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GOVERNMENT POLICY, CREDIT MARKETS AND ECONOMIC ACTIVITY

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ABSTRACT

The US government has recently conducted large scale purchases of assets and implemented policies that reduced the cost of funds to financial institutions. Arguably these policies have helped to correct credit market dysfunctions, allowing interest rate spreads to shrink and output to begin a recovery. We study four models of financial frictions which explore different channels by which these effects might have occured. Recent events have sparked a renewed interest in leverage restrictions and the consequences of bailouts of the creditors of banks with under-performing assets. We use two of our models to consider the welfare and other effects of these policies.

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1. Introduction

1.1. Preliminary Observations

The recession that began in late 2007 poses new challenges for macroeconomic modeling. Asset values collapsed, initially in housing and then in equity (see Figure 1a). In late 2008, interest rate spreads suddenly jumped to levels not seen in over 70 years (see Figure 1b).[1] There was widespread concern among policymakers that financial markets had become dysfunctional because of a deterioration in financial firm balance sheets associated with the fall in asset values.[2] These concerns were reinforced by the dramatic fall in investment in late 2008 (see Figure 1c), which suggested that a serious breakdown in the intermediation sector might have occurred. The US Treasury and Federal Reserve (Fed) reacted forcefully. The Fed's actions had the effect of reducing the cost of funds for financial institutions. For example, the Federal Funds rate was driven to zero (see Figure 1d) and the interest rate on the three month commercial paper of financial firms also fell sharply. In addition, the Fed took a variety of unconventional actions by acquiring various kinds of financial claims on financial and non-financial institutions. Standard macroeconomic models are silent on the rationale and on the effects of the Fed's unconventional monetary policies.

Still, there is casual evidence that suggests the Fed's unconventional monetary policy helped.[3] The Fed began to purchase financial assets in late 2008 and financial firm commercial paper spreads dissipated quickly thereafter. In March 2009 the Fed expanded its asset purchase program enormously and corporate bond spreads also began to come down (Figure 1b). Soon, aggregate output began to recover and the National Bureau of Economic Research declared an end to the recession in June, 2009 (Figure 1c). Of course, it is difficult to say what part of the recovery (if any) was due to the Fed's policies, what part was due to the tax and spending actions in the American Recovery and Reinvestment Act of 2009, and what part simply reflects the internal dynamics of the business cycle. Many observers suppose that the Fed's policies had at least some effect.

These observations raise challenging questions for macroeconomics:

- What are the mechanisms whereby a fall in asset values leads to a drop in financial intermediation and a jump in interest rate spreads?
- How do sharp reductions in short term interest rates and large scale government asset purchases correct these financial market dysfunctions? What are the effects of these actions on economic efficiency?

The answers to these questions are important for determining which asset market program should be undertaken and at what scale. Traditional macroeconomic models used in policy analysis in central banks have little to say about these questions.

[1] We examined monthly data on the interest rate on BAA and AAA rated bonds taken from the St. Louis Federal Reserve bank website. In December, 2008 the interest rate spread on BAA over AAA bonds peaked at 3.38 percent, at an annual rate. This is a higher spread than was observed in every month since 1933.

[2] The extent to which balance sheets became imparied was hard to assess because there did not exist clear market values for many of the financial assets in the balance sheets of financial institutions.

[3] See, for example, Gagnon, Raskin, Remache, and Sack (2010). For a less sanguine perspective on the effectiveness of the Fed's policy, see Krishnamurthy and Vissing-Jorgensen (2010) and Taylor and Williams (2009).

We survey the answers to the above questions from the perspective of four standard models borrowed from the banking literature and inserted into a general equilibrium environment. In each case, we drastically simplify the model environment so that we can focus sharply on the main ideas. Accordingly, the kind of details that are required to ensure that models fit quarterly time series data well are left out. For example, the models have only two periods, most shocks are left out of the analysis and we abstract from such things as labor effort, capital utilization, habit persistence, nominal variables, money, price and wage-setting frictions, etc. We also abstract from the distortionary effects of seigniorage and the other mechanisms by which governments and central banks acquire the purchasing power to finance their acquisition of private assets. We abstract from these complications by assuming revenues are raised with non-distorting, lump sum taxes. Finally, we make assumptions that allow us to abstract from the effects of changes in the distribution of income in the population. For the reasons described in section 2 below, it is important to relax this assumption in more general analyses of unconventional monetary policy.

Ultimately, the questions raised above must be addressed in fully specified dynamic, stochastic general equilibrium (DSGE) models. Only then can we say with confidence which of the financial frictions discussed below is quantitatively important. Similarly, we require a DSGE model if we are to quantify the magnitude of the required policy interventions. Work on the task of integrating financial frictions into DSGE models is well under way.[4] Our hope is that this paper may be useful in this enterprise by providing a bird's eye view of the qualitative properties of the different models, in terms of their implications for the questions raised above.

Our survey does not examine all models of financial frictions. For example, we do not review models that can be used to think about the effects of government asset purchases on a liquidity shortage (see, e.g., Moore (2009) and Kiyotaki and Moore (2008)).[5] Instead, we review models that are in the spirit of Mankiw (1986), Bernanke, Gertler and Gilchrist (1999) (BGG), Gertler and Karadi (2009) and Gertler and Kiyotaki (2011) (GK^2). We review four models. The first three focus on financial frictions in the banking sector. Among these, the first two feature moral hazard problems and the third features adverse selection. The fourth model features asymmetric information and monitoring costs along the lines stressed in BGG.

Our models capture in different ways the hypothesis that a drop in net worth caused the rise in interest rate spreads and the fall in investment and intermediation that occurred in 2007 and 2008. In our version of the BGG model, we also allow for an increase in microeconomic uncertainty to help account for the events of 2007 and 2008.[6] In each of our

[4] There is now a large literature devoted to constructing quantitative dynamic, stochastic general equilibrium models for evaluating the consequences of government asset purchase policies. For a partial list of this work, see Ajello (2010), Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997), Christiano, Motto and Rostagno, (2003,2010), Curdia and Woodford (2009), Del Negro, Eggertsson, Ferrero and Kiyotaki, (2010), Dib (2010), Fisher (1999), Gertler and Karadi (2009), Gertler and Kiyotaki (2011), Hirakata, Sudo and Ueda (2009a,2009b,2010), Liu, Wang and Zha (2010), Meh and Moran (2010), Ueda (2009), Zeng (2010).

[5] The Moore and Kiyotaki and Moore ideas are pursued quantitatively in Ajello (2010), and Del Negro, Eggertsson, Ferrero and Kiyotaki (2010).

[6] To our knowledge, the first papers to consider the economic effects of variations in microeconomic uncertainty are Williamson (1987) and the early drafts of Christiano, Motto and Rostagno (2010). More

model analyses, we investigate the consequences for economic efficiency of the following tax-financed government interventions: (i) reductions in the cost of funds to financial firms, (ii) equity injections into financial firms, (iii) loans to financial and nonfinancial firms, and (iv) transfers of net worth to non-financial firms. Regarding (ii), we define an equity injection as a tax-financed transfer of funds to a bank in which all the resulting profits are repaid to the government.[7] In the case of (iii), we define a government bank loan as a tax-financed commitment of funds that must be repaid on the same terms as those received by ordinary depositors.

All the models suggest that (i) helps to alleviate the dysfunctions triggered by a fall in net worth, though the precise mechanisms through which this happens varies. There is less agreement among the models in the case of (ii) and (iii). Whether these policies work depend on the details of the financial frictions. All the models suggest that (iv) helps. This is perhaps not surprising, since (iv) in effect undoes what we assume to be the cause of the trouble. However, policy (iv) entails a redistribution of wealth and income among the population. Since we abstract from the consequences of wealth redistribution, our analysis of (iv) must be viewed as incomplete.

Some policies are best analyzed in only a subset of our models. Examples of such policies include leverage restrictions on banks, as well as a policy of bailing out the creditors of banks experiencing losses on their portfolios. We study the first of these policies in only two of our models, the ones in sections 4 and 6 below. We study creditor bailouts in section 4.

1.2. Overview of the Model Analysis

1.2.1. Moral Hazard I: 'Running Away' Model

We first describe a simplified version of the analysis in GK[2], which focuses on a particular moral hazard problem in the financial sector.[8] This problem stems from the fact that bankers have the ability to abscond with a fraction of the assets they have under management. A repeated version of the one period model that we study provides a crude articulation of the post 2007 events. Before 2007, interest rate spreads were at their normal level (actually, zero according to the model) and the financial system functioned smoothly in that the first-best allocations were supported in equilibrium. Then with the collapse in banking net worth, interest rate spreads jumped and financial markets became dysfunctional, in the sense that the volume of intermediation and investment fell below their first-best levels.

According to the model, banks responded to the decline in their own net worth by restricting the amount of deposits that they issued.[9] Banks did so out of a fear that if they tried to maintain the level of deposits in the face of the decline in their net worth, depositors would lose confidence and take their money elsewhere.[10] Depositors would do so in the

recently, this type of shock has also been considered in Arellano, Bai and Kehoe (2010), Bigio (2011), Ikeda (2011a,b), Jermann and Quadrini (2010) and Kurlat (2010).

[7]By a 'bank' we mean any institution that intermediates between borrowers and lenders.

[8]For other analyses in this spirit, see Holmstrom and Tirole (1997), and Meh and Moran (2010).

[9]Throughout the analysis, we assume that banks cannot raise funds by issuing equity. In a private communication, Jamie McAndrews showed us data which suggests that equity is not a flexible source of revenues for banks. Our models are silent on the reasons for this.

[10]The model (and others in this manuscript) assumes that banks cannot increase their net worth. The

(correct) anticipation that a higher level of bank leverage would cause bankers to abscond with bank assets. From this perspective, a sharp cut in the cost of funds to banks calms the fears of depositors by raising bank profits and providing bankers with an incentive to continue doing business normally.

In the case of direct equity injections and loans, we follow GK^2 in assuming that the government can prevent banks from absconding with government funds.[11] Under these circumstances, it is perhaps not surprising that government equity injections and loans, (iii) and (iv), are effective. With the government taking over a part of the economy's intermediation activity, the amount of intermediation handled by the banking system is reduced to levels that can be handled efficiently with the reduced level of banking net worth. Of course, if the nature of the financial market frictions are not something that can be avoided by using the government in this way, then one suspects (iii) and (iv) are less likely to be helpful. This is the message of our second model.

1.2.2. Moral Hazard II: Unobserved Banker Effort

Our second model captures moral hazard in banking in a different way. We suppose that bankers must exert a privately observed and costly effort to identify good investment projects. The problem here is not that bankers may abscond with funds. Instead, it is that bankers may exert too little effort to make sure that assets under management are invested wisely. Bankers must be given an incentive to exert the efficient amount of effort. One way to accomplish this is for bank deposit rates to be independent of the performance of bank portfolios, so that bankers receive the full marginal return from exerting extra effort. But, bankers must have sufficient net worth of their own if the independence property of deposit rates is to be feasible. This is because we assume that bankers cannot hold a perfectly diversified portfolio of assets. As a result, bankers - even those that exert high effort - occasionally experience a low return on their assets. For deposit rates to be independent of the performance of banker portfolios, bankers with poorly performing portfolios must have sufficient net worth to pay the return on their deposits. We show that when bankers have a sufficiently high level of net worth, then bank deposit rates are independent of the performance of bank portfolios and equilibrium supports the first-best allocations.

Financial markets become dysfunctional when the banks whose assets perform poorly have too little net worth to cover their losses. Depositors in such banks must in effect share in the losses by receiving a low return. To be compensated for low returns from banks with poor assets, depositors require a relatively high return from banks with good assets. But, when deposit rates are linked to the performance of bank assets in this way, bankers have less incentive to exert effort. Reduced effort by bankers pushes down the average return on bank assets and, hence, deposit rates for savers. With lower deposit rates, household deposits - and, hence, investment - are reduced below their efficient levels.

model offers no explanation for this. The assumption does appear to be roughly consistent with observations. In private communication, James McAndrews shared the results of his research with Tobias Adrian. That work shows that bond issuance by financial firms declined sharply in the recent crisis, while equity issuance hardly rose.

[11] In addition, we assume that - unlike the bankers in the model - government employees do not have the opportunity to abscond with tax revenues.

Consider the implications for policy. Interest rate subsidies, policy (i), help by reducing the cost of funds to banks. This policy reduces banks' liabilities in the bad state and so increases the likelihood that deposit rates can be decoupled from bank asset performance. This result is of more general interest, because it conflicts with the widespread view that interest rate subsidies to banks cause them to undertake excessive risk. In our environment, an interest rate subsidy increases bankers' incentive to undertake effort, leading to a rise in the mean return on their portfolios and a corresponding reduction in variance. Interest rate subsidies have this effect by raising the marginal return on banker effort.

Government equity injections and loans, policies (ii) and (iii), have no effect in the model. Although the proof of this finding involves details, the result is perhaps not surprising. The government equity injections and bank loans that we consider do not offer any special opportunity to avoid financial frictions in the way that our first model of moral hazard does. It is not obvious (at least, to us) what unique advantage the government has in performing intermediation, when that activity involves a costly and hidden effort. Our hidden effort model illustrates the general principle that the sources of moral hazard matter for whether a particular government asset purchase program is effective.

Our hidden action model is well suited to studying the effects of leverage restrictions and bailouts of creditors to banks with poorly performing assets. We have noted above that when net worth is low, it may not be possible for deposits rates to be decoupled from the performance of bank assets. Obviously, if the quantity of deposits were sufficiently low, then deposits rates could be fixed and independent of bank asset performance even if net worth is low. We show that when binding leverage restrictions are placed on banks when net worth is low, social welfare is increased.

1.2.3. Adverse Selection

Our third model focuses on adverse selection as a source of financial market frictions.[12] To many observers, adverse selection is a natural framework for thinking about the financial market turmoil of recent years. Before 2007 owners of mortgage backed securities (MBS) had little difficulty selling these assets because most thought they were very secure. When housing prices collapsed, a cloud of uncertainty settled over the whole market as people wondered which MBS had been most affected. Market participants suspected that institutions selling MBS did so because they knew their MBS to be of poor quality. As a result, the market assigned a low price to these assets. This ensured that the only MBS traded was in fact bad MBS, with holders of good MBS not willing to sell at effectively fire-sale prices.[13] These general considerations motivate us to include a model of adverse selection in our analysis.

In our adverse selection model there are entrepreneurs who are endowed with net worth and with an investment project. Each entrepreneur must choose to either invest its net worth in its investment project, or deposit the net worth in a bank. Operating an investment

[12] There are several analyses of the recent credit crisis that focus on adverse selection in credit markets. See, for example, Chari, Shourideh and Zetlin-Jones (2010), Fishman and Parker (2010), House (2006), Ikeda (2011a,b), Kurlat (2010).

[13] This is a summary of the argument in Shimer, http://gregmankiw.blogspot.com/2008/09/case-against-paulson-plan.html. See Eisfeldt (2004) for a theoretical analysis that blends adverse selection and liquidity problems.

project requires a fixed amount of resources that exceeds the net worth of an entrepreneur. An entrepreneur that chooses to operate its investment project must acquire a bank loan. An adverse selection problem arises because the success probability of an investment project is private information for the entrepreneur to which the project belongs.

When entrepreneurial net worth drops, as in the period after 2007, then the adverse selection problem becomes more severe as entrepreneurs' dependence on external finance increases. The model predicts that in times like this, interest rate spreads increase and the shortfall in equilibrium investment relative to its first best level grows larger. We investigate what policies can correct these dysfunctions in credit markets. A subsidy to banks' cost of funds improves the situation. We also consider the effect of government deposits in banks. This type of intervention has no effect in our model.

1.2.4. Asymmetric Information and Monitoring Costs

Our fourth model of financial frictions is a two-period version of the one proposed in BGG. The BGG model is an important model of financial frictions, as there is a large literature analyzing it in fully specified, DSGE models. The model has proved useful in the analysis of macroeconomic data.[14]

In the model, entrepreneurs have access to investment projects. The entrepreneurs have their own net worth, but it is desirable for them to borrow additional funds from banks. Banks in turn obtain funds from households. The payoff on each entrepreneur's project is subject to an idiosyncratic shock. The realization of the shock is known ex post to an entrepreneur and the entrepreneur's household, but not to its banker. The banker must pay a monitoring cost if it wishes to observe the realization of an entrepreneur's idiosyncratic shock. We assume that banks offer entrepreneurs a competitive equilibrium 'standard debt contract'. The contract specifies a loan amount and an interest rate. The entrepreneur repays the loan with interest, if it can. If the entrepreneur cannot repay the loan because its idiosyncratic shock is too low, then it declares bankruptcy and is monitored by the bank which takes everything the entrepreneur has.[15]

Our environment is sufficiently simple that we obtain an analytic characterization of the inefficiency of equilibrium. We show that in the model the marginal social return on loans exceeds the average return, and it is the latter that is communicated to depositors by the deposit rate of interest. Lending is inefficiently low in the equilibrium because a planner prefers that households saving decision is made in response to a signal about the marginal return on loans. This problem is exacerbated when the net worth of entrepreneurs is reduced. Not surprisingly, we find that a policy of subsidizing bank interest rate costs improves welfare. Also, the optimal subsidy is higher when entrepreneurial net worth is low.

[14] See the Christiano, Motto and Rostagno (2003,2010) analysis of the US Great Depression, and of the past three decades' business cycles in the Euro Area and the US. An earlier DSGE model application of the costly state verification and monitoring cost idea can be found in the influential contribution by Carlstrom and Fuerst (1997). For another contribution of this idea in a DSGE model, see Jonas Fisher's doctoral dissertation, published as a 1996 Federal Reserve Bank of Chicago working paper and also in Fisher (1999).

[15] A feature of the BGG model is that there is no risk in banking. Recent extensions of the model introduce risk in banking. See, for example, Hirakata, Sudo and Ueda (2009a,2009b,2010), Ueda (2009), and Zeng (2010).

In addition, we study the effects of direct government loans to entrepreneurs, but find that this has no impact on the equilibrium.

The rest of the paper is organized as follows. Section 2 below describes what we call the Barro-Wallace irrelevance proposition, which sets out a basic challenge that any model of government asset purchases must address. The following two sections describe the two models of moral hazard. Section 5 studies the model of adverse selection. Second 6 describes the two-period version of the BGG model of financial frictions and its implications for policy. A final section presents concluding remarks.

2. The Barro-Wallace Irrelevance Proposition

Any analysis of unconventional policy must confront a basic question. If the government acquires privately issued assets by levying taxes (either in the present or the future), then the ownership of the asset passes from private agents to the government which later reduces households' tax obligations as the asset bears fruit. The question any analysis of asset purchases by the government has to answer why it makes a difference whether private agents hold assets themselves or the government holds them on taxpayers' behalf. In the simplest economic settings, households' intertemporal consumption opportunities are not affected by government asset purchases, so that such purchases are irrelevant for allocations and prices. We refer to this irrelevance result as the Barro-Wallace irrelevance proposition, because it is closely related to the Ricardian equivalence result emphasized by Barro (1974) and extended by Wallace (1981) to open market operations.[16] Any analysis in which government asset purchases have real effects must explain what assumptions have been made to defeat the Barro-Wallace irrelevance result.

One way to defeat Barro-Wallace irrelevance builds on heterogeneity in the population. For example, suppose that a subset of the population has a special desire to hold a certain asset (for example, 30 year Treasury bonds). If the government engages in a tax financed purchase of that bond, then in effect the bond is transferred from the subset of the population that holds it initially, to all taxpayers. Such a redistribution of assets among heterogeneous agents may change prices and allocations. This type of logic may be useful for interpreting the recent substantial changes that have occurred in the Federal Reserve's balance sheet.[17] We do not pursue this line of analysis further here, since we abstract from changes in the distribution of income in the population.

There are other ways in which tax financed purchases of private securities may have real effects. In the examples we explore, this can happen by changing the market rate of interest.

[16] What we are calling the Barro-Wallace irrelevance proposition is applied to government purchases of long term debt in Eggertsson and Woodford (2003).

[17] The logic in the text may also provide the foundation for a theory of the effectiveness of sterilized interventions in the foreign exchange markets.

3. Moral Hazard I: 'Running Away' Model[18]

We construct a two-period model. In the first period, households make deposits in banks. Bankers combine these deposits with their own net worth and provide funds to firms. In the second period, households purchase the goods produced by firms using income generated by bank profits and interest payments on bank deposits. The source of moral hazard is that bankers have an option to default by absconding with an exogenously fixed fraction of their total assets, leaving the rest to depositors. When a sufficiently large fraction of a bank's assets are purchased with bankers' own net worth, then a bank simply hurts itself by defaulting and it chooses not to do so. We show that, when the net worth of banks is sufficiently large that the option to default is not relevant, then the equilibrium allocations correspond to the first-best efficient allocations. We refer to this scenario as a 'normal time'. When banks' net worth is sufficiently low, banks restrict the supply of deposits. Banks do this because they know that if they planned a higher level of deposits, depositors would rationally lose confidence and take their deposits elsewhere. With the supply of deposits reduced in this way, and no change in demand, the market-clearing interest rate on deposits is low. Because the return on bank assets is fixed by assumption, the result is an increase in banks' interest rate spreads.[19] We refer to the situation in which bank net worth is so low that the banking system is dysfunctional and conducts too little intermediation as a 'crisis time'. Thus, the model articulates one view about what happened in the past few years: "a fall in housing prices and other assets caused a fall in bank net worth and initiated a crisis. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity was reduced." In contemplating such a scenario we imagine a version of our two-period model, repeated many times.

Government policy can push the economy out of crisis and back to normal by undoing the underlying cause of the problem. One way the government can do this is by purchasing bank assets. In the Gertler-Karadi and Gertler-Kiyotaki analysis it is assumed that the government has the ability to prevent banks from absconding with bank assets financed by equity or deposit liabilities to the government. We show that sufficiently large government purchases of bank assets can restore the banking system to normal. In particular, government asset purchases cause interest rate spreads to disappear and total intermediation to return to its first best level. Interest rate spreads disappear because government-financed purchases of assets induce a fall in household demand for deposits. If the government purchases are executed on a large enough scale, the fall in the demand for deposits is sufficient to push the deposit interest rate back up to the efficient level where it equals banks' return on their funds. The logic of the Barro-Wallace irrelevance result does not hold in a crisis time because tax-financed government purchases of bank assets have an impact on the interest rate.

Another policy that can resolve a crisis is one in which the government provides tax-financed loans to firms. Under this policy the government returns the proceeds of its investment in firms to households in the form of lower taxes in the second period. Households understand that this government policy is a substitute for their bank deposits and so they reduce the supply of deposits. With the supply and demand for bank deposits both reduced,

[18] This section is based on joint work with Tao Zha.

[19] So, profits per unit of bank deposits rises when banker net worth is low. However, total bank profits may be low because of the lower net worth of the banks.

the deposit interest rate rises back up and the interest rate spread is wiped out. Total intermediation returns to its normal level because, though household deposits are relatively low, this is matched by a corresponding increase in government provision of funds. In this way, government purchases of the assets of nonfinancial business can resolve a crisis.

Finally, we show that a policy of subsidizing banks' cost of funds can push the economy out of a crisis. Such a policy works by increasing banks' profits during a crisis and so reducing their temptation to abscond with bank assets. Understanding that their depositors are aware of this, banks expand their deposits back to the first best level.

We first describe the model. We then formally establish the properties of government policy just reviewed.

3.1. Model

There are many identical households, each with a unit measure of members. Some members are 'bankers' and others are 'workers'. There is perfect insurance inside households, so that all household members consume the same amount, c, in period 1 and C in period 2. In period 1, workers are endowed with y goods and the representative household makes a deposit, d, in a bank subject to its period 1 budget constraint:

$$c + d \leq y.$$

The representative household's period 2 budget constraint is:

$$C \leq Rd + \pi.$$

Here, R represents the gross return on deposits and π denotes the profits brought home by bankers. The household treats π as lump sum transfers. The intertemporal budget constraint is constructed using period 1 and period 2 budget constraints in the usual way:

$$c + \frac{C}{R} \leq y + \frac{\pi}{R}. \tag{3.1}$$

The representative household chooses c and C to maximize

$$u(c) + \beta u(C), \; u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \; \gamma > 0. \tag{3.2}$$

subject to R, π and (3.1). The solution to the household problem is:

$$c = \frac{y + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}, \; d = y - c, \; C = Rd + \pi. \tag{3.3}$$

We can see the basic logic of the Barro-Wallace irrelevance proposition from (3.1). Suppose the government raises taxes, T, in period 1, uses the proceeds to purchase T deposits and gives households a tax cut, RT, in period 2. The periods 1 and 2 budget constraints are replaced by:

$$c + d \leq y - T, \; C \leq Rd + \pi + RT. \tag{3.4}$$

Using these two equations to substitute out for $d + T$ we obtain (3.1) and T is irrelevant for the determination of c and C. Deposits are determined residually by $d = y - T$. If the government increases T, then d drops by the same amount. Of course, if we change the environment in some way, then the Barro-Wallace irrelevance proposition may no longer be true. This could happen, for example, if T affected R. To investigate this, we need to flesh out the rest of the model.

Bankers in period 1 are endowed with N goods. They accept deposits from households and purchase securities, s, from firms. Firms issue securities in order to finance the capital they use to produce consumption goods in period 2. Intermediation is crucial in this economy. If firms receive no resources from banks in period 1, then there can be no production, and therefore no consumption, in period 2.

We first consider the benchmark case in which there are no financial frictions and the banking sector helps the economy to achieve the first best allocations. We suppose that the gross rate of return on privately issued securities is technologically fixed at R^k. Bankers combine their own net worth, N, with the deposits received, d, to purchase s from firms. Firms use the proceeds from s to purchase an equal quantity of period 1 goods which they turn into capital. The quantity of goods produced by firms in period 2 using this capital is sR^k. Goods producing firms make no profits, so sR^k is the revenue they pass back to the banks. Banks pay Rd on household deposits in period 2. Bankers solve the following problem:

$$\pi = \max_{d} \left[sR^k - Rd \right], \tag{3.5}$$

where $s = N + d$ and N is the banker's state.

An equilibrium is defined as follows:

Benchmark Equilibrium: R, c, C, d, π such that
(i) the household and firm problems are solved
(ii) the bank problem, (3.5), is solved
(iii) markets for goods and deposits clear
(iii) $c, C > 0$

Condition (iii) indicates that we only consider interior equilibria, both here and elsewhere in the paper. A property of a benchmark equilibrium is $R = R^k$. To see this, suppose it were not so. If $R > R^k$ the bank would set $d = 0$ and if $R < R^k$ the bank would set $d = \infty$, neither of which is consistent with the equilibria that we study. Thus, in the benchmark case the interest rate faced by households in equilibrium coincides with the actual rate of return on capital. It is therefore not surprising that the first best allocations are achieved in this version of the model. That is, the allocations in the efficient, benchmark equilibrium coincide with the allocations that solve the following planning problem:[20]

$$\max_{c,C,k} u(c) + \beta u(C) \tag{3.6}$$

$$\text{subject to: } c + k \leq y + N, \ C \leq R^k k.$$

[20]We assume the environment is such that $c < y$.

The interest rate spread in this economy is defined as $R^k - R$. In the benchmark equilibrium the interest rate spread is zero. This makes sense, since there are no costs associated with intermediation and there is no default. We summarize this result as follows:

Proposition 3.1. *A benchmark equilibrium has the properties:*
(i) the interest rate spread, $R^k - R$, is zero
(ii) d takes on its first-best value.

