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ANIMAL SPIRITS, FINANCIAL CRISES AND PERSISTENT UNEMPLOYMENT

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Animal Spirits, Financial Crises and Persistent Unemployment
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ABSTRACT

This paper develops a rational expectations model with multiple equilibrium unemployment rates and uses it to explain financial crises. In contrast to earlier work on this topic, the model has equilibria where asset prices are unbounded above. I argue that this is an important feature of any rational-agent explanation of a financial crisis, since for the expansion phase of the crisis to be rational, investors must credibly believe that asset prices could keep increasing forever with positive probability. I explain the sudden crash in asset prices that precipitates a financial crisis as a switch from a non-stationary equilibrium, to an alternative stationary equilibrium, with a high and inefficient unemployment rate. I also explain how variations in beliefs about future wealth cause movements in the stock market. These wealth movements are transmitted to the unemployment rate through variations in aggregate demand.

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1 Introduction

This paper develops a rational expectations model with multiple equilibrium unemployment rates and uses it to explain financial crises. In contrast to earlier work on this topic (Farmer, 2008a,b, 2012a, 2010a,b,c,d; Farmer and Plotnikov, 2010), the model has equilibria where asset prices are unbounded above. I argue that this is an important feature of any rational-agent explanation of a financial crisis, since for the expansion phase of the crisis to be rational, investors must credibly believe that asset prices could keep increasing forever with positive probability.

The stock market boom of the 1920s, the Japanese land boom of the 1980s and the U.S. housing bubble of the 2000s were all characterized by dramatic increases in the value of asset prices, a high growth rate of consumption and GDP, and a falling unemployment rate. Following each of these episodes, the economy entered a period of stagnation. The most severe of these was the Great Depression of the 1930s when the U.S. unemployment rate increased from 2% to 25% and remained above 15% for a decade. The Japanese economy has still not fully recovered more than twenty years after Japanese property prices collapsed in 1989: And the Great Recession that followed the 2008 financial crisis was declared over by the NBER in June of 2009, but U.S. unemployment has remained above 8% for 30 months.

For the past thirty years, the dominant macroeconomic paradigm, real business cycle theory, has explained fluctuations in economic activity as the optimal response of a representative agent to random productivity shocks. Its close cousin, the new-Keynesian paradigm, adds additional shocks and nominal frictions to a real business cycle framework. Neither of these conventional models of macroeconomic fluctuations can explain financial crises.

Conventional models explain economic data as the unique equilibrium response of optimizing agents to shocks to fundamentals; these include the preferences of the agents, the state of technology, and the economy's endowments of labor and land. Financial crises are preceded by a period of rapid expansion in economic activity followed by a crash in asset prices and a sharp increase in the unemployment rate. They are difficult to explain with conventional models because the increase in asset prices during the expansion phase that precedes a financial crisis is not accompanied by any observable movement in economic fundamentals. And conventional models cannot explain what causes a crash.

Models of multiple equilibria provide a plausible alternative to conventional models of financial crises because they allow real economic events to be influenced by the beliefs of market participants. Observed unemployment, consumption, GDP, and asset prices can move, even when preferences, technology and endowments are fixed.

The multiple equilibrium model I construct in this paper explains a financial crisis as the movement from a non-stationary equilibrium, where asset prices and consumption are increasing and unemployment is falling, to a stationary equilibrium where consumption and asset prices are low, unemployment is high and the economy enters a period of stagnation.

2 Explaining a Financial Crisis with a Multiple-Equilibrium Model

A multiple equilibrium model that can account for the growth phase of a financial crisis must have two features. First, the model must have multiple equilibria. Second, the equilibria must be capable of explaining explosive growth in asset prices. In Farmer (2012a), I constructed a model with search and matching frictions in the labor market. That model contains a continuum of steady state unemployment rates: But it cannot explain the growth phase of the cycle because the asset price is bounded above and every bull market must come to an end at a predictable future date. As in conventional models, explosive growth in asset prices is ruled out by the assumption that actors are rational and forward looking.

In this paper, I extend my earlier work by allowing for richer preferences and technologies. I provide an example of a multiple-equilibrium model that can account for financial crises. In this model, it is rational for investors to keep bidding up asset prices since there are no physical or behavioral constraints that prevent the price from going even higher. The expansion phase of the crisis, in my model, is fully rational.[1]

But although asset prices *could* continue to rise; there is nothing to ensure that they *will* continue to rise other than the collective beliefs of market participants. Asset prices are moved by what George Soros has called 'the mood of the market'. If market participants lose confidence in the markets, there are many other equilibria that are consistent with alternative beliefs. I explain the end of the expansion, the *Minsky moment*, as a switch from an initial non-stationary equilibrium, characterized by rising asset prices and increasing economic activity, to one of many possible stationary equilibria where the unemployment rate may take any value in a bounded interval.[2] I also explain how variations in beliefs about future wealth cause movements in the stock market. These wealth movements are transmitted to the unemployment rate through variations in aggregate demand.

3 The Anatomy of a Financial Crisis

Conventional economic theory views the business cycle as the equilibrium response of a rational maximizing agent to a series of random shocks. In real business cycle theory, these shocks are innovations to an exogenous random productivity process. In modern medium scale new-Keynesian models of the kind pioneered by Smets and Wouters (2003), there are also preference shocks, liquidity shocks and money supply shocks. But the existence of ad-

[1] This is in contrast to the popular notion that the expansion phase of a financial crisis is an asset price bubble, fueled by 'irrational exuberance'.

[2] The term "Minsky moment", named after the economist Hyman Minsky, was coined in 1998 by Paul McCulley of PIMCO, to describe the 1998 Russian financial crisis.

ditional shocks does not alter the message. New-Keynesian and real business cycle models have the same roots and in both theories, the economy is a self-stabilizing system. Left to itself, the unemployment rate will converge to the level that would be chosen by a benevolent social planner whose goal was to maximize social welfare.

In his widely cited book, *Stabilizing an Unstable Economy*, Minsky (2008) lays out an alternative vision of business cycles in which the natural tendency of a free market economy is to swing between bouts of expansion and stagnation. In Minsky's vision of the market, it is the stabilizing forces of fiscal and monetary interventions by government that have prevented post-war business cycles from replicating the worst excesses of nineteenth century capitalism. In my view, this is correct, but Minsky's implementation of his vision, is overly dismissive of conventional economic theory. I do not believe that we must jettison two hundred and fifty years of economic thought to accommodate his ideas.

The dynamic stochastic general equilibrium models, (DSGE models), that were developed over the past thirty years are wrong in some dimensions. But they made important advances, many of which are worth retaining. These include the representative agent version of the rational actor model, the assumption of complete financial markets and the rational expectations assumption. These are all convenient fictions that make analysis tractable.

In Farmer (2012a) I introduced search and recruiting costs into an otherwise conventional DSGE model and I showed that the model has multiple equilibrium steady-state unemployment rates. That work reconciles Keynesian economics with general equilibrium theory in a new way. Unemployment does not arise because some prices are 'sticky' as in the new-Keynesian explanation. It arises as an equilibrium phenomenon in a model with missing markets. My work explains how rational actors, following price signals in a market economy, can fail to exploit all of the gains from trade.

But although my (2012a) paper can account for periods of persistently high unemployment, it cannot account for the expansion phase of the cycle where asset prices increase rapidly with no apparent end in sight. In Farmer (2012a), as in conventional models of business cycles, asset prices are bounded above. It follows that an explosive price path is inconsistent with equilibrium in a model with rational actors. This paper modifies the environment in my earlier work to allow for equilibria where asset prices are unbounded above.

4 Financial Crises in the Data

At the end of the 1990s, the repeal of the Glass-Steagall Act was followed by a period of financial deregulation and the creation of new financial instruments that allowed speculators to place bets on the housing market that were previously unavailable. Mortgages that had traditionally been held by the banks that originated them were packaged and sold as mortgage-backed-securities in organized financial markets.

