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ABSTRACT

We study optimal portfolio choice in a two-country model where assets represent claims on future consumption and facilitate trade in markets with imperfect credit. Assuming that foreign assets trade at a cost, agents hold relatively more domestic assets. Consequently, agents have larger claims to domestic over foreign consumption. Moreover, foreign assets turn over faster than domestic assets because the former have desirable liquidity properties, but represent inferior saving tools. Our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets.

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An online appendix is available at:
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1 Introduction

Agents' international portfolio choices have been a major topic of research in macroeconomics over the past two decades. The vast majority of papers emphasize the role that foreign assets play in helping agents diversify domestic income risk. While the role of assets in hedging consumption risk is admittedly crucial, an equally important characteristic of assets, their liquidity, has been overlooked by the literature. In this paper, we develop a theoretical framework that allows us to study how liquidity properties of assets shape agents' international portfolios. A concrete concept of asset liquidity is, therefore, required.

To that end, we employ a model in the tradition of monetary-search theory, extended to include real assets and trade between countries. Agents have access to alternating rounds of centralized, or Walrasian, markets and decentralized markets, where trade occurs in a bilateral fashion and credit is imperfect. In each country's Walrasian market, domestic and foreign agents can buy assets of that country at the ongoing market price. More interestingly, agents can bring a portfolio of assets to each country's decentralized market and trade it for locally-produced goods. Hence, assets serve a double function. First, they are claims to future consumption, as is standard in finance. Second, they serve as media of exchange, as is standard in monetary theory. It is precisely this second function that captures the notion of asset liquidity.

Assuming that agents incur a per-unit cost to trade foreign claims, we characterize equilibria in which different assets arise as media of exchange in different types of bilateral meetings. If agents' trading opportunities abroad are sufficiently frequent, then assets circulate as media of exchange locally. This means that domestic assets facilitate trade at home and foreign assets facilitate trade abroad. Hence, one contribution of our paper is to endogenize the commonly-assumed existence of "currency areas" in international macroeconomics. At the other extreme, if trading opportunities abroad are scarce, agents use their domestic assets to acquire consumption goods both at home and abroad. Finally, in an intermediate case, either type of equilibrium described above can arise, depending on other parameters that govern asset returns.

We use our model to address a long-standing puzzle in international finance as described in Tesar and Werner (1995) and Lewis (1999): a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets. In particular, developed countries obey three empirical regularities. First, in the average OECD country, agents' portfolios are heavily biased toward domestic assets. Second, economies that exhibit higher asset home bias also consume more goods and services from domestic sources. Third, turnover rates of foreign assets are significantly higher than those of domestic assets.

The model predicts that, as long as trading opportunities abroad are not much more frequent than at home, agents' portfolios exhibit home bias. Since agents hold larger amounts of domestic over foreign assets, they have larger claims to domestic over foreign consumption goods. Hence, our suggested mechanism positively links asset and consumption home bias.

More importantly, under additional parameter restrictions, foreign assets turn over faster than domestic assets because, while the former have desirable liquidity properties, they yield lower future consumption and are undesirable saving tools. In the absence of the liquidity factor, foreign asset trading costs, which are necessary to generate consumption and asset home bias, would yield lower turnover of foreign relative to domestic assets. Therefore, the unique aspect of our model is the ability to capture the three stylized facts simultaneously.

Amadi and Bergin (2008) offer an explanation for the coexistence of asset home bias and a higher turnover rate of foreign over domestic assets. The authors argue that a portfolio-choice model with a heterogeneous per-unit trading cost and a homogenous fixed entry cost produces an environment that is consistent with this stylized fact. While this mechanism may be in part responsible for the observations in the data, we argue that there is considerable room for an explanation that builds on our liquidity channel. First, unlike our framework, Amadi and Bergin's (2008) model does not link consumption and asset home bias, which is a regularity observed in the data. Second, our model generates a unique testable prediction that relates bilateral asset turnover rates to bilateral trade in goods, which is not shared by the alternative framework.

In our model, the rate at which foreign assets turn over is driven by the role that the assets play in facilitating trade abroad. Hence, an implication of our model is that a given importer turns over faster the assets of the country from which it imports more. To test this prediction, we combine bilateral goods trade and asset holding data for the 2002-2007 period with annual gross flows of foreign assets between the US and each of its OECD trading partners. We find that the correlation between the US import shares by source and the US turnover rate of assets from the same source ranges between 0.3364 and 0.5466 and it is statistically significant. The result provides direct support for the liquidity mechanism over the alternative.

The notion of liquidity that we employ draws upon the monetary search literature. Within that spirit, we assume that assets serve as means of payments in markets with imperfect credit.[1] However, our mechanism intends to capture the broader notion of assets as facilitators of trade. For example, sellers may require assets as collateral in order to deliver goods to buyers.[2] Alternatively, agents may rely on repurchase agreements to acquire goods and services. As Lagos (2011) points out, contractual details aside, in all of these environments assets effectively act as media of exchange in that they facilitate trade between sellers and untrustworthy buyers.

The liquidity mechanism that we propose reconciles the three stylized facts discussed earlier. However, in order to remain tractable, the model assumes away the risky nature of equity returns. Thus, the mechanism is complementary to existing models of aggregate uncertainty that relate asset and consumption home bias. For example, Heathcote and Perri (2007) use a standard international business cycle framework and show that, when preferences are biased

[1] This is in contrast with the finance literature, which typically relates the liquidity of an asset to the speed with which it can be converted into consumption. See for example Duffie, Garleanu, and Pedersen (2005).

[2] This notion of liquidity is closely related to the concept of pledgeability, which was introduced to the literature by Kiyotaki and Moore (1997) and Holmström and Tirole (1998).

toward domestic goods, asset home bias arises because endogenous international relative price fluctuations make domestic stocks a good hedge against non-diversifiable labor income risk. Their model generates a tight link between a country's degree of openness to trade and level of asset diversification. This positive link is also explored by Collard, Dellas, Diba, and Stockman (2009) in an endowment economy with separable utility between traded and non-traded goods. In their model, an agent's optimal portfolio includes the entire stock of home firms that produce domestic non-traded goods and a fully diversified portfolio of equities of firms that produce tradable goods. The authors show that, if the share of non-traded goods in consumption is large, the model can generate substantial portfolio home bias.

In other related literature, Michaelides (2003) solves for optimal international portfolios in the presence of liquidity constraints and undiversifiable labor income risk. The author shows that substantial asset home bias can be generated in a buffer stock saving model where agents face higher costs to invest abroad than at home. Assets, however, play a very different role in his model relative to ours. In the framework of Michaelides (2003), assets help smooth consumption in the presence of liquidity constraints, while in ours, assets help facilitate bilateral trade. Finally, Hnatkovska (2010) demonstrates that asset home bias and a high foreign asset turnover rate can arise in the presence of non-diversifiable non-traded consumption risk when each country specializes in production, preferences exhibit consumption home bias, and asset markets are incomplete. However, in this setting, domestic asset flows are also high, thus yielding equally high turnover rates of domestic assets.

The present paper is the first to deliver higher foreign over domestic asset turnover rates in a model where foreign asset trading costs naturally link consumption and asset home bias. The liquidity mechanism that generates these desirable predictions relates closely to the growing literature that focuses on the liquidity properties of objects other than fiat money. Lagos and Rocheteau (2008) assume that part of the economy's physical capital can be used as a medium of exchange along with money. Their goal is to study the issue of over-investment and how it is affected by inflation. Geromichalos, Licari, and Suarez-Lledo (2007) study the coexistence of money and a real financial asset as media of exchange, with special focus on the relationship between asset prices and monetary policy. Lester, Postlewaite, and Wright (2008) take this framework a step further and endogenize the acceptability (of various media of exchange) decisions of agents. Finally, Lagos (2010) considers a similar framework, enriched with uncertainty, in order to address the equity premium and risk-free rate puzzles. The present paper, however, is the first to introduce liquidity of assets in a multi-country environment and to explore the implications of liquidity on the distribution of asset holdings across countries.

Lastly, our paper relates to the literature of money-search models applied to international frameworks. In their pioneering work, Matsuyama, Kiyotaki, and Matsui (1993) employ a two-country, two-currency money-search model, with indivisible money and goods, and study conditions under which the two currencies serve as media of exchange in different countries.

Wright and Trejos (2001) maintain the assumption of good indivisibility in Matsuyama, Kiyotaki, and Matsui (1993), but they endogenize prices using bargaining theory. Head and Shi (2003) also consider a two-country, two-currency search model and show that the nominal exchange rate depends on the stocks and growth rates of the two monies. Finally, Camera and Winkler (2003) use a search-theoretic model of monetary exchange in order to show that the absence of well-integrated international goods markets does not necessarily imply a violation of the law of one price. In this paper, we employ a model with divisible assets and goods. Moreover, by endowing assets, other than fiat money, with certain liquidity properties, we bring the money-search literature closer to questions related to international portfolio diversification.

The remainder of the paper is organized as follows. Section 2 describes the modeling environment. Section 3 discusses the optimal behavior of agents in the economy. Section 4 characterizes the media of exchange that arise in a symmetric two-country model of international trade in goods and assets. Section 5 describes the model's predictions regarding asset and consumption home bias as well as asset turnover rates. Section 6 documents the stylized facts and provides empirical support for the proposed mechanism. Section 7 discusses the robustness of the results to different modeling assumptions and extensions. Finally, Section 8 concludes.

2 Physical Environment

Time is discrete with an infinite horizon. Each period consists of two sub-periods. During the first sub-period, trade occurs in decentralized markets (DM henceforth), which we describe in detail below. In the second sub-period, economic activity takes place in traditional Walrasian or centralized markets (CM henceforth). There is no aggregate uncertainty. There are two countries, A and B. Each country has a unit measure of buyers and a measure ξ of sellers who live forever. The identity of agents (as sellers or buyers) is permanent. During the first sub-period, a distinct DM opens within each country and anonymous bilateral trade takes place. We refer to these markets as DM_i, $i = A, B$. Without loss of generality, assume that DM_A opens first. Sellers from country i are immobile, but buyers are mobile. Therefore, in DM_i, sellers who are citizens of country i meet buyers who could be citizens of either country. During the second sub-period, all agents are located in their home country.

All agents discount the future between periods (but not sub-periods) at rate $\beta \in (0, 1)$. Buyers consume in both sub-periods and supply labor in the second sub-period. Their preferences, which are independent of their citizenship, are given by $\mathcal{U}(q_A, q_B, X, H)$, where q_i is consumption in DM_i, $i = A, B$, and X, H are consumption and labor in the (domestic) CM. Sellers consume only in the CM and they produce in both the DM and the CM. Sellers' preferences are given by $\mathcal{V}(h, X, H)$, where the only new variable, h, stands for hours worked in the DM.

In line with Lagos and Wright (2005), we adopt the functional forms

$$\begin{aligned}\mathcal{U}(q_{A}, q_{B}, X, H) &= u(q_{A}) + u(q_{B}) + U(X) - H, \\ \mathcal{V}(h, X, H) &= -c(h) + U(X) - H.\end{aligned}$$

We assume that u and U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $U' > 0$, $u'' < 0$, and $U'' \leq 0$. For simplicity, we set $c(h) = h$, but this is not crucial for any of our results. Let $q^* \equiv \{q : u'(q^*) = 1\}$, i.e. q^* denotes the optimal level of production in any bilateral meeting. Also, suppose that there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$, with $U(X^*) > X^*$.

During the round of decentralized trade, sellers and buyers are matched randomly according to a matching technology that is identical in both DM's. Let $\mathcal{B}, \mathcal{S}$ denote the total number of buyers and sellers in a certain DM.[3] The total number of matches in this market is given by $M(\mathcal{B}, \mathcal{S}) \leq \min\{\mathcal{B}, \mathcal{S}\}$, where M is increasing in both arguments. Since only the aggregate measure of buyers (as opposed to the individual measures of local and foreign buyers) appears in M, the matching technology is unbiased with respect to the citizenship of the buyer. The arrival rate of buyers to an arbitrary seller is $a_{\mathcal{S}} = M(\mathcal{B}, \mathcal{S})/\mathcal{S}$, and the arrival rate of sellers to an arbitrary buyer is $a_{\mathcal{B}} = M(\mathcal{B}, \mathcal{S})/\mathcal{B}$. In both DM's, $\mathcal{S} = \xi$. Buyers get to visit the domestic DM with probability $\sigma_{H} \in (0, 1)$ and the foreign DM with probability $\sigma_{F} \in (0, 1)$, so that in both DM's $\mathcal{B} = \sigma_{H} + \sigma_{F}$. The relative magnitude of σ_{F} to σ_{H} captures the degree of economic integration. In any bilateral meeting, the buyer makes a take-it-or-leave-it offer to the seller. Any sale to a citizen of country j will count as imports of country j from country i.

As mentioned earlier, during the second sub-period agents trade in centralized markets, CM_i, $i = A, B$. All agents consume and produce a general good or fruit which is identical in both countries. Thus, the domestic and the foreign general goods enter as perfect substitutes in the utility function. Agents are located in the home country and have access to a technology that can transform one unit of labor into one unit of the fruit. Furthermore, we assume that there are two trees, one in each country, that produce fruit, as in Lucas (1978). Shares of the tree in country i are traded in CM_i, but due to perfect financial integration, agents from country j can place any order and buy shares of this tree at the ongoing price ψ_i. Let $T_i > 0$ denote the total supply of the tree in country i and d_i the per-period dividend of tree i. Since in this paper we focus on symmetric equilibria, we assume that $T_{A} = T_{B} = T > 0$ and $d_{A} = d_{B} = d > 0$. T and d are exogenously given and constant.

Except from consuming the general good and trading shares of the trees in the CM's, buyers can also carry some claims into the DM's in order to trade them for a special good produced by local sellers. Hence, assets serve not only as stores of value, but also as media of exchange. The necessity for a medium of exchange arises due to anonymity and a double coincidence of

[3] To avoid the possibility of confusion, variables indexed by the term B are related to "country B". On the other hand, variables indexed by the term $\mathcal{B}$ are related to "Buyers".

wants problem that characterizes trade in the DM's (see Kocherlakota (1998) for an extensive discussion). Assets can serve as media of exchange as long as they are portable, storable, divisible, and recognizable by all agents. We assume that all these properties are satisfied. As we explain in more detail below, we do not place any *ad hoc* restrictions on which assets can serve where as means of payments.

