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ABSTRACT

We use a panel of Brazilian exporters, their products, and destination markets to document a set of regularities for multi-product exporters: (i) few top-selling products account for the bulk of a firm's exports in a market, (ii) the distribution of exporter scope (the number of products per firm in a market) is similar across markets, and (iii) within each market, exporter scope is positively associated with average sales per product. Our data also show that firms systematically export their highest-sales products across multiple destinations. To account for these regularities, we develop a model of firm-product heterogeneity with entry costs that depend on exporter scope. Estimating this model for the within-firm sales distribution we identify the nature and components of product entry costs. We find that firms face a strong decline in product sales with scope but also that market-specific entry costs drop fast. Counterfactual experiments with globally falling entry costs indicate that a large share of the simulated increase in trade is attributable to declines in the firm's entry cost for the first product.

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An online appendix is available at:
http://www.nber.org/data-appendix/w16641

1 Introduction

Market-specific entry costs are an important ingredient in recent trade theory. Combined with firm heterogeneity, entry costs serve as a key explanation for exporter behavior and the size distribution of firms.[1] After rounds of tariff reductions and drops in transport costs, local entry costs from technical barriers to trade and regulatory protection are thought to be major remaining impediments to trade (Baldwin 2000, Maskus and Wilson 2001).[2] Micro-econometric estimates suggest that firm entry costs are a substantive fraction of export sales (Das, Roberts, and Tybout 2007, Maskus, Otsuki, and Wilson 2005). Market-specific fixed costs do not only limit firm entry, they also hinder the expansion of prolific multi-product exporters.

To study the nature and components of entry costs, we use comprehensive data on multi-product firms and their destinations. Beyond the extensive margin of firm presence, we decompose, destination by destination, an exporter's sales into the extensive margin of the number of products—the *exporter scope*—and the remaining intensive margin of the exporter's average sales per product, which we call *exporter scale*. We use a structural approach to quantify the relevance of multi-product exporters in a general-equilibrium framework for the first time. To do so, we separately identify sources of within-firm heterogeneity in product sales and economies of scope in local product entry costs. Based on our estimates, we assess the general-equilibrium effects of reduced market-specific fixed costs on the expansion of incumbent multi-product exporters and new exporters. Our simulations suggest that new products of incumbent exporters contribute less to bilateral trade than new exporters.

A number of key regularities emerge from our Brazilian exporter data and discipline the analysis.[3] First, a few top-selling products explain the bulk of a firm's exports in a market, whereas wide-scope exporters sell their lowest-selling products in minor amounts.[4] Second, within des-

[1]See for example Melitz (2003), Chaney (2008) and Eaton, Kortum, and Kramarz (2010).

[2]The World Bank estimates in its *Global Economic Prospects 2004* report that trade facilitation would increase world trade by approximately $377 billion overall, to which an improvement in the regulatory environment would contribute $83 billion and services sector infrastructure and e-business usage another $154 billion (World Bank 2003). The World Trade Organization's *World Trade Report 2008* asserts that "with respect to product standards, technical regulations and sanitary and phytosanitary (SPS) measures, considerable opportunities exist for reducing trade costs" (WTO 2008, p. 149).

[3]To assess robustness for another country, we use panel data of Chilean exporters in 2000 (see Álvarez, Faruq, and López 2007). We find the regularities confirmed and estimates to be similar, and report them in our online Data Appendix.

[4]Bernard, Redding, and Schott (2010a) document a similar pattern for worldwide shipments by U.S. firms. We show that the pattern is repeated market by market.

tinations, there are few wide-scope and large-sales firms but many narrow-scope and small-sales firms. Third, within destinations, mean exporter scope and mean exporter scale are positively associated. These three regularities occur repeatedly destination by destination. Comparing across destinations but within firms, we find that exporters are likely to sell their highly successful products in many destinations and in large amounts. We interpret this body of regularities as evidence of heterogeneity in product efficiency, or consumer appeal, and as evidence of entry costs that vary by product and destination.

Guided by these facts, we propose a model of firm-product heterogeneity where firms face destination-specific entry costs for each of their products. The model rests on a single source of firm heterogeneity (productivity) and firms face declining efficiency in supplying their less successful products, similar to Eckel and Neary (2010) and Mayer, Melitz, and Ottaviano (2009).[5] Our specification of local entry costs accommodates the cases of both economies or diseconomies of scope.[6] The setup offers a tractable extension of the Melitz (2003) framework to multiple products where the firm decides along three export margins: its presence at export destinations, its exporter scope at a destination, and its individual product sales at the destination.

We use our model's structural implications to obtain novel estimates for parameters that govern separate entry cost components, fitting the within-firm heterogeneity under the first regularity. We check the fitted model's prediction for the remaining two regularities across firms and show that the model approximates well the scope and sales distributions and generates the observed positive association between exporter scope and exporter scale. Estimates point to a strong decline in product efficiency with scope. So only highly productive firms choose a wide scope. But local entry costs exhibit economies of scope for the introduction of additional products within a market, consistent with the fact that wide-scope exporters sell their lowest-selling products in minor amounts.

Having parameterized the model, we simulate a 25-percent reduction in entry costs and their effect on global trade. We distinguish between a decline in firm entry costs for the first product and a decline in entry costs that a multi-product exporter incurs for additional export products. We

[5]Bernard, Redding, and Schott (2010a) develop a multi-product firm model with products that have idiosyncratic country-specific demand shocks. Departing from CES demand, Eckel, Iacovone, Javorcik, and Neary (2010) study the firm's investment in product appeal. Exporter scope is socially optimal in these models. Thomas (2010) proposes an agency approach to product adoption and documents inefficient variation in firm scope for detergent manufacturers across local markets in Western Europe.

[6]Seminal references on economies of scope are Panzar and Willig (1977) and (1981). Formally, there are economies of scope if the cost function satisfies $C(x+y) < C(x) + C(y)$ (if the cost function is subadditive).

find that most of the simulated trade increase is due to falling entry costs for the first product—such as one-shot startup costs for information acquisition, the setup of certified and accredited testing facilities, investments in technology acquisition for export development, and perhaps brand marketing costs. In contrast, trade is less sensitive to falling entry costs for subsequent products—such as compliance with an individual product's technical requirements, mandatory or voluntary product safety standards, and packaging and labelling procedures, or expenditures for extending marketing and the distribution network to additional products.

Overall, a simulated 25-percent reduction in entry costs only results in a less than 1-percent welfare increase. Firm entry costs are typically found to influence sales little because only small exporters around the entry threshold respond (Das, Roberts, and Tybout 2007, di Giovanni and Levchenko 2010). But incumbent exporters add products when entry costs fall, suggesting potentially salient changes to trade flows because multi-product exporters dominate trade. Our simulations show, however, that the elasticity of trade with respect to product entry costs is even smaller than with respect to firm entry costs. The reason for the surprisingly small contribution of the extensive margin of adding products is the estimated combination of strongly declining product efficiencies and economies of scope in local entry, so that even highly productive wide-scope exporters add only products that sell minor amounts. We confirm the small response at the extensive margin of exporting products also for a simulated 25-percent drop in variable trade costs.

Our analysis is related to an emerging literature that documents the dominance of multi-product firms in the economy and multi-product exporters in international trade (Bernard, Redding, and Schott 2010b, Goldberg, Khandelwal, Pavcnik, and Topalova 2010).[7] Beyond existing evidence, we show systematic and recurrent exporter behavior market by market and the correlation of product sales within firms across markets.

For our model, we use a conventional demand system with constant elasticity of substitution (CES) and embed the Eckel and Neary (2010) production setup, by which firms can take up additional products (away from their core competency) only at lower marginal efficiency. This setup implies that a firm's product sales are perfectly correlated across the markets where a product is sold, which reflects features of our data but also distinguishes our approach from the stochastic

[7] Bernard, Jensen, and Schott (2009) show for U.S. trade data in 2000, for instance, that firms that export more than five products at the HS 10-digit level make up 30 percent of exporting firms but account for 97 percent of all exports. In our Brazilian exporter data for 2000, 25 percent of all manufacturing exporters ship more than ten products at the internationally comparable HS 6-digit level and account for 75 percent of total exports. Similar findings are shared by Iacovone and Javorcik (2008) for Mexico and Álvarez, Faruq, and López (2007) for Chile.

firm-product model of Bernard, Redding, and Schott (2010b and 2010a).[8] Mayer, Melitz, and Ottaviano (2009) analyze additional properties of the within-firm sales distribution through country-pair comparisons and explain them with an Eckel and Neary (2010) production setup under a varying demand elasticity.[9]

While intentionally parsimonious, our model is qualitatively consistent with the empirical regularities under a set of mild and empirically confirmed restrictions on product efficiency and entry costs. Moreover, under the common assumption of Pareto distributed firm productivities, the model preserves desirable predictions of previous trade theory: at the firm level the model generates a total sales distribution that is Pareto-shaped in the upper tail as in Chaney (2008), and at the country level it results in a general equilibrium gravity relationship resembling the one in Anderson and van Wincoop (2003) and Eaton and Kortum (2002).

Arkolakis, Costinot, and Rodríguez-Clare (2009) show for a wide family of models, which includes ours, that conditional on identical observed trade flows these models predict identical ex-post welfare gains irrespective of firm turnover and product-market reallocation. Their findings also imply, however, that models in that family differ in their predictions for trade flows and welfare with respect to ex-ante changes in entry costs. Our model provides market-specific microfoundations for these entry costs. The model's tractable setup can be used to compute the impact of rich policy experiments on trade flows and welfare.

The organization of this paper is in six more sections. In Section 2 we describe the data and present key regularities. We introduce the general model in Section 3 and show how it generates the regularities. In Section 4 we derive equilibrium and bilateral trade under a Pareto distribution and adopt parametric functional forms for estimation. We obtain structural estimates of entry cost parameters in Section 5, and simulate their cross sectional predictions. Section 6 applies these estimates to simulate a drop in entry costs. Section 7 concludes.

[8]Incomplete correlation can readily be built into our model, using random sales shocks per product as developed by Eaton, Kortum, and Kramarz (2010). The benchmark specification in Bernard, Redding, and Schott (2010b) makes product heterogeneity market specific so that product sales are uncorrelated across markets. For product sales covariation to be built into their model, a correlated distribution of product efficiencies would need to be specified.

[9]Feenstra and Ma (2008), Nocke and Yeaple (2006) and Dhingra (2010) study multi-product exporters but do not generate a within-firm sales distribution, which lies at the heart of our analysis.

2 Data

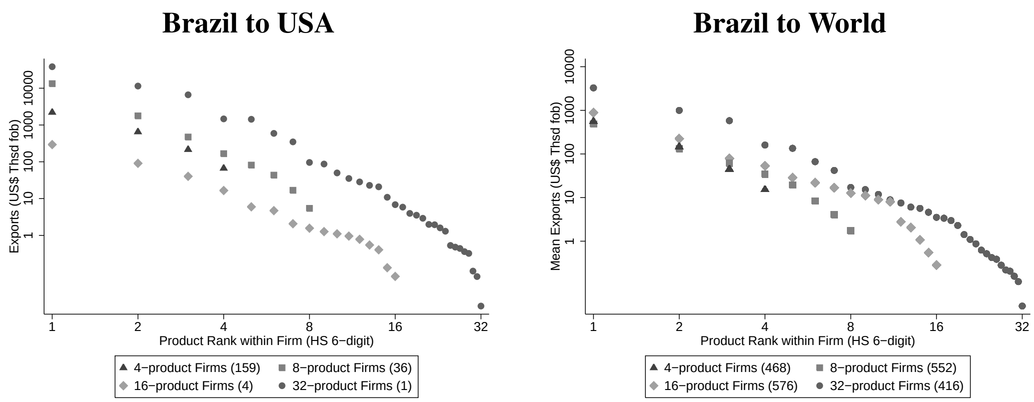
Our Brazilian exporter data derive from the universe of customs declarations for merchandize exports during the year 2000 by any firm. From these customs records, we construct a three-dimensional panel of exporters, their respective destination countries, and their export products at the Harmonized System (HS) 6-digit level. We briefly discuss the data sources and characteristics, and then present three main stylized facts that emerge from the data.

2.1 Data sources and sample characteristics

In our pristine exports data from SECEX (*Secretaria de Comércio Exterior*), product codes are 8-digit numbers (under the common Mercosur nomenclature), of which the first six digits coincide with the first six HS digits. We aggregate the monthly exports data to the HS 6-digit product, firm and year level. To relate our data to product-market information for destination countries and their sectors, we map the HS 6-digit codes to ISIC revision 2 at the two-digit level and link our data to World Trade Flow (WTF) data for the year 2000 (Feenstra, Lipsey, Deng, Ma, and Mo 2005). In 2000, our SECEX data for manufactured merchandize sold by Brazilian firms from any sector (including commercial intermediaries) reaches a coverage of 95.9 percent of Brazilian exports in WTF.

We restrict our sample to manufacturing firms and their exports of manufacturing products, removing intermediaries and their commercial resales of manufactures. The restriction to manufacturing firms and their manufactured products makes our findings closely comparable to Eaton, Kortum, and Kramarz (2004) and Bernard, Redding, and Schott (2010a), for example. The group of manufacturing firms covers a substantial fraction of exports (81.7 percent of the WTF manufactures exports).[10] The resulting manufacturing firm sample has 10,215 exporters shipping 3,717 manufacturing products at the 6-digit HS level to 170 foreign destinations, and a total of 162,570 exporter-destination-product observations. Multi-product exporters sell more than 90 percent of all exports from Brazil.

[10]Exporter behavior in Brazil is strikingly similar to that in leading export countries such as France and the United States (see our online Data Appendix). Appendix D.1 reports summary statistics and documents the dominance of multi-product exporters in total exports. In our online Data Appendix we also report findings from the complementary group of commercial intermediary firms and their exports of manufactures.



Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at the HS 6-digit level. World average in right-hand graph from pooling destinations where firms in a given exporter-scope group ship.

Figure 1: **Within-firm Sales Distribution**

2.2 Three regularities

To describe the extensive margin of exporting products, we look at the number of products that a firm sells at each destination. We decompose a firm ω's total exports to destination d, $t_d(\omega)$, into the number of products $G_d(\omega)$ sold at d (the *exporter scope* in d) and the average sales per export product $a_d(\omega) \equiv t_d(\omega)/G_d(\omega)$ in d (the *exporter scale* in d). We elicit three major stylized facts from the data at three levels of aggregation, moving from less to more aggregation.

Fact 1 *Within firms and destinations, exports are concentrated in few top-selling products. Wide-scope exporters sell small amounts of their lowest-selling products.*

Figure 1 depicts the distribution of sales of firms for different products within the firm.[11] We consider firms with the same number of products and rank the products of each firm from top-selling (rank 1) to lowest-selling at a given destination. We then take the average across firms of each product at a given product rank and plot the logarithm of this value against the logarithm of the rank of the product. The figure depicts the results for manufacturers that sell exactly 4, 8, 16 or 32 products to Brazil's top export destination in 2000, the United States, or worldwide over all destinations. The worldwide figures here treat the rest of the world as if it were a single destination (individual plots are similar destination by destination). The elasticity of individual product sales

[11] Bernard, Redding, and Schott (2010a) present evidence for U.S. firms' sales worldwide comparable to the right-hand side graph in Figure 1. Our analysis shifts attention to regularities by destination.

with respect to the rank of the product is about -2.8 in the United States and -2.6 worldwide implying that sales fall sharply with rank. As expected, the contribution of the top-selling products in the total sales of firms is large: for firms with 32 products in the USA or Argentina, the top 3 products account, on average, for more than 85 percent of their total sales. This number is 76 percent for the world as one destination.