The creation of mortgage-backed-securities and other forms of collateralized debt obligations was accompanied by a new instrument; the credit default swap, that allowed investors to speculate on the direction of house price movements. At the same time these new securities were created, there was an unprecedented increase in house prices in the United States that was not accompanied by increases in rents or by changes in any of the fundamentals one might normally associate with the housing market.[3] Popular accounts of the 2008 financial crisis attribute the increase and subsequent collapse of house prices to the bursting of an asset price bubble.[4]

Figure 1 illustrates the history of house prices, the stock market and unemployment in the U.S. since 1990. From 1990 through 1995 the Case-Shiller home price index increased at a rate of 1% per year, from 76 in the first quarter of 1990 to 80 in the fourth quarter of 1995.[5] Beginning in

[3] Kashiwagi (2010) documents these facts in his Ph.D. thesis.

[4] See, for example, Shiller (2008) for one such account.

[5] Data is from Robert Shiller, http://www.econ.yale.edu/~shiller/data.htm.

the first quarter of 1996, house price appreciation began to accelerate and the Case-Shiller index reached a peak of 190 in the second quarter of 2006. Between 1995 and 2006 the index grew at an annualized rate of 8%. In the third quarter of 2006, U.S. house prices began to fall for the first time in a century of data and by 2009Q1 the Case Shiller index had fallen by 42% to 129, a value it last attained in 2003.[6]

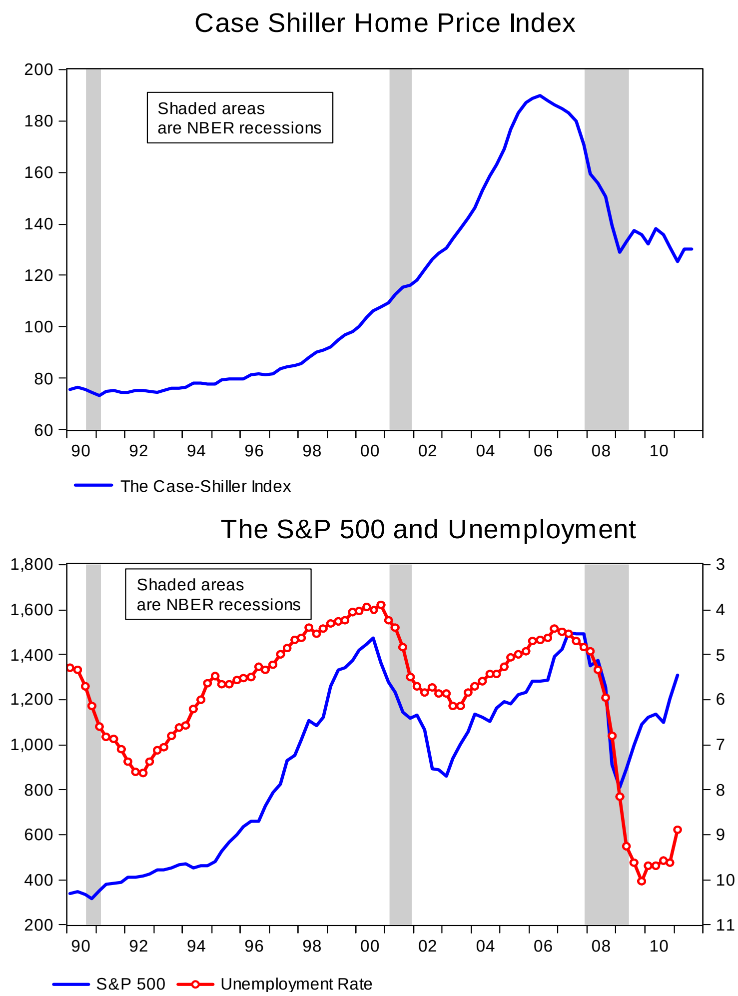

Figure 1: A House Price Index, a Stock Market Index and Unemployment

[6]Farmer (2012b) shows that the correlation between the stock market and unemployment has remained remarkably stable in the post-war data.

The fall in house prices was soon followed by a stock market crash as the S&P 500 fell from a peak of 1540 in October of 2007 to a trough of 757 in March of 2009, a drop of 56%. The collapse in the value of U.S. financial assets was accompanied by a doubling of the unemployment rate which went from 4.8% in October of 2007 to 10.1% in November of 2009.

Macroeconomic theory, of the kind that has dominated the profession for the past thirty years, cannot explain either the huge fall in asset values or the increase in unemployment that followed. This paper can explain both.

5 A Dynamic Equilibrium Model

In Section 5, I describe a dynamic equilibrium model that generalizes the assumptions of my earlier work (Farmer, 2012a) in two directions. I relax the assumption that workers are fired and rehired every period and I allow for more general preferences and technology. In Section 6, I characterize the properties of the equilibria of this more general model and I prove two important results.

First, I show that there is a continuum of equilibria. Second, I show that there is a stationary equilibrium in which the price of the asset is unbounded above. This result is important because it implies that there will also exist a *non-stationary equilibrium* in which the price of the asset grows without bound. An equilibrium of this kind is a natural candidate to explain the expansionary phase of a business cycle that precedes a financial crisis.

5.1 Households

There is a continuum of identical households, each of whom derives utility from consumption of a unique commodity, C_t, Households maximize utility,

$$J = E_s \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \frac{C_t^{1-\eta}}{1-\eta} \right\}, \tag{1}$$

subject to the constraints

$$p_{k,t}K_{t+1} + p_t C_t \leq (p_{k,t} + r_t) K_t + w_t L_t, \tag{2}$$

$$H_t \leq 1 - L_t, \tag{3}$$

$$L_{t+1} = L_t (1 - \delta) + \tilde{q}_t H_t. \tag{4}$$

Here, K_t is capital, L_t is employment, w_t is the money wage, p_t is the money price of commodities, $p_{k,t}$ is the money price of capital and and r_t is the rental rate. Equation (4) represents the assumption that if H_t workers search, $\tilde{q}_t H_t$ of them will find a job where the fraction $\tilde{q}_t$ is determined in equilibrium by the aggregate search technology.

Since we will need to value streams of payments I will assume that there exists a complete set of Arrow securities, one for each realization of S_t. The price at date t of a dollar delivered for sure at date τ in history $S^\tau \equiv \{S_t, S_{t+1}, ...S_\tau\}$ is given by the expression

$$Q_t^\tau = \frac{\beta^{\tau-t} p_t}{p_\tau} \left(\frac{C_\tau}{C_t} \right)^{-\eta}, \tag{5}$$

where I have suppressed the dependence of Q_t^τ on the history of shocks.

Using this definition, the transversality condition can be written as

$$\lim_{T \to \infty} Q_t^T p_{k,T} K_{T+1} = 0, \text{ for all histories } S^T. \tag{6}$$

Since leisure does not yield utility, households will choose,

$$H_t = 1 - L_t, \tag{7}$$

which implies that all unemployed workers search for a job. Substituting this expression into (4) gives

$$L_{t+1} = L_t (1 - \delta) + \tilde{q}_t (1 - L_t). \tag{8}$$

In addition, the household will allocate resources through time optimally. That assumption leads to the following consumption Euler equation,

$$C_t^{-\eta} = E_t \left\{ \beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}} \left(\frac{p_{k,t+1} + r_{t+1}}{p_{k,t}} \right) \right\}. \tag{9}$$

5.2 The Production Technology

The consumption commodity is produced by the technology

$$C_t = \left[b\left(S_t X_t\right)^{\rho} + a K_t^{\rho} \right]^{\frac{1}{\rho}}, \qquad a + b = 1 \tag{10}$$

where X_t is labor used in production, K_t is capital and S_t is a labor augmenting technology shock.

