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ABSTRACT

A rise in the sex ratio (increasing relative surplus of men in the marriage market) in China and several other economies, in theory, can simultaneously generate a decline in the real exchange rate (RER) and a rise in the current account surplus. We demonstrate this logic through both a savings channel and an effective labor supply channel. In this model, a low RER is not a cause of the current account surplus, nor is it a consequence of currency manipulations. Empirically, those economies with a high sex ratio tend to have a low real exchange rate, beyond what can be explained by the Balassa-Samuelson effect, financial underdevelopment, dependence ratio, and exchange rate regime classifications. Once these factors are accounted for, the Chinese real exchange rate is estimated to be undervalued by only a relatively trivial amount.

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1 Introduction

China and several other economies in Asia are experiencing an increasingly more severe relative surplus of men in the pre-marital age cohort. While the existing literature on the sex ratio has examined its social impact such as crime, we aim to explore neglected implications of the sex ratio imbalance for the real exchange rate. Real exchange rate undervaluation due to currency manipulation is a frequent topic in international economic policy discussions. Two commonly used criteria by researchers and international financial institutions for judging undervaluations are deviations from the purchasing power parity (PPP) and large and persistent current account surpluses. The goal of this paper is to demonstrate that a rise in the sex ratio can generate both phenomena. In other words, a low real exchange rate need not be the cause of a current account surplus. (Given a current account surplus, foreign exchange reserve accumulation could be a passive outcome of a country's capital account controls, rather than exchange rate interventions. In other words, if a country has no capital controls, e.g., Japan, a current account surplus shows up as an addition to its private sector's holding of foreign assets. With capital controls, which typically require compulsory surrender of foreign exchange earnings by firms or households, a current account surplus has to be converted into additional holding of foreign exchange reserves by the official sector.)

We highlight two channels through which a sex ratio imbalance could lead to an appearance of currency undervaluation. The first is a savings channel. If an economy experiences a shock that raises its savings rate, then the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect can be persistent if there are frictions that impede the reallocation of factors between the tradable and nontradable sectors.

How would a rise in this imbalance trigger a significant increase in the savings rate? In the case of China, the sex ratio at birth rose from being approximately balanced in the early 1980s to about 120 boys/100 girls by 2007. As the competition for brides intensifies, young men and their parents raise their savings rate in order to improve their relative standing in the marriage market. If the biological desire to have a female partner is strong, the response of the savings rate to a rise in the sex ratio can also be quantitatively large. Of course, our theory has to investigate why the behavior by women or their parents does not undo the competitive savings story.

The empirical motivation for the savings channel comes from Wei and Zhang (2009). They provide evidence from China at both the household level and regional level. First, across rural households with a son, they document that the savings rate tends to be higher in regions with a higher sex ratio imbalance (holding constant family income, age, gender, and educational level of the household head and other household characteristics). In comparison, for rural households with a daughter, their savings rate appears to be uncorrelated with the local sex ratio. Across cities, both households with a son and households with a daughter tend to have a higher savings rate in regions with a more skewed sex ratio, although the elasticity of the savings rate with respect to the sex ratio tends to be bigger for son families. Second, across Chinese provinces, they find a strong positive correlation between the local savings rate and the local sex ratio, after controlling for the age structure of the local population, income level, inequality, recent growth rate, local enrollment rate in the social safety net, and other factors. Third, to go from correlation to causality, they explore regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, and confirm the findings in the OLS regressions. The sex ratio effect is both economically and statistically significant. While the Chinese household savings rate approximately doubled from 16% (of disposable income) in 1990 to 31% in 2007, Wei and Zhang (2009) estimate that the rise in the sex ratio could explain about half of the increase in the household savings rate.

The second theoretical channel works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER. There is evidence from China that the effective labor supply is indeed larger in regions with a higher sex ratio (Wei and Zhang, 2010).

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity. Of course, if there are structural factors, other than a rise in the sex ratio, that have also triggered an increase in the aggregate savings rate (e.g., an increase in the government savings rate) or an increase in the effective labor supply (e.g., peculiar patterns of the rural-urban migration within a country), they would reinforce the mechanisms discussed in this paper, causing the real exchange rate to fall further.

A desire to enhance one's prospect in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The strict family planning policy in China, introduced in the early

1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2009, for more detail). It may not be a coincidence that the Chinese real exchange rate started to garner international attention around 2003 just when the first cohort born after the implementation of the strict family planning policy were entering the marriage market.

Throughout the model, we assume an exogenous sex ratio. While it is surely endogenous in the long-run as parental preference should evolve, the assumption of an exogenous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohort's savings decisions, but not those who have passed half of their working careers. Second, in terms of cross country experience, most countries with a skewed sex ratio have not shown a sign of reversal. This suggests that, if the sex ratio follows a mean reversion process, the speed of reversion is very low.

There are four bodies of work that are related to the current paper. First, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. Sarno and Taylor (2002) and Chinn (2011) provide recent surveys. Second, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men, it is a favorable shock to women. Could the women strategically reduce their savings so as to completely offset whatever increments in savings men may have? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in macroeconomic policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by US officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for exchange rates from a change in the sex ratio. The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause for the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily have a lower fertility rate than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows. In Section 2, we construct a simple overlapping generations (OLG) model with only one gender, and show that structural shocks, by raising the savings rate, can simultaneously produce a real exchange rate depreciation and a current account surplus. In Section 3, we present an OLG model with two genders, and demonstrate that a rise in the sex ratio could lead to a rise in both the aggregate savings rate and the current account, and a fall in the value of the real exchange rate. In Section 4, we calibrate the model to see if the sex ratio imbalance can produce changes in the real exchange rate and current account whose magnitudes are economically significant. In section 5, we provide some empirical evidence on the connection between the sex ratio and the real exchange rate. Section 6 offers concluding remarks and discusses possible future research.

2 A benchmark model with one gender

We start with a simple benchmark model with one gender. This allows us to see the savings channel in a transparent way. The setup is standard, and the discussion will pave the way for a model in the next section that features two genders and an unbalanced sex ratio.

There are two types of agents: consumers and producers. Consumers consume and make the saving decisions to maximize their intertemporal utilities. Producers choose capital and labor input to maximize the profits.

2.1 Consumers

Consumers live for two periods: young and old. They receive labor income in the first period and nothing in the second period after retiring. In the first period, consumers consume a part of the labor income in the first period and save the rest for the second period.

The final good C_t consumed by consumers consists of two parts: a tradable good C_{Tt} and a nontradable good C_{Nt}.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

We normalize the price of the tradable good to be one, and let P_{Nt} denote the relative price of the nontradable good. The consumer price index is $P_t = P_{Nt}^{\gamma}$.

Consumers earn labor income when they are young and retire when they are old. The optimization problem for a representative consumer is

$$\max \ u(C_{1t}) + \beta u(C_{2,t+1})$$

with the intertemporal budget constraint

$$P_t C_{1t} = (1 - s_t) y_t \text{ and } P_{t+1} C_{2,t+1} = R s_t y_t$$

where y_t is the disposable income and s_t is the savings rate of the young cohort. R is the gross interest rate in terms of the tradable good.

The optimal condition for the representative consumer's problem is

$$\frac{u'_{1t}}{P_t} = \beta R \frac{u'_{2,t+1}}{P_{t+1}} \tag{2.1}$$

We start with the case of a small open economy, and assume that the law of one price for the tradable good holds. The price of the tradable good is determined by the world market, and is set to be one in each period. The interest rate R in units of the tradable good is also a constant.

2.2 Producers

There are two sectors in the economy: a tradable good sector and a non-tradable good sector. Both markets are perfectly competitive. For simplicity, we make the same assumption as in Obstfeld and Rogoff (1996) that only the tradable good can be transformed into capital used in production.[1]

2.2.1 Tradable good producers

For simplicity, we assume a complete depreciation of capital at the end of every period. Tradable producers maximize

$$\max \ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - K_{T,t+\tau+1} \right]$$

where the production function is

$$Q_{Tt} = \frac{A_{Tt} K_{Tt}^{\alpha_T} L_{Tt}^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1 - \alpha_T)^{1-\alpha_T}}$$

[1] Relaxing this assumption will not change any of our results qualitatively.

Without any unanticipated shocks, the factor demand functions are, respectively,

$$R = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt}\left(\frac{L_{Tt}}{K_{Tt}}\right)^{1-\alpha_T} \quad (2.2)$$

$$w_t = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}}{L_{Tt}}\right)^{\alpha_T} \quad (2.3)$$

It is useful to note that when there is an unanticipated shock in period t, (2.2) does not hold since K_{Tt} is a predetermined variable.

2.2.2 Nontradable good producers

Nontradable good producers maximize the following objective function:

$$\max\ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau}\left[P_{N,t+\tau}Q_{N,t+\tau} - w_{t+\tau}L_{N,t+\tau} - K_{N,t+\tau+1}\right]$$

with the production function given by

$$Q_{Nt} = \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}$$

Without unanticipated shocks, we have

$$R = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_N} \quad (2.4)$$

$$w_t = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \quad (2.5)$$

If there is an unanticipated shock in period t, (2.4) does not hold.

In equilibrium, the market clearing condition for the nontradable good pins down the price of the nontradable good,

$$Q_{Nt} = \frac{\gamma P_t\left(C_{2t}+C_{1t}\right)}{P_{Nt}} \quad (2.6)$$

The labor market clearing condition is given by

$$L_{Tt} + L_{Nt} = 1 \quad (2.7)$$

Assuming no labor income tax (for simplicity), $y_t = w_t$.

Definition 1 *An equilibrium in the small open economy is a set* $\{s_t, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The households' savings rates, $s_t = \{s_{it}, s_{-i,t}\}$, maximize the household's welfare

$$s_t = \arg\max \{ V_t |\, s_{-i,t}, K_{Tt+1}, K_{Nt+t}, L_{Tt}, L_{Nt}, P_{Nt} \}$$

(ii) The allocation of capital stock and labor, and the output of the non-tradable good clear the factor and the output markets, and maximize the firms' profit. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).

2.3 A shock to the savings rate and the effect on the exchange rate

To illustrate the idea that a shock that raises the savings rate could lower the value of the real exchange rate, we now consider an unanticipated increase in the discount factor β that makes the young cohort more patient. In period t, (2.3) and (2.5) hold, but (2.2) and (2.4) fail.

The market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt} A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} = \gamma \left(R s_{t-1} w_{t-1} + (1 - s_t) w_t \right)$$

We can solve (2.1), (2.6), (2.3) and (2.5) to obtain the equilibrium in period t. Let $R = \frac{RP_t}{P_{t+1}}$ denote the real interest rate. We assume that the utility function is of the CRRA form, i.e., $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$. Following Obstfeld and Rogoff (1996) and assuming that the nontradable good sector is relatively more labor-intensive, i.e., $\alpha_N < \alpha_T$, we can obtain the following proposition.

Proposition 1 *With an increase in the discount factor β of the young cohort, the aggregate savings rate rises, and the price of the nontradable good falls. As a result, the real exchange rate depreciates and the current account increases.*

Proof. See Appendix A. ■

In the period in which the shock occurs, as a representative consumer becomes more patient, he would save more and consume less. The reduction in aggregate consumption leads to a decrease in the relative price of nontradable good (and a depreciation of the real exchange rate). As the rise in savings is not accompanied by a corresponding rise in investment, the country's current account increases. In summary, without currency manipulations, real factors that lead to a rise in a country's savings rate can simultaneously produce a fall in the real exchange rate and a rise in the current account. The low value of the real exchange rate is not the cause of the current account surplus.

Note that the effect on the RER and the current account last for one period. In period $t+1$, since the shock has been observed and taken into account by consumers and firms, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

In other words, the price of the nontradable good and the consumer price index go back to their initial levels. Later in the paper, we will demonstrate how frictions in the factor market can produce longer-lasting effects on the real exchange rate and the current account.

3 Unbalanced sex ratios and real exchange rates

In this section, we extend our benchmark model to a two-sex overlapping generations model. Within each cohort, there are women and men. A marriage can take place at the beginning of a cohort's second period, but only between a man and a women in the same cohort. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income. Everyone is endowed with an ability to give his/her spouse some additional emotional utility (or "love"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when an individual enters the marriage market. There are no divorces.

Each generation is characterized by an exogenous ratio of men to women $\phi(\geq 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects - in particularly, men do not have an intrinsic tendency to save more - except that the sex ratio may be unbalanced.

3.1 A small open economy

We start from a small open economy. As in the benchmark model, the price of the tradable good is always one and the interest rate in units of the tradable good is a constant R. As in Obstfeld and Rogoff (1995), we assume that only tradable goods can be converted into capital used in production.