In this economy, the Barro-Wallace irrelevance proposition is satisfied. Tax financed government purchases of private assets have no impact.

We now introduce the moral hazard problem studied by Gertler-Karadi and Gertler-Kiyotaki. A bank has two options: 'default' and 'not default'. Not defaulting means that a bank simply does what it does in the benchmark version of the model. In this case, the bank earns profits

$$\pi = R^k (N + d) - Rd. \tag{3.7}$$

The option to default means that the banker can take a fraction, θ, of the assets and leave whatever is left for the depositors. A defaulting bank receives $\theta R^k (N + d)$ and its depositors receive $(1 - \theta) R^k (N + d)$. The bank chooses the no default option if, and only if doing so increases its profits:

$$(N + d) R^k - Rd \geq \theta (N + d) R^k. \tag{3.8}$$

By rearranging terms, we see that (3.8) is equivalent with

$$(1 - \theta)(N + d) R^k \geq Rd. \tag{3.9}$$

That is, a bank chooses the no default option if, and only if, doing so reduces what depositors receive.

Each bank takes the interest rate on deposits as given, and sets its own level of deposits, d. Banks are required to post their intended values of d at the start of the period, so that households can assess whether or not a bank will default. We consider symmetric equilibria in which no bank chooses to default and the d posted by banks satisfy (3.8). In such an equilibrium an individual bank has no incentive to choose a level of deposits that violates (3.8) because depositors would in this case prefer to take their deposits to another bank, where they obtain a higher return (see (3.9)). In this setting, the banker solves the following problem:

$$\pi = \max_d \left[sR^k - Rd \right], \text{ subject to (3.8)}. \tag{3.10}$$

The definition of equilibrium we use in the case that the banker has a default option is:

Financial Equilibrium: R, c, C, d, π such that
(i) the household and firm problems are solved
(ii) the bank problem, (3.10), is solved
(iii) markets for goods and deposits clear
(iii) $c, C, d > 0$.

The difference between a financial equilibrium and a benchmark equilibrium lies in the definition of the banker problem.

When the bank's incentive constraint, (3.8), is non-binding, then $R^k = R$ and the no default condition reduces to:

$$NR^k \geq \theta (N + d) R^k.$$

If N is sufficiently large, (3.8) is non-binding and the equilibrium has the property that d is at its first-best level and the interest rate spread is zero.

Now suppose that N is sufficiently small (consider, for example, the case, $N = 0$) that (3.8) strictly binds.[21] In this case, the financial equilibrium would not be characterized by $R^k = R$. The only equilibrium is one in which R is below R^k. To see why, note that a reduction in R directly helps to restore (3.8) by increasing the term on the left of the inequality. In addition, the fall in R reduces d and this reduces both the left and right sides of (3.8).[22] We summarize this result in the following proposition:

Proposition 3.2. *When (3.8) is non-binding, the financial market equilibrium allocations are first-best and the interest rate spread is zero. When (3.8) binds, then the equilibrium values of d and R are below their first-best levels and the interest rate spread is positive.*

A sequentially repeated version of this model economy provides a rough characterization of events before and after 2007. Suppose that N was large in the early period, so that the economy was operating at its efficient level and no part of actual spreads was due to the type of default considerations addressed here. Then, in late 2007 the net worth of banks suddenly began to fall as a consequence of the collapse in housing prices. When the participation constraint began to bind, spreads opened up. The volume of intermediation - and the investment it supported - then collapsed.

3.2. Implications for Policy

We now consider the effects of four kinds of tax-financed unconventional monetary policies: injections of equity into banks, deposits in banks, direct loans to entrepreneurs and subsidies to banks' cost of funds. In each case, the policy is financed by lump sum taxes, T, in the first period. In the case of the asset purchase policies, the government transfers the proceeds back to households in the form of a second period tax reduction.

3.2.1. Equity Injections into Banks

In the case of an equity injection, the government transfers T to each bank. The government requires the banks to repay the earnings, $R^k T$, on the assets financed by the equity. The government transfers the $R^k T$ back to households in period 2 in the form of a tax reduction.

[21] That is, the multiplier on (3.8) in the Lagrangian representation of (3.10) is non-zero.

[22] Here, we use the fact that d is increasing in R. To see this, substitute out for π in (3.3) using (3.7) and solve for c to obtain:

$$c = \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k} (N + y).$$

Evidently, equilibrium consumption is strictly decreasing in R, so that d is strictly increasing in R.

We assume that unlike the household, the government has the power to prevent the bank from absconding with any part of the assets financed by T. Thus, for a bank that receives an equity injection of T, the incentive to default is still the object on the right of the inequality in (3.8). An equity injection also has no impact on a bank's profits:

$$(N+T+d)\,R^k - Rd - R^kT = (N+d)\,R^k - Rd.$$

Thus, for a given level of deposits, d, an equity injection has no effect on a bank's decision to default. However, the government's equity injection does affect the representative household's choice of d.

To understand how the representative household responds to the tax implications of an equity injection, a suitable adjustment of (3.3) implies:

$$c = \frac{y - T + \frac{\pi}{R} + \frac{R^kT}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Note that T does not directly cancel in the numerator because the rate of interest enjoyed by the government when it does an equity injection is different from the household's rate of return on deposits when (3.8) binds and $R^k \neq R$. To understand the general equilibrium impact of T on c it is necessary to substitute out for π (3.7):

$$c = \frac{y - T + \frac{R^k(N+d)-Rd}{R} + \frac{R^kT}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

The household's period 1 budget constraint implies $d = y - T - c$. Using this to substitute out for d in the above expression and rearranging:

$$\begin{aligned} c &= \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k}\,(N+y) \\ d &= y - c - T. \end{aligned} \tag{3.11}$$

Interestingly, the general equilibrium effect of T on consumption is nil, despite the difference between the government's and the household's interest rate. From the latter expression, we see that a rise in T has no impact on c and so it has a one-for-one negative impact on d. If (3.8) is non-binding, then the equity injection is irrelevant. There is no impact on total intermediation, $d+T$, and the interest rate spread remains unchanged at zero.

Now suppose that (3.8) is binding. The fall in d with a rise in T increases the right side of (3.8) and reduces the left side, thus making the incentive constraint less binding. With T large enough, the incentive constraint ceases to bind altogether and an analogous argument to the one leading up to proposition 3.2 establishes that the interest rate spread is eliminated, $R = R^k$, while total intermediation, $T+d$, achieves its first best level.

To see what level of T achieves the first-best, let d^* denote the level of deposits in a benchmark equilibrium (d^* can be found by solving (3.6) and setting $d^* = k - N$). Our assumption that (3.8) is strictly binding implies that

$$NR^k < \theta\,(N + d^*)\,,$$

so that d^* is not part of a financial equilibrium. Set T to the value, T^*, that solves

$$NR^k = \theta \left(N + d^* - T^* \right). \tag{3.12}$$

We summarize the preceding results in the form of a proposition:

Proposition 3.3. *When (3.8) is non-binding tax-financed equity injections have no impact on total intermediation, $d+T$, and on the interest rate spread, $R^k - R$. When (3.8) binds, tax financed equity injections reduce the interest rate spread and increase total intermediation. A sufficiently large injection restores spreads and total intermediation to their first-best level.*

We can express the equations of the model in words as follows. When N falls enough, the supply of deposits by banks decreases because the incentive constraint binds on the banks. This creates an interest rate spread by reducing the deposit rate (recall, the return on assets is fixed in this model). A tax-financed government purchase of assets causes the demand for deposits by households to decrease, pushing the deposit rate back up and reducing the interest rate spread. The decrease in deposits is somewhat offset by the rise in the deposit rate and this is why $d + T$ increases with the government intervention. The intervention is welfare improving because it pushes the economy back up to the first best allocations.

3.2.2. Government Deposits in Banks and Loans to Firms

Suppose the government makes tax-financed deposits, T, in banks in period 1. In period 2 it returns the proceeds to households in the form of a tax cut in the amount, RT. It is easy to verify that c and d are determined according to (3.11) in this case. As a result, total deposits, $d + T$, are invariant to T, for a given R.

If we assume that banks can as easily default on the government as on households, then total deposits, $d + T$ enter the incentive constraint and the tax-financed deposits are irrelevant. However, suppose that the government can prevent banks from defaulting on any part of the government's deposits. In that case, the profits earned by banks on government deposits, $\left(R^k - R\right) T$, are not counted in the incentive constraint, (3.8). With only household deposits in the incentive constraint, the analysis is identical to the analysis of equity injections.

Now consider the case where the government makes tax-financed loans directly to entrepreneurs. This case is formally identical to the case of tax-financed equity injections. For a given R, $d + T$ is invariant to T. However, because only d enters the incentive constraint, (3.8), the reduction in d that occurs with a rise in T relaxes the incentive constraint in case it is binding. This results in an increase in R and, hence, a rise in total intermediation. We summarize these results in the following proposition:

Proposition 3.4. *If the government can prevent bank defaults on its own bank deposits, then the effects of tax-financed government deposits in banks resemble the effects of equity injections summarized in proposition 3.3. Direct government loans to firms have the same effects as those of equity injections.*

3.2.3. Interest Rate Subsidies and Net Worth Transfers to Banks

We now consider a policy in which the government subsidizes the interest rate that banks pay on deposits. Suppose that the equilibrium is such that the incentive constraint, (3.8), is binding. As in the previous subsection this implies that the first-best level of deposits (i.e., the one that solves (3.6) with 'deposits' identified with $k - N$) violates (3.8):

$$(N + d^*)\, R^k - Rd^* < \theta\, (N + d^*)\, R^k, \tag{3.13}$$

when the deposit rate, R, is at its efficient level, R^k. Let $\tau > 0$ be the solution to:

$$(N + d^*)\, R^k - R^k\, (1 - \tau)\, d^* = \theta\, (N + d^*)\, R^k. \tag{3.14}$$

Note that there exists a unique value of $\tau > 0$ that solves this equation because the left side is increasing in τ and the left exceeds the right when $\tau = 1$. To finance the transfer, $\tau R^k d^*$, to banks the government levies taxes, $T = \tau R^k d^*$, on households in the second period. We now verify that this policy, together with $d = d^*$, $R = R^k$ and c, C at their first-best levels, c^*, C^*, satisfies all the equilibrium conditions. Bank profits in the second period are:

$$\pi = (N + d^*)\, R^k - R^k\, (1 - \tau)\, d^* = (N + d^*)\, R^k - R^k d^* + R^k \tau d^*.$$

Total household income is

$$Rd + \pi - T = (N + d^*)\, R^k.$$

The latter result and the assumption that c^*, C^* solve (3.6) imply that the household problem is solved. The fact that the incentive constraint is satisfied implies that the bank problem, (3.10), is solved. We summarize these findings as follows:

Proposition 3.5. *Suppose (3.8) binds in equilibrium, so that deposits are strictly below their first-best level in a financial equilibrium. Then, a subsidy to bank deposit liabilities at the rate defined by (3.14) ensures that the first best allocations are supported as a financial equilibrium.*

Next, we consider the case in which taxes are levied on households in the first period and the proceeds are given to bankers as a supplement to their net worth. The net worth transfer is financed by taxes on households in period 1. Suppose the equilibrium is such that the incentive constraint, (3.8), is binding. This implies that the first-best level of deposits, d^*, violates (3.8) and that (3.13) is satisfied with R at its efficient level, R^k. Let T denote the tax-financed transfer of net worth to bankers. The pre-tax level of banker net worth is N and after taxes it is $N + T$. We conjecture, and then verify, as for T sufficiently large, the financial equilibrium has the property that deposits equal $d^* - T$, the incentive constraint is non-binding, and c, C coincide with their first-best values. Let T be the solution to:

$$(N + d^*)\, R^k - R^k\, (d^* - T) = \theta\, (N + d^*)\, R^k. \tag{3.15}$$

Note that $N + d^*$ is unaffected under the tax policy and the conjecture about the equilibrium. A unique $T > 0$ that solves (3.15) is guaranteed to exist because the left side is monotonically increasing in T and the left side is assumed to be smaller than the right when $T = 0$.

To understand how the representative household responds to the tax financed equity injection, a suitable adjustment of (3.3) implies:

$$c = \frac{y - T + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Under our conjecture, $R = R^k$ and π is given by the expression on the left of the equality in (3.15). Substituting, we obtain (3.11), the level of consumption in the first-best equilibrium. This verifies our conjecture about the period 1 level of consumption. It is straightforward to verify that the first-best level of period 2 consumption satisfies the period 2 household budget constraint. We summarize our findings as follows:

Proposition 3.6. *Suppose (3.8) binds in equilibrium, so that deposits are strictly below their first-best level in a financial equilibrium. Then, a tax-financed transfer of net worth to bankers at a level defined in (3.15) ensures that the first best allocations are supported as a financial equilibrium.*

4. Moral Hazard II: Unobserved Banker Effort

The basic framework of the model used here is similar to the one in the previous section. The difference lies in the source of moral hazard. To make a high return for their depositors, we assume that bankers must exert an unobserved and costly effort. As in the case of the model in the previous section, the model used here can articulate the idea that the banking system supported efficient allocations prior to 2007, but then became dysfunctional as a consequence of a fall in bank net worth. As in the previous section, the fall in net worth pushes the economy against a non-linearity, which causes an increase in interest rate spreads, a fall in intermediation and in the activities that intermediation supports.

Despite the similarities, there are some important differences between the models in terms of their implications for policy. For example, the model used here implies that equity injections into banks during a crisis have no impact on equilibrium allocations. The model of the previous section implies that injections of bank equity can move the economy to the efficient allocations. In addition we use the model of this section to study a broader range of policy interventions. We consider the effects of government bailouts of the creditors of banks whose assets perform poorly. The model is also useful for thinking about the benefits of imposing leverage restrictions on banks.

The following section provides an intuitive summary of the analysis. After that comes the formal presentation.

4.1. Overview

There are two periods. There is a large number of households. Each household has many bankers and workers. Bankers are endowed in the first period with their own net worth and they combine this with deposits to acquire securities from firms.[23] There is a large number

[23] Whether the 'securities' take the form of loans or equity is the same in our model.

of firms, each having access to one investment project. The investment project available to some firms is a good one in that it has a high (fixed) gross rate of return. If these firms invest one unit of goods in period 1, they are able to produce R^g goods in period 2. The investment project available to other firms is bad and we denote the gross rate of return on these investment projects by R^b, where $R^b < R^g$. The rates of return, R^g and R^b, are exogenous and technologically determined.

Empirically, we observe that some banks enjoy higher profits than others, and we interpret this as reflecting that banks cannot hold a fully diversified portfolio of assets. This could be because there are many different types of investment projects - differentiated according to industry, geographic location, etc. - and there are gains to specializing in the identification of good projects of a particular type. In the model, these observations are captured by the assumption that banks can purchase the securities of at most one firm. Similarly, a firm can issue securities to at most one bank. Production for a firm is costless, and the rate of return on bank securities is identical to the rate of return on the underlying investment.[24] The task of a banker is to exert an unobserved and costly effort, e, to identify a firm with a good project. The ex post rate of return on the banker's securities is observed, but this does not reveal the banker's effort. This is because e only affects the probability, $p(e)$, that a banker identifies a good firm.

We define the efficient level of effort and of intermediation as those that occur in competitive markets in the special case that the efforts exerted by bankers are fully observed. For a banker to have the incentive to exert the efficient level of effort when effort is not observed requires that it receive a reward that is linked in the right way to the performance of its securities. Let R_b^d and R_g^d denote the interest rate on bank deposits when the bank's securities pay R^b and R^g, respectively. We show that a banker sets effort to its efficient level when $R_b^d = R_g^d$, i.e., when its cost of funds is independent of the performance of its securities. We characterize the situation in which $R_b^d = R_g^d$ as one in which the banker's creditors (i.e., the depositors) do not share in the losses when a banker's securities do not perform well. A banker exerts the efficient level of effort when $R_b^d = R_g^d$ because it fully internalizes the marginal benefit of increased effort.

For the arrangement, $R_b^d = R_g^d$, to be feasible it is necessary that the banker have a sufficiently large amount of net worth. Otherwise the banker would not have enough funds to pay depositors in the probability $1 - p(e)$ event that its loan turns out to be bad.[25] When net worth is too low in this sense, then a bank's depositors share in the loss that occurs when their bank's securities generate a bad return. In this case, depositors must receive a relatively high return, $R_g^d > R_b^d$, in the good state as compensation. But, with this cross-state pattern in deposit rates the banker does not fully capture the marginal product of increased effort. Thus, when banker net worth is not sufficiently high to permit an uncontingent deposit rate, the banker's incentive to exert effort is reduced. This reduced effort has a consequence that relatively more low quality projects are funded. As a result, the overall rate of return on deposits falls and so the quantity of deposits falls too. With the fall in deposits, intermediation and investment are reduced.

[24] Given that a bank undertakes a costly search to find a good firm, it would be interesting to explore an alternative model formulation in which the firm and bank that find each other engage in bilateral negotiations.

[25] Bankers do not have access to funds other than their own and those provided by depositors.

We now briefly discuss the concept of the 'interest rate spread'. We can loosely think of the bad state as a bankruptcy state, a state that occurs with relatively low probability. For the purpose of defining the interest rate spread, we think of the 'interest rate' paid by a bank on its source of funds as the rate it pays, R_g^d, when the good state is realized. This notion of the interest rate is similar to that of the face value of a bond, which specifies what the holder receives as long as nothing goes wrong with the issuing firm. Households are the ultimate source of funds for banks and they receive an interest rate, R, that is risk free. This is so, because the representative household is perfectly diversified across banks (it accomplishes this using a mutual fund) and so it receives the average rate of return across all banks. With these considerations in mind, we define the interest rate spread as follows:

$$R_g^d - R. \tag{4.1}$$

When bank net worth is sufficiently high, then $R_g^d - R = 0$, so that the interest rate spread is zero. When net worth falls enough, then R_b^d must be low in the bad state and thus R_g^d must be relatively high in the good state. As a result, the interest rate spread is positive when bank net worth is low.

In sum, when bank net worth is high (we refer to this as 'normal times'), then the interest rate spread is zero and effort and deposits are at their efficient levels. When bank net worth is low (a 'crisis'), then there is a positive interest rate spread and deposits are below their efficient levels. In this sense, the financial system is dysfunctional when net worth is sufficiently low. From this perspective, the model implications are qualitatively similar to those of the model in the previous section.

Still, the economics of the two models differ. For example, in the model considered here, the interest rate spread compensates for the low returns paid by banks with bad investments. In principle, one could perform an empirical study to measure the bank losses that are reflected in the high risk spread. In the model of section 3 the interest rate spread reflects a fear of out-of-equilibrium misbehavior by banks. As such, the fear is about something that does not actually happen.

The two models also differ in terms of their implications for policy. In the model of the previous section equity injections have no effect in normal times and they improve the efficiency of the economy in a crisis. In the model here, equity injections in normal times are counterproductive because they reduce bankers' incentives to exert effort. The intuition is simple. We treat an equity injection as a 'loan' from the government that must be repaid according to the actual return that the bank receives as a consequence of the government loan. The direct impact of this sort of loan on the bank is nil since it generates zero net cash flow regardless of whether the bank identifies a good or bad firm. However, there is a general equilibrium effect that matters. From the point of view of the household, an equity injection corresponds to a tax hike in the first period, followed by a tax reduction in the second period. Because this pattern of taxes satisfies part of the household's desire to save, the household responds by reducing its own deposits. With fewer deposits, the banker has less incentive to exert effort. With less effort, the average quality of bank securities falls. This produces a fall in the risk free interest rate paid to households and causes them to save less. The net effect is that intermediation falls below its ideal level.

It turns out that in a crisis, an equity injection has *no* effect in the model. This is because in a crisis there is an additional positive effect from equity investments which cancels the negative effects in normal times that were discussed in the previous paragraph. Recall, the definition of a crisis time is that net worth is too low to permit a state-non contingent interest rate on deposits. When household deposits with banks are reduced in response to an equity injection, it becomes possible to reduce the degree of state contingency in deposit rates. This is because, with lower deposits the amount of money owed by banks in the bad state is smaller and more likely to be manageable with bank net worth. The reduced state contingency in deposit rates improves the incentive of banks to exert effort. This positive effect exactly cancels the negative effects that occur in a normal time.

We also investigate other policies. For example, we study the effects of placing tax-financed government deposits in banks during a crisis. Such a policy has no effect because, consistent with the Barro-Wallace proposition, households respond by reducing their deposits by the same amount. Subsidizing banks' cost of funds in a crisis is helpful, because this policy improves the likelihood that a bank can cover losses with its own net worth. Bailing out the creditors of banks whose loans perform badly is also welfare-increasing in a crisis. Finally, we find that leverage restrictions improve welfare in a crisis. The reason for this is that by forcing banks to reduce their level of deposits, a leverage restriction increases the likelihood that a bank can cover its losses with its own net worth, thus increasing its incentive to exert effort. This is welfare-improving in a crisis, when banker effort is below its efficient level, absent government intervention.

The following subsections present the formal description of the model and the results, respectively.

4.2. Model

There are many identical households, each composed of many workers and bankers. The workers receive an endowment, y, in period 1 and the households allocate the endowment between period 1 consumption, c, and period 1 deposits in mutual funds, d. All quantity variables are expressed in per household member terms. The gross rate of return on deposits is risk free and is denoted by R. The preferences of the representative household are as in the previous example, in (3.2). Optimality of the deposit decision is associated with the usual intertemporal Euler equation. This Euler equation and the first period budget constraint are given by:

$$u'(c) = \beta R u'(C). \tag{4.2}$$

$$c + d = y \tag{4.3}$$

In the second period, households receive Rd and profits from their bankers, π. In the interior equilibria that we study, the second period budget constraint is satisfied as a strict equality:

$$C = Rd + \pi.$$

We impose the following restriction on the curvature parameter in the utility function (see (3.2)):

$$0 < \gamma < 1. \tag{4.4}$$

The upper bound on γ ensures that the equilibrium response of d to R is positive, which we view as the interesting case.

Bankers receive an endowment, N, in the first period. They combine N with deposits received from mutual funds and buy securities that finance the investment of a firm. Firms are perfectly competitive and costless to operate, so the bank receives the entire return on its firm's investment project. The probability, $p(e)$, that the firm whose securities the bank buys are good is specified as follows:

$$p(e) = a + be,\ \ b > 0, \tag{4.5}$$

so that $p'(e) = b$, $p''(e) = 0$. We only consider model parameter values that imply $0 < p(e) < 1$ in equilibrium.

The mean, $m(e)$, and variance, $V(e)$, of a bank's asset are given by

$$\begin{aligned} m(e) &= p(e) R^g + (1 - p(e)) R^b \\ V(e) &= p(e)(1 - p(e)) \left(R^g - R^b\right)^2, \end{aligned} \tag{4.6}$$

respectively. Note that

$$V'(e) = (1 - 2p(e)) \left(R^g - R^b\right)^2 b.$$

This expression is negative for $p(e) > 1/2$. In our analysis, we assume $p(e)$ satisfies this condition. Thus, when a bankers increase effort, the mean of the return on their securities increases and the variance decreases.

Our primary interest is in the scenario with 'financial frictions', in which the mutual fund does not observe the effort, e, made by the banker. To this end, it is of interest to first discuss the observable effort version of the model in which e is observed by the mutual fund. Throughout, we assume that e is observed by the banker's own household. Absent this assumption, a banker would always set $e = 0$ because e is costly to the banker and because a banker's consumption while at home is independent of the return on the banker's portfolio.

4.3. Observable Effort Benchmark

A loan contract between a banker and a mutual fund is characterized by four numbers, $\left(d, e, R_g^d, R_b^d\right)$. Here, R_g^d, R_b^d denote the gross returns on d paid by bankers whose firms turn out to be good and bad, respectively. All four elements of the contract are assumed to be directly verifiable to the mutual fund in the observable effort version of the model. Throughout, we assume that sufficient sanctions exist so that verifiable deviations from a contract never occur.

The representative competitive mutual fund itself takes deposits, d, from households and commits to paying households a gross rate of return, R. The mutual fund is competitive in that it treats R as exogenous. Because the representative mutual fund is perfectly diversified, its revenues from deposits, d, are $p(e) R_g^d d + [1 - p(e)] R_b^d d$. The mutual fund must repay Rd to depositors, so that the profits of the mutual fund are $p(e) R_g^d d + [1 - p(e)] R_b^d d - Rd$. Because mutual funds are competitive, profits must be zero:[26]

$$p(e) R_g^d d + [1 - p(e)] R_b^d d = Rd. \tag{4.7}$$

[26] If instead profits were positive, bankers would set $d = \infty$, but this exceeds the resources of households who make the deposits. If a positive value of d produced negative profits, then profit maximizing bankers would earn zero profits by setting $d = 0$. But, this would be less than the positive amount of deposits supplied by households in the interior equilibria that we study.

We assume the banker's only source of funds for repaying the mutual fund is the earnings on its investment. In each state of nature the banker must earn enough to pay its obligation to the mutual fund in that state of nature:

$$R^g (N+d) - R_g^d d \geq 0, \;\; R^b (N+d) - R_b^d d \geq 0.$$

In practice, these constraints will either never bind or they will only bind in the bad state of nature. Thus, an additional restriction on the menu of contracts, $\left(d, e, R_g^d, R_b^d\right)$, available to a bank is

$$R^b (N+d) - R_b^d d \geq 0. \tag{4.8}$$

The problem of the banker is to select a contract, $\left(d, e, R_g^d, R_b^d\right)$, from the menu defined by (4.7) and (4.8).

A banker's ex ante reward from a loan contract is:

$$\lambda \left\{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1-p(e)) \left[R^b (N+d) - R_b^d d \right] \right\} - \frac{1}{2} e^2, \tag{4.9}$$

where $e^2/2$ is the banker's utility cost of expending effort and λ denotes the marginal value of consumption for the household of the banker. In addition, d denotes the deposits issued by the banker and is distinct from the deposit decision of the banker's household. As part of the terms of the banker's arrangement with its household, the banker is required to seek a contract that maximizes (4.9). Throughout the analysis we assume the banker's household observes all the variables in (4.9) and that the household has the means to compel the banker to do what the household requires of it.

The Lagrangian representation of the banker's problem is

$$\max_{e,d,R_g^d,R_b^d} \lambda \{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1-p(e)) \left[R^b (N+d) - R_b^d d \right] \} - \frac{1}{2} e^2 \tag{4.10}$$

$$+ \mu \left[p(e) R_g^d d + (1-p(e)) R_b^d d - Rd \right] + \nu \left[R_b^d d - R^b (N+d) \right]$$

where μ is the Lagrange multiplier on (4.7) and $\nu \leq 0$ is the Lagrange multiplier on (4.8).

An interior equilibrium for this economy is:

Observable Effort Equilibrium: c, C, e, d, R, λ, R_g^d, R_b^d such that
(i) the household maximization problem is solved
(ii) mutual funds earn zero profits
(iii) the banker problem, (4.10), is solved
(iv) markets clear
(v) $c, C, d, e > 0$

We now study the properties of this equilibrium.

