its pure or absolute form is that no compulsory taxation is legitimate..." I read Nozick (1974), the most influential modern expositor of Libertarianism, as saying much the same thing, if not so directly, when he writes: "if it would be illegitimate for a tax system to seize some of a man's leisure (forced labor) for the purpose of serving the needy, how can it be legitimate for a tax system to seize some of a man's goods for that purpose?" The natural inference from such statements is that the allocation with no taxation is strictly preferred to all others according to the Libertarian criterion. For clarity, I will refer to the allocation with no taxation as the *laissez-faire* allocation and formally define it as follows.

**Definition 2** *The laissez-faire allocation,  $\{c_{lf}^i, y_{lf}^i\}_{i=1}^I \in \mathbb{F}$ , where  $G = 0$ , satisfies the following conditions (where  $U_x(c, y/w)$  denotes the partial derivative of individual utility with respect to  $x$ ):*

1.  $U_{c_{lf}^i}(c_{lf}^i, y_{lf}^i/w^i) = U_{y_{lf}^i}(c_{lf}^i, y_{lf}^i/w^i)/w^i$
2.  $c_{lf}^i = y_{lf}^i$ .

These conditions are simply that each individual maximizes utility and there are no interpersonal transfers. In the statement of the definition, I clarify that  $G = 0$ , as this is the allocation with no taxation and, therefore, no government spending.

The Libertarian preference relation, denoted  $\succeq_{Lib}$ , is then defined as follows:

$$\{c_{lf}^i, y_{lf}^i\}_{i=1}^I \succeq_{Lib} \{c^i, y^i\}_{i=1}^I, \quad (5)$$

for all possible  $\{c^i, y^i\}_{i=1}^I \in \mathbb{F}$ , where  $G = 0$ .

Moving from the preference relation  $\succeq_{Lib}$  to the  $\phi$ -optimal feasible allocation for Libertarianism is complicated by a well-known conceptual issue with the idea of the laissez-faire allocation.<sup>20</sup> Any economy is, in reality, inseparable from the existing set of taxes that fund the government and state institutions. The laissez-faire allocation is, therefore, not well-defined, because  $G = 0$  implies a very different economy than that the status quo. Whether or not this challenge is surmountable by Libertarians in general, it has a natural solution in this paper. The key is the use of the principle of equal sacrifice as the core idea of the Libertarian approach to taxation.

To see how the equal sacrifice principle implies a  $\phi$ -optimal feasible allocation for the Libertarian framework, consider the following thought experiment. Suppose that the public goods necessary to support the current economy are sustained without any cost to the economy, so that  $G = 0$  but the status quo economic system is feasible. To a Libertarian, the (no tax) laissez-faire outcome in this scenario is surely optimal. Now, suppose that sustaining those public goods is costly, so that  $G > 0$ . The equal sacrifice principle implies that the cost of the public goods will be distributed across individuals such that the utility loss is identical (and as small as possible) for all.

Formally, define  $\mathbb{ES}$  as the set of all feasible allocations that satisfy the principle of equal sacrifice relative to the laissez-faire allocation:

$$\mathbb{ES} = \left\{ \{c^i, y^i\}_{i=1}^I \in \mathbb{F} : U(c_{lf}^i, y_{lf}^i/w^i) - U(c^i, y^i/w^i) = U(c_{lf}^j, y_{lf}^j/w^j) - U(c^j, y^j/w^j) \text{ for all } i, j \in \{1, 2, \dots, I\} \right\}. \quad (6)$$

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<sup>20</sup>Another contentious issue, which I will abstract from, is whether that allocation reflects any past injustices that ought to be rectified. If so, even the Libertarian framework may justify government intervention. If not, then the laissez-faire allocation can be taken as just under the Libertarian framework.

The Libertarian-optimal feasible allocation is therefore that which achieves the smallest, but equal, sacrifice while funding  $G$ . Formally, we define  $\{c_{Lib}^i, y_{Lib}^i\}_{i=1}^I$  as follows:

$$\begin{aligned} \{c_{Lib}^i, y_{Lib}^i\}_{i=1}^I \in \mathbb{ES} : & U(c_{lf}^i, y_{lf}^i/w^i) - U(c_{Lib}^i, y_{Lib}^i/w^i) \leq U(c_{lf}^i, y_{lf}^i/w^i) - U(c^i, y^i/w^i), \\ & \text{for any } i \in \{1, 2, \dots, I\} \text{ and for all possible } \{c^i, y^i\}_{i=1}^I \in \mathbb{ES}. \end{aligned}$$

The next step is to specify the loss functions for the planner.

### 3.1 Loss functions

The Utilitarian loss function  $\mathcal{L}_{Util}$  is directly implied by the Utilitarian preference relation in expression (4):

$$\mathcal{L}_{Util}(\{c_{Util}^i, y_{Util}^i\}_i, \{c_*^i, y_*^i\}_i) = \sum_{i=1}^I p^i [U(c_{Util}^i, y_{Util}^i/w^i) - U(c_*^i, y_*^i/w^i)]. \quad (7)$$

In words, it is the sum of individuals' utility losses from having the equilibrium allocation  $\{c_*^i, y_*^i\}_i$  deviate from the Utilitarian-optimal feasible allocation. This loss function has the appealing property that it directly adopts the cardinal welfare comparisons underlying the Utilitarian preference relation.

The Libertarian loss function is a generalization of the principle of equal sacrifice. In its pure form, the equal sacrifice principle is too narrow for the purposes of this paper because it cannot rank allocations that deviate from exactly equal sacrifice (which the equilibrium allocation will do if the Utilitarian component of the social objective carries enough weight). Therefore, I specify a loss function that satisfies the following three properties: first, deviations of individual utility below the Libertarian-optimal feasible allocation are costly but deviations above the Libertarian-optimal feasible allocation yield little or no offsetting benefits<sup>21</sup>; second, losses increase more than proportionally with the size of the deviation of individual utility below the Libertarian-optimal feasible allocation; third, gains are concave in the size of the deviation of individual utility above the Libertarian-optimal feasible allocation. I formalize these properties as follows:

$$\mathcal{L}_{Lib}(\{c_{Lib}^i, y_{Lib}^i\}_i, \{c_*^i, y_*^i\}_i) = \sum_{i=1}^I p^i V(U(c_{Lib}^i, y_{Lib}^i/w^i), U(c_*^i, y_*^i/w^i)), \quad (8)$$

where

$$V(U_{Lib}^i, U_*^i) = \begin{cases} -(\delta [U_*^i - U_{Lib}^i])^\theta & \text{if } U_{Lib}^i < U_*^i \\ [\lambda (U_{Lib}^i - U_*^i)]^\rho & \text{if } U_{Lib}^i \geq U_*^i \end{cases}, \quad (9)$$

for scalars  $\{\delta \geq 0, \lambda > \delta, \theta \in (0, 1], \rho \geq 1\}$ .

Consistent with the first property, the loss function in expressions (8) and (9) applies weights  $\delta$  and  $\lambda$ , where  $0 \leq \delta < \lambda$ , to deviations of individual utility above and below the Libertarian-optimal feasible allocation. The asymmetric punishment of downward deviations from the Libertarian-optimal feasible allocation implied by  $\delta < \lambda$  respects the Pareto criterion but rejects the Utilitarian idea that the distribution of utility across individuals is irrelevant. Consistent with the second and third properties, the parameters  $\rho \geq 1$  and  $\theta \in (0, 1]$  imply losses that increase (weakly) more than proportionally with deviations below and gains that increase (weakly) less than proportionally for deviations above the Libertarian-optimal feasible allocation.

