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THE SOCIAL COST OF NEAR-RATIONAL INVESTMENT

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ABSTRACT

We show that the stock market may fail to aggregate information even if it appears to be efficient and that the resulting decrease in the information content of stock prices may drastically reduce welfare. We solve a macroeconomic model in which information about fundamentals is dispersed and households make small, correlated errors when forming expectations about future productivity. As information aggregates in the market, these errors amplify and crowd out the information content of stock prices. When stock prices reflect less information, the conditional variance of stock returns rises. This increase in financial risk distorts the long-run level of capital accumulation, and causes costly (first-order) distortions in the long-run level of consumption.

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1 Introduction

Efficient markets incorporate all available information into stock prices. As a result, investors can learn from equilibrium prices and update their expectations accordingly. But if investors learn from equilibrium prices, *anything* that moves prices has an impact on expectations held by *all* market participants. We explore the implications of this basic dynamic in a world in which people are less than perfect – a world in which they make small correlated errors when forming their expectations about the future.

We find that relaxing the rational paradigm in this minimal way results in a drastically different equilibrium with important consequences for financial markets, capital accumulation, and welfare: if information is dispersed across investors, the private return to making diligent investment decisions is orders of magnitude lower than the social return. If we allow for individuals an economically small propensity to make common errors in their investment decisions, information aggregation endogenously breaks down precisely when it is most socially valuable (i.e., when information is highly dispersed). This endogenous informational inefficiency results in higher conditional variance of asset returns and socially costly first-order distortions in the level of capital accumulation, labor supply, output, and consumption.

Our model builds on the standard real business cycle (RBC) model in which a consumption good is produced from capital and labor. Households supply labor to a representative firm and invest their wealth by trading claims to capital ("stocks") and bonds. The consumption good can be transformed into capital, and vice versa, by incurring a convex adjustment cost. The accumulation of capital is thus governed by its price relative to the consumption good (Tobin's Q). The only source of real risk in the economy are shocks to total factor productivity. We extend this standard setup by assuming each household receives a private signal about productivity in the next period, and solve for equilibrium expectations.

As a useful benchmark, we first examine two extreme cases in which the stock market has no role in aggregating information. In the first case, the private signal is perfectly accurate such that all households know next period's productivity without having to extract any information from the equilibrium price. In this case, our model is similar to the "News Shocks" model of Jaimovich and Rebelo (2009), in which all information about the future is common. The opposite extreme is the case in which the private signal consists only of noise. In this case, our model resembles the standard RBC model in which no one in the economy has any information about the future and there is consequently nothing to learn from the equilibrium stock price. Households face less financial risk in the former case than in the latter: the more households know about the future, the more information is reflected in the equilibrium price, and the lower the conditional variance of equilibrium stock returns.

The paper centers on the more interesting case in which households' private signals are neither perfectly accurate nor perfectly inaccurate: private signals contain both information about future productivity and some idiosyncratic noise (information is dispersed). In this case, households' optimal behavior is to look at the equilibrium stock price and use it to learn about the future. When information is dispersed, the stock market thus serves to aggregate information.

If all households are perfectly rational, the stock market is an effective aggregator of information: as long as the noise in the private signal is purely idiosyncratic, the equilibrium stock price becomes perfectly revealing about productivity in the next period (Grossman, 1976). When the stock price is perfectly revealing, the conditional variance of stock returns is just as low as it is in the case in which the private signal is perfectly accurate.

We then show that the economy behaves very differently if we allow households to deviate slightly from their fully rational behavior. In the "near-rational expectations equilibrium", households make small, cross-sectionally correlated errors when forming their expectations about future productivity. They are slightly too optimistic in some states of the world and slightly too pessimistic in others but, on average, do not have a bias in their expectations. When the average household is slightly too optimistic, it wants to invest slightly more of its wealth in stocks, and the stock price must rise. Households that observe this higher stock price may interpret it in one of two ways: as either a result of errors made by their peers or, with some probability, a reflection of more positive information about future productivity received by other market participants. Rational households should thus revise their expectations of future productivity upwards whenever they see a rise in the stock price. As households revise their expectations upward, the stock price must rise further, triggering yet another revision in expectations, and so on. Small errors in the expectation of the average household may thus lead to much larger, non-fundamental, deviations in the equilibrium stock price. The more dispersed information is across households, the stronger this feedback effect, because households rely more heavily on the stock price when their private signal is relatively uninformative. In fact, we show that small correlated errors in household expectations may destroy the stock market's capacity to aggregate information if information is sufficiently dispersed. The stock market's ability to aggregate information is thus most likely to break down precisely when it is most socially valuable. As small common errors reduce the amount of information reflected in the equilibrium stock price, the conditional variance of stock returns increases, and thus in the amount of financial risk households face rises.

The fact that we allow for small common deviations from fully rational behavior is central to these results. Small common errors in household expectations affect information aggregation through an information externality: an individual household does not internalize that making a small error that is correlated with the common error affects other households' ability to learn about the future. This externality is more severe when information is more dispersed, and it endogenously determines the market's capacity to aggregate information. By contrast, common noise in the signals households observe has no such external effects on the market's capacity to aggregate information.

We show in a simple application of the envelope theorem that households have little economic incentive to avoid small common errors in their expectation of future productivity – the expected utility cost to an individual household due to small errors in its expectation is economically small. A large literature in behavioral finance has developed a wide range of psychologically founded mechanisms that prompt households to make common errors in their investment decisions.[1] We thus remain open to many possible interpretations of the small common errors households make in our model. The idea is simply that households make small mistakes, and the integral over these mistakes has a non-degenerate distribution.

Although households have little incentive to avoid such "near-rational" errors in their behavior, these errors entail a first-order cost to society. A collapse in the stock market's capacity to aggregate information increases the financial risk investors face, prompting them to demand a higher equity premium. A rise in the equity premium in turn distorts the *level* of capital accumulation, labor supply, output, and consumption in the long run (at the stochastic steady state). Near-rational behavior thus has a level effect on output and consumption at the stochastic steady state. In addition, amplified near-rational errors result in non-fundamental volatility in stock prices, capital investment, labor supply, and all other economic aggregates.

We estimate our model to match key macroeconomic and financial data. Our results suggest that stock prices aggregate a significant amount of dispersed information, but that much of this information is crowded out in equilibrium.

We quantify the aggregate welfare losses attributable to near-rational behavior as the percentage rise in consumption that would make households indifferent between remaining in an economy in which the conditional variance of stock returns is high (the near-rational expectations equilibrium) and transitioning to the stochastic steady state of an economy in which all households behave fully rationally until the end of time. Our estimates of aggregate welfare losses attributable to near-rational behavior range between 3.59% and 69.71% of lifetime consumption, while the incentive to an individual household to avoid small common errors in its expectations is economically small (0.01% of lifetime consumption). In all cases, the social cost of near-rational behavior is two to three orders of magnitude larger than the private cost.

Related Literature To our knowledge, this paper is the first to address the welfare costs of pathologies in information aggregation within a full-fledged dynamic stochastic general

[1] Some examples are Odean (1998), Odean (1999), Daniel, Hirshleifer, and Subrahmanyam (2001), Barberis, Shleifer, and Vishny (1998), Bikhchandani, Hirshleifer, and Welch (1998), Hong and Stein (1999) and Allen and Gale (2003). In Hassan and Mertens (2011) we give one such interpretation in which households that want to insulate their investment decisions from the errors their peers make ("market sentiment") have to pay a small mental cost. In equilibrium, households then choose to make small, common errors of the type we assume in this paper.

equilibrium model.

In our model, investors' near-rational errors are endogenously amplified and result in a deterioration of the information content of asset prices. The equilibrium of the economy thus behaves as if irrational noise traders are de-stabilizing asset prices, although all individuals are almost perfectly rational. In this sense, our paper relates to the large literature on noisy rational expectations equilibria following Hellwig (1980) and Diamond and Verrecchia (1981), in which exogenous noise trading (or, equivalently, a stochastic supply of the traded asset) impede the aggregation of information.[2] Relative to this literature, we make progress on two dimensions. First, we are able to make statements about social welfare because the introduction of near-rational behavior puts discipline on the amount of noise in equilibrium asset prices, which is consistent with the notion that the losses accruing to individual households that cause this noise must be economically small. Second, we show that a given amount of near-rational errors has a more detrimental effect on the aggregation of information when information is more dispersed.[3]

The recent literature on pathologies in information aggregation in financial markets has focused on information externalities arising either from strategic complementarities or from higher-order uncertainty:[4] Amador and Weill (2012) and Goldstein et al. (2013) study models in which individuals have an incentive to overweight public information relative to private information due to a strategic complementarity. In their models, noise in public signals is endogenously amplified due to this over weighting. In Allen et al. (2006), Bacchetta and Van Wincoop (2008), and Qiu and Wang (2010), agents have differing information sets about multi-period returns and therefore must form expectations about the expectations of others. The dynamics of these higher-order expectations drive a wedge between asset prices and their fundamentals. This paper highlights a third, more basic type of information externality that arises even when no strategic complementarities are present and first-order expectations fully determine asset prices:[5] -individuals do not internalize how errors in their investment decisions affect others' equilibrium expectations. Pathologies similar to those outlined in this paper are thus likely to arise in any setting in which households observe asset prices that aggregate dispersed information.

We also contribute to a large literature that studies the welfare cost of pathologies in stock markets, including Stein (1987), Chauvin et al. (2011), and Lansing (2012). Most closely related are DeLong, Shleifer, Summers, and Waldmann (1989), who analyze the general equilibrium effects of noise-trader risk in an overlapping generations model with endogenous capital accumula-

[2]Most closely related are Wang (1994), where noise in asset prices arises endogenously from time-varying private investment opportunities, and Albagli (2011), where noise trader risk is amplified due to liquidity constraints on traders.

[3]The notion of near-rationality is due to Akerlof and Yellen (1985) and Mankiw (1985). Our application is closest to Cochrane (1989) and Chetty (2012), who use the utility cost of small deviations around an optimal policy to derive "economic standard errors." Other recent applications include Woodford (2010) and Dupor (2005).

[4]For an approach to pathologies in social learning based on social dynamics rather than on information externalities, see Burnside et al. (2011).

[5]The provision of public information thus always raises welfare in our framework (see Appendix B.2).

tion. A large literature in macroeconomics and in corporate finance focuses on the sensitivity of firms' investment to a given mispricing in the stock market. Some representative papers in this area are Blanchard, Rhee, and Summers (1993), Baker, Stein, and Wurgler (2003), Gilchrist, Himmelberg, and Huberman (2005), and Farhi and Panageas (2006).[6] One conclusion from this literature is that investment responds only moderately to mispricings in the stock market or that the stock market is a "sideshow" with respect to the real economy (Morck, Shleifer, and Vishny (1990)). We provide a new perspective on this finding: in our model, welfare losses are driven mainly by a distortion in the stochastic steady state, rather than an intertemporal misallocation of capital. In all of our calibrations, the observed sensitivity of the capital stock with respect to stock prices is uninformative about the welfare consequences of non-fundamental volatility in stock prices. Pathologies in the stock market may thus have large welfare consequences even if the stock market appears to be a "sideshow. "

This finding also relates to a large literature on the costs of business cycles:[7] First, we emphasize that macroeconomic fluctuations affect the *level* of consumption if they create financial risk. Standard cost-of-business cycles calculations in the spirit of Lucas (1987) do not capture this level effect.[8] Second, our model suggests this level effect may cause economically large welfare losses if uncertainty about macroeconomic fluctuations indeed cause the large amounts of financial risk which we observe in the data.

At a methodological level, an important difference from existing work is that our model requires solving for equilibrium expectations under dispersed information in a non-linear general equilibrium framework. Although a large body of general equilibrium models with dispersed information exist, such existing models feature policy functions that are (log) linear in the expectation of the shocks agents learn about (e.g. Hellwig (2005), Lorenzoni (2009), Angeletos et al. (2012), and Angeletos and La'O (2013)). However, in the standard RBC model with capital accumulation and decreasing returns to scale, households' policies are non-linear functions of the average expectation of future productivity. We are able solve our model due to recent advances in computational economics. We follow the solution method in Mertens (2009), which builds on Judd (1998) and Judd and Guu (2001) in using an asymptotically valid higher-order expansion in all state variables around the deterministic steady state of the model in combination with a nonlinear change of variables (Judd (2002)).[9]

In a closely related paper, Mertens (2009) derives welfare-improving policies for economies

[6] Also see Galeotti and Schiantarelli (1994); Polk and Sapienza (2004); Panageas (2005); and Chirinko and Schaller (2006)

[7] See Barlevy (2005) for an excellent survey.

[8] This finding is similar to the level effect of uninsured idiosyncratic investment risk on capital accumulation in Angeletos (2007).

[9] Closely related from a methodological perspective are Tille and van Wincoop (2008), who solve for portfolio holdings of international investors, using an alternative approximation method developed in Tille and van Wincoop (2010) and Devereux and Sutherland (2011).

in which distorted beliefs create too much volatility in asset markets. He shows that the stabilization of asset markets enhances welfare and that history-dependent policies may improve the information content of asset prices.

The remainder of the paper is structured as follows. Section 2 derives the main theoretical insights of the model in a simple three-period model. Section 3 discusses the robustness of the main insights and compares our near-rational approach to the standard noisy rational expectations model. Section 4 introduces our mechanism into a quantitative RBC model with endogenous capital accumulation. Section 5 estimates the model and presents quantitative results.

2 Static Model

The model economy exists at three time periods $t = 0, 1, 2$. At $t = 1$, an endowment of a numéraire good can be stored until $t = 2$, or converted into a unit of capital at adjustment cost $\frac{1}{2\kappa}K^2$, where $\kappa \geq 0$. At $t = 2$, each unit of capital returns η units of the numéraire:

$$Y = \eta K,\ \ \eta \sim N\left(\bar{\eta}, \sigma_\eta^2\right). \tag{1}$$

The capital adjustment technology is operated by an investment goods sector that performs instant arbitrage between the price of capital traded in a Walrasian stock market at $t = 1$, Q, and the number of units of capital in circulation:

$$\max_K\ \Pi = QK - K - \frac{1}{2\kappa}K^2.$$

Its first-order condition requires that it supply

$$K = \kappa(Q - 1) \tag{2}$$

units of capital to maximize arbitrage profits.

A continuum of identical households indexed by $i \in [0, 1]$ populates the economy. At $t = 1$, each household observes Q and receives a private signal about productivity:

$$s_i = \eta + \nu_i, \tag{3}$$

where ν_i represents *i.i.d.* draws from a normal distribution with zero mean and variance σ_ν^2.[10] Given s_i, each household chooses to purchase z_i units of capital ("stocks") to maximize expected

[10] In section 3, we show that the conclusions of our model hold for more general information structures in which the noise in the private signal is correlated across households and households observe a public signal in addition to their private signal.

utility from terminal wealth, $\mathcal{E}_{1i}[U_i]$, where

$$U_i = w_{2i} - \frac{\rho}{2} V_1[w_{2i}], \tag{4}$$

with $\rho > 0$. $V_1[w_{2i}]$ is the posterior variance given s_i and Q, and terminal wealth is given by

$$w_{2i} = z_i(\eta - Q) + \Pi, \tag{5}$$

where Π is a lump-sum transfer of profits from the investment goods sector.

When forming their expectation about productivity, households make a small error. This small error shifts the posterior probability density function of η by $\mu_i(\epsilon + \hat{\epsilon}_i)$, where the constant $\mu_i \geq 0$ measures household i's exposure to the error $\epsilon + \hat{\epsilon}_i$. We refer to a household as near-rational if $\mu_i = 1$ and as fully rational if $\mu_i = 0$. The expectational error is positively correlated across households, where $\epsilon \sim N(0, \sigma_\epsilon)$ is the common component that is the same across all households, $\hat{\epsilon}_i \sim N(0, \hat{\mu}\sigma_\epsilon)$ is the idiosyncratic component, and $\hat{\mu}$ calibrates the size of the correlation across households. The expectations operator $\mathcal{E}_{1i}$ is thus the rational expectations operator, except that it allows households to make small mistakes about the conditional mean of η:

$$\mathcal{E}_{1i}[\eta] = E_{1i}[\eta] + \mu_i(\epsilon + \hat{\epsilon}_i), \tag{6}$$

where $E_{1i}[\eta] = E[\eta|Q, s_i]$ is the rational expectations operator conditional on the information available to the household at time 1. Although near-rational households make correlated mistakes when forming their expectations, they understand the structure of the economy, understand the equilibrium mapping of information into Q, and have the correct perception of all higher moments of the conditional distribution of η.[11] Importantly, households know they and others load on the common error, ϵ.

The parameter σ_ϵ calibrates the size of households' expectational errors. In the limit in which $\sigma_\epsilon \to 0$, all households behave fully rationally and thus fully maximize their utility. A crucial assumption of the model is that σ_ϵ is small enough such that the utility gain to a near-rational household of eliminating the expectational error from its own behavior is economically small.

Definition 2.1
A household's behavior is near-rational if its time-zero willingness to pay to set $\mu_i = 0$ rather than $\mu_i = 1$ is below a threshold level, $\bar{\lambda} > 0$:

$$\lambda^i \equiv -\left(E_0\left[U_i|_{\mu_i=1}\right] - E_0\left[U_i|_{\mu_i=0}\right]\right) \leq \bar{\lambda}. \tag{8}$$

[11]Formally,

$$\mathcal{E}_{1i}\left[(\eta - \mathcal{E}_{1i}(\eta))^k\right] = E_{1i}\left[(\eta - E_{1i}(\eta))^k\right] \ \forall\ k > 1. \tag{7}$$

We thus pick σ_ϵ such that a near-rational households' incentive to eliminate the expectational error from its own behavior is economically small, $0 \leq \sigma_\epsilon \leq \bar{\sigma}_\epsilon$, where $\bar{\sigma}_\epsilon$ is the value of σ_ϵ for which (8) holds with equality.

Market clearing requires that aggregate demand for stocks is equal to the number of units of capital in circulation:

$$\int_0^1 z_i di = K. \tag{9}$$

We focus on symmetric equilibria such that $\mu_i = 1 \; \forall i$ and thus

$$\mathcal{E}_{1i}[\eta] = E[\eta|Q, s_i] + \epsilon + \hat{\epsilon}_i. \tag{10}$$

2.1 Solving the Model

Plugging (5) into (4) and taking the derivative with respect to z_i yields households' optimal demand for stocks:

$$z_i = \frac{\mathcal{E}_{1i}[\eta] - Q}{\rho V_1[\eta]}. \tag{11}$$

We can then use the market-clearing condition (9) and plug in (11) and (2) to show that the market price of capital is a linear function of the average expectation of η:

$$Q = \frac{\int_0^1 \mathcal{E}_{1i}[\eta]\, di + \kappa\rho V_1[\eta]}{1 + \kappa\rho V_1[\eta]}. \tag{12}$$

Households can thus directly infer $\int \mathcal{E}_{1i}[\eta]\, di$ from observing Q. We may guess that the equilibrium price function is linear in η and ϵ

$$Q = \pi_0 + \pi_1\eta + \gamma\epsilon. \tag{13}$$

Assuming this guess is correct, we can write the rational expectation of η given the private signal and Q as

$$E_{1i}[\eta] = \alpha_0 + \alpha_1 s_i + \alpha_2 \int_0^1 \mathcal{E}_{1i}[\eta]\, di, \tag{14}$$

where the constants α_0, α_1, and α_2 are the weights households give to the prior, the private signal, and the average expectation, respectively. Using (10), substituting (3), and taking the integral across individuals gives

$$\int_0^1 \mathcal{E}_{1i}[\eta]di = \frac{\alpha_0}{1-\alpha_2} + \frac{\alpha_1}{1-\alpha_2}\eta + \frac{1}{1-\alpha_2}\epsilon, \tag{15}$$

where the noise in private signals, ν_i, as well as the idiosyncratic errors, $\hat{\epsilon}_i$, integrate to zero. Plugging this expression back into (12) and matching coefficients with (13) verifies that the equilibrium price function is indeed linear.

It follows that in addition to s_i, households can extract an independent and unbiased signal about η, $\frac{(1-\alpha_2)}{\alpha_1}\int_0^1 \mathcal{E}_{1i}[\eta]di - \frac{\alpha_0}{\alpha_1} = \eta + \frac{1}{\alpha_1}\epsilon$, from observing Q. Bayes' rule then implies the posterior variance is the inverse of the sum of the precision of the prior and the two signals:

$$V[\eta|s_i, Q] = V_1[\eta] = \left(\sigma_\eta^{-2} + \sigma_\nu^{-2} + \alpha_1^2\sigma_\epsilon^{-2}\right)^{-1}. \tag{16}$$

Moreover, the conditional expectation of η is the precision-weighted sum of the signals and the prior mean divided by the posterior precision:[12]

$$E[\eta|s_i, Q] = \frac{\sigma_\eta^{-2}\bar{\eta} + \sigma_\nu^{-2}s_i + \alpha_1^2\sigma_\epsilon^{-2}\left(\eta + \frac{1}{\alpha_1}\epsilon\right)}{V_1[\eta]^{-1}}. \tag{17}$$

Matching coefficients with (14) yields

$$\alpha_0 = \frac{\bar{\eta}V_1(\eta)\sigma_\epsilon^2}{\sigma_\eta^2\left(\alpha_1 V_1(\eta) + \sigma_\epsilon^2\right)}, \quad (18) \qquad \alpha_1 = \frac{V_1[\eta]}{\sigma_\nu^2}, \quad (19) \qquad \alpha_2 = \frac{\alpha_1 V_1(\eta)}{\alpha_1 V_1(\eta) + \sigma_\epsilon^2}. \quad (20)$$

Solving the system equations consisting of the matched coefficients in (13), (16), (18), (19), and (20) leads to expressions in terms of parameters. Because the closed-form solution for $V_1[\eta]$ is somewhat cumbersome, it is convenient to rewrite the solution of all endogenous variables as a function of $V_1[\eta]$ and parameters.