A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she fails to get married, her second-period consumption is $P_{t+1}C^{w,n}_{2,t+1} = Rs^w_t y^w_t$, where R, y^w_t and s^w_t are the gross interest rate of an international bond, her endowment, and her savings rate, respectively, all in units of the tradable good. If she is married, her second-period consumption is $P_{t+1}C^w_{2,t+1} = \kappa\left(Rs^w_t y^w_t + Rs^m_t y^m_t\right)$, where y^m_t and s^m_t are her husband's first period endowment and savings rate, respectively. κ ($\frac{1}{2}$ =< κ =< 1) represents the notion that consumption within a marriage is a public good with congestion. As an example, if two spouses buy a car, both can use it. In contrast, were they single, they would have to buy two cars. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. When $\kappa = 1$, then all the consumption is a public good with no congestion[2].

The optimal savings rate is chosen to maximize the following objective function:

$$V_t^w = \max_{s_t^w} u(C_{1t}^w) + \beta E_t \left[u(C_{2,t+1}^w) + \eta^m \right]$$

subject to the budget constraints that

$$P_t C_{1t}^w = (1 - s_t^w) y_t^w \tag{3.1}$$

$$P_{t+1} C_{2,t+1}^w = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right) & \text{if married} \\ R s_t^w y_t^w & \text{otherwise} \end{cases} \tag{3.2}$$

where E_t is the conditional expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Bhaskar (2011) also introduces a similar "love" variable.

A Representative Man's Optimization Problem

A representative man's problem is symmetric to a women's problem. In particular, if he fails to get married, his second period consumption is $P_{t+1} C_{2,t+1}^{m,n} = R s_t^m y^m$. If he is married, his second period consumption is $P_{t+1} C_{2,t+1}^m = \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right)$. He will choose his savings rate to maximize the following value function

$$V_t^m = \max_{s_t^m} u(C_{1t}^m) + \beta E_t \left[u(C_{2,t+1}^m) + \eta^w \right]$$

subject to the budget constraints that

$$P_t C_{1t}^m = (1 - s_t^m) y_t^m \tag{3.3}$$

$$P_{t+1} C_{2,t+1}^m = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right) & \text{if married} \\ R s_t^m y_t^m & \text{otherwise} \end{cases} \tag{3.4}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w.

The Marriage Market[3]

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combi-

[2] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of both the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

[3] We use the word "market" informally here. The pairing of husbands and wives is not done through prices.

nation of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" she/he can obtain from her/his spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. To ensure that the preference is strict for both men and women, whenever there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same. Note that "love" is not in the eyes of a beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not have rejected him. Each available women in round k holds the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made, and the women accept the most attractive proposals.[4]

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men respectively. We normalize I^w and let $I^w = (0, 1)$. Since the sex ratio is ϕ, the set of men $I^m = (0, \phi)$. Men and women are ordered in such a way that a higher value in the set means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market, $\pi^w : I^w \to I^m$. That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the savings rates and emotional utility draws. This implies a mapping from a combination (s_i^w, η_i^w) to another combination (s_j^m, η_j^m). Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband (s_j^m, η_j^m) is also a random variable. Woman i's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\, i | s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right)}, \quad u\left(R s_i^w y_i^w\right) \; \right] dF^w\left(\eta_i^w\right) \\ = & \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\, i | s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right)} \right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(R s_i^w y_i^w\right) dF^w\left(\eta_i^w\right) \end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

[4] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0, \phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The measure for women in the marriage market remains one. In equilibrium, there exists an unique mapping for men in the marriage market: $\pi^m : I^m \to \widetilde{I}^w$, where π^m maps man j ($j \in I^m$) to woman i ($i \in I^w$). Those men with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = Rs^m_j y^m_j$. In general, man j's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\,j|s^m_j, \eta^m_j, s^m_{-j}, \eta^m_{-j}, s^w, \eta^w\right)}, \quad u\left(Rs^m_j y^m_j\right) \; \right] dF^m\left(\eta^m_j\right) \\ = & \int_{\bar{\pi}^m_j} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\,j|s^m_j, \eta^m_j, s^m_{-j}, \eta^m_{-j}, s^w, \eta^w\right)} \right] dF^m\left(\eta^m_j\right) + \int^{\bar{\pi}^m_j} u\left(Rs^m_j y^m_j\right) dF^m\left(\eta^m_j\right) \end{aligned}$$

where $\bar{\pi}^m_j$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi^m_j < \bar{\pi}^m_j$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a *positive assortative matching* emerges for those men and women who are married. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= \left(F^m\right)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. Furthermore, the lowest possible value of the emotional utility $\eta^{\min}$ is sufficiently large that everyone desires to be married. We also assume that there exists a small and exogenous possibility p that a woman may not find a marriage partner due to frictions in the marriage market. The last assumption plays no role in the analytical part of the model but will help the quantitative calibrations later. In equilibrium, given all her rivals' saving decisions and η^w, woman i's second period utility is

$$(1-p)\left[u\left(\frac{\kappa\left(Rs^w_t(i)y^w_t + Rs^m_t y^m_t\right)}{P_{t+1}}\right) + \int_{\eta^{\min}}^{\eta^{\max}} M(\tilde{\eta}^w_i)d\tilde{F}(\tilde{\eta}^w_i)\right] + pu\left(\frac{Rs^w_t y^w_t}{P_t}\right)$$

where $\tilde{\eta}^w_i = u\left(\frac{\kappa(Rs^w_t(i)y^w_t + Rs^m_t y^m_t)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w_t y^w_t + Rs^m_t y^m_t)}{P_{t+1}}\right) + \eta^w$.

Due to symmetry, we drop the sub-index i. A representative woman's first order condition, given men's savings decisions, is

$$-u'_{1w}\frac{y^w_t}{P_t} + \beta(1-p)\left[u'_{2w}\frac{\partial C^w_{2,t+1}}{\partial s^w_t} + \frac{\partial \int M\left(\tilde{\eta}^w\right)d\tilde{F}^w(\tilde{\eta}^w)}{\partial s^w_t}\right] + pu'_{2w,n}\frac{y^w_t}{P_{t+1}} = 0 \qquad (3.5)$$

where
$$\frac{\partial \int M\left(\tilde{\eta}^{w}\right) d\tilde{F}(\tilde{\eta}^{w})}{\partial s_t^w} = \kappa u_{2w}' R \frac{y_t^w}{P_{t+1}} \left[\int M'(\eta^w) dF(\eta^w) + M(\eta^{\min}) f(\eta^{\min}) \right]$$

Similarly, a representative man's second-period utility is

$$(1-p) \left[\tilde{\delta}_j^m u \left(\frac{\kappa \left(R s_t^w y_t^w + R s_t^m(j) y_t^m \right)}{P_{t+1}} \right) + \int_{M(\eta^{\min})}^{\eta^{\max}} M^{-1}\left(\tilde{\eta}_j^m\right) d\tilde{F}(\tilde{\eta}_j^m) \right] + \left[(1-p)(1-\tilde{\delta}_j^m) + p \right] u \left(\frac{R s_t^m(j) y_t^m}{P_{t+1}} \right)$$

where $\tilde{\eta}_j^m = u\left(\frac{\kappa(Rs_t^w y_t^w + Rs_t^m(j) y_t^m)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs_t^w y_t^w + Rs_t^m y_t^m)}{P_{t+1}}\right) + \eta_j^m$ and $\tilde{\delta}_j^m$ is the probability he gets married

$$\begin{aligned} \tilde{\delta}_j^m &= \Pr\left(u(C_{2,t+1}^w(j)) - u(C_{2,t+1}^w) + \eta_j^m \geq M(\eta^{\min}) \middle| R s^w y^w, R s^m y^m \right) \\ &= 1 - F\left(M(\eta^{\min}) - u(C_{2,t+1}^w(j)) + u(C_{2,t+1}^w) \right) \end{aligned} \tag{3.6}$$

His first order condition is

$$-u_{1m}' \frac{y_t^m}{P_t} + \beta(1-p) \left[\begin{array}{c} \delta^m u_{2m}' \frac{\partial C_{2,t+1}^m}{\partial s_t^m} + \int_{M(\eta^{\min})} \frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s_t^m} d\tilde{F}^m(\tilde{\eta}^m) \\ + f\left(M(\eta^{\min})\right) u_{2w}' \frac{\partial C_{2,t+1}^m}{\partial s_t^m} \left(u_{2m} - u_{2m,n} + \eta^{\min} \right) \end{array} \right] + \left[(1-\delta^m)(1-p) + p \right] u_{2m,n}' \frac{y_t^m}{P_{t+1}} = 0 \tag{3.7}$$

where
$$\int_{M(\eta^{\min})} \frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s^m} d\tilde{F}^m(\tilde{\eta}^m) = \kappa u_{2w}' R \frac{y_t^m}{P_{t+1}} \int_{M(\eta^{\min})} \left(M^{-1}\right)'(\eta^m) dF(\eta^m)$$

For simplicity, we assume that women and men will earn the same first period labor income and that there is no tax, i.e., $y_t^w = y_t^m = w_t$. We now define an equilibrium in this economy.

Definition 2 *An equilibrium is a set of savings rates, capital and labor allocation by sector, and the relative price of nontradable good* $\{s_t^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings rates by the representative woman and the representative man, conditional on other women and men's savings rates, $s_t^w = \left\{s_{it}^w, s_{-i,t}^w\right\}$ *and* $s_t^m = \left\{s_{jt}^w, s_{-j,t}^m\right\}$*, maximize their respective utilities*

$$\begin{aligned} s_{it}^w &= \arg\max\left\{ V_t^w \middle| s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \right\} \\ s_{jt}^w &= \arg\max\left\{ V_t^m \middle| s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \right\} \end{aligned}$$

(ii) The markets for capital, labor, and tradable and nontradable goods clear, and firms maximize their profits. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).*

Shocks to the sex ratio We now consider an unanticipated shock to the young cohort's sex ratio, i.e., the sex ratio rises from one to $\phi(>1)$ from period t onwards. The nature of the shock is motivated by the facts about the sex ratio imbalance in China. Since a severe sex ratio imbalance for the pre-marital age cohort is a relatively recent phenomenon, the older generations' savings decisions were largely made when there was no severe sex ratio imbalance. As the shock is unanticipated, (2.2) and (2.4) do not hold in period t.

As in the benchmark model, the market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}w_{t-1} + (1-s_t)w_t\right) \tag{3.8}$$

where $s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ is the aggregate savings rate by the young cohort in period t.

By (2.3) and (2.5), we have

$$w_t = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{3.9}$$

We can solve (3.5), (3.7), (3.8) and (3.9) to obtain the equilibrium in period t. If the utility function is of log form, $u(C) = \ln C$, for all men and women, and η is drawn from a uniform distribution, we have the following proposition.

Proposition 2 *As the sex ratio in the young cohort rises, a representative man weakly increases his savings rate while a representative woman weakly reduces her savings rate. However, the economy-wide savings rate increases unambiguously. The real exchange rate depreciates and the current account rises.*

Proof. See Appendix B. ■

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio because the need to compete in the marriage market becomes greater. Why does the representative woman reduce her savings rate? Because she anticipates a higher savings rate from her future husband, she does not need to sacrifice her first-period consumption as much as she otherwise would have to.

Second, why does the aggregate savings rate rise in response to a rise in the sex ratio? In other words, why is the increment in men's savings greater than the decline in women's savings? Intuitively, a representative man raises his savings rate for two reasons: in addition to improving his relative standing in the marriage market, he raises his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. In addition, since men save more, the rising share of men in the population would also raise the aggregate savings rate. While both channels contribute to a rise in the aggregate savings rate, it is easy to verify that the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different saving propensities).

Third, once we obtain an increase in the aggregate savings rate, the logic from the previous one-gender benchmark model applies. In particular, the relative price of the non-tradable good declines (and hence the real exchange rate depreciates), and the current account rises.

Similar to the benchmark model with a single gender, once the shock is observed and taken into account in period $t+1$, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

This means that the real exchange rate and the current account will return to the previous values after one period.

3.2 Capital adjustment costs

Without additional frictions, a shock to the sex ratio can only affect the real exchange rate for one period. If there are capital adjustment costs in each sector, the effect on the real exchange rate can be prolonged. We assume that the capital accumulation in each sector is as following:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where δ is the depreciation rate and I_t is investment. $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the adjustment cost as in Chari, Kehoe and McGrattan (2002).