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<sup>21</sup>This property is consistent with the classic "loss aversion" of Kahneman and Tversky (1979). However, equal sacrifice is not consistent with the diminishing sensitivity to losses that is part of classic prospect theory.

### 3.2 Planner's problem

With the loss functions defined by expressions (7), (8) and (9), the planner in this case chooses  $\{c_*^i, y_*^i\}_{i=1}^I$  to solve the following problem.

**Problem 2** *Social Planner's Problem (specific case)*

$$\min_{\{c_*^i, y_*^i\}_{i=1}^I \in \{\mathbb{F} \cap \mathbb{IC}\}} \left\{ \begin{array}{l} \alpha_{Util} \sum_{i=1}^I p^i [U(c_{Util}^i, y_{Util}^i/w^i) - U(c_*^i, y_*^i/w^i)] \\ + \alpha_{Lib} \sum_{i=1}^I p^i V(U(c_{Lib}^i, y_{Lib}^i/w^i), U(c_*^i, y_*^i/w^i)) \end{array} \right\}, \quad (10)$$

where

$$\alpha_{Util} + \alpha_{Lib} = 1,$$

$V(\cdot)$  is defined in (9),  $\mathbb{F}$  is defined in (2), and  $\mathbb{IC}$  is defined in (3).

This planner's problem is equivalent to the conventional approach if  $\alpha_{Lib} = 0$ .

In the next two sections, the optimal policy generated by this planner's problem will be analyzed in depth. First, however, to illustrate the effect of positive  $\alpha_{Lib}$  on optimal policy, I simulate a simple model with two types of workers and show how this form of normative diversity affects the well-being of individuals in the economy.

### 3.3 Example with two types

Individual income-earning ability is either  $w^1 = 10$  or  $w^2 = 50$ , each of which makes up half the population, so  $p^1 = p^2 = 0.5$ . The Libertarian loss function's parameters<sup>22</sup> are  $\delta = 0.5$ ,  $\lambda = 20$ ,  $\rho = 2.0$ ,  $\theta = 1.0$ , and the social loss function's weight on the Libertarian loss function is  $\alpha_{Lib} = 0.20$ . Government spending  $G$  is set to zero. The individual utility function is

$$U(c^i, y^i/w^i) = \frac{(c^i)^{1-\gamma} - 1}{1-\gamma} - \frac{1}{\sigma} \left(\frac{y^i}{w^i}\right)^\sigma,$$

where  $\gamma = 1.5$ ,  $\sigma = 3$ .

This simple example is most useful for making plain the effect of normative diversity on the allocation of utility across types of individuals. Figure 1 plots the utility of the high-ability type against the utility of the low-ability type. The bold solid line in the figure shows the utility possibilities frontier (UPF): that is, the highest incentive-compatible, feasible utility for the low-ability type given a utility level for the high-ability type. The thin solid and dotted lines are the indifference curves passing through the  $\phi$ -optimal feasible (but not necessarily incentive compatible) allocations for the Utilitarian and Libertarian frameworks, respectively. The dashed line is the indifference curve for the social planner that chooses (by tangency with the UPF) the optimal allocation for the economy. Also shown are the optimal allocations chosen by each framework.

<sup>22</sup>Note that  $\delta > 0$  is used here while  $\delta = 0$  is used in the detailed numerical simulations below. The indifference curves for the Libertarian are degenerate if  $\delta = 0$  because no gain for one type is worth a loss to another in that case. Therefore, I use  $\delta > 0$  so that the Libertarian indifference curve can be more intuitively compared to the Utilitarian.

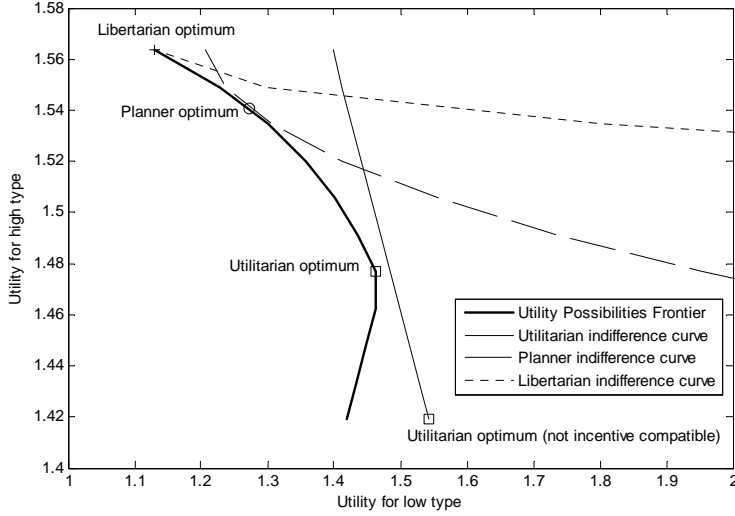


Figure 1: The Utility Possibilities Frontier and Indifference Curves

Figure 1 shows how the Libertarian loss function,  $\mathcal{L}_{Lib}$ , differs from the Utilitarian,  $\mathcal{L}_{Util}$ . To remain indifferent while moving away from its optimal allocation,  $\mathcal{L}_{Lib}$  requires a greater gain for the low-ability individual in exchange for a given loss for the high-ability individual. Moreover,  $\mathcal{L}_{Lib}$  increases more than proportionally with these deviations, while  $\mathcal{L}_{Util}$  is linear. The impact of incorporating this loss function in the planner's decisions is as expected: the planner compromises between the competing normative frameworks, implementing some redistribution but stopping well short of what the Utilitarian framework would choose and leaving the final allocation between it and the Libertarian-optimal feasible allocation. By varying  $\alpha_{Util}$ , we can shift the planner's chosen allocation along the UPF.

## 4 Analysis of optimal policy with normative diversity

In this section, I examine analytically the characteristics of optimal policy with normative diversity as formalized in Section 3. I consider two effects of increasing the weight on equal sacrifice in the social loss function. First, I show that it reduces the optimal extent of redistribution through tagging. Second, I show that it has a theoretically ambiguous impact on the pattern of optimal marginal tax rates.

### 4.1 Optimal tagging

To analyze optimal tagging, I modify the social planner's problem so that individuals differ in two characteristics: unobservable ability  $w$  indexed by  $i$ , and an observable, tagged variable indexed by  $m = \{1, 2, \dots, M\}$ . Therefore, allocations are denoted  $\{c^{i,m}, y^{i,m}\}_{i=1, m=1}^{I, M}$  and the population proportion of the individual with ability  $i$  and tagged variable value  $m$  is denoted  $p^{i,m}$  where  $\sum_{i=1}^I \sum_{m=1}^M p^{i,m} = 1$ . The modified planner's problem is as follows.

