NBER WORKING PAPER SERIES

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Working Paper 17325
http://www.nber.org/papers/w17325

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2011

This paper is based on work supported under a Netspar research grant. The Health and Retirement Study is sponsored by the National Institute on Aging (grant U01-AG009740) and is conducted by the University of Michigan. For comments and discussions, we thank Peter Bossaerts, Jiajia Cui, Frank de Jong, Liran Einav, Michael Gallmeyer, Ben Heijdra, Robin Lumsdaine, Alex Michaelides, Olivia Mitchell, Theo Nijman, Sam Schulhofer-Wohl, Pascal St-Amour, Mogens Steffensen, and seminar participants at APG, Australian National University, Columbia University, Erasmus University, Federal Reserve Bank of Chicago and Minneapolis, Financial Engines, Georgetown University, Georgia State University, Maastricht University, New York University, Northwestern University, Princeton University, Tilburg University, University of Chicago, University of Minnesota, University of New South Wales, University of Technology Sydney, University of Tokyo, University of Utah, Vanderbilt University, the 2011 Netspar International Pension Workshop, the 2011 SED Annual Meeting, the 2011 UBC Summer Finance Conference, the 2011 Conference on Economic Decisionmaking, the 2012 AEA Annual Meeting, the 2012 Utah Winter Finance Conference, the 2012 LAEF Conference on Health and Mortality, the 2012 Wharton Conference on Household Portfolio Choice and Investment Decisions, the 2012 NBER Summer Institute Economics of Household Saving Workshop, the 2012 EFA Annual Meeting, the 2012 Systemic Risk Conference: Economists Meet Neurologists, the 2012 NBER-Oxford Saïd-CFS-EIEF Conference on Household Finance, and the 2012 Q-Group Fall Seminar. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System, or the National Bureau of Economic Research.

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NBER Working Paper No. 17325
August 2011, Revised June 2013
JEL No. D14,D91,G11,G22,I10

ABSTRACT

We develop a pair of risk measures for the universe of life and health insurance products. Health delta measures the differential payoff that a product delivers in poor health, while mortality delta measures the differential payoff that a product delivers at death. A life-cycle model of insurance choice simplifies to replicating the optimal health and mortality delta through a portfolio of insurance products. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of life insurance, annuities including private pensions, and long-term care insurance. We then compare them to the optimal health and mortality delta implied by the life-cycle model. For the median household aged 51 to 57, the lifetime welfare cost of market incompleteness and suboptimal choice is 4 percent of total wealth. Both observed household characteristics and unobserved preference heterogeneity fail to explain this welfare cost.

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Retail financial advisors and insurance companies offer a wide variety of insurance products that includes life insurance, annuities, and long-term care insurance. They offer each of these products in a full range of maturities and payout structures. Examples include term life insurance with guaranteed term up to 30 years, universal or whole life insurance, immediate annuities, and deferred annuities whose income is deferrable for a year or longer. This variety begs for a risk measure that allows households to assess the degree of complementarity and substitutability between various products and, ultimately, to choose an optimal portfolio of products. Such risk measures already exist in other parts of the retail financial industry. For example, beta measures an equity product's exposure to aggregate market risk, while duration measures a fixed-income product's exposure to interest-rate risk. The existence of such risk measures, based on sound economic theory, has proven to be tremendously valuable in quantifying and managing financial risk for both households and institutions alike.

This paper develops a pair of risk measures for the universe of life and health insurance products, which we refer to as health and mortality delta. Health delta measures the differential payoff that a product delivers in poor health, while mortality delta measures the differential payoff that a product delivers at death. A life-cycle model of insurance choice implies optimal consumption as well as optimal health and mortality delta, which depend on preferences (e.g., risk aversion and bequest motive) and state variables (e.g., birth cohort, age, wealth, and health). An optimal portfolio of insurance products, not necessarily unique, aggregates health and mortality delta over individual products to replicate the optimal health and mortality delta implied by the life-cycle model.

We use our risk measures to assess whether the observed demand for insurance is close to the optimal demand, given the provision of public insurance through Social Security and Medicare. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions (i.e., defined benefit plans), and long-term care insurance. We estimate household preferences so that the observed demand most closely matches the optimal demand implied by the life-cycle model. We achieve sharp identification of relative risk aversion, the bequest motive, and the complementarity of consumption and health. Insurance choice, which contains valuable information about the desired path of wealth in future health states, is much more informative than the realized path of savings for identifying these preference parameters.

For each household, we estimate the welfare cost of deviations from the optimal demand, which we interpret to be the joint cost of market incompleteness (e.g., due to adverse selection as in Hendren (2012)) and suboptimal choice. For the median household aged 51 to 57, the lifetime welfare cost is 4 percent of total wealth, which is the sum of financial wealth and the present value of future income in excess of out-of-pocket health expenses. Our estimate is an order of magnitude larger than the welfare cost of under-diversification in stock and mutual fund portfolios (e.g., Calvet, Campbell, and Sodini, 2007, estimate it to be 0.5 percent of disposable income for the median Swedish household). Most of the welfare cost is explained by the difference between observed and optimal mortality delta, rather than by the difference between observed and optimal health delta. That is, choices over life insurance and annuities have a much larger impact on welfare than do choices over long-term care insurance.

The large welfare cost arises from the fact that the life-cycle model generates significant variation in the optimal health and mortality delta along its state variables (i.e., birth cohort, age, wealth, and health), which is not matched by the observed health and mortality delta implied by household insurance choice. The variation in the observed health and mortality delta is mostly driven by heterogeneity and inertia that arises from passive annuitization through private pensions. Moreover, observed household characteristics, which capture potential preference heterogeneity or private information about health, fail to explain the difference between observed and optimal demand. Even unobserved preference heterogeneity cannot fully rationalize the observed demand because households fail to optimally rebalance their portfolio over the life cycle (e.g., from life insurance to annuities). We uncover a new puzzle that is distinct from the so-called annuity puzzle in the literature. The unexplained variation in the degree to which households are annuitized, rather than the average level at which they are annuitized, is puzzling from the perspective of life-cycle theory.

This paper is not the first attempt to understand the demand for insurance such as life insurance (Bernheim, 1991; Inkmann and Michaelides, 2012), annuities (Brown, 2001; Inkmann, Lopes, and Michaelides, 2011), and long-term care insurance (Brown and Finkelstein, 2008; Lockwood, 2013). Relative to the previous literature, an important methodological contribution is to examine insurance choice comprehensively as a portfolio-choice problem, instead of one product at a time. By collapsing insurance choice into a pair of risk measures, we explicitly account for the complementarity and the substitutability between various products. In particular, annuities and private pensions can partially substitute for long-term care insurance, by insuring that households have sufficient income to cover late-life health expenses as long as they live. Therefore, one cannot study the demand for long-term care insurance without simultaneously thinking about annuities and private pensions.

The remainder of the paper is organized as follows. In Section 1, we develop a life-cycle model in which households face health and mortality risk and save in a complete set of insurance products that includes life insurance, annuities, and supplemental health insurance. In Section 2, we derive the optimal demand for insurance and a key formula for measuring the welfare cost of deviations from the optimal demand. In Section 3, we calibrate the life-cycle model based on the Health and Retirement Study. In Section 4, we compare the observed demand to the optimal demand implied by the life-cycle model. We then estimate the welfare cost of deviations from the optimal demand. In Section 5, we show that our findings are robust to potential model misspecification along both observed and unobserved dimensions. In Section 6, we illustrate how a portfolio of existing insurance products can replicate the optimal health and mortality delta implied by the life-cycle model. Section 7 concludes with practical implications of our study for retail financial advisors and insurance companies.

1. A Life-Cycle Model with Health and Mortality Risk

In this section, we develop a life-cycle model in which a household faces health and mortality risk that affects life expectancy, health expenses, and the marginal utility of consumption or wealth. The household can save in a bond as well as a complete set of insurance products that includes life insurance, annuities, and supplemental health insurance.

Complete markets is a natural starting point, given the rich menu of insurance products that retail financial advisors and insurance companies already offer. In Section 6, we show that a realistic portfolio of existing insurance products replicates the optimal health and mortality delta implied by a calibrated version of the life-cycle model. Even if actual markets are incomplete, we view complete markets as a useful benchmark for quantifying the importance of market incompleteness that may arise for various reasons, including adverse selection and borrowing constraints.

1.1. Health and Mortality Risk

In our model, health refers to any information that is verifiable through medical underwriting that involves a health examination and a review of medical history. For tractability, we do not model residual private information, such as self-assessments of health, that might affect the demand for insurance. In Section 5, however, we show that residual private information does not explain the difference between observed and optimal demand implied by the life-cycle model.

1.1.1. Health Transition Probabilities

A household consists of an insured and other members who share common resources. The insured lives for at most T periods and dies with certainty in period $T + 1$. In each period

$t \in \{1, \dots, T\}$, the insured's health is in one of three states, indexed as $h_t \in \{1, 2, 3\}$.[1] The health states are ordered so that $h_t = 1$ corresponds to death, $h_t = 2$ corresponds to poor health, and $h_t = 3$ corresponds to good health.

The insured's health evolves from period t to $t+1$ according to a Markov chain with a 3×3 transition matrix π_t. We denote the (i, j)th element of the transition matrix as

$$\pi_t(i, j) = \Pr(h_{t+1} = j | h_t = i). \tag{1}$$

Conditional on being in health state i in period t, $\pi_t(i, j)$ is the probability of being in health state j in period $t+1$. Death is an absorbing state so that $\pi_t(1, 1) = 1$. Let $\mathbf{e}_i$ denote a 3×1 vector whose ith element is one and whose other elements are zero. We define an n-period transition probability as

$$\pi_t^n(i, j) = \mathbf{e}_i' \prod_{s=0}^{n-1} \pi_{t+s} \mathbf{e}_j. \tag{2}$$

Conditional on being in health state i in period t, $\pi_t^n(i, j)$ is the probability of being in health state j in period $t + n$.

We define an n-period mortality rate as

$$p_t(n|i) = \begin{cases} \mathbf{e}_i' \pi_t \mathbf{e}_1 & \text{if } n = 1 \\ \mathbf{e}_i' \prod_{s=0}^{n-2} \pi_{t+s} \begin{bmatrix} \mathbf{0} & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix} \pi_{t+n-1} \mathbf{e}_1 & \text{if } n > 1 \end{cases}. \tag{3}$$

Conditional on being in health state i in period t, $p_t(n|i)$ is the probability of being alive in period $t + n - 1$ but dead in period $t + n$. We also define an n-period survival probability as

$$q_t(n|i) = 1 - \pi_t^n(i, 1). \tag{4}$$

Conditional on being in health state i in period t, $q_t(n|i)$ is the probability of being alive in period $t + n$.

1.1.2. Out-of-Pocket Health Expenses

The household has employer-provided health insurance while working and Medicare in retirement, which cover the basic health expenses. However, the household may face out-of-pocket

[1] The three-state model can be interpreted as a discrete-time analog of a continuous-time model in which a continuous process drives health risk, and a jump process drives mortality risk. While three states is appropriate for our empirical application, it is conceptually straightforward to extend our framework to more than three states (Hoem, 1969).

health expenses that are not covered by these policies, for which it could purchase supplemental health insurance. For example, Medicare does not cover nursing home care, for which the household could purchase long-term care insurance.

In the absence of supplemental health insurance, the household faces an out-of-pocket health expense M_t in each period t. The distribution of out-of-pocket health expenses depends on age and health, where $M_t(j)$ denotes its realization for health state j.[2] Naturally, poor health is associated with higher out-of-pocket health expenses. We assume that end-of-life health expenses incur in the last period prior to death. There is no health expense at death so that $M_t(1) = 0$.

1.2. Life and Health Insurance Products

In each period t, the household can save in a one-period bond, which earns gross interest R. In addition, the household can save in life insurance, deferred annuities, and supplemental health insurance of maturities one to $T - t$.

1.2.1. Life Insurance

Let $\mathbf{1}_t(j)$ denote an indicator function that is equal to one if the insured is in health state j in period t. Term life insurance of maturity n, issued in period t, pays out a death benefit of

$$D_{L,t+s}(n - s|h_{t+s}) = \mathbf{1}_{t+s}(1), \tag{5}$$

upon death of the insured in any period $s \in \{1, \ldots, n\}$. In each period t, $T-t$ is the maximum maturity because the insured dies with certainty in period $T+1$. For our purposes, universal or whole life insurance is a special case of term life insurance with the maximum maturity.

The pricing of life insurance depends on the insured's age and health at issuance of the policy. Naturally, younger and healthier individuals with longer life expectancy pay a lower premium.[3] Conditional on being in health state h_t in period t, the price of n-period life

[2] To focus on insurance choice, we abstract from the endogenous choice of health expenditure (see Picone, Uribe, and Wilson, 1998; Hugonnier, Pelgrin, and St-Amour, 2013; Yogo, 2009).

[3] The insurer could charge a premium that is independent of health in a pooling equilibrium (e.g., group life insurance). In that case, we would have to solve for a pooling price at which the insurer breaks even, given the aggregate demand for a given product. While a conceptually straightforward extension of our framework, such an exercise would be computationally challenging. We refer to a related literature that examines the welfare implications of pooled pricing and private information in annuity (Einav, Finkelstein, and Schrimpf, 2010) and health insurance markets (Einav, Finkelstein, and Cullen, 2010; Bundorf, Levin, and Mahoney, 2012).

insurance per unit of death benefit is

$$P_{L,t}(n|h_t) = \sum_{s=1}^{n} \frac{p_t(s|h_t)}{R_L^s}, \tag{6}$$

where $R_L \leq R$ is the discount rate. The pricing of life insurance is actuarially fair when $R_L = R$, while $R_L < R$ implies a markup.

1.2.2. Deferred Annuities

A deferred annuity of maturity n, issued in period t, pays out a constant income of

$$D_{A,t+s}(n - s|h_{t+s}) = \begin{cases} 0 & \text{if } s < n \\ 1 - \mathbf{1}_{t+s}(1) & \text{if } s \geq n \end{cases}, \tag{7}$$

in each period $s \in \{1, \ldots, T - t\}$ that the insured is alive. In each period t, $T - t$ is the maximum maturity because the insured dies with certainty in period $T+1$. For our purposes, an immediate annuity is a special case of deferred annuities with the minimum maturity (i.e., $n = 1$).

The pricing of annuities depends on the insured's age and health at issuance of the policy.[4] Naturally, younger and healthier individuals with longer life expectancy pay a higher premium. Conditional on being in health state h_t in period t, the price of an n-period annuity per unit of income is

$$P_{A,t}(n|h_t) = \sum_{s=n}^{T-t} \frac{q_t(s|h_t)}{R_A^s}, \tag{8}$$

where $R_A \leq R$ is the discount rate.