Then (2.2) and (2.4) become, respectively,

$$\begin{aligned} R &= 1-\delta + \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt}\left(\frac{L_{Tt}}{K_{Tt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Tt}}{K_{Tt}} - \delta\right) - \frac{b}{2}\left(\left(\frac{I_{Tt}}{K_{Tt}}\right)^2 - \delta^2\right) \qquad (3.10) \\ R &= 1-\delta + \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Nt}}{K_{Nt}} - \delta\right) - \frac{b}{2}\left(\left(\frac{I_{Nt}}{K_{Nt}}\right)^2 - \delta^2\right) \qquad (3.11) \end{aligned}$$

Without capital adjustment cost, i.e., $b = 0$, the price of the nontradable good will go back to its equilibrium level in period $t+1$. If $b>0$, then

$$P_{Nt} = \frac{\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt+1}\left(\frac{L_{Tt+1}}{K_{Tt+1}}\right)^{1-\alpha_T} - bR\left(\frac{I_{Tt+1}}{K_{Tt+1}} - \frac{I_{Nt+1}}{K_{Nt+1}}\right) - \frac{b}{2}\left(\left(\frac{I_{Tt+1}}{K_{Tt+1}}\right)^2 - \left(\frac{I_{Nt+1}}{K_{Nt+1}}\right)^2\right)}{\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\alpha_N A_{Nt+1}\left(\frac{L_{Nt+1}}{K_{Nt+1}}\right)^{1-\alpha_T}}$$

P_{Nt} is now a function of $\frac{I_{Tt+1}}{K_{Tt+1}}$ and $\frac{I_{Nt+1}}{K_{Nt+1}}$. If $\frac{I_{Tt+1}}{K_{Tt+1}} \neq \frac{I_{Nt+1}}{K_{Nt+1}}$, P_{Nt} is not a constant. This means that, with capital adjustment costs, the price of the nontradable good does not return immediately to its long-run equilibrium level. As a result, a rise in the sex ratio can have a long-lasting and depressing effect on the real exchange rate.

3.3 Two large countries

We now turn to a world with two large countries: Home and Foreign. Assume that they are identical in every respect except for their sex ratios. Specifically, in period t, the sex ratio of the young cohort in Home rises from one to ϕ ($\phi > 1$), while Foreign always has a balanced sex ratio. Households in each country consume a tradable good and a nontradable good.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \text{ and } C_t^* = \frac{(C_{Nt}^*)^{\gamma}(C_{Tt}^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where C_t and C_t^* represent home and foreign consumption indexes, respectively. Since we choose the tradable good as the numeraire, the consumer price index is $P_t = P_{Nt}^{\gamma}$, where P_{Nt} is the price of the home produced nontradable good. Similarly, the consumer price index in Foreign is $P_t^* = (P_{Nt}^*)^{\gamma}$.

The rise in Home's sex ratio in period t is assumed to be unanticipated. As a result, (2.2) and (2.4) fail in both Home and Foreign. By the same reasoning, Home experiences a real exchange rate depreciation in period t, but a real appreciation in period $t+1$. We can write the current account in Home and Foreign as follows:

$$CA_t = s_t w_t - s_{t-1}w_{t-1} + K_t - K_{t+1} \text{ and } CA_t^* = s_t^* w_t^* - s_{t-1}^* w_{t-1}^* + K_t^* - K_{t+1}^*$$

Before the shock, we had

$$s_{t-1} = s_{t-1}^*,\ w_{t-1} = w_{t-1}^* \text{ and } K_t = K_t^*$$

In period $t+1$, we have

$$P_{Nt} = P_{Nt},\ w_{t+1} = w_{t+1}^*, \text{ and } P_{t+1} = P_{t+1}^*$$

and the demand for the nontradable good is

$$Q_{N,t+1} = \frac{\gamma w_{t+1} \left((R-1) s_t + 1 \right)}{P_{Nt}} \text{ and } Q^*_{N,t+1} = \frac{\gamma w^*_{t+1} \left((R-1) s^*_t + 1 \right)}{P_{Nt}}$$

Since Home now has a higher sex ratio than Foreign, we have $s_t > s_t^*$, and therefore

$$Q_{N,t+1} > Q^*_{N,t+1}$$

We assume that the nontradable sector is more labor-intensive, i.e., $\alpha_N < \alpha_T$. Given the same technologies and the same labor endowments in the two countries, we have

$$K_{t+1} < K^*_{t+1}$$

In period t, since nothing changes in Foreign, it must be the case that $s_t^* w_t^* = s_{t-1} w_{t-1}$. Following the same steps as in the case of a small open economy, we can show that $s_t w_t > s_{t-1} w_{t-1} = s_t^* w_t^*$. Then it is easy to show that $CA_t > 0 > CA_t^*$. In other words, Home exhibits a current account surplus while Foreign experiences a current account deficit.

3.4 Endogenous labor supply

We turn to the case of endogenous labor supply. Just as a male raises his savings rate to gain a competitive advantage in the marriage market, he may choose to increase his supply of labor for the same reason in response to a rise in the sex ratio. This can translate into an increase in the effective aggregate labor supply if women do not decrease their labor supply too much. If the production of the nontradable good is more labor-intensive, the increase in the effective labor supply can reduce the relative price of the non-tradable good (and the value of the real exchange rate). Therefore, endogenous labor supply could reinforce the savings channel from the sex ratio shock, leading to an additional reduction in the real exchange rate.

We allow each person to endogenously choose the first period labor supply and the utility function of the first period is $u(C) + v(1-L)$, where L is the labor supply and $v(1-L)$ is the utility function of leisure. As in the standard literature, we assume that $v' > 0$ and $v'' < 0$. Again, for simplicity, we assume no tax on the labor income. The utility function governing the leisure-labor choice is the same for men and women. In other words, by assumption, men and women are intrinsically symmetric except for their ratio in the society.

We can rewrite the optimization problem for a representative woman as following:

$$\max \; u(C^w_{1t}) + v(1 - L^w_t) + \beta E_t \left[u(C^w_{2,t+1}) + \eta^m \right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^w &= (1-s_t^w) w_t L_t^w \\ P_{t+1} C_{2,t+1}^w &= \begin{cases} \kappa \left(R s_t^w L_t^w + R s_t^m L_t^m\right) w_t & \text{if married} \\ R s_t^w w_t L_t^w & \text{otherwise} \end{cases} \end{aligned}$$

The first order condition with respect to her labor supply is

$$u_{1w}' \frac{(1-s_t^w) w_t}{P_t} + \left[u_{2w}' \frac{\partial C_{2,t+1}^w}{\partial L_t^w} + \frac{\partial \int M(\tilde{\eta}^w)\, d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} \right] + \beta p u_{2w}' \frac{\kappa R s_t^w w_t}{P_{t+1}} - v_w' = 0$$

Notice that $\frac{\partial C_{2,t+1}^w}{\partial L_t^w} = \frac{\partial C_{2,t+1}^w}{\partial s_t^w} \frac{s_t^w}{L_t^w}$ and $\frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} = \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial s_t^w} \frac{s_t^w}{L_t^w}$. Combining the equation above with (3.5), we have

$$\frac{w_t}{P_t} = \frac{v_w'}{u_{1w}'} \tag{3.12}$$

The optimization problem for a representative man is similar:

$$\max \; u(C_{1t}^m) + v(1 - L_t^m) + \beta E_t \left[u(C_{2,t+1}^m) + \eta^w \right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^m &= (1-s_t^m) w_t L_t^m \\ P_{t+1} C_{2,t+1}^m &= \begin{cases} \kappa \left(R s_t^m L_t^m + R s_t^m L_t^m\right) w_t & \text{if married} \\ R s_t^m w_t L_t^m & \text{otherwise} \end{cases} \end{aligned}$$

The optimization condition for his labor supply is

$$\frac{w_t}{P_t} = \frac{v_m'}{u_{1m}'} \tag{3.13}$$

On the supply side, all equilibrium conditions remain the same except for the labor market clearing condition, which now becomes

$$L_{Tt} + L_{Nt} = \frac{1}{1+\phi} L_t^w + \frac{\phi}{1+\phi} L_t^m \tag{3.14}$$

We now define an equilibrium for such an economy.

Definition 3 *An equilibrium is a set* $\{(s_t^w, L_t^w), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings and labor supply decisions by women and men, $(s_t^w, L_t^w) = \left\{ s_{it}^w, s_{-i,t}^w, L_{it}^w, L_{-i,t}^w \right\}$

and $(s_t^m, L_t^m) = \left\{s_{it}^m, s_{-i,t}^m, L_{it}^m, L_{-i,t}^m\right\}$*, maximize their utilities, respectively,*

$$\begin{aligned}
\left(s_{it}^w, L_{it}^w\right) &= \arg\max\left\{V_t^w | \left(s_{-i,t}^w, L_{-i,t}^w\right), \left(s_t^m, L_t^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \\
\left(s_{jt}^m, L_{jt}^m\right) &= \arg\max\left\{V_t^m | \left(s_t^w, L_t^w\right), \left(s_{-j,t}^m, L_{-j,t}^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\}
\end{aligned}$$

(ii) The markets for both goods and factors clear, and firms' profits are maximized. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (3.14).*

As before, we assume that $u(C) = \ln C$. We let L_t denote the aggregate labor supply in period t, and assume that $\frac{v'' L}{v'}$ is non-decreasing in L.

Proposition 3 *As the sex ratio (in the young cohort) rises, a representative man weakly increases both his labor supply and his savings rate, while a representative woman weakly reduces both her labor supply and her savings rate. However, the economy-wide labor supply and savings rate both increase unambiguously. The real exchange rate depreciates, and the current account rises.*

Proof. See Appendix C. ■

In response to a rise in the sex ratio, for the same reason that men may cut their consumption and increase their savings rate, they may cut down their leisure and increase their labor supply. Similarly, for women, for the same reason that induce them to reduce their savings, they may reduce their labor supply (and increase leisure). In the aggregate, for the same reason that the increase in savings by men is more than enough to offset the decrease in savings by women, the increase in labor supply by men is also larger than the decrease in labor supply by women. Therefore, the aggregate labor supply rises in response to a rise in the sex ratio.

With a fixed labor supply, it is worth remembering that the nontradable sector shrinks after a rise in the sex ratio. The reason is that a decline in the relative price of the nontradable goods (due to the savings channel) makes it less attractive for labor and capital to stay in the nontradable sector. Now, with an endogenous labor supply, the total effective labor supply increases after a rise in the sex ratio according to Proposition 3. By a logic similar to the Rybzinksy theorem, this by itself has a tendency to induce an expansion of the nontradable sector if the production of the nontradable good is more labor intensive. Relative to the case of a fixed labor supply, adding the effect of endogenous labor supply leads to either an expansion of the nontradable sector, or at least a smaller reduction in the size of the nontradable sector. The exact scenario depends on parameter values. However, regardless of what happens to the size of the nontradable sector, the price of the nontradable good (and the value of the real exchange rate) must fall by a greater amount when the endogenous labor supply effect is added to the savings effect.

4 Calibrations

We start from a simple OLG model in which every cohort lives two periods and there are no capital adjustment costs. We then add some more realisms by (1) assuming a 50-period life and (2) introducing capital adjustment costs.

4.1 Parameters

We take the annual interest rate $R = 1.04$ and $\beta = R^{-1}$. We assume the tradable sector has a higher capital intensity, $\alpha_T = 0.6$ and the nontradable sector has a lower capital intensity $\alpha_N = 0.3$. The share of the nontradable good consumption in the aggregate consumption is set to be 0.7, $\gamma = 0.7$. Within a family, the congestion for family consumption, $\kappa = 0.8$.

The emotional utility η needs to follow a continuously differential distribution. We assume a truncated normal distribution which might be more realistic than the uniform distribution used in the analytical model. We choose a standard deviation that is relatively tight, $\sigma = 0.01$. This limits the extent of heterogeneity among women (or men) in the eye of the opposite sex. We truncate the distribution at the 1% in the left tail and at the 99% in the right tail. To choose the mean value of the emotional utility, we perform the following thought experiment. Holding all other factors constant, we can compute the income compensation needed to a life-time bachelor that can makes him indifferent between being married and being single.

$$u\left(\frac{1}{1+\beta}(1+x)y\right) = u\left(\frac{1}{1+\beta}y\right) + E\left(\eta\right)$$

where xy is the compensation paid to a life-time bachelor for being single and $\frac{1}{1+\beta}(1+x)y$ is his second period consumption. We calculate the value of x based on Blanchflower and Oswald (2004). Regressing self-reported well-being scores on income, marriage status, and other determinants, they estimate that a lasting marriage is, on average, worth $100,000 (in 1990 dollars) per year in the United States (compared to being widowed or separated) during 1972-1998. Since GDP per person employed is about $48,000 during the same period, this implies that a marriage is worth more than twice the average income for employed people in the U.S. We take the ratio $x = 2$ as the benchmark and then the mean value of the emotional utility/love is:

$$E\left(\eta\right) = u\left(\frac{3y}{1+\beta}\right) - u\left(\frac{y}{1+\beta}\right)$$

As a robustness check, we will also consider $x = 0.5$.

Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986), discount factor takes value 0.96 based on annual frequency. We take 20 years as one period, then $\beta = 0.96^{20} \simeq 0.45$
Share of nontradable good in the consumption basket	$\gamma = 0.7$	Burstein et al (2001)
Nontradable sector capital-intensity	$\alpha_N = 0.3$	Burstein et al (2001)
Tradable sector capital-intensity	$\alpha_T = 0.6$	Burstein et al (2001)
Share of capital input	$\alpha = 0.35$	Bernanke, Gertler and Gilchrist (1999)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Marriage market friction[5]	$p = 0.02$	$p = 0.05$ in the robustness checks
Love, standard deviation	$\sigma = 0.01$	$\sigma = 0.05$ in the robustness checks
Love, mean	$x = 2$	$x = 0.5$ in the robustness checks

4.2 Results for the 2-period OLG model

In Figure 1, we set parameter κ equal to 0.8. The benchmark case sets $x = 2$, $\sigma = 0.01$, and $p = 0.02$. With an unbalanced sex ratio ($\phi>1$), the real exchange rate depreciates. As the sex ratio rises from 1 to 1.5, the extent of real exchange rate depreciation increases from 0% to about 8%. At the same time, the economy-wide savings rate rises from 12% to 20%, and the current account surplus rises from 0% to 9% of GDP. As the first set of robustness checks, we experiment with different combinations of m=0.5 or 2, $\sigma = 0.05$ or 0.01, and $p = 0.02$ or 0.05. The results are also reported in Figure 1, and generally do not deviate from the benchmark very much.

