NBER WORKING PAPER SERIES

ASSET LIQUIDITY AND INTERNATIONAL PORTFOLIO CHOICE

Athanasios Geromichalos
Ina Simonovska

Working Paper 17331
http://www.nber.org/papers/w17331

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2011

We are grateful to Marco Bassetto, Jeffrey Campbell, Mariacristina De Nardi, Robert Feenstra, Guillaume Rocheteau, Peter Rupert, Kevin Salyer, Christopher Waller, Randall Wright, and seminar participants at the SED 2011, Atlanta Fed, SHUFE, HKUST, Midwest International Trade Spring 2011 Meeting, First Meeting of the West Coast Search and Matching Group, Organization of Markets Conference at UC Santa Barbara, AEA 2011, UC Santa Barbara, 2010 Search and Matching Conference at Penn, Philadelphia Fed, 2010 Money, Payments and Banking Workshop at Chicago Fed, and UC Davis for their feedback. This paper circulated under the title "Asset Liquidity and Home Bias". The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Asset Liquidity and International Portfolio Choice
Athanasios Geromichalos and Ina Simonovska
NBER Working Paper No. 17331
August 2011
JEL No. E44,F15,F36,G11

ABSTRACT

We study optimal asset portfolio choice in a two-country search-theoretic model of monetary exchange. We allow assets not only to represent claims on future consumption, but also to serve as means of payment. Assuming foreign assets trade at a cost, we characterize equilibria in which different countries' assets arise as media of exchange in different types of trades. More frequent trading opportunities at home result in agents holding proportionately more domestic over foreign assets. Consequently, agents have larger claims to domestic over foreign consumption goods. Moreover, foreign assets turn over faster than home assets because the former have desirable liquidity properties, but unfavorable returns over time. Our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

Athanasios Geromichalos
Department of Economics
University of California, Davis
One Shields Avenue
Davis, CA 95616
ageromich@ucdavis.edu

Ina Simonovska
Department of Economics
University of California, Davis
One Shields Avenue
Davis, CA 95616
and NBER
inasimonovska@ucdavis.edu

An online appendix is available at:
http://www.nber.org/data-appendix/w17331

1 Introduction

Agents' international portfolio choices have been a major topic of research in macroeconomics over the past two decades. The vast majority of papers emphasize the role that foreign assets play in helping agents diversify domestic income risk. While the role of assets in hedging consumption risk is admittedly crucial, an equally important characteristic of assets, their liquidity, has been overlooked by the literature. In this paper, we develop a theoretical framework that allows us to study how liquidity properties of assets shape agents' international portfolios. A concrete concept of asset liquidity is, therefore, required.

To that end, we employ a model in the tradition of monetary-search theory, extended to include real assets and trade between countries. Agents have access to alternating rounds of centralized, or Walrasian, markets and decentralized markets, where agents meet in a bilateral fashion and credit is imperfect. In each country's Walrasian market, domestic and foreign agents can buy assets of that country at the ongoing market price. More interestingly, agents can bring a portfolio of assets to each country's decentralized market and trade it for locally-produced goods. Hence, assets serve a double function. First, they are claims to future consumption, as is standard in finance. Second, they serve as media of exchange, as is standard in monetary theory. It is precisely this second function that captures the notion of asset liquidity.

Assuming that agents incur a small cost to liquidate foreign claims, we characterize equilibria in which different assets arise as media of exchange in different types of bilateral meetings. If agents' trading opportunities abroad are sufficiently frequent, assets circulate as media of exchange locally. This means that agents use domestic assets to trade at home and foreign assets to trade abroad. In this case, our liquidity mechanism endogenizes the commonly-assumed existence of currency areas in international macroeconomics. At the other extreme, if trading opportunities abroad are scarce, agents use domestic assets to trade both at home and abroad. Finally, there is an intermediate case in which either type of equilibrium described above can arise, depending on other parameters that govern asset returns.

As long as trading opportunities abroad are not much more frequent than at home, agents' portfolios exhibit home bias. Since agents hold larger amounts of domestic over foreign assets, they have larger claims to domestic over foreign consumption goods. Hence, our suggested mechanism positively links asset and consumption home bias. Moreover, foreign assets turn over faster than home assets because, while the former have desirable liquidity properties, they yield lower future consumption and are undesirable savings tools. This result is very particular to our mechanism. In the absence of the liquidity factor, foreign asset trading costs, which are necessary to generate consumption and asset home bias, would yield lower turnover of foreign relative to domestic assets. Thus, our model offers an answer to a long-standing puzzle in international finance as described in Lewis (1999): a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

We use the tight link between consumption and asset home bias to assess the model's quantitative ability to explain asset-allocation and turnover-rate patterns observed in the data. We calibrate the model's parameters to match the US import-to-GDP ratio and domestic asset turnover rate over the past decade. We then let the model endogenously generate asset portfolios and turnover rates of foreign assets. The calibrated model predicts a home asset share of seventy-six percent, even if foreign asset liquidation costs are arbitrarily small. It also generates a turnover rate for foreign assets that is 1.2 times higher than the rate for domestic assets. These results favor well with the data. In particular, developed countries obey three empirical regularities. First, bilateral equity investment shares are strongly positively correlated with bilateral import shares. Second, the average OECD country exhibits asset home bias, since its citizens hold seventy-six percent of their wealth in domestic equities. Third, Amadi and Bergin (2008) document that turnover rates of foreign assets are twice as high as those of domestic assets.

The liquidity mechanism described in this paper is fairly successful, both qualitatively and quantitatively, at explaining the three stylized facts discussed above. However, in order to remain tractable, the model assumes away the risky nature of equity returns. Thus, the mechanism is complementary to existing theories that relate asset and consumption home bias in models of aggregate uncertainty. For example, Heathcote and Perri (2007) use a standard international business cycle framework and show that, when preferences are biased toward domestic goods, asset home bias arises because endogenous international relative price fluctuations make domestic stocks a good hedge against non-diversifiable labor income risk. Their model generates a tight link between openness to trade and the level of asset diversification. This positive link is also explored by Collard, Dellas, Diba, and Stockman (2009) in an endowment economy with separable utility between traded and non-traded goods. In their model, an agent's optimal portfolio includes the entire stock of home firms that produce domestic non-traded goods and a fully diversified portfolio of equities of firms that produce tradable goods. The authors show that, if the share of non-traded goods in consumption is large, the model can generate substantial portfolio home bias.

In other related literature, Michaelides (2003) solves for optimal international portfolios in the presence of liquidity constraints and undiversifiable labor income risk. The author shows that substantial asset home bias can be generated in a buffer stock saving model where agents face higher costs to invest abroad than at home. Assets, however, play a very different role in his model relative to ours. In the framework of Michaelides (2003), assets help smooth consumption in the presence of liquidity constraints, while in ours, assets help facilitate bilateral trade. Finally, Hnatkovska (2010) demonstrates that asset home bias and a high foreign asset turnover rate can arise in the presence of non-diversifiable non-traded consumption risk when each country specializes in production, preferences exhibit consumption home bias, and asset markets are incomplete. However, in this setting, domestic asset flows are also high, thus yielding equally high turnover rates of domestic assets.

The present paper is the first to deliver higher foreign over domestic asset turnover rates in a model where foreign asset trading costs naturally link consumption and asset home bias. The liquidity mechanism that generates these desirable predictions relates closely to the growing literature that focuses on the liquidity properties of objects other than fiat money. In their pioneering work, Lagos and Rocheteau (2008) assume that part of the economy's physical capital can be used as a medium of exchange along with money. Their goal is to study the issue of overinvestment and how it is affected by inflation. Geromichalos, Licari, and Suarez-Lledo (2007) introduce a real financial asset and study the co-existence of money and the asset as media of exchange, with special focus on the relationship between asset prices and monetary policy. Lester, Postlewaite, and Wright (2008) take this framework one step further and endogenize the acceptability (of various media of exchange) decisions of agents. Finally, Lagos (2010) considers a similar framework, enriched with uncertainty, in order to address the equity premium and risk-free rate puzzles. The present paper, however, is the first to introduce liquidity of assets in a multi-country environment and to explore the implications of liquidity on the distribution of asset holdings across countries.

Lastly, our model relates to the literature of money-search models applied to international frameworks. In their pioneering work, Matsuyama, Kiyotaki, and Matsui (1993) employ a two-country, two-currency money-search model, with indivisible money and goods, and study conditions under which the two currencies serve as media of exchange in different countries. Wright and Trejos (2001) maintain the assumption of good indivisibility in Matsuyama, Kiyotaki, and Matsui (1993), but they endogenize prices using bargaining theory. Head and Shi (2003) also consider a two-country, two-currency search model and show that the nominal exchange rate depends on the stocks and growth rates of the two monies. Finally, Camera and Winkler (2003) use a search-theoretic model of monetary exchange in order to show that the absence of well-integrated international goods markets does not necessarily imply a violation of the law of one price. In this paper, we employ a model with divisible assets and goods. Moreover, by endowing assets, other than fiat money, with certain liquidity properties, we bring the money-search literature closer to questions related to international portfolio diversification.

The remainder of the paper is organized as follows. Section 2 describes the modeling environment. Section 3 discusses the optimal behavior of agents in the economy. Section 4 characterizes the media of exchange that arise in a symmetric two-country model of international trade in goods and assets. Section 5 describes the model's predictions regarding asset and consumption home bias as well as asset turnover rates, and provides empirical support for the mechanism. Section 6 evaluates the quantitative ability of the model to account for the observed patterns in asset portfolios and turnover rates. Section 7 concludes.

2 Physical Environment

Time is discrete with an infinite horizon. Each period consists of two sub-periods. During the first sub-period, trade occurs in decentralized markets (DM henceforth), which we describe in detail below. In the second sub-period, economic activity takes place in traditional Walrasian or centralized markets (CM henceforth). There is no aggregate uncertainty. There are two countries, A and B. Each country has a unit measure of buyers and a measure ξ of sellers who live forever. The identity of agents (as sellers or buyers) is permanent. During the first sub-period, a distinct DM opens within each country and anonymous bilateral trade takes place. We refer to these markets as DM_i, $i = A, B$. Without loss of generality, assume that DM_A opens first. Sellers from country i are immobile, but buyers are mobile. Therefore, in DM_i, sellers who are citizens of country i meet buyers who could be citizens of either country. During the second sub-period, all agents are located in their home country.

All agents discount the future between periods (but not sub-periods) at rate $\beta \in (0, 1)$. Buyers consume in both sub-periods and supply labor in the second sub-period. Their preferences, which are independent of their citizenship, are given by $\mathcal{U}^B(x_{_A}, x_{_B}, X, H)$, where x_i is consumption in DM_i, $i = A, B$, and X, H are consumption and labor in the (domestic) CM. Sellers consume only in the CM and they produce in both the DM and the CM. Sellers' preferences are given by $\mathcal{U}^S(h, X, H)$, where the only new variable is h, which stands for hours worked in the DM. In line with Lagos and Wright (2005), we adopt the functional forms

$$\begin{aligned} \mathcal{U}^B(x_{_A}, x_{_B}, X, H) &= u(x_{_A}) + u(x_{_B}) + U(X) - H, \\ \mathcal{U}^S(h, X, H) &= -c(h) + U(X) - H. \end{aligned}$$

We assume that u and U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $U' > 0$, $u'' < 0$, and $U'' \leq 0$. For simplicity we assume that $c(h) = h$, but this is not important for any of our results. Let $q^* \equiv \{q : u'(q^*) = 1\}$ and suppose that there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$ with $U(X^*) > X^*$.

During the round of decentralized trade, sellers and buyers are matched randomly, according to a matching technology that is identical in both DM's. Let B, S be the total number of buyers and sellers in some DM and define market tightness as $\theta = B/S$. The total number of matches in this market is given by $M(B, S)$, where M is assumed to be increasing in both arguments, concave, and homogeneous of degree one. The arrival rate of buyers to an arbitrary seller is $a_{_S} = M(B, S)/S = M(B/S, 1) \equiv f(\theta)$. Homogeneity of degree one implies that the arrival rate of sellers to an arbitrary buyer is $a_{_B} = M(B, S)/B = a_{_S}/\theta$. In both DM's, $S = \xi$. Buyers get to visit the domestic DM with probability $\sigma_{_H} \in (0, 1)$ and the foreign DM with probability $\sigma_{_F} \in (0, 1)$, which captures the degree of economic integration. The measure of *active* buyers in both DM's is given by $B = \sigma_{_H} + \sigma_{_F}$ and market tightness is given by

$\theta = \xi(\sigma_H + \sigma_F)^{-1}$. In any bilateral meeting, the buyer makes a take-it-or-leave-it offer to the seller. Any sale to a buyer from country j will count as imports of country j from country i.

As mentioned earlier, during the second sub-period agents trade in centralized markets, CM_i, $i = A, B$. All agents consume and produce a general good or fruit which is identical in both countries. Thus, the domestic and the foreign general goods enter as perfect substitutes in the utility function. Agents are located in the home country and have access to a technology that can transform one unit of labor into one unit of the fruit. Furthermore, we assume that there are two trees, one in each country, that produce fruit, as in Lucas (1978). Shares of the tree in country i are traded in CM_i, but due to perfect financial integration, agents from country j can place any order and buy shares of this tree at the ongoing price ψ_i. Let $T_i > 0$ denote the total supply of the tree in country i and R_i the dividend of tree i. Since in this paper we focus on symmetric equilibria, we assume that $T_A = T_B = T > 0$ and $R_A = R_B = R > 0$. T and R are exogenously given and constant.

Except from consuming the general good and trading shares of the trees in the CM's, buyers can also carry some claims into the DM's in order to trade them for a special good produced by local sellers. Hence, assets serve not only as stores of value, but also as media of exchange.[1] The necessity for a medium of exchange arises due to anonymity and a double coincidence of wants problem that characterizes trade in the decentralized markets (see Kocherlakota (1998) for an extensive discussion). Assets can serve as media of exchange as long as they are portable, storable, divisible, and recognizable by all agents. We assume that all these properties are satisfied. As we explain in more detail below, we do not place any *ad hoc* restrictions on which assets can serve where as means of payments.

In the absence of any frictions in the physical environment, due to the symmetry assumption made earlier, the model would predict that agents' share of foreign assets in their portfolios is anywhere between 0 and 100%. In order to derive sharper predictions, additional assumptions are necessary. One friction that is very common in the international macroeconomics literature is the so-called *currency areas* assumption, which dictates that trading in country i requires the use of that country's currency (in our case the asset) as a medium of exchange. This assumption is extremely appealing and empirically relevant. However, since our paper is in the spirit of modern monetary theory, we consider such a restriction undesirable, and insist that agents should choose for themselves which objects (assets) will be used as media of exchange and which will not.

We now introduce the main friction of our model. We assume that whenever an agent from

[1]We model trade as quid-pro-quo (good-for-asset), but one can think of assets serving as collateral for trade in decentralized markets. Moreover, our analysis is robust to the introduction of fiat money in the economy. In this case, agents hold assets as store of value due to positive asset returns, since fiat money has negative or zero return, depending on the rate of inflation. Suppose that trade in DM_i necessitates the use of i's currency. If it is costly to convert foreign assets into home currency, due to a spread in the exchange rate, agents will hold i's asset in order to trade in i's currency. Hence, our results would be preserved. Nevertheless, we do not model fiat money explicitly in order to keep the analysis simple and to abstract from policy considerations.

country i holds one share of country j's tree, $j \neq i$, she has a claim to $R - \kappa$ units of fruit, with $\kappa \in (0, R)$. Sometimes, we refer to κ as the cost of liquidating the foreign asset. The most straightforward way to interpret κ is the following: when an agent from country i holds one share of tree j, R units of the general good have to be physically delivered from country j to its claimant in country i. Therefore, κ represents a transportation cost. Alternatively, κ can be thought of as an information friction or cost that agents have to pay in order to participate in the foreign asset market.[2] Finally, κ may capture policy frictions such as tariffs on goods imports or taxes on foreign dividend returns, both of which are commonly observed across many countries. It is important to highlight that all the results presented in this paper hold for arbitrarily small values of κ.