In the absence of any frictions in the physical environment, due to the symmetry assumption made earlier, the model would predict that agents' share of foreign assets in their portfolios is anywhere between 0 and 100%. In order to derive sharper predictions, additional assumptions are necessary. One friction that is very common in the international macroeconomics literature is the so-called *currency areas* assumption, which dictates that trading in country i requires the use of that country's currency (in our case the asset) as a medium of exchange. This assumption is extremely appealing and empirically relevant. However, since our paper is in the spirit of modern monetary theory, we consider such a restriction undesirable, and we insist that agents should choose which assets to use as media of exchange.

We now introduce the main friction of our model. We assume that whenever an agent from country i holds one share of country j's tree, $j \neq i$, she has a claim to $d - \kappa$ units of fruit, with $\kappa \in (0, d)$. We label κ with the general term *transaction cost*. One can think of κ as an information friction or a cost that agents have to pay in order to participate in the foreign asset market.[4] Alternatively, κ may capture a policy friction, such as a tax on foreign dividend returns, which is commonly observed across many countries. Another way to think about κ follows from a more literal interpretation of the Lucas tree model: when an agent from country i holds one share of tree j, d units of fruit (general good) have to be physically delivered from country j to its claimant in country i. Therefore, κ could also represent a transportation cost. It is important to highlight that all the results presented in this paper hold for arbitrarily small values of κ.

3 Value Functions and Optimal Behavior

We begin with the description of the value functions in the CM. For a buyer from country $i = A, B$, the Bellman's equation is given by

$$W_i^{\mathcal{B}}(t) = \max_{X,H,\hat{t}} \left\{ U(X) - H + \beta V_i^{\mathcal{B}}(\hat{t}\,) \right\}$$

$$\text{s.t. } X + \psi_i \left(\hat{t}_{ii} + \hat{t}_{ij}\right) + \psi_j \left(\hat{t}_{ji} + \hat{t}_{jj}\right) = H + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj}).$$

[4] Similarly, Michaelides (2003) assumes that agents face a higher transaction cost when investing abroad.

The state variables are summarized by $t \equiv (t_{ii}, t_{ij}, t_{ji}, t_{jj})$, where t_{ij} is the amount of asset i that is used for trade in DM_j, and variables with hats denote next period's choices.[5] We have also defined $d_\kappa = d - \kappa$. It can be easily verified that, at the optimum, $X = X^*$. Using this fact and replacing H from the budget constraint into $W_i^{\mathcal{B}}$ yields

$$\begin{aligned} W_i^{\mathcal{B}}(t) &= U(X^*) - X^* + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj}) \\ &+ \max_{\hat{t}} \left\{ -\psi_i \left(\hat{t}_{ii} + \hat{t}_{ij} \right) - \psi_j \left(\hat{t}_{ji} + \hat{t}_{jj} \right) + \beta V_i^{\mathcal{B}}(\hat{t}) \right\}. \end{aligned} \tag{1}$$

Two observations are in order. First, since $\mathcal{U}$ is quasi-linear, the optimal choice of $\hat{t}$ does not depend on t, i.e. there are no wealth effects. Second, $W_i^{\mathcal{B}}$ is linear, and we can write

$$W_i^{\mathcal{B}}(t) = \Lambda_i^{\mathcal{B}} + (\psi_i + d)(t_{ii} + t_{ij}) + (\psi_j + d_\kappa)(t_{ji} + t_{jj}),$$

where the definition of $\Lambda_i^{\mathcal{B}}$ is obvious.

We now focus on sellers. As it is standard in monetary models, carrying assets across periods of time comes at a cost (in the case of fiat money, the cost is just the nominal interest rate). The only agents who are willing to incur this cost are the agents who benefit from the asset's liquidity, i.e. the buyers. Since in our model the identity of agents, as buyers or sellers, is fixed, sellers will always choose to leave the CM with zero asset holdings. For a more detailed proof of this result, see Rocheteau and Wright (2005).[6] Noting that sellers also choose $X = X^*$ in every period, we can write the value function of a seller from country $i = A, B$ as

$$\begin{aligned} W_i^{\mathcal{S}}(t_i, t_j) &= U(X^*) - X^* + (\psi_i + d)\, t_i + (\psi_j + d_\kappa)\, t_j + \beta V_i^{\mathcal{S}}(0,0) \\ &\equiv \Lambda_i^{\mathcal{S}} + (\psi_i + d)\, t_i + (\psi_j + d_\kappa)\, t_j. \end{aligned}$$

Like in the buyers' case, $W_i^{\mathcal{S}}$ is linear. Sellers leave the CM with zero asset holdings, but they might enter this market with positive amounts of both assets, which they received as means of payments in the preceding DM.

We now turn to the terms of trade in the DM's. As explained earlier, we place no restrictions on which assets can be used as media of exchange. Consider meetings in DM_i, and as a first case

[5] Hence, the buyer can bring any amount of local or foreign asset to the local DM, and the same is true for the foreign DM. This modeling choice guarantees that the order in which the DM's open is inessential. If the buyer was choosing some $(\hat{t}_i, \hat{t}_j)$, which she could bring to either DM, the amount of assets that she would choose to trade in, say, DM_i would crucially depend on whether she got/will get matched in DM_j. That is, the buyer's behavior would depend on the order in which the DM's open. We consider this an undesirable feature.

[6] In Rocheteau and Wright (2005), the nominal interest rate is assumed to be strictly positive. Hence, holding no money is the unique optimal choice for sellers. In our model, there will be a case in which the cost of carrying the home asset is zero (this will never be the case for the foreign asset because of the transaction cost). In this case, sellers are indifferent between holding zero or some positive amount of the home asset. Nevertheless, holding zero asset is *always* optimal. Moreover, if one assumes that there is a cost, $c > 0$, of participating in the asset markets, then holding zero assets is the unique optimal choice for the seller, even for a tiny c.

let the buyer be a citizen of country i (a local) with asset holdings denoted by t. The solution to the bargaining problem is a list (q_i, x_{ii}, x_{ji}), where q_i is the amount of special good, x_{ii} is the amount of asset i, and x_{ji} is the amount of asset j that changes hands. With take-it-or-leave-it offers by the buyer, the bargaining problem is

$$\begin{aligned} &\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) + W_i^{\mathcal{B}}(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) - W_i^{\mathcal{B}}(t)\right] \\ &\text{s.t.} \quad -q_i + W_i^{\mathcal{S}}(x_{ii}, x_{ji}) - W_i^{\mathcal{S}}(0,0) = 0 \\ &\text{and} \quad x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned} \tag{2}$$

Exploiting the linearity of the W's, we can re-write this problem as

$$\begin{aligned} &\max_{q_i, x_{ii}, x_{ji}} [u(q_i) - (\psi_i + d)x_{ii} - (\psi_j + d_\kappa)x_{ji}] \\ &\text{s.t.} \quad q_i = (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji} \\ &\text{and} \quad x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned}$$

The following lemma describes the bargaining solution in detail.

Lemma 1. *Define* $\pi_i \equiv (\psi_i + d)t_{ii} + (\psi_j + d_\kappa)t_{ji}$.

$$If \quad \pi_i \geq q^*, \quad then \quad \begin{cases} q_i = q^*, \\ (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji} = q^*. \end{cases}$$

$$If \quad \pi_i < q^*, \quad then \quad \begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii},\ x_{ji} = t_{ji}. \end{cases}$$

Proof. It can be easily verified that the suggested solution satisfies the necessary and sufficient conditions for maximization. □

All that matters for the bargaining solution is whether the buyer's asset balances, i.e. π_i, are sufficient to buy the optimal quantity q^*. If the answer to that question is yes, then $q_i = q^*$, and the buyer spends amounts of assets t_{ii}, t_{ji} such that $\pi_i = q^*$. Notice that in this case t_{ii}, t_{ji} cannot be pinned down separately.[7] Conversely, if $\pi_i < q^*$, the buyer gives up all her asset holdings and purchases as much q as her balances allow.

Now consider a meeting in DM_i when the buyer is from $j \neq i$ (a foreigner) with asset holdings t. Again, denote the solution by (q_i, x_{ii}, x_{ji}). The bargaining problem to be solved is the same as in (2), after replacing $W_i^{\mathcal{B}}$ with $W_j^{\mathcal{B}}$. Using the linearity of the value functions, one can write the bargaining problem as

[7] This does not cause any indeterminacy issues because, as we show later, buyers set $t_{ji} = 0$.

$$\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) - (\psi_i + d_\kappa)x_{ii} - (\psi_j + d)x_{ji}\right]$$
$$\text{s.t.} \quad q_i = (\psi_i + d)x_{ii} + (\psi_j + d_\kappa)x_{ji}$$
$$\text{and} \quad x_{ii} \le t_{ii}, x_{ji} \le t_{ji}.$$

Substituting for $(\psi_i + d)x_{ii} + (\psi_j + d)x_{ji}$ from the constraint, we can re-write the objective as $u(q_i) - q_i - \kappa x_{ji} + \kappa x_{ii}$. Notice that for every unit of asset i that goes from the buyer to the seller, two positive effects are generated. First, trade is facilitated, i.e. the seller produces q for the buyer in exchange for the asset. Second, the social surplus increases because the asset goes to the hands of the agent who has a higher valuation for it. The next Lemma describes the solution to the bargaining problem in detail.

Lemma 2. *Define* $\overline{q}(\psi) \equiv \left\{q : u'(q) = \frac{\psi + d_\kappa}{\psi + d}\right\}$ *and* $\underline{q}(\psi) \equiv \left\{q : u'(q) = \frac{\psi + d}{\psi + d_\kappa}\right\}$, *with* $\overline{q}(\psi) > q^* > \underline{q}(\psi)$, *for all* $\psi < \infty$. *The bargaining solution is the following*

a) If $t_{ii} \ge \frac{\overline{q}(\psi_i)}{\psi_i + d}$, *then* $\begin{cases} q_i = \overline{q}(\psi_i), \\ x_{ii} = \frac{\overline{q}(\psi_i)}{\psi_i + d}, \\ x_{ji} = 0. \end{cases}$

b) If $t_{ii} \in \left[\frac{\underline{q}(\psi_j)}{\psi_i + d}, \frac{\overline{q}(\psi_i)}{\psi_i + d}\right)$, *then* $\begin{cases} q_i = t_{ii}(\psi_i + d), \\ x_{ii} = t_{ii}, \\ x_j = 0. \end{cases}$

c1) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i + d}$ *and* $\pi_i \ge \underline{q}(\psi_j)$, *then* $\begin{cases} q_i = \underline{q}(\psi_j), \\ x_{ii} = t_{ii}, \\ x_{ji} = \frac{\underline{q}(\psi_j) - (\psi_i + d)t_{ii}}{\psi_j + d_\kappa}. \end{cases}$

c2) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i + d}$ *and* $\pi_i < \underline{q}(\psi_j)$, *then* $\begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii}, \\ x_{ji} = t_{ji}. \end{cases}$

Proof. See Appendix A. □

The solution is very intuitive. The buyer, citizen of j, should use only asset i whenever possible. If her asset-i holdings are unlimited, she should buy the quantity defined as $\overline{q}(\psi)$. This quantity is larger than q^*, the maximizer of $u(q) - q$, because of the second positive effect of using asset i described above (the wedge between the buyer's and the seller's valuation). In similar spirit, if the buyer's balances allow her to buy $\underline{q}(\psi)$ or more, she should not use any amount of asset j for trade. Positive amounts of asset j will change hands, only if the asset-i holdings are such that the buyer cannot purchase $\underline{q}(\psi)$. In that case, the buyer will use the amount of asset j that, together with all of her asset-i holdings, buys the quantity $\underline{q}(\psi)$.

Our last task in this section is to discuss the optimal portfolio choice of the buyer. To that end, we first describe the buyer's value function in the DM. Once this function has been established, we can plug it into equation (1) and characterize this agent's objective function. Define $p_H \equiv a_{\mathcal{B}}\sigma_H$ and $p_F \equiv a_{\mathcal{B}}\sigma_F$, i.e. p_H is the probability of matching in the local DM, and p_F is the analogous expression for the foreign DM. Then, the value function for a buyer from country i who enters the round of decentralized trade with asset holdings $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$, is given by

$$\begin{aligned} V_i^{\mathcal{B}}(t) &= p_H p_F \Big\{ u(q_i) + u(q_j) + W_i^{\mathcal{B}}(t_{ii} - x_{ii}, t_{ij} - x_{ij}, t_{ji} - x_{ji}, t_{jj} - x_{jj}) \Big\} \\ &+ p_H(1 - p_F) \Big\{ u(q_i) + W_i^{\mathcal{B}}(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) \Big\} \\ &+ p_F(1 - p_H) \Big\{ u(q_j) + W_i^{\mathcal{B}}(t_{ii}, t_{ij} - x_{ij}, t_{ji}, t_{jj} - x_{jj}) \Big\} \\ &+ (1 - p_H)(1 - p_F) W_i^{\mathcal{B}}(t), \end{aligned} \tag{3}$$

where q, x are determined by the bargaining protocols described in Lemmata 1 and 2. It is understood that (q_i, x_{ii}, x_{ji}) are functions of (t_{ii}, t_{ji}) and (q_j, x_{ij}, x_{jj}) are functions of (t_{ij}, t_{jj}).