For some purposes I will assume that

$$0 < \rho < 1. \tag{11}$$

This assumption places the technology on the linear side of the CES class and I will show in Proposition 2 that the assumption that inequality (11) holds is sufficient to guarantee that the function that links steady state asset prices with steady state employment, is invertible.[7]

5.3 Firms in a Search Model

Each firm solves the following problem,

$$\max_{\{K_t, V_t, X_t, L_t\}} E_s \left\{ \sum_{t=s}^{\infty} Q_s^t \left(\left[b\left(S_t X_t\right)^{\rho} + a K_t^{\rho} \right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{r_t}{p_t} K_t, \right) \right\} \tag{12}$$

[7] In an earlier version of this paper I asserted that this assumption is necessary in order for there to exist an equilibrium in which the asset price is unbounded. I am indebted to Mingming Jiang of the University of California Riverside, for pointing out that this assertion is incorrect. In the model where labor is a state variable, there always exists an equillibrium with an unbounded asset price.

subject to the constraints,

$$L_t = X_t + V_t, \tag{13}$$

$$L_{t+1} = L_t (1 - \delta) + q_t V_t. \tag{14}$$

Constraints (13) and (14) hold for all $t = s, \ldots$. The sequences of money prices $\{p_t\}$, money wage $\{w_t\}$, money rental rates $\{r_t\}$ and the present value prices $\{Q_s^t\}$, are taken as given where all variables are contingent on the histories of shocks. In addition, the firm takes the sequence of search efficiencies of a recruiter, $\{q_t\}$ as given.

Using equations (12) – (14) we may write the following Lagrangian for problem (12).

$$\max E_s \sum_{t=s}^{\infty} \left\{ Q_s^t \left(\left[bS_t^\rho (L_t - V_t)^\rho + aK_t^\rho \right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{r_t}{p_t} K_t \right.\right.$$
$$\left.\left. + \ \psi_t \left[(1-\delta) L_t + q_t V_t - L_{t+1} \right] \right) \right\}.$$

This expression is maximized when

$$a \left(\frac{C_t}{K_t} \right)^{1-\rho} = \frac{r_t}{p_t}, \tag{15}$$

$$bS_t^\rho \left(\frac{C_t}{L_t - V_t} \right)^{1-\rho} = \psi_t q_t, \tag{16}$$

and

$$\psi_t = E_s \left\{ Q_t^{t+1} \left(bS_{t+1}^\rho \left(\frac{C_{t+1}}{(L_{t+1} - V_{t+1})} \right)^{1-\rho} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1} (1-\delta) \right) \right\}. \tag{17}$$

The equations

$$C_t = \left[bS_t^\rho (L_t - V_t)^\rho + aK_t^\rho \right]^{\frac{1}{\rho}}, \tag{18}$$

and

$$L_{t+1} = L_t (1 - \delta) + q_t V_t, \tag{19}$$

must also hold. In addition, any optimal path must satisfy the transversality condition

$$\lim_{T \to \infty} Q_t^T \psi_T = 0 \text{ for all histories } S^T. \tag{20}$$

5.4 How $\tilde{q}$ and q are Determined in a Search Model

The variables $\tilde{q}_t$ and q_t, are determined in equilibrium by market clearing in the markets for search inputs. Let a variable with a bar denote an economy-wide average. Using this notation, $\bar{L}_t$ is the measure of aggregate employment and L_t is the measure of workers hired by the representative firm. These variables are conceptually distinct although they turn out to be equal in equilibrium.

Each period I assume that in aggregate, a measure

$$\bar{m}_t = \left(\Gamma \bar{V}_t\right)^{\theta} \left(1 - \bar{L}_t\right)^{1-\theta}, \tag{21}$$

of workers is hired, where Γ measures the efficiency of the match process and θ measures the elasticity of the recruiting effort by firms. This parameter can be identified in data from estimates of the Beveridge curve. Using U.S. data, Blanchard and Diamond (1990) found estimates of θ to be between 0.3 and 0.5. Since setting $\theta = 0.5$ will simplify some of the algebra of the model, I will make that assumption from this point on. In addition, I assume that a measure δL_t of workers lose their jobs for exogenous reasons.

Together, these assumptions imply that the labor force in period $t + 1$ will be given by the expression

$$\bar{L}_{t+1} = \bar{L}_t (1 - \delta) + \left(\Gamma \bar{V}_t\right)^{\frac{1}{2}} \left(1 - \bar{L}_t\right)^{\frac{1}{2}}. \tag{22}$$

Since (8) and (19) must also hold in a symmetric equilibrium it follows that

$$q_t = \Gamma^{\frac{1}{2}} \left(\frac{1 - \bar{L}_t}{\bar{V}_t} \right)^{\frac{1}{2}}, \tag{23}$$

and

$$\tilde{q}_t = \Gamma^{\frac{1}{2}} \left(\frac{\bar{V}_t}{1 - \bar{L}_t} \right)^{\frac{1}{2}}. \tag{24}$$

6 Characterizing Equilibria

In this section I will lay out the equations that characterize behavior in a symmetric equilibrium of the model and I will study the behavior of a special class of steady state equilibria. I will show that this model possesses a continuum of steady state equilibria for any level of employment in an open interval $L \in [0, \mu)$ where $\mu < 1$.

6.1 The Equations of the Model

The following eight equations characterize the competitive equilibrium conditions. Equations (25) and (26) represent the Euler equation and the pricing kernel.

$$C_t^{-\eta} = E_t \left\{ \beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}} \left(\frac{p_{k,t+1} + r_{t+1}}{p_{k,t}} \right) \right\}, \tag{25}$$

$$Q_t^{t+1} = \frac{\beta p_t}{p_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\eta}. \tag{26}$$

The next four equations combine optimizing behavior by firms with the search equilibrium condition (23),

$$\psi_t = E_t \left\{ Q_t^{t+1} \left(\psi_{t+1} q_{t+1} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1} (1 - \delta) \right) \right\}, \tag{27}$$

$$\frac{r_{t+1}}{p_{t+1}} = a \left(C_{t+1} \right)^{1-\rho}, \tag{28}$$

$$bS_t^{\rho}\left(\frac{C_t}{L_t - V_t}\right)^{1-\rho} = \psi_t q_t, \tag{29}$$

$$L_{t+1} = L_t (1-\delta) + (\Gamma V_t)^{\frac{1}{2}} (1-L_t)^{\frac{1}{2}}. \tag{30}$$

Here, ψ_t is the shadow price of labor and q_t is given by the labor market search technology as

$$q_t = \Gamma^{\frac{1}{2}} \left(\frac{1-L_t}{V_t}\right)^{\frac{1}{2}}. \tag{31}$$

Finally, since I assume that $K_t = 1$, the production function,

$$C_t = \left[bS_t^{\rho}(L_t - V_t)^{\rho} + a\right]^{\frac{1}{\rho}}, \tag{32}$$

must hold in aggregate.

These eight equations must determine the nine unknowns,

$$y_t \equiv \left\{C_t, \quad L_t, \quad V_t, \quad \frac{r_t}{p_t}, \quad \frac{w_t}{p_t}, \quad \frac{p_{k,t}}{p_t}, \quad Q_t, \quad q_t, \quad \psi_t\right\}. \tag{33}$$

The fact that there is one less equation than unknown arises from the absence of markets to allocate search intensity between the time of searching workers and the recruiting activities of firms, a point first made by Greenwald and Stiglitz (1988).

Farmer (2012a) proposed closing the model with the function

$$E_t\left[\frac{p_{k,t+1}}{p_{t+1}}\right] = x_t, \tag{34}$$

where x_t is a process that represents how beliefs are influenced by economic events. In this paper I modify that approach and I assume instead that

$$E_t\left[\frac{p_{k,t+1}}{w_{t+1}}\right] = x_t. \tag{35}$$

I will show that this model admits equilibria in which there may exist self-fulfilling asset market bubbles and crashes, where bubbles are in real asset prices using the money wage as the numeraire.

6.2 Steady State Equilibria

In Farmer (2012a) I showed, in a static version of this model with a Cobb-Douglas technology and logarithmic preferences, there is a steady state equilibrium for any value of L in the interval $[0, 1]$. In that model, for each equilibrium value of L, there is a different asset price $p_{k,t}$, but asset prices are bounded above.

The following definitions and propositions extend my previous work to the dynamic model with more general preferences and technologies and show that, in equilibrium, asset prices are unbounded. I begin by defining a steady state equilibrium.