We also set κ to be 0.7 or 0.9, respectively, and experiment with different combinations of other parameters. The results are reported in Figures 2 and 3. Generally speaking, the real exchange rate always depreciates more with a higher sex ratio. Both the savings rate and the current account (as a share of GDP) rise in response to a rise in the sex ratio.

We now consider endogenous labor supply in Figure 4. With $\kappa = 0.8$, $x = 2$, $\sigma = 0.01$, and $p = 0.02$, we obtain a much stronger exchange rate depreciation. As the sex ratio rises from 1 to 1.5, the extent of the real exchange rate depreciation also rises from 0% to about 35%. The aggregate savings rate rises from 12% to 24%, while the current account surplus rises from 0% first to close to 6% of GDP and reverse slightly to 4% of GDP. Robustness checks with other combinations of the parameters are reported in Figures 5 and 6. The results are broadly in line with the benchmark calibration. In particular, with an endogenous labor supply, a given rise in the sex ratio leads to a greater response in the real exchange rate.

[5] p is the exogenous possibility that any individual (a women or a man) entering the marriage market is bumped off the market independent of the sex ratio.

While the aggregate savings rate always rises with the sex ratio, the modest non-monotonic picture of the current account response deserves some comments. With a fixed labor supply, a rise in the sex ratio leads to an expansion of tradable good production but a contraction of nontradable good production. This leads to very little change in the aggregate (domestic) investment rate. As a result, a higher sex ratio leads to a higher savings rate, which produces an increase in the current account balance. In contrast, with an endogenous labor supply, a higher sex ratio leads to an increase in the effective labor supply. Both the tradable good and the nontradable good sectors could expand (or at least the non-tradable sector shrinks by a smaller amount than in the case of a fixed labor supply), which leads to an increase in aggregate domestic investment. As Figures 4-6 show, for the initial rise in the sex ratio (from 1 to 1.15), the current account surplus increases monotonically, indicating that the increase in the aggregate savings rate outpaces the increase in the aggregate investment rate. After that point, any additional increase in the sex ratio leads to a smaller current account surplus, indicating that the incremental savings rate is smaller than the incremental investment rate. Since virtually all economies in the real world have sex ratios (for the pre-marital age cohort) less than 1.15, we do not expect to see the turning point in the current account in the data.

4.3 An OLG model in which a cohort lives 50 periods

We now extend our benchmark model by assuming that every cohort lives 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage take place in the τth period. While differences in the savings rates by parents with a son versus parents with a daughter are an important feature of the data (Wei and Zhang, 2009), we are not able to solve the problem that features simultaneously parental savings for children and a nontradable sector. [See Du and Wei (2010) for a three-period model with parental savings for their child but without a nontradable sector.] Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data, we choose $\tau = 20$ as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married and the typical number of working years by a young person when he/she gets married. Generally speaking, the greater the value of τ, the stronger is the aggregate savings response to a given rise in the sex ratio.

A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + \eta^m \right) \right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R \left(A_t + y_t^w - P_t c_t^w \right)$$

where A_t is the wealth held by the woman at the beginning of period t. $y_t^w = w_t L_t^w$ is her labor income at the age t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + w_t L_t^w - P_t c_t\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^w\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held by wife and husband) at the beginning of period t. c_t is the public good consumption by wife and husband, which takes the same form as in the two period OLG model. The optimization problem for a representative man is similar. To simplify the calculation and generate interesting results, we assume that there is a lower bound of labor supply $\bar{L}$, $L_t^i \geq \bar{L}$ ($i = w, m$).

As before, we take $R = 1.04$ as the annual gross interest rate. The subjective discount factor now takes the value of $\beta = 1/R$. We assume capital accumulation evolves in the following way:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the quadratic capital adjustment cost. Following Chari, Kehoe and McGrattan (2002), we assume $\delta = 0.1$ and $b = 2.72$.

For the nature of the sex ratio shock, we use demographic changes in China over the last two decades as a guide. As the data exhibits a steady increase in the sex ratio in the pre-marital age cohort since 2002, we let the sex ratio at birth in the model rise continuously and smoothly until it reaches 1.2 in period 20. The sex ratio then stays at that level in all subsequent periods.

The calibration results are shown in Figures 7. As the sex ratio rises from 1 in period 0 to 1.2 in period 20, the real exchange rate depreciates by more than 10 percent. The economy-wide savings rate and the current account rise by more than 9 percent of GDP. As a robustness check, if capital adjusts more slowly, i.e., with a higher cost of capital adjustment, the real exchange rate depreciates by about 11 percent. The converse is true when the adjustment cost is lower.

5 Some suggestive empirics

Since the sex ratio effect is novel, it is useful to present and discuss some empirical evidence. We recall first the evidence in Wei and Zhang (2009) that a higher sex ratio has led to a rise in the household savings rate in China. Chinese households with a son in both rural and urban areas tend to save more in regions with a more skewed sex ratio. The savings rate by urban households with a daughter also tend to rise with the local sex ratio, although the savings rate by rural households with a daughter appears to be insensitive to the local sex ratio. The savings behavior by daughter-households is consistent with the notion that intra-household bargaining is sufficiently important that they do not cut down savings rate in response to a higher sex ratio (The model of Du and Wei,, 2010, formalizes this intuition). Using regional variations in the eneforcement of the family planning policy as instruments for the local sex ratio, Wei and Zhang (2009) suggest that the positive correlation reflects a causal effect from a higher sex ratio to a higher savings rate. Based on the IV regressions, they estimate that the rise in the sex ratio may explain about half of the observed rise in the household savings rate in the last two decades.

Some evidence that a higher sex ratio has increased effective labor supply is provided in Wei and Zhang (2011). In particular, the number of days a rural migrant worker chooses to work away from home tends to rise with the local sex ratio, especially if the migrant worker has a son at home. Similarly, migrant workers with a son from a region with a more skewed sex ratio are also more willing to work in a job that are more dangerous and less pleasant, such as in minining or construction, or with exposure to extreme heat, cold or hazardous material, presumably for a better wage.

We now provide some suggestive cross-country evidence on how the sex ratio imbalance may affect the real exchange rate. We first run regressions based on the following specification:

$$\ln RER_i = \alpha + \beta \cdot \text{sex ratio} + \gamma \cdot Z + \varepsilon_i$$

where RER_i is the real exchange rate for country i. Z is the set of control variables. We consider a sequentially expanding list of control variables including log GDP per capita, financial development index, government fiscal deficit, dependence ratio, and *de facto* exchange rate regime classifications.

The data for the real exchange rate and real GDP per capita are obtained from Penn World Table 6.3. The "price level of GDP" in the Penn World Table is equivalent to the inverse of the real exchange rate in the model: A higher value of the "price level of GDP" means a lower value of the real exchange rate. The sex ratio data is obtained from the World Factbook. As we are not able to find the sex ratio for the age cohort 10-25 for a large number of countries, we use age group 0-15 instead to maximize the country coverage.

We use two proxies for financial development. The first is the ratio of private credit to GDP, from the World Bank's WDI dataset. This is perhaps the most commonly used proxy in the standard literature. There is a clear outlier with this proxy: China has a very high level of bank credit, exceeding 100% of GDP. However, 80% of the bank loans go to state-owned firms, which are potentially less efficient than private firms (see Allen, Qian, and Qian, 2004). To deal with this problem, we modify the index by multiplying the credit to GDP ratio for China by 0.2. Because this measure is far from being perfect, we also use a second measure, which is the level of financial system sophistication as perceived by a survey of business executives reported in the Global Competitiveness Report (GCR).

For exchange rate regimes, we use two de facto classifications. The first comes from Reinhart and Rogoff (2004), who classify all regimes into four groups: peg, crawling peg, managed floating and free floating. The second classification comes from Levy-Yeyati and Sturzenegger (2005), who use three groups: fix, intermediate and free float.

For the dependent variable, logRER, and most regressors where appropriate, we use their average values over the period 2004-2008. The averaging process is meant to smooth out business cycle fluctuations and other noises. The period 2004-2008 is chosen because it is relatively recent, and the data are available for a large number of countries. (We have also examined a single year, 2006, and obtained similar results).

Table 1 provides summary statistics for the key variables. The log RER ranges from -2.22 to 0.41 in the sample, with a mean of -0.74 and a standard deviation of 0.59. The value of log RER for China indicates a substantial undervaluation on the order of 45% when compared to the simple criterion of purchasing power parity.

For the sex ratio for the age cohort 0-15, both the mean and the median across countries are 1.04, and the standard deviation is 0.02. For this age cohort, all countries in the sample have a sex ratio that is at least 1. The sex ratio for most of the countries is between 1 and 1.07. The following economies have a sex ratio that is 1.07 or higher: China (1.13), Macao (1.11), Korea (1.11), Singapore (1.09), Switzerland (1.08), Hong Kong (1.08), Vietnam (1.08), Jordan (1.07), Portugal (1.07) and India (1.07). They represent the most skewed sex ratios in the sample. China, by far, has the most unbalanced sex ratio in the world. If the same sex ratio persists into the marriage market, then at least one out of every eight young men cannot get married. As wives are typically a few years younger than their husbands, the actual probability of not being able to marry is likely to be modestly better in a country with a growing population (for which later cohorts are slightly larger). Nonetheless, the relative tightness of the marriage market for men across countries should still be highly correlated with this sex ratio measure. In addition, unlike most other countries, China exhibits a progressively smaller age cohort over time as a result of its strict family planning policy. As a result, the relative tightness of the marriage market for Chinese men when compared to their counterparts in other countries is likely to be worse than what is represented by this sex ratio. Furthermore, the Chinese sex ratios at birth in 1990 and 2005 are estimated to be 1.15 and 1.20, respectively (see Wei and Zhang, 2009). This implies that the sex ratio for the pre-marital age cohort will likely worsen in the foreseeable future.

We present a series of regressions in Table 2. The first column shows that the real exchange rate tends to be lower in poorer countries. This is commonly interpreted as confirmation of the Balassa-Samuelson effect. In Column 2, we add a proxy for financial development by the ratio of private sector credit to GDP. The positive coefficient on the new regressor indicates that countries with a poorer financial system tend to have a lower RER. In Column 3, we add the sex ratio. The coefficient on the sex ratio is negative and statistically significant, indicating that countries with a higher sex ratio tend to have a lower RER. Since oil exporting countries have a current income that is likely to be substantially higher than their permanent income (until they run out of the oil reserve), their current account and RER patterns may be different from other economies. In Column 4, we exclude major oil exporters and re-do the regression. This turns out to have little effect on the reult. In particuar, countries with a higher sex ratio continue to exhibit a lower RER.

In Column 5 of Table 2, we add several additional control variables: government fiscal deficit, terms of trade, capital account openness, and dependency ratio. Due to missing values for some of these variables, the sample size is dramatically smaller (a decline from 123 in Column 4 to 75 Column 5). Of these variables, the dependence ratio is the only significant variable. The positive coefficient on the dependence ratio (0.0093) means that countries with a low dependency ratio (fewer children and retirees as a share of the population) tend to have a low RER. By the logic of the life-cycle hypothesis, a lower dependency ratio produces a higher savings rate. By the model in Section 2, this could lead to a reduction in the value of the real exchange rate. It is noteworthy, however, even with these additional controls and in a smaller sample, the sex ratio effect is still statistically significant, although its point estimate is slightly smaller.

In Column 6 of Table 2, we take into account exchange rate regimes using the Reinhart-Rogoff (2004) de facto regime classifications. Relative to the countries on a fixed exchange rate regime (the left out group), those on a crawling peg appear to have a lower RER. Countries on other currency regimes do not appear to have a systematically different RER. With these controls, the negative effect of the sex ratio on the RER is still robust. In Column 7, we measure exchange rate regimes by the de facto classifications proposed by Levy Yeyati and Sturzenneger (2003). It turns out this does not affect the relationship between the sex ratio and the real exchange rate.

In Table 3, we re-do the regressions in Table 2 except that we now measure a country's financial development by the financial system system sophistication index from the Global Competitiveness Report. The results are broadly similar to Table 2. In particular, the coefficients on the sex ratio are negative in all five cases, and are significant in four of the five cases. The sex ratio coefficient is (marginally) not significant in Column 6 of Table 3, where the Reinhart-Rogoff exchange rate classifications are used as controls. We note, however, that this regression also has far fewer observations (35 only), which also reduces the power of the test. In any case, when the LYS exchange rate classifications are used instead (reported in Column 7), the sex ratio coefficient becomes significant again.