1.2.3. Supplemental Health Insurance

Supplemental health insurance of maturity n, issued in period t, covers

$$D_{H,t+s}(n - s|h_{t+s}) = \mathbf{1}_{t+s}(2)(M_{t+s}(2) - M_{t+s}(3)), \tag{9}$$

in each period $s \in \{1, \ldots, n\}$ that the insured is in poor health. Insofar as health expenses include nursing home and home health care, we also interpret this product as long-term care

[4] In the United States, annuities can be purchased without medical underwriting at a price that depends only on gender and age. However, those with a serious medical condition can purchase medically underwritten annuities at a lower price that reflects their impaired mortality.

insurance. A unit of this product represents full coverage, equating health expenses across all health states in which the insured is alive. In each period t, $T-t$ is the maximum maturity because the insured dies with certainty in period $T+1$.

The pricing of supplemental health insurance depends on the insured's age and health at issuance of the policy. Naturally, younger and healthier individuals with lower expected health expenses pay a lower premium. Conditional on being in health state h_t in period t, the price of n-period supplemental health insurance per unit of coverage is

$$P_{H,t}(n|h_t) = \sum_{s=1}^{n} \frac{\pi_t^s(h_t,2)(M_{t+s}(2) - M_{t+s}(3))}{R_H^s}, \tag{10}$$

where $R_H \leq R$ is the discount rate.

1.3. Health and Mortality Delta for Insurance Products

For each product $i = \{L, A, H\}$ of maturity n, we define its health delta in period t as

$$\Delta_{i,t}(n) = P_{i,t+1}(n-1|2) + D_{i,t+1}(n-1|2) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{11}$$

Health delta measures the differential payoff that a policy delivers in poor health relative to good health in period $t+1$. Similarly, we define its mortality delta in period t as

$$\delta_{i,t}(n) = D_{i,t+1}(n-1|1) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{12}$$

Mortality delta measures the differential payoff that a policy delivers at death relative to good health in period $t+1$.

Figure 1 illustrates the relation between the payoffs of a policy and its health and mortality delta. Section 3 explains how we estimate the payoffs based on the Health and Retirement Study, which is not important for the purposes of this illustration. The solid line represents the payoffs of a policy in the three possible health states in the subsequent period. Health delta is the payoff of a policy in poor health relative to good health, which is minus the slope of the dashed line if the horizontal distance between good and poor health is one. Mortality delta is the payoff of a policy at death relative to good health, which is minus the slope of the dotted line if the horizontal distance between good health and death is one.

Long-term life insurance and supplemental health insurance have positive health delta, while deferred annuities have negative health delta. That is, long-term life insurance is a substitute for supplemental health insurance in terms of health delta. This is because the expected payoff from long-term life insurance rises in poor health when the insured has shorter life expectancy, just like supplemental health insurance. In contrast, deferred annuities are complements of supplemental health insurance in terms of health delta. This is because the expected payoff from deferred annuities falls in poor health when the insured has shorter life expectancy, which is the opposite of supplemental health insurance.

Life insurance has positive mortality delta, while deferred annuities and long-term health insurance have negative mortality delta. That is, deferred annuities and long-term health insurance are complements of life insurance in terms of mortality delta. This is because deferred annuities and long-term health insurance lose their value entirely at death, which is the opposite of life insurance. Therefore, deferred annuities and long-term health insurance are both effective ways to transfer wealth to future states in which the insured remains alive and faces high health expenses.

Figure 1 highlights the fact that one must study insurance products together, instead of one product at a time. Long-term life insurance not only insures mortality risk, but also has exposure to health delta. Deferred annuities not only insure longevity risk, but also have exposure to health delta. Finally, long-term health insurance not only insures health risk, but also has exposure to mortality delta.

1.4. Budget Constraint

In each period t that the insured is alive, the household starts with initial wealth A_t. The household receives labor or retirement income Y_t, pays health expenses M_t, and consumes C_t. The household saves the wealth remaining after health expenses and consumption in bonds, life insurance, deferred annuities, and supplemental health insurance. Let B_t denote the total face value of bonds, and let $B_{i,t}(n) \geq 0$ denote the total face value of policy i of maturity n. The household's savings in period t is

$$A_t + Y_t - M_t - C_t = \frac{B_t}{R} + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} P_{i,t}(n) B_{i,t}(n). \tag{13}$$

We assume that the household can borrow from its savings in insurance products at the gross interest rate R. Therefore, a loan from insurance policies is a negative position in bonds. For our purposes, a loan from insurance policies is a simple way to model actual features of these products. The premium for long-term life insurance and long-term care insurance is typically paid through constant periodic payments, instead of a lump-sum payment up front. The option to pay through periodic payments is essentially equivalent to borrowing against the value of the policy because the present value of the periodic payments is equal to the value of the policy at issuance. Whole life insurance typically has an explicit option to borrow from the cash surrender value of the policy. Finally, households can take out a loan from annuities in a defined contribution plan.

The intertemporal budget constraint is

$$A_{t+1} = B_t + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} (P_{i,t+1}(n-1) + D_{i,t+1}(n-1))B_{i,t}(n). \tag{14}$$

That is, wealth in the subsequent period is equal to the face value of maturing bonds plus the (realized and expected) payoffs from life insurance, deferred annuities, and supplemental health insurance. Let $A_{t+1}(j)$ denote wealth if health state j is realized in period $t+1$. In particular, wealth that is bequeathed if the insured dies in period $t+1$ is

$$A_{t+1}(1) = B_t + \sum_{n=1}^{T-t} B_{L,t}(n). \tag{15}$$

That is, wealth at the insured's death is equal to the face value of maturing bonds plus the death benefit from life insurance. The household must have non-negative wealth at the insured's death, that is, $A_{t+1}(1) \geq 0$.

1.5. *Objective Function*

The household maximizes expected utility over consumption while alive and the bequest upon death. The household's objective function in health state $h_t \in \{2, 3\}$ is

$$U_t(h_t) = \left\{ \omega(h_t)^\gamma C_t^{1-\gamma} + \beta \left[\pi_t(h_t, 1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) U_{t+1}(j)^{1-\gamma} \right] \right\}^{1/(1-\gamma)}, \tag{16}$$

with the terminal value

$$U_T(h_T) = \omega(h_T)^{\gamma/(1-\gamma)} C_T. \tag{17}$$

The parameter $\beta \in (0, 1)$ is the subjective discount factor, and $\gamma > 1$ is relative risk aversion. The health state-dependent utility parameter $\omega(h_t) \geq 0$ allows the marginal utility of consumption or wealth to vary across health states. The presence of a bequest motive is parameterized as $\omega(1) > 0$, in contrast to its absence $\omega(1) = 0$. Consumption and health are complements if the marginal utility of consumption is lower in poor health, which is parameterized as $\omega(2) < \omega(3)$. Otherwise, consumption and health are substitutes if $\omega(2) > \omega(3)$.

2. Optimal Demand for Insurance

In this section, we derive the optimal demand for insurance under complete markets. When markets are complete, there are potentially many combinations of insurance products that achieve the same consumption and wealth allocations. Therefore, we characterize the unique solution to the life-cycle problem in terms of optimal consumption and optimal health and mortality delta. We also derive a key formula for measuring the welfare cost of deviations from the optimal demand.

2.1. *Optimal Health and Mortality Delta*

We define health delta in period t as the difference in realized wealth between poor and good health in period $t+1$:

$$\Delta_t = A_{t+1}(2) - A_{t+1}(3). \tag{18}$$

Similarly, we define mortality delta in period t as the difference in realized wealth between death and good health in period $t+1$:

$$\delta_t = A_{t+1}(1) - A_{t+1}(3). \tag{19}$$

Proposition 1. *The solution to the life-cycle problem under complete markets is*

$$C_t^* = c_t(h_t)\left(A_t + \sum_{s=0}^{T-t} \frac{\mathbf{E}_t[Y_{t+s} - M_{t+s}|h_t]}{R^s}\right), \tag{20}$$

$$\Delta_t^* = \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)}\left(\frac{\omega(2)}{c_{t+1}(2)} - \frac{\omega(3)}{c_{t+1}(3)}\right) - \left(\sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|2]}{R^{s-1}} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}\right), \tag{21}$$

$$\delta_t^* = \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)}\left(\omega(1) - \frac{\omega(3)}{c_{t+1}(3)}\right) + \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}. \tag{22}$$

The average propensity to consume in health state $h_t \in \{2, 3\}$ is

$$c_t(h_t) = \left[1 + \frac{\pi_t(h_t, 1)(\beta R)^{1/\gamma}\omega(1)}{R\omega(h_t)} + \sum_{j=2}^{3} \frac{\pi_t(h_t, j)(\beta R)^{1/\gamma}\omega(j)}{R\omega(h_t)c_{t+1}(j)}\right]^{-1}, \tag{23}$$

with the terminal value $c_T(h_T) = 1$.

As shown in Appendix A, the optimal policy equates the marginal utility of consumption or wealth across all future health states (Yaari, 1965). The expression for the optimal health delta (i.e., Δ_t^*) shows that three forces drive the household's desire to insure poor health relative to good health. First, the household would like to deliver relatively more wealth to the health state in which the marginal utility of consumption is high, determined by the relative magnitudes of $\omega(2)$ and $\omega(3)$. Second, the household would like to deliver relatively more wealth to the health state in which the average propensity to consume is low, determined by the relative magnitudes of $c_{t+1}(2)$ and $c_{t+1}(3)$. Naturally, the household consumes more slowly out of wealth in good health associated with longer life expectancy. Finally, the household would like to deliver relatively more wealth to the health state in which lifetime disposable income (i.e., income in excess of out-of-pocket health expenses) is low. Naturally, the household has lower lifetime disposable income in poor health associated with shorter life expectancy, higher health expenses, and potentially lower income.

The same three forces also explain the expression for the optimal mortality delta (i.e., δ_t^*). First, the household would like to deliver relatively more wealth to death if the bequest motive (i.e., $\omega(1)$) is strong. Second, the household would like to deliver relatively more wealth to death if the average propensity to consume in good health (i.e., $c_{t+1}(3)$) is high. Finally, the household would like to deliver relatively more wealth to death if lifetime disposable income is high in good health.

2.2. *Optimal Portfolio of Insurance Products*

Proposition 2. *Given an optimal consumption policy, a feasible portfolio policy that satisfies the budget constraint* (13) *is optimal if it satisfies the equations*

$$\Delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \Delta_{i,t}(n) B_{i,t}(n), \tag{24}$$

$$\delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \delta_{i,t}(n) B_{i,t}(n). \tag{25}$$

Proposition 2 shows that health and mortality delta are sufficient for constructing an optimal portfolio of insurance products. Health delta $\Delta_{i,t}(n)$ measures the marginal contribution that policy i of maturity n has to the overall health delta. A portfolio, not necessarily unique, that satisfies equation (24) delivers the optimal amount of wealth to poor health in period $t+1$. Similarly, mortality delta $\delta_{i,t}(n)$ measures the marginal contribution that policy i of maturity n has to the overall mortality delta. A portfolio, not necessarily unique, that satisfies equation (25) delivers the optimal amount of wealth to death in period $t+1$.

2.3. Welfare Cost of Deviations from the Optimal Health and Mortality Delta

Suppose the household's demand for insurance were to deviate from the optimal demand in Proposition 1. As shown in Appendix A, we estimate the welfare cost of such deviations from the optimal demand through a second-order Taylor approximation around the known value function under complete markets. By the envelope theorem, the welfare cost is second order for sufficiently small deviations from the optimal demand (Cochrane, 1989).

Proposition 3. *Let V_t^* denote the value function associated with the sequence $\{\Delta^*_{t+s-1}(i), \delta^*_{t+s-1}(i)\}_{s=1}^n$ of optimal health and mortality delta under complete markets. Let V_t denote the value function associated with an alternative sequence $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=1}^n$ of health and mortality delta that satisfies the budget constraint. The welfare cost of deviations from the optimal health and mortality delta is*

$$\begin{aligned}
L_t(n) =& \frac{V_t}{V_t^*} - 1 \\
\approx& \frac{1}{2} \sum_{s=1}^{n} \sum_{i=2}^{3} \left[\frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i)^2} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))^2 \right. \\
&+ \frac{\partial^2 L_t(n)}{\partial \delta_{t+s-1}(i)^2} (\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i))^2 \\
&\left. +2 \frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i) \partial \delta_{t+s-1}(i)} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))(\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i)) \right],
\end{aligned} \tag{26}$$

where the expressions for the second partial derivatives are given in Appendix A.

The observed demand for insurance may deviate from the optimal demand for two reasons. First, the observed demand may be suboptimal, given the complexity of the portfolio-choice problem and the lack of clear academic guidance. Second, markets may be incomplete due to adverse selection, borrowing constraints, or other frictions that are outside the model. Because these two hypotheses are not mutually exclusive and difficult to distinguish based on the available data, we do not quantify their relative importance. Instead, we focus on estimating the joint cost of market incompleteness and suboptimal choice in this paper.

3. Calibrating the Life-Cycle Model

3.1. Health and Retirement Study

We calibrate the life-cycle model based on the Health and Retirement Study, which is a representative panel of older households in the United States since 1992. This household survey is uniquely suited for our study because it contains household-level data on health outcomes, health expenses, income, and wealth as well as ownership of life insurance, annuities, private pensions, and long-term care insurance. Some of these critical variables are missing in other household surveys such as the Panel Study of Income Dynamics and the Survey of Consumer Finances. We focus on households whose primary respondent is male and aged 51 or older at the time of interview. We also require that households have both positive income and net worth to be included in our sample.

We also restrict our sample based on health insurance coverage to reduce potential heterogeneity in out-of-pocket health expenses. The spirit of our screening criteria is that we isolate households that have adequate health insurance coverage, for whom the primary out-of-pocket health expense is nursing home care. We first eliminate households whose primary respondent is on Medicaid. We then select only those households whose primary respondent has employer-provided or individual health insurance. For respondents aged 65 and older, this criterion includes those that have supplemental coverage through Medicare Advantage (Part C), Medicare Part D, Medigap, or long-term care insurance. However, it excludes those that are solely on traditional Medicare (Parts A and B). Overall, this criterion eliminates only 17 percent of otherwise eligible households at age 51, and 29 percent of otherwise eligible households at age 65. We believe that the uniformity of health insurance coverage within the resulting sample trades off favorably with a narrower concept of health risk and smaller sample size. We also refer the reader to an earlier version of this paper, in which we did not screen households based on health insurance coverage.

Life insurance is written on the life of an insured, while resources like income and wealth are shared by the members of a household. Because the male respondent is typically married at the time of first interview, we must make some measurement assumptions when mapping the data to the model. We measure health outcomes and the ownership of life insurance, annuities including private pensions, and long-term care insurance for only the male respondent. We measure health expenses, income, and wealth at the household level. These measurement assumptions are consistent with the model insofar as the budget constraint holds for the household, and the male respondent buys life insurance to leave a bequest for surviving household members when he dies.

