NBER WORKING PAPER SERIES

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Working Paper 17639
http://www.nber.org/papers/w17639

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2011

We gratefully acknowledge the helpful comments of Craig Garthwaite, Jin Li, Mike Mazzeo, and seminar participants at Stanford GSB, UC-Berkeley (Haas), University of Cologne, University of Heidelberg, Northwestern-Toulouse Joint Workshop on Industrial Organization, Workshop on Natural Experiments and Controlled Field Studies, Organizational Economics workshop at Queen's School of Business, and the Tournaments, Contests and Relative Performance Evaluation conference at North Carolina State University. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 17639
December 2011
JEL No. J01,J3,J31

ABSTRACT

We consider how an elimination tournament's ability to select the most skilled competitor as the winner is shaped by past, current, and future competition. We present a two-stage model that yields the following main results: (1) a shadow effect — the weaker the expected future competitor, the greater the probability that the stronger player wins in the current stage and (2) an effort spillover effect — previous effort reduces the probability that the stronger player wins in the current stage. We test our theory predictions using data from high-stakes tournaments and betting markets. Empirical results suggest that shadow and spillover effects influence match outcomes.

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Competition for employment and education, innovation funding, and design opportunities can all be framed as multi-stage elimination tournaments in which players are knocked out over successive stages of the event. These contests are often designed to increase player effort—indeed, much of the theoretical and empirical literature focuses on contests as incentive mechanisms. Yet, tournaments may also serve as selection mechanisms, identifying the "best" candidates as overall winners. In labor tournaments where employees' latent talents are not directly observable, firms may organize contests to reveal their workers' relative abilities.[1] For example, in searching for a CEO, a firm may use a tournament to identify and promote the highest-ability candidate, not simply the one who puts forth the most effort.

In this paper, we study how the strategies of heterogeneous players in match-pair elimination tournaments are shaped by past, current, and future competition. More specifically, we examine how these intertemporal effects influence a tournament's ability to reveal the strongest player as the winner. Negative spillover from past stages may make current effort more costly and depress performance, while the shadow of tough future competition decreases a player's expected future payoffs and also may lead to lower current effort. The differential impact of past and future competition across players in a given match changes the effectiveness of tournaments as a selection mechanism—both negative spillover and tough future competition increase the probability that a weak candidate wins overall. Our results have practical implications; whether the contest aims to encourage effort, select a strong winner, or both, we find evidence suggesting that firms, educators, and other contest designers may need to consider the role of past and future competition in structuring incentives.

In personnel tournaments, workers risk elimination as they advance through corporate management levels. In most contexts, retention of the highest quality worker is most desirable. For example, GE's former CEO, Jack Welch, designed an explicit elimination tournament to select his successor (Konrad 2009).[2]

Competition between firms may also be knockout events. In 2010, GE announced a three-stage elimination tournament, the Ecomagination Challenge, to award $200 million to the firm that developed the best smart grid technologies. More commonly, architectural firms may compete for large contracts and investment banks may compete for new clients over several stages of proposals and commitments. Political races also may involve elimination stages—a candidate must win his party's primary election to compete in the general election to hold office. Many sporting events are also structured as elimination tournaments.

[1] In contrast, Lazear (1986) discusses how performance pay may attract higher quality workers *into the firm* when the firm cannot readily observe innate worker ability.

[2] Lemieux, MacLeod and Parent (2009) discuss the growing importance of performance pay.

In each of these examples, effort is clearly important; firms want to hire designers, bankers and innovators who will invest heavily in the activity at hand, voters want their representatives to work hard on their behalf, and spectators enjoy high action games. However, selection may also be a prime objective of the contest organizer—a client may desire the most creative design firm, voters may value the most skilled politician, and a board may want the smartest executive to lead the company.

We explore elimination tournaments as selection mechanisms with a two-stage match-pair model. Our analysis yields two main results: First, we identify a shadow effect of future competition—the weaker the expected competitor in the next stage, the greater the probability that the stronger player wins in the current match. Second, we find an effort spillover effect—with negative spillover, effort in earlier stages lowers the probability that the stronger player wins. We also find that noise around effort and a flatter prize structure reduce the probability that the stronger player wins. Finally, our analysis of an underdog advantage broadly rationalizes the conflicting findings in the existing literature.

One particular strength of our model is that its predictions are framed in terms of outcomes—effort alone is notoriously difficult to measure in the field. We test our theoretical predictions using the outcomes of high-stakes matches; we exploit the random assignment of players in professional tennis tournament draws. Examining the effect of changes in the skill of the expected competitor in the next round, we find evidence of a shadow effect in all but the last rounds of play. Spillover in tennis tournaments appears to have a negative impact on the probability of winning, particularly for the stronger player. We also examine tennis betting markets and find that bookmakers' prices reflect both spillover from past competition and the shadow of future opponents.

In early work on knock-out tournaments, Rosen (1986) models a multi-stage contest where players have Tullock-style contest success functions. Rosen's main result explains the skewed compensation distributions found in many firms—extra rewards are required in late stages of these elimination tournaments to maintain equal levels of effort across stages. Harbaugh and Klumpp (2005) study a special case of Rosen's model with a single prize and where the total supply of effort is fixed across two periods and equal for all players. Because effort is costless in their model, players exhaust all of their remaining effort in the final period. As a result, the weaker player always exert more effort than the stronger player in the first stage—the stronger player conserves his effort in anticipation of stiff competition in a final stage match against an equally skilled opponent. In this context, "spillover" between stages disadvantages the low-skill players in the final round.

Searls (1963) compares the statistical properties of single- and double-elimination contests and predicts that single-elimination events are most likely to select the highest ability player as the winner. Groh *et al.* (2008) describe the optimal seeding of heterogeneous players according to the contest designer's objective. Modeling contests as all-pay auctions, they find that common seeding rules that match weakest to strongest players in the semifinals maximize the probability that the strongest player wins overall.

Ryvkin (2009) considers the elasticities of a player's equilibrium effort with respect to his own ability and the abilities of his opponents across several tournament formats. In elimination tournaments with weakly heterogeneous players, he finds that the abilities of opponents in the more distant future have a lower impact on a player's equilibrium effort than does the ability of the current opponent. Ryvkin also shows that, when players' relative abilities are uniformly distributed, a "balanced" seeding can eliminate the dependence of a player's equilibrium effort on his opponents' abilities.

Our effort spillover prediction relates to previous work on fatigue in dynamic competition. Ryvkin (2011) presents a winner-take-all model where homogeneous players face a binary effort decision and effort has no explicit cost—these features are in stark contrast to our model where player are heterogeneous and effort is a continuous and costly choice variable in a multi-prize tournament. In his work, fatigue accumulates across stages and players have no opportunity to refresh their effort resources. Among other results, he finds that equilibrium effort is decreasing in fatigue. Our contribution complements and extend his theoretical and experimental fatigue result to a more flexible and descriptive context. Moreover, we consider explicitly—theoretically and in the field—the interaction between negative spillover from the past and the shadow of expected future competition.

Sunde (2009) tests the incentive effect of player heterogeneity using data from selected professional tennis tournaments. He finds that heterogeneity impacts the effort choice of the stronger player more than it changes the effort of the weaker player in a match. In his analysis, this means that the weaker player wins fewer games per set and the stronger player wins more games per set as heterogeneity increases. However, these effects are not symmetric: for an equal change in rank disparity, the increase in the number of games won by the stronger player is smaller than the decrease in the number of games lost by the weaker player. In contrast to Sunde's work, we study the role of skill heterogeneity across multiple stages of an event—that is, we examine the incentive impact of ability differences with past, current, and (expected) future opponents. The effects of player heterogeneity on effort in one-shot tournaments has been studied both theoretically (e.g. Baik, 1994; Moldovanu and Sela, 2001; Szymanski and Valletti, 2005; Minor, 2011) and empirically (e.g. Knoeber and Thurman, 1994; Brown, forthcoming).

The paper is organized as follows: Section 1 presents a two-stage model of an elimination tournament. We derive several propositions and outline the testable hypotheses. In Section

2, we describe our data and empirical strategy for testing these predictions. Section 3 describes the results and Section 4 discusses the spillover and shadow effects in the context of betting markets. We conclude in Section 5 and discuss the implications of our findings for contest designers.

1 Theory

We study a new theoretical version of knockout tournaments that we describe as "sequentially-resolved elimination tournaments." Matches in each stage are staggered across time; within a stage, players in later matches learn the identity of their potential future opponent from outcomes of earlier matches. Sequential play is often found in practice; for example, in firm-level tournaments, simultaneous promotions to division vice-president may be rare. Instead, the identity of the new appointee is known to other workers still competing for a parallel executive spot—the hopeful workers now know their future opponent for advancement beyond vice-president. To our knowledge, we are the first to consider such a format theoretically.

We use an additive noise model, as in Lazear and Rosen's (1981) foundational work on one-shot labor tournaments, to focus on the dynamics of a multi-stage elimination tournament. This structure is in contrast with other models of elimination tournaments where all matches in a given stage occur simultaneously (for example, see Stracke (2011)) and the contest success function takes on a Tullock form (see Rosen (1986)).[3]

In the following section, we explore the role of past and future competition on tournament outcomes. We present a model that is simple enough to clearly inform our empirical tests, yet rich enough to capture common features of high-stakes, multi-stage tournaments. Namely, we model an elimination tournament with heterogeneously skilled players competing in sustained competition—one could imagine professionals of varying abilities competing over months or years for a prized position within the firm.[4] In the main text, we consider these spillover and shadow effects separately for expositional ease; however, in the appendix, we present an analysis of the effects operating simultaneously. Combining the effects does not change the general predictions of the model.

Our theory results describe the probability that the stronger player wins in different stages of the elimination event. These predictions speak directly to our broader research question of "selecting the best." That is, our comparative statics results provide predictions about when the strongest player is most likely to advance to future rounds of competition

[3] Unlike the Tullock model where competitors exert the equal effort in the final stage regardless of their relative abilities, the additive noise model allows for heterogeneous efforts in all stages.

[4] In a sports context, our model better reflects the dynamics of an endurance event (e.g. tennis) than competition requiring a short burst of effort (e.g. power lifting).

and, ultimately, win the tournament.

1.1 Model Set-Up

Consider a two-stage elimination tournament with four players, where the players who win in the first stage advance to the final stage. The overall tournament winner receives a prize of V_W, while the second-place competitor receives a prize V_L. Let $V_W > V_L > 0$ and define the prize spread $\Delta V = V_W - V_L$. Let player i's total cost be a function of his effort x_i and his cost type c_i.[5] We denote player i's costs as $c_i \gamma(x_i)$, where $\gamma'(x_i) > 0$, $\gamma'(0) = 0$ and $\gamma''(x_i) > 0$. We assume that cost types, c_i, vary across all players and are commonly known amongst competitors.

