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PEGS AND PAIN

Stephanie Schmitt-Grohé
Martin Uríbe

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ABSTRACT

We identify a disconnect between historical and model-based assessments of the costs of currency pegs due to nominal rigidities. While the former attribute major contractions and massive unemployment to currency pegs, the latter find miniscule welfare losses. The goal of this paper is to reconcile these two assessments. We refocus attention to downward wage inflexibility as the central source of nominal rigidity. More importantly, our model departs from existing sticky wage models in the Calvo-Rotemberg tradition in that employment is not always demand determined. This departure creates an endogenous connection between macroeconomic volatility and the average level of unemployment and in this way opens the door to large welfare gains from stabilization policy. In a calibrated version of the model, an external crisis, defined as a two-standard-deviation decline in tradable output and a two-standard-deviation increase in the country interest rate premium, causes the unemployment rate to rise by more than 20 percentage points under a peg. Currency pegs are shown to be highly costly also during regular business-cycle fluctuations. The median welfare cost of a currency peg is 4 and 10 percent of consumption per period.

Stephanie Schmitt-Grohé
Department of Economics
Columbia University
New York, NY 10027
and NBER
stephanie.schmittgrohe@columbia.edu

Martin Uríbe
Department of Economics
Columbia University
International Affairs Building
New York, NY 10027
and NBER
martin.uribe@columbia.edu

1 Introduction

Fixed exchange rate arrangements are easy to adopt but difficult to maintain. Countries can find themselves confined to a currency peg in a number of ways. For instance, a country could have adopted a currency peg as a way to stop high or hyperinflation in a swift and nontraumatic way. A classical example is the Argentine Convertibility Law of April 1991, which, by mandating a one-to-one exchange rate between the Argentine peso and the U.S. dollar, painlessly eliminated hyperinflation in less than a year. Another route by which countries arrive at a currency peg is the joining of a monetary union. Recent examples include emerging countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries that joined the Eurozone. Most of these countries experienced an initial transition into the Euro characterized by low inflation, low interest rates, and economic expansion.

The Achilles' heel of currency pegs is, however, that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade or hikes in the country interest-rate premium. The reason is that such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both. The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, if nominal prices, especially factor prices, are downwardly rigid, the real depreciation will take place only slowly, causing recession and unemployment along the way. This narrative is well known. It goes back at least to Keynes (1925) who argued that Britain's 1925 decision to return to the gold standard at the 1913 parity despite the significant increase in the aggregate price level that took place during World War I, would cause deflation in nominal wages with deleterious consequences for unemployment and economic activity. Similarly, Friedman's (1953) seminal essay points at downward nominal wage rigidity as the central argument against fixed exchange rates.

This paper is motivated by the apparent disconnect between historical and model-based assessments of the welfare costs of currency pegs. Economic history studies tend to attribute devastating economic outcomes to currency pegs. For example, in his authoritative treatment, Eichengreen (1992) argues that the Gold Standard is the key to understanding the Great Depression. Eichengreen and Sachs (1985) go further and point to the combination of downward wage rigidity and the Gold Standard as essential for understanding the duration of the Great Depression across countries. Specifically, they argue that countries that went off gold earlier had lower real wages and a more speedy recovery than countries that insisted on defending the gold standard. Bernanke and Carey (1996) make a similar point, namely, that a fixed exchange rate regime cum nominal wage rigidity was the key propagating factor of the Great Depression. More recently, this thesis has been put forth by Krugman to explain the roots of the Great Recession in peripheral Europe.[1] In light of these historical accounts that attribute major economic contractions and massive unemployment to the combination of currency pegs and nominal rigidities, one might expect that existing model-based quantitative studies would predict large welfare losses of currency pegs. However, this is not the case. For example, Kollmann (2002) and Galí and Monacelli (2005) perform welfare-cost calculations in the context of small, open economy models with Calvo-type sticky product prices. They find that the welfare cost of currency pegs is less than one tenth of one percent of consumption per period. The historical and model based assessments of the welfare costs of currency pegs are, therefore, miles apart. According to the former, currency pegs are extremely painful, whereas according to the latter they are a minor inconvenience.

The goal of this paper is to reconcile the historical and model-based assessments of the pains of pegs. In doing so, we refocus attention on downward wage rigidity as the central nominal friction and propose a different model of the labor market. An important feature that distinguishes our theoretical framework from standard new Keynesian models of nominal wage rigidity is that our formulation implies an endogenous connection between macroeconomic volatility and the secular level of unemployment. This connection is due to the nature of the labor contract implicit in our model, according to which employment is demand determined during contractions but supply and demand determined during booms. As a result, involuntary unemployment emerges during downturns and full employment during booms. Consequently, aggregate fluctuations cause unemployment on average. More importantly, the average level of unemployment is increasing in the amplitude of the business cycle, opening the door to large welfare gains from macroeconomic stabilization policy. This connection between aggregate volatility and the average level of unemployment is absent in more standard sticky-wage models, such as those following in the Calvo or Rotemberg tradition. In this class of models, employment is always demand determined. As a result increases in unemployment during recessions are roughly offset by reductions in unemployment (or increases in involuntary overtime) during booms, annihilating any sizable link between the amplitude of the cycle and the average level of unemployment and output. As a result, typically, this class of models delivers miniscule welfare gains from optimal stabilization

[1] See, for instance, his New York Times column of April 29, 2010 and his New York Times Magazine article of January 12, 2011.

policy.

The paper embeds downward nominal wage rigidity into a dynamic, stochastic model of a small open economy with traded and nontraded goods. The model is driven by stochastic disturbances to the traded endowment (which can be interpreted as variations in the terms of trade) and by disturbances to the country interest-rate premium. We estimate the joint law of motion of these two exogenous variables using Argentine data and feed it into a calibrated version of the model, which is then solved using global methods.

We show that in our model during an external crisis, defined as a two-standard-error increase in the country interest-rate premium and an equally large drop in the terms of trade, unemployment rises by more than twenty percentage points under a peg. The model predicts that currency pegs are costly not only during crises but also over regular business-cycle fluctuations. We find that in our baseline model the unconditional mean of involuntary unemployment is 14 percent in the fixed exchange-rate regime. In turn, large levels of unemployment translate into large welfare losses. We measure the welfare cost of a currency peg as the difference between the level of welfare enjoyed by households under a currency peg and the level of welfare associated with the optimal exchange rate policy. We find that the median welfare cost of living under a currency peg is enormous, between 4 and 10 percent of lifetime consumption.

This paper is related to the extensive literature on optimum currency areas initiated by Mundell (1961). More recently, substantial research activity has been devoted to studying the extent to which Europe forms an optimum currency area (see, for instance, Baldwin and Wyplosz; 2009; and Mongelli, 2005). Of the many criteria for an optimum currency area that have been identified in the literature, the four most problematic ones for Europe appear to be: lack of labor mobility in spite of regulation that allows for the physical movement of people across the eurozone; asymmetric shocks across states; nominal rigidities; and lack of fiscal federalism. Our theoretical framework incorporates these departures from the conditions for an optimum currency area and quantifies the cost for an individual country of being part of the resulting suboptimal currency union. We wish to stress, however, that we do not consider our analysis a full assessment of the desirability of currency unions or currency pegs. For, by construction, currency pegs are suboptimal in our setup, as they are in the model-based welfare evaluations cited above. Rather, our goal is to overcome the disconnect between historical and model-based assessments of the costs of pegs stemming from the presence of nominal rigidities.

The remainder of the paper is organized as follows: Section 2 develops a dynamic stochastic model of a small open economy with involuntary unemployment due to downward nominal wage rigidity. This section also characterizes aggregate macroeconomic dynamics under a currency peg and under the optimal exchange rate policy. Section 3 presents empirical evidence on downward nominal wage rigidity and uses it to calibrate the model's structural parameter governing this source of nominal friction. Section 4 presents a quantitative analysis of external crises under currency pegs and under the optimal exchange rate policy. Section 5 calculates the welfare benefits of switching from a currency peg to the optimal exchange rate policy. Section 6 analyzes how exchange rate policy affects the long-run distribution of external debt. Section 7 augments the model to allow for a demand for money as a second source of nominal friction. Section 8 extends the model to allow for an endogenous labor supply decision. Section 9 allows for production in the traded sector. Section 10 studies the robustness of our findings to alternative parameterizations of the baseline model. Section 11 presents a fiscal policy that implements the efficient allocation when the exchange rate is fixed. Finally, section 12 concludes.

2 Downward Wage Rigidity in a Small Open Economy

In this section, we develop a model of a small open economy in which nominal wages are downwardly rigid. In the model, the labor market is perfectly competitive. As a result, even though all participants understand that wages are nominally rigid, they do not act strategically in their pricing behavior. Instead, workers and firms take factor prices as given. The model features a traded and a nontraded sector and aggregate fluctuations are driven by stochastic movements in the value of tradable output and in the country-specific interest rate.

2.1 Households

The economy is populated by a large number of infinitely-lived households with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c_t denotes consumption, U is a utility index assumed to be increasing and strictly concave, $\beta \in (0,1)$ is a subjective discount factor, and $\mathbb{E}_t$ is the expectations operator conditional on information available in period t. Consumption is assumed to be a composite good made of tradable and nontradable goods via the aggregation technology

$$c_t = A(c_t^T, c_t^N), \tag{2}$$

where c_t^T denotes consumption of tradables, c_t^N denotes consumption of nontradables, and A, defined over positive values of both of its arguments, denotes an aggregator function assumed to be homogeneous of degree one, increasing, concave, and to satisfy the Inada conditions. These assumptions imply that tradables and nontradables are normal goods.

One source of business-cycle fluctuations in our model are variations in the value of tradable output. Specifically, we assume that households are endowed with an exogenous and stochastic amount of tradable goods, y_t^T. Movements in y_t^T can be interpreted either as caused by innovations in the terms of trade or by productivity shocks in the traded sector. The rationale for introducing this type of shock is based on existing studies suggesting that they are an important source of fluctuations in emerging economies (see, for instance, Mendoza, 1995). In section 9, we relax the assumption that y_t^T is exogenous by assuming that tradables are produced with a technology that uses labor as an endogenous input and is buffeted by productivity (or terms-of-trade) shocks. Households are also assumed to be endowed with a constant number of hours, $\bar{h}$, which they supply inelastically to the labor market. The assumption of an inelastic labor supply is motivated in part by microeconometric evidence (e.g., Blundell and MaCurdy, 1999) and macroeconometric evidence from models with nominal rigidities (e.g., Justiniano, Primiceri, and Tambalotti, 2010; and Smets and Wouters, 2007) suggesting that the labor supply elasticity is near zero. A second reason for assuming an inelastic labor supply is that it makes the workings of our two-sector model more transparent. In section 10, we relax this assumption by endogenizing the labor-supply decision.

Because of the presence of nominal wage rigidities in the labor market, households will in general only be able to work $h_t \leq \bar{h}$ hours each period. Households take h_t as exogenously determined. Households also receive profits from the ownership of firms, denoted Φ_t, and expressed in terms of domestic currency. Households have access to an internationally traded risk-free pure discount bond that pays the country-specific interest rate, r_t, in terms of foreign currency when held between periods t and $t+1$. We assume that r_t is exogenous and stochastic. It represents one of the two sources of aggregate uncertainty in the model. Allowing for country interest-rate shocks is motivated by existing studies that find that this type of disturbance represents a significant source of business-cycle fluctuations in emerging countries (see Neumeyer and Perri, 2005; and Uribe and Yue, 2006).[2] The household's sequential budget constraint in period t is then given by

$$P_t^T c_t^T + P_t^N c_t^N + d_t E_t \leq P_t^T y_t^T + W_t h_t + \frac{d_{t+1}}{1+r_t} E_t + \Phi_t, \tag{3}$$

[2] A potentially fruitful extension of the present study could be to endogenize the country interest rate by, for example, allowing for limited commitment to repay debt.

where P_t^T denotes the nominal price of tradables, P_t^N denotes the nominal price of nontradables, W_t denotes the nominal wage rate, and E_t denotes the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. The variable d_{t+1} denotes the amount of debt assumed in period t and maturing in period $t+1$. We assume that all of the external liabilities of the household are denominated in foreign currency. This assumption is motivated by the empirical literature on the 'original sin,' which documents that virtually all of the debt issued by emerging countries is denominated in foreign currency (see, for example, Eichengreen, Hausmann, and Panizza, 2005). Households are subject to a no-Ponzi game constraint of the form

$$d_{t+1} \leq \bar{d}, \quad \text{for } t \geq 0, \tag{4}$$

where $\bar{d} > 0$ is the natural debt limit. We assume that the law of one price holds for traded goods and that the foreign-currency price of traded goods is constant and normalized to unity. This assumption implies that

$$P_t^T = E_t.$$

The optimization problem of the household consists in choosing contingent plans c_t, c_t^T, c_t^N, and d_{t+1} to maximize (1) subject to the aggregation technology (2), the sequential budget constraint (3), and the borrowing limit (4). Letting λ_t/E_t and μ_t denote the Lagrange multipliers associated with (3) and (4), respectively, the optimality conditions associated with this dynamic maximization problem are (2), (3) holding with equality, (4), and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \tag{5}$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N), \tag{6}$$

$$\frac{\lambda_t}{1+r_t} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{7}$$

$$\mu_t \geq 0, \tag{8}$$

$$\mu_t (d_{t+1} - \bar{d}) = 0, \tag{9}$$

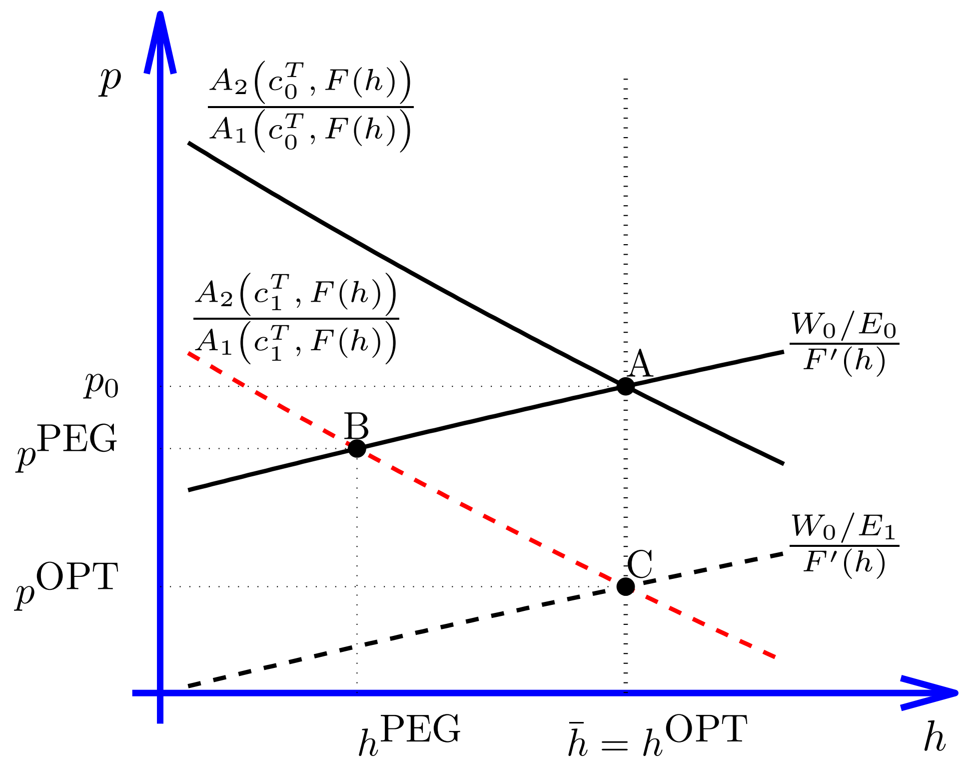
where A_i denotes the partial derivative of A with respect to its i-th argument, and

$$p_t \equiv \frac{P_t^N}{E_t}$$

denotes the relative price of nontradables in terms of tradables.