We calibrate the life-cycle model so that each period corresponds to two years, matching the frequency of interviews in the Health and Retirement Study. The model starts at age 51 to correspond to the youngest age at which respondents enter the survey. We assume that respondents die with certainty at age 111, so that there are a total of 30 periods (60 years) in the life-cycle model. We set the riskless interest rate to 2 percent annually, which is roughly the average real return on the one-year Treasury note during our sample period.

3.2. *Definition of the Health States*

In this section, we categorize health into three states including death, which is the minimum number of states necessary to model both health and mortality risk. For our purposes, the relevant criteria for poor health are that both the mortality rate and health expenses are high. This is precisely the state in which life insurance and long-term care insurance are valuable.

In Table 1, we use a probit model to predict future mortality based on doctor-diagnosed health problems and its interaction with age. Doctor-diagnosed health problems are statistically significant predictors of future mortality. For example, the marginal effect of cancer on the mortality rate is 10.43 with a t-statistic of 7.10. This means that respondents with cancer are 10.43 percentage points more likely to die within two years, holding everything else constant. Past age 51, each additional ten years is associated with an increase of 2.28 percentage points in the mortality rate.

Based on the estimated probit model, we calculate the predicted mortality rate for each male respondent at each interview. We then define the following health states.

1. Death.
2. Poor health: The predicted mortality rate is higher than the median conditional on birth cohort and age. In addition, out-of-pocket health expenses are higher than the median conditional on birth cohort, age, and the ownership of long-term care insurance.
3. Good health: Alive and not in poor health.

Our definition conditions on birth cohort and age because mortality rates and health expenses vary substantially across these groups.

To verify that our definition of the health states are reasonable, Panel A of Table 2 reports health problems that respondents face by age group and health state. Within each age group, respondents in poor health have a higher prevalence of doctor-diagnosed health problems. For example, among respondents aged 65 to 78, 27 percent of those in poor health have had cancer, which is higher than 10 percent of those in good health. Older respondents, especially those in poor health, have a higher prevalence of difficulty with activities of daily living. For example, among respondents aged 79 and older, 24 percent of those in poor health have some difficulty dressing, which is higher than 13 percent of those in good health.

Panel B of Table 2 reports health care utilization by age group and health state. Within each age group, respondents in poor health are more likely to have used health care in the two years prior to the interview. For example, among respondents aged 79 and older, 13 percent of those in poor health have stayed at a nursing home, which is higher than 5 percent of those in good health. This is consistent with the fact that respondents in poor health have higher out-of-pocket health expenses than those in good health.

Panel C of Table 2 reports the ownership rates of life insurance, annuities including private pensions, and long-term care insurance by age group and health state. Among respondents aged 51 to 64, 86 percent of those in poor health own some type of life insurance, which is comparable to 85 percent of those in good health. Although the ownership rate for life insurance falls in age, it remains remarkably high for older respondents. Among respondents aged 65 to 78, 56 percent of those in poor health receive annuity income from a private source that is not Social Security, which is comparable to 59 percent of those in good health.

Panel D of Table 2 reports the face value of life insurance and net worth by age group and health state. Among respondents aged 51 to 64 that own some type of life insurance, the median face value is \$90.1k for those in poor health, which is comparable to \$87.7k for those in good health. Among respondents aged 65 to 78, the median net worth excluding life insurance and annuities is \$230.8k for those in poor health, which is comparable to \$252.1k for those in good health.

3.3. Health and Mortality Risk

3.3.1. Health Transition Probabilities

After defining the three health states, we estimate the transition probabilities between the health states through an ordered probit model. The outcome variable is the health state at two years from the present interview. The explanatory variables include dummy variables for present health state and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with age, and cohort dummies. The dummy variable for 65 or older accounts for potential changes in household behavior that arise from eligibility for Social Security and Medicare. Our estimated transition probabilities for each cohort are the predicted probabilities from the ordered probit model.

To get a sense for these transition probabilities, Panel A of Table 3 reports the health distribution by age for a population of respondents who were born between 1936 and 1940 and are in good health at age 51. By age 79, 50 percent of the population are dead, and 15 percent are in poor health. Panel B reports the average life expectancy conditional on age and health. Respondents in poor health at age 51 are expected to live for 25 more years, which is shorter than 27 years for those in good health. The difference in life expectancy between poor and good health remains relatively constant for older respondents.

3.3.2. Out-of-Pocket Health Expenses

As explained in Appendix B, we use a panel regression model to estimate how out-of-pocket health expenses depend on birth cohort, age, health, and income. Our measure of out-of-pocket health expenses is comprehensive, including nursing home and end-of-life health expenses. We exclude households that own long-term care insurance in our estimation because the relevant concept of out-of-pocket health expenses in the life-cycle model is that in the absence of additional coverage.

Panel C of Table 3 reports average annual out-of-pocket health expenses by age and health for the cohort born between 1936 and 1940. For comparison, Panel D reports average annual income by age, which includes Social Security but excludes annuities and private pensions.[5] Households in poor health at age 51 have annual out-of-pocket health expenses of \$2.4k, which is higher than \$0.4k for those in good health. Out-of-pocket health expenses rise rapidly in old age (De Nardi, French, and Jones, 2010). Households in poor health at age 93 have annual out-of-pocket health expenses of \$33.4k, which is higher than \$5.1k for those in good health. Since annual income at age 93 is \$15.9k, households in poor health must partly cover health expenses through savings.

Panel E of Table 3 reports the present value of future disposable income by age and health. Households in good health at age 93 have −\$89.0k in lifetime disposable income because the present value of future health expenses exceeds the present value of future income. A younger household can insure this late-life risk by purchasing deferred annuities or long-term care insurance.

3.4. Pricing of Insurance

We do not observe the premiums that households actually pay for life insurance, annuities, and long-term care insurance. Therefore, in our baseline calibration, we assume that insurance is actuarially fair conditional on birth cohort, age, and health. That is, we set the discount rate on insurance products to be the same as the riskless interest rate of 2 percent (i.e., $R_L = R_A = R_H = R$). Insurance may not be actuarially fair in practice for various reasons including adverse selection, moral hazard, imperfect competition, regulation, and financial market frictions (Koijen and Yogo, 2012). To capture these possibilities, Section 5 reports an alternative calibration in which insurance is more expensive than actuarially fair.

The impact of private information on the pricing of insurance is ambiguous because adverse selection on health may be offset by advantageous selection on another dimension

[5] As explained in Appendix B, we use a panel regression model to estimate how income depends on birth cohort and age. Our specification does not include health because we found that those coefficients are statistically insignificant.

of private information such as preferences (de Meza and Webb, 2001). In life insurance markets, Cawley and Philipson (1999) find no evidence for private information. Although the pricing of annuities depends on gender and age only, Finkelstein and Poterba (2004) find evidence for separation along contract dimensions such as payout structure. In long-term care insurance markets, Finkelstein and McGarry (2006) find no significant relation between the ownership of insurance and future long-term care utilization, consistent with the absence of adverse selection or moral hazard. However, they argue that this may be due to private information about health offsetting unobserved preferences for insurance. Given the ambiguous nature of both the theoretical predictions and the empirical findings, the absence of private information is a natural starting point for our baseline calibration. However, we consider private information as a potential explanation for the heterogeneity in the demand for insurance products in Section 5.

3.5. Insurance Ownership

Figure 2 reports the ownership rates for term and whole life insurance, annuities including private pensions, and long-term care insurance. The ownership rate for term life insurance exceeds 70 percent for households aged 51 to 57. The ownership rate for annuities including private pensions is nearly 60 percent for households aged 65 to 71, while the ownership rate for long-term care insurance is just below 20 percent for the same group.

We do not observe the maturity of term life insurance or the exact coverage amount for long-term care insurance. Therefore, we must make some measurement assumptions to map these insurance products to their counterparts in the life-cycle model. We assume that term life insurance matures in two years and that whole life insurance matures at death. The assumption that term life insurance is short term is motivated by the fact that (annually renewable) group policies account for a large share of these policies. We assume that annuity income starts at age 65, which is the full Social Security retirement age, and terminates at death. We assume that the ownership of long-term care insurance corresponds to owning one unit of short-term supplemental health insurance. Therefore, a household that owns long-term care insurance is fully insured against uncertainty in health expenses for the subsequent period.

Conditional on ownership, households report the face value of term and whole life insurance. Measurement error in the face value of these policies could contaminate our estimates of health and mortality delta. As explained in Appendix B, we use a panel regression model to estimate how the face values of term and whole life insurance depend on birth cohort, age, health, and income. Instead of the observed face values, we use the predicted values with household fixed effects under the assumption that measurement error is transitory. We apply the same procedure to annuity and pension income.

We model all payoffs from insurance products as real. We normalize the death benefit of life insurance and the income from annuities to be \$1k in 2005 dollars. Modeling nominal payments for insurance products would introduce inflation risk, which is beyond the scope of this paper. Moreover, a cost-of-living-adjustment rider that effectively eliminates inflation risk is sometimes available for life insurance, annuities, and long-term care insurance. In the data, we deflate the face value of life insurance as well as pension and annuity income by the consumer price index to 2005 dollars.

3.6. Health and Mortality Delta Implied by Household Insurance Choice

For each household at each interview, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions, and long-term care insurance. The household's health delta is determined by positive health delta from whole life insurance and long-term care insurance, which is offset by negative health delta from annuities including private pensions. The household's mortality delta is determined by positive mortality delta from term and whole life insurance, which is offset by negative mortality delta from annuities including private pensions.

Figure 3 reports the health and mortality delta for each household-interview observation, together with the mean and the standard deviation at each age. Average health delta is negative throughout the life cycle. This implies that annuities have a predominant effect on the average household's health delta. Average mortality delta is positive for younger households and negative for older households. This implies that life insurance has a predominant effect on younger households' mortality delta, while annuities have a predominant effect for older households. The cross-sectional variation in mortality delta is significantly higher than that in health delta throughout the life cycle.

When we calculate the health delta for each household based solely on its ownership of annuities including private pensions, it explains 98 percent of the variation in the overall health delta. When we calculate the mortality delta for each household in a similar way, it explains 56 percent of the variation in the overall mortality delta. In addition, Panel C of Table 2 reports that private pensions, rather than the active purchase of individual annuities, account for most of private annuitization. Together, these facts imply that most of the variation in the observed health and mortality delta is driven by passive annuitization through private pensions.

4. Welfare Cost of Household Insurance Choice

In this section, we first estimate household preferences based on the observed demand for insurance. We then compare the observed demand to the optimal demand implied by the life-cycle model. Finally, we estimate the welfare cost of deviations from the optimal demand.

4.1. Estimating Household Preferences

Proposition 1 shows that the subjective discount factor is not separately identified from relative risk aversion since it enters through the term $(\beta R)^{1/\gamma}$. Therefore, we calibrate the subjective discount factor to $\beta = 0.96$ annually, which is a common choice in the life-cycle literature. We also normalize the utility weight for good health to $\omega(3) = 1$. We then stack the remaining preference parameters in a column vector as $\theta = [\gamma, \omega(1), \omega(2)]'$. For each household-interview observation $i \in \{1, \ldots, I\}$, let $L_i(\theta)$ denote the per-period welfare cost, implied by equation (26) for $n = 1$. We estimate household preferences to minimize the average per-period welfare cost, $\sum_{i=1}^{I} L_i(\theta)/I$. By construction, the welfare cost implied by the estimated preference parameters is a lower bound for the true welfare cost, under the identifying assumption of identical preferences. In Section 5, we relax this assumption and allow preferences to be household-specific.

We implement our estimation problem through continuous-updating generalized method of moments. Define the moment function

$$m(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{\partial L_i(\theta)}{\partial \theta} \tag{27}$$

and the weighting matrix

$$W(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{\partial L_i(\theta)}{\partial \theta} \frac{\partial L_i(\theta)}{\partial \theta'}. \tag{28}$$

Then our estimator for household preferences is

$$\widehat{\theta} = \arg\min_{\theta} m(\theta)' W(\theta)^{-1} m(\theta). \tag{29}$$

Table 4 reports our estimates of household preferences. Our estimate of relative risk aversion is 2.12 with a standard error of 0.01. Our point estimate of relative risk aversion is somewhat lower, and our standard errors are much smaller than previous estimates based on the Health and Retirement Study. In particular, our point estimate is in the lower range of the confidence interval in De Nardi, French, and Jones (2010), which is estimated from the realized path savings, instead of insurance choice (i.e., the desired path of wealth in future health states). Our point estimate is also lower than that in Barsky et al. (1997), which is based on responses to income gamble questions. Note that higher risk aversion necessarily implies higher welfare cost of insurance choice because Proposition 3 implies that the welfare cost is approximately linear in relative risk aversion.

Our estimate of the bequest motive is 5.17 with a standard error of 0.03. That is, households have a strong bequest motive that is equivalent to more than 5 periods (10 years) of consumption. The presence of a bequest motive is consistent with the survey evidence (Laitner and Juster, 1996; Ameriks et al., 2011). The right panel of Figure 3 explains why we find such a strong bequest motive. Average mortality delta is positive for younger households because many own life insurance, and only slightly negative for older households because many do not own annuities or private pensions. As emphasized by Bernheim (1991) and Brown (2001), an intentional bequest motive can simultaneously justify a strong demand for life insurance and a weak demand for annuities. The fact that our sample includes married men, who may want to leave wealth for a surviving spouse, partly explains why we find such a strong bequest motive. To allow for the possibility that the bequest motive may vary by martial status, or household characteristics more generally, we estimate the bequest motive separately by household in Section 5.

Our estimate of the utility weight for poor health is 0.76 with a standard error of 0.01. The left panel of Figure 3 explains why we find that consumption and health are complements. Average health delta is negative throughout the life cycle because few households own long-term care insurance, and many more households own annuities including private pensions. These ownership patterns reveal that the average household desires to deliver less wealth to poor future health states, which must be justified through a low marginal utility of consumption in poor health. Put differently, we should see a lot more demand for long-term care insurance if consumption and health were not strong complements. Finkelstein, Luttmer, and Notowidigdo (2013) also find evidence for complementarity of consumption and health, based on the relation between realized permanent income (proxy for consumption) and health, instead of insurance choice (i.e., the desired path of consumption in future health states).

4.2. Observed versus Optimal Demand for Insurance

The left panel of Figure 4 is a scatter plot of the observed health delta for each household-interview observation against the optimal health delta implied by the life-cycle model. The right panel is an analogous scatter plot for mortality delta. In both panels, the slope of the regression line is significantly less than one. That is, the life-cycle model generates significant variation in the optimal health and mortality delta that is not matched by the data. By construction, the variation in the optimal health and mortality delta depends only on the state variables of the life-cycle model, which are birth cohort, age, wealth, and health. Hence, the key takeaway is that even though the observed health and mortality delta vary significantly across households, they do not vary sufficiently along the state variables of the life-cycle model.