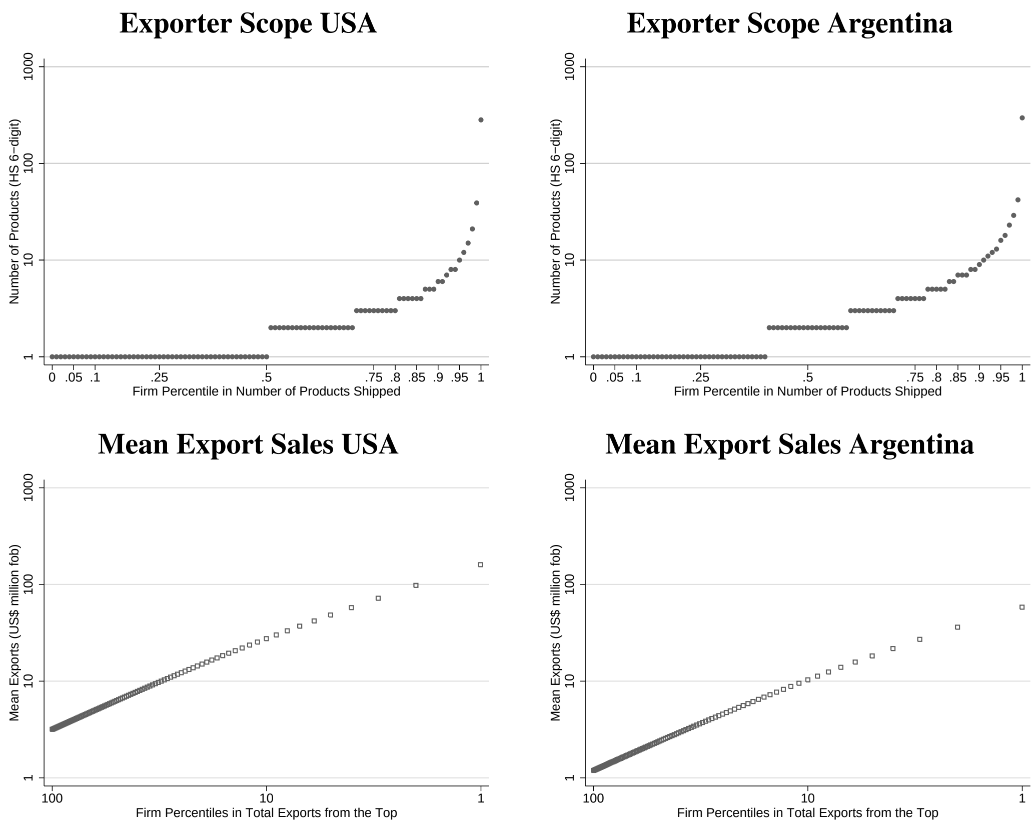
For shipments to the United States in Figure 1, the top-selling product (rank 1) sells on average US$ 38 million at 32-product firms but only US$ 2.2 million at 4-product firms.[12] On average, the top-selling product of multi-product exporters accounts for 70 percent of their sales to a destination. For the lowest-selling product, in contrast, narrower-scope firms have far higher average sales per product than wide-scope exporters. The lowest-selling product of 32-products exporters to the United States, for instance, sells for merely US$ 12 in 2000 (rank 32) and 16-products exporters ship just US$ 77 of their lowest-selling product (rank 16). In contrast, the lowest-selling product of 8 and 4-products exporters (rank 8 and rank 4) sells for US$ 5,400 and US$ 67,000 respectively. Thus, the findings in Figure 1 suggest that wide-scope exporters have higher sales for their first product than narrow-scope firms. At the same time, wide-scope exporters tolerate lower sales for their lowest-selling products than narrow-scope firms.[13]

Fact 2 *Within destinations, there are few wide-scope and large-sales firms but many narrow-scope and small-sales firms.*

To graph the exporter scope distribution, we rank firms according to their exporter scopes in a destination market. The upper panel of Figure 2 plots exporter scope against the scope percentiles for Brazil's top two exporting markets, the United States and Argentina. These plots too are similar for most Brazilian destinations. For instance, the median Brazilian exporter sells one or two products per destination and the mean number of products is around three to four products in individual destinations (see also Table D.1 in the Appendix). Exporter scope is a discrete variable but the overall shape of the distributions approximately resembles that of a power-law distributed variable.

[12]There is considerable small-sample variability within single destinations so that top-product sales may not generally increase between firms with increasing scope. In Figure 1 for the United States, for instance, the (four) 16-product firms exhibit untypically low top-product sales compared to 8-product firms, whereas the (nine) 17-product firms do exhibit higher top-product sales compared to the (22) 9-product firms as expected. Destination aggregates do not exhibit such small-sample variability.

[13]Beyond Fact 1, Mayer, Melitz, and Ottaviano (2009) document that the slope of the graphs in Figure 1 is steeper in larger destination markets.

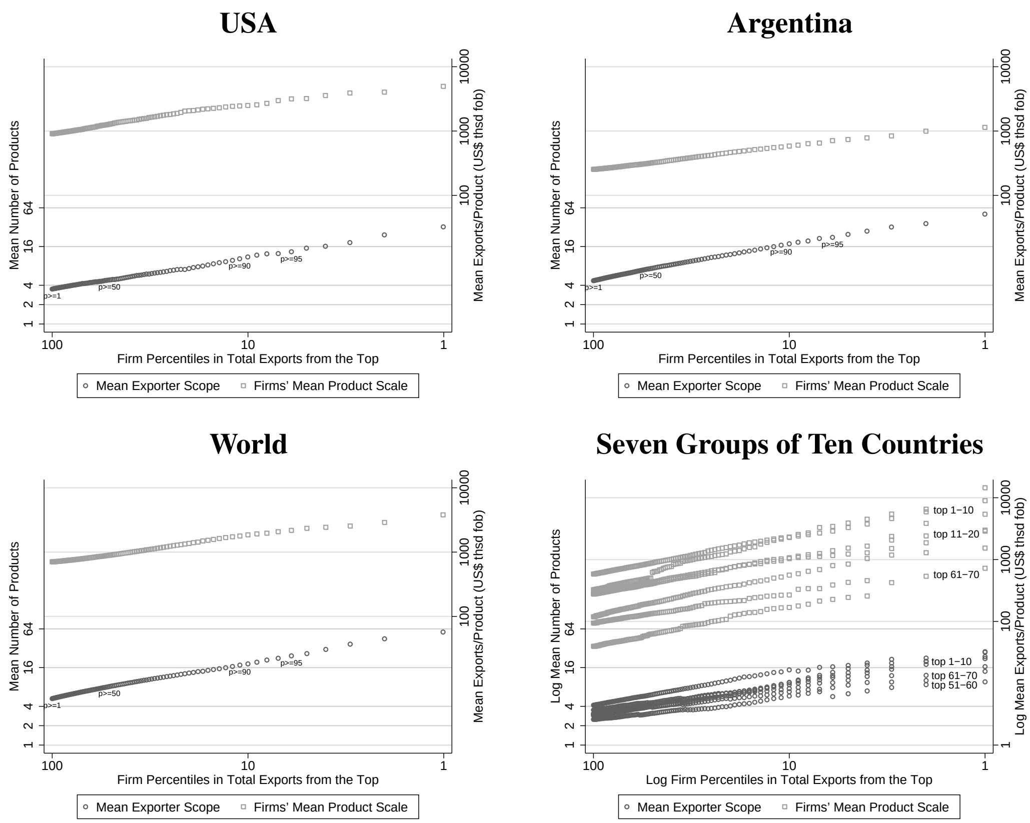


Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at HS 6-digit level. In the lower panel, the left-most observations are all exporters; at the next percentile are exporter observations with sales in the top 99 percentiles; up to the right-most observations with exporters whose sales are in the top percentile.

Figure 2: **Exporter Scope and Export Sales Distributions**

To graph the export sales distribution, we use cumulative plots in the lower panel of Figure 2. A cumulative plot naturally relates to later regularities for exporter scope and exporter scale. On the horizontal axis, we now group firms at *or above* a given total exports percentile. At the origin, we cumulate all firms and plot mean total sales $\bar{t}_{sd}$. Then we step one percentile to the right along the horizontal axis and restrict the sample to all those firms that are in the top 99 percentiles, depicting mean total sales for that group of firms. We continue to move up in the total-exports ranking of firms and graph mean total sales by percentile group until we reach the top-percentile group of firms. Such a cumulative plot puts the emphasis on the mean exporter by percentile group and weights down deviant behavior of small-scale exporters.[14] It is the signature of a power law dis-

[14]Introducing the marketing cost mechanism of Arkolakis (2010) is a straightforward extension of our model and would allow us to match the size distribution of smaller firms as well. For our focus is on the multi-product firm, we abstain from an exploration of small-firm deviations.



Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: World graph is based on pooling all markets. The groups-of-ten graph shows 70 markets (with 100 or more Brazilian exporters), where markets are first ranked by total sales and then lumped to seven groups of ten countries by total-sales rank. Products at the Harmonized-System 6-digit level. Left-most observations are all exporters; at the next percentile are exporter observations with sales in the top 99 percentiles; up to the right-most observations with exporters whose sales are in the top percentile.

Figure 3: **Mean Exporter Scale and Mean Exporter Scope**

tribution that the log-log relationship in such a cumulative plot is linear. A power-law distribution implies that less frequent outcomes command a more than proportionate share of the total while the most frequent outcomes represent only a subordinate fraction. So there are few large-sales firms but many small-sales firms. (Plots of total exports against the total exports percentiles confirm the inference and resemble findings in Eaton, Kortum, and Kramarz (2010).) The plots for sales are similar across Brazilian destinations. Next we investigate the relationship between exporter scale and exporter scope.

Fact 3 *Within destinations, mean exporter scope and mean exporter scale are positively associated.*

One might expect from Fact 1 that wide-scope firms would have low average sales per product because they adopt more products with minor sales. The opposite is the case. In Figure 3 we plot firms' mean scope and scope-weighted mean exporter scale at a destination against these firms' rank in total exports at that destination.[15]

The means in Figure 3 are computed in the same way as mean exports before and are linear decompositions of their counterparts in the lower panel of Figure 2, by construction. On the horizontal axis, we group firms at or above a given total exports percentile. At the origin, we cumulate all firms and plot their mean scope $\bar{G}_{sd}$ and the scope-weighted mean exporter scale $\bar{a}_{sd}$ so that the product of the two means yields mean total sales $\bar{t}_{sd}$. Then we move upwards in the total-exports ranking of firms, percentile by percentile, dropping from the sample all those firms that are below the next higher total-exports percentile and depict mean exporter scope and mean average scale for the higher-ranked group of firms.

The log mean scope and log mean scale both increase in the firms' log percentile. The increases are close to linear in the two export markets United States and Argentina and, on average, in the world (treating all destinations as a single market). This log-linear pattern is also visible in groups of ten similarly ranked destinations (70 destinations with at least 100 Brazilian exporters where destinations are first ranked by total sales and then lumped to seven groups of ten countries). Overall, Figure 3 strongly suggests that there is a systematically positive relationship between *average* exporter scale and exporter scope.

2.3 Product shipments across destinations

Before we turn to a firm-level model, we investigate whether firms systematically sell their most successful products across destinations.[16] We present evidence that a firm's successful products in one market are also its leading products in other markets, and that a firm's successful products reach a larger number of markets. To document systematic sales patterns by product and firm across markets, we use the United States as our reference country. The United States is Brazil's top export destination in 2000.

[15] Scope-weighted mean exporter scale is $[\sum_\omega G_d(\omega) a_d(\omega)]/\sum_\omega G_d(\omega) = \sum_\omega t_d(\omega)/\sum_g G_d(\omega)$. For unweighted mean exporter scale, a similar positive association as depicted in Figure 3 arises. We present those figures and numerous additional results in our online Data Appendix.

[16] With the critique by Armenter and Koren (2010) and their balls-and-bins model of trade in mind, we pursue this analysis also to show that systematic patterns in product entry are inconsistent with a simple stochastic model where exporters are just random collections of products.

Table 1: Overlaps between Reference Countries and Rest of World by Product Rank

Product rank in Ref. country	Reference country: USA				Reference country: Argentina			
	Overlap (1)	Overlap top prd. (2)	#Dest./ firm (3)	#Firms (4)	Overlap (5)	Overlap top prd. (6)	#Dest./ firm (7)	#Firms (8)
1	.83	.83	8.9	2,280	.77	.77	7.8	3,071
2	.54	.77	13.0	1,033	.54	.76	10.7	1,672
4	.36	.73	18.9	368	.38	.67	14.2	797
8	.34	.69	24.1	137	.30	.63	18.5	307
16	.26	.59	24.3	63	.24	.54	22.6	136
32	.24	.53	30.2	22	.22	.50	29.7	48
64	.15	.49	38.9	10	.29	.40	35.9	19
128	.13	.69	42.4	5	.11	.33	43.8	12

Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Destination counts in columns 3 and 7 are mean numbers of destinations to which firms with at least as many products as reported for a rank ship. Overlap in columns 1 and 5 is the proportion of destinations that a product of reported rank reaches relative to the overall destination counts (in columns 3 and 7). Overlap in columns 2 and 6 is the proportion of destinations that the top-selling product of firms with at least as many products as reported for a rank reaches relative to the overall destination counts (in columns 3 and 7). Products at the HS 6-digit level, ranked by decreasing export value within firm in reference country. Sample restricted to firm-products that ship to reference country and at least one other destination.

First, within a firm, the leading products have systematically higher sales market by market. We number products within a firm and a destination by their rank in the firm's local sales, assigning rank one to the firm's top-selling product at a destination, rank two to the firm's second-to-top product at the destination, and so forth. For each given HS 6-digit product that a firm sells in the United States we correlate the firm-product's rank elsewhere with the firm-product's U.S. rank. We find a correlation coefficient of .747 and a Spearman's rank correlation coefficient of .837.[17]

Second, lower ranked products reach systematically fewer destinations. Table 1 documents that the number of destinations where a firm ships a product drops with the product's rank in the reference country. Consider the top-ranked product in the United States for the 2,280 firms that ship at least one product to the United States (including single-product exporters to the United States). These firms reach 8.9 destinations on average and their top-selling product ships to 83 percent of the destinations that the firms reach with any product. Firms that sell at least two products in the United States reach on average 13.0 destinations but their second-ranked product only ships to a fraction of 54 percent of the destinations reached with any product. This fraction drops to

[17] When we repeat the exercise for Argentina (the second most important Brazilian export destination) as reference country, we find an even higher correlation coefficient of .785 and a Spearman's rank correlation coefficient of .860 for the same firm's and same product's ranks elsewhere.

36 percent for the fourth-ranked product for firms with at least 4 products in the United States and to 13 percent for the 128th ranked product. For Argentina as a reference country, the fraction drops systematically from 77 percent for the top-selling product to 11 percent for the 128th ranked product.

Finally, we report evidence that export scale per product is positively associated at the individual firm-product level, within industries and destinations (and not just across groups of firms as Fact 3 showed). For our sample of manufacturing exporters and their individual manufactured products, a regression of the log sales per product in a market on the seller's log exporter scope in the same market, controlling for industry and destination fixed effects, documents a coefficient that is positive (0.072) and statistically significantly different from zero at the 1-percent level. So, wide-scope exporters in a destination also receive systematically higher revenues for each individual product. This finding refutes the hypothesis that a firm is a random collection of products. For a random collection of products, the exporter scale would be independent of the exporter scope in a market.[18]

We turn to a model of exporting that generates the three stylized facts, and then revisit the data to empirically evaluate the derived relationships. The model strives to explain the behavior of multi-product exporters and to be quantitatively meaningful when matching the multi-product facts at successive levels of aggregation. The characteristic log-linear relationships in the data will motivate the choice of functional forms later.

3 A Model of Exporter Scope and Exporter Scale

Our model rests on a single source of firm heterogeneity. Firms sell one or multiple products in the markets where they enter. There are three key ingredients: a firm's overall productivity that affects all products of the firm worldwide; firm-product specific efficiency that determines individual product sales worldwide; and local fixed entry costs that depend on the number of products that a firm sells in each destination market.

[18] If firms drew their product sizes from the same distribution (even if this distribution were truncated so that only a fraction of the firm's products made it to a given market), then the scale of each firm-product would not be related to the firm's scope.

3.1 Consumers

There are N countries. We label the source country of an export shipment with s and the export destination with d. There is a measure of L_d consumers at destination d. Consumers have symmetric preferences with a constant elasticity of substitution σ over a continuum of varieties. In our multi-product setting, a conventional "variety" offered by a firm ω from source country s to destination d is the product composite

$$X_{sd}(\omega) \equiv \left(\sum_{g=1}^{G_{sd}(\omega)} x_{sdg}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $G_{sd}(\omega)$ is the number of products that firm ω sells in country d and $x_{sdg}(\omega)$ is the quantity of product g that consumers consume. In marketing terminology, the product composite is often called a firm's product line or product mix. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations.