**Problem 3** *Social Planner's Problem with Tagging*

$$\min_{\{c_*^{i,m}, y_*^{i,m}\}_{i=1, m=1}^{I, M} \in \{\mathbb{F} \cap \mathbb{IC}\}} \left\{ \begin{array}{l} \alpha_{Util} \sum_{i=1}^I \sum_{m=1}^M p^{i,m} \left[ U \left( c_{Util}^{i,m}, y_{Util}^{i,m} / w^i \right) - U \left( c_*^{i,m}, y_*^{i,m} / w^i \right) \right] \\ + \alpha_{Lib} \sum_{i=1}^I \sum_{m=1}^M p^{i,m} V \left( U \left( c_{Lib}^{i,m}, y_{Lib}^{i,m} / w^i \right), U \left( c_*^{i,m}, y_*^{i,m} / w^i \right) \right) \end{array} \right\}, \quad (11)$$

where

$$\alpha_{Util} + \alpha_{Lib} = 1,$$

$V(\cdot)$  is a modified version of (9),

$$V \left( U_{Lib}^{i,m}, U_*^{i,m} \right) = \begin{cases} - \left( \delta \left[ U_*^{i,m} - U_{Lib}^{i,m} \right] \right)^\theta & \text{if } U_{Lib}^{i,m} < U_*^{i,m} \\ \left[ \lambda \left( U_{Lib}^{i,m} - U_*^{i,m} \right) \right]^\rho & \text{if } U_{Lib}^{i,m} \geq U_*^{i,m} \end{cases}, \quad (12)$$

for scalars  $\{\delta \geq 0, \lambda > \delta, \theta \in (0, 1], \rho \geq 1\}$ .

the feasibility set is a natural modification of expression (2),

$$\mathbb{F} = \left\{ \{c^{i,m}, y^{i,m}\}_{i=1, m=1}^{I, M} : \sum_{i=1}^I \sum_{m=1}^M p^{i,m} (y^{i,m} - c^{i,m}) \geq G \right\}, \quad (13)$$

and the set of incentive compatible allocations  $\mathbb{IC}$  is:

$$\mathbb{IC} = \left\{ \{c^{i,m}, y^{i,m}\}_{i=1, m=1}^{I, M} : U \left( c^{i,m}, y^{i,m} / w^i \right) \geq U \left( c^{j,m}, y^{j,m} / w^i \right) \text{ for all } i, j \in \{1, 2, \dots, I\} \text{ and } m \in \{1, 2, \dots, M\} \right\}. \quad (14)$$

In this problem the incentive constraints (14) are  $m$ -specific. That is, the planner can restrict each individual to the allocations within his or her tagged group, whereas if tagging were excluded the planner would be required to ensure that each individual preferred his or her allocation to that of any individual in any tagged group.

The following proposition is implied by the first-order conditions of this planner's problem, assuming separable utility between consumption and labor effort. The proof can be found in the Appendix.

**Proposition 1** *If  $U_{c, y/w}(c, y/w) = 0$ , the solution to the Social Planner's Problem with Tagging satisfies:*

$$\frac{E_i \left[ \left( U_{c_*^{i,m}} \right)^{-1} \right]}{E_i \left[ \left( U_{c_*^{i,n}} \right)^{-1} \right]} = \frac{E_i \left[ \alpha_{Util} - \alpha_{Lib} \frac{\partial V \left( U_{Lib}^{i,m}, U_*^{i,m} \right)}{\partial U_*^{i,m}} \right]}{E_i \left[ \alpha_{Util} - \alpha_{Lib} \frac{\partial V \left( U_{Lib}^{i,n}, U_*^{i,n} \right)}{\partial U_*^{i,n}} \right]}, \quad (15)$$

where  $U_*^{i,m}$  denotes  $U \left( c_*^{i,m}, y_*^{i,m} / w^i \right)$  and  $U_{c_*^{i,m}}$  denotes  $\partial U \left( c_*^{i,m}, y_*^{i,m} / w^i \right) / \partial c_*^{i,m}$ .

The left-hand side of (15) is the ratio of the expected inverse marginal utilities of consumption across tagged types. This equals the ratio of the cost in consumption units of an incentive-compatible marginal increase in utility across all individuals with tagged value  $m$  versus  $n$ . The following corollary makes plain why this ratio is of interest.



**Corollary 1** *If  $\alpha_{Lib} = 0$ , equation (15) simplifies to:*

$$\frac{E_i \left[ \left( U_{c_*^{i,m}} \right)^{-1} \right]}{E_i \left[ \left( U_{c_*^{i,n}} \right)^{-1} \right]} = 1. \quad (16)$$

This result, also shown in Weinzierl (2011) for age-dependent taxes and labeled the Symmetric Inverse Euler equation in that context, shows that the Utilitarian planner with access to tagging will equalize the cost of providing utility to tagged groups. Intuitively, the planner has full information about the tag, so any opportunity to raise overall welfare by transfers across tag values will be exploited.

Next, I derive a condition analogous to (16) for positive  $\alpha_{Lib}$ . I make two assumptions to provide a clean benchmark case.<sup>23</sup>

**Assumption 1:** *All tagged groups can be ordered  $\{1, 2, \dots, M\}$  so that for any pair of groups  $(m, n) \in \{1, 2, \dots, M\}$ ,  $m < n$  implies that the solution to the Social Planner's Problem with Tagging when  $\alpha_{Lib} = 0$  satisfies*

$$V \left( U_{Lib}^{i,m}, U_*^{i,m} \right) \leq V \left( U_{Lib}^{i,n}, U_*^{i,n} \right) \text{ for all } i = \{1, 2, \dots, I\}, \quad (17)$$

and

$$V \left( U_{Lib}^{i,m}, U_*^{i,m} \right) < V \left( U_{Lib}^{i,n}, U_*^{i,n} \right) \text{ for at least one } i = \{1, 2, \dots, I\}. \quad (18)$$

In words, Assumption 1 holds that tagged groups can be "ranked", for instance by some function of the mean and variance of wages within each group, so that higher-ranked groups fare no better, and in some cases worse, than lower-ranked groups when the planner is a pure Utilitarian. That is, individuals of any given ability obtain allocations that generate greater losses or smaller gains when they are members of a higher-ranked group.<sup>24</sup>

Assumption 1 is closely related to a well-known result from previous optimal tax analyses that an "advantaged" tagged group is taxed heavily by a conventional Utilitarian-optimal tax policy. Mankiw and Weinzierl (2010) show this numerically for the optimal height tax in the United States, under which a tall taxpayer ends up with lower utility than a short taxpayer of the same ability. Intuitively, the planner treats those with the advantaged tag as higher-skilled workers on average, requiring them to produce more income than others. Mirrlees (1971) showed much the same result for higher ability individuals in the full information case (which is the relevant analogue) of his optimal tax problem.

**Assumption 2:** In the function  $V(\cdot)$  specified in expression (12),  $\delta = 0$ .

Assumption 2 specifies that the Libertarian loss function takes the (arguably natural) value of zero for any allocations in which the individual's utility exceeds his or her utility in the Libertarian-optimal policy. I also apply this assumption in the numerical simulations below, as it seems to represent most accurately the distinction between a concern for equal sacrifice and the conventional Utilitarian objective.

With these assumptions, the following corollary to Proposition 1 can be derived and compared with Corollary 1 above. The proof is in the Appendix.

<sup>23</sup>These assumptions are sufficient, but not necessary, for the result in Corollary 2. Numerical simulations show that tagged groups may be less distinct than under Assumption 1, and the Libertarian loss function may be more similar to the Utilitarian loss function than under Assumption 2, while the optimal extent of tagging remains lower than in the conventional theory.

<sup>24</sup>Assumption 1 can also be seen as the outcome of two sub-assumptions. Namely,  $U_*^{i,m} \geq U_*^{i,n}$  for all  $i = \{1, 2, \dots, I\}$ , with a strict inequality for at least one  $i$ , when  $\alpha_{Lib} = 0$ ; and  $U_{Lib}^{i,m} = U_{Lib}^{i,n}$  for all  $i = \{1, 2, \dots, I\}$ .