The next step is to lead eqution (3) by one period in order to obtain $V_i^{\mathcal{B}}(\hat{t}\,)$. Then, plug the latter into (1) and focus only on the terms that contain the control variables $\hat{t} = (\hat{t}_{ii}, \hat{t}_{ij}, \hat{t}_{ji}, \hat{t}_{jj})$. This is the objective function of the buyer from country i, and it can be written as

$$J_i^{\mathcal{B}}(\hat{t}\,) = J_{i,H}^{\mathcal{B}}(\hat{t}_{ii}, \hat{t}_{ji}) + J_{i,F}^{\mathcal{B}}(\hat{t}_{ij}, \hat{t}_{jj}), \tag{4}$$

where we have defined

$$\begin{aligned} J_{i,H}^{\mathcal{B}}(\hat{t}_{ii}, \hat{t}_{ji}) &= \Big[-\psi_i + \beta \Big(\hat{\psi}_i + d \Big) \Big] \hat{t}_{ii} + \Big[-\psi_j + \beta \Big(\hat{\psi}_j + d_\kappa \Big) \Big] \hat{t}_{ji} \\ &+ \beta p_H \Big\{ u(q_i(\hat{t}_{ii}, \hat{t}_{ji})) - \Big(\hat{\psi}_i + d \Big) x_{ii}(\hat{t}_{ii}, \hat{t}_{ji}) - \Big(\hat{\psi}_j + d_\kappa \Big) x_{ji}(\hat{t}_{ii}, \hat{t}_{ji}) \Big\}, \end{aligned} \tag{5}$$

$$\begin{aligned} J_{i,F}^{\mathcal{B}}(\hat{t}_{ij}, \hat{t}_{jj}) &= \Big[-\psi_i + \beta \Big(\hat{\psi}_i + d \Big) \Big] \hat{t}_{ij} + \Big[-\psi_j + \beta \Big(\hat{\psi}_j + d_\kappa \Big) \Big] \hat{t}_{jj} \\ &+ \beta p_F \Big\{ u(q_j(\hat{t}_{ij}, \hat{t}_{jj})) - \Big(\hat{\psi}_i + d \Big) x_{ij}(\hat{t}_{ij}, \hat{t}_{jj}) - \Big(\hat{\psi}_j + d_\kappa \Big) x_{jj}(\hat{t}_{ij}, \hat{t}_{jj}) \Big\}. \end{aligned} \tag{6}$$

The term $J_{i,n}^{\mathcal{B}}$ is the part of the objective that reflects the choice of assets to be traded in DM_n, with $n = H$ for home or $n = F$ for foreign.

Equation (4) highlights that the optimal choice of asset holdings to be traded in the home and foreign DM ($(\hat{t}_{ii}, \hat{t}_{ji})$ and $(\hat{t}_{ij}, \hat{t}_{jj})$, respectively) can be studied in isolation. The term $-\psi_i + \beta(\hat{\psi}_i + d)$ represents the net gain from carrying one unit of the domestic asset from today's CM into tomorrow's CM. Sometimes we refer to the negative of this term as the cost of carrying the asset across periods. Similarly, $-\psi_j + \beta(\hat{\psi}_j + d_\kappa)$ is the net gain from carrying one unit of the foreign asset across consecutive CM's. In both (5) and (6), the second line represents the expected (discounted) surplus of the buyer in each DM. The following Lemma states an important result regarding the sign of the cost terms that appear in the agents' objective functions.

Lemma 3. *In any equilibrium,* $\psi_l \geq \beta(\hat{\psi}_l + d)$*,* $l = A, B$*.*

Proof. This is a standard result in monetary theory. If $\psi_l < \beta(\hat{\psi}_l + d)$ for some l, agents of country l have an infinite demand for this asset, and so equilibrium is not well defined. □

According to the Lemma, the net gain from carrying home assets across periods is non-positive and the net gain from carrying foreign assets across periods is strictly negative due to the term κ. This result assigns an intuitive interpretation to the objective function established above: a buyer wishes to bring assets with her in the DM in order to facilitate trade. However, she faces a trade-off because carrying these assets is not free (equations (5) and (6)).[8] We are now ready to discuss the optimal portfolio choice of the buyer and, consequently, equilibrium.

4 Equilibrium in the Two-Country Model

We begin this section with a general definition of equilibrium and we explain why focusing on symmetric, steady-state equilibria can make the analysis more tractable.

Definition 1. An equilibrium for the two-country economy is a list of solutions to the bargaining problems in DM_i, $i = A, B$ described by Lemmata 1 and 2 and bounded paths of ψ_A, ψ_B, such that buyers maximize their objective function (described by (5) and (6)) under the market clearing conditions $\sum_{l=A,B}\sum_{i=A,B} t^i_{Al} = T = \sum_{l=A,B}\sum_{i=A,B} t^i_{Bl}$. The term t^i_{jl} denotes demand of a buyer from country i for asset j to trade in DM_l.

In the remainder of the paper we focus on symmetric, steady-state equilibria. In our model, p_H, p_F are the same in both countries. This fact, in combination with the strict concavity of $J^{\mathcal{B}}_{i,H}$ and $J^{\mathcal{B}}_{i,F}$ (which is standard in the Lagos-Wright model), implies the following: each buyer has a certain (degenerate) demand for the home asset and a certain (degenerate) demand for the foreign asset, but these demand functions do not depend on the agent's citizenship. In other words, $t^A_{AA} = t^B_{BB}$, $t^A_{AB} = t^B_{BA}$, $t^A_{BA} = t^B_{AB}$, and $t^A_{BB} = t^B_{AA}$.[9] Two important implications follow. First, both assets have equal aggregate supply (T) and aggregate demand. Therefore, their equilibrium price has to be equal, $\psi_A = \psi_B = \psi$. Second, by the bargaining protocols, the

[8] Lemma 3 also clarifies our claim that it is always optimal for sellers to hold zero assets. In the case of foreign assets, this is the unique optimal choice.

[9] For example, $t^A_{AA} = t^B_{BB}$ means that the demand for asset A of a buyer from A in order to trade in her local DM is equal to the demand of a buyer from B for asset B used for trade in her local DM. The remaining equations admit similar interpretations.

amount of special good that changes hands in any DM depends only on whether the buyer is a local or a foreigner, but not on the label of the DM. These facts lead to the following definition.

Definition 2. A symmetric, steady-state equilibrium for the two-country economy can be summarized by the objects $\{t_{HH}, t_{HF}, t_{FH}, t_{FF}, q_H, q_F, \psi\}$. The term t_{ij} is the equilibrium asset holdings of the representative buyer (of any country) for asset i to be used for trade in DM_j, with $i, j = H$ for home or $i, j = F$ for foreign. For future reference also define the total home and foreign asset holdings of buyers, $t_H = t_{HH} + t_{HF}$ and $t_F = t_{FH} + t_{FF}$. The term q_i stands for the amount of special good that changes hands in any DM when the buyer is local ($i = H$) or foreign ($i = F$). Finally, ψ is the symmetric, steady-state equilibrium asset price. Equilibrium objects are such that agents maximize their respective objective functions and markets clear.

We now proceed to a more careful discussion of the optimal portfolio choice of buyers and, consequently, equilibrium. Notice that the symmetric, steady-state version of Lemma 3 dictates that $\psi \geq \beta d/(1-\beta) \equiv \psi^*$. The term ψ^* is the so-called *fundamental* value of the asset, i.e. the unique price that agents would be willing to pay for one unit of this asset if we were to shut down the DM's (in which case the model would coincide with a two-country Lucas-tree model). We will examine the choice of (t_{HH}, t_{FH}) and (t_{HF}, t_{FF}) separately, by looking at the symmetric, steady-state versions of (5) and (6).

Lemma 4. *A buyer's optimal choice of asset holdings for trade in the local DM satisfies $t_{FH} = 0$. Moreover, if $\psi > \psi^*$, t_{HH} solves*

$$\psi = \beta(\psi + d)\left\{1 + p_H[u'\left((\psi + d)t_{HH}\right) - 1]\right\}, \tag{7}$$

and if $\psi = \psi^*$, $t_{HH} \geq q^*/(\psi + d)$.

Proof. See Appendix A. ☐

Lemma 4 reveals that buyers never carry foreign assets to trade in the local DM. The intuition behind this result is straightforward. Recall from Lemma 1 that any combination of assets for which $(\psi+d)t_{HH} + (\psi+d_\kappa)t_{FH} = \pi$, for some given π, buys the same amount of special good. Since the foreign asset has a higher holding cost, it is always optimal for the buyer to purchase any desired quantity using t_{HH} only. When the cost to carry assets falls to zero, optimality requires that the buyer bring any amount of assets that buy her q^* in the local DM (this amount maximizes the buyer's surplus). By Lemma 1, any $t_{HH} \geq (1-\beta)q^*/d$ does that job.

Next, we consider the optimal choice of assets used for trade in the foreign DM. There are three scenarios (or regimes), that depend on the values of the following parameters: β, p_F, and κ/d. In the first scenario, regardless of asset prices, the agent uses only foreign assets to trade in the foreign DM. In the second scenario, again regardless of asset prices, the agent chooses

$t_{FF} = 0$ and trades in the foreign DM with her home asset. Finally, there is a third regime, in which the agent uses either t_{HF} or t_{FF} as means of payment (except from a knife-edge case), depending on asset prices. The following Lemma describes the details. Figure 1 summarizes the parameter values that constitute the various regions described in Lemma 5.

Lemma 5. *Case 1: Assume $(p_F, \beta) \in R_1$, or $(p_F, \beta) \in R_2$ and $\frac{\kappa}{d} > \frac{(1-2p_F)}{(1-p_F)(1-\beta)} \equiv \tilde{\kappa}$. Then the buyer uses only the foreign asset as a medium of exchange in the foreign DM. The optimal t_{FF} satisfies*

$$\psi = \beta\left[(1-p_F)(\psi + d_\kappa) + p_F(\psi + d)\, u'\left((\psi + d)t_{FF}\right)\right]. \tag{8}$$

If $\psi > \psi^*$, then $t_{HF} = 0$, and if $\psi = \psi^*$, then $t_{HF} \in \mathbb{R}_+$. Also, $q_F = (\psi + d)t_{FF}$.

Case 2: Assume $(p_F, \beta) \in R_4$. The buyer sets $t_{FF} = 0$ and uses only the domestic asset as a medium of exchange in the foreign DM. If $\psi > \psi^*$, then t_{HF} satisfies

$$\psi = \beta\left[(1-p_F)(\psi + d) + p_F(\psi + d_\kappa)\, u'\left((\psi + d_\kappa)t_{HF}\right)\right], \tag{9}$$

with $q_F = (\psi + d_\kappa)t_{HF}$. If $\psi = \psi^*$, then any $t_{HF} \geq \underline{q}(\psi^*)/(d/(1-\beta) - \kappa)$ is optimal. In this case, $q_F = \underline{q}(\psi^*)$.

Case 3: Assume $(p_F, \beta) \in R_3$, or $(p_F, \beta) \in R_2$ and $\kappa/d \leq \tilde{\kappa}$. Also, define $\psi_c \equiv \beta(1-p_F)(2d - \kappa)/[1 - 2\beta(1-p_F)]$. The following sub-cases arise: a) If $\psi > \psi_c$, then the optimal t_{HF}, t_{FF}, q_F are as in Case 1 above. b) If $\psi < \psi_c$, then the optimal t_{HF}, t_{FF}, q_F are as in Case 2 above. c) In the knife-edge case $\psi = \psi_c$, $t_{HF}, t_{FF} > 0$ and both (8) and (9) hold. The optimal choices t_{HF}, t_{FF} cannot be uniquely pinned down, but q_F is uniquely given by $q_{F,c} \equiv \{q : u'(q) = (1-p_F)/p_F\}$.

Proof. See Appendix A. □

Lemma 5 has an intuitive explanation. The term κ creates a "wedge" between the asset valuations of a seller and a buyer who come from different countries. The buyer knows that if she meets with a foreign seller, she will have higher purchasing power if she carries the foreign asset (which represents a home asset to the seller). On the other hand, meeting the seller is not guaranteed, and if the buyer stays unmatched in the foreign DM, she will receive the lower dividend associated with the foreign asset. A reverse story applies regarding the optimal choice of t_{HF}. If the buyer carries a large amount of the home asset and matches in the foreign DM, she will have lower purchasing power because, for every unit of t_{HF} that she passes to the seller, the latter will receive a lower dividend in the forthcoming CM. Not surprisingly, when p_F is high, the buyer decides to carry out trades in the foreign DM using the foreign asset. On the other hand, when it is less likely to match abroad (low p_F), the buyer prefers to carry her home asset, even when trading in the foreign country. In intermediate cases, either regime can arise, depending on the asset price, which affects the holding costs.[10]

[10] An interesting point to notice is that in Case 3, for ψ in the neighborhood of ψ_c, the nature of the agent's

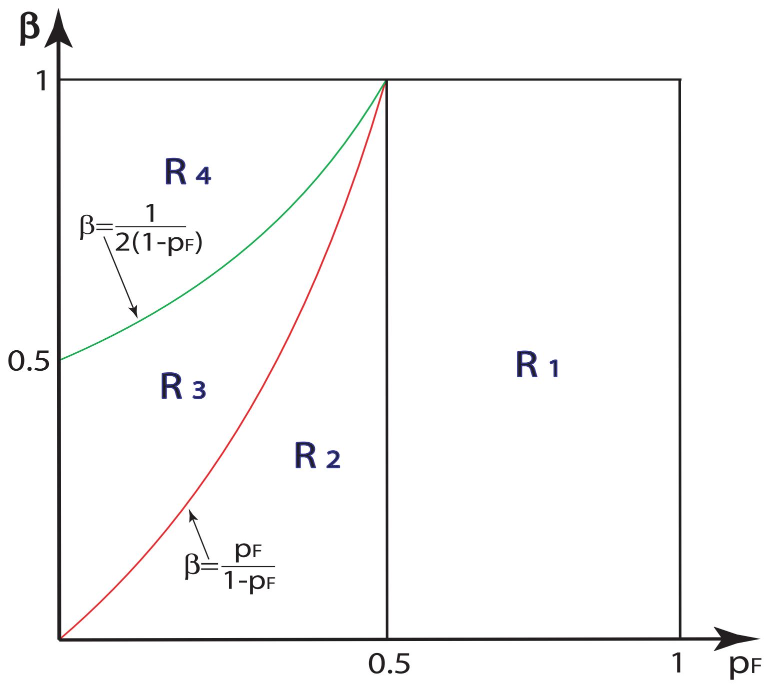

Figure 1: Parameter Values and Regions

The relationship between the optimal t_{HF} and t_{FF} with p_F is straightforward. What is perhaps more surprising is the fact that, in R_2, the buyer chooses to trade with the foreign asset when κ is relatively high. This might seem counterintuitive at first, given that a high κ means a low dividend for a buyer who did not dispose of the foreign asset. But one should not forget that a high κ also implies low purchasing power for the buyer, if she carries only t_{HF}. These forces have opposing effects. Whether it is optimal to trade with t_{HF} or t_{FF} depends on the value of β. When κ is high, the buyer realizes that she might receive a lower dividend, but this will happen *tomorrow*. On the other hand, when the buyer is trying to buy some q and pay with t_{HF}, the seller will receive a lower dividend in the current period's CM, i.e. *tonight*. Thus, a high κ dictates the use of t_{FF} in foreign meetings, as long as agents are not very patient. As an extreme case, consider points in R_2 that are close to the origin, and suppose κ is very high. Although p_F is tiny, the buyer chooses to trade with t_{FF} because, in this region, β is also tiny.