The consumer's utility at destination d is

$$\left(\sum_{k=1}^{N} \int_{\omega \in \mathbf{\Omega}_{kd}} X_{kd}(\omega)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \qquad \text{for } \sigma > 1, \tag{1}$$

where $\mathbf{\Omega}_{kd}$ is the set of firms that ship from source country k to destination d. For simplicity we assume that the elasticity of substitution across a firm's products is the same as the elasticity of substitution between varieties of different firms.[19]

The representative consumer earns a wage w_d from inelastically supplying her unit of labor endowment to producers in country d and receives a per-capita dividend distribution π_d equal to her share $1/L_d$ in total profits at national firms. We denote total income with $Y_d = (w_d + \pi_d)L_d$. The consumer's first-order conditions of utility maximization imply a product demand

$$x_{sdg}(\omega) = \left(\frac{p_{sdg}}{P_d} \right)^{-\sigma} \frac{T_d}{P_d}, \tag{2}$$

[19] In Appendix C (and in our companion paper Arkolakis and Muendler (2010) for a continuum of products), we generalize the model to consumer preferences with two nests. The inner nest contains the products of a firm, which are substitutes with an elasticity of ε. The outer nest aggregates those firm-level product lines over firms and source countries, where the product lines are substitutes with a different elasticity $\sigma \neq \varepsilon$. In this paper we set $\varepsilon = \sigma$. The general case of $\varepsilon \neq \sigma$ is fully consistent with the key regularities that we uncover but it introduces additional degrees of freedom into the model that cannot be disciplined with three-dimensional firm-product-destination data such as ours.

Allanson and Montagna (2005) adopt a similar nested CES form to study the product life-cycle and market structure, and Atkeson and Burstein (2008) use a similar nested CES form in a heterogeneous-firms model of trade but do not consider multi-product firms.

where p_{sdg} is the price of product g in market d and we denote by T_d the total spending of consumer in country d. In the calibration, we will allow for the possibility that total spending T_d is different from country output Y_d so that we use different notation for the two terms. We define the corresponding ideal price index P_d as

$$P_d \equiv \left[\sum_{k=1}^{N} \int_{\omega \in \boldsymbol{\Omega}_{kd}} \sum_{g=1}^{G_{kd}(\omega)} p_{kdg}(\omega)^{-(\sigma-1)} \, \mathrm{d}\omega \right]^{-\frac{1}{\sigma-1}}. \tag{3}$$

3.2 Firms

Following Chaney (2008), we assume that there is a continuum of potential producers of measure J_s in each source country s. Productivity is the only source of firm heterogeneity so that, under the model assumptions below, firms of the same type ϕ from country s face an identical optimization problem in every destination d. Since all firms with productivity ϕ will make identical decisions in equilibrium, it is convenient to name them by their common characteristic ϕ from now on.

A firm of type ϕ chooses the number of products $G_{sd}(\phi)$ to sell to a given market d. The firm makes each product $g \in \{1, 2, \ldots, G_{sd}(\phi)\}$ with a linear production technology, employing local labor with efficiency ϕ_g. When exported, a product incurs a standard iceberg trade cost so that $\tau_{sd} > 1$ units must be shipped from s for one unit to arrive at destination d. We normalize $\tau_{ss} = 1$ for domestic sales. Note that this iceberg trade cost is common to all firms and to all firm-products shipping from s to d.

Without loss of generality we order each firm's products in terms of their efficiency so that $\phi_1 \geq \phi_2 \geq \ldots \geq \phi_{G_{sd}}$. Ranking products by consumer appeal would generate isomorphic results for within-firm product sales heterogeneity. A firm will enter export market d with the most efficient product first and then expand its scope moving up the marginal-cost ladder product by product. Under this convention we write the efficiency of the g-th product of a firm ϕ as

$$\phi_g \equiv \frac{\phi}{h(g)} \qquad \text{with} \quad h'(g) > 0. \tag{4}$$

We normalize $h(1) = 1$ so that $\phi_1 = \phi$. We think of the function $h(g) : [0, +\infty) \to [1, +\infty)$ as a continuous and differentiable function but we will consider its values at discrete points $g = 1, 2, \ldots, G_{sd}$ as appropriate.[20]

[20]Considering the function in its whole domain allows us to express various conditions in a general form as we will illustrate later on. The function $h(g)$ could be considered destination specific but such generality would introduce degrees of freedom that are not required for our analysis.

By varying firm-product efficiencies, some products will sell systematically more across markets (as empirically documented in Section 2.3 above). In turn, the assumption that the firm faces a drop in efficiency for each additional product when its exporter scope widens is a common assumption in multi-product models of exporters. Similar models are Eckel and Neary (2010), who define the product with the highest efficiency as the “core competency” of the firm, and Mayer, Melitz, and Ottaviano (2009). Nocke and Yeaple (2006), in contrast, assume that wider scope reduces efficiency for all infra-marginal products.

Related to the marginal-cost schedule $h(g)$ we define firm ϕ's product efficiency index as

$$H(G_{sd}) \equiv \left(\sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}} . \tag{5}$$

This efficiency index will play an important role in the firm's optimality conditions for scope choice. Since the marginal-cost schedule strictly increases in exporter scope, a firm's product efficiency index strictly decreases as its exporter scope widens, resembling the insight from the stochastic firm-product model of Bernard, Redding, and Schott (2010a).

As the firm widens its exporter scope, it also faces a product-destination specific incremental local entry cost $f_{sd}(g)$ that is zero at zero scope and strictly positive otherwise:[21]

$$f_{sd}(0) = 0 \quad \text{and} \quad f_{sd}(g) > 0 \quad \text{for all } g = 1, 2, \ldots, G_{sd}, \tag{6}$$

where $f_{sd}(g)$ is a continuous function in $[1, +\infty)$.

The incremental local entry cost $f_{sd}(g)$ accommodates fixed costs of production (e.g. with $0 < f_{ss}(g) < f_{sd}(g)$). In a market, the incremental local entry costs $f_{sd}(g)$ may increase or decrease with exporter scope. But a firm's local entry costs

$$F_{sd}\left(G_{sd}\right) = \textstyle\sum_{g=1}^{G_{sd}} f_{sd}(g)$$

necessarily increase with exporter scope G_{sd} in country d because $f_{sd}(g) > 0$.[22] We assume that the incremental local entry costs $f_{sd}(g)$ are paid in terms of importer (destination country) wages

[21] Brambilla (2009) adopts a related specification but its implications are not explored in an equilibrium firm-product model.

[22] As long as the firm's first product causes a nontrivial fixed local entry cost, we do not need any additional fixed local entry cost. In continuous product space with nested CES utility, in contrast, local entry costs must be non-zero at zero scope because a firm would otherwise export to all destinations worldwide (see Arkolakis and Muendler 2010, Bernard, Redding, and Schott 2010a).

so that $F_{sd}(G_{sd})$ is homogeneous of degree one in w_d. Combined with the preceding varying firm-product efficiencies, this local entry cost structure allows us to endogenize the exporter scope choice at each destination d.

In summary, there are two scope-dependent cost components in our model, the marginal cost schedule $h(g)$ and the incremental local entry cost $f_{sd}(g)$. Suppose for a moment that the incremental local entry cost is constant and independent of g with $f_{sd}(g) = f_{sd}$. Then a firm in our model faces diseconomies of scope because the marginal-cost schedule $h(g)$ strictly increases with the product index g. But, if incremental local entry costs decrease sufficiently strongly with g, there could be overall economies of scope.

A firm with a productivity ϕ from country s faces the following optimization problem for selling to destination market d

$$\pi_{sd}(\phi) = \max_{G_{sd}, p_{sdg}} \sum_{g=1}^{G_{sd}} \left(p_{sdg} - \tau_{sd} \frac{w_s}{\phi / h(g)} \right) \left(\frac{p_{sdg}}{P_d} \right)^{-\sigma} \frac{T_d}{P_d} - F_{sd}(G_{sd}).$$

The firm's first-order conditions with respect to individual prices p_{sdg} imply product prices

$$p_{sdg}(\phi) = \tilde{\sigma}\, \tau_{sd}\, w_s\, h(g)/\phi \tag{7}$$

with an identical markup over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma - 1) > 1$ for $\sigma > 1$.[23] A firm's choice of optimal prices implies optimal product sales for product g

$$p_{sdg}(\phi) x_{sdg}(\phi) = \left(\frac{P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \frac{\phi}{h(g)} \right)^{\sigma-1} T_d. \tag{8}$$

Summing (8) over the firm's products at destination d, firm ϕ's optimal total exports to destination d are

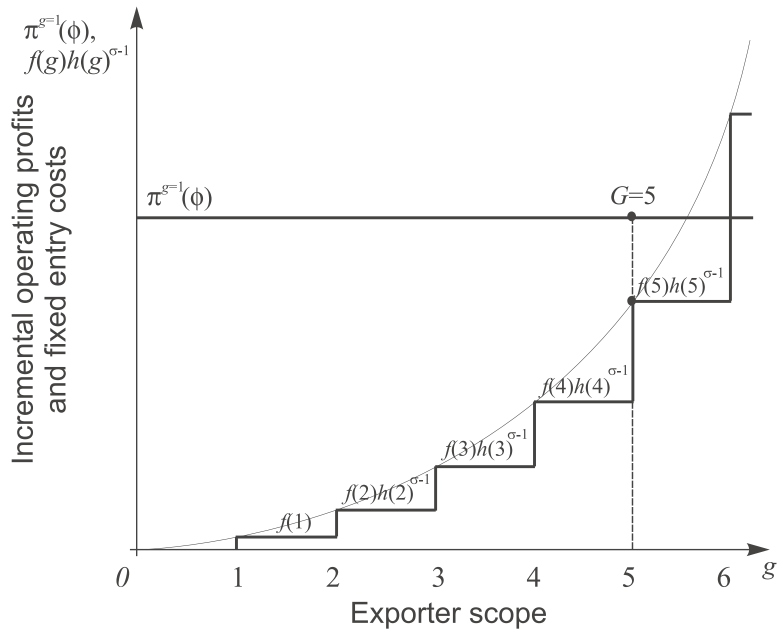
$$t_{sd}(\phi) = \sum_{g=1}^{G_{sd}(\phi)} p_{sdg}(\phi)\, x_{sdg}(\phi) = \left(\frac{P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \phi \right)^{\sigma-1} T_d\, H\big(G_{sd}(\phi)\big)^{-(\sigma-1)}, \tag{9}$$

where $H(G_{sd})$ is a firm's product efficiency index from (5). The term $H(G_{sd}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{sd}(\phi)$.

Given constant markups over marginal cost, profits at a destination d for a firm ϕ selling G_{sd} are

$$\pi_{sd}(\phi) = \left(\frac{P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \phi \right)^{\sigma-1} \frac{T_d}{\sigma} H\big(G_{sd}\big)^{-(\sigma-1)} - \sum_{g=1}^{G_{sd}} f_{sd}(g).$$

[23] Similarly, in continuous product space (Arkolakis and Muendler 2010) the optimal markup does not vary with exporter scope for constant elasticities of substitution under monopolistic competition (even in the general case of different elasticities of substitution $\varepsilon \neq \sigma$).



Note: Operating profits for the core product are $\pi^{g=1}(\phi) = [P\,\phi/(\tilde{\sigma}\,\tau\,w)]^{\sigma-1}T/\sigma$. Combined incremental scope costs $z(g) \equiv f(g)\,h(g)^{\sigma-1}$ strictly increase in g by Assumption 1, with $f(0) = 0$ and $h(1) = 1$.

Figure 4: **Optimal Exporter Scope**

For profit maximization with respect to exporter scope to be well defined, we impose the following condition.

Assumption 1 (Strictly increasing combined incremental scope costs). *Combined incremental scope costs $z_{sd}(G) \equiv f_{sd}(G)\,h(G)^{\sigma-1}$ strictly increase in exporter scope G.*

Under this assumption, the optimal choice for $G_{sd}(\phi)$ is the largest $G \in \{0, 1, \ldots\}$ such that operating profits from that product equal (or still exceed) the incremental local entry costs:

$$\left(\frac{P_d}{\tilde{\sigma}\,\tau_{sd}\,w_s}\,\frac{\phi}{h(G)}\right)^{\sigma-1}\frac{T_d}{\sigma} \;\geq\; f_{sd}(G) \quad \Longleftrightarrow$$

$$\pi_{sd}^{g=1}(\phi) \equiv \left(\frac{P_d\,\phi}{\tilde{\sigma}\,\tau_{sd}\,w_s}\right)^{\sigma-1}\frac{T_d}{\sigma} \;\geq\; f_{sd}(G)\,h(G)^{\sigma-1} \equiv z_{sd}(G). \tag{10}$$

Operating profits from the core product are $\pi_{sd}^{g=1}(\phi)$, and operating profits from each additional product g are $\pi_{sd}^{g=1}(\phi)/h(g)^{\sigma-1}$.

Figure 4 depicts the choice of optimal exporter scope. A firm will keep widening its exporter scope as long as adding products does not reduce total profits. Equivalently, a firm will keep widening scope as long as incremental scope costs $z_{sd}(g)$ are weakly less than the firm's core operating profits $\pi_{sd}^{g=1}(\phi)$. In this optimality condition, incremental local entry cost and costs from declining product efficiency enter multiplicatively and their product must increase in scope for a well defined optimum to exist. Thus, Assumption 1 is comparable to a second-order condition (for perfectly divisible scope in the continuum version of the model, Assumption 1 is equivalent to the second order condition). When Assumption 1 holds we will say that a firm faces *overall diseconomies of scope*.

We can express the condition for optimal scope more intuitively and evaluate the optimal scope of different firms. Firm ϕ exports from s to d iff $\pi_{sd}(\phi) \geq 0$. At the break-even point $\pi_{sd}(\phi) = 0$, the firm is firm is indif and only iferent between selling its first product to market d and remaining absent. Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold ϕ_{sd}^* for exporting from s to d is given by

$$(\phi_{sd}^*)^{\sigma-1} \equiv \frac{\sigma f_{sd}(1)}{T_d} \left(\frac{\tilde{\sigma}\, \tau_{sd}\, w_s}{P_d} \right)^{\sigma-1}. \tag{11}$$

In general, we can define the productivity threshold $\phi_{sd}^{*,G}$ such that firms with $\phi \geq \phi_{sd}^{*,G}$ sell at least G_{sd} products as

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} \equiv \frac{\sigma z_{sd}(G)}{T_d} \left(\frac{\tilde{\sigma}\, \tau_{sd}\, w_s}{P_d} \right)^{\sigma-1},$$

where $z_{sd}(G) \equiv f_{sd}(G)\, h(G)^{\sigma-1}$, or more succinctly, using (11), as

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{z_{sd}(G)}{f_{sd}(1)} \left(\phi_{sd}^*\right)^{\sigma-1}, \tag{12}$$

under the convention that $\phi_{sd}^* \equiv \phi_{sd}^{*,1}$. Note that if Assumption 1 holds then $\phi_{sd}^* < \phi_{sd}^{*,2} < \phi_{sd}^{*,3} < \ldots$ so that more productive firms introduce more products in a given market. So $G_{sd}(\phi)$ is a step-function that weakly increases in ϕ.

Using the above definitions, we can rewrite individual product sales (8) and total sales (9) as

$$\begin{aligned} p_{sdg}(\phi) x_{sdg}(\phi) &= \sigma\, f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} h(g)^{-(\sigma-1)} \\ &= \sigma\, z_{sd}(G_{sd}(\phi)) \left(\frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} h(g)^{-(\sigma-1)} \end{aligned} \tag{13}$$

and

$$t_{sd}(\phi) = \sigma\, f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} H\big(G_{sd}(\phi)\big)^{-(\sigma-1)}. \tag{14}$$

The following proposition summarizes the findings.