**Corollary 2** *If Assumptions 1 and 2 hold, then the solution to Social Planner's Problem with Tagging satisfies*

$$\frac{E_i \left[ \left( U_{c_*^i, m} \right)^{-1} \right]}{E_i \left[ \left( U_{c_*^i, n} \right)^{-1} \right]} < 1. \quad (19)$$

Corollary 2 is the main analytical result of the paper. It states that the planner who puts positive weight on the Libertarian framework allocates consumption in a way that leaves the cost of raising utility for the disadvantaged group (i.e.,  $m$  in this example) lower than that for the advantaged group. As shown in result (16), a purely Utilitarian planner would transfer additional resources to the disadvantaged group, but the planner with this more diverse normative framework stops short, redistributing less. The numerical simulations below reinforce this lesson.

Intuitively, taxing the advantaged tagged group to aid the disadvantaged group generates costs in unequal sacrifice to this planner. A Utilitarian planner ignores the distribution of sacrifice, caring only about total sacrifice (which tagging helps to minimize). This disparity in the treatment of transfers across tagged groups causes an optimal policy based in part on equal sacrifice to use tagging less than in conventional theory.

## 4.2 Optimal marginal distortions

The classic topic of optimal income tax analysis is marginal distortions to labor supply. To analyze these distortions in this paper's generalized model, I return to the Social Planner's Problem stated in (10), where individuals differ only in ability  $w$ . Denote with  $\mu^{j|i}$  the multiplier on the incentive constraint indicating that the allocation to ability type  $j$  is not preferred by the individual of ability type  $i$ . The first-order conditions of this planner's problem generate the following proposition.

**Proposition 2** *The optimal marginal distortion to the labor supply decision of an individual with ability type  $i$ , denoted  $\tau_*^i$ , satisfies the following condition.*

$$1 - \tau_*^i = \frac{U_{y_*^i}(c_*^i, y_*^i/w^i)}{w^i U_{c_*^i}(c_*^i, y_*^i/w^i)} = \frac{p^i \left( 1 + \alpha_{Lib} \left( \frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} - 1 \right) \right) + \sum_{j=1}^I p^j (\mu^{j|i} - \mu^{i|j})}{p^i \left( 1 + \alpha_{Lib} \left( \frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} - 1 \right) \right) + \sum_{j=1}^I p^j \left( \mu^{j|i} - \frac{w^i}{w^j} \frac{U_{y_*^i}(c_*^i, y_*^i/w^j)}{U_{y_*^i}(c_*^i, y_*^i/w^i)} \mu^{i|j} \right)}, \quad (20)$$

where  $U_x$  is the partial derivative of individual utility with respect to  $x$  and  $U_*^i$  denotes  $U(c_*^i, y_*^i, w^i)$ .

To interpret condition (20), start with the conventional case in which  $\alpha_{Lib} = 0$ . Then, the result simplifies to:

$$1 - \tau_*^i = \frac{U_{y_*^i}(c_*^i, y_*^i/w^i)}{w^i U_{c_*^i}(c_*^i, y_*^i/w^i)} = \frac{p^i + \sum_{j=1}^I p^j (\mu^{j|i} - \mu^{i|j})}{p^i + \sum_{j=1}^I p^j \left( \mu^{j|i} - \frac{w^i}{w^j} \frac{U_{y_*^i}(c_*^i, y_*^i/w^j)}{U_{y_*^i}(c_*^i, y_*^i/w^i)} \mu^{i|j} \right)}. \quad (21)$$

In result (21), the term  $\frac{w^i}{w^j} \frac{U_{y_*^i}(c_*^i, y_*^i/w^j)}{U_{y_*^i}(c_*^i, y_*^i/w^i)}$  is less than one for  $w^i < w^j$ , so that binding incentive constraints on higher skill types (i.e.,  $\mu^{i|j} > 0$ ) drive the optimal distortion  $\tau_*^i$  above zero in the conventional model.

A positive marginal distortion on type  $i$  has a benefit and a cost in conventional theory. The benefit of such a distortion is that it allows the planner to offer a more generous tax treatment to  $i$  without tempting

higher-skilled individuals to claim it. The greater the gain in social welfare due to this redistribution from higher earners to  $i$  (measured by  $\mu^{i|j}$  for  $w^j > w^i$  in both expressions), the greater is the optimal distortion to  $i$ . The conventional cost of such a distortion is the reduced effort and, therefore, output from type  $i$ . The size of this cost increases with the share of  $i$  in the population,  $p^i$ , explaining why  $p^i$  enters expression (21) as the first term in both the numerator and denominator, pushing the distortion toward zero as  $p^i$  increases.

If  $\alpha_{Lib} > 0$ , comparing results (20) and (21) shows that both the benefits and costs of optimal marginal distortions are affected, yielding an ambiguous overall impact of normative diversity on marginal tax rates.

First, with  $\alpha_{Lib} > 0$  marginal distortions have a second cost, namely the deviations from the Libertarian-optimal allocations that they cause. The social cost of this deviation for individual  $i$  is measured by the expression  $\alpha_{Lib} \left( \frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} - 1 \right)$ , and it affects the size of the first terms in the numerator and denominator of (20),  $p^i \left( 1 + \alpha_{Lib} \left( \frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} - 1 \right) \right)$ . A larger  $\alpha_{Lib}$  will increase these terms and decrease the optimal distortion on  $i$  if  $\frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} > 1$ , while a larger  $\alpha_{Lib}$  will decrease these terms and raise the optimal distortion on  $i$  if  $\frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i} \in (0, 1)$ . Note that  $\frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i}$  measures the marginal reduction in social loss from raising the allocated utility of type  $i$ . Starting from the Utilitarian allocation, where  $\alpha_{Lib} = 0$ , this reduction in loss will be greater for the high-skilled, as their allocated utilities will be far below the laissez-faire allocation. Raising the utilities of lower-skilled individuals will, in contrast, have smaller effects on social loss because they are already receiving higher utilities than in the laissez-faire. Formally,  $\frac{-\partial V(U_{Lib}^i, U_*^i)}{\partial U_*^i}$  is likely to be increasing in type because losses increase more than proportionally with deviations below the laissez-faire allocation and gains are concave in deviations above it<sup>25</sup>. This effect of increasing  $\alpha_{Lib}$  will tend to be a decrease in the optimal distortions on higher-skilled workers relative to lower-skilled workers.

Second, the benefits of redistribution change when the planner puts weight on equal sacrifice. In particular, the social value of redistributing from higher-skilled to lower-skilled and moderate-skilled individuals ( $\mu^{i|j}$  for  $w^j > w^i$ ) is less, because the planner values proximity to the laissez-faire allocation and places less value on individuals enjoying utilities that exceed the laissez faire (e.g., low earners). With smaller benefits from redistributing to the low- and moderate-skilled individuals, the required distortions on them are smaller. Therefore, this effect of increasing  $\alpha_{Lib}$  will tend to decrease the optimal distortions on low- and moderate-skilled workers relative to higher-skilled workers.

The ambiguity in the effects of positive  $\alpha_{Lib}$  on optimal marginal tax rates indicates the general difficulty in obtaining definitive results from condition (20). For a more comprehensive characterization of optimal income taxes with normative diversity, I turn to calibrated numerical simulations in the next section.

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<sup>25</sup> There is a technical complication to this statement, given the specific form for the Libertarian loss function assumed here, as allocations approach the laissez-faire allocation from above. With the assumed loss function, an increase in utility for an individual just above the laissez-faire can generate a larger gain for the planner than can an increase in utility for an individual just below it. If  $\delta = 0$  in (12), as in the numerical simulations below, this complication is avoided. This is the same reason for imposing Assumption 2 above.

