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ABSTRACT

We develop a pair of risk measures for the universe of health and longevity products that includes life insurance, annuities, and supplementary health insurance. Health delta measures the differential payoff that a product delivers in poor health, while mortality delta measures the differential payoff that a product delivers at death. Optimal insurance choice simplifies to the problem of choosing a portfolio of health and longevity products that replicates the optimal health and mortality delta. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of life insurance, annuities including private pensions, supplementary health insurance, and long-term care insurance. For the median household aged 51 to 58, the lifetime welfare cost of market incompleteness and suboptimal insurance choice is 17 percent of total wealth.

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1. Introduction

Retail financial advisors and insurance companies offer a wide variety of health and longevity products that includes life insurance, annuities, Medigap insurance, and long-term care insurance. Each of these products comes in a potentially confusing variety of maturities and payout structures. Examples include term life insurance with guaranteed terms of one to 30 years, universal or whole life insurance, immediate life annuities, and deferred annuities whose income is deferrable for any period greater than a year. This variety begs for a risk measure that allows households to assess to what extent these products are complements or substitutes and, ultimately, to choose an optimal portfolio of products. Such risk measures already exist in other parts of the retail financial industry. For example, beta measures an equity product's exposure to aggregate market risk, while duration measures a fixed-income product's exposure to interest-rate risk. The existence of such risk measures, based on sound economic theory, has proven to be tremendously valuable in quantifying and managing financial risk for both households and institutions alike.

This paper develops a pair of risk measures for health and longevity products, which we refer to as health and mortality delta. Health delta measures the differential payoff that a product delivers in poor health, while mortality delta measures the differential payoff that a product delivers at death. Standard life-cycle theory implies an optimal health and mortality delta that depends on preferences (e.g., risk aversion and bequest motive) and state variables (e.g., birth cohort, age, wealth, and health). Optimal insurance choice then simplifies to the problem of choosing a portfolio of health and longevity products, not necessarily unique, that replicates the optimal health and mortality delta.

We use our risk measures to assess how close the observed demand is to the optimal private demand for health and longevity insurance, given the provision of public insurance through Social Security and Medicare. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions, supplementary health insurance, and long-term care insurance. We estimate household preferences so that the optimal demand predicted by the life-cycle model most closely matches the observed demand for health and longevity insurance. We achieve sharp identification of relative risk aversion, the bequest motive, and the degree of complementarity between consumption and health. Household insurance choice, which contains information about the desired path of savings in future health states, is much more informative than the realized path of savings for identifying these preference parameters.

We estimate the welfare cost for each household as a function of the deviations from the optimal health and mortality delta. For the median household aged 51 to 58, the lifetime welfare cost is 17 percent of total wealth, which includes the present value of future income in excess of out-of-pocket health expenses. The lifetime welfare cost drops to 8 percent of total wealth for households aged 67 to 74. We interpret this welfare cost as the joint cost of market incompleteness and suboptimal insurance choice. Most of the welfare cost is explained by the deviations from the optimal mortality delta, rather than by the deviations from the optimal health delta. In other words, choices over life insurance and annuities have a much larger impact on the welfare cost than do choices over supplementary health insurance and long-term care insurance.

The large welfare cost arises from the fact that the life-cycle model predicts large variation in the optimal health and mortality delta along its state variables (i.e., birth cohort, age, wealth, and health), which is not matched by the observed health and mortality delta. The variation in the observed health and mortality delta is mostly driven by heterogeneity and inertia that arises from passive annuitization through private pensions. Moreover, the deviations from the optimal health and mortality delta remain mostly unexplained by observed household characteristics that capture potential preference heterogeneity or private information about health. Therefore, our finding is not driven by model mis-specification along these observable dimensions.

Our work is not the first attempt to understand the demand for health and longevity insurance such as life insurance (Bernheim, 1991; Inkmann and Michaelides, 2012), annuities (Brown, 2001; Inkmann, Lopes, and Michaelides, 2011), and long-term care insurance (Brown and Finkelstein, 2008; Lockwood, 2012). Relative to the previous literature, an important methodological contribution is to examine household insurance choice comprehensively as a portfolio choice problem, instead of one product at a time. By collapsing household insurance choice into a pair of risk measures, we explicitly account for the complementarity and the substitutability between different types of health and longevity products. We uncover a new puzzle that is distinct from the so-called annuity puzzle in the literature. The unexplained variation in the degree to which households are annuitized, rather than the average level at which households are annuitized, is puzzling from the perspective of standard life-cycle theory.

The remainder of the paper is organized as follows. In Section 2, we develop a life-cycle model in which a household faces health and mortality risk and saves in a complete set of health and longevity products that includes life insurance, annuities, and supplementary health insurance. In Section 3, we derive the optimal demand for health and longevity insurance and a key formula for measuring the welfare cost of deviations from the optimal demand. In Section 4, we calibrate the life-cycle model using the Health and Retirement Study. In Section 5, we compare the observed demand for health and longevity insurance to the optimal demand predicted by the life-cycle model. We then estimate the welfare cost of deviations from the optimal demand for health and longevity insurance. In Section 6, we illustrate how a household can replicate the optimal health and mortality delta through existing health and longevity products. Section 7 concludes with practical implications of our study for retail financial advisors and insurance companies.

2. A Life-Cycle Model with Health and Mortality Risk

In this section, we develop a life-cycle model in which a household faces health and mortality risk that affects life expectancy, health expenses, and the marginal utility of consumption or wealth. The household can save in a bond as well as a complete set of health and longevity products that includes life insurance, annuities, and supplementary health insurance. We view complete markets as a fairly realistic assumption, given the wide variety of health and longevity products that retail financial advisors and insurance companies already offer. In fact, Section 6 shows that a portfolio of existing health and longevity products replicates the optimal health and mortality delta predicted by the calibrated life-cycle model. Therefore, the optimal demand under complete markets is a natural benchmark for evaluating the observed demand for health and longevity insurance.

2.1 Health and Mortality Risk

In our model, health refers to any information that is verifiable through medical underwriting that involves a health examination and a review of medical history. For tractability, we do not model residual private information, such as self assessments of health, that might affect the demand for health and longevity insurance. In Section 5, however, we show that residual private information does not explain the deviations of the observed demand from the optimal demand predicted by the life-cycle model.

2.1.1 Health Transition Probabilities

A household consists of an insured and other members who share the same resources. The insured lives for at most T periods and dies with certainty in period $T+1$. In each period $t \in \{1, \dots, T\}$, the insured's health is in one of three states, indexed as $h_t \in \{1, 2, 3\}$.[1] The health states are ordered so that $h_t = 1$ corresponds to death, $h_t = 2$ corresponds to poor

[1] While three states is appropriate for our empirical application, it is conceptually straightforward to extend our framework to more than three states.

health, and $h_t = 3$ corresponds to good health. The three-state model can be interpreted as a discrete-time analog of a continuous-time model in which a continuous process drives health risk and a jump process drives mortality risk (Milevsky and Promislow, 2001).

The insured's health evolves from period t to $t+1$ according to a Markov chain with a 3×3 transition matrix π_t. We denote the (i, j)th element of the transition matrix as

$$\pi_t(i, j) = \Pr(h_{t+1} = j | h_t = i). \tag{1}$$

Conditional on being in health state i in period t, $\pi_t(i, j)$ is the probability of being in health state j in period $t+1$. Death is an absorbing state so that $\pi_t(1, 1) = 1$. Let $\mathbf{e}_i$ denote a 3×1 vector whose ith element is one and whose other elements are zero. We define an n-period transition probability as

$$\pi_t^n(i, j) = \mathbf{e}_i' \prod_{s=0}^{n-1} \pi_{t+s} \mathbf{e}_j. \tag{2}$$

Conditional on being in health state i in period t, $\pi_t^n(i, j)$ is the probability of being in health state j in period $t+n$.

We define an n-period mortality rate as

$$p_t(n|i) = \begin{cases} \mathbf{e}_i' \pi_t \mathbf{e}_1 & \text{if } n = 1 \\ \mathbf{e}_i' \prod_{s=0}^{n-2} \pi_{t+s} \begin{bmatrix} \mathbf{0} & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix} \pi_{t+n-1} \mathbf{e}_1 & \text{if } n > 1 \end{cases}. \tag{3}$$

Conditional on being in health state i in period t, $p_t(n|i)$ is the probability of being alive in period $t+n-1$ but dead in period $t+n$. We also define an n-period survival probability as

$$q_t(n|i) = 1 - \pi_t^n(i, 1). \tag{4}$$

Conditional on being in health state i in period t, $q_t(n|i)$ is the probability of being alive in period $t+n$.

2.1.2 Out-of-Pocket Health Expenses

Although most households are covered by employer-provided health insurance or Medicare, they still face the risk of high out-of-pocket health expenses, especially in old age. Many health plans only cover basic or in-network care, have capped benefits, or do not cover entire categories of health expenses. For example, Medicare does not cover nursing home care, and Medicaid only covers a limited and capped amount of nursing home care for those that qualify. Moreover, a household can lose health insurance through a layoff or a divorce. Health insurance must specify coverage for each type of future health contingency and treatment, some of which are not known to exist in advance. The fact that health insurance coverage can be short term or incomplete is perhaps a natural consequence of the complexity of these policies.

We model the consequences of imperfect health insurance as follows. In each period, the household faces an exogenous out-of-pocket health expense whose distribution depends on age and health.[2] We denote the out-of-pocket health expense in period t as M_t, or as $M_t(h_t)$ to denote its realization for a particular health state. Naturally, poor health is associated with higher out-of-pocket health expenses. There is no health expense at death so that $M_t(1) = 0$. Since the model is in discrete time, we assume that end-of-life health expenses incur in the last period prior to death.

2.2 Health and Longevity Products

In each period t, the household can save in a one-period bond, which earns gross interest R. In addition, the household can save in life insurance, annuities, and supplementary health insurance of maturities one through $T - t$.

[2]We assume that health expenses are exogenous to focus on the household's choice over a rich menu of health and longevity products. We refer to Picone, Uribe, and Wilson (1998), Yogo (2009), and Hugonnier, Pelgrin, and St-Amour (2012) for a life-cycle model in which health expenditure is endogenous.

2.2.1 Term Life Insurance

Let $\mathbf{1}_{t+s}(j)$ denote an indicator function that is equal to one if the insured is in health state j in period $t+s$. Life insurance with maturity n, issued in period t, pays out a death benefit of

$$D_{L,t+s}(n-s|h_{t+s}) = \mathbf{1}_{t+s}(1), \tag{5}$$

upon death of the insured in any period $t+s \in \{t+1,\ldots,t+n\}$. In each period t, $T-t$ is the maximum available maturity since the insured dies with certainty in period $T+1$. For our purposes, universal or whole life insurance is a special case of term life insurance with the maximum available maturity.

The pricing of life insurance depends on the insured's age and health at issuance of the policy. Naturally, younger and healthier individuals with longer life expectancy pay a lower premium.[3] Conditional on being in health state h_t in period t, the price of n-period life insurance per unit of death benefit is

$$P_{L,t}(n|h_t) = \sum_{s=1}^{n} \frac{p_t(s|h_t)}{R_L^s}, \tag{6}$$

where $R_L \leq R$ is the discount rate. The pricing of life insurance is actuarially fair when $R_L = R$, while $R_L < R$ implies that life insurance is sold at a markup.

[3]The insurer could charge a premium that is independent of health in a pooling equilibrium (e.g., group life insurance). In that case, we would have to solve for a pooling price at which the insurer breaks even, given the aggregate demand for a given product. While a conceptually straightforward extension of our framework, such an exercise would be computationally challenging. We refer to a related literature that examines the welfare implications of pooled pricing and private information in annuity (Einav, Finkelstein, and Schrimpf, 2010) and health insurance markets (Einav, Finkelstein, and Cullen, 2010; Bundorf, Levin, and Mahoney, 2012).

2.2.2 Annuities

An annuity with maturity n, issued in period t, pays out a constant income of

$$D_{A,t+s}(n-s|h_{t+s}) = 1 - \mathbf{1}_{t+s}(1), \tag{7}$$

in each period $t+s \in \{t+1,\ldots,t+n\}$ while the insured is alive. In each period t, $T-t$ is the maximum available maturity since the insured dies with certainty in period $T+1$.

The pricing of annuities depends on the insured's age and health at issuance of the policy.[4] Naturally, younger and healthier individuals with longer life expectancy pay a higher premium. Conditional on being in health state h_t in period t, the price of an n-period annuity per unit of income is

$$P_{A,t}(n|h_t) = \sum_{s=1}^{n} \frac{q_t(s|h_t)}{R_A^s}, \tag{8}$$

where $R_A \leq R$ is the discount rate.

An n-period deferred annuity, issued in period t, pays out one unit of income starting in period $t+n+1$, while the insured is alive. Such a deferred annuity can be synthesized through a portfolio of life insurance and annuities that we have introduced already. Specifically, an n-period deferred annuity has the same payoffs as a portfolio consisting of one unit each of life insurance with terms one through n, one unit each of annuities with terms one through $n-1$ and $T-t$, and a loan with face value of $\sum_{s=1}^{n}(n-s+1)/R^s$.

[4] In the United States, annuities can be purchased without medical underwriting at a price that depends only on gender and age. However, those with a serious medical condition can purchase medically underwritten annuities at a lower price that reflects their impaired mortality.

2.2.3 Supplementary Health Insurance

Supplementary health insurance with maturity n, issued in period t, covers

$$D_{H,t+s}(n-s|h_{t+s}) = \mathbf{1}_{t+s}(2)(M_{t+s}(2) - M_{t+s}(3)), \tag{9}$$

in each period $t+s \in \{t+1, \dots, t+n\}$ if the insured is in poor health. Insofar as out-of-pocket health expenses include nursing home or home health care expenses, we can also interpret this product as long-term care insurance. A unit of this product represents full insurance, equating out-of-pocket health expenses across all health states in which the insured is alive. In each period t, $T-t$ is the maximum available maturity since the insured dies with certainty in period $T+1$.

The pricing of supplementary health insurance depends on the insured's age and health at issuance of the policy. Naturally, younger and healthier individuals with lower expected health expenses pay a lower premium. Conditional on being in health state h_t in period t, the price of n-period health insurance per unit of coverage is

$$P_{H,t}(n|h_t) = \sum_{s=1}^{n} \frac{\pi_t^s(h_t, 2)(M_{t+s}(2) - M_{t+s}(3))}{R_H^s}, \tag{10}$$

where $R_H \leq R$ is the discount rate.

2.3 Health and Mortality Delta for Health and Longevity Products

For each product $i = \{L, A, H\}$ with maturity n, we define its health delta in period t as

$$\Delta_{i,t}(n) = P_{i,t+1}(n-1|2) + D_{i,t+1}(n-1|2) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{11}$$

Health delta measures the differential payoff that a policy delivers in poor health relative to good health in period $t+1$. Similarly, we define its mortality delta in period t as

$$\delta_{i,t}(n) = D_{i,t+1}(n-1|1) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{12}$$

Mortality delta measures the differential payoff that a policy delivers at death relative to good health in period $t+1$.