So far we have analyzed the optimal behavior of agents for given (symmetric, steady state) asset prices. To complete the model, we incorporate the exogenous supply of assets in the analysis and treat ψ as an equilibrium object. For the reader's convenience, before we state the proposition that characterizes equilibrium, we repeat the definitions of some objects introduced above, and we define a few new objects.[11]

optimal choice transforms fully. For $\psi = \psi_c - \epsilon$, $\epsilon > 0$, $t_{FF} = 0$ and $t_{HF} > 0$. But for $\psi = \psi_c + \epsilon$, $t_{HF} = 0$ and the agent uses only foreign assets as a medium of exchange. Despite this dramatic change in the agent's portfolio composition, as ψ crosses the critical value ψ_c, the amount of special good purchased is continuous in ψ. To see this point, just set $\psi = \psi_c + \epsilon$ in (8) and $\psi = \psi_c - \epsilon$ in (9). It is easy to show that, as $\epsilon \to 0$, $q_F \to q_{F,c}$ in both cases.

[11] A comment on notation: when we write $q^*_{F,1}$, the asterisk refers to the fact that this is the highest value that q_F

$$
\begin{aligned}
q^*_{F,1} &\equiv \left\{ q : u'(q) = 1 + \frac{\kappa}{d}\frac{(1-\beta)(1-p_F)}{p_F} \right\} \\
q^*_{F,2} &\equiv \underline{q}(\psi^*) = \left\{ q : u'(q) = \frac{d}{d-(1-\beta)\kappa} \right\} \\
q_{H,c} &\equiv \left\{ q : u'(q) = \frac{d(1-2p_F) - \kappa(1-\beta)(1-p_F)}{p_H[d - \beta\kappa(1-p_F)]} \right\} \\
q_{F,c} &\equiv \left\{ q : u'(q) = \frac{1-p_F}{p_F} \right\} \\
T^*_1 &\equiv \frac{(1-\beta)(q^* + q^*_{F,1})}{d} \\
T^*_2 &\equiv (1-\beta)\left(\frac{q^*}{d} + \frac{\underline{q}(\psi^*)}{d - \kappa(1-\beta)} \right) \\
T_c &\equiv \frac{1-2\beta(1-p_F)}{d - \kappa\beta(1-p_F)}(q_{H,c} + q_{F,c}) < T^*_2 \\
\psi_c &\equiv \frac{\beta(1-p_F)(2d-\kappa)}{1-2\beta(1-p_F)} \\
\tilde{\kappa} &\equiv \frac{(1-2p_F)}{(1-p_F)(1-\beta)}
\end{aligned}
$$

Proposition 1. *For all parameter values, in equilibrium, $t_{FH} = 0$. Moreover:*

a) If $(p_F, \beta) \in R_1$, or $(p_F, \beta) \in R_2$ and $\kappa/d \geq \tilde{\kappa}$, equilibrium is characterized by "local asset dominance" and agents trade in the foreign DM using only foreign assets. Hence, $t_{HF} = 0$, $t_H = t_{HH}$, and $t_F = t_{FF}$. If $T \geq T^*_1$, then $\psi = \psi^*$, $q_H = q^*$, $q_F = q^*_{F,1} < q^*$, and $t_H = T - q^*_{F,1}(1-\beta)/d$. If $T < T^*_1$, then $\psi > \psi^*$, $q_H < q^*$, and $q_F < q^*_{F,1}$. For any T in this region, $\partial\psi/\partial T < 0$, $\partial q_H/\partial T > 0$, and $\partial q_F/\partial T > 0$.

b) If $(\beta, p_F) \in R_4$, an "international asset" equilibrium arises and agents use their home assets in all DM's, i.e. $t_{FF} = 0$, so $t_F = 0$ and $t_H = T$. If $T \geq T^*_2$, then $\psi = \psi^*$, $q_H = q^*$, and $q_F = \underline{q}(\psi^*) < q^*$. If $T < T^*_2$, then $\psi > \psi^*$, $q_H < q^*$, and $q_F < \underline{q}(\psi^*)$. For any T in this region, $\partial\psi/\partial T < 0$, $\partial q_H/\partial T > 0$, and $\partial q_F/\partial T > 0$.

c) If $(\beta, p_F) \in R_3$, or $(\beta, p_F) \in R_2$ and $\kappa/d < \tilde{\kappa}$, a "mixed regime" equilibrium arises in the sense that either local asset dominance or international asset could arise depending on T. If $T > T_c$, we are in the international asset regime. For T's in this region, $\psi \in [\psi^*, \psi_c)$, $q_H \in (q_{H,c}, q^*]$, and $q_F \in (q_{F,c}, q^*_{F,2}]$. If $T < T_c$, we switch to a local asset dominance equilibrium and $\psi > \psi_c$, $q_H < q_{H,c}$, and $q_F < q_{F,c}$. In the knife-edge case where $T = T_c$, buyers purchase $q_F = q_{F,c}$ in the foreign DM using any combination of home and foreign assets.

Proof. See Appendix A. □

can reach and the number 1 refers to the case, in particular Case 1.

A unique steady state equilibrium exists for all parameter values. Buyers never carry foreign assets in order to trade in their home DM. Under parameter values summarized as Case 1 in Lemma 5, an equilibrium with local asset dominance arises. Buyers choose to trade in the foreign DM using the foreign asset only, because the positive effect (high purchasing power) of holding foreign assets dominates the negative effect (low dividend). There exists a critical level T_1^* that captures the *liquidity needs* of the economy. For $T < T_1^*$, increasing T helps buyers purchase more special good in both DM's. Hence, the marginal valuation of one unit of the claim is higher than ψ^*, i.e. the price of the asset in a world where the Lucas-tree serves only as a store of value. The difference $\psi - \psi^*$ represents a *liquidity premium* because it reflects a premium in the valuation of the asset that stems from its second role (as a medium of exchange). When $T \geq T_1^*$, the asset's liquidity properties have been exploited (increasing T does not help buyers purchase more good) and $\psi = \psi^*$. When this is true, the cost of carrying the home asset is zero, so local buyers absorb all the excess supply, $t_H = T - q^*_{F,1}(1-\beta)/d$.

A similar analysis applies when parameter values are as in Case 2 in Lemma 5. We refer to this case as an international asset equilibrium because agents use their home asset as means of payment everywhere in the world. The asset supply that captures the liquidity needs of the economy is given by T_2^*. When $T \geq T_2^*$, the asset price is down to its fundamental value and q_H, q_F have reached their upper bounds. Notice that in this case, buyers do not purchase any foreign assets in the CM, i.e. $t_F = 0$ and $t_H = T$. However, this does not mean that no agent ever holds any foreign assets. Sellers from country i who got matched with foreign buyers get payed with, and therefore hold some, asset j (which represents a home asset to the buyers). This will be important in the next section where we will compute asset home bias.

When parameter values are as in Case 3 in Lemma 5, we can end up with local asset dominance or international asset equilibrium depending on T, which directly affects ψ. Equilibria with local asset dominance arise if and only if $T > T_c$. When T is large, equilibrium prices are relatively low, which means that the cost of carrying the asset is relatively low. When the cost of carrying assets is low, the term κ becomes relatively more important. Recall from the discussion of Lemma 5 that it is precisely when κ is relatively high that agents choose to use the foreign asset in order to trade in the foreign DM (this becomes obvious when $(\beta, p_F) \in R_2$). On the other hand, if T is very small, the cost of carrying assets is large, so the term κ becomes less relevant. For intermediate values of (β, p_F) (remember we are in Case 3), this leads the buyers to optimally choose their home asset to trade in all DM's.

5 Home Bias and High Turnover

In this section we derive three predictions of our model. These include the existence of asset home bias, the positive correlation between asset and consumption home bias, and the higher turnover rate of foreign over domestic assets. We focus on the local asset dominance equilibrium, which is empirically relevant as it corresponds to a currency area. We treat the case of international asset equilibrium in the accompanying Online Appendix.

5.1 Preliminaries

In this section we establish some useful properties of the equilibrium values of q_H, q_F and t_H, t_F, which will lead us to our discussion of asset and consumption home bias as well as asset turnover rates. For given ψ, the first-order conditions (7) and (8) implicitly define q_H, q_F as functions of the probabilities p_H, p_F. We have

$$\begin{aligned} q_H &= q_H(p_H) \equiv \left\{ q : u'(q) = 1 + \frac{\psi - \beta(\psi + d)}{\beta p_H(\psi + d)} \right\}, \\ q_F &= q_F(p_F) \equiv \left\{ q : u'(q) = \frac{\psi - \beta(\psi + d_\kappa)(1 - p_F)}{\beta p_F(\psi + d)} \right\}. \end{aligned}$$

The produced quantities can be used to characterize the volumes of assets that change hands in bilateral meetings. From the bargaining solution, we have

$$\begin{aligned} t_H(p_H) &= \frac{q_H(p_H)}{\psi + d}, \\ t_F(p_F) &= \frac{q_F(p_F)}{\psi + d}. \end{aligned}$$

Lemma 6. *a) If $T < T_1^*$, the following results hold: i) The functions $q_H(p_H)$, $q_F(p_F)$ and $t_H(p_H)$, $t_F(p_F)$ are strictly increasing. ii) For every $p_H = p_F = p \in (0,1)$, $q_H(p) > q_F(p)$. iii) For any $p_F \in (0,1)$, define $\tilde{p}_H(p_F) \equiv \{p : q_H(p) = q_F(p_F)\}$. Then, $\tilde{p}_H(p_F) < p_F$, and for any $p_F \in (0,1)$, $p_H > \tilde{p}_H(p_F)$ implies $q_H(p_H) > q_F(p_F)$. iv) For any $p_F \in (0,1)$, $p_H > \tilde{p}_H(p_F)$ implies $t_H(p_H) > t_F(p_F)$.*

b) If $T \geq T_1^*$, $q_H = q^* > q_{F,1}^* = q_F$. Also, $t_H = T - q_{F,1}^*(1-\beta)/d > q_{F,1}^*(1-\beta)/d = t_F$.

Proof. See Appendix A. □

When the economy is liquidity constrained, i.e. $T < T_1^*$, the equilibrium quantity of special good purchased in a certain DM, and the volume of the traded asset, are increasing in the probability of visiting that DM. Since, $q_H(p)$ is strictly larger than $q_F(p)$, for all $p \in (0,1)$, agents purchase greater amounts of the home special good, even when the probability of trading in the local DM is smaller than the probability of trading in the foreign DM (but not much smaller). Moreover, since trade in a certain DM is facilitated by the local asset, the condition which guarantees that $q_H > q_F$ (namely $p_H > \tilde{p}_H(p_F)$), will also guarantee that $t_H > t_F$. On the other hand, if $T \geq T_1^*$, we have $q_H > q_F$ and $t_H > t_F$, regardless of p_H, p_F.

5.2 Asset Home Bias

In this section, we derive sufficient conditions under which the model's predicted asset portfolio is biased toward domestic assets. The result is summarized in Proposition 2.

Proposition 2. *a) Assume that $T < T_1^*$. For any $p_F \in (0,1)$, let $p_H > \tilde{p}_H(p_F)$. Then agents' portfolios exhibit home bias in the sense that the home asset share in the entire portfolio is greater than fifty percent. Formally, the home asset share is*

$$HA = \frac{2t_H + p_F t_F}{2T} > 0.5. \tag{10}$$

b) Assume that $T \geq T_1^*$. Agents' portfolios exhibit home bias for any p_H, p_F. Formally,

$$HA = \frac{2\left(T - \frac{1-\beta}{d} q_{F,1}^*\right) + p_F \frac{1-\beta}{d} q_{F,1}^*}{2T} > 0.5. \tag{11}$$

Proof. See Appendix A. □

The details of the derivation of the asset home bias formula can be found in Appendix B. To understand why the left-hand side of (10) represents the home asset share in the agents' portfolio, focus on the citizens of a certain country. In the denominator, T stands for the citizens' total asset holdings. It is multiplied by two because we account for asset holdings in both the DM and the CM (we assume equal weight). The numerator represents the home asset holdings. The term $2t_H$ stands for the buyers' asset holdings in the CM and in the DM. The term $p_F t_F$ is the amount of the home asset held by sellers who matched with foreign buyers. The formula in (11) admits a similar interpretation.

To understand the result, first consider the case in which $T < T_1^*$. Then, portfolios exhibit home bias as long as trading opportunities at home are not significantly less abundant than trading opportunities abroad. This follows directly from Lemma 6. Intuitively, when the economy is liquidity constrained, agents incur positive costs to hold the domestic and the foreign asset. Hence, they choose the optimal mix of assets in anticipation of their purchasing needs for special goods, which are governed by the trading opportunities. The higher the probability for a successful match in a market, the higher the demand for the asset that facilitates trade there.

When $T \geq T_1^*$, agents purchase the quantities q^* and $q_{F,1}^*$. The term q^* is independent of the matching probabilities, while $q_{F,1}^*$ is rising in p_F but never exceeds q^*. Hence, for the purpose of acquiring special goods, the demand for foreign assets cannot exceed the demand for domestic assets. Moreover, as soon as the liquidity needs of the economy are satisfied, asset prices fall to fundamental values. Then, domestic assets become useful saving tools, while foreign assets remain costly to hold. Hence, agents hold the domestic asset both as a saving tool and in order to engage in trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad.

5.3 Consumption and Asset Home Bias

In this model, consumption and asset home bias coexist. Proposition 3 states the result.

Proposition 3. *Define C_F, C_T, and C_H as the value of foreign (or imported) consumption, total consumption, and consumption produced at home, respectively.*

a) Assume $T < T_1^*$. For any $p_F \in (0, 1)$, let $p_H > \tilde{p}_H(p_F)$. Then $C_H > C_F$, implying $\frac{C_H}{C_T} > 0.5$.

b) Assume $T \geq T_1^*$. For any p_H, p_F, we have $\frac{C_H}{C_T} > 0.5$.

Proof. See Appendix B. □

The details of the accounting of consumption can be found in Appendix B. For each agent in the model, imported consumption consists of special goods bought from foreign sellers as well as fruit obtained from abroad when net claims to foreign trees are positive. On the contrary, domestic consumption includes special goods bought in domestic DM meetings as well as fruit obtained not only when net claims to the domestic tree are positive, but also via work.