Proposition 1 *If Assumption 1 holds, then for all $s, d \in \{1, \ldots, N\}$*

- *exporter scope $G_{sd}(\phi)$ is positive and weakly increases in ϕ for $\phi \geq \phi^*_{sd}$;*
- *total firm exports $t_{sd}(\phi)$ are positive and strictly increase in ϕ for $\phi \geq \phi^*_{sd}$.*

Proof. The first statement follows directly from the discussion above. The second statement follows because $H(G_{sd}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{sd}(\phi)$ and $G_{sd}(\phi)$ weakly increases in ϕ so that $t_{sd}(\phi)$ strictly increases in ϕ by (14). ■

The firm's equilibrium choices for total sales $t_{sd}(\phi)$ and the number of products sold $G_{sd}(\phi)$ determine its *exporter scale* in market d,

$$a_{sd}(\phi) \equiv \frac{t_{sd}(\phi)}{G_{sd}(\phi)} = \sigma\, f_{sd}(1) \left(\frac{\phi}{\phi^*_{sd}}\right)^{\sigma-1} \frac{H\big(G_{sd}(\phi)\big)^{-(\sigma-1)}}{G_{sd}(\phi)}, \tag{15}$$

conditional on exporting from s to d. In Section 2, we presented scope-weighted mean exporter scale. Exporter scale $a_{sd}(\phi)$ is tightly related to scope-weighted mean exporter scale: if $a_{sd}(\phi)$ is a monotonic function of productivity then scope-weighted mean exporter scale is a monotonic function of productivity.[24] In our model, it is easy to work with $a_{sd}(\phi)$ so we will characterize its analytical properties to describe scope-weighted mean exporter scale. Under an additional restriction, $a_{sd}(\phi)$ increases in firm productivity ϕ and therefore also in firm total sales:

Restriction 1 (Strong overall diseconomies of scope). *Combined incremental scope costs $z_{sd}(G) \equiv f_{sd}(G)\, h(G)^{\sigma-1}$ strictly increase in G with an elasticity*

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1.$$

Restriction 1 is more stringent than Assumption 1 in that the restriction not only requires z_{sd} to increase with G but that the increase be more than proportional. We can then state the following result.[25]

Proposition 2 *If $z_{sd}(G)$ satisfies Restriction 1, then exporter scale $a_{sd}(\phi)$ strictly increases in ϕ at the thresholds $\phi = \phi^*_{sd}, \phi^{*,2}_{sd}, \phi^{*,3}_{sd}, \ldots, \phi^{*,G}_{sd}$.*

[24] To see this note that $(t(\phi) + x) \,/\, (G(\phi) + y) \leq x/y \iff t(\phi)/G(\phi) \leq x/y$. So if t/G declines then excluding lower percentiles in scope-weighted mean exporter scale leads to increases in its value.

[25] Whereas the proposition demonstrates that the function $a_{sd}(\phi)$ generically increases in ϕ, this statement is not true for all ϕ. The simple reason is that the choice of products (the denominator of the function $a_{sd}(\phi)$) is a step function that depends on combined incremental scope costs $z_{sd}(G)$. The summation in the numerator of $a_{sd}(\phi)$ is also a step function but one that only depends on $h(G)$, thus rendering stronger statements about $a_{sd}(\phi)$ elusive.

Proof. See Appendix A.1. ■

This proposition is particularly informative in situations where $f_{sd}(g)$ is a strictly decreasing function. In such situations a highly productive firm adds many low-selling products because the firm can generate additional profits from these products as $f_{sd}(g)$ declines. So it is possible in such situations that wide-scope firms would have low exporter scale. Restriction 1, however, suffices to guarantee that scale increases even if $f_{sd}(g)$ is a strictly decreasing function: it implies that the efficiencies of marginal products decline so fast that only highly productive firms introduce them. These productive firms have high sales for their top selling products, which means that their overall scale is larger.[26]

The model can also parsimoniously generate the concentration of a firm's sales in its core products. To do that we need to introduce an additional sufficient restriction on $h(g)$.

Restriction 2 (Bounded firm-product efficiency). *The marginal-cost schedule* $h(\cdot)$ *results in bounded firm-product efficiency*

$$\lim_{G\to\infty} H(G)^{-(\sigma-1)} = \sum_{g=1}^{\infty} h(g)^{-(\sigma-1)} \in (0, +\infty) \, .$$

A number of conventional real analysis tests (e.g. the root test or the ratio test, see Rudin 1976, ch. 3) can be used to determine whether the sum converges by looking at the limiting terms $h(g)^{-(\sigma-1)}$ as $g \to \infty$. Formally, when this sum converges, the minimum share of a product g' is bounded from below by the finite number $h(g')^{-(\sigma-1)} / \sum_{g=1}^{+\infty} h(g)^{-(\sigma-1)}$. Intuitively, Restriction 2 implies that the "core" products account for a significant share of total sales, which remain bounded even if many additional products are added.

4 Model Equilibrium and Model Predictions

To derive clear predictions for the model equilibrium we specify a Pareto distribution of firm productivity following Helpman, Melitz, and Yeaple (2004) and Chaney (2008). A firm's productivity ϕ is drawn from a Pareto distribution with a source-country dependent location parameter b_s and a shape parameter θ over the support $[b_s, +\infty)$ for $s = 1, \ldots, N$. So the cumulative distribution

[26] Restriction 1 is a sufficient condition for the proposition. Examples can be found where Restriction 1 fails but $a_{sd}(\phi)$ generically still increases in ϕ. The result that scale increases with scope is not trivial and does not necessarily generalize to other setups. In the Bernard, Redding, and Schott (2010a) multi-product model, for instance, it can be shown that $a_{sd}(\phi)$ is constant under a Pareto distribution of product-specific demand shocks.

function of ϕ is $\Pr = 1 - (b_s)^\theta/\phi^\theta$ and the probability density function is $\theta(b_s)^\theta/\phi^{\theta+1}$, where more advanced countries are thought to have a higher location parameter b_s. Therefore the measure of firms selling to country d, that is the measure of firms with productivity above the threshold ϕ^*_{sd}, is

$$M_{sd} = J_s(b_s)^\theta/(\phi^*_{sd})^\theta. \tag{16}$$

The probability density function of the conditional distribution of entrants is given by

$$\mu_{sd}(\phi) = \begin{cases} \theta(\phi^*_{sd})^\theta/\phi^{\theta+1} & \text{if } \phi \geq \phi^*_{sd}, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

4.1 Equilibrium and the gravity equation of trade

Under the Pareto assumption we can compute several aggregate statistics for the model. We denote aggregate bilateral sales of firms from s to country d with T_{sd}. The corresponding average sales per firm are defined as $\bar{T}_{sd}$, so that $T_{sd} = M_{sd}\bar{T}_{sd}$ and

$$\bar{T}_{sd} = \int_{\phi^*_{sd}} t_{sd}(\phi)\, \mu_{sd}(\phi)\, \mathrm{d}\phi. \tag{18}$$

Similarly, we define average local entry costs as

$$\bar{F}_{sd} = \int_{\phi^*_{sd}} F_{sd}(G_{sd}(\phi))\, \mu_{sd}(\phi)\, \mathrm{d}\phi.$$

To compute $\bar{F}_{sd}$ in terms of fundamentals we need two further necessary assumptions.

Assumption 2 (Pareto probability mass in low tail). *The Pareto shape parameter satisfies* $\theta > \sigma - 1$.

Assumption 3 (Bounded local entry costs and product efficiency). *Incremental local entry costs and product efficiency satisfy* $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} \in (0, +\infty)$, *where* $\tilde{\theta} \equiv \theta/(\sigma-1)$.

Assumptions 2 and 3 guarantee that average sales per firm are positive and finite.

Proposition 3 *Suppose Assumptions 1, 2 and 3 hold. Then for all* $s, d \in \{1, \ldots, N\}$, *average sales per firm are a constant multiple of average local entry costs:*

$$\bar{T}_{sd} = \frac{\tilde{\theta}\,\sigma}{\tilde{\theta}-1} f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} = \frac{\tilde{\theta}\,\sigma}{\tilde{\theta}-1}\, \bar{F}_{sd}, \tag{19}$$

where $\tilde{\theta} \equiv \theta/(\sigma-1)$.

Proof. See Appendix A.2. ■

The share of total local entry costs in total exports $\bar{F}_{sd}/\bar{T}_{sd}$ only depends on the model's parameters θ and σ, even though local entry costs vary by source and destination country. So, despite firm-product heterogeneity, bilateral average sales can be summarized with a function only of the parameters θ and σ and the properties of average local entry costs $\bar{F}_{sd}$.

Finally, we can use definition (16) of M_{sd} together with definition (11) of ϕ^*_{sd} and expression (19) for average sales to derive bilateral expenditure shares of country d on products from country s

$$\lambda_{sd} = \frac{M_{sd}\bar{T}_{sd}}{\sum_k M_{kd}\bar{T}_{kd}} = \frac{J_s(b_s)^\theta (w_s\tau_{sd})^{-\theta}\, f_{sd}(1)^{-\tilde{\theta}}\bar{F}_{sd}}{\sum_k J_k(b_k)^\theta (w_k\tau_{kd})^{-\theta}\, f_{kd}(1)^{-\tilde{\theta}}\bar{F}_{kd}}, \tag{20}$$

where $\tilde{\theta} \equiv \theta/(\sigma-1)$, and $f_{sd}(1)^{-\tilde{\theta}}\bar{F}_{sd} = \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)}h(G)^{-\theta}$ by equation (19).

Remarkably, the elasticity of trade with respect to variable trade costs is $-\theta$, as in Eaton and Kortum (2002) and Chaney (2008).[27] Thus, our framework is consistent with bilateral gravity. The difference between our model, in terms of aggregate bilateral trade flows, and the framework of Eaton and Kortum (2002) is that fixed costs affect bilateral trade similar to Chaney (2008). Beyond previous work, we provide a micro-foundation as to how entry cost components affect aggregate bilateral trade through the weighted sum $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)}h(G)^{-\theta}$. So our model offers a tool to evaluate the responsiveness of overall trade to changes in individual entry cost components.

The partial elasticity $\eta_{\lambda,f(g)}$ of trade with respect to a product g's entry cost component is $-(\tilde{\theta}-1)$ times the product's share in the weighted sum. To assess the relative importance of the extensive margin of exporting products, relative to firm entry with the core product, we can compare elasticities using the ratio

$$\frac{\eta_{\lambda,f(g)}}{\eta_{\lambda,f(1)}} = \frac{f_{sd}(g)^{-(\tilde{\theta}-1)}h(g)^{-\theta}}{f_{sd}(1)^{-(\tilde{\theta}-1)}} \tag{21}$$

for $g = 2, \ldots$ and the standardization $h(1) = 1$. Our model does not restrict this ratio to increase or decrease with g as a product becomes less important in the within-firm sales distribution. It therefore remains an empirical matter to quantify the importance of product entry relative to firm entry when entry costs change.

We can also compute mean exporter scope in a destination. For the average number of products to be finite we will need the necessary assumption that

[27] In our model, the elasticity of trade with respect to trade costs is the negative Pareto shape parameter, whereas it is the negative Fréchet shape parameter in Eaton and Kortum (2002).

Assumption 4 (Strongly increasing combined incremental scope costs). *Combined incremental scope costs satisfy* $\sum_{G=1}^{\infty} z_{sd}(G)^{-\tilde{\theta}} \in (0, +\infty)$.

This assumption is in general more restrictive than Assumption 1. It requires that combined incremental scope costs $Z(G)$ do not just increase in G, but increase at a rate faster than $1/\tilde{\theta}$.[28]

Mean exporter scope in a destination is

$$\bar{G}_{sd} = \int_{\phi^*_{sd}} G_{sd}(\phi)\,\theta\,\frac{(\phi^*_{sd})^\theta}{(\phi)^{\theta+1}}\,\mathrm{d}\phi = (\phi^*_{sd})^\theta\,\theta \left[\int_{\phi^*_{sd}}^{\phi^{*,2}_{sd}} \phi^{-(\theta+1)}\,\mathrm{d}\phi + \int_{\phi^{*,2}_{sd}}^{\phi^{*,3}_{sd}} 2\,\phi^{-(\theta+1)}\,\mathrm{d}\phi + \ldots\right].$$

Completing the integration, rearranging terms and using equation (12), we obtain

$$\bar{G}_{sd} = f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} z_{sd}(G)^{-\tilde{\theta}}. \tag{22}$$

The expression implies that mean exporter scope is invariant to destination market characteristics other than local entry costs. A priori there is no reason why mean exporter scope $\bar{G}_{sd}$ should be related to bilateral distance between s and d or to the size of the destination market. This implication resonates with the evidence of highly robust scope distributions across destinations as presented in Section 2.2.[29]

We turn to the model's equilibrium. Notice that total manufacturing output of a country s equals its total sales across all destinations:

$$Y_s = \sum_{k=1}^{N} \lambda_{sk}\,T_k. \tag{23}$$

Additionally, Proposition 3 implies that a country's total spending on fixed local entry costs is a constant (source country invariant) share of bilateral exports. This result implies that the share of wages in total income is constant (source country invariant). To see why observe that the share of net profits from bilateral sales is the share of gross variable profits in total sales $1/\sigma$ less the fixed costs paid and divided by total sales $(\tilde{\theta}-1)/\tilde{\theta}\,\sigma$. Thus, using the result of Proposition 3, $\pi_{sd}L_d/T_{sd} = 1/\sigma - (\tilde{\theta}-1)/(\tilde{\theta}\sigma) = 1/(\tilde{\theta}\,\sigma) = 1/(\theta\,\tilde{\sigma})$. Total profits for country s are $\pi_s L_s = \sum_k \lambda_{sk}\,T_k/(\tilde{\theta}\sigma)$, where $\sum_k \lambda_{sk}T_k$ is the country's total income by (23). So profit income and

[28]To see that, rearrange the expression to $\sum_{G=1}^{\infty}[f_{sd}(G)h(G)^{\sigma-1}]^{-\tilde{\theta}} = \sum_{G=1}^{\infty}[Z(G)]^{-\tilde{\theta}}$ and notice that the ratio rule (see Rudin 1976, ch. 3) requires that $Z(G)$ increases at a rate faster than $1/\tilde{\theta}$ so that the sum converges.

[29]A regression of Brazil's mean exporter scope on two main source-destination characteristics—market share and import market size (see Appendix D.1)—shows that mean exporter scope responds relatively little to country characteristics, whereas the number of firms shipping to a destination is closely related to those characteristics (similar to Eaton, Kortum, and Kramarz 2004).

Table 2: Parametric Functional Forms

Condition		Parameter values
Ass. 1	Strictly increasing combined incremental scope costs	$\delta+\alpha(\sigma-1)>0$
Ass. 2	Pareto probability mass in low tail	$\theta>\sigma-1$
Ass. 3	Bounded local entry costs	$\delta+\alpha(\sigma-1)>(\delta+1)/\tilde{\theta}$
Ass. 4	Strongly increasing combined incremental scope costs	$\delta+\alpha(\sigma-1)>1/\tilde{\theta}$
Restr. 1	Strong overall diseconomies of scope	$\delta+\alpha(\sigma-1)>1$
Restr. 2	Bounded firm-product efficiency	$\alpha(\sigma-1)>1$

Note: Functional forms $f_{sd}(g)=f_{sd}\cdot g^{\delta}$ and $h(g)=g^{\alpha}$ by (25).

wage income can be expressed as constant shares of total income:

$$\pi_s L_s = \frac{1}{\tilde{\theta}\sigma} Y_s \quad \text{and} \quad w_s L_s = \frac{\tilde{\theta}\sigma-1}{\tilde{\theta}\sigma} Y_s. \tag{24}$$

We can now define an equilibrium in this economy, assuming for simplicity that trade is balanced with $Y_d = T_d$. (We will relax this assumption in the calibration.) Given τ_{sd}, J_s, b_s and definitions (16) and (17) for all $s,d=1,\ldots,N$, an equilibrium is a set of firm-product consumption allocations for the representative consumer $x_{sdg}(\phi)$ and prices and exporter scopes for the representative firms $[p_{sdg}(\phi), G_{sd}(\phi)]$ for $g=1,\ldots,G_{sd}(\phi)$ and $\phi\in\mathbf{\Omega}_{sd}$, and a set of wages w_s, such that (i) equation (2) is the solution of the representative consumer optimization program, (ii) equations (7) and (10) solve the firm profit maximization programs, (iii) the current account balance condition (23) holds in every country s where λ_{sd} is given by (20), and (iv) P_d and $\phi^{*,G}_{sd}$ jointly satisfy equations (3) and (11) with Y_s given by equation (24).