Figure 1 illustrates the relation between the payoffs of a policy and its health and mortality delta. In this illustration, short-term policies have maturity of two years (i.e., the frequency of interviews in the Health and Retirement Study), while long-term policies mature at death. We normalize the death benefit of life insurance and the income from annuities to be \$1k. Section 4 contains details about how we calibrate the prices of long-term policies, which are not important for the purposes of this illustration.

The solid line represents the payoffs of a policy in the three health states. Health delta is the payoff of a policy in poor health relative to good health, which is minus the slope of the dashed line if the horizontal distance between good and poor health is one. Mortality delta is the payoff of a policy at death relative to good health, which is minus the slope of the dotted line if the horizontal distance between good health and death is one.

Short-term life insurance pays out \$1k only if the insured dies. Therefore, short-term life insurance has zero health delta and a mortality delta of \$1k. Even if the insured remains alive, long-term life insurance is worth the present value of \$1k in the event of future death, which is higher in poor health when he has impaired mortality. Therefore, long-term life insurance has both positive health delta and positive mortality delta.

The short-term annuity pays out \$1k only if the insured remains alive. Therefore, the short-term annuity has zero health delta and a mortality delta of −\$1k. In addition to the income if the insured remains alive, the long-term annuity is worth the present value of \$1k in each future period that he remains alive, which is higher in good health when he has longer life expectancy. Therefore, the long-term annuity has both negative health delta and negative mortality delta.

Short-term health insurance covers only poor health when the insured has high out-of-pocket health expenses. Therefore, short-term health insurance has positive health delta and zero mortality delta. In addition to the coverage in poor health, long-term health insurance is worth the present value of coverage in the event of future poor health, which is higher in poor health when the insured has higher expected health expenses. Therefore, long-term health insurance has positive health delta and negative mortality delta.

2.4 Budget Constraint

In each period t that the insured is alive, the household receives labor or retirement income Y_t and pays out-of-pocket health expenses M_t. The realization of both income and health expenses can depend on age and health. Let W_t denote the household's cash-on-hand in period t, which is its wealth after receiving income and paying health expenses. The household consumes from cash-on-hand and saves the remaining wealth in bonds, life insurance, annuities, and supplementary health insurance. Let B_t denote the total face value of bonds, and let $B_{i,t}(n) \geq 0$ denote the total face value of policy i with maturity n. The household's savings in period t is

$$W_t - C_t = \frac{B_t}{R} + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} P_{i,t}(n|h_t) B_{i,t}(n). \tag{13}$$

We assume that the household can borrow from its savings in health and longevity products at the gross interest rate R. Therefore, a loan from health and longevity products is a negative position in bonds. For our purposes, a loan from health and longevity products is a simple way to model actual features of these policies. As we have mentioned already, a deferred annuity is equivalent to a portfolio of life insurance, annuities, and an embedded loan. The premium for long-term life or health insurance is typically paid through constant periodic payments over the term of the policy, instead of as an upfront lump-sum payment. The option to pay through periodic payments is essentially equivalent to borrowing against the value of the policy because the present value of the periodic payments is equal to the value of the policy at issuance. Whole life insurance typically has an explicit option to borrow from the cash surrender value of the policy. Finally, households can take out a loan from annuities in a defined contribution plan.

Let

$$A_{t+1}(j) = B_t + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} (P_{i,t+1}(n-1|j) + D_{i,t+1}(n-1|j)) B_{i,t}(n) \tag{14}$$

denote the household's wealth, prior to receiving income and paying health expenses, if health state j is realized in period $t+1$. In particular,

$$A_{t+1}(1) = B_t + \sum_{n=1}^{T-t} B_{L,t}(n) \tag{15}$$

is the wealth that is bequeathed if the insured dies in period $t+1$. The household must have non-negative wealth at the time of the insured's death, that is, $A_{t+1}(1) \geq 0$. The household's intertemporal budget constraint is

$$W_{t+1} = A_{t+1} + Y_{t+1} - M_{t+1}. \tag{16}$$

2.5 Objective Function

For each health state $h_t \in \{2,3\}$ in period t, we define the household's objective function recursively as

$$U_t(h_t) = \left\{ \omega(h_t)^\gamma C_t^{1-\gamma} + \beta \left[\pi_t(h_t,1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t,j) U_{t+1}(j)^{1-\gamma} \right] \right\}^{1/(1-\gamma)}, \tag{17}$$

with the terminal value

$$U_T(h_T) = \omega(h_T)^{\gamma/(1-\gamma)} C_T. \tag{18}$$

The parameter $\beta \in (0, 1)$ is the subjective discount factor, and $\gamma > 1$ is relative risk aversion. The health state-dependent utility parameter $\omega(h_t) \geq 0$ allows the marginal utility of consumption or wealth to vary across health states. The presence of a bequest motive is parameterized as $\omega(1) > 0$, in contrast to its absence $\omega(1) = 0$. The parameterization $\omega(2) < \omega(3)$ makes consumption and health complements in the sense that the marginal utility of consumption is lower in poor health.

3. Optimal Demand for Health and Longevity Insurance

In this section, we derive the optimal demand for health and longevity insurance under complete markets. When markets are complete, there are potentially many combinations of health and longevity products that achieve the same consumption and wealth allocations. Therefore, we characterize the solution to the life-cycle problem as an optimal consumption policy and an optimal pair of health and mortality delta. We also derive a key formula for measuring the welfare cost of deviations from the optimal demand for health and longevity insurance.

3.1 Optimal Health and Mortality Delta

To simplify notation, we define disposable income as income in excess of out-of-pocket health expenses. We then define total wealth as cash-on-hand plus the present value of future disposable income:

$$\widehat{W}_t = W_t + \sum_{s=1}^{T-t} \frac{\mathbf{E}_t[Y_{t+s} - M_{t+s}|h_t]}{R^s}. \tag{19}$$

We define health delta in period t as the difference in realized wealth between poor and good health in period $t+1$:

$$\Delta_t = A_{t+1}(2) - A_{t+1}(3). \tag{20}$$

Similarly, we define mortality delta in period t as the difference in realized wealth between death and good health in period $t+1$:

$$\delta_t = A_{t+1}(1) - A_{t+1}(3). \tag{21}$$

Proposition 1. *The solution to the life-cycle problem under complete markets is*

$$C_t^* = c_t(h_t)\widehat{W}_t, \tag{22}$$

$$\begin{aligned} \Delta_t^* =& \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)} \left(\frac{\omega(2)}{c_{t+1}(2)} - \frac{\omega(3)}{c_{t+1}(3)} \right) \\ &- \left(\sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|2]}{R^{s-1}} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}} \right), \end{aligned} \tag{23}$$

$$\delta_t^* = \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)} \left(\omega(1) - \frac{\omega(3)}{c_{t+1}(3)} \right) + \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}. \tag{24}$$

The average propensity to consume in health state $h_t \in \{2, 3\}$ is

$$c_t(h_t) = \left[1 + \frac{\pi_t(h_t, 1)(\beta R)^{1/\gamma} \omega(1)}{R\omega(h_t)} + \sum_{j=2}^{3} \frac{\pi_t(h_t, j)(\beta R)^{1/\gamma} \omega(j)}{R\omega(h_t) c_{t+1}(j)} \right]^{-1}, \tag{25}$$

with the terminal value $c_T(h_T) = 1$.

As shown in Appendix A, the optimal policy equates the marginal utility of consumption or wealth across all health states in period $t+1$. The expression for the optimal health delta (i.e., Δ_t^*) shows that there are three forces that drive the household's desire to insure poor health relative to good health. First, the household would like to deliver relatively more wealth to the health state in which the marginal utility of consumption is high, determined by the relative magnitudes of $\omega(2)$ and $\omega(3)$. Second, the household would like to deliver relatively more wealth to the health state in which the average propensity to consume is low, determined by the relative magnitudes of $c_{t+1}(2)$ and $c_{t+1}(3)$. Naturally, the household consumes more slowly out of wealth in good health associated with longer life expectancy. Finally, the household would like to deliver relatively more wealth to the health state in which lifetime disposable income is low. Naturally, the household has lower lifetime disposable income in poor health associated with shorter life expectancy, higher health expenses, and potentially lower income.

The same three forces also explain the expression for the optimal mortality delta (i.e., δ_t^*). First, the household would like to deliver relatively more wealth to death if the marginal utility of the bequest (i.e., $\omega(1)$) is high. Second, the household would like to deliver relatively more wealth to death if the average propensity to consume in good health (i.e., $c_{t+1}(3)$) is high. Finally, the household would like to deliver relatively more wealth to death if lifetime disposable income is high in good health.

3.2 Replicating the Optimal Health and Mortality Delta through Health and Longevity Products

Proposition 2. *Given an optimal consumption policy, a feasible portfolio policy that satisfies the budget constraint (13) is optimal if it satisfies the equations*

$$\Delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \Delta_{i,t}(n) B_{i,t}(n), \tag{26}$$

$$\delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \delta_{i,t}(n) B_{i,t}(n). \tag{27}$$

Proposition 2 shows that health and mortality delta are sufficient for constructing an optimal portfolio of health and longevity products. Health delta $\Delta_{i,t}(n)$ measures the marginal contribution that policy i with maturity n has to the household's health delta. Mortality delta $\delta_{i,t}(n)$ measures the marginal contribution that policy i with maturity n has to the household's mortality delta. A portfolio of health and longevity products, not necessarily unique, that satisfies equation (26) delivers the optimal amount of wealth to poor health in period $t+1$. Similarly, a portfolio of health and longevity products, not necessarily unique, that satisfies equation (27) delivers the optimal amount of wealth to death in period $t+1$.

3.3 Welfare Cost of Deviations from the Optimal Health and Mortality Delta

Suppose the household's health and mortality delta were to deviate from the optimal health and mortality delta given in Proposition 1. As shown in Appendix A, we estimate the welfare cost of such deviations from the optimal health and mortality delta through a second-order Taylor approximation around the known value function under complete markets. By the envelope theorem, the welfare cost is second order for sufficiently small deviations from the optimal health and mortality delta (Cochrane, 1989).

Proposition 3. *Let V_t^* denote the value function associated with the sequence $\{\Delta_{t+s-1}^*(i), \delta_{t+s-1}^*(i)\}_{s=1}^n$ of optimal health and mortality delta under complete markets. Let V_t denote the value function associated with an alternative sequence $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=1}^n$ of health and mortality delta that satisfies the budget constraint. The welfare cost of deviations from the optimal health and mortality delta is*

$$
\begin{aligned}
L_t(n) =& \frac{V_t}{V_t^*} - 1 \\
\approx& \frac{1}{2} \sum_{s=1}^{n} \sum_{i=2}^{3} \left[\frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i)^2} (\Delta_{t+s-1}(i) - \Delta_{t+s-1}^*(i))^2 \right. \\
& + \frac{\partial^2 L_t(n)}{\partial \delta_{t+s-1}(i)^2} (\delta_{t+s-1}(i) - \delta_{t+s-1}^*(i))^2 \\
& \left. + 2 \frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i) \partial \delta_{t+s-1}(i)} (\Delta_{t+s-1}(i) - \Delta_{t+s-1}^*(i))(\delta_{t+s-1}(i) - \delta_{t+s-1}^*(i)) \right], \qquad (28)
\end{aligned}
$$

where the expressions for the second partial derivatives are given in Appendix A.

A household may not achieve the optimal health and mortality delta under complete markets for two reasons. First, markets may be incomplete due to borrowing constraints, or the menu of health and longevity products may be incomplete for some demographic groups. Second, a nearly rational household may hold a suboptimal portfolio of health and longevity products, even though markets are complete (Calvet, Campbell, and Sodini, 2007). This explanation is especially plausible for health and longevity products because there is no clear guidance on optimal portfolio choice, unlike for equity and fixed-income products. Because these two reasons are not mutually exclusive and difficult to distinguish based on the available data, we do not quantify the relative importance of these two hypotheses. Instead, we focus on estimating the joint cost of market incompleteness and suboptimal insurance choice in this paper.

4. Calibrating the Life-Cycle Model

4.1 Health and Retirement Study

We use the Health and Retirement Study to calibrate the life-cycle model, which is a representative panel of older households in the United States since 1992. This household survey is uniquely suited for our study because it contains household-level data on health outcomes, health expenses, income, and wealth as well as ownership of life insurance, annuities, supplementary health insurance, and long-term care insurance. Some of these critical variables are missing in other household surveys such as the Panel Study of Income Dynamics or the Survey of Consumer Finances. We focus on households whose male respondent is aged 51 and older at the time of interview. We also require that households have both positive income and net worth to be included in our sample. Appendix B contains details on the construction of the relevant variables for our analysis.

Life insurance is written on the life of an insured, while resources like income and wealth are shared by the members of a household. Because the male respondent is typically married at the time of first interview, we must make some measurement assumptions when mapping the data to the model. We measure health outcomes and the ownership of life insurance, annuities, supplementary health insurance, and long-term care insurance for only the male respondent. We measure health expenses, income, and wealth at the household level. These measurement assumptions are consistent with the model insofar as the budget constraint holds for the household, and the male respondent buys life insurance to leave a bequest for surviving household members when he dies.

We calibrate the life-cycle model so that each period corresponds to two years, matching the frequency of interviews in the Health and Retirement Study. The model starts at age 51 to correspond to the youngest age at which respondents enter the survey. We assume that respondents die with certainty at age 111, so that there are a total of 30 periods (60 years) in the life-cycle model. We set the riskless interest rate to 2 percent annually, which is roughly the average real return on the one-year Treasury note during our sample period.

4.2 Definition of the Health States

In this section, we categorize health into three states including death, which is the minimum number of states necessary to model both health and mortality risk. For our purposes, the relevant criteria for poor health are that both the mortality rate and health expenses are high. This is precisely the state in which life insurance and supplementary health insurance are valuable.

In Table 1, we use a probit model to predict future mortality based on observed health problems. The explanatory variables include dummy variables for doctor-diagnosed health problems, age, the interaction of the health problems with age, and cohort dummies. The marginal effect of high blood pressure on the mortality rate is 1.66 with a t-statistic of 3.52. This means that respondents with high blood pressure are 1.66 percentage points more likely to die within two years, holding everything else constant. Respondents with cancer are 13.62 percentage points more likely to die, while those with lung disease are 8.21 percentage points more likely to die. Past age 51, each additional ten years is associated with an increase of 3.26 percentage points in the mortality rate.

Using the estimated probit model, we calculate the predicted mortality rate for each male respondent at each interview. We also calculate the ratio of out-of-pocket health expenses to income at each interview. We then define the following three health states.

1. Death.

2. Poor health: The predicted mortality rate is higher than the median conditional on cohort and age. In addition, the ratio of out-of-pocket health expenses to income is higher than the median conditional on cohort, age, and the ownership of supplementary health insurance and long-term care insurance.