Then, the results follow from Proposition 2 and Lemma 6. Suppose that the parameters are restricted such that the economy exhibits asset home bias. Since, at the country level, more domestic assets are held relative to foreign ones, net aggregate claims to foreign trees fall short of net claims to domestic trees. With respect to the DM, Lemma 6 ensures that the quantity (and value) exchanged (and consumed) in a domestic meeting exceeds the quantity (and value) exchanged in a foreign meeting. Thus, for any non-negative work effort, domestic consumption exceeds imported consumption. Finally, Lemma 6 and Proposition 3 suggest that agents buy more consumption goods from countries whose assets they hold in higher amounts. Thus, the model yields the following testable prediction: bilateral imports are positively correlated with bilateral asset positions.

The above discussion demonstrates that consumption and asset home bias are intimately related in the model. When the economy is liquidity constrained, asset and consumption home bias arise as long as trading opportunities at home are not significantly less abundant than trading opportunities abroad. The latter occurs if the probability with which agents visit the domestic relative to the foreign market is higher. It is reasonable to argue that such parameter restriction would hold if agents have a stronger preference for domestic goods, experience more frequent positive shocks to domestic consumption, or simply possess superior information over the quality of domestic goods. These assumptions are standard in the home bias literature (see

Heathcote and Perri (2007), Collard, Dellas, Diba, and Stockman (2009) and Hnatkovska (2010)).

However, when the liquidity needs of the economy are satisfied, consumption and asset home bias coexist regardless of the relative trading opportunities in domestic and foreign markets. The key insight from this case is that domestic assets are useful saving tools, while foreign assets are not due to the existence of transaction costs. Hence, agents hold the domestic asset both as a saving tool and in order to engage in trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad. This very mechanism lies at the heart of the model's ability to reconcile the coexistence of asset and consumption home bias with a higher turnover rate of foreign over domestic assets.

5.4 Domestic and Foreign Asset Turnover Rates

The model's predictions on asset and consumption home bias discussed above are shared with the frameworks of Heathcote and Perri (2007), Collard, Dellas, Diba, and Stockman (2009), and Hnatkovska (2010). However, the additional prediction that relates domestic and foreign asset turnover rates discussed below is unique to the present model.

Proposition 4. *Define the turnover rates of home and foreign assets as*

$$TR_{H} = \frac{3p_{H}t_{H} + 2p_{F}t_{F}}{2t_{H} + p_{F}t_{F}}, \ \ \forall T < T_1^*, \tag{12}$$

$$TR_{H} = \frac{\frac{1-\beta}{d}\left(3p_{H}q^* + 2p_{F}q^*_{F,1}\right)}{2\left(T - \frac{1-\beta}{d}q^*_{F,1}\right) + \frac{1-\beta}{d}p_{F}q^*_{F,1}}, \ \ \forall T \geq T_1^*, \tag{13}$$

$$TR_{F} = \frac{2p_{F}}{2 - p_{F}}, \ \ \forall T. \tag{14}$$

There exists a level of asset supply $\tilde{T}$, with $T_1^* \leq \tilde{T} < \infty$, such that $T \geq \tilde{T}$ implies $TR_{F} > TR_{H}$.

Since the derivations of turnover rates are essential for the understanding of the Proposition, we present the proof in the main text.

Proof of Proposition 4. Following Warnock (2002), we define the foreign turnover rate as the ratio of the total volume of the foreign asset *traded* by the citizens of the domestic country (numerator) over the total volume of the foreign asset *held* by the citizens of the domestic country (denominator).[12] For theoretical consistency, we define the domestic turnover rate as the ratio of the total volume of the domestic asset *traded* by the citizens of the domestic country over the total volume of the domestic asset *held* by the citizens of the domestic country. We continue to

[12] Warnock's (2002) definition uses values rather than volumes, but the two are identical in the steady state.

count asset holdings in both the CM and the DM, assuming equal weights for the two markets. It is understood that we count asset holdings at the end of each subperiod.

a) Suppose $T < T_1^*$. TR_F is given by (14). The numerator reflects all the trades of the foreign asset carried out by the citizens of the domestic country.[13] A measure p_F of buyers match in the foreign DM and exchange t_F units of the foreign asset to acquire the special good. They also purchase t_F units of the foreign asset in the CM in order to re-balance their portfolios. The denominator consists of the total holdings of the foreign asset, which amount to $(2-p_F)t_F$. This expression includes the holdings of buyers in both the CM and the DM, $2t_F$, net of the amount given up by buyers of measure p_F who matched abroad. Since the term t_F appears in both the numerator and the denominator, it cancels out, yielding the constant turnover rate in (14).

TR_H is given by (12). The numerator includes the amount of domestic assets that buyers buy, $p_H t_H$, and sellers sell, $p_H t_H + p_F t_F$, in order to re-balance their portfolios in the CM after successfully matching in the DM. In addition, the numerator reflects the amount of domestic assets exchanged in the DM between sellers and domestic and foreign buyers, respectively, $p_H t_H + p_F t_F$. The denominator corresponds to total domestic assets held, which can be found in the numerator of the home-asset share expression in (10).

b) Suppose $T \geq T_1^*$. Clearly, TR_F is still given by (14), but TR_H differs. The logic of the calculations is identical. The main difference lies in the denominator. When $T \geq T_1^*$, not all domestic asset holdings are used as media of exchange in the DM because they have desirable saving properties. Following the same strategy as above and substituting asset holdings with the corresponding quantities of special goods exchanged in the DM obtains (13). TR_F is constant, so it is unaffected by the asset supply. However, TR_H is decreasing in T, for all T, and $TR_H \to 0$ as $T \to \infty$. Therefore, there exists $\tilde{T}$, with $T_1^* \leq \tilde{T} < \infty$, such that $T \geq \tilde{T}$ necessarily implies $TR_F > TR_H$. $\square$

As in Propositions 2 and 3, there are two distinct cases to consider. If $T < T_1^*$, the domestic and foreign turnover rates are given by (12) and (14), respectively. However, unlike the home asset and consumption shares, the domestic turnover rate is *not* monotone in the trading probability at home. Hence, an unambiguous ranking between domestic and foreign turnover rates cannot be made in this case.

In contrast, when T exceeds T_1^*, agents purchase the quantities q^* and $q_{F,1}^*$, so increases in T have no effect on the quantities of special goods bought. Instead, increases in the asset supply raise the agents' domestic asset holdings, but have no effect on the amounts of domestic assets that change hands during a period, leading to a decrease in the turnover rate of the domestic as-

[13] Throughout, we adopt the following accounting procedure: (i) In the DM, we count each transaction only once, since, by definition, the meeting is bilateral in the sense that the buyer and the seller trade with each other; (ii) In the CM, we count both the amount of assets bought by the buyers and the amount sold by the sellers because the market is Walrasian and, therefore, agents trade against the market and not with each other. In fact, the latter procedure is applied when accounting for stock-market transactions in the data.

set. Intuitively, when the liquidity needs of the economy are satisfied, domestic assets are useful saving tools, while foreign assets are not due to the presence of transaction costs. Hence, agents hold the domestic asset as a saving tool and use it to trade at home, while they only acquire enough foreign assets to satisfy their purchasing needs abroad. The domestic asset's desirable saving property results in a reduction in its turnover rate. On the other hand, agents unload foreign assets (to foreign sellers) at the first given opportunity, which leads to a relatively higher turnover rate of the foreign asset. Thus, the mechanism that drives the coexistence of asset and consumption bias also yields a higher turnover rate of foreign over domestic assets.

5.4.1 Unique Testable Prediction: Asset Turnover and Trade

In our model, the rate at which *foreign* assets turn over is driven by the role that the assets play in facilitating trade abroad. This is apparent in expression (14), where the foreign turnover rate is an increasing function of the probability to trade abroad, p_F. Moreover, Lemma 6 ensures that the value of imported consumption, as defined in Proposition 3, is increasing in this trading probability. Hence, a natural implication of our model is that a given importer turns over faster the assets of the (foreign) country from which it imports more. This prediction is unique to our model and it will ultimately serve as a test of the liquidity mechanism.

5.4.2 Alternative Definition of Domestic Turnover Rate

In the theoretical analysis above, we adopted Warnock's (2002) definition of the foreign turnover rate, which is standard in the empirical literature. Given this definition, we constructed a theoretically-consistent definition of the domestic turnover rate. However, since the existing empirical literature has not considered the liquidity mechanism developed in this paper, it does not report measures of the theoretically-consistent domestic turnover rate. Instead, Warnock (2002) defines domestic turnover as the ratio of annual transactions on a market to its capitalization. The market for which estimates are reported is the stock exchange. Consequently, we re-establish the turnover results using the empirically-relevant definition of domestic turnover.

In the model, the CM represents the stock exchange. Its market capitalization is simply the total asset supply T. The transactions that constitute the numerator are all the trades (purchases and sales) of domestic claims by both domestic and foreign agents. Domestic agents who successfully matched in the domestic DM re-balance their portfolios as follows: a measure p_H of buyers buy t_H units, and measures p_H and p_F of sellers sell t_H and t_F units, respectively. In addition, a measure p_F of foreign buyers who matched in the domestic DM purchase t_F units of the domestic asset. Hence, the empirically-relevant turnover rate of home assets is

$$TR_H = \frac{2(p_H t_H + p_F t_F)}{T}, \qquad \forall T < T_1^*,$$
$$TR_H = \frac{2^{\frac{1-\beta}{d}} \left(p_H q^* + p_F q^*_{F,1} \right)}{T}, \qquad \forall T \geq T_1^*, ,$$

where the second line is obtained by substituting out the asset holdings using the appropriate formulas for the liquidity-unconstrained case. Once again, the foreign turnover rate in (14) is constant, so it is unaffected by the asset supply. On the other hand, if $T \geq T_1^*$, TR_H is decreasing in T, and $TR_H \to 0$ as $T \to \infty$. Therefore, there exists $\tilde{\tilde{T}}$, with $T_1^* \leq \tilde{\tilde{T}} < \infty$, such that $T \geq \tilde{\tilde{T}}$ implies $TR_F > TR_H$.

In sum, we can provide sufficient conditions such that countries exhibit asset and consumption home bias as well as higher foreign over domestic asset turnover rates.

6 Empirical Analysis

In this section, we document three stylized facts that in unison represent a puzzle for the international finance literature. Throughout the section, we focus our analysis on equities for comparison to the existing literature and due to data availability.

6.1 Asset Home Bias

In section 5.2, we discussed the first prediction of the model: countries' asset portfolios exhibit home bias under reasonable parameter restrictions.

Home bias represents an empirical regularity in cross-country data. In their seminal paper, French and Poterba (1991) document that, in 1989, the five largest economies held ninety percent of their wealth in domestic equities. Although home bias is not as severe in recent decades, it does persist in the data. We document this fact using cross-country data on international asset and liability positions provided by Lane and Milesi-Ferretti (2007) and data on stock market capitalization from the WDI.

Table 1 shows that, during the 1997-2007 period, the mean domestic asset share among 27 OECD countries was 75 percent. To measure asset home bias, we employ a methodology similar to Collard, Dellas, Diba, and Stockman (2009). First, we compute international diversification,

$$\text{Int'l Dvrsf.} = \frac{\text{Foreign Portfolio Equity Assets}}{\text{Stock Mkt Cap + Foreign Portfolio Equity Assets - Foreign Portfolio Equity Liab.}},$$

where Foreign Portfolio Equity Asset refers to a less-than-ten-percent share ownership of a firm headquartered abroad. Then, we capture asset home bias through the home asset share

$$\text{HA} = 1 - \text{Int'l Dvrsf.}$$

Table 1: Average Home Equity Share of Portfolio, 1997-2007

Country	HA	Country	HA	Country	HA
Australia	0.8374	Germany	0.5951	Norway	0.5493
Austria	0.4279	Greece	0.9536	Poland	0.9850
Belgium	0.5720	Hungary	0.9450	Portugal	0.7011
Canada	0.7432	Italy	0.6364	Spain	0.8589
Chile	0.8366	Japan	0.8972	Sweden	0.6290
Czech Republic	0.8382	Korea, Republic of	0.9538	Switzerland	0.5659
Denmark	0.6218	Mexico	0.9280	Turkey	0.9772
Finland	0.7206	Netherlands	0.4018	United Kingdom	0.6906
France	0.7523	New Zealand	0.6502	United States	0.8531
		OECD Average	*0.7452*		

With this definition in mind, all but one country in our sample exhibit asset home bias, since average home asset shares (HA) over the period are in excess of the fifty-percent benchmark that characterizes a fully-diversified portfolio in our model.

6.2 Consumption and Asset Home Bias

In section 5.3, we showed that consumption and asset home bias coexist in the model under reasonable parameter restrictions. This prediction is supported by the empirical literature. Heathcote and Perri (2007) and Collard, Dellas, Diba, and Stockman (2009) document that more open economies (in terms of trade as a fraction of GDP) exhibit lower asset home bias.

In section 5.3, we also derived the second prediction of the model: bilateral imports are positively correlated with bilateral asset positions. To test it, we obtain bilateral equity holding data from the Coordinated Portfolio Investment Survey (CPIS) database provided by the World Bank. The advantage of the database is that it contains bilateral asset holdings for the set of OECD countries considered above during the second half of the studied decade, 2002-2007. The disadvantage is that a number of positive bilateral observations are not made publicly available for security reasons. Thus, domestic asset shares cannot be computed, since missing observations will bias home asset shares upward. Consequently, we use the database to simply test the positive correlation between bilateral imports and asset positions.

To compute bilateral asset holding shares, we combine the CPIS data with stock market capitalization data from WDI. Then, we merge these statistics with annual bilateral imports obtained from Stats.Oecd.[14] We compute bilateral import shares using GDP data from Stats.Oecd.

Table 2: Bilateral Import and Equity Holding Shares, 2002-2007

year	correlation	p-value	# obs
2002	0.4504	0.0000	570
2003	0.3833	0.0000	584
2004	0.3581	0.0000	602
2005	0.3707	0.0000	593
2006	0.3898	0.0000	617
2007	0.3973	0.0000	614

Note: # obs = # available bilateral equity holding data points

Table 2 shows the cross-country correlation between bilateral equity holding and import shares for the 2002-2007 period. The correlation ranges between 0.3581 and 0.4504 and it is highly statistically significant. Hence, it is reasonable to conclude that bilateral equity holding and import shares are positively correlated in the data. These results are supported by the existing literature. Aviat and Coeurdacier (2007) find that a 10% increase in bilateral trade raises bilateral asset holdings by 6% to 7% in a simultaneous gravity-equations framework.