Lemma 2.2
The unique linear symmetric equilibrium of this economy is characterized as follows:

$$\pi_0 = \frac{V_1[\eta]\left(\bar{\eta} + \kappa\rho\sigma_\eta^2\right)}{\left(1 + \kappa\rho V_1[\eta]\right)\sigma_\eta^2}, \quad (21) \qquad \pi_1 = \frac{\sigma_\eta^2 - V_1[\eta]}{\left(1 + \kappa\rho V_1[\eta]\right)\sigma_\eta^2}, \quad (22) \qquad \gamma = \frac{\left(\sigma_\eta^2 - V_1[\eta]\right)\sigma_\nu^2}{V_1[\eta]\left(1 + \kappa\rho V_1[\eta]\right)\sigma_\eta^2} \quad (23)$$

where the conditional variance is implicitly defined by

$$V_1[\eta] = \left(\sigma_\eta^{-2} + \sigma_\nu^{-2} + \frac{V_1[\eta]^2}{\sigma_\nu^4}\sigma_\epsilon^{-2}\right)^{-1}. \tag{24}$$

Proof *See Appendix A.2.*

These results have two immediate implications. First, note the standard deviation of the idiosyncratic component of expectational errors, $\hat{\mu}$, appears in none of the solutions above and is thus irrelevant for the stock price, the conditional variance of η, and all aggregate variables.

[12] See Appendix A.1 for a more detailed proof.

Second, although the sensitivity of the capital stock with respect to the stock price, κ, influences the response of the stock price to η and ϵ, it does not affect households' ability to learn about η. To see this, note that the conditional variance of η in (24) is independent of κ. Similarly, plugging (19) into (17) shows that the loadings of households' expectation operators on η and ϵ depend only on σ_ϵ, σ_ν, and σ_ϵ, and not on κ. How much households can learn about the future is thus unrelated to the function linking stock prices to the real economy. We will show in section 4 that this insight carries over to a quantitative model in which the equilibrium expectations operator continues to be a function of the same three variables, despite much more complex and non-linear macroeconomic dynamics.

2.2 External Effect on Information Aggregation

The last two terms on the right-hand side of (15) reflect two channels through which near-rational behavior affects equilibrium expectations. The last term shows that the small common error is amplified with the multiplier $1/(1-\alpha_2)$. Because α_2 is a number between 0 and 1, the multiplier is always larger than 1, reflecting the fact that the stock price also transmits the common error whenever it transmits information. The extent of amplification depends on how much households (rationally) rely on Q when forming their expectation of η. The bigger the weight they place on the stock price, α_2, the larger the error in equilibrium expectations relative to ϵ. The second term on the right-hand side reflects the indirect effect of near-rational behavior, which arises due to the fact that households optimally calculate the coefficients α_1 and α_2. When the market price of capital transmits an amplified common error in addition to information about η, households rationally lower α_2 and, as a result, decrease the equilibrium information content of stock prices.

To understand these two effects, it is useful to consider the limiting case in which all households are fully rational ($\sigma_\epsilon \to 0$). In this case, the model coincides with a standard rational expectations equilibrium in which the stock price is perfectly revealing about future productivity (commonly referred to as the "Grossman equilibrium"). Appendix A.3 shows that

$$\lim_{\sigma_\epsilon \to 0} \left[V_1\left[\eta\right]\right] = 0 \tag{25}$$

and

$$\lim_{\sigma_\epsilon \to 0} \left[\pi_1\right] = 1. \tag{26}$$

In this case, households put all weight on the stock price ($\alpha_2 = 1$) and no weight on their private signal ($\alpha_1 = 0$). In this limit, a marginally small common error is infinitely amplified, $\gamma = 1/(1-\alpha_2) = \infty$. The direct effect of near-rational behavior is thus to generate large non-fundamental errors in the market price of capital and large common errors in equilibrium expectations. The indirect effect of near-rational behavior is households' rational reaction to this fact. When the stock price transmits amplified common errors, households rationally reduce α_2, such that the multiplier on ϵ becomes finite. However, a reduction of α_2 also reduces the elasticity of the stock price with respect to information, π_1. The following proposition formalizes this intuition.

Proposition 2.3
Near-rational behavior globally decreases the elasticity of the stock price with respect to future productivity:

$$\frac{\partial \pi_1}{\partial \sigma_\epsilon} < 0 \ \forall \sigma_\epsilon > 0. \tag{27}$$

As the standard deviation of the small common error approaches 0, the marginal effect of near-rational behavior on this elasticity becomes infinitely large:

$$\lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\epsilon} \right] = -\infty. \tag{28}$$

Proof *See Appendix A.4.*

It follows that near-rational behavior has a first-order detrimental effect on the stock market's capacity to transmit information. This result contrasts sharply with the utility considerations of an individual household that loads on the small common error. At $t = 0$ (i.e., before the arrival of information about η), a household would be willing to pay

$$\lambda^i \equiv -\left(E_0 \left[U_i|_{\mu_i=1} \right] - E_0 \left[U_i|_{\mu_i=0} \right] \right) = -\left(E_0 \left[w_{2i}|_{\mu_i=1} \right] - E_0 \left[w_{2i}|_{\mu_i=0} \right] \right)$$

units of the numéraire to avoid loading on the near-rational error at $t = 1$. Note that for the purposes of calculating this compensating variation, we use the rational utility measure, i.e. the willingness to pay of a fully rational household that knows it has the opportunity to take advantage of its near-rational peers' errors at $t = 1$.

Proposition 2.4
As σ_ϵ goes to zero, the marginal disutility accruing to an individual household from increasing its loading on the near-rational error goes to zero:

$$\lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \lambda^i}{\partial \mu_i} \right] = 0.$$

Proof *See Appendix A.5.*

This result follows directly from the envelope theorem. Both the demand schedule for stocks in (11) and the expectations operator (14) are the result of an optimization. Near-rational errors, even if they contain a common component, represent a small deviation from the household's optimal program. Because the slope of the utility function at the optimum is 0, the marginal effect of these deviations on utility is also 0.

The key to understanding this result is that a hypothetical rational household ($\mu_i = 0$) that lives in an economy populated by near-rational households $\left(\mu_{j\neq i} = 1\right)$ has no informational advantage. Near-rational households understand fully that they and other households are making errors when forming their expectations. They react to this fact by lowering α_2 in response to a rise in the equilibrium amount of non-fundamental volatility in the market price of capital. Because learning about η is isomorphic to learning about ϵ, near-rational households thus already do everything possible to learn about the common component of the near-rational error. (From (13), it is clear that knowing η and Q is the same as knowing ϵ and Q; we could thus rewrite the entire optimal program using a signal extraction about ϵ and obtain the same result.) Absent an informational advantage, the optimal behavior of a fully rational household is then simply to implement the same optimal program, but without the near-rational error. The utility gain from behaving fully rationally instead of near rationally is thus 0 at the margin.

By continuity, propositions 2.3 and 2.4 imply that for small σ_ϵ near-rational behavior represents an externality that has a first-order detrimental effect on the market's ability to aggregate information, whereas households have only a negligible (lower-order) incentive to avoid making these errors. The stock market thus fails to aggregate information even though it is efficient in the sense that a fully rational household cannot systematically outperform a near-rational household with the same information set.

We next consider the comparative static of this result when private information becomes more dispersed in the economy.

Proposition 2.5

1. *Near-rational errors of a given size, σ_ϵ, have a more detrimental effect on information aggregation the more private information is more dispersed:*

$$\frac{\partial \pi_1}{\partial \sigma_\nu} = -\frac{\left(1+\kappa\rho\sigma_\eta^2\right)\left(2\sigma_\nu^2\sigma_\epsilon^2 V_1\left[\eta\right]^2 + 4V_1\left[\eta\right]^4\right)}{\sigma_\eta^2(1+\kappa\rho V_1[\eta])^2\left(\sigma_\nu^5\sigma_\epsilon^2 + 2\sigma_\nu V\left[\eta\right]^3\right)} \begin{cases} < 0 & \text{if } \sigma_\epsilon > 0 \\ = 0 & \text{in the limit } \sigma_\epsilon \to 0. \end{cases} \tag{29}$$

2. *Any strictly positive σ_ϵ may destroy the stock market's capacity to aggregate information as the dispersion of private information goes to infinity:*

$$\lim_{\sigma_\nu\to\infty}\left[\frac{V_1\left[\eta\right]}{\sigma_\eta^2}\right] = \begin{cases} 1 & \text{if } \sigma_\epsilon > 0 \\ 0 & \text{in the limit } \sigma_\epsilon \to 0. \end{cases} \tag{30}$$

Proof *See Appendix A.6.*

Figure 1 illustrates this point. It plots the ratio of the conditional variance of η to its unconditional variance over the level of dispersion of private information. To facilitate the interpretation of the results, we scale all standard deviations with the standard deviation of the productivity shock, σ_η. With this scaling, all standard deviations have a natural interpretation. In particular, the ratio $(\frac{\sigma_\nu}{\sigma_\eta})^2$ measures the level of dispersion of information in the economy as the number of individuals who, in the absence of a market price, would need to pool their private information in order to reduce the conditional variance of η by one half. A value of 0 on the vertical axis indicates households can perfectly predict tomorrow's realization of η, whereas a value of 1 indicates η is completely unpredictable.

The solid red line shows that when all households behave fully rationally ($\sigma_\epsilon = 0$), η is perfectly predictable, regardless of how dispersed information is in the economy. In this case, the market price of capital perfectly transmits all available information in the economy. This situation changes drastically when $\frac{\sigma_\epsilon}{\sigma_\eta} > 0$. The thick blue line plots the results for the case in which the standard deviation of the common component of the near-rational error is 1% of the standard deviation of η. The curve rises steeply and converges to 1. To the very left of the graph, when the private signal is more precise and households thus rely relatively little on the stock price when learning about the future, near-rational behavior has a relatively small detrimental effect on information aggregation. However, when we move to the right of the graph, households rely more on the stock price, and near-rational behavior has a larger detrimental effect. When information is highly dispersed, near-rational behavior results in the total collapse of information aggregation.

The second statement in Proposition 2.5 implies this qualitative result does not depend on *how* near-rational households are. Figure 1 plots the comparative statics for near-rational errors that are an order of magnitude larger ($\frac{\sigma_\epsilon}{\sigma_\eta} = 0.1$) and an order of magnitude smaller ($\frac{\sigma_\epsilon}{\sigma_\eta} = 0.001$) for comparison. In each case, the productivity shock becomes completely unpredictable if information is sufficiently dispersed.

The implication of this finding is that information aggregation in financial markets is most likely to break down precisely when it is most socially valuable – when information is highly dispersed. If the private signal households receive is sufficiently noisy, *any given amount of near-rational errors* in investor behavior may completely destroy the market's capacity to aggregate information.

Proposition 2.6

For a given level of σ_ϵ, the utility loss accruing to an individual household due to its own near-rational behavior decreases with the dispersion of private information in the economy:

$$\frac{\partial \lambda^i}{\partial \sigma_\nu} = -\sigma_\epsilon^2 \frac{\sigma_\nu^2 \sigma_\epsilon^2 (1+\hat{\mu}^2) + 2\hat{\mu}^2 V_1[\eta]^2 + 2\sigma_\nu^2 V_1[\eta]}{\rho(\sigma_\epsilon^2 \sigma_\nu^5 + 2\sigma_\nu V_1[\eta]^3)} < 0. \tag{31}$$

Proof *See Appendix A.7.*

Near-rational behavior thus becomes "cheaper" precisely when it is more harmful, because, again, learning about ϵ is isomorphic to learning about η. When households' private signals are more precise, they know more about η and thus more easily detect the common component of the near-rational error, which makes near-rational behavior costlier. When private signals are noisier and information aggregation breaks down, no one can learn much about the future. The less households can learn about the future, the less they can distinguish movements in the stock prices that are due to η from movements that are due to ϵ.

A crucial feature of this result is again that any collapse in the aggregation of information affects everyone in the economy: a fully rational household would do only a marginally better job at predicting η than the near-rational households. Conditional on receiving the same private signal, the difference in the expectation of the rational and near-rational household is small, $\epsilon + \hat{\epsilon}_i$. In fact, the posterior variance we plotted in Figure 1 is the conditional variance of such a fully rational household. We can write it as

$$\frac{V_1[\eta]}{\sigma_\eta^2} = \frac{1}{\sigma_\eta^2}\left(\alpha_1^2\sigma_\nu^2 + (1-\pi_1)^2\sigma_\eta^2 + (\gamma-1)^2\sigma_\epsilon^2\right). \tag{32}$$

The expression for the precision of the forecast of a near-rational household is identical, except the third term in brackets is then $\gamma^2\sigma_\epsilon^2$.[13]

Figure 2 decomposes the conditional variance (32) into its three components. The thick blue line in Figure 2 is the same as the thick blue in Figure 1. It plots the ratio of the conditional variance of η to its unconditional variance over the level of dispersion of private information for the case in which $\frac{\sigma_\epsilon}{\sigma_\eta} = 0.01$. The dotted line plots the first term on the right-hand side of (32), which is the error households make in their forecast of η_{t+1} due to the noise in their private signal. It is close to 0 throughout, reflecting the fact that households reduce α_1 when the private signal contains more noise, such that differences of opinion remain small in equilibrium. The broken line plots the second term, which is the error households make in their forecast because the stock price does not reflect all information about η_{t+1} (the "indirect" effect of near-rational behavior), and the third component is the error households make due to amplified common errors in the stock price (the "direct" effect of near-rational behavior).

At low levels of σ_ν, amplified small common errors are the main source of households' forecast errors. As private information becomes more dispersed, the amplification rises and eventually peaks as households, confronted with noisy private signals and a noisy stock price, begin to rely

[13] See Appendix A.1 for a formal derivation. By "precision" of the forecasts of near-rational household, we refer to $\frac{E_{it}\left[(\eta_{t+1}-\mathcal{E}_{it}(\eta_{t+1}))^2\right]}{\sigma_\eta^2}$. Note, however, that from (7), the near-rational household has the same "perceived" conditional variance as a rational household,
$\mathcal{E}_{it}\left[(\eta_{t+1}-\mathcal{E}_{it}(\eta_{t+1}))^2\right] = E_{it}\left[(\eta_{t+1}-E_{it}(\eta_{t+1}))^2\right].$

more on their priors. At the same time, the information content of the stock price begins to fall. In the region in which the broken line approaches 1, small common errors result in a complete collapse of information aggregation.

2.3 Real Effects of Near-rational Behavior

Now that we understand the aggregation of information in our model, we can ask how near-rational behavior affects the economy as a whole. Using (13), we can write equilibrium stock returns as

$$\eta - Q = -\pi_0 + \eta\left(1 - \pi_1\right) - \gamma\epsilon. \tag{33}$$

Near-rational behavior affects each of the three terms on the right-hand side. The third term on the right-hand side shows that it induces non-fundamental volatility in stock returns. Second, from (27), near-rational behavior reduces π_1 and thus causes a rise in the volatility of equilibrium stock returns and "excess volatility" (in the sense that the volatility of stock returns is 0 when all households behave fully rationally and positive if and only if $\sigma_\epsilon > 0$):

$$V_0\left[\eta - Q\right] = \gamma^2\sigma_\epsilon^2 + \left(1 - \pi_1\right)^2\sigma_\eta^2.$$

Near-rational behavior thus results in an increase in the amount of financial risk households face that invest in the stock market. Third, this increase in financial risk in turn induces a rise in the equity premium (the first term on the right-hand side). Taking time-zero expectations of (33) and using (21) and (22) to substitute for π_0 yields

$$E_0\left[\eta - Q\right] = \kappa\rho\left(\bar{\eta} - 1\right)\left(\frac{\left(1 - \pi_1\right)\sigma_\eta^2}{1 + \kappa\rho\sigma_\eta^2}\right). \tag{34}$$

These three effects of near-rational behavior on equilibrium stock returns are mirrored in the effect of near-rational behavior on the equilibrium capital stock.

Proposition 2.7
Near-rational behavior lowers the covariance between capital accumulation and productivity and reduces the expected level of capital accumulation by impeding the stock market's capacity to aggregate information:

$$\frac{\partial E_0\left[K\right]}{\partial\sigma_\epsilon} = \frac{\partial E_0\left[K\right]}{\partial\pi_1}\frac{\partial\pi_1}{\partial\sigma_\epsilon} < 0 \text{ and } \frac{\partial Cov_0\left[K,\eta\right]}{\partial\sigma_\epsilon} = \frac{\partial Cov_0\left[K,\eta\right]}{\partial\pi_1}\frac{\partial\pi_1}{\partial\sigma_\epsilon} < 0\ . \tag{35}$$

Proof *Plug (13) into (2) and to get*

$$K = \kappa\left(\pi_0 + \pi_1\eta + \gamma\epsilon - 1\right). \tag{36}$$

Taking time-zero expectations of (36) and using (21) and (22) to substitute for π_0 yields

$$E_0\left[K\right] = \kappa\left(\bar{\eta} - 1\right)\left(\frac{1 + \pi_1 \kappa \rho \sigma_\eta^2}{1 + \kappa \rho \sigma_\eta^2}\right). \tag{37}$$

In addition, from (36) and (33), we have

$$Cov_0\left(K, \eta\right) = \kappa \pi_1 \sigma_\eta^2. \tag{38}$$

It follows directly that $\frac{\partial E_0[K]}{\partial \pi_1} > 0$ and $\frac{\partial Cov_0[K,\eta]}{\partial \pi_1} > 0$. The remainder of the proof follows from Proposition 2.3.

By impeding the market's capacity to aggregate information, near-rational behavior thus depresses the level of capital accumulation and induces a mis-allocation of capital across states. Our model allows us to assess the welfare effects of these distortions while avoiding two common difficulties in noisy rational expectations models. First, because the model features only a single class of agents, we can calculate the ex-ante (time zero) utility of these agents without having to consider the utility of non-maximizing "noise" or "liquidity" traders that are commonly used as a modeling device to induce noise in the equilibrium stock price. Second, as near-rational households are, by definition, "nearly" maximizing their utility, whether we consider welfare under the fully rational or the near-rational measure is inconsequential. Because the biases in the behavior of near-rational households are small, it does not matter whether we respect them for the purposes of our utility calculations, that is, $\mathcal{E}_0\left[U_i\right] \approx E_0\left[U_i\right]$ (see, e.g., Brunnermeier et al. (2012)). Using these two insights, we can show the following lemma:

Lemma 2.8
The ex-ante utilitarian social welfare function can be written as:

$$SWF = E_0\left[U_i\right] = E_0\left[w_a\right] - \left(\frac{1}{2\rho\sigma_\nu^2} + \frac{\rho}{2}E_0\left[K^2\right]\right)V_1[\eta], \tag{39}$$

where

$$E_0\left[w_a\right] = \left(\bar{\eta} - 1\right)E_0\left[K\right] + Cov_0\left(K, \eta\right) - \frac{1}{2\kappa}E_0\left[K^2\right]. \tag{40}$$

Proof *See Appendix A.8.*

The first statement shows that the utilitarian social welfare function depends on the expected level of aggregate wealth, $w_a \equiv K\left(\eta - Q\right) + \frac{K^2}{2\kappa}$, and the expected variance of portfolio returns. The expected level of aggregate wealth depends on the expected level of capital accumulation, the covariance of capital with productivity, and the expected capital adjustment costs. The expected variance depends on two terms. The first reflects the cross-sectional variance induced by any dispersion of portfolio holdings across households. The second term reflects the variance of the average portfolio held by households (recall that $\int z_i di = K$). Welfare increases monotonically in the expected level of wealth and decreases in the two variance terms.

Proposition 2.9

Near-rational behavior lowers welfare by lowering the time-zero expected level and raising the time-zero expected variance of wealth. This effect represents a negative externality of near-rational behavior that transmits itself through the effect of near-rational errors on the stock market's capacity to transmit information:

$$\lim_{\sigma_\epsilon \to 0}\left[\frac{\partial SWF}{\partial \sigma_\epsilon} - \frac{\partial E_0\left[U_i\right]}{\partial \mu_i}\right] = \lim_{\sigma_\epsilon \to 0}\left[\frac{\partial SWF}{\partial \pi_1}\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] = \sigma_\eta^2 \frac{\sigma_\nu^2 \rho\kappa\left(1+\kappa\rho\left(\left(1-\bar{\eta}\right)^2+\sigma_\eta^2\right)\right)+1}{2\sigma_\nu^2\rho\left(1+\kappa\rho\sigma_\eta^2\right)} \lim_{\sigma_\epsilon \to 0}\left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0.$$

Proof *See Appendix A.9.*

By reducing the market's capacity to transmit information, near-rational behavior thus has a large (first-order) external effect on welfare. Using Lemma 2.8, it is easy to see that this effect works through four channels:

$$\frac{\partial SWF}{\partial \sigma_\epsilon} = \left[\left(\bar{\eta}-1\right)\frac{\partial E_0\left[K\right]}{\partial \pi_1} + \frac{\partial Cov_0\left(K,\eta\right)}{\partial \pi_1} - \left(\frac{1}{2\kappa}+\frac{\rho}{2}V_1[\eta]\right)\frac{\partial E_0\left[K^2\right]}{\partial \pi_1} - \left(\frac{1}{2\rho\sigma_\nu^2}+\frac{\rho}{2}E_0\left[K^2\right]\right)\frac{\partial V_1[\eta]}{\partial \pi_1}\right]\frac{\partial \pi_1}{\partial \sigma_\epsilon}. \tag{41}$$

From Proposition 2.7, we know that the first two terms in the square brackets are positive. In addition, from (22), it follows that $\frac{\partial V_1[\eta]}{\partial \pi_1} < 0$, such that the fourth term is also positive. Near-rational behavior thus lowers welfare by depressing the equilibrium capital stock, decreasing $Cov_0\left(K,\eta\right)$, increasing the dispersion of wealth, and increasing the variance of the average portfolio held by households. The only term with an ambiguous sign is the third term, which reflects an ambiguous effect of near-rational behavior on expected capital adjustment costs. We show in Appendix A.9 that for small σ_ϵ, near-rational behavior lowers expected adjustment costs. However, the three other terms swamp this positive effect on welfare.