In the left panel of Figure 4, the 45-degree line divides the sample into two groups. Above the 45-degree line are households that have too much whole life insurance or long-term care insurance, whose health delta is higher than the optimal health delta. Below the 45-degree line are households that have too much annuities including private pensions, whose health delta is lower than the optimal health delta.

In the right panel of Figure 4, the 45-degree line divides the sample into two groups. Above the 45-degree line are households that are under-annuitized, whose mortality delta is higher than the optimal mortality delta. Below the 45-degree line are households that are over-annuitized, whose mortality delta is lower than the optimal mortality delta. This figure uncovers a new puzzle that is distinct from the so-called annuity puzzle in the literature. The unexplained variation in the degree to which households are annuitized, rather than the average level at which they are annuitized, is puzzling from the perspective of life-cycle theory.

4.3. Per-Period Welfare Cost

We now estimate the per-period welfare cost by applying Proposition 3 for $n = 1$. Conceptually, the per-period welfare cost assumes that the household deviates from the optimal health and mortality delta in the present period, then returns to the optimal demand for the remaining lifetime. While the per-period welfare cost is not our primary measure of interest, we can estimate it based on the observed ownership of insurance alone, without an auxiliary model for how this ownership may evolve over time.

Panel A of Table 5 reports the median per-period (two-year) welfare cost by age group. The per-period welfare cost for households aged 51 to 57 is 0.35 percent of total wealth with a standard error of 0.02 percent. Through equation (26) for $n = 1$, we can decompose this welfare cost into the sum of three parts. The difference between observed and optimal mortality delta essentially explains the entire welfare cost. This simply reflects the fact that mortality delta has significantly higher cross-sectional variation, as shown in Figure 3. The per-period welfare cost is relatively constant in age, which implies that the life-cycle model fits uniformly well over the life cycle.

4.4. Lifetime Welfare Cost

We now estimate the lifetime welfare cost by applying Proposition 3 for $n = T - t$. This is essentially a present-value calculation that accumulates the per-period welfare cost over the life cycle. This calculation requires an auxiliary model for how insurance ownership evolves over time. For simplicity, our baseline model assumes perfect persistence in ownership, which is a fairly close approximation to the data. Our results are similar with an alternative model based on estimated transition probabilities of ownership, which is described in Appendix C.

Panel B of Table 5 reports the median lifetime welfare cost by age group. The lifetime welfare cost for households aged 51 to 57 is 3.89 percent of total wealth with a standard error of 0.07 percent. This is a large welfare cost that is equivalent to almost 4 percent reduction in lifetime consumption, as implied by the homogeneity of preferences. Through equation (26) for $n = T - t$, we can decompose this welfare cost into the sum of three parts. The difference between observed and optimal health delta explains 0.23 percent of the welfare cost, while the difference between observed and optimal mortality delta explains 3.83 percent. The interaction between health and mortality delta explains the remainder of the welfare cost, which is -0.17 percent. The lifetime welfare cost is higher for younger households, for whom the per-period welfare cost accumulates over a longer expected lifetime.

5. Testing for Model Misspecification

In this section, we show that observed household characteristics fail to explain the deviations from the optimal demand for insurance. We then show that even unobserved heterogeneity in bequest motives cannot fully rationalize the observed demand. Finally, we show that our estimated welfare costs are robust to alternative assumptions about the pricing of insurance.

5.1. Misspecification along Observed Household Characteristics

In Table 6, we examine whether observed household characteristics, which capture potential preference heterogeneity or private information about health, explain the difference between observed and optimal health and mortality delta, scaled by total wealth. If these unmodeled characteristics are important determinants of insurance choice, they should have significant explanatory power for the residuals generated by the life-cycle model. Overall, we find little evidence for such model misspecification. That is, observed household characteristics have little explanatory power for the difference between observed and optimal health and mortality delta.

Married and more educated households have higher health delta than the optimal health delta predicted by the life-cycle model. However, children and race are not significant determinants of the difference between observed and optimal health delta. The coefficient on poor self-reported health is positive and significant. That is, households in worse health tend to own more whole life insurance and long-term care insurance and less annuities, which is consistent with adverse selection. However, these household characteristics ultimately explain little of the difference between observed and optimal health delta, as implied by an R^2 of 4 percent.

Households that are married, have living children, and are more educated have higher mortality delta than the optimal mortality delta predicted by the life-cycle model. This is consistent with the hypothesis that the bequest motive is stronger for these households. The coefficients on poor and fair self-reported health are positive and significant, while the coefficient on excellent self-reported health is negative and significant. That is, households in worse health tend to own more life insurance and less annuities, which is consistent with adverse selection. However, these household characteristics ultimately explain little of the difference between observed and optimal mortality delta, as implied by an R^2 of 6 percent.

In specifications that are not reported in Table 6, we have ruled out significant explanatory power for other variables that capture potential preference heterogeneity or private information about health. They are variables that capture heterogeneity in bequest motives (i.e., self-reported probability of leaving a bequest), risk aversion (i.e., responses to income gamble questions), and private information about health (i.e., difficulty with activities of daily living, self-reported probability of living to age 75, and self-reported probability of moving to a nursing home). Overall, the evidence suggests that the life-cycle model is not misspecified along these observed dimensions.

5.2. Unobserved Preference Heterogeneity

Unobserved heterogeneity in bequest motives is a potential explanation for the unexplained variation in mortality delta across households (Fang and Kung, 2012). According to this hypothesis, households with high mortality delta in Figure 3 simply have strong bequest motives, and those with low mortality delta have weak bequest motives. As discussed in Section 3.6, most of the variation in the observed health and mortality delta arises from heterogeneity in the ownership of private pensions. Therefore, this hypothesis requires an argument that households with weak bequest motives actively choose jobs that offer private pension benefits.

For each household, we estimate the bequest motive (i.e., $\omega(1)$) that minimizes the average per-period welfare cost. Figure 5 shows that there is considerable heterogeneity in the estimated bequest motive from none to over 10 periods (20 years) of consumption.

The left panel of Figure 6 is a scatter plot of the observed health delta for each household-interview observation against the optimal health delta implied by the life-cycle model with household-specific bequest motives. The right panel is an analogous scatter plot for mortality delta. In both panels, the slope of the regression line is much closer to one than in Figure 4. That is, household-specific bequest motives can rationalize the insurance choice of a given household in a given period. However, it does not necessarily rationalize the insurance choice of a given household over its entire lifetime, as we will discuss next.

As reported in Panel A of Table 7, the median per-period (two-year) welfare cost for households aged 51 to 57 is 0.02 percent of total wealth. However, as reported in Panel B, the lifetime welfare cost for the same group is 3.22 percent of total wealth. In comparison to Table 5, the per-period welfare cost is significantly lower, but the lifetime welfare cost remains large. The reason for this result is that the welfare cost of insurance choice is not only generated by the variation across households, but also by the variation within a household over the life cycle. As shown in Section 6, the optimal insurance choice requires that households actively rebalance from positive to negative mortality delta (i.e., life insurance to annuities) over the life cycle. As discussed in Section 3, actual insurance choice is much more persistent, due to inertia that arises from passive annuitization through private pensions.

5.3. Actuarially Unfair Insurance

The baseline estimates in Table 5 are based on the assumption that insurance is actuarially fair. In Table 8, we consider an alternative scenario in which insurance is more expensive than actuarially fair. We assume that the discount rate (or the expected return) on life insurance, annuities, and long-term care insurance is 0 percent, while the riskless interest rate is 2 percent. This is a fairly extreme assumption that corresponds to the upper range of estimates for deviations from actuarially fair pricing in the annuity market (Mitchell et al., 1999). Both the per-period and the lifetime welfare costs are essentially the same as the baseline estimate in Table 5. Therefore, our findings are robust to alternative assumptions about the pricing of insurance.

6. Optimal Portfolio of Existing Insurance Products

In this section, we illustrate how a portfolio of existing insurance products can replicate the optimal health and mortality delta implied by the life-cycle model. Our illustration is for a male, who was born between 1936 and 1940 and is in good health at age 51. The household faces the health transition probabilities, out-of-pocket health expenses, and income that are reported in Table 3. The household's initial wealth is $87.8k at age 51, which is chosen to match average net worth excluding life insurance and annuities for this cohort. In addition to bonds, the household can save in short-term life insurance, deferred annuities, and long-term care insurance (i.e., short-term supplemental health insurance). Figure 1 reports the health and mortality delta for these insurance products at age 51. The household's preference parameters are those that we estimate in the Health and Retirement Study, reported in Table 4.

Panel A of Table 9 reports the optimal health and mortality delta, which we calculate through Proposition 1. The optimal health delta is \$1.8k at age 51, which implies that the household desires an additional \$1.8k in poor health relative to good health at age 53. As equation (21) shows, three offsetting forces determine the optimal health delta. First, the household has preference for consumption in good health over poor health (i.e., $\omega(2) < \omega(3)$ in Table 4), which lowers the optimal health delta. Second, the household saves less in poor health because of shorter life expectancy (i.e., $c_{t+1}(2) > c_{t+1}(3)$), which lowers the optimal health delta. Third, the household has lower lifetime disposable income in poor health, which raises the optimal health delta. The third force more than offsets the first two, so that the optimal health delta is overall positive at age 51.

The optimal mortality delta is \$264.4k at age 51, which implies that the household desires to leave an additional \$264.4k at death relative to good health at age 53. As equation (22) shows, three offsetting forces determine the optimal mortality delta. First, the household has preference for bequest over consumption in good health (i.e., $\omega(1) > \omega(3)$ in Table 4), which raises the optimal mortality delta. Second, the household must save for future consumption in good health (i.e., $c_{t+1}(3) < 1$), which lowers the optimal mortality delta. Third, the household has higher lifetime disposable income in good health, which raises the optimal mortality delta. The first and third forces more than offset the second, so that the optimal mortality delta is overall positive at age 51.

Panel B of Table 9 reports a portfolio of life insurance, deferred annuities, and long-term care insurance that replicates the optimal health and mortality delta, which we calculate through Proposition 2. The optimal portfolio at age 51 consists of 264.4 units (i.e., death benefit of \$264.4k) of life insurance, no deferred annuities, 0.4 units of long-term care insurance, and 81.7 units of bonds. Panel C reports the cost of the optimal portfolio, which is the sum of \$8.8k in life insurance, \$0.5k in long-term care insurance, and \$78.6k in bonds.

The left panel of Figure 7 shows that the optimal health delta has a slightly U-shaped profile over the life cycle. To replicate the optimal health delta, the household needs long-term care insurance at age 86 and older when out-of-pocket health expenses start to rise dramatically. Since one unit of long-term care insurance eliminates all uncertainty in health expenses in the subsequent period, the positions reported in Panel B of Table 9 imply that the household demands only partial coverage throughout the life cycle. This is partly explained by the fact that consumption and health are complements. Moreover, higher health expenses in poor health are offset by shorter life expectancy, lowering the optimal health delta relative to full coverage.

The right panel of Figure 7 shows that the optimal mortality delta falls over the life cycle. To replicate the optimal mortality delta, the household needs life insurance when young to generate positive mortality delta, then switch to deferred annuities when old to generate negative mortality delta. The optimal position in deferred annuities increases from 5.5 units at age 58 to 63.1 units at age 93. A practical implication of Figure 7 is that an insurance company may want to package life insurance and deferred annuities into a product that automatically replicates the life-cycle profile for optimal mortality delta, eliminating the need for active rebalancing.

In this illustration, the household is exposed to reclassification risk because it has access to only short-term life and supplemental health insurance. For example, a household in good health at age 51 has to pay a higher premium for life and supplemental health insurance at age 53 if its health declines. As emphasized by Cochrane (1995), the household can insure reclassification risk in a world with health state-contingent securities. Our illustration here shows that an optimal portfolio of short-term insurance products essentially replicates health state-contingent securities, thereby insuring reclassification risk.

7. Conclusion

In this paper, we find large welfare cost of deviations from the optimal demand for insurance. We have reasons to believe that this is a consequence of suboptimal choice for many households. First, the variation in the observed demand is mostly driven by heterogeneity and inertia that arises from passive annuitization through private pensions. Second, we calibrate the life-cycle model to the Health and Retirement Study and find that a typical household can replicate the optimal health and mortality delta through existing insurance products. Finally, there has been little academic guidance on optimal portfolio choice for insurance products, unlike for equity and fixed-income products. Because of this lack of guidance, existing financial calculators (available from insurance companies) make recommendations for life insurance, annuities, and long-term care insurance in isolation, instead of as a comprehensive financial decision.

To improve household insurance choice, retail financial advisors and insurance companies should report the health and mortality delta of their insurance products, just as mutual fund companies already report the market beta of their equity products and the duration of their fixed-income products. We hope that these risk measures will facilitate standardization, identify overlap between existing products, identify risks that are not insured by existing products, and ultimately lead to new product development. One such product that we find particularly promising is a life-cycle product that automatically shifts from life insurance to annuities as a function of age, so that households achieve the optimal mortality delta over the life cycle without active rebalancing. This product would be analogous to life-cycle funds that automatically shift from equity to fixed income as a function of age, which have proven to be tremendously successful in the mutual fund industry.

Smarter default plans for employer-provided insurance and retirement accounts is yet another way to improve household insurance choice, especially for the financially illiterate. The default plan for group life insurance could start with a higher death benefit for younger workers, and let it gradually decline to no coverage at retirement age. Defined contribution plans could annuitize a share of savings by default, thereby mimicking a defined benefit plan. Thus, a combination of group life insurance and annuitization through retirement accounts could replicate the optimal mortality delta over the life cycle, without active decisions on the part of employees. These simple changes to the default plan only affect the allocation of wealth across future health states, without changing the overall level of savings. Hence, these changes are essentially cost neutral to the employer, and the resulting improvement in the employees' welfare comes for free.