The first order conditions associated with the banker problem in equilibrium are:

$$e : \lambda p'(e) \left[\left(R^g - R^b \right) (N+d) - \left(R_g^d - R_b^d \right) d \right] - e + \mu p'(e) \left(R_g^d - R_b^d \right) d = 0$$

$$d : \lambda \left\{ p(e) \left(R^g - R_g^d \right) + (1-p(e)) \left(R^b - R_b^d \right) \right\} + \mu \left[p(e) R_g^d + (1-p(e)) R_b^d - R \right] + \nu \left(R_b^d - R^b \right) = 0$$

$$R_g^d : -\lambda p(e) d + \mu p(e) d = 0$$

$$R_b^d : -\lambda (1-p(e)) d + \mu (1-p(e)) d + \nu d = 0$$

$$\mu : p(e) R_g^d d + (1-p(e)) R_b^d d = Rd$$

$$\nu : \nu \left[R_b^d d - R^b (N+d) \right] = 0, \;\; \nu \leq 0, R_b^d d - R^b (N+d) \leq 0,$$

where "$x:$ " indicates the first order condition with respect to the variable, x. Adding the R_g^d and R_b^d equations, we obtain:

$$\mu = \lambda - \nu. \tag{4.11}$$

Substituting (4.11) back into the R_g^d equation, we find

$$\nu = 0,$$

so that the cash constraint is non-binding. Substituting the latter two results back into the system of equations, they reduce to:

$$e : e = \lambda p'(e)\left(R^g - R^b\right)(N + d) \tag{4.12}$$

$$d : R = p(e) R^g + (1 - p(e)) R^b \tag{4.13}$$

$$\mu : R = p(e) R_g^d + (1 - p(e)) R_b^d. \tag{4.14}$$

Note from (4.12) that in setting effort, e, the banker looks only at the sum, $N+d$, and not at how this sum breaks down into the component reflecting banker's own resources, N, and the component reflecting the resources, d, supplied by the mutual fund. By committing to care for d as if these were the banker's own funds, the banker is able to obtain better contract terms from the mutual fund. The banker is able to commit to the level of effort in (4.12) because e is observable to the mutual fund and throughout the analysis we assume that all actions which are verifiable are enforceable.

The profits, π, brought home by the bankers in the representative household in period 2 are:

$$\pi = p(e)\left[R^g(N + d) - R_g^d d\right] + (1 - p(e))\left[R^b(N + d) - R_b^d d\right] = RN, \tag{4.15}$$

using the zero profit condition of mutual funds. Thus, the representative household's second period budget constraint is:

$$C = R(N + d). \tag{4.16}$$

The five equilibrium conditions, (4.12), (4.13), (4.3), (4.2) and (4.16), can be used to determine values for

$$c, C, e, d, R.$$

Also,

$$\lambda = \beta u'(C). \tag{4.17}$$

Although the observable effort version of the model uniquely determines variables like c, C, d and R, it does not uniquely determine the values of the state contingent return on deposits, R_g^d, R_b^d. These are restricted only by (4.14) and (4.8). For example, there is an equilibrium in which deposits have the following state contingent pattern: $R_g^d = R^g$, $R_b^d = R^b$. There may also be an equilibrium in which deposit rates are not state contingent, so that $R_g^d = R_b^d = R$. However, for the latter to be an equilibrium requires that N be sufficiently large. The equilibrium values of c, C, e, d, R, λ, are the same across all state contingent returns on deposits that are consistent with (4.14) and (4.8).

4.4. Unobservable Effort

We now suppose that the banker's effort, e, is not observed by the mutual fund. Thus, whatever d, R_g^d, R_b^d is specified in the contract, a banker always chooses e to maximize:

$$\lambda \left\{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1-p(e)) \left[R^b (N+d) - R_b^d d \right] \right\} - \frac{1}{2} e^2.$$

The first order condition necessary for optimality is:

$$e : e = \lambda p'(e) \left[\left(R^g - R^b \right)(N+d) - \left(R_g^d - R_b^d \right) d \right]. \tag{4.18}$$

Note that $R_g^d > R_b^d$ reduces the banker's incentive to exert effort. This is because in this case the banker receives a smaller portion of the marginal increase in expected profits caused by a marginal increase in effort. Understanding that e will be selected according to (4.18), a mutual fund will only offer contracts, $\left(d, e, R_g^d, R_b^d\right)$, that satisfy not just (4.8), but also (4.18).

In light of the previous observations, the Lagrangian representation of the banker's problem is:

$$\begin{aligned} \max_{\left(e,d,R_g^d,R_b^d\right)} \; & \lambda \left\{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1-p(e)) \left[R^b (N+d) - R_b^d d \right] \right\} - \frac{1}{2} e^2 \\ & + \mu \left[p(e) R_g^d d + (1-p(e)) R_b^d d - R d \right] \\ & + \eta \left(e - \lambda p'(e) \left[\left(R^g - R^b \right)(N+d) - \left(R_g^d - R_b^d \right) d \right] \right) \\ & + \nu \left[R_b^d d - R^b (N+d) \right]. \end{aligned} \tag{4.19}$$

where η is the Lagrange multiplier on (4.18).

The equilibrium concept used here is:

Unobservable Effort Equilibrium: c, C, e, d, R, λ, R_g^d, R_b^d such that
(i) the household maximization problem is solved
(ii) mutual funds earn zero profits
(iii) the banker problem, (4.19), is solved
(iv) markets clear
(v) $c, C, d, e > 0$

To understand the properties of this equilibrium, consider the first order necessary conditions associated with the banker problem, (4.19):

$$\begin{aligned} & e : \lambda p'(e) \left[\left(R^g - R^b \right)(N+d) - \left(R_g^d - R_b^d \right) d \right] - e + \mu p'(e) \left(R_g^d - R_b^d \right) d \\ & \quad + \eta \left(1 - \lambda p''(e) \left[\left(R^g - R^b \right)(N+d) - \left(R_g^d - R_b^d \right) d \right] \right) = 0 \\ & d : 0 = \lambda p(e) \left[R^g - R_g^d \right] + \lambda (1-p(e)) \left[R^b - R_b^d \right] + \mu \left[p(e) R_g^d + (1-p(e)) R_b^d - R \right] \\ & \quad - \eta \lambda p'(e) \left[\left(R^g - R^b \right) - \left(R_g^d - R_b^d \right) \right] + \nu \left(R_b^d - R^b \right) \\ & R_g^d : -\lambda p(e) + \mu p(e) + \eta \lambda p'(e) = 0 \\ & R_b^d : -\lambda (1-p(e)) + \mu (1-p(e)) - \eta \lambda p'(e) + \nu = 0 \\ & \mu : R = p(e) R_g^d + (1-p(e)) R_b^d \\ & \eta : e = \lambda p'(e) \left[\left(R^g - R^b \right)(N+d) - \left(R_g^d - R_b^d \right) d \right] \\ & \nu : \nu \left[R_b^d d - R^b (N+d) \right] = 0, \; \nu \leq 0, \left[R_b^d d - R^b (N+d) \right] \leq 0. \end{aligned} \tag{4.20}$$

We refer to these equations - and their counterparts below - by their names to the left of the colon. Add the R_g^d and R_b^d equations, to obtain (4.11). After using (4.11) to substitute out for μ in (4.20), making use of (4.5) and rearranging:[27]

$$\begin{aligned} e &: (\lambda-\nu)\, b \left(R_g^d - R_b^d\right) d + \eta = 0 \\ d &: R = p(e)\, R^g + (1-p(e))\, R^b \\ R_g^d &: \nu p(e) = \eta \lambda b \\ \mu &: R = p(e)\, R_g^d + (1-p(e))\, R_b^d \\ \eta &: e = \lambda b \left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right) d\right] \\ \nu &: \nu \left[R_b^d d - R^b (N+d)\right] = 0,\ \nu \leq 0, \left[R_b^d d - R^b (N+d)\right] \leq 0. \end{aligned} \tag{4.21}$$

We distinguish two cases. Equilibrium in a 'normal time' corresponds to the case when N is sufficiently large that the cash constraint is nonbinding, so that $\nu = 0$. Equilibrium in a 'crisis time' corresponds to the case when $\nu < 0$.

We first consider the properties of equilibrium in a normal time. Substituting $\nu = 0$ into the R_g^d equation, we deduce that in an interior equilibrium with $d, \lambda > 0$, the multiplier on the incentive constraint, η, is zero. With $\eta = 0$ and the fact, $\lambda - \nu > 0$, the e and μ equations imply:

$$R_g^d = R_b^d = R. \tag{4.22}$$

It then follows from the η equation that:

$$e : e = \lambda b \left(R^g - R^b\right)(N+d). \tag{4.23}$$

Equations (4.23) and the μ equation in (4.21), together with the three household equilibrium conditions, (4.3), (4.2) and (4.16), represent 5 conditions. These conditions can be used to determine the following 5 variables:

$$c, C, e, d, R.$$

A notable feature of the equilibrium in a normal time is that the incentive constraint, (4.18), is non-binding and the allocations are efficient in the sense that they coincide with the allocations in the version of the model in which effort is observable. The level of effort exerted by the banker in the unobservable effort equilibrium coincides with what it is in the observable effort equilibrium because the loan contract transfers the full marginal product of effort to the banker. This is accomplished by making the rate of interest on banker deposits not state contingent (see (4.22)). The interest rate spread in this equilibrium (see (4.1)) is zero in a normal time. We state these results as a proposition:

[27] The d equation in (4.21) is a simplified version of the d equation in (4.20), obtained as follows. Substitute from (4.11) and the R_g^d and μ equations in (4.20) into the d equation in (4.20) to obtain, after some algebra:

$$0 = (\lambda - \nu)\left[p(e)\, R^g + (1-p(e))\, R^b - R\right].$$

The result follows from the observation that $(\lambda - \nu)$ is strictly positive since $\nu \leq 0$ and $\lambda > 0$ in an interior equilibrium. The e equation in (4.21) is a simplified version of the e equation in (4.20), after making use of (4.11) and the ν equation in (4.20).

Proposition 4.1. *When the cash constraint, (4.8), does not bind (i.e., $\nu = 0$), then the allocations in the unobserved effort equilibrium coincide with those in the observed effort equilibrium and the interest rate spread is zero.*

We now turn to the case in which the cash constraint is binding, so that $\nu < 0$ and

$$\nu : R_b^d d = R^b (N + d) . \tag{4.24}$$

In this case, the observed and unobserved effort equilibria diverge, since the cash constraint never binds in the observed effort equilibrium. The R_g^d equation in (4.21) implies $\eta < 0$, so that according to the e equation in (4.21),

$$R_g^d > R_b^d \tag{4.25}$$

in an interior equilibrium with $d > 0$. It follows from the η equation in (4.21) that

$$e < \lambda b \left(R^g - R^b \right) (N + d) .$$

That is, the banker in a crisis equilibrium exerts less effort, for given $N + d$, than it does in the observed effort equilibrium. The reason is that with (4.25), the banker does not capture the full marginal return from effort. With reduced effort, equation d in (4.21) shows that equilibrium R is smaller. Given (4.4), the household equilibrium conditions, (4.3), (4.2) and (4.16), imply a lower d, reinforcing the low e. We summarize these findings in the following proposition:

Proposition 4.2. *When the cash constraint, (4.8), binds (i.e., $\nu < 0$), then e, d and R in the unobserved effort equilibrium are lower than they are in the observed effort equilibrium, and the interest rate spread is positive.*

4.5. Implications for Policy

In this section, we consider the impact of government deposits and equity injections into banks, and show that these are not helpful in a crisis. We then show that bank deposit rate subsidies and transfers of net worth to banks can solve the crisis completely by eliminating the interest rate spread and moving allocations to their efficient levels. Finally, we study the effects of bailing out the creditors of banks with poor-performing securities and the effects of leverage restrictions.

4.5.1. Government Deposits into Mutual Funds

Consider the case where the government raises taxes, T, and deposits the proceeds in the mutual fund. The household's period 1 budget constraint is given by:

$$c + \tilde{d} = y, \tag{4.26}$$

where $\tilde{d}$ denotes $d + T$, and d denotes deposits placed by households in the mutual fund. The intertemporal condition, (4.2) is unaffected by the change. The household's second period budget constraint is unaffected by the change, except that d is replaced by $\tilde{d}$. Similarly, the equilibrium conditions associated with the banker problem, (4.19), are unchanged, with the exception that d is replaced by $\tilde{d}$. In particular, if the government deposits taxpayer money into the mutual funds, taxpayers reduce their deposits by the same amount and there is no change. From the point of view of households in the economy, it is the same whether deposits are held in their capacity as taxpayers or directly in their own name. That is, the policy considered in this section does not overcome the Barro-Wallace irrelevance proposition. This conclusion makes use of the assumption we use throughout our analysis, that an equilibrium is interior. In the present context, this implies that T is not so large that the constraint, $d \geq 0$, is non-binding. We summarize these results in the form of a proposition:

Proposition 4.3. *In an interior equilibrium, the level of tax-financed government deposits are irrelevant for the equilibrium levels of* $c, C, R, d + T, e$.

4.5.2. Equity Injections into Banks

In this section we adopt the same interpretation of equity injections as in section 3.2.1. That is, the government raises taxes, T, and hands these over to the banks in period 1. The government requires that the banks repay the earnings they actually make on these funds in period 2. Under this policy, the expected profits of the bank are

$$p(e)\left[R^g(N+T+d) - R_g^d d - R^g T\right] + (1-p(e))\left[R^b(N+T+d) - R_b^d d - R^b T\right].$$

Note that taxes enter revenues symmetrically with deposits and the bank's own net worth. On the cost side, equity injections require that banks pay the government the actual rate of return on its securities. Thus, equity injections have no direct impact on bank profits, because they enter revenues and costs in exactly the same way. For the same reason, equity injections also do not change the banker's cash requirement in the bad state. That is, the bank requirement that revenues be no smaller than costs is, in the presence of equity injections,

$$R^b(N+d+T) \geq R_b^d d + R^b T,$$

so that T cancels from both sides and thus coincides with (4.8). We conclude that the banker's problem, (4.19), is completely unaltered by the presence of equity injections.

Now consider the household problem. The period 1 budget constraint is:

$$c + d \leq y - T, \tag{4.27}$$

reflecting that equity injections, T, are financed with taxes on households. The government transfers the revenues from equity injections back to households in period 2. In this way, the period 2 household budget constraint is:

$$\begin{aligned} C &= Rd + p(e)\left[R^g(N+d) - R_g^d d\right] + (1-p(e))\left[R^b(N+d) - R_b^d d\right] \\ &\quad + \left[p(e)R^g + (1-p(e))R^b\right]T \\ &= R(N+d+T). \end{aligned} \tag{4.28}$$

The last terms on the right reflects that the government's distribution of equity among banks is completely diversified. The intertemporal Euler equation, (4.2), is unchanged.

>From the household problem we see that an increase in T induces an equal reduction in d, for a given value of R. Note that although T does not enter the banker's problem, d does. Thus, it is possible that T has an indirect effect on the equilibrium.

Consider first the case when the cash constraint in the bad state is not binding, $\nu = 0$. In this case, the problem solved by the banker's contract is given by (4.19) with $\nu = 0$, so that (4.22) and (4.23) are satisfied. In this case, increased equity injections, for a given interest rate, R, reduce deposits and so reduce the banker's incentives to exert effort, e (see (4.23)). This in turn produces a fall in R, so that d falls some more. Thus, $d+T$ falls with a tax-financed equity injection in a normal time when the cash constraint is not binding. The intuition for this result is described in section 4.1. We summarize this result in the form of a proposition:

Proposition 4.4. *If the cash constraint, (4.8), is not binding, then an equity injection produces a fall in effort, e, the interest rate, R, and total intermediation, $d+T$.*

In a crisis time when the cash constraint in the bad state is binding, the fall in $d+T$ that occurs with an equity injection is offset by a second effect. The two cancel, and so equity injections are irrelevant in a crisis. The second effect occurs because a fall in deposits, d, loosens the cash constraint, (4.8), in the bad state. This relaxation of (4.8) requires an increase in the rate of return on deposits for banks in the bad state. The reduction in the state contingency of deposit rates enhances bankers' incentives to exert effort. As a result, the fraction of good projects that are identified is increased, so that the risk free rate rises, leading to a rise in deposits. Formally,

Proposition 4.5. *If the cash constraint is binding, then an equity injection has no impact on consumption, c, C, the interest rate, R, and the volume of intermediation, $d+T$.*

See the appendix for a proof of this proposition.

We summarize our two propositions as follows. In a normal time when the cash constraint is binding, equity injections are counterproductive, as they lead to a reduction in effort by bankers. In crisis times, an equity injection satisfies the Barro-Wallace irrelevance result, so that they have no impact on $c, C, R, d+T$ as long as the cash constraint remains binding. Once equity injections reach a sufficient scale, then the cash constraint ceases to bind and Proposition 4.4 is relevant. That is, equity injections that are large enough to render the cash constraint non-binding are counterproductive in that they reduce effort.

4.5.3. Interest Rate Subsidies and Net Worth Transfers to Banks

As we have emphasized, the heart of the problem in a crisis is that banks with poor-performing securities do not have enough resources to fully absorb their losses. Equilibrium in this case requires that deposit rates covary positively with the return on the bank portfolio. But, this positive covariance leads to welfare reducing allocations by reducing banks' incentive to exert effort. State non-contingency in the banks' deposit rate is crucial if they are to have enough incentive to exert the efficient level of effort. This reasoning suggests two policies that can help solve the problem. First, by reducing the costs of their deposits, a tax-financed subsidy on banks' cost of funds makes it possible for banks to cover their losses in bad states and for bank deposit rates to be state non-contingent. Second, a tax-financed transfer of equity to banks also allows them to cover their losses in bad states with state non-contingent deposit rates.

Consider first the case of interest rate subsidies. Suppose we have the allocations and returns in the observable effort equilibrium. The assumption that we are in a crisis implies that if $R = R_b^d = R_g^d$, where R solves (4.13), then the cash constraint, (4.8), is violated:

$$Rd > R^b (N + d). \tag{4.29}$$

Let τ solve

$$(1 - \tau) Rd = R^b (N + d). \tag{4.30}$$

A value of $\tau > 0$ is guaranteed to exist because the left side of this expression is monotonically decreasing in τ and it is zero when $\tau = 1$. All the equilibrium conditions associated with the banker problem (see (4.21)) are satisfied, with $\nu = 0$. As a result, the banker exerts the level of effort that occurs in the observed effort equilibrium (see (4.23)). The key thing is that state non-contingency of deposit rates causes the banker to exert effort as though the deposits belonged to the banker. The fact that the level of deposit rates is lower across the realized returns of its securities is irrelevant to the effort exerted by the banker.

It remains only to verify that the household decisions in the observable effort equilibrium also solve their problem in the unobservable effort equilibrium with an interest rate subsidy. The households' period 1 budget constraint, (4.3), is unaffected. The household's intertemporal Euler equation, (4.2), is also not affected. The only household equilibrium condition that requires attention is its second period budget constraint, because the tax subsidy to banks is financed by period 2 taxes, $T = \tau Rd$,

$$C = Rd + \pi - T.$$

Bank profits, π, are higher under the interest rate subsidy than they are in the observable effort equilibrium. However, they are higher by exactly T. So, the assumption that the period 2 household budget constraint is satisfied in an observable effort equilibrium implies that the allocations in that equilibrium also satisfy the above budget constraint with taxes. We summarize these findings as follows:

Proposition 4.6. *Suppose the cash constraint in an unobservable effort equilibrium is binding. The interest rate subsidy, (4.30), financed by a period 2 tax on households causes the allocations in the unobservable effort equilibrium to coincide with those in the observable effort equilibrium.*

The interest subsidy policy is of wider interest because it allows us to address a common view that interest rate subsidies to banks lead them to undertake excessive risk. In the environment here, an interest rate subsidy in a crisis induces bankers to exert greater effort, e. This leads to a rise in the mean return on assets, and a fall in their variance. Thus, this environment does not rationalize the common view about the impact of interest subsidies on risk taking by banks.

We now turn to tax financed transfers of net worth to banks. Suppose again that the cash constraint is binding in the unobservable effort equilibrium. The government raises taxes,

T, in period 1 and transfers the proceeds to banks. If the transfer is sufficiently large, then the cash constraint in the unobservable effort equilibrium ceases to bind. To establish this result, suppose we have the allocations in the observable effort equilibrium in hand. The assumption that we are in a crisis implies that if $R = R_b^d = R_g^d$, where R solves (4.13), then the cash constraint, (4.8), is violated, as in (4.29).

We first consider the response of the observable effort equilibrium to $T > 0$. Banks' pretax net worth is N and after taxes their net worth is $N + T$. We conjecture, and then verify, that with $T > 0$, deposits decline one-for-one in the observable effort equilibrium and period 1 and period 2 consumption allocations do not change. Suppose that T satisfies:

$$R\left(d-T\right)=R^{b}\left(N+d\right). \tag{4.31}$$

The value of T that satisfies this equation exists and is unique because the left side is monotonically decreasing and continuous in T and it is zero when $T = d$. According to (4.31) the cash constraint is (marginally) non-binding. It is easily verified that the household period 1 budget constraint and Euler equations in the observable effort equilibrium are satisfied (see (4.3) and (4.2)). It is also easily verified that households' second period income is invariant to T.[28] Finally, the bank equilibrium conditions, (4.12), (4.13), (4.14), easily seen to be satisfied. We conclude that we have an observable effort equilibrium. Because in addition the cash constraint is satisfied, it follows that we have an unobserved effort equilibrium too. We summarize our finding as follows:

Proposition 4.7. *Suppose the cash constraint in an unobservable effort equilibrium is binding. The net worth subsidy, (4.31), financed by a period 1 tax on households causes the allocations in the unobservable effort equilibrium to coincide with those in the observable effort equilibrium.*

4.5.4. Creditor Bailouts

In this subsection we explore another policy that can increase welfare in a crisis. This policy subsidizes bank creditors (i.e., the mutual funds) when their portfolios perform poorly (i.e., when banks earn R^b). This policy is helpful because it goes to the heart of the problem. The problem when net worth is too low is that creditors must share in the losses when bank portfolios perform poorly. Under these circumstances creditors require R_g^d to be high to compensate them for the losses associated with the low R_b^d. This increase in $R_g^d - R_b^d$ causes bankers to reduce effort below the efficient level (recall (4.18)). By subsidizing creditors in the bad state, $R_g^d - R_b^d$, is reduced and effort moves back in the direction of its efficient level. We explore the quantitative magnitude of these effects in this section.

Let R_b^d denote, as before, the bank's payment in the bad state. The amount the mutual fund actually receives is $(1+\tau) R_b^d$. We assume that the bailout, τR_b^d, is financed by a lump sum tax on households in the second period. The zero profit condition of the mutual fund is:

$$p\left(e\right)R_g^d d+\left(1-p\left(e\right)\right)\left(1+\tau\right)R_b^d d=Rd.$$

[28] This requires performing substitutions similar to those in (4.16).

With this change, the equilibrium loan contract is the $\left(e, d, R_g^d, R_b^d\right)$ that solves the following analog of (4.19):

$$\max_{\left(e,d,R_g^d,R_b^d\right)} \lambda \left\{p\left(e\right)\left[R^g\left(N+d\right)-R_g^d d\right]+\left(1-p\left(e\right)\right)\left[R^b\left(N+d\right)-R_b^d d\right]\right\}-\frac{1}{2}e^2 \tag{4.32}$$
$$+\mu\left[p\left(e\right)R_g^d d+\left(1-p\left(e\right)\right)\left(1+\tau\right)R_b^d d-Rd\right]$$
$$+\eta\left(e-\lambda p'\left(e\right)\left[\left(R^g-R^b\right)\left(N+d\right)-\left(R_g^d-R_b^d\right)d\right]\right)$$
$$+\nu\left[R_b^d d-R^b\left(N+d\right)\right].$$

Note that τ only enters the zero profit condition of mutual funds. Because τ does not explicitly enter the banks' own profits, the incentive constraint on bank effort is not affected. For a detailed characterization of the loan contract and an algorithm for computing the equilibrium, see section A.3 in the Appendix.

We compute the socially optimal value of τ in a numerical example. The social welfare function aggregates the utility of everyone in the household:

$$u\left(c\right)+\beta u\left(C\right)-\frac{1}{2}e^2.$$

We do the computations for a crisis situation, one in which N is sufficiently low that $\nu \neq 0$. We construct an example by first selecting an equilibrium with $\tau = 0$ in which the cash constraint is non-binding, i.e., $\nu = 0$. We then reduce N sufficiently so that the cash constraint is binding and we then compute equilibria for a range of values of τ.

We must assign values to the following parameters:

$$\beta, \gamma, R^g, R^b, a, b, y, N,$$

where a and b are the parameters of $p\left(e\right)$ (see (4.5)), and γ, β are parameters that govern household utility (see (3.2)). We set $\beta = 0.97$, $\gamma = 0.9$, $a = 0.5$ and $N = 1$ and we set the other four parameters, R^g, R^b, b and y to achieve $R = 1/\beta$ and the following three calibration targets:

$$p\left(e\right)=0.99,\ V\left(e\right)=0.0036,\ \frac{d}{y}=0.26,$$

where $V\left(e\right)$ denotes the variance, across banks, of returns (see (4.6)). The equilibrium associated with this parameterization is characterized by a non-binding cash constraint. In this equilibrium, $R_b^d = R_g^d = R$ when $\tau = 0$. We verified numerically, that $\tau = 0$ corresponds to a local maximum of the social welfare function.

We reduced the value of N to 0.70, in which case the cash constraint is binding. Figure 2 displays features of the equilibrium for values of $\tau \in (0, 2)$. The optimal value of τ is roughly 0.7282. Note that equilibrium effort, e, is increasing in τ. As indicated in the introduction to this section, this result reflects that $R_g^d - R_b^d$ is falling in τ. The rise in equilibrium effort gives rise to an increase in the return, R, generated by the financial system and hence produces a rise in deposits, d.

4.5.5. Leverage Restrictions

In normal times, a binding leverage restriction on banks reduces welfare because the equilibrium is efficient. However, bankers make inefficiently low effort in a crisis because their cost of funds is positively correlated with the performance of their assets. This correlation reflects that bankers' net worth is too low for them to insulate creditors from losses when banks experience a low return, R^b. Obviously, if banks had a sufficiently low level of deposits when net worth is low, then bankers' net worth would be sufficient to cover losses. This raises the possibility that leverage restrictions may be welfare improving when net worth is low. However, recall that bank incentives are not only a function of $R_g^d - R_b^d$, but also of the level of deposits, d (see (4.18)). So, it is not so obvious, ex ante, that leverage restrictions are desirable in a crisis. For this reason, we investigate the desirability of leverage restrictions in a numerical example. In the example, we use the same parameter values as the ones in the previous subsection. We find that leverage restrictions indeed are desirable in crisis times.

We suppose that the government imposes a restriction on the equilibrium contract, which prohibits banks from exceeding a specified level of leverage, $\bar{L}$:

$$\frac{N+d}{N} \leq \bar{L}.$$

This leads to the following alternative formulation of the problem solved by the equilibrium contract:

$$\begin{aligned} &\max_{\left(e,d,R_g^d,R_b^d\right)} \lambda \left\{p\left(e\right)\left[R^g\left(N+d\right)-R_g^d d\right]+\left(1-p\left(e\right)\right)\left[R^b\left(N+d\right)-R_b^d d\right]\right\} \qquad (4.33) \\ &-\frac{1}{2}e^2 \\ &+\mu\left[p\left(e\right)R_g^d d+\left(1-p\left(e\right)\right)\left(1+\tau\right)R_b^d d-Rd\right] \\ &+\eta\left(e-\lambda p'\left(e\right)\left[\left(R^g-R^b\right)\left(N+d\right)-\left(R_g^d-R_b^d\right)d\right]\right) \\ &+\nu\left[R_b^d d-R^b\left(N+d\right)\right]+\delta\left[\bar{L}N-\left(N+d\right)\right], \end{aligned}$$

where $\delta \geq 0$ is the multiplier on the leverage constraint. We assume the last two constraints are binding, so that $\delta > 0$, $\nu < 0$.