This result has important implications beyond its immediate application to the present paper. A large literature on the real effects of stock market dysfunctionality has traditionally focused on estimating either the sensitivity of capital investment with respect to stock prices, κ, or the covariance of capital investment with non-fundamental movements of stock prices, $Cov_0\left(K,\epsilon\right) = \kappa\gamma\epsilon$. Although the latter expression is related to (38), how these two variables map into the four channels outlined above is unclear. In particular, depending on the parameters of the model, the distortion of the level of capital accumulation or the cross-sectional dispersion of wealth may be costly even if the stock market appears as a "sideshow" (Morck, Shleifer, and Vishny (1990)) in the sense that κ is low.

3 Robustness and Extensions of the Static Model

3.1 Amplification of Other Shocks and Alternative Information Environments

The key result of the analysis above is that small correlated errors in household expectations have a lower-order effect on an individual household's utility but a first-order external effect on social welfare. In this section, we show that the small deviations from rationality as specified in (10) are crucial to this result. Using (9), (11), (14), and (10), we can rewrite the market-clearing condition as

$$\frac{\alpha_0 + \alpha_1 \int s_i di + \alpha_2 Q + \epsilon - Q}{\rho V_1[\eta]} = K, \tag{42}$$

where for simplicity we consider the case in which the capital stock is exogenous ($\kappa = 0$), such that from (12), we have $Q = \int \mathcal{E}_{1i}[\eta]\, di$. There are two obvious alternative avenues to introducing small disturbances into this relationship. We may introduce small common noise in the private signal of the form

$$s_i = \eta + \nu_i + \zeta, \tag{43}$$

where $\zeta \sim N\left(0, \sigma_\zeta^2\right)$ (on the left-hand side) or small inelastic shocks to the supply of stocks induced by noise traders (on the right-hand side). In this section, we show that both of these alternative types of small disturbances do not result in a negative externality. In Appendix B.4, we also consider near-rational errors about the conditional variance of η, which we show to be isomorphic to the near-rational errors about the conditional expectation in (10) under some additional assumptions. In addition, Appendix B.2 discusses the case in which households observe an exogenous public signal in addition to the equilibrium stock price. To simplify the exposition, we consider the case in which $\kappa = 0$ throughout.

3.1.1 Aggregate Noise in the Private Signal

Consider a model identical to the one given in section 2, with the exception that instead of (3), we now have aggregate noise in the private signal (43).

Proposition 3.1

As the standard deviation of the small common error approaches 0, the marginal effect of near-rational behavior on the elasticity of the stock price with respect to productivity becomes infinitely large:

$$\lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] = -\infty.$$

By contrast, the marginal effect of aggregate noise in the private signal on this elasticity goes to

0 as the standard deviation of aggregate noise in the private signal goes to 0.

$$\lim_{\sigma_\zeta \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\zeta} \right] = 0.$$

Proof *See Appendix B.1.1.*

This proposition has two direct implications. First, small common noise in the private signal is not amplified in equilibrium and thus has only a lower-order effect on the market's capacity to aggregate information. To see this, solve (42) for Q and plug in (43) to get

$$Q = \frac{\alpha_0}{1-\alpha_2} + \frac{\alpha_1}{1-\alpha_2} (\eta + \zeta) + \frac{1}{1-\alpha_2} \epsilon - \frac{\rho V_1[\eta]}{1-\alpha_2} K. \tag{44}$$

The result follows from the fact that $\frac{\alpha_1}{1-\alpha_2} = \pi_1$ is a number between 0 and 1. Small common noise in the private signal thus has only a lower-order effect on welfare and thus does not result in the kind of externality shown in Proposition 2.9.

Second, it demonstrates that none of the conclusions of the model in section 2 rely on the stock price becoming perfectly revealing about η if all households are fully rational. Near-rational behavior continues to have a first-order effect on information aggregation even if there is (large) common noise in the private signal.

Figure 3 illustrates these results. The thick blue line plots the now familiar effect of a small common error in household expectations with $\frac{\sigma_\epsilon}{\sigma_\eta} = 0.01$. The red horizontal line plots the effect of an identical amount of small common noise in the private signal (i.e. $\frac{\sigma_\zeta}{\sigma_\eta} = 0.01$). The red line has an intercept of 0.01^2 and is perfectly horizontal. The common noise in the private signal is not amplified, and does the fact that an individual household observes a signal with common noise does not have an external effect on the market's capacity to aggregate information. The effect of common noise in the private signal is thus invariant to how dispersed information is in the economy.

The broken lines in Figure 3 show the same comparative static, but in the presence of large common noise in the private signal ($\frac{\sigma_\zeta}{\sigma_\eta} = 1$). Both lines retain their shape but now have a higher intercept, reflecting the fact that less information is now available to aggregate, even if the stock price is fully revealing. However, for the remaining dispersed information, the information externality of near-rational behavior operates in the same way as in the model in section 2. The externality is thus relevant whenever financial markets play an important role in aggregating dispersed information, regardless of the exact information structure. Appendix B.2 shows the same is true in a model in which households observe an exogenous public signal in addition to the stock price.

3.1.2 Noise Trading

Consider two modifications to the model in section 2: First, households have rational expectations:

$$\mu_i = 0 \ \forall i.$$

Second, in addition to the unit interval of rational households, the economy is inhabited by a unit interval of noise traders $j \in [0, 1]$ inhabit the economy. Noise traders are identical to rational households in that they have the same preferences (4), budget constraint (5), and information set (they receive the signal (3) and observe the equilibrium stock price Q). However, when making their portfolio decisions, noise traders do not maximize their utility but exogenously and inelastically demand

$$z_j = \mu_j \vartheta, \tag{45}$$

where $\vartheta \sim N\left(0, \sigma_\vartheta^2\right)$. This behavior makes the supply of stocks stochastic from the perspective of rational households.

Because $\kappa = 0$ implies $K = 0$, market clearing requires that the sum of rational households' and noise traders' stock demands equals zero:

$$\int_0^1 z_i di + \int_0^1 \mu_j \vartheta \ dj = 0, \tag{46}$$

where $\mu_j = 1 \ \forall j$.

Proposition 3.2
Shocks to noise-trader demand lower the utility of noise traders but raise the welfare of rational households. Noise traders' demand shocks thus represent a positive externality on rational households:

$$\frac{\partial SWF}{\partial \sigma_\vartheta} > 0 \ \forall \sigma_\vartheta > 0 \ \textit{and} \ \frac{\partial E_0[U_j]}{\partial \mu_j} < 0 \ \forall \mu_j > 0.$$

Proof *See Appendix B.3.1.*

The intuition behind this result is a redistribution of wealth between the two types of agents in the model. Although rational households incur some losses due to the increased variability of their portfolios, the market compensates them for the higher risk they take in the form of a higher risk premium. Their welfare increases because they can "lean against" noise traders' demand and thus earn higher expected returns on their investments.[14] Noise-trader demand shocks thus represent a positive rather than negative externality on the welfare of rational households.

[14] With endogenous capital accumulation ($\kappa > 0$), there also exist parameter combinations for which the deadweight loss from distortions in the capital stock outweighs the redistribution of wealth from noise traders to rational households such that the marginal effect on rational households' utility becomes negative.

In addition, the size of this externality shrinks to 0 in the limit in which noise-trader demand shocks become small.

Proposition 3.3
As the standard deviation of noise-trader demand approaches 0, its marginal effect on the elasticity of the stock price with respect to productivity goes to 0

$$\lim_{\sigma_\vartheta \to 0} \frac{\partial \pi_1}{\partial \sigma_\vartheta} = 0.$$

Proof *See Appendix B.3.2.*

To see the intuition for this result, replace K with ϑ in (44). Noise-trader demand shocks are multiplied with $\frac{\rho V_1[\eta]}{1-\alpha_2}$. For small σ_ϑ, both the numerator and the denominator go to 0, such that the fraction as a whole remains a finite number. (In Appendix B.3.2, we show that the multiplier on noise traders' demand shocks is always strictly smaller than $\rho\sigma_\nu^2$.) Small common shocks to noise traders' demand thus have no first-order effect on the equilibrium informativeness of stock prices. As a result, they affect neither noise traders' own utility nor the welfare of rational households. We show in the appendix that

$$\lim_{\sigma_\vartheta \to 0} \left[\frac{\partial SWF}{\partial \sigma_\vartheta} \right] = \lim_{\sigma_\vartheta \to 0} \left[\frac{\partial E_0[U_j]}{\partial \mu_j} \right] = 0.$$

Small shocks to noise traders' demand thus do not give rise to the type of externality we derive in section 2. In addition, allowing for large shocks to noise-trader demand actually gives rise to a positive rather than a negative externality.

3.2 Alternative Counterfactual

A guiding principle in our analysis of a near-rational household's incentive to become fully rational in section 2 was that households have the same information set, regardless of whether they behave fully rationally or near-rationally. In particular, a rational household can condition its decisions on s_i and Q, but does not know the small correlated error it would have made, had it been near-rational.

We can relax this assumption by considering the willingness to pay of a rational household at $t = 0$ for observing $\epsilon + \hat{\epsilon}_i$ at $t = 1$. A rational household can benefit from observing this error by extracting the information it conveys about η (and equivalently about the common component in the error, ϵ). Using (13), we can define

$$\hat{s}_i \equiv \frac{Q - \gamma\left(\epsilon + \hat{\epsilon}_i\right) - \pi_0}{\pi_1} = \eta - \frac{\gamma \hat{\epsilon}_i}{\pi_1}, \tag{47}$$

where $\hat{s}_i$ is the un-biased signal about η conveyed by $\epsilon + \hat{\epsilon}_i$.

Proposition 3.4

As the standard deviation of the near-rational error goes to 0, a rational household's willingness to pay to observe the near-rational error it would have made had it been near-rational goes to

$$\lim_{\sigma_\epsilon \to 0} \left[E_0 \left[U_i|_{\mu_i=0,\hat{s}_i} \right] - E_0 \left[U_i|_{\mu_i=0} \right] \right] = \frac{1}{2\hat{\mu}^2}. \tag{48}$$

Proof *See Appendix B.5.*

The potential gain of observing this additional signal thus goes to one half of the ratio of common variance to idiosyncratic variance in the error in household expectations. Since none of the results in section 2 place restrictions on $\hat{\mu}$, the potential incentive to observe $\epsilon + \hat{\epsilon}_i$ is thus small for a large range of plausible parameters.

4 Quantitative Model

In this section, we set up a quantitative DSGE model that allows us to estimate the welfare effects and the equilibrium impact of near-rational behavior. To this end, we use a de-centralization of the DSGE model by Croce (2013). We choose this model mainly because it performs well in matching both macroeconomic and asset-pricing moments and because it can be readily solved using perturbation methods.

4.1 Setup

Technology is characterized by a linear homogeneous production function that uses capital, K_t, and labor, N_t, as inputs:

$$Y_t = K_t^{\alpha}(e^{a_t} N_t)^{1-\alpha}, \tag{49}$$

where Y_t stands for output of the consumption good. The productivity of labor, a_t, has a long-run component, ω, and a short-run component, φ:

$$\Delta a_{t+1} = \mu_a + \omega_t + \varphi_{t+1}, \tag{50}$$

where the long-run component follows

$$\omega_t = \rho \omega_{t-1} + \eta_t. \tag{51}$$

Both shocks to productivity, φ and η, are *i.i.d.* normally distributed with mean zero and standard deviations σ_φ and σ_η, respectively.

The equation of motion of the capital stock is

$$K_{t+1} = (1-\delta_k)K_t + I_t - G_t K_t, \tag{52}$$

where I_t denotes aggregate investment and δ_k is the rate of depreciation. Furthermore, there are convex adjustment costs to capital following Jermann (1998):

$$G_t = \frac{I_t}{K_t} - \left(\frac{v_1}{1-\frac{1}{\xi}} \left(\frac{I_t}{K_t} \right)^{1-\frac{1}{\xi}} + v_0 \right), \tag{53}$$

where v_1 and v_2 are positive constants and the parameter ξ determines the equilibrium elasticity of the capital stock with respect to the stock price.

A representative firm purchases capital and labor services from households. Because it rents services from an existing capital stock, the firm's objective collapses to a period-by-period maximization problem:

$$\max_{K_t, N_t} Y_t - d_t K_t - w_t N_t, \tag{54}$$

where K_t and N_t denote factor demands for capital and labor respectively. First-order conditions with respect to capital and labor, pin down the market-clearing wage,

$$w_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{55}$$

and the rental rate of capital,

$$d_t = \alpha \frac{Y_t}{K_t}. \tag{56}$$

Both factors thus receive their marginal product. Because the production function is linear homogeneous, the representative firm makes zero economic profits from producing the consumption good.

The representative firm owns an investment goods sector that converts the consumption good into units of capital, while incurring adjustment costs. It takes the price of capital as given and then performs instantaneous arbitrage:

$$\max_{I_t} Q_t \left(I_t - G_t K_t \right) - I_t. \tag{57}$$

Taking the first-order condition of (57) gives us the equilibrium price of capital (Tobin's Q):

$$Q_t = \frac{1}{1-G_t'}. \tag{58}$$

Since there are decreasing returns to scale in converting consumption goods to capital, the investment goods sector makes positive profits in each period. Profits are paid to shareholders as a part of dividends per share:[15]

$$D_t = \alpha \frac{Y_t}{K_t} + Q_t \left(G'_t \frac{I_t}{K_t} - G_t \right). \tag{59}$$

A continuum of households on the interval $i \in [0, 1]$ has Epstein and Zin (1989) preferences over the consumption bundle $\tilde{C}_{it}$:

$$U_{it} = \left((1-\delta) \tilde{C}_{it}^{1-\frac{1}{\psi}} - \pi(b_{it}) + \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{60}$$

where the parameters ψ and γ measure the households' intertemporal elasticity of substitution and relative risk aversion, respectively. $\pi(b_{it})$ is a small penalty for holding bonds that ensures a well-defined portfolio choice at the deterministic steady state (Judd and Guu, 2001).[16] The consumption bundle $\tilde{C}_{it}$ is a Cobb-Douglas aggregate of consumption and leisure:

$$\tilde{C}_{it} = C_{it}^{o} (e^{a_{t-1}} (1 - n_{it}))^{1-o}, \tag{61}$$

where leisure scales with aggregate productivity to ensure the existence of a balanced growth path.

At the beginning of every period, each household receives a private signal about the shock to long-run productivity:

$$s_{it} = \eta_{t+1} + \nu_{it}, \tag{62}$$

where ν_{it} again represents *i.i.d.* draws from a normal distribution with zero mean and variance σ_ν^2. Households observe all prices and aggregate state variables at time t and understand the structure of the economy as well as the equilibrium mapping of dispersed information into prices and economic aggregates. The rational expectations operator, conditional on all the information available to household i at time t, is

$$E_{it}[\cdot] = E\left[\cdot | s_{it}, Q_t, r_t, d_t, w_t, C_t, N_t, K_t, Y_t, I_t, G_t, R_t, a_t, \omega_{t-1}\right]. \tag{63}$$

The only sources of uncertainty are thus the two innovations, η_{t+1} and φ_{t+1}. While households have no information about the short-run shock ($E_{it}\left[\varphi_{t+1}\right] = 0$), they must form a conditional expectation of η_{t+1}. As in the model in section 2, we assume households make a small common error when forming this expectation. The expectations operator $\mathcal{E}_{it}$ in (60) is thus the rational

[15] Alternatively, profits may be paid to households as a lump-sum transfer; this assumption matters little for the quantitative results of the model.

[16] We use the simple quadratic form $\pi(b_{it}) = \frac{1}{2000} e^{a_{t-1}(1-\frac{1}{\psi})} \left(\frac{b_{it}}{e^{a_{t-1}}} \right)^2$.

expectations operator with the only exception that

$$\mathcal{E}_{it}\left[\eta_{t+1}\right] = E_{it}\left[\eta_{t+1}\right] + \mu_i\left(\epsilon_t + \hat{\epsilon}_{it}\right), \tag{64}$$

where again $\epsilon_t \sim N(0, \sigma_\epsilon)$ and $\hat{\epsilon}_{it} \sim N(0, \hat{\mu}\sigma_\epsilon)$.

Given s_{it} and their knowledge about the state of the economy, households maximize lifetime utility (60) by choosing a time path for consumption and labor, and their holdings of stocks and bonds $\{\tilde{C}_{it}, n_{it}, k_{it}, b_{it}\}_{t=0}^{\infty}$. Each household's optimization is subject to a budget constraint:

$$Q_t k_{it+1} + b_{it} = Q_{t-1} R_t k_{it} + (1 + r_{t-1}) b_{it-1} + H_{it} - C_{it} + w_t n_{it}, \tag{65}$$

where Q_t is again the price of capital, H_{it} are transfers from state-contingent claims discussed below, and w_t is the wage rate. The returns to capital are defined as

$$R_{t+1} = \frac{(1-\delta_k)Q_{t+1} + \alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}\left(G'_{t+1}\frac{I_{t+1}}{K_{t+1}} - G_{t+1}\right)}{Q_t}. \tag{66}$$

The market-clearing conditions for the stock, bonds, labor, and goods markets are

$$K_{t+1} = \int k_{it+1} di, \tag{67}$$

$$0 = \int b_{it} di, \tag{68}$$

$$N_t = \int n_{it} di, \tag{69}$$

and

$$Y_t = C_t + I_t. \tag{70}$$

Finally, the payments from contingent claims, H_{it}, avoid having to keep track of the evolution of wealth across households.[17] At the beginning of each period (and before receiving their private signal), households can trade claims that are contingent on the state of the economy and on the realization of the noise they receive in their private signal, ν_{it}. These claims are in zero net supply and pay off at the beginning of the next period. Because the claims are traded before any information about η_{t+1} is known, their prices cannot reveal any information about future productivity. Contingent-claims trading thus completes markets between periods, without affecting households' signal-extraction problem. In equilibrium, all households choose to hold

[17] See Mertens and Judd (2013) for a perturbation-based approach to solving incomplete markets models with substantial heterogeneity.

these securities with net payoff

$$H_{it} = \begin{cases} Q_{t-1}R_tK_t - Q_{t-1}R_tk_{it} - (1+r_{t-1})b_{it-1} & \text{if } \{C_{it}, n_{it}, k_{it}, b_{it}\} = \arg\max\ (60)|_{H_{it}=0} \\ 0 & \text{otherwise} \end{cases}, \tag{71}$$

such that all households enter each period with the same amount of wealth. From (67) and (68), it follows immediately that these claims are in zero net supply:

$$\int H_{it} di = 0. \tag{72}$$

Definition 4.1

Given a time path of shocks $\{\eta_t, \epsilon_t, \varphi_t, \{\nu_{it}, \hat{\epsilon}_{it} : i \in [0,1]\}\}_{t=0}^{\infty}$*, an equilibrium in this economy is a time path of quantities* $\{\{C_{it}, b_{it}, n_{it}, k_{it} : i \in [0,1]\}, C_t, N_t, K_t, Y_t, I_t, G_t, R_t, a_t, \omega_t\}_{t=0}^{\infty}$*, signals* $\{s_{it} : i \in [0,1]\}_{t=0}^{\infty}$*, and prices* $\{Q_t, r_t, d_t, w_t\}_{t=0}^{\infty}$ *with the following properties:*

1. $\{\{C_{it}\}, \{b_{it}\}, \{n_{it}\}, \{k_{it}\}\}_{t=0}^{\infty}$ *maximize households' lifetime utility (60) given the vector of prices, and the random sequences* $\{\epsilon_t, \{\nu_{it}, \hat{\epsilon}_{it}\}\}_{t=0}^{\infty}$*;*

2. *The demand for capital and labor services solves the representative firm's maximization problem (54) given the vector of prices;*

3. $\{I_t\}_{t=0}^{\infty}$ *is the investment goods sector's optimal policy, maximizing (57) given the vector of prices;*

4. $\{w_t\}_{t=0}^{\infty}$ *clears the labor market,* $\{Q_t\}_{t=0}^{\infty}$ *clears the stock market,* $\{r_t\}_{t=0}^{\infty}$ *clears the bond market, and* $\{d_t\}_{t=0}^{\infty}$ *clears the market for capital services;*

5. $\{Y_t\}_{t=0}^{\infty}$ *is determined by the production function (49), and* $\{K_t\}_{t=0}^{\infty}$*,* $\{G_t\}_{t=0}^{\infty}$*,* $\{a_t\}_{t=0}^{\infty}$*,* $\{R_t\}_{t=0}^{\infty}$*, and* $\{\omega_t\}_{t=0}^{\infty}$ *evolve according to (52), (53), (50), (66), and (51), respectively;*

6. $\{C_t, N_t\}_{t=0}^{\infty}$ *are given by the identities*

$$X_t = \int_0^1 X_{it} di \ , \ X = C, N. \tag{73}$$

The *rational expectations equilibrium* is the economy in which $\sigma_\epsilon = 0$, such that the expectations operator $\mathcal{E}$ in equation (60) coincides with the rational expectation in (63). The *near-rational expectations equilibrium* posits that $\sigma_\epsilon > 0$; households make small errors as given in (64).

We compute the first-order and envelope conditions for households in Appendix C.2.