3 Value Functions and Optimal Behavior

We begin with the description of the value functions in the Walrasian market. For a seller from country $i = A, B$, the Bellman's equation is given by

$$W_i^S(t) = \max_{X,H,\hat{t}} \left\{U(X) - H + \beta V_i^S(\hat{t})\right\}$$
$$\text{s.t. } X + \psi_i \hat{t}_i + \psi_j \hat{t}_j = H + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j,$$

where $t \equiv (t_i, t_j)$, and t_l denotes the amount of asset $l = A, B$ held by the agent when she enters CM_i. We have also defined $R_\kappa = R - \kappa$. Variables with hats denote next period's choices. It can be easily verified that, at the optimum, $X = X^*$. Using this fact and replacing H from the budget constraint into W_i^S yields

$$W_i^S(t) \;=\; U(X^*) - X^* + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j + \max_{\hat{t}} \left\{-\psi_i \hat{t}_i - \psi_j \hat{t}_j + \beta V_i^S(\hat{t})\right\}. \tag{1}$$

Some results are worth highlighting here. First, the choice of $\hat{t}$ does not depend on t. In other words, there are no wealth effects, which follows from the quasi-linearity of $\mathcal{U}$. Second, the function W_i^S is linear and we can write[3]

$$W_i^S(t) = \Lambda_i^S + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j,$$

where the definition of Λ_i^S is obvious.

Next, consider a buyer from country i. The state variables for this agent are summarized

[2] Similarly, Michaelides (2003) assumes that agents face a higher transaction cost when investing abroad.

[3] The term Λ_i^S consists of constant terms (like X^*, $U(X^*)$) and terms that depend on $\hat{t}$. By the no-wealth-effect property, the latter does not depend on t. Hence, W_i^S is linear.

by $t \equiv (t_{ii}, t_{ij}, t_{ji}, t_{jj})$, where t_{ij} is the amount of asset i that is used for trade in DM_j. In other words, we allow buyers to choose any amount of assets they wish, but we do not allow them to use t_{ij} and t_{jj} for trade in DM_i. This assumption makes the agent's maximization problem more tractable.[4] The Bellman's equation for a buyer from country i is given by

$$
\begin{aligned}
&W_i^B(t) = \max_{X,H,\hat{t}} \left\{U(X) - H + \beta V_i^B(\hat{t})\right\} \\
&\text{s.t. } X + \psi_i\left(\hat{t}_{ii} + \hat{t}_{ij}\right) + \psi_j\left(\hat{t}_{ji} + \hat{t}_{jj}\right) = H + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}).
\end{aligned}
$$

Once again, $X = X^*$ at the optimum. This allows us to write

$$
\begin{aligned}
W_i^B(t) &= U(X^*) - X^* + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}) \\
&+ \max_{\hat{t}} \left\{-\psi_i\left(\hat{t}_{ii} + \hat{t}_{ij}\right) - \psi_j\left(\hat{t}_{ji} + \hat{t}_{jj}\right) + \beta V_i^B(\hat{t})\right\}. \qquad (2)
\end{aligned}
$$

As in the case of sellers, there are no wealth effects in the choice of $\hat{t}$, and W_i^B is linear,

$$
W_i^B(t) = \Lambda_i^B + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}),
$$

where the definition of Λ_i^B is obvious.

We now turn to the terms of trade in the DM's. As explained earlier, we place no restrictions on which assets can be used as means of payments. Consider meetings in DM_i, and as a first case let the buyer be a citizen of country i (a local). Suppose that the asset holdings of the buyer are t and those of the seller are $\tilde{t}$. The solution to the bargaining problem is a list (q_i, x_{ii}, x_{ji}), where q_i is the amount of special good, x_{ii} is the amount of asset i, and x_{ji} is the amount of asset j that changes hands. With take-it-or-leave-it offers by the buyer, the bargaining problem is

$$
\begin{aligned}
&\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) + W_i^B(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) - W_i^B(t)\right] \\
&\text{s.t. } \; - q_i + W_i^S(\tilde{t}_i + x_{ii}, \tilde{t}_j + x_{ji}) - W_i^S(\tilde{t}) = 0 \qquad (3) \\
&\text{and } \; x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}.
\end{aligned}
$$

Exploiting the linearity of the W's, we can re-write this problem as

$$
\begin{aligned}
&\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) - (\psi_i + R)x_{ii} - (\psi_j + R_\kappa)x_{ji}\right] \\
&\text{s.t. } \quad q_i = (\psi_i + R)x_{ii} + (\psi_j + R_\kappa)x_{ji} \\
&\text{and } \; x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}.
\end{aligned}
$$

[4] Moreover, given our model specification, if agents only choose some (t_i, t_j), which they can trade in any market, the order in which markets open clearly matters. We consider this an undesirable feature.

The following lemma describes the bargaining solution in detail.

Lemma 1. *Define* $\pi_i \equiv (\psi_i + R)t_{ii} + (\psi_j + R_\kappa)t_{ji}$.

$$If \quad \pi_i \geq q^*, \quad then \quad \begin{cases} q_i = q^*, \\ (\psi_i + R)x_{ii} + (\psi_j + R_\kappa)x_{ji} = q^*. \end{cases}$$

$$If \quad \pi_i < q^*, \quad then \quad \begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii},\, x_{ji} = t_{ji}. \end{cases}$$

Proof. It can be easily verified that the suggested solution satisfies the necessary and sufficient conditions for maximization. □

This bargaining solution is standard in models where multiple assets can serve as media of exchange.[5] All that matters for the bargaining solution is whether the buyer's real balances of assets used for trade in DM_i, i.e. π_i, are sufficient to buy the first-best quantity. If the answer to that question is yes, then $q_i = q^*$, and the buyer spends amounts of assets t_{ii}, t_{ji} such that $\pi_i = q^*$. Notice that in this case t_{ii}, t_{ji} cannot be pinned down separately.[6] Conversely, if $\pi_i < q^*$, the buyer gives up all her asset holdings and purchases as much q as her real balances allow.

Now consider a meeting in DM_i when the buyer is from $j \neq i$ (a foreigner) with asset holdings t. Again, denote the solution by (q_i, x_{ii}, x_{ji}). The bargaining problem to be solved is the same as in (3), after replacing W_i^B with W_j^B. Using the linearity of the value functions, one can re-write the bargaining problem as[7]

$$\begin{aligned} &\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) - (\psi_i + R_\kappa)x_{ii} - (\psi_j + R)x_{ji}\right] \\ &\text{s.t.} \quad q_i = (\psi_i + R)x_{ii} + (\psi_j + R_\kappa)x_{ji} \\ &\text{and} \ \ x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}. \end{aligned}$$

Substituting for $(\psi_i + R)x_{ii} + (\psi_j + R)x_{ji}$ from the constraint, we can re-write the objective as $u(q_i) - q_i - \kappa x_{ji} + \kappa x_{ii}$. Notice that for every unit of asset i that goes from the buyer to the seller, two positive effects are generated. First, trade is facilitated, i.e. the seller produces q for the

[5] See for example Geromichalos, Licari, and Suarez-Lledo (2007) and Lester, Postlewaite, and Wright (2008).

[6] This does not cause any indeterminacy issues because, as we show later, the agent sets $t_{ji} = 0$.

[7] Here we have silently assumed that the buyer and the seller can only exchange goods for assets and not assets for assets. Given that the two parties have different valuations for the two assets, it is not at all clear why this should be the case. The justification for our assumption is as follows. Since sellers have no bargaining power, it is always optimal to bring zero assets into the match, which is what they will do in equilibrium. Hence, although for the sake of generality we describe the bargaining solution for any asset holding $\tilde{t}$ by the seller, we know that in equilibrium $\tilde{t} = 0$. So our assumption simplifies the bargaining problem without affecting equilibrium results.

buyer in exchange for the asset. Second, the social surplus increases because the asset goes to the hands of the agent who has a higher valuation for it (the seller does not incur the liquidation cost κ for asset i). These facts determine the spirit of the solution to the bargaining problem, which is now stated in detail.

Lemma 2. *Define* $\overline{q}(\psi) \equiv \left\{q : u'(q) = \frac{\psi+R_\kappa}{\psi+R}\right\}$ *and* $\underline{q}(\psi) \equiv \left\{q : u'(q) = \frac{\psi+R}{\psi+R_\kappa}\right\}$*, with* $\overline{q}(\psi) > q^* > \underline{q}(\psi)$*, for all* $\psi < \infty$*. The bargaining solution is the following*

a) If $t_{ii} \geq \frac{\overline{q}(\psi_i)}{\psi_i+R}$*, then* $\begin{cases} q_i = \overline{q}(\psi_i), \\ x_{ii} = \frac{\overline{q}(\psi_i)}{\psi_i+R}, \\ x_{ji} = 0. \end{cases}$

b) If $t_{ii} \in \left[\frac{\underline{q}(\psi_j)}{\psi_i+R}, \frac{\overline{q}(\psi_i)}{\psi_i+R}\right)$*, then* $\begin{cases} q_i = t_{ii}(\psi_i + R), \\ x_{ii} = t_{ii}, \\ x_j = 0. \end{cases}$

c1) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i+R}$ *and* $\pi_i \geq \underline{q}(\psi_j)$*, then* $\begin{cases} q_i = \underline{q}(\psi_j), \\ x_{ii} = t_{ii}, \\ x_{ji} = \frac{\underline{q}(\psi_j)-(\psi_i+R)t_{ii}}{\psi_j+R_\kappa}. \end{cases}$

c2) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i+R}$ *and* $\pi_i < \underline{q}(\psi_j)$*, then* $\begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii}, \\ x_{ji} = t_{ji}. \end{cases}$

Proof. See Appendix A. □

The solution to the bargaining problem is very intuitive and it is driven by the amount of foreign assets held by the citizen of country j. The buyer should use only asset i whenever possible. If her asset-i holdings are unlimited, she should buy the quantity defined as $\overline{q}(\psi)$. This quantity is larger than q^*, the maximizer of $u(q) - q$, because of the second positive effect of using asset i, described above (the wedge between the buyer's and the seller's valuation). In similar spirit, if the buyer's balances allow her to buy $\underline{q}(\psi)$ or more, she should not use any amount of asset j for trade. Positive amounts of asset j will change hands, only if the asset-i holdings are such that the buyer cannot purchase $\underline{q}(\psi)$. In that case, the buyer will use the amount of asset j that, together with all of her asset-i holdings, buys the quantity $\underline{q}(\psi)$.

We now proceed to the description of the value functions in the DM. Once these functions have been established, we can plug them into the expressions for the CM value functions (equations (1) and (2)) and characterize the optimal behavior of the agents. Consider first a seller from country i. Since buyers make take-it-or-leave-it offers, $V_i^S(t) = W_i^S(t)$. Use this result in (1) to obtain

$$W_i^S(t) = K_i^S + \max_{\hat{t}} \left\{ J_i^S(\hat{t}) \right\},$$

where

$$J_i^S(\hat{t}) = \left[-\psi_i + \beta\left(\hat{\psi}_i + R\right)\right]\hat{t}_i + \left[-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)\right]\hat{t}_j, \tag{4}$$

and K_i^S is a term that does not depend on $\hat{t}$.[8] We refer to J_i^S as the objective function of seller i. Maximization of this function with respect to $\hat{t}$ fully describes the optimal asset holdings of this agent in every period. We will return to discuss the optimal choice after completing the description of the objective functions for all agents.

Consider a buyer from i. The value function for this agent, when she enters the decentralized-trade round with asset holdings $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$, is given by[9]

$$\begin{aligned} V_i^B(t) &= \sigma_H \sigma_F a_B^2 \Big\{ u(q_i) + u(q_j) + W_i^B\left(\hat{t}_{ii} - x_{ii}, \hat{t}_{ij} - x_{ij}, \hat{t}_{ji} - x_{ji}, \hat{t}_{jj} - x_{jj}\right) \Big\} \\ &+ \sigma_H a_B (1 - \sigma_F a_B) \Big\{ u(q_i) + W_i^B\left(\hat{t}_{ii} - x_{ii}, \hat{t}_{ij}, \hat{t}_{ji} - x_{ji}, \hat{t}_{jj}\right) \Big\} \\ &+ \sigma_F a_B (1 - \sigma_H a_B) \Big\{ u(q_j) + W_i^B\left(\hat{t}_{ii}, \hat{t}_{ij} - x_{ij}, \hat{t}_{ji}, \hat{t}_{jj} - x_{jj}\right) \Big\} \\ &+ (1 - \sigma_H a_B)(1 - \sigma_F a_B) W_i^B(t), \end{aligned}$$

where the q and x terms are determined through the bargaining protocols described in Lemmata 1 and 2. Also, it is understood that (q_i, x_{ii}, x_{ji}) are functions of $(\hat{t}_{ii}, \hat{t}_{ji})$ and (q_j, x_{ij}, x_{jj}) are functions of $(\hat{t}_{ij}, \hat{t}_{jj})$. Following the same steps as in the case of a seller, one can conclude that the objective function for the buyer is given by

$$J_i^B(\hat{t}) = J_{i,H}^B(\hat{t}) + J_{i,F}^B(\hat{t}), \tag{5}$$

where

$$\begin{aligned} J_{i,H}^B(\hat{t}_{ii}, \hat{t}_{ji}) &= \left[-\psi_i + \beta\left(\hat{\psi}_i + R\right)\right]\hat{t}_{ii} + \left[-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)\right]\hat{t}_{ji} \\ &+ \beta p_H \Big\{ u(q_i(\hat{t}_{ii}, \hat{t}_{ji})) - \left(\hat{\psi}_i + R\right) x_{ii}(\hat{t}_{ii}, \hat{t}_{ji}) - \left(\hat{\psi}_j + R_\kappa\right) x_{ji}(\hat{t}_{ii}, \hat{t}_{ji}) \Big\}, \qquad (6) \\ J_{i,F}^B(\hat{t}_{ij}, \hat{t}_{jj}) &= \left[-\psi_i + \beta\left(\hat{\psi}_i + R\right)\right]\hat{t}_{ij} + \left[-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)\right]\hat{t}_{jj} \\ &+ \beta p_F \Big\{ u(q_j(\hat{t}_{ij}, \hat{t}_{jj})) - \left(\hat{\psi}_i + R\right) x_{ij}(\hat{t}_{ij}, \hat{t}_{jj}) - \left(\hat{\psi}_j + R_\kappa\right) x_{jj}(\hat{t}_{ij}, \hat{t}_{jj}) \Big\}. \qquad (7) \end{aligned}$$

In the expressions above, we have defined $p_H \equiv a_B \sigma_H$ and $p_F \equiv a_B \sigma_F$, i.e. p_H is the probability of being an active and matched buyer in the local DM and p_F is the analogous expression for

[8] More precisely, one can show that, $K_i^S = (U(X^*) - X^*)(1+\beta) + (\psi_i + R)t_i + (\psi_j + R_\kappa)t_j + \beta \max_{\hat{\hat{t}}} \left\{ -\hat{\psi}_i \hat{\hat{t}}_i - \hat{\psi}_j \hat{\hat{t}}_j + \beta V_i^S\left(\hat{\hat{t}}\right) \right\}$. By the no-wealth-effect result, the choice of $\hat{\hat{t}}$ does not depend on $\hat{t}$.