For ease of exposition, we model heterogeneity through players' cost types. However, several alternative models produce identical results: for example, if we instead capture heterogeneity across valuations by defining a player's prize value as $\frac{V}{c_i}$ or by allowing the impact of an additional unit of effort on a player's probability of winning to vary across competitors. It can also be shown that capturing heterogeneity by varying cost function convexity leads to similar results.

Recall that matches in the first-stage are sequential. Assume that players 3 and 4 compete first. Then, player 1 faces player 2 knowing the outcome of the previous match. Without loss of generality, we assume that player 3 won his match against player 4.

1.1.1 Final Stage

Assume that player 1 won his first-stage match. To find the equilibrium of the multi-stage game, we begin by analyzing the strategies of player 1 and his opponent player 3 in the *final* stage. Define player 1's expected payoff function as

$$\pi_{1,final} = P_1(x_1, x_3)\,\Delta V - c_1 \gamma(x_1) + V_L \tag{1}$$

where his probability of winning takes the following form:

$$P_1(x_1, x_3) = \begin{cases} 1 \text{ if } x_1 + \varepsilon_1 > x_3 + \varepsilon_3 \\ \frac{1}{2} \text{ if } x_1 + \varepsilon_1 = x_3 + \varepsilon_3 \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

[5] One could define a mapping $E : \mathbb{R}_+^N \to \mathbb{R}_+^1$ that collapses levels of N effort-generating activities to the real line. The overall cost of effort is then strictly increasing in the resultant scalar x_i.

where $x_i + \varepsilon_i$ is player i's level of output. Output is a function of both effort x_i and a random noise term ε_i. In definition (2), the probability that player 1 wins is increasing in his own effort and decreasing in the effort of his opponent.

Define $\varepsilon = \varepsilon_3 - \varepsilon_1$ and let ε be distributed according to some distribution G such that probability (2) can be written as

$$P_1(x_1, x_3) = P_1(x_1 - x_3 > \varepsilon) = G(x_1 - x_3) \tag{3}$$

Now, player 1's payoff function (1) can be written as

$$\pi_{1,final} = G(x_1 - x_3)\,\Delta V - c_1\gamma(x_1) + V_L \tag{4}$$

and his first order condition is

$$\frac{\partial \pi_{1,final}}{\partial x_1} = G'(x_1 - x_3)\,\Delta V - c_1\gamma'(x_1) = 0 \tag{5}$$

Following Konrad (2009) and Ederer (2010), we assume that G is distributed uniformly with the following support[6]

$$G \sim U\left[-\frac{1}{2}a, \frac{1}{2}a\right]$$

and, therefore,

$$G' = \frac{1}{a}$$

The assumption that G is uniformly distributed removes the strategic interdependence of players' current period effort choices (Konrad, 2009). This allows us to isolate the consequences of past effort choices and potential future competition on current-stage effort. In a firm context, this would assume that a worker's optimal effort choice is independent of the identity of his current opponent; of course, in earlier stages, his optimal effort depends on his expectations about future opponents' identities. In the Appendix, we relax this assumption of same-stage independence and allow players' optimal effort choices to depend on both their current *and* future opponents. We show that the general predictions of the model continue to hold with more general distributions that allow for same-stage interdependence, including the normal distribution.

Rewriting the first order condition (5) yields:

$$\frac{\partial \pi_{1,final}}{\partial x_1} = \frac{\Delta V}{a} - c_1\gamma'(x_1) = 0$$

[6] See the Appendix for conditions on the primitives of the model that assure that G is well-defined.

which we can rearrange as the following expression:

$$\gamma'(x_i) = \frac{\Delta V}{ac_i} \text{ for } i = 1, 3 \tag{6}$$

Assume for the remainder of the analysis that player 1 is the stronger player $(c_1 < c_3)$. Then, expression (6) implies player 1 exerts more equilibrium effort in the final stage $(x_1^* > x_3^*)$. This inequality implies that the stronger player is more likely to win in the final stage, relatively to his weaker opponent—that is, the better player is more likely to be "selected" as the overall tournament winner.

In the final round, since both players are guaranteed at least the second prize V_L, increasing the first prize amounts to increasing the stakes of the contest. As expected, higher stakes leads to more effort from both players, though the stronger player increases his effort more than the weaker player. Also, increasing the noise around effort (i.e., increasing a, the width of the support of G) reduces equilibrium effort, particularly for the stronger player. Finally, as expected, effort choices are increasing in ability.

1.1.2 First Stage

Define z_1 and z_2 as the efforts of players 1 and 2 in the first stage. Player 1's expected payoff function in the first stage is

$$\pi_{1,first} = P_1(z_1, z_2)\tilde{V}_1 - c_1\gamma(z_1) \tag{7}$$

where $\tilde{V}_1$ is his continuation value (i.e., his payoff in the final stage):

$$\tilde{V}_1 = \pi_1(x_1^*, x_3^*(c_3)).$$

Equation (7) yields the first order condition

$$\frac{\partial \pi_{1,first}}{\partial z_1} = \frac{\tilde{V}_1}{a} - c_1\gamma'(z_1) = 0$$

which we can rearrange, for either player, as the following expression:

$$\gamma'(z_i) = \frac{\tilde{V}_i}{ac_i} \text{ for } i = 1, 2 \tag{8}$$

As in the final stage, equilibrium effort is increasing in ability and the continuation value. Increasing the noise around effort has an adverse effect on first stage effort.

Recall that, at the start of their match, players 1 and 2 already know the outcome of the other first-stage match between players 3 and 4. Of course, this means that players 3 and 4 did not know exactly the identity of *their* future opponent. Instead, we assume that they formed an expectation of their continuation value as follows:

$$\mathrm{E}\left[\tilde{V}_i\right] = p_{1|i}\tilde{V}_i\left(x_i^*, x_1^*\right) + \left(1 - p_{1|i}\right)\tilde{V}_i\left(x_i^*, x_2^*\right) \text{ for } i = 3, 4$$

where $p_{1|i}$ is the equilibrium probability that player 1 wins knowing that he will face player i in the final stage.[7] Note that player i cannot influence this probability $p_{1|i}$ because it is a function of the *realized* outcome of the completed match between players 3 and 4. This simplifies our analysis because player i's first-stage effort z_i does not change this probability $p_{1|i}$. Thus, for players 3 and 4, we can restate the expression of their equilibrium effort (8) as

$$\gamma'\left(z_i\right) = \frac{\mathrm{E}\left[\tilde{V}_i\right]}{ac_i} \text{ for } i = 3, 4$$

and the analysis described above for players 1 and 2 applies similarly.

1.2 Shadow of Future Competition

We can use the model to understand the impact of known or expected future competition on the likelihood that strong players advance to future stages of the tournament—of course, this influences the likelihood that a high-skill player is selected as the overall winner.

Consider a decrease in the skill of the future opponent, player 3 (i.e. c_3 increases). This change has the effect of increasing the continuation value for both players 1 and 2 in the *first* stage. Since player 1 has a lower cost of effort than player 2, player 1 will increase his first-stage effort more than player 2 because his return to a change in the continuation value is greater.

From the final-stage first order condition, equation (6), it follows that x_3^* decreases as c_3 increases. To understand the effect of decreasing x_3^* on the final stage payoff $\pi_{1,final}$,we take the derivative of equation (4)

$$\frac{\partial \pi_{1,final}^*}{\partial x_3^*} = G'\left(x_1^* - x_3^*\right)\Delta V = -\frac{\Delta V}{a} < 0$$

That is, as the final stage opponent's effort decreases, the player 1's final stage equilibrium payoff increases. A change in the final round opponent's skill will have an equal effect on player 2: $\frac{\partial \pi_{1,final}^*}{\partial x_3^*} = \frac{\partial \pi_{2,final}^*}{\partial x_3^*} = -\frac{\Delta V}{a}$.

[7] When player 1 is stronger than player 2, $\tilde{V}_i\left(x_i^*, x_1^*\right) < \mathrm{E}\left[\tilde{V}_i\right] < \tilde{V}_i\left(x_i^*, x_2^*\right)$ for $i = 3, 4$.

Since the change in the continuation value is the same for players 1 and 2, the stronger player will increase his effort (z_1^*) more than player 2 will increase his effort (z_2^*) .To see this, add a term W to represent the (equal) change in the continuation value to equation (8):

$$\gamma'(z_i) = \frac{\tilde{V}_i + W}{ac_i}$$

Note that the impact of the increase in the continuation value is larger for player 1 relative to player 2 $\left(\frac{W}{ac_1} > \frac{W}{ac_2}\right)$. We require that both players' effort choices are sufficiently sensitive to a change in marginal benefit—players must be able to scale up their current equilibrium effort levels in response to increased tournament rewards. Mathematically, this requires that $\frac{\gamma''(z_1^*)}{\gamma''(z_2^*)} < \frac{c_2}{c_1}$. This condition on the slope of the marginal cost curve insures that the stronger player is sufficiently sensitive to a change in marginal benefit induced by a weaker future opponent; this further improves the stronger player's probability of winning in the first stage. For example, it can be shown that this inequality is satisfied with all cost functions of the form cx^p where $p > 1$. More generally, a sufficient but slack condition for the inequality is $\gamma''' \leq 0$.[8] Note that cost functions of the form cx^p with $p > 2$ fail to satisfy $\gamma''' \leq 0$, but still meet the necessary condition of $\frac{\gamma''(z_1^*)}{\gamma''(z_2^*)} < \frac{c_2}{c_1}$.

This analysis gives us the following proposition:

Proposition 1 *Assuming that* $\frac{\gamma''(z_1^*)}{\gamma''(z_2^*)} < \frac{c_2}{c_1}$, *as the skill of the future competitor in the final stage declines (increases), the stronger player becomes even more (less) likely to win in the first stage and thus more (less) likely to be selected as the overall tournament winner.*

1.3 Effort Spillover

We can also examine effort spillover between stages of the tournament. Spillover can take either a positive or negative form. Positive spillover might reflect learning (by doing), skill building or momentum within a firm. For example, an innovation team whose proposal advances to a second stage of funding might benefit from its first-stage experience, both technical and relational. With positive spillover, second-stage effort is less costly than first stage effort. In contrast, negative spillover might reflect fatigue or reduced resources in later stages. For example, architects competing in design competitions might exhaust their creative resources in early stages and have only limited energy for second-stage proposals. In this case, second-stage effort is more costly than first-stage effort.[9]

[8] Ederer (2010) discusses models where the results instead depend critically on the sign of γ'''.