4.2 Solving the model

We use the solution method developed in Mertens (2009) to transform the equilibrium conditions of the model into a form we can solve with standard techniques. The key to this approach is to show that all prices and economic aggregates are a function of the usual "macroeconomic" state variables of the model $S_t = \{K_t, \omega_{t-1}, \eta_t, \varphi_t\}$ as well as households' average expectation of η_{t+1}:

$$\hat{q}_t = \int \mathcal{E}_{it}\left[\eta_{t+1}\right] di = \int E_{it}\left[\eta_{t+1}\right] di + \epsilon_t. \tag{74}$$

Lemma 4.2
A recursive equilibrium exists satisfying the system of equations in definition 4.1 with the following properties:
(1) A household's optimal behavior depends on the current (commonly known) state of the economy, S_t, the household's conditional expectation of next period's innovation to productivity, $\mathcal{E}_{it}[\eta_{t+1}]$, and the average expectation of this innovation, $\hat{q}_t$. The conditional expectation, in turn, depends on the private signal s_{it} as well as $\hat{q}_t$. We can thus write the set of state variables that determine individual behavior as

$$x_{it} = x(S_t, \hat{q}_t, \mathcal{E}_{it}[\eta_{t+1}]), x = C, n, k_{+1}, b. \tag{75}$$

(2) All prices and economic aggregates depend on the current state of the economy and $\hat{q}_t$:

$$X_t = X(S_t, \hat{q}_t), X = C, N, K_{+1}, Y, I, G, R, Q, r, d, w. \tag{76}$$

Proof: *See Appendix C.3.*

Given this lemma, we are able to use standard perturbation methods to solve for households' equilibrium policies as a function of the vector $\{S_t, \hat{q}_t, \mathcal{E}_{it}[\eta_{t+1}]\}$ and for all economic aggregates as a function of $\{S_t, \hat{q}\}$. In other words, we can separate the solution of the non-linear model from the information microstructure by simply treating $\mathcal{E}_{it}[\eta_{t+1}]$ and $\hat{q}_t$ as state variables. The final step of the solution is then to solve for $\mathcal{E}_{it}[\eta_{t+1}]$.

Condition 1 The equilibrium stock price, Q, or at least one other economic aggregate or price is a strictly monotonic function of $\hat{q}$.

An immediate implication of Lemma 4.2 is that all prices and economic aggregates have the same information content. Given condition 1, $\hat{q}$ is simply a monotonic transformation of Q. Households can thus infer $\hat{q}$ from observing the equilibrium stock price (or any other economic aggregate that is monotonic in $\hat{q}$). Learning from the stock price is then just as good as learning from its monotonic transformation, because Q and $\hat{q}$ span the same σ-algebra. Although we cannot solve for the mapping of $\hat{q}$ into Q in closed form, we can easily check for monotonicity using the numerical solution of the model.

Lemma 4.3

Households' equilibrium expectations of η_{t+1} are independent of the aggregate dynamics of the model. Conditioning on s_{it} and $\hat{q}_t$ results in expectations of the form

$$E_{it}[\eta_{t+1}] = \frac{\sigma_\nu^{-2}\left(\eta_{t+1}+\nu_{it}\right)+\alpha_1^2\sigma_\epsilon^{-2}\left(\eta_{t+1}+\frac{1}{\alpha_1}\epsilon_t\right)}{V_t[\eta_{t+1}]^{-1}}, \tag{77}$$

$$V_t[\eta_{t+1}] = \left(\sigma_\eta^{-2}+\sigma_\nu^{-2}+\alpha_1^2\sigma_\epsilon^{-2}\right)^{-1}, \tag{78}$$

where

$$\alpha_1 = \frac{V_t[\eta_{t+1}]}{\sigma_\nu^2}. \tag{79}$$

Proof *See Appendix C.4.*

Households' equilibrium expectations thus take exactly the same form as in the static model. As a result, all the qualitative results concerning the effect of the near-rational error on equilibrium expectations derived in section 2.2 readily carry over to the quantitative model.

5 Estimation and Results

Our model depends on 16 parameters. Table 1 lists nine standard macroeconomic parameters that we set equal to the values used in Croce (2013). In particular, α is set to match the capital income share and δ_k is set to match the annualized capital depreciation rate in the US economy (6%). In addition, the average annual growth rate of productivity, μ_a, is 1.8%. The relative risk aversion and the intertemporal elasticity of substitution are set to values of 10 and 2, respectively. The annualized subjective discount factor, δ, is fixed at 0.965. The parameters v_1 and v_2 in the adjustment-cost function are set such that, at the deterministic steady state, $G_t = 0$ and $\partial G_t/\partial\left(I_t/K_t\right) = 0$. (This implies $v_0 = \left(\frac{1}{1-\xi}\right)(\delta + e^\mu - 1)$ and $v_1 = (\delta + e^\mu - 1)^{\frac{1}{\xi}}$.)

We also follow Croce (2013) in calculating excess stock returns as the excess returns on a levered claim to capital:

$$R_{ex,t}^{LEV} = (R_t - r_{t-1})^{\phi_{lev}}\,. \tag{80}$$

This practice is standard in the finance literature because, in the data, most claims to equity are levered, where we set $\phi_{lev} = 2$, consistent with the amount of financial leverage measured by Rauh and Sufi (2012).

In addition, we set the standard deviation of the idiosyncratic component of near-rational errors to 0 for simplicity, $\hat{\mu} = 0$.

We estimate the remaining six parameters according to two criteria. First, we choose σ_ϵ such that a near-rational household's willingness to pay to set $\mu_i = 0$ (and thus become fully rational) is equal to 0.01% of permanent consumption. We maintain this condition throughout all comparative statics shown below, such that across all specifications, the private cost of near-rational behavior is held constant.

Second, we estimate the remaining five parameters ($\sigma_\varphi, \sigma_\eta, \rho, \xi$, and σ_ν) to minimize the loss function:

$$(m - \theta)' W (m - \theta), \tag{81}$$

where m is a vector of moments generated by the model and θ is the vector of data targets for these moments. W is a diagonal matrix where each entry is $\frac{1}{N_j \theta_j^2}$, where θ_j is the jth entry in θ and N_j is the total number of moments in θ that are of the same type as j (distinguishing between macroeconomic, asset-pricing, and information-related moments in the data). Our estimation thus minimizes a weighted sum of squared deviations from the target vector, adjusting for the number of each type of moment in the vector.

In all specifications below, we measure the private cost of near-rational behavior as the compensating variation in terms of permanent consumption for individual i of setting $\mu_i = 0$ in all future periods. Similarly, we measure the social cost of near-rational behavior as the compensating variation for individual i of setting $\mu_j = 0 \ \forall j \neq i$. Both calculations are performed at the stochastic steady state of the model, such that our measure for the social cost of near-rational behavior takes into account that the abolition of near-rational behavior today induces an adjustment process to a new stochastic steady-state level of capital. See Appendix D.1 for details on this calculation.

5.1 Results

The first column of Table 2 shows 16 data targets constructed from annual US data (1929-2008), listing first the information-related, then the macroeconomic, and then the asset-pricing moments. The data sources used for the latter two types of moments are standard. We construct the two information-related moments from the Survey of Professional Forecasters 1969-2008. The first is the ratio of conditional to unconditional variance of productivity growth, which we take to correspond to $V_t[\eta_{t+1}]/\sigma_\eta^2$ in the model.[18] The second is the variance of forecasts of GDP growth across professional forecasters, which we take to correspond to the cross-sectional dispersion in the equilibrium expectations of GDP growth across households in the model. See

[18] We have also attempted to separately identify the conditional variance of short-run vs. long-run shocks to productivity growth in the data by estimating separate ARMA processes for forecasts and realizations of GDP growth. However, due to the limited number of observations, these estimates, too noisy to allow a meaningful distinction.

the caption of Table 2 and Appendix D.2 for details.[19]

Specification (1) gives our benchmark estimation – the best fit to the data according to (81). It returns standard deviations of the short-run and long-run shocks to productivity of 2.82% and 0.50%, respectively, where the latter has a persistence of $\rho = 0.995$. The adjustment-cost parameter is $\xi = 4$ and the dispersion of private information is $\sigma_\nu/\sigma_\eta = 182$, implying that in the absence of a stock price, $182^2 = 33,124$ households would have to pool their private signals to reduce the conditional variance of η by one half.

The standard deviation of the common error in household expectations is 0.003% of the standard deviation of the shock to long-term productivity, but is amplified by a factor of 14,046, such that a one-standard-deviation shock to ϵ results in a 0.42 σ_η rise in the average expectation of η. Consequently, equilibrium stock prices transmit more noise than information, with a noise-to-signal ratio of 2.37. In this situation, a given household in the model would be willing to give up 66.86% of its lifetime consumption in order to abolish other households' near-rational behavior. The social cost of near-rational behavior is thus three orders of magnitude larger than the private cost (0.01% of permanent consumption).

The model delivers a good fit to the data. In particular, the standard deviation of output is estimated at 4.33, and thus only slightly higher than the data target (3.34, s.e.=0.39). Similarly, the model estimates an equity premium that is slightly too high (4.16 vs. 3.89, s.e.=2.17 in the data). Although the model almost matches the standard deviation of stock prices (33.65%), it falls short of matching the standard deviation of stock returns.[20] Overall, the performance of the model in matching standard moments is similar to that of standard long-run risk models. In addition, the model matches almost perfectly the conditional variance of productivity shocks in the data but falls short of generating a large dispersion in equilibrium expectations across households (0.001% vs. 0.27% in the data). An obvious reason for the latter failure is that in the data, we only observe disagreement about next year's GDP growth, whereas private information about shocks to long-run productivity growth generates differences in opinion about GDP growth many years in the future. (We return to this issue below.)

Specification (2) shows the case in which all households are fully rational, $\sigma_\epsilon = 0$. In this case, the dispersion of private information is irrelevant because the stock market perfectly reveals η_{t+1}, such that any specification with $\sigma_\nu < \infty$ returns the same moments. Comparing the results of specifications (1) and (2) shows that near-rational behavior has relatively small effects on standard business-cycle moments, and larger but still moderate effects on asset-pricing moments. For example, the equity premium is 4.16% when households are near-rational and 3.96% when households are fully rational. Nevertheless, the estimated effect of near-rational

[19]Note that we deviate from standard practice in the long-run risk literature in that we calibrate the model directly at the annual frequency to match annual moments.

[20]Note that in the interest of parsimony, we do not artificially raise the standard deviation of stock returns by introducing an additional cash-flow shock as is standard in the literature.

behavior on capital accumulation is sizable. When households are near-rational, the stochastic steady-state level of capital accumulation is 0.88% higher than when they are fully rational. Note that this distortion in the level of capital accumulation has the opposite sign of the distortion we found in the static model. The reason near-rational behavior increases rather than decreases the level of capital in the quantitative model is the precautionary savings motive that arises in general equilibrium. An increase in the conditional variance of η makes households more reluctant to invest in stocks versus bonds, but also induces them to save more, such that the risk-free rate falls. The sign of the overall distortion thus depends on the size of these opposing forces.

Despite this sizable distortion in the level of capital accumulation, the effects of near-rational behavior on the moments of the model appear small compared to its effect on welfare (66.86% of permanent consumption). The reason for this apparent discrepancy is that with an intertemporal elasticity of substitution larger than 1, households have a preference for early resolution of uncertainty. Aside from its effects on the level and the dynamics of economic activity, the availability of information about the future thus has an additional, direct effect on households' utility. The same mechanism that allows the model to produce a relatively large equity premium (Bansal and Yaron, 2004) and large costs of business cycles (Epstein et al., 2013; Croce, 2012) thus also results in a surprisingly large social cost of near-rational behavior when it prevents households from learning about long-term shocks to productivity.

Figure 4 shows a comparative static of selected moments over the level of dispersion in private information, σ_ν, corresponding to specification (1) in Table 2. In all panels, the black dotted line shows the limit $\sigma_\nu \to \infty$. The red dashed line shows the results corresponding to specification (2), where the stock price perfectly aggregates all private information about η ($\sigma_\epsilon = 0$). Throughout, we adjust σ_ϵ to fix the private cost of near-rational behavior at 0.01% of permanent consumption.

Consistent with its counterpart in the static model, the first panel of Figure 4 shows that a given amount of near-rational behavior (now fixed in terms of its utility cost to the individual) has a larger effect on the stock market's capacity to aggregate information when private information is more dispersed. The larger this external effect, the larger the social cost of near-rational behavior and the distortion in the level of capital accumulation (up to 1.25% if information aggregation is completely destroyed).

The following panel shows the multiplier on the near-rational error, $1/(1-\alpha_2)$. Consistent with Figure 2 in the static model, it shows the amplification peaks around factor 15,000 for intermediate values of σ_ν. As we move farther to the right in the graph, the noise-to-signal ratio in stock prices increases to the point that households give up learning from stock prices, begin to lower α_2, and instead rely on their priors. During this process, the multiplier decreases and eventually converges to 1, while the noise-to-signal ratio and the conditional variance of stock returns continue to increase monotonically.

The remaining panels show that a higher conditional variance of stock returns results in a higher equity premium of up to 4.25%. While the conditional variance of η, and thus the conditional variance of stock returns, rises monotonically as we move from left to right in the graph, the unconditional variance of stock returns is initially lower and only rises above the level it obtains under full rationality at high levels of σ_ν. The same is true for the standard deviation of output and investment. The reason for this "dip" is in the dynamic effects of near-rational behavior.

The solid blue lines in all panels of Figure 5 show impulse-response functions based on specification (1) in Table 2. The black dotted lines again show the case in which information aggregation breaks down completely ($\sigma_\nu \to \infty$), and the red dashed line shows the results corresponding to full rationality (specification (2)). The left panels depict responses to a two-standard-deviation common error in household expectations in period 10. They show that the near-rational error induces noise in stock returns and all other economic aggregates.[21] Of particular interest is the size of the response in stock returns. Recall that in this specification, the standard deviation of the common error is 0.003% of σ_η, which means households' expectations of η in period 11 are $0.00003 * \sigma_\eta$, although they should be zero. Through its external effect on household expectations, this extremely small disturbance moves stock returns by approximately 1.1 percentage points, consumption growth by 0.3 percentage points, and output growth by 0.25 percentage points in equilibrium. The positive excess return in period 10 is then followed by a negative excess return of similar magnitude in period 11.

The right panels of Figure 5 show responses to a two-standard-deviation shock to η in period 11. The graphs show that learning about the shock in advance allows households to adjust their behavior ex ante. Interestingly, however, the amplitude of the solid blue line is smaller than the amplitudes of the dashed and dotted lines. When stock prices transmit both noise and information, households partially adjust their behavior in both periods 10 and 11, instead of fully adjusting in a single period (period 10 in the full-information case and period 11 in the no-information case). As a result, near-rational behavior also dampens the dynamic response to fundamental shocks. Depending on which of the dynamic effects dominates, near-rational behavior may thus lower the unconditional variance of stock returns and other economic aggregates for some range of parameters, explaining the "dip" in some of the plots in Figure 4.

[21]Note that, due to Jensen's inequality, the level of the excess stock return in the absence of shocks shown in Figure 5 differs from the unconditional mean excess stock return shown in Table 2.

5.2 Robustness

An important insight from Figure 4 is that the quantitative effects of near-rational behavior depend crucially on the degree of dispersion in private information across households. Specifications (3) and (4) in Table 2 thus reestimate σ_ν using alternative vectors of data targets. Holding constant our benchmark estimates of σ_φ, σ_η, ρ, and ξ, specification (3) estimates σ_ν using only our seven macroeconomic moments as data targets. The result, $\sigma_\nu/\sigma_\eta = 190$, is almost identical to the estimate in specification (1), yielding a social cost of near-rational behavior of 69.71% and a large amplification of near-rational errors (factor 13,186). The model thus delivers a better fit to macroeconomic moments when economic aggregates show a relatively large response to non-fundamental shocks.

Specification (4) repeats the same exercise but now also includes the seven asset-pricing moments in the target vector. The estimation returns a lower number, $\sigma_\nu/\sigma_\eta = 48$, resulting in an amplification of near-rational errors of factor 10,083 and a social cost of near-rational behavior of 17.47% of permanent consumption.

Specifications (5) and (6) in Table 2 repeat the analysis of specifications (1) and (2), but for an alternative model in which households receive private signals about the short-run rather than about the long-run shock to productivity,

$$s_{it} = \varphi_{t+1} + \nu_{it}.$$

Specification (5) again maximizes the fit of the alternative model according to the loss function (81). It shows a social cost of near-rational behavior of 3.59% of consumption and an effect on the stochastic steady-state level of capital accumulation of 0.58%.

Despite this now much smaller external effect of near-rational behavior on social welfare, the estimates in specification (5) and (6) show much larger effects of near-rational behavior on all business-cycle and asset-pricing moments. Figure 6 depicts the effects graphically. It shows that, depending on the dispersion of private information, near-rational behavior may raise the equity premium by up to 0.5 percentage points and may lower the standard deviation of output by 0.4 percentage points. Because households now learn about the *i.i.d.* component of the productivity shock, the standard deviation of investment falls from 3.6 times the standard deviation of output in the full-information case to 2.3 times the standard deviation of output in the no-information case. The less households learn about the future, the less they adjust their capital holdings over time. In this case, near-rational behavior thus indirectly dampens the volatility of the capital stock, such that the unconditional variance of stock returns actually falls when stock prices transmit relatively more noise. Near-rational behavior can thus affect conditional and unconditional moments in opposite directions, depending on the parameters of the model.

5.3 The Stock Market: Not a "Sideshow"

Figure 7 revisits the question of whether non-fundamental volatility in stock prices can be socially costly even if capital investment is relatively unresponsive to stock prices. It shows a comparative static of the social cost of near-rational behavior over the elasticity of the capital stock with respect to stock prices, ξ. The calculations in the top and bottom panels are based on specifications (1) and (5) in Table 2, respectively. In both cases, we show results for values of ξ ranging from one half to double our benchmark estimate. The right graphs in each of the two panels show the same comparative static for the standard deviation of investment for comparison.

The plots show that the social cost of a breakdown in the stock market's capacity to aggregate information is almost completely invariant to variation in the elasticity of the capital stock with respect to stock prices. If households have private information about the long-run shock to productivity, our point estimate of the social cost of near-rational investment is stable at around 67% of permanent consumption, regardless of whether this elasticity is 2 or 8. The social cost of a breakdown in the stock market's capacity to aggregate information is therefore almost completely independent of the size of the dynamic response of the capital stock to a given mispricing.

6 Conclusion

This paper shows that the stock market may fail to aggregate information even if it appears to be efficient in the sense that rational investors cannot systematically outperform the market. The resulting decrease in the information content of stock prices may drastically reduce welfare even if the elasticity of the capital investment with respect to stock prices is low.

In our model, each household has some private information about future productivity. If all households behave perfectly rationally, the equilibrium stock price reflects the information held by all market participants and directs resources to their most efficient use. This core function of financial markets may break down if we allow for the possibility that households do not respond to economically small incentives (i.e., on the order of 0.01% of consumption). In particular, if households make small, cross-sectionally correlated, errors when forming their expectations about future productivity, these errors give rise to an information externality: households do not internalize how errors in their investment decisions affect others' equilibrium expectations. This information externality leads information aggregation to break down precisely when it is most socially valuable, that is, when private information is highly dispersed.

The resulting collapse of the information content of stock prices increases the amount of financial risk households face and thus induces them to demand higher risk premia for holding stocks. Higher risk premia in turn distort the level of capital accumulation, output, and consumption in the long run. The social return to diligent investor behavior is thus orders of magnitude larger than the private return.

Our quantitative estimates for the social cost of near-rational behavior range between 3.6% and 69.7% of permanent consumption. The social value of the stock market's capacity to aggregate information is particularly large if it prevents households from learning about long-run shocks to productivity.

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Table 1: Calibrated Parameters

Parameter	Value	Parameter	Value
α	0.34	δ_k	0.060
δ	0.965	γ	10.
μ_a	0.018	ψ	2.0
o	0.20	ϕ_{lev}	2.0

Notes: Calibrated parameters. α: capital share; δ_k: capital depreciation rate; μ_a: average growth of productivity; σ_η: standard deviation of shock to the long-run component in productivity growth; σ_φ: standard deviation of short-run component in productivity growth; o consumption share; γ: relative risk aversion; ψ: intertemporal elasticity of substitution; δ: subjective discount factor; ϕ_{lev}: leverage of market return.

Table 2: Estimation Restults

Model			$s_{\text{it}} = \eta_{t+1} + \nu_{\text{it}}$				$s_{\text{it}} = \varphi_{t+1} + \nu_{\text{it}}$	
	Data	Std.Err.	(1)	(2)	(3)	(4)	(5)	(6)
Data Targets Used			All moments	Full rationality	Macro moments	Macro&AP moments	All moments	Full rationality
$\sigma_\epsilon/\sigma_{\eta,\varphi}(\%)$			0.003	0.	0.003	0.004	0.576	0.
$\sigma_\nu/\sigma_{\eta,\varphi}$			182.	$<\infty$	190.	48.	54.	$<\infty$
$\sigma_\varphi(\%)$			2.82	2.82	2.82	2.82	3.02	3.02
$\sigma_\eta(\%)$			0.5	0.5	0.5	0.5	0.5	0.5
ρ			0.995	0.995	0.995	0.995	0.995	0.995
ξ			4.	4.	4.	4.	4.	4.
$\alpha_1/(1\text{-}\alpha_2)$			0.3	1.	0.27	0.82	0.	1.
$1/(1\text{-}\alpha_2)$			14,046.	0.	13,168.	10,083.	11.	0.
Noise to Signal			2.37	0.	2.74	0.22	286.57	0.
$\sigma(b_i/(K_{\text{SS}}))(\%)$			0.62	0.	0.62	0.62	1.32	0.
$\Delta K_{SS}(\%)$			0.88	0.	0.92	0.24	0.58	0.
Social Loss (%)			66.86	0.	69.71	17.47	3.59	0.
Private Loss (%)			0.01	0.	0.01	0.01	0.01	0.
$V_t[\eta,\varphi]/\sigma_{\eta,\varphi}{}^2$	0.74	0.05	0.7	0.	0.73	0.18	1.	0.
$\sigma_{\text{xs}}(\text{E[dy]})(\%)$	0.27	0.15	0.001	0.	0.001	0.001	0.047	0.
σ(dy)(%)	3.34	0.39	4.33	4.33	4.33	4.32	4.42	4.81
σ(dc)/σ(dy)	0.65	0.04	0.9	0.9	0.9	0.9	0.89	0.79
σ(di)/σ(dy)	4.45	0.44	2.16	2.18	2.17	2.16	2.31	3.6
cor(dc, di)	0.68	0.11	0.59	0.58	0.59	0.59	0.57	0.29
ACF[dc]	0.51	0.14	0.87	0.87	0.87	0.87	0.85	0.95
E[r](%)	0.64	0.34	0.2	0.34	0.2	0.3	0.29	0.54
σ(r)(%)	3.82	0.46	4.39	4.38	4.39	4.38	4.47	4.56
cor(dc, $r_{\text{ex}}{}^{\text{lev}}$)	0.15	0.13	0.12	0.12	0.12	0.13	0.13	−0.05
E[$r_{\text{ex}}{}^{\text{lev}}$](%)	3.89	2.17	4.16	3.96	4.16	4.01	3.58	3.12
σ[$r_{\text{ex}}{}^{\text{lev}}$](%)	21.21	1.33	7.16	7.17	7.17	7.14	7.52	9.15
σ(q)(%)	41.	3.	33.65	33.59	33.65	33.6	33.94	34.
ACF[$r_{\text{ex}}{}^{\text{lev}}$]	0.05	0.11	0.05	0.03	0.05	0.04	0.03	−0.37
ACF[r]	0.7	0.08	0.57	0.58	0.57	0.58	0.5	0.06
ACF[q]	0.9	0.02	0.98	0.98	0.98	0.98	0.98	0.93

Note: Lowercase letters denote logs. $E[.]$, $\sigma(.)$, and $cor(.,.)$ denote means, standard deviations, and correlations, respectively. d stands for the first difference in the time series (e.g., $\sigma(dy)$ stands for the standard deviation of output growth). $ACF[.]$ refers to the first-order autocorrelation. K_{SS} stands for the stochastic steady state level of capital accumulation. Data targets constructed from annual US data 1929-2008. $V_t[\eta, \varphi_{t+1}]/\sigma^2_{\eta,\varphi}$ and $\sigma_{\text{xs}}(\text{E[dy]})$ are the ratio of conditional to unconditional variance of productivity growth and dispersion cross-sectional standard deviation of one-year-ahead forecasts of GDP growth, respectively. Both are estimated from the Survey of Professional Forecasters 1969-2008. See Appendix D.2 for details.