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Table 1: Predicting Future Mortality with Observed Health Problems

A probit model is used to predict death within two years from the present interview. The table reports the marginal effects on the mortality rate (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The omitted cohort is respondents born prior to 1911. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Explanatory variable	Marginal effect	t-statistic
Doctor-diagnosed health problems:		
High blood pressure	0.70	(1.64)
Diabetes	4.49	(5.32)
Cancer	10.43	(7.10)
Lung disease	6.07	(4.57)
Heart problems	1.98	(3.39)
Stroke	3.62	(2.84)
$(\text{Age} - 51)/10$	2.28	(11.16)
$\times$ High blood pressure	-0.03	(-0.18)
$\times$ Diabetes	-0.61	(-2.85)
$\times$ Cancer	-1.39	(-6.29)
$\times$ Lung disease	0.03	(0.11)
$\times$ Heart problems	0.08	(0.44)
$\times$ Stroke	-0.02	(-0.08)
Birth cohort:		
1911–1915	-1.24	(-3.77)
1916–1920	-1.83	(-6.73)
1921–1925	-2.56	(-10.94)
1926–1930	-3.02	(-12.63)
1931–1935	-3.34	(-10.54)
1936–1940	-3.62	(-9.38)
1941–1945	-3.11	(-10.29)
1946–1950	-3.20	(-13.49)
1951–1955	-2.84	(-9.99)
Correctly predicted (percent):		
Both outcomes	94	
Dead only	66	
Alive only	94	
Observations	38,913	

Table 2: Health Problems, Health Care Utilization, and Insurance Ownership

Panel D reports the median of total face value conditional on ownership, deflated by the consumer price index to 2005 dollars. Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. Net worth excludes the value of life insurance, annuities, and private pensions. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Age	51–64		65–78		79–	
Health	Poor	Good	Poor	Good	Poor	Good
Panel A: Doctor-diagnosed health problems and difficulty with activities of daily living (percent)						
High blood pressure	57	28	65	46	63	45
Diabetes	19	8	34	13	26	14
Cancer	8	3	27	10	32	20
Lung disease	7	3	19	5	21	6
Heart problems	23	9	53	21	73	33
Stroke	5	2	15	4	30	9
Some difficulty bathing	3	1	5	2	21	8
Some difficulty dressing	6	3	10	5	24	13
Some difficulty eating	1	0	3	1	12	4
Panel B: Health care utilization (percent)						
Monthly doctor visits	9	3	16	5	21	11
Hospital stay	25	11	42	22	55	33
Outpatient surgery	22	16	28	23	27	26
Nursing home stay	0	0	2	1	13	5
Home health care	3	1	10	4	22	9
Special facilities and services	7	4	10	6	14	9
Prescription drugs	79	49	95	76	97	84
Panel C: Life insurance, annuities, and long-term care insurance (percent)						
All life insurance	86	85	77	77	68	69
Term life insurance	72	70	56	58	46	48
Whole life insurance	35	34	33	31	28	28
Annuities including private pensions	46	49	56	59	58	63
Annuities excluding private pensions	1	1	4	4	6	7
Long-term care insurance	8	9	18	20	17	18
Panel D: Face value of life insurance and net worth (median in thousands of 2005 dollars)						
All life insurance	90.1	87.7	27.8	26.9	13.9	11.7
Term life insurance	75.0	78.5	22.2	22.3	10.5	9.7
Whole life insurance	42.8	40.9	23.0	23.0	16.3	13.9
Net worth	152.6	173.1	230.8	252.1	229.5	234.5
Observations	7,360	11,901	4,250	9,516	1,512	3,488

Table 3: Health Dynamics, Out-of-Pocket Health Expenses, and Income

Panels A and B are based on the ordered probit model for health transition probabilities. Panel C is based on the panel regression model for out-of-pocket health expenses. Panel D is based on the panel regression model for income. Panel E reports the present value of future disposable income (i.e., income in excess of out-of-pocket health expenses), based on the health transition probabilities and a riskless interest rate of 2 percent. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

	Age						
Health	51	58	65	72	79	86	93
Panel A: Health distribution (percent)							
Dead	0	12	26	36	50	65	88
Poor	0	24	20	16	15	14	7
Good	100	64	54	49	35	21	5
Panel B: Remaining life expectancy (years)							
Poor	25	22	17	13	8	6	4
Good	27	24	20	16	12	9	6
Mean	27	24	19	16	11	8	5
Panel C: Out-of-pocket health expenses (thousands of 2005 dollars per year)							
Poor	2.4	2.9	3.5	4.1	6.8	12.0	33.4
Good	0.4	0.7	0.9	1.2	1.8	2.6	5.1
Mean	0.4	1.3	1.6	1.9	3.3	6.4	22.2
Panel D: Income (thousands of 2005 dollars per year)							
Mean	55.8	48.4	30.2	24.8	20.0	17.8	15.9
Panel E: Present value of future disposable income (thousands of 2005 dollars)							
Poor	538.8	375.7	214.2	127.9	41.2	-5.7	-55.1
Good	582.0	423.3	261.1	169.1	66.3	-0.9	-89.0
Mean	582.0	410.2	248.5	159.1	58.8	-2.9	-68.5

Table 4: Estimates of Household Preferences

The subjective discount factor is calibrated to 0.96 annually, and the utility weight for good health is normalized to one. The remaining preference parameters are estimated by continuous-updating generalized method of moments with heteroskedasticity-robust standard errors in parentheses. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Parameter	Symbol	Value
Subjective discount factor	β	0.96
Relative risk aversion	γ	2.12
		(0.01)
Bequest motive	$\omega(1)$	5.17
		(0.03)
Utility weight for poor health	$\omega(2)$	0.76
		(0.01)
Utility weight for good health	$\omega(3)$	1.00
Observations		27,792

Table 5: Welfare Cost of Household Insurance Choice

The welfare cost for each household is measured by the difference between observed and optimal health and mortality delta. The lifetime welfare cost assumes perfect persistence in insurance ownership. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in percentage of total wealth)						
Total cost	0.35	0.25	0.21	0.26	0.32	0.37
	(0.02)	(0.02)	(0.02)	(0.04)	(0.10)	(0.70)
Cost due to health delta	0.00	0.01	0.03	0.08	0.13	0.12
	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)	(0.68)
Cost due to mortality delta	0.35	0.24	0.17	0.20	0.28	0.39
	(0.02)	(0.02)	(0.03)	(0.06)	(0.16)	(0.59)
Panel B: Lifetime welfare cost (median in percentage of total wealth)						
Total cost	3.89	3.76	3.65	2.99	2.09	1.35
	(0.07)	(0.06)	(0.08)	(0.16)	(0.37)	(0.95)
Cost due to health delta	0.23	0.30	0.44	0.59	0.53	0.34
	(0.00)	(0.00)	(0.01)	(0.03)	(0.20)	(0.85)
Cost due to mortality delta	3.83	3.72	3.67	3.23	2.30	1.39
	(0.08)	(0.07)	(0.08)	(0.14)	(0.38)	(0.83)

Table 6: Explaining the Difference between Observed and Optimal Health and Mortality Delta

A linear regression model is estimated to explain the difference between observed and optimal health and mortality delta, scaled by total wealth. The table reports the regression coefficients with heteroskedasticity-robust t-statistics in parentheses. The omitted categories for the dummy variables are non-high school graduate, white, good self-reported health, and born prior to 1911. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Explanatory variable	Health delta		Mortality delta	
65 or older	0.13	(0.32)	0.87	(0.37)
Poor health	-0.04	(-0.73)	1.20	(1.37)
Married	0.33	(2.84)	10.00	(5.35)
Has living children	-0.04	(-0.43)	4.29	(2.08)
Education:				
High school graduate	0.42	(3.14)	5.27	(2.91)
College graduate	0.65	(4.99)	11.70	(6.64)
Race:				
Black	0.05	(0.47)	-5.77	(-2.55)
Hispanic and other	0.13	(0.99)	-7.60	(-2.79)
Self-reported health status:				
Poor	0.22	(2.64)	2.82	(2.26)
Fair	0.10	(1.43)	2.27	(1.98)
Very good	-0.06	(-0.38)	-1.42	(-0.74)
Excellent	-0.09	(-0.53)	-8.20	(-1.94)
$(\text{Age} - 51)/10$	-1.10	(-2.68)	2.87	(0.60)
× 65 or older	-0.04	(-0.10)	2.15	(0.57)
× Poor health	0.39	(3.56)	2.46	(2.63)
× Married	0.10	(0.48)	-5.25	(-2.90)
× Has living children	0.40	(1.96)	-2.35	(-1.11)
× High school graduate	-0.16	(-0.71)	-2.27	(-1.34)
× College graduate	-0.37	(-1.72)	-8.42	(-5.04)
× Black	-0.33	(-1.59)	4.78	(1.86)
× Hispanic and other	-0.94	(-3.26)	1.47	(0.45)
× Poor	-0.47	(-2.70)	-4.11	(-2.85)
× Fair	-0.07	(-0.49)	-2.22	(-1.79)
× Very good	0.05	(0.19)	0.73	(0.40)
× Excellent	-0.28	(-1.14)	5.26	(1.34)
$(\text{Age} - 51)^2/100$	0.32	(1.97)	2.83	(1.25)
× 65 or older	-0.17	(-1.26)	-3.23	(-1.51)
× Poor health	-0.07	(-1.95)	-0.83	(-3.89)
× Married	-0.04	(-0.66)	0.72	(1.85)
× Has living children	-0.13	(-2.37)	0.30	(0.65)
× High school graduate	0.03	(0.48)	0.21	(0.58)
× College graduate	0.10	(1.58)	1.52	(4.27)
× Black	0.08	(1.27)	-1.03	(-1.71)
× Hispanic and other	0.23	(2.66)	0.20	(0.24)
× Poor	0.11	(2.04)	0.94	(2.76)
× Fair	0.00	(0.05)	0.46	(1.60)
× Very good	-0.02	(-0.22)	-0.09	(-0.23)
× Excellent	0.07	(1.12)	-0.87	(-1.08)
Birth cohort:				
1911–1915	-0.21	(-1.30)	0.24	(0.94)
1916–1920	-0.38	(-2.31)	0.53	(2.25)
1921–1925	-0.51	(-2.75)	0.80	(3.29)
1926–1930	-0.93	(-4.66)	-0.43	(-1.52)
1931–1935	-1.38	(-6.72)	-2.12	(-5.70)
1936–1940	-1.83	(-8.72)	-5.02	(-10.01)
1941–1945	-2.22	(-10.25)	-8.13	(-11.96)
1946–1950	-2.20	(-10.14)	-8.74	(-10.87)
1951–1955	-2.39	(-10.98)	-19.60	(-11.41)
R^2	0.04		0.06	
Observations	27,458		27,458	

Table 7: Welfare Cost of Household Insurance Choice under Household-Specific Bequest Motives

This table reports the welfare cost under an alternative assumption that the bequest motive is household-specific. Figure 5 reports the distribution of the estimated bequest motives. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in percentage of total wealth)						
Total cost	0.02	0.03	0.03	0.03	0.02	0.04
	(0.00)	(0.00)	(0.00)	(0.01)	(0.03)	(0.63)
Cost due to health delta	0.01	0.01	0.02	0.02	0.01	0.02
	(0.01)	(0.01)	(0.02)	(0.03)	(0.08)	(0.65)
Cost due to mortality delta	0.01	0.01	0.01	0.01	0.01	0.02
	(0.01)	(0.01)	(0.02)	(0.03)	(0.08)	(0.45)
Panel B: Lifetime welfare cost (median in percentage of total wealth)						
Total cost	3.22	3.02	2.88	1.91	1.14	0.87
	(0.28)	(0.22)	(0.26)	(0.30)	(0.46)	(1.01)
Cost due to health delta	0.38	0.38	0.48	0.38	0.26	0.34
	(0.08)	(0.06)	(0.08)	(0.09)	(0.26)	(0.96)
Cost due to mortality delta	3.20	3.02	2.95	1.98	1.07	0.50
	(0.30)	(0.23)	(0.27)	(0.29)	(0.46)	(0.94)

Table 8: Welfare Cost of Household Insurance Choice under Actuarially Unfair Insurance
This table reports the welfare cost under an alternative assumption that insurance is more expensive than actuarially fair. The discount rates on life insurance, annuities, and long-term care insurance are calibrated to 0 percent annually, while the riskless interest rate is 2 percent. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in percentage of total wealth)						
Total cost	0.35	0.25	0.21	0.27	0.35	0.40
	(0.02)	(0.02)	(0.02)	(0.04)	(0.10)	(0.69)
Cost due to health delta	0.00	0.01	0.03	0.09	0.14	0.14
	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)	(0.67)
Cost due to mortality delta	0.35	0.24	0.18	0.21	0.30	0.43
	(0.02)	(0.02)	(0.03)	(0.06)	(0.16)	(0.58)
Panel B: Lifetime welfare cost (median in percentage of total wealth)						
Total cost	3.93	3.79	3.74	3.16	2.19	1.46
	(0.07)	(0.06)	(0.08)	(0.16)	(0.37)	(0.93)
Cost due to health delta	0.23	0.30	0.44	0.61	0.56	0.37
	(0.00)	(0.01)	(0.01)	(0.03)	(0.20)	(0.84)
Cost due to mortality delta	3.90	3.80	3.80	3.40	2.40	1.51
	(0.08)	(0.07)	(0.08)	(0.14)	(0.37)	(0.82)

Table 9: Optimal Portfolio of Insurance Products

Panel A reports the optimal health and mortality delta by age, implied by the life-cycle model with the preference parameters in Table 4. Panel B reports a portfolio of short-term life insurance, deferred annuities, long-term care insurance (i.e., short-term supplemental health insurance), and bonds that replicates the optimal health and mortality delta. Short-term policies mature in two years, and the income from deferred annuities start at age 65. Panel C reports the cost of the optimal portfolio in thousands of 2005 dollars, averaged across the health distribution at the given age. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

	Age						
	51	58	65	72	79	86	93
Panel A: Optimal health and mortality delta (thousands of 2005 dollars)							
Health delta	1.8	-4.3	-13.2	-18.9	-24.3	-23.9	6.3
Mortality delta	264.4	125.5	15.6	-26.2	-66.2	-96.9	-153.6
Panel B: Optimal portfolio (units)							
Life insurance	264.4	156.0	93.4	60.1	19.0	0.0	0.0
Deferred annuities	0.0	5.5	10.0	13.2	17.8	26.8	63.1
Long-term care insurance	0.4	0.0	0.0	0.0	0.0	0.3	0.7
Bonds	81.7	170.1	207.8	223.8	243.2	247.1	228.2
Panel C: Cost of the optimal portfolio (thousands of 2005 dollars)							
Life insurance	8.8	6.0	4.0	3.0	1.7	0.0	0.0
Deferred annuities	0.0	27.2	68.5	74.4	67.4	66.8	75.0
Long-term care insurance	0.5	0.0	0.0	0.0	0.0	2.3	20.7
Bonds	78.6	163.5	199.8	215.1	233.8	237.5	219.4
Total cost	87.8	196.6	272.3	292.4	302.8	306.6	315.1