3. Good health: Alive and not in poor health.

To verify that our definition of the health states are reasonable, Panel A of Table 2 reports specific health problems that respondents face by age group and health state. Within each age group, respondents in poor health have higher prevalence of doctor-diagnosed health problems. For example, among respondents aged 51 to 66, 28 percent of those in poor health have had heart problems, which is higher than 11 percent of those in good health. Older respondents, especially those in poor health, have higher prevalence of difficulty with activities of daily living. For example, among respondents aged 83 or older, 16 percent of those in poor health have some difficulty eating, which is higher than 7 percent of those in good health.

Panel B of Table 2 reports health care utilization by age group and health state. Within each age group, respondents in poor health are more likely to have used health care in the two years prior to the interview. For example, among respondents aged 51 to 66, 79 percent of those in poor health use prescription drugs regularly, which is higher than 52 percent of those in good health. Among respondents aged 83 or older, 19 percent of those in poor health have stayed at a nursing home, which is higher than 8 percent of those in good health. These facts explain why respondents in poor health have higher out-of-pocket health expenses than those in good health.

Panel C of Table 2 reports health insurance coverage by age group and health state. Among respondents aged 51 to 66, 22 percent of those in poor health are covered by Medicare, which is higher than 17 percent of those in good health. This difference is explained by the fact that some respondents who are disabled become eligible for Medicare prior to age 65. Almost all respondents aged 67 or older are covered by Medicare. Among respondents aged 51 to 66, 58 percent of those in poor health are covered by an employer-provided health plan, which is lower than 63 percent of those in good health. Within each age group, the ownership rates of supplementary health insurance and long-term care insurance are remarkably similar across health states.

Panel D of Table 2 reports the ownership rate of life insurance, the ownership rate of annuities including private pensions, and net worth by age group and health state. Among respondents aged 51 to 66, 78 percent of those in poor health own some type of life insurance, which is comparable to 80 percent of those in good health. Although the ownership rate for life insurance declines in age, it remains remarkably high for older respondents. Among respondents aged 67 to 82, 56 percent of those in poor health receive annuity income that is not from Social Security, which is comparable to 54 percent of those in good health. Among respondents aged 67 to 82, the median net worth excluding life insurance and annuities is $186k for those in poor health, which is comparable to $187k for those in good health.

4.3 Health and Mortality Risk

4.3.1 Health Transition Probabilities

Once we have defined the three health states, we estimate the transition probabilities between the health states using an ordered probit model. The outcome variable is the health state at two years from the present interview. The explanatory variables include dummy variables for present health state and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with age, and cohort dummies. The dummy variable for 65 or older allows for potential changes in household behavior when they become eligible for Social Security and Medicare. Our estimated transition probabilities for each cohort are the predicted probabilities from the ordered probit model.

To get a sense for these transition probabilities, Panel A of Table 3 reports the health distribution by age for a population of respondents who were born between 1936 and 1940 and are in good health at age 51. By age 67, 30 percent of the population are dead, and 18 percent are in poor health. By age 83, 62 percent of the population are dead, and 14 percent are in poor health. Panel B reports the average life expectancy conditional on age and health.[5] Respondents in poor health at age 51 are expected to live for 24 more years, which

[5]For comparison, the corresponding estimates of life expectancy from the Social Security cohort life tables are 28 years at age 51, 21 years at age 59, 16 years at age 67, 11 years at age 75, 7 years at age 83, 4 years at age 91, and 2 years at age 99.

is shorter than 26 years for those in good health. The difference in life expectancy between poor and good health remains relatively constant for older respondents. Respondents in poor health at age 83 are expected to live for 8 more years, which is shorter than 10 years for those in good health.

4.3.2 Out-of-Pocket Health Expenses

We use a comprehensive measure of out-of-pocket health expenses that includes payments of health insurance premiums and end-of-life health expenses. As explained in Appendix B, we use a panel regression model to estimate how out-of-pocket health expenses depend on cohort, age, health, and income.

Panel C of Table 3 reports average annual out-of-pocket health expenses by age and health for the cohort born between 1936 and 1940. For comparison, Panel D reports average annual income by age, which includes Social Security but excludes annuities and private pensions.[6] Households in poor health at age 51 have annual out-of-pocket health expenses of $2k, which is higher than $0.4k for those in good health. Out-of-pocket health expenses rise rapidly in old age (De Nardi, French, and Jones, 2010). Households in poor health at age 83 have annual out-of-pocket health expenses of $21.1k, which is higher than $7.4k for those in good health. Since annual income at age 83 is $18k, households in poor health must dissave in order to consume and pay health expenses.

Households in poor health not only face higher health expenses today, but they also face higher future health expenses. Panel E of Table 3 reports the present value of future disposable income (i.e., income in excess of out-of-pocket health expenses) by age and health. Households in poor health at age 59 have $232k in lifetime disposable income that they can consume or bequeath, which is lower than $270k for those in good health. A respondent in good health at age 51 is unlikely to be in poor health or die, at least in the near future.

[6] For simplicity, our calibration assumes that income depends on cohort and age, but not on health. While there is some evidence that income varies with health, such variation is much smaller than the variation in out-of-pocket health expenses, which is our main focus.

However, poor health or death can have an important impact on lifetime resources. Health and longevity insurance allows households to insure this uncertainty in lifetime resources across health states.

4.4 Pricing of Health and Longevity Products

In the baseline calibration, we set the discount rate on health and longevity products to be the same as the riskless interest rate of 2 percent (i.e., $R_L = R_A = R_H = R$). In other words, we assume that the pricing of health and longevity products is actuarially fair conditional on age and health. This assumption is necessary because we do not observe the premiums that households actually pay for life insurance, supplementary health insurance, and long-term care insurance. There are various reasons why the pricing of health and longevity products may not be actuarially fair in practice: rents arising from imperfect competition, discounts reflecting the poor credit quality of insurers, risk premia arising from aggregate health and mortality risk, and the presence of private information. To capture these scenarios, we consider an alternative calibration in which health and longevity products are more expensive than actuarially fair in Section 5.6.

The impact of private information on the pricing of insurance is ambiguous because adverse selection on health may be offset by advantageous selection on another dimension of private information such as risk aversion (de Meza and Webb, 2001). In life insurance markets, there is no evidence for private information about health (Cawley and Philipson, 1999). Because the pricing of annuities depends on gender and age but not on health, annuity markets may be in a separating equilibrium along contract dimensions like payout structure (Finkelstein and Poterba, 2004). In long-term care and Medigap insurance markets, private information about health appears to be offset by advantageous selection on risk aversion and cognitive ability (Finkelstein and McGarry, 2006; Fang, Keane, and Silverman, 2008). Given the ambiguous nature of both the theoretical predictions and the empirical findings, the absence of private information serves as a satisfactory starting point for the baseline calibration. However, we examine private information as a potential explanation for the heterogeneity in demand for health and longevity insurance in our empirical work.

4.5 Ownership of Health and Longevity Products

Figure 2 reports the ownership rates for term and whole life insurance, annuities including private pensions, supplementary health insurance, and long-term care insurance. The ownership rate for term life insurance exceeds 60 percent for households aged 51 to 58, while the ownership rate for annuities including private pensions exceeds 50 percent for households aged 67 to 74. In comparison, the ownership rates for supplementary health insurance and long-term care insurance are much lower. For example, the ownership rate for long-term care insurance is only slightly above 10 percent for households aged 67 to 74.

We do not have information about the maturity of term life insurance or the exact coverage of supplementary health insurance or long-term care insurance. Therefore, we must make some measurement assumptions in order to map these health and longevity products to their counterparts in the life-cycle model. We assume that term life insurance matures in two years and that whole life insurance matures at death. The assumption that term life insurance is short term is motivated by the fact that (annually renewable) group policies account for a large share of these policies. We assume that annuity income starts at age 65, which is the full Social Security retirement age, and terminates at death. We assume that the ownership of supplementary health insurance corresponds to owning half a unit of short-term health insurance in the life-cycle model. Similarly, the ownership of long-term care insurance corresponds to owning half a unit of short-term health insurance. Therefore, a household that owns both supplementary health insurance and long-term care insurance is fully insured against out-of-pocket health expenses for one period. This assumption is based on estimates that nursing home expenses account for approximately half of out-of-pocket health expenses for older households (Marshall, McGarry, and Skinner, 2011).

Conditional on ownership, households report the face value of term and whole life insurance. Measurement error in the face value of these policies would contaminate our estimates of health and mortality delta. As explained in Appendix B, we use a panel regression model to estimate how the face values with term and whole life insurance depend on cohort, age, and income. Instead of using the observed face values, we use the predicted values with household fixed effects under the assumption that measurement error is transitory. We apply the same procedure to annuity and pension income.

We model all health and longevity products as policies with real payments. We normalize the death benefit of life insurance and the income from annuities to be \$1k in 2005 dollars. Modeling nominal payments for health and longevity products would introduce inflation risk, which is beyond the scope of this paper. Moreover, a cost-of-living-adjustment rider that effectively eliminates inflation risk is sometimes available for life insurance, annuities, and long-term care insurance. In the data, we deflate the face value of life insurance as well as pension and annuity income by the consumer price index to 2005 dollars.

4.6 Health and Mortality Delta Implied by Household Insurance Choice

For each household at each interview, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions, supplementary health insurance, and long-term care insurance. The household's health delta is determined by positive health delta from whole life insurance, supplementary health insurance, and long-term care insurance, which is offset by negative health delta from annuities including private pensions. The household's mortality delta is determined by positive mortality delta from term and whole life insurance, which is offset by negative mortality delta from annuities including private pensions.

Figure 3 reports the health and mortality delta for each household-interview observation, together with the mean and the standard deviation at each age. Average health delta is slightly negative throughout the life cycle. This implies that annuities have a predominant effect on the average household's health delta. Average mortality delta is positive for younger households and negative for older households. This implies that life insurance has a predominant effect on younger households' mortality delta, while annuities have a predominant effect for older households. The cross-sectional variation in mortality delta is significantly higher than that in health delta throughout the life cycle.

When we calculate the health delta for each household based solely on its ownership of annuities including private pensions, it explains 98 percent of the variation in the overall health delta. When we calculate the mortality delta for each household in a similar way, it explains 59 percent of the variation in the overall mortality delta. In addition, Panel D of Table 2 reports that private pensions, rather than the purchase of individual annuities, explain almost all of private annuitization. Together, these facts imply that most of the variation in the observed health and mortality delta is driven by heterogeneity and inertia that arises from passive annuitization through private pensions.

5. Welfare Cost of Household Insurance Choice

In this section, we first estimate household preferences based on the observed demand for health and longevity insurance. We then compare the observed demand for health and longevity insurance to the optimal demand predicted by the life-cycle model. We show that the deviations from the optimal demand remain mostly unexplained by observed household characteristics that capture potential preference heterogeneity or private information about health. Finally, we apply Proposition 3 to estimate the welfare cost of deviations from the optimal demand for health and longevity insurance.

5.1 Estimating Household Preferences

We calibrate the subjective discount factor to $\beta = 0.96$ annually, which is a common choice in the life-cycle literature. We normalize the utility weight for good health to $\omega(3) = 1$. We then stack the remaining preference parameters in a column vector as $\theta = [\gamma, \omega(1), \omega(2)]'$. For each household-interview observation $i \in \{1, \dots, I\}$, we denote the per-period welfare cost, implied by equation (28) for $n = 1$, as $L_i(\theta)$. We estimate household preferences to minimize the average per-period welfare cost:

$$\frac{1}{I} \sum_{i=1}^{I} L_i(\theta). \tag{29}$$

By construction, the welfare cost implied by the estimated preference parameters is a lower-bound estimate for the welfare cost under the true preference parameters.

We implement our estimation problem through continuous-updating generalized method of moments. Define the moment function

$$m(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{\partial L_i(\theta)}{\partial \theta} \tag{30}$$

and the weighting matrix

$$W(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{\partial L_i(\theta)}{\partial \theta} \frac{\partial L_i(\theta)}{\partial \theta'}. \tag{31}$$

Then our estimator for household preferences is

$$\widehat{\theta} = \arg\min_{\theta} m(\theta)' W(\theta)^{-1} m(\theta). \tag{32}$$

Table 4 reports our estimates of household preferences. While our point estimates are consistent with the previous literature, our standard errors are much smaller. Our estimate of relative risk aversion is 2.51 with a standard error of 0.01. Our point estimate of relative risk aversion is in the lower end of the confidence interval reported by De Nardi, French, and Jones (2010). Our estimate of the utility weight for death is 4.64 with a standard error of 0.03. This means that households have a strong bequest motive that is equivalent to

4.64 periods (more than 9 years) of consumption. Ameriks et al. (2011) also find evidence for a strong bequest motive. Finally, our estimate of the utility weight for poor health is 0.71 with a standard error of 0.01. Viscusi and Evans (1990) and Finkelstein, Luttmer, and Notowidigdo (2012) also find evidence that consumption and health are complements.

5.2 Observed versus Optimal Demand for Health and Longevity Insurance

The left panel of Figure 4 is a scatter plot of the observed health delta for each household-interview observation against the optimal health delta predicted by the life-cycle model. The right panel is an analogous scatter plot for mortality delta. In both panels, the linear regression line through the data points is significantly flatter than 45 degrees. This implies that the life-cycle model generates large variation in the optimal health and mortality delta that is not matched by the data. By construction, the variation in the optimal health and mortality delta depends only on the state variables of the life-cycle model, which are birth cohort, age, wealth, and health. Hence, the key takeaway is that while the observed health and mortality delta vary significantly across households, they do not vary sufficiently along the state variables of the life-cycle model.

In the right panel of Figure 4, the 45 degree line divides the sample into two groups. Above the 45 degree line are households that are under-annuitized, whose mortality delta is higher than the optimal mortality delta. Below the 45 degree line are households that are over-annuitized, whose mortality delta is lower than the optimal mortality delta. This figure shows that the unexplained variation in the degree to which households are annuitized, rather than the average level at which households are annuitized, is puzzling from the perspective of standard life-cycle theory.

5.3 Testing for Mis-specification

In Table 5, we regress the deviations from the optimal health and mortality delta, normalized by total wealth, onto observed household characteristics that capture potential preference heterogeneity or private information about health. If these factors that are missing from the life-cycle model are important determinants of household insurance choice, they should have significant explanatory power for the residuals generated by the model. Overall, we find little evidence for such mis-specification. The deviations from the optimal health and mortality delta remain mostly unexplained by observed household characteristics.

In the baseline specification in column (1) of Table 5, we regress the deviations from the optimal health delta onto the key life-cycle variables: dummy variables for poor health and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with age, and cohort dummies. Altogether, these life-cycle variables explain only 0.34 percent of the deviations from the optimal health delta. This implies that the life-cycle model adequately captures the variation in the observed health delta along the life-cycle dimensions.

In column (2) of Table 5, we include additional explanatory variables that capture potential preference heterogeneity or private information about health. The positive and significant coefficient on marital status implies that the deviations from the optimal health delta are higher for married households. Similarly, the deviations from the optimal health delta are higher for high school and college graduates, compared to households with no high school education. The coefficients on poor and fair self-reported health are positive and significant, while the coefficients on very good and excellent self-reported health are negative and significant. The sign of these coefficients on self-reported health are consistent with the presence of adverse selection, that households in worse health tend to own more health insurance. However, an R^2 of 5.54 percent implies that these variables ultimately explain very little of the deviations from the optimal health delta.