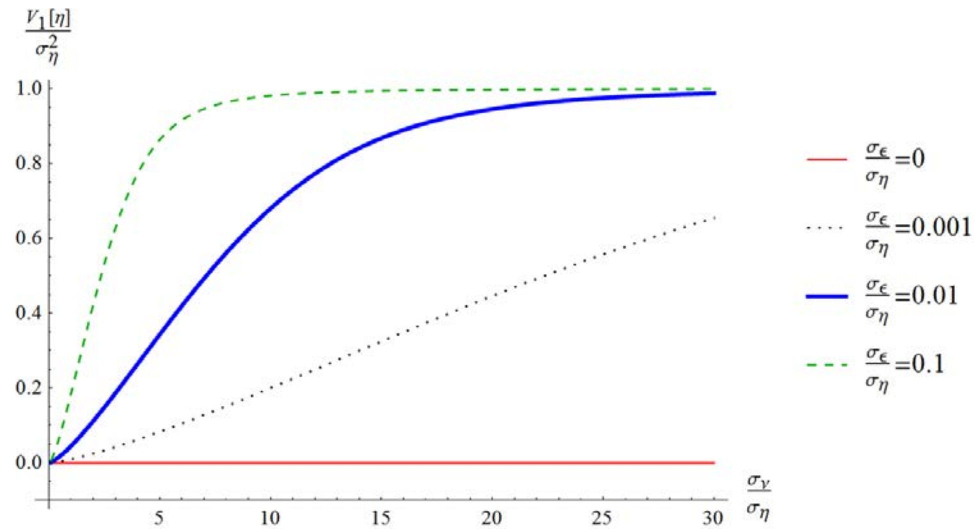

Figure 1: Ratio of the conditional variance of η to its unconditional variance plotted over the level of dispersion of private information, σ_ν/σ_η.

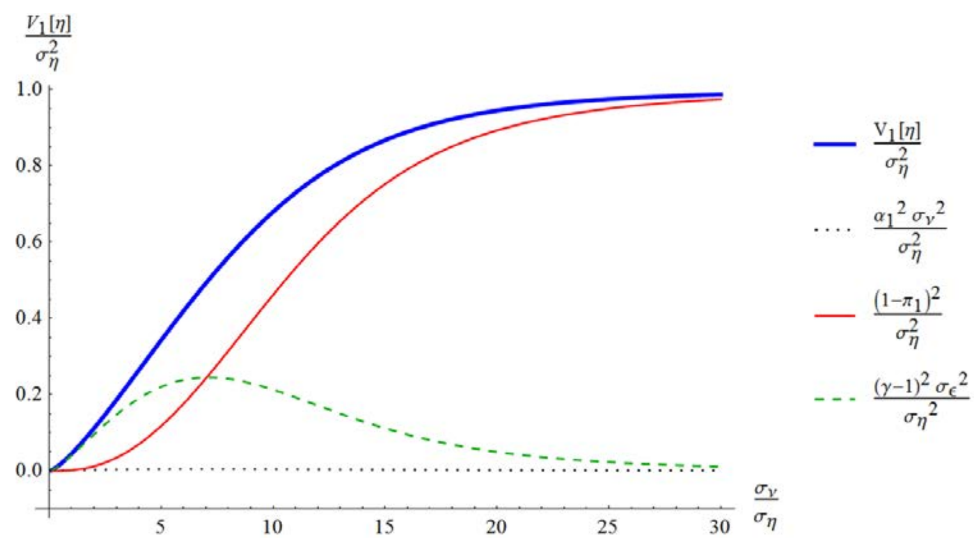

Figure 2: Decomposition of the ratio of the conditional variance of η to its unconditional variance plotted over the level of dispersion of private information.

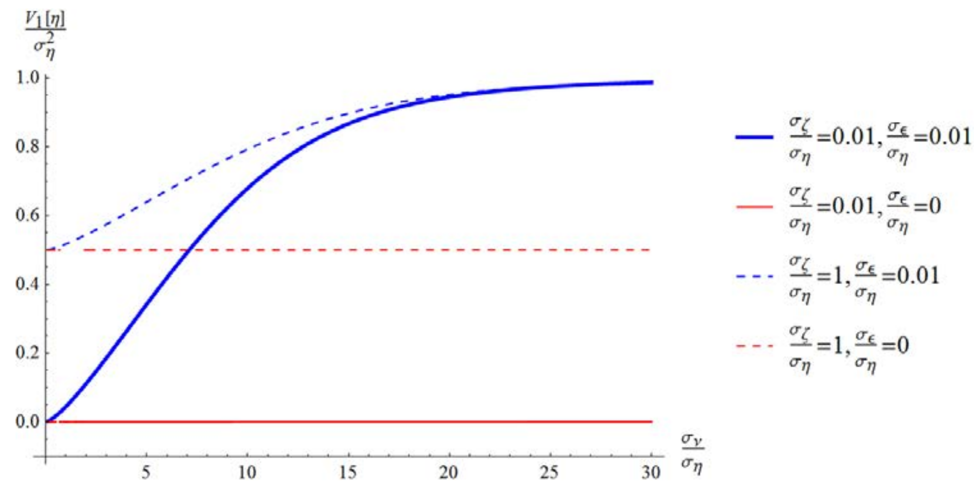

Figure 3: Comparison of the effects of small common errors in household expectations with the effects of small common noise in the private signal (solid lines) and large common noise in the private signal (broken lines).

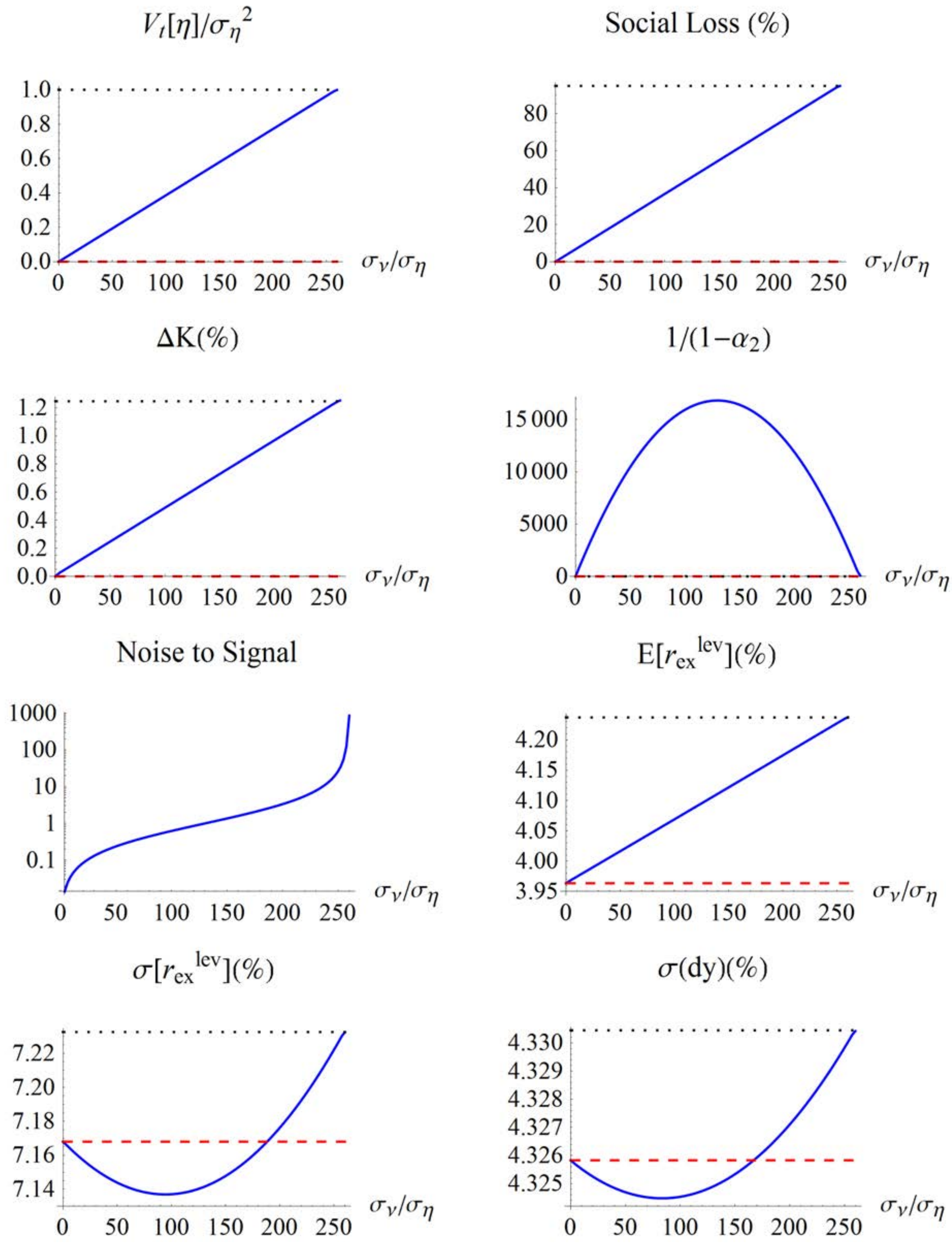

Figure 4: Comparative statics over σ_ν for selected business-cycle and asset-pricing moments. All plots are based on specification (1) in Table 2. Lowercase letters denote logs. $E[.]$ and $\sigma(.)$ denote means and standard deviations, respectively. d stands for the first difference in the time series.

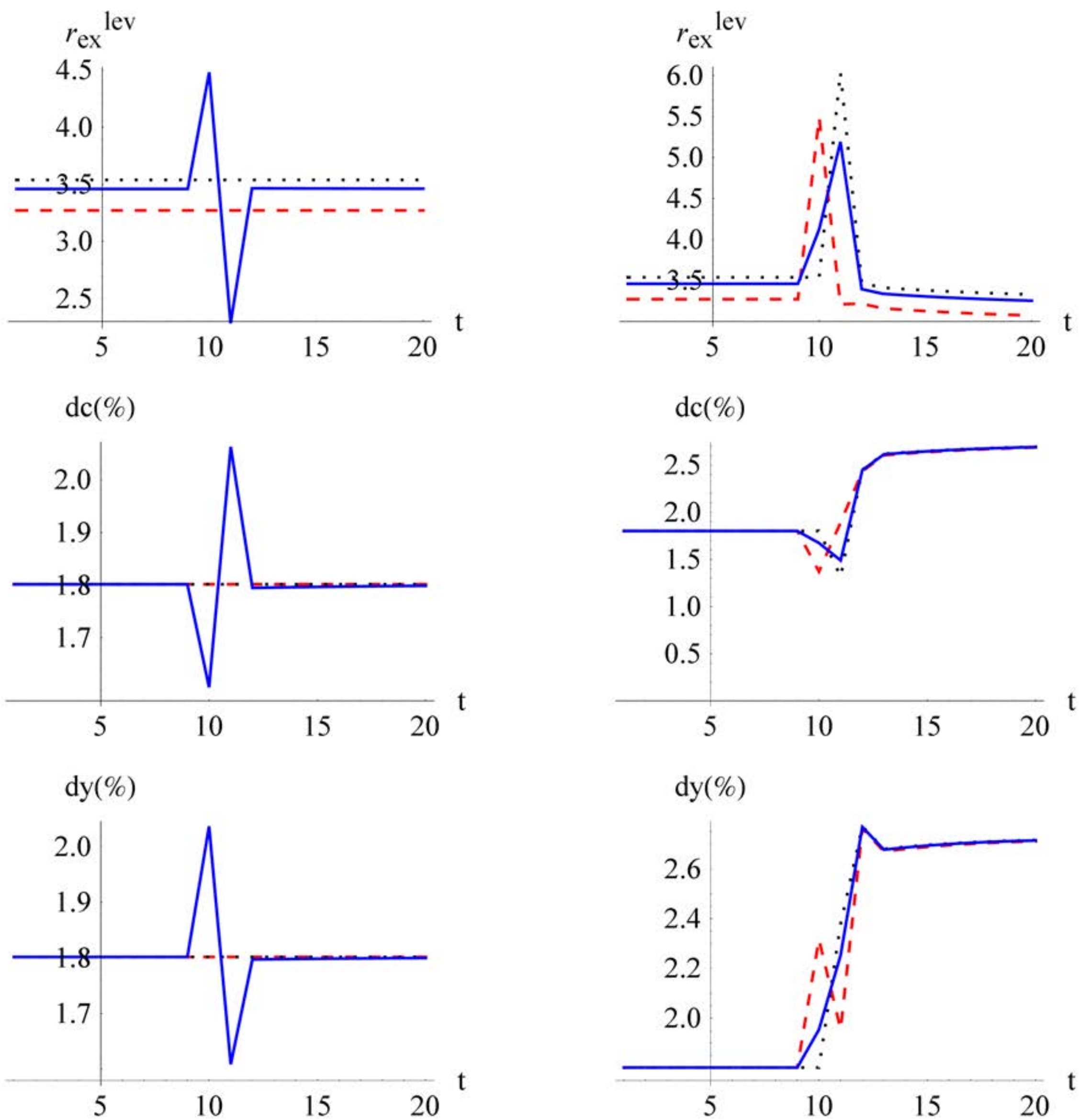

Figure 5: The solid blue lines in all panels of Figure 5 show impulse-response functions based on specification (1) in Table 2. The black dotted lines show the case in which information aggregation breaks down completely ($\sigma_\nu \to \infty$), and the red dashed line shows the results corresponding to full rationality (specification (2)). Left panels: responses to a two-standard-deviation common error in household expectations in period 10. Right panels: responses to a two-stand-deviation shock to η in period 11. Lowercase letters denote logs. d stands for the first difference in the time series.

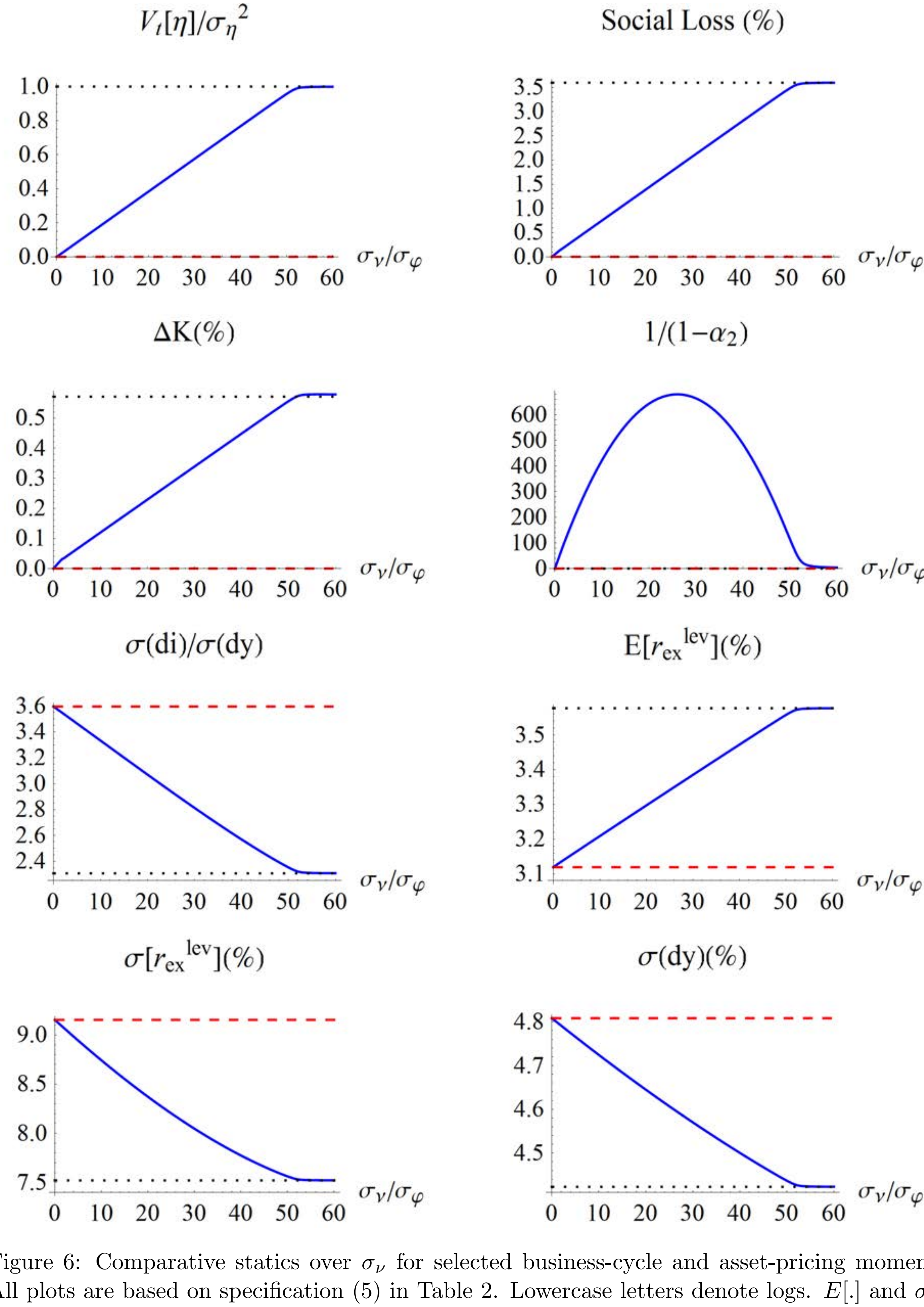

Figure 6: Comparative statics over σ_ν for selected business-cycle and asset-pricing moments. All plots are based on specification (5) in Table 2. Lowercase letters denote logs. $E[.]$ and $\sigma(.)$ denote means and standard deviations, respectively. d stands for the first difference in the time series.

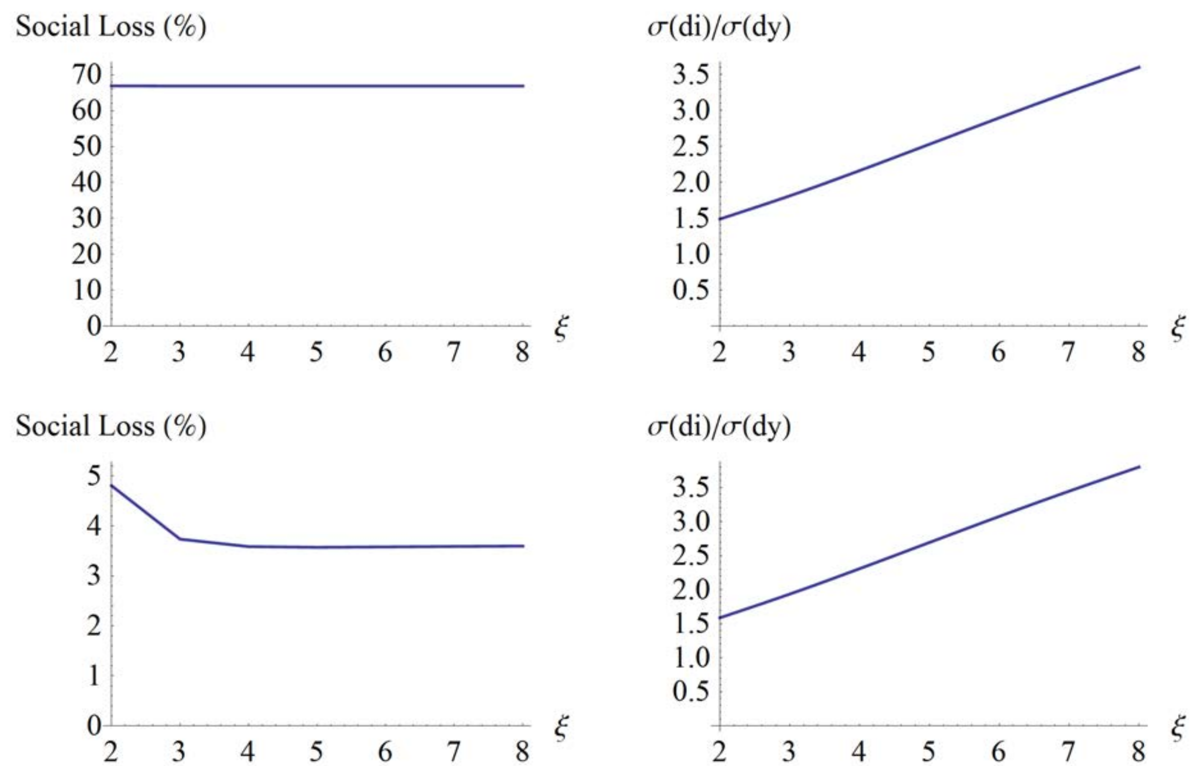

Figure 7: Comparative statics for aggregate welfare losses and the standard deviation of investment over the elasticity of capital investment with respect to the stock price, ξ. Top panels: based on specification (1) of Table 2. Bottom panels: based on specification (5) of Table 2. Lowercase letters denote logs. $\sigma(.)$ denotes means and standard deviations. d stands for the first difference in the time series.