4.2 Predictions for the cross section of firms

Parametrizing the model allows us to quantitatively match the patterns that we observe in the Brazilian data. Guided by the various log-linear relationships observed in Section 2.2 we set

$$\begin{aligned} f_{sd}(g) &= f_{sd}\cdot g^{\delta} \quad \text{for } \delta\in(-\infty,+\infty), \\ h(g) &= g^{\alpha} \qquad\quad \text{for } \alpha\in[0,+\infty). \end{aligned} \tag{25}$$

We first consider the necessary assumptions for equilibrium existence. Table 2 lists the conditions for the parametric functions. Assumption 4 implies Assumption 1. It depends on the sign of δ whether Assumption 3, which is needed to generate finite aggregate exports, implies Assumption 1 (or Assumption 4).[30]

Additional model restrictions generate desired facts. Restriction 1, by which the combined incremental scope costs increase in scope with an elasticity of more than one, translates into $\delta + \alpha(\sigma-1) > 1$. So Restriction 1 is more stringent than Assumption 1 (and than Assumption 4). Finally, Restriction 2 implies that $\alpha(\sigma-1) > 1$ and its relationship to Restriction 1 depends on the sign of δ.

The optimal number of products for firms with $\phi \geq \phi^*_{sd}$ is given by equation (10) and can be written as

$$G_{sd}(\phi) = \text{integer}\left\{(\phi/\phi^*_{sd})^{\frac{\sigma-1}{\delta+\alpha(\sigma-1)}}\right\}. \quad (26)$$

Using this relationship and equation (13) we can express optimal sales of a firm ϕ's g-th product in market d as a function of the total number of products that the firm sells in d:

$$p_{sdg}(\phi)x_{sdg}(\phi) = \sigma f_{sd}(1)\, G_{sd}(\phi)^{\delta+\alpha(\sigma-1)} \left(\frac{\phi}{\phi^{*,G}_{sd}}\right)^{\sigma-1} g^{-\alpha(\sigma-1)}. \quad (27)$$

So total sales of a firm with G_{sd} products are

$$t_{sd}(\phi) = \sigma f_{sd}(1)\, G_{sd}(\phi)^{\delta+\alpha(\sigma-1)} \left(\frac{\phi}{\phi^{*,G}_{sd}}\right)^{\sigma-1} H\big(G_{sd}(\phi)\big)^{-(\sigma-1)} \quad (28)$$

where $H\big(G_{sd}(\phi)\big)^{-(\sigma-1)} = \sum_{g=1}^{G_{sd}(\phi)} g^{-\alpha(\sigma-1)}$, while exporter scale becomes

$$a_{sd}(\phi) = \sigma f_{sd}(1)\, G_{sd}(\phi)^{\delta+\alpha(\sigma-1)-1} \left(\frac{\phi}{\phi^{*,G}_{sd}}\right)^{\sigma-1} H\big(G_{sd}(\phi)\big)^{-(\sigma-1)}. \quad (29)$$

Note that the restriction $\delta + \alpha(\sigma-1) > 1$ (Restriction 1) is a sufficient condition for $a_{sd}(\phi)$ to increase with G_{sd} but not a necessary one since $H(G_{sd})^{-(\sigma-1)}$ also increases in scope.

Under the parametrization, the partial elasticity of trade (21) with respect to an additional product' entry cost, relative to the core product, becomes

$$\frac{\eta_{\lambda,f(g)}}{\eta_{\lambda,f(1)}} = g^{-\delta(\tilde{\theta}-1)-\alpha\theta}$$

[30]The necessary conditions for equilibrium existence can be summarized as

$$\min\left\{\delta(\tilde{\theta}-1), \delta\tilde{\theta}\right\} + \alpha\theta > 1 \quad \text{and} \quad \tilde{\theta} > 1.$$

By parametrization (25), the combined fixed cost function $f_{sd}(1)^{-\tilde{\theta}}\bar{F}_{sd}(\nu) \equiv f_{sd}(1)^{\tilde{\theta}-1}\sum_{G=1}^{\infty}(G)^{-\nu}$ contains a Riemann zeta function $\zeta(\nu) \equiv \sum_{G=1}^{\infty} G^{-\nu}$ for a real parameter $\nu \equiv \tilde{\theta}[\delta + \alpha(\sigma-1)] + \delta$.

for $g = 2, \ldots$. The power is strictly negative if and only if $\delta + \alpha(\sigma-1) > \delta/\tilde{\theta}$. So it depends on the sign and magnitude of δ whether the elasticity of trade with respect to an additional product's fixed cost is higher or lower than the elasticity of firm entry.

4.3 Relation to regularities

We relate the model's predictions to Facts 1 through 3 as presented in Section 2.2. By Fact 1 (Figure 1), a firm's sales within a destination are concentrated in few core products. In the model, the degree of concentration is regulated by how fast expression $h(g)$ increases with g (the elasticity $\alpha(\sigma-1)$, which we will estimate in the next section). Thus, this fact is intimately related to Restriction 2. Note that Figure 1 also implies that wide-scope exporters sell more of their top-selling products than firms with few products. The model's equation (27) matches this fact under Assumption 1. Finally, the fact that wide-scope exporters sell their lowest-ranked products for tiny amounts suggests that product efficiency (or consumer appeal) strongly declines for those products and that fixed entry costs for these products are low. Our estimation will quantify the magnitudes.

Fact 2 (Figure 2) documents a high frequency of narrow-scope and small-sales firms in the distributions of Brazilian exporters. The model's predictions for the scope distribution of exporters depend critically on the functional forms that we specify for $f_{sd}(g)$ and $h(g)$. Our parametrization (25) implies that $G_{sd}(\phi)$ is Pareto distributed in the upper tail with shape parameter $\tilde{\theta}[\delta+\alpha(\sigma-1)]$. Under this parametrization the model can potentially match the fast decline in the number of firms selling products to a destination market. Predictions regarding the overall sales distribution are less dependent on the functional form choices for $f_{sd}(g)$ or $h(g)$. Restriction 2 is a sufficient condition for equation (28) to exhibit a Pareto distribution in the upper tail. In that case total firm exports are Pareto distributed with shape parameter $-\tilde{\theta}$ in the upper tail, which is reminiscent of results in the Chaney (2008) and Eaton, Kortum, and Kramarz (2010) models.[31]

Last, we consider conditions under which the model generates Fact 3 (Figure 3), the positive relationship between exporter scope and scope-weighted mean exporter scale. Restriction 1 on

[31] To see that sales are Pareto distributed notice that the sales percentile Pr of a firm with productivity ϕ is given by $1 - \Pr = (\phi^*_{sd}/\phi)^{\theta}$. So we can express equation (14) as

$$\Pr = 1 - t_{sd}(\phi)^{-\tilde{\theta}} \left(\sigma f_{sd}(1) H\big(G_{sd}(\phi)\big)^{-(\sigma-1)}\right)^{\tilde{\theta}}.$$

Since a firm's product efficiency index converges to a constant as $t_{sd}(\phi) \to \infty$ and $\Pr \to 1$, this expression means that sales are Pareto distributed in the upper tail with a shape parameter $\tilde{\theta}$.

the combined incremental scope costs $z_{sd}(G)$ is sufficient for this to happen, as summarized by Proposition 2. Under our parametrization (25), exporter scale $a_{sd}(\phi)$ (and thus scope-weighted mean exporter scale $\bar{a}_{sd}(\phi)$) and exporter scope $G_{sd}(\phi)$ are positively associated if $\delta+\alpha(\sigma-1) > 1$, that is if there is a sufficiently strong decline in product efficiency with scope.

We now apply the model to the Brazilian data and estimate parameters for the functional forms (25). Our estimation target is the within-firm product sales distribution, using the same data that we used to generate Figure 1. To then evaluate the model's performance, we use the within-firm estimates to simulate cross-firm relationships regarding exporter scope and exporter scale as in Figures 2 and 3.

5 Estimation

Equation (27) is the basis for our estimation. We augment the equation by a multiplicative sales disturbance ϵ_{sdg}, which may be due to unanticipated demand shocks after pre-determined scope choice, or optimization and measurement error. We estimate the equation in its log form:

$$\begin{aligned} \ln p_{sdg}(\phi)x_{sdg}(\phi) \quad &= \quad [\delta + \alpha(\sigma-1)] \ln G_{sd}(\phi) \,-\, \alpha(\sigma-1) \ln g \\ &\qquad + \,(\sigma-1) \ln \left(\phi/\phi_{sd}^{*,G}\right) \,+ \ln \sigma f_{sd}(1) \,+\, \ln \epsilon_{sdg}(\phi). \end{aligned} \tag{30}$$

This equation is closely related to Figure 1 (Fact 1). Whereas Figure 1 presents the averages across firms for products with the same rank, equation (27) relates individual product sales directly to exporter scope and the product's rank within the firm. Consistent with that Figure, equation (27) implies that product sales increase in exporter scope and drop with the product's rank within the firm. Our identification of parameters δ and $\alpha(\sigma-1)$ relies on these two relationships between firm scope and individual product sales.

Beyond the disturbance $\ln \epsilon_{sdg}$, there are two more unobserved components in the estimation equation. The first unobserved component, $\ln \sigma$, is common to all firms. The second unobserved component, $(\sigma-1)\ln(\phi/\phi_{sd}^{*,G})$, varies by firm and destination and we capture it with firm-destination fixed effects.[32] To assess the robustness of the relationship we estimate equation (27) with alternative sets of fixed effects.

[32]In theory, this latter term varies little for large exporters since the maximum variation $(\phi_{sd}^{*,G+1}/\phi_{sd}^{*,G})^{\sigma-1}$ tends to one as G_{sd} becomes large.

Table 3: Individual Product Sales

Log Exp./prod.	Firm-destination-prod. data			
estimator	Ind. FE	Ind. FE	Firm FE	Firm-dest. FE
controls		Dest.	Dest.	
	(1)	(2)	(3)	(4)
Log # Products	1.396	1.319	1.557	1.273
	(.007)	(.007)	(.008)	(.007)
Log Product Rank	-2.558	-2.574	-2.624	-2.656
	(.007)	(.007)	(.006)	(.006)
Scope elast. of local entry cost (δ)	-1.162	-1.256	-1.067	-1.383
Scope elast. of prod. efficiency ($\alpha(\sigma-1)$)	2.558	2.574	2.624	2.656
Observations	162,570	162,570	162,570	162,570
Panels	259	259	10,215	27,362
R^2 (within)	.462	.510	.582	.618
Corr. Firm FE, $X'\beta$			.085	-.055

Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at the Harmonized-System 6-digit level. Constant (as well as industry and destination fixed effects in columns 1 through 3) included but not reported. R^2 is within fit (relative to industry and firm fixed effects). Standard errors in parentheses. Regression equation

$$\ln p_{d\phi g}\, x_{d\phi g} = [\delta + \alpha(\sigma-1)] \ln G_{d\phi} \;-\; [\alpha(\sigma-1)] \ln g_{d\phi} \;+ \ln\sigma \;+\; (\sigma-1)\ln\left(\phi/\phi_d^{*,G}\right) \;+\; \ln\epsilon_{d\phi g}.$$

5.1 Estimates

Table 3 documents that different specifications result in strikingly robust estimates of $\delta + \alpha(\sigma-1)$ and $\alpha(\sigma-1)$ for Brazilian manufacturers and their manufacturing products. Across specifications in Table 3, δ falls in the range from -1.07 to -1.38 and $\alpha(\sigma-1)$ in the range from 2.56 to 2.66.[33] The magnitude of the δ estimate implies that incremental local entry costs drop at an elasticity of up to around -1.4 when manufacturers introduce additional products in a market with a presence. But firm-product efficiency drops off even faster with an elasticity of around 2.6 or more. Adding the two fixed scope cost coefficients suggests that there are net overall diseconomies of scope with a scope elasticity of 1.2 or more. The coefficient estimates also suggest that both Restriction 1 and 2 are satisfied in our data.

To assess the robustness of our estimates we perform two more estimation exercises. First, we aggregate the data as in Figure 1 (Fact 1) and fit the according regression equation with non-linear least squares under literature-guided calibrations of the free parameter $\tilde{\theta}$ (see Appendix D.2).

[33]When we allow the coefficients on log exporter scope and log product rank to vary across 29 industries (ISIC 3-digit) in the specification of column 4, then δ falls in the range from -1.19 to -1.59 for 25 out of 29 industries, and $\alpha(\sigma-1)$ in the range from 2.23 to 3.38 (see our online Data Appendix).

Regardless of free parameter choice, we find for $\delta+\alpha(\sigma–1)$ estimates of 1.60 to 1.61 and for $\alpha(\sigma–1)$ estimates of 2.55, close to the estimates in Table 3. Second, we move on to estimate parameters also from the cross section of firms. Note that the slopes of the graphs in Figure 3 (Fact 3) are equal to the respective Pareto shape parameters and observe that our parametrization implies that $G_{sd}(\phi)$ is Pareto distributed in the upper tail with shape parameter $\tilde{\theta}[\delta+\alpha(\sigma-1)]$ by equation (26) and $a_{sd}(\phi)$ is Pareto distributed in the upper tail with shape parameter $\tilde{\theta}[\delta+\alpha(\sigma-1)]/[\delta+\alpha(\sigma-1)-1]$ by equation (29). So the ratio of the two Pareto shape parameters also yields an estimate of $\delta+\alpha(\sigma-1)$. For the United States and Argentina in Figure 3, for instance, we fit the graphs to linear relationships as implied by the Pareto distribution and find estimates for $\delta+\alpha(\sigma-1)$ of 1.88 and 1.66, respectively, with R^2 above 97 percent in the individual regression. These are reasonably close to the implied estimates of $\delta+\alpha(\sigma-1)$ between 1.27 and 1.56 in Table 3.

Supportive evidence on our main mechanisms comes from empirical studies in industrial organization. As regards declining product sales with wider scope, there is evidence that production costs increase for firms that introduce more products (e.g. Bayus and Putsis 1999 for PCs). Our finding of economies of scope in market-specific entry costs echoes related evidence of falling marketing costs with scope in the consumer goods industry (e.g. Morgan and Rego 2009, who define a market segment by NAICS operating code similar to our definition of an HS-6 digit product) and falling distribution costs in the finance industry (e.g. Cummins, Weiss, Xie, and Zi 2010). Beyond a quantification of diseconomies of scope, we are interested in their relationship to exporter entry and implications for the firm size distribution and global trade.

Using our estimates, the power in the partial elasticity ratio (21) is strictly negative because $\delta+\alpha(\sigma-1)>0>\delta/\tilde{\theta}$. So the partial elasticity of trade with respect to an additional product's fixed cost is lower than the elasticity with respect to the core product. In other words, our estimates imply that product entry at multi-product exporters should matter less than firm entry with the core product. We will return to the magnitudes in our simulation.