Columns (3) and (4) of Table 5 repeats the same exercise for the deviations from the optimal mortality delta. In the baseline specification in column (3), we first regress the deviations from the optimal mortality delta onto the key life-cycle variables. Altogether, these life-cycle variables explain only 1.51 percent of the deviations from the optimal mortality delta. This implies that the life-cycle model adequately captures the variation in the observed mortality delta along the life-cycle dimensions.

In column (4) of Table 5, we include additional explanatory variables that capture potential preference heterogeneity or private information about health. The deviations from the optimal mortality delta are higher for married households and households with living children. Similarly, the deviations from the optimal mortality delta are higher for high school and college graduates, compared to households with no high school education. These findings are consistent with the hypothesis that the bequest motive is stronger for households that are married, have living children, and are more educated. The coefficients on poor and fair self-reported health are positive and significant, while the coefficients on very good and excellent self-reported health are negative and significant. The sign of these coefficients on self-reported health are consistent with the presence of adverse selection, that households in worse health tend to own more life insurance. However, an R^2 of 8.19 percent implies that these variables ultimately explain very little of the deviations from the optimal mortality delta.

In specifications that are not reported in Table 5, we have ruled out significant explanatory power for other variables that capture potential preference heterogeneity or private information about health. They are variables that capture heterogeneity in bequest motives (i.e., self-reported probability of leaving a bequest), risk aversion (i.e., responses to income gamble questions), and private information about health (i.e., difficulty with activities of daily living, self-reported probability of living to age 75, and self-reported probability of moving to a nursing home).

5.4 Per-Period Welfare Cost

In this section, we estimate the welfare cost of deviating from the optimal health and mortality delta in the current period, then following the optimal policy for the remaining lifetime. That is, we estimate the per-period welfare cost by applying Proposition 3 for $n = 1$. While the per-period welfare cost is not our primary measure of interest, we can estimate it based on the observed ownership of health and longevity products alone, without an auxiliary model for how this ownership evolves over time.

Panel A of Table 6 reports the median per-period (two-year) welfare cost by age group. The per-period welfare cost for households aged 51 to 58 is 0.54 percent with a standard error of 0.09 percent. Through equation (28) for $n = 1$, we can decompose this welfare cost into the sum of three parts. The deviations from the optimal health delta explain 0.01 percent of the welfare cost, while the deviations from the optimal mortality delta explain 0.54 percent. The interaction between health and mortality delta explains the remainder of the welfare cost, which is -0.01 percent.

The top three panels of Figure 5 reports the per-period welfare cost and its decomposition for each household-interview observation, together with the median at each age. The figure shows large variation in the per-period welfare cost across households of the same age. The median per-period welfare cost is relatively constant in age. This implies that the life-cycle model does not fit certain parts of the life cycle worse than others.

5.5 Lifetime Welfare Cost

The per-period welfare cost is based on the assumption that the household deviates from the optimal health and mortality delta for the current period, then follows the optimal policy for the remainder of its lifetime. In reality, a household that deviates from the optimal policy for one period is likely to persist in the suboptimal policy for many periods. In this section, we estimate the lifetime welfare cost by applying Proposition 3 for $n = T - t$.

In order to measure the lifetime welfare cost, we must first model how the ownership of health and longevity products evolves over time, exploiting the panel dimension of the data. In Table 7, we use a probit model to predict the ownership of a given policy at two years from the present interview. The key explanatory variable is whether the household is a present policy owner. Households aged 51 that are present owners of term life insurance are 46 percentage points more likely to own it at the next interview. Similarly, households aged 51 that are present owners of whole life insurance are 67 percentage points more likely to own it at the next interview. Households aged 51 that are present owners of annuities including private pensions are 53 percentage points more likely to own them at the next interview. Households aged 51 that are present owners of supplementary health insurance are 33 percentage points more likely to own it at the next interview. Finally, households aged 51 that are present owners of long-term care insurance are 24 percentage points more likely to own it at the next interview.

Based on the predicted probabilities from the probit model, we estimate the joint transition matrix for the health state and the ownership of health and longevity products. For each household, we then calculate the most likely sequence of future ownership of health and longevity products, conditional on the realized health state. Finally, we calculate the sequence of future health and mortality delta implied by the ownership of health and longevity products (i.e., $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=2}^{T-t}$ in Proposition 3).

Panel B of Table 6 reports the median lifetime welfare cost by age group. The lifetime welfare cost for households aged 51 to 58 is 17.43 percent with a standard error of 2.98 percent. This is a large welfare cost that is equivalent to a 17 percent reduction in lifetime consumption, as implied by the homogeneity of preferences. Through equation (28) for $n = T - t$, we can decompose this welfare cost into the sum of three parts. The deviations from the optimal health delta explain 0.71 percent of the welfare cost, while the deviations from the optimal mortality delta explain 18.27 percent. The interaction between health and mortality delta explains the remainder of the welfare cost, which is -1.55 percent. The lifetime welfare cost for households aged 67 to 74 drops to 7.75 percent with a standard error of 0.21 percent.

The bottom three panels of Figure 5 reports the lifetime welfare cost and its decomposition for each household-interview observation, together with the median at each age. The median lifetime welfare cost is higher for younger households, for whom the per-period welfare cost accumulates over a longer expected lifetime. The median lifetime welfare cost declines rapidly until age 83 and becomes small for older households with shorter life expectancy. Mortality delta explains almost all of the lifetime welfare cost because, as shown in Figure 3, its cross-sectional variation is significantly higher than that in health delta.

5.6 Robustness under Alternative Assumptions

5.6.1 Actuarially Unfair Insurance

The baseline estimates in Table 6 assume that the pricing of health and longevity products is actuarially fair. In Table 8, we consider an alternative scenario in which the pricing of health and longevity products is more expensive than actuarially fair. We assume that the discount rate (or the expected return) on life insurance, annuities, and supplementary health insurance is 0 percent, while the riskless interest rate is 2 percent. This is a fairly extreme assumption that corresponds to the upper range of the deviations from actuarially fair pricing that is estimated for the annuity market (Mitchell et al., 1999). Both the per-period and the lifetime welfare costs are essentially the same as the baseline estimate in Table 6. Therefore, our findings are robust to alternative assumptions about the pricing of health and longevity products.

5.6.2 Preference Heterogeneity

A natural hypothesis for the cross-sectional variation in the observed mortality delta is that households have heterogenous preferences with respect to bequest motives. In Table 5, we have already ruled out the importance of heterogeneity in bequest motives along observable dimensions. As a robustness check, we also experimented with heterogeneity in bequest motives that is uncorrelated with observed household characteristics (Fang and Kung, 2012). In particular, we have tried specifications where the utility weight on death (i.e., $\omega(1)$) is drawn from a log-normal or a binomial distribution that is independent of observed household characteristics.

We found that such preference heterogeneity did not reduce our estimate of the welfare cost. In fact, our point estimate for the variance of the bequest motive converged to zero. Intuitively, the life-cycle model with homogeneous preferences already generates large variation in the optimal health and mortality delta along its state variables, which is not matched by the observed health and mortality delta. Introducing preference heterogeneity only further increases the unconditional variance of the optimal health and mortality delta, overshooting the unconditional variance in the data.

6. Replicating the Optimal Health and Mortality Delta

In this section, we illustrate how a household can replicate the optimal health and mortality delta through existing health and longevity products. Our illustration is for a male, who was born between 1936 and 1940 and is in good health at age 51. The household's initial wealth is \$65k at age 51, which is chosen to match average net worth excluding life insurance and annuities for this cohort. The household can buy short-term life insurance, deferred annuities, short-term health insurance, and bonds. The household does not face borrowing constraints, so that it achieves the optimal health and mortality delta under complete markets. The household's preference parameters are those given in Table 4.

Panel A of Table 9 reports the optimal health and mortality delta, which we calculate by applying Proposition 1. The optimal health delta is $-\$2$k at age 51, which implies that the household needs \$2k less in poor health relative to good health at age 53. As equation (23) shows, there are three offsetting forces that determine the optimal health delta. First, the household has preference for consumption in good health over poor health (i.e., $\omega(2) < \omega(3)$), which pushes the optimal health delta to be more negative. Second, the household saves less in poor health because of shorter life expectancy (i.e., $c_{t+1}(2) > c_{t+1}(3)$), which pushes the optimal health delta to be more negative. Third, the household has lower lifetime disposable income in poor health, which pushes the optimal health delta to be more positive. The first two forces dominate the third, so that the optimal health delta is negative at age 51.

The optimal mortality delta is \$190k at age 51, which implies that the household needs an additional \$190k at death relative to good health at age 53. As equation (24) shows, there are three offsetting forces that determine the optimal mortality delta. First, the household has preference for bequest over consumption in good health (i.e., $\omega(1) > \omega(3)$), which pushes the optimal mortality delta to be more positive. Second, the household must save for future consumption in good health (i.e., $c_{t+1}(3) < 1$), which pushes the optimal mortality delta to be more negative. Third, the household has higher lifetime disposable income in good health, which pushes the optimal mortality delta to be more positive. The first and third forces dominate the second, so that the optimal mortality delta is positive at age 51.

Panel B of Table 9 reports a portfolio of short-term life insurance, deferred annuities, and short-term health insurance that replicates the optimal health and mortality delta, which we calculate by applying Proposition 2. The optimal portfolio at age 51 consists of 211 units (i.e., death benefit of \$211k) of short-term life insurance, 5 units of deferred annuities (i.e., annuity income of \$5k over two years), no short-term health insurance, and 41 units of bonds. Panel C reports the cost of the optimal portfolio, which is the sum of \$6k in short-term life insurance, \$19k in deferred annuities, and \$39k in bonds.

The left panel of Figure 6 shows that the optimal health delta has a U-shaped profile over the life cycle. To replicate the optimal health delta, the households does not need short-term health insurance through age 67. Thereafter, the household must gradually increase its position in short-term health insurance to 0.81 units at age 99. Since one unit of short-term health insurance eliminates all uncertainty in out-of-pocket health expenses in the next period, these positions imply that the household demands only partial health insurance for most of the life cycle. The intuition for this result is that higher out-of-pocket health expenses in poor health are offset by shorter life expectancy, lowering the optimal health delta relative to full health insurance.

The right panel of Figure 6 shows that the optimal mortality delta declines over the life cycle. To replicate the optimal mortality delta, the household must hold short-term life insurance when young to generate positive mortality delta, then switch to deferred annuities when old to generate negative mortality delta. The optimal position in deferred annuities increases from 5 units at age 51 to 91 units at age 99. A practical implication of Figure 6 is that an insurance company may want to package life insurance and deferred annuities into a product that automatically replicates the life-cycle profile for optimal mortality delta, eliminating the need for active rebalancing.

In this illustration, the household is exposed to reclassification risk because it has access to only short-term life insurance and health insurance. In other words, a household in good health at age 51 has to pay a higher premium for life insurance and health insurance at age 53 if its health deteriorates. As emphasized by Cochrane (1995), the household can insure reclassification risk in a world with health state-contingent securities. Our illustration here shows that an optimal portfolio of short-term life insurance and health insurance essentially replicates health state-contingent securities, thereby insuring reclassification risk.

7. Conclusion

In this paper, we find significant welfare cost of deviations from the optimal demand for health and longevity insurance under complete markets. We have reasons to believe that suboptimal insurance choice, rather than market incompleteness, is the source of this welfare cost for most households. First, the variation in the observed demand for health and longevity insurance is mostly driven by heterogeneity and inertia that arises from passive annuitization through private pensions. Second, there has been little academic guidance on optimal portfolio choice for health and longevity products, unlike for equity and fixed-income products. Finally, we calibrate our life-cycle model to the Health and Retirement Study and find that a typical household can replicate the optimal health and mortality delta over the life cycle through existing health and longevity products.

To improve household insurance choice, retail financial advisors and insurance companies should report the health and mortality delta of their health and longevity products, just as mutual fund companies already report the market beta of their equity products and the duration of their fixed-income products. We hope that these risk measures will facilitate standardization, identify overlap between existing products, identify risks that are not insured by existing products, and ultimately lead to new product development. One such product that we find particularly promising is a life-cycle product that automatically shifts from life insurance to deferred annuities as a function of age, so that a household achieves the optimal mortality delta over the life cycle without active rebalancing. This product would be analogous to life-cycle funds that automatically shift from equity to fixed income as a function of age, which have proven to be tremendously successful in the mutual fund industry.

Appendix A. Proof of the Propositions

A.1 Proof of Proposition 1

We rewrite savings in period t as

$$W_t - C_t = \sum_{j=1}^{3} \frac{\pi_t(h_t, j)}{R} A_{t+1}(j). \tag{A1}$$

The household maximizes the objective function (17) subject to equation (A1) and the intertemporal budget constraint (16). In each period $t \in [1, T-1]$, the Bellman equation is

$$\begin{aligned} V_t(h_t, W_t) = \max_{C_t, A_{t+1}(1), A_{t+1}(2), A_{t+1}(3)} & \Big\{ \omega(h_t)^\gamma C_t^{1-\gamma} \\ & + \beta \left[\pi_t(h_t, 1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) V_{t+1}(j, W_{t+1}(j))^{1-\gamma} \right] \Big\}^{1/1-\gamma}. \end{aligned} \tag{A2}$$

The proposition claims that the optimal health state-contingent wealth policies are given by

$$A^*_{t+1}(1) = \frac{(\beta R)^{1/\gamma} \omega(1) C^*_t}{\omega(h_t)}, \tag{A3}$$

$$A^*_{t+1}(j) = \frac{(\beta R)^{1/\gamma} \omega(j) C^*_t}{\omega(h_t) c_{t+1}(j)} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s} | j]}{R^{s-1}} \ \forall j \in \{2, 3\}. \tag{A4}$$

The proof proceeds by backward induction.