Online Appendix

A Appendix to Section 2

A.1 Derivation of (16), (17), and (32)

Plugging (15) back into (12) and matching coefficients with (13) yields

$$\pi_0 = \frac{\alpha_0 + \rho V_1[\eta]\kappa(1-\alpha_2)}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}, \quad (82) \qquad \pi_1 = \frac{\alpha_1}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}, \quad (83) \qquad \gamma = \frac{1}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}. \quad (84)$$

Using (3) and (13), the vector (η, s_i, Q) has unconditional expectation $(\bar{\eta}, \bar{\eta}, \pi_0 + \pi_1\bar{\eta})$ and the following variance covariance matrix:

$$\Sigma = \begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}.$$

Thus, by the property of the conditional variance of the multi-normal distribution,

$$\begin{aligned} V_1[\eta] &= \sigma_\eta^2 - \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 \end{pmatrix} \\ &= \frac{1}{\sigma_\eta^{-2} + \left(\pi_1^2\gamma^{-2}\sigma_\epsilon^{-2} + \sigma_\nu^{-2}\right)}. \end{aligned} \quad (85)$$

Plugging (83) and (84) into this expression gives (16).
Similarly, by the properties of the multi-normal distribution,

$$E[\eta|s_i, Q] = \bar{\eta} + \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}^{-1} \begin{pmatrix} s_i - \bar{\eta} \\ Q - (\pi_0 + \pi_1\bar{\eta}) \end{pmatrix}.$$

Replacing Q by (12) and plugging in (15) and (85) gives (17).
Matching the coefficients of (17) with (14)

$$\begin{pmatrix} \alpha_1 \\ \alpha_2(1+\rho V_1[\eta]\kappa) \end{pmatrix} = \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}^{-1},$$

and solving for α_1, α_2 yields

$$\alpha_1 = \frac{\gamma^2\sigma_\eta^2\sigma_\epsilon^2}{\gamma^2\sigma_\nu^2\sigma_\epsilon^2 + \sigma_\eta^2\left(\pi_1^2\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2\right)}, \tag{86}$$

$$\alpha_2 = \frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\left(\gamma^2\sigma_\nu^2\sigma_\epsilon^2 + \sigma_\eta^2\left(\pi_1^2\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2\right)\right)(1+\rho V_1[\eta]\kappa)}. \tag{87}$$

Combining (87) with (83), (84), and (16) yields (32).

A.2 Proof of Lemma 2.2

Use the law of total variance and (13) and (14) to get

$$\begin{aligned}
\sigma_\eta^2 &= V_1[\eta] + V_0[E_{1i}[\eta]] \\
&= V_1[\eta] + V_0[\alpha_1\nu_i + (\alpha_1 + \alpha_2\pi_1(1+\rho V_1[\eta]\kappa))\,\eta + \alpha_2\gamma\epsilon(1+\rho V_1[\eta]\kappa)] \\
&= V_1[\eta] + \alpha_1^2\sigma_\nu^2 + (\alpha_1 + \alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2\sigma_\eta^2 + \alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2. \qquad (88)
\end{aligned}$$

Now note from (19) and (20) that

$$\alpha_1^2\sigma_\nu^2 + \alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2 = \frac{V_1[\eta]^2}{\sigma_\nu^2} + \frac{V_1[\eta]^4}{\sigma_\nu^4\sigma_\epsilon^2} = \frac{V_1[\eta]^2}{\sigma_\nu^2} + \frac{V_1[\eta]^2\alpha_1^2}{\sigma_\epsilon^2},$$

and from (16) that $\frac{\alpha_1^2}{\sigma_\epsilon^2} = \frac{1}{V_1[\eta]} - \left(\sigma_\eta^{-2} + \sigma_\nu^{-2}\right)$ such that

$$\alpha_1^2\sigma_\nu^2 + \alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2 = V_1[\eta] - V_1[\eta]^2\sigma_\eta^{-2}.$$

In addition, using (83) we can show that

$$\begin{aligned}
(\alpha_1 + \alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2 &= ((1-\alpha_2)\pi_1(1+\rho V_1[\eta]\kappa) + \alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2 \\
&= \pi_1^2(1+\rho V_1[\eta]\kappa)^2.
\end{aligned}$$

Substituting these two expressions back into (88) yields

$$\sigma_\eta^2 = 2V_1[\eta] - V_1[\eta]^2\sigma_\eta^{-2} + \pi_1^2(1+\rho V_1[\eta]\kappa)^2\sigma_\eta^2.$$

Solving this expression for $V_1[\eta]$ gives

$$V_1[\eta] = \frac{\sigma_\eta^2(1-\pi_1)}{1+\kappa\rho\pi_1\sigma_\eta^2}. \tag{89}$$

Now take the market-clearing condition (9), plug in (10) and (11) on the left-hand side and (2) on the right to get

$$\frac{\int_0^1 E_{1i}[\eta]di - Q + \epsilon}{\rho V_1[\eta]} = \kappa(Q-1).$$

Take the unconditional expectation on both sides:

$$E_0[\eta - Q] = \rho\kappa V_1[\eta](E_0[Q]-1).$$

Now note from (13) that $E_0[Q] = \pi_0 + \pi_1\bar{\eta}$ and therefore:

$$-\pi_0 + (1-\pi_1)\bar{\eta} = \rho\kappa V_1[\eta](E_0[Q]-1).$$

Solving for π_0 and plugging in (89) yields

$$\pi_0 = \frac{(1-\pi_1)(\bar{\eta}+\kappa\rho\sigma_\eta^2)}{1+\kappa\rho\sigma_\eta^2}. \tag{90}$$

Similarly, from (83), (84), and (19), it follows that

$$\gamma = \pi_1 \frac{\sigma_\nu^2}{V_1\left[\eta\right]}. \tag{91}$$

Again plugging in (89) yields

$$\gamma = \frac{\pi_1\sigma_\nu^2\left(1+\kappa\rho\pi_1\sigma_\eta^2\right)}{\sigma_\eta^2(1-\pi_1)}. \tag{92}$$

To solve for π_1, substitute (19), (20), and (91) into (83) to get

$$\pi_1 = \sigma_\nu^{-2}\left(V_1\left[\eta\right]^{-1} - \frac{V_1\left[\eta\right]^2}{\pi_1\sigma_\nu^4\sigma_\epsilon^2} + \rho\kappa\right)^{-1}. \tag{93}$$

Combining this expression with (89) and solving yields (22). Plugging (22) into (90) and (91) separately gives (21) and (23). And substituting α_1 using (19) in (16) yields (24).

A.3 Deriving (25) and (26)

Solving (24) for $V_1[\eta]$ yields three roots, one of which is real and in the interval $[0,\sigma_\eta^2]$:

$$V_1[\eta] = \frac{\sqrt[3]{2}\left(9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2 + \sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2 + 4\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\right)}\right)^{2/3} - 2\sqrt[3]{3}\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)}{6^{2/3}\sigma_\eta^2\sqrt[3]{9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2 + \sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2 + 4\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\right)}}}. \tag{94}$$

Rewrite this expression in order form of σ_ϵ:

$$V_1(\eta) = \frac{O(1)\left(O(1)O(\sigma_\epsilon^2) + \sqrt{O(1)O(\sigma_\epsilon^4) + O(1)O(\sigma_\epsilon^6)}\right)^{2/3} - O(1)O(\sigma_\epsilon^2)}{O(1)\left(O(1)O(\sigma_\epsilon^2) + \sqrt{O(1)O(\sigma_\epsilon^4) + O(1)O(\sigma_\epsilon^6)}\right)^{1/3}} \tag{95}$$
$$= O(1)\frac{O(\sigma_\epsilon^2)}{O(\sigma_\epsilon)} = O(1)O(\sigma_\epsilon),$$

where we denote $y = O(x)\ if\ \frac{y}{x} = constant$ as $\sigma_\epsilon \to 0$. It follows directly that

$$\lim_{\sigma_\epsilon \to 0} [V_1[\eta]] = 0.$$

Plugging this into (22) yields (26).

A.4 Proof of Proposition 2.3

Solve (89) for π_1 and differentiate with respect to $V_1[\eta]$ to get

$$\frac{\partial \pi_1}{\partial V_1[\eta]} = -\frac{1 + \kappa\rho\sigma_\eta^2}{\sigma_\eta^2(1 + \kappa\rho V_1[\eta])^2} < 0. \tag{96}$$

In addition, differentiate both sides of (24) with respect to σ_ϵ and rearrange to get

$$\frac{\partial V_1[\eta]}{\partial \sigma_\epsilon} = \frac{2V_1[\eta]^4}{2\sigma_\epsilon^2 V_1[\eta]^3 + \sigma_\epsilon^3\sigma_\nu^4} > 0.$$

Then the fact that $\frac{\partial \pi_1}{\partial \sigma_\epsilon} = \frac{\partial \pi_1}{\partial V_1[\eta]}\frac{\partial V_1[\eta]}{\partial \sigma_\epsilon}$ yields (27), and applying (25) with (95) yields (28).

A.5 Proof of Proposition 2.4

Lemma A.1

A near-rational household's ex-ante willingness to pay for eliminating the near-rational error from its own behavior is

$$\lambda^i = \frac{1}{2}\mu_i^2\frac{\sigma_\epsilon^2(1 + \hat{\mu}^2)}{\rho V_1[\eta]} + \frac{1}{\rho}\mu_i\frac{V_1[\eta]}{\sigma_\nu^2}. \tag{97}$$

Proof *Taking (4), plugging in (5), taking time-zero expectations, and rearranging yields*

$$E_0U_i = E_0\left[z_i(\eta - Q) + \Pi\right] - \frac{\rho}{2}E_0\left[z_i^2\right]V_1[\eta],$$

where $z_i = \frac{E_{1i}[\eta] + \mu_i(\epsilon + \hat{\epsilon}_i) - Q}{\rho V_1[\eta]}$ *from (11) and (6). It follows that the compensating variation from* *the perspective of a near-rational individual who considers becoming fully rational is*

$$\begin{aligned}
\lambda^i &\equiv -\left(E_0\left[U_i\right] - E_0\left[U_i|_{\mu_i=0}\right]\right) &(98)\\
&= -E_0\left[\frac{\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}(\eta-Q)\right] + \frac{\rho}{2}E_0\left[\left(\frac{\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}\right)^2 + 2\frac{E_{1i}\left[\eta\right]-Q}{\rho V_1[\eta]}\frac{\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}\right]V_1[\eta]\\
&= -E_0\left[\frac{\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}\eta\right] + E_0\left[\frac{\rho}{2}\left(\frac{\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}\right)^2 V_1[\eta]\right] + E_0\left[\frac{E_{1i}\left[\eta\right]\mu_i\left(\epsilon+\hat{\epsilon}_i\right)}{\rho V_1[\eta]}\right]\\
&= \frac{1}{\rho V_1[\eta]}\left(E_0\left[\frac{1}{2}\mu_i^2\left(\epsilon+\hat{\epsilon}_i\right)^2\right] + E_0\left[E_{1i}\left[\eta\right]\mu_i\left(\epsilon+\hat{\epsilon}_i\right)\right]\right).
\end{aligned}$$

Plugging in (12), (13), and (14) and simplifying yields

$$\begin{aligned}
\lambda^i &= \frac{1}{\rho V_1[\eta]}\left(\frac{1}{2}\mu_i^2\sigma_\epsilon^2(1+\hat{\mu}^2) + (1+\rho V_1[\eta]\kappa)E_0\left[\left(\alpha_2\gamma\mu_i\epsilon^2 + \alpha_2\gamma\epsilon\mu_i\hat{\epsilon}_i\right)\right]\right)\\
&= \frac{1}{\rho V_1[\eta]}\left(\frac{1}{2}\mu_i^2\sigma_\epsilon^2(1+\hat{\mu}^2) + \mu_i\alpha_2\gamma\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)\right)\\
&= \frac{1}{2}\mu_i^2\frac{\sigma_\epsilon^2\left(1+\hat{\mu}^2\right)}{\rho V_1[\eta]} + \mu_i\frac{\alpha_2\gamma\sigma_\epsilon^2}{\rho V_1[\eta]}(1+\rho V_1[\eta]\kappa).
\end{aligned}$$

In addition, note that from (19), (20), (84), and (92), we have that $\frac{\alpha_2\gamma\sigma_\epsilon^2}{V_1[\eta]} = \frac{V_1[\eta]}{\sigma_\nu^2(1+\rho V_1[\eta]\kappa)}$. *Plugging this in yields the expression given in the proposition.*

Using this lemma, the proof of Proposition 2.4 proceeds as follows: taking the derivative of (97) yields

$$\frac{\partial\lambda^i}{\partial\mu_i} = \mu_i\frac{\sigma_\epsilon^2\left(1+\hat{\mu}^2\right)}{\rho V_1[\eta]} + \frac{V_1[\eta]}{\rho\sigma_\nu^2}.$$

From (95), we have that the numerator of the first term on the right-hand side approaches 0 at a faster rate than the denominator. The second term collapses to 0 as $V_1\left[\eta\right]$ *approaches 0 as* $\sigma_\epsilon \to 0$.

A.6 Proof of Propsition 2.5

For the first part of the proposition, differentiate both sides of (24) with respect to σ_ν and rearrange to get

$$\frac{\partial V_1[\eta]}{\partial\sigma_\nu} = \frac{2\sigma_\nu^2\sigma_\epsilon^2 V_1\left[\eta\right]^2 + 4V_1\left[\eta\right]^4}{\sigma_\nu^5\sigma_\epsilon^2 + 2\sigma_\nu V_1\left[\eta\right]^3} > 0. \tag{99}$$

Combing this with (96) proves the first equality and the inequality for strictly positive σ_ϵ. The proof of the case $\sigma_\epsilon \to 0$ follows directly from (25).

For the second part of the proposition, similar to (95), rewriting $\frac{V_1[\eta]}{\sigma_\eta^2}$ in the order form of σ_ν

and σ_ϵ for them going to infinity and 0 respectively,

$$\begin{aligned}\frac{V_1[\eta]}{\sigma_\eta^2} =& \frac{\sqrt[3]{2}\left(9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2+\sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2+4\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\right)}\right)^{2/3}-2\sqrt[3]{3}\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)}{6^{2/3}\sigma_\eta^4\sqrt[3]{9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2+\sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2+4\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\right)}}} \\ =& \frac{O(1)\left(O(1)O(\sigma_\nu^4)O(\sigma_\epsilon^2)+\sqrt{O(1)O(\sigma_\nu^{12})O(\sigma_\epsilon^6)}\right)^{2/3}-O(1)O(\sigma_\nu^2)O(\sigma_\epsilon^2)-O(1)O(\sigma_\nu^4)O(\sigma_\epsilon^2)}{O(1)\left(O(1)O(\sigma_\nu^4)O(\sigma_\epsilon^2)+\sqrt{O(1)O(\sigma_\nu^{12})O(\sigma_\epsilon^6)}\right)^{1/3}} \\ =& O(1)\frac{O(\sigma_\nu^4)O(\sigma_\epsilon^2)}{O(\sigma_\nu^2)O(\sigma_\epsilon)} = O(1)O(\sigma_\nu^2)O(\sigma_\epsilon)\end{aligned}$$

gives the second line of (30) and the first line follows directly from (24).

A.7 Proof of Proposition 2.6

Taking the derivative with respect to σ_ν on both sides of (97) with $\mu_i = 1$ and simplifying yields

$$\frac{\partial\lambda^i}{\partial\sigma_\nu} = -2\frac{V_1[\eta]}{\sigma_\nu^3\rho} - \frac{1}{2\rho}\left(-\frac{2}{\sigma_\nu^2} + \frac{\sigma_\epsilon^2\left(1+\hat{\mu}^2\right)}{V_1[\eta]^2}\right)\frac{\partial V_1[\eta]}{\partial\sigma_\nu}.$$

Plugging in (99) and simplifying yields (31).

A.8 Proof of Lemma 2.8

Combine (3), (6), (9), (10), (11), (14), and (19) to show that

$$z_i - K = \frac{\nu_i}{\rho\sigma_\nu^2}. \tag{100}$$

From (2), equilibrium profits are

$$\Pi = \kappa\frac{(Q-1)^2}{2}. \tag{101}$$

Taking (4), plugging in (5), and substituting Π using (101) and (2) yields

$$U_i = z_i(\eta - Q) + \frac{K^2}{2\kappa} - \frac{\rho}{2}z_i^2V_1[\eta].$$

Replacing $z_i = \frac{\nu_i}{\rho\sigma_\nu^2} + K$, applying the definition $w_a = K(\eta - Q) + \frac{K^2}{2\kappa}$, and taking time-zero expectations on both sides yields

$$E_0[U_i] = E_0\left[\frac{\nu_i}{\rho\sigma_\nu^2}(\eta - Q) + K(\eta - Q) + \frac{K^2}{2\kappa}\right] - \frac{\rho}{2}E_0\left[\left(\frac{\nu_i}{\rho\sigma_\nu^2} + K\right)^2\right]V_1[\eta].$$

The second equality in (39) follows from the fact that $E_0[\nu_i]=0$ and ν_i is uncorrelated η, K, and Q. As a result, the first term in the left square brackets drops out and $E_0\left[2\frac{\nu_i}{\rho\sigma_\nu}K\right]=0$ in the right square brackets.

The first equality follows from noting that $E_0[U_i]$ does not depend on ν_i. It is thus independent of i, and we have that

$$SWF \equiv \int_0^1 E_0[U_i]\,di = E_0[U_i].$$

Finally, use (2) to substitute Q out of (5):

$$w_a = K(\eta-1) - \frac{K^2}{2\kappa} = (K\eta - E_0[K]\bar{\eta}) + (E_0[K]\bar{\eta} - K) - \frac{K^2}{2\kappa}. \tag{102}$$

Taking time-zero expectations on both sides yields (40) .

A.9 Proof of Proposition 2.9

From (98), we have $\frac{\partial\lambda^i}{\partial\mu_i} = -\frac{\partial E_0[U_i]}{\partial\mu_i}$. Thus it follows directly from Proposition 2.4 that

$$\lim_{\sigma_\epsilon\to 0}\left[-\frac{\partial E_0[U_i]}{\partial\mu_i}\right] = 0.$$

For the second and third equality, note that the social welfare function (39) depends on three terms: the level of expected wealth, the idiosyncratic component in the expected volatility of portfolio returns, and the aggregate component in the expected volatility of portfolio returns. We first solve each of the three components as a function of the parameters of the model and π_1. Equating (16) and (89) gives

$$\gamma = \sqrt{\frac{(1-\pi_1)\,\pi_1^2\sigma_\nu^2\sigma_\eta 2}{\sigma_\epsilon^2\left(\pi_1\sigma_\nu^2 + \sigma_\eta^2\left(\pi_1\sigma_\nu^2\kappa\rho + (\pi_1 - 1)\right)\right)}}. \tag{103}$$

Squaring both sides of (36) and taking expectations gives $E_0[K^2]$. Plugging $E_0[K^2]$, (37), and (38) into (40) and substituting in (90) and (103) yields

$$\begin{aligned} E_0[w_a] \quad &= \quad -\frac{1}{2}\kappa\{2\bar{\eta}\left(1 - \frac{(\pi_1-1)^2\left(\bar{\eta}+\kappa\rho\sigma_\eta^2\right)}{\kappa\rho\sigma_\eta^2+1}\right) + \frac{(\pi_1-1)^2\left(\bar{\eta}+\kappa\rho\sigma_\eta^2\right)^2}{\left(\kappa\rho\sigma_\eta^2+1\right)^2} \\ &\quad + (\pi_1-2)\,\pi_1^2\bar{\eta}^2 + \frac{(1-\pi_1)\,\pi_1^2\sigma_\nu^2\sigma_\eta^2}{\pi_1\sigma_\nu^2+\sigma_\eta^2\left(\pi_1\left(\sigma_\nu^2\kappa\rho+1\right)-1\right)} + (\pi_1-2)\,\pi_1\sigma_\eta^2 - 1\}. \end{aligned}$$

We can then show that

$$\lim_{\sigma_\epsilon\to 0}\left[\frac{\partial E_0[w_a]}{\partial\sigma_\epsilon}\right] = \lim_{\sigma_\epsilon\to 0}\left[\frac{\partial E_0[w_a]}{\partial\pi_1}\frac{\partial\pi_1}{\partial\sigma_\epsilon}\right] = \frac{\kappa\sigma_\eta^2}{2\left(1+\kappa\rho\sigma_\eta^2\right)}\lim_{\sigma_\epsilon\to 0}\left[\frac{\partial\pi_1}{\partial\sigma_\epsilon}\right] < 0,$$

where the last equality uses (27). Using (27) and (96) from Proposition 2.3,

$$\lim_{\sigma_\epsilon \to 0}\left[-\frac{1}{2\rho\sigma_\nu^2}\frac{\partial V_1[\eta]}{\partial \sigma_\epsilon}\right] = -\frac{1}{2\rho\sigma_\nu^2}\lim_{\sigma_\epsilon \to 0}\left(\frac{\partial \pi_1}{\partial V_1[\eta]}\right)^{-1}\lim_{\sigma_\epsilon \to 0}\left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0.$$

Similarly, taking time-zero expectations of the third term and plugging in (90), (103), and (89) yields

$$E_0\left[K^2\right]V_1[\eta] = (1-\pi_1)\,\kappa^2\sigma_\eta^2\frac{\sigma_\eta^2\left(\frac{\kappa\rho\left(\pi_1^2(\bar{\eta}-1)^2-1\right)}{\kappa\rho\sigma_\eta^2+1}+\frac{\kappa\rho\left(-(\pi_1-2)\pi_1(\bar{\eta}-1)^2-1\right)}{\left(\kappa\rho\sigma_\eta^2+1\right)^2}-\frac{(\pi_1-1)\pi_1^2\sigma_\nu^2}{\sigma_\eta^2\left(\pi_1\left(\sigma_\nu^2\kappa\rho+1\right)-1\right)+\pi_1\sigma_\nu^2}+\pi_1^2\right)+\frac{(\bar{\eta}-2)\bar{\eta}}{\left(\kappa\rho\sigma_\eta^2+1\right)^2}+1}{\pi_1\kappa\rho\sigma_\eta^2+1}.$$

Again taking the derivative with respect to σ_ϵ, taking the limit as σ_ϵ goes to zero and using (27) yields

$$\lim_{\sigma_\epsilon \to 0}\left[-\frac{\rho}{2}\frac{\partial E_0\left[K^2\right]V_1[\eta]}{\partial \sigma_\epsilon}\right] = \frac{\kappa^2\rho\sigma_\eta^2\left((1-\bar{\eta})^2+\sigma_\eta^2\right)}{2\left(1+\kappa\rho\sigma_\eta^2\right)}\lim_{\sigma_\epsilon \to 0}\left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0$$

and concludes the proof.