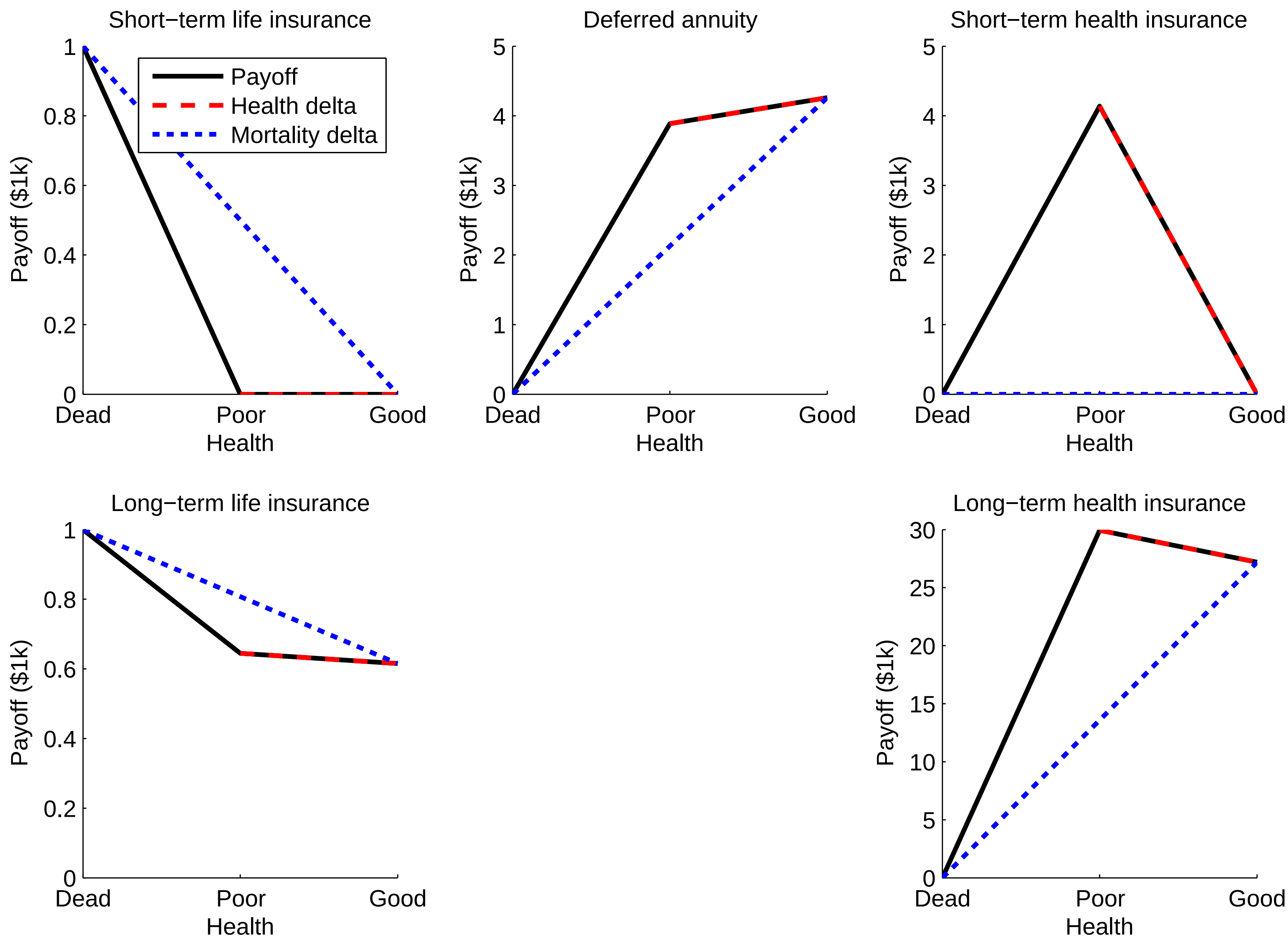

Figure 1: Health and Mortality Delta for Insurance Products

This figure reports the health and mortality delta for life insurance, deferred annuities, and supplemental health insurance. The solid line represents the payoff of each policy for the three possible health states in two years, reported in thousands of 2005 dollars. Short-term policies mature in two years (i.e., the frequency of interviews in the Health and Retirement Study), while long-term policies mature at death. The income from deferred annuities start at age 65. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

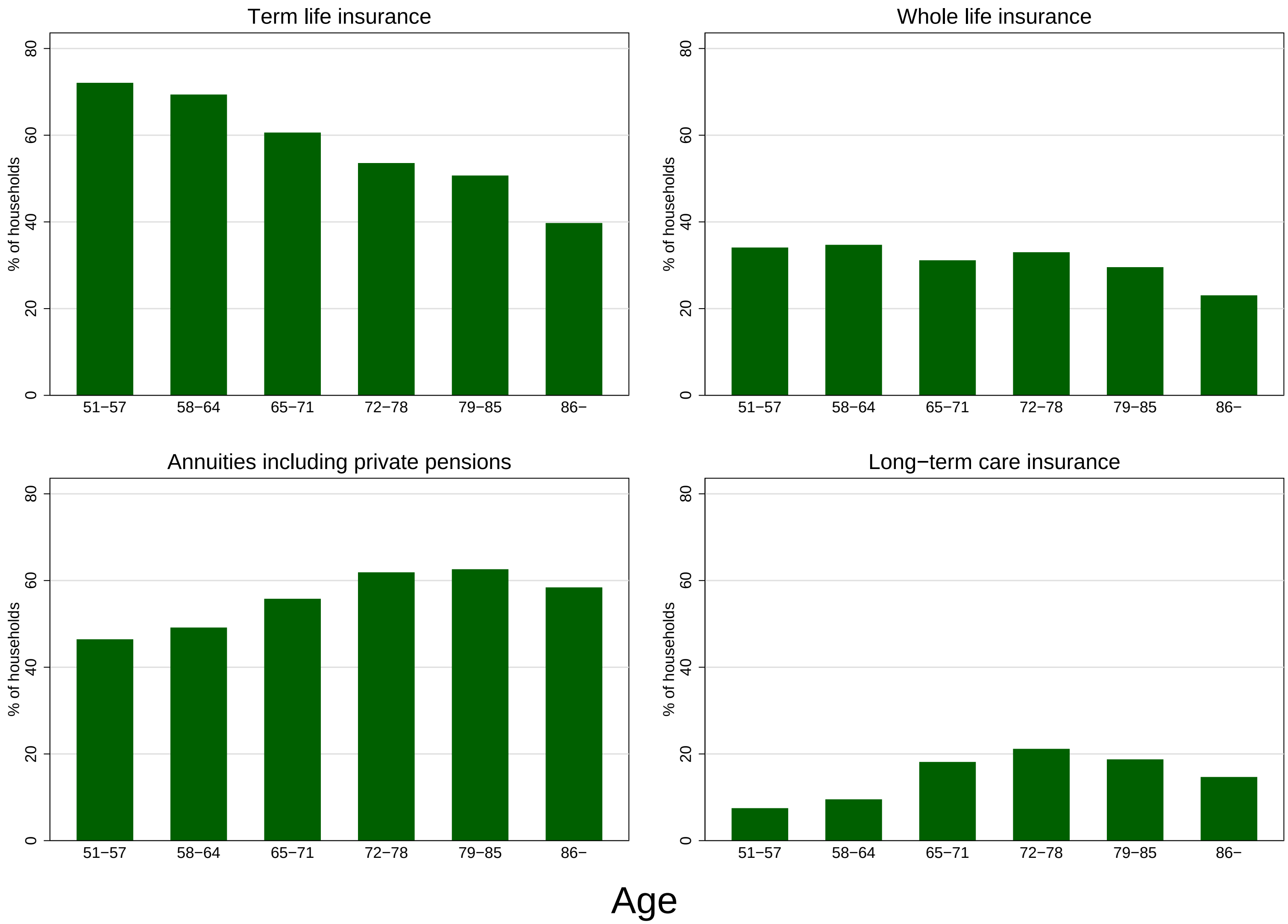

Figure 2: Rates of Insurance Ownership

Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

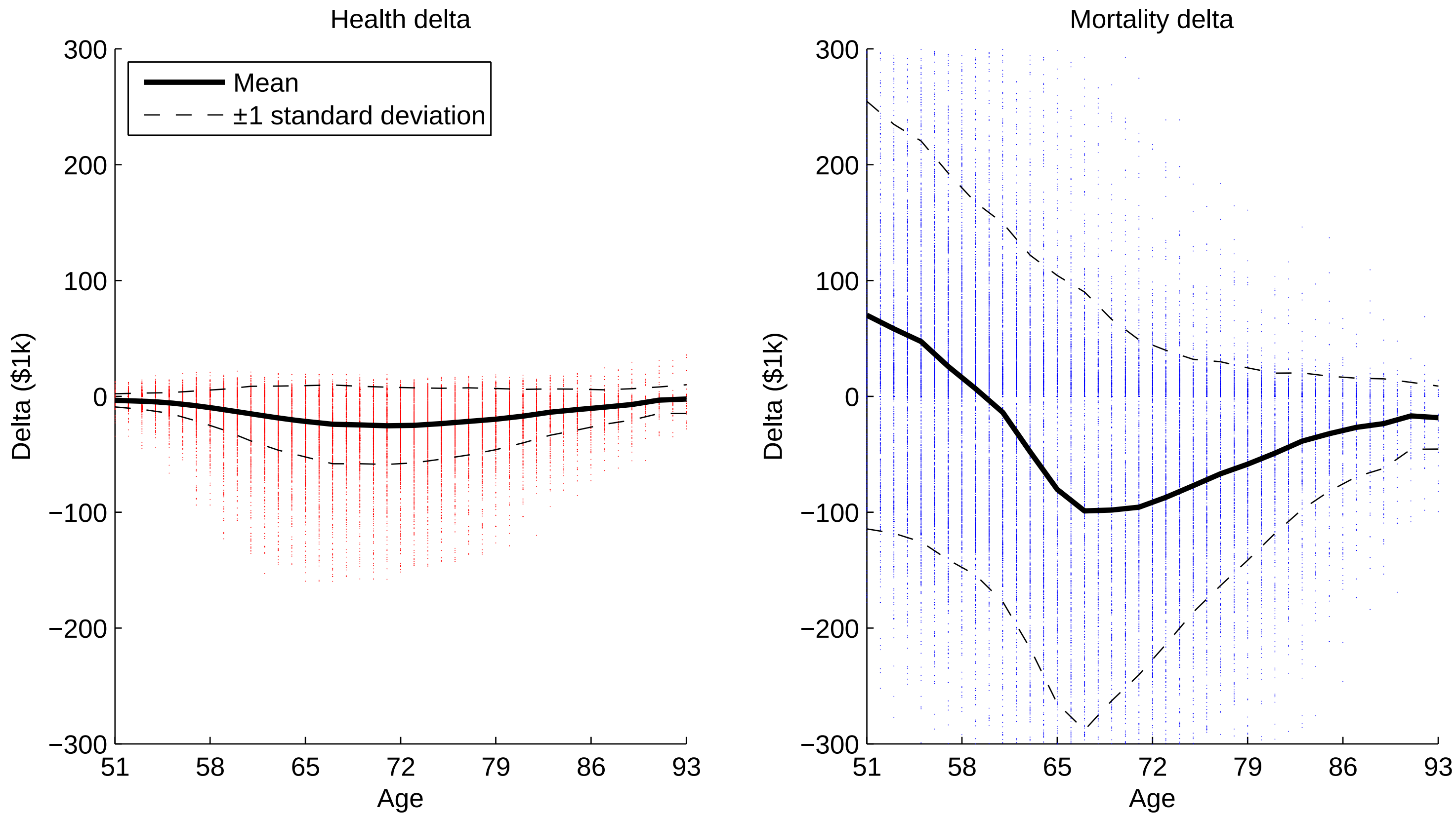

Figure 3: Observed Health and Mortality Delta over the Life Cycle
Each dot in the left (right) panel represents a household-interview observation of health (mortality) delta. The figure also reports the mean and the standard deviation by age, smoothed around a plus or minus one-year window. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

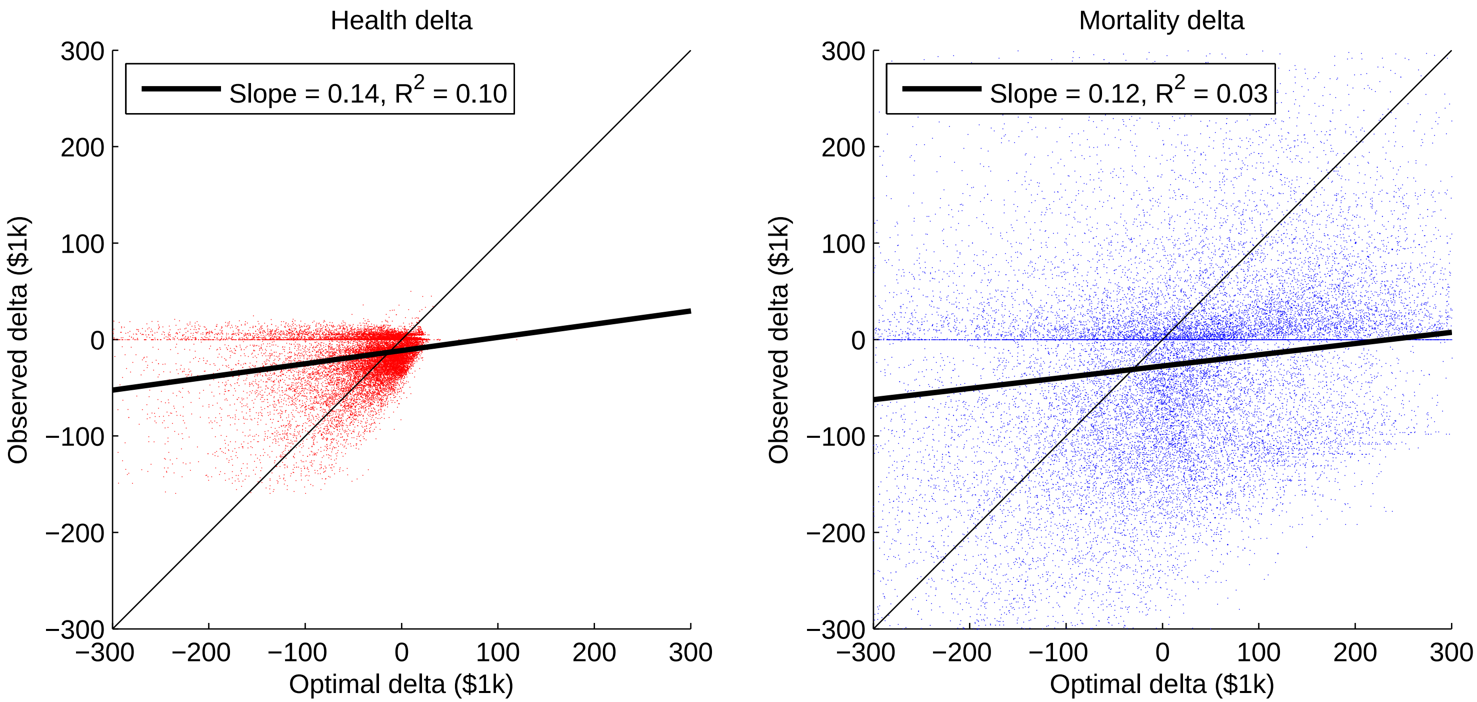

Figure 4: Observed versus Optimal Health and Mortality Delta
The left (right) panel is a scatter plot of the observed versus optimal health (mortality) delta. The optimal health and mortality delta are implied by the life-cycle model with the preference parameters in Table 4. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