[9] Different notions of spillover have been explored in the literature in settings where players with exogenous, fixed resources make effort allocation decisions over multiple periods of play. For recent examples, see Sela and Erez (2011) and Harbaugh and Klumpp (2005).

Consider a scenario where effort expended by a player in the first stage influences his effort in the final stage. We can rewrite player 1's final-stage payoff as

$$\pi_{1,final} = G\left(x_1 - x_3\right)\Delta V - c_1\gamma\left(x_1, z_1\right) + V_L$$

where the cost function reflects current *and* past effort.

First, consider the case where a player's marginal cost of effort in the final stage is unaffected by previous stage effort: $\frac{\partial\gamma(x_1,z_1)}{\partial x_1\partial z_1} = 0$. If previous effort appears only as a fixed cost in the final stage, we would expect no change in final-stage effort. For example, a design team that submits an innovative proposal in the first stage might require specialized equipment to complete the building phase in the second stage.

To study a negative spillover effect, we let a player's marginal cost of effort in the final stage be increasing in first-stage effort: $\frac{\partial\gamma(x_1,z_1)}{\partial x_1\partial z_1} > 0$. Consider again expression (6).We see that final stage equilibrium effort is strictly decreasing in first stage effort because the marginal cost of final-stage effort is increasing in first-stage effort. With positive spillover, a player's marginal cost of effort in the final stage is decreasing in first-stage effort: $\frac{\partial\gamma(x_1,z_1)}{\partial x_1\partial z_1} < 0$. Now, from expression (6), final-stage equilibrium effort is strictly increasing in first-stage effort.

We can re-write player 1's marginal cost of effort with spillover as $\gamma'\left(x_1, z_1\right) = \gamma'\left(x_1, 0\right)k$ where $k > 1$ for net negative spillover and $k < 1$ for net positive spillover. Revisiting expression (6) ,we can rewrite marginal cost as

$$\gamma'\left(x_1\right)k = \frac{\Delta V}{ac_1} \quad \text{or} \quad \gamma'\left(x_1\right) = \frac{\Delta V}{a\tilde{c}_1} \tag{9}$$

where $\tilde{c}_1 = c_1 k$. Straightforward calculations show that $\frac{\partial\pi_{1,final}}{\partial c_1} < 0$. Therefore, when $\tilde{c}_1 > c_1$, final period profit $\tilde{V}_1$ is less than when there is no effort spillover. Conversely, when $\tilde{c}_1 < c_1$, final period profit $\tilde{V}_1$ is greater than when there is no spillover.

The presence of the negative (positive) spillover also reduces (increases) *first-stage* effort; in expression (8) , lower (higher) $\tilde{V}_1$ leads to lower (higher) effort.

Since negative spillover decreases final-stage equilibrium effort, an increase in first-stage effort implies a lower probability of success in the final stage, holding the opponent's effort and skill constant. Of course, the opposite is true for positive spillover. On net, when both players in a match suffer negative spillover, the increase in effort costs in the current stage reduces the probability that the stronger player wins. The direction and impact of spillover depends on the context and, thus, is an empirical question.

As seen in expressions (9) , an increase in noise a has a similar impact to an increase in negative spillover. That is, an increase in the noise of the effort-to-output relationship reduces the disparity between players' efforts, reducing the probability that the stronger player wins. We collect these findings in the following proposition:

Proposition 2 *In any stage, a common proportional increase in effective cost type or an increase in noise decreases the probability that the stronger player is selected as the winner. That is, when player i's effective cost type is kc_i, $G(z_1^* - z_2^*) \to 0.5$ and $G(x_1^* - x_3^*) \to 0.5$ as the degree of negative spillover $k \to \infty$ or the noise parameter $a \to \infty$.*

Proof. From expressions (6) and (8) ,we can see that as cost types converge to infinity, effort becomes so costly that no effort is exerted. Thus, the stronger player should fare worse in events that are proportionally more costly for all players in every round. With increased noise, a similar logic applies—as the support of G goes to infinity, equilibrium effort converges to zero—the marginal return to effort declines. In the limit, effort has no impact on a player's probability of success and, therefore, no effort is exerted. ■

This proposition suggests that, with negative spillover, weaker players might support costlier competitive conditions—for example, a weaker player might advocate for more stringent common standards or more difficult tasks.

Our result that negative spillover levels the playing field in both stages is in contrast to Harbaugh and Klumpp's (2005) finding that intertemporal tradeoffs level the playing field for the first stage but do the opposite effect in the final stage. Their result is sensitive to the assumptions that effort is costless and that players' total efforts are equally constrained.

1.4 Prize Effect

We can explore how changes in the prize structure affect equilibrium effort choice and tournament outcomes. In particular, we study the effect of increasing the spread, ΔV, between first and second prize. To simplify our exposition, we assume that participants' costs are quadratic and hold constant the skill level of players in each stage.

1.4.1 Prize Effect - Final Stage

From equation (6) , we know that final period effort is

$$x_i^* = \frac{\Delta V}{2ac_i}$$

Since $c_1 < c_2$, increasing ΔV increases the effort of the stronger player more than for the weaker player. That is, the effort disparity between players increases as the relative stakes increase.

1.4.2 Prize Effect - First stage

From our analysis above, we can write out player i's continuation value when he faces quadratic costs and player j:

$$\widetilde{V}_i = \left(\frac{\frac{\Delta V}{2ac_i} - \frac{\Delta V}{2ac_j} + \frac{a}{2}}{a} \right) \Delta V - c_i \left(\frac{\Delta V}{2ac_i} \right)^2 + V_L$$

Using equation (8) and our expression for $\widetilde{V}_i$,we can solve explicitly for first-stage equilibrium effort:

$$z_i^* = \frac{\left(\frac{\frac{\Delta V}{2ac_i} - \frac{\Delta V}{2ac_j} + \frac{a}{2}}{a} \right) \Delta V - c_i \left(\frac{\Delta V}{2ac_i} \right)^2 + V_L}{2c_i a}$$

The following expression describes the change in equilibrium effort resulting from a change in ΔV:

$$\begin{aligned} \frac{\partial z_i^*}{\partial \Delta V} &: \frac{\frac{\Delta V}{a^2} \left(\frac{1}{c_i} - \frac{1}{c_j} \right) + \frac{1}{2} - \frac{\Delta V}{2a^2 c_i}}{2c_i a} \\ &= \frac{\frac{\Delta V}{a^2} \left(\frac{\frac{1}{2}}{c_i} - \frac{1}{c_j} \right) + \frac{1}{2}}{2c_i a} \end{aligned}$$

This leads to the following proposition:

Proposition 3 *With quadratic costs, for a given prize spread increase, the stronger player is even more likely to be selected as the winner in either stage.*

Proof. See Appendix 6.3. ■

This proposition suggests that tournaments with steeper prize structures will more often lead to the success and selection of the most able player.

1.5 Underdog Advantage

Now, we can compare the difference in effort choices between the stronger and weaker player across stages to determine differences in their probabilities of winning, holding fixed the skill differential between players. For example, we can compare the outcome of a final-stage match between players of cost types 1 and 3 with the outcome of a first-stage match between players of these same cost types.

Assume that a player of cost type 1 is stronger than a player of cost type 3 $(c_1 < c_3)$. As long as the final-stage losing prize V_L is large enough relative to ΔV, then effort disparity across stages is ordered: $x_1^* - x_3^* < z_1^* - z_3^*$. Since effort disparity is greater in the first stage than in the final stage, expression (3) necessarily means that the stronger player has a greater probability of winning in the first stage relative to the final stage. That is, when certain prize conditions are satisfied, the weaker player has a *lesser* probability of *losing* against a stronger player in the final stage relative to his probability of losing against that same opponent in the first stage of an otherwise identical tournament. The intuition is as follows: As we increase the losing prize V_L while holding prize spread ΔV constant, the difference in first-stage efforts $(z_1^* - z_3^*)$ increases because the stronger player is more sensitive to changes in the continuation value. In contrast, the difference in final stage efforts $(x_1^* - x_3^*)$ remains the same, since the losing prize does not enter the first order condition for the final stage.

While the contest success functions are not directly comparable, this result broadly captures conflicting findings in the existing literature: While Rosen (1986) finds that there is always an "underdog advantage" in his Tullock-style contest with multiple prizes, Harbaugh and Klumpp (2005) find the opposite is true in winner-take-all contests when the supply of effort is fixed and unused effort is valueless.[10] We find that the presence or absence of the underdog advantage will depend critically on the relative sizes of the first- and second-prizes.

This leads to our fourth proposition:

Proposition 4 $P_{1,final}(x_1^*, x_3^*(c_3)) < P_{1,first}(z_1^*, z_2^*(c_2))$ *when* $c_2 = c_3$ *and* $V_L > \Delta V + c_2\gamma\left(h\left(\frac{\Delta V}{ac_2}\right)\right)$. *With a sufficiently large second place prize relative to the first place prize, the probability that the weaker player wins in the final stage (and thus is selected as the overall tournament winner) is greater than the probability that he wins in the first stage, holding opponent skill constant.*

Proof. See Appendix 6.3. ■

1.6 Model Predictions

The theory model outlined above provides the following main predictions:

1. **Shadow of Future Competitors:** The worse-ranked the expected competitor in the next stage, the greater the probability that the stronger player is selected as the winner in the current stage.

2. **Effort Spillover between Stages:** In equilibrium, increased negative (positive) spillover decreases (increases) the probability that the stronger player is selected as the winner in the final stage.

[10] Harbaugh and Klumpp's "underdog disadvantage" is reduced (and, in the limit, eliminated) by the introduction of a sufficiently large second prize.

We also explore some additional comparative static results:

Noise in Effort: The noisier the effort-to-output relationship, the lower the probability that the stronger player wins in either stage.

Prize Spread: A steeper prize structure improves the stronger player's probability of success in all stages.

Underdog Advantage in Final Stage: Fixing the competitors' abilities and given a sufficiently large (small) second-place prize, the probability of winning is greater (smaller) for the weaker player in the final stage, relative to the first stage.

One strength of this particular model is that, while these hypotheses emerge from differences in the abilities and efforts of players, the testable implications can be framed in terms of outcomes. This means that although effort is notoriously difficult to assess in field data, we can test the two main predictions of the model by observing tournament *outcomes*. That is, we can identify readily whether changes in the strength of future competition or the degree of negative spillover impacts whether the most able competitor is selected as the tournament winner. In the following sections, we describe our data and empirical analysis.