Optimality condition (5) can be interpreted as a demand function for nontradables. In-

Figure 1: Currency Pegs and Unemployment



tuitively, it states that an increase in the relative price of nontradables induces households to engage in expenditure switching by consuming relatively less nontradables. Our maintained assumptions regarding the form of the aggregator function A guarantee that, for a given level of c_t^T, the left-hand side of (5) is a decreasing function of c_t^N. Figure 1 displays this downward sloping relationship with a solid line. The figure is drawn in the space (h_t, p_t) by taking into account market clearing in the nontraded sector, $c_t^N = F(h_t)$, which will be introduced later on. The level of c_t^T acts as a shifter of the demand schedule for nontradables. A decline in c_t^T (from c_0^T to $c_1^T < c_0^T$ in the figure) causes the demand schedule to shift downward and to the left (see the broken downward sloping line in figure 1).

Optimality condition (7) equates the marginal benefits and the marginal costs of borrowing $1/(1+r_t)$ units of tradables in period t. The left-hand side of this expression indicates the marginal utility of $1/(1+r_t)$ units of tradables consumed in period t. The right-hand side presents the marginal cost of borrowing $1/(1+r_t)$ units of tradables in period t. This cost is the sum of two components. One is $\beta \mathbb{E}_t \lambda_{t+1}$, which indicates the decline in utility caused by the reduction in tradable consumption in period $t+1$ necessary to repay the extra debt assumed in period t. The second component of the marginal cost is μ_t, which measures the shadow cost of tightening the borrowing limit. This second component is nil in periods in which $d_{t+1} < \bar{d}$, that is, in periods in which the natural borrowing limit does not bind.

2.2 Firms

The nontraded good is produced using labor as the sole factor input by means of an increasing and concave production function, $F(h_t)$. The firm operates in competitive product and labor markets. Profits, Φ_t, are given by

$$\Phi_t = P_t^N F(h_t) - W_t h_t.$$

The firm chooses h_t to maximize profits taking the nominal price, P_t^N, and the nominal wage rate, W_t, as given. The optimality condition associated with this problem is

$$P_t^N F'(h_t) = W_t. \tag{10}$$

This first-order condition implicitly defines the firm's demand for labor. Alternatively, writing this expression as $p_t = \frac{W_t/E_t}{F'(h_t)}$, it can be interpreted as the supply schedule of nontradables. Given the wage rate, an increase in p_t induces firms to hire more hours and hence supply more nontradable goods. This supply schedule is shown with a solid upward sloping line in figure 1. Given the nominal wage rate, W_t, a devaluation of the domestic currency (an increase in E_t from E_0 to E_1 in the figure) acts as a shifter of the supply schedule. Specifically, given W_t, an increase in E_t lowers the real labor cost, inducing firms to expand employment. This shift in the supply schedule is shown with an upward sloping dashed line in figure 1.

In the present model, firms are always on their labor demand curve. Put differently, firms never display unfilled vacancies nor are forced to employ more workers than desired. As we will see shortly, this will not be the case for workers, who will at times be off their labor supply schedule and will experience involuntary unemployment. We note that the fact that firms are always on their labor demand schedule does not mean that employment is solely demand determined. As we will see, in this model, employment is sometimes demand determined (typically during downturns), and sometimes supply and demand determined (typically during expansions). This feature of the present model distinguishes it from sticky-wage models in the Calvo-Rotemberg tradition, in which employment is always demand determined.

An assumption that distinguishes the present setup from the existing related literature is that nominal wages are assumed to be downwardly rigid. Specifically, we impose that

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \tag{11}$$

This setup nests the cases of absolute downward rigidity, when $\gamma \geq 1$, and full wage flexibility, when $\gamma = 0$.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. Actual employment must satisfy

$$h_t \leq \bar{h} \tag{12}$$

at all times. Finally, we assume that at any point in time, wages and employment must satisfy the slackness condition

$$(\bar{h} - h_t)\,(W_t - \gamma W_{t-1}) = 0. \tag{13}$$

This condition states that periods of unemployment must be accompanied by a binding wage constraint. It also states that when the wage constraint is not binding, the economy must be in full employment.

2.3 General Equilibrium

Market clearing in the nontraded sector requires that

$$c_t^N = F(h_t). \tag{14}$$

Combining this market clearing condition, the definition of firm profits, and the household's sequential budget constraint, equation (3), yields

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}. \tag{15}$$

Letting $w_t \equiv W_t/E_t$ denote the real wage in terms of tradables and $\epsilon_t \equiv E_t/E_{t-1}$ denote the gross devaluation rate, we can rewrite equations (10), (11), and (13), respectively, as

$$p_t F'(h_t) = w_t, \tag{16}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{17}$$

and

$$(\bar{h} - h_t)\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0. \tag{18}$$

We can then define the general equilibrium of the economy as follows:

Definition 1 (General Equilibrium Dynamics) *The general equilibrium dynamics are given by stochastic processes c_t, c_t^T, c_t^N, h_t, p_t, w_t, d_{t+1}, λ_t, and μ_t satisfying (2) and (4)-(9), (12), (14)-(18) given the exogenous stochastic processes y_t^T and r_t, an exchange rate policy, and initial conditions w_{-1} and d_0.*

We consider two polar cases for the yet unspecified exchange rate policy: the optimal exchange rate policy and a currency peg.

2.4 Currency Pegs

Under a currency peg the government commits to keeping the nominal exchange rate constant over time, $E_t = E_{t-1}$ for $t \geq 0$. As a result, the gross rate of devaluation equals unity at all times:

$$\epsilon_t = 1, \tag{19}$$

for $t \geq 0$. Aggregate equilibrium dynamics under a currency peg are then given by the exchange rate policy (19) and the conditions listed in Definition 1.

An important assumption implicit in equation (19) is that currency pegs are fully credible and that the government has a commitment technology at its disposal to ensure the peg permanently. This is a conservative assumption. For the existing empirical literature has cast serious doubts on the hypothesis that pegs are a successful tool for making low-inflation regimes credible and long lasting. In their authoritative survey, Calvo and Végh (1999, page 1553) write "A notable aspect of exchange-rate-based stabilization programs is that, as noted in table 1, a vast majority has ended in balance of payment crises. In fact, of all the major programs listed in table 1, the Argentine 1991 Convertibility Plan is so far the only successful plan, which has maintained the exchange rate at the level chosen at the inception of the program. Eight of the twelve plans ended in full-blown crisis." With the benefit of hindsight, we should also include the Convertibility plan among the failed fixed-exchange-rate experiments. We conjecture that, were we to embrace the empirically more compelling assumption that pegs suffer from temporariness and lack of credibility, as modeled, for instance in Calvo (1986), their associated welfare costs could be even larger than the numbers reported in the present study.

According to the present model, under a currency peg, the economy is subject to two nominal rigidities. One is policy induced: The nominal exchange rate, E_t, is kept fixed by the monetary authority. The second is structural and is given by the downward rigidity of the nominal wage W_t. The combination of these two nominal rigidities results in a real rigidity. Specifically, under a currency peg, the real wage expressed in terms of tradables, w_t, is downwardly rigid, and adjusts only sluggishly, at the rate $(1-\gamma)$, to negative demand shocks. The labor market is, therefore, in general, in disequilibrium and features involuntary unemployment. The magnitude of the labor market disequilibrium is a function of the amount by which the past real wage exceeds the current full-employment real wage. It follows that under a currency peg, the past real wage, w_{t-1}, becomes a relevant state variable for the economy.

Figure 1 depicts qualitatively the general disequilibrium for a given value of the aggregate domestic absorption of tradable goods, c^T. Assume that the initial desired level of tradable absorption is c_0^T. Market clearing in the nontraded sector occurs at point A, where the demand function, $A_2(c_0^T, F(h))/A_1(c_0^T, F(h))$, intersects the supply schedule, $(W_0/E_0)/F'(h)$. At point A, the economy enjoys full employment ($h = \bar{h}$) and the relative price of nontradables is equal to p_0. At point A, firms are on their labor demand schedule and households are on their labor supply schedule.

Suppose now that a negative demand shock, such as a deterioration in the terms of trade or a rise in the country premium, causes a decline in the desired aggregate absorption of tradables from c_0^T to $c_1^T < c_0^T$. This adverse shock causes the demand function to shift down and to the left. This is because, at each level of nontraded consumption, households are willing to consume less traded goods only if nontraded goods become relatively cheaper. At the same time the supply schedule does not shift.[3] This is because the combination of a currency peg and downward rigidity of nominal wages prevents the real wage from adjusting downward. The new intersection of the demand and supply schedules occurs at point B. At this point, firms are on their labor demand schedule, but households are off their labor supply schedule. Employment is equal to $h^{PEG} < \bar{h}$ and the economy suffers from involuntary unemployment at the rate $\bar{h} - h^{PEG}$, where h^{PEG} denotes the level of employment that obtains under the currency peg.

Alternatively, consider an exchange rate policy that preserves full employment when the economy is hit by the negative external shock. Specifically, suppose that in response to the negative external shock, the central bank were to devalue the domestic currency so as to deflate the purchasing power of nominal wages to a point consistent with full employment. That is, suppose that the central bank sets the exchange rate at the optimal level $E_1 > E_0$ satisfying $(W_0/E_1)/F'(\bar{h}) = A_2(c_1^T, F(\bar{h}))/A_1(c_1^T, F(\bar{h}))$. In this case the supply schedule would shift down and to the right intersecting the new demand schedule at point C, where unemployment is nil ($h = \bar{h}$). So, unlike the currency peg, the optimal exchange rate policy deflates the real value of wages, and in this way avoids involuntary unemployment. A further difference in the macroeconomic adjustment under a currency peg and the optimal

[3] For expositional convenience, we are assuming here that $\gamma = 1$. A value of γ close to but less than unity would imply a small displacement of the supply schedule down and to the right.

exchange rate policy is that under a currency peg the relative price of non-tradables falls by less than under the optimal exchange rate policy. Specifically, in figure 1 the relative price of nontradables falls to p^{PEG}, whereas under the optimal policy, the relative price falls to $p^{OPT} < p^{PEG}$. This insufficient decline in the relative price of nontradables stands in the way of households switching expenditures away from tradables and toward nontradables, which is the root of the unemployment problem associated with currency pegs.

We now use the graphical apparatus developed here to show that under a currency peg the present model implies an endogenous connection between the amplitude of the cycle and the average levels of involuntary unemployment and output. This connection opens the door to large welfare gains from optimal stabilization policy. To visualize this connection, note that the currency-peg economy adjusts asymmetrically to negative and positive external shocks. As explained above, in response to a negative external shock, employment is demand determined, households are forced off their labor supply schedule, involuntary unemployment emerges, and production of nontradables is inefficiently low. Consider now the adjustment to a positive external shock, which can be represented as moving from point C to point A in figure 1. In sharp contrast to what happens in response to a negative shock, in response to a positive shock, households are not forced off their labor supply schedule. That is, employment is no longer solely demand determined, but demand and supply determined. Thus, the adjustment of the economy to a positive external shock results in full employment and in an efficient level of production of nontradables. It follows that over the business cycle, the model economy fluctuates between periods of full employment and an efficient level of production and periods of involuntary unemployment and inefficiently low levels of production. Therefore, the average levels of involuntary unemployment and nontraded output depend on the amplitude of the cycle. This feature of our model is an important difference with existing sticky-wage models in the Calvo-Rotemberg tradition. In this class of models, employment is always demand determined. As a result increases in involuntary unemployment during recessions are roughly offset by reductions in unemployment during booms. It follows that in the standard new Keynesian framework the average levels of unemployment and output do not depend in a quantitatively relevant way on the amplitude of the business cycle. This is the root cause of the small welfare gains from optimal stabilization policy predicted by this class of models.

2.5 Optimal Exchange Rate Policy

Consider an exchange rate arrangement in which the central bank always sets the devaluation rate to ensure full employment in the labor market, that is, to ensure that

$$h_t = \bar{h},$$

for all $t \geq 0$. We refer to this monetary arrangement as the full-employment exchange rate policy. This policy amounts to setting the devaluation rate to ensure that the real wage equals the full-employment real wage rate at all times. Formally, the optimal policy ensures that

$$w_t = \omega(c_t^T),$$

where $\omega(c_t^T)$ denotes the full-employment real wage rate and is given by

$$\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \tag{20}$$

The assumed properties of the aggregator function A ensure that the function $\omega(\cdot)$ is strictly increasing in the domestic absorption of tradables, c_t^T,

$$\omega'(c_t^T) > 0.$$

The optimal exchange rate policy stipulates that should the nominal value of the full-employment real wage evaluated at last period's nominal exchange rate, $\omega(c_t^T)E_{t-1}$, fall below the lower bound γW_{t-1}, then the central bank devalues the domestic currency to ensure that $\omega(c_t^T)E_t \geq \gamma W_{t-1}$. That is, the devaluation rate makes the nominal wage, W_t, greater than or equal to its lower bound, γW_{t-1}, and at the same time guarantees that the real wage, w_t, equal the full-employment real wage $\omega(c_t^T)$. If, in this case, the central bank chose not to devalue, the economy would experience unemployment, because downward wage rigidities would prevent the real wage from falling to the full-employment level $\omega(c_t^T)$.

In general, any exchange rate policy satisfying

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{\omega(c_t^T)} \tag{21}$$

ensures full employment at all times. All exchange rate policies pertaining to this family deliver the same real allocation and are therefore equivalent from a welfare point of view.

From this class of full-employment policies, the one that minimizes movements in the nominal exchange rate is given by

$$\epsilon_t = \max\left\{1, \gamma \frac{w_{t-1}}{\omega(c_t^T)}\right\}. \tag{22}$$

This policy is of interest because it will inform us about the minimum devaluation required to maintain full employment during an external crisis. Another full-employment exchange rate policy of interest is

$$\epsilon_t = \gamma \frac{w_{t-1}}{\omega(c_t^T)}. \tag{23}$$

This full-employment policy targets nominal wage growth, for it implies that in equilibrium W_t/W_{t-1} equals γ at all times.

The complete set of equilibrium conditions under the optimal exchange rate regime is then given by Definition 1 and the exchange rate policy given in equation (21).

Because under the optimal exchange rate policy the real wage rate is always equal to the full-employment real wage, equation (21) implies that for all $t > 0$ the devaluation rate satisfies

$$\epsilon_t \geq \gamma \frac{\omega(c_{t-1}^T)}{\omega(c_t^T)}; \quad t > 0.$$

Recalling that $\omega(\cdot)$ is a strictly increasing function of tradable absorption, this expression states that the central bank must devalue the domestic currency when tradable expenditure falls. Data stemming from our model may lead non-microfounded econometric analysis to erroneously conclude that devaluations are contractionary. See, for instance, the empirical literature surveyed in section 3.4 of Frankel (2011). However, the role of devaluations under the full-employment exchange rate policy is precisely the opposite, namely, to prevent the contraction in the tradable sector to spill over into the nontraded sector. It follows that under the full-employment exchange rate policy, devaluations are indeed expansionary in the sense that should they not take place, aggregate contractions would be even larger. Thus, under the full-employment exchange rate regime, our model with downward nominal-wage rigidities turns the view that 'devaluations are contractionary' on its head and instead predicts that 'contractions are *devaluatory*.'