In Tables 4 and 5, we examine the relationship between the sex ratio and the (private-sector) current account. Because our theory does not discuss government savings behavior, we choose to define the dependent variable as a country's current account account (as a share of GDP) minus the government savings (as a share of GDP). Otherwise, the regression specifications are similar to those in Tables 2 and 3. The sex ratio has a positive coefficient which is statistically significant in almost all cases except when the sample size becomes very small.

In sum, we find that the sex ratio has a significant impact on the real exchange rate and current account in a way consistent with our theory: as the sex ratio rises, a country tends to have a real exchange rate depreciation and a current account surplus. (An important caveat is that we do not have a clever idea to instrument for the sex ratio in the cross country context; future research will have to investigate the causality more thoroughly.)

To be clear, as the sex ratio imbalance is a severe problem only in a subset of countries, it is not a key fundamental for the real exchange rate in most countries. Nonetheless, for those countries with a severe sex ratio imbalance, including China, one might not have an accurate view on the equilibrium exchange rate unless one takes it into account. To illustrate the quantitative significance of the empirical relations, we compute the extent of the Chinese real exchange rate undervaluation (or the value of the RER relative to what can be predicted based on the fundamentals) by taking the point estimates in Columns 1-2 and 5 of Tables 2-5, respectively, at their face value. The results are tabulated in Table 6. As noted earlier, relative to the simple-minded PPP, the Chinese exchange rate is undervalued by about 45%. Once we adjust for the Balassa-Samuelson effect, the extent of the undervaluation becomes 55% (column 1 of Table 6) - apparently the Chinese RER is even lower than other countries at the comparable income level. If we additionally consider financial underdevelopment (proxied by the ratio of private sector loans to GDP), the Chinese RER undervaluation is reduced to 43% (column 2, row 1 of Table 6), which is still economically significant. If we also take into account government deficit, terms of trade, and capital account openness, the extent of the RER undervaluation is 35% (column 3, row 1). If we further take into account the dependency ratio, the extent of undervaluation drops to 18% (column 4, row 1). Finally, if we add the sex ratio effect, the extent of undervaluation becomes 8% (column 5, row 1 of Table 6). The last number represents a relatively trivial amount of undervaluation since major exchange rates (e.g., the euro/dollar rate or the yen/dollar rate) could easily fluctuate by more than 8% in a year. If we proxy financial development by the rating of financial system sophistication, and also take into account the sex ratio effect and other structural variables, the extent of the Chinese RER undervaluation becomes 2% (column 5, row 2 of Table 6), an even smaller amount.

We can do similar calculations for the Chinese (private sector) current account (as a share of GDP) in excess of the fundamentals. If we only take into the regularity that poorer countries tend to have a lower current account balance, the Chinese excess CA is on the order of 14%. If we take into account the sex ratio effect as well as financial underdevelopment, the dependency ratio and other variables in the regressions, the excess amount of current account becomes somewhere between 0.3% and 2.0%, depending on which proxy for financial development is used. These numbers illustrate that the sex ratio is a quantitatively important structural factor, though it is not the only one. In particular, the dependency ratio is also a very important factor. In any case, if these structural factors are not taken into account, one might mistakenly exaggerate the role of currency manipulation in affecting both the RER and the current account.

6 Conclusion

A low value of the real exchange rate (i.e., deviations from the PPP from below), a large current account surplus, and accumulation of foreign exchange reserve are the commonly used criteria for judging currency undervaluation or manipulation. We argue that none of them is a logically sound criterion. Instead, a dramatic rise in the sex ratio for the premarital age cohort in China since 2002, could generate both a depreciation of the real exchange rate and a rise in the current account surplus. With capital controls (including mandatory surrender of foreign exchange earnings), a persistent current account surplus can mechanically be converted into a rise in a country's foreign exchange reserve.

The usual narrative about the Chinese external economy connects the three variables in the following way: The authorities intervene aggressively in the currency market in order to generate an artificial undervaluation of its currency. This generates a rise in the foreign exchange reserve holdings and a fall in the real exchange rate. As a result of the currency undervaluation, the country manages to produce a current account surplus. The model and the evidence in this paper encourage the reader to consider an alternative way to connect the three variables: structural factors, such as a rise in the sex ratio, simultaneously generate a rise in the current account (through a rise in the savings rate) and a fall in the real value of the exchange rate. The low real exchange rate is not the cause of the current account surplus. With mandatory surrender of foreign exchange earnings required of by the country's capital control regime, the current account surplus is converted passively into an increase in the central bank's foreign exchange reserve holdings.

If other factors, in addition to a rise in the sex ratio, have also contributed to a rise in the Chinese savings rate, such as a reduction in the dependency ratio, or a rise in the corporate and government savings rates, they can complement the sex ratio effect and reinforce an appearance of an undervalued currency even when there is no manipulation. To be clear, this paper is not saying that no manipulations have occured. Instead, it illustrates potential pitfalls in assessing the equilibrium exchange rate when important structural factors are not accounted for.

Empirically, countries with a high sex ratio do appear to have a low value of the real exchange rate and a current account surplus. If we take the econometric point estimates at face value, it appears that the Chinese real exchange rate has only a relatively small amount of undervaluation (2-8%) once we take into account the sex ratio effect and other structural factors.

In future research, the model could be extended to allow for endogenous adjustment of the sex ratio. This will help us to assess the speed of the reversal of the sex ratio and the unwinding of the current account surplus and currency "undervaluation."

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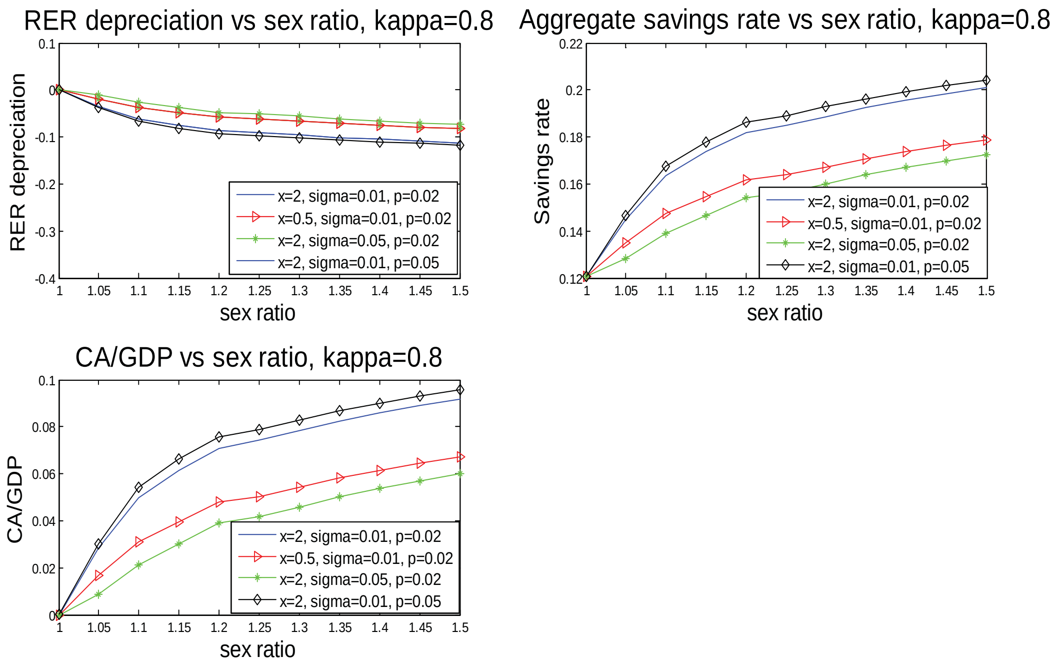