5.2 Model predictions

To illustrate model predictions, we choose $\tilde{\theta}=1.28$. This value falls within the range of parameterizations in the literature, so we use it as a benchmark in later simulations. In particular, Luttmer (2007) and di Giovanni and Levchenko (2010) use $\tilde{\theta}=1.06$ as reported by Axtell (2001) and close to Zipf's Law. Arkolakis (2010) estimates $\tilde{\theta}=1.49$ using the Chaney (2008) version of the Melitz

Table 4: Benchmark Parameter Values

Parameter		Value	Source
Scope elasticity of local entry costs	δ	-1.38	Estimate (Table 3, col. 4)
Scope elasticity of product-introd. cost	$\alpha(\sigma-1)$	2.66	Estimate (Table 3, col. 4)
Pareto shape parameter of productivity	θ	8.28	Eaton and Kortum (2002)
Elasticity of substitution betw. varieties	σ	7.49	Broda and Weinstein (2006)
Pareto shape parameter of total sales	$\tilde{\theta}$	1.28	$\tilde{\theta} \equiv \theta/(\sigma-1)$

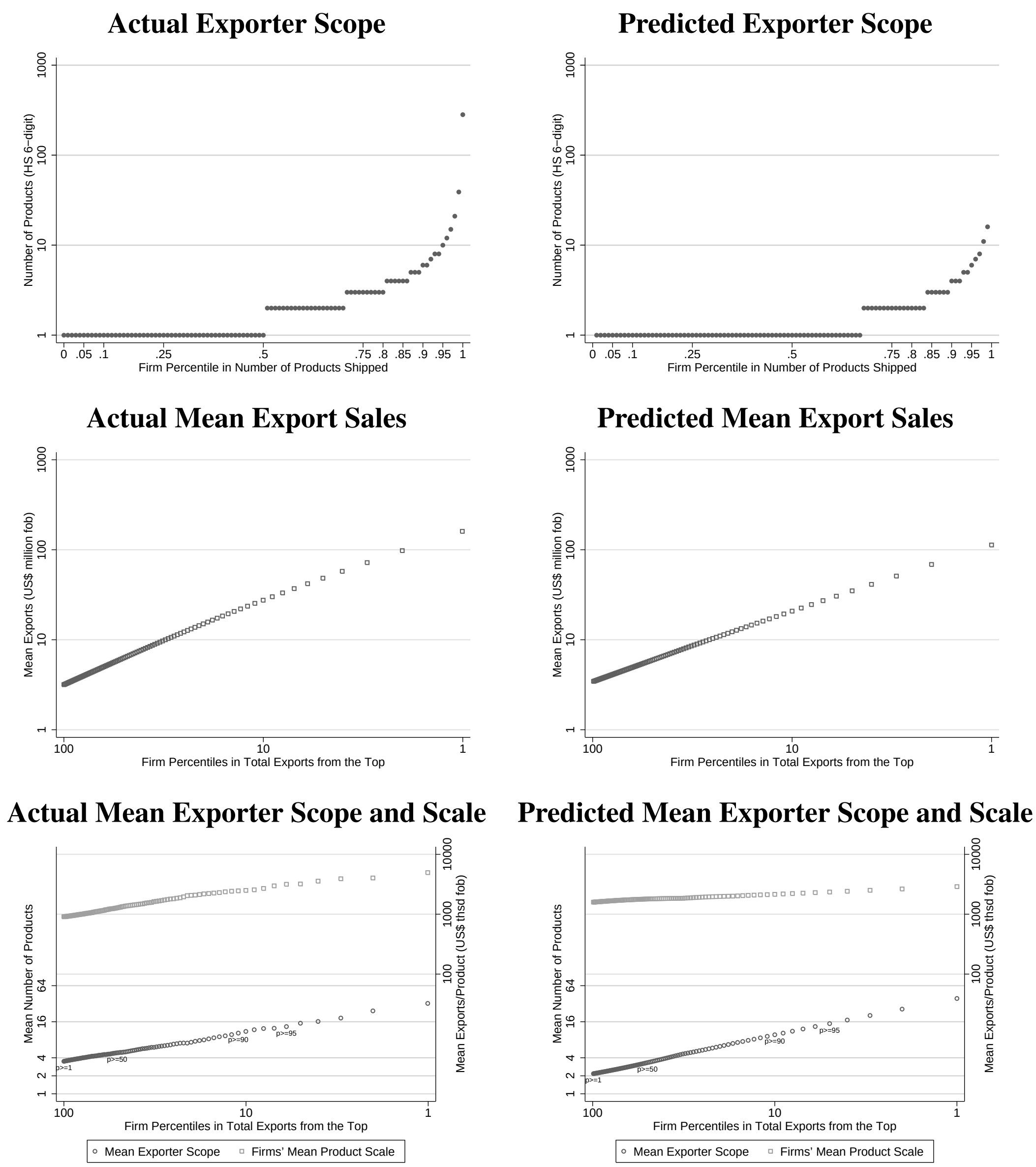
(2003) model with Pareto distributed productivity.[34] For $\theta = 8.28$ from Eaton and Kortum (2002), for instance, a value of $\tilde{\theta} = 1.28$ implies $\sigma = 7.49$, which is in the range of demand elasticity estimates reported by Broda and Weinstein (2006).

Based on our coefficient estimates for Fact 1 on the within-firm size distribution, we evaluate the calibrated model for Facts 2 and 3. We view these predictions as over-identifying checks on the model in the cross-section of firms. We simulate data from our estimated model and generate statistics exactly as we did using the pristine data for Facts 2 and 3 discussed in Section 2.

The upper and middle panels of Figure 5 show the actual (left-hand side) and simulated (right-hand side) exporter scope and mean sales distributions. Our estimate of $\delta + \alpha(\sigma - 1) = 1.27$ implies overall diseconomies scope so that firms introduce more products only if they are much more productive. Thus, unless the distribution of firms is strongly heterogeneous in sales (low $\tilde{\theta}$), the model cannot fully replicate the number of products that highly productive firms adopt. From the discussion in Section 4.2 and the fact that the coefficient estimates imply $\alpha(\sigma-1) = 2.66 > 1$ (in support of Restriction 2), it follows that the distribution of total sales is Pareto in the upper tail with a coefficient $-\tilde{\theta}$ (set to 1.28). The model simulation matches the upper tail well in our graph for mean sales at or a above a given percentile, which weights down deviant small-firm behavior in the low tail.

The lower panel of Figure 5 shows the model predictions for mean exporter scope and exporter scale as functions of the overall firm size percentile. The model tracks reasonably well the distribution of mean exporter scope. This success comes despite the fact that the parameters δ and $\alpha(\sigma-1)$

[34] The observation that the model exhibits a good fit with a lower $\tilde{\theta}$ than the estimate from more aggregate trade data (as in Arkolakis 2010) is indicative of unmodelled within-firm heterogeneity (in addition to unmodelled firm heterogeneity as in Arkolakis 2010). For we strive to explain multi-product firms' optimal choices, we abstract from idiosyncratic sales shocks per product and choose a value of θ at the midpoint of the range of existing estimates.



Source: *SECEX* 2000, manufacturing firms and their manufactured products to the United States.
Note: Products at HS 6-digit level. For simulations, we set $\alpha(\sigma-1) = 2.66$ and $\delta = -1.38$ from Table 3 (column 4), $\theta = 8.28$ following Eaton and Kortum (2002), and $\sigma = 7.49$ so that $\tilde{\theta} = 1.28$.

Figure 5: **Exporter Scope and Exporter Scale and Their Model Predictions for the USA**

are estimated to match the within-firm heterogeneity of sales. A reason for the close prediction is that averaging over all upper percentiles in cumulative graphs assigns little weight to small-scope exporters. The estimated model can also generate the increase in the exporter scale with firm size but falls short of delivering the exact magnitude of that increase. Adding random random sales shocks per product to the model could improve its predictions in this dimension. Overall, we find the model's quantitative performance satisfactory. The fact that parameter estimates to fit the within-firm sales heterogeneity (Fact 1) also deliver a good approximation to the cross-firm distribution of exporter scope and a plausible approximation to the scope-weighted mean exporter scale distribution reassures us of our estimates.

Having queried the predictions of the calibrated model in several dimensions we proceed to perform counterfactual experiments of a hypothetical trade liberalization with respect to local entry costs.

6 Counterfactuals

We conduct a counterfactual simulation to quantify the implied impact of our estimates for changes to bilateral trade when market-specific entry costs drop. Brazil being close to the median country in exports per capita, we consider our parameter estimates informative for global trade.[35]

To perform counterfactual experiments we add three ingredients to the model following Eaton, Kortum, and Kramarz (2010). (i) We introduce intermediate inputs as in Eaton and Kortum (2002). In particular, we assume that the production of each product uses a Cobb-Douglas aggregate of labor and a composite of all other manufacturing products with cost P_d. The labor share in manufacturing production is β, and the share of intermediate inputs $1-\beta$. So the total input cost is $w_d = W_d^{\beta} P_d^{1-\beta}$, where we now think of w_d as the input cost and W_d as the wage. (ii) There is a non-manufacturing sector in each country as in Alvarez and Lucas (2007) that combines manufactures with labor, in a Cobb-Douglas fashion, where manufactures have a share γ in GDP. The price of final output in country d is proportional to $P_d^{\gamma} W_d^{1-\gamma}$. We state the resulting equations in Appendix A. (iii) We allow for a manufacturing trade deficit D_d, and for an overall trade deficit D_d^T in goods and services. Both deficits are set to their observed levels in 2000.

[35]By the WTF and WDI data for all industries and countries, Brazil ranks at the 48th percentile (top 100th out of 192) in terms of exports per capita in 2000. In 2000, Brazil's total exports are at the 88th percentile worldwide (top 27th out of 205).

We compute the share of manufacturing in GDP for each country using data on GDP, manufacturing production and trade (as described in Appendix D.3). We set the labor share in manufacturing production to $\beta = .330$, the sample average for countries with available information (Appendix D.3). To compute the impact of a counterfactual change in entry costs, we use the Dekle, Eaton, and Kortum (2007) methodology (details in Appendix B). The merit of this method is that it requires no information on the initial level of technology, iceberg trade costs, and entry costs. Instead, we can compute the changes in all equilibrium variables using as a simple input the percentage change in the underlying parameter of interest (entry cost parameters in our case).

We conduct two experiments with entry costs. The first experiment is a 25-percent drop in the entry costs of exporting the first product $\hat{f}_{sd}(1) = .75$ for $s \neq d$. We interpret this experiment as representing a decline in firm-level exporting cost such as one-time costs for information acquisition, search and matching costs for wholesale or retail representatives abroad, expenses for trade fairs, the setup of certified and accredited testing facilities, investments in technology acquisition for export development, one-time costs of product re-design and building up logistics for export, and perhaps brand marketing costs.

The second experiment is a 25-percent drop in entry costs of exporting $\hat{f}_{sd}(g) = .75$ for all products $g = 1, 2, \ldots$ and $s \neq d$. We view this exercise as a counterfactual decrease of non-variable trade cost barriers that apply to all products repeatedly, such as technical barriers to trade.[36] Examples of such trade barriers are product re-designs to meet local technical requirements or energy-saving regulations, costs to satisfy performance requirements, costs of compliance with voluntary safety rules or mandatory sanitary regulations including administrative and legal fees, product-specific fixed costs of labelling and re-packaging, access costs to additional wholesale representatives and retail shelve space, or labelling and marking costs for particular exporting markets.

[36]The perceived importance of technical barriers to trade for individual products is reflected in dispute settlement cases at the World Trade Organization. As of October 2010, out of the total 418 dispute settlement cases ever brought, 41 cases cite the Technical Barriers to Trade (TBT) and 37 cases the Sanitary and Phytosanitary Measures (SPS) agreements in their request for consultations. In numbers, these case counts are comparable to the 84 cases that cite anti-dumping (Article VI of GATT 1994) in the consultations request. The first dispute settlement case ever brought against the United States (DS2) cites the Technical Barriers to Trade agreement with regards to gasoline standards.

Maskus, Otsuki, and Wilson (2005) find substantive fixed costs of compliance with technical barriers to trade, approximately $425,000 per firm in survey data from 16 developing countries in the World Bank's Technical Barriers to Trade (TBT) Survey Database. Deardorff and Stern (1998) classify non-tariff trade barriers into six broad categories; two of their categories correspond closely to entry costs: "technical barriers to trade" (quality standards, safety regulations, packaging and labelling requirements) and specific "customs procedures and administrative practices" (startup procedures and documentation requirements).

Table 5: Simulation of Real-Wage Increase in Percent due to Decline in Entry Cost

25-percent decline	$\theta = 8.28$		$\theta = 4.87$			$\theta = 8.28$		$\theta = 4.87$	
product range	1st	all	1st	all		1st	all	1st	all
Armenia	.06	.07	.11	.14	Kuwait	.22	.27	.44	.52
Australia	.18	.22	.33	.40	Kyrgyzstan	.12	.14	.23	.27
Austria	.36	.43	.67	.80	Latvia	.50	.60	.86	1.02
Azerbaijan	.13	.16	.25	.30	Lithuania	.34	.41	.62	.75
BelgmLuxNthl	.45	.53	.85	1.01	Malta	1.0	1.20	1.74	2.07
Bolivia	.10	.12	.19	.23	Mexico	.45	.54	.77	.92
Brazil	.07	.08	.14	.16	Morocco	.21	.25	.40	.48
Bulgaria	.30	.36	.58	.69	Norway	.23	.28	.44	.52
Canada	.36	.43	.66	.79	Oman	.36	.43	.64	.76
Chile	.18	.22	.35	.42	Poland	.22	.27	.44	.52
ChinaHKG	.20	.23	.39	.47	Portugal	.28	.34	.53	.64
Colombia	.10	.12	.20	.24	Romania	.27	.32	.52	.63
CostaRica	.03	.04	.11	.13	RussianFed	.11	.13	.24	.29
Cyprus	.42	.50	.77	.93	Senegal	.18	.21	.33	.40
Ethiopia	.04	.05	.09	.11	Slovenia	.47	.57	.90	1.07
Finland	.17	.21	.38	.45	SouthKorea	.19	.23	.40	.48
FranceMonaco	.21	.25	.41	.48	Spain	.20	.24	.39	.47
Germany	.21	.25	.42	.50	SriLanka	.18	.22	.35	.41
Hungary	.69	.82	1.26	1.50	Sweden	.16	.20	.36	.43
IndMalSgThai	.47	.57	.94	1.13	TrinidadTbg	.06	.07	.13	.15
India	.31	.38	.57	.68	Tunisia	.34	.41	.65	.78
Iran	.08	.10	.17	.20	Turkey	.15	.18	.29	.35
Ireland	.26	.31	.67	.80	UK	.19	.23	.38	.45
Israel	.21	.25	.39	.47	USA	.09	.10	.18	.21
Italy	.16	.19	.33	.39	Ukraine	.14	.17	.31	.38
Japan	.05	.06	.11	.13	Uruguay	.12	.15	.23	.28
Jordan	.17	.21	.33	.40	VietNam	.33	.40	.63	.76
Kazakhstan	.28	.34	.51	.61	Yemen	.10	.12	.22	.26
Kenya	.05	.06	.12	.14	RestOfWorld	.65	.77	1.13	1.35
					World Avg.	.25	.29	.47	.56

Notes: Own calculations, real wage change in percentage points. Values of $\theta = 8.28$ as in Eaton and Kortum (2002) and $\theta = 4.87$ as in Eaton, Kortum, and Kramarz (2010). See Appendix D.3 for data construction. Following Dekle, Eaton, and Kortum (2007), we collapse (i) Hong Kong, Macão and mainland China, (ii) Belgium, Luxembourg and the Netherlands, and (iii) Indonesia, Malaysia, Singapore, and Thailand into single entities.

Table 5 summarizes the results for increases in real wages. In our model, relative real-wage increases are proportional to relative increases in bilateral trade flows. Our simulated real-wage gains from declining entry costs are smaller than the ones typically found for falling variable trade costs (e.g. Eaton, Kortum, and Kramarz 2010). The finding that declining fixed costs may not have a large impact on welfare is also similar to simulation results for home-market entry by di Giovanni and Levchenko (2010). Our explanation differs, however. At the firm entry margin, we confirm the di Giovanni and Levchenko (2010) home-market simulation by which only small firms surpass the entry threshold as fixed costs decline; these firms create these firms create little additional trade. For multi-product firms, however, the effect could potentially dif and only ifer if a reduction of fixed local entry costs induced the highly productive wide-scope firms to expand fast. But our structural a strong decline in competency with scope when they take up additional products, so there is also no strong increase in bilateral trade from the extensive margin of exporting products at wide-scope firms.

Overall, the elasticity of trade with respect to fixed costs $-(\tilde{\theta}-1)$ is close to zero and therefore makes the aggregate impact of changes in fixed costs small. If we use the lower value of $\theta = 4.87$ for our simulations (the value considered by Eaton, Kortum, and Kramarz 2010), while keeping $\tilde{\theta} = 1.28$, then the welfare gains roughly double for all countries. So the value of $\theta = 4.87$ strongly alters the gains from changes in fixed costs. Reliable estimates of θ, or equivalently of σ given $\tilde{\theta}$, are an arguably important aspect of confidence in trade simulations (see for example Helpman, Melitz, and Rubinstein 2008).

As a final check on the extensive margin of exporting products, we simulate a 25-percent drop in variable trade costs $\hat{\tau}_{sd} = .75$ for all products. Real-wage gains from trade with respect to changes in variable trade costs are much larger and within the magnitudes reported by Eaton, Kortum, and Kramarz (2010). However, these magnitudes are affected only to a minor degree by changes in θ, reminiscent of results in Atkeson and Burstein (2010) and Bergstrand, Egger, and Larch (2010). We decompose the gains from trade into the contributions at each margin, by holding the other two margins constant at a time. The extensive margin of firm entry contributes 31 percent of the welfare gains from reduced variable trade costs. The extensive margin of product entry from expanding exporter scope contributes only 5 percent. The bulk of welfare gains accrues at the remaining intensive margin of the exporter's average sales per product with 64 percent.