2 Data

Professional tennis offers an ideal environment in which to test the empirical implications of the theory.[11] Tennis events are single-elimination tournaments—only winning players advance to successive stages until two players meet in the final stage to determine the overall winner. Prizes increase across stages with the largest prize going to the overall winner. The distribution of prizes is known in advance of all tournaments. The financial stakes are substantial and vary across events—for example, the total purse for the 2009 US Open singles competition was \$16 million with a \$1.7 million prize for first place, while the total purse for the 2009 SAP Open was \$531,000 and the winner received \$90,925.

Our empirical analysis exploits the random nature of the initial tournament draw. By ATP rules, the top 20 to 25% of players in an event (the "seeds") are distributed across the draw: the top two seeds are placed on opposite ends of the draw; the next two seeds are randomly assigned to interior slots on the draw; the next four seeds are randomly assigned to other slots; etc. After the seeded players have been assigned, the remaining players are then randomly placed to matches prior to the start of the event.[12] This variation provides the

[11] While tennis tournament organizers may various objections beyond selection, it is the *structure* of these tournaments that lends itself to our empirical tests. That is, one would expect tournament competitors to respond to the structure and incentives, not the *reason* for that contest design

[12] Note that the seeding is done according to rank within a tournament; the top seed in one event may have a different skill level than the top seed in another tournament.

identification for our empirical approach—we can observe the same skilled player compete against a variety of randomly-assigned opponents. For example, in our data, we can observe the fourth best player in the world play against competitors ranked 50^{th}, 100^{th}, and 250^{th} in the first round of similar tournaments in a single year.

The structure of tennis tournaments is particularly conducive to studying the shadow of future competition—both players (and the econometrician) know the competitors in the parallel match. In some cases, players know exactly who they would face in the next round; in other cases, they can make reasonable predictions about upcoming opponents. Moreover, player ability is also observable to players and researchers—past performance, as well as world rankings statistics, are widely available. Figure 1 presents the draw from the 2007 Swiss Indoors tournament in Basel. In the first round, Del Potro and Russell knew that their next opponent would be either Federer or Berrer. Of course, given the ability difference between these possible future opponents, Del Potro and Russell were likely predicting that their second-round opponent would be Federer.

Data from professional tennis has been used in other research: Walker and Wooders (2001) used video footage and data from the finals of 10 Grand Slam events to identify mixed strategies. Malueg and Yates (2010) study best-of-three contests using four years of data from professional tennis matches with evenly-skilled opponents. They find that the winner of the first set of a match tends to exert more effort in the second set than does the loser and, in the event of a third set, players exert equal effort. Forrest and McHale (2007) use professional tennis tour and bookmaking data and find a modest long-shot bias. Gonzalez-Diaz *et al.* (2010) use data from US Open tournaments to assess individual players' abilities to adjust their performance depending on the importance of the competitive situation. They find that heterogeneity in this ability drives differences in players' long-term success. Using detailed data from the men's and women's professional tennis circuits, Gilsdorf and Sukhatme (2008a and 2008b) find that larger marginal prizes increase the probability that the stronger player wins.

2.1 Professional Tennis Match Data

To test the predictions outlined in the theory, we examine the behavior of professional tennis players in 615 international tournaments on the ATP World Tour between January 2001 and June 2010. The data include game-level scores and player attributes for men's singles matches (available at http://www.tennis-data.co.uk). The four "Grand Slam" events—the Australian, French, and US Opens, and Wimbledon—are included in the data. All of the tournaments are multi-round, single-elimination events played over several days.

Tournament draws may include 28, 32, 48, 56, 96 or 128 players. Of the 615 events in the data, 433 tournaments consist of five rounds of play—rounds 1 and 2, quarterfinals, semifinals, and the final. Six rounds are played in 128 events. Fifty-four tournaments, including the Grand Slam events, consist of seven rounds of play—rounds 1 to 4, quarterfinals, semifinals, and the final. Most ATP events are best-of-three sets, while the Grand Slam events are best-of-five sets.[13] Figure 1 is a typical draw for a five-round, 32-player tournament. Depending on the number of competitors, first-round byes may be awarded to the top-ranked players.[14]

World rankings (officially called the South African Airways ATP Rankings) are based on points that players accumulate over the previous 12 months. The ATP points directly reflect the pyramid structure of tournaments. More points are awarded to players who advance in top tournaments; for example, a Grand Slam winners earns the maximum points awarded for a single event.[15] ATP rankings are simply a rank-order of all players by their accumulated points. In our analysis, we use the ATP rankings to account for players' skill levels.[16]

Table 1 presents summary statistics from over 28,000 men's professional tennis matches. On average, matches are decided after 23 games; however, players play more games on average in the final round than in the first or semifinal rounds (p-$value < 0.01$). Match winners are significantly more skilled than losers (p-$value < 0.01$). Tournament winners typically rank 30th in the world, while second-place finishers are 45th in the rankings. Tournaments' seeding formats generally pair the weakest players against the strongest players in the first round. Consequently, the disparity in rankings decreases as players advance. To consider the competitive balance of matches across rounds, we also report the rankings ratio (worse rank divided by better rank). While mean rankings ratios remain relatively stable across rounds, the variance appears to decline. The skill-related summary statistics suggest that while high-skill players do not *always* win their matches, on average, opponents become closer in ability as tournaments progress.

[13] To win a set, a player must win at least six games and at least two games more than his opponent. A game is won by the player who wins at least four points and at least two more than his opponent. Set tie-break rules vary by tournament.

[14] Byes automatically advance a player to the next round.

[15] For details of the world ranking system, see the 2011 ATP World Tour Rulebook, available online at www.atpworldtour.com.

[16] Klaassen and Magnus (2003) suggest a transformation of rankings to account for differences in ability between high- and low-skilled players. They calculate a player's ability as $R = K + 1 - \log_2 (ranking)$,where K is the total number of rounds in the tournament and $ranking$ is the player's tournament seed. All of our analyses are robust to this alternative measure of skill heterogeneity—results with the Klaassen-Magnus transformation are qualitatively very similar to the results using ATP rankings and are not reported.

3 Results

In this section, we present empirical tests of the theoretical predictions. We first examine performance data from professional tennis matches, presenting empirical results supporting both spillover and shadow effects. We then report additional evidence relating to the model's predictions about an underdog advantage. In Section (4), we ask whether shadow and spillover effects have been priced into betting markets. Although this additional analysis is not a direct test of the theory, it does provide further support for the importance of understanding these phenomena.

3.1 Spillover and Shadow Effects

Proposition 1 states that weaker future competition will increase the stronger competitor's probability of success in the current stage, thus increasing the probability that the tournament selects the best player as the winner. This prediction follows from the observation that, while weaker future competition will cause both players to increase their effort in the current period, the current effort of the better-ranked player increases even more than the current effort of his worse-ranked opponent. Proposition 2 considers the role of spillover in effort choice. The direction of the spillover effect is often an empirical question; however, one might expect negative spillover in events that require intense effort exertion over a short period of time. In professional tennis, players may face a higher cost of effort if their total exertion in previous matches induced lasting fatigue.

The following specification allows us to study the effects of shadow and spillover simultaneously:

$$\begin{aligned} strongwins_{mt} = \beta_0 + \beta_1 Future_{mt} + \beta_2 Current_{mt} \\ + \beta_3 SPastGames_{it} + \beta_4 WPastGames_{it} + \gamma X_t + \varepsilon_{mt} \end{aligned} \tag{10}$$

where $strongwins_{mt}$ is a binary indicator of whether the better-ranked player in match m won in a stated round of tournament t, $Future_{mt}$ represents the expected ability of the opponent in the next round, $Current_{mt}$ represents the heterogeneity of players' skills in the current match, $SPastGames_{it}$ is the number of games played in all previous rounds of the tournament by the better-ranked player, $WPastGames_{it}$ is the number of games played in all previous rounds of the tournament by the worse-ranked player, X_t is a matrix of tournament-level controls, and ε_{mt} is the error term. We estimate all equations using OLS with a robust variance estimator; results are quantitatively very similar for a probit specification and are not reported.

In the reported regression, $Current_{mt}$ is the ratio of the rank of the worse player and the better player. $Future_{mt}$ is the rank associated with the stronger player in the parallel match. For example, for the 2007 Swiss Indoors tournament (see Figure 1), the expected future opponent for the match between Del Potro and Russell would be Federer. For the Del Potro-Russell match, $Current_{mt} = \frac{71}{49}$ and $Future_{mt} = 1$. Note that this construction of $Future_{mt}$ is a conservative one—we are assuming that the future competitor will always be the better of the two potential opponents in the next round. This means that, on average, we are understating the continuation value for players in the current round. Consequently, our coefficient estimates on $Future_{mt}$ will understate the actual shadow effect.[17] Results are qualitatively similar if we instead use an average of future opponents' rankings.

Tournament-specific fixed effects capture average event-level characteristics and control for differences between tournaments (e.g. media attention). Total purse size for any given event has not varied substantially across time. For example, the purse for the US Open has grown by an average of 3% each year from 1997 to 2011. Over the same period, the inflation rate was roughly the same. Therefore, real purse size was relatively stable and the purse effect is captured by tournament dummies. Additionally, because some tournaments have changed venues over time, we include additional controls for surface and court type.

Results: Table 2

Table 2 reports the estimated coefficients for regression (10) by tournament round.[18] The round-by-round analysis overcomes the confounding influence of changes in marginal prizes across rounds within a tournament, while still controlling for the differences in prizes for a given round across events. Moreover, it allows us to cleanly account for players' past exertion (and the hypothesized spillover effect) without concerns about the autocorrelation of the same player's performance and effort over a single tournament.

In all rounds before the semifinals, the coefficient on the shadow effect ($Future_{mt}$) is positive and statistically significant (*p-values* < 0.05). That is, the weaker the future opponent (i.e. a larger rank), the greater the probability that the stronger player is selected as the winner in the current round. The estimated effects for the quarterfinals and semifinals are also positive, although not statistically significant at conventional levels—this may re-

[17] While our data do not report start and end times for all matches, we can determine the order of play in some tournaments (e.g. 1,730 out of 10,812 first-round matches). Restricting our sample to only those data, we find that, in general, the coefficient on $Future_{mt}$ is larger when the parallel match has been resolved relative to case where the winner of the parallel match has not been determined. This difference conforms to our theory prediction; however, the coefficients are not statistically different from each other with these much smaller samples.

[18] Note that the first and last columns of Table 2 omit estimates for spillover and shadows, respectively—there is no spillover for players in the first round of a tournament and players face no shadow in the final round.

flect both small sample sizes and limited variation in opponents' skills at advanced stages of these tournaments. The magnitude of the coefficients may also be interpreted—we include mean and standard deviation values of future opponent rank in the table. For a one standard-deviation increase in future opponent's rank (decrease in ability), we estimate that the probability that the stronger player wins in the current round increases by 2.0 to 5.7 percentage points. Given that the average probability that the stronger player wins is approximately 65%, on average, the shadow effect represents a 5% increase in the probability of winning.