Figure 1: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.8

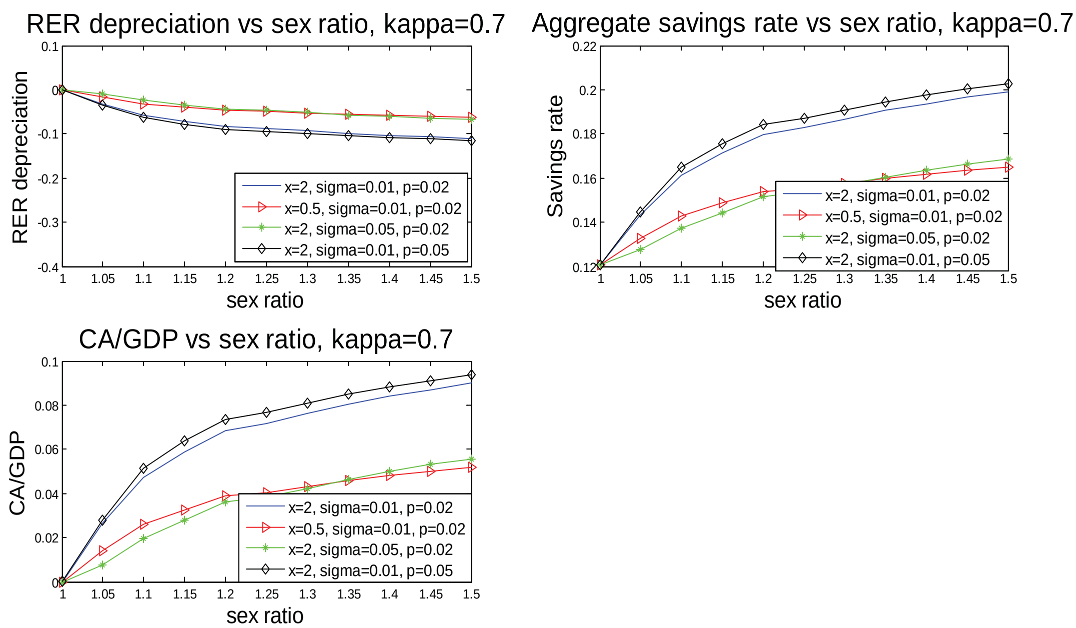

Figure 2: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.7

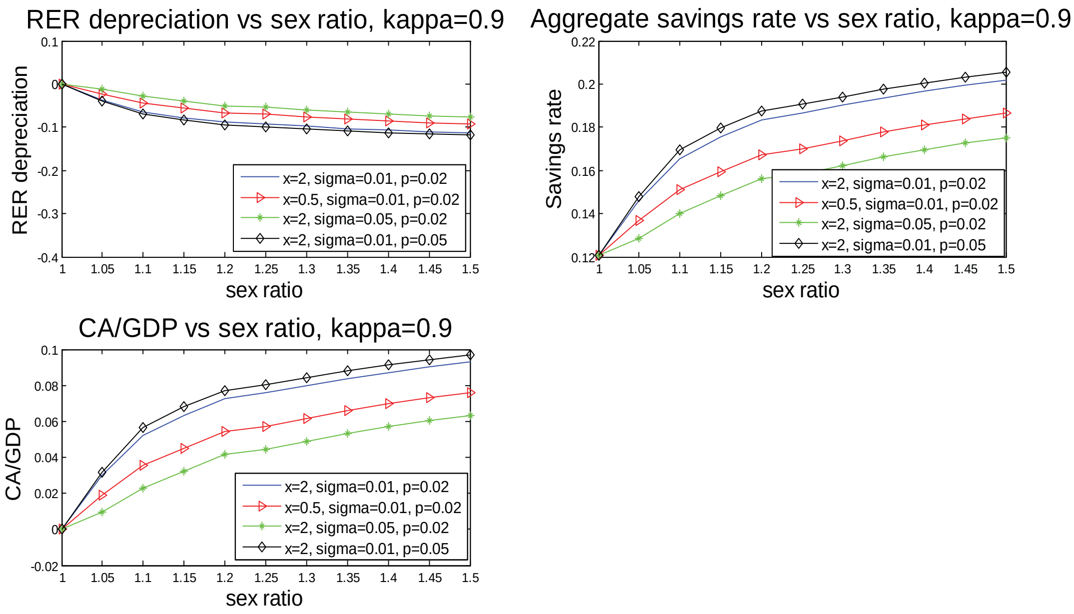

Figure 3: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.9

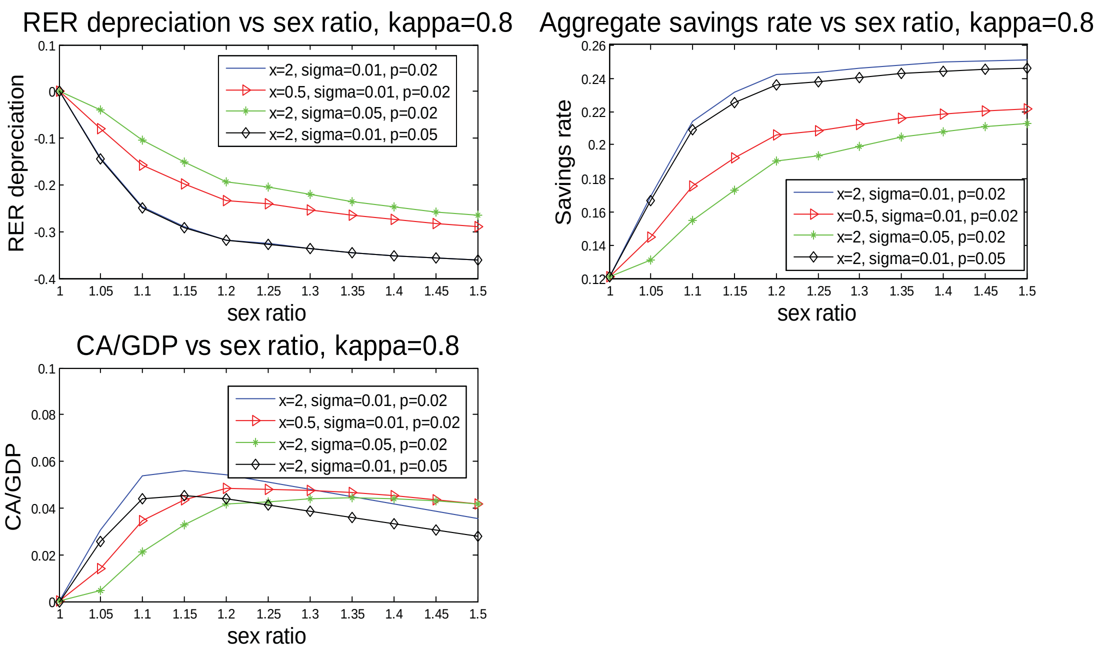

Figure 4: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.8

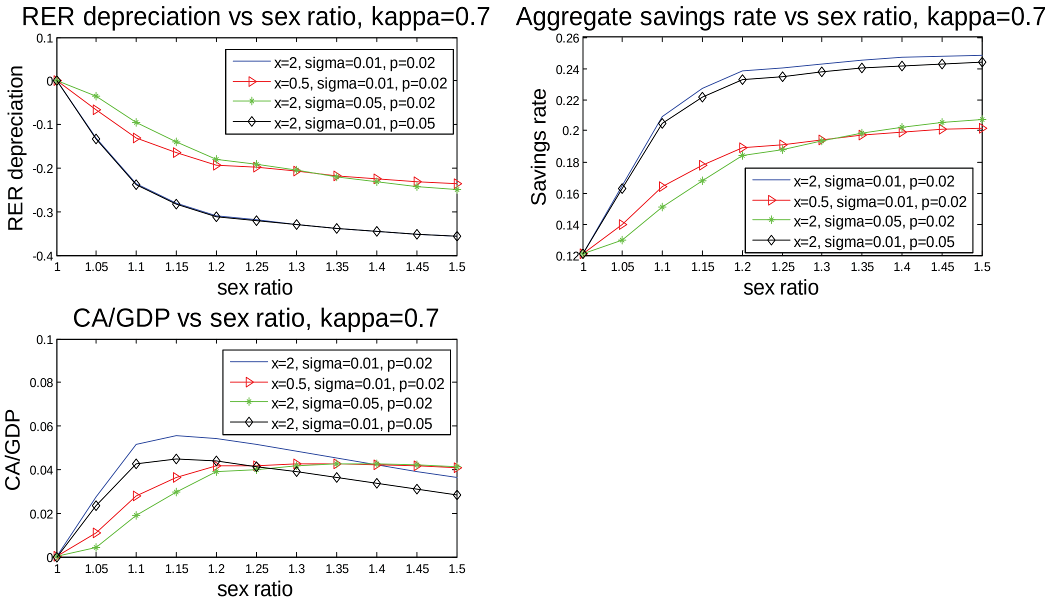

Figure 5: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.7

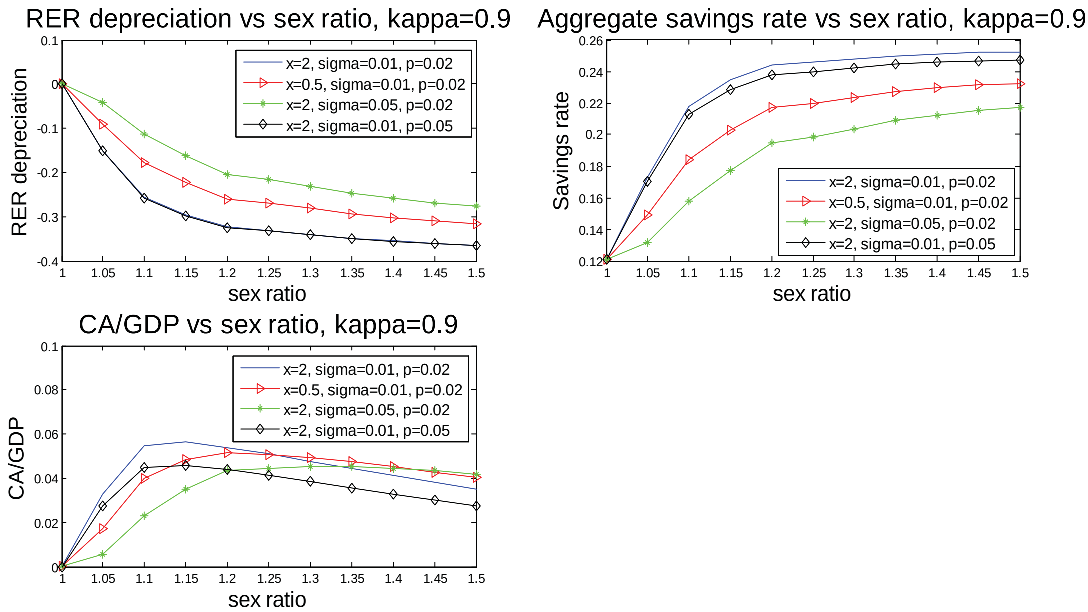

Figure 6: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.9

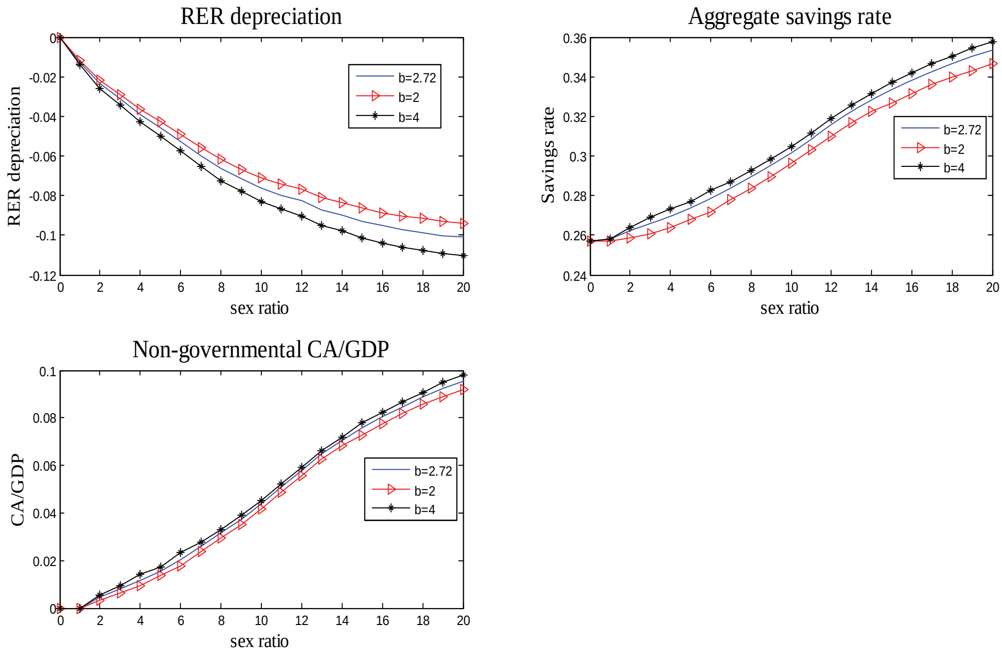

Figure 7: Impulse responses of RER, aggregate savings rate and CA/GDP, τ=20

Table 1: Summary statistics, 2004-2008 average

Variable	Mean	Median	Standard deviation	Min value	Max value
Ln(RER)	-0.74	-0.80	0.59	-2.22	0.41
(Private Sector) Current account	-3.63	-2.93	9.32	-31.51	26.91
Real GDP per capita (US$)	12986	7747	13733	367	77057
Private credit (% of GDP)	56.63	38.70	52.26	2.08	319.72
Financial system sophistication	3.78	3.66	0.79	2.52	5.28
Sex ratio	1.04	1.04	0.02	1.00	1.13
Fiscal deficit (% of GDP)	-1.47	-0.37	5.98	-25.98	11.38
Terms of trade	113	102	33.8	70.0	205.8
Capital account openness	0.53	0.118	1.64	-1.83	2.50
Dependency ratio	60.75	54.84	17.64	28.47	107.60

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable "*p*" (called "price level of GDP") in the Penn World Tables is equivalent to the real exchange rate relative to the US dollar: A lower value of *p* means a depreciation in the real exchange rate.
- Private Sector Current account = current account to GDP ratio minus the government savings to GDP ratio.
- For the ratio of private credit (% of GDP), we follow Allen, Qian and Qian (2004) and modify the measure for China by multiplying 0.2 to the credit to GDP ratio. This is to correct for the fact that only 20% of the bank loans go to private firms. Financial system sophistication from the Global Competitiveness Report is another measure for the financial development.
- Fiscal deficit data is obtained from IFS database. Terms of trade index is defined as the ratio of export price index to the import price index, which is from Worldbank database. We use the capital account openness index in Chinn and Ito (2008) to measure the degree of capital controls. A higher value means less capital control. Dependency ratio data can be obtained from Worldbank database.

Table 2: Ln(real exchange rate) and the sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-4.290**	-4.012**	-3.193*	-3.408**	-3.500**
			(1.667)	(1.713)	(1.797)	(1.568)	(1.754)
Ln(GDP per capita)	0.318**	0.190**	0.236**	0.233**	0.360**	0.402**	0.359**
	(0.030)	(0.038)	(0.041)	(0.044)	(0.073)	(0.063)	(0.073)
Private credit (% of GDP)		0.004**	0.004**	0.004**	0.003**	0.002**	0.002**
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fiscal deficit					-0.007	0.002	-0.005
					(0.009)	(0.008)	(0.009)
Terms of trade					0.0002	-0.001	0.0003
					(0.001)	(0.001)	(0.001)
Capital account openness					0.060**	0.029	0.058**
					(0.027)	(0.024)	(0.027)
Dependency ratio					0.009**	0.010**	0.008*
					(0.004)	(0.004)	(0.004)
Crawling peg (RR)						-0.397**	
						(0.075)	
Managed floating (RR)						-0.036	
						(0.077)	
Free floating (RR)						-0.081	
						(0.119)	
Intermediate (LYS)							-0.078
							(0.092)
Float (LYS)							-0.145*
							(0.085)
Observations	142	132	132	123	92	89	92
R-squared	0.444	0.542	0.564	0.579	0.706	0.801	0.716

Dependent variable = ln(RER). Standard errors are in parentheses, ** p<0.05, * p<0.1

Table 3: Ln(real exchange rate) and the sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-6.192**	-6.255**	-5.051*	-4.664*	-4.430
			(1.964)	(1.995)	(2.500)	(2.802)	(2.908)
Ln(GDP per capita)	0.318**	0.480**	0.443**	0.447**	0.529**	0.526**	0.531**
	(0.030)	(0.082)	(0.077)	(0.088)	(0.123)	(0.119)	(0.127)
Financial system sophistication		0.170*	0.252**	0.245**	0.099	0.034	0.086
		(0.089)	(0.086)	(0.099)	(0.110)	(0.121)	(0.116)
Fiscal deficit					-0.022	-0.014	-0.025
					(0.015)	(0.015)	(0.017)
Terms of trade					-0.004	-0.006**	-0.005
					(0.003)	(0.003)	(0.003)
Capital account openness					0.063	0.058	0.073
					(0.042)	(0.047)	(0.047)
Dependency ratio					0.014**	0.017**	0.017*
					(0.007)	(0.007)	(0.008)
Crawling peg (RR)						-0.285*	
						(0.147)	
Managed floating (RR)						0.045	
						(0.102)	
Free floating (RR)						0.053	
						(0.173)	
Intermediate (LYS)							-0.052
							(0.137)
Float (LYS)							0.044
							(0.125)
Observations	142	54	54	49	43	42	43
R-squared	0.444	0.748	0.791	0.797	0.844	0.866	0.845

- Dependent variable = log(RER). Standard errors are in parentheses, ** p<0.05, * p<0.1