6.3 Domestic and Foreign Asset Turnover Rates

In section 5.4, we derived the third and key prediction of the model: if assets are abundant, then, not only do asset and consumption home bias coexist, but also foreign assets turn over faster than domestic assets. This result is robust to different definitions of domestic asset turnover rates. The key insight behind it is that asset flows used to compute *foreign* turnover rates do not only account for portfolio rebalancing, but also reflect the use of assets to facilitate trade.

Empirically, Tesar and Werner (1995) were the first to document considerably higher foreign over domestic turnover rates (as defined in section 5.4.2) for the year 1989 for five major economies: United States, Japan, Germany, United Kingdom, and Canada. Warnock (2002) revised the authors' estimates for the United States and Canada and in addition presented new estimates for the pair of countries for the year 1997. Finally, Amadi and Bergin (2008) repeated Warnock's (2002) exercise using CPIS data for the United States, Japan, Germany, and Canada during the 1993-2001 period, and concluded that, on average, foreign turnover rates are twice

[14] We use the Grand Total Import statistic, which includes imports across all commodity categories. Bilateral import data for the service sector are not available. In any case, including service trade should not affect our results, since services are mostly non-tradable and they account for a tiny portion of overall trade.

as high as those of home assets. It is the coexistence of higher foreign over domestic turnover rates with consumption and asset home bias that puzzles the international finance literature.

Amadi and Bergin (2008) offer an explanation for the coexistence of asset home bias and a higher turnover rate of foreign over domestic assets. The authors argue that a portfolio-choice model with a heterogeneous per-unit trading cost and a homogenous fixed entry cost produces an environment that is consistent with this stylized fact. While this mechanism may be in part responsible for the observations in the data, we argue that there is considerable room for an explanation that builds on our liquidity channel. First, unlike our framework, Amadi and Bergin's (2008) model does not link consumption and asset home bias, which is a regularity observed in the data. Second, our model generates a unique testable prediction that relates bilateral asset turnover rates to bilateral trade in goods, which is not shared by the alternative framework.

6.3.1 Unique Testable Prediction: Asset Turnover and Trade

In section 5.4.1, we derived a unique prediction of our model: a given importer turns over faster the assets of the country from which it imports more. To test this prediction, we supplement the data on bilateral import and equity holding shares from section 6.2 with US Treasury data on annual gross flows of foreign assets between the US and each of its trading partners in our sample. This allows us to compute bilateral foreign turnover rates for the US during the 2002-2007 period. According to our model, there should be a positive correlation between US import shares by source and the US turnover rate of assets from the same source.

Table 3: Logged Bilateral Import Shares and Turnover Rates, 2002-2007

year	correlation	p-value	# obs
2002	0.5466	0.0039	26
2003	0.4223	0.0316	26
2004	0.4662	0.0164	26
2005	0.3364	0.0929	26
2006	0.4210	0.0322	26
2007	0.4521	0.0204	26

Note: # obs = # US trading partners in sample

Table 3 shows the correlation between logged bilateral import shares and logged bilateral turnover rates for the US and its trading partners during the 2002-2007 period.[15] The correlation ranges between 0.3364 and 0.5466 and it is statistically significant at the 5% (10%) level (in 2005). These findings provide direct support for the liquidity mechanism over the alternative.

[15] We conduct the analysis in logs because import shares and turnover rates have different units.

7 Discussion

In sections 5 and 6, we documented three stylized facts and we demonstrated that the simple model proposed in this paper can reconcile them. However, in order to remain tractable, the model assumed away the risky nature of equity returns. We argue that incorporating stochastic asset returns into the model will not affect the qualitative results.

Suppose that asset returns are stochastic, but not perfectly correlated across countries. In a symmetric two-country world without a liquidity mechanism, agents will hold equal shares of domestic and foreign assets in their portfolios. Adding the liquidity mechanism and foreign asset transaction costs will tilt agents' portfolios toward domestic assets—the higher the cost, the higher the degree of asset home bias. For given probability to trade abroad, higher transaction costs will also increase consumption home bias. Finally, foreign turnover will continue to exceed domestic turnover because of the liquidity mechanism. Hence, the frameworks will differ in the parameter restrictions necessary in order to generate the desirable predictions.

Moreover, our analysis is robust to the introduction of fiat money in the economy. In that case, agents will hold assets as store of value due to positive asset returns since fiat money has negative or zero return, depending on the rate of inflation. Suppose that we impose the standard currency-area assumption in the model; namely, trade in DM_i necessitates the use of i's currency. If it is costly to convert foreign assets into home currency, due to a spread in the exchange rate, agents will hold i's asset in order to trade in i's currency. Hence, all of our results would be preserved. Moreover, since fiat money will serve as a medium of exchange in the economy, the model would predict that total domestic assets—the sum of real assets and fiat money—will turn over faster than total foreign assets. This is likely the case in the US, where the velocity of money is high. We abstract from fiat currency considerations because the valuation of fiat money is dependent on policy, which is beyond the scope of our paper.

Finally, one may question the assumption that both domestic and international credit markets are imperfect and therefore media of exchange are necessary. To preserve the three key predictions of the model when the economy is not liquidity constrained, it is sufficient to assume that credit is less perfect in cross-border trade. Admittedly, credit is likely better enforced in domestic relative to international exchanges of goods and services. While, to the best of our knowledge, direct evidence on domestic credit enforcement is not available, there exists considerable evidence that international trade is credit-constrained. Amiti and Weinstein (2009) describe in detail the financial aspects of international trade and convincingly argue that credit is imperfect in cross-border exchange of goods.

In sum, the theoretical results presented in this paper are robust to several extensions of the model. However, the potential quantitative fit of the model will be sensitive to different modeling assumptions. We leave it to future quantitative work to incorporate the effects of asset liquidity on asset and consumption home bias as well as asset turnover rates.

8 Conclusion

In this paper, we study optimal asset portfolio choice in a two-country search-theoretic model of monetary exchange. We allow assets to not only represent claims on future consumption, but to also serve as media of exchange. In the model, trading in a certain country involves the exchange of locally produced goods for a portfolio of assets. Assuming foreign assets trade at a cost, we provide sufficient conditions for the existence of two types of exchange regimes: local asset dominance and international asset equilibrium.

The model predicts that agents' portfolios exhibit home bias as long as trading opportunities abroad are not much more frequent than at home. Since agents hold larger amounts of domestic over foreign assets, they have larger claims to domestic over foreign consumption goods. Hence, the mechanism positively links asset and consumption home bias. More importantly, under additional parameter restrictions, foreign assets turn over faster than home assets because, while the former have desirable liquidity properties, they yield lower future consumption and are undesirable savings tools. Therefore, our theory offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets.

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A Theory Appendix

Proof of Lemma 2. The term κ creates a wedge in the valuation of asset i between the seller (citizen of i) and the buyer (citizen of j). A positive amount of asset j will change hands only if all units of asset i have already been traded. Hence, the solution to the bargaining problem follows directly from answering the following two questions: 1) If the buyer carried unlimited amounts of asset i (which is foreign to him but local to the seller), what would be the optimal level of q to be traded? 2) If the buyer carried zero units of asset i, what would be the optimal level of q to be traded? It can be easily verified that the answer to the first question is given by $\overline{q}(\psi) \equiv \{q : u'(q) = (\psi + d_\kappa)/(\psi + d)\}$, which is clearly larger than q^*. Similarly, the answer to the second question is given by $\underline{q}(\psi) \equiv \{q : u'(q) = (\psi + d)/(\psi + d_\kappa)\} < q^*$.

Given these observations, the suggested solution follows naturally. When $t_{ii} \geq \underline{q}(\psi_j)/(\psi_i + d)$, asset j never changes hands (cases a and b). If $t_{ii} \geq \overline{q}(\psi_j)/(\psi_i + d)q_i$ (case a), q_i is equal to the optimum. However, if $t_{ii} \in \big[\underline{q}(\psi_j)/(\psi_i + d), \overline{q}(\psi_i)/(\psi_i + d)\big)$ (case b), q_i is bounded by the real value of the buyer's asset-i holdings, i.e. $t_{ii}(\psi_i + d)$. If $t_{ii} < \underline{q}(\psi_j)/(\psi_i + d)$ (case c), some asset j is traded as well. If the total asset balances are such that $\pi_i \geq \underline{q}(\psi_j)$ (case c1), then the buyer gives up all her asset i and enough asset j so that the quantity $\underline{q}(\psi_j)$ can be purchased. On the other hand, if $\pi_i < \underline{q}(\psi_j)$ (case c2), the buyer gives up all her asset holdings and purchases $q_i = \pi_i = (\psi_i + d)t_{ii} + (\psi_j + d_\kappa)t_{ji}$. □

Proof of Lemma 4. Focus on the symmetric, steady-state version of (5). Pick any (t^0_{HH}, t^0_{FH}), with $t^0_{FH} > 0$ and define $q^0 \equiv \{q : (\psi + d)t^0_{HH} + (\psi + d_\kappa)t^0_{FH}\}$, i.e. the quantity of the special good that (t^0_{HH}, t^0_{FH}) can buy. The buyer can purchase q^0 by setting $t_{FH} = 0$ and increasing her domestic asset holdings to $t^1_{HH} = q^0/(\psi + d)$. We claim that the buyer always achieves a higher value if she purchases (the arbitrarily chosen) q^0 with asset holdings $(t^1_{HH}, 0)$ rather than (t^0_{HH}, t^0_{FH}). To see this point notice that

$$
\begin{aligned}
V^{\mathcal{B}}_H(t^1_{HH}, 0) &= [-\psi + \beta(\psi + d)]\, t^1_{HH} + \beta p_H[u(q^0) - q^0], \\
V^{\mathcal{B}}_H(t^0_{HH}, t^0_{FH}) &= [-\psi + \beta(\psi + d)]\, t^0_{HH} + [-\psi + \beta(\psi + d_\kappa)]\, t^0_{FH} + \beta p_H[u(q^0) - q^0].
\end{aligned}
$$

After some algebra, one can show that $V^{\mathcal{B}}_H(t^1_{HH}, 0) > V^{\mathcal{B}}_H(t^0_{HH}, t^0_{FH})$ will be true if and only if $t^1_{HH} < t^0_{HH} + t^0_{FH}$. Multiply the last expression by $\psi + d$ and add and subtract the term κt^0_{FH} on the right-hand side. Then, one can conclude that $V^{\mathcal{B}}_H(t^1_{HH}, 0) > V^{\mathcal{B}}_H(t^0_{HH}, t^0_{FH})$ is true if and only if $\kappa t^0_{FH} > 0$, which is true by assumption. Thus, the optimal strategy for the buyer is to purchase any desired q in the local DM by carrying only the domestic asset.

When $\psi = \psi^*$, the cost of carrying home assets is zero. We have $t_{HH} \geq q^*/(\psi + d)$, because the buyer carries any t_{HH} which is greater than the amount that buys her q^*. □

Proof of Lemma 5. The description of the optimal choice of t_{HF} is more complex than the one of t_{FF} for the following reason. If the buyer does not use foreign assets as media of exchange, we know that $t_{FF} = 0$. This is true because the cost of holding foreign assets is always strictly positive. However, with home assets, if $\psi = \psi^*$, the buyer might want to hold some home assets even if she is not using them to carry out transactions in the DM. To avoid these complications, first we focus on the case where $\psi > \psi^*$. Once we have established which assets serve as means of payments in the various cases, we let $\psi = \psi^*$ and conclude the description of the optimal choice of home assets.

The symmetric, steady-state version of the buyer's objective function, i.e. equation (6), is

$$
\begin{aligned}
V^{\mathcal{B}}_F(t_{HF}, t_{FF}) &= [-\psi + \beta(\psi + d)]\, t_{HF} + [-\psi + \beta(\psi + d_\kappa)]\, t_{FF} \\
&+ \beta p_F\left[u\left(q_F(t_{HF}, t_{FF})\right) - (\psi + d)\, x_H(t_{HF}, t_{FF}) - (\psi + d_\kappa)\, x_F(t_{HF}, t_{FF})\right], \quad \text{(a.1)}
\end{aligned}
$$

where x_H denotes the amount of domestic assets (with respect to the buyer's citizenship) and x_F the amount of foreign assets that change hands in a DM meeting in the foreign country. Since different asset holdings t_{HF}, t_{FF} lead to different expressions for the terms q_F, x_H and x_F (determined in Lemma 2), it is of no use to take first-order conditions in (a.1). Instead, we look into different combinations of t_{HF}, t_{FF} holdings. We start by ruling out several regions of asset holdings as strictly dominated. This allows us to narrow down the set of possibilities and eventually take first-order conditions in the remaining relevant regions. Figure 2 depicts the regions described in this proof. The proof proceeds in several steps.

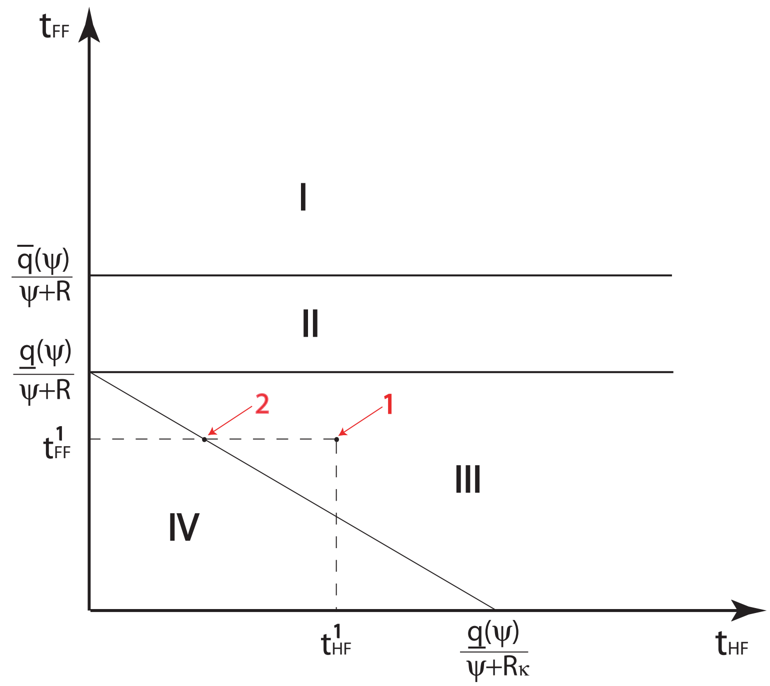

Figure 2: Regions

Step 1: The optimal t_{HF}, t_{FF} can never be such that $t_{FF} > \overline{q}(\psi)/(\psi + d)$. This represents region I in Figure 2 and case (a) in Lemma 2. To see why this claim is true, just notice that for any t_{HF}, t_{FF} in this region the buyer is already buying the highest possible quantity of special good, namely $\overline{q}(\psi)$. Hence, increasing t_{FF} has a strictly positive cost and no benefit.