B Appendix to Section 3

B.1 Dispersed Information with Aggregate Noise in Private Signal

We may guess that

$$Q = \pi_0 + \pi_1(\eta+\zeta) + \gamma\epsilon,$$

where both the expectation (14) and the coefficients π_0, π_1, and γ are the ones given in the main text. However, the variance-covariance matrix of the vector (η, s_i, Q) changes to

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\zeta^2+\sigma_\eta^2+\sigma_\nu^2 & \pi_1\left(\sigma_\zeta^2+\sigma_\eta^2\right) \\ \pi_1\sigma_\eta^2 & \pi_1\left(\sigma_\zeta^2+\sigma_\eta^2\right) & \pi_1^2\left(\sigma_\zeta^2+\sigma_\eta^2\right)+\gamma^2\sigma_\epsilon^2 \end{pmatrix}.$$

Applying the projection theorem yields

$$\begin{aligned} \alpha_1 &= \frac{\gamma^2\sigma_\eta^2\sigma_\epsilon^2}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_2 &= \frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2} \end{aligned} \tag{104}$$

and

$$V_1[\eta] = \frac{\sigma_\eta^2\left(\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2}.$$

B.1.1 Proof of Proposition 3.1

Combining (83), (84), and (104) yields

$$\pi_1 = \sigma_\eta^2 \left(\sigma_\zeta^2 + \sigma_\eta^2\right)^{-1} - 2^{1/3}3^{-1/3}\sigma_\nu^2 \left(\sigma_\zeta^2 + \sigma_\eta^2 + \sigma_\nu^2\right)\sigma_\epsilon^2 \Phi^{-1} + \frac{2^{-1/3}3^{-2/3}\Phi}{\left(\sigma_\zeta^2 + \sigma_\eta^2\right)^3}, \tag{105}$$

where

$$\Phi = \left(-9\sigma_\eta^2\left(\sigma_\zeta^2+\sigma_\eta^2\right)^5\sigma_\nu^4\sigma_\epsilon^2 + \sqrt{3}\sqrt{\left(\sigma_\zeta^2+\sigma_\eta^2\right)^9\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^4\left(\sigma_\zeta^2+\sigma_\eta^2\right)\sigma_\nu^2 + 4\left(\sigma_\zeta^2+\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_\epsilon^2\right)}\right)^{1/3}.$$

Rewriting this expression in order form of σ_ϵ yields

$$\pi_1 = O(1) - 2^{1/3}3^{-1/3}O(1)O(\sigma_\epsilon^2)\Phi^{-1} + 2^{-1/3}3^{-2/3}O(1)\Phi$$

and

$$\Phi = \left(-9O(\sigma_\epsilon^2) + \sqrt{3\left(O(\sigma_\epsilon^2) + 4O(\sigma_\epsilon^6)\right)}\right)^{\frac{1}{3}} = O(\sigma_\epsilon),$$

where we denote $y = O(x)$ *if* $\frac{y}{x} = constant$ as $\sigma_\epsilon \to 0$. Taking the derivative with respect to σ_ϵ yields

$$\frac{\partial \pi_1}{\partial \sigma_\epsilon} = -2^{4/3}3^{-1/3}O(\sigma_\epsilon)\Phi^{-1} + 2^{1/3}3^{-1/3}O(\sigma_\epsilon^2)\Phi^{-2}\frac{\partial \Phi}{\partial \sigma_\epsilon} + 2^{-1/3}3^{-2/3}O(1)\frac{\partial \Phi}{\partial \sigma_\epsilon}$$

and

$$\frac{\partial \Phi}{\partial \sigma_\epsilon} = \frac{1}{3}\Phi^{-2}\left(-18O(\sigma_\epsilon) + \frac{1}{2}\left(O(\sigma_\epsilon^2) + 4O(\sigma_\epsilon^6)\right)^{-\frac{1}{2}}\left(2O(\sigma_\epsilon) + 24O(\sigma_\epsilon^5)\right)\right).$$

Cancelling coefficients and taking the limit on both sides yields the proof of the first statement:

$$\begin{aligned}
\lim_{\sigma_\epsilon \to 0}\frac{\partial \pi_1}{\partial \sigma_\epsilon} &= -\lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon)\Phi^{-1} + \lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon^2)\Phi^{-2}\frac{\partial \Phi}{\partial \sigma_\epsilon} + \lim_{\sigma_\epsilon \to 0}\frac{\partial \Phi}{\partial \sigma_\epsilon} \\
&= -\lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon)O(\sigma_\epsilon^{-1}) + \lim_{\sigma_\epsilon \to 0}\left(O(\sigma_\epsilon^2)O(\sigma_\epsilon^{-4}) + 1\right)O(\sigma_\epsilon^{-2})\left(-O(\sigma_\epsilon) + O(\sigma_\epsilon^2)\right) \\
&= -\infty.
\end{aligned}$$

Similarly, rewriting (105) in order form of σ_ζ yields

$$\pi_1 = O(1)O(\sigma_\zeta^{-2}) - 2^{1/3}3^{-1/3}O(1)O(\sigma_\zeta^2)\Phi^{-1} + 2^{-1/3}3^{-2/3}O(\sigma_\zeta^{-6})\Phi$$

and

$$\Phi = \left(-9O(1)O(\sigma_\zeta^{10}) + \sqrt{3\left(27O(1)O(\sigma_\zeta^{20}) + 4O(\sigma_\zeta^{24})\right)}\right)^{\frac{1}{3}} = O(\sigma_\zeta^4).$$

Taking the derivative with respect to σ_ζ yields

$$\begin{aligned}\frac{\partial \pi_1}{\partial \sigma_\zeta} \quad = \quad & -O(1)O(\sigma_\zeta^{-3}) - 2^{4/3}3^{-1/3}O(1)O(\sigma_\zeta)\Phi^{-1} + 2^{1/3}3^{-1/3}O(1)O(\sigma_\zeta^2)\Phi^{-2}\frac{\partial \Phi}{\partial \sigma_\zeta} \\ & -2^{2/3}3^{1/3}O(\sigma_\zeta^{-7})\Phi + 2^{-1/3}3^{-2/3}O(\sigma_\zeta^{-6})\frac{\partial \Phi}{\partial \sigma_\zeta}\end{aligned}$$

and

$$\frac{\partial \Phi}{\partial \sigma_\zeta} = \frac{1}{3}\Phi^{-2}\left(-90\ O(1)O(\sigma_\zeta^9) + \frac{\sqrt{3}}{2}\left(27\ O(1)O(\sigma_\zeta^{20}) + 4\ O(\sigma_\zeta^{24})\right)^{-1/2}\left(540\ O(1)O(\sigma_\zeta^{19}) + 96O(\sigma_\zeta^{23})\right)\right).$$

The proof of the second statement follows from applying L'Hopital's rule to this expression. Because the analytical expressions become rather cumbersome, we refer the reader to the Mathematica file provided on the authors' websites for the remainder of the proof of the second statement.

B.2 Dispersed Information with an Exogenous Public Signal

Consider a model identical to the one given in section 2 with the exception that in addition to their private signal, households also observe a public signal about future productivity,

$$g = \eta + \zeta,$$

where $\zeta \sim N(0, \sigma_\zeta^2)$. We may then guess that the solution for Q is some linear function of $\eta,\ \zeta,$ and ϵ:

$$Q = \pi_0 + \pi_1\eta + \pi_2\zeta + \gamma\epsilon,$$

where the rational expectation of η given Q and the private and public signals is

$$E_{it}\left(\eta_{t+1}\right) = \alpha_0 + \alpha_1 s_i + \alpha_2 Q + \alpha_3 g.$$

A matching coefficients algorithm parallel to that in section 2.1 gives

$$\pi_1 = \frac{\alpha_1 + \alpha_3}{1-\alpha_2},\ \pi_2 = \frac{\alpha_3}{1-\alpha_2},\ \gamma = \frac{1}{1-\alpha_2}.$$

The amplification of near-rational errors is thus influenced only in so far as the presence of public information may induce households to put less weight on the market price of capital when forming their expectations.

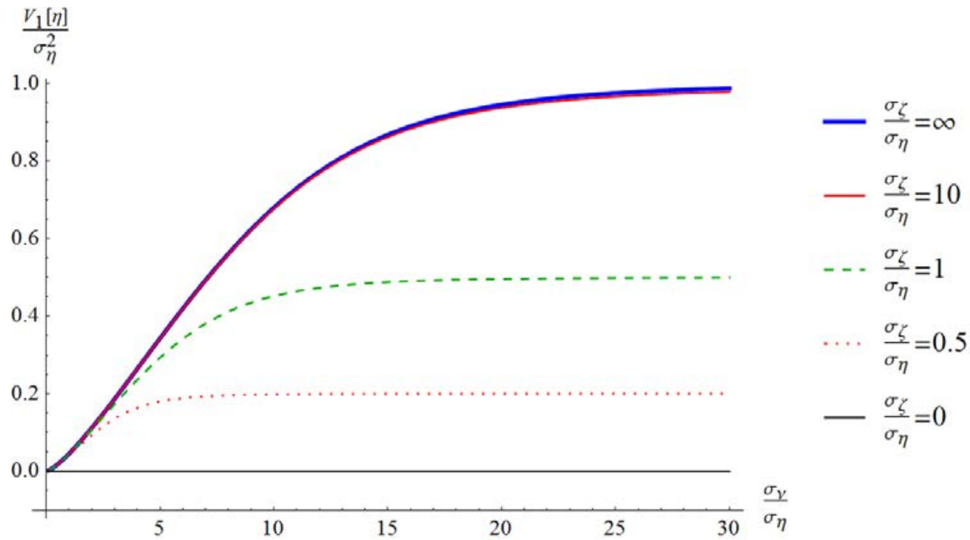

Figure 8: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η, and for varying precisions of the public signal. In each case, $\sigma_\epsilon/\sigma_\eta$ is set to 0.01.

The vector (η, s_i, Q, g) has the following variance covariance matrix:

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2+\sigma_\nu^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_2^2\sigma_\zeta^2+\pi_1^2\sigma_\eta^2+\gamma^2\sigma_\epsilon^2 & \pi_2\sigma_\zeta^2+\pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 & \pi_2\sigma_\zeta^2+\pi_1\sigma_\eta^2 & \sigma_\zeta^2+\sigma_\eta^2 \end{pmatrix}.$$

Solving the signal-extraction problem returns

$$\begin{aligned} \alpha_1 &= \frac{\gamma^2\sigma_\zeta^2\sigma_\eta^2\sigma_\epsilon^2}{\sigma_\zeta^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_2 &= \frac{(\pi_1-\pi_2)\sigma_\zeta^2\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_3 &= \frac{\sigma_\eta^2\sigma_\nu^2\left(\gamma^2\sigma_\epsilon^2+\pi_2(\pi_2-\pi_1)\sigma_\zeta^2\right)}{\sigma_\zeta^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \end{aligned}.$$

Based on these results, Figure 8 plots the conditional variance of η for the rational and near-rational expectations equilibrium and for varying levels of precision of the public signal.

In the absence of near-rational behavior, the provision of public information makes no difference, because households are already fully informed from the outset. When households are near-rational, the presence of the public signal is relevant only insofar as a collapse of information aggregation affects only the subset of information that is dispersed across households and not the information that is publicly available. If the public information provided is relatively precise, $\frac{V_1[\eta]}{\sigma_\eta^2}$ now converges to values less than 1 as σ_ν goes to infinity.

B.3 Comparison with Noise-Trader Model

B.3.1 Proof of Proposition 3.2

Because households are now fully rational, their demand schedule is

$$z_i = \frac{E_{1i}[\eta] - Q}{\rho V_1[\eta]}. \tag{106}$$

Taking time-zero expectations of (4), plugging in (5) and (106), and simplifying by law of iterated expectations yields

$$\begin{aligned} E_0\left[U_i\right] &= E_0\left[\frac{E_{1i}[\eta-Q](\eta-Q)}{\rho V_1[\eta]}\right] - \frac{\rho}{2}E_0\left[\frac{\left(E_{1i}[\eta-Q]\right)^2}{\rho^2 V_1[\eta]}\right] = \frac{1}{2}E_0\left[\frac{\left(E_{1i}[\eta-Q]\right)^2}{\rho V_1[\eta]}\right] \\ &= \frac{1}{2\rho V_1[\eta]}\left(V_0\left[E_{1i}[\eta-Q]\right] + \left(E_0\left[\eta-Q\right]\right)^2\right), \end{aligned}$$

where we have used that $\Pi = 0$ when $\kappa = 0$. Using the law of total variance, we can then replace $V_0\left[E_{1i}[\eta-Q]\right] = V_0\left[\eta-Q\right] - V_1\left[\eta\right]$ and simplify to get

$$E_0\left[U_i\right] = \frac{\left(E_0[\eta-Q]\right)^2 + V_0[\eta-Q]}{2\rho V_1[\eta]} - \frac{1}{2\rho} = SWF,$$

where the second equality uses the fact that $E_0\left[U_i\right]$ is no longer a function of i and thus $SWF = \int E_0\left[U_i\right] di = E_0\left[U_i\right]$.

Plugging in (13) and the expressions from (108) yields

$$SWF = \frac{1}{2}\sigma_\nu^2\sigma_\vartheta^2\rho - \frac{1}{2}\frac{\sigma_\nu^6\sigma_\vartheta^4\rho^3}{\sigma_\nu^4\sigma_\vartheta^2\rho^2 + \sigma_\eta^2\left(\sigma_\nu^2\sigma_\vartheta^2\rho^2+1\right)}.$$

It follows immediately that

$$\frac{\partial SWF}{\partial\sigma_\vartheta} = \frac{\sigma_\nu^8\sigma_\vartheta^5\rho^6\sigma_\eta^2 + \sigma_\vartheta\sigma_\eta^4\left(\sigma_\nu^3\sigma_\vartheta^2\rho^3+\sigma_\nu\rho\right)^2}{\rho\left(\sigma_\eta^2\left(\sigma_\nu^2\sigma_\vartheta^2\rho^2+1\right)+\sigma_\nu^4\sigma_\vartheta^2\rho^2\right)^2} > 0.$$

To calculate expected utility of noise traders, again take time-zero expectations of (4), plug in (5) and (45), and simplify to get

$$\begin{aligned} E_0[U_j] &= E_0\left[\mu_j\vartheta(\eta-Q)\right] - \frac{\rho}{2}E_0\left[\mu_j^2\vartheta^2\right]V_1\left[\eta\right] \\ &= -\mu_j\gamma\sigma_\vartheta^2 - \frac{\rho}{2}\mu_j^2\sigma_\vartheta^2 V_1\left[\eta\right]. \end{aligned}$$

Taking the derivative with respect to μ_j yields

$$\frac{\partial E_0[U_j]}{\partial \mu_j} = -\gamma\sigma_\vartheta^2 - \rho\mu_j\sigma_\vartheta^2 V_1[\eta] < 0. \tag{107}$$

B.3.2 Proof of Proposition 3.3

Substituting $E_{1i}[\eta]$ in (106) with $E_{1i}[\eta] = \alpha_0 + \alpha_1 s_i + \alpha_2 Q$ and (3), plugging the resulting expression into (46), and simplifying yields

$$\alpha_0 + \alpha_1\left(\eta + \int_0^1 \nu_i di\right) + (\alpha_2 - 1)\,Q = \rho V_1[\eta]\vartheta.$$

Solving this expression for Q and matching coefficients with (13) yields

$$\pi_0 = \frac{\alpha_0}{1-\alpha_2},\ \pi_1 = \frac{\alpha_1}{1-\alpha_2},\ \gamma = \frac{\rho V_1[\eta]}{1-\alpha_2}.$$

Note that the expressions π_0 and π_1 are identical to (82) and (83). Similarly, repeating the steps in section 2.1, we find that the expressions for (18), (19), and (20) are identical to those in the near-rational model. However, the expression for γ is now multiplied with $\rho V_1[\eta]$ relative to its counterpart in (84). Solving the system yields

$$\pi_0 = \frac{\sigma_\eta^{-2}\bar{\eta}}{\sigma_\eta^{-2} + \sigma_\nu^{-2} + \rho^{-2}\sigma_\vartheta^{-2}\sigma_\nu^{-4}},\ \pi_1 = \frac{\sigma_\nu^{-2} + \rho^{-2}\sigma_\vartheta^{-2}\sigma_\nu^{-4}}{\sigma_\eta^{-2} + \sigma_\nu^{-2} + \rho^{-2}\sigma_\vartheta^{-2}\sigma_\nu^{-4}},\ \gamma = \rho\sigma_\nu^2\pi_1. \tag{108}$$

Taking the derivative of π_1 with respect to σ_ϑ in (108) and simplifying yields

$$\frac{\partial \pi_1}{\partial \sigma_\vartheta} = -\frac{2\sigma_\nu^4\sigma_\vartheta\rho^2\sigma_\eta^2}{\left(\sigma_\eta^2\left(\sigma_\nu^2\sigma_\vartheta^2\rho^2 + 1\right) + \sigma_\nu^4\sigma_\vartheta^2\rho^2\right)^2}.$$

As σ_ϑ approaches 0 the denominator approaches σ_η^4 while the numerator approaches 0.

B.4 Errors about Higher Moments

Rather than making near-rational errors about the conditional mean of η, we may consider a model identical to the one in section 2, but in which households make a small common error about the second conditional moment rather than about the first conditional moment. We could then rewrite (42) as

$$\frac{\alpha_0 + \alpha_1 \int s_i di + \alpha_2 Q - Q}{\rho V_1[\eta] + \epsilon_V} = K.$$

Solving for Q yields

$$\frac{\alpha_0 - K\rho V_1[\eta]}{1-\alpha_2} + \frac{\alpha_1}{1-\alpha_2}\eta - \frac{K}{1-\alpha_2}\epsilon_V = Q.$$

In a model with an exogenous and strictly positive supply of capital, near-rational errors about the first and second conditional moments are thus isomorphic. However, with an endogenous capital stock, errors about the second conditional moment break the Gaussian structure of the model and are more complicated to analyze.

B.5 Proof of Proposition 3.4

Lemma B.1

A rational household would pay

$$E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] - E_0\left[U_i|_{\mu_i=0}\right] = \frac{\pi_1^2\left(\left((\pi_1-1)\,\bar{\eta}+\pi_0\right)^2 + \gamma^2\sigma_\epsilon^2 + (\pi_1-2)\,\pi_1\sigma_\eta^2 + \sigma_\eta^2\right)}{2\gamma^2\sigma_\epsilon^2\hat{\mu}^2} \tag{109}$$

to observe the near-rational error it would have made, had it been near-rational.

Proof *First, a household using additional signal $\hat{s}_i$ has a conditional variance of*

$$V[\eta|s_i,Q,\hat{s}_i] \equiv \hat{V}_1[\eta] = \left(\sigma_\eta^{-2} + \sigma_\nu^{-2} + \pi_1^2\gamma^{-2}\sigma_\epsilon^{-2}\left(1+\hat{\mu}^{-2}\right)\right)^{-1} \tag{110}$$

and holds the posterior expectation

$$E[\eta|s_i,Q,\hat{s}_i] \equiv \hat{E}_{i1}[\eta] = \frac{\sigma_\eta^{-2}\bar{\eta} + \sigma_\nu^{-2}s_i + \pi_1^2\sigma_\epsilon^{-2}\gamma^{-2}(\eta+\frac{\gamma}{\pi_1}\epsilon) + \pi_1^2\sigma_\epsilon^{-2}\gamma^{-2}\hat{\mu}^{-2}\hat{s}_i}{\hat{V}_1[\eta]^{-1}}. \tag{111}$$

Second, plugging (5) into (4), taking time-zero expectations, and rearranging yields

$$E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] = E_0\left[z_i(\eta-Q)+\Pi\right] - \frac{\rho}{2}E_0\left[z_i^2\right]\hat{V}_1[\eta],$$

where $z_i = \frac{\hat{E}_{1i}[\eta]-Q}{\rho\hat{V}_1[\eta]}$ from (11). It follows that a rational household's willingness to pay to observe $\hat{s}_i$ is

$$\begin{aligned} E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] - E_0\left[U_i|_{\mu_i=0}\right] \quad = \quad & E_0\left[\frac{\hat{E}_{1i}[\eta]-Q}{\rho\hat{V}_1[\eta]}(\eta-Q) + \kappa\frac{(Q-1)^2}{2}\right] - \frac{\rho}{2}E_0\left[\left(\frac{\hat{E}_{1i}[\eta]-Q}{\rho\hat{V}_1[\eta]}\right)^2\right]\hat{V}_1[\eta] \\ & - \left(E_0\left[\frac{E_{1i}[\eta]-Q}{\rho V_1[\eta]}(\eta-Q) + \kappa\frac{(Q-1)^2}{2}\right] - \frac{\rho}{2}E_0\left[\left(\frac{E_{1i}[\eta]-Q}{\rho V_1[\eta]}\right)^2\right]V_1[\eta]\right). \end{aligned} \tag{112}$$

Plugging in (3), (13), (16), (17), (47), (110), and (111) and applying the expectations operator yields the expression in the proof. Note that this calculation is somewhat involved.