Definition 1 *A Non-Stochastic Steady State Equilibrium is a vector* $\left\{C, L, V, \frac{r}{p}, \frac{w}{p}, \frac{p_k}{p}, Q, q, \psi\right\}$ *that solves the equations*

$$\frac{p_k}{w} = x, \tag{36}$$

$$\frac{C^{1-\rho}p}{p_k} = \frac{1-\beta}{a\beta}, \tag{37}$$

$$Q = \beta, \tag{38}$$

$$\psi\left(1-\beta\left(1-\delta\right)\right) = \beta q\psi - \beta\frac{w}{p}, \tag{39}$$

$$\frac{r}{p} = aC^{1-\rho}, \tag{40}$$

$$b\left(\frac{C}{L-V}\right)^{1-\rho} = \psi q, \tag{41}$$

$$\delta^2 L^2 = \Gamma V\left(1-L\right), \tag{42}$$

$$q = \Gamma^{\frac{1}{2}} \left(\frac{1-L}{V} \right)^{\frac{1}{2}}, \tag{43}$$

$$C = \left[b \left(L - V \right)^{\rho} + a \right]^{\frac{1}{\rho}}. \tag{44}$$

These equations are derived from Equations (25) – (32) and Equation (34) by assuming that $S_t = 1$ for all t and solving the resulting non-stochastic equations for a steady state.

Proposition 1 *Define the constants λ, μ and Ω as follows*

$$\lambda = \frac{\Gamma}{\Gamma + \delta^2}, \quad \mu = \frac{\beta \Gamma}{\beta \Gamma + \delta \left(1 - \beta \left(1 - \delta \right) \right)},$$
$$\Omega = \left(\frac{a \beta}{1 - \beta} \right) \frac{\Gamma^{\rho} \left(\Gamma + \delta^2 \right)^{1-\rho}}{\beta \Gamma + \delta \left(1 - \beta \left(1 - \delta \right) \right)} \left(\frac{\beta}{b} \right). \tag{45}$$

For all $L \in [0, \mu)$, there exists a steady state equilibrium. The values of the endogenous variables Q, C, V and q, for each value of L are given by the expressions

$$Q = \beta, \qquad C = \left(b L^{\rho} \left(1 - \frac{\delta^2 L}{\Gamma \left(1 - L \right)} \right)^{\rho} + a \right)^{\frac{1}{\rho}},$$
$$V = \frac{\delta^2 L^2}{\Gamma \left(1 - L \right)}, \qquad q = \Gamma^{\frac{1}{2}} \left(\frac{1-L}{V} \right)^{\frac{1}{2}}, \tag{46}$$

and the values of the variable $\frac{r}{p}, \psi$ and $\frac{w}{p}$ are computed from (28), (29) and (27). The price of capital, measured in wage units is described by a continuous function: $g(L) : [0, \mu) \to \tilde{P} \subset R_{+}$ where

$$\frac{p_k}{w} = g(L) \equiv \frac{\Omega L^{1-\rho} \left(1 - L \right)^{\rho} \left[\lambda - L \right]^{1-\rho}}{\mu - L}. \tag{47}$$

Proposition 2 *If $0 < \rho < 1$, $\tilde{P} \equiv R_{+,}$ and the function g is strictly increasing with*

$$g(0) = 0, \quad g(\mu) = \infty. \tag{48}$$

By the inverse function theorem there exists a function $h(x) = R_+ \rightarrow [0, \mu)$ such that for all $x \in R_+$ there exists a steady state equilibrium where

$$L = h(x). \tag{49}$$

The vector of endogenous variables y_t defined in (33) is determined as in Proposition 1.

Proposition 1 establishes that the equations that define a steady state equilibrium have a solution for a set of values of L less than or equal to some maximum value μ.[8] Proposition 2 goes further. It shows that, if $0 < \rho < 1$, L and p_k are related by a monotonically increasing function. When $L = 0$, $p_k = 0$ and p_k becomes infinite as L attains it's upper bound.

7 A Simplified Version of the Model

In this section, I will simplify the model developed in the previous two sections by assuming that labor is fired and rehired every period. I will show that this model contains an equilibrium where asset prices increase and where the increase could potentially continue forever. But there are also many equilibria where the economy stagnates. The non-stationary equilibrium and all but one of the stationary equilibria are inefficient. Unless the economy fortuitously finds its way to the social planning optimum, the end of a bull market will be followed by a period of stagnation with high unemployment and low economic welfare.

7.1 Allowing Workers to Recruit Themselves

Consider a version of the model where the labor force is fired and rehired in every period. This assumption leads to the following expression for aggregate

[8] The parameter Γ measures the efficiency of the match process. As Γ approaches ∞, the set of sustainable equilibrium employment rates approaches the interval $[0, 1]$.

employment,

$$\bar{L}_t = \left(\Gamma \bar{V}_t\right)^{\frac{1}{2}} \left(H_t\right)^{\frac{1}{2}}, \tag{50}$$

where

$$H_t = 1. \tag{51}$$

In this version of the model, firms decide each period on a plan $\{V_t, X_t\}$ that specifies how many workers will be allocated to recruiting and how many to production. Notice that since firms begin each period with no workers; I am allowing workers to recruit themselves. This fiction is well worth adopting since it allows me to describe equilibria in closed form.

7.2 The Theory of Aggregate Supply

In Farmer (2012a) I studied the case of this model where $\rho = 0$ and $\eta = 1$. By choosing $w_t = 1$ as the numeraire, I showed that one can derive an 'aggregate supply' equation that describes how the value of the output of the economy, measured in wage units, is related to aggregate employment.[9] That equation took the form,

$$Z_t = \frac{1}{b} L_t, \tag{52}$$

where, in Farmer (2012a), I defined

$$Z_t = \frac{p_t C_t}{w_t}, \tag{53}$$

to be equal to GDP, measured in wage units. The generalization of Equation (53) to an economy with more general preferences and technology (derived in Appendix D), is the expression,

$$Z_t = g\left(L_t\right), \tag{54}$$

[9] See Farmer (2008a) for a discussion of this concept of aggregate supply and it's relationship to Keynes' definition in *The General Theory* (Keynes, 1936).

where

$$g(L_t) \equiv \frac{\Gamma^\rho L_t^{1-\rho}}{bS_t^\rho (\Gamma - L_t)^\rho}, \tag{55}$$

and

$$Z_t \equiv \frac{p_t C_t^{1-\rho}}{w_t}. \tag{56}$$

I will continue to use the terms *aggregate demand* and *aggregate supply* to refer to the functions that relate Z_t to employment although the variable Z_t is no longer identical to GDP once one moves beyond the economy with logarithmic preferences and Cobb-Douglas technology.

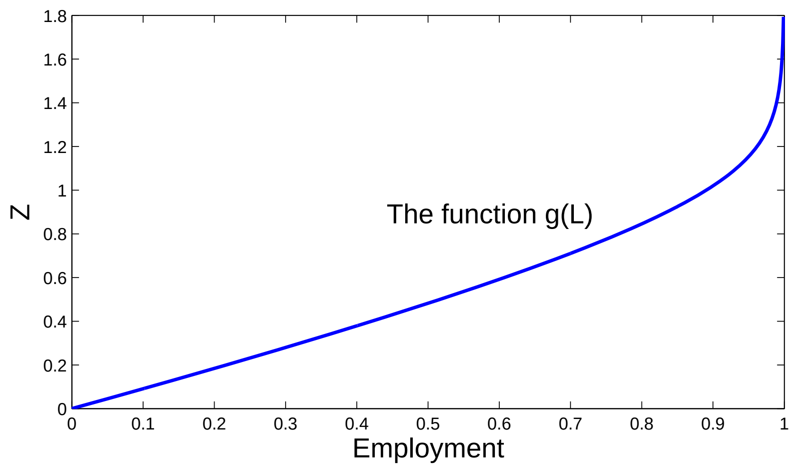

Figure 2: The Function $g(L)$

The function $g(L)$ is graphed in Figure 2 for the case $\Gamma = 1$ and $\rho = 0.5$.[10] This function is approximately linear for most of its domain, but asymptotes to infinity as L_t approaches Γ, where Γ is the maximum feasible value of employment.

The fact that Z_t may be unbounded will be important for a theory of asset market bubbles since I will show that there are equilibria where Z_t is

[10] To construct an example of the model where the asset price may be unbounded, the parameter, ρ, must be is between 0 and 1.The more general model developed in sections 5 and 6, contain an equilibrium where the asset price is unbounded for all values of ρ. It follows that the feature of my example, that asset prices can grow without bound, extends to more general assumptions.

determined by a sequence of self-fulfilling beliefs about asset prices. The fact that there exists a steady state equilibrium for all non-negative values of p_k/w means that it is rational for investors to keep bidding up the price of an asset: For any value of p_k/w, there is a higher value that is consistent with rational behavior and market clearing.