## 5 Numerical results

In this section I study the effect of normative diversity on optimal tagging and income taxation through numerical simulations calibrated to micro-level data for the United States. First, I consider three prominent potential tags—height, gender, and race—and show that this paper’s approach can reconcile the rejection of all of these tags with the acceptance of redistributive income taxes driven by differences in income-earning ability.<sup>26</sup> I then choose a model parameterization that rejects these tags and use it to simulate optimal income taxes using a more detailed ability distribution. This latter simulation yields substantial redistribution, progressive average tax rates, and a profile of marginal tax rates resembling that in Saez (2001) but with a lower maximum rate. This last finding relates to the puzzle identified in Diamond and Saez (2011) that the conventional model implies substantially higher peak marginal rates than those prevailing in current policy.

### 5.1 Optimal tagging with normative diversity

For the optimal policy simulations, I will use the following parameter values in the Social Planner’s Problem with Tagging:

$\alpha_{Lib}$	$\rho$	$\theta$	$\delta$	$\lambda$	$\gamma$	$\frac{1}{\sigma-1}$	$G$
{0.10, 0.20}	2.0	1.0	0	{10, 20}	1.5	0.5	20

The parameter  $\alpha_{Lib} = 1 - \alpha_{Util}$  is the weight on the Libertarian loss function, i.e., the principle of equal sacrifice, in the social objective function. Recall that one interpretation of  $\alpha_{Lib}$  is as the weight of the equal sacrifice principle in the median voter’s normative preferences. As noted in the Introduction, researchers have found that the proportion of individuals whose opinions correspond to traditional Libertarian views is approximately 10 to 20 percent (Boaz and Kirby 2007, Cappelen et al. 2011). Consistent with that finding, Konow (2003) reports the results of a survey testing an updated version of Robert Nozick’s famous Wilt Chamberlain example, finding that the case in which a talented basketball player earns (and keeps) an extraordinarily high income due to voluntary payments by his fans is considered "fair" by at least 24 percent of respondents.

The next four parameters determine the shape of that loss function as specified in expression (12):  $\rho$  and  $\theta$  determine its concavity, while  $\delta$  and  $\lambda$  determine the extent of loss aversion. A larger  $\lambda$  relative to  $\delta$  means that the social loss function interprets downward deviations from equal sacrifice as more costly. The final three parameters are familiar from conventional models:  $\gamma$  and  $\frac{1}{\sigma-1}$  are the coefficient of relative risk aversion and the labor supply elasticity, and  $G$  is required government revenue. I choose  $\gamma$  and  $\frac{1}{\sigma-1}$  to match mainstream estimates and  $G$  to approximate the current value (as a share of total income) in the United States, but the results below are robust to choosing different values of these three parameters.

The data required for the simulation of the optimal height, gender, and race taxes are ability distributions by tagged type. I classify respondents to the National Longitudinal Survey of Youth into three height categories, two gender categories, and two race categories.<sup>27</sup> For height, I use gender-dependent ranges, as the height distributions of males and females are substantially different: for men the thresholds are 70 and 72 inches; for women the thresholds are 63 and 66 inches. Table 2 lists the twelve tagged groups that these divisions generate in descending order of their mean wage, where the wage is reported earnings divided by

<sup>26</sup>For simplicity, I do not consider differences in preferences or elasticities across these groups, though such differences provide an alternative justification for tagging.

<sup>27</sup>I omit individuals who report negative wages or earnings or who report less than 500 or more than 4,000 hours of annual work. The results are not sensitive to these restrictions, which are likely to remove misreported data.

reported hours in 1996.<sup>28</sup> The table also shows the mean and standard deviation of each group's reported wages, and each group's sample size in the NLSY.

	1	2	3	4	5	6	7	8	9	10	11	12
	Tall	Med.	Short	Tall	Tall	Med.	Short	Med.	Tall	Short	Med.	Short
	M	M	M	F	M	M	M	F	F	F	F	F
	White	White	White	White	NW	NW	NW	White	NW	White	NW	NW
Mean wage	17.2	16.4	15.8	13.7	13.6	12.7	12.5	12.4	12.0	12.0	10.8	10.3
SD wage	11.2	10.6	10.3	11.4	9.0	7.8	8.4	11.3	10.1	9.5	7.3	5.9
Obs.	411	507	785	340	226	314	994	557	223	405	469	653

The differences in wages among these twelve tagged groups are substantial. The highest-earning group in Table 2 earns a mean wage nearly 70 percent greater than the lowest-earning group. Overall, average wages are higher for those who are tall, male, and white. Appendix Table 1 provides more detail than Table 2, reporting the distributions of the members of the tagged groups across ten wage bins. These wage distributions are the second key input to the numerical simulations (in addition to the assumed parameters in Table 1).

For each of the four parameter vectors implied by Table 1, I report measures of the optimal extent of tagging and income tax progressivity in Table 3 and Table 4, respectively.

To measure the extent of tagging, Table 3 reports the "extra" average tax paid by or transfer made to the members of each tagged group as a share of their income when the planner can use tagging as compared to when it cannot.<sup>29</sup> More specifically, this is the ratio of total tax payments to total income for each group under the optimal policy less the same ratio under the constrained-optimal policy with no tagging. If that difference is positive, the group is paying taxes in addition to what they would pay if tagging were prohibited. If that difference is negative, they are receiving an extra transfer. For reference, I report the same statistic for the policy solution when  $\alpha_{Lib} = 0$ , the fully Utilitarian (conventional Mirrleesian) planner.

		1	2	3	4	5	6	7	8	9	10	11	12
		Tall	Med.	Short	Tall	Tall	Med.	Short	Med.	Tall	Short	Med.	Short
		M	M	M	F	M	M	M	F	F	F	F	F
$\alpha_{Lib}$	$\lambda$	White	White	White	White	NW	NW	NW	White	NW	White	NW	NW
0	10	13.6	11.4	9.3	0.5	0.8	-2.9	-4.8	-6.8	-7.3	-9.0	-15.4	-17.8
0.10	10	2.2	1.9	1.7	0.7	0.4	-0.3	-0.7	-0.9	-0.7	-1.4	-2.8	-4.1
0.20	10	0.9	0.7	0.7	0.6	0.2	-0.1	-0.3	-0.3	0.0	-0.4	-1.1	-2.2
0.10	20	0.4	0.1	0.1	0.2	-0.1	-0.1	-0.2	0.1	0.0	0.1	-0.2	-0.8
0.20	20	0.3	0.1	0.1	0.6	-0.3	0.2	-0.2	0.1	-0.2	-0.1	0.0	-0.7

<sup>28</sup>Using all three tags in concert maximizes the power of tagging in the conventional model. Analyses of each component tag applied in isolation yield results consistent with, and in fact a bit stronger than, the results of applying them together.

<sup>29</sup>The planner's problem when it cannot tag differs from the Social Planner's Problem with Tagging in that each individual  $i, m$  must prefer its bundle to any other bundle  $j, n$ . In that problem, tagged groups with higher wage distributions will pay greater average tax rates because the tax system is progressive. The "extra" taxes and transfers reported in Table 4 isolate the direct effects of tags on taxes.

To gauge the progressivity of the optimal income tax, Table 4 reports the average tax rate paid by the members of each wage range under each parameterization.