[9] The first line represents the case in which the buyer is matched in both DM's. The second and third lines represent the cases in which she matches only in the home and foreign DM, respectively. The last line stands for the case in which the buyer does not trade in any DM and walks into the CM with her original asset holdings.

the foreign DM. $J^B_{i,n}$ is the part of the objective that reflects the choice of assets to be traded in DM_n, with $n = H$ for home or $n = F$ for foreign.

Equation (5) highlights that the optimal choice of asset holdings to be traded in the home and foreign DM (($\hat{t}_{ii}, \hat{t}_{ji}$) and ($\hat{t}_{ij}, \hat{t}_{jj}$), respectively) can be studied in isolation. This leads to a much more tractable optimization problem and it is the main reason for choosing the state space of a buyer to be $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$. The term $-\psi_i + \beta(\hat{\psi}_i + R)$ represents the net gain of carrying one unit of the domestic asset from today's CM into tomorrow's CM. Sometimes we refer to the negative of this term as the cost of carrying the asset across periods. Similarly, $-\psi_j + \beta(\hat{\psi}_j + R_\kappa)$ is the net gain from carrying one unit of the foreign asset across consecutive CM's. In both (6) and (7), the second line represents the expected (discounted) surplus of the buyer in each DM. The following Lemma states an important result regarding the sign of the cost terms that appear in the agents' objective functions.

Lemma 3. *In any equilibrium,* $\psi_k \geq \beta(\hat{\psi}_k + R)$, $k = A, B$.

Proof. A formal proof can be found in Geromichalos, Licari, and Suarez-Lledo (2007). To see the result intuitively, just notice that if $\psi_k < \beta(\hat{\psi}_k + R)$ for some k, agents of country k will have an infinite demand for this asset, and so equilibrium is not well defined. □

According to the Lemma, for any agent from country i, the net gain of carrying home assets across periods is non-positive and the net gain of carrying foreign assets across periods is strictly negative due to the term κ. The non-negative sign of the cost terms assigns a very intuitive interpretation to the objective functions of agents: a buyer wishes to bring assets with her in the DM in order to facilitate trade. However, she faces a trade-off because carrying these assets is not free (equations (6) and (7)). On the other hand, sellers have no benefit from carrying assets into the DM. This is why in (4) only the cost terms are present. We are now ready to discuss the optimal portfolio choices of agents and, consequently, equilibrium.

4 Equilibrium in the Two-Country Model

We begin this section with a general definition of equilibrium and we explain why focusing on symmetric, steady-state equilibria can make the analysis more tractable.

Definition 1. An equilibrium for the two-country economy is a list of solutions to the bargaining problems in DM_i, $i = A, B$ described by Lemmata 1 and 2 and bounded paths of ψ_A, ψ_B, such that agents maximize their objective functions (described by (4), (6), and (7)) under the market clearing conditions $T_A = \sum_{k=A,B} \sum_{i=A,B} t^i_{Ak} + \xi \sum_{i=A,B} t^i_A$ and $T_B = \sum_{k=A,B} \sum_{i=A,B} t^i_{Bk} + \xi \sum_{i=A,B} t^i_B$. The term t^i_{jk} denotes demand of a buyer from country i for asset j to trade in DM_k, and the term t^i_j denotes the demand of a seller from country i for asset j.

In the remainder of the paper we focus on steady-state equilibria. Moreover, the two-country environment we consider is completely symmetric in the following sense: $T_A = T_B = T$, $R_A = R_B = R$, and σ_H, σ_F are the same in both countries. Given the concavity of the buyers' objective functions, each buyer has a certain (degenerate) demand for the home asset and a certain (degenerate) demand for the foreign asset, but these demand functions do not depend on the agent's citizenship. In terms of equilibrium objects, this implies that $t^A_{AA} = t^B_{BB}$, $t^A_{AB} = t^B_{BA}$, $t^A_{BA} = t^B_{AB}$, and $t^A_{BB} = t^B_{AA}$.[10] The demand of sellers need not be degenerate, but given asset prices, we can always choose it conveniently in order to clear the market (see Lemma 4 below for details). Two important implications follow. First, both assets have equal aggregate supply (T) and aggregate demand. Therefore, their equilibrium price has to be equal, $\psi = \psi_A = \psi_B$. Second, by the bargaining protocols, the amount of special good that changes hands in any DM depends only on whether the buyer is a local or a foreigner, but not on the label of the DM. These facts lead to the following definition.

Definition 2. A symmetric steady-state equilibrium for the two-country economy can be summarized by the objects $\{t_{HH}, t_{HF}, t_{FH}, t_{FF}, t^S_H, t^S_F, q_H, q_F, \psi\}$. The term t_{ij} is the equilibrium asset holdings of the representative buyer (of any country) for asset i to be used for trade in DM_j, with $i, j = H$ for home or $i, j = F$ for foreign. For future reference also define the total home and foreign asset holdings of buyers, $t_H = t_{HH} + t_{HF}$ and $t_F = t_{FH} + t_{FF}$. The term t^S_i, $i = H, F$, represents equilibrium asset-i holdings for the representative seller, and q_i stands for the amount of special good that changes hands in any DM when the buyer is local ($i = H$) or foreign ($i = F$). Finally, ψ is the symmetric, steady-state equilibrium asset price. Equilibrium objects are such that agents maximize their respective objective functions and markets clear.

We now proceed to a more careful discussion of the optimal portfolio choice of agents and, consequently, equilibrium. Notice that the symmetric, steady-state version of Lemma 3 dictates that $\psi \geq \beta R/(1-\beta) \equiv \psi^*$. The term ψ^* is the so-called *fundamental* value of the asset, i.e. the unique price that agents would be willing to pay for one unit of this asset if we were to shut down the DM's (in which case the model would coincide with a two-country Lucas-tree model). The optimal portfolio choice of a seller is straightforward.

Lemma 4. *A seller's optimal choice of asset holdings satisfies* $t^S_F = 0$ *and*

$$t^S_H = \begin{cases} 0, & \textit{if } \psi > \psi^*, \\ \in \mathbb{R}_+, & \textit{if } \psi = \psi^*. \end{cases}$$

[10] For example, $t^A_{AA} = t^B_{BB}$ means that the demand for asset A of a buyer from A in order to trade in her local DM is equal to the demand of a buyer from B for asset B used for trade in her local decentralized market. The remaining equations admit similar interpretations.

Proof. The result follows directly from inspection of (4) and Lemma 3. ☐

The seller never wishes to hold a positive amount of the foreign asset, given that the cost of carrying this asset across periods is strictly negative, and she has no benefit from bringing assets into the DM. A seller might demand some home assets, if the holding cost is zero. This is true when $\psi = \psi^*$. We now turn to the optimal portfolio choice of the buyer. We will examine the choice of (t_{HH}, t_{FH}) and (t_{HF}, t_{FF}) separately, by looking at the symmetric, steady-state versions of (6) and (7).

Lemma 5. *A buyer's optimal choice of asset holdings for trade in the local DM satisfies $t_{FH} = 0$. Moreover, if $\psi > \psi^*$, t_{HH} solves*

$$\psi = \beta(\psi + R)\left\{1 + p_H[u'((\psi + R)t_{HH}) - 1]\right\}, \tag{8}$$

and if $\psi = \psi^*$, $t_{HH} \geq q^*/(\psi + R)$.

Proof. See Appendix A. ☐

Lemma 5 reveals that buyers never carry foreign assets to trade in the local DM. The intuition behind this result is straightforward. Recall from Lemma 1 that any combination of assets for which $(\psi + R)t_{HH} + (\psi + R_\kappa)t_{FH} = \pi$, for some given π, buys the same amount of special good. Since the foreign asset has a higher holding cost, it is always optimal for the buyer to purchase any desired quantity using t_{HH} only. When the cost to carry assets falls to zero, optimality requires that the buyer bring any amount of assets that buy her q^* in the local DM (this amount maximizes the buyer's surplus). By Lemma 1, any $t_{HH} \geq (1-\beta)q^*/R$ does that job.

Next, we consider the optimal choice of assets used for trade in the foreign DM. There are three scenarios (or regimes), that depend on the values of the following parameters: β, p_F, and κ/R. In the first scenario, regardless of asset prices, the agent uses only foreign assets to trade in the foreign DM. In the second scenario, again regardless of asset prices, the agent chooses $t_{FF} = 0$ and trades in the foreign DM with her home asset. Finally, there is a third regime, in which the agent uses either t_{HF} or t_{FF} as means of payment (except from a knife-edge case), depending on asset prices. The following Lemma describes the details. Figure 1 summarizes the parameter values that constitute the various regions described in Lemma 6.

Lemma 6. *<u>Case 1</u>: Assume $(p_F, \beta) \in R_1$, or $(p_F, \beta) \in R_2$ and $\frac{\kappa}{R} > \frac{(1-2p_F)}{(1-p_F)(1-\beta)} \equiv \tilde{\kappa}$. Then the buyer uses only the foreign asset as a medium of exchange in the foreign DM. The optimal t_{FF} satisfies*

$$\psi = \beta\left[(1-p_F)(\psi + R_\kappa) + p_F(\psi + R)\, u'((\psi + R)t_{FF})\right]. \tag{9}$$

If $\psi > \psi^*$, then $t_{_{HF}} = 0$, and if $\psi = \psi^*$, then $t_{_{HF}} \in \mathbb{R}_+$. Also, $q_{_F} = (\psi + R)t_{_{FF}}$.

Case 2: Assume $(p_{_F}, \beta) \in R_4$. The buyer sets $t_{_{FF}} = 0$ and uses only the domestic asset as a medium of exchange in the foreign DM. If $\psi > \psi^*$, then $t_{_{HF}}$ satisfies

$$\psi = \beta \left[(1 - p_{_F})(\psi + R) + p_{_F}(\psi + R_\kappa)\, u'\left((\psi + R_\kappa)t_{_{HF}}\right) \right], \tag{10}$$

with $q_{_F} = (\psi + R_\kappa)t_{_{HF}}$. If $\psi = \psi^*$, then any $t_{_{HF}} \geq \underline{q}(\psi^*)/(R/(1-\beta) - \kappa)$ is optimal. In this case, $q_{_F} = \underline{q}(\psi^*)$.

Case 3: Assume $(p_{_F}, \beta) \in R_3$, or $(p_{_F}, \beta) \in R_2$ and $\kappa/R \leq \tilde{\kappa}$. Also, define $\psi_c \equiv \beta(1 - p_{_F})(2R - \kappa)/[1 - 2\beta(1 - p_{_F})]$. The following sub-cases arise: a) If $\psi > \psi_c$, then the optimal $t_{_{HF}}, t_{_{FF}}, q_{_F}$ are as in Case 1 above. b) If $\psi < \psi_c$, then the optimal $t_{_{HF}}, t_{_{FF}}, q_{_F}$ are as in Case 2 above. c) In the knife-edge case $\psi = \psi_c$, $t_{_{HF}}, t_{_{FF}} > 0$ and both (9) and (10) hold. The optimal choices $t_{_{HF}}, t_{_{FF}}$ cannot be uniquely pinned down, but $q_{_F}$ is uniquely given by $q_{_{F,c}} \equiv \{q : u'(q) = (1 - p_{_F})/p_{_F}\}$.

Proof. See Appendix A. □

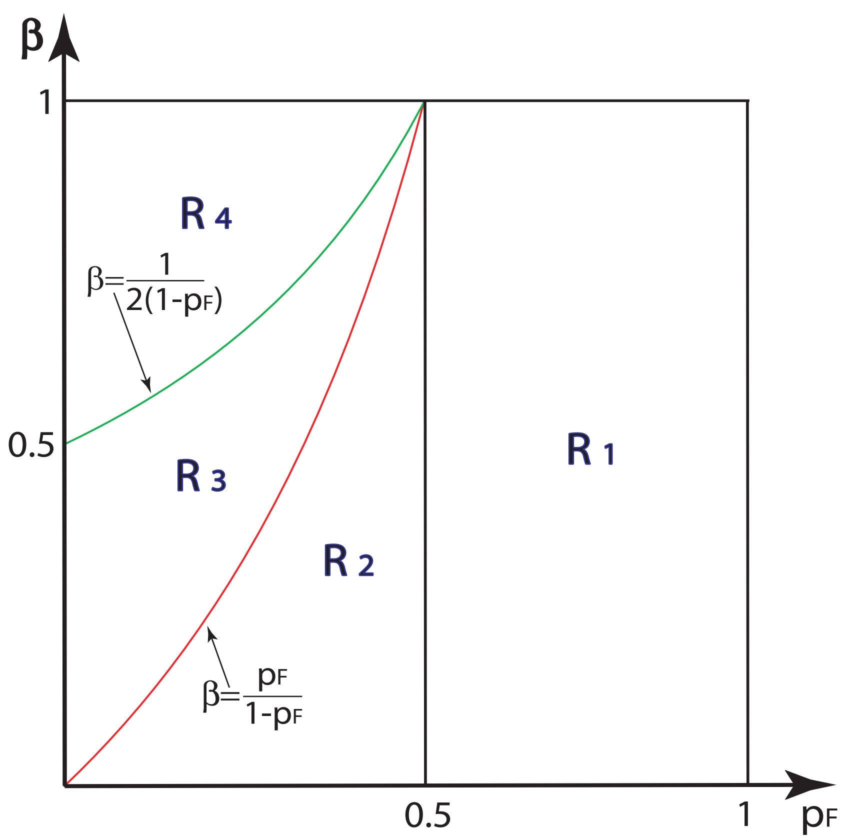

Figure 1: Parameter Values and Regions

Lemma 6 has an intuitive explanation. The liquidation cost κ creates a "wedge" between the asset valuations of a seller and a buyer who come from different countries. The buyer knows that if she meets with a foreign seller, she will have higher purchasing power if she carries the foreign asset (which represents a home asset to the seller). On the other hand, meeting the seller is not guaranteed, and if the buyer stays unmatched in the foreign DM, she will be stuck with the "toxic" asset that incurs a liquidation cost. A reverse story applies regarding the optimal choice of $t_{_{HF}}$. If the buyer carries a large amount of this asset and matches in the foreign DM, she will have lower purchasing power because, for every unit of $t_{_{HF}}$ that she passes to the seller, the latter will suffer a liquidation cost in the forthcoming CM. Not surprisingly, when $p_{_F}$ is high, the buyer decides to carry out trades in the foreign DM using the foreign asset. On the other hand, when it is less likely to match abroad (low $p_{_F}$), the buyer prefers to carry her home asset, even when trading in the foreign country. In intermediate cases, either regime can arise, depending on the asset price, which affects the holding costs.[11]

The relationship between the optimal $t_{_{HF}}$ and $t_{_{FF}}$ with $p_{_F}$ is straightforward. What is perhaps more surprising is the fact that, in R_2, the buyer chooses to trade with the foreign asset when κ is relatively high. This might seem counterintuitive at first, given that a high κ means high liquidation cost for a buyer who did not dispose of the foreign asset. But one should not forget that a high κ also implies low purchasing power for the buyer, if she carries only $t_{_{HF}}$. These forces have opposing effects. Whether it is optimal to trade with $t_{_{HF}}$ or $t_{_{FF}}$ depends on the value of β. When κ is high, the buyer realizes that she might incur a high liquidation cost, but this will happen *tomorrow*. On the other hand, when the buyer is trying to buy some q and pay with $t_{_{HF}}$, the seller will have to incur a cost in the current period's CM, i.e. *tonight*. Thus, a high κ dictates the use of $t_{_{FF}}$ in foreign meetings, as long as agents are not very patient. As an extreme case, consider points in R_2 that are close to the origin, and suppose κ is very high. Although $p_{_F}$ is tiny, the buyer chooses to trade with $t_{_{FF}}$ because, in this region, β is also tiny.