Step 2: The optimal t_{HF}, t_{FF} can never be such that $t_{FF} \in (\underline{q}(\psi)/(\psi + d), \overline{q}(\psi)/(\psi + d)]$ and $t_{HF} > 0$. This represents region II, excluding the vertical axis, in Figure 2 and case (b) in Lemma 2. To see why the claim is true, recall from Lemma 2 that for this region of asset holdings $x_H = 0$. Therefore, there is no benefit from carrying the home asset (but there is a cost). Hence, in this region the objective function becomes

$$V_F^{\mathcal{B}}(t_{HF}, t_{FF}) = -\beta\kappa t_{FF} + \beta p_F \left[u\left((\psi + d)t_{FF}\right) - (\psi + d_\kappa)\, t_{FF}\right]. \tag{a.2}$$

The optimal t_{FF} is determined by taking the first-order condition in (a.2).

Step 3: The optimal t_{HF}, t_{FF} cannot lie in the interior of region III in Figure 2 (and case (c1) in Lemma 2). To see why this is true, consider any point in the interior of region III, such as point 1 with coordinates (t^1_{HF}, t^1_{FF}). The objective function for the buyer, evaluated at point 1, is

$$\begin{aligned} V_F^{\mathcal{B}}(t^1_{HF}, t^1_{FF}) &= \left[-\psi + \beta\left(\psi + d\right)\right] t^1_{HF} + \left[-\psi + \beta\left(\psi + d_\kappa\right)\right] t^1_{FF} \\ &+ \beta p_F \left[u\left(\underline{q}(\psi)\right) - (\psi + d)\left(\frac{\underline{q}(\psi) - (\psi + d)}{\psi + d_\kappa}\right) - (\psi + d_\kappa)\, t^1_{FF}\right]. \end{aligned} \tag{a.3}$$

Now consider the following experiment. Leave t_{FF} unchanged, but reduce t_{HF} so that the buyer can still purchase $\underline{q}(\psi)$. This is represented by point 2 in the Figure, with coordinates $(t^2_{HF}, t^2_{FF}) = \left((\underline{q}(\psi) - (\psi + d)t^1_{FF})/(\psi + d_\kappa), t^1_{FF}\right)$. The objective function evaluated at point 2 is

$$\begin{aligned} V_F^{\mathcal{B}}(t^2_{HF}, t^2_{FF}) &= \left[-\psi + \beta\left(\psi + d\right)\right] \frac{\underline{q}(\psi) - (\psi + d)t^1_{FF}}{\psi + d_\kappa} + \left[-\psi + \beta\left(\psi + d_\kappa\right)\right] t^1_{FF} \\ &+ \beta p_F \left[u\left(\underline{q}(\psi)\right) - (\psi + d)\left(\frac{\underline{q}(\psi) - (\psi + d)}{\psi + d_\kappa}\right) - (\psi + d_\kappa)\, t^1_{FF}\right]. \end{aligned} \tag{a.4}$$

It follows from (a.3) and (a.4) that

$$V_F^{\mathcal{B}}(t^2_{HF}, t^2_{FF}) - V_F^{\mathcal{B}}(t^1_{HF}, t^1_{FF}) = \left[-\psi + \beta\left(\psi + d\right)\right]\left(t^2_{HF} - t^1_{HF}\right),$$

which is strictly positive by Lemma 3 and the fact that $t^2_{HF} < t^1_{HF}$. Hence, the agent will never choose a point (t_{HF}, t_{FF}) in the interior of region III.

Step 4: Finally consider the region of t_{HF}, t_{FF} represented by region IV (case (c2) of Lemma 2). The objective function in this region becomes

$$\begin{aligned} V_F^{\mathcal{B}}(t_{HF}, t_{FF}) &= \left[-\psi + \beta\left(\psi + d\right)\left(1 - p_F\right)\right] t_{HF} + \left[-\psi + \beta\left(\psi + d_\kappa\right)\left(1 - p_F\right)\right] t_{FF} \\ &+ \beta p_F u\left((\psi + d_\kappa)t_{HF} + (\psi + d)t_{FF}\right) \\ &= -a_1 t_{HF} - a_2 t_{FF} + a_3 u\left(a_4 t_{HF} + a_5 t_{FF}\right), \end{aligned} \tag{a.5}$$

where we defined $a_1 \equiv \psi - \beta\left(\psi + d\right)\left(1 - p_F\right)$, $a_2 \equiv \psi - \beta\left(\psi + d_\kappa\right)\left(1 - p_F\right)$, $a_3 \equiv \beta p_F$, $a_4 \equiv \psi + d_\kappa$, and $a_5 \equiv \psi + d$. Notice that $a_i > 0$ for all i, $a_1 < a_2$, and $a_4 < a_5$. The first order-conditions in (a.5) with respect to t_{HF}, t_{FF}, are

$$-a_1 + a_3 u'\left(a_4 t_{HF} + a_5 t_{FF}\right) a_4 \leq 0, \;\; = 0 \textit{ if } \; t_{HF} > 0, \tag{a.6}$$

$$-a_2 + a_3 u'\left(a_4 t_{HF} + a_5 t_{FF}\right) a_5 \leq 0, \;\; = 0 \textit{ if } \; t_{FF} > 0. \tag{a.7}$$

Hence, the optimal choice of t_{HF}, t_{FF} is a corner solution, with the exception of the knife-edge case in which $a_2/a_1 = a_5/a_4$. If $a_2/a_1 > a_5/a_4$, then $t_{HF} > 0$ and $t_{FF} = 0$. On the other hand, if $a_2/a_1 < a_5/a_4$, then $t_{HF} = 0$ and $t_{FF} > 0$.

Using the definitions of the a_i terms, one can show that $a_2/a_1 < a_5/a_4$ if and only if

$$\beta(1 - p_F)(2d - \kappa) < \psi\left[1 - 2\beta(1 - p_F)\right]. \tag{a.8}$$

First, notice that if $\beta \geq 1/(2(1 - p_F))$ (region R_4 in Figure 1), the condition in (a.8) can never hold, since the left-hand side is positive. In this case, $t_{FF} = 0$ and the value of t_{HF} is given by (a.6), which after replacing for the a_i terms is equivalent to (9).

From now on let $\beta < 1/(2(1-p_{_F}))$. The condition in (a.8) becomes

$$\psi > \frac{\beta(1-p_{_F})(2d-\kappa)}{1-2\beta(1-p_{_F})} \equiv \psi_c. \tag{a.9}$$

Using the definition of ψ_c above and recalling the definition of the "fundamental value" of the asset, $\psi^* \equiv \beta d/(1-\beta)$, one can show that $\psi^* > \psi_c$ if and only if

$$\kappa(1-\beta)(1-p_{_F}) > d(1-2p_{_F}). \tag{a.10}$$

After some algebra, it turns out that the inequality in (a.10) holds if either: i) $p_{_F} \geq 1/2$ (region R_1 in Figure 1) or ii) $p_{_F} < 1/2$, $\beta < p_{_F}/(1-p_{_F})$ and $\kappa/d > (1-2p_{_F})[(1-\beta)(1-p_{_F})]^{-1} \equiv \tilde{\kappa}$ (the values of β and $p_{_F}$ described here are represented by region R_2 in Figure 1). However, Lemma 3 indicates that any equilibrium price satisfies $\psi \geq \psi^*$. Therefore, if $\psi^* > \psi_c$, the inequality in (a.9) is always true. In this case, $t_{_{HF}} = 0$ and the value of $t_{_{FF}}$ is given by (a.7). After replacing for the a_i terms, this is equivalent to (8).

We are left with the case in which either $\beta, p_{_F}$ fall in region R_3 of Figure 1 or they fall in region R_2 and also $\kappa/d \leq \tilde{\kappa}$. Here, $\psi^* \leq \psi_c$, and there will exist equilibrium prices that can be larger or smaller than ψ_c (this will be determined later when we introduce the supply of the assets, T). For ψ's that are large enough, so that (a.9) holds, the optimal choice of the agent satisfies $t_{_{HF}} = 0$, $t_{_{FF}} > 0$. For $\psi \in [\psi^*, \psi_c)$, we have $t_{_{HF}} > 0$, $t_{_{FF}} = 0$. In the knife-edge case where $\psi = \psi_c$, we have $t_{_{HF}}, t_{_{FF}} > 0$, and both (a.6), (a.7) hold with equality. The choices of $t_{_{HF}}, t_{_{FF}}$ cannot be separately pinned down, but they are such that the quantity bought in the DM satisfies $u'(q_{_F}) = p_{_F}/(1-p_{_F})$.

Step 5: In this step we establish the statements of Lemma 5 for $\psi > \psi^*$ (which we have assumed so far). The case in which $\psi = \psi^*$ is discussed in Step 6. Let us summarize the results derived so far. From Step 1, we know that the agent will never choose $t_{_{HF}}, t_{_{FF}}$ in region I. From Step 2, we know that if the agent wishes to purchase $q_{_F} > \underline{q}(\psi)$, she will do so by using $t_{_{FF}}$ only. From Step 3, we know that points in the interior of region III are dominated by asset holdings that purchase the same quantity, i.e. $\underline{q}(\psi)$, using fewer home assets. Finally, from Step 4, we know that if the agent wishes to purchase $q_{_F} \leq \underline{q}(\psi)$, she will do so by using either $t_{_{FF}}$ or $t_{_{HF}}$, but not both, as a medium of exchange.

<u>Case 1:</u> If parameter values are as in Case 1 of the lemma, the agent uses only $t_{_{FF}}$ as a medium of exchange. Hence, the objective function is given by (a.2), and the optimal choice of $t_{_{FF}}$ is described by (8).

<u>Case 2:</u> Next, consider parameter values as in Case 2 of the lemma. This case is less straightforward than Case 1. We have shown (Step 4) that if the agent wishes to purchase some $q_{_F} \leq \underline{q}(\psi)$, she is better off by setting $t_{_{FF}} = 0$. But from Step 1 we also know that if $q_{_F} > \underline{q}(\psi)$ the agent is better of using the foreign asset as a medium of exchange. To establish Case 2 of

the lemma, we need to exclude the latter possibility. Suppose, by way of contradiction, that the agent wants to purchase $q_F > \underline{q}(\psi)$ and, therefore, chooses $t_{FF} > 0$. Under the contradictory assumption, the quantity of special good purchased is

$$q_F(\psi) \equiv \Big\{ q : \psi = \beta \left[(1 - p_F)(\psi + d_\kappa) + p_F (\psi + d)\, u'(q) \right] \Big\}. \quad \text{(a.11)}$$

Notice three important facts. First, $q_F(\psi)$ is strictly decreasing in ψ. Second, $\underline{q}(\psi)$ is strictly increasing in ψ. Third, we claim that $q_F(\psi^*) < \underline{q}(\psi^*)$. To see why this is true, observe that

$$\begin{aligned} u'(\underline{q}(\psi^*)) &= \frac{d}{d - \kappa(1-\beta)}, \\ u'(q_F(\psi^*)) &= 1 + \frac{\kappa(1-p_F)(1-\beta)}{dp_F}. \end{aligned}$$

Our claim that $q_F(\psi^*) < \underline{q}(\psi^*)$ will be true if and only if $u'(q_F(\psi^*)) > u'\left(\underline{q}(\psi^*)\right)$, which is true if and only if $1 + \kappa(1-p_F)(1-\beta)/(dp_F) > d/[d - \kappa(1-\beta)]$, which after some more manipulations can be written as

$$\frac{\kappa}{d} < \frac{(1-2p_F)}{(1-p_F)(1-\beta)}. \quad \text{(a.12)}$$

Since here $\beta \geq p_F/(1-p_F)$, the right-hand side of (a.12) is greater than or equal to 1. Hence, the inequality in (a.12) is always satisfied. Combining these three facts implies that $q_F(\psi) < \underline{q}(\psi)$ for all ψ, which is a straightforward contradiction to our assumption that the agent purchases $q_F > \underline{q}(\psi)$ and, therefore, chooses $t_{FF} > 0$. Hence, the agent chooses to use only the home asset as a medium of exchange, and (given that $\psi > \psi^*$) the optimal t_{HF} solves (9).

Case 3: Finally, consider parameters such as in Case 3 of the lemma. We know that if the agent wishes to purchase $q_F \leq \underline{q}(\psi)$, we have $t_{FF} > 0$ if and only if $\psi \geq \psi_c$. Using an argument identical to the one we used in Case 2, one can show that the agent will never wish to purchase $q_F > \underline{q}(\psi)$.[16] Thus, the analysis in Step 4 fully characterizes the optimal choice of the agent, and Case 3 of the lemma follows directly.

Step 6: We only need to conclude the description of the optimal choice of t_{HF} when $\psi = \psi^*$. In Case 1, the home asset is not used as a medium of exchange. If $\psi = \psi^*$, the cost of carrying the home asset is zero. Hence, any $t_{HF} \in \mathbb{R}_+$ is optimal. In Case 2, the buyer uses the home asset as a means of payment. Thus, with $\psi = \psi^*$, she is willing to carry any amount of t_{HF} that allows her to buy the q_F which maximizes the buyer's surplus in the foreign DM. This quantity is given by $\underline{q}(\psi^*)$, implying that any $t_{HF} \geq \underline{q}(\psi^*)/(d/(1-\beta) - \kappa)$ is optimal. Finally, in Case 3, $\psi = \psi^*$ means that $\psi < \psi_c$. Therefore, the optimal behavior of the buyer coincides with Case 2 described above. □

[16] This follows directly from (a.12), since the term on the right-hand side is just $\tilde{\kappa}$.

Proof of Proposition 1. The fact that $t_{FH} = 0$ follows immediately from Lemma 4. The proofs for parts (a), (b), and (c) are similar. Hence, we only prove part (a) in detail.