$\alpha_{Lib}$	$\lambda$	Average wage rate in range									
		2.81	6.50	10.03	13.82	17.80	21.70	27.28	43.25	62.06	95.96
0	10	-337	-44	5	23	31	36	41	51	53	55
0.10	10	-252	-23	11	22	27	30	34	44	48	51
0.20	10	-217	-15	13	21	25	27	30	41	45	49
0.10	20	-188	-8	15	19	23	25	28	39	43	47
0.20	20	-145	0	15	18	21	23	26	35	39	45

Finally, Table 5 shows the welfare gain obtainable from tagging in each case. To compute this welfare gain, I calculate the increase in consumption for all individuals that would lower the total social loss under the policy without tagging to the level of total social loss obtained by the optimal policy.

$\alpha_{Lib}$	$\lambda$	Percent of aggregate consumption
0	10	0.93
0.10	10	0.17
0.20	10	0.09
0.10	20	0.05
0.20	20	0.03

The results in these three tables show that incorporating a concern for equal sacrifice can explain the simultaneous rejection of tagging and acceptance of substantial income redistribution through progressive taxes observed in policy. Table 3 shows that equal sacrifice dramatically reduces the appeal of tagging according to height, gender, and race, despite the substantial information that these three tags carry about income-earning ability. While large group-specific taxes and transfers are optimal when  $\alpha_{Lib} = 0$  and none are optimal once  $\alpha_{Lib} = 1$  (not shown), even seemingly modest values for  $\alpha_{Lib}$  generate a steep decline in the use of tags. At the same time, for these values of  $\alpha_{Lib}$ , Table 4 shows that in all cases the extent of redistribution and progressivity remains quite high when measured by either the maximal average tax rate or the gap between the maximal and minimal average tax rates. Table 5 shows that the welfare gains one might achieve through tagging are estimated to be large in the conventional case of  $\alpha_{Lib} = 0$  but are negligible in all other cases.

As a specific example, consider the parameterization in which  $\alpha_{Lib} = 0.20$  and  $\lambda = 10$ . The optimal tag-based tax is 0.9 percent of the highest-earning group's total income in this parameterization, whereas the conventional model suggests a tax of 13.6 percent. Consistent with this reduced role for tagging, the welfare gain from tagging in this parameterization is negligible: translated into the magnitudes of the current U.S. economy, it is equivalent to under \$10 billion, while the conventional model implies a gain worth nearly \$100 billion. Assuming some costs from false tagging and administration (Akerlof 1978), these tags would likely be welfare-reducing, on net, in this parameterization. Nevertheless, in this parameterization top earners pay an average tax rate of 49 percent, close to the 55 percent recommended by the conventional model, and a

substantial transfer is made to the poor. To see this more clearly, consider Figure 2, which plots the schedule of average tax rates for this parameterization and two polar cases: the fully Utilitarian ( $\alpha_{Lib} = 0$ ) and the fully Libertarian, or Equal Sacrifice, ( $\alpha_{Lib} = 1.0$ ) policies.<sup>30</sup>

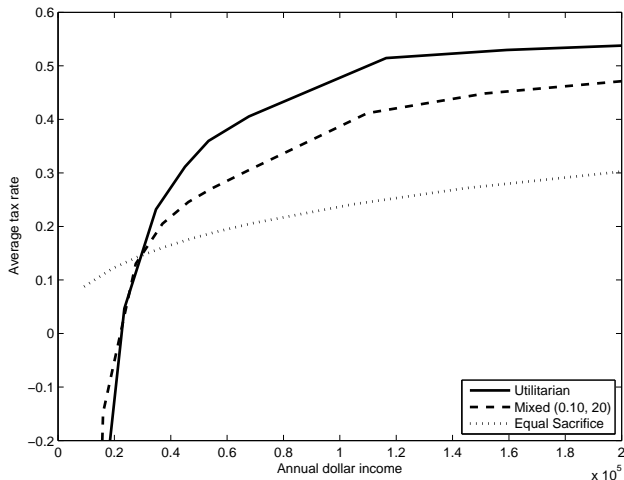


Figure 2: Optimal Average Tax Rates

As Figure 2 makes clear, the optimal policy according to the mixed social loss function is substantially redistributive and much more closely resembles the pure Utilitarian, conventional optimal policy than the policy that prioritizes only equal sacrifice.

The intuition for these results is as follows. The principle of equal sacrifice is consistent with the use of progressive taxes to pay for public goods if the utility sacrifice caused by any given rate of taxation is smaller for a higher-income individual than a lower-income one. But, that principle places no value on redistribution (the average tax rate on the lowest-ability type is positive when  $\alpha_{Lib} = 1.0$ ). Similarly, while both Utilitarians and Libertarians value the efficiency advantage of tagging, it violates the equal sacrifice principle because tagged personal characteristics have no bearing on individual utility. Altogether, the introduction of equal sacrifice considerations into the evaluation of outcomes causes optimal policy to move away from redistribution and, especially, tagging. For the range of parameters considered here, those effects are enough to make the optimal extent of tagging on height, gender, and race negligible but leave substantial redistribution and progressivity intact.

As this intuitive explanation suggests, the key forces determining the optimal extent of tagging in this model will apply to different degrees for different tags. Most important, the costs that tagging generates from the perspective of the equal sacrifice principle will be smaller when a tag is closely correlated with ability. If a tag were a perfect indicator of ability, it would generate no costs according to equal sacrifice. Given that such a tag would continue to generate efficiency gains by being inelastic to taxation, it would be more valuable to the social planner. In other words, the model suggests that personal characteristics are more likely to be used as tags when they provide stronger and more reliable signals of income-earning ability.

This conclusion is consistent with the limited ways in which tagging is currently used. Blindness, disability, and dependent children are three tags for which income-earning ability (per person) is highly likely to

<sup>30</sup>In this and the following figures that show annual dollar income, I convert the results of the simulations to annual figures as follows. The average labor effort in the more detailed simulation with  $\alpha_{Lib} = 0.80$  and  $\lambda = 10$  (below) is approximately 0.60 of total time available. The mean worker in the sample works approximately 2200 hours per year. That implies  $2200/0.6=3667$  hours as the appropriate multiplicative factor for the earnings returned by the policy simulation.

be low. Benefit programs for the elderly may not appear at first glance as well-suited to the theory, but it is important to recall that improvements in medical care have made the elderly of today much healthier than their predecessors. In fact, the current debate over raising the retirement age because of increased lifespans and plausible working lives is consistent with the lessons of this model.

## 5.2 Optimal income taxes

In this section, I explore the effects on optimal income taxes of assigning a sufficiently large role to equal sacrifice in the social loss function that optimal tagging is negligible for the three prominent characteristics examined in the previous section, height, gender, and race. In particular, I assume  $\alpha_{Lib} = 0.20$ , reflecting a weight of 20 percent on the Libertarian framework, and  $\lambda = 10$  as the loss aversion factor in the Libertarian loss function from Table 1. All other parameters are as in Table 1.<sup>31</sup> In the previous simulation, this combination of parameters yielded an optimal policy with minimal tagging and a top average income tax rate of 49 percent (this is also the case shown in Figure 2). For the data, I use a lognormal-Pareto calibration of the U.S. wage distribution that extends to a wage of \$150, originally calculated by Mankiw, Weinzierl, and Yagan (2009).<sup>32</sup> Average tax rates are  $(y - c)/y$  and marginal tax rates are as in (20).