Table 4: Non-governmental CA/GDP vs sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			66.43*	78.43**	134.7**	111.6**	94.24
			(37.09)	(36.65)	(37.52)	(56.43)	(56.51)
Ln(GDP per capita)	2.025**	3.683**	2.964**	2.050**	4.941**	4.035	3.834
	(0.639)	(0.876)	(0.957)	(0.975)	(1.529)	(3.415)	(3.115)
Private credit (% of GDP)		-0.048**	-0.046**	-0.030*	-0.054**	-0.053**	-0.051**
		(0.018)	(0.018)	(0.018)	(0.018)	(0.025)	(0.024)
Fiscal deficit					0.079	-0.031	0.101
					(0.187)	(0.379)	(0.345)
Terms of trade					0.021	0.127	0.131*
					(0.029)	(0.076)	(0.076)
Capital account openness					-0.315	-0.017	-0.081
					(0.563)	(1.508)	(1.353)
Dependency ratio					0.175*	0.209	0.439
					(0.089)	(0.745)	(0.720)
Share of working age people						0.163	0.797
						(1.884)	(1.885)
Social security expenditure (% of GDP)						0.137	0.115
						(0.250)	(0.233)
Crawling peg (RR)						3.413	
						(3.694)	
Managed floating (RR)						0.957	
						(2.840)	
Free floating (RR)						1.556	
						(5.730)	
Intermediate (LYS)							1.789
							(2.911)
Float (LYS)							0.117
							(2.469)
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	127	127	120	91	47	48
R-squared	0.073	0.125	0.147	0.121	0.275	0.543	0.532

Table 5: Non-governmental CA/GDP vs sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			103.3**	77.57**	129.6**	51.98	13.34
			(43.29)	(37.42)	(52.89)	(96.33)	(80.39)
Ln(GDP per capita)	2.025**	1.715	2.327	-0.470	2.009	3.752	-0.194
	(0.639)	(1.744)	(1.688)	(1.641)	(2.613)	(5.031)	(3.845)
Financial system sophistication		-0.925	-2.290	0.876	-0.477	4.029	3.480
		(1.889)	(1.896)	(1.861)	(2.337)	(3.336)	(2.951)
Fiscal deficit					-0.185	0.081	0.385
					(0.313)	(0.533)	(0.512)
Terms of trade					0.039	-0.070	0.0772
					(0.055)	(0.118)	(0.105)
Capital account openness					-0.103	-1.300	-0.569
					(0.879)	(2.619)	(1.765)
Dependency ratio					0.251	0.167	-1.807
					(0.150)	(5.527)	(2.375)
Share of working age people						0.125	-3.446
						(12.23)	(5.400)
Social security expenditure (% of GDP)						0.056	-0.092
						(0.302)	(0.257)
Crawling peg (RR)						7.405	
						(5.893)	
Managed floating (RR)						2.020	
						(3.055)	
Free floating (RR)						-8.510	
						(7.890)	
Intermediate (LYS)							7.626**
							(3.571)
Float (LYS)							-3.593
							(4.050)
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	54	54	49	43	32	33
R-squared	0.073	0.023	0.123	0.118	0.200	0.478	0.505

Table 6: Real exchange rate undervaluation and excess current account: The Case of China

	% of RER undervaluation					Excess (non-governmental) current account				
	(1) Only BS	(2) FD+BS	(3) Add GD +TT+KA	(4) Add DR	(5) Add SR	(1) Only BS	(2) FD+BS	(3) Add GD +TT+KA	(4) Add DR	(5) Add SR
Financial development index										
Private credit (% of GDP)	55.26	43.45	35.44	17.91	7.86	13.52	12.06	11.39	8.97	2.01
Financial system sophistication	55.26	46.38	31.31	16.78	2.24	13.52	10.26	10.11	7.97	0.37

Notes:

A. Excess RER undervaluation = model prediction – actual log RER. (A positive number describes % undervaluation).

B. Excess current account = private sector current account (i.e., current account net of government savings) – model prediction;

C. The five columns include progressively more regresors:

- (1) The only regressor (other than the intercept) is log income, a proxy for the Balassa-Samuelson (BS) effect;
- (2) Add financial development (FD) to the list of regressors;
- (3) Add government fiscal deficit (GD), terms of trade (TT), and capital account openness (KA);
- (4) Add the dependence ratio (DR);
- (5) Add the sex ratio (SR)

D. The last two rows correspond to estimates when two different proxies for financial development are used. The first row uses the ratio of credit to the private sector to GDP, and the second row uses an index of local financial system sophistication from the Global Competitiveness Report.

A Proof of Proposition 1 (not for publication)

Proof. We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= (u_1'' + \beta R u_2'') \frac{w_t}{P_t}, \, \Omega_{12} = \Omega_{13} = \Omega_{14} = 0 \\
\Omega_{21} &= \gamma w_t, \Omega_{22} = -\gamma(1-s_t), \, \Omega_{23} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}, \, \Omega_{24} = \frac{P_{Nt}(1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{31} &= \Omega_{33} = 0, \, \Omega_{32} = -1, \, \Omega_{34} = \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1-L_{Nt})^{-\alpha_T - 1} \\
\Omega_{41} &= 0, \, \Omega_{42} = -1, \, \Omega_{43} = \frac{w_t}{P_{Nt}}, \, \Omega_{44} = -\left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1-L_{Nt})^{-\alpha_N - 1}
\end{aligned}$$

and

$$z_1 = -R u_2', \, z_2 = z_3 = z_4 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \Omega_{11} \cdot \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}$$

and

$$\begin{aligned}
\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} &= \text{negative} \, .terms + \gamma(1-s_t) \left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T} \\
&\quad - \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) C_{Nt}
\end{aligned}$$

Since the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, it follows that $\gamma(1-s_t)w_t < P_{Nt} C_{Nt}$. Therefore,

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

and $\det(\Omega) > 0$

Then it is easy to show that

$$\frac{ds_t}{d\beta} = \frac{\det\begin{pmatrix} z_1 & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ z_2 & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ z_3 & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ z_4 & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1}{\Omega_{11}} > 0$$

and the price of the nontradable good

$$\frac{dP_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & z_1 & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & z_2 & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & z_3 & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & z_4 & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1\Omega_{21}\Omega_{32}(\Omega_{44} - \Omega_{34})}{\det(\Omega)} < 0$$

The labor input in the nontradable sector

$$\frac{dL_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & z_1 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & z_2 \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & z_3 \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & z_4 \end{pmatrix}}{\det(\Omega)} = -\frac{z_1\Omega_{21}\Omega_{32}\Omega_{34}}{\det(\Omega)} < 0$$

In period $t+1$, the shock has been observed, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

which means that after one period the shock occurs, the price of the nontradable good and the consumer price index will go back to their initial levels. As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$. Since

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_tw_t - s_{t-1}w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w\left(\left(R-1\right)s_t + 1\right)}{P_N}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w\left(\left(R-1\right)s_t + 1\right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N)\left[(R-1)s_t + 1\right]}{(1-\alpha_T)R} w$$

and then

$$CA_t = s_t w_t - s_{t-1} w + \frac{(\alpha_T - \alpha_N)(R-1)\left(s_t - s_{t-1}\right)}{(1-\alpha_T)R} w$$

To show $\frac{dCA_t}{d\beta} > 0$, we only need to show $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\beta} > 0$. One sufficient condition for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\beta} + P_{Nt} \frac{ds_t}{d\beta} > 0$$

which means

$$\frac{dP_{Nt}/d\beta}{ds_t/d\beta} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\beta}$ and $\frac{ds_t}{d\beta}$, we have

$$\begin{aligned}
\frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t) w_t C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) + P_{Nt} C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term \\
&= \frac{\left(P_{Nt} C_{Nt} - \gamma(1-s_t) w_t\right)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term
\end{aligned}$$

As shown above, $P_{Nt} C_{Nt} - \gamma(1-s_t) w_t > 0$, then $\frac{dCA_t}{d\beta} > 0$, in period t, the country will experience a current account surplus. ■

B Proof of Proposition 2 (not for publication)

Proof. If emotional utilities are large enough, when $\phi = 1$, or ϕ is close to one, we have $V^i > V_n^i$ $(i = w, m)$. Since $\frac{1}{2} \leq \kappa \leq 1$,

$$\kappa(Rs^m w_t + Rs^w w_t) > \max(Rs^w w_t,\ Rs^m w_t)$$

which means that within the neighborhood of $\phi = 1$, we have $\kappa u'_{2m} < u'_{2m,n}$.

We proceed in two steps. In the first step, we assume that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of ϕ, and prove that a higher sex ratio leads to a higher savings rate. In the second step, we prove by contradiction that the inequality indeed holds for all values of ϕ.

We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= \left[u''_{1w} + \beta(1-p)\kappa^2 R u''_{2w}\left(1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right) + pRu''_{2w,n}\right]\frac{w_t}{P_t} \\
\Omega_{12} &= \beta(1-p)\kappa^2 R u''_{2w}\left[1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right]\frac{w_t}{P_t} \\
\Omega_{14} &= (1-\sigma)\,\beta\left((1-p)\kappa u'_{2w}\left[1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right] + pu'_{2w,n}\right)\frac{R\gamma P_{Nt}}{P_{t+1}},\ \ \Omega_{13} = \Omega_{15} = 0 \\
\Omega_{21} &= \beta(1-p)\kappa^2\left[\left(\delta^m + 1\right) + f\left(M(\eta^{\min})\right)\left(u_{2m} + \eta^{\min} - u_{2m,n}\right)\right) u''_{2m}R + R\frac{u'_{2w}u'_{2m}}{\eta^{\max} - \eta^{\min}}\right]\frac{w_t}{P_t} \\
\Omega_{22} &= \frac{w_t}{P_t}\left[\begin{array}{c} \beta\kappa^2 R\delta^m u''_{2m} + \beta\kappa^2 u''_{2m}R + \beta R\kappa u'_{2w} f\left(M(\eta^{\min})\right)\left(\kappa u'_{2m} - u'_{2m,n}\right) \\ +\beta\kappa^2 R u''_{2w} f\left(M(\eta^{\min})\right)\left(u_{2m} + \eta^{\min} - u_{2m,n}\right) + u''_{1m} \end{array}\right] \\
&\quad +\beta\left[p + (1-p)(1-\delta^m)\right] R u''_{2m,n}\frac{w_t}{P_t} \\
\Omega_{23} &= \beta R\kappa u'_{2w} f\left(M(\eta^{\min})\right)\left(u_{2m} - u_{2m,n}\right)\frac{1-\sigma}{w_t} \\
\Omega_{24} &= (1-\sigma)\left(\begin{array}{c} (1-p)\left[\kappa\delta^m u'_{2m} + \kappa u'_{2w} f\left(M(\eta^{\min})\right)\left(u_{2m} + \eta^{\min} - u_{2m,n}\right)\right] \\ +\left[\left(1-\delta^m\right)(1-p) + p\right] u'_{2m,n} \end{array}\right)\frac{R\gamma P_{Nt}}{P_{t+1}},\ \Omega_{25} = 0
\end{aligned}$$

$$\Omega_{31} = \frac{\gamma w_t}{1+\phi}, \Omega_{32} = \frac{\gamma\phi w_t}{1+\phi}, \Omega_{33} = -\gamma(1-s_t), \Omega_{34} = \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}, \Omega_{35} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}$$

$$\Omega_{41} = \Omega_{42} = 0, \Omega_{43} = -1, \Omega_{44} = 0, \Omega_{45} = \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T}(1-\alpha_T)A_{Tt}K_{Tt}^{\alpha_T}(1-L_{Nt})^{-\alpha_T-1}$$

$$\Omega_{51} = \Omega_{52} = 0, \Omega_{53} = -1, \Omega_{54} = \frac{w_t}{P_{Nt}}, \Omega_{55} = -\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_T}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}{}^{-\alpha_N-1}$$

and

$$z_1 = 0,\ z_2 = (1-p)\frac{\partial\delta^m}{\partial\phi}\left(u'_{2m}(n) - \kappa u'_{2m}\right),\ z_3 = -\frac{\gamma w_t\left(s_t^m - s_t^w\right)}{1+\phi},\ z_4 = z_5 = 0$$

If we assume the utility function is log, $u(C) = \ln(C)$, then $\Omega_{14} = \Omega_{23} = \Omega_{24} = 0$, and

$$\det(\Omega) = \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}\cdot\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}$$

It is easy to show that

$$\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix} > 0$$

and

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} = \text{negative}\,.terms + \gamma(1-s_t)\left(\frac{w_t}{P_{Nt}}\right)^2\frac{1-\alpha_T}{L_T} - \left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)C_{Nt}$$