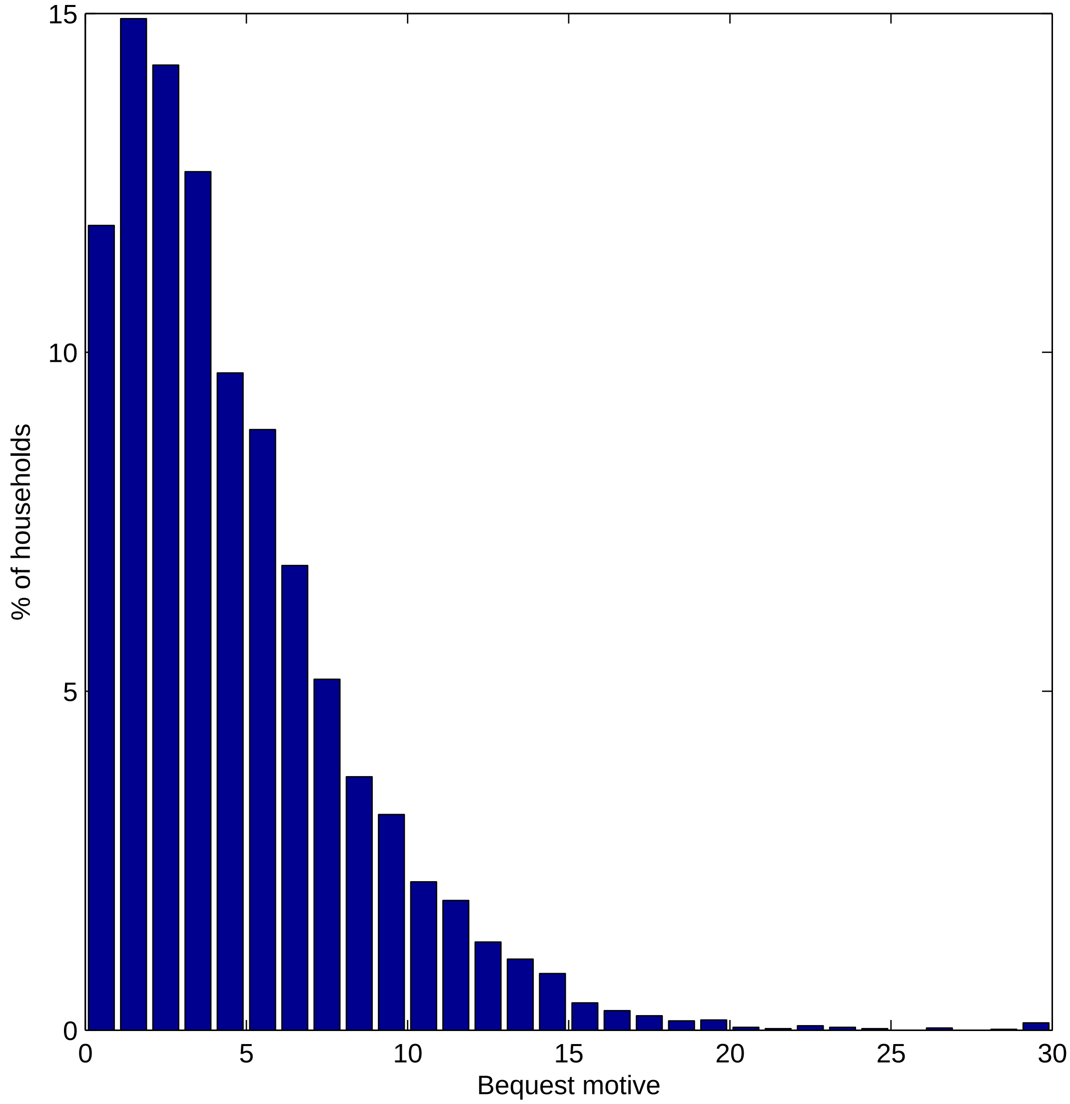

Figure 5: Distribution of Estimated Bequest Motives
The bequest motive (i.e., $\omega(1)$) is estimated for each household to minimize the average per-period welfare cost. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

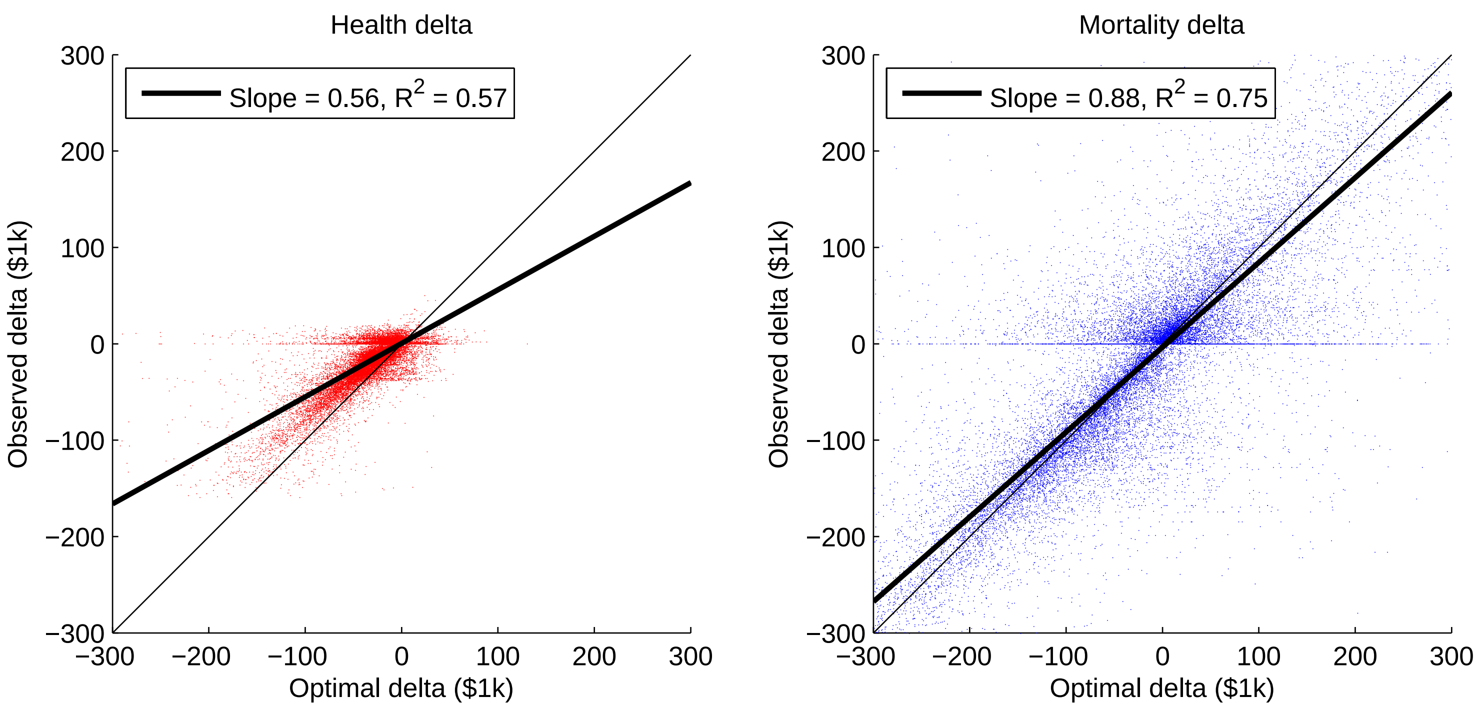

Figure 6: Observed versus Optimal Health and Mortality Delta under Household-Specific Bequest Motives
The left (right) panel is a scatter plot of the observed versus optimal health (mortality) delta. The optimal health and mortality delta are implied by the life-cycle model with household-specific bequest motives, whose distribution is reported in Figure 5. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

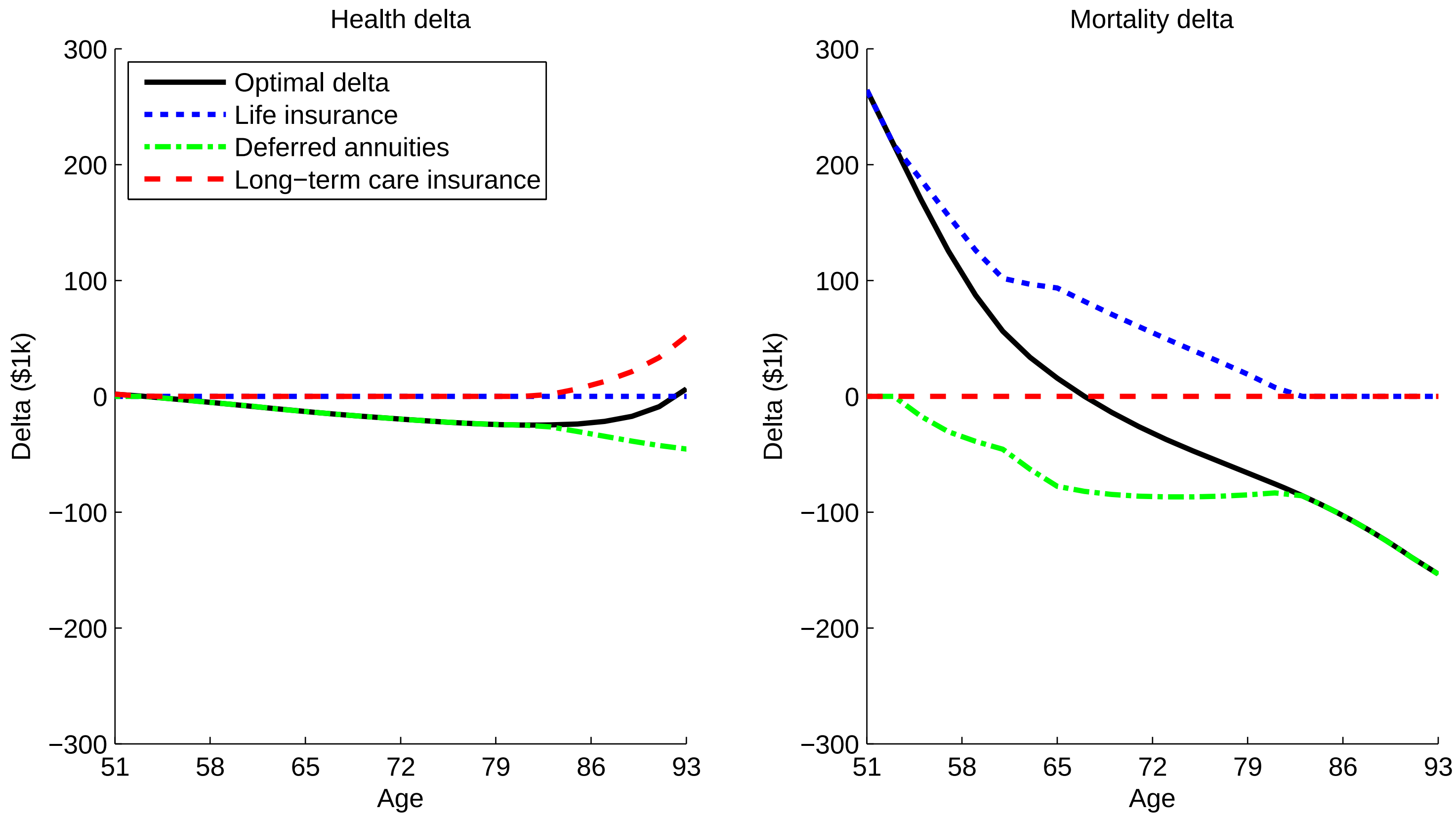

Figure 7: Optimal Health and Mortality Delta over the Life Cycle
The sum of health (mortality) delta for short-term life insurance, deferred annuities, and long-term care insurance (i.e., short-term supplemental health insurance) equals the optimal health (mortality) delta at each age. Short-term policies mature in two years, and the income from deferred annuities start at age 65. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

Appendix

A. Proof of the Propositions

A.1. Proof of Proposition 1

The household maximizes the objective function (16) subject to the intertemporal budget constraint, which we rewrite as

$$A_t + Y_t - M_t - C_t = \sum_{j=1}^{3} \frac{\pi_t(h_t, j)}{R} A_{t+1}(j). \tag{A1}$$

The Bellman equation in period t is

$$\begin{aligned} V_t(h_t, A_t) = \max_{C_t, A_{t+1}(1), A_{t+1}(2), A_{t+1}(3)} & \Big\{\omega(h_t)^\gamma C_t^{1-\gamma} \\ & + \beta \left[\pi_t(h_t, 1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) V_{t+1}(j, A_{t+1}(j))^{1-\gamma} \right] \Big\}^{1/1-\gamma}. \end{aligned} \tag{A2}$$

The proposition claims that the optimal health state-contingent wealth policies are given by

$$A^*_{t+1}(1) = \frac{(\beta R)^{1/\gamma} \omega(1) C^*_t}{\omega(h_t)}, \tag{A3}$$

$$A^*_{t+1}(j) = \frac{(\beta R)^{1/\gamma} \omega(j) C^*_t}{\omega(h_t) c_{t+1}(j)} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|j]}{R^{s-1}} \; \forall j \in \{2, 3\}. \tag{A4}$$

The proof proceeds by backward induction.