7.3 The Theory of Aggregate Demand

To close this model I will show that Z_t can be described as a linear function of the asset price, and I will provide a theory that explains the evolution of asset prices as the outcome of self-fulfilling beliefs. This function is found by combining the representative agent's Euler equation and the firm's first-order condition for renting capital.[11]

In general, the asset price will be a function of the entire sequence of expected future rental payments. But there is a special case that is analogous to assuming that preferences are logarithmic and technology is Cobb-Douglas. When preferences and technologies are linked by the parametric restriction,

$$\eta = 1 - \rho, \tag{57}$$

the asset pricing equation that links $p_{k,t}$ to aggregate demand is represented by the following linear function,

$$\frac{p_{k,t}}{w_t} = \theta Z_t, \tag{58}$$

where

$$\theta \equiv \frac{\alpha\beta}{1-\beta}. \tag{59}$$

Equation (58) is derived in Appendix E. The following section imposes the

[11] In the case where the technology is Cobb-Douglas, Z_t is the same as aggregate demand; it is equal in that case to the value of GDP in wage units. In the case of a CES technology, the price level and the value of aggregate demand are determined by recognizing that output in physical units is determined by the production function.

parametric restriction implied by Equation (57). I will use this assumption to illustrate the properties of equilibria in this simple model.

7.4 Explaining Financial Crises with a Model of Aggregate Demand and Supply

Figure 3 shows how a financial crisis can be explained using the model developed in this paper. The figure plots the aggregate supply curve, Equation (54), and the aggregate demand curve, Equation (58). The vertical axis represents the variable Z and the horizontal axis measures employment.

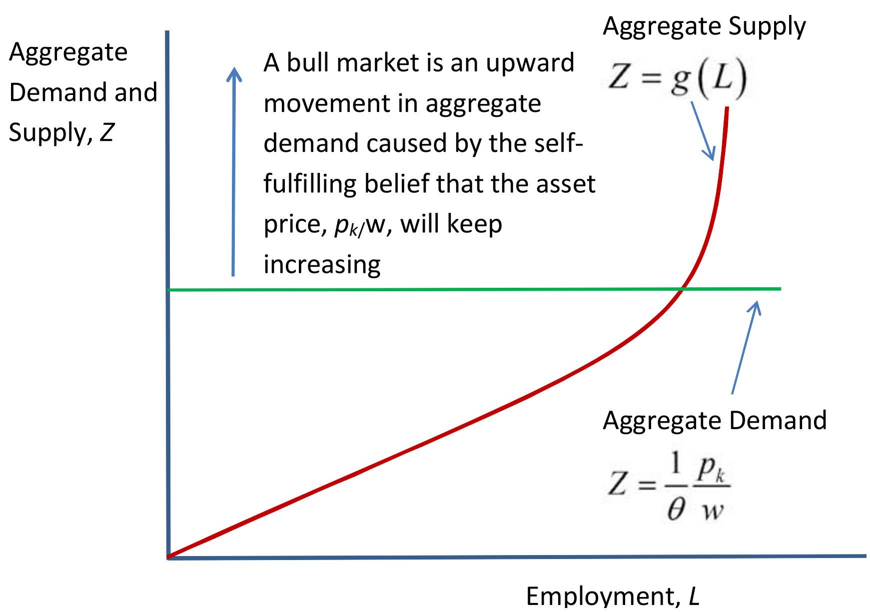

Figure 3: Aggregate Demand and Supply

As in Farmer (2012a), the model is closed by assuming that asset market participants form self-fulfilling beliefs about the value of the stock market. And for every value of the asset price, there is a different labor market equilibrium. What is new in this paper, is that the aggregate supply curve, Equation (54), asymptotes to infinity as employment approaches its upper bound, Γ. That is an important modification because it implies that it is rational for asset market participants to believe that the value of the stock market can keep increasing forever; a bull market is rational in the sense that there are no physical or behavioral constraints that would cause it to end.

The expansion phase of the crisis is represented on Figure 3 as an upward movement in the horizontal line that I have labeled 'aggregate demand', driven by the belief that prices will continue to rise. It is a wave of confidence amongst market participants. But if market participants lose confidence, the asset price can equally well tumble to a lower value with high unemployment. Importantly, any unemployment rate in the interval $[0, \Gamma]$ can persist as a steady state equilibrium.[12]

8 The Mood of the Market

How might one use this model to transmit the effects of self-fulfilling beliefs about asset prices through the real economy? The key is that *any* sequence of non-negative asset prices is consistent with a rational expectations equilibrium. If agents believe that the asset price will increase by 3% per year forever then that is an equilibrium belief. Similarly, if agents believe that the price will remain fixed at an arbitrary level; that too is an equilibrium. Since the real value of the S&P 500 is well described as a random walk, a theory which asserts that anything goes in equilibrium is consistent with observation.

What causes a financial crisis? Recall that I have defined x_t to be the

[12]What explains how changing beliefs cause a switch from one equilibrium to another? One promising explanation is in models developed by social psychologists to describe collective action. For example Granovetter (1978) describes a model where the choices of a group of actors depends in a sensitive way on the distribution of the characteristics of a population. Models like this have their origins in theories of the spread of infectious disease (Bailey, 1976) and have been exploited recently by economists (Burnside, Eichenbaum, and Rebelo, 2011) to describe the formation and collapse of bubbles in asset markets.

expected real value of the asset price in period $t+1$,

$$E_t\left[\frac{p_{k,t+1}}{w_{t+1}}\right]=x_t. \tag{60}$$

I propose to model the determination of x_t by a *belief function* $F(\cdot)$ that describes how the forecasts of market participants depend on past observations.

The belief function might depend only on past beliefs as in Equation (61),

$$x_t=F(x_{t-1}), \tag{61}$$

or it might involve feedback from the economy, as in

$$x_t=F(x_{t-1},L_{t-1})+\varepsilon_t, \tag{62}$$

where ε_t is a shock to beliefs.

It is important to recognize that the function $F(\cdot)$ is not an alternative to the rational expectations assumption. *It is in addition to it.* The rational expectations assumption is captured by equations (63) and (64),

$$\frac{p_{k,t}}{w_t}=E_{t-1}\left[\frac{p_{k,t}}{w_t}\right]+v_t, \tag{63}$$

$$E_{t-1}[v_t]=0. \tag{64}$$

In a rational expectations equilibrium, the forecast error, v_t, will be a function of shocks to all of the fundamentals including shocks to the belief function.

9 Switching Between Equilibria

A simple example of a belief function that exhibits expansions and crashes is given by Equation (65)

$$x_t = \alpha_{s_t} + \gamma_{s_t} x_{t-1} + \varepsilon_t, \tag{65}$$

where $\{\alpha_{s_t}, \gamma_{s,t}\}$ are governed by a Markov chain s_t. Suppose for example, that s_t switches with known probabilities between an expansion state in which this equation is explosive, and a crash state, in which it returns to a stable steady state. In that case, we would observe periods of explosive growth in consumption and asset prices, accompanied by falling unemployment, that were associated with the expansion phase of the cycle. But if investors were to lose confidence, represented by a switch to the crash state, we would observe a collapse in asset prices, and an increase in the unemployment rate, just as we observe after the collapse of asset prices in the real world data.

In the model, there is an optimal allocation of labor between production and search. During the later stages of the expansion, very high employment is associated with too many people involved in the search activity. This excess allocation of individuals to search is reminiscent of observations from periods of hyperinflation, where popular accounts report that search for goods to buy or sell becomes the dominant form of economic activity, and the real quantity of goods produced falls. Although there are no search costs in the goods markets in my model, there *is* an excess allocation of labor to the process of search during the later stages of the expansion.

Although there are many equilibria, all but one of them are inefficient. During the expansion, the unemployment rate falls, prices rise, and consumption and GDP grow. But although consumption measured in wage units will grow during the entire expansion phase of the cycle, once employment moves past the social planning optimum, the physical production of consumption goods will begin to decline.