The nine panels in Figure 3 display selected characteristics of the equilibrium, for a range of values of $\bar{L}$ and for two values of the bailout rate, τ. The two values of τ are $\tau = 0$ and $\tau = 0.7282$, which is its optimal value when there are no leverage restrictions. When $\tau = 0$ and 0.7282, leverage in the absence of leverage restrictions is 2.0453 and 2.0684, respectively. The highest value of $\bar{L}$ reported in Figure 3 is 2.0453.

Consider the case, $\tau = 0$, first. Note from Panel b that social welfare initially rises as $\bar{L}$ is reduced from $\bar{L} = 2.0453$. The optimal value of $\bar{L}$ is 1.9980. This represents a 2.3 percent cut in leverage, which translates into a reasonably substantial cut of roughly 5 percent in deposits, d. Consistent with the intuition provided above, the reduction in $\bar{L}$ reduces the state contingency in banks' costs of credit, $R_g^d - R_b^d$ (see Panel i). According to Panel e and Panel h, the fall in $R_g^d - R_b^d$ results in higher effort, e, despite the lower level of deposits. As a result, the leverage restriction produces an increase in the cross-section average return on bank portfolios (Panel d), as well as a fall in the cross-section variance, $V(e)$ in (4.6). To be consistent with clearing in the market for deposits, the deposit rate, R, must fall as the leverage restriction becomes more binding (see Panel c). With the fall in the deposit rate and the increase in the average return on assets, there is a rise in the net profits earned by banks on deposits. In the absence of government intervention, competition drives these profits to zero.[29] In effect, the government reduction in $\bar{L}$ causes the banking sector to behave as a monopsonist. Restricting $\bar{L}$ raises banker utility, (4.9), according to Panel a.

We now turn to the case, $\tau = 0.7282$. The results suggest that bailouts are to some extent a substitute for leverage restrictions. To see this, note that when τ is positive, then the optimal level of leverage is raised. This property of the model starkly contradicts conventional wisdom, which maintains that leverage restrictions are required to undo the bad side effects of bailout commitments.

The conventional wisdom on leverage can perhaps be paraphrased as follows: "bailouts reduce the incentive for creditors to play a socially important role in monitoring bankers, and this leads bankers to choose overly risky portfolios". In our model, creditors have no ability to monitor bankers and so this monitoring channel is not present. However, based on comparing the unobservable and observed effort versions of our model, we can conjecture what would happen if we modified our model so that creditors could decide, at a cost, whether and how much to monitor banks. Recall that in the observed effort version of our model studied in section 4.4 creditors perfectly monitor the activities of the banker. In that model, even if net worth is so low that creditors must share in banker losses, the effort level of bankers is efficient. Thus, suppose net worth is low, so that the observable and unobservable effort equilibria differ. Suppose further that the economy is repeated twice, with the observable effort equilibrium occurring in a first date and the unobservable effort equilibrium occurring in the second date. Loosely, one can interpret this two-date economy (each date has two subperiods) as one in which creditors monitor in the first date but do not monitor in the second date. In this model, the effort level of bankers is inefficiently low in the second date (recall Proposition 4.2) and the cross-sectional variance of their portfolios increases as a result (see (4.6)). This reasoning suggests to us that our model would be consistent with the conventional wisdom if creditor monitoring of bankers were endogenized. Of course an important empirical question is whether in fact creditors do have the ability to monitor banks apart from observing the performance of banker securities.

The other results corresponding to the case, $\tau = 0.7282$, are consistent with the idea that leverage complements bailouts in this model. For example, at every level of $\bar{L}$, banker effort is higher with $\tau > 0$ than with $\tau = 0$. Finally, note that the response of d (Panel e) and c (Panel f) are invariant to τ. This is because net worth and y are fixed in the figure and in this case leverage immediately determines d and c.

5. Adverse Selection

In this section, we consider credit market frictions that occur when there is a lemons problem such as the one emphasized in Akerlof (1970) and Stiglitz and Weiss (1981). Each of a large number of entrepreneurs has access to a single investment project. Because their own net worth is not sufficient to finance their investment project, entrepreneurs must rely on

[29] See the d and μ equations in (4.21).

external finance from banks. Some entrepreneurial projects are relatively safe and others are riskier, but the bank cannot differentiate between them. To compensate for losses from the riskier entrepreneurs the interest rate spread - the difference between the rate charged by the bank to borrowers and the cost of funds to the bank - must be positive.[30] The price distortions associated with the interest rate spread have the consequence that intermediation and investment are below their efficient levels. A drop in entrepreneurial net worth aggravates the distortions because entrepreneurs become more dependent on external finance. In this respect, the model resembles the ones in the previous two sections.

We insert the entrepreneurs and bankers into the type of general equilibrium environment considered in the previous sections of this paper. When entrepreneurial net worth falls and R falls, interest rate spreads jump and the supply of saving from the rest of the economy drops and investment drops. In this way, the environment rationalizes the type of observations that motivate this paper, about the behavior of interest rates spreads and intermediation in the recent market turmoil.

Consistent with the analysis of Mankiw (1986) and Bernanke and Gertler (1990), who consider a similar environment, we find that a subsidy to banks' cost of funds can ameliorate the problem.[31] Indeed, a suitable choice of the interest rate subsidy can make the market allocations coincide with the efficient allocations. This is so, even though the subsidy policy does not require observing borrowers' individual risks while our efficient allocations are defined for a benevolent planner who does observe those risks. A subsidy to entrepreneurs, by reducing their dependence on external finance, can also improve allocations. We consider government deposits in banks, but these do not overcome the Barro-Wallace irrelevance result. That is, they have no effect on the allocations.

5.1. Model

We adopt the same basic environment as in the rest of the paper. The economy is populated by many large and identical households. The representative household has a unit measure of members composed of workers, bankers and entrepreneurs. The measure of entrepreneurs is $e < 1$.[32] All these individuals receive perfect consumption insurance from households. Workers and entrepreneurs receive endowments of y and N, respectively, in the first period. Here, $y > 0$ is measured in household per capita terms. We find it convenient to measure $N < 1$ in entrepreneur per capita terms. Thus, in household per capita terms the quantity of entrepreneurial net worth is eN. The conditions that characterize household optimization are as in the other parts of this paper, but are reproduced here for convenience:

[30] Our model has the property that the only equilibrium is a pooling equilibrium, i.e., one in which entrepreneurs with different risks receive the same loan contract. For additional discussion of pooling and separating equilibria under adverse selection in credit markets, see Stiglitz and Weiss (1992) and the references they cite.

[31] Our model is most closely related to the one in Mankiw (1986).

[32] The letter, e, is not to be confused with effort in the previous section.

$$c + d = y \tag{5.1}$$
$$c^{-\gamma} = \beta R C^{-\gamma},\ \gamma > 0 \tag{5.2}$$
$$C = Rd + e\pi. \tag{5.3}$$

Here, c denotes first period household consumption, d denotes deposits, and C denotes second period consumption. These three variables are measured in household per capita terms. The object, π, denotes earnings, in entrepreneur per capita terms, brought home in period 2 by entrepreneurs. Finally, R denotes the gross rate of interest on deposits. The Euler equation in (5.2) indicates that the household's deposit decision is unchanged from what it was in previous sections. We now discuss the problems of the entrepreneurs and the bankers.[33]

Each entrepreneur must select between one of two options. It can deposit its net worth in a bank and earn RN. Alternatively, it can borrow funds from a bank and operate its investment project. The entrepreneur makes its decision after observing the values of θ and p which characterize its investment project, and which are drawn independently from the cumulative distribution function (cdf), $F(\theta, p)$. An entrepreneur's realized values of $\theta > 0$ and $p \in [0, 1]$ are known only to the entrepreneur and to the household to which it belongs. The distribution, F, is known to all. All investment projects are indivisible and require an investment of one good in period 1. We explain the reason for our assumption that there is an upper bound on the scale of investment in Proposition 5.3, section 5.2.1 below. In period 2, the investment project yields θ goods with probability p and zero goods with probability $1 - p$. Our analysis is greatly simplified by placing the following restriction on F :

$$\theta p = \bar{\theta}. \tag{5.4}$$

Here, $\bar{\theta}$ is a non-random parameter known to all. Thus, each entrepreneur's investment project generates the same expected return, but differs in terms of riskiness. We characterize F by specifying a distribution for p and then setting $\theta = \bar{\theta}/p$. We assume that p is drawn from a uniform distribution with support, $[0, 1]$.

Because $N < 1$, an entrepreneur must obtain a bank loan if it is to operate its investment technology. After the entrepreneur and banker leave the household, their location is randomized. Because there is a continuum of households, the probability that an entrepreneur meets a banker from its own household is zero. So, the fact that an entrepreneur's own household knows everything about the entrepreneur's investment project implies nothing for what the entrepreneur's banker knows. We suppose that the entrepreneur's banker can only observe whether the entrepreneur's project succeeds or fails. In case the project succeeds, the banker cannot tell ex post what that project's value of θ was. As a result, the payment made by the entrepreneur to its bank can only be contingent on whether or not the entrepreneur's project was successful. We denote the interest rate paid by the entrepreneur in the event that the project succeeds by r. Because the entrepreneur has no resources in the event that the project fails, the interest rate in that event must be zero.

Entrepreneurs which choose not to activate their investment projects earn RN with certainty by depositing their net worth in banks. For an entrepreneur that decides to operate

[33] Our model is an adaptation on the model in Mankiw (1986), especially the example on page 463.

its project, with probability p it earns $\theta - r(1-N)$ and with the complementary probability it earns nothing. It is in the household's interest that each of its entrepreneurs make the project activation decision with the objective of maximizing expected earnings. The law of large numbers and the fact that there are many entrepreneurs in each household, guarantees that if each entrepreneur behaves in this way, the total resources brought home by all entrepreneurs in a family is maximized. Households are assumed to be able to compel entrepreneurs to maximize expected returns and not divert any profits by: (i) the threat to withhold perfect consumption insurance and (ii) the fact that households observe everything (including θ and p) about their entrepreneurs. Thus, a given entrepreneur invests its net worth, N, in its project and borrows $1-N$ if and only if its realized value of p satisfies:

$$\bar{\theta} - pr(1-N) \geq RN. \tag{5.5}$$

The values of p that satisfy (5.5) are as follows:

$$0 \leq p \leq \bar{p}(r), \ \bar{p}(r) \equiv \frac{\bar{\theta} - RN}{r(1-N)}. \tag{5.6}$$

The object, $\bar{p}(r)$, in (5.6) summarizes several interesting features of the equilibrium. For example, note that when r increases, entrepreneurs with higher values of p decide to not activate their investment project (i.e., $\bar{p}(r)$ is decreasing in r). The reason is that under our assumptions expected investment income is fixed at $\bar{\theta}$ while entrepreneurs with high p projects are more likely to experience success and so to pay r to their banker. As a result, the expected return on investment is lower for entrepreneurs with less risky investment projects, i.e., those with high p. Also, $\bar{p}(r)$ is the fraction of entrepreneurs that invest:

$$\int_0^{\bar{p}(r)} dp = \bar{p}(r). \tag{5.7}$$

Here, we have used the fact that the density of entrepreneurs is unity for each $p \in [0,1]$. Similarly, because the quantity of goods used in each investment project is unity, $\bar{p}(r)$ also corresponds to the total quantity of goods invested by all entrepreneurs in a household. The average value of p among the entrepreneurs that invest is denoted $\Pi(r)$, where

$$\Pi(r) = \frac{\int_0^{\bar{p}(r)} pdp}{\bar{p}(r)} = \frac{1}{2}\bar{p}(r). \tag{5.8}$$

Expression (5.8) reflects that, among the entrepreneurs that operate their investment technologies, the density of entrepreneurs with $p \in [0, \bar{p}(r)]$ is $1/\bar{p}(r)$. Finally, we show below that $\bar{p}(r)$ is inversely proportional to the interest rate spread, the difference between the interest rate, r, paid by entrepreneurs with successful projects and the risk free rate, R. We restrict our attention to model parameterizations that imply the efficient allocations (see section 5.2.1 below) and the equilibrium allocations are interior. This means the usual non-negativity constraints on quantities and also, $0 < \bar{p}(r) < 1$.

We now turn to the bankers. Because each banker is fully diversified across entrepreneurs, its revenues are non-random. Because we also assume banks are competitive, it follows that their profits must be zero. A banker's average earnings per unit of loan is $\Pi(r)r$. The cost of a unit of deposits for a bank is R, so that the each banker's zero profit condition is:

$$\Pi(r)r = R. \tag{5.9}$$

Much of the economics of the model are summarized in (5.9). For example, multiplying (5.8) by r and using (5.9), we obtain a simple expression for the interest rate spread:

$$\text{'interest rate spread'} = \frac{r}{R} = \frac{2}{\bar{p}(r)}. \tag{5.10}$$

According to this expression, the interest rate spread is at least 2, and can be much higher. The intuition for (5.10) is simple. Suppose all entrepreneurs activated their investment project, so that $\bar{p} = 1$. In this case, the average probability of success is 1/2 (see (5.8)). With half the entrepreneurs unable to pay, the ones that do pay must pay $2R$ if the bank is to be able to pay R to its depositors.[34] From (5.6) it is evident that a plausible parameterization of the model can be constructed, with the property that a fall in N produces a rise in the interest rate spread.

Interestingly, (5.9) completely determines the rate of interest, R, in the model. To see this, substitute (5.6) into (5.8), to obtain:

$$\Pi(r)r = \frac{1}{2}\frac{\bar{\theta} - RN}{1-N} = R, \tag{5.11}$$

where the second equality reflects (5.9). Evidently, R is determined exclusively by variables specific to the loan market and not by, for example, households' intertemporal preferences. That the zero profit condition is compatible with only one interest rate is a striking result, though well known in the literature on adverse selection. To understand the result requires understanding why bank revenues per loan are independent of the interest rate on bank loans, r. Note that a higher r implies bankers earn more revenues from entrepreneurs who borrow and repay their loan. However, this positive impact on revenues is canceled by an adverse selection effect. Recall that when a bank raises r, entrepreneurs with a high probability of repaying their loan decide to become inactive.[35] As a result, the average probability that an entrepreneur repays the loan falls (see (5.6) and (5.7)). In principle, this need not be a problem because the lower probability entrepreneurs also enjoy a better outcome when they are successful. However, this is little comfort to the bankers in the model, because they must charge the same interest rate, r, to everyone. A fixed interest rate on loans prevents the bank from sharing in the huge payoffs experienced by low p entrepreneurs when they are successful. This is why a bank's revenues are independent of r.

[34] Clearly, an empirically plausible version of our model would require a density function for p that places greater mass on higher p.

[35] This phenomenon is captured by a quote from Adam Smith's Wealth of Nations, cited in Stiglitz and Weiss (1992). According to Stiglitz and Weisee, Adam Smith wrote that if the interest rate was fixed too high, "... the greater part of the money which was to be lent, would be lent to prodigals and profectors ... Sober people, who will give for the use of money no more than a part of what they are likely to make by the use of it, would not venture into the competition..."

Adverse selection also explains why the revenue function, $\Pi(r)r$, is decreasing in R. As R increases, high p entrepreneurs switch to being inactive and this reduces the average p among borrowers, reducing bank revenues per unit of loan extended.

Because the interest rate is determined by the zero profit condition, in equilibrium the quantity of saving by households adjusts passively to the R that is implied by (5.11). If, for example, the supply of saving were perfectly elastic at an interest rate that is different from the one that solves (5.11), then a small perturbation in the variables (such as N) that determine $\Pi(r)r$ would have an enormous impact on intermediation. In a one-sector model such as ours, the notion that the supply of saving is highly inelastic seems implausible. However, in a multi-sector (or, open economy) version of the model, the situation would be different. Thus, suppose that the zero profit condition in (5.9) pertained to bankers specializing in the supply of funds to a particular sector that is small enough that it takes the economy-wide deposit rate, R, as given. In this case, the supply of funds to the particular sector could be expected to be perfectly elastic at the interest rate, R. If a decrease in net worth, N, among the entrepreneurs of the given sector drives revenues per loan (i.e., the object to the right of the first equality in (5.11)) down, then a fall in N could cause intermediation in that sector to collapse entirely. We do not explore the multisector version of our model more here, though this would clearly be of interest.

Clearing in the loan market requires that the quantity of investment, $e\bar{p}(r)$, equals the quantity of household deposits, d, plus the quantity of net worth, eN, in the hands of entrepreneurs:

$$e\bar{p}(r) = d + eN. \tag{5.12}$$

We now obtain a simplified expression for period 2 household income. Averaging entrepreneurial earnings over all entrepreneurs, we obtain:

$$\pi = \int_0^{\bar{p}(r)} \left\{\bar{\theta} - pr(1-N)\right\} dp + \int_{\bar{p}(r)}^1 NR = \bar{p}(r)\left[\bar{\theta} - \Pi(r)r(1-N)\right] + (1-\bar{p}(r))NR. \tag{5.13}$$

Adding $e\pi$ to household earnings on deposits yields the equilibrium expression for total household income in the second period:

$$Rd + e\bar{p}(r)\left[\bar{\theta} - \Pi(r)r(1-N)\right] + e(1-\bar{p}(r))NR = e\bar{p}(r)\bar{\theta}.$$

The expression after the equality is obtained after substituting out for R and d using (5.9) and (5.12). The object, $e\bar{p}(r)\bar{\theta}$, represents the total period 2 output from investment projects, in household per capita terms. Replacing total household income with its equilibrium value of $e\bar{p}(r)\bar{\theta}$ and evaluating (5.3) at equality, we obtain the household's second period budget constraint in equilibrium:

$$C = e\bar{p}(r)\bar{\theta}. \tag{5.14}$$

Consistent with Walras' law, (5.14) is also the second period resource constraint.

We have the following definition of equilibrium:

Adverse Selection Equilibrium: $c, C, d, r, R, \pi,\ \bar{p}(r)$ such that
(i) c, C, d solve the household problem given R, π
(ii) banks earn zero profits
(iii) entrepreneurs maximize expected revenues

An equilibrium is straightforward to compute for this economy. The five equilibrium conditions, (5.1), (5.2), (5.9), (5.12), and (5.14), as well as the definitions of π and $\bar{p}(r)$ in (5.13) and (5.6), respectively, are sufficient to determine the seven equilibrium variables. Evaluate (5.9) using (5.6) and the definition in (5.8) and solve the resulting expression for R :

$$R = \frac{\bar{\theta}}{2-N}. \tag{5.15}$$

Because $N < 1$ it follows that R is less than the social return, $\bar{\theta}$, on loans. Combine (5.14) and (5.2) and use (5.6):

$$c = (\beta R)^{-\frac{1}{\gamma}} \frac{\bar{\theta} - RN}{r(1-N)} \bar{\theta} e. \tag{5.16}$$

Use the latter expression and (5.12) to substitute out for c and d in (5.1). Solving the resulting expression for r, and making use of (5.15), we obtain:

$$r = 2eR \frac{(\beta R)^{\frac{-1}{\gamma}} \bar{\theta} + 1}{y + eN}. \tag{5.17}$$

In this way, all the equilibrium variables can be computed uniquely as long as the model parameters are such that an interior equilibrium - $\bar{p}(r) < 1$ and $c, C > 0$ - exists.[36]

The ratio of equations (5.15) and (5.17) provide a convenient expression for the interest rate spread, r/R :

$$\frac{r}{R} = 2e \frac{(\beta R)^{\frac{-1}{\gamma}} \bar{\theta} + 1}{y + eN}. \tag{5.18}$$

According to (5.15), R falls with a decrease in N. According to (5.18), this fact alone drives the spread up. The total effect of a decrease in N on the interest rate spread also involves the denominator in (5.18), and this drives the interest rate spread up too. We summarize these results as follows:

Proposition 5.1. *When an interior adverse selection equilibrium exists, it is unique and characterized by (5.15), (5.16) and (5.17). The interest rate spread, given by (5.18), rises with a reduction in N.*

5.2. Implications for Policy

The first subsection below discusses a planner problem for our model economy. In addition, we use this subsection to explain why we assume the existence of an upper bound on the scale of entrepreneurial projects. In the second subsection we show that two types of subsidy schemes improve the equilibrium allocations: a tax-financed transfer of net worth to entrepreneurs and a tax subsidy to banks. Tax financed government deposits with the banks do not overcome the Barro-Wallace proposition. They have no effect because they do not affect the equilibrium interest rate on bank deposits. Households respond to the increase in taxes by reducing their deposits one-for-one with the increase in taxes and government deposits.

[36] A sufficient condition is that, in addition to $N < 1$, the parameters satisfy

$$(y + eN)\left(\bar{\theta}\beta\right)^{\frac{1}{\gamma}} \leq e\left[\left(\bar{\theta}\beta\right)^{\frac{1}{\gamma}} + \bar{\theta}\right].$$

5.2.1. Efficient Allocations

We consider the allocations selected by a benevolent planner who observes an entrepreneur's p, though not the outcome of its project. We use these allocations as a benchmark from which to evaluate the adverse selection equilibrium and various policy interventions studied in the next section. Although here we assume the planner observes each project's p, the policy interventions studied in subsequent subsections below do not require that policymakers observe p.

The planner faces the period 1 resource constraint,

$$c + d \leq y. \tag{5.19}$$

To describe the planner's decisions about which and how many projects to activate and to derive the planner's period 2 resource constraint, we find it useful to describe the model environment using a particular figure.

Figure 4 arranges all the agents in the economy in the unit square. Each point in the square corresponds to a particular household (vertical dimension) and member of household (horizontal dimension). There is a unit measure of households and a unit measure of members of any given household. We suppose that the box is constructed in period 1, just after each entrepreneur has drawn its value of p. A horizontal line inside the box highlights one particular household. The points on the line to the left of e correspond to the entrepreneurs. The points to the right of e correspond to the workers and bankers. The entrepreneurs are ordered according to their value of p, from $p = 0$ to $p = 1$ passing from left to right. For any particular $p \in [0, 1]$, the entrepreneur with that investment project is indicated by the point, pe, on the horizontal axis. Each point on a vertical line through pe corresponds to the entrepreneurs with the given p in the cross section of households. Each of those entrepreneurs has the value of θ that is given by (5.4).

The planner must decide how many entrepreneurs in the interval, 0 to e, to activate. If the planner elects to activate an entrepreneur with a particular p, it instructs all the entrepreneurs in the cross section of households with that p to activate their project. The planner is indifferent about which projects (i.e., which p's) to activate. Each project is the same to the planner because each has the same mean productivity, $\bar{\theta}$, and entrepreneurs suffer no cost to activate their project. As a result, there is no loss of generality in simply assuming that the planner selects entrepreneurs with p's extending from $p = 0$ to $p = \bar{p}$, for some $\bar{p} \leq 1$. This corresponds to the mass of entrepreneurs in the interval, 0 to $e\bar{p}$, in the figure.

Consider a mass of entrepreneurs on an arbitrary interval, Δ, inside $[0, e]$. The resource cost of activating these entrepreneurs in the cross section of households is the area of the rectangle with base Δ inside the unit square. The latter area is just Δ itself. This reflects the assumption that there is a unit mass of households and that each project costs one unit of resources to activate. The available net worth, N per entrepreneur, is sufficient to operate the entrepreneurs corresponding to the interval, 0 to eN. Since these resources have no alternative use, the planner applies them. Activating additional entrepreneurs is costly to the planner because this requires suppressing consumption in period 1. Suppose the planner considers activating an additional mass, d, of entrepreneurs. This corresponds to the entrepreneurs extending from the point, eN to the point, $eN + d$ in the figure. Activating these entrepreneurs requires d resources. So, if the planner wishes to activate a measure, $e\bar{p}$, of entrepreneurs, then $d + eN$ resources are needed, subject to:

$$e\bar{p} \leq d + eN. \tag{5.20}$$

When the planner activates entrepreneurs from 0 to $e\bar{p}$, the total amount of goods available in period 2 is $e\bar{p}\bar{\theta}$. Thus, the second period resource constraint for the planner is:

$$C \leq e\bar{p}\bar{\theta}. \tag{5.21}$$

The planner's problem is to solve:

$$\max_{c,C,\bar{p},d} u(c) + \beta u\left(C\right),$$

subject to $0 \leq \bar{p} \leq 1$, (5.19), (5.20) and (5.21) and $c, C \geq 0$. The unique interior solution is characterized by the first order conditions evaluated at equality. Solving these, we obtain:

$$c = \frac{y + eN}{\left(\beta\bar{\theta}\right)^{\frac{1}{\gamma}} + \bar{\theta}}\bar{\theta}, \tag{5.22}$$

$$e\bar{p} = \frac{y + eN}{1 + \bar{\theta}\left(\beta\bar{\theta}\right)^{-\frac{1}{\gamma}}}, \tag{5.23}$$

$$C = c\left(\beta\bar{\theta}\right)^{\frac{1}{\gamma}}, \tag{5.24}$$

with d given by solving (5.19) with equality. It is convenient to compare these allocations with the allocations in the adverse selection equilibrium. Substituting (5.17) into (5.16) and using (5.15), we obtain that first period consumption in the equilibrium is:

$$c = \frac{y + eN}{\left(\beta R\right)^{\frac{1}{\gamma}} + \bar{\theta}}\bar{\theta}.$$

Using (5.6) and making use of (5.15) and (5.17), we find that total resource use in the adverse selection equilibrium is:

$$e\bar{p}\left(r\right) = \frac{y + eN}{1 + \bar{\theta}\left(\beta R\right)^{-\frac{1}{\gamma}}} \tag{5.25}$$

According to (5.2), second period consumption in equilibrium is:

$$C = c\left(\beta R\right)^{\frac{1}{\gamma}}.$$

Evidently, the sole factor preventing the equilibrium from replicating the planner's allocations is that the interest rate, R, is too low. In the adverse selection equilibrium R is $\bar{\theta}/\left(2 - N\right)$, but the social rate of return on investment is $\bar{\theta}$. With the market sending the wrong signal to households about the return on investment, saving and investment are too low. The problem is more severe, the smaller is N.

The ratio of investment in equilibrium to its first best level is given by dividing (5.25) with (5.23):

$$\frac{1 + \bar{\theta}\left(\bar{\theta}\beta\right)^{-\frac{1}{\gamma}}}{1 + \bar{\theta}\left(\beta R\right)^{-\frac{1}{\gamma}}}.$$

$>$From this expression, we see that equilibrium investment falls relatively more than the first best level of investment when N decreases (here, we have used the relation between R and N in (5.15)).