Coefficient estimates for the two spillover variables take on predicted signs and, in general, are statistically significant—more previous games for the stronger player decreases the probability he wins in the current match, while in general more previous games for the weaker player increases the chance that the stronger player wins. The magnitudes of the coefficients suggest that, on average, a one standard-deviation increase in the number of previous games is associated with a 10 and 4 percentage point decline in the probability of winning in the current match for the stronger and weaker player, respectively. For the stronger player in an average match, this represents a 15% decline in the probability of winning; for weaker players, this reflects a 11% decline in success.

The history of the stronger player appears to drive his current success more than the history of his opponent—from expression (11) of our model, we expect the stronger player to be more adversely affected than the weaker player for a given increase in spillover. That is, we predict that more negative spillover will reduce the likelihood that the stronger player is selected as the winner. Indeed, in the data, the effect of previous games played by the stronger player is often larger than the effect of the weaker player's previous games. T-tests comparing the magnitude of the spillover estimates ($H_o : \beta_3 = -\beta_4$) reject equality in the 2nd round, 4th round, quarter- and semi-finals (*p-values* < 0.05). We cannot reject the null of equal effects in the final round, perhaps because players tend to be relatively well-matched in terms of ability.

Comparing across rounds, the effects of spillover from both stronger and weaker players' histories are smaller in the final periods of tournaments relative to early rounds. This may be because many events provides additional rest periods for players between the later rounds of play, while early-round schedules often have players competing on consecutive days.

As expected, the coefficient on skill disparity in the current match is also positive and statistically significant (*p-value* < 0.01), indicating that increased heterogeneity between the players increases the probability that the stronger player wins.

We estimate, but do not report, results for regression (10) using ATP points as a measure of player skill. Estimates are qualitatively similar to results using ATP rankings.

3.2 Underdog Advantage

Rosen (1986) predicts that the weaker competitor's likelihood of success should always be higher in the final round relative to the probability that he wins earlier in an elimination tournament. Our proposition 4 provides a critical caveat, requiring that the second-place prize be sufficiently large relative to the gap between first and second-place prizes in order to induce an underdog advantage.

In the ATP match data, the weaker competitor (underdog) wins 34.1% of the matches in final rounds and 32.6% of matches in earlier rounds. However, this difference is not statistically significant at conventional levels. For robustness, we also ran a comparison while controlling for player skill and tournament-level heterogeneities. Again, we failed to find statistically-significant differences between the weaker players' probability of winning in final and non-final rounds. Although not conclusive, our analysis suggests that the losing prizes may not be sufficiently large (relative to the winning prizes) to induce an underdog advantage in the final round.

4 Betting Markets, Spillover and Shadows

While spillover and shadow effects have been relatively understudied in the literature, in this section, we explore whether active markets already recognize these dynamics of multi-stage competition. Indeed, by examining data from professional betting markets, we find compelling evidence that subtle spillover and shadow effects have been incorporated into prices.

The efficiency of prediction and betting markets has been studied extensively in the literature; for examples, see the survey by Vaughn Williams (1999). Prediction markets are founded on the argument that by aggregating information, competitive markets should result in prices that reflect all available information (Fama 1970). Therefore, driven by aggregated information and expectations, prediction market prices may offer good forecasts of actual outcomes (Spann and Skeira 2003).

Similarly, betting odds reflect bookmakers' predictions of future outcomes. Betting odds may change as new public or private information becomes available to the bookmaker and with changes in the volume of bets that may be driven by individual bettors' private information. As with formal prediction markets, we might expect betting odds to provide good forecasts.

Spann and Skeira (2008) compare forecasts from prediction markets and betting odds using data for German premier soccer league matches. They find that prediction markets and betting odds provide equally accurate forecasts. This result seems reasonable, since betting companies with inaccurate and inefficient odds should not survive.

To examine whether betting markets incorporate information about the effects of shadow and spillover, we estimate a regression similar to equation (10). Now, instead of a binary indicator of the actual outcome, the dependent variable is the probability that the stronger player wins the match as implied by betting markets.

Our data include closing odds from professional bookmakers for pre-match betting.[19] Woodland and Woodland (1999) note that bookmakers adjust odds based on the volume of bets, making the odds available as the betting market closes particularly rich in information. In our analysis, we use the median of the available odds data since the data from no single firm covered all matches.[20] Overall, there was little variation between odds posted by different bookmakers for the same match, perhaps because participants in tennis betting markets tend to be specialists and there is little casual betting (Forrest and McHale 2007). [21]

The accuracy of odds market predictions suggests that information beyond simple rankings are being priced in the market. Between 2001 and 2010, predictions from the market are correct for 69% of the 25,633 matches for which betting data are available. Given that the stronger player actually wins in 65% of matches, one might not be surprised by this accuracy if the market always predicted that the stronger player wins. However, in 18% of the matches, the betting odds imply that the weaker player is expected to win. Interestingly, these market predictions are accurate nearly 63% of the time. That is, these betting markets do almost as well predicting an upset as they do predicting a win by the stronger player. This is particularly notable since a naive assessment of the ATP rankings in these matches might suggest that the odds are still solidly against the weaker player—on average, the weaker player's rank is 2.1 times higher (worse) than his opponent.

Results: Table 3

Table 3 reports results for round-level regressions where the dependent variable is the probability that the stronger player wins as implied by the betting market. Overall, coefficient estimates suggest that the betting predictions incorporate information about players' past and expected future competition.

[19] Data from 11 betting firms (Bet365, Bet&Win, Centrebet, Expekt, Ladbrokes, Gamebookers, Interwetten, Pinnacles, Sportingbet, Stan James, and Unibet) are included in our main dataset obtained from www.tennis-data.co.uk. Several betting firms also offer in-play betting, but we focus our analysis on pre-match bets only.

[20] We calculate the probability odds from the decimal odds in the original data. Probability odds are 1/(decimal odds-1).

[21] A positive long-shot bias—where the market undervalues the true favorite and overvalues the long-shot—has been documented in tennis odds by Forrest and McHale (2007). However, the authors find this small bias is consistent over a broad range of match-ups. In contrast to some markets, they do not find any range with a negative long-shot bias.

Coefficient estimates for the effect of a stronger future opponent are positive and statistically significant for all rounds except for the shadow of the final round on the semifinals (*p-values* range from < 0.01 to 0.1). It is not surprising that we do not identify the effect of the shadow of the final on the semifinal, given the compressed distribution of skill at the end of the tournament.

Since the betting market closes only at the start of the match (and after the end of earlier rounds), players' past exertion information is readily available. Indeed, coefficient estimates for the stronger and weaker players' previous number of games are statistically significant (*p-values* < 0.01) and take on the expected signs. More previous games played by the stronger player is associated with a decrease in expectations of his success, while more previous games played by the weaker player is associated with an increase in expectations that the stronger player wins. As in Section 3.1 and as predicted by theory, the magnitude of these coefficients suggests that stronger players are more adversely affected by a given level of spillover relative to weaker players.

Greater heterogeneity in players' abilities may increase the market's expectation that the stronger player wins—the coefficient on rank ratio is positive and statistically significant in all rounds (*p-values* < 0.01).

Overall, we find strong evidence that prices in tennis betting markets reflect both the shadow and spillover effects predicted by our model. Interestingly, we again find no evidence of an underdog advantage—higher predicted odds for the weaker player in the final relative to earlier rounds—in the betting data.

5 Conclusion

In this paper, we explore a class of contests we call "sequentially-resolved elimination tournaments." We present a two-stage, match-pair tournament model that provides two sharp results: (a) a shadow effect of future competition—the weaker the expected competitor in the final stage, the greater the probability that the stronger player is selected as the winner in the first match; (b) an effort spillover effect—increased negative (positive) spillover decreases (increases) the probability that the stronger player wins in the final stage. We also identify a noise effect, whereby increasing noise around effort reduces the probability that the stronger player wins in either stage, and a prize spread effect, whereby increasing stakes improves the stronger player's probability of success in both stages. We provide an analysis of the underdog advantage suggesting that its presence depends on the distribution of prizes.

We test our two main theoretical hypotheses using data from professional tennis matches. We find evidence of a substantial shadow effect, where a weaker future competitor increases the probability that the stronger player wins the current match. We also identify a negative spillover effect in tennis tournaments—more effort exertion in the previous rounds is associated with significantly less success in the current round. We do not find support for an underdog advantage in the final tournament stage, suggesting that the prize conditions for this hypothesized phenomenon may not be satisfied in the data. Unfortunately, these data do not allow us to test our secondary theoretical results—in tennis, the plausible measures of noisiness (best-of-three vs. best-of-five matches) and the steepness of the prize distribution are highly correlated and thus the effects cannot be identified cleanly. However, our theoretical model provides clear noise- and prize-related predictions.

In a supplemental analysis, we use probability odds data from bookmakers to show that betting markets recognize and price in the spillover and shadow effects.

5.1 Discussion

Our findings have implications in terms of the structure of elimination tournaments. Tournaments are often designed to identify high-ability candidates in environments where the contest organizer cannot readily observe innate talent. Our results suggest ways by which a contest designer can improve the likelihood that the strongest candidate succeeds. Limiting negative spillover by allowing competitors opportunities to refresh their resources between stages increases the probability that the stronger type wins. Firms may also want to encourage positive spillover through learning. For example, in an innovation contest, firms should be given adequate time between stages to raise additional funds and pursue more advanced technology improvements. Similarly, a firm may wish to institute a "work-life balance" program that promotes employee wellness, discourages career-related burnout, and improves the probability that the firm's labor tournament promotes the strongest workers.

Contest organizers may also wish to avoid "noisy" competition where the effort-to-output technology is less precise; for example, a larger panel of decision-makers in innovation, design or personnel tournaments may yield more discriminating selection.

Higher powered prizes also enhance selection across stages—large prize spreads, as well as small loser prizes, will reduce the chance of selecting the lower-ability candidate.

In contrast, if a contest designer is concerned with the unevenness of competition, it can design a more balanced contest with more negative spillover, noisier effort-to-output activities, and a flatter prize structure.