Using this lemma, we now proof the Proposition. From (89), we have

$$1-\pi_1=\frac{V_1[\eta]\left(\kappa\rho\sigma_\eta^2+1\right)}{\sigma_\eta^2\left(\kappa\rho V_1\left[\eta\right]+1\right)}. \tag{113}$$

From (94), we have

$$V_1\left[\eta\right]=\frac{O(\sigma_\epsilon^2)}{O(\sigma_\epsilon)}-O(\sigma_\epsilon)=O(\sigma_\epsilon). \tag{114}$$

Combining (113) and (114) yields $1-\pi_1=O(\sigma_\epsilon)$. Thus, using (90) and (103), we have

$$\begin{aligned}\pi_0 &= O(1-\pi_1) \ = \ O(\sigma_\epsilon),\\ \gamma &= O(\sqrt{\frac{\pi_1(1-\pi_1)}{\sigma_\epsilon^2}})=O(\sqrt{\frac{\pi_1\sigma_\epsilon}{\sigma_\epsilon^2}}) \ = \ O(\pi_1^{\frac{1}{2}}{\sigma_\epsilon}^{-\frac{1}{2}}).\end{aligned}$$

With these two facts, taking the limit of (109) of Lemma B.1 as $\sigma_\epsilon\to0$ yields

$$\begin{aligned}\lim_{\sigma_\epsilon\to0}\left[E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right]-E_0\left[U_i|_{\mu_i=0}\right]\right] &= \lim_{\sigma_\epsilon\to0}\frac{\pi_1^2\left(O\left(\sigma_\epsilon^2\right)+\left(\pi_1-2\right)\pi_1\sigma_\eta^2+\sigma_\eta^2\right)}{2\ O\left(\frac{\pi_1}{\sigma_\epsilon}\right)\sigma_\epsilon^2\hat{\mu}^2}+\lim_{\sigma_\epsilon\to0}\pi_1^2\frac{1}{2\hat{\mu}^2}\\ &= \lim_{\sigma_\epsilon\to0}\pi_1^2\frac{O\left(\sigma_\epsilon^2\right)}{2\ O(\pi_1\sigma_\epsilon)\hat{\mu}^2}+\lim_{\sigma_\epsilon\to0}\pi_1^2\frac{\left(\pi_1-2\right)\pi_1\sigma_\eta^2+\sigma_\eta^2}{2\ O(\pi_1\sigma_\epsilon)\hat{\mu}^2}+\lim_{\sigma_\epsilon\to0}\pi_1^2\frac{1}{2\hat{\mu}^2}\end{aligned}$$

Then using (26) and simply plugging in $\pi_1=1$ gives (48).

C Appendix to Section 4

C.1 Equation of Motion for Capital

Plugging (71) into (65) and integrating over individuals on both sides with market-clearing conditions (67), (68), and (69) gives

$$Q_tK_{t+1}=Q_{t-1}R_tK_t-C_t+w_tN_t. \tag{115}$$

Plugging in (55), (66), and (70) yields (52).

C.2 Deriving the Equilibrium Conditions

After taking the ratio of the first-order conditions with respect to labor and consumption, we get the marginal rate of substitution between labor and consumption:

$$\frac{1-o}{o}\frac{\left(1-n_{it}\right)^{-1}}{C_{it}^{-1}}=w_t. \tag{116}$$

The optimal choice of stock holdings is determined by the familiar asset-pricing equation,

$$\mathcal{E}_{it}[M_{it+1}R_{t+1}] = 1, \tag{117}$$

where the stochastic discount factor $M_{i,t+1}$ is given by

$$M_{it+1} = \delta \left(\frac{C_{it+1}}{C_{it}}\right)^{-1} \left(\frac{\tilde{C}_{it+1}}{\tilde{C}_{it}}\right)^{1-\frac{1}{\psi}} \left(\frac{U_{it+1}}{\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}, \tag{118}$$

and returns R_{t+1} are defined in (66).

Similarly, by combining the first-order and envelope conditions for bonds, the optimal choice of bonds holdings is determined by

$$\mathcal{E}_{it}[M_{it+1}](1+r_t) - \frac{\pi'(b_{it})}{o(1-\delta)(1-\frac{1}{\psi})\tilde{C}_{it}^{1-\frac{1}{\psi}}C_{it}^{-1}} = 1. \tag{119}$$

Given these conditions of optimality, capital and labor markets clear when conditions (67) and (69) hold, and the optimal consumption follows from the household's budget constraint (65).

C.2.1 Detailed Derivation

Agents maximize utility (60) subject to budget constraint (65). State variables in individual optimization are the holdings of capital and bonds, namely, $U_{it} = U_{it}(k_{it}, b_{it-1})$. We denote the derivatives of the value function with respect to k_{it} and b_{it-1} by U_{ikt} and U_{ibt} respectively. Thus the first-order conditions and envelope conditions are as follows:
First-order condition with respect to consumption:

$$(1-\delta)\tilde{C}_{it}^{-\frac{1}{\psi}}\tilde{C}_{it}oC_{it}^{-1} = \delta\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}[U_{it+1}^{-\gamma}U_{ikt+1}\frac{1}{Q_t}]. \tag{120}$$

First-order condition with respect to bonds:

$$\delta\mathcal{E}_{it}[U_{it+1}^{1-\gamma}]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}\left[U_{it+1}^{-\gamma}\left(U_{ikt+1}\frac{1}{Q_t} - U_{ibt+1}\right)\right] + (1-\frac{1}{\psi})^{-1}\pi'(b_{it}) = 0. \tag{121}$$

First-order condition with respect to labor:

$$(1-\delta)\tilde{C}_{it}^{-\frac{1}{\psi}}\tilde{C}_{it}(1-o)(1-n_{it})^{-1} = \delta\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}[U_{it+1}^{-\gamma}U_{ikt+1}\frac{w_t}{Q_t}]. \tag{122}$$

Envelope condition for capital:

$$U_{ikt} = U_{it}^{\frac{1}{\psi}} \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{ikt+1} \frac{Q_{t-1}}{Q_t} R_t]. \tag{123}$$

Envelope condition for bonds:

$$U_{ibt} = U_{it}^{\frac{1}{\psi}} \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{ikt+1} \frac{1}{Q_t} (1 + r_{t-1})]. \tag{124}$$

Taking the ratio of first-order conditions with respect to labor (122) and consumption (120) gives (116), where w_t is given by (55).

Plugging the first-order condition with respect to consumption (120) into the right-hand side of the envelope condition for capital (123) gives

$$U_{ikt} = U_{it}^{\frac{1}{\psi}} (1 - \delta) \tilde{C}_{it}^{1 - \frac{1}{\psi}} o C_{it}^{-1} Q_{t-1} R_t. \tag{125}$$

Iterating (125) to $t+1$, plugging $\frac{U_{ikt+1}}{Q_t}$ into the first-order condition with respect to consumption (120), and rearranging yields

$$\tilde{C}_{it}^{-\frac{1}{\psi}} \tilde{C}_{it} o C_{it}^{-1} = \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{it+1}^{\frac{1}{\psi}} \tilde{C}_{it+1}^{1-\frac{1}{\psi}} o C_{it+1}^{-1} R_{t+1}]. \tag{126}$$

Using (118) in (126) yields (117).

Analogously, for bond holdings, combining first-order conditions with respect to bonds (121) and consumption (120) gives

$$(1 - \delta) \tilde{C}_{it}^{-\frac{1}{\psi}} \tilde{C}_{it} o C_{it}^{-1} = \delta \mathcal{E}_{it} [U_{it+1}^{1-\gamma}]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it} \left[U_{it+1}^{-\gamma} U_{ibt+1} \right] - \frac{\pi'(b_{it})}{1 - \frac{1}{\psi}}. \tag{127}$$

Combining the first-order condition with respect to consumption (120) and the envelope condition for bond holdings (124) gives

$$U_{ibt} = U_{it}^{\frac{1}{\psi}} (1 - \delta) \tilde{C}_{it}^{1 - \frac{1}{\psi}} o C_{it}^{-1} (1 + r_{t-1}). \tag{128}$$

Substituting (128) into (127) for U_{ibt+1} simplifies to (119).

C.3 Proof of Lemma 4.2

We proceed in three steps that demonstrate the consistency of the two statements in Lemma 4.2.

First, individual state variables are functions of the set of commonly known state variables S_t as they would be in a representative agent economy. Furthermore, households form beliefs about next period's innovation to productivity using their private signal and the market price of capital. Any individual choice by households x_i (where x can be consumption c, labor n, or capital holdings k') is thus a function of the state space $x_i(S_{it})$, where $S_{it} = \left\{S_t, \hat{q}_t, \mathcal{E}_{it}[\eta_{t+1}]\right\}$. Plugging this structure into our equilibrium condition results in a form

$$g_l(S_{it}) = \mathcal{E}_{it}\left[g_r(S_{it}, S_{it+1})\right]. \tag{129}$$

Note here that S_{it} contains all possible state variables in period t, and hence aggregate variables can be determined by a subset of this state vector as well.

Now we show that given the structure on the right-hand side of the equation, the left-hand side is a function of the state space S_{it}. We replace the function inside the expectation on the right-hand side by its Taylor series:

$$\begin{aligned} g_r &\left[S_{it}, K_{t+1}, \omega_t, \eta_{t+1}, \varphi_{t+1}, \hat{q}_{t+1}, \mathcal{E}_{it+1}\right] \\ &= \sum_{\mathbf{j}} \frac{c_{\mathbf{j}}(S_{it})}{\mathbf{j}!}(K_{t+1} - K_0)^{j_1}\omega_t^{j_2}\eta_{t+1}^{j_3}\varphi_{t+1}^{j_4}\hat{q}_{t+1}^{j_5}\mathcal{E}_{it+1}^{j_6}, \end{aligned} \tag{130}$$

where K_0 is the level of capital at the deterministic steady state, $\mathcal{E}_{it} = \mathcal{E}_{it}[\eta_{t+1}]$, $c_{\mathbf{j}}(S_{it})$ denotes the (state-t dependent) coefficients of the Taylor series, and $\mathbf{j} = (j_1, j_2, j_3, j_4, j_5, j_6)$ a multi-index for the expansion.

Now we take near-rational expectations conditional on s_{it} and $\hat{q}_t$. As Lemma 4.3 shows, the conditional expectation is a sufficient statistic for the entire posterior distribution due to normality and a constant conditional variance. The terms depending on K_{t+1} and ω_t are known at time t and can thus be taken outside the expectations operator. Moreover, we get a series of terms depending on the conditional expectation of φ_{t+1}. Because φ_{t+1} is unpredictable for an investor at time t and all shocks are uncorrelated with each other, the first-order term is 0, and all the higher-order terms depending on $\mathcal{E}_{it}[\varphi_{t+1}]$ are just moments of the unconditional distributions of φ. The same is true for the terms depending on $\hat{q}_{t+1}$, and $\mathcal{E}_{it+1}$. The only terms remaining inside the expectations operator are then those depending on η_{t+1}. We can thus write

$$\begin{aligned} \mathcal{E}_{it}\left[g_r[S_{it}, S_{it+1}]\right] &= \sum_{j=0}^{\infty} \frac{\hat{c}_j(S_{it}, K_{t+1}, \rho\omega_{t-1} + \eta_t)}{j!}\mathcal{E}_{it}[\eta_{t+1}^j] \\ &= g_l(K_t, \omega_{t-1}, \eta_t, \varphi_t, \hat{q}_t, \mathcal{E}_{it}), \end{aligned} \tag{131}$$

where the coefficients $\hat{c}_j(S_{it}, K_{t+1}, \omega_t)$ collect all the terms depending on the K_{t+1}, ω_t, and higher moments of the shocks η_{t+1} and $\mathcal{E}_{it+1}$. The third line follows from the second since all expectations of higher-order monomials of η_{t+1} are known. This step again follows from the conditional normality with constant variance and known (deterministic) higher moments. Hence we only need to keep track of the expectation of the innovation to productivity but its higher conditional moments are constant.

Finally, in deriving the set of individual state variables, we notice that contingent-claims trading eliminates any meaningful distribution of capital across time, and thus show the consistency of the individual state space.

Second, we show that aggregate quantities depend on known state variables as well as the average expectation of next period's innovation to productivity $\hat{q}$. Therefore, consider an aggregate variable of the form

$$\bar{X}(\bar{S}) = \int x_i(S_i) di, \tag{132}$$

where $\bar{X}$ can represent labor (as in (69)), consumption (73), or capital (67). Again, we plug in the Taylor series representation for individual state variables:

$$\int x_i(S_i) di = \int \sum_{\mathbf{j}} \frac{c_{\mathbf{j}}}{\mathbf{j}!} (K_t - K_0)^{j_1} \omega_{t-1}^{j_2} \eta_t^{j_3} \varphi_t^{j_4} \hat{q}_t^{j_5} \mathcal{E}_{it}^{j_6} di. \tag{133}$$

Only the last term differs across households, and thus all other variables can be taken outside the integral. Integrating over individual expectations can be rewritten as

$$\int \mathcal{E}_{it}^{j} di = \int (\mathcal{E}_{it} - \hat{q}_t + \hat{q}_t)^j di = \sum_{k=0}^{j} \begin{pmatrix} j \\ k \end{pmatrix} \int (\mathcal{E}_{it} - \hat{q}_t)^k di \hat{q}_t^{j-k}. \tag{134}$$

Again, all moments of $\mathcal{E}_{it} - \hat{q}_t$, which only depends on ν_{it}, are known and thus the integral only depends on $\hat{q}$. Therefore, equation (132) holds.

Using these insights, we solve the model using standard perturbation techniques. Perturbation methods approximate equilibrium policy functions by their Taylor series around the deterministic steady state. To arrive at the coefficients of the Taylor series, we bring all equilibrium conditions into the appropriate form shown in equation (129). Successively differentiating the equation, evaluating at the steady state, and solving the resulting system of equations for the coefficients in the Taylor series delivers the approximate solutions for the equilibrium policy functions and prices.

C.4 Proof of Lemma 4.3

Given Lemma 4.2, it follows immediately that

$$E_{it}\left[\eta_{t+1}\right] = E\left[\eta_{t+1}|s_{it}, S_t\right] = E\left[\eta_{t+1}|s_{it}, \hat{q}_t\right], \tag{135}$$

where $\hat{q}_t$ is defined by (74).

We can thus guess that the rational expectation of η_{t+1} is the linear function

$$E_{it}[\eta_{t+1}] = \alpha_0 + \alpha_1 s_{it} + \alpha_2 \hat{q}_t, \tag{136}$$

where α_0, α_1, and α_2 are the optimal weights on the prior, the private signal, and the average expectation, respectively. Substituting in (74), taking the integral across individuals, and solving for $\int E_{it}\left[\eta_{t+1}\right] di$ gives

$$\int E_{it}[\eta_{t+1}]di = \frac{\alpha_0}{1-\alpha_2} + \frac{\alpha_1}{1-\alpha_2}\eta_{t+1} + \frac{\alpha_2}{1-\alpha_2}\epsilon_t. \tag{137}$$

Adding ϵ_t on both sides of the equation, substituting (74) and simplifying yields

$$\frac{1-\alpha_2}{\alpha_1}\hat{q}_t - \frac{\alpha_0}{\alpha_1} = \eta_{t+1} + \frac{1}{\alpha_1}\epsilon_t. \tag{138}$$

Thus with the normality of the fundamental shock ϵ_t and the demand statistic $\hat{q}_t$, (77) and (78) follows directly from Bayes' rule. Matching coefficients of (77) with (136) gives (79), which concludes Lemma 4.3.

D Appendix to Section 5

D.1 Welfare Calculations

Lemma D.1

The share increase in lifetime consumption that makes a household indifferent with respect to the implementation of a given policy experiment at time 0 *can be written as*

$$\lambda = \frac{\log\left(\hat{U}_0\right) - \log\left(\bar{U}_0\right)}{o},$$

where $\hat{U}_0 = E_0\left[U\left(\left\{\hat{C}_{it}, \hat{n}_{it}\right\}_{t=1}^{\infty}\right)\right]$, $\bar{U}_0 = E_0\left[U\left(\left\{\bar{C}_{it}, \bar{n}_{it}\right\}_{t=1}^{\infty}\right)\right]$, *and the sequences* $\left\{\hat{C}, \hat{n}\right\}$ *refer to the household's sequences of consumption and labor if the policy is implemented, and* $\left\{\bar{C}, \bar{n}\right\}$ *are the corresponding sequences if the policy is not implemented.*

Proof *First note that the utility function (60) is homogeneous of degree o in consumption:*

$$U\left(\left\{e^{\lambda}C_{it},n_{it}\right\}_{t=1}^{\infty}\right)=e^{o\lambda}U\left(\{C_{it},n_{it}\}_{t=1}^{\infty}\right).$$

Using this property, it follows that the share increase in consumption, λ, that compensates the household for not adopting the policy can be written as

$$\hat{U}_0=e^{o\lambda}\bar{U}_0.$$

The lemma follows from solving this equation for λ.

D.2 Data Sources

Consumption (C_t). Per-capita consumption data are from the National Income and Product Accounts (NIPA) annual data reported by the Bureau of Economic Analysis (BEA). The data are constructed as the sum of consumption expenditures on nondurable goods and services (Table 1.1.5, Lines 5 and 6) deflated by corresponding price deflators (Table 1.1.9, Lines 5 and 6).

Physical Investment (I_t). Per-capita physical investment data are also from the NIPA tables. We measure physical investment by fixed investment (Table 1.1.5, Line 8) minus information-processing equipment (Table 5.5.5, Line 3) deflated by its price deflator (Table 1.1.9, Line 8). Information-processing equipment is interpreted as investment in intangible capital and is therefore subtracted from fixed investment.

Output (Y_t). It is the sum of total consumption and investment, that is, $C_t + I_t$. We exclude government expenditure and net export because they are not explicitly modeled in our economy.

Labor (N_t). It is measured as the total number of full-time and part-time employees as reported in the NIPA Table 6.4. Data are annual.

Stock market return (R_t) and **Risk-free rate**. (r_t) The stock market returns are from the Fama-French dataset available online on K. French′s webpage at

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors.zip.
The nominal risk-free rate is measured by the annual three-month T-bill return. The real stock market returns and risk-free rate are computed by subtracting realized inflation (annual CPI through FRED) from the nominal risk-free rate.

Price-dividends (pd_t) ratio and **Tobin′s Q** (Q_t). Data on annual price-dividend ratio, and dividend are obtained from CRSP. Annual dividends are obtained by time-aggregating monthly dividends. Nominal dividends are turned into real dividends using the CPI index. Data on Tobin′s Q are from the Flow of Funds (FoF) and are obtained directly from the St. Louis Fed by dividing the variable MVEONWMVBSNNCB (Line 35 of Table B.102 in the FoF report) by TNWMVBSNNCB (Line 32 of table B.102 in the FoF report).

GDP Forecast ($\hat{y}_{it}$ and $\hat{y}_t$). GDP forecast data during 1969-2010 are from the Survey of Professional Forecasters provided by the Philadelphia Federal Reserve at

http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/ where $\hat{y}_{it}$ is the forecast of forecaster i for time t and $\hat{y}$ is the mean forecast of all forecasters for time t.

D.3 Moments Generation

Annual data from 1929 to 2008 are used to generate the target moments in Table 2. Lowercase letters denote log-units, dx denotes growth rate of variable x, and $E[.]$, $\sigma(.)$, and corr(., .) denote the mean, volatility, and correlation, respectively. r_f is the risk-free rate and r_{ex}^{lev} is the excess return of the stock market. All returns and growth rates are in percentages. For instance, $\sigma(dy)(\%)$ is the standard deviation of $log(GDP_{t+1}) - log(GDP_t)$ in percentage terms. Standard errors of moments and moment ratios are calculated by bootstrapping.

$\sigma_{\text{xs}}(\text{E[dy]})$ is the dispersion in GDP forecasts across forecasters, calculated as the time-series average of the cross-sectional standard deviation of one-year-ahead forecasts (1969-2008).

$\frac{V_t[\eta]}{\sigma_\eta^2}$ is calculated by $\frac{1}{n}\Sigma_{i=1}^{n}\frac{var_i(da_{t+1}-E_{it}[da_{t+1}])}{var(da_{t+1})}$, the cross-sectional average of time-series variances of individual forecast errors of productivity growth, standardized by the variance of realized productivity growth.

Productivity growth da_{t+1} and its forecasts $E_{it}[da_{t+1}]$ are calculated by Solow residuals. Taking logs of the production function and differencing across time gives $d\,y_{t+1} = \alpha dk_{t+1} + (1-\alpha)\,dn_{t+1} + (1-\alpha)\,da_{t+1}$. Regress dy_{t+1} on dk_{t+1} and dn_{t+1} to obtain the residual da_{t+1}. Similarly, using individual forecasts $E_{it}[dy_{t+1}]$ and assuming dk_{t+1} and dn_{t+1} are known at t gives individual forecasts of productivity growth $E_{it}[da_{t+1}]$.

The standard error is calculated from one million random draws of $var_i\,(da_{t+1} - E_{it}[da_{t+1}])$ and $var(da_{t+1})$, assuming each of them is normally distributed with the variance itself as the mean and its bootstrapped standard error as the standard deviation, among which negative draws are omitted.

To get reasonable forecasts, only professional forecasters who participated in the survey for more than 20 years between 1969 and 2008 are used to calculate $var_{it}\,(da_{t+1} - E_{it}[da_{t+1}])$.