7 Concluding Remarks

We have used three-dimensional panel data of Brazilian exporters, their products and their destination markets to uncover the costs that exporters face in selling additional products and expanding in individual markets. The level of detail of the Brazilian data and the multi-product model that we develop allow us to follow a reductionist approach, relating bilateral trade in the aggregate back to the product adoption decision at individual firms. Building up from a firm's choices of destination markets and local product lines, our model leads to relationships that are consistent with the disaggregated trade data but also with previous theories of trade such as Chaney (2008), Eaton, Kortum, and Kramarz (2010), and Arkolakis (2010), which consider more aggregate trade flows.

Estimation of this micro-founded model allows us to quantify the responsiveness of bilateral trade to fixed entry cost components. We find in simulations that declines in fixed entry costs only lead to relatively small increases in trade from new products at incumbents because multi-product exporters face relatively low incremental entry costs but strong declines in competency when they take up additional products. Though our approach allows us to analyze trade data at various levels of aggregation, it nevertheless leaves recently available information from micro-trade data unexplored, including information on unit prices and time series information. Our approach suggests that such additional information may prove valuable in understanding more precisely the barriers that determine the choice of products and the span of exporters.

Appendix

A Proofs

A.1 Proof of Proposition 2

The optimal choice for $G_{sd}(\phi)$ is the largest $G_{sd} \in \{0, 1, \ldots\}$ such that inequality (10),

$$\left(\frac{P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \, \frac{\phi}{h(G_{sd})} \right)^{\sigma-1} \frac{T_d}{\sigma} \geq f_{sd}(G_{sd}),$$

is weakly satisfied, as shown in the text. In discrete product space, G_{sd} is an integer. For simplicity, define $\tilde{G}_{sd}$ as the continuous variable that solves

$$z_{sd}(\tilde{G}_{sd}) = \left(\frac{P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \, \phi \right)^{\sigma-1} \frac{T_d}{\sigma},$$

where $z_{sd}(G) \equiv f_{sd}(G)\, h(G)^{\sigma-1}$. $z_{sd}(G)$ strictly increases in G by Restriction 1. So $z_{sd}(G)$ is invertible. Hence $\tilde{G}_{sd} = \tilde{G}_{sd}\big((\phi/\phi^*_{sd})^{\sigma-1}\big)$ can also be expressed as the inverse function

$$\tilde{G}_{sd} = z^{-1}_{sd}\Big(f_{sd}(1)\, (\phi/\phi^*_{sd})^{\sigma-1} \Big). \tag{A.1}$$

Note that $\tilde{G}_{sd} = G_{sd}$ for $\phi = \phi^*_{sd}$, $\phi^{*,2}_{sd}$, $\phi^{*,3}_{sd}$, and so forth.

As a shorthand, define the argument $x \equiv f_{sd}(1)\, (\phi/\phi^*_{sd})^{\sigma-1}$. Restriction 1 is equivalent to

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1 \iff \frac{Z(G)}{Z'(G)\, G} < 1 \iff \frac{[Z^{-1}(G)]'\, Z(G)}{G} < 1,$$

where the last step follows by the inverse function theorem since $z_{sd}(G)$ strictly increases in G. By equation (A.1), $\tilde{G}_{sd} = z^{-1}_{sd}(x)$, so Restriction 1 is also equivalent to

$$\frac{\partial \ln z_{sd}(G)}{\partial \ln G} > 1 \iff \frac{\tilde{G}'(x)\, x}{\tilde{G}(x)} < 1 \iff \tilde{G}(x) - \tilde{G}'(x)\, x > 0. \tag{A.2}$$

We want to show that, if Restriction 1 holds, exporter scale $a_{sd}(\phi)$ strictly increases in ϕ when evaluated at the points $\phi = \phi^*_{sd}, \phi^{*,2}_{sd}, \phi^{*,3}_{sd}$, and so forth. So, by (15) and with the shorthand $x = f_{sd}(1)\, (\phi/\phi^*_{sd})^{\sigma-1}$, consider

$$a_{sd}(x) = \sigma\, x\, H\big(\tilde{G}(x)\big)^{-(\sigma-1)} / \tilde{G}(x).$$

The term $H(\tilde{G}(x))^{-(\sigma-1)}$ strictly increases in $\tilde{G}(x)$ and x at $\phi = \phi^*_{sd}, \phi^{*,2}_{sd}, \phi^{*,3}_{sd}$, and so forth. Since we are only interested in sufficiency of the Restriction and since $H(\cdot)^{-(\sigma-1)}$ strictly increases at the evaluation points, it suffices to show that the result is true for $x/\tilde{G}(x)$. $x/\tilde{G}(x)$ is differentiable at the points $\phi = \phi^*_{sd}, \phi^{*,2}_{sd}, \phi^{*,3}_{sd}$, and so forth, so exporter scale $a_{sd}(\cdot)$ strictly increases if

$$\left(x/\tilde{G}(x)\right)' > 0 \iff \frac{\tilde{G}(x) - \tilde{G}'(x)\,x}{[\tilde{G}(x)]^2} > 0 \iff \tilde{G}(x) - \tilde{G}'(x)\,x > 0.$$

The last step is an implication of Restriction 1 as shown in (A.2). This establishes the result.

A.2 Proof of Proposition 3

Average sales of firms in s shipping to d are

$$\bar{T}_{sd} = \int_{\phi^*_{sd}} t_{sd}(\phi)\cdot\mu_{sd}(\phi)\,\mathrm{d}\phi = \int_{\phi^*_{sd}} \sigma\, f_{sd}(1)\left(\frac{\phi}{\phi^*_{sd}}\right)^{\sigma-1} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)}\cdot\theta\,\frac{(\phi^*_{sd})^{\theta}}{(\phi)^{\theta+1}}\,\mathrm{d}\phi$$

by optimal total exports (14) and the definition of a firm's product efficiency index (5). Consider the term $\int_{\phi^*_{sd}} \phi^{\sigma-1-(\theta+1)} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)}\,\mathrm{d}\phi$. Rewrite the term as a piecewise integral

$$\begin{aligned}
\int_{\phi^*_{sd}} \sum_{g=1}^{G_{sd}(\phi)} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}}\,\mathrm{d}\phi &= \int_{\phi^*_{sd}}^{\phi^{*,2}_{sd}} \sum_{g=1}^{1} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}}\,\mathrm{d}\phi + \int_{\phi^{*,2}_{sd}}^{\phi^{*,3}_{sd}} \sum_{g=1}^{2} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}}\,\mathrm{d}\phi + \ldots \\
&= \frac{1}{h(1)^{\sigma-1}} \int_{\phi^*_{sd}}^{\infty} \phi^{\sigma-1-(\theta+1)}\,\mathrm{d}\phi + \frac{1}{h(2)^{\sigma-1}} \int_{\phi^{*,2}_{sd}}^{\infty} \phi^{\sigma-1-(\theta+1)}\,\mathrm{d}\phi + \ldots \\
&= \frac{1}{h(1)^{\sigma-1}} \frac{(\phi^*_{sd})^{(\sigma-1)-\theta}}{\theta-(\sigma-1)} + \frac{1}{h(2)^{\sigma-1}} \frac{\left(\phi^{*,2}_{sd}\right)^{(\sigma-1)-\theta}}{\theta-(\sigma-1)} + \ldots
\end{aligned}$$

for $\theta > \sigma - 1$. Using the definitions of $\phi^*_{sd}, \phi^{*,2}_{sd}$ etc. from (12), we have

$$\int_{\phi^*_{sd}} \sum_{g=1}^{G_{sd}(\phi)} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}}\,\mathrm{d}\phi = \frac{1}{\theta-(\sigma-1)}\left(\frac{f_{sd}(1)}{(\phi^*_{sd})^{\sigma-1}}\right)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} \frac{f_{sd}(G)^{-(\tilde{\theta}-1)}}{h(G)^{\theta}}. \tag{A.3}$$

So $\bar{T}_{sd} = [\tilde{\theta}\,\sigma/(\tilde{\theta}-1)]\, f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$, proving the first equality in (19). The expression is finite by Assumption 3.

Average fixed costs paid by firms in s selling to d are

$$\begin{aligned}
\bar{F}_{sd} &= \int_{\phi^*_{sd}}^{\phi^{*,2}_{sd}} F_{sd}(1)\,\theta\,\frac{(\phi^*_{sd})^{\theta}}{\phi^{\theta+1}}\,\mathrm{d}\phi + \int_{\phi^{*,2}_{sd}}^{\phi^{*,3}_{sd}} F_{sd}(2)\,\theta\,\frac{(\phi^*_{sd})^{\theta}}{\phi^{\theta+1}}\,\mathrm{d}\phi + \\
&= F_{sd}(1)\,(\phi^*_{sd})^{\theta}\left[(\phi^*_{sd})^{-\theta} - \left(\phi^{*,2}_{sd}\right)^{-\theta}\right] + F_{sd}(2)\,(\phi^*_{sd})^{\theta}\left[\left(\phi^{*,2}_{sd}\right)^{-\theta} - \left(\phi^{*,3}_{sd}\right)^{-\theta}\right] + \ldots\,.
\end{aligned}$$

Using the definition $F_{sd}(G_{sd}) = \sum_{g=1}^{G_{sd}} f_{sd}(g)$ and collecting terms with a common $\phi_{sd}^{*,G}$ we can rewrite the above expression as

$$\bar{F}_{sd} = f_{sd}(1) + \left(\phi_{sd}^{*,2}\right)^{-\theta} \left(\phi_{sd}^{*}\right)^{\theta} f_{sd}(2) + \left(\phi_{sd}^{*,3}\right)^{-\theta} \left(\phi_{sd}^{*}\right)^{\theta} f_{sd}(3) + \dots .$$

Using the definition of $\phi_{sd}^{*,G}$ from equation (12) in the above equation we get

$$\begin{aligned}
\bar{F}_{sd} &= f_{sd}(1) + \left(\frac{f_{sd}(2)^{1/(\sigma-1)}h(2)}{f_{sd}(1)^{1/(\sigma-1)}h(1)}\right)^{-\theta} f_{sd}(2) + \dots \\
&= \left[f_{sd}(1) + f_{sd}(1)^{\tilde{\theta}} \left(f_{sd}(2)^{1/(\sigma-1)}h(2)\right)^{-\theta} f_{sd}(2) + \dots\right] \\
&= f_{sd}(1)^{\tilde{\theta}} \left[f_{sd}(1)^{-(\tilde{\theta}-1)} + f_{sd}(2)^{-(\tilde{\theta}-1)}h(2)^{-\theta} + \dots\right] \\
&= f_{sd}(1)^{\tilde{\theta}} \textstyle\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)}h(G)^{-\theta} \\
&= \frac{\tilde{\theta}-1}{\tilde{\theta}\,\sigma}\,\bar{T}_{sd}.
\end{aligned}$$

This proves the second equality in (19). So $\bar{F}_{sd}/\bar{T}_{sd}$ is a destination invariant constant.

A.3 Mean sales per product

Figure 1 (Fact 1) depicts average sales of the g-th product for firms with an exporter scope of exactly G_{sd} products in market d. To compute this statistic, we integrate the sales of the g-th product $p_{sdg}(\phi)x_{sdg}(\phi)$ over the probability density of firms with ϕ such that $\phi_{sd}^{*,G} \leq \phi \leq \phi_{sd}^{*,G+1}$. We thus have

$$\begin{aligned}
&\textstyle\int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} p_{sdg}(\phi)x_{sdg}(\phi)\theta \frac{\left[\left(\phi_{sd}^{*,G}\right)^{-\theta} - \left(\phi_{sd}^{*,G+1}\right)^{-\theta}\right]^{-1}}{\phi^{\theta+1}}\,\mathrm{d}\phi = \\
&= \sigma f_{sd}(1)\, G_{sd}^{\delta+\alpha(\sigma-1)} \textstyle\int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} \left(\frac{\phi}{\phi_{sd}^{*,G}}\right)^{\sigma-1} g^{-\alpha(\sigma-1)}\theta \left(\frac{\left[\left(\phi_{sd}^{*,G}\right)^{-\theta} - \left(\phi_{sd}^{*,G+1}\right)^{-\theta}\right]^{-1}}{\phi^{\theta+1}}\right) \mathrm{d}\phi \\
&= -f_{sd}(1)\, G_{sd}^{\delta+\alpha(\sigma-1)} g^{-\alpha(\sigma-1)} \frac{\sigma\theta}{\theta-(\sigma-1)} \frac{\left(\phi_{sd}^{*,G+1}\right)^{(\sigma-1)-\theta} - \left(\phi_{sd}^{*,G}\right)^{\sigma-1-\theta}}{\left(\phi_{sd}^{*,G}\right)^{(\sigma-1)-\theta} - \left(\phi_{sd}^{*,G}\right)^{\sigma-1}\left(\phi_{sd}^{*,G+1}\right)^{-\theta}} \\
&= \frac{\sigma\theta}{\theta-(\sigma-1)}\, f_{sd}(1)\, G_{sd}^{\delta+\alpha(\sigma-1)} \frac{1-\left(\frac{G_{sd}+1}{G_{sd}}\right)^{[1-\tilde{\theta}][\delta+\alpha(\sigma-1)]}}{1-\left(\frac{G_{sd}+1}{G_{sd}}\right)^{-\tilde{\theta}[\delta+\alpha(\sigma-1)]}} g^{-\alpha(\sigma-1)},
\end{aligned}$$

where we used the definition from equation (11) in the last equality.

B Counterfactuals and Calibration

For the counterfactuals we set $w_s = W_s^{\beta} P_s^{1-\beta}$ and, introducing auxiliary notation, we replace $f_{sd}(1)\bar{F}_{sd}$ with $W_d^{-(\tilde{\theta}-1)}\tilde{F}_{sd}$ since fixed costs are homogeneous of degree one in the import country's wage (and thus $f_{sd}(1)\bar{F}_{sd}$ is homogeneous of degree $1-\tilde{\theta}$). We denote future variables with a prime, and we consider technological parameters and labor endowments as time invariant. Using expression (20) for current trade shares λ_{sd}, we can express counterfactual trade shares as

$$\lambda'_{sd} = \frac{\lambda_{sd}\left(\hat{W}_s^{\beta}\hat{P}_s^{1-\beta}\right)^{-\theta}\hat{\tau}_{sd}^{-\theta}\widehat{\tilde{F}_{sd}}}{\sum_k \lambda_{kd}\left(\hat{W}_k^{\beta}\hat{P}_k^{1-\beta}\right)^{-\theta}\hat{\tau}_{kd}^{-\theta}\widehat{\tilde{F}_{kd}}} \tag{B.4}$$

The price index (3) can be re-written as

$$\begin{aligned} P_d^{1-\sigma} &= \sum_k \int_{\phi^*_{kd}} M_{kd}\left[\sum_{g=1}^{G_{kd}(\phi)}\left(\tilde{\sigma}\frac{W_k^{\beta}P_k^{1-\beta}}{\phi/h(g)}\tau_{kd}\right)^{1-\sigma}\right]\frac{\theta\left(\phi^*_{kd}\right)^{\theta}}{\phi^{\theta+1}}\mathrm{d}\phi \\ &= \sum_k \left(W_k^{\beta}P_k^{1-\beta}\tilde{\sigma}\tau_{kd}\right)^{1-\sigma} J_k(b_k)^{\theta}\theta\int_{\phi^*_{kd}}\sum_{\upsilon=1}^{G_{kd}(\phi)}\frac{\phi^{\sigma-\theta-2}}{h(g)^{\sigma-1}}\mathrm{d}\phi. \end{aligned}$$

By Proposition 3, we can replace the integral term so that

$$P_d^{-\theta} = \left(T_d\right)^{\tilde{\theta}-1}\frac{\left(\sigma\right)^{-(\tilde{\theta}-1)}\left(\tilde{\sigma}\right)^{-\theta}}{1-1/\tilde{\theta}}\sum_k J_k b_k^{\theta}\left(W_k^{\beta}P_k^{1-\beta}\right)^{-\theta}\tau_{kd}^{-\theta}W_d^{-(\tilde{\theta}-1)}\tilde{F}_{kd}, \tag{B.5}$$

which can be restated as[37]

$$\hat{P}_d = \left[\sum_k \lambda_{kd}\left(\hat{W}_k^{\beta}\hat{P}_k^{1-\beta}\right)^{-\theta}\hat{\tau}_{kd}^{-\theta}\widehat{\tilde{F}_{kd}}\right]^{-1/\theta}\left(\frac{\hat{T}_d}{\hat{W}_d}\right)^{1/\theta-1/(\sigma-1)}. \tag{B.6}$$

The final equation for our counterfactuals follows Eaton, Kortum, and Kramarz (2010, Appendix E). We allow the share γ_d of manufacturing value added in GDP to be country specific.