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6 Appendix

6.1 Conditions for $G(\cdot)$

To ensure that probabilities are well-defined, we require two conditions on the primitives:

$$0 < \frac{h\left(\frac{\Delta V}{ac_1}\right) - h\left(\frac{\Delta V}{ac_3}\right) + \frac{a}{2}}{a} < 1 \quad \text{and} \quad 0 < \frac{h\left(\frac{\tilde{V}_1}{ac_1}\right) - h\left(\frac{\tilde{V}_2}{ac_2}\right) + \frac{a}{2}}{a} < 1$$

where $h \equiv [\gamma'(\cdot)]^{-1}$. These conditions ensure that $G(\cdot) \in (0, 1)$ for both stages in equilibrium. Depending on the model parameters, one condition will determine the upper bound of $G(\cdot)$ and the other condition will determine the lower bound.

6.2 Combined Shadow and Spillover Effects

Our main analysis considers separately the effects of effort spillover and the shadow of future competition. Here, we present an analysis when both effects are at play. Combining the effects does not change the general predictions of the previous analysis—spillover continues to even the playing field, while weaker future competition does the opposite.

6.2.1 Spillover and Shadow - Final Stage

We begin with the final stage and fix players' abilities across the stages. For illustration purposes and computational ease, we again assume quadratic costs. Our first order condition for the final stage yields equilibrium effort choice

$$x_i^* = \frac{\Delta V}{2ac_i k_i(z_i)}$$

where $k_i(\cdot)$ reflects the degree of spillover from the previous stage and is an increasing function of first stage effort z_i. As expected, greater first-stage effort results in lower equilibrium effort in the final stage. Further, this effect is amplified for the stronger type since $c_1 < c_2$. The final stage spillover effect is therefore

$$\frac{\partial x_i^*}{\partial k_i(z_i)} = \frac{-\Delta V}{2ac_i k_i(z_i)^2} < 0$$

Therefore, a given level of spillover ($k_1 = k_2 = k$) reduces the disparity between participants' efforts in the final stage, since $\frac{\partial x_1^*}{\partial k(z_1)} < \frac{\partial x_2^*}{\partial k(z_2)} < 0$.

6.2.2 Spillover and Shadow - First Stage

Next, we consider effort decisions in the first stage and write player i's payoff as[22]

$$\pi_i = G_{first}(\cdot)\left(\left(\frac{\frac{\Delta V}{2ac_ik_i} - \frac{\Delta V}{2ac_jk_j} + \frac{a}{2}}{a}\right)\Delta V - c_ik_i\left(\frac{\Delta V}{2ac_ik_i}\right)^2 + V_L\right) - c_iz_i^2$$

The first order condition for the first stage is

$$\frac{\partial \pi_i}{\partial z_i} = \frac{\left(\frac{\frac{\Delta V}{2ac_ik_i} - \frac{\Delta V}{2ac_jk_j} + \frac{a}{2}}{a}\right)\Delta V - c_ik_i\left(\frac{\Delta V}{2ac_ik_i}\right)^2 + V_L}{a} + G_{first}(\cdot)\left(\frac{-\Delta V^2}{2a^2c_ik_i^2} + \left(\frac{\Delta V^2}{4a^2c_ik_i^2}\right)\right)\frac{\partial k_i}{\partial z_i} - 2c_iz_i = 0$$

which then gives us the following expression for first-stage equilibrium effort:

$$z_i^* = \underbrace{\frac{\left(\frac{\frac{\Delta V}{2ac_ik_i} - \frac{\Delta V}{2ac_jk_j} + \frac{a}{2}}{a}\right)\Delta V - c_ik_i\left(\frac{\Delta V}{2ac_ik_i}\right)^2 + V_L}{2c_ia}}_{\text{shadow effect}} + \underbrace{\frac{G_{first}(\cdot)\left(\frac{-\Delta V^2}{2a^2c_ik_i^2} + \left(\frac{\Delta V^2}{4a^2c_ik_i^2}\right)\right)\frac{\partial k_i}{\partial z_i}}{2c_i}}_{\text{spillover effect}} \quad (11)$$

With no spillover $(k = 1)$, the left term is precisely the shadow effect we described in Section (1.2). The right term reflects spillover. When $k = 1$, this spillover term is greater for the stronger player and, thus, the stronger player reduces his effort more than the lesser player (since $G_{first}(\cdot) \geq \frac{1}{2}$ and $c_1 < c_2$). Since the stronger player exerts more equilibrium effort in the first stage, he will necessarily suffer more spillover in the final stage (assuming players face a common $k(z_i)$ function). Thus, spillover has the effect of evening the playing field in both stages. That is, *ceteris paribus*, spillover increases the chance of an upset.

6.3 Proofs

Proposition 3: *With quadratic costs, for a given prize spread increase, the stronger player is even more likely to be selected as the winner in either stage.*

Proof. Let player 1 be the stronger player and player 2 be the weaker player so that $c_1 < c_2$. The result for the final stage effort follows immediately from equation (6). To compare the effect of changing the prize spread ΔV on players' first stage equilibrium efforts, we consider

[22] We write $k_i(z_i)$ as k_i to simplify the notation in this section.

the following expression:

$$\begin{aligned}\frac{\partial z_1^*}{\partial \Delta V}-\frac{\partial z_2^*}{\partial \Delta V} &= \frac{\frac{\Delta V}{a^2}\left(\frac{\frac{1}{2}}{c_1}-\frac{1}{c_2}\right)+\frac{1}{2}}{2c_1 a}-\frac{\frac{\Delta V}{a^2}\left(\frac{\frac{1}{2}}{c_2}-\frac{1}{c_1}\right)+\frac{1}{2}}{2c_2 a}\\ &= \frac{\Delta V}{2a^3}\left(\frac{\frac{1}{2}}{c_1^2}-\frac{1}{c_1c_2}\right)+\frac{1}{4c_1a}-\frac{\Delta V}{2a^3}\left(\frac{\frac{1}{2}}{c_2^2}-\frac{1}{c_1c_2}\right)-\frac{1}{4c_2a}\\ &> \frac{\Delta V}{2a^3}\left(\frac{\frac{1}{2}}{c_1^2}-\frac{\frac{1}{2}}{c_2^2}\right)>0\end{aligned}$$

The final inequality is always met, since $c_1 < c_2$ by assumption. Therefore, the stronger player increases his effort more than the weaker player for a given increase in ΔV. Consequently, this increased effort disparity increases the probability that the stronger player wins in the either stage. ■

Proposition 4: $P_{1,final}\left(x_1^*, x_3^*\left(c_3\right)\right) < P_{1,first}\left(z_1^*, z_2^*\left(c_2\right)\right)$ *when* $c_2 = c_3$ *and* $V_L > \Delta V + c_2\gamma\left(h\left(\frac{\Delta V}{ac_2}\right)\right)$. *With a sufficiently large second place prize relative to the first place prize, the probability that the weaker player wins in the final stage (and thus is selected as the overall tournament winner) is greater than the probability that he wins in the first stage, holding opponent skill constant.*

Proof. Proposition 4 identifies an underdog advantage in the final stage—that is, the weaker player has a greater probability of winning in the final stage over the first stage. This occurs when $G_{first} > G_{final}$ where

$$\begin{aligned}G_{final} &= \frac{h\left(\frac{\Delta V}{ac_1}\right)-h\left(\frac{\Delta V}{ac_3}\right)+\frac{a}{2}}{a}\\ G_{first} &= \frac{h\left(\frac{\tilde{V}_1}{ac_1}\right)-h\left(\frac{\tilde{V}_2}{ac_2}\right)+\frac{a}{2}}{a}\end{aligned}$$

in other words,

$$h\left(\frac{\tilde{V}_1}{ac_1}\right)-h\left(\frac{\tilde{V}_2}{ac_2}\right)>h\left(\frac{\Delta V}{ac_1}\right)-h\left(\frac{\Delta V}{ac_2}\right) \tag{12}$$

Assume that the skill level of player 1's opponents in the first and final stages are equal, $c_2 = c_3$.

We must show that $\tilde{V}_2 > \Delta V$. This will prove the inequality in expression (12) since the difference in the pairs of h functions is increasing in that stage's prize and we know that

$\tilde{V}_1 > \tilde{V}_2$.

$$\begin{aligned} \tilde{V}_2 &> \Delta V \\ (1 - G_{final}(\cdot)) \Delta V - c_2 \gamma (x_2) + V_L &> \Delta V \\ V_L &> G_{final}(\cdot) \Delta V + c_2 \gamma (x_2) \\ V_L &> G_{final}(\cdot) \Delta V + c_2 \gamma \left(h \left(\frac{\Delta V}{a c_2} \right) \right) \end{aligned}$$

The final inequality provides a sufficient condition for an underdog advantage. Note that when we fix ΔV the value of the RHS of the inequality is also fixed. Then, holding ΔV fixed, we can find some $V_L > 0$ that satisfies the inequality. However, recall that G_{first} must be less than 1; yet, G_{first} is increasing in V_L. To see that some values satisfy $G_{first} \leq 1$, we allow the prize levels to converge: $V_L \to V_W$. In this case, as $\Delta V \to 0$, the RHS of the inequality approaches 0 and the inequality is then easily satisfied since $V_L > 0$. ■

For a simple example, set: $c_1 = 0.7$; $c_2 = 1$; $V_L = 0.1$; $\Delta V = 1$; $a = 1.5$. With quadratic costs, our inequality fails—i.e., the weaker player is *even less* likely to win in the final stage. However, with $V_L = 0.5$, the weaker player is *more* likely to win in the final stage. Thus, given a large enough second-place prize relative to the first-place prize, the weaker player is more likely to win against the same opponent in the final stage.

6.4 A General Distribution Case

6.4.1 Final Stage

The model in the body of the paper presents results when the noise in players' output is distributed uniformly; recall that, in section (1), we define $\varepsilon = \varepsilon_3 - \varepsilon_1$ and assume that ε is distributed according to $G \sim U\left[-\frac{1}{2}a, \frac{1}{2}a\right]$. In fact, similar results can be derived for any unimodal and symmetric distribution $G(\bullet)$ with mean zero. For illustrative purposes, let players face quadratic costs.[23] Again assume that the first-stage matches are resolved sequentially; players 1 and 2 know that player 3 won his parallel match to advance to the final stage.