Because the household dies with certainty in period $T+1$, optimal consumption in period T is $C^*_T = W_T$. Thus, the value function in period T is

$$V_T(h_T, W_T) = \omega(h_T)^{\gamma/(1-\gamma)} W_T. \tag{A5}$$

The first-order conditions in period $T-1$ are

$$\begin{aligned}\omega(h_{T-1})^{\gamma}C_{T-1}^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_{T}^{*}(1)^{-\gamma}\\ =&\beta R\omega(h_{T})^{\gamma}(A_{T}^{*}(j)+Y_{T}(j)-M_{T}(j))^{-\gamma}\ \forall j\in\{2,3\}.\end{aligned} \tag{A6}$$

These equations, together with equation (A1), imply the policy functions (22), (A3), and (A4) for period $T-1$. Substituting the policy functions into the Bellman equation, the value function in period $T-1$ is

$$V_{T-1}(h_{T-1},W_{T-1}) = \left(\frac{\omega(h_{T-1})}{c_{T-1}(h_{T-1})}\right)^{\gamma/(1-\gamma)}\widehat{W}_{T-1}. \tag{A7}$$

Suppose that the value function in each period $t+1$ is

$$V_{t+1}(h_{t+1},W_{t+1}) = \left(\frac{\omega(h_{t+1})}{c_{t+1}(h_{t+1})}\right)^{\gamma/(1-\gamma)}\widehat{W}_{t+1}. \tag{A8}$$

The first-order conditions in each period t are

$$\begin{aligned}\omega(h_{t})^{\gamma}C_{t}^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_{t+1}^{*}(1)^{-\gamma}\\ =&\frac{\beta R\omega(j)^{\gamma}}{c_{t+1}(j)^{\gamma}}\left(A_{t+1}^{*}(j)+\sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s}-M_{t+s}|j]}{R^{s-1}}\right)^{-\gamma}\quad\forall j\in\{2,3\}.\end{aligned} \tag{A9}$$

These equations, together with equation (A1), imply the policy functions (22), (A3), and (A4) for each period t. Substituting the policy functions into the Bellman equation, the value function in each period t is

$$V_{t}(h_{t},W_{t}) = \left(\frac{\omega(h_{t})}{c_{t}(h_{t})}\right)^{\gamma/(1-\gamma)}\widehat{W}_{t}. \tag{A10}$$

A.2 Proof of Proposition 3

To simplify notation, let $\pi_t^0(h_t, i) = \mathbf{1}_t(i)$. Iterating forward on the budget constraint (A1),

$$
\begin{aligned}
W_t - C_t = & \sum_{s=1}^{n-1}\sum_{i=2}^{3} \frac{\pi_t^s(h_t,i)}{R^s}(C_{t+s}(i) - Y_{t+s}(i) + M_{t+s}(i)) \\
& + \sum_{s=1}^{n}\sum_{i=2}^{3} \frac{\pi_t^{s-1}(h_t,i)\pi_{t+s-1}(i,1)}{R^s}(\delta_{t+s-1}(i) + A_{t+s}(i)) \\
& + \sum_{i=2}^{3}\left[\frac{\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)}{R^n}(\Delta_{t+n-1}(i) + A_{t+n}(i)) \right. \\
& \left. + \frac{\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,3)}{R^n} A_{t+n}(i)\right].
\end{aligned} \tag{A11}
$$

We consider perturbations of health and mortality delta that satisfy the budget constraint:

$$\pi_{t+n-1}(i,2)\partial\Delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0, \tag{A12}$$

$$\pi_{t+n-1}(i,1)\partial\delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0. \tag{A13}$$

We write the value function under complete markets as

$$
\begin{aligned}
& V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i)) = \Bigg\{ \omega(h_t)^\gamma C_t^{1-\gamma} + \sum_{s=1}^{n-1} \beta^s \sum_{i=2}^{3} \pi_t^s(h_t,i)\omega(i)^\gamma C_{t+s}(i)^{1-\gamma} \\
& + \sum_{s=1}^{n} \beta^s \sum_{i=2}^{3} \pi_t^{s-1}(h_t,i)\pi_{t+s-1}(i,1)\omega(1)^\gamma(\delta_{t+s-1}(i) + A_{t+s}(i))^{1-\gamma} \\
& + \beta^n \sum_{i=2}^{3} \big[\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{1-\gamma} \\
& + \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,3)V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{1-\gamma}\big]\Big\}^{1/(1-\gamma)}.
\end{aligned} \tag{A14}
$$

Iterating forward on the first-order conditions (A9),

$$
\begin{aligned}
&\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{\gamma/(1-\gamma)} V_t^{*-\gamma} = (\beta R)^n \omega(1)^\gamma (\delta^*_{t+n-1}(i) + A^*_{t+n}(i))^{-\gamma} \\
&= (\beta R)^n \left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta^*_{t+n-1}(i) + A^*_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma} \\
&= (\beta R)^n \left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A^*_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}. \qquad \text{(A15)}
\end{aligned}
$$

Taking the partial derivative of equation (A14) with respect to $\Delta_{t+n-1}(i)$,

$$
\begin{aligned}
&\frac{\partial V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = \beta^n \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2) V_t^\gamma \\
&\times \Big[-\pi_{t+n-1}(i,1)\omega(1)^\gamma (\delta_{t+n-1}(i) + A_{t+n}(i))^{-\gamma} \\
&+ (1 - \pi_{t+n-1}(i,2)) \left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma} \\
&- \pi_{t+n-1}(i,3) \left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}\Big]. \qquad \text{(A16)}
\end{aligned}
$$

Evaluating at the optimal policy,

$$
\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = 0. \qquad \text{(A17)}
$$

Similarly, the first partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$
\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)} = 0. \qquad \text{(A18)}
$$

Taking the partial derivative of equation (A16) with respect to $\Delta_{t+n-1}(i)$ and evaluating at the optimal policy,

$$
\begin{aligned}
&\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\gamma\beta^n \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2 V_t^{*\gamma} \\
&\times \big[\pi_{t+n-1}(i,1)\omega(1)^\gamma(\delta^*_{t+n-1}(i) + A^*_{t+n}(i))^{-1-\gamma} \\
&+ \frac{(1-\pi_{t+n-1}(i,2))^2}{\pi_{t+n-1}(i,2)}\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(2, \Delta^*_{t+n-1}(i) + A^*_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-1-\gamma} \\
&+\pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(3, A^*_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-1-\gamma}\Bigg].
\end{aligned}
\tag{A19}
$$

Substituting the first-order conditions (A15),

$$
\begin{aligned}
\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} &= -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)} \\
\times \left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]&.
\end{aligned}
\tag{A20}
$$

Similarly, the second partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$
\begin{aligned}
\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} &= -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)} \\
\times \left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]&.
\end{aligned}
\tag{A21}
$$

Finally, the cross-partial derivative of the value function with respect to health and mortality delta, evaluated at the optimal policy, is

$$
\begin{aligned}
\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial \delta_{t+n-1}(i)} &= -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2)}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)} \\
\times \left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2)) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]&.
\end{aligned}
\tag{A22}
$$

Dividing by V_t^* and substituting the value function (A10),

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A23}$$
$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A24}$$
$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i) \partial \delta_{t+n-1}(i)} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2) \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2)) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A25}$$

Appendix B. Health and Retirement Study

The Health and Retirement Study is a panel survey designed to study the health and wealth dynamics of the elderly in the United States. The data consist of five cohorts: the Study of Assets and Health Dynamics among the Oldest Old (born before 1924), the Children of Depression (born 1924 to 1930), the initial HRS cohort (born 1931 to 1941), the War Baby (born 1942 to 1947), and the Early Baby Boomer (born 1948 to 1953). Many of the variables that we use are from the RAND HRS (Version I), which is produced by the RAND Center for the Study of Aging with funding from the National Institute on Aging and the Social Security Administration. Whenever necessary, we use variables from both the core and exit interviews to supplement the RAND HRS. The data consist of eight waves, covering every two years between 1992 and 2006.

The Health and Retirement Study continues to interview respondents that enter nursing homes. However, any respondent that enters a nursing home receives a zero sampling weight because these weights are based on the non-institutionalized population of the Current Population Survey. Therefore, the use of sampling weights would lead us to underestimate nursing home expenses, which account for a large share of out-of-pocket health expenses for older households. Because nursing home expenses are important for this paper, we do not use sampling weights in any of our analysis.

Since wave 3, the survey asks bracketing questions to solicit a range of values for questions that initially receive a non-response. Based on the range of values implied by the bracketing questions, we use the following methodology to impute missing observations. For each missing observation, we calculate the minimum and maximum values that are implied by the responses to the bracketing questions. For each non-missing observation, we set the minimum and maximum values to be the valid response. We then estimate the mean and the standard deviation of the variable in question through interval regression, under the assumption of log-normality. Finally, we fill in each missing observation as the conditional mean of the distribution in the bracketed range.

B.1 Out-of-Pocket Health Expenses

Out-of-pocket health expenses from the RAND HRS are the total amount paid for hospitals, nursing homes, doctor visits, dentist visits, outpatient surgery, prescription drugs, home health care, and special facilities. Payments of health insurance premiums from the core interviews are the sum of premiums paid for Medicare/Medicaid HMO, private health insurance, long-term care insurance, and prescription drug coverage (i.e., Medicare Part D). We convert the premium reported at monthly, quarterly, semi-annual, or annual frequency to the total implied payment over the previous two years.

Since wave 3, out-of-pocket health expenses at the end of life are available through the exit interviews. Without end-of-life expenses, we would underestimate the true cost of poor health in old age, especially in the upper tail of the distribution (Marshall, McGarry, and Skinner, 2011). Out-of-pocket health expenses from the exit interviews are the total amount paid for hospitals, nursing homes, doctor visits, prescription drugs, home health care, other health services, other medical expenses, and other non-medical expenses.

We measure out-of-pocket health expenses as the sum of out-of-pocket health expenses from the RAND HRS and payments of health insurance premiums from the core interviews. For the last core interview prior to death of the primary respondent, we also add out-of-pocket health expenses at the end of life from the exit interviews. We measure out-of-pocket health expenses at the household level as the sum of these expenses for both the male respondent and his wife, if married.

We estimate the life-cycle profile for out-of-pocket health expenses through a panel regression with household fixed effects. We model the logarithm of real out-of-pocket health expenses as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. The dummy variable for 65 or older allows for potential changes in household behavior when they become eligible for Social Security and Medicare. We use the estimated regression model, averaging the household fixed effects by cohort and ownership of supplementary health insurance and long-term care insurance, to predict out-of-pocket health expenses in the absence of these policies by cohort, age, and health.

B.2 Income

Income includes labor income, Social Security disability and supplemental security income, Social Security retirement income, and unemployment or workers compensation. Income excludes pension and annuity income and capital income. We calculate after-tax income by subtracting federal income tax liabilities, estimated through the NBER TAXSIM program (Version 9). We measure household income as the sum of income for both the male respondent and his wife, if married.

We estimate the life-cycle profile for income through a panel regression with household fixed effects. We model the logarithm of real after-tax income as a function of a dummy variable for 65 or older, a quadratic polynomial in age, and the interaction of the dummy variable with age. We use the estimated regression model, averaging the household fixed effects by cohort, to predict income by cohort and age.

B.3 Life Insurance

We measure the ownership and the face value of life insurance using the core interviews. Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. In waves 1 through 3, we measure the total face value of all policies as the sum of the face value of term and whole life insurance. In wave 4, only the total face value of all policies, and not the breakdown between term and whole life insurance, is available. In waves 5 through 8, we measure the total face value of term life insurance as the difference between the face value of all policies and whole life insurance.

We estimate the life-cycle profile for the face value of life insurance through a panel regression with household fixed effects. We model the logarithm of the real face value of life insurance as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict the face value of life insurance by household fixed effect, age, and health.

B.4 Annuities including Private Pensions

We define ownership of annuities including private pensions as either participation in a defined-benefit plan at the present employer or positive reported pension and annuity income.

We estimate the life-cycle profile for pension and annuity income through a panel regression with household fixed effects. We model the logarithm of real pension and annuity income as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict pension and annuity income by household fixed effect, age, and health.

B.5 Net Worth

Household assets include checking, savings, and money market accounts; CD, government savings bonds, and T-bills; bonds and bond funds; IRA and Keogh accounts; businesses; stocks, mutual funds, and investment trusts; and primary and secondary residence. Household liabilities include all mortgages for primary and secondary residence, other home loans for primary residence, and other debt. Net worth is the value of assets minus the value of liabilities.

We estimate the life-cycle profile for net worth through a panel regression with household fixed effects. We model the logarithm of real net worth as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, income, and the interaction of the dummy variables with age and income. We use the estimated regression model to predict net worth by household fixed effect, age, and health.

References

Ameriks, John, Andrew Caplin, Steven Laufer, and Stijn Van Nieuwerburgh. 2011. "The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Bequest and Precautionary Motives." *Journal of Finance* 66 (2):519–561.

Bernheim, B. Douglas. 1991. "How Strong Are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities." *Journal of Political Economy* 99 (5):899–927.

Brown, Jeffrey R. 2001. "Private Pensions, Mortality Risk, and the Decision to Annuitize." *Journal of Public Economics* 82 (1):29–62.

Brown, Jeffrey R. and Amy Finkelstein. 2008. "The Interaction of Public and Private Insurance: Medicaid and the Long-Term Care Insurance Market." *American Economic Review* 98 (3):1083–1102.

Bundorf, M. Kate, Jonathan D. Levin, and Neale Mahoney. 2012. "Pricing and Welfare in Health Plan Choice." *American Economic Review* :forthcoming.

Calvet, Laurent E., John Y. Campbell, and Paolo Sodini. 2007. "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes." *Journal of Political Economy* 115 (5):707–747.

Cawley, John and Tomas Philipson. 1999. "An Empirical Examination of Information Barriers to Trade in Insurance." *American Economic Review* 89 (4):827–846.

Cochrane, John H. 1989. "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives." *American Economic Review* 79 (3):319–337.

———. 1995. "Time-Consistent Health Insurance." *Journal of Political Economy* 103 (3):445–473.

de Meza, David and David C. Webb. 2001. "Advantageous Selection in Insurance Markets." *RAND Journal of Economics* 32 (2):249–262.

De Nardi, Mariacristina, Eric French, and John Bailey Jones. 2010. "Why Do the Elderly Save? The Role of Medical Expenses." *Journal of Political Economy* 118 (1):39–75.

Einav, Liran, Amy Finkelstein, and Mark R. Cullen. 2010. "Estimating Welfare in Insurance Markets Using Variation in Prices." *Quarterly Journal of Economics* 125 (3):877–921.

Einav, Liran, Amy Finkelstein, and Paul Schrimpf. 2010. "Optimal Mandates and the Welfare Cost of Asymmetric Information: Evidence from the U.K. Annuity Market." *Econometrica* 78 (3):1031–1092.

Fang, Hanming, Michael P. Keane, and Dan Silverman. 2008. "Sources of Advantageous Selection: Evidence from the Medigap Insurance Market." *Journal of Political Economy* 116 (2):303–350.

Fang, Hanming and Edward Kung. 2012. "Why Do Life Insurance Policyholders Lapse? The Roles of Income, Health and Bequest Motive Shocks." NBER Working Paper No. 17899.

Finkelstein, Amy, Erzo F. P. Luttmer, and Matthew J. Notowidigdo. 2012. "What Good Is Wealth without Health? The Effect of Health on the Marginal Utility of Consumption." *Journal of the European Economic Association* :forthcoming.

Finkelstein, Amy and Kathleen McGarry. 2006. "Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market." *American Economic Review* 96 (4):938–958.

Finkelstein, Amy and James Poterba. 2004. "Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market." *Journal of Political Economy* 112 (1):183–208.

Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour. 2012. "Health and (Other) Asset Holdings." *Review of Economic Studies* :forthcoming.

Inkmann, Joachim, Paula Lopes, and Alexander Michaelides. 2011. "How Deep Is the Annuity Market Participation Puzzle?" *Review of Financial Studies* 24 (1):279–319.