We summarize the preceding results in the form of a proposition:

Proposition 5.2. *The equilibrium household deposit rate, R, is less than the social return on investment, $\bar{\theta}$, and R falls with a reduction in N. Equilibrium investment falls relatively more than the first-best level of investment with a reduction in N.*

In our adverse selection model we suppose that there is an upper bound on the resources that entrepreneurs can invest in their projects. In section B in the appendix we consider a version of the model which does not impose an upper bound on the scale of entrepreneurial projects. For that version of the model we find:

Proposition 5.3. *Suppose entrepreneurial projects have constant returns and can be operated at any scale. If there is an equilibrium, then (i) only entrepreneurs with the lowest value of p operate their projects, (ii) the aggregate profits of these entrepreneurs is zero, (iii) the allocations in equilibrium coincide with the first-best efficient allocations and $R = \bar{\theta}$.*

To help ensure the existence of an equilibrium under the assumption of Proposition 5.3 we modify the distribution of p slightly by supposing that it has positive mass at the lower bound of its support, and that the lower bound is a (very) small positive number. To us, property (i) renders the equilibrium of this version of our model uninteresting. Also, we suspect that in reality investment projects have diminishing returns to scale. Although the diminishing returns to scale implicit in our upper bound assumption for investment projects is extreme, we find this assumption more interesting than the constant returns to scale alternative addressed in proposition 5.3. Intermediate scenarios are presumably also of interest, but we do not examine these here.

5.2.2. Interest Rate Subsidies

According to the analysis in previous subsections, the problem with the adverse selection equilibrium is that the deposit rate, R, is too low. In addition, when net worth drops, the problem is aggravated as R falls even more and investment, relative to first-best, drops (see Proposition 5.3). Consistent with the empirical phenomenon we seek to understand, the interest rate spread also rises with a drop in N (see Proposition 5.1). The inefficiently low deposit rate, R, reflects that banks do not recover the full returns that their loans make possible. This suggests two direct ways to repair the market mechanism: subsidize bank earnings or, equivalently, their cost of funds. We consider the latter here. We show that an appropriate interest rate subsidy can make the allocations in the adverse selection equilibrium coincide with the efficient allocations. Significantly, implementation of this policy does not require that the government observe any characteristics of the banks' borrowers. This stands in contrast with the planner in the previous subsection, which was assumed to observe each entrepreneur's p.

Denoting the pre-tax cost of funds to the bank by R, the after subsidy cost under our policy is $R/(1+\nu) < R$. We suppose that this subsidy is financed by a lump sum tax on households in the period 2, in the amount

$$T = (1-N)\, e\bar{p}(r)\left[R - \frac{R}{1+\nu}\right].$$

Here, the terms in front of the square bracket represent the total amount of loans made by the banks to the active entrepreneurs. The banks fund these loans with deposits taken from inactive entrepreneurs and households. The amount of the subsidy is $R - R/(1+\nu)$ per unit of loans made. The household's second period budget constraint, (5.3), is replaced by

$$C \leq Rd + e\pi - T.$$

Repeating the substitutions leading up to equation (5.14), taking account of the modified second period budget constraint of the household, we find that (5.14) continues to hold.

The impact of the tax subsidy on the equilibrium value of R is determined by studying the appropriately modified bank zero profit condition, (5.9):

$$\Pi(r)\, r = \frac{R}{1+\nu}. \tag{5.26}$$

Substituting (5.6) and (5.8) into the latter expression and solving for R :

$$R = \frac{\bar{\theta}(1+\nu)}{2-N+N\nu}.$$

Evidently, to achieve $R = \bar{\theta}$ requires $\nu = 1$.

Recall that the seven conditions determining $c, C, d, r, R, \pi,\ \bar{p}(r)$ are (5.1), (5.2), (5.9), (5.12), (5.14), (5.13) and (5.6). We have verified that (5.14) continues to hold. Apart from (5.9), the other conditions are obviously unaffected by T. The only equilibrium condition that must be adjusted is the bank zero profit condition, (5.9), which we replace by (5.26). With $\nu = 1$, the interest rate that solves (5.26) is the efficient one, $R = \bar{\theta}$. Given that the other equations are unaffected, it follows from the discussion in the previous subsection that the efficient allocations are supported by the subsidy policy. We summarize this result in a proposition:

Proposition 5.4. *With an interest cost subsidy, $\nu = 1$, the allocations in the adverse selection equilibrium are efficient.*

5.2.3. Tax Financed Transfers to Entrepreneurs

Here, we consider a policy of raising lump sum taxes, T, on households in period 1 and transferring T/e to each entrepreneur. By setting

$$T = e(1-N), \tag{5.27}$$

the transfer ensures that each entrepreneur has enough funds to fully finance its investment project. With this policy the banking system is circumvented and so there are no frictions.

A problem with this policy is that it presses hard on a feature of the model that we have little confidence in. In particular, we assume that it is known how much net worth each entrepreneur has and how much they need for their investment project. In practice, these assumptions are not satisfied. In addition, our environment abstracts from any problems associated with distributing wealth between entrepreneurs and other agents. These would have to be considered in case such a policy were actually implemented.

We now establish that the above tax-transfer scheme accomplishes what we claim it does. The value of R that satisfies the bank zero profit condition, (5.9), is still the one in (5.15), except that N is replaced by the post-transfer level of entrepreneurial net worth. Since that is unity under the tax-transfer scheme, we have that $R = \bar{\theta}$, its value in the efficient equilibrium. It is also straightforward to verify that c, C and $\bar{p}$ satisfy (5.23). We have assumed that model parameters imply $\bar{p} \leq 1$. When $\bar{p} < 1$, then $d < 0$. That is, in this case some of the net worth transferred to entrepreneurs is recycled back to households through the loan market.

We summarize these results as follows:

Proposition 5.5. *Under the tax-transfer scheme in (5.27), the allocations in the adverse selection equilibrium are efficient.*

6. Asymmetric Information and Monitoring Costs

This section presents an asymmetric information model along the lines proposed in BGG. We adapt the model so that it fits into the same simple setting studied in the previous sections. As in the previous section, the financial friction in this model reflects the circumstances of entrepreneurs rather than banks. In terms of recent data, we can perhaps think of the financial frictions in this environment as pertaining to home buyers or firms whose net worth has declined during the recent financial market turmoil (see Figure 1). In principle, the model could also be interpreted as applying to actual banks. Real world banks resemble entrepreneurs in the BGG model. The entrepreneurs in the model have their own net worth, they accept loans (i.e., they 'take deposits') and they acquire assets.[37]

As in the previous sections, we assume the trigger for the crisis is a fall in the net worth of the financially constrained agents in the economy. Our environment is well suited to contemplating the effects of an increase in microeconomic uncertainty, and so we consider this as an additional trigger for the crisis. Like the model in the previous section (though unlike our first two models), the model analyzed here does not display a sharp dichotomy between 'normal' and 'crisis' times. This is because equilibrium conditions of the model do not include equations that are satisfied as strict equalities for some values of the state and strict inequalities for other values of the state. As a result, we simply define a normal time as one in which net worth is high and a crisis time as one in which net worth is substantially lower.

We now provide a sketch of the model. There are two periods. A large number of workers, entrepreneurs and bankers with identical utility functions live in a representative household.

[37] For asymmetric information and monitoring cost models applied specifically to banking, see Hirakata, Sudo and Ueda (2009a,b, 2010) and Zeng (2010).

Workers have an exogenous endowment, y, of income in the first period, while entrepreneurs possess N units of net worth. Households allocate y to first period household consumption and to bank deposits. Entrepreneurs combine their net worth with loans from banks to create the capital used to produce goods in period 2. Each entrepreneur's production function is perturbed by an idiosyncratic shock which is privately observed. The entrepreneur's bank can only observe the realization of the entrepreneur's technology shock by paying a monitoring cost. The measure of microeconomic uncertainty referred to above pertains to the variance across entrepreneurs in their technology shock. As is usual in an environment like this, we assume that banks provide entrepreneurs with a standard debt contract. Household consumption in the second period is financed out of income on period 1 deposits as well as entrepreneurial profits.

The model implies that a fall in entrepreneurial net worth causes interest rate spreads to rise and investment to fall. We consider various policy responses. We find that a policy that subsidizes the cost of funds to banks is welfare improving in both normal and crisis times. Moreover, the optimal value of the subsidy is greater in crisis times, so that the model suggests a more aggressive policy stance then. We also find that, absent government intervention, bank leverage is too low. In this sense, the model does not include the kind of features that rationalize the current interest in imposing leverage restrictions on economic agents. The reason for our 'underborrowing' result is that the marginal return on loans is higher than the average return, while the economics of the model implies that households focus on the average return.

Next, we show that the Barro-Wallace irrelevance result applies for loans made by the government to entrepreneurs. Thus, the financial market frictions in the model do not rationalize a policy of government purchases of nonfinancial business assets. Finally, we show that a sufficiently large tax-financed transfer of net worth to entrepreneurs allows the economy to support the first-best equilibrium outcomes. Since the model abstracts from the income distribution consequences of this type of policy, we only think of the result as illustrating the logic of the model. The latter result is reported in Proposition C.4 in Appendix C.2.6.

Although the model framework into which we insert the BGG-type financial frictions in this section has the virtue of consistency with the other models in the paper, it has one potential drawback. In our framework, the price of capital is always unity, while in the literature (see, e.g., BGG or CMR), the price of capital is endogenous. Moreover, the recent literature on pecuniary externalities (see Bianchi (2011), Korinek (2010), Lorenzoni (2008) and Mendoza (2009)) raise the possibility that there is overborrowing when entrepreneurial net worth is in part a function of the market price of an asset like capital (see, e.g., Bianchi (2011) and Mendoza (2009)). These findings motivate us to investigate the robustness of our underborrowing result to endogenizing the price of capital the way it is done in BGG. We do so by introducing curvature into the technology for converting consumption goods into capital. Because capital is the only source of net worth for entrepreneurs, the change introduces the type of pecuniary externality studied in the literature. We display numerical results which suggest that our underborrowing result in fact is robust to this change.

6.1. Entrepreneurs and Banks

The typical entrepreneur takes its net worth, N, and approaches a bank for a loan, B. It combines its net worth and the loan to produce output in period 2 using the following production function:

$$\omega \left(N + B\right) r^k, \tag{6.1}$$

where r^k is a fixed parameter of technology. Also, ω is an idiosyncratic productivity shock,

$$\omega \ \sim \ F, \ \int_0^\infty dF\left(\omega\right) = 1, \tag{6.2}$$

where F is the cdf of ω.

Before the realization of an entrepreneur's productivity shock, the entrepreneur and its bank have the same information about ω. Both know the shock will be drawn from F. After the realization of the shock, the bank and the entrepreneur are asymmetrically informed. An entrepreneur observes the realization of its ω, but the entrepreneur's bank can only observe the shock by paying a monitoring cost. Townsend (1979) showed that under these circumstances a 'standard debt contract' works well. Under such a contract, the entrepreneur pays the bank an amount, ZB, in period 2 if it is able to do so. Given F, some entrepreneurs experience such a low ω that they are not be able to repay ZB. Under a standard debt contract, those entrepreneurs are 'bankrupt'. Banks verify this by monitoring those entrepreneurs at a cost,

$$\mu\omega \left(N + B\right) r^k,$$

where $\mu > 0$ is a parameter. A bankrupt entrepreneur transfers everything it has to its bank.

The cutoff level of productivity, $\bar{\omega}$, that separates the bankrupt and non-bankrupt entrepreneurs is defined by:

$$\bar{\omega} \left(N + B\right) r^k = ZB. \tag{6.3}$$

According to (6.3):

$$\bar{\omega} = \frac{ZB}{\left(N + B\right) r^k} = \frac{Z\frac{B}{N}}{\frac{(N+B)}{N} r^k} = \frac{Z}{r^k}\frac{L-1}{L},$$

where L denotes the leverage of the entrepreneur,

$$L \equiv \frac{N + B}{N}. \tag{6.4}$$

Evidently, as $L \rightarrow \infty$, $\bar{\omega}$ converges to a constant, i.e., the value of ω that an entrepreneur with no net worth needs to be able to pay back its debt.

>From the perspective of period 1, an individual entrepreneur's expected profits, π, in period 2 are given by:

$$\pi = \int_{\bar{\omega}}^\infty \left[\omega \left(N + B\right) r^k - ZB\right] dF\left(\omega\right) = NLr^k \left(\int_{\bar{\omega}}^\infty \left[\omega - \bar{\omega}\right] dF\left(\omega\right)\right), \tag{6.5}$$

using (6.3). The expression, (6.5), is written in compact notation as follows:

$$\pi = NLr^k \int_{\bar{\omega}}^\infty \left[\omega - \bar{\omega}\right] dF\left(\omega\right) = NLr^k \left(1 - \Gamma\left(\bar{\omega}\right)\right), \tag{6.6}$$

where

$$\Gamma(\bar{\omega}) \equiv G(\bar{\omega}) + \bar{\omega}\left[1 - F(\bar{\omega})\right], \; G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega). \tag{6.7}$$

The entrepreneur maximizes (6.6) and remits all its profits to its household. It does so in exchange for perfect consumption insurance. It is in the interest of the household that each of its entrepreneurs maximizes expected returns because, by the law of large numbers, this implies that entrepreneurs as a group maximize the total resources brought home to the household in period 2. Households can observe everything about their own member entrepreneurs (including ω) and we make the (standard) assumption that what is observable is enforceable. That is, the household has the means to make sure that each of its entrepreneurs actually maximizes (6.6) and does not, for example, divert any proceeds towards itself.

>From (6.6) and the observation about $\bar{\omega}$ for large L, we see that the entrepreneur's objective is unbounded above in L, for any given Z and r^k. As a result, we cannot use the classic demand and supply paradigm in which the entrepreneur takes Z as given and chooses a loan amount, B. To describe our market arrangement, we must first discuss the situation of the banks.

There is a large number of competitive banks, each of which makes loans to entrepreneurs and takes deposits from households. Because banks are diversified, there is no risk on the asset side of their balance sheet. Although a bank does not know whether any particular entrepreneur will fully repay its loan, the bank knows exactly how much it will receive from its entrepreneurs as a group. Because there is no risk on the asset side of the balance sheet, it is feasible for banks to commit in period 1 to paying households a fixed and certain gross rate of interest, R, on their deposits in period 2. Because banks are competitive, they take R as given. A bank that makes size B loans to each of a large number of entrepreneurs earns the following per entrepreneur:

$$\left[1 - F(\bar{\omega})\right] ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)\, r^k (N + B).$$

Here, the term before the plus sign corresponds to the revenues received from entrepreneurs that are not bankrupt, i.e., those with $\omega \geq \bar{\omega}$. The term after the plus sign indicates receipts, net of bank monitoring costs, received from entrepreneurs that cannot fully repay their loan. Since the cost of funds is RB, the bank's zero profit condition is (using (6.3)):

$$\left[1 - F(\bar{\omega})\right] \bar{\omega} (N + B)\, r^k + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)\, r^k (N + B) = RB,$$

or,

$$\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \frac{r^k (N + B)}{B} = R. \tag{6.8}$$

An important feature of this environment is that the interest rate paid to households is proportional to the average return, $r^k (N + B)/B$, on loans and not, say, to the marginal return. We discuss the policy implications of this feature, evident from (6.8), below. Banks are indifferent between loan contracts, as long as they satisfy the above zero profit condition.

Expression (6.8) motivates the market arrangement that we adopt. Let the combinations of $\bar{\omega}$ and B that satisfy (6.8) define a 'menu' of loan contracts that is available to entrepreneurs, for given R.[38] It is convenient to express this menu in terms of L and $\bar{\omega}$. Rewriting (6.8):

$$L = \frac{1}{1 - \frac{r^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}. \tag{6.9}$$

Entrepreneurs take N, R and r^k as given and select the contract, $(L, \bar{\omega})$, from this menu which maximizes expected profits, (6.6). Using (6.9) to substitute out for L in the entrepreneur's objective, the problem reduces to one of choosing $\bar{\omega}$ to maximize:

$$Nr^k \frac{1 - \Gamma\left(\bar{\omega}\right)}{1 - \frac{r^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}.$$

The first order necessary condition for optimization problem is:

$$\frac{1 - F\left(\bar{\omega}\right)}{1 - \Gamma\left(\bar{\omega}\right)} = \frac{\frac{r^k}{R}\left[1 - F\left(\bar{\omega}\right) - \mu\bar{\omega}F'\left(\bar{\omega}\right)\right]}{1 - \frac{r^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}, \tag{6.10}$$

which can be solved for $\bar{\omega}$ given R.[39] Given the solution for $\bar{\omega}$, L solves (6.9) and Z solves (6.3).

A notable feature of the contract is that L and Z are independent of net worth, N. That is, if entrepreneurs had different levels of net worth, the theory as stated predicts that each entrepreneur in the cross section receives a loan contract specifying the same leverage and rate of interest. This feature of the model reflects the assumption that all entrepreneurs draw ω from the same distribution, F. In a more realistic setting, different entrepreneurs would draw from different F's and they would receive different debt contracts.

6.2. Households and Government

In period 1 the household budget constraint is:

$$c + B \leq y, \tag{6.11}$$

where c, B, y denote consumption, bank deposits and an endowment of output, y. Expression (6.11) is also the period 1 resource constraint. The second period budget constraint of the household is

$$C \leq (1 + \tau) RB + \pi - T,$$

where C denotes period 2 consumption, T denotes lump sum taxes, τ denotes a subsidy on household saving and π denotes the profits brought home by entrepreneurs.[40] Households maximize utility,

$$u(c) + \beta u(C), \tag{6.12}$$

[38] Note that a collection, $(\bar{\omega}, B)$, is equivalent to a collection, (Z, B) by $\bar{\omega} = ZB/\left[(N + B) r^k\right]$ and the fact that N and r^k are exogenous to the entrepreneurs at the time the contract is undertaken.

[39] Here, we have used $\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega})$ and $G'(\bar{\omega}) = \bar{\omega}F'(\bar{\omega})$.

[40] As noted previously, by the law of large numbers the expected profits of individual entrepreneurs, what we defined in (6.6) as π, also corresponds to aggregate profits (in per capita terms) for all the entrepreneurs in the household.

subject to their periods 1 and 2 budget constraints. In practice, we assume,

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha},\ \alpha > 0.$$

The government's budget constraint is:

$$T = \tau R B. \tag{6.13}$$

The second period resource constraint is obtained from the household budget constraint by substituting out for π from (6.6), RB from (6.8) and T from (6.13), to obtain the second period resource constraint:

$$C \leq r^k (N + B) [1 - \mu G(\bar{\omega})]. \tag{6.14}$$

According to (6.14), period 2 consumption is no greater than total output, net of the output used up in monitoring by banks.

6.3. Equilibrium

We define an equilibrium as follows:

Definition 6.1. *For given τ, a private sector equilibrium is a $(C, c, R, \bar{\omega}, B, T)$ such that*
(i) The household problem is solved (see section 6.2)
(ii) The problem of the entrepreneur is solved (see section 6.1)
(iii) Bank profits are zero (see section 6.1)
(iv) The government budget constraint is satisfied (see section 6.2)
(v) The first and second period resource constraints are satisfied.

For convenience, we collect the equations that characterize a private sector equilibrium here:

Equation number	Equation	Economic description
	$C = c(\beta[1+\tau]R)^{\frac{1}{\alpha}}$	household first order condition
(6.14)	$C = r^k [N+B][1-\mu G(\bar{\omega})]$	period 2 resource constraint
(6.11)	$c + B = y$	period 1 resource constraint
(6.10)	$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{r^k}{R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{r^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	contract efficiency condition
(6.9)	$\frac{N+B}{N} = \frac{1}{1-\frac{r^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	bank zero profit condition

(6.15)

These equations represent 5 equations in 5 private sector equilibrium objects:

$$C, c, R, \bar{\omega}, B. \tag{6.16}$$

The equilibrium value of T can be backed out by imposing either the household or government budget constraint. Note too that the rate of interest on entrepreneurs, Z, is determined from (6.3).

6.4. Implications for Policy

The first part of this section shows that the equilibrium in our economy is characterized by too little borrowing, so that subsidizing the cost of funds to banks is welfare improving. We then show that a policy of extending loans directly to entrepreneurs fails to overcome the Barro-Wallace irrelevance result, and so has no impact.

6.4.1. Subsidizing the Cost of Funds to Banks and Leverage Restrictions

Interest rate subsidies are desirable from a welfare point of view because they correct a particular inefficiency in the model. Households make their deposit decision treating R as the marginal return on a deposit. However, the structure of financial markets is such that R corresponds to the average, not the marginal, return on loans. Not surprisingly, a planner prefers that household deposit decisions be made based on the marginal return. We show that in our environment, the marginal return on loans is higher than the average return, so that the market signal received by the households, R, does not provide them with an appropriately strong incentive to save. An interest rate subsidy corrects this problem.

In the second subsection we turn to quantitative simulations. In view of the results in the previous paragraph, it is perhaps not surprising that restrictions on leverage in the model reduce welfare. We find that the market inefficiency in the model - the excess of the marginal return on loans over the average return - increases in a crisis time. As a result, the interest rate subsidy ought to be expanded substantially then. In addition, efficient policy expands leverage by a greater percent in a crisis time than in normal times.

Qualitative Analysis A private sector equilibrium is defined conditional on a particular value of τ. If we treat τ as unknown, then the system is underdetermined: there are many equilibria, one for each possible value of τ. The Ramsey equilibrium is defined as the best of these equilibria in terms of social welfare, (6.12). That is, the Ramsey equilibrium solves

$$\max_{c,C,B,\bar{\omega},R,\tau} u(c) + \beta u(C),$$

subject to (6.14), (6.11), (6.10), (6.9) and the household intertemporal first order condition. The latter is non-binding as it can simply be used to define τ without placing any limitation on the maximization problem. Making use of the latter observation, and substituting out for c and C using (6.14), (6.11), the Ramsey equilibrium allocations can be found by solving:

$$\max_{B,\bar{\omega}} u(y-B) + \beta u\left(r^k [N+B][1-\mu G(\bar{\omega})]\right), \tag{6.17}$$

subject to (6.10), (6.9). It is convenient to further simplify the problem by solving (6.9) for r^k/R and using the result to substitute out for r^k/R in (6.10):

$$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{L(B)-1}{[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})], \tag{6.18}$$

where $L(B)$ denotes the private sector equilibrium level of leverage, as a function of B :

$$L(B) = \frac{N+B}{N}. \tag{6.19}$$

Equation (6.18) defines a mapping from B to $\bar{\omega}$. Denote this mapping by $\bar{\omega}(B)$.[41] Substitute $\bar{\omega}(B)$ into (6.17) and the Ramsey problem reduces to the following:[42]

$$\max_B u(y-B) + \beta u\left(r^k [N+B][1-\mu G(\bar{\omega}(B))]\right).$$

The first order necessary condition for an (interior) optimum is:

$$\frac{u'(c)}{\beta u'(C)} = r^k \{1-\mu [G(\bar{\omega}(B)) + (N+B) G'(\bar{\omega}(B)) \bar{\omega}'(B)]\} \equiv R^m, \tag{6.20}$$

say. The term, R^m, denotes the marginal social return on loans.

Once the Ramsey problem is solved, the remaining objects in Ramsey equilibrium can be found as follows. The cost of funds to banks, R, is obtained by computing the average return on loans:

$$R = \frac{r^k L(B)}{L(B)-1}[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})].$$

Here, we have used (6.8) and (6.19). The saving subsidy, τ, is computed so that the household's saving decision is based on the marginal return on loans, R^m, and not the average return, R:

$$R^m = R(1+\tau). \tag{6.21}$$

Two features of (6.20) deserve emphasis. First, r^k is the return to loans in the 'first best' version of our economy. The first best allocations are those that solve the problem:

$$\max_{c,C,B} u(c) + \beta u(C)$$
$$\text{subject to: } c+B \leq y, \quad C = r^k [N+B].$$

This is the problem in which allocations are chosen by a planner that can observe each entrepreneur's productivity shock, ω. In this problem, there are no monitoring costs. The object in braces in (6.20) represents a wedge introduced by the presence of asymmetric information. A second interesting feature of (6.20) is that the solution to the Ramsey problem implies $\tau > 0$. This is because the intertemporal marginal rate of substitution in consumption is equated with the marginal return to loans in the Ramsey problem, R^m, while in the private sector equilibrium with $\tau = 0$ it is equated to the average return on loans, R, (recall the discussion surrounding (6.8).)[43] Proposition C.1 in section C.1 of the Appendix establishes that under a certain regularity condition, $R^m > R$, so that marginal return on loans exceeds the corresponding average return. Thus, by (6.21), $\tau > 0$. To define the regularity condition, let the hazard rate be denoted as follows:

$$h(\bar{\omega}) \equiv \frac{F'(\bar{\omega})}{1-F(\bar{\omega})}. \tag{6.22}$$

[41] In the numerical examples we have studied, we have found that when there exists a value of $\bar{\omega}$ that solves (6.18) for a given B, that value of $\bar{\omega}$ is unique.

[42] We generally ignore imposing the usual non-negativity constraints such as $B \geq 0$, $y - B \geq 0$, to minimize clutter and in the hope that this does not generate confusion. Throughout, we assume that model parameters are set so that non-negativity constraints are non-binding.

[43] In his analysis of costly state verification problems, Fisher (1999) also emphasizes that the quantity of lending is determined by the average, not the marginal, return on a loan.

The regularity condition is

$$\bar{\omega}h(\bar{\omega}) \text{ increasing in } \bar{\omega}. \tag{6.23}$$

BGG study (6.23) and argue that it is satisfied when F corresponds to the lognormal distribution.

Although we assume $\mu > 0$ in our model, it is interesting to consider the limiting case, $\mu \to 0$. In this case, $R = r^k$ according to (6.10). But, (6.20) implies $R^m = r^k$ when $\mu = 0$, so we conclude that in this case the average and marginal returns coincide. That is, if $\mu = 0$ then $\tau = 0$ in the Ramsey equilibrium.

We summarize the preceding results in the form of a proposition:

Proposition 6.2. *Suppose $\mu > 0$ and condition (6.23) holds. Then, the interest rate subsidy, τ, is positive in a Ramsey equilibrium. This reflects that (i) the household bases its saving decision on the after tax average return on deposits, $(1+\tau)R$, while the Ramsey planner wishes that the household base its saving decision on the corresponding marginal return, R^m, and (ii) $R^m > R$. Suppose $\mu = 0$. Then $R = R^m$; $\tau = 0$ in a Ramsey equilibrium; and the allocations in a private sector equilibrium with $\tau = 0$ are first-best.*

Quantitative Analysis We construct several numerical examples to illustrate the observations in the previous subsection. We suppose that the cumulative distribution of ω, denoted by F, is lognormal. This distribution has two parameters, $E\log\omega$ and

$$\sigma^2 \equiv Var(\log\omega).$$

The assumption, $E\omega = 1$, implies $E\log\omega = -\sigma^2/2$ so σ is the only free parameter in F. The baseline values of our model parameters are displayed in Table 1. These parameter values were chosen in part to ensure a bankruptcy rate of 4 percent, i.e., $F(\bar{\omega}) = 0.04$, a leverage ratio, $L = 2$, and $R = 1.01$ when $\tau = 0$. The value of $F(\bar{\omega})$ that we use is about one percentage point higher than what appears in the literature (see Christiano, Motto and Rostagno (2010) for a review). That literature uses models that are specified at a quarterly frequency. Given our setting of β we are tempted to interpret the period in the model as one year, in which case our specification of $F(\bar{\omega})$ is somewhat low relative to the literature. However, given that the model has only two periods it is unclear how to compare the time dimension of the model with the quarterly time dimension in empirical models. In light of these considerations, we decided to use a relatively conventional value for $F(\bar{\omega})$ in our calibration. Our value of μ is also within the range used in the literature. We select values for σ, r^k and $\bar{\omega}$ so that (6.10), (6.9) and the calibrated value of $F(\bar{\omega})$ are satisfied. The resulting value of σ, reported in Table 1, is within the range used in the literature.