10 Low Frequency Facts

How well does this model fit with stylized facts? Consider first, the prediction that the value of consumption, measured in wage units, should move one for one with the stock market.

Figure 4 displays the log of consumption per person and the value of the S&P 500. Both series have been deflated by a measure of the money wage. This figure shows that at low frequency, consumption and the stock market move together. A more formal analysis reveals that each of these series is non-stationary and that they are cointegrated, a feature of the data that has been known at least since the work of Lettau and Ludvigson (2004). I have taken logs of both variables since the transformed variables are unbounded above and below, a property that is necessary for a series to be non-stationary in a formal statistical sense.

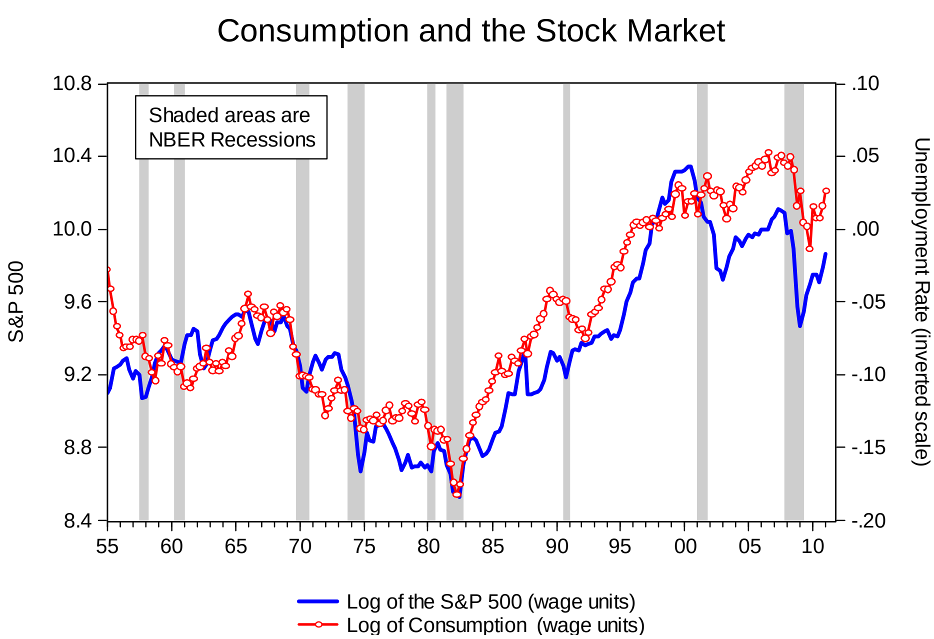

Figure 4: Consumption and the Stock Market

In the model described in this paper, variations in beliefs about future wealth cause movements in the stock market. These wealth movements are transmitted to the unemployment rate by movements in aggregate demand.

Figure 5, shows the connection between wealth and unemployment since 1955. I have plotted the logarithm of the S&P 500, measured in wage units, against the log of a logistic transformation of the unemployment rate. Once again, these transformations lead to unbounded series that are non-stationary but cointegrated in the data.

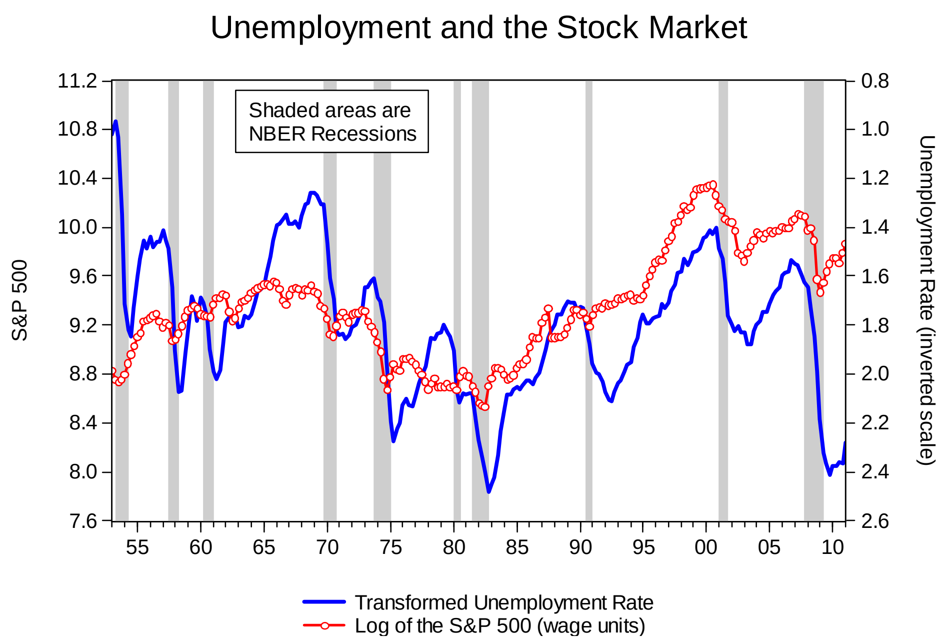

Figure 5: Unemployment and the Stock Market

Although it may not be apparent by inspection, the connection between the real value of the S&P and the transformed unemployment rate is remarkably stable. Farmer (2012b) shows that the two series are well described by a cointegrated vector autoregression and that an equation estimated on data from 1955q1 through 1979q3 does a very good job of explaining the one-step ahead forecast errors for the sample period form 1979q4 through 2011q1.

A further prediction of the theory, is that the real wage should move procyclically with GDP. Figure 6 shows that this prediction is in line with data at low frequencies. The figure plots GDP in wage units on the left axis and a measure of the consumption real wage on the right axis.

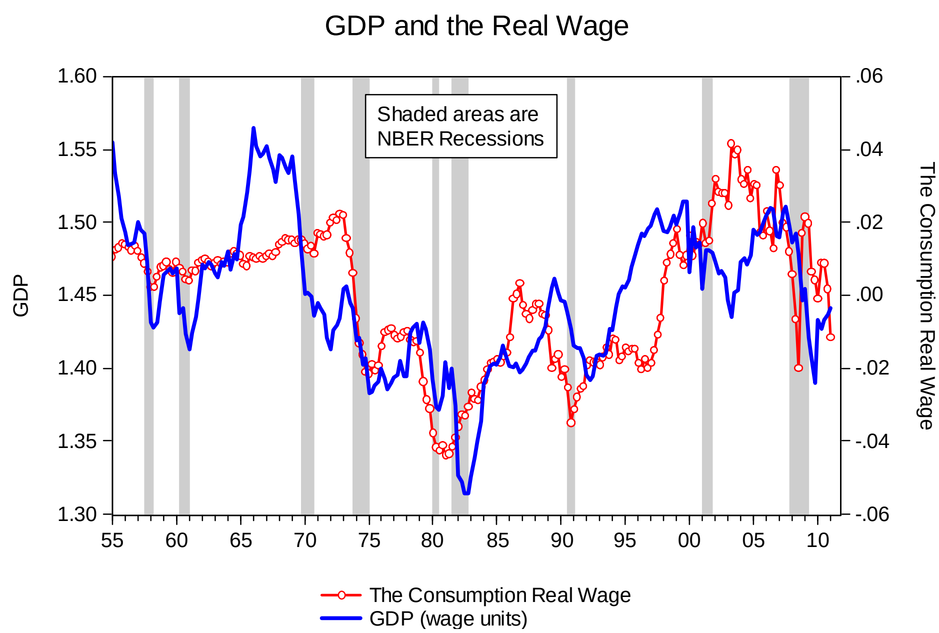

Figure 6: GDP and the Real Wage

The consumption real wage is constructed by dividing the wage series by the consumption deflator and taking deviations from a linear trend.[13] In the data, movements in the real wage precede movements in GDP. In the model, which abstracts from labor market dynamics, these movements are coincident.

11 High Frequency Facts

Figure 7 plots the S&P 500 and Shiller's measure of smoothed corporate earnings for quarterly data from 1960 through 2011. Both series are measured in logs. This figure demonstrates that business cycles are associated not just with big runups in asset prices, but also with long periods over which prices grow faster or slower than earnings.