Inkmann, Joachim and Alexander Michaelides. 2012. "Can the Life Insurance Market Provide Evidence for a Bequest Motive?" *Journal of Risk and Insurance* :forthcoming.

Lockwood, Lee M. 2012. "Incidental Bequests: Bequest Motives and the Choice to Self-Insure Late-Life Risks." Working paper, Northwestern University.

Marshall, Samuel, Kathleen M. McGarry, and Jonathan S. Skinner. 2011. "The Risk of Out-of-Pocket Health Care Expenditure at the End of Life." In *Explorations in the Economics of Aging*, edited by David A. Wise, chap. 3. Chicago: The University of Chicago Press, 101–128.

Milevsky, Moshe A. and David S. Promislow. 2001. "Mortality Derivatives and the Option to Annuitise." *Insurance: Mathematics and Economics* 29 (3):299–318.

Mitchell, Olivia S., James M. Poterba, Mark J. Warshawsky, and Jeffrey R. Brown. 1999. "New Evidence on the Money's Worth of Individual Annuities." *American Economic Review* 89 (5):1299–1318.

Picone, Gabriel, Martin Uribe, and R. Mark Wilson. 1998. "The Effect of Uncertainty on the Demand for Medical Care, Health Capital and Wealth." *Journal of Health Economics* 17 (2):171–185.

Viscusi, W. Kip and William N. Evans. 1990. "Utility Functions that Depend on Health Status: Estimates and Economic Implications." *American Economic Review* 80 (3):353–374.

Yogo, Motohiro. 2009. "Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets." NBER Working Paper No. 15307.

Table 1: Predicting Future Mortality with Observed Health Problems

A probit model is used to predict death within two years from the present interview. The explanatory variables include dummy variables for doctor-diagnosed health problems, age, the interaction of the health problems with age, and cohort dummies. The omitted cohort is those born prior to 1911. The table reports the marginal effects on the mortality rate (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

Explanatory variable	Marginal effect	t-statistic
Doctor-diagnosed health problems:		
High blood pressure	1.66	(3.52)
Diabetes	5.66	(6.41)
Cancer	13.62	(8.61)
Lung disease	8.21	(6.17)
Heart problems	2.60	(4.18)
Stroke	5.57	(4.40)
(Age − 51)/10	3.26	(12.71)
× High blood pressure	-0.44	(-2.31)
× Diabetes	-0.72	(-3.07)
× Cancer	-1.79	(-7.25)
× Lung disease	-0.28	(-1.05)
× Heart problems	0.04	(0.18)
× Stroke	-0.32	(-1.17)
Birth cohort:		
1911–1915	-1.69	(-5.23)
1916–1920	-2.39	(-8.16)
1921–1925	-3.32	(-12.17)
1926–1930	-3.58	(-11.69)
1931–1935	-3.74	(-8.79)
1936–1940	-4.08	(-8.37)
1941–1945	-3.46	(-8.33)
1946–1950	-3.51	(-9.47)
1951–1955	-3.03	(-4.97)
Correctly predicted (percent):		
Both outcomes	92.87	
Dead only	55.13	
Alive only	93.01	
Observations	43,452	

Table 2: Health Problems, Health Care Utilization, and Health Insurance Coverage

Panel A reports the percent of respondents who have ever had doctor-diagnosed health problems or have some difficulty with activities of daily living at the time of interview. Panel B reports the percent of respondents who have used health care in the two years prior to the interview. Panel C reports the percent of respondents who have health insurance at the time of interview. Panel D reports the percent of respondents who own life insurance or annuities including private pensions at the time of interview. It also reports the median of total face value conditional on ownership, deflated by the consumer price index to 2005 dollars. Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. Supplementary health insurance includes Medigap insurance and refers to any coverage that is not government, employer-provided, or long-term care insurance. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

Age	51–66		67–82		83–	
Health	Poor	Good	Poor	Good	Poor	Good
Panel A: Health problems (percent)						
Doctor-diagnosed health problems:						
High blood pressure	59	32	65	46	57	44
Diabetes	22	9	34	14	20	14
Cancer	9	4	30	12	28	20
Lung disease	10	4	21	7	20	7
Heart problems	28	11	56	26	77	31
Stroke	7	3	21	6	33	11
Some difficulty with						
Bathing	5	1	10	4	27	15
Dressing	9	4	14	8	30	18
Eating	2	1	5	2	16	7
Panel B: Health care utilization (percent)						
Monthly doctor visits	11	4	17	8	21	12
Hospital stay	27	14	43	26	54	37
Outpatient surgery	21	17	26	21	24	21
Nursing home stay	1	0	4	2	19	8
Home health care	4	2	12	6	24	13
Special facilities and services	7	4	10	6	15	11
Prescription drugs	79	52	94	76	97	80
Panel C: Health insurance (percent)						
Medicare	22	17	98	97	99	98
Medicaid	3	2	3	5	5	7
Employer-provided health insurance	58	63	36	32	30	26
Supplementary health insurance	10	11	33	32	38	38
Long-term care insurance	7	7	12	13	10	9
Panel D: Life insurance, annuities including private pensions, and net worth (thousands of 2005 dollars)						
Ownership rate (percent):						
All life insurance	78	80	71	71	59	57
Term life insurance	62	65	50	51	38	38
Whole life insurance	29	33	30	28	23	19
Annuities including private pensions	40	45	56	54	52	53
Annuities excluding private pensions	1	1	4	4	6	6
Median face value conditional on ownership:						
All life insurance	57	72	18	20	10	10
Term life insurance	50	67	12	14	7	7
Whole life insurance	35	40	20	20	11	11
Net worth excluding life insurance and annuities	120	161	186	187	170	153

Table 3: Health Distribution, Life Expectancy, and Out-of-Pocket Health Expenses
Panel A reports the health distribution at each age for a population of respondents who are in good health at age 51. Panel B reports the remaining life expectancy by age and health. Panel C reports annual out-of-pocket health expenses by age and health in thousands of 2005 dollars. Panel D reports annual income by age in thousands of 2005 dollars. Panel E reports the present value of future income in excess of out-of-pocket health expenses by age and health in thousands of 2005 dollars. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

	Age						
Health	51	59	67	75	83	91	99
Panel A: Health distribution (percent)							
Dead	0	15	30	45	62	83	97
Poor	0	22	18	16	14	9	2
Good	100	63	52	39	23	8	1
Panel B: Remaining life expectancy (years)							
Poor	24	20	15	11	8	5	4
Good	26	23	19	14	10	7	4
Mean	26	22	18	13	9	6	4
Panel C: Out-of-pocket health expenses (thousands of 2005 dollars per year)							
Poor	2.0	4.5	7.6	12.5	21.4	37.9	69.5
Good	0.4	1.2	2.6	4.6	7.4	10.6	13.7
Mean	0.4	2.1	3.8	6.8	12.6	25.2	53.8
Panel D: Income (thousands of 2005 dollars per year)							
Mean	51	38	26	21	18	16	14
Panel E: Present value of future disposable income (thousands of 2005 dollars)							
Poor	428	232	107	18	-42	-78	-96
Good	467	270	135	31	-49	-106	-135
Mean	467	260	128	27	-46	-91	-107

Table 4: Estimated Household Preferences

The subjective discount factor is calibrated to 0.96 annually, and the utility weight for good health is normalized to one. The remaining preference parameters are estimated by continuous-updating generalized method of moments with heteroskedasticity-robust standard errors in parentheses. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

Parameter	Symbol	Value
Subjective discount factor	β	0.96
Relative risk aversion	γ	2.51
		(0.01)
Utility weight for death	$\omega(1)$	4.64
		(0.03)
Utility weight for poor health	$\omega(2)$	0.71
		(0.01)
Utility weight for good health	$\omega(3)$	1.00

Table 5: Explaining the Deviations from the Optimal Health and Mortality Delta

A linear regression model is used to explain the deviations from the optimal health and mortality delta, normalized by total wealth. The explanatory variables in the first specification include dummy variables for poor health and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with age, and cohort dummies. The omitted cohort is those born prior to 1911. Additional explanatory variables in the second specification include dummy variables for marital status, living children, education, and self-reported general health status. The omitted categories are no high school education and good self-reported health status. The table reports the regression coefficients with heteroskedasticity-robust t-statistics in parentheses. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

	Health delta				Mortality delta			
Explanatory variable	(1)		(2)		(3)		(4)	
Poor health	-0.23	(-4.88)	-0.11	(-2.28)	-3.53	(-5.06)	-1.49	(-2.25)
65 or older	-0.25	(-1.21)	-0.55	(-2.42)	3.44	(2.73)	5.13	(3.17)
Married			0.61	(6.69)			11.29	(8.59)
Has living children			0.12	(1.21)			3.12	(1.96)
Education:								
High school graduate			0.20	(2.43)			6.44	(5.06)
College graduate			0.44	(5.13)			9.24	(7.41)
Self-reported health status:								
Poor			0.15	(2.76)			2.72	(3.63)
Fair			0.13	(2.44)			2.44	(3.73)
Very good			-0.43	(-4.19)			-8.00	(-5.14)
Excellent			-0.68	(-3.93)			-10.65	(-4.59)
$(\text{Age} - 51)/10$	-0.67	(-4.60)	-1.00	(-4.58)	6.48	(2.84)	17.94	(5.09)
× Poor health	0.28	(4.22)	0.26	(3.97)	3.45	(4.89)	2.27	(3.40)
× 65 or older	0.46	(2.07)	0.73	(3.14)	-5.43	(-2.29)	-7.92	(-2.92)
× Married			0.28	(2.51)			-5.13	(-4.17)
× Has living children			-0.01	(-0.04)			-1.98	(-1.28)
× High school graduate			0.30	(2.84)			-2.80	(-2.33)
× College graduate			0.22	(2.05)			-4.58	(-3.90)
× Poor			-0.07	(-0.75)			-2.35	(-2.95)
× Fair			0.00	(-0.03)			-1.43	(-2.15)
× Very good			0.09	(0.70)			5.46	(3.71)
× Excellent			-0.10	(-0.51)			5.39	(2.47)
$(\text{Age} - 51)^2/100$	0.24	(2.27)	0.38	(3.55)	-2.29	(-1.58)	-4.35	(-2.71)
× Poor health	-0.05	(-2.53)	-0.05	(-2.93)	-0.67	(-4.29)	-0.51	(-3.41)
× 65 or older	-0.20	(-1.85)	-0.27	(-2.50)	2.31	(1.60)	3.23	(2.17)
× Married			-0.10	(-3.67)			0.60	(2.33)
× Has living children			-0.02	(-0.63)			0.28	(0.84)
× High school graduate			-0.06	(-2.46)			0.36	(1.42)
× College graduate			-0.05	(-1.85)			0.68	(2.75)
× Poor			0.01	(0.21)			0.47	(2.56)
× Fair			-0.01	(-0.35)			0.22	(1.43)
× Very good			0.00	(0.07)			-0.94	(-3.04)
× Excellent			0.06	(1.30)			-0.73	(-1.61)
Birth cohort:								
1911–1915	0.17	(1.84)	0.16	(1.73)	0.46	(1.84)	0.37	(1.49)
1916–1920	0.20	(2.17)	0.14	(1.53)	0.81	(3.28)	0.66	(2.68)
1921–1925	0.28	(2.87)	0.17	(1.73)	1.32	(4.73)	1.09	(3.91)
1926–1930	0.20	(1.90)	0.02	(0.18)	1.46	(4.51)	0.94	(2.89)
1931–1935	0.13	(1.22)	0.00	(0.04)	1.31	(3.54)	1.13	(3.07)
1936–1940	0.14	(1.24)	-0.01	(-0.09)	2.17	(5.07)	1.75	(4.11)
1941–1945	0.09	(0.80)	-0.06	(-0.53)	2.66	(5.15)	2.20	(4.33)
1946–1950	-0.05	(-0.38)	-0.21	(-1.77)	4.34	(7.15)	3.64	(6.08)
1951–1955	-0.40	(-2.91)	-0.48	(-3.53)	7.22	(9.32)	7.67	(9.50)
R^2 (percent)	0.34		5.54		1.51		8.19	
Observations	32,778		32,341		32,778		32,341	

Table 6: Welfare Cost of Household Insurance Choice

The welfare cost for each household is measured by the deviations from the optimal health and mortality delta. The lifetime welfare cost is based on the probability of future ownership of health and longevity products, implied by the probit model in Table 7. The median welfare cost for each age group is reported as percent of total wealth. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

	Age					
	51–58	59–66	67–74	75–82	83–90	91–
Panel A: Per-period welfare cost (percent of total wealth)						
Total cost	0.54	0.35	0.34	0.42	0.49	0.49
	(0.09)	(0.03)	(0.03)	(0.06)	(0.17)	(0.86)
Cost due to health delta	0.01	0.03	0.06	0.11	0.15	0.14
	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)	(1.11)
Cost due to mortality delta	0.54	0.33	0.29	0.36	0.51	0.75
	(0.09)	(0.03)	(0.04)	(0.08)	(0.28)	(1.70)
Panel B: Lifetime welfare cost (percent of total wealth)						
Total cost	17.43	13.59	7.75	3.02	1.63	1.23
	(2.98)	(0.68)	(0.21)	(0.16)	(0.28)	(1.31)
Cost due to health delta	0.71	0.52	0.49	0.48	0.42	0.28
	(0.42)	(0.01)	(0.01)	(0.02)	(0.11)	(2.35)
Cost due to mortality delta	18.27	14.69	8.67	3.57	2.14	1.80
	(2.89)	(0.65)	(0.19)	(0.17)	(0.46)	(2.53)

Table 7: Predicting the Future Ownership of Health and Longevity Products

A probit model is used to predict the ownership of a given policy at two years from the present interview. The explanatory variables include dummy variables for present policy owner, poor health, and 65 or older; a quadratic polynomial in age; the interaction of the dummy variables with age; and cohort dummies. The omitted cohort is those born prior to 1911. The table reports the marginal effects on the probability of ownership (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