As in other sections of the paper, we capture the onset of the crisis with an exogenous drop in N.[44] The quantitative properties of the model are displayed in Table 2. Panel A in that table displays the properties of the model under the benchmark parameterization. This is our characterization of the economy in a 'normal' time. Panels B and C display two representations of our model economy in a 'crisis' time. Panel B corresponds to the

[44] See Gertler and Kiyotaki (2011) and Shimer (2010) for other studies that model the shock that triggers the recent financial crisis as a drop in wealth.

case where N is reduced. Our second representation of a crisis is that the drop in N is accompanied by a 20 percent rise in σ. We are interested in this representation as a way to capture casual evidence that there was a general increase in 'uncertainty' during the crisis. The results for this case are reported in Panel C. In each panel, the column labelled '$\tau = 0$' displays the private sector equilibrium with $\tau = 0$. The column, 'Ramsey', indicates the equilibrium with the Ramsey-optimal τ. Finally, the column marked 'First Best' indicates the first best allocations.

Note from Panel A that in normal times the Ramsey interest rate subsidy, τ, is 0.3 percent. Thus, consistent with Proposition 6.2, in normal times the marginal return on investment exceeds the average return. Because equilibrium saving increases in the subsidy, saving and investment are both higher in the Ramsey equilibrium than they are with $\tau = 0$. The increased supply of saving reduces the equilibrium interest rate, R, so that τ is effectively a subsidy to banks' cost of funds. The interest rate spread is slightly higher in the Ramsey equilibrium than it is in the private sector equilibrium with $\tau = 0$. This reflects that loans to entrepreneurs are larger in the Ramsey equilibrium, so that monitoring costs associated with bankruptcies are larger.

We now turn to Panel B. With the drop in N the economy is poorer and so we expect substantial effects, even in the first best allocations. According to Panel B, the first best level of consumption in the first and second periods drops. There is a rise in household saving in response to the shock, but the rise is smaller than the fall in N so that investment drops. In terms of the response in the private sector equilibrium with $\tau = 0$, note that there is a substantial jump in the interest rate spread, from 1.23 percent to 7.83 percent. This jump is associated with a very large rise in bankruptcies, from 4 percent to 21 percent. In addition, consumption in both periods and investment all drop by large amounts. In terms of interest rates and quantities, the drop in N generates a response qualitatively similar to what we found in the previous models.

In terms of policy, the optimal interest rate subsidy rises more than 5-fold in response to the drop in N. The intervention reduces the cost of funds to banks (R falls an additional 9 basis points in the Ramsey equilibrium, compared to the $\tau = 0$ equilibrium). Thus, this model shares the implication of the other models in this paper which indicate that a 'crisis' triggered by a fall in net worth justifies a policy of (increased) interest rate subsidies to banks.

For the results in Panel C, we set $N = 1/2$ and $\sigma = 1.2 \times 0.37$, where 0.37 is the value of σ in the baseline parameterization (see Table 1). Of course, this change in our experiment has no impact on the first best allocations.

The jump in σ causes the interest rate spread to jump by nearly 5 percentage points in both the $\tau = 0$ and Ramsey equilibria. In addition, investment is reduced, though only by a small amount. Note that the increase in σ produces a period 1 rise in consumption. This happens because the increase in σ exacerbates the financial frictions and induces substitution away from activities (investment) that involve finance and towards activities (period 1 consumption) that do not. Christiano, Motto and Rostagno (2010), who incorporate the financial frictions described here into a dynamic model, find that fluctuations in σ account for a substantial portion of business cycle fluctuations. This is because other features of their dynamic general equilibrium model reverse our model's prediction that consumption and investment comove negatively in response to disturbances in σ.

Turning to the implications for policy, note in Panel C that the σ shock magnifies the rise in the Ramsey tax subsidy rate. Thus, an increase in uncertainty calls for a greater subsidy to banks' cost of funds and, hence, more leverage.

In sum, the numerical analysis shows that, at a qualitative level, the model of this section rationalizes the view that a drop in net worth reduces investment, raises interest rate spreads and warrants interest rate subsidies to banks.

6.4.2. Government Loans to Entrepreneurs

We consider a government policy that raises a lump sum tax, T, on households in the first period. It then lends T to entrepreneurs using the same technology available to banks and transfers the proceeds to households in the second period in the form of a lump sum tax rebate. We show that this policy has no impact on equilibrium allocations because it simply displaces, one-for-one, private saving. That is, the Barro-Wallace irrelevance result holds for government purchases of the financial assets of non-financial business.

We suppose that the government and banks offer the same menu of loan contracts to entrepreneurs. Because entrepreneurs are all identical, each chooses the same loan contract, regardless of whether they borrow from private banks or the government. That is, each receives the same leverage and interest rate, characterized by (6.9) and (6.10). Denote the fraction of entrepreneurs which receives their loans from private banks by ν, while the complementary fraction receives their loans from the government. Thus, the net worth of entrepreneurs receiving their loans from private banks and the government is νN and $(1-\nu)N$, respectively.[45]

In the first period, the representative household deposits b with the banking system. The household's first-period budget constraint is

$$c + b + T \leq y. \tag{6.24}$$

Expression (6.24) is also the period 1 resource constraint. The household's second period budget constraint is:

$$C \leq Rb + \text{government lump sum taxes+entrepreneurial earnings}$$

By the zero profit condition of private banks, household deposits generate the following return in equilibrium:

$$Rb = r^k (\nu N + b) [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})].$$

The entrepreneurs financed by banks return the following profits to the households:

$$r^k (\nu N + b) [1 - \Gamma(\bar{\omega})].$$

Thus, household income from deposits plus the profits generated by the entrepreneurs financed by those deposits is:

$$Rb + r^k (\nu N + b) [1 - \Gamma(\bar{\omega})] = r^k (\nu N + b) [1 - \mu G(\bar{\omega})].$$

[45] Recall that all variables are in household per capita terms.

Similarly, the households receive a tax rebate from the government plus profits from the entrepreneurs financed by the government, in the amount:

$$r^k \left(\left(1 - \nu \right) N + T \right) \left[1 - \mu G \left(\bar{\omega} \right) \right].$$

Then, total household income in the second period is:

$$\begin{aligned} & r^k \left[\left(\nu N + b \right) + \left(\left(1 - \nu \right) N + T \right) \right] \left[1 - \mu G \left(\bar{\omega} \right) \right] \\ & = r^k \left(N + b + T \right) \left[1 - \mu G \left(\bar{\omega} \right) \right] \end{aligned}$$

Combining the household's budget constraint with the equilibrium expressions for entrepreneurial profits and deposit interest, we obtain the second period resource constraint:

$$C \leq r^k \left(N + b + T \right) \left[1 - \mu G \left(\bar{\omega} \right) \right]. \tag{6.25}$$

The necessary and sufficient conditions for an (interior) household optimum are that the first and second period budget constraints (i.e., (6.24) and (6.25)) hold with equality and that the intertemporal marginal rate of substitution in consumption be equated to the household deposit rate, R. These three conditions coincide with the first three equations in (6.15) if we identify B with $b + T$ and we set $\tau = 0$.

Thus, the equilibrium allocations of the model in this subsection are determined recursively. The five equations in (6.15) with $\tau = 0$ determine the five equilibrium objects in (6.16). Private lending, b, is then determined by $b = B - T$. If the government increases loans, T, to entrepreneurs, then household loans to entrepreneurs through the banking system are reduced by the same amount. Of course, this assumes (as we do throughout this paper) that we only consider interior equilibria. For example, if $B < T$ then the non-negativity constraint, $b \geq 0$ would be binding and we expect the tax policy to have real effects.[46] We state this result in the form of a proposition:

Proposition 6.3. *Suppose the government has the same technology for making loans to entrepreneurs as banks. Then in an interior equilibrium, tax financed government loans to entrepreneurs have no impact on rates of return, asset prices, consumption and investment.*

Of course, this proposition is not true if there are differences between the loans provided by the government and those provided by banks. We suspect that interesting asymmetries would involve the government having a less efficient technology for making loans than the private sector. If so, then we conjecture that social welfare would be reduced with an increase in T. Thus the environment of this section, in contrast to the one in section 3, appears to provide little rationale for government purchases of securities issued by non-financial business.

6.5. Pecuniary Externalities

We consider a sequence of economies, starting with our baseline specification in which the technology for converting entrepreneurial resources, $N + B$, into productive capital is linear.

[46] We suspect that the environment of this section is not an interesting one for considering equilbria in which the constraint, $b \geq 0$ is binding.

When there is curvature in the technology for producing capital the price of capital becomes endogenous and this potentially introduces a particular pecuniary externality. We established in Proposition 6.2 above that in our baseline linear case the equilibrium is characterized by underborrowing. We find that when curvature in the production of capital is increased, the optimal value of the subsidy at first increases, so that underborrowing becomes more severe. For higher levels of curvature, the optimal value of the subsidy converges slowly to zero from above, so that the underborrowing result is attenuated. On net, our numerical results indicate that our underborrowing result is robust to all but the very highest levels of curvature. And even then, we do not obtain overborrowing.

To further explain our results, we now indicate the key features of the modified model, which we inherit from BGG. In particular, in the version of the model with endogenous capital prices entrepreneurs are endowed with a quantity of capital, k, at the beginning of the period. As in BGG, they sell this capital to capital producers for a price, P_k. This price defines the net worth of entrepreneurs, $N = P_k k$. Capital producers operate a technology which combines k with investment goods to produce and sell new capital, K, at a price, P_K. The technology operated by capital producers is

$$K = k + \left(\frac{I}{k}\right)^{\gamma} k,\ 0 \leq \gamma \leq 1.$$

Entrepreneurs go to banks with their net worth and obtain a standard debt contract with loan amount, B. They purchase K using this loan and their net worth, subject to:

$$P_K K \leq B + N.$$

The entrepreneur operates a production technology analogous to the one in (6.1) and (6.2):

$$\omega K r^k,$$

where ω is distributed as in (6.2) and r^k is a fixed parameter. With our modification, the prices of old and new capital, P_k and P_K, are endogenous. The rate of return on capital, R^k, is also endogenous, with:

$$R^k \equiv \frac{r^k}{P_K}.$$

Now, suppose a bank deviates from the standard debt contract and makes an additional loan to one particular entrepreneur. This has two effects that involve pecuniary externalities. First, when the entrepreneur uses the loan to purchase new capital, the entrepreneur drives its price, P_K, up by a (small) amount. This encourages capital production and leads to a rise in P_k, thus raising the net worth of other entrepreneurs and loosening their collateral constraint. As a result, the other entrepreneurs are able to borrow more. The second effect of extending a loan to an entrepreneur is that the things the entrepreneur buys (in this case, K) become more expensive (i.e., P_K increases). Note that the two effects work against each other. Depending on the relative strengths of the two effects, various results could occur in principle. The details of the modified model economy are presented in Appendix C.

We now report our quantitative experiment. We solved for the Ramsey optimal subsidy rate, τ, for $0.01 \leq \gamma \leq 1$, holding all other parameters fixed at their baseline level (see Table

1). Figure 5 displays the computed values of τ. Note that as γ decreases, the inefficiency of the economy initially increases since τ increases. However, for γ below roughly 0.85, the inefficiency of equilibrium decreases monotonically with additional reductions in γ. For γ near 0.3, the inefficiency is virtually completely eliminated since the Ramsey optimal τ is close to zero.

7. Concluding Remarks

In the past decade, DSGE models have been constructed that have proved useful for analyzing questions of interest to policy makers.[47] In recent years, the Federal Reserve has undertaken various actions - a large scale asset purchase problems, reductions in banks' cost of funds - with the objective of correcting dysfunctions in credit markets. The DSGE models developed to place structure on the policy discussions before 2007 are silent on the rationale, design and appropriate scale of recent policy actions. DSGE models must integrate the right sort of financial frictions to be useful given the new policy questions. This paper surveys four candidate models and summarizes some of their implications for recent policy actions.

[47] See Christiano, Trabant and Walentin (2011) for a review.

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A. Notes on the Unobserved Effort Model

A.1. Computational Strategy for Solving the Baseline Model

Computing an equilibrium when the cash constraint, (4.8), is not binding (i.e., $\nu = 0$) is straightforward. Here, we describe an algorithm for computing an equilibrium when the cash constraint is binding, so that $\nu < 0$. There are ten equilibrium conditions. This includes the six conditions associated with the banks, (4.21), the three conditions associated with household optimization, (4.3), (4.2) and (4.16) and the definition of λ, the marginal utility of second period consumption, (4.17). The ten variables to be solved for are:

$$\lambda, c, C, R_g^d, R_b^d, R, e, d, \nu, \eta.$$

When $\nu < 0$, the e and ν equilibrium conditions associated with the banks can simplified and we do so first.

That $\nu \neq 0$ implies the ν equation in (4.21) can be replaced by:

$$\nu : R_b^d d = R^b \left(N + d\right).$$

Note that the μ equation implies

$$R_g^d - R_b^d = \frac{R - R_b^d}{p\left(e\right)}.$$

Use this expression to substitute out for $R_g^d - R_b^d$ in the η equation:

$$e = \lambda b \left[\left(R^g - R^b\right)\left(N + d\right) - \frac{R - R_b^d}{p\left(e\right)} d\right]. \tag{A.1}$$

Note that the d equation implies

$$R^g - R^b = \frac{R - R^b}{p\left(e\right)}.$$

Use this expression to substitute out for $p\left(e\right)$ in (A.1) and use the ν equation to substitute out for R_b^d in (A.1), to obtain:

$$e = \lambda b \left[\left(R^g - R^b\right)\left(N + d\right) - \left(R^g - R^b\right) \frac{R - R^b \frac{(N+d)}{d}}{R - R^b} d\right].$$

Factor $\left(R^g - R^b\right)$, collect terms in d and rearrange, to obtain the adjustment to the η equation that is possible when $\nu \neq 0$:

$$\eta : e = \lambda b \left(R^g - R^b\right) \frac{R}{R - R^b} N. \tag{A.2}$$

The equilibrium conditions associated with the banks, with the appropriate adjustments that are possible when $\nu \neq 0$, are:

$$\begin{aligned} e &: (\lambda - \nu)\, b \left(R_g^d - R_b^d\right) d + \eta = 0 \\ d &: R = p(e)\, R^g + (1 - p(e))\, R^b \\ R_g^d &: \nu p(e) = \eta \lambda b \\ \mu &: R = p(e)\, R_g^d + (1 - p(e))\, R_b^d \\ \eta &: e = \lambda b \left(R^g - R^b\right) \frac{R}{R - R^b} N \\ \nu &: R_b^d d = R^b (N + d)\,. \end{aligned} \tag{A.3}$$

To solve this system, fix $R > R^b$. Solve for λ, c, C, d using (4.17) and the household equations, (4.3), (4.2), (4.16). Compute R_b^d using the ν equation. Compute e using the η equation. Compute R_g^d using the μ equation. Adjust the value of R until the d equation is satisfied. To investigate the possibility of multiple equilibria, we considered values of R on a fine grid over a wide range of values and it appeared that there is only one value of R that satisfies the d equation. Finally, the e and R_g^d equations can be used to solve linearly for ν and η. For example, with some algebra we find

$$\nu = \frac{b^2 \left(R_g^d - R_b^d\right) \lambda^2 d}{b^2 \left(R_g^d - R_b^d\right) \lambda d - p(e)}.$$

This completes the discussion of computing the equilibrium.

A.2. Proof of Proposition 4.5

In this appendix, we prove a slightly more precise statement of Proposition 4.5 in section 4.5.2. Let

$$B \equiv d + T.$$

Then:

Proposition A.1. *Let T and T' denote two different levels of tax-financed equity finance.*

- *Equilibrium is characterized by the following invariance property. If c, C, e, R, λ, B satisfies the equilibrium conditions under T, then c, C, e, R, λ, B also satisfies the equilibrium conditions under T' as long as $\nu < 0$ in both cases.*

- *In addition, (i) while $\nu < 0$, ν is monotone increasing in T and (ii) there is a T large enough, say T^*, such $T^* < B$ and $\nu = 0$ for $B > T \geq T^*$.*

It is worth stressing that the invariance property applies only to c, C, e, R, λ, B and not to all the endogenous variables of the model. These include, in addition:

$$\nu, \eta, R_g^d \text{ and } R_b^d. \tag{A.4}$$

The equilibrium values of the variables in (A.4) do change with T, when $\nu < 0$. We now prove the above proposition.

The four equilibrium conditions associated with the household are given by (4.17), (4.27), (4.28) and (4.2):

$$\begin{aligned} C &= R(N + d + T) \\ y &= c + d + T \\ c^{-\gamma} &= \beta R C^{-\gamma} \\ \lambda &= u'(C). \end{aligned}$$

Note that if c, C, R, λ, B solve these equations for one value of T, then the same c, C, R, λ, B solve these equations for another value of T. That is, the household equilibrium conditions satisfy the invariance property.

Now consider the equilibrium conditions associated with the banks. Recall from section 4.5.2 that only private deposits, d, (i.e., not T) enter the bank equilibrium conditions. The equilibrium conditions for the case, $\nu \neq 0$, are stated in (A.3). For convenience, we rewrite the equations here, replacing d with $B - T$:

$$\begin{aligned} e &: (\lambda - \nu) b \left(R_g^d - R_b^d\right)(B - T) + \eta = 0 \\ d &: R = p(e) R^g + (1 - p(e)) R^b \\ R_g^d &: \nu p(e) = \eta \lambda b \\ \mu &: R = p(e) R_g^d + (1 - p(e)) R_b^d \\ \eta &: e = \lambda b \left(R^g - R^b\right) \frac{R}{R - R^b} N \\ \nu &: R_b^d (B - T) = R^b (N + B - T). \end{aligned} \tag{A.5}$$

The d and η equations clearly satisfy the invariance property. It remains to verify that the four equations, e, R_g^d, μ and ν, do so too. For given B, λ, e, R these four equations represent four linear equations in the four unknowns in (A.4). We now verify that these equations have a unique solution. With this result, our invariance property is established.

Using the μ equation to substitute out for $R_g^d - R_b^d$ in the e equation and the R_g^d equation to substitute out for η, the remaining two equations are:

$$\begin{aligned} e &: (\lambda - \nu) b \frac{R - R_b^d}{p(e)} (B - T) + \nu \frac{p(e)}{\lambda b} = 0 \\ \nu &: R_b^d (B - T) = R^b (N + B - T) \end{aligned}$$

Use the second of these equations to substitute out for R_b^d in the e equation:

$$e : (\lambda - \nu) b \frac{R(B - T) - (N + B - T) R^b}{p(e)} + \nu \frac{p(e)}{\lambda b} = 0.$$

Solving for ν :

$$-\nu = \frac{\lambda^2}{\frac{\left(\frac{p(e)}{b}\right)^2}{R(B-T)-(N+B-T)R^b} - \lambda b}. \tag{A.6}$$

Given the value of $\nu < 0$ in (A.6), we can now uniquely solve for η, R_g^d and R_b^d in (A.4). The invariance property is established. Note that the invariance property applies only to the variables in

We now turn to properties (i) and (ii) in the proposition. According to (A.6) our assumption that the cash constraint is binding in the bad state, $\nu < 0$, implies

$$R(B-T) - (N+B-T)R^b > 0.$$

Also, note that $R > R^b$ according to the d equation in (A.5). As a result, for fixed B, R, the above expression is linear and decreasing in T. Let T^* denote the value of T where the above expression passes through zero. Note that $T^* < B$, so that deposits, $B - T^*$, are positive. We can see from (A.6) that as $T \to T^*$, $-\nu \to 0$. That is, the cash constraint is marginally non-binding for $T = T^*$. For $T > T^*$ the cash constraint is non-binding and $\nu = 0$. This completes the proof of the proposition.

A.3. Solving the Version of the Model with Bailouts and Leverage

We describe algorithms for solving the versions of the model studied in sections 4.5.4 and 4.5.5 in the main text. We begin with the version of the model in 4.5.4. The first order conditions associated with the problem in (4.19) are a suitable adjustment on (4.20):

$$\begin{aligned}
&e: \lambda p'(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right] - e + \mu p'(e)\left(R_g^d - (1+\tau)R_b^d\right)d \qquad \text{(A.7)}\\
&\quad + \eta\left(1 - \lambda p''(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right]\right) = 0\\
&d: 0 = \lambda p(e)\left[R^g - R_g^d\right] + \lambda(1-p(e))\left[R^b - R_b^d\right] + \mu\left[p(e)R_g^d + (1-p(e))(1+\tau)R_b^d - R\right]\\
&\quad - \eta\lambda p'(e)\left[\left(R^g - R^b\right) - \left(R_g^d - R_b^d\right)\right] + \nu\left(R_b^d - R^b\right)\\
&R_g^d: -\lambda p(e) + \mu p(e) + \eta\lambda p'(e) = 0\\
&R_b^d: -\lambda(1-p(e)) + \mu(1-p(e))(1+\tau) - \eta\lambda p'(e) + \nu = 0\\
&\mu: R = p(e)R_g^d + (1-p(e))(1+\tau)R_b^d\\
&\eta: e = \lambda p'(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right]\\
&\nu: R_b^d d - R^b(N+d) = 0.
\end{aligned}$$

Here, we assume $\nu \neq 0$, so that the cash constraint is binding.

We eliminate the multipliers, μ and η, and two equations from this system. Add the R_g^d equation to the R_b^d equation and solve for μ :

$$\mu = \frac{\lambda - \nu}{1 + (1-p(e))\tau}$$

>From the R_g^d equation:

$$\begin{aligned}
\eta\lambda b &= \left[\lambda - \frac{\lambda - \nu}{1 + (1-p(e))\tau}\right]p(e)\\
&= \tilde{\nu}p(e),
\end{aligned}$$

where

$$\tilde{\nu} \equiv \frac{\nu + (1 - p(e)) \tau \lambda}{1 + (1 - p(e)) \tau}. \tag{A.8}$$

Substitute the expressions for μ and $\eta\lambda b$, as well as the μ equation, into the d equation:

$$d : 0 = \lambda p(e) \left[R^g - R_g^d\right] + \lambda (1 - p(e)) \left[R^b - R_b^d\right] - \tilde{\nu} p(e) \left[\left(R^g - R^b\right) - \left(R_g^d - R_b^d\right)\right] + \nu \left(R_b^d - R^b\right)$$

or, after rearranging,

$$d : 0 = (\lambda - \tilde{\nu}) \left[p(e) \left(R^g - R_g^d\right) + (1 - p(e)) \left(R^b - R_b^d\right)\right] + (\nu - \tilde{\nu}) \left(R_b^d - R^b\right).$$

Note that when $\tau = 0$, then $\tilde{\nu} = \nu$ and this equation together with the μ equation in (4.21) reduces to d in (4.21).[48]

We must adjust the equilibrium conditions of the household to accommodate the taxes required to finance the bank bailouts. The profits, π, brought home by the bankers in the representative household in period 2 are:

$$\pi = p(e) \left[R^g (N + d) - R_g^d d\right] + (1 - p(e)) \left[R^b (N + d) - R_b^d d\right].$$

The representative household's second period budget constraint is:

$$C = Rd + \pi - T,$$

where T denotes the lump sum taxes required to finance τ :

$$T = (1 - p(e)) \tau R_b^d d.$$

Substituting out for T and π in the second period budget constraint,

$$\begin{aligned}
C &= Rd + p(e) \left[R^g (N + d) - R_g^d d\right] + (1 - p(e)) \left[R^b (N + d) - R_b^d d\right] - (1 - p(e)) \tau R_b^d d \\
&= Rd + p(e) R^g (N + d) + (1 - p(e)) R^b (N + d) - \left[p(e) R_g^d d + (1 - p(e)) R_b^d d\right] - (1 - p(e)) \tau R_b^d d \\
&= Rd + p(e) R^g (N + d) + (1 - p(e)) R^b (N + d) \\
&- \left[p(e) R_g^d d + (1 + \tau)(1 - p(e)) R_b^d d - (1 - p(e)) \tau R_b^d d\right] - (1 - p(e)) \tau R_b^d d \\
&= Rd + p(e) R^g (N + d) + (1 - p(e)) R^b (N + d) - Rd
\end{aligned}$$

where the fourth equality makes use of the zero profit condition of mutual funds. Then,

$$C = \left[p(e) R^g + (1 - p(e)) R^b\right] (N + d). \tag{A.9}$$

>From the household intertemporal Euler equation, $c^{-\gamma} = \beta R C^{-\gamma}$, we have

$$C = c (\beta R)^{\frac{1}{\gamma}}.$$

Substitute d out using the first and second period budget constraints:

$$c + \frac{C}{p(e) R^g + (1 - p(e)) R^b} = N + y$$

[48] Recall, $\lambda - \nu > 0$ because $\lambda, -\nu > 0$.

Use the intertemporal first order condition to substitute out for C, so that the equilibrium conditions for the household are:

$$c = \frac{N + y}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{p(e)R^g + (1-p(e))R^b}}, \; d = y - c.$$

We now collect the equilibrium conditions. Replace the d equation in (A.7), and make use of the expressions for η and μ to obtain the following system:

$$\begin{aligned}
e &: \frac{\lambda - \nu}{1 + (1 - p(e))\tau} b \left(R_g^d - (1+\tau) R_b^d \right) d + \frac{\tilde{\nu}}{\lambda b} p(e) = 0 \\
d &: 0 = (\lambda - \tilde{\nu}) \left[p(e) \left(R^g - R_g^d \right) + (1 - p(e)) \left(R^b - R_b^d \right) \right] + (\nu - \tilde{\nu}) \left(R_b^d - R^b \right) \\
\mu &: R = p(e) R_g^d + (1 - p(e)) (1 + \tau) R_b^d \\
\eta &: e = \lambda b \left[\left(R^g - R^b \right) (N + d) - \left(R_g^d - R_b^d \right) d \right] \\
\nu &: R_b^d d - R^b (N + d) = 0 \\
c &= \frac{N + y}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{p(e)R^g + (1-p(e))R^b}} \\
d &= y - c \\
c^{-\gamma} &= \beta R C^{-\gamma}
\end{aligned} \tag{A.10}$$

where the last three equations in (A.10) are the equilibrium conditions associated with the household. It is understood that λ is determined according to (4.17) and $\tilde{\nu}$ according to (A.8). In addition, the functional form for $p(e)$ has been used. The expressions in (A.10) represents 8 equations in 8 unknowns:

$$\nu, e, R_g^d, R_b^d, d, c, C, R.$$

These equations reduce to (4.21) when $\tau = 0$ in the case $\nu \neq 0$. We solve the above equations by solving one equation in R.

Fix a value for R. We now defined a mapping from e into itself. Fix a value for e. Use the last three equations in (A.10) to solve for c, C, d. Then, use the ν equation to solve for R_b^d :

$$R_b^d = R^b \left(\frac{N + d}{d} \right).$$

Use the zero profit condition for mutual funds to solve for R_g^d :

$$R_g^d = \frac{R - (1 - p(e)) (1 + \tau) R_b^d}{p(e)}.$$

Finally, use the η equation to solve for e :

$$\eta : e = \lambda b \left[\left(R^g - R^b \right) (N + d) - \left(R_g^d - R_b^d \right) d \right]$$

Adjust the value of e until a fixed point is obtained.

