THE EFFECT OF UNCERTAINTY ON INVESTMENT:
EVIDENCE FROM TEXAS OIL DRILLING

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ABSTRACT

Despite widespread application of real options theory in the literature, the extent to which firms actually delay irreversible investments following an increase in the uncertainty of their environment is not empirically well-known. This paper estimates firms' responsiveness to changes in uncertainty using detailed data on oil well drilling in Texas and expectations of future oil price volatility derived from the NYMEX futures options market. Using a dynamic model of firms' investment problem, I find that oil companies respond to changes in expected price volatility by adjusting their drilling activity by a magnitude consistent with the optimal response prescribed by theory.

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1. Introduction

How do firms make decisions regarding irreversible investments in uncertain economic environments? Such situations are common in a variety of industries: American Electric Power must commence construction of new plants before knowing future demand for electricity, Boeing must sink costs into new airplane designs before orders from customers are realized, and ExxonMobil must drill wells in the midst of a fluctuating price of oil. Each of these investments is at least partially irreversible because the assets created cannot be fully appropriated to an alternative use. In other words, these investments, once complete, become sunk costs.

The real options literature, beginning with Marschak (1949) and Arrow (1968) and developed in Bernanke (1983), Pindyck (1991), and Dixit and Pindyck (1994), explains how firms should time such investments. Real options theory views an irreversible investment as an option in that, at any point in time, a firm may choose to either invest immediately or delay and observe the evolution of the investment's payoff. A key insight is that the option to delay has value when future states of the world with positive returns to investing and states with negative returns are both possible, even holding the expected future return constant at its current level. Thus, in the presence of irreversibility and uncertainty, a naïve investment timing rule—proceed with an investment if its expected benefit even slightly exceeds its cost—is suboptimal because it does not account for the value of continuing to hold the option. Instead, firms should delay irreversible investments until a significant gap develops between the investments' expected benefits and costs. Moreover, as uncertainty increases, real options theory tells us that the incentive to delay should grow stronger and the gap between the expected benefit and cost necessary to trigger investment should widen.

While real options theory therefore prescribes how firms should carry out irreversible investments in uncertain environments, it is not empirically well-known how firms actually proceed in such situations. In particular, the theory's central prediction that firms should be more likely to delay investment if uncertainty increases, all else equal, has received only limited empirical scrutiny. The primary aim of this paper is therefore to assess the extent to which firms'

responses to changes in uncertainty align with the theory, using data on oil drilling activity in Texas coupled with market expectations of the volatility of the future price of oil.

The need for empirical work in the real options literature is underscored by the existence of numerous applications that assume firms optimally make decisions in the presence of uncertainty. In industrial organization, Pakes (1986), Dixit (1989), Grenadier (2002), Aguerrevere (2003), and Collard-Wexler (2008) model the implications of uncertainty and sunk costs for investment, entry, and research and development in several settings and under various forms of competition. The general dynamic oligopoly model of Ericson and Pakes (1995) is built on a framework in which firms treat many decisions as options. In macroeconomics, Bernanke (1983), Hassler (1996), Bloom (2009), and Bloom *et al.* (2007, 2009) construct models that emphasize the importance of changes in economy-wide uncertainty in determining the level of aggregate investment. Finally, in the environmental and resource economics literature, Arrow and Fisher (1974), among others, discuss the role of uncertainty in dictating when "green" investments should be undertaken.

I empirically examine the extent to which investments in oil wells respond to changes in uncertainty using a unique dataset of well-level drilling activity in Texas. I combine these drilling data with information from the New York Mercantile Exchange (NYMEX) on the expected future price of oil and the expected future price volatility. The expected volatility is derived from the NYMEX futures options market, in which volatility is implicitly traded and priced. Under a hypothesis that the market is an efficient aggregator of information, the implied volatility from futures options will incorporate more information than an expected volatility measure derived from price histories alone.

I conduct my analysis using an econometric model of firms' optimal drilling investment in the presence of time-varying uncertainty. The model is based on Rust's (1987) nested fixed point approach but allows the volatility of the process governing state transitions to vary over time. The use of this model allows me to do more than carry out a simple "yes/no" test of

whether or not firms respond to changes in expected oil price volatility: I can also compare the magnitude of firms' responses in the data to the magnitude prescribed by the model.

I find that the response of investment to changes in implied volatility is broadly consistent with optimal decision-making. In the reference case specification, in which the model's auxiliary parameters and assumptions most closely match the data and institutional setting, I find that the magnitude of firms' collective response to volatility shocks aligns very closely with theory. Alternative specifications and assumptions lead to estimates of different magnitudes, though these estimates remain qualitatively similar to the optimal response so long as volatility expectations are measured using implied volatility from futures options. When I instead measure expectations using historical price volatility, the estimated response of investment to changes in volatility is attenuated and imprecise, reflecting the relatively weak forecasting power of this measure.