So far we have analyzed the optimal behavior of agents for given (symmetric and steady state) asset prices. To complete the model, we need to incorporate the exogenous supply of assets in the analysis and treat ψ as an equilibrium object. Before we can proceed with this task, the following issue has to be handled: when T is plentiful, in a way that will be made precise below, $\psi = \psi^*$ and the cost of carrying home assets is zero. In this case, the optimal choices of $t_{_{HH}}, t_{_{HF}}, t^S_{_H}$ are not uniquely determined. There are many ways to break this indeterminacy and this choice does not affect our results. We assume that it is buyers who absorb the excess supply.[12] Hence, in all equilibria, $t^S_{_H} = 0$.

[11] An interesting point to notice is that in Case 3, for ψ in the neighborhood of ψ_c, the nature of the agent's optimal choice transforms fully. For $\psi = \psi_c - \epsilon$, $\epsilon > 0$, $t_{_{FF}} = 0$ and $t_{_{HF}} > 0$. But for $\psi = \psi_c + \epsilon$, $t_{_{HF}} = 0$ and the agent uses only foreign assets as a medium of exchange. Despite this dramatic change in the agent's portfolio composition, as ψ crosses the critical value ψ_c, the amount of special good purchased is continuous in ψ. To see this point, just set $\psi = \psi_c + \epsilon$ in (9) and $\psi = \psi_c - \epsilon$ in (10). It is easy to show that, as $\epsilon \to 0$, $q_{_F} \to q_{_{F,c}}$ in both cases.

[12] This result could arise endogenously, if we imposed a cost, $c > 0$, of participating in the asset markets. Since sellers get a zero payoff by holding any amount of assets in $\mathbb{R}_+$, they would choose to hold zero even for a tiny c. On the other hand, buyers want to hold strictly positive amounts of assets in order to trade in the DM. Since the cost is sunk, if $\psi = \psi^*$, they would be happy to hold any amount of assets that exceeds the amount used for trade.

For the reader's convenience, before we state the proposition that characterizes equilibrium, we repeat the definitions of some objects introduced above and we define a few new objects.[13]

$$\begin{aligned}
q^*_{F,1} &\equiv \left\{q : u'(q) = 1 + \frac{\kappa}{R}\frac{(1-\beta)(1-p_F)}{p_F}\right\} \\
q^*_{F,2} &\equiv \underline{q}\,(\psi^*) = \left\{q : u'(q) = \frac{R}{R-(1-\beta)\kappa}\right\} \\
q_{H,c} &\equiv \left\{q : u'(q) = \frac{R(1-2p_F)-\kappa(1-\beta)(1-p_F)}{p_H[R-\beta\kappa(1-p_F)]}\right\} \\
q_{F,c} &\equiv \left\{q : u'(q) = \frac{1-p_F}{p_F}\right\} \\
T^*_1 &\equiv \frac{(1-\beta)(q^*+q^*_{F,1})}{R} \\
T^*_2 &\equiv (1-\beta)\left(\frac{q^*}{R} + \frac{\underline{q}(\psi^*)}{R-\kappa(1-\beta)}\right) \\
T_c &\equiv \frac{1-2\beta(1-p_F)}{R-\kappa\beta(1-p_F)}(q_{H,c}+q_{F,c}) < T^*_2 \\
\psi_c &\equiv \frac{\beta(1-p_F)(2R-\kappa)}{1-2\beta(1-p_F)} \\
\tilde{\kappa} &\equiv \frac{(1-2p_F)}{(1-p_F)(1-\beta)}
\end{aligned}$$

Proposition 1. *For all parameter values, in equilibrium, $t_{FH}=0$. Moreover:*

a) If $(p_F,\beta)\in R_1$, or $(p_F,\beta)\in R_2$ and $\kappa/R \geq \tilde{\kappa}$, equilibrium is characterized by "local currency dominance" and agents trade in the foreign DM using only foreign assets. Hence, $t_{HF}=0$, $t_H=t_{HH}$, and $t_F=t_{FF}$. If $T\geq T^*_1$, then $\psi=\psi^*$, $q_H=q^*$, $q_F=q^*_{F,1}<q^*$, and $t_H=T-q^*_{F,1}(1-\beta)/R$. If $T<T^*_1$, then $\psi>\psi^*$, $q_H<q^*$, and $q_F<q^*_{F,1}$. For any T in this region, $\partial\psi/\partial T<0$, $\partial q_H/\partial T>0$, and $\partial q_F/\partial T>0$.

b) If $(\beta,p_F)\in R_4$, an "international currencies" equilibrium arises and agents use their home assets in all DM's, i.e. $t_{FF}=0$, so $t_F=0$ and $t_H=T$. If $T\geq T^*_2$, then $\psi=\psi^*$, $q_H=q^*$, and $q_F=\underline{q}(\psi^*)<q^*$. If $T<T^*_2$, then $\psi>\psi^*$, $q_H<q^*$, and $q_F<\underline{q}(\psi^*)$. For any T in this region, $\partial\psi/\partial T<0$, $\partial q_H/\partial T>0$, and $\partial q_F/\partial T>0$.

c) If $(\beta,p_F)\in R_3$, or $(\beta,p_F)\in R_2$ and $\kappa/R<\tilde{\kappa}$, a "mixed regime" equilibrium arises in the sense that either local currency dominance or international currencies could arise depending on T. If $T>T_c$, we are in the international currencies regime. For T's in this region, $\psi\in[\psi^*,\psi_c)$, $q_H\in(q_{H,c},q^*]$, and $q_F\in(q_{F,c},q^*_{F,2}]$. If $T<T_c$, we switch to a local currency dominance equilibrium and $\psi>\psi_c$, $q_H<q_{H,c}$, and $q_F<q_{F,c}$. In the knife-edge case where $T=T_c$, buyers purchase $q_F=q_{F,c}$ in the foreign DM using any combination of home and foreign assets.

[13] A comment on notation: when we write $q^*_{F,1}$, the asterisk refers to the fact that this is the highest value that q_F can reach and the number 1 refers to the case, in particular Case 1.

Proof. See Appendix A. □

A unique steady state equilibrium exists for all parameter values. Buyers never carry foreign assets in order to trade in their home DM. Under parameter values summarized as Case 1 in Lemma 6, an equilibrium with local currency dominance arises. Buyers choose to trade in the foreign DM using the foreign asset only, because the positive effect (high purchasing power) of holding foreign assets dominates the negative effect (liquidation cost). There exists a critical level T_1^* that captures the *liquidity needs* of the economy. For $T < T_1^*$, increasing T helps buyers purchase more special good in both DM's. Hence, the marginal valuation of one unit of the claim is higher than ψ^*, i.e. the price of the asset in a world where the Lucas-tree serves only as a store of value. Sometimes we refer to the distance $\psi - \psi^*$ as the *liquidity premium* because it reflects a premium in the valuation of the asset that stems from its second role (as a medium of exchange). When $T \geq T_1^*$, the asset's liquidity properties have been exploited (increasing T does not help buyers purchase more good) and $\psi = \psi^*$. When this is true, the cost of carrying the home asset is zero, so local buyers absorb all the excess supply, $t_H = T - q_{F,1}^*(1-\beta)/R$.

A similar analysis applies when parameter values are as in Case 2 in Lemma 6. We refer to this case as an international currencies equilibrium because agents use their home asset as means of payment everywhere in the world. The asset supply that captures the liquidity needs of the economy is given by T_2^*. When $T \geq T_2^*$, the asset price is down to its fundamental value and q_H, q_F have reached their upper bounds. Notice that in this case, buyers do not purchase any foreign assets in the CM, i.e. $t_F = 0$ and $t_H = T$. However, this does not mean that no agent ever holds any foreign assets. Sellers from country i, who got matched with foreign buyers, get payed with, and therefore hold some, asset j (which represents a home asset to the buyers). This will be important in the next section where we will compute asset home bias.

When parameter values are as in Case 3 in Lemma 6, we can end up with local currency dominance or international currencies equilibrium depending on T, which directly affects ψ. Equilibria with local currency dominance arise if and only if $T > T_c$. The intuition behind this result is as follows: when T is large, equilibrium prices are relatively low, which means that the cost of carrying the asset is relatively low. When the cost of carrying assets is low, the term κ becomes relatively more important. Recall from the discussion of Lemma 6 that it is precisely when κ is relatively high that agents choose to use the foreign asset in order to trade in the foreign DM (this becomes obvious when $(\beta, p_F) \in R_2$). On the other hand, if T is very small, the cost of carrying assets is large, so the term κ becomes less relevant. For intermediate values of (β, p_F) (remember we are in Case 3), this leads the buyers to optimally choose their home asset to trade in all DM's.

5 Home Bias and High Turnover

In this section we explore three predictions of our model that are supported in the data. These include the existence of asset home bias, the positive correlation between asset and consumption home bias, and the higher turnover rates of foreign over home assets. We characterize the properties of the local currency dominance equilibrium.[14] We focus on this case because the quantitative exercise in Section 6 confirms that developed countries' data suggest parameter values that support this very equilibrium. We describe the properties of the international currencies equilibrium in the accompanying Web Appendix.

5.1 Preliminaries

We begin this section by comparing the equilibrium values of q_H, q_F and t_H, t_F, which will lead us to our discussion of asset and consumption home bias as well as asset turnover rates. For given ψ, the first-order conditions (8) and (9) implicitly define q_H, q_F as functions of the probabilities p_H, p_F. We have

$$\begin{aligned} q_H &= G_H(p_H) \equiv \left\{ q : u'(q) = 1 + \frac{\psi - \beta(\psi + R)}{\beta p_H(\psi + R)} \right\}, \\ q_F &= G_F(p_F) \equiv \left\{ q : u'(q) = \frac{\psi - \beta(\psi + R_\kappa)(1 - p_F)}{\beta p_F(\psi + R)} \right\}. \end{aligned}$$

The following lemma summarizes some useful properties of these functions.

Lemma 7. *a) For $p_H = p_F = 0$, $G_H(p) = G_F(p) = 0$. For every $p_H = p_F = p \in (0, 1]$, $G_H(p) > G_F(p)$.*

b) For any $p_F \in (0, 1]$, define $\tilde{p}_H(p_F) \equiv \{p : G_H(p) = G_F(p_F)\}$ and notice that $\tilde{p}_H(p_F) < p_F$. Then, for any $p_F \in (0, 1]$, $p_H > \tilde{p}_H(p_F)$ implies $q_H > q_F$.

c) For any $p_F \in (0, 1]$, $p_H > \tilde{p}_H(p_F)$ implies $t_H > t_F$.

Proof. See Appendix A. □

The basic results of Lemma 7 are depicted in Figure 2 below. For any given $p_H = p_F = p > 0$, $G_H > G_F$. For any given p_F, we can find a critical $\tilde{p}_H(p_F)$, such that $p_H > \tilde{p}_H(p_F)$ implies $q_H > q_F$. In the local currency dominance regime, $t_H = t_{HH}$ and $t_F = t_{FF}$. Also, from bargaining, $q_H = \min\{(\psi + R)t_{HH}, q^*\}$ and $q_F = (\psi + R)t_{FF}$. Hence, if $p_H > \tilde{p}_H(p_F)$, not only $q_H > q_F$, but also $t_H > t_F$. In words, agents buy greater amounts of home versus foreign assets, even when the probability of trading in the local DM is smaller than the probability of trading in the foreign

[14] Hence, one of the following is true: i) $(p_F, \beta) \in R_1$, or ii) $(p_F, \beta) \in R_2$ and $\kappa/R \geq \tilde{\kappa}$, or iii) $T > T_c$ and either $(p_F, \beta) \in R_3$, or $(p_F, \beta) \in R_2$ and $\kappa/R < \tilde{\kappa}$.

DM (but not much smaller). Notice that we do not claim that $t_H > t_F$ is equivalent to asset home bias (although this turns out to be a sufficient condition), since we have not formally defined home bias yet. Also, notice that these results apply only to the $T < T_1^*$ case. If $T \geq T_1^*$, we know that $q_H = q^* > q_{F,1}^* = q_F$ regardless of p_H, p_F.

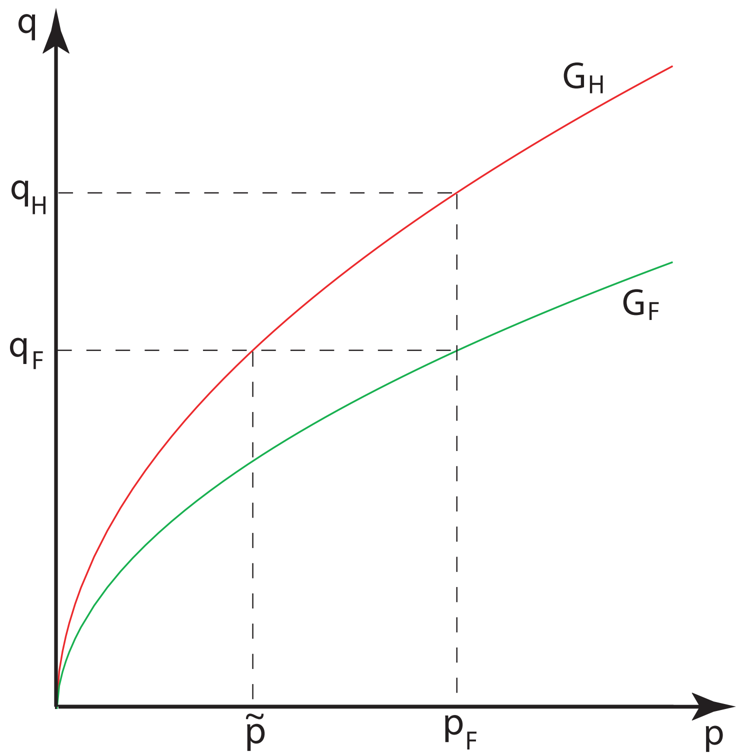

Figure 2: Trading Opportunities and Quantities Traded

5.2 Asset Home Bias

In this section, we argue that the model's predicted asset portfolio is biased toward domestic assets. The result is summarized in Proposition 2.

Proposition 2. *a) Assume that $T < T_1^*$. For any $p_F \in (0, 1]$, let $p_H > \tilde{p}_H(p_F)$. Then agents' portfolios exhibit home bias in the sense that the home asset's share in the entire portfolio is greater than fifty percent. Formally, the home asset share is*

$$HA = \frac{2t_H + p_F t_F}{2(t_H + t_F)} > 0.5. \tag{11}$$

b) Assume that $T \geq T_1^*$. Agents' portfolios exhibit home bias for any p_H, p_F. Formally,

$$HA = \frac{2\left(T - \frac{1-\beta}{R} q_{F,1}^*\right) + p_F \frac{1-\beta}{R} q_{F,1}^*}{2T} > 0.5. \tag{12}$$

Proof. See Appendix A. ☐

For $T < T_1^*$, as long as trading opportunities at home are not significantly less than trading opportunities abroad, a reasonable assumption for most countries engaged in international trade, the countries' portfolios will exhibit home bias. This follows directly from Lemma 7. The details of the derivation of the home asset bias formula can be found in Appendix B. To understand why the left-hand side of (11) represents the home asset's share in the agents' portfolio, focus on country i. The term $t_{_H} + t_{_F}$, in the denominator, stands for the total asset holdings. It is multiplied by 2 because we account for asset holdings in both the DM and the CM (we assume equal weight). The numerator represents the home asset holdings. The term $p_{_F} t_{_F}$ is the amount of asset i held by sellers who matched in the DM with foreign buyers who were carrying $t_{_F}$ units of asset i, which to them is foreign. The formula in (12) admits a similar interpretation.