To simplify notation, we define total wealth as cash-on-hand plus the present value of future disposable income:

$$W_t = A_t + \sum_{s=0}^{T-t} \frac{\mathbf{E}_t[Y_{t+s} - M_{t+s}|h_t]}{R^s}. \tag{A5}$$

Because the household dies with certainty in period $T+1$, optimal consumption in period T is $C^*_T = W_T$. Thus, the value function in period T is

$$V_T(h_T, A_T) = \omega(h_T)^{\gamma/(1-\gamma)} W_T. \tag{A6}$$

The first-order conditions in period $T-1$ are

$$\begin{aligned}\omega(h_{T-1})^{\gamma}C_{T-1}^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_T^*(1)^{-\gamma}\\ =&\beta R\omega(j)^{\gamma}(A_T^*(j)+Y_T(j)-M_T(j))^{-\gamma}\ \forall j\in\{2,3\}.\end{aligned} \tag{A7}$$

These equations, together with equation (A1), imply the policy functions (20), (A3), and (A4) for period $T-1$. Substituting the policy functions into the Bellman equation, the value function in period $T-1$ is

$$V_{T-1}(h_{T-1},A_{T-1})=\left(\frac{\omega(h_{T-1})}{c_{T-1}(h_{T-1})}\right)^{\gamma/(1-\gamma)}W_{T-1}. \tag{A8}$$

Suppose that the value function in period $t+1$ is

$$V_{t+1}(h_{t+1},A_{t+1})=\left(\frac{\omega(h_{t+1})}{c_{t+1}(h_{t+1})}\right)^{\gamma/(1-\gamma)}W_{t+1}. \tag{A9}$$

The first-order conditions in period t are

$$\begin{aligned}\omega(h_t)^{\gamma}C_t^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_{t+1}^*(1)^{-\gamma}\\ =&\frac{\beta R\omega(j)^{\gamma}}{c_{t+1}(j)^{\gamma}}\left(A_{t+1}^*(j)+\sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s}-M_{t+s}|j]}{R^{s-1}}\right)^{-\gamma}\quad\forall j\in\{2,3\}.\end{aligned} \tag{A10}$$

These equations, together with equation (A1), imply the policy functions (20), (A3), and (A4) for period t. Substituting the policy functions into the Bellman equation, the value function in period t is

$$V_t(h_t,A_t)=\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{\gamma/(1-\gamma)}W_t. \tag{A11}$$

A.2. Proof of Proposition 3

To simplify notation, let $\pi_t^0(h_t, i) = \mathbf{1}_t(i)$. Iterating forward on the budget constraint (A1),

$$\begin{aligned}
A_t + Y_t - M_t - C_t = & \sum_{s=1}^{n-1} \sum_{i=2}^{3} \frac{\pi_t^s(h_t, i)}{R^s} (C_{t+s}(i) - Y_{t+s}(i) + M_{t+s}(i)) \\
& + \sum_{s=1}^{n} \sum_{i=2}^{3} \frac{\pi_t^{s-1}(h_t, i) \pi_{t+s-1}(i, 1)}{R^s} (\delta_{t+s-1}(i) + A_{t+s}(i)) \\
& + \sum_{i=2}^{3} \left[\frac{\pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 2)}{R^n} (\Delta_{t+n-1}(i) + A_{t+n}(i)) \right. \\
& \left. + \frac{\pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 3)}{R^n} A_{t+n}(i) \right]. \qquad \text{(A12)}
\end{aligned}$$

We consider perturbations of health and mortality delta that satisfy the budget constraint:

$$\pi_{t+n-1}(i, 2) \partial \Delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0, \qquad \text{(A13)}$$

$$\pi_{t+n-1}(i, 1) \partial \delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0. \qquad \text{(A14)}$$

We write the value function under complete markets as

$$\begin{aligned}
V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i)) = & \left\{ \omega(h_t)^\gamma C_t^{1-\gamma} + \sum_{s=1}^{n-1} \beta^s \sum_{i=2}^{3} \pi_t^s(h_t, i) \omega(i)^\gamma C_{t+s}(i)^{1-\gamma} \right. \\
& + \sum_{s=1}^{n} \beta^s \sum_{i=2}^{3} \pi_t^{s-1}(h_t, i) \pi_{t+s-1}(i, 1) \omega(1)^\gamma (\delta_{t+s-1}(i) + A_{t+s}(i))^{1-\gamma} \\
& + \beta^n \sum_{i=2}^{3} \left[\pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 2) V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{1-\gamma} \right. \\
& \left. \left. + \pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 3) V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{1-\gamma} \right] \right\}^{1/(1-\gamma)}. \qquad \text{(A15)}
\end{aligned}$$

Iterating forward on the first-order conditions (A10),

$$\begin{aligned}
& \left(\frac{\omega(h_t)}{c_t(h_t)} \right)^{\gamma/(1-\gamma)} V_t^{*-\gamma} = (\beta R)^n \omega(1)^\gamma (\delta_{t+n-1}^*(i) + A_{t+n}^*(i))^{-\gamma} \\
& = (\beta R)^n \left(\frac{\omega(2)}{c_{t+n}(2)} \right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}^*(i) + A_{t+n}^*(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma} \\
& = (\beta R)^n \left(\frac{\omega(3)}{c_{t+n}(3)} \right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}^*(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}. \qquad \text{(A16)}
\end{aligned}$$

Taking the partial derivative of equation (A15) with respect to $\Delta_{t+n-1}(i)$,

$$\frac{\partial V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = \beta^n \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)V_t^{\gamma}$$
$$\times \Big[-\pi_{t+n-1}(i,1)\omega(1)^{\gamma}(\delta_{t+n-1}(i) + A_{t+n}(i))^{-\gamma}$$
$$+ (1 - \pi_{t+n-1}(i,2))\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma}$$
$$-\pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}\Bigg]. \tag{A17}$$

Evaluating at the optimal policy,

$$\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = 0. \tag{A18}$$

Similarly, the first partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)} = 0. \tag{A19}$$

Taking the partial derivative of equation (A17) with respect to $\Delta_{t+n-1}(i)$ and evaluating at the optimal policy,

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\gamma\beta^n \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2 V_t^{*\gamma}$$
$$\times \Big[\pi_{t+n-1}(i,1)\omega(1)^{\gamma}(\delta^*_{t+n-1}(i) + A^*_{t+n}(i))^{-1-\gamma}$$
$$+ \frac{(1 - \pi_{t+n-1}(i,2))^2}{\pi_{t+n-1}(i,2)}\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(2, \Delta^*_{t+n-1}(i) + A^*_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-1-\gamma}$$
$$+\pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(3, A^*_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-1-\gamma}\Bigg]. \tag{A20}$$

Substituting the first-order conditions (A16),

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2}{\beta^{n/\gamma} R^{n(1+1/\gamma)} V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times \left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1 - \pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A21}$$

Similarly, the second partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)^2}{\beta^{n/\gamma} R^{n(1+1/\gamma)} V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2)c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A22}$$

Finally, the cross-partial derivative of the value function with respect to health and mortality delta, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial \delta_{t+n-1}(i)} = -\frac{\gamma \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2))c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A23}$$

Dividing by V_t^* and substituting the value function (A11),

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2\omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times\left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A24}$$

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)^2\omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times\left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2)c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A25}$$

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial \delta_{t+n-1}(i)} = -\frac{\gamma \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2)\omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times\left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2))c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A26}$$

B. Health and Retirement Study

The Health and Retirement Study is a panel survey designed to study the health and wealth dynamics of the elderly in the United States. The data consist of five cohorts: the Study of Assets and Health Dynamics among the Oldest Old (born before 1924), the Children of Depression (born 1924 to 1930), the initial HRS cohort (born 1931 to 1941), the War Baby (born 1942 to 1947), and the Early Baby Boomer (born 1948 to 1953). Many of the variables that we use are from the RAND HRS (Version L), which is produced by the RAND Center for the Study of Aging with funding from the National Institute on Aging and the Social Security Administration. Whenever necessary, we use variables from both the core and exit interviews to supplement the RAND HRS. The data consist of ten waves, covering every two years between 1992 and 2010.

The Health and Retirement Study continues to interview respondents that enter nursing homes. However, any respondent that enters a nursing home receives a zero sampling weight because these weights are based on the non-institutionalized population of the Current Population Survey. Therefore, the use of sampling weights would lead us to underestimate nursing home expenses, which account for a large share of out-of-pocket health expenses for older households. Because nursing home expenses are important for this paper, we do not use sampling weights in any of our analysis.

Since the third wave, the survey asks bracketing questions to solicit a range of values for questions that initially receive a non-response. Based on the range of values implied by the bracketing questions, we use the following methodology to impute missing observations. For each missing observation, we calculate the minimum and maximum values that are implied by the responses to the bracketing questions. For each non-missing observation, we set the minimum and maximum values to be the valid response. We then estimate the mean and the standard deviation of the variable in question through interval regression, under the assumption of log-normality. Finally, we fill in each missing observation as the conditional mean of the distribution in the bracketed range.

B.1. Out-of-Pocket Health Expenses

Out-of-pocket health expenses from the RAND HRS consist of the total amount paid for hospitals, nursing homes, doctor visits, dentist visits, outpatient surgery, prescription drugs, home health care, and special facilities. We measure out-of-pocket health expenses at the household level as the sum of these expenses for both the male respondent and his spouse, if married.

Since the third wave, out-of-pocket health expenses at the end of life are available through the exit interviews. Without end-of-life expenses, we would underestimate the true cost of poor health in old age, especially in the upper tail of the distribution (Marshall, McGarry, and Skinner, 2011). Out-of-pocket health expenses from the exit interviews consist of the total amount paid for hospitals, nursing homes, doctor visits, prescription drugs, home health care, other health services, other medical expenses, and other non-medical expenses. For the last core interview prior to death of the primary respondent, we add out-of-pocket health expenses at the end of life from the exit interviews.

We estimate the life-cycle profile for out-of-pocket health expenses, on the subsample of households without long-term care insurance, through a panel regression with household fixed effects. We model the logarithm of real out-of-pocket health expenses as a function of dummy variables for 65 or older and poor health, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. The dummy variable for 65 or older accounts for potential changes in household behavior that arise from eligibility for Social Security and Medicare. We use the estimated regression model to predict out-of-pocket health expenses in the absence of long-term care insurance by cohort, age, and health.

B.2. Income

Our measure of income includes labor income, Social Security disability and supplemental security income, Social Security retirement income, and unemployment or workers compensation. It excludes pension and annuity income and capital income. We calculate after-tax income by subtracting federal income tax liabilities, estimated through the NBER TAXSIM program (Version 9). Household income is the sum of income for both the male respondent and his spouse, if married.

We estimate the life-cycle profile for income through a panel regression with household fixed effects. We model the logarithm of real after-tax income as a function of a dummy variable for 65 or older, a quadratic polynomial in age, and the interaction of the dummy variable with age. We use the estimated regression model to predict income by cohort and age.

B.3. Life Insurance

The ownership and the face value of life insurance are from the core interviews. Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. In the first to third waves, the total face value of all policies is the sum of the face value of term and whole life insurance. In the fourth wave, only the total face value of all policies, and not the breakdown between term and whole life insurance, is available. In fifth to tenth waves, the total face value of term life insurance is the difference between the face value of all policies and that of whole life insurance.

We estimate the life-cycle profile for the face value of life insurance through a panel regression with household fixed effects. We model the logarithm of the real face value of life insurance as a function of dummy variables for 65 or older and poor health, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict the face value of life insurance by household fixed effect, age, and health.

B.4. Annuities Including Private Pensions

We define the ownership of annuities including private pensions as either participation in a defined-benefit plan at the present employer or positive reported pension and annuity income.

We estimate the life-cycle profile for pension and annuity income through a panel regression with household fixed effects. We model the logarithm of real pension and annuity income as a function of dummy variables for 65 or older and poor health, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict pension and annuity income by household fixed effect, age, and health.

B.5. Net Worth

Household assets include checking, savings, and money market accounts; CD, government savings bonds, and T-bills; bonds and bond funds; IRA and Keogh accounts; businesses; stocks, mutual funds, and investment trusts; and primary and secondary residence. Household liabilities include all mortgages for primary and secondary residence, other home loans for primary residence, and other debt. Net worth is the value of assets minus the value of liabilities.

We estimate the life-cycle profile for net worth through a panel regression with household fixed effects. We model the logarithm of real net worth as a function of dummy variables for 65 or older and poor health, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict net worth by household fixed effect, age, and health.

C. Transition Probabilities for Insurance Ownership

In Table C1, we use a probit model to predict the ownership of a given policy at two years from the present interview. The key explanatory variable is whether the household is a present policyholder. Households aged 51 that are present policyholders of term life insurance are 43 percentage points more likely to be a policyholder at the next interview. Similarly, households aged 51 that are present policyholders of whole life insurance are 68 percentage points more likely to be a policyholder at the next interview. Households aged 51 that are present policyholders of annuities including private pensions are 51 percentage points more likely to be a policyholder at the next interview. Finally, households aged 51 that are present policyholders of long-term care insurance are 28 percentage points more likely to be a policyholder at the next interview.

Based on the predicted probabilities from the probit model, we estimate the joint transition matrix for the health state and insurance ownership. For each household, we then calculate the most likely sequence of future insurance ownership, conditional on the realized health state. Finally, we calculate the sequence of future health and mortality delta implied by insurance ownership (i.e., $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=2}^{T-t}$ in Proposition 3).

Table C1: Predicting Future Insurance Ownership

A probit model is used to predict the ownership of a given policy at two years from the present interview. The table reports the marginal effects on the probability of ownership (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The omitted cohort is those born prior to 1911. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Explanatory variable	Term life insurance		Whole life insurance		Annuities including private pensions		Long-term care insurance	
Policyholder	43.27	(20.64)	68.41	(41.39)	51.14	(40.32)	28.35	(11.59)
65 or older	-14.24	(-1.83)	-27.08	(-3.67)	2.50	(0.37)	3.35	(0.93)
Poor health	0.00	(0.00)	-2.05	(-0.98)	0.33	(0.20)	0.28	(0.30)
(Age – 51)/10	1.96	(0.29)	-5.21	(-0.83)	-19.17	(-4.09)	-0.78	(-0.31)
× Policyholder	7.01	(2.40)	-1.68	(-0.59)	5.66	(2.52)	12.43	(8.36)
× 65 or older	5.47	(0.61)	25.25	(2.88)	14.49	(1.99)	-1.26	(-0.32)
× Poor health	0.15	(0.05)	3.69	(1.25)	-0.90	(-0.38)	-0.50	(-0.38)
$(\text{Age} - 51)^2/100$	-6.23	(-1.42)	-1.80	(-0.42)	10.65	(3.45)	1.74	(1.02)
× Policyholder	-1.33	(-1.66)	-0.26	(-0.32)	0.51	(0.78)	-1.71	(-4.14)
× 65 or older	4.76	(1.05)	-3.55	(-0.81)	-11.85	(-3.60)	-1.36	(-0.75)
× Poor health	-0.24	(-0.28)	-1.05	(-1.21)	-0.18	(-0.25)	0.23	(0.58)
Birth cohort:								
1911–1915	2.16	(0.50)	-10.12	(-3.09)	-1.00	(-0.29)	8.34	(1.96)
1916–1920	10.16	(2.67)	-13.61	(-4.72)	-3.63	(-1.03)	7.12	(1.75)
1921–1925	10.55	(2.71)	-14.93	(-5.17)	-6.96	(-1.91)	14.60	(2.95)
1926–1930	12.65	(3.20)	-18.51	(-6.81)	-8.76	(-2.33)	16.64	(3.25)
1931–1935	15.34	(3.83)	-20.12	(-6.88)	-13.80	(-3.64)	17.03	(3.51)
1936–1940	17.06	(4.18)	-24.66	(-8.42)	-18.28	(-4.83)	17.03	(3.69)
1941–1945	20.00	(5.38)	-24.01	(-10.32)	-19.50	(-5.15)	20.17	(3.64)
1946–1950	24.86	(7.69)	-25.02	(-12.85)	-23.54	(-6.38)	23.55	(3.82)
1951–1955	21.51	(6.25)	-24.33	(-15.80)	-29.05	(-8.24)	26.14	(3.89)
Correctly predicted (percent):								
Both outcomes	77		85		80		91	
Policyholder only	80		77		81		68	
Non-policyholder only	71		89		78		94	
Observations	18,536		18,799		35,966		35,376	