The full-employment exchange rate policy completely eliminates all real effects stemming from nominal wage rigidities. Indeed, one can show that the equilibrium under the full-employment exchange rate policy is identical to the equilibrium of an economy with full wage flexibility. Since wage rigidity is the only source of distortion in the present model, it follows that the equilibrium under the full-employment exchange rate policy is Pareto optimal. The equilibrium dynamics under the optimal exchange rate policy can therefore be characterized as the solution to the following value function problem:

$$v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h}))) + \beta \mathbb{E}_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\} \tag{24}$$

subject to (4) and (15), where the function $v^{OPT}(y_t^T, r_t, d_t)$ represents the welfare level of the representative agent under the full-employment exchange rate policy in state (y_t^T, r_t, d_t). The equilibrium processes of all other endogenous variables of the model can be readily obtained from (21) and the conditions listed in Definition 1. The following proposition provides a formal statement of these results:

Proposition 1 *Any exchange rate policy satisfying condition (21) delivers a real allocation that exhibits full employment ($h_t = \bar{h}$) at all dates and states and, furthermore, is Pareto optimal.*

Proof: See appendix A.

The fact that the aggregate dynamics under optimal exchange rate policy can be described as the solution to a Bellman equation greatly facilitates the quantitative characterization of the model's predictions. As we will see shortly, this is not the case under a fixed-exchange-rate policy.

3 Evidence On Downward Nominal Wage Rigidity

A central assumption in our theoretical framework is that nominal wages are downwardly rigid. This assumption is embodied in the parameter γ, defining an upper bound on the rate of nominal wage decreases. In this section, we present and review empirical evidence in support of this assumption. To stress the ubiquitousness of downward wage frictions, we consider evidence based on data from developed, emerging, and poor regions of the world, as well as from formal and informal labor markets.

One way in which the real effects of wage rigidity can be identified is by exploiting the seasonal dimension of wage adjustments. Olivei and Tenreyro (2007) document that in the United States nominal wage adjustments tend to be decided in the second half of the year and take effect at the very beginning of each year. This means that nominal wages are more rigid in the first than in the second half of the year. Olivei and Tenreyro exploit this seasonal pattern in wage adjustment and show, using VAR techniques, that monetary shocks that take place in the first half of the year have larger effects on aggregate activity than monetary shocks that occur in the second half of the year. Olivei and Tenreyro (2010) extend this result by observing that the seasonal component of nominal wage adjustments varies widely from country to country. They find that in countries in which wage setting is seasonal (e.g., the United States and Japan), the real effects of monetary policy also depend on the season in which they occur. By contrast, in countries in which wage setting does not display a pronounced seasonal pattern (e.g., France, Germany, and the United Kingdom), the real effect of monetary policy innovations do not significantly depend on the season in which they are implemented. The findings of Olivei and Tenreyro (2007, 2010) provide evidence in favor of the hypothesis that nominal wage rigidity has significant economic effect on economic activity.

A further question is whether nominal wage rigidity is asymmetric as assumed in our theoretical model. The existing empirical evidence strongly supports this assumption. Gottschalk (2005), for example, uses SIPP panel data to estimate the probability of wage declines, increases, and no changes for male and female hourly workers working for the same employer over the period 1986-1993 in the United States. He finds that for males the probabilities of wage increases, wage constancy, and wage declines are, respectively, 41.2, 53.7, and 5.1 percent per year. The corresponding probabilities for females are 46.5, 49.2, and 4.3 percent. These findings suggest that over the course of one year a very small fraction of workers experiences a decline in nominal wages, while about half of workers experience no change. For the purpose of our argument, it is important to note that the sample period used by Gottschalk comprises the 1991 U.S. recession, for it implies that the observed scarcity of nominal wage cuts took place in the context of elevated unemployment. Barattieri, Basu, and Gottschalk (2010) report similar findings using data from the 1996-2000 SIPP panel. Interestingly, this study is not restricted to workers working for the same employer. A similar pattern of downward nominal wage rigidity is documented by Holden and Wulfsberg (2008) using industry-level wage data in 19 OECD countries over the period 1973-1999.

The evidence referenced above is based on data from formal labor markets in developed economies. However, a similar pattern of asymmetry in nominal wage adjustments emerges in informal labor markets located in poor areas of the world. Kaur (2012), for example, studies the behavior of nominal wages in casual daily agricultural labor markets in rural India. Specifically, she examines market-level wage and employment responses to local rainfall shocks in 500 Indian districts from 1956 to 2008. She finds that nominal wage adjustment is asymmetric. In particular, nominal wages rise in response to positive rain shocks but fail to fall during droughts. In addition, negative rain shocks cause labor rationing and unemployment. More importantly, inflation, which is uncorrelated with local rainfall shocks, moderates these effects. During periods of relatively high inflation, local droughts are more likely to result in lower real wages and less labor rationing.

The empirical literature surveyed thus far establishes that nominal wage rigidity is asymmetric and has significant economic effects. However, because it does not provide information on the speed of nominal downward wage adjustments, it does not lend itself to calibrating the wage-rigidity parameter γ. For this reason, we now propose an empirical strategy for identifying this parameter. It consists in observing the behavior of nominal wages during periods of rising unemployment. We focus on episodes in which an economy undergoing a severe recession keeps the nominal exchange rate fixed. Two prominent examples are Argentina during the second half of the Convertibility Plan (1996-2001) and the periphery of Europe during the great recession of 2008.

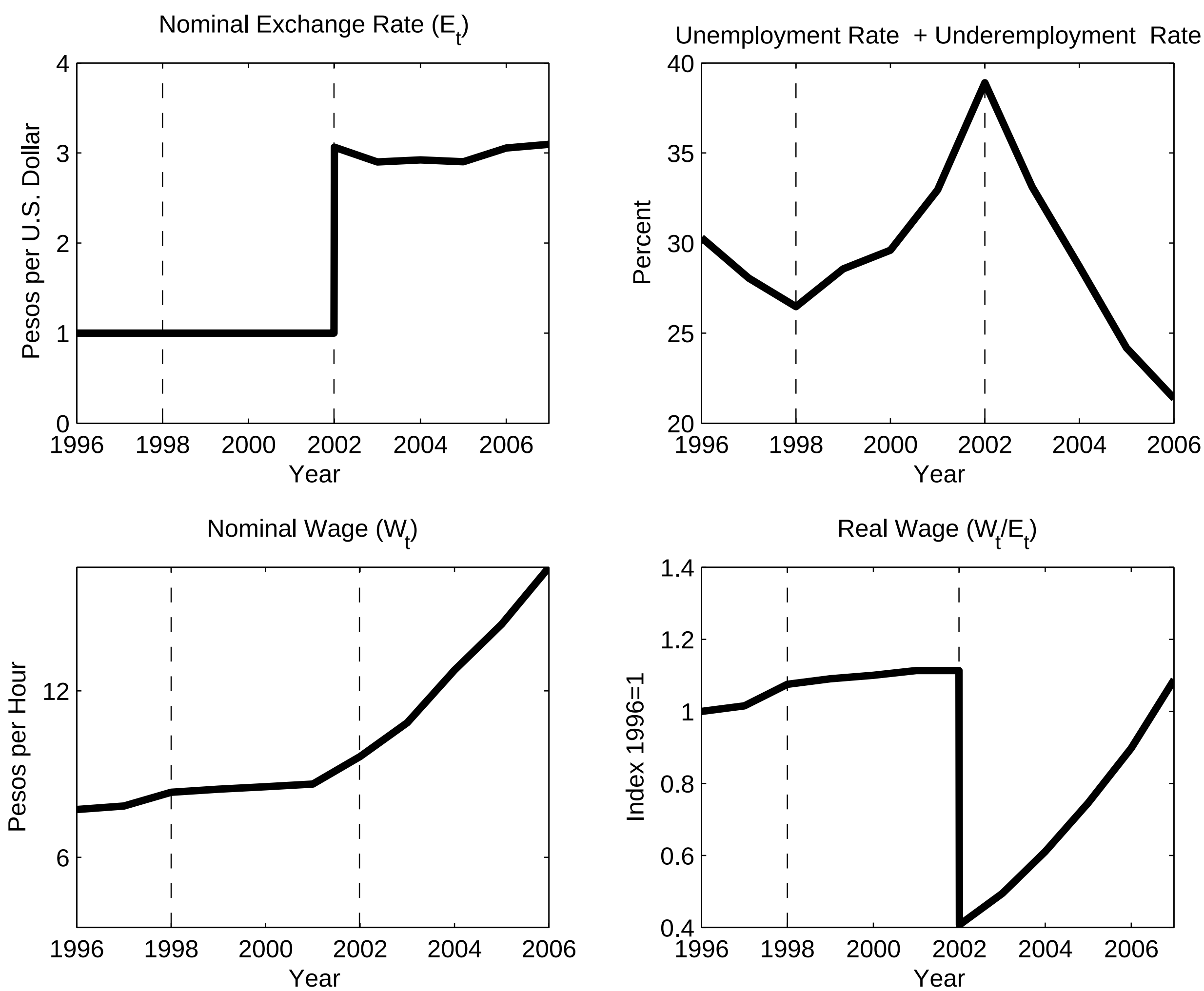
Figure 2 displays nominal (peso) wages, real (dollar) wages, the nominal exchange rate, and subemployment (defined as the sum of unemployment and underemployment) for Argentina during the period 1996-2006. The subperiod 1998-2001 is of particular interest because the Argentine central bank was holding on to the currency peg in spite of the fact that the economy was undergoing a severe contraction and both unemployment and underemployment were in a steep ascent. In the context of a flexible-wage model, one would expect that the rise in unemployment would be associated with falling real wages. However, during this period, the average nominal wage never fell. Indeed, because the Argentine peso was pegged to the dollar, the dollar wage rose throughout the 1998-2001 contraction. We interpret this behavior of nominal wages as suggesting a value of γ greater than or equal to unity. Additionally, the fact that real wages fell significantly and persistently with the devaluation of 2002 suggests that the 1998-2001 period was one of censored wage deflation, which further strengthens the view that nominal wages suffer from downward inflexibility.

We note that during the 1998-2001 Argentine contraction, consumer prices, unlike nominal wages, did fall significantly. The CPI rate of inflation was on average -0.86 percent per year over the period 1998-2001. It follows that real wages rose not only in dollar terms but also in terms of CPI units.[4]

The second episode that we use to infer the value of γ is the great recession of 2008 in the periphery of Europe. Table 1 presents a rough estimate of γ for eleven European economies that are either on the euro or pegging to the euro. Our model predicts that nominal wages should fall at the rate γ whenever the unemployment rate is above trend. The table shows the unemployment rate in 2008:Q1 and 2011:Q2. The starting point of this period corresponds to the beginning of the great recession in Europe according to the CEPR Euro Area Business Cycle Dating Committee, and the end point corresponds to the most recent data available at the time of the writing of this paper. The 2008 crisis caused unemployment rates to rise sharply across all eleven countries. The table also displays

[4]In addition, this evidence provides some support for our assumption that downward nominal rigidities are less stringent for product prices than for factor prices.

Figure 2: Nominal Wages and Unemployment in Argentina, 1996-2006



Source. Nominal exchange rate and nominal wage, BLS. Subemployment, INDEC.

Table 1: Unemployment, Nominal Wages, and γ: Evidence from the Eurozone

Country	Unemployment Rate 2008Q1 (in percent)	Unemployment Rate 2011Q2 (in percent)	Wage Growth $\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent)	Implied Value of γ
Bulgaria	6.1	11.3	43.3	1.028
Cyprus	3.8	6.9	10.7	1.008
Estonia	4.1	12.8	2.5	1.002
Greece	7.8	16.7	-2.3	0.9982
Ireland	4.9	14.3	0.5	1.0004
Lithuania	4.1	15.6	-5.1	0.996
Latvia	6.1	16.2	-0.6	0.9995
Portugal	8.3	12.5	1.91	1.001
Spain	9.2	20.8	8.0	1.006
Slovenia	4.7	7.9	12.5	1.009
Slovakia	10.2	13.3	13.4	1.010

Note. W is an index of nominal average hourly labor cost in manufacturing, construction, and services. Unemployment is the economy-wide unemployment rate. Source: EuroStat.

the total growth of nominal hourly labor cost in manufacturing, construction and services (including the public sector) over the thirteen-quarter period 2008:Q1-2011:Q2.[5] Despite the large surge in unemployment, nominal wages grew in most countries and in those in which it fell, the decline was modest. The implied value of γ, shown in the last column of table 1, is given by the average growth rate of nominal wages over the period considered (that is, $\gamma = (W_{2011:Q2}/W_{2008:Q1})^{1/13}$). The estimated values of γ range from 0.996 for Lithuania to 1.028 for Bulgaria. Again, this evidence suggests a value of γ close to or even larger than unity.

In light of the above empirical evidence, we choose as the baseline value of γ 0.99. This value means that nominal wages can fall frictionlessly by up to 4 percent per year. We regard this choice as conservative in the sense that it allows for more downward wage flexibility than what is suggested by the evidence presented above. Consider, for instance, the evidence presented in table 1. The largest fall in wages over the 13-quarter period since the start of the 2008 crisis is 5.1 percent and corresponds to Lithuania. Under our calibration, however, wages would have been allowed to fall by more than twice this amount, namely 13 percent. This conservative value allows for effects not explicitly captured by our model, such as foreign inflation and long-run growth.[6] We also consider even more conservative values of γ in the

[5]The public sector is not included for Spain due to data limitations.

[6]For example, over the thirteen-quarter sample period considered in table 1 inflation in Germany was 3.6

interval 0.96 to 0.99. The lower end of this range allows for a 52 percent decline in nominal wages over a 13-quarter period, a figure that vastly exceeds the observed declines in nominal wages in the eurozone since the onset of the great recession.

4 A Quantitative Analysis of External Crises

We wish to quantitatively characterize the response of our model to a large negative external shock. We have in mind extraordinary contractions like the 1989 or 2001 crises in Argentina, or the 2008 great recession in peripheral Europe. To this end, we estimate the joint stochastic process of our assumed exogenous driving forces (traded output and the country interest rate) using Argentine data. We calibrate the remaining structural parameters of the model to match salient aspects of the Argentine economy. Finally, we compute the economy's response to a large negative shock to traded output under a currency peg and under the optimal exchange rate policy (22).

4.1 Exogenous Driving Process and Definition of Crisis

The law of motion of tradable output and the country-specific interest rate is assumed to be given by the following autoregressive process:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \tag{25}$$

where ϵ_t is a white noise of order 2 by 1 distributed $N(\emptyset, \Sigma_\epsilon)$. The parameter r denotes the deterministic steady-state value of r_t. We estimate this system using Argentine data over the period 1983:Q1 to 2001:Q4. We exclude the period post 2001 because Argentina was in default between 2002 and 2005 and excluded from international capital markets. The default was reflected in excessively high country premia (see figure 3(b)). Excluding this period is in order because interest rates were not allocative, which is at odds with our maintained assumption that the country never loses access to international financial markets. This is a conservative choice, for inclusion of the default period would imply a more volatile driving force accentuating the real effects of currency pegs on unemployment.