Home bias represents an empirical regularity in cross-country data. We document this fact using cross-country data on international asset and liability positions provided by Lane and Milesi-Ferretti (2007). During the years 1997-2007, the average domestic asset share among 31 OECD countries was 76 percent (see Table 2 in Appendix C). To measure asset home bias, we employ a methodology similar to Collard, Dellas, Diba, and Stockman (2009). First, we compute international diversification as follows

$$\text{Int'l Dvrsf.} = \frac{\text{Foreign Portfolio Equity Assets}}{\text{Stock Mkt Cap + Foreign Portfolio Equity Assets - Foreign Portfolio Equity Liab.}}$$

Then, we capture asset home bias through the home asset share

$$\text{HA} = 1 - \text{Int'l Dvrsf.}$$

With this definition in mind, the countries in our sample exhibit asset home bias, since the average home asset share (HA) is well in excess of the fifty-percent benchmark that characterizes a fully-diversified portfolio in our model.

5.3 Consumption and Asset Home Bias

In this model, consumption and asset home bias coexist. Proposition 3 states the result.

Proposition 3. *Define $C_{_F}$, $C_{_T}$, and $C_{_H}$ as the value of foreign (or imported) consumption, total consumption, and consumption produced at home, respectively.*

a) Assume $T < T_1^*$. For any $p_{_F} \in (0, 1]$, let $p_{_H} > \tilde{p}_{_H}(p_{_F})$. Then $C_{_H} > C_{_F}$, implying $\frac{C_H}{C_T} > 0.5$.

b) Assume $T \geq T_1^*$. For any $p_{_H}, p_{_F}$, we have $\frac{C_H}{C_T} > 0.5$.

Proof. See Appendix B. □

To understand Proposition 3, notice that for each agent in the model, imported consumption consists of special goods bought from foreign sellers as well as fruit obtained from abroad when net claims to foreign trees are positive. On the contrary, domestic consumption includes special goods bought in domestic DM meetings as well as fruit obtained not only when net claims to the domestic tree are positive, but also via work. From Proposition 2, as long as trading opportunities abroad are not much higher than at home, the economy exhibits home bias in asset holdings. Since, at the country level, more domestic assets are held relative to foreign ones, net aggregate claims to foreign trees fall short of net claims to domestic trees. With respect to the DM, Lemma 7 ensures that the quantity exchanged (and consumed) in a domestic meeting exceeds the quantity exchanged in a foreign meeting. Since expenditure is increasing in the quantity purchased, the value of domestic consumption exceeds the value of imported consumption in the DM. Thus, for any non-negative work effort, domestic consumption exceeds imported consumption, ensuring that domestic goods' share of total consumption is well in excess of fifty percent.

The above discussion demonstrates that consumption and asset home bias are interrelated in the model. Moreover, Lemma 7 and Propositions 2 and 3 suggest that agents will buy more consumption goods from countries whose assets they hold in higher amounts. Thus, the model predicts that bilateral imports are positively correlated with bilateral investment positions.

To test this prediction, we obtain bilateral equity portfolio investment data for the year 2007 from the Coordinated Portfolio Investment Survey (CPIS) database provided by the World Bank. The advantage of this database is that it documents bilateral asset holdings for the 31 OECD countries considered earlier. The disadvantage, on the other hand, is that a number of positive bilateral observations are not made publicly available for security reasons. Thus, domestic asset shares cannot be computed, since missing observations will bias home asset shares upward. Consequently, we use the database to simply test the positive correlation between bilateral imports and investment positions. After dropping missing entries, we are left with 784 observations on bilateral asset holdings. Using stock market capitalization data for 2007, we compute bilateral investment shares.

We merge these data with bilateral import data for the same year, obtained from Stats.Oecd.[15] We compute bilateral import shares using GDP data for 2007 from the WDI. The correlation between the bilateral equity investment and import shares is 0.4186 and it is highly statistically significant. Moreover, a simple OLS regression of asset investment shares on import shares (and a constant term) yields a slope coefficient of 0.4309, with standard error of 0.0334. These

[15]We use the Grand Total Import statistic, which includes imports across all commodity categories. Bilateral import data for the service sector are not available. In any case, including service trade should not affect our results, since services are mostly non-tradable and they account for a tiny portion of overall trade.

findings provide strong evidence that consumption and asset home bias are positively related not only at the aggregate, but also at the bilateral level, much like predicted by the model.

5.4 Domestic and Foreign Asset Turnover Rates

The two predictions discussed above are in line with the existing literature (see Heathcote and Perri (2007), Collard, Dellas, Diba, and Stockman (2009) and Hnatkovska (2010)). However, the additional prediction that relates domestic and foreign asset turnover rates discussed below is unique to the present model.

Proposition 4. *Define the turnover rates of home and foreign assets as*

$$TR_H = \frac{3p_H t_H + 2p_F t_F}{2t_H + p_F t_F}, \qquad \forall T < T_1^*. \tag{13}$$

$$TR_H = \frac{\frac{1-\beta}{R}\left(3p_H q^* + 2p_F q^*_{F,1}\right)}{2\left(T - \frac{1-\beta}{R} q^*_{F,1}\right) + \frac{1-\beta}{R} p_F q^*_{F,1}}, \qquad \forall T \geq T_1^*. \tag{14}$$

$$TR_F = \frac{2p_F}{2 - p_F}, \qquad \forall T. \tag{15}$$

There exists a level of asset supply $\tilde{T} < \infty$, such that $T \geq \tilde{T}$ implies $TR_F > TR_H$.

Since the derivations of turnover rates are essential for the understanding of the Proposition, we present the proof in the main text.

Proof of Proposition 4. We define the turnover rate of an asset as the ratio of the total volume of that asset *traded* by citizens of a certain country (numerator) over the total volume of that asset *held* by the citizens of the same country (denominator).

a) Suppose $T < T_1^*$. TR_F is given by (15). The numerator reflects all the trades of the foreign asset carried out by the citizens of a certain country. A measure p_F of buyers match in the foreign DM and use t_F units of the foreign asset to buy the special good. These same buyers also purchase t_F units of the foreign asset in the CM in order to re-balance their portfolios. The denominator consists of the total holdings of the foreign asset, which amount to $(2-p_F)t_F$. This expression includes the holdings of buyers in both the CM and the DM, $2t_F$, net of the amount given up by buyers of measure p_F who matched abroad. Since the term t_F appears in both the numerator and the denominator, it cancels out, yielding the constant turnover rate in (15).

TR_H is given by (13). The numerator includes the amount of local assets buyers buy, $p_H t_H$, and sellers sell, $p_H t_H + p_F t_F$, in order to re-balance their portfolios in the CM after successfully matching in the DM. In addition, the numerator reflects the amount of local assets exchanged in the DM between sellers and domestic and foreign buyers, respectively, $p_{H}t_{H} + p_{F}t_{F}$. The denominator corresponds to total domestic assets held, which can be found in the numerator of the home-asset share expression in (11).

b) Suppose $T \geq T_1^*$. Clearly, TR_{F} is still given by (15), but TR_{H} differs. The logic of the calculations is identical. The main difference lies in the denominator since, with $T \geq T_1^*$, not all asset holdings are used as media of exchange in the DM. Following the same strategy as above and substituting asset holdings with the corresponding quantities of special goods exchanged obtains (14).

Clearly TR_{F} is constant, so it is unaffected by the asset supply. Moreover, it is easy to verify that TR_{H} is strictly decreasing in t_{H}. Since, for $T > T_1^*$, $t_{H} = T - T_1^*$, TR_{H} is decreasing in T, for all T, and $TR_{H} \to 0$ as $T \to \infty$. Therefore, there exists $\tilde{T} < \infty$, such that $T \geq \tilde{T}$ necessarily implies $TR_{F} > TR_{H}$. $\square$

Mechanically, when T exceeds T_1^*, q_{H} reaches the first-best q^*, so increasing T has no effect on the quantity of special good bought. Therefore, in this range, a higher T increases the home asset holdings of agents, but has no effect on the amount of home assets that changes hands during a period, leading to a relatively small turnover ratio for the home asset.

Intuitively, in the local currency dominance equilibrium, claims to the home Lucas-tree can serve both as store of value and as a medium of exchange. However, claims to the foreign tree are only valued for their services as media of exchange: agents do not hold these assets as a store of value (their rate of return is negative), but they do hold them in order to facilitate trade in the foreign DM. Therefore, when $T > T_1^*$, the cost of carrying the home asset is zero and agents keep the home asset as a store of value, which reduces its turnover rate. On the other hand, agents unload foreign assets (to foreign sellers) at the first given opportunity, which leads to a relatively higher turnover rate.

Empirically, Amadi and Bergin (2008) document that turnover rates of foreign assets are twice as high as those of home assets for four countries over a large period of time. The co-existence of high turnover rates of foreign assets and bias toward domestic assets in agents' portfolios has been a long-standing puzzle in the international finance literature. As the results in this section establish, the liquidity mechanism, coupled with the assumption of costly foreign asset trade, can reconcile these observations.[16]

[16] Empirically, foreign equity turns over faster than domestic equity. However, it is worth noting that if one were to compute the turnover rate of combined assets, including equity, bonds and money, it is likely that domestic assets will have higher turnover rates than foreign assets. Adding fiat currencies to the model and assuming that it is costly to convert foreign assets into home currency due to the standard exchange-rate spread would allow the model to capture the two observations. We abstract from fiat currency considerations because the valuation of fiat money is dependent on policy, which is beyond the scope of our paper.

6 Quantitative Analysis

To complete the analysis, we ask the following question: Can the model generate substantial asset home bias and foreign asset turnover rate when its parameters are calibrated to match the moments of a typical economy? In order to answer this question, we engage in a calibration exercise using data on the US economy. In particular, we choose the key parameters in the model in order to match the import-to-GDP ratio and the turnover rate of domestic assets in the US over the past decade. We then derive international asset portfolios and turnover rates of foreign assets predicted by the model, and we discuss how these moments relate to actual data.

6.1 Calibration

In order to proceed with the quantitative exercise, we choose functional forms that prevail in the modern monetary literature. We follow Lagos and Wright (2005) and let the utility function in the decentralized market be $u(q) = \log(q + b) - \log(b)$, which ensures that $u(0) = 0$, as necessary in order to obtain a solution to the bargaining problem. Further, we assume that the arrival rate of a buyer takes on the functional form $a_{_B} = (1 - \exp(\theta))\theta$, where θ was defined to be market tightness.

Next, we discuss a technical issue that arises in this adaptation of the Lagos and Wright (2005) model in a multi-country/international trade environment. Recall that all agents consume X^* in the CM. This parameter reflects the relative size of the CM compared to the DM.[17] Since, in our model, all the interesting economic activity takes place in the DM, we would like a relatively small X^*. However, if X^* is too small, agents who enter the CM with a lot of assets, for example sellers who just traded in the DM, might be induced to work negative hours.

To deal with this issue, in the quantitative exercise we assume that $U(X) = X$. Linear utility is a sub-case of the quasi-linearity assumed throughout the paper. Therefore, all the nice properties of the value functions described earlier will continue to hold. Moreover, with linear utility, agents are indifferent between working many hours and consuming more, or working few hours and consuming less. Although there is a continuum of consumption and labor choices that are all optimal for the agent, different choices lead to different values of GDP, which is essential for accounting. We take advantage of this indeterminacy and assume that the agent always picks the pair of (optimal) X, H that involves the minimum amount of hours worked. This guarantees that the size of the CM sector is the minimum possible.[18]

[17] More precisely, a larger X^* implies that, for given asset holdings, agents will need to work more in the CM in order to be able to consume their desired amount of general good. Suppose we are interested in the percentage of total production that takes place in the CM. A higher X^* implies that the CM sector is relatively larger, since all the hours agents work in order to reach X^* contribute to the CM production.

[18] To see this point, consider an agent who enters the CM with assets whose value, in terms of the numéraire good, is y. From the no-wealth-effect property of W, we know that this agent will choose an amount of assets

With the functional forms in mind, we need to choose values for the following parameters $(b, \beta, R, T, \xi, \sigma_H, \sigma_F, \kappa)$. Notice a few points. First, as in any Lucas-tree model, the product RT, which represents the total dividend in a country, determines all equilibrium real variables. Thus, we need not take a stand on each one of these parameters individually. Consequently, we fix $R = 1$, and calibrate T below. Second, below we will be choosing a parameter value for σ_F in order to match observed US import-to-GDP ratio. Since this statistic has been roughly fifteen percent over the past decade, it is reasonable to expect that, in any equilibrium, σ_F will never exceed σ_H. Thus, we normalize σ_H to unity and calibrate σ_F relative to σ_H. If σ_F is zero, then a country is in complete autarky. Since σ_F is a probability, the maximum value it can take on is one. Under these restrictions on the parameters, the measure of active buyers in a country can vary between one and two. Recall that the measure of sellers is fixed at ξ. Then, a rise in σ_F, which is interpreted as trade liberalization, will result in an increase in the measure of buyers in a country. If ξ is relatively small, then domestic buyers can be crowded out and left unmatched, which will decrease the welfare of such agents. In order to avoid this situation, we set ξ equal to two, which is the maximum measure of buyers that can arise in the model. In this case, we ensure that opening up to trade is an optimal policy.

Following Lagos and Wright (2005), we let $b = 0.0001$, which is positive, but small enough to minimize deviations from the log utility function. We let $\beta = 0.97$, since we will work with annual data. Finally, in order to illustrate the fact that the results in the paper are preserved for arbitrarily small values of the cost to liquidate foreign assets, we set κ to 0.0001.

We are left with the following two parameters: σ_F and T. These two parameters are the most important ones in the model since they determine the type of equilibrium that arises. σ_F governs whether the equilibrium will be of the local-currency-dominance type, in which case local assets circulate as local media of exchange, or of the international-currencies type, where agents use their respective domestic asset to trade everywhere. Moreover, conditional on being in a particular type of equilibrium, the value of T determines whether the economy is liquidity constrained or not, which further determines equilibrium prices and allocations. Consequently, we let the US data tell us what the values of these two parameters should be. In particular, we use the model's equations and the parameters discussed in previous paragraphs in order to find the unique values for σ_F and T, so that the model matches the following two moments in the US data: import-to-GDP ratio and turnover rate of domestic assets.[19]

---

to carry into the next period's DM independently of the value of y. For simplicity, let the market value of assets carried into the next period be z. Because of linear utility, if $y > z$, the agent is equally happy between: (a) consuming $y - z$ units of the CM good and working zero hours or (b) consuming $y - z + \alpha$, for any $\alpha \in \mathbb{R}_+$, and working α hours. We assume that $X = y - z$ and $H = 0$. If $z > y$, in a similar fashion we have $X = 0$ and $H = z - y$.