[13]The consumption real wage is the author's measure of the money wage divided by the non-durable consumption-goods deflator from the NIPA accounts. This is an appropriate measure of the real wage in a model that abstracts from investment. I have detrended this series since the model abstracts from productivity growth.

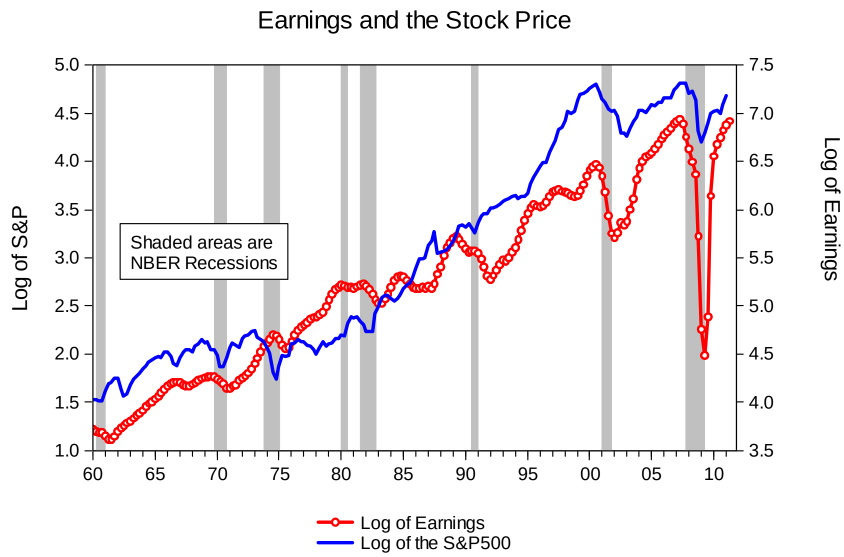

Figure 7: Earnings and the S&P 500

The S&P 500 and Shiller's earnings series, are well represented by cointegrated random walks, a prediction that is consistent with the model developed in this paper. But the theory predicts much more.

In the model I have constructed, earnings, the stock price, consumption and employment should all move in tandem, not just at low frequencies, but also at high frequencies. Figures 4 through 7 demonstrate that this prediction is violated. It is clear from these figures that there are significant high-frequency movements in the stock price that do not mirror movements either in consumption or in earnings. Indeed, given the model's simplicity, it would be surprising if that were not the case.[14]

Part of the reason that the example cannot account for short-run deviations of consumption and earnings from the stock price is that I have chosen a very special example that can be solved in closed form. This example, where $1 - \rho = \eta$, implies that the price-earnings ratio is a constant. Relaxing this

[14]These high frequncy correlations are beyond the scope of the current paper and will be dealt with in separate forthcoming work.

parametric restriction allows the model to capture movements in the price-earnings ratio and with richer parameterizations, consumption is no longer tied to the stock price in the short run.

12 Concluding Comments

How does my work fit into other recent theories of the crisis? There is a large literature that tries to understand the role of credit in a financial collapse. I do not want to deny that leverage plays a role in financing the expansion phase that precedes a financial crisis. But in the model I have developed in this paper, leverage is a symptom of the process that ends in financial collapse, and not the cause. In a high employment equilibrium, asset prices will be high and households and firms will be able to finance a relatively high amount of debt. When the bull market ends and asset prices crash, the debt that was accrued during the expansion phase of the cycle will appear burdensome to those who are indebted.

The two most recent recessions look a lot more like the 1929 contraction than any of the other post-war recessions. Each of them was accompanied by a boom and subsequent bust in asset prices, a feature that was not present in the other nine post-war recessions. In my view, the deregulation of financial markets in the 1990s had a lot to do with that. But what allowed asset prices to grow so fast in the first place, and why was asset price growth not arbitraged away by efficient financial markets?

The answer is that a rapid expansion in asset prices is part of a rational expectations equilibrium in a world of multiple equilibria. It is not an example of 'irrational exuberance'. There is nothing in the economic environment to dictate that a bull market *must* come to an end in any given time period. But, equally, there is no reason why it should persist forever. Financial crises result from changing moods in the financial markets. But although they are equilibrium phenomena, in the sense of modern macroeconomic theory, they are not socially optimal. In the model I have constructed, not all equilibria are efficient, and that has important implications. My work suggests that economic policies designed to reduce the volatility of asset market movements will significantly increase economic welfare.

Appendix A

This appendix describes the problem that would be faced by a social planner whose goal was to maximize the utility of the representative household.

The economy satisfies all of the assumptions of standard general equilibrium theory. There are two convex technologies and preferences are assumed to be concave, hence the programming problem, defined as

$$\max_{\{V_t, L_{t+1}\}} E_s \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \left(\frac{\left[b S_t^{\rho} \left(L_t - V_t \right)^{\rho} + a \right]^{\frac{1-\eta}{\rho}}}{1-\eta} + \psi_t \left[L_t \left(1 - \delta \right) + \left(\Gamma V_t \right)^{\frac{1}{2}} \left(1 - L_t \right)^{\frac{1}{2}} - L_{t+1} \right] \right) \right\} \quad \text{(A1)}$$

has a unique solution.

Proposition 3 *Define the constants A, B, and C as follows,*

$$A = \frac{\beta \Gamma^{\frac{1}{2}}}{2}, \quad B = 1 - \beta \left(1 - \delta \right), \quad C = \frac{\beta \Gamma^{\frac{1}{2}}}{2}. \quad \text{(A2)}$$

Let $\bar{X}$ be the unique positive root of the quadratic

$$AX^2 + BX - C = 0, \quad \text{(A3)}$$

where $\bar{X}$ is given by the expression

$$\bar{X} = \frac{-\left[1 - \beta \left(1 - \delta \right) \right] + \sqrt{\left[1 - \beta \left(1 - \delta \right) \right]^2 + \Gamma \beta^2}}{\beta \Gamma^{\frac{1}{2}}}. \quad \text{(A4)}$$

For values of β close to 1, the optimal sequences $\{V_s, L_s\}_{s=t}^{\infty}$ that solve (A1) converge asymptotically to a pair of numbers $\{L, V\}$ where

$$L = \frac{\Gamma^{\frac{1}{2}} \bar{X}}{\delta + \Gamma^{\frac{1}{2}} \bar{X}}, \qquad V = \left(\frac{\delta}{\delta + \Gamma^{\frac{1}{2}} \bar{X}} \right) \bar{X}^2. \quad \text{(A5)}$$

Proof of Proposition 3

Proof. A solution to Problem (A1) must satisfy the following first order conditions,

$$b\frac{\left[bS_t^{\rho}\left(L_t-V_t\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}S_t^{\rho}}{\left(L_t-V_t\right)^{1-\rho}}=\frac{1}{2}\psi_t\Gamma^{\frac{1}{2}}\left(\frac{1-L_t}{V_t}\right)^{\frac{1}{2}}, \tag{A6}$$

$$\psi_t=E_t\left\{\beta\left(b\frac{\left[bS_{t+1}^{\rho}\left(L_{t+1}-V_{t+1}\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}S_{t+1}^{\rho}}{\left(L_{t+1}-V_{t+1}\right)^{1-\rho}}\right.\right.$$
$$\left.\left.+\psi_{t+1}\left[\left(1-\delta\right)-\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V_{t+1}}{1-L_{t+1}}\right)^{\frac{1}{2}}\right]\right)\right\}, \tag{A7}$$

$$L_{t+1}=L_t\left(1-\delta\right)+\left(\Gamma V_t\right)^{\frac{1}{2}}\left(1-L_t\right)^{\frac{1}{2}}. \tag{A8}$$

These equations must be obeyed by the optimal path $\{L_{s+1},V_s,\psi_s\}_{s=t}^{\infty}$ where L_t is given by an initial condition. Since the problem is concave, the solution is unique.