Explanatory variable	Term life insurance		Whole life insurance		Annuities including private pensions		Supplementary health insurance		Long-term care insurance	
Present owner	46.36	(24.39)	66.80	(38.66)	53.19	(43.64)	33.46	(15.28)	23.84	(9.13)
Poor health	-2.39	(-1.05)	-1.23	(-0.62)	-5.78	(-3.55)	3.38	(2.67)	-1.42	(-1.97)
65 or older	-11.83	(-1.42)	-26.47	(-3.44)	2.85	(0.40)	19.82	(4.98)	1.80	(0.58)
$(\text{Age} - 51)/10$	17.02	(2.43)	-10.46	(-1.73)	-23.87	(-5.05)	-21.56	(-6.33)	-1.57	(-0.75)
$\times$ Present owner	2.97	(1.03)	-2.01	(-0.74)	10.83	(4.95)	6.38	(3.99)	9.90	(7.45)
$\times$ Poor health	-0.04	(-0.01)	0.03	(0.01)	5.57	(2.36)	-0.29	(-0.19)	0.82	(0.76)
$\times$ 65 or older	-4.35	(-0.45)	29.90	(3.41)	13.96	(1.81)	3.84	(0.83)	1.28	(0.37)
$(\text{Age} - 51)^2/100$	-12.95	(-2.77)	3.51	(0.85)	11.83	(3.81)	19.02	(8.60)	2.26	(1.61)
$\times$ Present owner	-0.48	(-0.60)	-0.16	(-0.21)	-0.53	(-0.84)	-1.17	(-2.86)	-1.42	(-3.88)
$\times$ Poor health	0.25	(0.29)	0.27	(0.34)	-1.21	(-1.70)	0.26	(0.62)	0.08	(0.25)
$\times$ 65 or older	11.03	(2.29)	-9.00	(-2.09)	-11.70	(-3.48)	-14.81	(-6.43)	-2.46	(-1.61)
Birth cohort:										
1911–1915	8.00	(2.11)	-9.47	(-3.55)	2.70	(0.87)	12.74	(5.09)	2.06	(0.92)
1916–1920	14.23	(3.86)	-11.61	(-4.54)	0.90	(0.27)	13.97	(5.07)	1.79	(0.76)
1921–1925	17.76	(4.75)	-13.74	(-5.32)	-2.59	(-0.72)	15.12	(5.11)	4.92	(1.68)
1926–1930	20.17	(5.25)	-16.40	(-6.53)	-3.83	(-1.03)	18.48	(5.76)	5.72	(1.85)
1931–1935	25.27	(6.54)	-18.78	(-6.93)	-6.80	(-1.78)	16.72	(5.49)	5.91	(1.98)
1936–1940	28.79	(7.45)	-22.52	(-8.44)	-10.14	(-2.63)	14.18	(4.80)	6.47	(2.16)
1941–1945	31.50	(9.66)	-20.74	(-9.92)	-11.95	(-3.08)	9.26	(2.98)	9.16	(2.42)
1946–1950	35.92	(14.07)	-22.17	(-14.75)	-14.27	(-3.62)	6.48	(1.97)	10.81	(2.48)
1951–1955	36.75	(16.66)	-21.57	(-17.05)	-21.89	(-5.39)	-0.89	(-0.27)	11.23	(2.29)
Correctly predicted (percent):										
Both outcomes	75.68		85.20		81.82		85.07		92.17	
Owner only	77.24		74.62		81.84		60.66		64.51	
Not owner only	73.56		89.52		81.79		89.84		94.37	
Observations	18,353		18,651		39,457		38,031		38,080	

Table 8: Welfare Cost under Actuarially Unfair Insurance

This table reports the welfare cost under an alternative assumption that health and longevity products are more expensive than actuarially fair. The discount rates on life insurance, annuities, and supplementary health insurance are calibrated to 0 percent annually, while the riskless interest rate is 2 percent. The median welfare cost for each age group is reported as percent of total wealth. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

	Age					
	51–58	59–66	67–74	75–82	83–90	91–
Panel A: Per-period welfare cost (percent of total wealth)						
Total cost	0.51	0.34	0.33	0.42	0.49	0.51
	(0.09)	(0.03)	(0.03)	(0.06)	(0.17)	(0.85)
Cost due to health delta	0.01	0.03	0.06	0.11	0.16	0.14
	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)	(1.11)
Cost due to mortality delta	0.51	0.32	0.29	0.36	0.51	0.74
	(0.09)	(0.03)	(0.04)	(0.08)	(0.28)	(1.69)
Panel B: Lifetime welfare cost (percent of total wealth)						
Total cost	16.93	13.30	7.50	2.97	1.64	1.22
	(2.91)	(0.67)	(0.20)	(0.16)	(0.28)	(1.30)
Cost due to health delta	0.76	0.54	0.48	0.47	0.41	0.28
	(0.42)	(0.01)	(0.01)	(0.02)	(0.11)	(2.35)
Cost due to mortality delta	17.91	14.40	8.42	3.51	2.14	1.81
	(2.83)	(0.64)	(0.19)	(0.17)	(0.45)	(2.52)

Table 9: An Optimal Portfolio of Health and Longevity Products

Panel A reports the optimal health and mortality delta by age, predicted by the life-cycle model with the preference parameters in Table 4. Panel B reports a portfolio of short-term life insurance, deferred annuities, short-term health insurance, and bonds that replicates the optimal health and mortality delta. Short-term policies have maturity of two years, and the income from deferred annuities start at age 65. Panel C reports the cost of the optimal portfolio in thousands of 2005 dollars, averaged across the health distribution at the given age. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

	Age						
	51	59	67	75	83	91	99
Panel A: Optimal health and mortality delta (thousands of 2005 dollars)							
Health delta	-2	-11	-20	-24	-20	6	80
Mortality delta	190	22	-66	-119	-155	-178	-179
Panel B: Optimal portfolio (units)							
Short-term life insurance	211	92	43	0	0	0	0
Deferred annuities	5	12	15	21	37	61	91
Short-term health insurance	0.00	0.00	0.00	0.12	0.58	0.74	0.81
Bonds	41	144	177	207	193	181	169
Panel C: Cost of the optimal portfolio (thousands of 2005 dollars)							
Short-term life insurance	6	4	2	0	0	0	0
Deferred annuities	19	62	96	99	116	109	80
Short-term health insurance	0	0	0	1	7	20	42
Bonds	39	138	170	198	186	174	162
Total cost	65	204	268	298	308	302	284

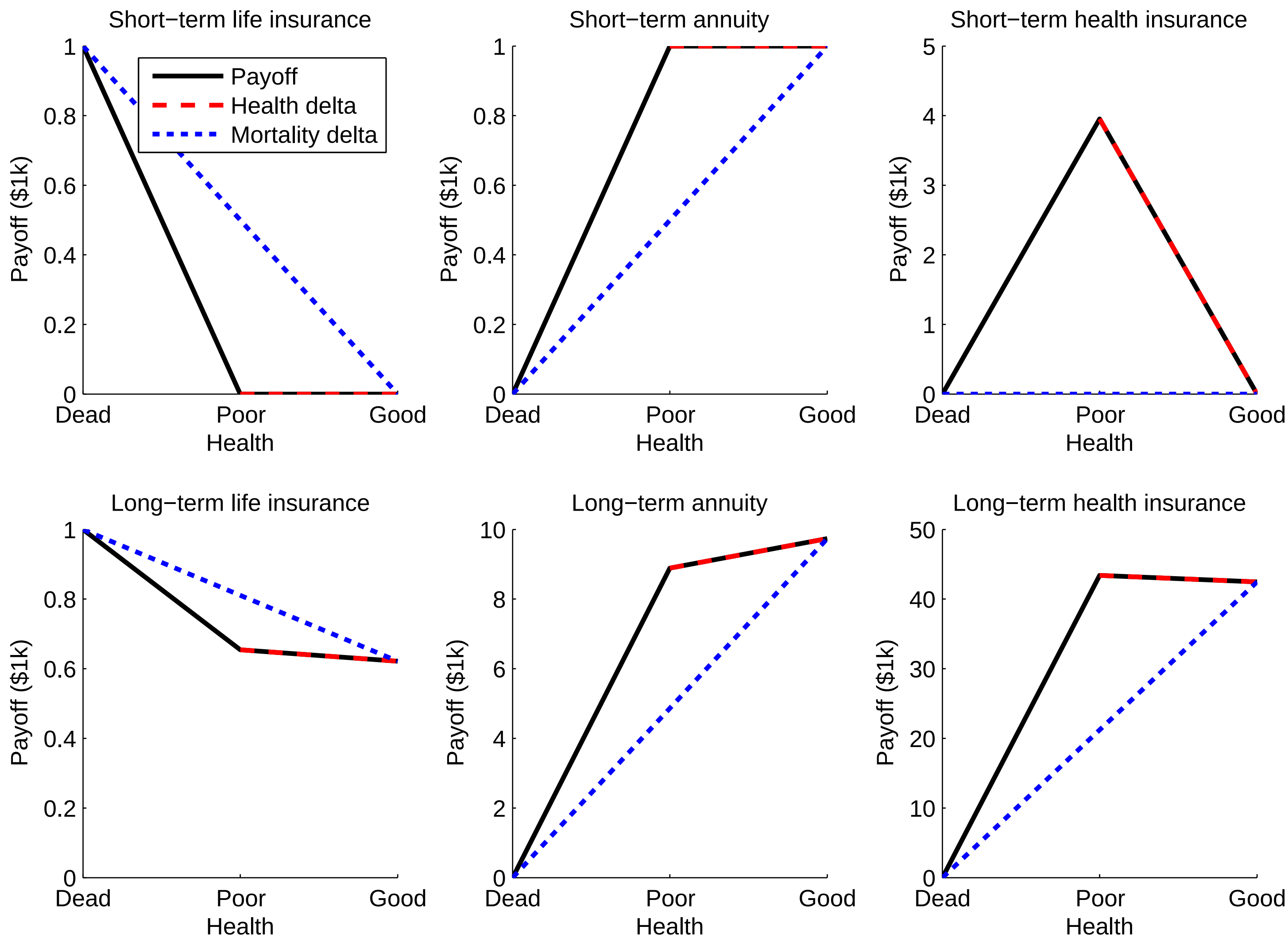

Figure 1: Health and Mortality Delta for Health and Longevity Products

This figure reports the health and mortality delta for life insurance, annuities, and health insurance. The solid line represents the payoff of each policy for the three possible health states in two years, reported in thousands of 2005 dollars. Health delta is minus the slope of the dashed line, normalizing the horizontal distance between good and poor health to one. Mortality delta is minus the slope of the dotted line, normalizing the horizontal distance between good health and death to one. Short-term policies have maturity of two years, while long-term policies mature at death. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.

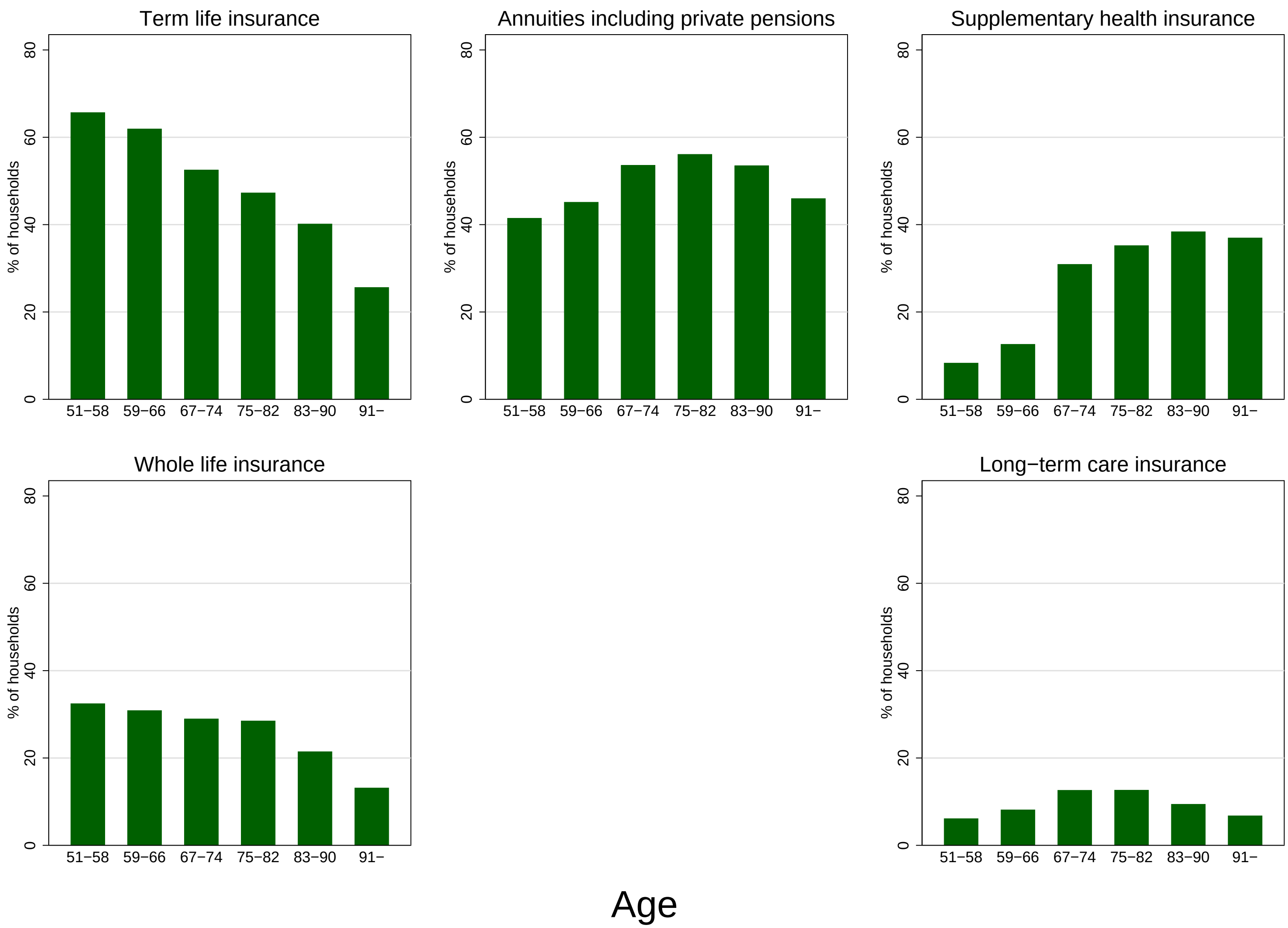

Figure 2: Ownership Rates of Health and Longevity Products

Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. Supplementary health insurance includes Medigap insurance and refers to any coverage that is not government, employer-provided, or long-term care insurance. The sample consists of male respondents aged 51 and older in the Health and Retirement Study from 1992 to 2006.