[19] In the model, all imports are consumed; therefore, imported consumption is equivalent to total imports. Moreover, GDP is straightforward to compute using the production approach. Thus, we follow Collard, Dellas, Diba, and Stockman (2009) and we approximate consumption home bias by the import-to-GDP ratio. In the data, imports include both consumption and intermediate goods. So, in order to directly measure consumption home

The mean import-to-GDP ratio of the US over the 1997-2007 period was 0.1475.[20] According to Table 1 in Amadi and Bergin (2008), the mean domestic asset turnover rate in the US during the 1993-2001 period was 0.73. We derive imports and GDP for the model in detail in Appendix B. The next section discusses the results of the calibration exercise. Table 1 below summarizes the calibration results.

Table 1: Parameter Values and US Data Moments, 1993-2007

Parameter	Moment	Source
N=2	# countries in sample	-
b=0.0001	-	Lagos and Wright (2005)
R=1	normalization	-
σ_H=1	normalization	-
ξ=2	ensures trade is welfare-improving	-
κ=0.0001	arbitrarily small	-
β=0.97	annual real interest rate = 3%	-
T=0.0853	domestic asset turnover rate = 0.73	Amadi and Bergin (2008)
σ_F=0.9671	Import/GDP = 0.1475	WDI

6.2 Quantitative Results

The calibrated model suggests a value for σ_F of 0.9671. Using the definitions of the probability of a match abroad p_F and market tightness θ, as well as the functional form for the arrival rate of the buyer, we obtain $p_F = \sigma_F[1-\exp(\xi/(\sigma_H+\sigma_F))][\xi/(\sigma_H+\sigma_F)]$. Substituting in the parameter values yields a value for p_F of 0.6156. According to Lemma 6, this parameter value suggests that the model's equilibrium is of the local-currency-dominance type. Thus, domestic assets serve as media of exchange at home only, which is consistent with the fact that most countries in the world constitute independent currency areas.

Given that we are in Case 1 of Lemma 6, the properties of the equilibrium of the model are summarized under bullet $a)$ in Proposition 1. In particular, if asset supply is above the critical level T_1^*, the economy is not liquidity constrained. This means that the asset price falls to the fundamental value and quantities reach their upper bounds. The quantity exchanged in a local meeting is given by q^* and the quantity exchanged in a foreign meeting is even lower and equals $q^*_{F,1}$, both of which are described in Definition 2 above. Using the parameter values discussed above, the critical asset supply value equals 0.06. The calibrated model suggests that the

bias, one would need to obtain disaggregate US import data and categorize goods into consumption and non-consumption goods. Then one would need to combine these data with total US consumption data in order to compute the share of consumption that is imported. We opt to use import-to-GDP ratios instead, purely in order to make our quantitative results comparable to the existing literature.

[20] Data are from WDI.

level of asset supply necessary in order to match the domestic asset turnover rate in the US is 0.0853. Hence, according to the model, over the past decade the US economy was not liquidity constrained.

Then, we can compute the asset portfolio and the foreign asset turnover rate predicted by the model in the unconstrained, local currency dominance equilibrium. Maintaining the notation corresponding to this type of equilibrium, the domestic asset share of a country's total portfolio is given by

$$HA_1^* = \frac{2T + \frac{1-\beta}{R}(p_F - 2)q_{F,1}^*}{2T}.$$

Applying this formula, the calibrated model yields a home asset share of seventy-six percent. The corresponding statistic for the US data over the 1997-2007 period is eighty-five percent. Thus, the model generates a substantial amount of home bias.

Furthermore, recall that the foreign asset turnover rate for this type of equilibrium is $2p_F/(2-p_F)$. Using the calibrated parameters, the model suggests that the foreign asset turnover rate is 0.8893. This rate is 1.22 times higher than the domestic asset turnover rate. The intuition for this result lies in the liquidity mechanism introduced in this paper. In equilibrium, domestic assets are held both for the purpose of future consumption and trade in domestic decentralized markets. Foreign assets, on the other hand, yield lower rates of return and hence they do not represent a useful saving technology. Thus, buyers acquire them purely for the purpose of trading abroad in the future and, if unlucky in the matching process, they use the first opportunity to liquidate them in the upcoming centralized market.

Quantitatively, the ratio of foreign to domestic asset turnover falls short of the one observed in the US data. Yet, the model makes a successful step toward reconciling the behavior observed in the data, as it is the first to generate a higher turnover rate of foreign over domestic assets, while maintaining a strong quantitative link between consumption and asset home bias.

7 Conclusion

In this paper, we study optimal asset portfolio choice in a two-country search-theoretic model of monetary exchange. We allow assets to not only represent claims on future consumption, but to also serve as media of exchange. In the model, trading in a certain country involves the exchange of locally produced goods for a portfolio of assets. Assuming foreign assets trade at a cost, we provide sufficient conditions for the existence of currency-area and international-currencies equilibria.

We further argue that the novel liquidity mechanism can help rationalize the existence of asset home bias, which is a well-known empirical regularity in the data. According to the model, more frequent trading opportunities at home result in agents holding proportionately more domestic over foreign assets. As international trade becomes more integrated, agents demand higher amounts of foreign assets. Moreover, foreign assets turn over faster than home assets because the former have desirable liquidity properties, but unfavorable returns over time. Thus, our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

The liquidity mechanism is fairly successful at replicating the US data over the past decade. When the model's parameters are calibrated to match the observed import-to-GDP ratio and the domestic asset turnover rate of the US economy, the model generates a domestic asset portfolio share of seventy-six percent. Moreover, the predicted foreign asset turnover rate is 1.22 times higher than its domestic counterpart. This result is unique to the model presented in this paper and is due to the novel liquidity mechanism we introduce to international economics.

Finally, the model we propose abstracts away from risk diversification considerations and rather explores liquidity properties of assets. As such, it is complementary to the large literature on home bias and can be incorporated into existing frameworks in order to further study asset and consumption home bias, both qualitatively and quantitatively.

References

AMADI, A. A., AND P. R. BERGIN (2008): "Understanding international portfolio diversification and turnover rates," *Journal of International Financial Markets, Institutions and Money*, 18(2), 191–206.

CAMERA, G., AND J. WINKLER (2003): "International monetary trade and the law of one price," *Journal of Monetary Economics*, 50(7), 1531 – 1553.

COLLARD, F., H. DELLAS, B. DIBA, AND A. STOCKMAN (2009): "Goods Trade and International Equity Portfolios," *The University of Adelaide School of Economics Working Paper*, (14).

GEROMICHALOS, A., J. M. LICARI, AND J. SUAREZ-LLEDO (2007): "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10(4), 761–779.

HEAD, A., AND S. SHI (2003): "A fundamental theory of exchange rates and direct currency trades," *Journal of Monetary Economics*, 50(7), 1555 – 1591.

HEATHCOTE, J., AND F. PERRI (2007): "The International Diversification Puzzle Is Not As Bad As You Think," Working Paper 13483, National Bureau of Economic Research.

HNATKOVSKA, V. (2010): "Home bias and high turnover: Dynamic portfolio choice with incomplete markets," *Journal of International Economics*, 80(1), 113–128.

KOCHERLAKOTA, N. R. (1998): "Money Is Memory," *Journal of Economic Theory*, 81(2), 232–251.

LAGOS, R. (2010): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57(8), 913–930.

LAGOS, R., AND G. ROCHETEAU (2008): "Money and capital as competing media of exchange," *Journal of Economic Theory*, 142(1), 247–258.

LAGOS, R., AND R. WRIGHT (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.

LANE, P. R., AND G. M. MILESI-FERRETTI (2007): "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004," *Journal of International Economics*, 73(2), 223–250.

LESTER, B., A. POSTLEWAITE, AND R. WRIGHT (2008): "Information, Liquidity and Asset Prices," PIER Working Paper Archive 08-039, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.

LEWIS, K. K. (1999): "Trying to Explain Home Bias in Equities and Consumption," *Journal of Economic Literature*, 37(2), 571–608.

LUCAS, ROBERT E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–45.

MATSUYAMA, K., N. KIYOTAKI, AND A. MATSUI (1993): "Toward a Theory of International Currency," *Review of Economic Studies*, 60(2), 283–307.

MICHAELIDES, A. (2003): "International portfolio choice, liquidity constraints and the home equity bias puzzle," *Journal of Economic Dynamics and Control*, 28(3), 555–594.

WRIGHT, R., AND A. TREJOS (2001): "International Currency," *The B.E. Journal of Macroeconomics*, 0(1).

A Theory Appendix

Proof of Lemma 2. The liquidation cost κ creates a wedge in the valuation of asset i between the seller (citizen of country i) and the buyer (citizen of j). A necessary condition for a positive amount of asset j to change hands is that all units of asset i have already been traded. Hence, the solution to the bargaining problem follows directly from answering the following two questions: 1) If the buyer carried unlimited amounts of asset i (which is foreign to him but local to the seller), what would be the first-best level of q to be traded? 2) If the buyer carried zero units of asset i, what would be the first-best level of q to be traded? It can be easily verified that the answer to the first question is given by $\overline{q}(\psi) \equiv \{q : u'(q) = (\psi + R_\kappa)/(\psi + R)\}$, which is clearly larger than q^*. Similarly, the answer to the second question is given by $\underline{q}(\psi) \equiv \{q : u'(q) = (\psi + R)/(\psi + R_\kappa)\} < q^*$.

Given these observations, the suggested solution follows naturally. When $t_{ii} \geq \underline{q}(\psi_j)/(\psi_i + R)$, asset j never changes hands (cases a and b). If $t_{ii} \geq \overline{q}(\psi_j)/(\psi_i + R)q_i$ (case a), q_i is equal to the first best. However, if $t_{ii} \in \left[\underline{q}(\psi_j)/(\psi_i + R), \overline{q}(\psi_i)/(\psi_i + R)\right)$ (case b), q_i is bounded by the real value of the buyer's asset-i holdings, i.e. $t_{ii}(\psi_i + R)$. If $t_{ii} < \underline{q}(\psi_j)/(\psi_i + R)$ (case c), some asset j is traded as well. If the total real balances are such that $\pi_i \geq \underline{q}(\psi_j)$ (case c1), then the buyer gives up all her asset i and enough asset j so that the quantity $\underline{q}(\psi_j)$ can be purchased. On the other hand, if $\pi_i < \underline{q}(\psi_j)$ (case c2), the buyer gives up all her asset holdings and purchases $q_i = \pi_i = (\psi_i + R)t_{ii} + (\psi_j + R_\kappa)t_{ji}$. □

Proof of Lemma 5. To describe the optimal behavior of the buyer with respect to asset holdings to be traded in her local DM, we focus on the symmetric, steady-state version of (6). Pick any (t^0_{HH}, t^0_{FH}), with $t^0_{FH} > 0$ and define $q^0 \equiv \{q : (\psi + R)t^0_{HH} + (\psi + R_\kappa)t^0_{FH}$, i.e. the quantity of the special good that asset holdings (t^0_{HH}, t^0_{FH}) can buy. The buyer under consideration can purchase q^0 by setting $t_{FH} = 0$ and increasing her domestic asset holdings to $t^1_{HH} = q^0/(\psi + R)$. We claim that the buyer always achieves a higher value if she purchases (the arbitrarily chosen) q^0 with asset holdings $(t^1_{HH}, 0)$ rather than (t^0_{HH}, t^0_{FH}). To see this point notice that

$$\begin{aligned} V^B_H(t^1_{HH}, 0) &= \left[-\psi + \beta\left(\psi + R\right)\right] t^1_{HH} + \beta p_H[u(q^0) - q^0], \\ V^B_H(t^0_{HH}, t^0_{FH}) &= \left[-\psi + \beta\left(\psi + R\right)\right] t^0_{HH} + \left[-\psi + \beta\left(\psi + R_\kappa\right)\right] t^0_{FH} + \beta p_H[u(q^0) - q^0]. \end{aligned}$$

After some algebra, one can show that $V^B_H(t^1_{HH}, 0) > V^B_H(t^0_{HH}, t^0_{FH})$ will be true if and only if $t^1_{HH} < t^0_{HH} + t^0_{FH}$. Multiply the last expression by $\psi + R$ and add and subtract the term κt^0_{FH} on the right-hand side. Then, one can conclude that $V^B_H(t^1_{HH}, 0) > V^B_H(t^0_{HH}, t^0_{FH})$ is true if and only if $\kappa t^0_{FH} > 0$, which is true by assumption. Thus, the optimal strategy for the buyer is to purchase any desired q in the local DM by carrying only the domestic asset. When $\psi = \psi^*$, the cost of carrying home assets is zero, hence the buyer can carry any amount of assets bigger than the amount that buys her the first-best quantity q^*. $\square$

Proof of Lemma 6. The description of the optimal choice of $t_{_{HF}}$ is more complex than the one of $t_{_{FF}}$, for the following reason. If the buyer decides to not use foreign assets as media of exchange, then we know that $t_{_{FF}} = 0$. This is true because the cost of holding foreign assets is always strictly positive. However, with home assets, if $\psi = \psi^*$, many different choices of $t_{_{HF}}$ are optimal. Hence, the buyer might want to hold some home assets even if she is not using them to carry out transactions in the DM. To avoid these complications, first we focus on the case where $\psi > \psi^*$. Then, once we have established which assets serve as means of payments in the various cases, we let $\psi = \psi^*$ and conclude the description of the optimal choice of home assets.

Let us start by re-writing the symmetric, steady-state version of the buyer's relevant objective function, i.e. equation (7). We have

$$\begin{aligned} V_F^B(t_{_{HF}}, t_{_{FF}}) &= [-\psi + \beta(\psi + R)]\, t_{_{HF}} + [-\psi + \beta(\psi + R_\kappa)]\, t_{_{FF}} \\ &+ \beta p_{_F}\left[u\left(q_{_F}(t_{_{HF}}, t_{_{FF}})\right) - (\psi + R)\, x_{_H}(t_{_{HF}}, t_{_{FF}}) - (\psi + R_\kappa)\, x_{_F}(t_{_{HF}}, t_{_{FF}})\right], \end{aligned} \tag{a.1}$$

where $x_{_H}$ denotes the amount of domestic assets (with respect to the buyer's citizenship) and $x_{_F}$ the amount of foreign assets that change hands in a DM meeting in the foreign country. Since different asset holdings $t_{_{HF}}, t_{_{FF}}$ lead to different expressions for the terms $q_{_F}, x_{_H}$ and $x_{_F}$ (determined in Lemma 2), it is of no use to take first-order conditions in (a.1). Instead, we look into different combinations of $t_{_{HF}}, t_{_{FF}}$ holdings. We start by ruling out several regions of asset holdings as strictly dominated. This allows us to narrow down the set of possibilities and eventually take first-order conditions in the remaining relevant regions. Figure 3 depicts the regions described in this proof. The proof proceeds in several steps.

Step 1: The optimal $t_{_{HF}}, t_{_{FF}}$ can never be such that $t_{_{FF}} > \overline{q}(\psi)/(\psi + R)$. This represents region A in Figure 3 and case (a) in Lemma 2. To see why this claim is true, just notice that for any $t_{_{HF}}, t_{_{FF}}$ in this region the buyer is already buying the highest possible quantity of special good, namely $\overline{q}(\psi)$. Hence, increasing $t_{_{FF}}$ has a strictly positive cost and no benefit.