Figure 3 shows the optimal schedule of marginal tax rates for this calibration, and Figure 4 shows the optimal schedule of average tax rates. All figures show allocations for annual earnings up to \$200,000. For comparison, each figure also shows the optimal results under a pure Utilitarian framework, that is when  $\alpha_{Lib} = 0$  as in the conventional optimal tax model.

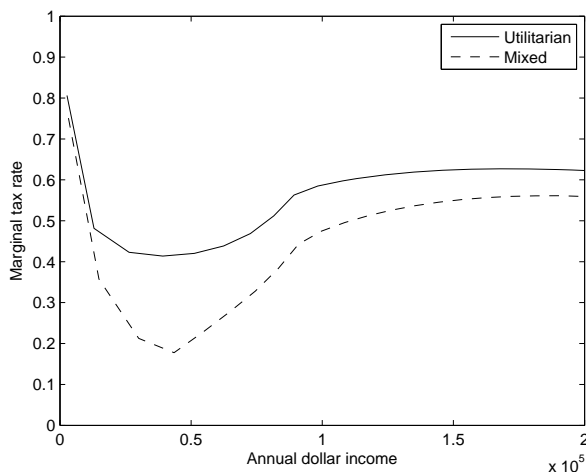


Figure 3: Optimal Marginal Tax Rates

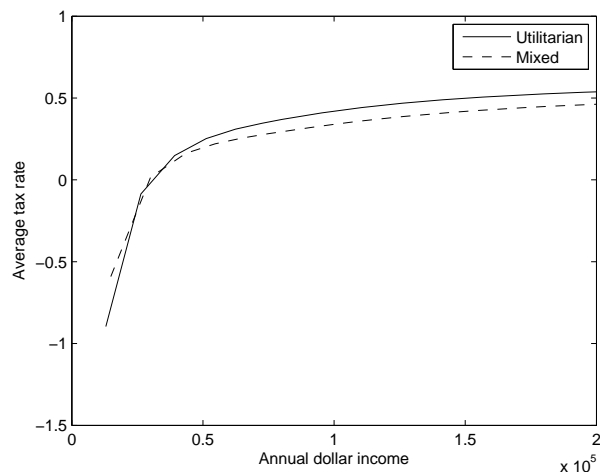


Figure 4: Optimal Average Tax Rates

Figure 3 shows that marginal tax rates have the U-shape introduced by Diamond (1998) and Saez (2001) whether  $\alpha_{Lib} = 0$  or 0.20, though positive  $\alpha_{Lib}$  does lead to lower rates for all workers. The effect of the second factor highlighted in the discussion of result (20) is particularly apparent, in that optimal marginal rates decline over a wider range of the ability distribution when  $\alpha_{Lib}$  is positive. The explanation for this pattern is that the planner with  $\alpha_{Lib} > 0$  redistributes less from high earners. This reduces the high earners' temptation to mimic moderate income earners and thus the required distortions on the latter.

<sup>31</sup>  $G$  differs but is chosen to approximate the same share of total output across simulations.

<sup>32</sup> The previous section's simulation used a calibration of the U.S. ability (i.e., wage) distribution that was limited by the availability of tagging data. Here, I use a more detailed calibration of the U.S. wage distribution.



The optimal marginal income tax rate at high incomes falls substantially, by about seven percentage points, with this role for the principle of equal sacrifice. Therefore, this paper’s explanation for the limited use of tagging may help address a second puzzling gap between conventional theory and existing policy: the low marginal tax rates at high incomes in policy relative to what theory recommends.<sup>33</sup> That gap is stated clearly by Diamond and Saez (2011), who derive a formula for the optimal marginal tax rate on high incomes as a function of utility parameters and the shape of the ability distribution. They conclude that the optimal top rate is "73 percent, substantially higher than the current 42.5 percent top US marginal tax rate (combining all taxes)." The top rate in the mixed policy shown in Figure 3 is 56 percent, compared to 63 percent under the conventional Utilitarian criterion.

Nevertheless, Figure 4 shows that substantial redistribution persists despite this role for the principle of equal sacrifice. The high-skilled continue to pay sizeable average tax rates of 46 percent, though this is less than the 54 percent under the Utilitarian policy. A related, unreported result is that the lowest-ability type enjoys a level of consumption worth 55 percent of average consumption in the economy under the policy with  $\alpha_{Lib} = 0.20$  compared to 64 percent under the Utilitarian policy with  $\alpha_{Lib} = 0$ .

These simulations show, therefore, that variation in unobserved income-earning ability remains a powerful force for redistribution in a modification of the conventional optimal tax model that includes sufficient weight on the principle of equal sacrifice to reject differentiated taxation according to height, gender, and race. At the same time, this form of normative diversity does lower optimal marginal distortions to levels closer to that which we observe in reality.

## 6 Conclusion

Modern optimal tax research is inherently normative. Though controversial, being normative is also key to the literature’s appeal, as it enables economists to apply their tools to a task of substantial importance: the design of the tax system.

A normative research agenda is only as relevant as its normative framework, however, and herein lies a problem. Optimal tax research has yet to adopt a normative framework that captures the diverse content of debates over taxes among the public, policymakers, or scholars of economic justice.<sup>34</sup>

This paper argues that one specific manifestation of this problem is the conventional optimal tax model’s recommendation of substantial tagging, the tailoring of taxes to personal characteristics. Though the theoretical case for widespread tagging has been clear for nearly four decades, policy includes tagging in only limited ways.

In this paper, I propose a way to expand the normative scope of optimal tax research to include moral frameworks other than the Utilitarianism that has dominated analysis since Mirrlees (1971). This expansion is intended to bring optimal tax research closer to, and therefore make it more relevant for, real-world debates in which normative heterogeneity plays a substantial role.

I then apply this general approach to include a specific alternative normative priority, the classic principle of equal sacrifice, and show that doing so can explain the limited role of tagging in policy. Equal sacrifice is a principle of long-standing importance in tax theory and is associated with influential Libertarian views of economic justice. With a social objective function that includes a role for equal sacrifice, the Utilitarian

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<sup>33</sup>Of course, a number of other potential explanations exist for why top marginal tax rates are not higher, such as a higher elasticity of taxable incomes at high income levels or preference heterogeneity (see Lockwood and Weinzierl, 2012).

<sup>34</sup>See the note in Section 1 on important exceptions to these statements in the work of Stiglitz (1987), Werning (2007), Saez and Stantcheva (2012), Fleurbaey and Maniquet (2006), Besley and Coate (1992), and Auerbach and Hassett (1999).

gains from tagging must be weighed against the costs it generates in unequal sacrifice.<sup>35</sup> Not all tags are equal in this model: a tag generates greater costs in unequal sacrifice the weaker is its correlation to ability. Thus, this model can explain not only the rejection of most tags but also the few cases in which tagging is used in real-world policy: i.e., in benefits for the elderly, disabled, and parents of young children.

In addition, this explanation for the limited role of tagging has implications for income taxes that further increase its appeal. First, it preserves the conventional theory's justification for redistribution based on unobserved ability. Second, it has the potential to explain why marginal income tax rates at high incomes are not as large in reality as conventional theory would recommend, a prominent puzzle in recent research.

In sum, incorporating into the theory of optimal taxation the principle of equal sacrifice substantially improves the match between that theory's recommendations and the reality of tax policy.

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<sup>35</sup>As discussed in Section 1, the model's conclusions on tagging are consistent with the more familiar critique based on the idea of horizontal equity, but they are based on a comprehensive, rigorous, and precise normative justification.