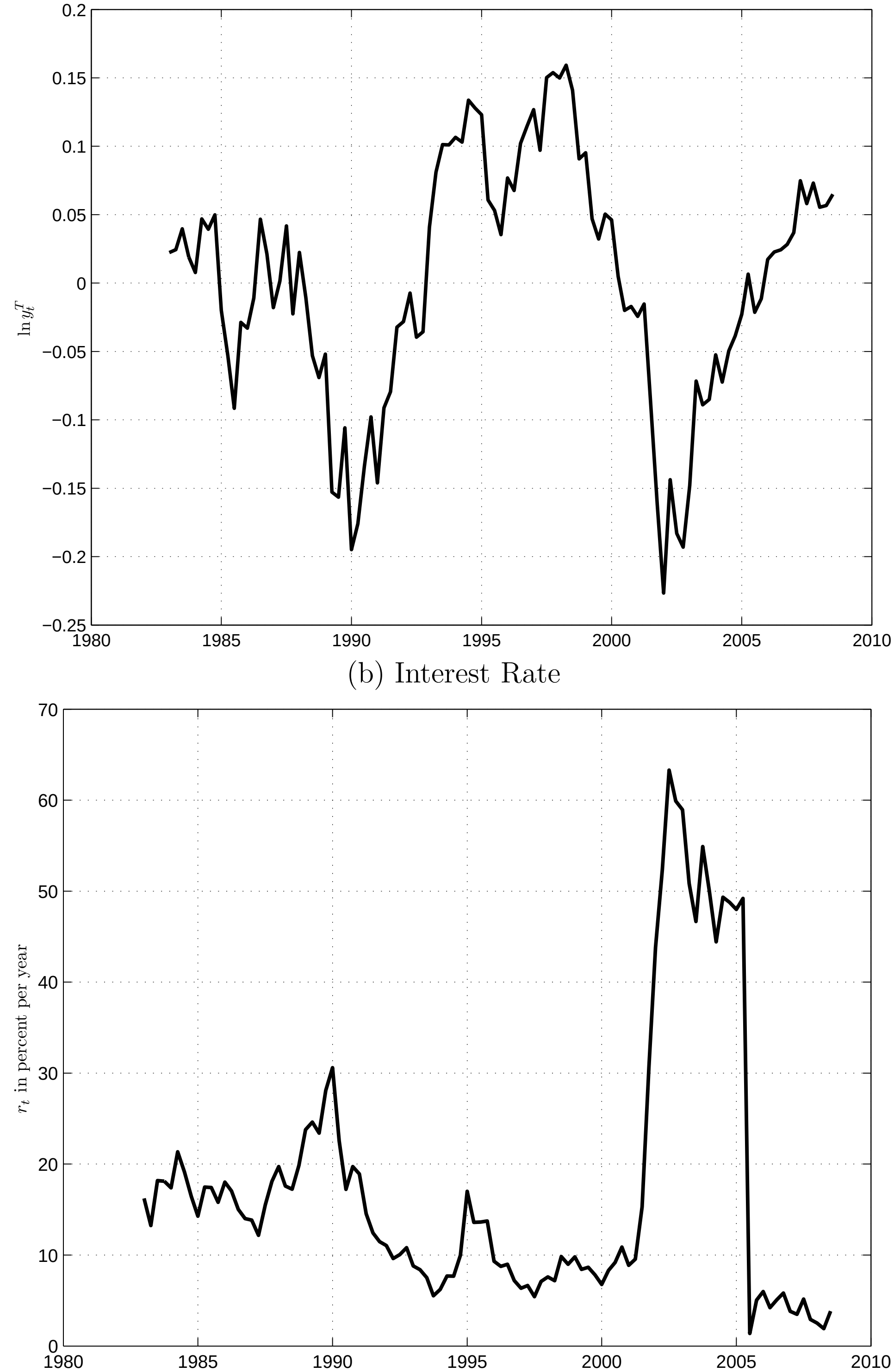
Our empirical measure of y_t^T is the cyclical component of Argentine GDP in agriculture, forestry, fishing, mining, and manufacturing.[7] We obtain the cyclical component by removing

percent, or about 0.3 percent per quarter. At the same time, long-run growth in per capita income in Europe is about 1.2 percent per year, or 0.3 percent per quarter. Allowing for these effects would entail lowering γ by 0.006. For this reason, we use a value of γ of 0.99 rather than 0.996 as suggested in table 1.

[7]The data were downloaded from www.indec.mecon.ar.

Figure 3: Traded Output and Interest Rate in Argentina, 1983:Q1-2008:Q3

(a) Traded Output



Note. Traded output is expressed in log-deviations from trend. Source: See the main text.

a log-quadratic time trend. Figure 3(a) displays the resulting time series. We measure the country-specific real interest rate as the sum of the EMBI+ spread for Argentina and the 90-day Treasury-Bill rate, deflated using a measure of expected dollar inflation.[8]

Our OLS estimates of the matrices A and Σ_ϵ and of the scalar r are

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \qquad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \qquad r = 0.0316.$$

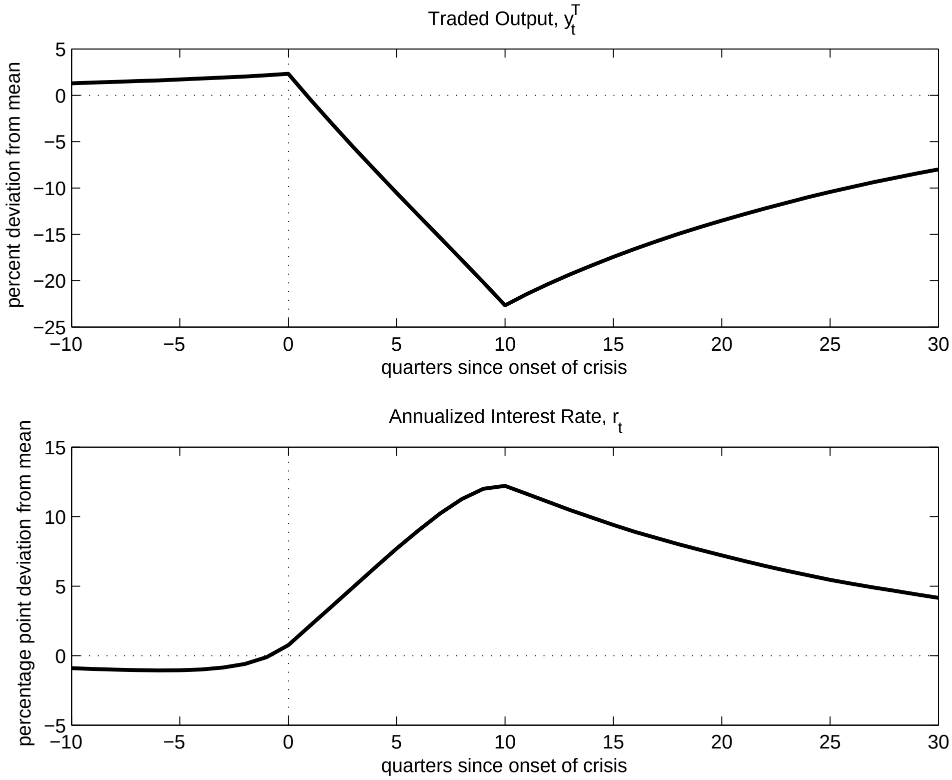
According to these estimates, both $\ln y_t^T$ and r_t are highly volatile, with unconditional standard deviations of 12.2 percent and 1.7 percent per quarter (6.8 percent per year), respectively. Also, the unconditional contemporaneous correlation between $\ln y_t^T$ and r_t is high and negative at -0.86. This means that periods of relatively low traded output are associated with high interest rates and vice versa. The estimated joint autoregressive process implies that both traded output and the real interest rate are highly persistent, with first-order autocorrelations of 0.95 and 0.93, respectively. Finally, we estimate a steady-state real interest rate of 3.16 percent per quarter, or 12.6 percent per year. This high average value reflects the fact that our sample covers a period in which Argentina underwent a great deal of economic turbulence.

We discretize the AR(1) process given in equation (25) using 21 equally spaced points for $\ln y_t^T$ and 11 equally spaced points for $\ln(1 + r_t)/(1 + r)$. We construct the transition probability matrix of the state $(\ln y_t^T, \ln((1+r_t)/(1+r)))$ by simulating a time series of length 1,000,000 drawn from the system (25). We associate each observation in the time series with one of the 231 possible discrete states by distance minimization. The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period. The resulting transition probability matrix captures well the covariance matrices of order 0 and 1.

During the great Argentine crises of 1989 and 2001 traded output fell by about two standard deviations within a period of two and a half years. Accordingly, we define an external crisis in our theoretical model as a situation in which tradable output is at or above trend in period t and at least two standard deviations below trend in period $t + 10$. To

[8]Specifically, we construct the time series for the quarterly real Argentine interest rate, r_t, as $1+r_t = (1+i_t)E_t\frac{1}{1+\pi_{t+1}}$, where i_t denotes the dollar interest rate charged to Argentina in international financial markets and π_t is U.S. CPI inflation. For the period 1983:Q1 to 1997:Q4, we take i_t to be the Argentine interest rate series constructed by Neumeyer and Perri (2005) and posted at www.fperri.net/data/neuperri.xls. For the period 1998:Q1 to 2001:Q4, we measure i_t as the sum of the EMBI+ spread and the 90-day Treasury bill rate, which is in line with the definition used in Neumeyer and Perri. We measure $E_t\frac{1}{1+\pi_{t+1}}$ by the fitted component of a regression of $\frac{1}{1+\pi_{t+1}}$ onto a constant and two lags. This regression uses quarterly data on the growth rate of the U.S. CPI index from 1947:Q1 to 2010:Q2.

Figure 4: The Source of a Crisis



characterize the economy's behavior during such episodes, we simulate the model for 20 million quarters and identify episodes in which movements in traded output conform to our definition of an external crisis. We then average the responses of all variables of interest across the crisis episodes and subtract their respective means taken over the entire sample of 20 million quarters. The beginning of a crisis is normalized at $t = 0$.

Figure 4 displays the predicted average behavior of the two exogenous variables, traded output and the country interest rate, during a crisis. The downturn in traded output can be interpreted either as a drastic fall in the quantity of tradables produced by the economy or as an exogenous collapse in the country's terms of trade. The figure shows that at the trough of the crisis (period 10), tradable output is 23 percent below trend. The contraction in tradable output is accompanied by a sharp increase in the interest rate that international financial markets charge to the emerging economy. The country interest rate peaks in quarter 10 at about 12 percentage points per annum above its average value. This behavior of the interest rate is dictated by the estimated negative correlation between tradable output and country interest rates.

Table 2: Calibration

Parameter	Value	Description
γ	0.99-0.95	Degree of downward nominal wage rigidity
σ	5	Inverse of intertemporal elasticity of consumption
y^T	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
a	0.26	Share of tradables
ξ	0.44	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
β	0.9375	Quarterly subjective discount factor

4.2 Calibration

We adopt a CRRA form for the period utility function, a CES form for the aggregator function, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}},$$

and

$$F(h) = h^{\alpha}.$$

We calibrate the model at a quarterly frequency using data from Argentina as shown in table 2. Reinhart and Végh (1995) estimate the intertemporal elasticity of substitution to be 0.21 using Argentine quarterly data. We therefore set σ equal to 5. We normalize the steady-state levels of output of tradables and hours at unity. Then, if the steady-state trade-balance-to-output ratio is small, the parameter a is approximately equal to the share of traded output in total output. We set this parameter at 0.26, which is the share of traded output (as defined above) observed in Argentine data over the period 1980:Q1-2010:Q1. Using time series data for Argentina over the period 1993Q1-2001Q3, González Rozada et al. (2004) estimate the elasticity of substitution between traded and nontraded consumption, ξ, to be 0.44. This estimate is consistent with the cross-country estimates of Stockman and Tesar (1995). These authors include in their estimation both developed and developing countries. Restricting the sample to include only developing countries yields a value of ξ of 0.43 (see Akinci, 2011). Following Uribe's (1997) evidence on the size of the labor share in the nontraded sector in Argentina, we set α equal to 0.75.

We set $\bar{d}$ at the natural debt limit, which we define as the level of external debt that can be supported with zero tradable consumption when the household perpetually receives the lowest possible realization of tradable endowment, ${y^T}^{\min}$, and faces the highest possible realization of the interest rate, $r^{\max}$. Formally, $\bar{d} \equiv {y^T}^{\min}(1 + r^{\max})/r^{\max}$. Given our discretized estimate of the exogenous driving process, $\bar{d}$ equals 8.34. In the numerical simulations presented below, we fix the upper bound of the debt grid at 8. The reason why we set this upper bound slightly below the natural debt limit is that our numerical algorithm requires evaluating the aggregator function for consumption at all points in the discretized state space. If we allowed the state to take the value $({y^T}^{\min}, r^{\max}, \bar{d})$, traded consumption would take zero or negative values for all possible choices of next-period debt in the grid. As a result, the aggregate level of consumption would not be defined at this particular state. Experimenting with values for the upper bound of the debt grid closer to the natural debt limit did not affect our results.

The final parameter we calibrate is the subjective discount factor β. We set this parameter so as to match a foreign-debt-to-output ratio of 26 percent per year, a value in line with that reported for Argentina over our calibration period by Lane and Milesi-Ferretti (2007). In the context of our model, the task of calibrating β is complicated by the fact that the debt-to-output ratio is highly sensitive to the assumed monetary regime. This is problematic because in emerging countries in general and in Argentina in particular, monetary regimes tend to change frequently and widely. A compromise is therefore in order. In calibrating β, we assume that the underlying monetary regime takes the form of a currency peg. This strategy results in a value of β of 0.9375. In section 10 we explore the sensitivity of our results to changes in β as well as all other calibrated parameters.

4.3 Crisis Dynamics Under A Currency Peg

At center stage in our analysis is the characterization of the costs of maintaining a currency peg. A currency peg is meant to capture, for example, the monetary policy in place in Argentina between April 1991 and December 2001 or the monetary restrictions faced by the small emerging economies that are members of the Eurozone, such as Greece, Portugal, and Ireland.

Approximating the dynamics of the model under a currency peg is computationally more demanding than doing so under optimal exchange rate policy due to the emergence of a fourth state variable, w_{t-1}. In addition, because of the distortions created by nominal rigidities, aggregate dynamics cannot be cast in terms of a Bellman equation without introducing additional state variables (such as the individual level of debt, which households perceive as distinct from its aggregate counterpart). We therefore approximate the solution by Euler equation iteration over a discretized version of the state space $(y_t^T, r_t, d_t, w_{t-1})$. Appendix B describes our numerical algorithm in more detail. The discretization of the exogenous states y_t^T and r_t is as described in section 4.1. For the discretization of the endogenous states, we use 501 equally spaced points for external debt, d_t, and 500 equally spaced points for the logarithm of w_{t-1}.

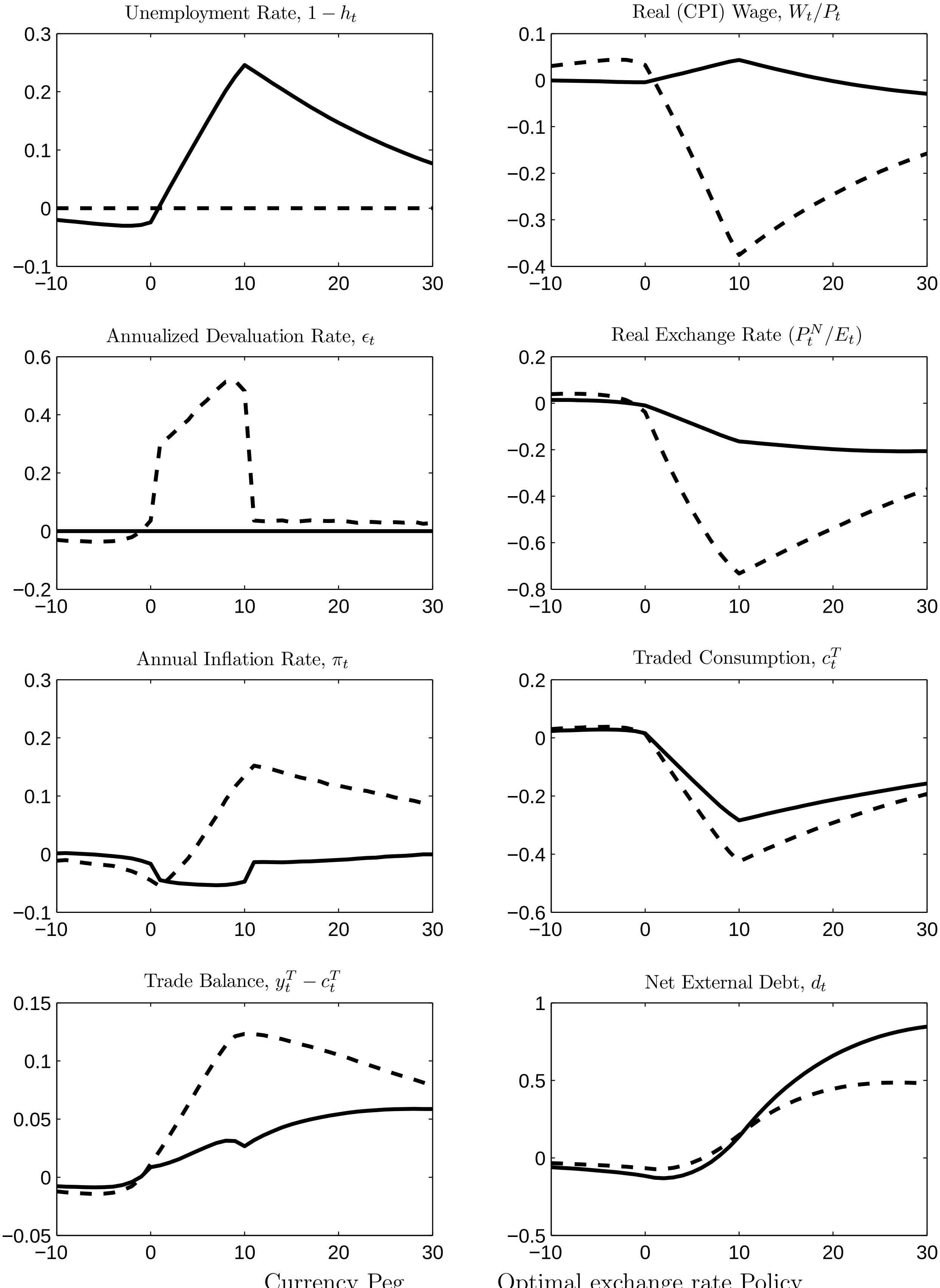
Figure 5 depicts with solid lines the dynamics induced by a large external crisis when the monetary policy takes the form of an exchange rate peg. Recall from section 4.1 that an external crisis is defined as a situation in which in period 0 tradable output is at or above trend and in period 10 it is at least two standard deviations below trend.