[37] We can use expression (B.5) together with equation (20) to obtain

$$P_d^{-\theta} = \left(T_d\right)^{\tilde{\theta}-1}\frac{\left(\sigma\right)^{-(\tilde{\theta}-1)}\left(\tilde{\sigma}\right)^{-\theta}}{1-1/\tilde{\theta}}\frac{J_d b_d^{\theta}\left(W_d^{\beta}P_d^{1-\beta}\right)^{-\theta}W_d^{-(\tilde{\theta}-1)}\tilde{F}_{dd}}{\lambda_{dd}}.$$

Thus changes in real wage are

$$\widehat{\left(\frac{W_d}{P_d}\right)} = \left(\widehat{\lambda_{dd}}\right)^{-1/\theta}\frac{\left(\widehat{T_d}/\widehat{W_d}\right)^{\frac{1-\tilde{\theta}}{\theta}}}{\left(\widehat{\tilde{F}_{dd}}\right)^{-1/\theta}}.$$

We consider $\widehat{\tilde{F}_{dd}} = 1$ in our counterfactual exercise, so this expression differs for domestic entry costs from a similar one in Arkolakis, Costinot, and Rodríguez-Clare (2009) inasmuch as changes in the ratio $\widehat{T_d}/\widehat{W_d}$ reflect changes in the ratio of total absorption to wages (which is not one due to non-zero deficits).

Total manufacturing absorption is

$$T_d = \gamma_d \cdot \underbrace{\left(Y_d^T + D_d^T\right)}_{\text{final demand (total labor income, profits)}} + (1-\beta) \cdot \underbrace{\frac{\sigma-1}{\sigma} Y_d}_{\substack{\text{demand for intermediates} \\ \text{by manufacturing sector}}},$$

where Y_d^T is total GDP of country d, including labor income and profits, D_d^T is the current account deficit and Y_d output of the manufacturing sector. Notice that manufacturing spending equals $T_d = Y_d + D_d$, where D_d is the trade deficit in the manufacturing sector. So we can solve for T_d and Y_d and obtain

$$\begin{aligned} T_d &= \frac{\gamma_d\left(Y_d^T + D_d^T\right) - (1-\beta)(1-1/\sigma)D_d}{1/\sigma + \beta(1-1/\sigma)}, \\ Y_d &= \frac{\gamma_d\left(Y_d^T + D_d^T\right) - D_d}{1/\sigma + \beta(1-1/\sigma)}. \end{aligned} \tag{B.7}$$

To summarize, using the Dekle, Eaton, and Kortum (2007) algorithm, we can compute how given changes in the fixed costs $\widehat{\tilde{F}_{kd}}$ lead to $\hat{\lambda}_{sd}$, $\hat{P}_d$, $\hat{W}_d$, and the new T'_d, Y'_d by inspecting equations (B.4), (B.5) and imposing the current account balance condition

$$Y'_s L_s = \sum_{k=1}^{N} \lambda'_{sk} T'_k, \tag{B.8}$$

where we use relationship (B.7) and $T'_d = Y'_d + D_d$.

We obtain individual γ_d by country. To do so, we solve equation (B.7) for γ_d and compute

$$\gamma_d = \frac{T_d - (1-\beta)(1-1/\sigma)\left(T_d - D_d\right)}{Y_d^T + D_d^T}. \tag{B.9}$$

for every country, using data on T_d, D_d, Y_d^T and D_d^T.

C Nested Utility

We generalize the model to consumer preferences

$$\left(\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} \left[\sum_{g=1}^{G_{sd}(\omega)} x_{sdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \varepsilon > 1,\ \sigma > 1, \varepsilon \neq \sigma.$$

In this case we redefine the product efficiency index as:

$$H(G_{sd}) \equiv \left(\sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\varepsilon-1)} \right)^{-\frac{1}{\varepsilon-1}} . \tag{C.10}$$

With this new definition, the expressions for firm product sales (9) and for aggregate bilateral trade (19) in Proposition 3 remain unaltered. Restrictions 1 and 2 take a generalized form but the expressions are similar. For remaining details on the generalized model see our online Technical Appendix.

With this generalization, a firm's individual products can be less substitutable among themselves than with outside products (if $\varepsilon < \sigma$) or more substitutable ($\varepsilon > \sigma$). So the introduction of additional products has a direct effect on the sales of infra-marginal products. The effect is symmetric for all products, so relative sales of a firm's existing products are not affected by the introduction of additional products. This constancy of relative sales in our model does not carry over to models with CES-preferences and a countable number of firms such as Feenstra and Ma (2008) or to models with non-CES preferences such as Mayer, Melitz, and Ottaviano (2009) and Dhingra (2010).

D Data

D.1 Exporter-product-destination data

We identify an exporter's sector from the firm's reported *CNAE* four-digit industry (for 654 industries across all sectors of the economy) in the administrative RAIS records (*Relação Anual de Informações Sociais*) at the Brazilian labor ministry. The level of detail in *CNAE* is roughly comparable to the *NAICS 2007* five-digit level. To map from the HS 6-digit codes to ISIC revision 2 at the two-digit level we use an extended SITC-to-ISIC concordance, augmenting an OECD concordance for select manufacturing industries to all industries.[38]

As Table D.1 shows in columns 5 and 6, our Brazilian manufacturer sample includes 10,215 firms with shipments of 3,717 manufacturing products at the 6-digit Harmonized System level to 170 destinations, and a total of 162,570 exporter-destination-product observations.[39] Exporters

[38]Our SITC-to-ISIC concordance is available at URL *econ.ucsd.edu/muendler/resource*.

[39]We remove export records with zero value from the Brazilian data, which include shipments of commercial

Table D.1: Sample Characteristics by Destination

	From Brazil				
to destination (d)	USA (1)	Argentina (2)	OECD[a] (3)	non-OECD[a] (4)	Agg. World[a] (5)
# of Firms (M)	3,083	4,590	5,041	8,664	10,215
# of Destinations (N)	1	1	23	147	170
# of HS-6 products (G)	2,144	2,814	2,772	3,537	3,717
# of Observations	10,775	21,623	36,359	126,211	162,570
Destination share in Tot. exp.	.257	.144	.559	.441	1
Firm shares in Total exports					
Single-prod. firms	.123	.086	.142	.069	.090
Multi-prod. firms' top prod.	.662	.555	.625	.573	.597
Multi-prod. firms' other prod.	.215	.359	.233	.359	.313
Median Total exp. ($T_d(m)$)	.120	.068	.137	.066	.089
Median Exp. scope ($G_d(m)$)	1	2	2	2	2
Median Exp. scale ($a_d(m)$)	.068	.031	.070	.028	.037
Mean Total exports ($\bar{t}_d$)	3.170	1.192	4.217	1.932	3.720
Mean Exp. scope ($\bar{G}_d$)	3.495	4.711	3.933	5.176	5.278
Mean Exp. scale ($\bar{a}_d$)	.907	.253	1.072	.373	.705

[a]Each aggregate region (world, OECD, non-OECD) treated as a single destination, collapsing product shipments to different countries into single product shipment.

Sources: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at the HS 6-digit level. Exports in US$ million fob. Firms' exporter scale ($\bar{a}_d$ in US$ million fob) is the scope-weighted arithmetic mean of exporter scales. OECD includes all OECD members in 1990. The United States is Brazil's top export destination in 2000, Argentina second to top.

shipping multiple products dominate. They ship more than 90 percent of all exports from Brazil, and their global top-selling product accounts for 60 percent of Brazilian exports worldwide. A firm's top product, however, is not the same in all destinations. On average, the firms' local top-selling products account for 70 percent of local sales by destination (not reported).

To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table D.1 (columns 3 to 6), we treat each aggregate as if it were a single destination and collapse all product shipments to different countries within the aggregate into a single product shipment. In most data treatments in the text, in contrast, we analyze these variables country by country, consistent with our main hypothesis that distribution-side determinants of trade matter repeatedly destination by destination.

The median exporter is a relatively small exporter, with sales to the rest of the world totalling

samples but also potential reporting errors, and lose 408 of initially 162,978 exporter-destination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count.

around US$ 89,000. The mean exporter, in contrast, sells around US$ 3.7 million abroad, more than 40 times as much as the median manufacturer. Exporter scope and exporter scale exhibit similarly stark differences between mean and median. The median Brazilian manufacturer sells two products worldwide, but the mean scope per firm is 5.3 products. The median Brazilian manufacturer has a product scale of around US$ 37,000 per product, but the exporter scale per exporter is US$ 705,000, or around 20 times as high as that for the median firm.[40]

The importance of the top-selling product at multi-product exporters and the mean-median ratios repeat across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations (United States and Argentina), as well as the OECD and non-OECD aggregates. Our theory emphasizes the importance of exporting behavior within destinations. Within single countries, the mean manufacturer's exports exceed the median manufacturer's exports by similarly large factors as in the aggregate, between 14 (in Argentina, column 2) and 26 (in the United States, column 1). In the OECD aggregate (column 3), exports of the mean firm exceed the exports of the median firm by a factor of about 30. Interestingly, the same mean-median ratio of about 30 prevails in the non-OECD aggregate.

We further investigate the striking similarity of firm scope choices across destinations by relating the mean number of products to market characteristics. For this purpose, we consider a further decomposition of the left-hand side variable: define $\bar{t}_{sd}$ as the mean sales of firms from s selling to d divided by the exporter scale of these firms in market d. Thus $\bar{t}_{sd} \equiv \bar{G}_{sd}\,\bar{t}_{sd}/\bar{G}_{sd} \equiv \bar{G}_{sd}\,\bar{a}_{sd}$, where $\bar{G}_{sd}$ is the exporter mean exporter scope and $\bar{a}_{sd} \equiv \bar{t}_d/\bar{G}_{sd}$ is the exporter mean product scale per product. We regress the log mean exporter scope $\bar{G}_d$ on the log of $\lambda_{sd}T_{sd}$.

$$\ln \bar{G}_{sd} = \underset{(.676)}{2.324} + \underset{(.057)}{.087} \ln \lambda_{sd} - \underset{(.046)}{.058} \ln T_{sd}.$$

The R^2 is .281 (standard errors in parentheses). Neither market share nor market size are statistically significant predictors of exporter scope at conventional levels.[41] So, most of the variation in firms' exports to a market is due to variation in their mean scale per product.

At the firm level, the Brazilian data exhibit market-presence patterns broadly similar to the

[40] The means in Table D.1 are calculated as follows. A source country's total exports T_d are decomposed into $T_d = M_d\,\bar{G}_d\,\bar{a}_d$, where M_d is the number of exporters to destination d, $\bar{G}_d \equiv \sum_{\phi=1}^{M_d} G_d(\phi)/M_d$ is the exporters' mean exporter scope, and $\bar{a}_d \equiv \bar{t}_d/\bar{G}_d$ is their products' exporter scale. Equivalently, $\bar{a}_d$ is the weighted arithmetic mean of $a_d(\phi)$ over all ϕ, with weights $G_d(\phi)$: $\bar{a}_d = \sum_{\phi=1}^{M_d} G_d(\phi)\,a_d(\phi)/[\sum_{\phi=1}^{M_d} G_d(\phi)] = \bar{t}_d/\bar{G}_d$. As the decomposition shows, scope weighting is necessary for the mean scope and the exporter scale to yield total exports when multiplied.

[41] The R^2 drops to .212 when including $\ln M_{sd}$ and industry fixed effects but coefficients become statistically significant at conventional levels except for market size, while magnitudes change little.

French and U.S. firm-destination data. Similar to Eaton, Kortum, and Kramarz (2004), for instance, the elasticity of the number of firms with respect to the number of export destinations is about -2.5, just as for French exporters.

D.2 Estimation of Figure 1 (Fact 1)

To generate data such as those in Figure 1 (Fact 1), we aggregate individual product sales by averaging over all products of a given rank and all firms with a given exporter scope in a destination.

As shown in Appendix A.3, our model implies that the regression equation for average product sales in Figure 1 is

$$\begin{aligned}\ln \bar{a}_{sd} &= \ln \tfrac{\theta\sigma}{\theta-(\sigma-1)} f_{sd}(1) + [\delta + \alpha(\sigma-1)] \ln G_{sd}(\phi) - [\alpha(\sigma-1)] \ln g \\ &\quad + \ln\left[1 - [G_{sd}/(G_{sd}+1)]^{(\tilde{\theta}-1)(\delta+\alpha(\sigma-1))}\right] - \ln\left[1 - [G_{sd}/(G_{sd}+1)]^{\tilde{\theta}(\delta+\alpha(\sigma-1))}\right]\end{aligned}$$

for $g \leq G_{sd}$, where $\bar{a}_{sd} \equiv \int_{\phi_{sd}^{*,G}}^{\phi_{sd}^{*,G+1}} p_{sdg}(\phi)\, x_{sdg}(\phi)[\theta/\phi^{\theta+1}][1/(\phi_{sd}^{*,G})^{-\theta} - (\phi_{sd}^{*,G+1})^{-\theta}]\, \mathrm{d}\phi$ is average product sales for a product of rank g over all firms with exporter scope G_{sd}.

We fit this regression equation with non-linear least squares under three choices of the free parameter: $\tilde{\theta} = 1.06$ (as used by Luttmer (2007) and di Giovanni and Levchenko (2010) and reported by Axtell (2001)); $\tilde{\theta} = 1.28$ (our middle-ground benchmark for counterfactual simulations); and $\tilde{\theta} = 1.49$ (as reported by Arkolakis (2010)). We find for $\delta + \alpha(\sigma-1)$ estimates of 1.60 and 1.61 under the three parametrizations and for $\alpha(\sigma-1)$ an estimate of 2.55 under all three parametrizations. These estimates closely resemble those in Table 3. Not surprisingly, the estimate for $\alpha(\sigma-1)$ is close to the slopes between 2.6 and 2.8 that were reported in Section 2.2 for Figure 1.

D.3 Data for simulations

For bilateral trade and trade balances in manufacturing products, we use World Trade Flow (WTF) data in U.S. dollars for the year 2000 (Feenstra, Lipsey, Deng, Ma, and Mo 2005). To mitigate the effect of entrepôt trade, we follow Dekle, Eaton, and Kortum (2007) and collapse (i) Hong Kong, Macão and mainland China, (ii) Belgium, Luxembourg and the Netherlands, and (iii) Indonesia, Malaysia, Singapore, and Thailand into single entities. In 2000, import information for India is missing from WTF. We obtain information for India in 2000 from UN Comtrade. We keep only manufactured products from the WTF data, using a concordance from the OECD at the SITC revision-2 4-digit level to determine manufactured products. So our final bilateral trade data set excludes agricultural and mining commodities. By our construction, the world's trade balance in manufactures is zero.

For information on GDP, manufacturing value added and the overall trade balances in goods and services in 2000 we use the World Bank's World Development Indicators 2009 (WDI). India included, our initial WTF sample has 132 countries that can be matched to the WDI data, and we collapse bilateral trade for the rest of the world by trade partner into a 133rd observation. We compute GDP and manufacturing value added for the rest of the world as the WDI reported world total less the sample total of our 132 matched countries. We set the overall trade balances in goods and services for the rest of the world so that the world total is zero.

We obtain β from the UNIDO Industrial Statistics Database at the 3-digit ISIC level (revision 2), which offers both manufacturing value added and manufacturing gross production for 51 of our sample countries and the rest of the world. Averaging the ratio of manufacturing value added to manufacturing output in 2000 over these countries yields $\beta = .330$. This worldwide β estimate enters our computation of γ_d by (B.9).

We finally need information on manufacturing absorption. Following Eaton, Kortum, and Kramarz (2004), we infer manufacturing absorption as manufacturing output (from the UNIDO Industrial Statistics Database 2005) plus the trade deficit (from WTF). The UNIDO data for manufacturing output are considerably less complete than either WTF or WDI. We obtain manufacturing output for Brazil from the Brazilian statistical agency IBGE (2010). Our final country sample for which we have manufacturing absorption contains 57 countries. By the model in Appendix B, γ_d is given as (B.9). We use our WTF-WDI-UNIDO data to calculate γ_d by country. For the rest of the world, we set γ_d to the sample average over our 57 countries (average $\gamma = .244$) and back out manufacturing absorption for the rest of the world from (B.9).

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