We now have $d, c, C, R, R_b^d, R_g^d, e$ in hand. It remains to determine a value for ν. Substituting out for $\tilde{\nu}$ in the e equation, and multiplying by $1+(1-p(e))\tau$:

$$(\lambda-\nu)\, b\left(R_g^d-(1+\tau) R_b^d\right) d+\frac{1}{\lambda b}[\nu+(1-p(e)) \tau \lambda]\, p(e)=0.$$

Note:

$$\begin{aligned}
&(\lambda-\nu)\, b\left(R_g^d-(1+\tau) R_b^d\right) d+\frac{1}{\lambda b}[\nu+(1-p(e)) \tau \lambda]\, p(e) \\
&=\lambda b\left(R_g^d-(1+\tau) R_b^d\right) d-\nu b\left(R_g^d-(1+\tau) R_b^d\right) d \\
&+\frac{1}{\lambda b} \nu p(e)+\frac{1}{b}(1-p(e))\, p(e)\, \tau \\
&=\nu\left[\frac{1}{\lambda b} p(e)-b\left(R_g^d-(1+\tau) R_b^d\right) d\right]+\lambda b\left(R_g^d-(1+\tau) R_b^d\right) d+\frac{1}{b}(1-p(e))\, p(e)\, \tau .
\end{aligned}$$

Then,

$$\nu=\frac{\lambda b\left(R_g^d-(1+\tau) R_b^d\right) d+\frac{1}{b}(1-p(e))\, p(e)\, \tau}{b\left(R_g^d-(1+\tau) R_b^d\right) d-\frac{1}{\lambda b} p(e)}.$$

Finally, adjust the value of R until the d equation is satisfied.

We now turn to the model considered in (4.5.5). Relative to the solution to (4.32) that we just analyzed, the solution to (4.33) involves only replacing the zero in the d equation in (A.10) with δ and adding $\bar{L}N=(N+d)$ as an additional equation. We now have 9 equations in 9 unknowns. We adapt the strategy just described to solve this model. We solve two equations in two unknowns, R, e. Fix values for R, e and solve for $d, c, C, R_b^d, \nu, R_g^d$ in the same way as before. These calculations do not involve the d equation. Then, solve for δ using the modified d equation:

$$\delta=(\lambda-\tilde{\nu})\left[p(e)\left(R^g-R_g^d\right)+(1-p(e))\left(R^b-R_b^d\right)\right]+(\nu-\tilde{\nu})\left(R_b^d-R^b\right).$$

Finally, adjust R and e until the η equation in (A.10) and $\bar{L}N=(N+d)$ are satisfied. In effect, this solution strategy replaces the d equation with the leverage constraint as an equality.

B. Notes on the Adverse Selection Model

In this appendix we prove Proposition 5.3, which underlies the reason for the assumption in section 5 that there is an upper bound on the scale of investment projects. Proposition 5.3 addresses what happens if we instead adopt an assumption at the opposite extreme, that investment projects have constant returns to scale without any upper bound. We show that if there is an equilibrium in this version of the model, then it must be that all borrowing is done by the entrepreneurs with the lowest value of p and the aggregate profits of those entrepreneurs are zero. Thus in this equilibrium the household receives the whole marginal product of its saving and the first best efficient allocations are supported. For equilibrium to be well defined in this case we obviously require that there be positive mass on the lower bound of the support for p, and that that lower bound be positive. Thus, we suppose that $p \in [\varepsilon, 1]$, where ε is a very small, positive number.

Suppose an entrepreneur with a particular $p^* \in [\varepsilon, 1]$ chooses to activate its project, and let B_{p^*} denote the amount borrowed by that entrepreneur. Such an entrepreneur's profits must be no less than what it can earn by simply depositing its net worth in the bank and not borrowing anything.[49] That is, the analog of (5.5) must hold:

$$\left(\bar{\theta} - p^* r\right) B_{p^*} + N\left(\bar{\theta} - R\right) \geq 0. \tag{B.1}$$

For each $p \leq p^*$ there exists a B_p such that (B.1) also holds, so that entrepreneurs with $p \leq p^*$ also choose to activate their projects. In equilibrium, it must be that

$$\bar{\theta} \leq pr, \ p \leq p^*, \tag{B.2}$$

for otherwise there is no choice of B_p that optimizes expected entrepreneurial profits.

For an interior equilibrium in which there is a positive supply of deposits to banks, it must be that $B_p > 0$ for some p. Entrepreneurs with lower probabilities of success also borrow positive amounts, and we conclude that in an interior equilibrium there exists a $p^u \in [\varepsilon, 1]$ such that entpreneurs with $\varepsilon \leq p \leq p^u$ choose $B_p > 0$. For p in this interval, it must be that

$$\bar{\theta} \geq pr, \ p \leq p^u, \tag{B.3}$$

for otherwise the supposition, $B_p > 0$, is contradicted. Let $p^+ \equiv \min(p^u, p^*)$. Combining (B.2) and (B.3), we conclude

$$\bar{\theta} = pr, \text{ for } \varepsilon \leq p \leq p^+.$$

But, this expression can only hold if $p^+ = \varepsilon$. We conclude that in an interior equilibrium only the entrepreneurs with the lowest probability of success operate their projects. Each of these entrepreneurs pay $r = \bar{\theta}/\varepsilon$ in interest and earn zero profits ex ante. Because banks are competitive and so make zero profits, $R = \bar{\theta}$. That is, in equilibrium households receive the actual social marginal return on loans. As a result, the allocations in equilibrium coincide with the first-best efficient allocations. This establishes Proposition 5.3.

C. Notes on the Asymmetric Information Model

C.1. Proof that the Marginal Return Exceeds the Average Return on Loans

For convenience, we repeat the expression for the marginal return on loans, the object to the right of the equality in (6.20), here:

$$R^m \equiv r^k \left\{1 - \mu\left[G\left(\bar{\omega}(B)\right) + (N + B) G'\left(\bar{\omega}(B)\right)\bar{\omega}'(B)\right]\right\}. \tag{C.1}$$

We wish to establish $R^m > R$. Here, R is the equilibrium cost of funds to banks, which we showed is also equal to the average return on bank loans (see (6.8)):

$$R = \left[\Gamma\left(\bar{\omega}(B)\right) - \mu G\left(\bar{\omega}(B)\right)\right] \frac{r^k (N + B)}{B}. \tag{C.2}$$

[49] We implicitly ignore another option for the entrepreneur: deposit N in the bank and then borrow from the bank and invest in the project. A property of equilibrium is that $R \leq \bar{\theta}$, so that no entrepreneur would ever choose this option.

Thus, we establish that (under a regularity condition stated below) the marginal return on loans exceeds the corresponding average return.

In (C.2) and (C.1), $\bar{\omega}(B)$ is defined by (6.18) and (6.19). We reproduce these expressions here (after substituting out for L in (6.18) using (6.19)) for convenience:

$$\frac{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})} = \frac{N}{B} \frac{\Gamma'(\bar{\omega})}{1 - \Gamma(\bar{\omega})}. \tag{C.3}$$

The mapping, $\bar{\omega}(B)$, characterizes how $\bar{\omega}$ changes with a change in B, given the zero profit condition of banks, (6.9) and the efficiency condition characterizing the solution to the entrepreneurial contracting problem, (6.10). For our analysis, we require the derivative of $\bar{\omega}(B)$ with respect to B :

$$\bar{\omega}'(B) = \frac{(1 - \Gamma)(\Gamma' - \mu G')}{\Gamma'(\Gamma' - \mu G')(N + B) + \Gamma''(\Gamma - \mu G)N - (1 - \Gamma)(\Gamma'' - \mu G'')B}, \tag{C.4}$$

where we have omitted the argument, $\bar{\omega}$, in Γ, G, Γ', G' for notational simplicity.

The loan contract between entrepreneurs and banks solves the following optimization problem:

$$\max_{\bar{\omega}, B} r^k (N + B)[1 - \Gamma(\bar{\omega})], \tag{C.5}$$

subject to

$$r^k (N + B) \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] - RB = 0. \tag{C.6}$$

Letting $\lambda \geq 0$ denote the Lagrange multiplier associated with constraint (C.6), the first order conditions of the problem are:

$$r^k[1 - \Gamma(\bar{\omega})] + \lambda \left\{ r^k[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] - R \right\} = 0, \tag{C.7}$$

$$\lambda = \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}. \tag{C.8}$$

It is easy to verify that

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0, \; G'(\bar{\omega}) = \bar{\omega} f(\bar{\omega}) > 0, \tag{C.9}$$

where $f(\bar{\omega}) \equiv dF(\bar{\omega})/d\bar{\omega}$. Conditions (C.8), (C.9) and $\lambda \geq 0$ imply[50]

$$\lambda > 1. \tag{C.10}$$

Solving (C.7) for λ and multiplying the numerator and denominator of (C.8) by $r^k(N + B)\bar{\omega}'(B)$, we obtain:

$$\lambda = \frac{r^k[1 - \Gamma(\bar{\omega})]}{R - r^k[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}, \tag{C.11}$$

$$\lambda = \frac{r^k(N + B)\Gamma'(\bar{\omega})\bar{\omega}'(B)}{r^k(N + B)[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]\bar{\omega}'(B)}, \tag{C.12}$$

[50] To see that $\lambda \geq 0$, suppose on the contrary that $\lambda < 0$. Note that $\Gamma(0) = G(0) = 0$, so that the solution to the Lagrangian representation of the problem solved by the loan contract is $B = \infty$ and $\bar{\omega} = 0$. This does not solve the problem of maximizing (C.5) subject to (C.6) because (C.6) is violated.

where $\bar{\omega}'(B)$ is defined in (C.4).

Combining (C.11) and (C.12), we obtain the following expression for λ:

$$\lambda = \frac{r^k[1-\Gamma] - r^k(N+B)\Gamma'\bar{\omega}'(B)}{R - r^k[\Gamma - \mu G] - r^k(N+B)[\Gamma' - \mu G']\bar{\omega}'(B)}. \tag{C.13}$$

The numerator of (C.13) consists of the numerator of (C.11) minus the numerator of (C.12). Similarly, the denominator of (C.13) consists of the denominator of (C.11) minus the denominator of (C.12).[51] Now we rewrite the marginal return, (C.1), making use of the expression for λ in (C.13):

$$\begin{aligned} R^m =& r^k\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) + 1 - \Gamma(\bar{\omega})\right] - r^k(N+B)\left[\Gamma'(\bar{\omega}) + \mu G'(\bar{\omega}) - \Gamma'(\bar{\omega})\right]\bar{\omega}'(B), \\ =& \left\{r^k\left[1-\Gamma(\bar{\omega})\right] - r^k(N+B)\Gamma'(\bar{\omega})\bar{\omega}'(B)\right\} \\ & - \left\{R - r^k\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] - r^k(N+B)\left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right]\bar{\omega}'(B)\right\} + R, \\ =& R + (\lambda - 1)\left\{R - r^k\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] - r^k(N+B)\left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right]\bar{\omega}'(B)\right\}, \end{aligned} \tag{C.14}$$

where R denotes the average return, (C.2). From (C.10), $\lambda > 1$. Hence, if the object in braces in (C.14) is positive then $R^m > R$. The object in braces in (C.14) is the denominator of λ in (C.13). Because $\lambda > 0$, we know that the denominator is positive if the numerator of λ is positive. Using the expression for $\bar{\omega}'(B)$, (C.4), we rewrite the numerator as follows:

$$\begin{aligned} & r^k[1-\Gamma] - r^k(N+B)\Gamma'\bar{\omega}'(I) \\ =& r^k[1-\Gamma] - \frac{r^k(N+B)\Gamma'(\bar{\omega})(1-\Gamma)(\Gamma' - \mu G')}{\Gamma'(\Gamma' - \mu G')(N+B) + \Gamma''(\Gamma - \mu G)N - (1-\Gamma)(\Gamma'' - \mu G'')B} \\ =& r^k[1-\Gamma]\left\{1 - \frac{1}{1 + \frac{\Gamma''(\Gamma - \mu G)N - (1-\Gamma)(\Gamma'' - \mu G'')B}{(N+B)\Gamma'(\bar{\omega})(\Gamma' - \mu G')}}\right\}. \end{aligned}$$

This object is positive if

$$\frac{\Gamma''(\Gamma - \mu G)N - (1-\Gamma)(\Gamma'' - \mu G'')B}{(N+B)\Gamma'(\Gamma' - \mu G')}, \tag{C.15}$$

is positive. A sufficient condition for (C.15) to be positive is that $\bar{\omega}h(\bar{\omega})$ is increasing in $\bar{\omega}$, where $h(\bar{\omega})$ denotes the hazard rate (see (6.22)). According to BGG, this implies that (i) $\Gamma' - \mu G' > 0$ in equilibrium, and (ii) $\Gamma' G'' - \Gamma'' G' > 0$ for all $\bar{\omega}$.[52] Condition (i) implies that the denominator of (C.15) is positive. The following result shows that (ii) implies the

[51] We have used the following result. Suppose

$$\lambda = \frac{A}{B} = \frac{C}{D}.$$

Then, $\lambda = (A+C)/(B+D)$.

[52] Condition (i) reflects a combination of the result at the top of page 1382 in BGG, as well as the observation at the top of page 1385. Condition (ii) is established on page 1382 in BGG.

numerator of (C.15) is positive:

$$\begin{aligned}
&\Gamma''(\Gamma-\mu G)N-(1-\Gamma)(\Gamma''-\mu G'')B\\
&=\Gamma''(\Gamma-\mu G)\frac{(1-\Gamma)(\Gamma'-\mu G')}{\Gamma'(\Gamma-\mu G)}B-(1-\Gamma)(\Gamma''-\mu G'')B\\
&=\left[\Gamma''(\Gamma'-\mu G')-\Gamma'(\Gamma''-\mu G'')\right]\frac{1-\Gamma}{\Gamma'}B\\
&=\mu\left[\Gamma' G''-G'\Gamma''\right]\frac{1-\Gamma}{\Gamma'}B>0.
\end{aligned}$$

Here, the first equality uses (C.3) to substitute out for N. This completes the proof of the proposition that the marginal return on B exceeds the corresponding average return. We state this proposition formally as follows:

Proposition C.1. *Suppose that $\bar{\omega}h(\bar{\omega})$ is increasing in $\bar{\omega}$. Then $R^m > R$.*

C.2. Model with Curvature in the Production of Capital

Here, we introduce the modifications to the model which cause the price of capital to be endogenous and possibly be the source of a pecuniary externality. We introduce a representative, competitive firm that produces capital. Rather than building its own capital (as we assume in the main text) the entrepreneur uses its net worth and bank loans to purchase capital from the representative capital producer. The following subsection describes the problem of that firm. We then derive all the model equilibrium conditions. The model is virtually identical to the one in the main text. Still, there are some differences in notation and so we spell out all the details at the risk of some overlap. After that, we describe the algorithm used to compute the equilibrium. This algorithm is the basis for the calculations discussed in the text. Finally, the last subsection describes the proposition that establishes that a sufficiently large net worth transfer to entrepreneurs supports the first-best allocations.

C.2.1. Capital Producers

At the start of period 1, each entrepreneur is endowed with an equal quantity, k, of capital goods. Each entrepreneur sells its capital to capital producers and receives $N = P_k k$, where P_k is the market price of capital in terms of the period 1 numeraire good, consumption. Here, N denotes an entrepreneur's net worth after selling its capital. In the main text, $P_k = 1$ always, and so $N = k$ there. The fact that N is a function of the market price, P_k, creates the potential for a pecuniary externality in this model.

A perfectly competitive, representative capital producer operates the following production function:

$$K = k + f\left(\frac{I}{k}\right)k,$$

where

$$f\left(\frac{I}{k}\right) = \left(\frac{I}{k}\right)^{\gamma},\ 0 < \gamma < 1.$$

Here, I denotes a quantity of investment goods, measured in units of the period 1 numeraire good and K denotes the quantity of new capital produced by the capital producer. Profits of the capital producer are given by:

$$P_K K - I - P_k k. \tag{C.16}$$

Here, P_K denotes the market price of new capital, in units of the period 1 numeraire good. The representative capital producer takes prices, P_K, P_k, as given. In an interior equilibrium, profit maximization leads to the following first order condition for k :

$$P_K \left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k} \right] = P_k.$$

The first order condition for I is:

$$P_K f'\left(\frac{I}{k}\right) = 1.$$

Combining the two first order conditions with the production function implies the capital producer's profits, (C.16), are zero.

C.2.2. Entrepreneurs and Banks

The typical entrepreneur take its net worth, N, and approaches a bank for a loan, B. It combines its net worth and the loan to purchase new capital:

$$P_K K = N + B.$$

In period 2 the entrepreneur uses its capital, K, to produce

$$\omega K r^k$$

goods, where r^k is a fixed parameter of technology. In addition, ω is an idiosyncratic productivity shock,

$$\omega \sim F, \quad \int_0^\infty dF(\omega) = 1,$$

where F is the cdf of a log-normal distribution.

A representative bank offers the entrepreneur a standard debt contract in period 1, before the realization of ω. Under the contract, the entrepreneur pays the bank an amount, ZB, in period 2 if it is able to do so. Entrepreneurs whose ω is too low to pay ZB in full are 'bankrupt'. Banks verify this by monitoring those entrepreneurs, at a cost of

$$\mu\omega K r^k$$

goods, where $\mu > 0$ is a parameter. Bankrupt entrepreneurs must transfer everything they have to their bank.

The cutoff level of productivity, $\bar{\omega}$, that separates the bankrupt and non-bankrupt entrepreneurs is defined by:

$$\bar{\omega} K r^k = ZB. \tag{C.17}$$

$>$From the perspective of period 1, an individual entrepreneur's expected profits in period 2 are given by:

$$\int_{\bar{\omega}}^{\infty} \left[\omega K r^k - ZB\right] dF(\omega) = \int_{\bar{\omega}}^{\infty} \left[\omega K r^k - \bar{\omega} K r^k\right] dF(\omega) = NR^k L \left(\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] \, dF(\omega) \right), \tag{C.18}$$

using (C.17). In (C.18), L denotes leverage (see (6.4)) and R^k denotes the rate of return on capital:

$$R^k \equiv \frac{r^k}{P_K}.$$

The expression, (C.18), is written in compact notation as follows:

$$NR^k L \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] \, dF(\omega) = NLR^k \left(1 - \Gamma(\bar{\omega})\right), \tag{C.19}$$

where

$$\Gamma(\bar{\omega}) \equiv G(\bar{\omega}) + \bar{\omega}\left[1 - F(\bar{\omega})\right], \; G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega). \tag{C.20}$$

An entrepreneur agrees to maximize (C.19) and remit all its profits to its household in period 2 in exchange for perfect consumption insurance. With entrepreneurs maximizing (C.19), the household ensures that its entrepreneurs as a group maximize the total resources available to the household in period 2. Households can observe everything about their own member entrepreneurs (including ω).

We now turn to the banks. There is a large number of competitive banks, each of which makes loans to entrepreneurs and takes deposits from households. Because there is no risk on the asset side of the balance sheet, it is feasible for banks to commit in period 1 to paying households a fixed and certain gross rate of interest, R, on their deposits in period 2. Because banks are competitive, they take R as given. A bank that makes size B loans to each of a large number of entrepreneurs earns the following per entrepreneur:

$$\left[1 - F(\bar{\omega})\right] ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) \, r^k K.$$

Here, the term before the plus sign indicates the revenues from entrepreneurs that are not bankrupt, i.e., those with $\omega \geq \bar{\omega}$. The term after the plus sign indicates receipts, net of bank monitoring costs, from entrepreneurs that cannot pay interest, Z. Since the cost of funds is RB, the bank's zero profit condition is (using (C.17)):

$$\left[1 - F(\bar{\omega})\right] \bar{\omega} K r^k + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) \, r^k K = RB,$$

or,

$$\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \frac{R^k P_K K}{B} = R. \tag{C.21}$$

As in the main text, the interest rate paid to households is proportional to the average return, $R^k P_K K / B$, on loans and not, say, to the marginal return.

Let the combinations of $\bar{\omega}$ and B that satisfy (C.21) define a 'menu' of loan contracts that is available to entrepreneurs, for given P_K and R^k.[53] It is convenient to express this menu in terms of L and $\bar{\omega}$. Rewriting (C.21):

$$L = \frac{1}{1 - \frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}. \tag{C.22}$$

Entrepreneurs take N and R^k as given and select the contract, $(L, \bar{\omega})$, from this menu which maximizes expected profits, (C.19). Using (C.22) to substitute out for L in the entrepreneur's objective, the problem reduces to one of choosing $\bar{\omega}$ to maximize:

$$NR^k \frac{1 - \Gamma\left(\bar{\omega}\right)}{1 - \frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}.$$

The first order necessary condition for optimization is:

$$\frac{1 - F\left(\bar{\omega}\right)}{1 - \Gamma\left(\bar{\omega}\right)} = \frac{\frac{R^k}{R}\left[1 - F\left(\bar{\omega}\right) - \mu \bar{\omega} F'\left(\bar{\omega}\right)\right]}{1 - \frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]},$$

which can be solved for $\bar{\omega}$ given R^k and R. Given the solution for $\bar{\omega}$, L and Z solve:

$$L = \frac{1}{1 - \frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}, \quad Z = R^k \bar{\omega} \frac{L}{L-1},$$

respectively. As in the text, L and Z are independent of net worth, N.

C.2.3. Households and Government

In period 1 the household budget constraint is:

$$c + B \leq y, \tag{C.23}$$

where c, B, y denote consumption, bank deposits and an endowment of output, y. In the second period, deposits generate an after tax return,

$$\left(1 + \tau\right) RB = \left(1 + \tau\right) r^k K \left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right],$$

where τ denotes a subsidy on household saving and (C.21) has been used. Total profits brought home by a household's entrepreneurs are denoted by π :

$$\pi = K r^k \left(1 - \Gamma\left(\bar{\omega}\right)\right).$$

Taking into account that banks have zero profits, the second period budget constraint is:

$$C \leq \left(1 + \tau\right) RB + \pi - T, \tag{C.24}$$

[53] Note that a collection, $(\bar{\omega}, B)$, is equivalent to a collection, (Z, B) by $\bar{\omega} = ZB / \left[\left(N + B\right) R^k\right]$ and the assumption that entrepreneurs view N and R^k as parametric.

where T denotes lump sum taxes and C denotes second period consumption. Substituting,

$$C \leq (1+\tau) r^k K [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] + K r^k (1 - \Gamma(\bar{\omega})) - T.$$

The government's budget constraint is:

$$T = \tau R B = \tau r^k K [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]. \tag{C.25}$$

If we combine the government's budget constraint with the household's second period budget constraint, we obtain the second period resource constraint:

$$C \leq r^k K [1 - \mu G(\bar{\omega})], \tag{C.26}$$

which is Walras' law in our environment. That is, period 2 consumption is no greater than total output, net of the output used up in monitoring by banks.

Households maximize

$$u(c) + \beta u(C)$$

subject to (C.23) and (C.24). The first order necessary and sufficient conditions corresponding to this problem are:

$$\frac{u'(c)}{\beta u'(C)} = (1+\tau) R, \; c + \frac{C}{(1+\tau) R} = y + \frac{\pi - T}{(1+\tau) R}.$$

In practice, we assume,

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}.$$

C.2.4. Equilibrium

We define an equilibrium as follows:

Definition C.2. *A private sector equilibrium is a* $(C, c, R, K, \bar{\omega}, B, P_k, P_K, I, T)$ *such that*

(i) The household problem is solved for given τ

(ii) The problem of the entrepreneur is solved

(iii) Bank profits are zero

(iv) The problem of the capital producer is solved (see section C.2.1)

(v) The government budget constraint is satisfied

(vi) The first and second period resource constraints are satisfied.

For convenience, we collect the equations that characterize a private sector equilibrium here. We divide these equilibrium conditions into a household block, and an entrepreneur/bank/capital producer block. The first block is:

Equation number	Household	Economic description
(1)	$C = c\left(\beta [1+\tau] R\right)^{\frac{1}{\alpha}}$	household first order condition
(2)	$C = r^k K [1 - \mu G(\bar{\omega})]$	period 2 resource constraint
(3)	$c + I = y$	period 1 resource constraint

Expression (3) is the household's first period budget constraint, with B replaced with I. This replacement is possible for the following reason. Total entrepreneurial assets, $P_K K$, can be written as follows:

$$P_K K = N + B = P_k k + B.$$

Here, the first equality is the entrepreneur's budget constraint and the second equality uses the definition, $N = P_k k$. Zero profits for the capital producers (see section C.2.1) implies $P_K K = I + P_k k$, and the fact,

$$B = I, \tag{C.27}$$

follows.

The impact of the household and government budget constraints is completely captured by the period 1 and period 2 resource constraints and expression (1), and so the budget constraints are not included among the equilibrium conditions associated with the household.

The set of equilibrium conditions associated with entrepreneurs, banks and capital producers is:

Equation number	Efficiency conditions for firms	Economic description of condition
(4)	$R^k = r^k / P_K$	rate of return on capital
(5)	$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{R^k}{R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	contract efficiency condition
(6)	$\frac{P_K K}{P_k k} = \frac{1}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	bank zero profit condition
(7)	$P_k = P_K \left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right]$	efficiency condition of capital producers
(8)	$1 = P_K f'\left(\frac{I}{k}\right)$	optimality of I choice by capital producers
(9)	$K = \left[1 + f\left(\frac{I}{k}\right)\right] k$	capital accumulation technology

Equations (1)-(9) represents 9 equations in 9 private sector equilibrium objects:

$$C, c, R, R^k, K, \bar{\omega}, P_k, P_K, I.$$

The equilibrium value of T can be backed out by imposing either the household or government budget constraint. Note too that the equilibrium rate of interest on entrepreneurs, Z, is determined from (6.3) and the facts, (C.27), $N = P_k k$.