The central characteristic of the response of the currency-peg economy to an external crisis is that the contraction in the traded sector spills over to the nontraded sector. This is because the monetary authority fails to devalue the domestic currency preventing real wages and thus also marginal costs in the nontraded sector from adjusting downward. With real wages too high, the labor market is in disequilibrium and unemployment emerges. At the same time, because marginal costs are high, firms are unwilling to cut the relative price of nontradables. Consequently, there is insufficient expenditure switching away from tradables toward nontradables. Put differently, the combination of downward nominal wage rigidity and a currency peg hinders the ability of the price system to signal to firms and consumers the relative aggregate scarcity of tradable goods and the relative aggregate abundance of nontradable goods.

The collapse in the demand for nontradables pushes the unemployment rate up by more than 20 percentage points. Moreover, unemployment is highly persistent. Even five years after the trough of the crisis, the unemployment rate remains more than 7 percentage points above average. This sluggishness in unemployment is due to the fact that real wages adjust slowly downward at the gross rate γ. The predicted unemployment is involuntary, in the sense that every unemployed individual would be willing to work at the going wage rate.

The predictions of our model suggest that a hands-off policy approach whereby the necessary real depreciation is let to occur via deflation, while maintaining a currency peg, is highly costly in terms of unemployment and forgone consumption of nontradables. Although the currency-peg economy displays a mild deflation during the crisis, the fall in nominal product prices is too small and too slow to bring about the efficient real depreciation required to avoid unemployment. It is interesting to point out that nominal product prices fail to decline to the level consistent with full employment in spite of the fact that they are fully flexible. The reason for the seemingly sticky behavior of product price inflation is that, because nominal wages are downwardly rigid, marginal costs remain high and therefore firms are unable to cut prices without making losses. Next, we show that this is a key difference between price

Figure 5: The Dynamics of a Crisis: Response to a Two-Standard-Deviation Fall in y_t^T



dynamics under a currency peg and the optimal exchange-rate regime.

4.4 Crisis Dynamics Under the Optimal Exchange Rate Policy

We numerically approximate the equilibrium dynamics under the optimal exchange rate policy by applying the method of value function iteration over a discretized state space. Under optimal exchange rate policy, the state of the economy in period $t \geq 0$ is the triplet $\{y_t^T, r_t, d_t\}$. In subsection 4.1, we explain the method employed for discretizing the exogenous state space $\{y_t^T, r_t\}$. In the discretization of the endogenous state d_t, we use 1001 equally spaced points.

Figure 5 displays with broken lines the average response of the economy to a large external crisis under the optimal exchange rate policy. By construction, the behavior of the exogenous variables, traded output and the country interest rate, is identical to that used to simulate the crisis in the currency-peg economy (see figure 4).

The central difference in the response to an external crisis between the optimal exchange-rate-economy and the currency-peg economy is that in the optimal-exchange-rate economy the monetary authority prevents the external crisis from spilling over to the nontraded sector. Indeed, as shown in proposition 1 the unemployment rate is nil under the optimal exchange-rate policy. To maintain full employment in the nontraded sector, the government engineers a significant expenditure switch away from tradables and toward nontradables. The instrument the government uses to accomplish this expenditure switch is a series of large devaluations of the domestic currency of about 40 percent per year during the contractionary phase of the crisis. The main purpose of these devaluations is to lower labor costs in the nontraded sector. In turn, the decline in labor costs allows firms to lower the relative price of nontradable goods in terms of tradable goods, which results in a depreciation of the real exchange rate. The purchasing power of wages in terms of the composite consumption good, W_t/P_t, falls by about 40 percent and the real exchange rate, $p_t \equiv P_t^N/E_t$, depreciates by about 70 percent, where P_t denotes the nominal price of one unit of the composite consumption good in period t, and P_t^N denotes the nominal price of one unit of nontradable consumption in period t.[9]

During the crisis, the constraint on nominal wage cuts is typically binding. Therefore, the median quarterly proportional decline in nominal wages is $1-\gamma$, or 1 percent per quarter, implying a cumulative decline of about 10 percent over the course of the crisis. The nominal price of nontradables mimics the behavior of nominal wages. To see this, note that under the optimal exchange rate policy P_t^N must satisfy $P_t^N F'(\bar{h}) = W_t$, which implies that the nominal price of nontradables falls at the same rate as nominal wages. It follows that the nominal

[9]Formally, $P_t \equiv \left[a^{\xi} E_t^{1-\xi} + (1-a)^{\xi} {P_t^N}^{1-\xi}\right]^{\frac{1}{1-\xi}}$.

price of nontradables remains relatively flat as the large nominal devaluations occur. This prediction of our model is remarkable because nominal prices of nontradables are assumed to be fully flexible. The predicted sluggish adjustment of the nominal price of nontradables is in line with the empirical findings of Burstein, Eichenbaum, and Rebelo (2005) who report that the primary force behind the observed large drop in the real exchange rate that occurred after the large devaluations in Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994), and Thailand (1997) was the slow adjustment in the nominal prices of nontradable goods.

In the tradable sector, the expenditure switch is reflected in a drastic fall in tradable consumption of about 40 percent. In fact the fall in the domestic absorption of tradable goods is larger than the contraction in the supply of tradables. This results in a significant improvement in the trade balance from slightly below trend to about 10 percent of tradable GDP above trend. However, because of the elevated debt-service cost stemming from the interest-rate hike, the large improvement in the trade balance is not sufficient to prevent external debt from growing during the crisis.

The large optimal devaluations that take place during the crises are prima facie an indication that rigidly adhering to a currency peg during times of duress might carry nontrivial real effects in terms of unemployment, output, and welfare. We turn to this issue next.

5 The Welfare Costs of Currency Pegs

In the present model, currency pegs are costly not only during crises but also over regular business-cycle fluctuations. For currency pegs make the economy vulnerable to persistent unemployment spells. Under our calibration the average unemployment rate under a currency peg is 14 percent. As a consequence the average supply of nontraded goods is lower than in the optimal exchange rate economy, in which unemployment is zero at all times.

We quantify the welfare cost of living in an economy in which the central bank pegs the currency by computing the percent increase in the consumption stream of the representative household living in the currency-peg economy that would make him as happy as living in the optimal exchange rate economy. This computation must take explicitly into account the transitional dynamics induced by the switch from a peg to the optimal policy. Specifically, one can express the value function associated with the currency-peg economy as

$$v^{PEG}(y_t^T, r_t, d_t, w_{t-1}) = E_t \sum_{s=0}^{\infty} \beta^s \frac{\left(c_{t+s}^{PEG}\right)^{1-\sigma} - 1}{1-\sigma},$$

where c_t^{PEG} denotes the stochastic process of consumption of the composite good in the currency-peg economy. Then, define the proportional compensation rate $\lambda(y_t^T, r_t, d_t, w_{t-1})$ implicitly as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\left[c_{t+s}^{PEG}(1+\lambda(y_t^T, r_t, d_t, w_{t-1}))\right]^{1-\sigma} - 1}{1-\sigma} = v^{OPT}(y_t^T, r_t, d_t),$$

where $v^{OPT}(y_t^T, r_t, d_t)$ denotes the value function associated with the optimal exchange-rate policy, defined in equation (24). Solving for the welfare cost $\lambda(y_t^T, r_t, d_t, w_{t-1})$, we obtain

$$\lambda(y_t^T, r_t, d_t, w_{t-1}) = \left[\frac{v^{OPT}(y_t^T, r_t, d_t)(1-\sigma) + (1-\beta)^{-1}}{v^{PEG}(y_t^T, r_t, d_t, w_{t-1})(1-\sigma) + (1-\beta)^{-1}}\right]^{1/(1-\sigma)} - 1.$$

This expression makes it clear that the compensation $\lambda(y_t^T, r_t, d_t, w_{t-1})$ is state dependent. Specifically, the distribution of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ depends upon the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$.

The ergodic distributions of debt under the currency peg and under the optimal policy do not have the same support (see figure 8 to be discussed below). Therefore, in order to compute $\lambda(y_t^T, r_t, d_t, w_{t-1})$ one must evaluate the welfare function $v^{OPT}(y_t^T, r_t, d_t)$ at levels of debt outside of the support of its ergodic distribution under the optimal policy. Having computed $\lambda(y_t^T, r_t, d_t, w_{t-1})$ for all values of the state in its ergodic distribution under the currency peg, we proceed to compute the probability density function of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ by sampling from the ergodic distribution of the state under the currency peg.

Figure 6 displays with a solid line the unconditional probability density function of $\lambda(y_t^T, r_t, d_t, w_{t-1})$ expressed in percentage points. The probability density is highly skewed to the right, implying that the probability of very high welfare costs is non-negligible. Table 3 reports the median and the mean of $\lambda(y_t^T, r_t, d_t, w_{t-1})$. The median welfare cost of a currency peg is 10.4 percent of the consumption stream. That is, households living in a currency peg economy require 10.4 percent more consumption in every date and state in order to be indifferent between staying in the currency-peg regime and switching to the optimal exchange rate regime. This is an enormous number as welfare costs go in monetary business-cycle theory. Even under the most favorable initial conditions, the welfare cost of a currency peg is large, 5.6 percent of consumption each period. (This figure corresponds to the lower bound of the support of the probability density of $\lambda(y_t^T, r_t, d_t, w_{t-1})$.)

The welfare consequences of unemployment under a currency peg are reflected in the implied average levels of consumption. Eventhough the currency-peg economy enjoys 8.5 percent higher average consumption of tradables, due to lower average debt holdings (see figure 8), aggregate consumption under the currency peg is 5 percent lower than under the

Figure 6: Probability Density Function of the Welfare Cost of Currency Pegs

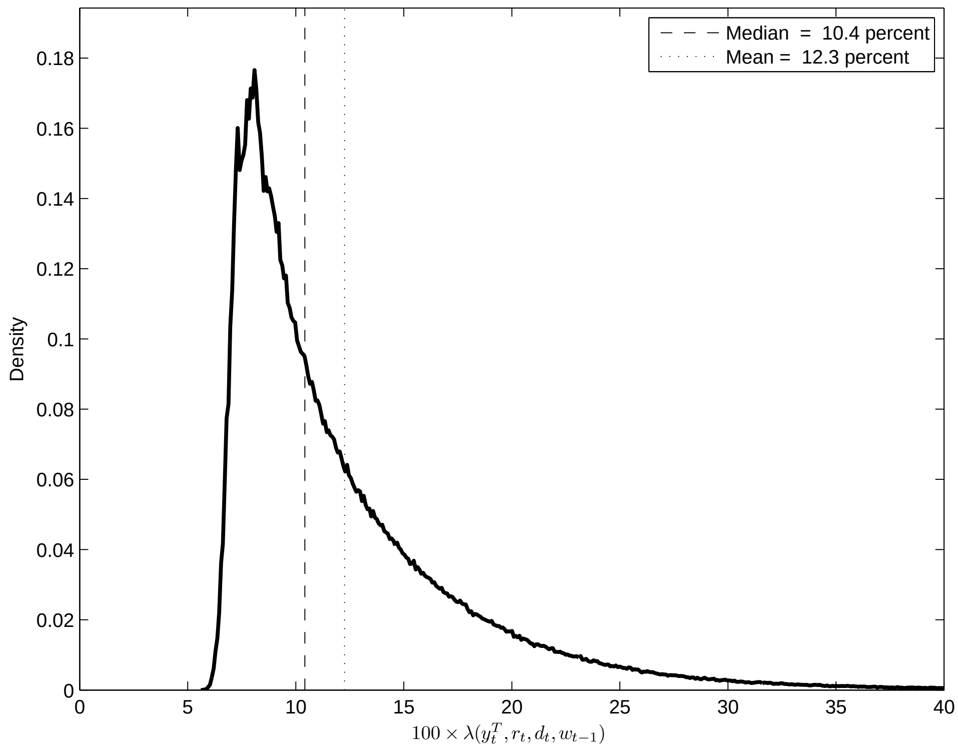


Table 3: The Welfare Costs of Currency Pegs

	Welfare Cost	
Parameterization	Median	Mean
Baseline	10.4	12.3
Endogenous labor supply: $\theta = 1$	4.5	6.2
Endogenous labor supply: $\theta = 6$	6.8	8.6
Production in Traded Sector: $\alpha_T = 0.5$	9.0	10.6
Less downward wage rigidity: $\gamma = 0.98$	6.7	8.1
Less downward wage rigidity: $\gamma = 0.97$	5.1	6.2
Less downward wage rigidity: $\gamma = 0.96$	4.3	5.2
Higher patience: $\beta = 0.945$	8.0	9.2
Higher intratemp. elast. of subst.: $\xi = 0.88$	8.6	10.8
Higher share of tradables: $a = 0.4$	9.1	10.4
Higher intertemp. elast. of subst.: $\sigma = 2$	9.9	10.8

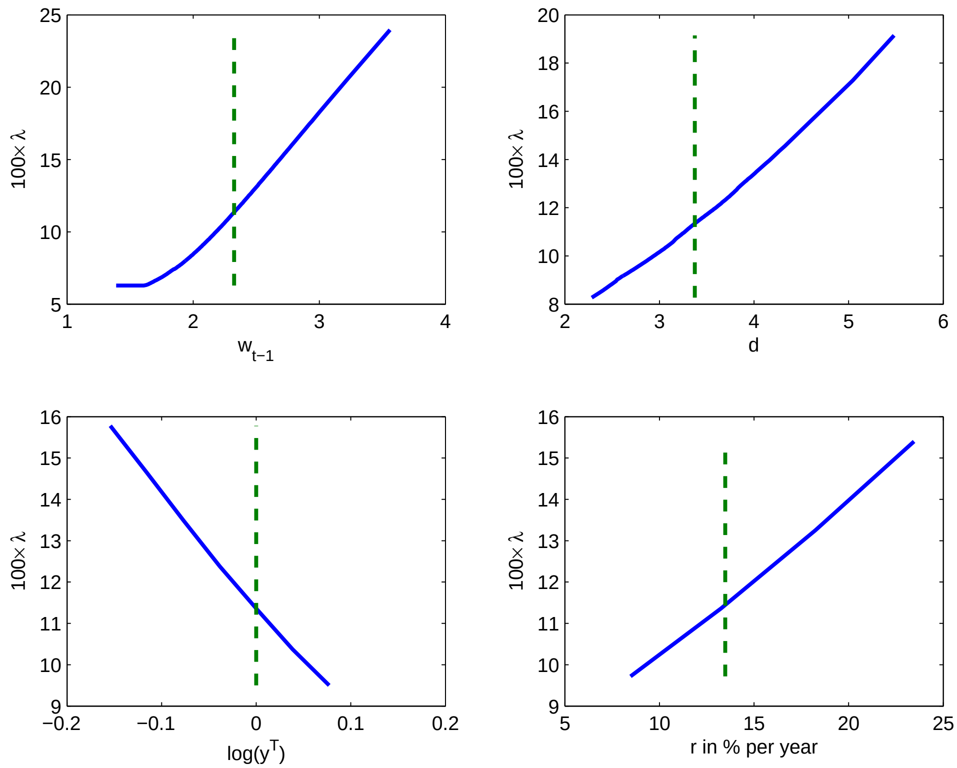
Note. The welfare cost of a currency peg is given by the percent increase in consumption that would make an individual living in the currency-peg economy as well off as an individual living in the optimal-exchange-rate economy for a given state of the economy $(y_t^T, r_t, d_t, w_{t-1})$.

optimal exchange rate policy. This is because, due to unemployment in the nontraded sector, consumption of nontradables under the currency peg is on average 11 percent lower than under the optimal exchange rate regime. As mentioned earlier, this is the main reason why the welfare costs of currency pegs are so high in our model.