Notice that the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} < 0$$

and $\det(\Omega) < 0$

Then

$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} < 0$$

and the aggregate savings rate by the young cohort $s_t = \frac{\phi}{1+\phi} s_t^m + \frac{1}{1+\phi} s_t^w$,

$$\frac{ds_t}{d\phi} = \frac{\phi}{1+\phi} \frac{ds_t^m}{d\phi} + \frac{1}{1+\phi} \frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3 \frac{w_t^2}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det \begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}} + \frac{z_2 \left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right) \frac{w_t}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31} < 0$, and since $z_2 < 0$, $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_t C_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1}w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. By Obstfeld and Rogoff (1995), if the sex ratio remains constant ϕ after period t, the price of the nontradable good will go back to its initial level, which means that real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1)\, s_t + 1\right)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As in Obstfeld and Rogoff (1995), we assume that $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt}K_{N,t+1}^{\alpha_N}L_{N,t+1}^{1-\alpha_N}=\frac{\gamma w\left(\left(R-1\right)s_t+1\right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1}=\left[\frac{\alpha_T}{(1-\alpha_T)}-\left(\frac{\alpha_T}{1-\alpha_T}-\frac{\alpha_N}{1-\alpha_N}\right)\gamma\left((R-1)s+1\right)\right]\frac{w}{R}$$

and then

$$\begin{aligned}\Delta K_{t+1} &= \gamma\left(\frac{\alpha_T}{1-\alpha_T}-\frac{\alpha_N}{1-\alpha_N}\right)\left(R-1\right)\left(s_t-s_{t-1}\right)\frac{w}{R}\\ CA_t &= s_tw_t-s_{t-1}w+\gamma\left(\frac{\alpha_T}{1-\alpha_T}-\frac{\alpha_N}{1-\alpha_N}\right)\left(R-1\right)\left(s_t-s_{t-1}\right)\frac{w}{R}\end{aligned}$$

To show $\frac{dCA_t}{d\phi}>0$, we only need to show $\frac{d(s_tw_t-s_{t-1}w_{t-1})}{d\phi}>0$. By (3.9), one sufficient condition is for the inequality is

$$s_tP_{Nt}>s_{t-1}P_{Nt}$$

To show this inequality, we just need to show

$$s_t\frac{dP_{Nt}}{d\phi}+P_{Nt}\frac{ds_t}{d\phi}>0$$

which means

$$\frac{dP_{Nt}/d\phi}{ds_t/d\phi}+\frac{P_{Nt}}{s_t}>0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned}\frac{dP_{Nt}}{ds_t}+\frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t)w_tC_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)+P_{Nt}C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)}{s_t\cdot\text{positive}\,.terms}+\text{positive}\,.term\\ &= \frac{\left(P_{Nt}C_{Nt}-\gamma(1-s_t)w_t\right)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)}{s_t\cdot\text{positive}\,.terms}+\text{positive}\,.term\end{aligned}$$

As shown above, $P_{Nt}C_{Nt}-\gamma(1-s_t)w_t>0$, then $\frac{dCA_t}{d\phi}>0$, in period t, the country will experience a current account surplus.

The impact of a rise in the sex ratio on the social welfare is

$$\begin{aligned}\frac{\partial U^w}{\partial\phi} &= w_t\left(\beta R\frac{P_{t+1}}{P_t}\left((1-p)\kappa u'_{2w}+pu'_{2w,n}\right)-u'_{1w}\right)\frac{ds^w}{d\phi}\\ &\quad+\beta(1-p)\left(w_tR\frac{P_{t+1}}{P_t}\kappa u'_{2w}\frac{ds^m}{d\phi}+\frac{E\left[\eta\right]}{\phi^2}\right)\\ &> (1-p)\left(w_t\kappa\beta R\frac{P_{t+1}}{P_t}u'_{2w}\left(\frac{ds^m}{d\phi}+\frac{ds^w}{d\phi}\right)+\frac{\beta}{\phi^2}E\left[\eta\right]\right)>0\end{aligned}\tag{B.1}$$

$$
\begin{aligned}
\frac{\partial U^m}{\partial \phi} &= w_t \left(-u'_{1m} + \beta R \frac{P_{t+1}}{P_t} \kappa \delta (1-p) u'_{2m} + \beta R \frac{P_{t+1}}{P_t} \left[(1-p)\delta + (1-\delta) \right] u'_{2m,n} \right) \frac{ds^m}{d\phi} \\
&\quad + \beta (1-p) \left\{ w_t R \frac{P_{t+1}}{P_t} \kappa \delta u'_{2w} \frac{ds^w}{d\phi} + \left[\begin{array}{c} \frac{\partial \delta}{\partial \phi} \left(u_{2m} - u_{2m,n} + \eta^{\min} \right) \\ + \frac{\partial \left(\int_{M(\eta^{\min})} M^{-1}(\eta^m) dF(\eta^m) \right)}{\partial \phi} \end{array} \right] \right\} \qquad \text{(B.2)} \\
&< (1-p) \left(w_t \beta R \frac{P_{t+1}}{P_t} \kappa \delta u'_{2w} \frac{ds^w}{d\phi} - \frac{\beta}{\phi^2} \left(u_{2m} - u_{2m,n} + \eta^{\min} \right) - \frac{\beta}{\phi^2} E\left[\eta \right] \right) < 0
\end{aligned}
$$

where the first inequality in (B.1) holds because

$$
-u'_{1w} + (1-p)\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2w} + p \beta R \frac{P_{t+1}}{P_t} u'_{2w,n} = -(1-p)\beta R \frac{P_{t+1}}{P_t} \kappa u'_{2m} \left[\frac{1}{\phi} + M(\eta^{\min}) f(\eta^{\min}) \right] < 0
$$

and the first inequality in (B.2) holds because

$$
\begin{aligned}
& -u'_{1m} + \beta R \frac{P_{t+1}}{P_t} \kappa (1-p)(1+\delta) u'_{2m} + \beta R \frac{P_{t+1}}{P_t} p (1-\delta) u'_{2m,n} \\
= \; & -(1-p) \beta R \frac{P_{t+1}}{P_t} \kappa u'_{2w} \left(\frac{u_{2m} + \eta^{\min} - u_{2m,n}}{\eta^{\max} - \eta^{\min}} \right) < 0
\end{aligned}
$$

We now show by contradiction that $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs. Suppose not, then $\kappa u'_{2m} < u'_{2m,n}$ may fail sometime. Due to continuity of z_2, there exists a level of sex ratio ϕ_0 at which $\kappa u'_{2m} = u'_{2m,n}$, which implies that $z_2 = 0$.

As in Du and Wei (2010), we can show that

$$
z_2 |_{\phi = \phi_0} = 0 \;\; \text{and} \;\; \left. \frac{d^k z_2}{d\phi^k} \right|_{\phi = \phi_0} = 0 \text{ for any } k > 0
$$

which means that $z_2 = 0$ for all ϕs. This contradicts the assumption that $z_2 < 0$ when $\phi = 1$. Therefore, the inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all ϕs. ■

C Proof of Proposition 3 (not for publication)

Proof. If $u(C) = \ln C$, for $\phi < \phi_1$, by the optimal labor supply condition, we have

$$
0 < \frac{dL^i_t}{ds^i_t} = \frac{1}{1 - s^i_t} \frac{v'_i L^i_t}{v'_i - v''_i L^i_t} \qquad \text{(C.1)}
$$

where $i = w, m$.

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= \left[u''_{1w} + \beta(1-p)\kappa^2 R u''_{2w}\left(1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right) + \beta p R u''_{2w,n}\right]\frac{w_t L_t^w}{P_t} \\
&\quad + \left[-(1-s_t)u''_{1w} + \beta(1-p)\kappa^2 R s_t^w u''_{2w}\left(1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right) + \beta p R s_t^w u''_{2w,n}\right]\frac{w_t}{P_t}\frac{dL_t^w}{ds_t^w} \\
\Omega_{12} &= \beta(1-p)\kappa^2 R u''_{2w}\left(1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right)\left(L_t^m + s_t^m \frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t},\ \Omega_{13} = \Omega_{14} = \Omega_{15} = 0 \\
\Omega_{21} &= \beta(1-p)\kappa^2\left[\left((\delta^m + 1) + f\left(M(\eta^{\min})\right)\left(u_{2m} + \eta^{\min} - u_{2m,n}\right)\right)u''_{2m}R + R\frac{u'_{2w}u'_{2m}}{\eta^{\max} - \eta^{\min}}\right]\left(L_t^w + s_t^w \frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
\Omega_{22} &= \frac{w_t}{P_t}\left[\begin{array}{c} \beta\kappa^2 u''_{2m}R + \beta R\kappa u'_{2w} f\left(M(\eta^{\min})\right)(\kappa u'_{2m} - u'_{2m,n}) \\ +\beta\kappa^2 R u''_{2w} f\left(M(\eta^{\min})\right)\left(u_{2m} + \eta^{\min} - u_{2m,n}\right) + u''_{1m} \end{array}\right]\left(L_t^m + s_t^m \frac{dL_t^m}{ds_t^m}\right) - (1 - s_t^m)u''_{1m}\frac{w_t}{P_t}\frac{dL_t^m}{ds_t^m} \\
&\quad + \beta\left[p + (1-p)(1-\delta^m)\right]R u''_{2m,n}\left(L_t^m + s_t^m \frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
\Omega_{23} &= \Omega_{24} = \Omega_{25} = 0
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi}\left(L_t^w + s_t^w \frac{dL_t^w}{ds_t^w}\right),\ \Omega_{32} = \frac{\gamma \phi w_t}{1+\phi}\left(L_t^m + s_t^m \frac{dL_t^m}{ds_t^m}\right) \\
\Omega_{33} &= -\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right],\ \Omega_{34} = \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\ \Omega_{35} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{41} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}}\frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w},\ \Omega_{42} = -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}}\frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m} \\
\Omega_{43} &= -1,\ \Omega_{44} = 0,\ \Omega_{45} = \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T}(1-\alpha_T)A_{Tt}K_{Tt}^{\alpha_T}\left(\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}\right)^{-\alpha_T - 1} \\
\Omega_{51} &= \Omega_{52} = 0,\ \Omega_{53} = -1,\ \Omega_{54} = \frac{w_t}{P_{Nt}},\ \Omega_{55} = -\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_N}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}{L_{Nt}}^{-\alpha_N - 1}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= 0,\ z_2 = (1-p)\frac{\partial \delta^m}{\partial \phi}\left(u'_{2m}(n) - \kappa u'_{2m}\right),\ z_3 = -\frac{\gamma w_t\left(s_t^m L_t^m - s_t^w L_t^w\right)}{1+\phi} \\
z_4 &= \frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}}\frac{L_t^m - L_t^w}{(1+\phi)^2},\ z_5 = 0
\end{aligned}$$

The determinant of matrix Ω is

$$\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

It is easy to show that

$$\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} = \text{negative}.terms + \gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right]\left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T}$$
$$-\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) C_{Nt}$$

Notice that the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right] w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and $\det(\Omega) < 0$

Then

$$\frac{ds_t^m}{d\phi} = -\frac{z_2\Omega_{11}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} < 0$$

By (C.1), we have

$$\frac{dL_t^m}{d\phi} > 0 \text{ and } \frac{dL_t^w}{d\phi} < 0$$

The aggregate savings rate by the young cohort $s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$,

$$\frac{ds_t}{d\phi} = \frac{\phi}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

The aggregate labor supply in period t

$$\frac{dL_t}{d\phi} = \frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w}\frac{ds_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2}$$

Under the assumption $\frac{v''L}{v'}$ is non-decreasing in L, by (C.1), $\frac{dL_t^m}{ds_t^m} > \frac{dL_t^w}{ds_t^w}$, then we have $\frac{dL_t}{d\phi} > 0$, which means the aggregate labor supply is increasing in the sex ratio.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3\left(\frac{w_t^2}{P_{Nt}}\right)\frac{\alpha_T}{L_{Tt}} + z_4\left(\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54}\right)}{\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}} + \frac{z_2\left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right)\frac{w_t^2}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54} < 0$, then $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1)\cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1}L_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = \left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)w_t - s_{t-1}w_{t-1}L_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. Following Obstfeld and Rogoff (1995), if the sex ratio remains constant at ϕ after period t, the price of the nontradable good will go back to its initial level, which means that the real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain the same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w\left((R-1)\left(\frac{s_{t+1}^w}{1+\phi} + \frac{\phi s_{t+1}^m}{1+\phi}\right) + 1\right)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

In period $t+1$,

$$A_{Nt}K_{N,t+1}^{\alpha_N}L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w\left((R-1)\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) + 1\right)}{P_{N,t+1}}$$

In equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N)\left[(R-1)\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) + 1\right]}{(1-\alpha_T)R} w$$

and then

$$CA_t = \left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) w_t - s_{t-1} w L_{t-1} + \frac{(\alpha_T - \alpha_N)(R-1)\left(\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) - s_{t-1}L_{t-1}\right)}{(1-\alpha_T)R} w$$

To show $\frac{dCA_t}{d\phi} > 0$, we only need to show $\frac{d\left(\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) w_t - s_{t-1} w L_{t-1}\right)}{d\phi} > 0$. By (3.9), one sufficient condition is for the inequality is

$$\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) P_{Nt} > s_{t-1} L_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right) \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plug the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{-\gamma(1-s_t) w_t C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) + P_{Nt}C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term \\ &= \frac{(P_{Nt}C_{Nt} - \gamma(1-s_t)w_t)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term \end{aligned}$$

As shown above, $P_{Nt}C_{Nt} - \gamma(1-s_t)w_t > 0$, then $\frac{dCA_t}{d\phi} > 0$, in period t, the country will experience a current account surplus. ■