The fact that agents use only foreign assets in order to trade in the foreign DM follows from Lemma 5. Moreover, applying total differentiation in (7) and (8) yields

$$\frac{\partial q_H}{\partial \psi} = \frac{d}{\beta p_H(\psi + d)^2 u''(q_H)} < 0,$$
$$\frac{\partial q_F}{\partial \psi} = \frac{d - \kappa\beta(1 - p_F)}{\beta p_F(\psi + d)^2 u''(q_F)} < 0.$$

Since q_H and q_F are strictly decreasing in ψ, there exists a critical level of T, T_1^*, such that $T \geq T_1^*$ implies $\psi = \psi^*$, which in turn implies $q_H = q^*$ and $q_F = q^*_{F,1}$ (the latter statement follows from (7) and (8)). To find this critical level use a) the bargaining solutions $(\psi + d)t_{HH} = q_H$, $(\psi + d)t_{FF} = q_F$, evaluated at $\psi = \psi^*$, and b) the market clearing condition, which under the specific parameter values becomes $T = t_{HH} + t_{FF}$. This yields $T_1^* = (1-\beta)(q^* + q^*_{F,1})/d$. If $T > T_1^*$, the demand of foreigners for a certain asset is given by $t_{FF} = (1 - \beta)q^*_{F,1}/d$. This allows them to purchase the maximum possible quantity of foreign special good, $q^*_{F,1}$. The rest of the supply, T, is absorbed by local buyers. Hence, $t_H = t_{HH} = T - (1 - \beta)q^*_{F,1}/d$.

When $T < T_1^*$, agents are not buying the maximum possible amount of the special good. Hence, in this range, $\psi > \psi^*$, $q_H < q^*$, $q_F < q^*_{F,1}$, and $\partial\psi/\partial T < 0, \partial q_H/\partial T > 0, \partial q_F/\partial T > 0$. As $T \to T_1^*$, the liquidity properties of the asset are exploited; ψ reaches the fundamental value, $\psi \to \psi^*$, and q_H, q_F reach their upper bounds, namely $q_H \to q^*$ and $q_F \to q^*_{F,1}$. □

Proof of Lemma 6. a) Let $T < T_1^*$. i) The monotonicity of $q_H(p_H)$ and $q_F(p_F)$ follows from the fact that $u'' < 0$. The monotonicity of $t_H(p_H)$ and $t_F(p_F)$ follows trivially from the bargaining solution. ii) For some $p_H = p_F = p \in (0, 1)$, $p_H(p) > p_F(p)$ will be true if and only if $u'(q_H) < u'(q_F)$. After some algebra, one can show that this is equivalent to $0 < \kappa(1 - p)$, which is always true. iii) The fact that $\tilde{p}_H(p_F) < p_F$ follows directly from parts (i) and (ii). Given the definition of $\tilde{p}_H(p_F)$ and the fact that $q_H(p_H)$ is increasing, for any p_F and $p_H > \tilde{p}_H(p_F)$, we have $q_H(\tilde{p}_H(p_F)) > q_F(p_F)$. iv) The result follows directly from part (iii) and the bargaining solution.

b) Let $T \geq T_1^*$. The facts $q_H = q^*$, $q_F = q^*_{F,1}$, $t_H = T - q^*_{F,1}(1 - \beta)/d$, and $t_F = q^*_{F,1}(1 - \beta)/d$ are already presented in Proposition 1. Also, $q^* > q^*_{F,1}$ follows directly from the definition of $q^*_{F,1}$. To see why $T - q^*_{F,1}(1 - \beta)/d > q^*_{F,1}(1 - \beta)/d$, recall that here

$$T > T_1^* = \frac{1 - \beta}{d}\left(q^* + q^*_{F,1}\right) > \frac{2(1 - \beta)}{d} q^*_{F,1},$$

where the last inequalty uses the fact that $q^* > q^*_{F,1}$. This concludes the proof. □

Proof of Proposition 2. We prove that the expressions on the left-hand sides of (10) and (11) are greater than 0.5. The derivation of these expressions is relegated to Appendix B.

a) If for a given p_F, $p_H > \tilde{p}_H(p_F)$, the inequality in (10) follows immediately from Lemma 6.

b) The left-hand side of (11) is bigger than 0.5 if and only if

$$T > \frac{1-\beta}{d} q_{F,1}^{*} \left(2 - p_F\right).$$

This is always satisfied since, by Lemma 6, $t_H = T - q_{F,1}^{*}(1-\beta)/d > t_F = q_{F,1}^{*}(1-\beta)/d$. □

B Accounting Appendix

Although agents are ex-ante identical, their decisions in the CM, regarding how many hours to work or whether they should visit the financial markets and re-balance their portfolios, depend on whether they got matched in the local and/or foreign DM. In this Appendix, we focus on the labor, consumption, and asset-holding decisions of the following seven groups: 1) sellers who got matched with a local buyer (group 1), 2) sellers who got matched with a foreign buyer (group 2), 3) sellers who did not get matched (group 3), 4) buyers who got matched in both DM's (group 4), 5) buyers who got matched only in the home DM (group 5), 6) buyers who got matched only in the foreign DM (group 6), and 7) buyers who did not get matched (group 7).

The measures of these groups are given by: $\mu_1 = \xi a_{\mathcal{S}} \sigma_H/(\sigma_H + \sigma_F)$, $\mu_2 = \xi a_{\mathcal{S}} \sigma_F/(\sigma_H + \sigma_F)$, $\mu_3 = \xi(1 - a_{\mathcal{S}})$, $\mu_4 = \sigma_H \sigma_F a_{\mathcal{B}}^2$, $\mu_5 = \sigma_H a_{\mathcal{B}} \left(1 - \sigma_F a_{\mathcal{B}}\right)$, $\mu_6 = \sigma_F a_{\mathcal{B}} \left(1 - \sigma_H a_{\mathcal{B}}\right)$, and $\mu_7 = \left(1 - \sigma_H a_{\mathcal{B}}\right)\left(1 - \sigma_F a_{\mathcal{B}}\right)$. For future reference notice that $\mu_1 = p_H$, $\mu_2 = p_F$, $\mu_4 = p_H p_F$, $\mu_5 = p_H \left(1 - p_F\right)$, $\mu_6 = p_F \left(1 - p_H\right)$, and $\mu_7 = \left(1 - p_H\right)\left(1 - p_F\right)$.

Derivation of Home Assets Bias formulas in (10) and (11). We define asset home bias as the ratio of the weighted sum of domestic asset holdings over the weighted sum of all asset holdings for all citizens of a certain country. We count asset holdings in both the CM and the DM and assume equal weights for the two markets. It is understood that we count asset holdings at the end of the sub-periods. For example, an agent of group 4 enters the CM with no assets, but re-balances her portfolio, so at the end of the CM she holds t_H of the home asset and t_F of the foreign asset. On the other hand, an agent of group 1 enters the CM with some home assets, but she sells these assets, so at the end of the CM she holds zero.

a) First consider $T \leq T_1^{*}$. Let the numerator of the asset home bias formula (weighted sum of domestic asset holdings) be denoted by A_H. Given the strategy described above,

$$A_H = t_H + p_H t_F + p_F t_F + \left(1 - p_H\right) t_H.$$

The first term is the total home asset holdings in the CM, i.e. t_H times the total measure of buyers (one). Sellers hold zero assets in the CM. The second term represents home asset holdings in the DM by sellers who got matched with local buyers. The third term represents home asset holdings in the DM by sellers who got matched with foreign buyers. Recall that we are describing the local asset dominance regime. Hence, foreign buyers pay sellers in local assets. The last term represents home asset holdings in the DM by the buyers who did not get matched in the local DM and, hence, hold some domestic assets (groups 6 and 7). After some algebra one can conclude that $A_H = 2t_{_H} + p_{_F} t_{_F}$.

Following similar steps, one can show that the denominator of the asset home bias formula (weighted sum of all asset holdings), is given by $A_T = 2\left(t_{_H} + t_{_F}\right)$. From market clearing, $t_{_H} + t_{_F} = T$. Therefore, (10) follows immediately.

b) Consider $T > T_1^*$. Now the cost of carrying home assets is zero. As a result, buyers carry more assets than what they use in the DM as media of exchange. Therefore, agents from groups 4 and 5, who previously did not show up in the expression A_H, will now carry some home asset leftovers. The numerator of the asset home bias formula is

$$\begin{aligned} A_H &= \left(T - \frac{1-\beta}{d} q^*_{_{F,1}}\right) + p_{_H} \frac{1-\beta}{d} q^* + p_{_F} \frac{1-\beta}{d} q^*_{_{F,1}} \\ &+ p_{_H} \left(T - \frac{1-\beta}{d} q^* - \frac{1-\beta}{d} q^*_{_{F,1}}\right) + (1-p_{_H}) \left(T - \frac{1-\beta}{d} q^*_{_{F,1}}\right). \end{aligned}$$

The first term is the total home asset holdings in the CM (held by buyers whose measure is one). The second term represents home asset holdings in the DM by sellers who got matched with local buyers. The third term represents home asset holdings in the DM by sellers who got matched with foreign buyers. The fourth term represents home asset holdings in the DM by buyers who got matched in the local DM (groups 4 and 5). Finally, the fifth term stands for the home asset holdings of buyers who did not trade in the local DM (groups 6 and 7). After some algebra, we obtain

$$A_H = 2\left(T - \frac{1-\beta}{d} q^*_{_{F,1}}\right) + p_{_F} \frac{1-\beta}{d} q^*_{_{F,1}}.$$

One can also show that the denominator of the asset home bias formula is given by $A_T = 2T$. Hence, (11) follows immediately. □

Proof of Proposition 3. a) Let $C^i_{_F}$, $C^i_{_H}$ be the value of foreign and domestic consumption for the typical agent in group i, $i = 1, ..., 7$, respectively. All consumption is denominated in terms of the general good. Moreover, let H^i denote the hours worked by the typical agent in group i, $i = 1, ..., 7$. All agents consume X^* in the CM, so agents with many assets will work fewer hours. Sometimes it will be useful to directly use the agents' budget constraint and replace X^* with more informative expressions. First, consider sellers. We have $C^1_H = C^2_H = C^3_H = X^*$ and $C^1_F = C^2_F = C^3_F = 0$. Clearly, even sellers who matched with foreign buyers have no imported consumption, since in the local asset dominance regime, sellers get payed in local assets.

Unlike sellers, buyers might also consume in the DM. For group 4, we have $C_H^4 = X^* + (\psi + d)t_H = X^* + q_H$, where q_F represents DM consumption. Also, $C_F^4 = (\psi + d)t_F = q_F$. Group 5 does not match abroad, hence these agents carry some foreign assets into the CM, which in turn implies that some of their CM consumption is imported. Clearly, $C_F^5 = d_\kappa t_F$. On the other hand, $C_H^5 = H^5 + dt_H$.[17] For group 6, all CM consumption (X^*) is domestic and all DM consumption, q_F, is imported. Finally, group 7 consumes only in the CM. We have $C_H^7 = H^7 - dt_H$ and $C_F^7 = d_\kappa t_F$.

The analysis in the previous paragraph reveals that $p_H > \tilde{p}_H(p_F)$, implies $C_H^i > C_F^i$ for all groups except 6, for which the inequality could be reversed, depending on the magnitude of X^*. To prove the desired result we contrast the domestic and foreign consumption for groups 2 and 6. $C_H > C_F$ will be true if and only if $\sum_{i=1}^{7} \mu_i C_H^i > \sum_{i=1}^{7} \mu_i C_F^i$. Since $C_H^i > C_F^i$ holds for $i = 1, 3, 4, 5, 7$, the latter statement will be true as long as

$$\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6. \tag{b.1}$$

From the budget constraint of group 2, we have $C_H^2 = X^* = H^2 + q_F$. Also, from the analysis above, $C_F^2 = 0$. Hence, (b.1) holds if and only if

$$\begin{aligned} p_F(H^2 + q_F) + p_F(1 - p_H)X^* &\geq p_F(1 - p_H)q_F \Leftrightarrow \\ p_F H^2 + p_F(1 - p_H)X^* &\geq -p_H p_F q_F, \end{aligned}$$

which is always true, since $X^*, H^i \geq 0$.

b) Like in part (a), $C_H^1 = C_H^2 = C_H^3 = X^*$ and $C_F^1 = C_F^2 = C_F^3 = 0$. For group 4 $C_H^4 = X^* + q^*$ and $C_F^4 = q_{F,1}^*$. For group 5 CM consumption (X^*) can be broken down into domestic, $H^5 + dT - q^* - (1-\beta)q_{F,1}^*$, and imported, $d_\kappa(1-\beta)q_{F,1}^*/d$. In the DM all consumption is domestic and equal to q^*. Hence, $C_H^5 = H^5 + dT - (1-\beta)q_{F,1}^*$ and $C_F^5 = d_\kappa(1-\beta)q_{F,1}^*/d$. For group 6 all CM consumption (X^*) is domestic and all DM consumption, $q_{F,1}^*$, is imported. Finally, group 7 only consumes in the CM. From this group's budget constraint, it follows that $C_H^7 = H^7 + dT - (1-\beta)q_{F,1}^*$ and $C_F^5 = d_\kappa(1-\beta)q_{F,1}^*/d$.

It is clear that for $i = 1, 2, 3, 4$, $C_H^i > C_F^i$. For groups 5 and 7 the same argument is true, but it is less obvious, hence we prove it formally below. For group 6 this argument need not be true. Consider group 5. $C_H^5 > C_F^5$ will be true if and only if

$$H^5 + dT > (1-\beta)q_{F,1}^* \left(2 - \frac{\kappa}{d}\right). \tag{b.2}$$

[17] To see this point, notice that this agent's budget constraint is $X^* + \psi(t_H + t_F) = H^5 + (\psi + d_\kappa)t_F$ or $X^* = H^5 - \psi t_H + d_\kappa t_F$, where $d_\kappa t_F$ is imported CM consumption and $H^5 - \psi t_H$ is domestic CM consumption. Since domestic DM consumption is given by $(\psi + d)t_H$, we have $C_H^5 = H^5 + dt_H$.

But here $T > T_1^*$, which implies

$$dT > (1-\beta)(q^* + q^*_{F,1}) \Leftrightarrow dT > 2(1-\beta)q^*_{F,1} \Leftrightarrow dT + H^5 > \left(2 - \frac{\kappa}{d}\right)(1-\beta)q^*_{F,1}.$$

This confirms that the inequality in (b.2) holds true. Following similar steps establishes that $C_H^7 > C_F^7$. Hence, $C_H^i > C_F^i$ for all i, except possibly group 6. However, as in part (a), one can easily verify that $\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6$, which implies that $C_H > C_F$ also holds. □