Step 2: The optimal $t_{_{HF}}, t_{_{FF}}$ can never be such that $t_{_{FF}} \in (\underline{q}(\psi)/(\psi + R), \overline{q}(\psi)/(\psi + R)]$ and $t_{_{HF}} > 0$. This represents region B, excluding the vertical axis, in Figure 3 and case (b) in Lemma 2. To see why the claim is true, recall from Lemma 2 that for this region of asset holdings $x_{_H} = 0$. Therefore, there is no benefit from carrying the home asset (but there is a cost). Hence, in this region the objective function becomes

$$V_F^B(t_{_{HF}}, t_{_{FF}}) = -\beta\kappa t_{_{FF}} + \beta p_{_F}\left[u\left((\psi + R)t_{_{FF}}\right) - (\psi + R_\kappa)\, t_{_{FF}}\right]. \tag{a.2}$$

The optimal $t_{_{FF}}$ is determined by taking the first-order condition in (a.2).

Step 3: The optimal $t_{_{HF}}, t_{_{FF}}$ cannot lie in the interior of region $C1$ in Figure 3. This region

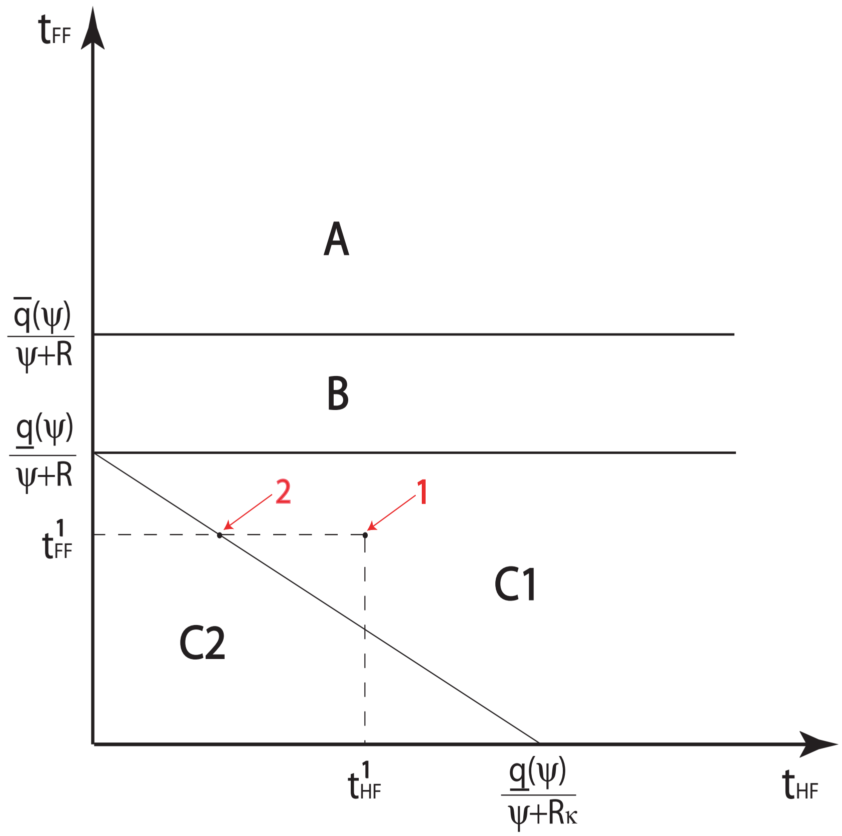

Figure 3: Regions

stands for case (c1) in Lemma 2. To see why the claim is true, consider any point in the interior of region $C1$, such as point 1 with coordinates (t^1_{HF}, t^1_{FF}). The objective function for the buyer, evaluated at point 1, is

$$\begin{aligned} V^B_F(t^1_{HF}, t^1_{FF}) &= [-\psi + \beta(\psi + R)]\, t^1_{HF} + [-\psi + \beta(\psi + R_\kappa)]\, t^1_{FF} \\ &+ \beta p_F \left[u\left(\underline{q}(\psi)\right) - (\psi + R)\left(\frac{\underline{q}(\psi) - (\psi + R)}{\psi + R_\kappa}\right) - (\psi + R_\kappa)\, t^1_{FF}) \right]. \end{aligned} \tag{a.3}$$

Now consider the following experiment. Leave t_{FF} unchanged, but reduce the t_{HF} holdings so that the buyer can still purchase $\underline{q}(\psi)$. This is represented by point 2 in the Figure, with coordinates $(t^2_{HF}, t^2_{FF}) = \left((\underline{q}(\psi) - (\psi + R)t^1_{FF})/(\psi + R_\kappa), t^1_{FF}\right)$. The objective function evaluated at point 2 is

$$\begin{aligned} V^B_F(t^2_{HF}, t^2_{FF}) &= [-\psi + \beta(\psi + R)]\, \frac{\underline{q}(\psi) - (\psi + R)t^1_{FF}}{\psi + R_\kappa} + [-\psi + \beta(\psi + R_\kappa)]\, t^1_{FF} \\ &+ \beta p_F \left[u\left(\underline{q}(\psi)\right) - (\psi + R)\left(\frac{\underline{q}(\psi) - (\psi + R)}{\psi + R_\kappa}\right) - (\psi + R_\kappa)\, t^1_{FF}) \right]. \end{aligned} \tag{a.4}$$

It follows from (a.3) and (a.4) that

$$V_F^B(t_{HF}^2, t_{FF}^2) - V_F^B(t_{HF}^1, t_{FF}^1) = [-\psi + \beta(\psi + R)]\left(t_{HF}^2 - t_{HF}^1\right),$$

which is strictly positive by Lemma 3 and the fact that $t_{HF}^2 < t_{HF}^1$. Hence, the agent will never choose a point (t_{HF}, t_{FF}) in the interior of region $C1$.

Step 4: Finally consider the region of t_{HF}, t_{FF} represented by region $C2$, which is equivalent to case (c2) of Lemma 2. The objective function in this region becomes

$$\begin{aligned} V_F^B(t_{HF}, t_{FF}) &= [-\psi + \beta(\psi + R)(1 - p_F)]\, t_{HF} + [-\psi + \beta(\psi + R_\kappa)(1 - p_F)]\, t_{FF} \\ &+ \beta p_F u\left((\psi + R_\kappa)t_{HF} + (\psi + R)t_{FF}\right) \\ &= -a_1 t_{HF} - a_2 t_{FF} + a_3 u\left(a_4 t_{HF} + a_5 t_{FF}\right), \end{aligned} \tag{a.5}$$

where we have defined

$$a_1 \equiv \psi - \beta(\psi + R)(1 - p_F),\ a_2 \equiv \psi - \beta(\psi + R_\kappa)(1 - p_F)$$
$$a_3 \equiv \beta p_F,\ a_4 \equiv \psi + R_\kappa,\ a_5 \equiv \psi + R.$$

Notice that $a_i > 0$ for all i, $a_1 < a_2$, and $a_4 < a_5$. Taking first order-conditions in (a.5) implies

$$-a_1 + a_3 u'\left(a_4 t_{HF} + a_5 t_{FF}\right) a_4 \leq 0,\ = 0 \textit{ if } t_{HF} > 0, \tag{a.6}$$
$$-a_2 + a_3 u'\left(a_4 t_{HF} + a_5 t_{FF}\right) a_5 \leq 0,\ = 0 \textit{ if } t_{FF} > 0. \tag{a.7}$$

Hence, the optimal choice of t_{HF}, t_{FF} is a corner solution, with the exception of the knife-edge case in which $a_2/a_1 = a_5/a_4$. If $a_2/a_1 > a_5/a_4$, then $t_{HF} > 0$ and $t_{FF} = 0$. On the other hand, if $a_2/a_1 < a_5/a_4$, then $t_{HF} = 0$ and $t_{FF} > 0$.

Using the definitions of the a_i terms, one can show that $a_2/a_1 < a_5/a_4$ if and only if

$$\beta(1 - p_F)(2R - \kappa) < \psi\left[1 - 2\beta(1 - p_F)\right]. \tag{a.8}$$

First, notice that if $\beta \geq 1/(2(1 - p_F))$ (region R_4 in Figure 1), the condition in (a.8) can never hold, since the left-hand side is positive. In this case, $t_{FF} = 0$ and the value of t_{HF} is given by (a.6), which after replacing for the a_i terms is equivalent to (10).

From now on let $\beta < 1/(2(1 - p_F))$. The condition in (a.8) becomes

$$\psi > \frac{\beta(1 - p_F)(2R - \kappa)}{1 - 2\beta(1 - p_F)} \equiv \psi_c. \tag{a.9}$$

Using the definition of ψ_c above and recalling the definition of the "fundamental value" of the asset, $\psi^* \equiv \beta R/(1-\beta)$, one can show that $\psi^* > \psi_c$ if and only if

$$\kappa(1-\beta)(1-p_{_F}) > R(1-2p_{_F}). \tag{a.10}$$

After some algebra, it turns out that the inequality in (a.10) holds if either: i) $p_{_F} \geq 1/2$ (region R_1 in Figure 1) or ii) $p_{_F} < 1/2$, $\beta < p_{_F}/(1-p_{_F})$ and $\kappa/R > (1-2p_{_F})[(1-\beta)(1-p_{_F})]^{-1} \equiv \tilde{\kappa}$ (the values of β and $p_{_F}$ described here are represented by region R_2 in Figure 1). However, Lemma 3 indicates that any equilibrium price satisfies $\psi \geq \psi^*$. Therefore, if $\psi^* > \psi_c$, the inequality in (a.9) is always true. In this case, $t_{_{HF}} = 0$ and the value of $t_{_{FF}}$ is given by (a.7). After replacing for the a_i terms, this is equivalent to (9).

We are left with the case in which either $\beta, p_{_F}$ fall in region R_3 of Figure 1 or they fall in region R_2 and also $\kappa/R \leq \tilde{\kappa}$. Here, $\psi^* \leq \psi_c$, and there will exist equilibrium prices that can be larger or smaller than ψ_c (this will be determined later when we introduce the supply of the assets, T). For ψ's that are large enough, so that (a.9) holds, the optimal choice of the agent satisfies $t_{_{HF}} = 0$, $t_{_{FF}} > 0$. For $\psi \in [\psi^*, \psi_c)$, we have $t_{_{HF}} > 0$, $t_{_{FF}} = 0$. In the knife-edge case where $\psi = \psi_c$, we have $t_{_{HF}}, t_{_{FF}} > 0$, and both (a.6), (a.7) hold with equality. The choices of $t_{_{HF}}, t_{_{FF}}$ cannot be separately pinned down, but they are such that the quantity bought in the DM satisfies $u'(q_{_F}) = p_{_F}/(1-p_{_F})$.

Step 5: In this step we establish the statements of Lemma 6 for $\psi > \psi^*$ (which we have assumed so far). The case in which $\psi = \psi^*$ is discussed in Step 6. Let us summarize the results derived so far. From Step 1, we know that the agent will never choose $t_{_{HF}}, t_{_{FF}}$ in region A. From Step 2, we know that if the agent wishes to purchase $q_{_F} > \underline{q}(\psi)$, she will do so by using $t_{_{FF}}$ only. From Step 3, we know that points in the interior of region $C1$ are dominated by asset holdings that purchase the same quantity, i.e. $\underline{q}(\psi)$, using fewer home assets. Finally, from Step 4, we know that if the agent wishes to purchase $q_{_F} \leq \underline{q}(\psi)$, she will do so by using either $t_{_{FF}}$ or $t_{_{HF}}$, but not both, as a medium of exchange.

<u>Case 1:</u> If parameter values are as in Case 1 of the lemma, the agent uses only $t_{_{FF}}$ as a medium of exchange. Hence, the objective function is given by (a.2), and the optimal choice of $t_{_{FF}}$ is described by (9).

<u>Case 2:</u> Next, consider parameter values as in Case 2 of the lemma. This case is less straightforward than Case 1. We have shown (Step 4) that if the agent wishes to purchase some $q_{_F} \leq \underline{q}(\psi)$, she is better off by setting $t_{_{FF}} = 0$. But from Step 1 we also know that if $q_{_F} > \underline{q}(\psi)$ the agent is better of using the foreign asset as a medium of exchange. To establish Case 2 of the lemma, we need to exclude the latter possibility. Suppose, by way of contradiction, that the agent wants to purchase $q_{_F} > \underline{q}(\psi)$ and, therefore, chooses $t_{_{FF}} > 0$. Under the contradictory assumption, the quantity of special good purchased is

$$q_{_F}(\psi) \equiv \Big\{ q : \psi = \beta \left[(1-p_{_F})(\psi + R_\kappa) + p_{_F}(\psi + R)\, u'(q) \right] \Big\}. \tag{a.11}$$

Notice three important facts. First, $q_F(\psi)$ is strictly decreasing in ψ. Second, $\underline{q}(\psi)$ is strictly increasing in ψ. Third, we claim that $q_F(\psi^*) < \underline{q}(\psi^*)$. To see why this is true, observe that

$$\begin{aligned} u'(\underline{q}(\psi^*)) &= \frac{R}{R-\kappa(1-\beta)}, \\ u'(q_F(\psi^*)) &= 1+\frac{\kappa(1-p_F)(1-\beta)}{Rp_F}. \end{aligned}$$

Our claim that $q_F(\psi^*) < \underline{q}(\psi^*)$ will be true if and only if $u'(q_F(\psi^*)) > u'\left(\underline{q}(\psi^*)\right)$, which is true if and only if $1+\kappa(1-p_F)(1-\beta)/(Rp_F) > R/[R-\kappa(1-\beta)]$, which after some more manipulations can be written as

$$\frac{\kappa}{R} < \frac{(1-2p_F)}{(1-p_F)(1-\beta)}. \tag{a.12}$$

Since here $\beta \geq p_F/(1-p_F)$, the right-hand side of (a.12) is greater than or equal to 1. Hence, the inequality in (a.12) is always satisfied. Combining these three facts implies that $q_F(\psi) < \underline{q}(\psi)$ for all ψ, which is a straightforward contradiction to our assumption that the agent purchases $q_F > \underline{q}(\psi)$ and, therefore, chooses $t_{FF} > 0$. Hence, the agent chooses to use only the home asset as a medium of exchange, and (given that $\psi > \psi^*$) the optimal t_{HF} solves (10).

Case 3: Finally, consider parameters such as in Case 3 of the lemma. We know that if the agent wishes to purchase $q_F \leq \underline{q}(\psi)$, we have $t_{FF} > 0$ if and only if $\psi \geq \psi_c$. Using an argument identical to the one we used in Case 2, one can show that the agent will never wish to purchase $q_F > \underline{q}(\psi)$.[21] Thus, the analysis in Step 4 fully characterizes the optimal choice of the agent, and Case 3 of the lemma follows directly.

Step 6: We only need to conclude the description of the optimal choice of t_{HF} when $\psi = \psi^*$. In Case 1, the home asset is not used as a medium of exchange. If $\psi = \psi^*$, the cost of carrying the home asset is zero. Hence, any $t_{HF} \in \mathbb{R}_+$ is optimal. In Case 2, the buyer uses the home asset as a means of payment. Thus, with $\psi = \psi^*$, she is willing to carry any amount of t_{HF} that allows her to buy the q_F which maximizes the buyer's surplus in the foreign DM. This quantity is given by $\underline{q}(\psi^*)$, implying that any $t_{HF} \geq \underline{q}(\psi^*)/(R/(1-\beta)-\kappa)$ is optimal. Finally, in Case 3, $\psi = \psi^*$ means that $\psi < \psi_c$. Therefore, the optimal behavior of the buyer coincides with Case 2 described above. □

Proof of Proposition 1. The fact that $t_{FH} = 0$ follows immediately from Lemma 5. The proofs for parts (a), (b), and (c) are similar. Hence, we only prove part (a) in detail.