Let $\{L,V,\psi\}$ be a non-stochastic steady state solution of (A1), defined as a solution to the equations,

$$b\frac{\left[b\left(L-V\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}}{\left(L-V\right)^{1-\rho}}=\frac{\psi}{2}\Gamma^{\frac{1}{2}}\left(\frac{1-L}{V}\right)^{\frac{1}{2}}, \tag{A9}$$

$$\psi=\beta b\frac{\left[b\left(L-V\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}}{\left(L-V\right)^{1-\rho}}+\beta\psi\left(1-\delta\right)-\beta\psi\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V}{1-L}\right)^{\frac{1}{2}}. \tag{A10}$$

Rearranging these expressions, defining

$$X=\left(\frac{V}{1-L}\right)^{\frac{1}{2}}, \tag{A11}$$

gives

$$AX^2 + BX - C = 0, \tag{A12}$$

where,

$$A = \frac{\beta\Gamma^{\frac{1}{2}}}{2}, \quad B = 1 - \beta(1-\delta), \quad C = \frac{\beta\Gamma^{\frac{1}{2}}}{2}. \tag{A13}$$

This establishes the quadratic defined in the proposition. The values of L and V are found by combining (A11) with the steady state value of (30), given by,

$$\delta L = (\Gamma V)^{\frac{1}{2}}(1-L)^{\frac{1}{2}}. \tag{A14}$$

The local existence and convergence of dynamic paths, when β is 'close enough' to 1, is a consequence of the turnpike property of optimal growth models. See, for example, Cass (1966). ■

Appendix B

Proof of Proposition 1

Proof. Since only real variables are determined in equilibrium we are free to choose the normalization $w = 1$. In a steady state equilibrium it follows from (36) that,

$$p_k = \frac{a\beta}{1-\beta} Z, \tag{B1}$$

where

$$Z \equiv pC^{1-\rho}. \tag{B2}$$

We now seek an expression for Z as a function of L.

Combining (39) with (41), using the normalization $w = 1$, we have,

$$\frac{(1-\beta(1-\delta))}{q} b \left(\frac{C}{L-V}\right)^{1-\rho} = \beta b \left(\frac{C}{L-V}\right)^{1-\rho} - \beta \frac{1}{p}. \tag{B3}$$

Combining (42) and (43) gives

$$q = \frac{\Gamma(1-L)}{\delta L}, \tag{B4}$$

and substituting for q from (B4) in (B3) gives

$$\frac{1}{p} = b\left(\frac{C}{L-V}\right)^{1-\rho}\left(\frac{\beta\Gamma(1-L) - \delta(1-\beta(1-\delta))L}{\beta\Gamma(1-L)}\right). \tag{B5}$$

Note that prices are non-negative since $V \leq L$, $L \leq 1$ and

$$\beta\Gamma > \delta(1-\beta(1-\delta))L + \beta\Gamma,$$

where this inequality follows since $\beta < 1$ and $\delta < 1$. We next seek an expression for V as a function of L. Substituting from (B4) into (43) gives

$$V = \frac{\delta^2 L^2}{\Gamma(1-L)}, \tag{B6}$$

and hence

$$L - V = L\left(1 - \frac{\delta^2 L}{\Gamma(1-L)}\right). \tag{B7}$$

Substituting from (B7) into (B5) and rearranging terms gives

$$\begin{aligned} p_k &\equiv \frac{a\beta}{1-\beta} p C^{1-\rho} \\ &= \left(\frac{a\beta}{1-\beta}\right)\frac{\beta L^{1-\rho}\Gamma^{p}(1-L)^{\rho}\left[\Gamma(1-L)-\delta^2 L\right]^{1-\rho}}{b\left(\beta\Gamma(1-L)-\delta(1-\beta(1-\delta))L\right)} \equiv g(L). \end{aligned} \tag{B8}$$

Finally, using the definitions of Ω, μ and λ from (45), we have

$$g(L) = \frac{\Omega L^{1-\rho}(1-L)^{\rho}[\lambda - L]^{1-\rho}}{\mu - L},$$

which establishes the form of the function g. ■

Appendix C

Proof of Proposition 2

Proof. We must show that, for $\rho > 0$, g is strictly increasing. First notice that from (45) that, since $0 < \beta < 1$,

$$\mu < \lambda < 1. \tag{C1}$$

Taking the logarithmic derivative of g gives

$$\frac{L}{g}\frac{\partial g'}{\partial L}\bigg|_L = (1-\rho) - \rho\frac{L}{1-L} - (1-\rho)\frac{L}{\lambda - L} + \frac{L}{\mu - L}. \tag{C2}$$

Rearranging terms

$$\overbrace{(1-\rho)}^{A_1} + \overbrace{\rho L\left(\frac{1}{\lambda - L} - \frac{1}{1-L}\right)}^{A_2} + \overbrace{L\left(\frac{1}{\mu - L} - \frac{1}{\lambda - L}\right)}^{A_3} > 0 \tag{C3}$$

where

$$A_1 > 0, \;\; A_2 > 0 \text{ and } \;A_3 > 0 \;\text{ for all }\; L \leq \mu. \tag{C4}$$

The first inequality follows since $0 \leq \rho \leq 1$, and the second two inequalities follow from the additional facts that $\mu < \lambda < 1$ and $L < \mu$. ■

Appendix D

Aggregate Supply in the Simplified Model

This Appendix derives the aggregate supply curve, Equation (55). By combining Equation (50) with Equation (D1),

$$L_t = q_t V_t, \tag{D1}$$

and imposing the symmetric equilibrium conditions, $\bar{L}_t = L_t$ and $\bar{V}_t = V_t$ we arrive at the following expression for q_t

$$q_t = \frac{\Gamma}{\bar{L}_t}. \tag{D2}$$

Each firm solves the static problem,

$$\max \left[bS_t^\rho \left(L_t - V_t\right)^\rho + aK_t^\rho\right]^{\frac{1}{\rho}} - \frac{w_t}{p_t}L_t - \frac{r_t}{p_t}K_t \tag{D3}$$

where

$$L_t = X_t + V_t, \tag{D4}$$

and

$$L_t = q_t V_t. \tag{D5}$$

Substituting from (D4) and (D5) into (D3) and using (D2) leads to the reduced form problem,

$$\max \left[bS_t^\rho L_t^\rho \left(1 - \frac{\bar{L}_t}{\Gamma}\right)^\rho + aK_t^\rho\right]^{\frac{1}{\rho}} - \frac{w_t}{p_t}L_t - \frac{r_t}{p_t}K_t, \tag{D6}$$

which is maximized when

$$bS_t^\rho \left(1 - \frac{\bar{L}_t}{\Gamma}\right)^\rho \left(\frac{C_t}{L_t}\right)^{1-\rho} = \frac{w_t}{p_t}, \tag{D7}$$

and

$$a\left(C_t\right)^{1-\rho} = \frac{r_t}{p_t}, \tag{D8}$$

where I have made use of the equilibrium assumption to set $K_t = 1$. The aggregate supply curve, Equation (55) is derived by setting $w_t = 1$, imposing

$L_t = \bar{L}_t$, and solving (D7) for $Z_t \equiv p_t C_t^{1-\rho}$,

$$Z_t = \frac{\Gamma^\rho L_t^{1-\rho}}{b S_t^\rho \left(\Gamma - L_t\right)^\rho}. \tag{D9}$$

Appendix E

Aggregate Demand in the Simplified Model

This Appendix derives the aggregate demand curve, Equation (58). Consider the Euler equation,

$$C_t^{-\eta} = E_t \left\{ \beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}} \left(\frac{p_{k,t+1} + r_{t+1}}{p_{k,t}} \right) \right\}. \tag{E1}$$

We may combine this expression with the first order condition for profit maximization,

$$\frac{r_t}{p_t} = \alpha C_t^{1-\rho}, \tag{E2}$$

to generate the expression,

$$\frac{p_{k,t}}{C_t^\eta p_t} = E_t \left\{ \frac{\beta p_{k,t+1}}{p_{t+1} C_{t+1}^\eta} + \alpha \beta C_{t+1}^{1-\rho-\eta} \right\}. \tag{E3}$$

When the restriction $1 - \rho = \eta$ holds, this reduces to

$$\frac{p_{k,t}}{C_t^\eta p_t} = E_t \left\{ \frac{\beta p_{k,t+1}}{p_{t+1} C_{t+1}^\eta} + \alpha \beta \right\}. \tag{E4}$$

which can be iterated forwards to give the expression in Equation (58).

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