Player 1's payoff function for the final stage can be written as

$$\pi_{1,final} = G(x_1 - x_3) \Delta V - c_1 (x_1)^2 + V_L$$

[23]More generally, players can face any cost function of the form x^a.

and his first order condition is

$$\frac{\partial \pi_{1,final}}{\partial x_1} = G'(x_1 - x_3)\,\Delta V - 2c_1 x_1 = 0$$

Similarly, player 3's first order condition is

$$G'(x_3 - x_1)\,\Delta V - 2c_3 x_3 = 0$$

Since $G(\cdot)$ is symmetric about its mean, it follows that $G'(x_1 - x_3) = G'(x_3 - x_1)$. This implies the following in equilibrium:

$$\begin{aligned} c_1 2x_1^* &= c_3 2x_3^* \\ \frac{x_1^*}{x_3^*} &= \frac{c_3}{c_1} \end{aligned} \tag{13}$$

Although changes in the prize spread or the noise around players' output affect equilibrium effort, the *ratio* of players' efforts is constant. It follows, for example, that an increase in the prize spread that leads to higher equilibrium effort from both competitors will necessarily increase the absolute spread between players' efforts. In turn, this increases the probability that the stronger player wins in the current stage since his probability of winning is $G(x_1 - x_3)$. In contrast, as equilibrium effort falls—for example, from the adverse effects of negative spillover—the absolute spread between players' efforts decreases. Here, the probability that the stronger player wins declines with equilibrium effort levels.

Since $G(\bullet)$ can be any unimodal symmetric distribution, the impact of changes in the variance of $G(\bullet)$ depends on the exact distribution and its parameters. The top panel of Figure A1 provides an illustration: consider two normal distributions centered at zero with standard deviations of 1 and 2, respectively.

First consider region A. When the players are relatively similar in ability and thus choose similar equilibrium efforts, reducing the variance means a "thickening" of the density. This provides greater incentives for both players, as the marginal return to effort is greater. Therefore, when players are similar in ability, the probability that the stronger player wins increases as the variance decreases.

Now consider region B where the ability difference between players is substantial and decreased variance means a "thinning" of the density. This weakens incentives for both players, as the marginal return to effort is reduced. Therefore, in this region, decreased variance reduces the probability that the stronger player wins.

6.4.2 First Stage Effort and Shadow Effect

Next, we consider players' effort choices in the first stage of competition. Here, player 1 faces a similar payoff function to his final-stage problem, but considers his continuation value $\widetilde{V}_1$ instead of the prize spread ΔV. This yields the following first order condition:

$$G'(z_1 - z_2)\widetilde{V}_1 - c_1 2 z_1 = 0$$

To study the impact of the ability of the future competitor on first-stage outcomes, we consider the case where the player 3 becomes a weaker opponent (i.e., c_3 increases). The following two conditions are sufficient for player 1 to weakly increase his effort relative to his current opponent's effort choice and thus improve his probability of winning in the first stage: (a) $G'(x_1^* - x_3^*) \geq G'(x_2^* - x_3^*)$ and (b) $\frac{\partial}{\partial c_3}\widetilde{V}_1 \geq \frac{\partial}{\partial c_3}\widetilde{V}_2$. In the following sections, we describe when each of these conditions holds.

Condition (a): $G'(x_1^* - x_3^*) \geq G'(x_2^* - x_3^*)$ When noise is distributed uniformly, $G'(x_1 - x_3) = G'(x_2 - x_3)$ and condition (a) is always met. For more general distributions, we must analyze this condition over several cases. Consider a contest where ε,the difference in players' additive noise terms, is drawn from a normal distribution, illustrated below in the bottom panel of Figure A1. Since the ordering of players' efforts is critical for the analysis, we outline three cases.

Ordering 1) When $x_3^* < x_2^* < x_1^*$, players 1 and 2 expect to face a future opponent who is weaker than both of them. For example, in the figure below, suppose that $x_1^* - x_3^*$ lies at C and $x_2^* - x_3^*$ lies between B and C. Here, we violate condition (a) since $G'(x_1 - x_3) < G'(x_2 - x_3)$. If player 1 and 2 are similar enough in ability, the current increase in player 1's effort can still be greater despite a smaller change in continuation value since $c_1 < c_2$. That is, player 1 is more sensitive to a change in continuation value so a smaller change in the value can still yield a greater change in effort for player 1.

Ordering 2) When $x_2^* < x_1^* < x_3^*$, the future opponent is always stronger than both current players. In this case, it is unambiguous that the stronger player has a greater increase of effort since now $x_1^* - x_3^*$ lies say between A and B and $x_2^* - x_3^*$ to the left of that. This means $G'(x_1 - x_3) > G'(x_2 - x_3)$.

Ordering 3) When $x_2^* < x_3^* < x_1^*$, the future opponent is stronger than player 2 but weaker than player 1. We can now have either $x_1^* - x_3^* > |x_2^* - x_3^*|$ or $x_1^* - x_3^* \leq |x_2^* - x_3^*|$. The first inequality leads to a violation of condition (a), since $G'(x_1 - x_3) < G'(x_2 - x_3)$; in contrast, condition (a) is satisfied for the second inequality. That is, condition (a) is satisfied when $x_1^* - x_3^*$ falls between B and C and $x_2^* - x_3^*$ falls below A.

Overall, we find that the condition is satisfied whenever the future opponent is sufficiently strong. That is, we find the shadow effect of future competition as long as the future opponent is sufficiently more skilled than the weaker player in the current match.

Assuming that the spreads between players' abilities are similar *across* parallel matches, we will observe most often the case where the expect future opponent is similar in ability to the stronger current player. In the event that the weaker player does win the parallel match, the marginal returns to effort may be reversed and the current weaker player may actually increase his effort more than his stronger opponent. However, in our empirical analysis, these less frequent cases simply work *against* finding a positive shadow effect.

Condition (b): $\frac{\partial}{\partial c_3}\widetilde{V}_1 \geq \frac{\partial}{\partial c_3}\widetilde{V}_2$ To consider the second condition, recall that player 1's final stage equilibrium payoff is

$$\pi_{1,final} = \widetilde{V}_1 = G\left(x_1 - x_3\right)\Delta V - c_1\left(x_1\right)^2 + V_L$$

The following expression describes how final stage profit changes as a function of player 1's opponent's cost type c_3 :

$$\frac{\partial \pi_{1,final}}{\partial c_3} = G'\left(x_1^* - x_3^*\right)\Delta V\left(\frac{\partial x_1^*}{\partial c_3} - \frac{\partial x_3^*}{\partial c_3}\right) - c_1 2x_1^*\frac{\partial x_1^*}{\partial c_3} \tag{14}$$

From equation (13) ,we know that $x_1^* = \frac{x_3^* c_3}{c_1}$ which implies that $\frac{\partial x_1^*}{\partial c_3} = \frac{x_3^*}{c_1}$and $x_3^* = \frac{x_1^* c_1}{c_3}$ which implies that $\frac{\partial x_3^*}{\partial c_3} = \frac{-x_1^* c_1}{c_3^2}$. Thus, we can rewrite expression (14) as

$$\begin{aligned}
&G'\left(x_1^* - x_3^*\right)\Delta V\left(\frac{x_3^*}{c_1} + \frac{x_1^* c_1}{c_3^2}\right) - c_1 2x_1^*\frac{x_3^*}{c_1} \\
&= G'\left(x_1^* - x_3^*\right)\Delta V\frac{x_1^* c_1}{c_3^2} > 0
\end{aligned}$$

Thus, for $\frac{\partial}{\partial c3}\widetilde{V}_1 \geq \frac{\partial}{\partial c3}\widetilde{V}_2$, we want to show $x_1^* c_1 \geq x_2^* c_2$. Recall that, in equilibrium,

$$x_1^* = \frac{G'(x_1^* - x_3^*)\Delta V}{2c_1}$$

Thus, $x_1^* c_1 = \frac{G'(x_1^* - x_3^*)\Delta V}{2}$ and $x_2^* c_2 = \frac{G'(x_2^* - x_3^*)\Delta V}{2}$. If our first condition $G'(x_1^* - x_3^*) \geq G'(x_2^* - x_3^*)$ is met, then $x_1^* c_1 \geq x_2^* c_2$ and $\frac{\partial}{\partial c3}\widetilde{V}_1 \geq \frac{\partial}{\partial c3}\widetilde{V}_2$. That is, $G'(x_1^* - x_3^*) \geq G'(x_2^* - x_3^*)$ is necessary and sufficient for meeting condition (b). In other words, the shadow effect is a function of both the distribution of noise and players' skill disparity.

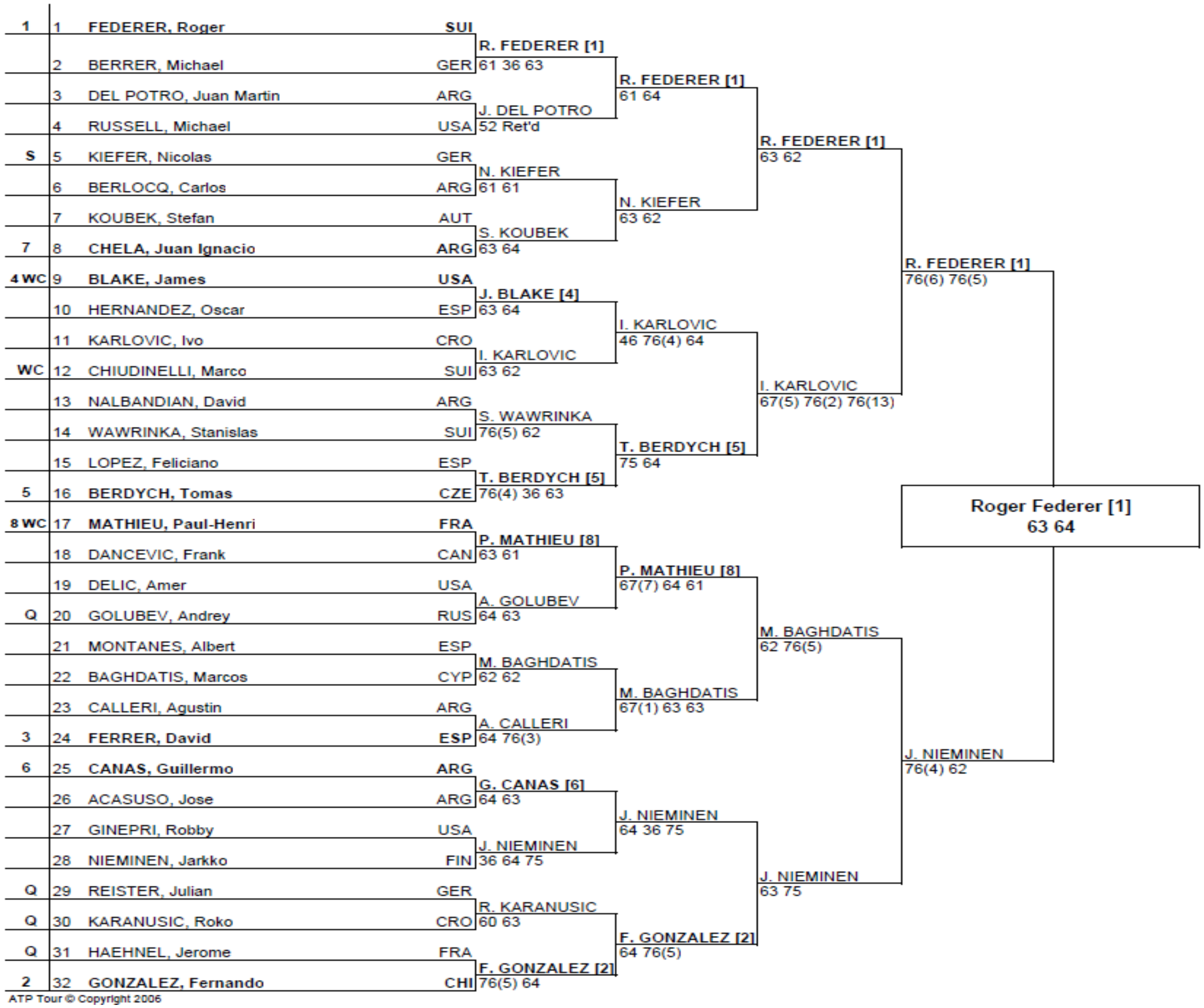
6.5 Spillover

In the section (1.3) , we describe negative spillover as increasing players' effective cost types. Assume that two players experience the same level of exertion in the first stage, leading to the same proportional increase in cost types in the final stage. The ratio of their efforts remains unchanged; however, final stage efforts are lower and thus the absolute spread in efforts is smaller and the stronger player is less likely to win the match. Therefore, as we found in the uniform case, spillover evens the playing field.