It is of interest to show that the equilibrium allocations are first-best when $\mu = 0$. In this case, equation (5) can only be satisfied with $R = R^k$ and $\bar{\omega}$ disappears from that equation. Combining equations (6), (7) and (9):

$$\frac{1}{1-\Gamma(\bar{\omega})} = \frac{1 + \left(\frac{I}{k}\right)^\gamma}{1 + (1-\gamma)\left(\frac{I}{k}\right)^\gamma}.$$

We can think of this equation as defining $\bar{\omega}$ and, hence, Z (i.e., the spread). But, this variable does not enter the other equations, and so it plays no role in determining the quantity allocations. The other equations are (1), (2) and (3). Expressing these with $\mu = 0$, and using (8), (9):

$$\text{(1)'} \quad \frac{1}{\beta}\left(\frac{C}{c}\right)^\alpha = (1+\tau)\, r^k \gamma \left(\frac{I}{k}\right)^{\gamma-1}$$

$$\text{(2)'} \quad C = r^k k \left[1 + \left(\frac{I}{k}\right)^\gamma\right]$$

$$\text{(3)} \quad c + I = y \ ,$$

where a prime indicates that the equation has been adjusted using (8) or (9). The above system represents three equations in three unknowns, c, C, I. Substitute out I using (3):

$$\begin{array}{ll} (1)\text{''} & \frac{1}{\beta}\left(\frac{C}{c}\right)^{\alpha} = (1+\tau)\, r^k \gamma \left(\frac{y-c}{k}\right)^{\gamma-1} \\ (2)\text{''} & C = r^k k \left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right] \;. \end{array}$$

In sum, the system can be solved as follows. First, solve (1)" and (2)" for C and c. Then, $I = y - c$ and P_K can be computed from (8). Finally, (4) and (5) imply

$$R = R^k = \frac{r^k}{P_K}.$$

We define the first-best problem as the problem for a planner who observes the entrepreneurs' ω realizations. Such a planner obviously does not have to pay monitoring costs. The problem of this planner is:

$$\begin{gathered} \max u(c) + \beta u(C) \\ C = r^k k \left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right]. \end{gathered} \tag{C.28}$$

Expressing this in Lagrangian form:

$$\max u(c) + \beta u(C) + \lambda \left(r^k k \left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right] - C \right)$$

The first order conditions are:

$$\begin{aligned} u'(c) &= \lambda r^k \gamma \left(\frac{y-c}{k}\right)^{\gamma-1} \\ \beta u'(C) &= \lambda \end{aligned}$$

so that the necessary and sufficient conditions for optimization reduce to:

$$\begin{aligned} \frac{u'(c)}{\beta u'(C)} &= \overbrace{r^k \gamma \left(\frac{y-c}{k}\right)^{\gamma-1}}^{\text{marginal return on investment}} \\ C &= r^k k \left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right] \end{aligned}$$

Evidently, these equations coincide with (1)"-(2)" when $\tau = 0$. We conclude that the equilibrium supports the first-best consumption and investment allocations. We summarize our findings as follows:

Proposition C.3. *Suppose* $0 < \gamma \leq 1$. *When monitoring costs are zero, then the equilibrium consumption and investment allocations coincide with the solution to the first-best problem.*

C.2.5. Computation of Private Sector Equilibrium

Here, we describe an algorithm for computing the private sector equilibrium conditional on an arbitrary value of τ. Unlike in the previous subsection, in this subsection we take the model parameters as given and compute values for the 9 model endogenous variables that satisfy the 9 model equations, (1)-(9) in section C.2.4. Our strategy is to find $\tilde{R} \equiv RP_K$ such that $g\left(\tilde{R}\right) = 0$. To define the mapping, g, from $\tilde{R}$ into the real line, fix a value for $\tilde{R}$. Solve for $\bar{\omega}$ using (5) and

$$\frac{R^k}{R} = \frac{r^k}{RP_K} = \frac{r^k}{\tilde{R}}.$$

Next, compute the object on the right side of (6), and call the result, X. Then, (6) is written:

$$\frac{P_K K}{P_k k} = \frac{1 + f\left(\frac{I}{k}\right)}{1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}} = \frac{1 + \left(\frac{I}{k}\right)^{\gamma}}{1 + (1-\gamma)\left(\frac{I}{k}\right)^{\gamma}} = X.$$

Rewriting this expression,

$$\left(\frac{I}{k}\right)^{\gamma} = \frac{X - 1}{1 - X(1-\gamma)},$$

and solve for I/k. Then, solve (9) for K/k, (8) for P_K and (7) for P_k. Finally,

$$R = \frac{\tilde{R}}{P_K}.$$

Combine (1), (2) and (3):

$$\begin{aligned} c\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k K\left[1 - \mu G\left(\bar{\omega}\right)\right] \\ \left[y - I\right]\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k K\left[1 - \mu G\left(\bar{\omega}\right)\right] \\ \left[\frac{y}{k} - \frac{I}{k}\right]\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k \frac{K}{k}\left[1 - \mu G\left(\bar{\omega}\right)\right]. \end{aligned}$$

Solve the latter for y/k :

$$\frac{y}{k} = \frac{\frac{I}{k}\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}} + r^k \frac{K}{k}\left[1 - \mu G\left(\bar{\omega}\right)\right]}{\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}}}.$$

Solve (2) and (3) for c, C. In this way, we define mappings, $c\left(\tilde{R}\right), C\left(\tilde{R}\right)$. Then, let

$$g\left(\tilde{R}\right) \equiv C\left(\tilde{R}\right) - c\left(\tilde{R}\right)\left(\beta\left[1+\tau\right]R\right)^{\frac{1}{\alpha}}.$$

Adjust $\tilde{R}$ until $g\left(\tilde{R}\right) = 0$, i.e., (1) is satisfied.

C.2.6. Net Worth Transfers to Entrepreneurs

Here, we consider a policy in which the government raises taxes in the first period and then simply gives the banks the proceeds as a subsidy. With this policy, the government in effect can cause the economy to fully circumvent the financial frictions and move to the first best allocations.

Suppose the government raises $T > 0$ in the first period and transfers the proceeds as a lump sum gift to entrepreneurs. Let I^* denote the first best level of investment (see (C.28)). The optimal policy is to set $T = I^*$. In this case, the entrepreneurs have enough net worth that they do not need to borrow from banks. In this way there are no monitoring costs.

It is easy to verify that with one exception the equilibrium conditions of the model considered here coincide with equations (1)-(9) in section C.2.4. The only change is that the object on the left of the equality in (6) is replaced by:

$$\frac{P_K K}{P_k k + T}.$$

This is the leverage ratio of the entrepreneurs and the above expression reflects that with the policy considered here, entrepreneurs' net worth consists of the value of the capital they are endowed with, plus the tax transfer received from the government. If we set $T = I$, then leverage is unity by the zero profit condition on capital producers, (C.16). But, if the left side of (6) is unity, the right side must be too. This can be accomplished by $\bar{\omega} = 0$, since in this case $\Gamma(\bar{\omega}) = F(\bar{\omega}) = G(\bar{\omega}) = 0$ (see (6.7)). Equation (5) then requires $R^k/R = 1$. The remaining equations coincide with the equations of the first best equilibrium, and so these can be satisfied by setting $T = I^*$. We summarize these results as follows:

Proposition C.4. *A policy of lump-sum tax financed transfers of net worth to entrepreneurs can achieve the first-best allocations.*

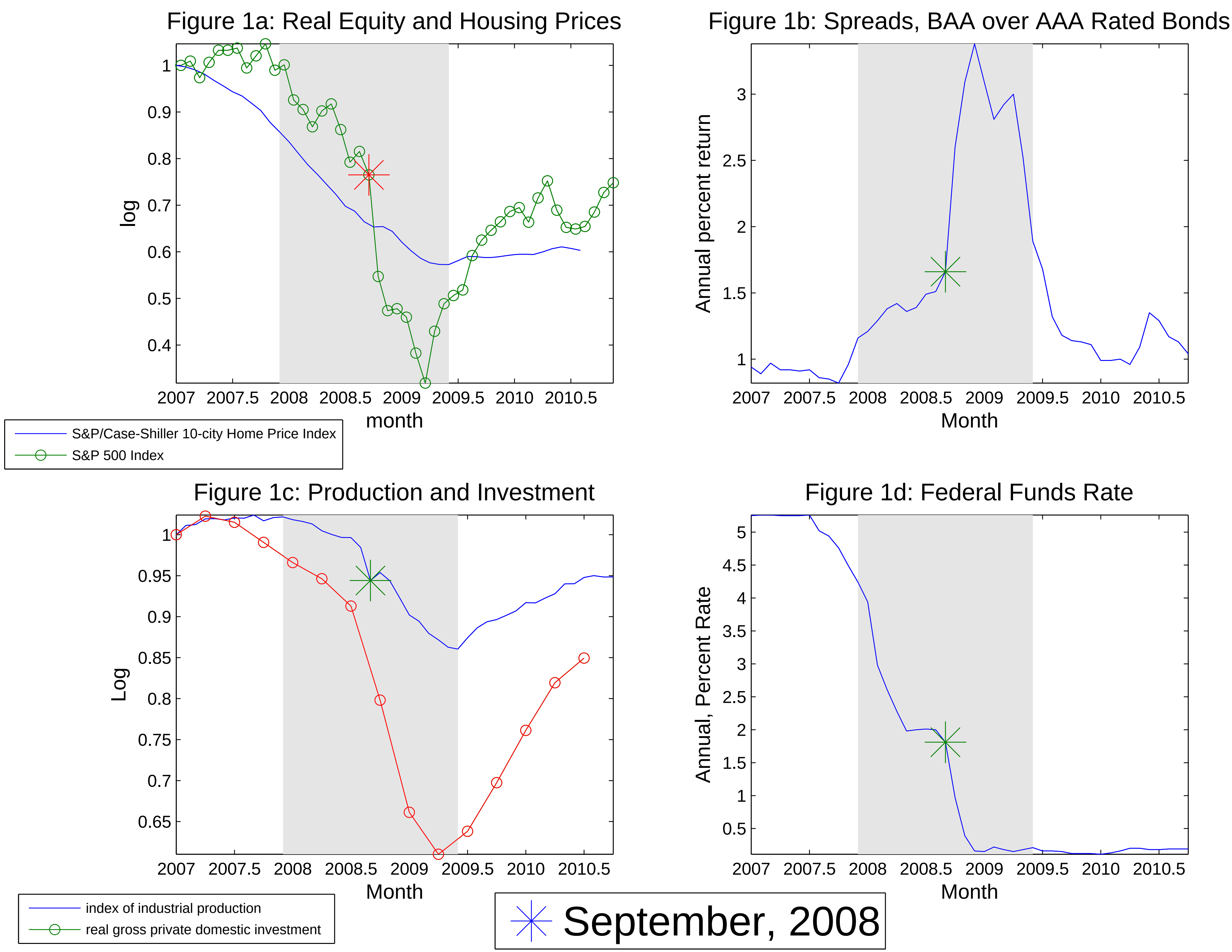

Figure 1a: Real Equity and Housing Prices
log
month
S&P/Case-Shiller 10-city Home Price Index
S&P 500 Index
Figure 1b: Spreads, BAA over AAA Rated Bonds
Annual percent return
Month
Figure 1c: Production and Investment
Log
Month
index of industrial production
real gross private domestic investment
Figure 1d: Federal Funds Rate
Annual, Percent Rate
Month
September, 2008

Figure 2: Hidden Effort Model Properties, Various τ

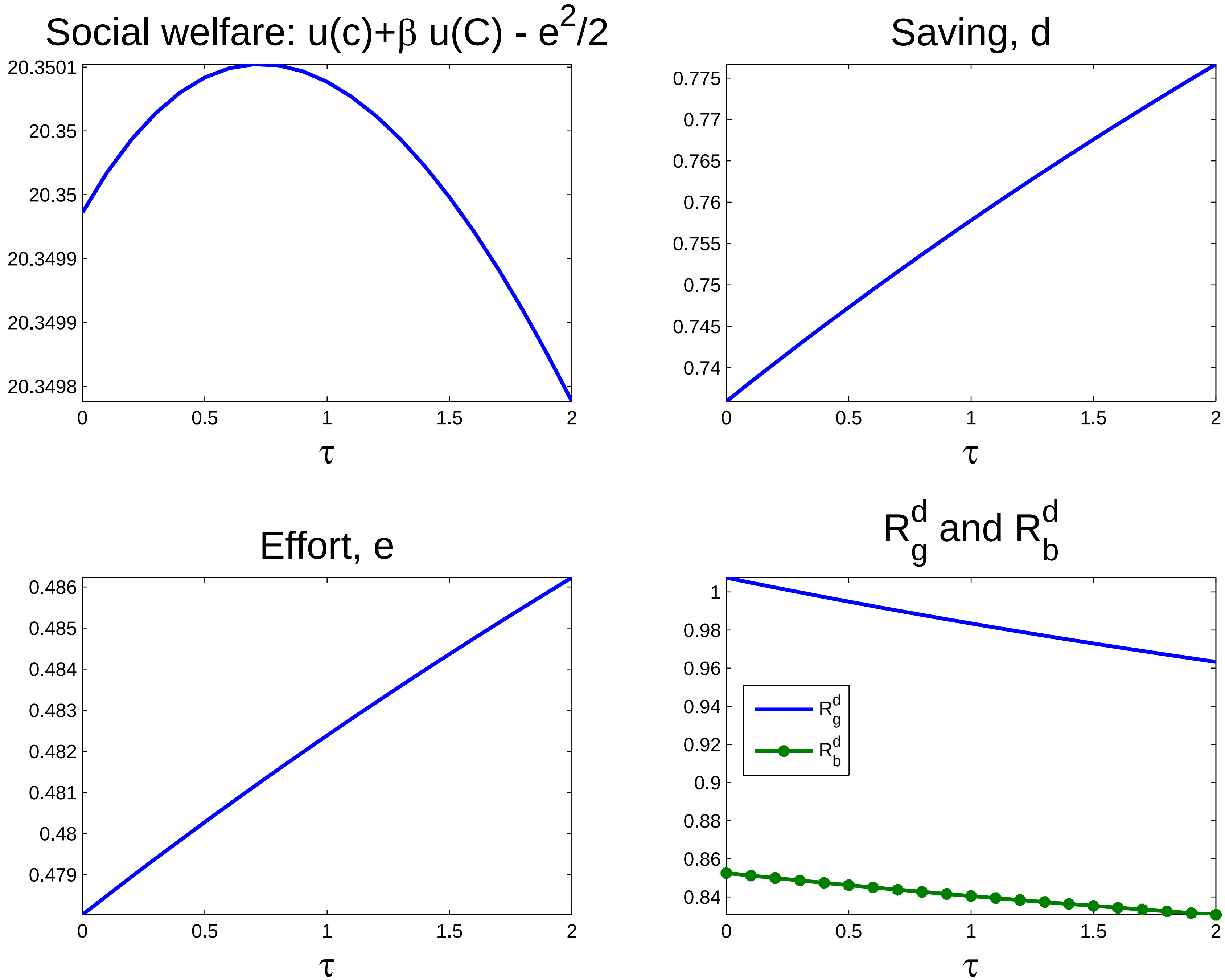

Figure 3: Hidden Effort Model Properties for Various Leverage Restrictions and Bailout Rates, τ

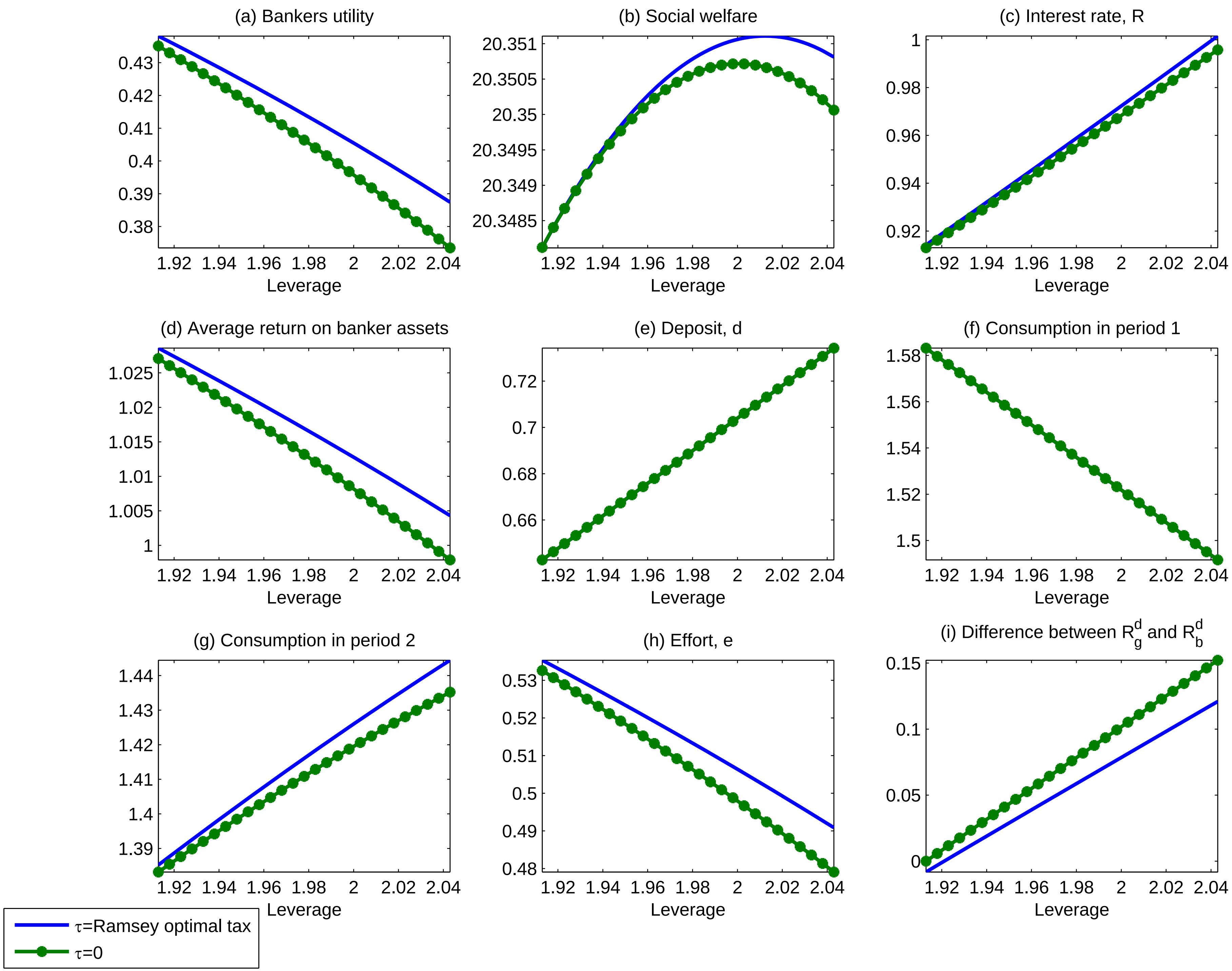

Figure 4: Agents in the Adverse Selection Model

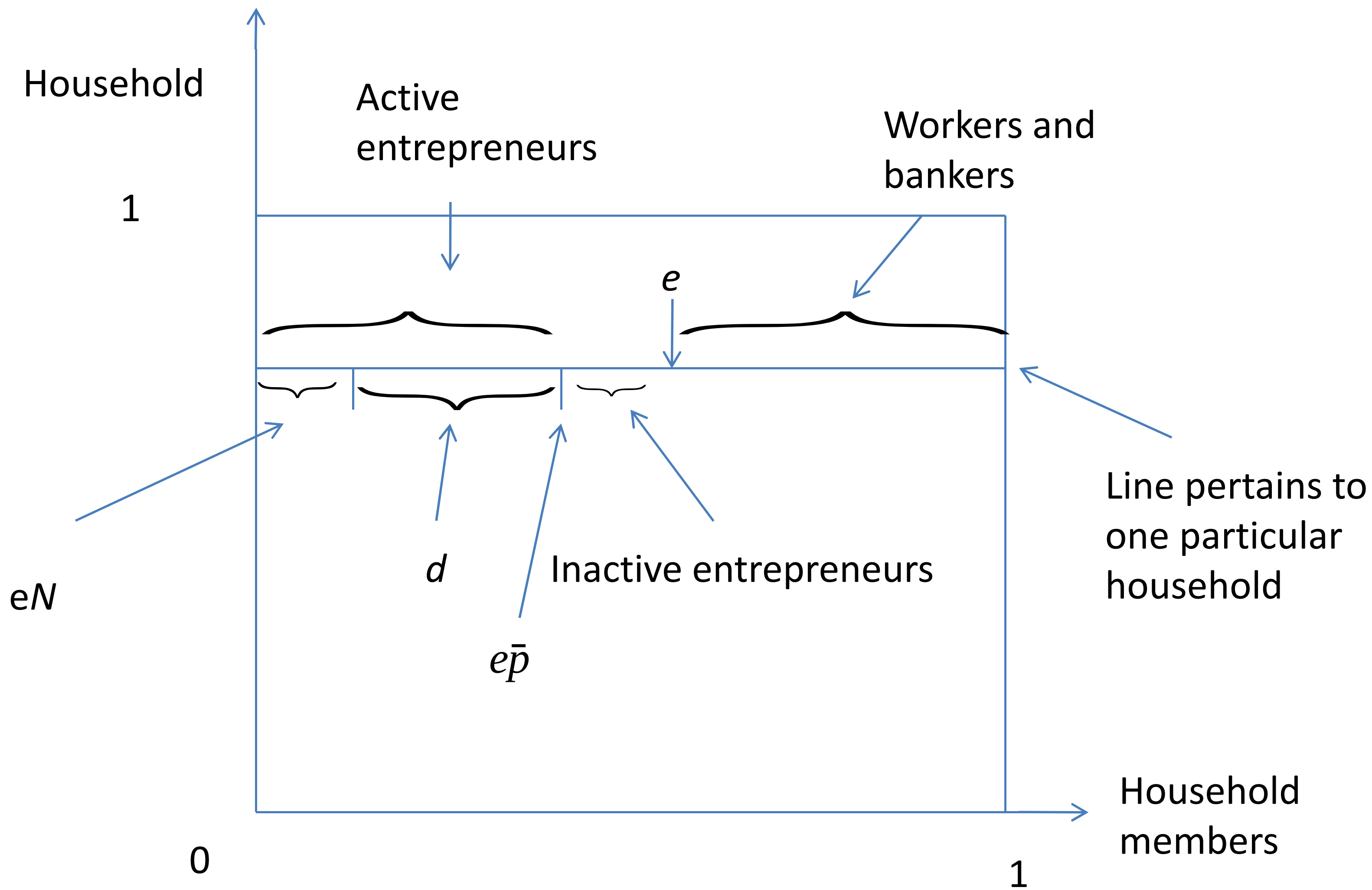

Figure 5: Optimal Subsidy Rate, in Presence of Pecuniary Externality

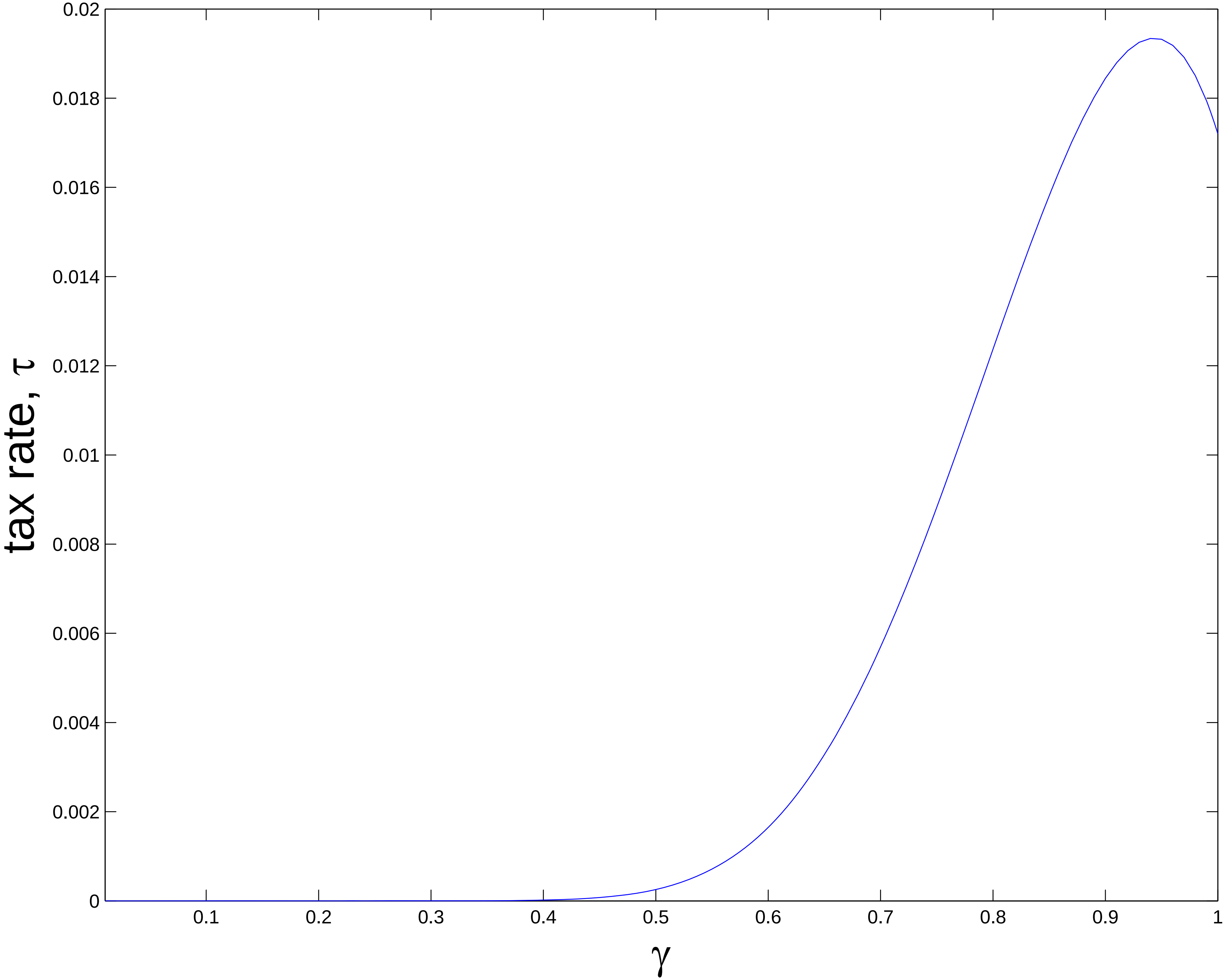

Table 1: Parameters of the Asymmetric Information Model

β	discount factor	0.97	α	relative risk aversion	1	r^k	return on capital	1.04
σ	standard deviation	0.37	μ	monitoring cost	0.2	y	household's endowment	3.11
N	entrepreneur's endowmer	1						

Table 2: Properties of the Asymmetric Information Model

		Panel A: Baseline			Panel B: Drop in N			Panel C: Drop in N and rise in σ		
		τ=0	Ramsey	First best	τ=0	Ramsey	First best	τ=0	Ramsey	First best
Interest rate subsidy, 100τ		0	0.32	–	0	1.84	–	0	1.95	–
Financial variables										
$(R$-1)100	risk-free rate	1.00	0.98	–	-5.36	-5.45	–	-6.99	-7.08	–
$(Z/R$-1)100	spread	1.23	1.24	–	7.68	7.83	–	12.32	12.53	–
N/y	net worth	0.321	0.321	–	0.161	0.161	–	0.161	0.161	–
$F(\bar{\omega})$100	bankruptcy rate	4.00	4.03	–	20.94	21.27	–	28.95	29.35	–
L	leverage ratio	2.000	2.003	–	3.434	3.466	–	3.418	3.452	–
Real variables										
c/y	time 1 consumption	0.679	0.678	0.671	0.609	0.604	0.589	0.612	0.606	0.589
C/y	time 2 consumption	0.665	0.666	0.676	0.559	0.564	0.594	0.552	0.557	0.594
$(B+N)/y$	investment	0.642	0.643	0.650	0.551	0.556	0.571	0.549	0.554	0.571

Note: (i) The columns headed `Panel A: Baseline' correspond to the baseline parameterization reported in the text. The columns headed `Panel B: Drop in N' correspond to the baseline parameterization with replaced by N=1/2. The columns headed `Panel C: Drop in $N$ and rise in σ' correspond to the baseline parameterization with $N$=1/2 and σ replaced the product of its value in the baseline parameterization and 1.2. (ii) The column headed `τ=0' reports a private sector equilibrium with zero interest rate subsidy. The column headed `Ramsey' reports the Ramsey equilibrium. The column headed `First best' reports the first best allocation.