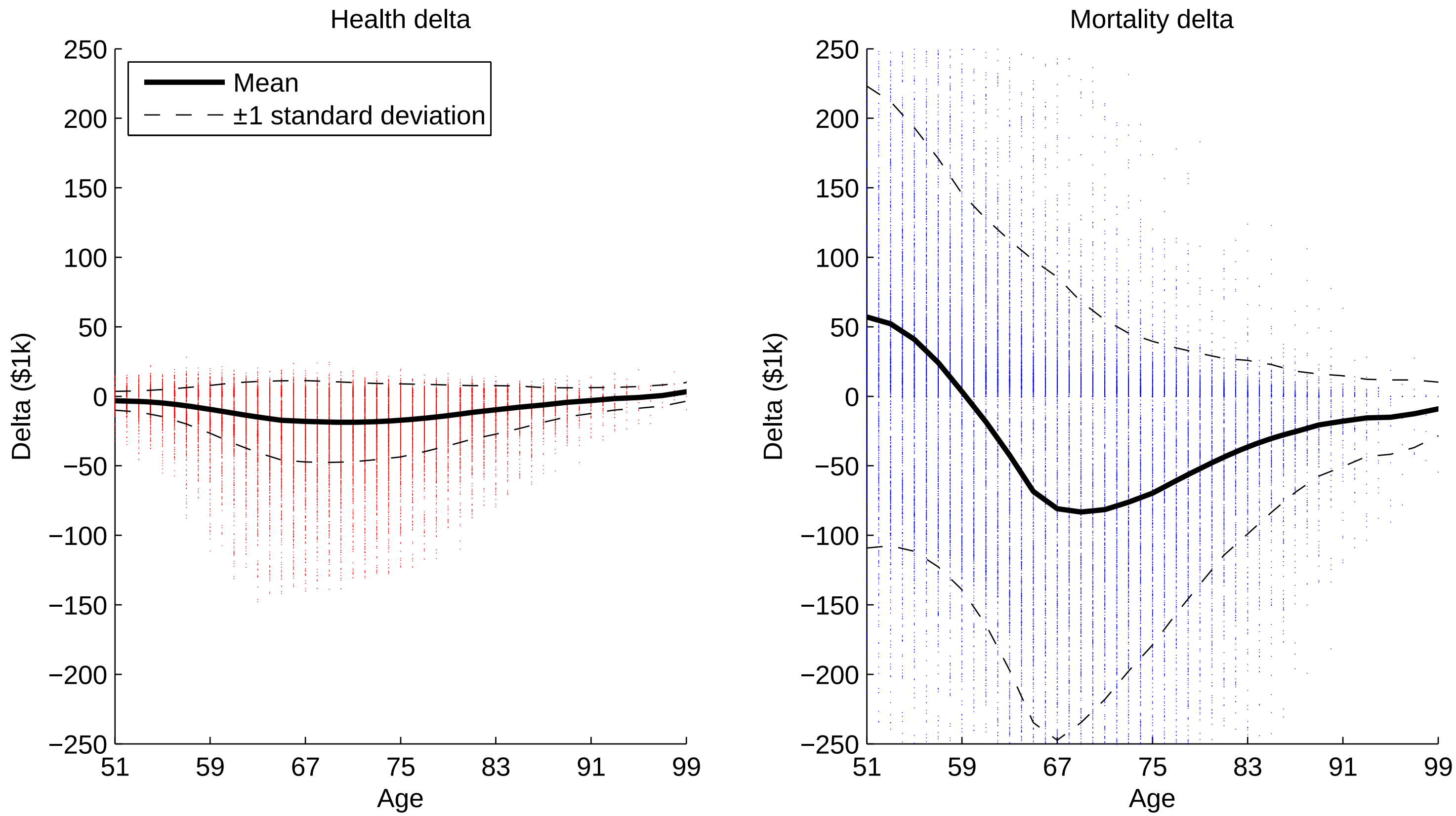

Figure 3: Observed Health and Mortality Delta over the Life Cycle
The left (right) panel reports the mean and the standard deviation of observed health (mortality) delta by age, smoothed around a plus or minus two-year window. Each dot represents one of 32,778 household-interview observations in the Health and Retirement Study from 1992 to 2006.

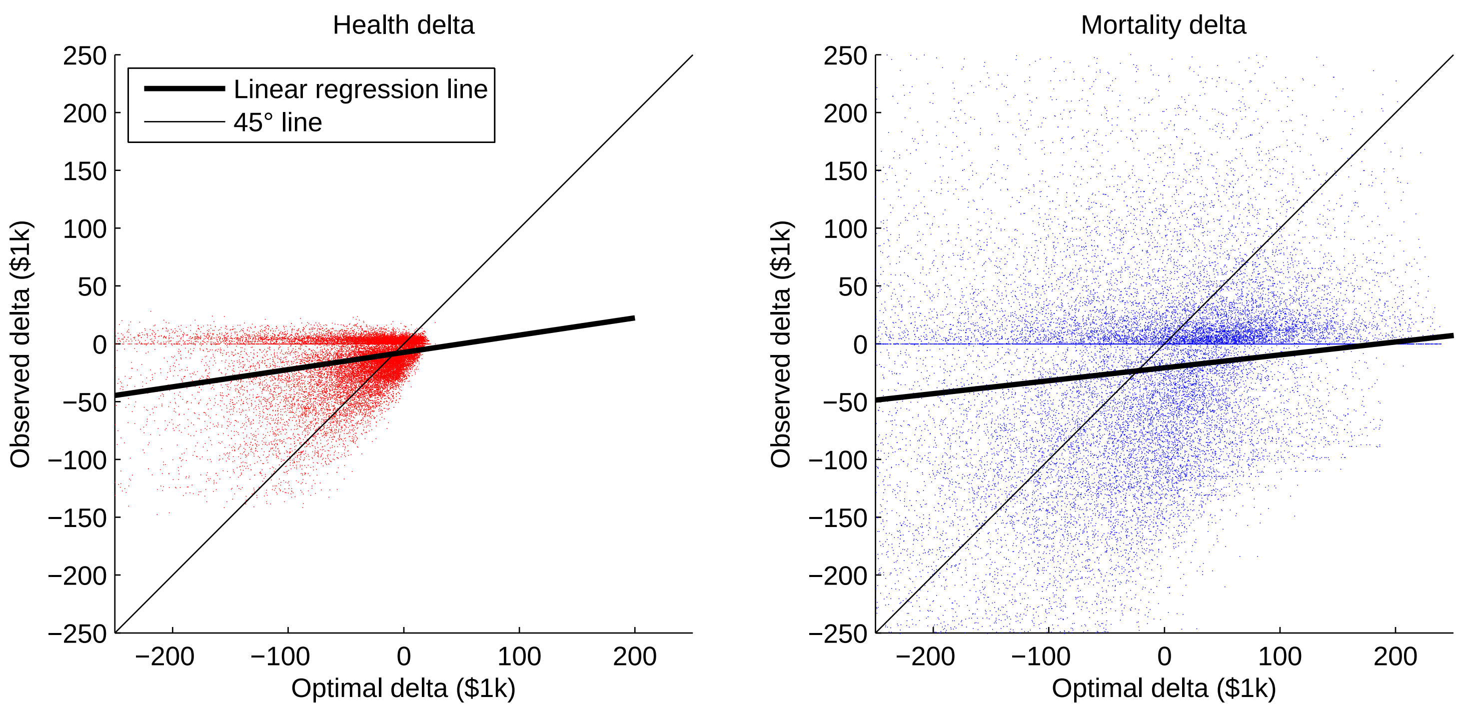

Figure 4: Observed versus Optimal Health and Mortality Delta
The left (right) panel is a scatter plot of the observed versus the optimal health (mortality) delta. The optimal health and mortality delta are predicted by the life-cycle model with the preference parameters in Table 4. Each dot represents one of 32,778 household-interview observations in the Health and Retirement Study from 1992 to 2006.

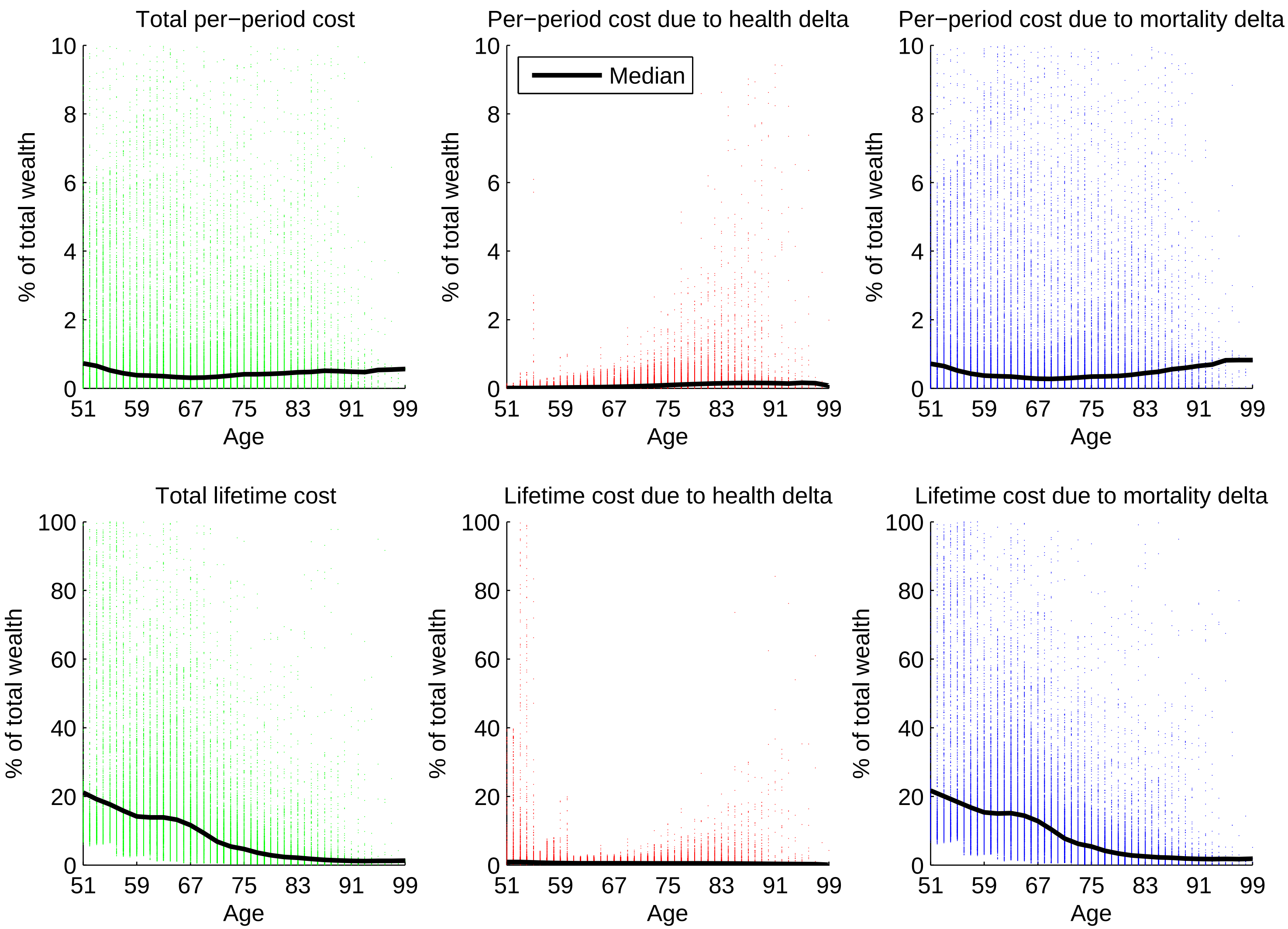

Figure 5: Welfare Cost of Household Insurance Choice

The welfare cost for each household is measured by the deviations from the optimal health and mortality delta. The lifetime welfare cost is based on the probability of future ownership of health and longevity products, implied by the probit model in Table 7. Each dot represents one of 32,778 household-interview observations in the Health and Retirement Study from 1992 to 2006. The median welfare cost is smoothed around a plus or minus two-year window and expressed as percent of total wealth.

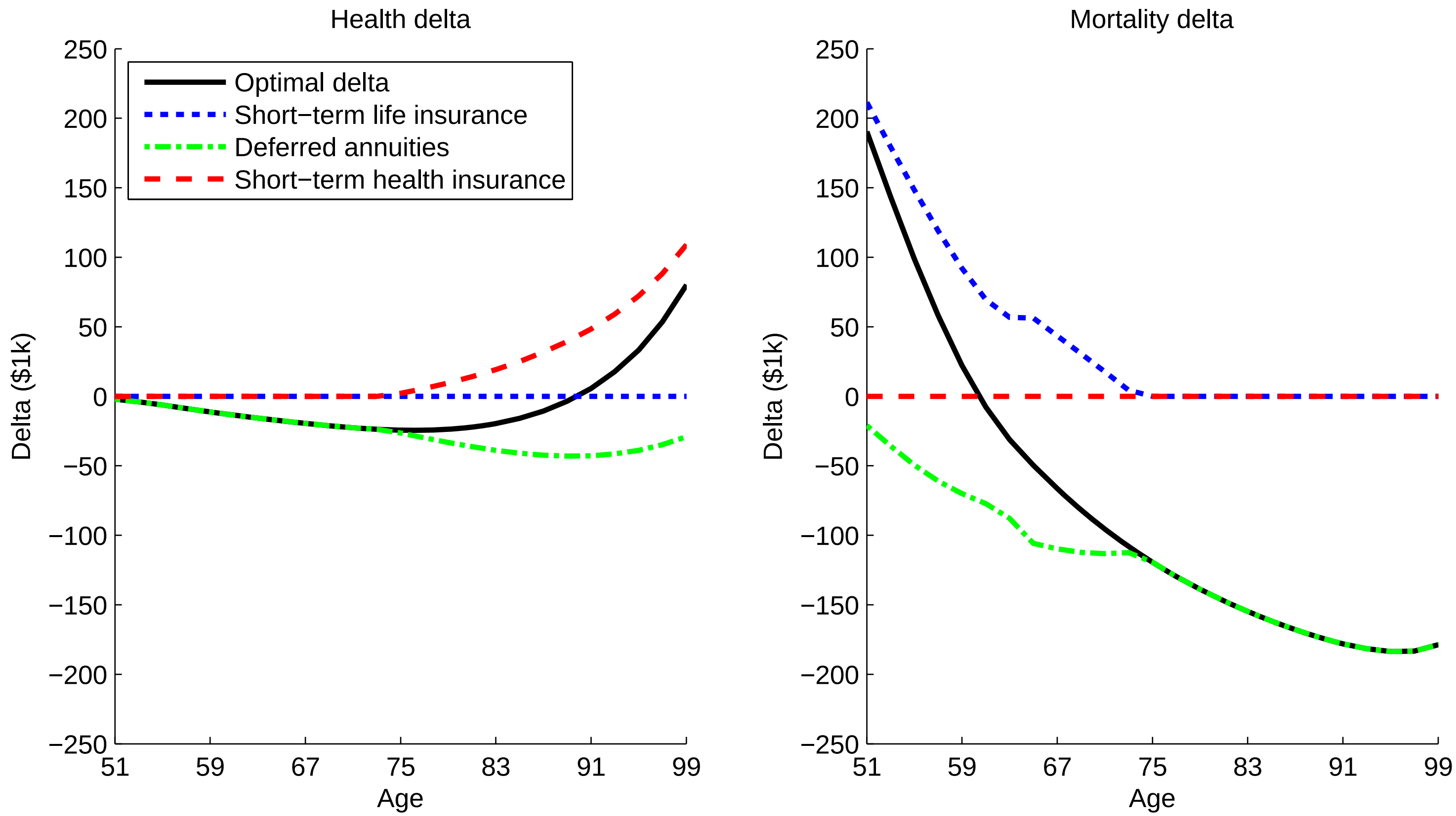

Figure 6: Optimal Health and Mortality Delta over the Life Cycle
The sum of health (mortality) delta for short-term life insurance, deferred annuities, and short-term health insurance equals the optimal health (mortality) delta at each age. Short-term policies have maturity of two years, and the income from deferred annuities start at age 65. The reported estimates are for male respondents in the Health and Retirement Study, who were born between 1936 and 1940 and are in good health at age 51.