Our finding of large welfare costs of currency pegs stands in stark contrast to a large body of work, pioneered by Lucas (1987), suggesting that the costs of business cycles (not just of suboptimal monetary policy) are minor. Lucas' approach to computing the welfare costs of business cycles consists in first removing a trend from a consumption time series and then evaluating a second-order approximation of welfare using observed deviations of consumption from trend. Implicit in this methodology is the assumption that the trend is unaffected by policy. In our model, however, suboptimal monetary policy, creates an endogenous connection between the amplitude of the business cycle and the average rate of unemployment. In turn, through its effect on the average level of unemployment, suboptimal exchange-rate policy has a significant effect on the average level of consumption. And indeed lower average consumption is the main reason currency pegs are so costly in our model. It follows that applying Lucas' methodology to data stemming from our model would overlook the effects of policy on trend consumption and therefore would result in spuriously low welfare costs.

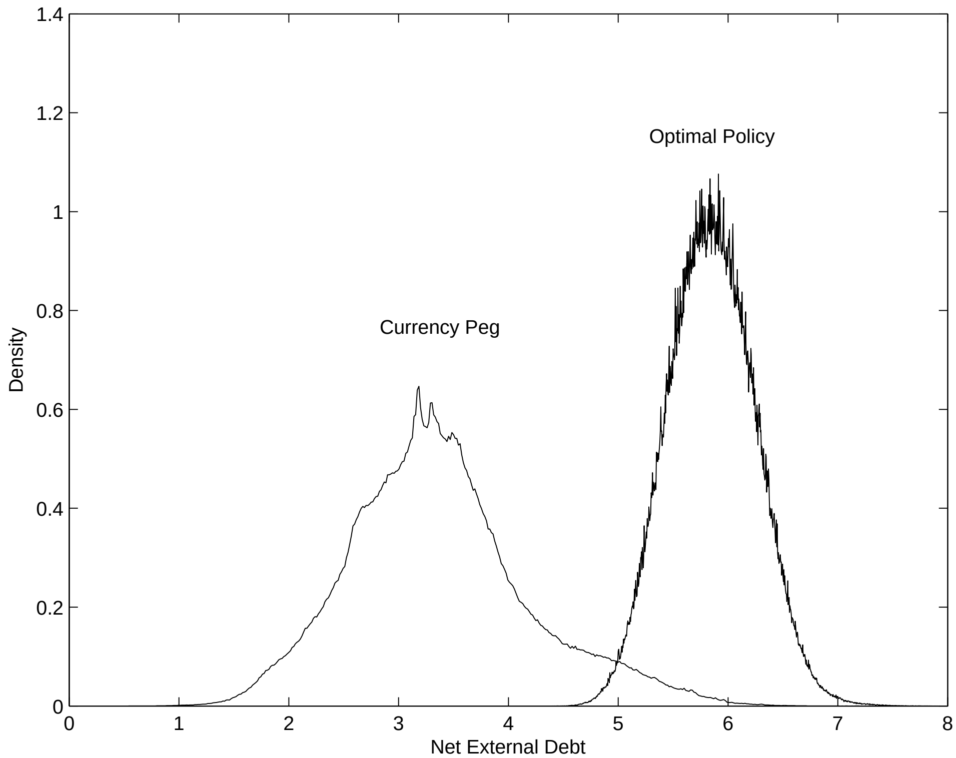
The marked skewness of the unconditional distribution of the welfare cost of currency pegs suggests that there are initial states at which an economy undergoing a currency peg is particularly vulnerable. Figure 7 sheds light on what these states are. It displays the welfare cost of currency pegs as a function of the four state variables. In each panel only one state variable is allowed to vary (along the horizontal axis) and the remaining three state variables are fixed at their respective unconditional means (under a currency peg, in the case of endogenous states). The figure shows that currency pegs are more painful when the country is initially more indebted, when it inherits higher past real wages, when the tradable sector is undergoing a contraction (due, for example, to unfavorable terms of trade), or when the country interest-rate premium is high. Viewing the recent debt crisis among eurozone emerging countries through the lens of our model, it is not difficult to understand why doubts about the optimality of European monetary union are the strongest for member countries like Greece, Portugal, and Spain: These are countries with highly inflexible labor markets that before the 2008 crisis experienced large increases in wages and sizable current account deficits.

Figure 7: Welfare Cost of Currency Pegs as a Function of the State Variables



Note. In each plot, all states except the one shown on the horizontal axis are fixed at their unconditional mean values. The dashed vertical lines indicate the unconditional mean of the state displayed on the horizontal axis (under a currency peg if the state is endogenous).

Figure 8: The Distribution of External Debt



6 Debt Dynamics

One consequence of the combination of downward wage rigidity and a currency peg is an excessive absorption of tradable goods during the crisis (see the right panel in the third row of figure 5). The excess consumption of tradables (relative to what is optimal) is driven by the fact that the exchange rate does not depreciate sufficiently in real terms. Under a currency peg the improvement in the trade balance during the crisis is more modest than under the optimal exchange rate policy, spurring a more rapid rise in external debt driven by the higher interest rates (see the two panels at the bottom of figure 5). Indeed, the suboptimal behavior of the external debt affects the country's long-run ability to accumulate external financial obligations. In this section, we explore the relationship between the exchange rate regime and the level and volatility of external debt.

Figure 8 displays the unconditional distribution of the external debt in the stochastic steady state of the currency-peg and optimal exchange rate economies. The debt distribution associated with the optimal exchange rate economy features a higher mean and is more concentrated about the mean than the debt distribution associated with the currency-peg economy. Specifically, the mean debt under the optimal exchange rate policy is 5.8 (or 146 percent of the annual steady-state endowment of traded goods) and the standard deviation is 0.4 (or 10 percent of annual traded endowment). By contrast, in the currency-peg economy the mean external debt is 3.4 (or 84 percent of annual steady-state tradable output) with a standard deviation of 0.8 (or 20 percent of annual tradable output).

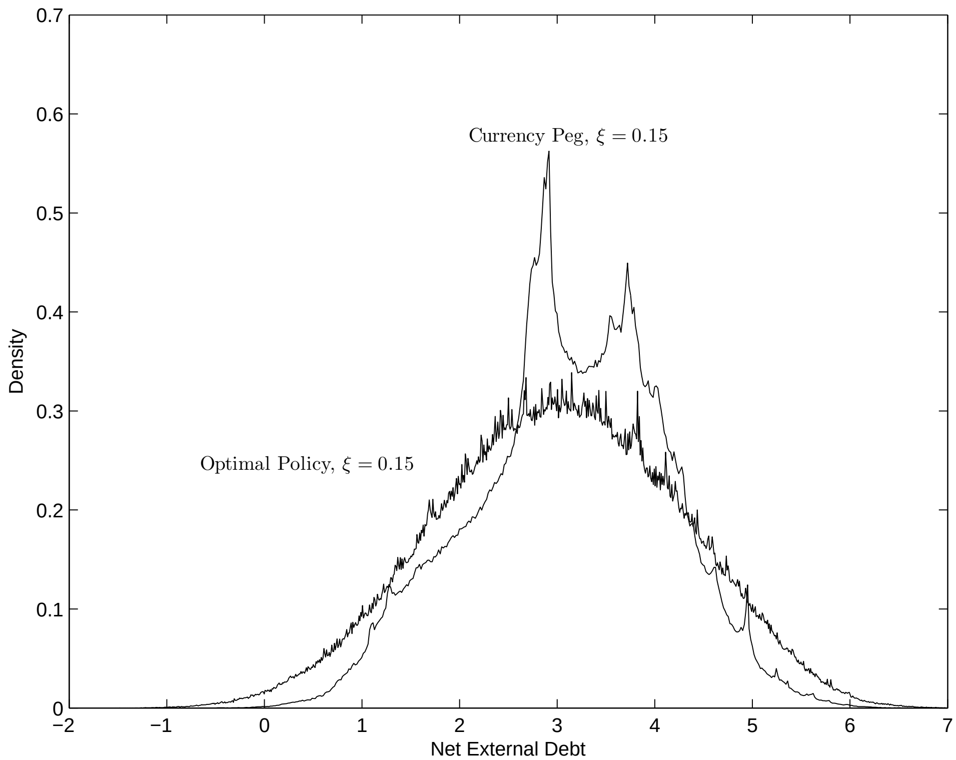
These differences in the distribution of the level of external debt are due to the fact that in the currency-peg economy the price mechanism that induces households to drastically cut their absorption of tradable goods during periods in which the traded endowment is low and the interest rate is high malfunctions due to the real rigidity imposed by the combination of downward nominal wage rigidity and a currency peg. During crises, the currency-peg economy takes on too much external debt. The resulting path of debt is excessively volatile. The only way such debt dynamics can be supported in the long-run is by lowering the average level of external debt. In summary, inefficient short-run volatility acts as a borrowing constraint and has long-run effects on the debt distribution.

The prediction that the average level of external debt is higher under the optimal exchange rate policy than under a currency peg depends, however, on the relative magnitudes of the intra- and intertemporal elasticities of substitution, ξ and $1/\sigma$, respectively. Our baseline calibration assumes that the intratemporal elasticity exceeds the intertemporal elasticity, 0.44 versus 0.2. If the intertemporal elasticity of substitution is set at a higher value than the intratemporal elasticity, it becomes possible that the average level of external debt is higher in the currency-peg economy than in the optimal exchange rate economy. Figure 9 illustrates this point. It displays the distribution of external debt under the optimal exchange rate policy and under the currency peg for the case that the intratemporal elasticity of substitution is smaller than the intertemporal elasticity of substitution, $\xi < 1/\sigma$. Specifically, in producing the figure we set ξ at 0.15 and maintain all other parameters at their baseline values, including $1/\sigma = 0.2$. Under this calibration, the mean external debt is larger under a peg than under the optimal exchange rate policy.

The reason why the difference in mean debt under the peg and the optimal policy depends on the difference between the intertemporal and intratemporal elasticities of substitution is that this difference governs the relative response of tradable consumption to disturbances in the marginal utility of wealth. In appendix C, we show that the elasticity of traded consumption with respect to the marginal utility of wealth satisfies the following expression:

$$\frac{d\ln(c_t^T)}{d\ln(\lambda_t)} = \begin{cases} \frac{-1}{\eta\sigma+(1-\eta)\frac{1}{\xi}} & \text{Under the optimal exchange rate policy or when wage rigidity does not bind in a peg} \\ \frac{-1}{\eta\sigma+(1-\eta)\frac{1}{\xi}+\left(\sigma-\frac{1}{\xi}\right)\frac{1-\eta}{1+\xi\left(\frac{1-\alpha}{\alpha}\right)}} & \text{when wage rigidity does bind in a peg} \end{cases},$$

Figure 9: The Distribution of External Debt when $\xi < \frac{1}{\sigma}$



where $\eta \in (0,1)$ is a measure of the share of tradable consumption in total consumption. This expression shows that in the special case in which the intratemporal and intertemporal elasticities of substitution are equal to each other ($\xi = 1/\sigma$), the wealth elasticity of traded consumption is the same under both exchange rate regimes. That is, a given rise in the marginal utility of wealth, due, for instance to a fall in tradable endowment or a rise in the world interest rate when households are net external debtors, causes the same proportional decline in the consumption of tradable goods under a peg and under the optimal exchange rate policy. However, when the intratemporal elasticity of substitution exceeds the intertemporal elasticity ($\xi > 1/\sigma$), as is the case in our baseline calibration, then the contraction in tradable consumption triggered by a given proportional increase in the marginal utility of wealth is smaller under a peg than under the optimal exchange rate policy. It follows that in this case, during a crisis, the economy experiences a smaller improvement in the trade balance and larger accumulation of external debt under the currency peg than it does under the optimal exchange rate policy. The reverse is true when the intratemporal elasticity is smaller than the intertemporal one.

The intuition behind this finding is as follows. One can show that tradable and nontradable goods are gross substitutes in consumption if the intratemporal elasticity of substitution, ξ, is larger than the intertemporal elasticity of substitution, $1/\sigma$. That is, if $\xi > 1/\sigma$, then traded consumption falls when the relative price of nontradables declines. During a crisis, the relative price of nontradables falls by much more under the optimal exchange rate regime than under a currency peg. Therefore, under gross substitutability, traded consumption falls by more under the optimal exchange rate regime than under the currency peg. This implies, in turn, that the economy displays a smaller improvement in the trade balance and a larger deterioration in the external debt position during a crisis under a currency peg than it does under the optimal exchange rate policy. The reverse is the case if tradables and nontradables are gross complements ($\xi < 1/\sigma$).

In the special case in which the intra- and intertemporal elasticities of substitution are equal to each other ($\xi = 1/\sigma$), one can show analytically that the stochastic processes of external debt, d_t and consumption of tradables, c_t^T, are independent of the exchange rate regime and, in particular, identical under a currency peg and the optimal exchange-rate policy. In other words, in this special case, the dynamics of d_t and c_t^T are independent of the presence of downward wage rigidity. We collect this result in the following proposition.

Proposition 2 *Suppose $\xi = 1/\sigma$. Then the dynamics of d_t and c_t^T are independent of the exchange-rate regime.*

Proof: Note that when $\xi = 1/\sigma$, we have that condition (6) becomes $\lambda_t = a(c_t^T)^{-1/\xi}$.

Now, consider the following subset of conditions describing aggregate dynamics (4), (6)-(9), and (15). These conditions can be solved for c_t^T, d_t, λ_t, and μ_t independently of the exchange-rate regime.$\blacksquare$

It is worth noting that this proposition does not imply the absence of welfare costs associated with currency pegs. For downward wage rigidity in combination with a fixed exchange rate will continue to cause unemployment and hence lower production and consumption of nontradable goods when the economy is hit by negative external shocks. The proposition does say, however, that unemployment in the nontraded sector will not disrupt the allocation of expenditures in tradable goods.

7 A Demand For Money

We now introduce money in the model. The motivation for allowing for real balances is that doing so creates a channel through which devaluations may become costly.