6.6 Underdog Advantage

An underdog advantage still exists with a more general noise distribution when the second prize is sufficiently large relative to the prize spread. This can be shown by holding ΔV fixed while increasing V_L. Both players' continuation values increase equally in the first stage; however, to maintain the ratio of equilibrium efforts, the stronger player must increase his effort more than the weaker player. Final stage efforts are unaffected by changes in V_L since the second prize does not enter into players' final stage first order conditions. Hence, with a sufficiently large V_L, the weaker player is more likely to win in the final stage relative to his chances in the first stage.

<u>Figure 1 - Example Draw from 2007 Davidoff Swiss Indoors in Basel</u>



source: ATPworldtour.com

Table 1 - Summary Statistics for ATP World Tour Events January 2001 to May 2010

	All Rounds	1st Round	2nd Round	3rd Round	4th Round	Quarterfinals	Semifinals	The Final
# of Matches Played	28370	14497	7237	1897	433	2461	1230	615
Average # of games played	23.1	21.6	24.6	26.7	31.7	23.3	23.5	25.3
	(10.4)	(11.4)	(8.6)	(10.1)	(11.5)	(7.7)	(7.6)	(8.7)
Average Rank of Winner	58.6	71.7	52.5	32.5	21.5	43.8	37.0	29.7
	(73.2)	(83.4)	(61.1)	(55.0)	(57.3)	(53.4)	(49.6)	(39.2)
Average Rank of Loser	95.5	119.7	90.9	54.9	39.9	62.0	50.5	43.9
	(121.0)	(147.7)	(97.0)	(64.4)	(58.1)	(65.3)	(56.3)	(55.2)
Average Rank Ratio	6.8	5.7	8.2	8.9	10.4	6.7	6.6	6.7
(Worse / Better by Rank)	(20.9)	(21.5)	(20.6)	(18.9)	(20.1)	(24.3)	(12.8)	(13.4)

Note: Data contain player and performance information for 615 tournaments. Values in parentheses are standard deviations.

Table 2 - Combined Spillover and Shadow Effects

Dependent Variable: **Stronger Player Wins in Current Period (0 or 1)**

	2nd on 1st Round	3rd on 2nd Round	Qfinals on 2nd Round	4th on 3rd Round	Qfinals on 3rd Round	Qfinals on 4th	Sfinals on Qfinals	Final on Sfinals	The Final (no shadow)
Expected Future Opponent Rank	0.079% ***	0.084% ***	0.071% ***	0.165% **	0.150% ***	0.421% ***	0.057%	0.077%	
	(0.0001)	(0.0003)	(0.0002)	(0.0008)	(0.0006)	(0.0015)	(0.0004)	(0.0006)	
Stronger Player's Previous Games		-0.350% ***	-0.435% ***	-0.442% ***	-0.513% ***	-0.581% *	-0.310% ***	-0.158% ***	-0.132% **
		(0.0009)	(0.0008)	(0.0013)	(0.0011)	(0.0031)	(0.0008)	(0.0004)	(0.0007)
Weaker Player's Previous Games		0.219% **	0.175% *	0.110%	0.094%	0.163%	0.139% *	0.062% *	0.087% *
		(0.0009)	(0.0009)	(0.0011)	(0.0010)	(0.0021)	(0.0008)	(0.0004)	(0.0005)
Current Rank Ratio (Worse / Better Rank)	0.161% ***	0.281% ***	0.194% ***	0.340% ***	0.265% ***	0.338% ***	0.089% **	0.543% ***	0.363%
	(0.0005)	(0.0004)	(0.0003)	(0.0006)	(0.0005)	(0.0008)	(0.0004)	(0.0012)	(0.0032)
# of observations	12575	3759	4891	858	1461	432	2450	1220	615

Notes: "Expected Future Opponent Rank" is the rank of the stronger player in the parallel event. That is, it is the rank of the stronger of the potential opponents in the next round. Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.

Table 3 - Predicting Betting Probability Odds by Spillovers and Shadows (Rankings)

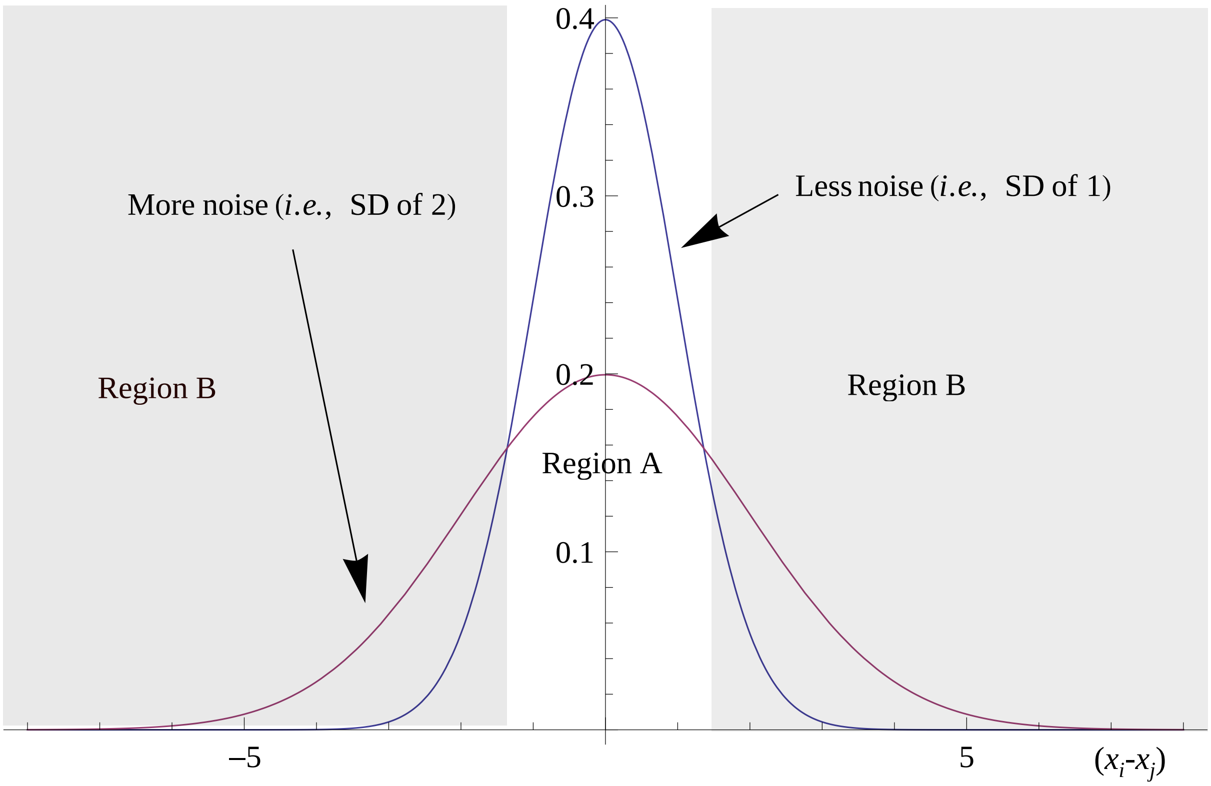
Dependent Variable: **Implied Probability the Stronger Player Wins (%)**

	2nd on 1st Round	3rd on 2nd Round	Qfinals on 2nd Round	4th on 3rd Round	Qfinals on 3rd Round	Qfinals on 4th	Sfinals on Qfinals	Final on Sfinals	The Final (no shadow)
Expected Future Opponent Rank	0.083% *** (0.0001)	0.095% *** (0.0001)	0.041% *** (0.0001)	0.075% *** (0.0003)	0.075% *** (0.0002)	0.054% (0.0006)	0.021% * (0.0001)	0.020% (0.0002)	
Stronger Player's Previous Games		-0.312% *** (0.0003)	-0.340% *** (0.0003)	-0.363% *** (0.0004)	-0.369% *** (0.0003)	-0.508% *** (0.0011)	-0.291% *** (0.0003)	-0.102% *** (0.0001)	-0.094% *** (0.0002)
Weaker Player's Previous Games		0.116% *** (0.0003)	0.114% *** (0.0003)	0.084% *** (0.0004)	0.091% *** (0.0003)	0.116% (0.0008)	0.069% *** (0.0002)	0.036% *** (0.0001)	0.053% *** (0.0002)
Current Rank Ratio (Worse / Better Rank)	0.147% *** (0.0005)	0.249% *** (0.0004)	0.178% *** (0.0002)	0.318% *** (0.0006)	0.248% *** (0.0005)	0.329% *** (0.0007)	0.117% (0.0007)	0.415% *** (0.0009)	0.338% *** (0.0009)
# of observations	11983	3617	4586	834	1414	425	2339	1169	591

Notes: "Expected Future Opponent Rank" is the rank of the stronger player in the parallel event. That is, it is the rank of the stronger of the potential opponents in the next round. Values in parentheses are robust standard errors. ***, ** and * denote statistical significance at *p-values* of 1%, 5% and 10%, respectively. Regressions include tournament-level fixed effects.

Figure A1: Example Densities of Joint Noise - Normal Distribution

Panel 1: Return to Effort as a Function of Different "Noise Levels"



Panel 2: Normal Distribution of Joint Noise