The fact that agents use only foreign assets in order to trade in the foreign DM follows from

[21] This follows directly from (a.12), since the term on the right-hand side is just $\tilde{\kappa}$.

Lemma 6. Moreover, applying total differentiation in (8) and (9) yields

$$\frac{\partial q_{H}}{\partial \psi} = \frac{R}{\beta p_{H}(\psi + R)^2 u''(q_{H})} < 0,$$
$$\frac{\partial q_{F}}{\partial \psi} = \frac{R - \kappa\beta(1 - p_{F})}{\beta p_{F}(\psi + R)^2 u''(q_{F})} < 0.$$

Since q_{H} and q_{F} are strictly decreasing in ψ, there exists a critical level of T, T_1^*, such that $T \geq T_1^*$ implies $\psi = \psi^*$, which in turn implies $q_{H} = q^*$ and $q_{F} = q^*_{F,1}$ (the latter statement follows from (8) and (9)). To find this critical level use a) the bargaining solutions $(\psi + R)t_{HH} = q_{H}$, $(\psi + R)t_{FF} = q_{F}$, evaluated at $\psi = \psi^*$, and b) the market clearing condition, which under the specific parameter values becomes $T = t_{HH} + t_{FF}$. This yields $T_1^* = (1 - \beta)(q^* + q^*_{F,1})/R$. If $T > T_1^*$, the demand of foreigners for a certain asset is given by $t_{FF} = (1 - \beta)q^*_{F,1}/R$. This allows them to purchase the maximum possible quantity of foreign special good, $q^*_{F,1}$. The rest of the supply, T, is absorbed by local buyers. Hence, $t_{H} = t_{HH} = T - (1 - \beta)q^*_{F,1}/R$.

When $T < T_1^*$, agents are not buying the maximum possible amount of the special good. Hence, in this range, $\psi > \psi^*$, $q_{H} < q^*$, $q_{F} < q^*_{F,1}$, and $\partial\psi/\partial T < 0, \partial q_{H}/\partial T > 0, \partial q_{F}/\partial T > 0$. As $T \to T_1^*$, the liquidity properties of the asset are exploited; ψ reaches the fundamental value, $\psi \to \psi^*$, and q_{H}, q_{F} reach their upper bounds, namely $q_{H} \to q^*$ and $q_{F} \to q^*_{F,1}$. $\square$

Proof of Lemma 7. a) The fact that $p_{H} = p_{F} = 0$ implies $G_{H}(p) = G_{F}(p) = 0$ is straightforward. For some $p_{H} = p_{F} = p \in (0, 1]$, $G_{H}(p) > G_{F}(p)$ will be true if and only if $u'(q_{H}) < u'(q_{F})$. After some algebra, one can show that this is equivalent to $0 < \kappa(1 - p)$, which is always true.

b) Follows directly from part (a) and inspection of Figure 2.

c) Follows directly from part (b) and the bargaining solution. $\square$

Proof of Proposition 2. We prove that the expressions on the left-hand sides of (11) and (12) are greater than 0.5. The derivation of these expressions is relegated to Appendix B.

a) If for a given p_{F}, $p_{H} > \tilde{p}_{H}(p_{F})$, then, by Lemma 7, $t_{H} > t_{F}$ and the inequality in (11) follows immediately.

b) The left-hand side of (12) is bigger than 0.5 if and only if

$$T > \frac{1 - \beta}{R} q^*_{F,1} \left(2 - p_{F}\right) \tag{a.13}$$

We have

$$T > T_1^* = \frac{1 - \beta}{R}\left(q^* + q^*_{F,1}\right) > \frac{2(1 - \beta)}{R} q^*_{F,1} > \frac{1 - \beta}{R} q^*_{F,1} \left(2 - p_{F}\right).$$

Hence, (a.13) is always satisfied. $\square$

B Accounting Appendix

Although sellers and buyers are ex-ante identical, their decisions in the CM, regarding how many hours to work or whether they should visit the financial markets and re-balance their portfolios, depend on whether they got matched in the local and/or foreign DM. Throughout this Appendix, we repeatedly focus on the labor, consumption, and asset-holding decisions of the following seven groups: 1) sellers who got matched with a local buyer (group 1), 2) sellers who got matched with a foreign buyer (group 2), 3) sellers who did not get matched (group 3), 4) buyers who got matched in both DM's (group 4), 5) buyers who got matched only in the home DM (group 5), 6) buyers who got matched only in the foreign DM (group 6), and 7) buyers who did not get matched (group 7).

The measures of these groups are given by: $\mu_1 = \xi a_S \sigma_H / (\sigma_H + \sigma_F)$, $\mu_2 = \xi a_S \sigma_F / (\sigma_H + \sigma_F)$, $\mu_3 = \xi(1 - a_S)$, $\mu_4 = \sigma_H \sigma_F a_B^2$, $\mu_5 = \sigma_H a_B (1 - \sigma_F a_B)$, $\mu_6 = \sigma_F a_B (1 - \sigma_H a_B)$, and $\mu_7 = (1 - \sigma_H a_B)(1 - \sigma_F a_B)$. For future reference notice that $\mu_1 = p_H$, $\mu_2 = p_F$, $\mu_4 = p_H p_F$, $\mu_5 = p_H (1 - p_F)$, $\mu_6 = p_F (1 - p_H)$, and $\mu_7 = (1 - p_H)(1 - p_F)$.

Derivation of Home Assets Bias formulas in (11) and (12). We define asset home bias as the ratio of the weighted sum of domestic asset holdings over the weighted sum of all asset holdings for all citizens of a certain country. We count asset holdings in both the CM and the DM and assume equal weights for the two markets. It is understood that we count asset holdings at the end of the sub-periods. For example, an agent of group 4 enters the CM with no assets, but re-balances her portfolio, so at the end of the CM she holds t_H of the home asset and t_F of the foreign asset. On the other hand, an agent of group 1 enters the CM with some home assets, but she sells these assets, so at the end of the CM she holds zero.

a) First consider $T \leq T_1^*$. Let the numerator of the asset home bias formula (weighted sum of domestic asset holdings) be denoted by A_H. Given the strategy described above,

$$A_H = t_H + p_H t_F + p_F t_F + (1 - p_H) t_H.$$

The first term is the total home asset holdings in the CM, i.e. t_H times the total measure of buyers (one). Sellers hold zero assets in the CM. The second term represents home asset holdings in the DM by sellers who got matched with local buyers. The third term represents home asset holdings in the DM by sellers who got matched with foreign buyers. Recall that we are describing the local currency dominance regime. Hence, foreign buyers pay sellers in local assets. The last term represents home asset holdings in the DM by the buyers who did not get matched in the local DM and, hence, hold some domestic assets (groups 6 and 7). After some algebra one can conclude that

$$A_H = 2t_{_H} + p_{_F} t_{_F}.$$

Following similar steps, one can show that the denominator of the asset home bias formula (weighted sum of all asset holdings), is given by $A_T = 2\left(t_{_H} + t_{_F}\right)$. Therefore, (11) follows immediately.

b) Now consider $T > T_1^*$. The difference with part (a) is that now the cost of carrying home assets is zero. As a result, buyers carry more assets than what they use in the DM as media of exchange. Therefore, agents from groups 4 and 5, who previously did not show up in the expression A_H, will now carry some home asset leftovers. The numerator of the asset home bias formula is

$$\begin{aligned} A_H &= \left(T - \frac{1-\beta}{R} q^*_{_{F,1}}\right) + p_{_H} \frac{1-\beta}{R} q^* + p_{_F} \frac{1-\beta}{R} q^*_{_{F,1}} \\ &+ p_{_H} \left(T - \frac{1-\beta}{R} q^* - \frac{1-\beta}{R} q^*_{_{F,1}}\right) + (1 - p_{_H}) \left(T - \frac{1-\beta}{R} q^*_{_{F,1}}\right). \end{aligned}$$

The first term is the total home asset holdings in the CM (held by buyers whose measure is one). The second term represents home asset holdings in the DM by sellers who got matched with local buyers. The third term represents home asset holdings in the DM by sellers who got matched with foreign buyers. The fourth term represents home asset holdings in the DM by buyers who got matched in the local DM (groups 4 and 5). Finally, the fifth term stands for the home asset holdings of buyers who did not trade in the local DM (groups 6 and 7). After some algebra, we obtain

$$A_H = 2\left(T - \frac{1-\beta}{R} q^*_{_{F,1}}\right) + p_{_F} \frac{1-\beta}{R} q^*_{_{F,1}}.$$

One can also show that the denominator of the asset home bias formula is given by $A_T = 2T$. Hence, (12) follows immediately. $\square$

Proof of Proposition 3. a) Let $C^i_{_F}$, $C^i_{_H}$ be the value of foreign and domestic consumption for the typical agent in group i, $i = 1, ..., 7$, respectively. All consumption is denominated in terms of the general good. Moreover, let H^i denote the hours worked by the typical agent in group i, $i = 1, ..., 7$. All agents consume X^* in the CM, so agents with many assets will work fewer hours. Sometimes it will be useful to directly use the agents' budget constraint and replace X^* with more informative expressions. First, consider sellers. We have $C^1_H = C^2_H = C^3_H = X^*$ and $C^1_F = C^2_F = C^3_F = 0$. Clearly, even sellers who matched with foreign buyers have no imported consumption, since in the local currency dominance regime, sellers get payed in local assets.

Unlike sellers, buyers might also consume in the DM. For group 4, we have $C^4_H = X^* +$

$(\psi + R)t_{H} = X^{*} + q_{H}$, where q_{F} represents DM consumption. Also, $C_F^4 = (\psi + R)t_{F} = q_{F}$. Group 5 does not match abroad, hence these agents carry some foreign assets into the CM, which in turn implies that some of their CM consumption is imported. Clearly, $C_F^5 = R_\kappa t_{F}$. On the other hand, $C_H^5 = H^5 + Rt_{H}$.[22] For group 6, all CM consumption (X^*) is domestic and all DM consumption, q_{F}, is imported. Finally, group 7 consumes only in the CM. We have $C_H^7 = H^7 - Rt_{H}$ and $C_F^7 = R_\kappa t_{F}$.

The analysis in the previous paragraph reveals that $p_{H} > \tilde{p}_{H}(p_{F})$, implies $C_H^i > C_F^i$ for all groups except 6, for which the inequality could be reversed, depending on the magnitude of X^*. To prove the desired result we contrast the domestic and foreign consumption for groups 2 and 6. $C_H > C_F$ will be true if and only if $\sum_{i=1}^7 \mu_i C_H^i > \sum_{i=1}^7 \mu_i C_F^i$. Since $C_H^i > C_F^i$ holds for $i = 1, 3, 4, 5, 7$, the latter statement will be true as long as

$$\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6. \tag{b.1}$$

From the budget constraint of group 2, we have $C_H^2 = X^* = H^2 + q_{F}$. Also, from the analysis above, $C_F^2 = 0$. Hence, (b.1) holds if and only if

$$\begin{aligned} p_{F}(H^2 + q_{F}) + p_{F}(1 - p_{H}) X^* &\geq p_{F}(1 - p_{H}) q_{F} \Leftrightarrow \\ p_{F} H^2 + p_{F}(1 - p_{H}) X^* &\geq -p_{H} p_{F} q_{F}, \end{aligned}$$

which is always true, since $X^*, H^i \geq 0$.

b) Like in part (a), $C_H^1 = C_H^2 = C_H^3 = X^*$ and $C_F^1 = C_F^2 = C_F^3 = 0$. For group 4 $C_H^4 = X^* + q^*$ and $C_F^4 = q^*_{F,1}$. For group 5 CM consumption (X^*) can be broken down into domestic, $H^5 + RT - q^* - (1-\beta)q^*_{F,1}$, and imported, $R_\kappa(1-\beta)q^*_{F,1}/R$. In the DM all consumption is domestic and equal to q^*. Hence, $C_H^5 = H^5 + RT - (1-\beta)q^*_{F,1}$ and $C_F^5 = R_\kappa(1-\beta)q^*_{F,1}/R$. For group 6 all CM consumption (X^*) is domestic and all DM consumption, $q^*_{F,1}$, is imported. Finally, group 7 only consumes in the CM. From this group's budget constraint, it follows that $C_H^7 = H^7 + RT - (1-\beta)q^*_{F,1}$ and $C_F^5 = R_\kappa(1-\beta)q^*_{F,1}/R$.

It is clear that for $i = 1, 2, 3, 4$, $C_H^i > C_F^i$. For groups 5 and 7 the same argument is true, but it is less obvious, hence we prove it formally below. For group 6 this argument need not be true. Consider group 5. $C_H^5 > C_F^5$ will be true if and only if

$$H^5 + RT > (1-\beta)q^*_{F,1}\left(2 - \frac{\kappa}{R}\right). \tag{b.2}$$

[22] To see this point, notice that this agent's budget constraint is $X^* + \psi(t_{H} + t_{F}) = H^5 + (\psi + R_\kappa)t_{F}$ or $X^* = H^5 - \psi t_{H} + R_\kappa t_{F}$, where $R_\kappa t_{F}$ is imported CM consumption and $H^5 - \psi t_{H}$ is domestic CM consumption. Since domestic DM consumption is given by $(\psi + R)t_{H}$, we have $C_H^5 = H^5 + Rt_{H}$.

But here $T > T_1^*$, which implies

$$RT > (1-\beta)(q^* + q^*_{F,1}) \Leftrightarrow RT > 2(1-\beta)q^*_{F,1} \Leftrightarrow RT + H^5 > \left(2 - \frac{\kappa}{R}\right)(1-\beta)q^*_{F,1}.$$

This confirms that the inequality in (b.2) holds true. Following similar steps establishes that $C_H^7 > C_F^7$. Hence, $C_H^i > C_F^i$ for all i, except possibly group 6. However, as in part (a), one can easily verify that $\mu_2 C_H^2 + \mu_6 C_H^6 \geq \mu_2 C_F^2 + \mu_6 C_F^6$, which is a sufficient condition for $C_H > C_F$ to also hold. $\square$

C Data Appendix

Table 2: Average Home Equity Share of Portfolio, 1997-2007

Country	HA	Country	HA	Country	HA
Australia	0.8374	Greece	0.9536	Poland	0.9850
Austria	0.4279	Hungary	0.9450	Portugal	0.7011
Belgium	0.5720	Iceland	0.7303	Slovenia	0.9366
Canada	0.7432	Israel	0.8577	Spain	0.8589
Chile	0.8366	Italy	0.6364	Sweden	0.6290
Czech Republic	0.8382	Japan	0.8972	Switzerland	0.5659
Denmark	0.6218	Korea, Republic of	0.9538	Turkey	0.9772
Estonia	0.8552	Mexico	0.9280	United Kingdom	0.6906
Finland	0.7206	Netherlands	0.4018	United States	0.8531
France	0.7523	New Zealand	0.6502		
Germany	0.5951	Norway	0.5493	*OECD Average*	*0.7581*

Notes
HA=1-International Diversification
International Diversification = Foreign Portfolio Equity Assets / (Stock Market Capitalization + Foreign Portfolio Equity Assets - Foreign Portfolio Equity Liabilities)
Foreign Portfolio Equity Asset is $< 10\%$ share ownership of firm headquartered abroad
Data Sources: Foreign Portfolio Equity Assets and Liabilities from Lane and Milesi-Ferretti (2007). Stock Market Capitalization from WDI.