We modify the period utility function as follows:

$$u(c_t, M_t/P_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \delta_0 \frac{(M_t/P_t)^{1-\delta_1} - 1}{1-\delta_1},$$

where M_t denotes domestic nominal money balances held in period t, and P_t denotes the domestic-currency price of consumption. The household's sequential budget constraint is now given by

$$c_t^T + p_t c_t^N + d_t + \frac{M_t}{E_t} \leq y_t^T + w_t h_t + \frac{d_{t+1}}{1+r_t} + \frac{M_{t-1}}{E_t} + \phi_t + \tau_t,$$

where τ_t denotes a transfer received from the government. The optimality condition of the household with respect to money implies the liquidity preference function

$$\frac{M_t}{P_t} = \left(\frac{i_t}{\delta_0(1+i_t)} \right)^{-1/\delta_1} c_t^{\sigma/\delta_1},$$

where i_t denotes the nominal interest rate on one-period, domestic-currency-denominated bonds. In turn, the nominal interest rate must satisfy

$$\frac{\lambda_t}{1+i_t} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\epsilon_{t+1}} + \mu_t.$$

All variables in this expression are as defined earlier. The remaining building blocks of the model are the same as in the baseline formulation developed in section 2. We assume that the opportunity cost of holding money faced by the household is bounded below by a positive parameter $\underline{i}$ close to zero. We cannot impose a strict zero bound on the opportunity cost of holding money because the above liquidity preference function does not display satiation. This lower bound could be implemented through Gesell-like taxes. We set $\underline{i}$ to 0.000625, which corresponds to 25 basis points per year. Guided by the empirical regularity that the expenditure elasticity of money demand is close to unity, we set $\delta_1 = \sigma$. Finally, we calibrate δ_0 so that the average money-to-output ratio implied by our currency-peg economy matches the 7.3 percent value observed in Argentina during the Convertibility plan (1991-2001). This calibration strategy yields $\delta_0 = 0.000053$.

As before, we compare two exchange-rate regimes. One is a peg and the other is a full-employment exchange-rate policy. Note that, because of the presence of money, the full-employment policy is no longer Pareto optimal, since the central bank now has only one instrument, the rate of devaluation, and two targets, unemployment and the opportunity cost of holding money. This means that the welfare costs of pegs reported here are lower bounds, in the sense that the welfare costs of pegs relative to the Pareto optimal policy must necessarily be at least as large. Finally, we must pick one among the infinite exchange-rate policies that belong to the family of full-employment exchange-rate regimes. This choice is inconsequential in the baseline model, because of the absence of money. However, it does matter in the present monetary economy, because each full-employment regime will in general be associated with different paths for the opportunity cost of holding money. We pick the following regime:

$$\epsilon_t = \gamma \frac{w_{t-1}}{\Omega(c_t^T)}.$$

Under this full-employment exchange-rate regime gross nominal-wage growth is constant over time and equal to γ. In this regime, the unconditional median of gross inflation is also given by γ. This exchange-rate regime exemplifies that the presence of nominal downward wage rigidity does not imply that a full-employment exchange-rate policy must be inflationary on average. This point is important for understanding that in general full-employment policies need not entail a loss of welfare due to higher average levels of inflation. The way the central bank manages to bring about full employment in a low-inflation environment is by aggressively devaluing the domestic currency during recessions (see figure 5), and at the same time revaluing the currency during booms. These revaluations entail no employment costs because they occur during periods in which the full employment real wage is increasing. So these revaluations accommodate the efficient increase in real wages while preventing nominal wage inflation.

Table 4 displays the welfare cost of a currency peg relative to the full-employment

Table 4: Welfare Costs of Currency Pegs In A Money-Demand Economy

Average Money-to-Output Ratio During Pegs %	δ_0	Welfare Cost Median	Welfare Cost Mean
0	0	10.4	12.3
7.4	0.000053	10.5	12.5
16.1	0.0026	10.6	12.6

Note. The welfare cost of a currency peg is calculated as the percent increase in consumption that would make an individual living in the currency-peg economy as well off as an individual living in the economy with a full-employment exchange-rate policy for a given state of the economy $(y_t^T, r_t, d_t, w_{t-1})$.

exchange-rate policy. The main insight from this table is that introducing a demand for money does not alter the size of the welfare cost of currency pegs in a significant way. The median welfare cost of a currency peg is 10.5 percent of consumption per period, which is essentially the same cost as the one obtained in the economy without a demand for money. The reason for this result is that the full-employment policy need not be associated with higher average inflation than a currency peg. Indeed, under the full-employment exchange-rate policy studied here, the average rate of inflation is lower than under the currency peg (-3.1 versus 0.1 percent per year). As a result, average money holdings are not significantly different in the two exchange-rate regimes. The last row of the table presents welfare calculations for an economy with a money-to-output ratio that is more than twice as large as the one observed in the calibration sample (7.4 versus 16.1 percent). We find that the welfare costs of a currency peg are robust to increasing households preference for liquidity. The reason for this result is again that the full-employment exchange-rate regime need not increase the average rate of inflation.

8 Endogenous Labor Supply

We now relax the assumption of an inelastic labor supply schedule. Specifically, we consider a period-utility specification of the form

$$U(c_t, h_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{(\bar{h} - h_t)^{1-\theta} - 1}{1-\theta},$$

where φ, θ, and $\bar{h}$, are positive parameters, and c_t and h_t denote, respectively, consumption of the composite good and hours worked. Under this specification, the household's optimization problem features a new efficiency condition of the form

$$\varphi(\bar{h} - h_t^s)^{-\theta} = w_t \lambda_t,$$

where h_t^s denotes the number of hours supplied to the market, which may exceed the number of hours actually worked, h_t. In addition, conditions (12) and (18) of the baseline economy are replaced, respectively, by

$$h_t^s \geq h_t,$$

and

$$(h_t^s - h_t)\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0.$$

All other conditions describing aggregate dynamics are as before. In the present economy, the variable $h_t^s - h_t$ measures the level of involuntary unemployment. We set $\varphi = 4.4$, $\bar{h} = 3$, and $\theta = 1.001$ or 6. The values of φ and $\bar{h}$ are chosen so that the average level of employment is about unity, which is the level of full-employment in the baseline economy with inelastic labor supply. (It is somewhat above unity for $\theta = 6$ and somewhat below unity for $\theta = 1.001$.) The two values of θ are meant to capture an environment with a relatively large labor supply elasticity ($\theta = 1.001$, or near logarithmic specification) and one with relatively low labor supply elasticity ($\theta = 6$). Specifically, business-cycle studies measure the Frisch labor supply elasticity for preference specifications of the type used here as $(\bar{h} - h_t)/h_t/\theta$ evaluated at some average level of employment. In our model, under a peg this measure of the labor supply elasticity equals 0.3 on average when θ equals 6 and 4 when θ equals 1.001. (The corresponding values under the optimal policy are 0.2 and 3.)

Table 3 shows the welfare cost of a currency peg implied by this model specification. The median welfare cost is 4.5 percent of consumption per period when θ equals 1.001 and 6.8 percent when θ equals 6. These welfare costs continue to be large compared to results in the related literature. The intuition behind this result is that with a higher labor supply elasticity wages rise by less during booms, which reduces the required fall during contractions, making the lower bound constraint on nominal wages less stringent.

It is worth noting that the welfare costs of currency pegs obtained under endogenous labor supply are likely to be biased downward. The reason is that the preference specification considered in this section treats voluntary leisure and involuntary leisure (i.e., involuntary unemployment) as perfect substitutes. There exists an extensive empirical literature on subjective well being when unemployed suggesting that the nonpecuniary costs of unemployment are substantial (see, Winkelmann and Winkelmann, 1998, and the references cited therein).

9 Production in the Traded Sector

Our baseline model specification assumes that the supply of tradables, y_t^T, is exogenous. We now relax this assumption and assume instead that tradables are produced with labor. Specifically, we assume that

$$y_t^T = e^{z_t} \left(h_t^T\right)^{\alpha_T},$$

where y_t^T denotes output of tradable goods, h_t^T denotes labor employed in the traded sector, and $\alpha_T \in (0,1)$ is a parameter. The variable z_t is assumed to be exogenous and stochastic. We interpret z_t either as a productivity shock in the traded sector or as a disturbance in the country's terms of trade. We assume that, as in the nontraded sector, firms in the traded sector are perfectly competitive in product and labor markets. Firms choose labor to maximize profits, which are given by $P_t^T e^{z_t} \left(h_t^T\right)^{\alpha_T} - W_t h_t^T$. The first-order condition associated with the firm's profit maximization problem is

$$\alpha_T P_t^T e^{z_t} \left(h_t^T\right)^{\alpha_T - 1} = W_t. \qquad (26)$$

Letting h_t^N denote hours employed in the nontraded sector, total hours worked, denoted by h_t, are given by

$$h_t = h_t^T + h_t^N.$$

All other conditions of the model are as in the baseline formulation.

We assume that z_t and r_t follow the joint stochastic process given in equation (25), with z_t taking the place of $\ln y_t^T$. This strategy for calibrating the law of motion of z_t results in a standard deviation of $\ln y_t^T$ of 0.14 under a peg which is slightly above the value in the baseline model. A more satisfactory approach to calibrating the parameters defining the process z_t would be to construct a time series for total factor productivity in the traded sector. We do not pursue this avenue here. Following Uribe (1997), we set $\alpha_T = 0.5$. All other parameters take the values indicated in table 2.

Table 3 shows that the welfare cost of currency pegs in the present economy is 10.6 percent of consumption per period on average. It follows that our main finding is robust to allowing for production in the traded sector. The intuition for why the welfare costs of currency pegs continue to be large even when the supply of tradables is endogenous, can be illustrated by considering the adjustment of the economy to negative shocks when the lower bound on wages is binding. Consider first a negative interest-rate shock (i.e., an increase in r_t). If the wage rigidity is binding, then, as in the baseline economy, employment in the nontraded sector falls because of a weaker demand for this type of goods. At the same time, optimality condition (26) indicates that employment in the traded sector is unchanged, since wages are downwardly rigid and the exchange rate is pegged. This means that the unemployment that emerges in the nontraded sector will not be absorbed by the traded sector. Consider now the effect of a deterioration in the terms-of-trade or a negative productivity shock in the traded sector (i.e., a decline in z_t). Suppose again that the lower bound on nominal wages is binding. In this case, optimality condition (26) implies that employment in the traded sector will fall. This is because the product wage is unchanged but the marginal product of labor falls at any given level of employment. In the nontraded sector, demand declines because the negative productivity shocks produces a negative income effect. It follows that the nontraded sector does not absorb the hours lost in the traded sector. On the contrary, employment in the nontraded sector will also decline due to a weaker demand. It follows that the introduction of production in the traded sector does not ameliorate the unemployment problem induced by the combination of nominal downward wage rigidity and a currency peg. Our results show that this is the case not only qualitatively but also quantitatively.

10 Sensitivity Analysis

In this section, we study how the welfare costs of currency pegs are affected by alternative parameterizations of the model. In particular, we study the sensitivity of the implied welfare costs of currency pegs to lower degrees of downward wage rigidity, a higher intertemporal elasticity of substitution, a higher intratemporal elasticity of substitution, a higher share of tradables in consumption, and preferences featuring more patient households.

10.1 Lower Downward Nominal Wage Rigidity

The fact that unemployment is the main source of welfare losses associated with currency pegs suggests that a key parameter determining the magnitude of these welfare losses should be γ, which governs the degree of downward nominal wage rigidity. Our baseline calibration ($\gamma = 0.99$) implies that nominal wages can fall frictionlessly up to four percent per year. As argued in section 3, this is a conservative value in the sense that it allows for falls in nominal wages during crises that are much larger than those observed either in the 2001 Argentine crisis or the ongoing crisis in peripheral Europe. We now consider alternative values that allow for nominal wage declines of up to 16 percent per year. Taking into account that the largest wage decline observed in Argentina in 2001 or in the periphery of Europe since the onset of the great recession was 1.6 percent per year (Lithuania, see table 1), it follows that we are considering degrees of wage rigidity substantially lower than those implied by observed wage movements during recent large contractions. Table 3 shows that the median welfare cost of a currency peg is 4.3 percent for the lowest value of γ considered, 0.96. This welfare cost, although smaller than the one obtained under our baseline calibration of γ, is still a large figure compared to existing results in monetary economics. The intuition why currency pegs are less painful when wages are more downwardly flexible is straightforward. A negative aggregate demand shock reduces the demand for nontradables which requires a fall in the real wage rate to avoid unemployment. Under a currency peg this downward adjustment must be brought about exclusively by a fall in nominal wages. The less downwardly rigid are nominal wages, the faster is the downward adjustment in both the nominal and the real wage and therefore the smaller is the resulting level of unemployment.

10.2 Patience

We now consider the case that households are more patient. A reason for considering this case is that in principle there exist two equivalent strategies to calibrate the subjective discount factor β. One strategy is to match the mean net external debt to output ratio. The other is to match the mean trade-balance-to-output ratio. When working with actual data, however, these two strategies are not equivalent because of the necessity to use finite samples and because of the presence of measurement error. In our baseline calibration we adopt the strategy of matching the average debt-to-output ratio in Argentina, which as mentioned earlier is 26 percent over the calibration period. This approach delivers a trade-balance-to-output ratio of 3.7 percent which is higher than the value of 2.2 percent observed over our calibration period. We now calibrate β so as to match this value of the trade-balance-to-output ratio. This requires raising β from its baseline value of 0.9375 to 0.945. The raising of the discount factor results in a decline in the average debt-to-output ratio in the currency peg economy from 26 to 15 percent of annual output. Table 3 shows that under this alternative calibration the welfare costs of a currency peg relative to the optimal exchange rate policy has a median of 8 percent and a mean of 9.2 percent of the consumption stream. These figures are smaller than those corresponding to the baseline calibration but still extremely large as costs of business cycles go. As discussed earlier, in our economy with downwardly rigid wages a currency peg acts like a borrowing constraint. This is because a currency peg induces a more dispersed debt distribution, which requires a lower mean debt (or higher precautionary savings) to ensure long-run solvency. More patient agents are in general less negatively affected by a given borrowing limit than are less patient agents. This explains why the welfare cost of a currency peg under the current calibration is smaller than under the baseline calibration.

10.3 Intratemporal Elasticity of Substitution

An important parameter in our model is ξ measuring the intratemporal elasticity of substitution between traded and non-traded goods. A reason why this parameter is important in our model is that the higher is the elasticity of substitution between tradables and nontradables, the smaller is the real depreciation required to bring about the necessary expenditure switch away from tradables and towards nontradables in response to negative aggregate demand shocks. It follows that the higher is ξ the less disruptive is the presence of nominal downward rigidities in wages for macroeconomic adjustment. We therefore consider a value of ξ that is twice as large as our baseline value. This value is in line with the one obtained by restricting the cross-country sample of Stockman and Tesar (1995) to include only developed countries, (see Akinci 2011). This higher value of ξ is also consistent with the estimates reported in Ostry and Reinhart (1992) for a panel of Latin American countries. Consistent with our intuition, we find that the average unemployment rate falls from 14 to 11 percent as ξ increases from its baseline value of 0.44 to 0.88. And the median welfare cost of a currency peg relative to the optimal exchange rate policy falls from 10.4 to 8.6 percent of the consumption stream. These figures show that even in an environment in which consumers can substantially more easily substitute tradables for nontradables, currency pegs continue to be extremely painful.

10.4 Share of Nontradables in Consumption

The inefficiency created by the combination of downward wage rigidity and a fixed exchange rate manifests itself in underutilization of labor services in the nontraded sector. It is then of interest to investigate the sensitivity of our findings to reducing the importance of the nontraded sector. We do so by lowering the parameter $1-a$, which weights the consumption of nontradables in the aggregator function. Lowering the share of nontradables should reduce the welfare costs of pegs, the inefficiently low level of nontradable consumption they induce is given a smaller weight in the utility function. Table 3 reports the welfare cost of currency pegs for an economy in which a takes the value 0.4 and all other parameters are kept at their baseline values. Under this calibration the share of traded consumption in total consumption implied by the model is 43 percent on average. This value is higher than the ones observed for Argentina and the peripheral European countries shown in table 1. The table shows that the welfare costs of currency pegs continue to be extremely high even for a small share of the nontraded sector.