There exist previous studies that have empirically examined whether investments respond to changes in uncertainty, though without linking the magnitudes of the estimated effects to theory. Several of these studies, like this one, focus on natural resource industries. Hurn and Wright (1994), Moel and Tufano (2002), and Dunne and Mu (2010) examine the impact of resource price volatility on offshore oil field investments, gold mine openings and closings, and refinery investments, respectively. None of these papers uses implied volatility to measure expected price volatility—the uncertainty measure is the historic realized variance of commodity prices—and they collectively find mixed evidence on whether increases in volatility reduce investment. Paddock, Siegel, and Smith (1988) shows that option pricing techniques yield more accurate predictions of oil lease valuations than do traditional net present value calculations, though without investigating the impact of changes in uncertainty over time. Other micro-empirical work includes Guiso and Parigi (1999), which finds evidence from a cross-sectional survey that Italian firms whose managers subjectively report high levels of expected demand uncertainty tend to have relatively low levels of investment. List and Haigh (2010) meanwhile

provides experimental evidence that investment timing decisions of agents (drawn from student and professional trader subject pools) are generally responsive to changes in payoff uncertainty.

Another set of papers in the macroeconomics literature measures the response of aggregate output and investment to changes in economy-wide uncertainty, as measured by the volatility of stock market returns or interest rates (Hassler 2001, Alexopoulos and Cohen 2009, Fernandez-Villaverde *et al.* 2009, and Bloom 2009). A related work is Leahy and Whited (1996), which examines firm-level investment and stock return volatilities. These papers generally find that increases in volatility are associated with decreases in output or investment. However, factors that influence the level of investments' expected payoffs are difficult to proxy for in this literature, and Bachmann *et al.* (2010) argues that a negative correlation between first and second moment shocks can lead to downward-biased estimates of the effects of an increase in uncertainty. Leahy and Whited (1996) also note that fluctuations in stock returns likely reflect the volatility of factors beyond those impacting the future revenues associated with new, marginal investment opportunities.

This paper's focus on the Texas onshore drilling industry as its object of study, combined with the econometric modeling of the firms' investment timing problem, confers valuable advantages relative to previous work. First, I possess data at the level of each individual investment—the drilling of each well—and need not rely on aggregate data or accounting data. Second, the NYMEX futures and futures options markets provide measures of the expected level and volatility of each investment's expected return that, in principle, incorporate all available information at the time of the investment. Such measures are not available in most industry settings, and here they allow for a separation of first and second moment shocks. Finally, I take advantage of the fact that oil production is a highly competitive industry, with no one firm able to influence the price of oil, and I focus on oil fields in which common pool issues are not a concern. I am therefore able to treat each firm's investment decision as a single-agent dynamic investment problem. This approach, which would be questionable in most other industries,

allows me to measure the magnitude of firms' response to uncertainty relative to the theoretical optimum, going beyond a simple test of whether or not firms respond to volatility shocks at all.

In what follows, I first discuss relevant institutional details of the Texas onshore drilling industry and the datasets I use. Section 3 follows with a descriptive analysis of the data. The remainder of the paper focuses on the estimation of a structural model of the drilling investment problem with time-varying uncertainty: section 4 presents the model, section 5 discusses the estimation procedure, and section 6 follows with the estimation results. Section 7 provides concluding remarks.

2. Institutional Setting and Data

2.1 Drilling description, types of wells used in this study, and drilling data

Oil and gas reserves are found in geologic formations known as fields that lie beneath the earth's surface, and the mission of an oil production company is to extract these reserves for processing and sale. To recover the reserves, the firm needs to drill wells into the field. Drilling is an up-front investment in future production; if a drilled well is successful in finding reserves, it will then produce oil for a period of several years, requiring relatively small operating expenses for maintenance and pumping. The firm does not know in advance how much oil will be produced (if any) from a newly drilled well, though it will form an expectation of this quantity based on available information, such as seismic surveys and the production outcomes of previously drilled wells. The price that the firm will receive for the produced oil is also not known with certainty at the time of drilling. Conversations with industry participants have indicated that some, though not all, firms use the NYMEX market to hedge at least some of their price risk. This use of the NYMEX indicates that risk aversion over future oil prices is unlikely to influence drilling decisions, since any firm that is risk averse can hedge the price risk away.

Drilling costs range from a few hundred thousand dollars for a relatively shallow well that is a few thousand feet deep to millions of dollars for a very deep well (as much as 20,000

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feet deep). Once drilled, these costs are almost completely sunk: the labor and drilling rig rental costs expended during drilling cannot be recovered, nor can the expensive steel well casing and cement that run down the length of the hole. Drilling can therefore be modeled as a fully irreversible investment.