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## 7 Appendix

### 7.1 Proof of Proposition 1

The first-order condition of the planner's problem with respect to  $c_*^{i,m}$  is:

$$-\alpha_{Util} p^{i,m} + \alpha_{Lib} p^{i,m} \frac{\partial V(U_{Lib}^{i,m}, U_*^{i,m})}{\partial U(c_*^{i,m}, y_*^{i,m}/w^i)} - \frac{\mu_F}{U_c(c_*^{i,m}, y_*^{i,m}/w^i)} p^{i,m} + \mu^{j|i,m} - \mu^{i|j,m} = 0,$$

where  $\mu_F$  is the multiplier on the feasibility constraint and  $\mu^{j|i,m}$  is the multiplier on the incentive constraint that type  $i$  prefers its allocation to any other type  $j$ , for any group  $m$ . In deriving this condition, I used separability in the utility function to set  $\frac{U_c(c_*^{i,m}, y_*^{i,m}/w^i)}{U_c(c_*^{i,m}, y_*^{i,m}/w^i)} = 1$ . Taking the sum across types and simplifying yields:

$$E_i \left[ -\alpha_{Util} + \alpha_{Lib} \frac{\partial V(U_{Lib}^{i,m}, U_*^{i,m})}{\partial U(c_*^{i,m}, y_*^{i,m}/w^i)} \right] = E_i \left[ \frac{\mu_F}{U_c(c_*^{i,m}, y_*^{i,m}/w^i)} \right].$$

The analogous condition applies for  $c_*^{i,n}$ , where  $n$  indicates a different tagged group:

$$E_i \left[ -\alpha_{Util} + \alpha_{Lib} \frac{\partial V(U_{Lib}^{i,n}, U_*^{i,n})}{\partial U(c_*^{i,n}, y_*^{i,n}/w^i)} \right] = E_i \left[ \frac{\mu_F}{U_c(c_*^{i,n}, y_*^{i,n}/w^i)} \right].$$

Combining these conditions, we can write:

$$\frac{E_i \left\{ \left[ U_c(c_*^{i,m}, y_*^{i,m}/w^i) \right]^{-1} \right\}}{E_i \left\{ \left[ U_c(c_*^{i,n}, y_*^{i,n}/w^i) \right]^{-1} \right\}} = \frac{E_i \left[ \alpha_{Util} - \alpha_{Lib} \frac{\partial V(U_{Lib}^{i,m}, U_*^{i,m})}{\partial U(c_*^{i,m}, y_*^{i,m}/w^i)} \right]}{E_i \left[ \alpha_{Util} - \alpha_{Lib} \frac{\partial V(U_{Lib}^{i,n}, U_*^{i,n})}{\partial U(c_*^{i,n}, y_*^{i,n}/w^i)} \right]}.$$

This is the result in Proposition 1.

### 7.2 Proof of Corollary 2

Specification (12) implies

$$\frac{\partial V(U_{Lib}^{i,m}, U_*^{i,m})}{\partial U(c_*^{i,m}, y_*^{i,m}/w^i)} \leq 0$$

Thus, by expression (15), the corollary holds if the following condition is satisfied as a weak inequality for all  $i \in \{1, 2, \dots, I\}$  and satisfied with a strict inequality for at least one  $i \in \{1, 2, \dots, I\}$ :

$$\frac{\partial V(U_{Lib}^{i,m}, U_*^{i,m})}{\partial U_*^{i,m}} \geq \frac{\partial V(U_{Lib}^{i,n}, U_*^{i,n})}{\partial U_*^{i,n}}. \quad (22)$$

Three possibilities must be considered, depending on the positions of the planner's chosen allocations relative to the Libertarian-optimal allocations.

First, by imposing Assumption 2, we know that for any allocation under which  $U_{Lib}^{i,m} < U_*^{i,m}$  and  $U_{Lib}^{i,n} < U_*^{i,n}$ ,  $V(U_{Lib}^{i,m}, U_*^{i,m}) = V(U_{Lib}^{i,n}, U_*^{i,n}) = 0$ , so condition (22) is satisfied as an equality. This leaves two

cases to consider.

Second, suppose  $U_{Lib}^{i,m} < U_*^{i,m}$  and  $U_{Lib}^{i,n} \geq U_*^{i,n}$ . Then, (12) implies that condition (22) reduces to:

$$\theta \delta \left( \delta \left[ U_*^{i,m} - U_{Lib}^{i,m} \right] \right)^{\theta-1} \leq \rho \lambda \left[ \lambda \left( U_{Lib}^{i,n} - U_*^{i,n} \right) \right]^{\rho-1}. \quad (23)$$

This holds immediately for  $\delta = 0$ .

Third, suppose  $U_{Lib}^{i,m} \geq U_*^{i,m}$  and  $U_{Lib}^{i,n} \geq U_*^{i,n}$ . In that case, (12) implies that condition (22) reduces to:

$$-\rho \lambda \left[ \lambda \left( U_{Lib}^{i,m} - U_*^{i,m} \right) \right]^{\rho-1} \geq -\rho \lambda \left[ \lambda \left( U_{Lib}^{i,n} - U_*^{i,n} \right) \right]^{\rho-1},$$

or

$$\lambda \left( U_{Lib}^{i,m} - U_*^{i,m} \right) \leq \lambda \left( U_{Lib}^{i,n} - U_*^{i,n} \right). \quad (24)$$

This holds due to Assumption 1.

Imposing Assumption 1 implies that for some  $i \in \{1, 2, \dots, I\}$ , either condition (23) or (24) will hold with a strict inequality.

Therefore, we have shown that condition (22) is satisfied as a weak inequality for all  $i \in \{1, 2, \dots, I\}$  and satisfied with a strict inequality for at least one  $i \in \{1, 2, \dots, I\}$ :

### 7.3 Wage distributions by tagged group

The following table lists the wage distributions by tagged group. The representative wage rate in each bin is the mean wage across the population within that bin range.

Appendix Table 1: Wage distributions by tagged group												
	1	2	3	4	5	6	7	8	9	10	11	12
	Tall	Med.	Short	Tall	Tall	Med.	Short	Med.	Tall	Short	Med.	Short
	M	M	M	F	M	M	M	F	F	F	F	F
Wage	White	White	White	White	NW	NW	NW	White	NW	White	NW	NW
2.81	0.02	0.03	0.04	0.04	0.08	0.06	0.08	0.09	0.10	0.12	0.10	0.13
6.50	0.11	0.13	0.15	0.25	0.17	0.25	0.24	0.28	0.22	0.26	0.32	0.30
10.03	0.21	0.22	0.22	0.24	0.26	0.24	0.25	0.23	0.28	0.25	0.25	0.23
13.82	0.23	0.20	0.23	0.18	0.23	0.22	0.20	0.20	0.17	0.16	0.17	0.18
17.80	0.15	0.14	0.14	0.12	0.11	0.11	0.10	0.08	0.12	0.09	0.06	0.10
21.70	0.11	0.12	0.09	0.08	0.06	0.06	0.05	0.04	0.05	0.05	0.05	0.03
27.28	0.10	0.09	0.08	0.05	0.07	0.05	0.06	0.03	0.03	0.05	0.03	0.02
43.25	0.05	0.05	0.05	0.02	0.02	0.02	0.01	0.02	0.01	0.005	0.01	0.003
62.06	0.01	0.01	0.01	0.01	0.01	0.003	0.01	0.01	–	0.01	0.002	–
95.96	0.005	0.002	0.001	0.003	–	–	0.001	0.004	0.004	0.005	0.002	–