10.5 Intertemporal Elasticity of Substitution

We also investigate the sensitivity of our findings to increasing the intertemporal elasticity of substitution, $1/\sigma$. Specifically, we lower σ from its baseline value of 5 to a value of 2. This latter value is widely used in emerging-country business-cycle analysis, see Uribe (2011) and the references cited therein. Raising the intertemporal elasticity of substitution makes households less risk averse and as a result more willing to assume external debt. Holding all parameters other than σ constant at their baseline values, the lowering of σ results in debt distributions (under both the currency peg regime and the optimal exchange rate regime) that pile up to the left of the natural debt limit. The implied debt-to-output ratios are many times larger than those observed over our calibration period. For this reason, we adjust the value of β from its baseline value of 0.9375 to 0.964 to ensure that together with a value of $\sigma = 2$, the currency-peg economy delivers an external debt share in line with that observed over the calibration period (about 26 percent of annual output). In contrast to the baseline calibration, under the present calibration the intertemporal elasticity of substitution, $1/\sigma$, exceeds the intratemporal elasticity, ξ. As a consequence and as suggested in section 6, the average debt to output ratio is higher in the currency peg economy than in the optimal exchange rate economy (0.24 versus 0.14, respectively), which represents a reversal of the predictions obtained under the baseline calibration. Table 3 shows that under this alternative calibration the welfare costs of currency pegs continue to be extremely high with a median of 9.9 percent of consumption per period. This figure is slightly smaller than its baseline counterpart. This is expected because less risk averse agents are more tolerant to economic fluctuations, and because consumption is more volatile in the currency peg economy than in the optimal exchange rate policy economy. However, in our model, the bulk of the welfare losses associated with a currency peg stems not from this second-order source but from the average unemployment induced by this type of policy, which is a first-order effect. And the high rate of unemployment induced by a currency peg appears to be robust to changes in σ. Specifically, when σ takes the value of 2, the average rate of unemployment continues to be high, above ten percent. Such a high rate of unemployment implies a permanent loss of nontradable output of about 12 percent per period, which given the weight of 0.74 of nontradable consumption in the aggregator function implies a permanent loss of total consumption of about 9 percent per period. This loss of consumption is entirely avoided under the optimal exchange rate policy because in that case the economy is always operating at full employment.

11 A Fiscal Alternative To Devaluations

Many observers have suggested the use of fiscal policy to ease the pains of currency pegs currently felt in the periphery of the European Union. However, advocates of active fiscal policy do not speak with a single voice. Some argue that the right medicine for emerging country members of the European Union is fiscal restraint via tax increases and cuts in public expenditures. Others hold diametrically opposed views and argue that only widespread increases in government spending and tax cuts can offer pain relief. Our model suggests that both of these extreme views are misguided. Instead, the model suggests that the way to ease the pain of a currency peg by means of fiscal policy is more sophisticated in nature. Specifically, optimal fiscal policy in the context of a currency peg consists in a time-varying labor income subsidy that targets industries with high degrees of downward wage rigidity. It can be shown that in our currency-peg economy the full-employment equilibrium can be reached by implementing a proportional wage subsidy at the rate τ_t, where

$$\tau_t = \max\left\{0, 1 - \frac{\omega(c_t^T)}{\gamma w_{t-1}}\right\}, \tag{27}$$

where $\omega(c_t^T)$ denotes the full-employment real wage and γw_{t-1} denotes the real wage that prevails when the wage rigidity is binding. If the combination of the currency peg and the downward nominal wage rigidity prevent the real wage from falling to the full-employment real wage, the subsidy is positive. Otherwise, the subsidy is zero. This subsidy scheme can be financed in a revenue neutral fashion, by an appropriate proportional tax on any source of income (labor income, $w_t h_t$, tradable income, y_t^T, profits, ϕ_t, or any combination thereof). We note that these financing schemes work even when the labor supply is elastic. The reason is that the subsidy is positive only in states of the world in which, in the absence of the subsidy, households are off their labor supply schedule, or involuntarily unemployed.[10] It is clear from equation (27) that the optimal subsidy inherits the stochastic properties of the optimal devaluation rate studied in previous sections (see equation (21)). Because the optimal devaluation rate is found to be highly volatile at business-cycle frequency, it follows that the fiscal alternative presented here may indeed introduce an impractically high level of volatility in the tax/subsidy regime. This conclusion gives credence to the dictum that nominal rigidities are best addressed by monetary policy.

[10] In a more recent contribution, Farhi et al. (2011) expand this idea to other economic environments.

12 Conclusion

In this paper we fill a gap between historical and model-based assessments of the costs of currency pegs. While the former attribute major macroeconomic failures to currency pegs, the latter find that their welfare consequences are negligible. Our theoretical approach is based on a familiar narrative that goes back to Keynes' (1925) and Friedman' (1953) reservations against fixed exchange-rate arrangements. We incorporate this narrative in the context of a dynamic stochastic model of the open economy amenable to welfare evaluations. We refocus attention on downward nominal wage rigidity as the central source of monetary nonneutrality. Our theoretical contribution is to depart from sticky-wage models in the Calvo-Rotemberg tradition by assuming that employment is not always demand determined. A novel implication of the model we propose is an endogenous connection between the amplitude of the business cycle and the average rate of unemployment. This connection opens the door to large welfare gains from optimal stabilization policy, and therefore has the potential to bring historical and model-based assessments of the pains of pegs closer together.

Using a calibrated version of our model, we find that external crises can be extremely contractionary when the exchange rate is fixed. Specifically, the model predicts that a large external shock, defined as a two-standard-deviation collapse in the value of tradable output and a two-standard-deviation increase in the country interest-rate premium, causes a massive increase in unemployment of about 20 percent of the labor force. This figure is consistent with the unemployment rates observed in the aftermath of recent large contractions in emerging market economies that followed a fixed exchange rate regime, including Argentina 1998-2001 and the periphery of the European Union (e.g., Latvia, Greece, Portugal, Spain, and Ireland) post 2008. Furthermore, we find that under a currency peg unemployment is highly persistent. Our model predicts that even five years after the trough of the crisis the unemployment rate remains about 7 percentage points above average.

Our model predicts that currency pegs are costly not only during crises but also over regular business-cycle fluctuations. We find that the unconditional welfare costs of currency pegs are enormous. At the median of the distribution of welfare costs, households living under a currency peg require a ten percent increase in consumption every period to be as well off as households living in an economy in which the central bank implements the optimal exchange rate policy. The source of these large welfare costs is primarily that currency pegs entail high levels of unemployment which affects negatively the supply and ultimately the domestic absorption of nontradable goods. We find that the welfare costs of currency pegs are larger than its median of ten percent when the initial state of the economy is characterized by weak fundamentals such as high external debt, high past real wages, high country premia, or weak terms of trade. These findings shed light on why pressures to abandon the currency peg emerged with force in Argentina in 2001 and across the emerging-market members of the European Union in the wake of the great contraction of 2008. Besides being on a fixed exchange rate, these countries had in common high country premia, high levels of external debt, weak terms of trade, and a highly unionized labor force that all but prevented nominal wage cuts.

Appendix A: Proof of Proposition 1

Consider a set of stochastic processes $\{c_t,\ c_t^T,\ c_t^N,\ h_t,\ p_t,\ w_t,\ d_{t+1},\ \lambda_t,\ \mu_t,\ \epsilon_t\}_{t=0}^{\infty}$ satisfying the conditions given in Definition 1 and the exchange rate policy (21). We first show that h_t must equal $\bar{h}$ at all times. This part of the proof is by contradiction. Suppose $h_t < \bar{h}$ for some $t \geq 0$. Then, by (18) we have that

$$w_t = \frac{\gamma w_{t-1}}{\epsilon_t}. \tag{28}$$

Using (21) to eliminate ϵ_t, implies that $w_t \leq \omega(c_t^T)$. Using (5), (16), and (20) we can write this inequality as

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Because the left-hand side of this expression is strictly decreasing in h_t, we have that h_t must equal $\bar{h}$, which is a contradiction. We have therefore shown that under the exchange rate policy given in (21), unemployment is nil at all dates and states. Evaluating (2), (4), (6)-(9), (14) and (15) at $h_t = \bar{h}$, we obtain

$$d_{t+1} \geq \bar{d}$$

$$\lambda_t = U'(A(c_t^T, \bar{h}))A_1(c_t^T, \bar{h})$$

$$\frac{\lambda_t}{1 + r_t} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t$$

$$\mu_t \geq 0,$$

$$\mu_t(d_{t+1} - \bar{d}) = 0,$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},$$

which are precisely the first-order necessary and sufficient conditions associated with the social planner's problem consisting in maximizing (24) subject to (4) and (15). The fact that the first-order conditions of the social planner's problem are necessary and sufficient follows directly from the strict concavity of the planner's objective and the convexity of the planner's constraint set.

Appendix B: Numerical Algorithm for Approximating the Aggregate Dynamics Under a Currency Peg

Define the discretized state as follows:

$$Y^T = \{y_1^T, y_2^T, \dots, y_{ny}^T\}$$

$$R = \{r_1, r_2, \dots, r_{nr}\}$$

$$D = \{d_1, d_2, \dots, d_{nd}\}$$

$$W = \{w_1, w_2, \dots, w_{nw}\}.$$

In iteration n, suppose the guess for the solution for the marginal utility of tradable goods is given by the function Λ^n, mapping $Y^T \times R \times D \times W$ into $\mathbb{R}$. To obtain the next guess Λ^{n+1}, proceed as follows:

(1) For a given state $(y_i^T, r_j, d_k, w_\ell)$ with $i \in \{1, \dots, ny\}$, $j \in \{1, \dots, nr\}$, $k \in \{1, \dots, nd\}$, $\ell \in \{1, \dots, nw\}$, denote the level of debt due next period by d_s for $s \in \{1, \dots, nd\}$.

(2) Use condition (15) to find the corresponding level of c^T as

$$c^T(d_s) = y_i^T + \frac{d_s}{1 + r_j} - d_k.$$

and use condition (20) to determine $\omega(c^T(d_s))$

$$\omega(c^T(d_s)) = \frac{A_2(c^T(d_s), F(\bar{h}))}{A_1(c^T(d_s), F(\bar{h}))} F'(\bar{h}).$$

(3) If the full-employment wage violates constraint (17) for $\epsilon_t = 1$, then the current wage must be equal to γw_ℓ. Therefore, we have that the current wage is given by

$$w' = \max\left\{\gamma w_\ell, \omega(c^T(d_s))\right\}.$$

Pick the current wage, which will be a state variable for the next period, so that the wage rate takes on one of the values in the set W. Formally, we have that

$$qq = \text{argmin}_{q \in \{1, \dots, nw\}}.|w_q - w'|$$

and then denote the current wage choice given d_s as w_{qq}.

(4) To find the level of employment associated with d_s, note that if $\omega(c^T(d_s)) \geq \gamma w_\ell$,

then $h(d_s) = \bar{h}$, else $h(d_s)$ solves

$$\gamma w_\ell = \frac{A_2(c^T(d_s), F(h(d_s))}{A_1(c^T(d_s), F(h(d_s)))} F'(h(d_s)).$$

(5) Find the level of nontraded consumption from (14)

$$c^N(d_s) = F(h(d_s)),$$

the level of consumption of the aggregate good from equation (2) as

$$c(d_s) = A(c^T(d_s), c^N(d_s)),$$

and the current value of the marginal utility of consumption of tradables from (6)

$$\lambda(d_s) = U'(c(d_s))A_1(c^T(d_s), c^N(d_s))$$

(6) Use equation (7) to construct μ as

$$\mu(d_s) = \frac{\lambda(d_s)}{1 + r_j} - \beta \sum_{ii=1}^{ii=ny} \sum_{jj=1}^{nr} Prob(y_{ii}^T, r_{jj} | y_i^T, r_j) \Lambda^n(y_{ii}^T, r_{jj}, d_s, w_{qq}).$$

If $\mu \geq 0$ and $s = nd$, then d_s is the optimal choice of debt in the current period and $s^* = nd$. In this case:

$$\Lambda^{n+1} = U'(c(d_{nd}))A_1(c^T(d_{nd}), c^N(d_{nd})).$$

Else construct μ for all $s \in \{1, \ldots, nd-1\}$. Find the optimal s as $s^* = \text{argmin}_{s\in\{1,\ldots,nd-1\}} |\mu(d_s)|$.

Construct

$$\Lambda^{n+1}(y_i^T, r_j, d_k, w_\ell) = U'(c(d_{s^*}))A_1(c^T(d_{s*}), c^N(d_{s^*})).$$

(7) Keep iterating in this way, until the maximum distance (taken over the $ny \times nr \times nd \times nw$ states between Λ^{n+1} and Λ^n is less than $1e - 8$.

Appendix C: The Wealth Elasticity of Traded Consumption

Using the assumed functional forms for preferences and technology, we can write expressions (2), (5), and (6) , respectively, as

$$c = \left[a(c^T)^{1-1/\xi} + (1-a)(c^N)^{1-1/\xi}\right]^{1/(1-1/\xi)},$$

$$p = \left(\frac{1-a}{a}\right)\left(\frac{c^T}{c^N}\right)^{1/\xi},$$

and

$$ac^{\sigma-1/\xi}{c^T}^{1/\xi} = \lambda^{-1}.$$

Throughout this analysis we drop time subscripts unless otherwise necessary. In the labor market, the situation depends on the monetary policy in place. Under the optimal policy, we have full employment at all times, which implies that

$$c^N = 1.$$

This expression will also hold under a currency peg in states in which the downward-wage-rigidity constraint is not binding. By contrast, when this constraint does bind under a currency peg, the labor market will be in disequilibrium. In this case, we have that $pF'(h) = \gamma w_{-1}$ and $c^N = F(h)$, where w_{-1} denotes the previous period's real wage rate. Using the assumed functional forms, we can combine these two expressions to obtain

$$\alpha p {c^N}^{(\alpha-1)/\alpha} = \gamma w_{-1}$$

We now proceed to log linearize all of the above expressions around some point $\tilde{c}^T$ for tradable consumption and unity or nontraded consumption. We denote with a circumflex accent log deviations from this reference point. The log-linearized version of the above expressions is

$$\hat{c} = \eta\hat{c}^T + (1-\eta)\hat{c}^N,$$

$$\hat{p} = \frac{1}{\xi}\left(\hat{c}^T - \hat{c}^N\right)$$

$$\left(\sigma - \frac{1}{\xi}\right)\hat{c} + \frac{1}{\xi}\hat{c}^T = -\hat{\lambda},$$

and

$$\hat{c}^N = \begin{cases} 0 & \text{Under the optimal exchange rate policy} \\ & \text{or when wage rigidity does not bind in a peg} \\ & \\ \frac{\alpha}{1-\alpha}\hat{p} & \text{when wage rigidity does bind in a peg} \end{cases}$$

where $\eta \equiv \frac{a(\tilde{c}^T)^{1-1/\xi}}{a(\tilde{c}^T)^{1-1/\xi}+(1-a)}$. Solving for $\hat{c}^T$ as a function of $\hat{\lambda}$, we obtain

$$\hat{c}^T = \begin{cases} \frac{-\hat{\lambda}}{\eta\sigma+(1-\eta)\frac{1}{\xi}} & \text{Under the optimal exchange rate policy} \\ & \text{or when wage rigidity does not bind in a peg} \\ & \\ \frac{-\hat{\lambda}}{\eta\sigma+(1-\eta)\frac{1}{\xi}+\left(\sigma-\frac{1}{\xi}\right)\frac{1-\eta}{1+\xi\left(\frac{1-\alpha}{\alpha}\right)}} & \text{when wage rigidity does bind in a peg} \end{cases}.$$

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