Wells can be one of three types: exploratory, development, or infill. Exploratory wells are drilled into new prospective fields, and if successful they can not only be productive themselves but also lead to additional drilling activity. Development wells are those that follow the exploratory well: they increase the number of penetrations into a recently discovered field in order to drain its reserves. Finally, infill wells are drilled late in a field's life to enhance an oil field's production by "filling in" areas of the reservoir that have not been fully exploited by the pre-existing well stock.

In this paper, I exclude exploratory and development wells and analyze only the subset of data corresponding to infill wells. This exclusion facilitates this study in two important ways. First, examining only infill wells constrains the set of available drilling options to those that exist within a finite, known set of fields. Thus, I need not be concerned with the creation of new options through new field discoveries or leasing activity. Second, the majority of production from a typical infill well takes place within the first year or two of the well's life: because infill wells tap only small isolated pools of oil that have been left behind by older wells in a field, their productive life is quite short. Thus, I may rely on liquid near-term futures to provide expected prices and volatilities that are relevant for these wells rather than less liquid long-term futures.

I also distinguish wells drilled in fields operated by a single firm from wells drilled in fields operated by multiple firms. The process by which production companies acquire leases—rights to drill on particular plots of land—often leads to situations in which several firms have the right to drill in and produce from a single field (see Wiggins and Libecap 1985). This division of operating rights leads to a common pool problem to the extent that each firm's actions leads to informational and extraction externalities for its neighbors, suggesting that in such situations a dynamic game is needed to model firms' drilling problem. This paper avoids

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this substantial complication by focusing exclusively on wells drilled in sole-operated fields, for which a single-agent model is sufficient to model drilling behavior.[1]

I obtained drilling data from the Texas Railroad Commission (TRRC), yielding information regarding every well drilled in Texas from 1977 through 2003.[2] These data identify when each well was drilled, which field it was drilled in, whether it was drilled for oil or for gas, and the identity of the production company that drilled it. During the 1993-2003 period for which I also observe data on drilling costs and expected oil prices, I observe a total of 23,279 oil wells.[3] Of these, 17,456 are infill wells and 1,150 are infill wells drilled in sole-operated fields.[4]

The time series of Texas-wide drilling activity is depicted in figure 1 as the number of wells drilled per month. These data appear to be noisy because they are integer count data ranging from 2 to 19 wells per month. The time series of drilling activity in a larger sample that includes wells drilled in common pool fields does not exhibit this noisiness, confirming that it is due to the integer count nature of the data rather than a systematic feature of the industry.

The drilled wells are spread over 663 sole-operated fields and 453 firms. The mean number of wells per field is 1.73, and I observe only one well drilled in the majority of fields in the data. The maximum number of wells I observe in any field is 31. In addition to the 663 fields

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[1] Industry participants have suggested that the degree of strategic interaction amongst firms drilling infill wells in common pool fields may be limited in practice because infill drilling targets tend to be small pools that are geologically isolated from other parts of the field. In addition, the TRRC regulates the minimum distance from a neighbor's lease at which a well may be drilled. Correspondingly, the time series of infill drilling in common pools is very similar to that shown for sole-operated fields in figure 1. I nonetheless focus my analysis on sole-operated fields to be conservative, though estimating the model using the full sample of infill wells yields similar results to those presented here (the estimate of β is 1.033).

[2] While drilling data exist beyond 2003, industry participants have indicated that the dramatic increase in oil and natural gas prices that began in 2004 increased drilling activity to the extent that the rig market became extremely tight. Long wait lists developed when large production companies locked up rigs on long-term contracts so that the spot rental market could not allocate rigs based on price. Because these unobservable wait lists disconnect drilling decisions from observed drilling, I only use data through 2003.

[3] I define an oil well as a well that is marked as a well for oil (rather than for "gas" or "both") on its TRRC drilling permit and is drilled into a field for which average oil production exceeds average natural gas production on an energy equivalence basis (1 barrel of oil is equivalent to 5.8 thousand cubic feet of gas).

[4] I define infill wells as those that are drilled into fields discovered prior to 1 January, 1990. I define a sole-operated field as one for which, in every year from 1993-2003, only a single firm is listed as a leaseholder in the field's annual production data. This definition allows a field to be traded from one firm to another but disallows fields in which several firms operate simultaneously on different leases.

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in which I observe drilling, I also observe 6,637 sole-operated oilfields in which no infill wells are drilled. The median number of wells per firm is 1, the mean is 2.54, and the maximum is 31. Thus, the majority of wells in the dataset can be characterized as having been drilled by small firms in relatively small, old fields with few remaining drilling opportunities.

2.2 Oil production

I acquired oil production data from the TRRC to assess the production that resulted from the observed drilling activity. The TRRC records monthly oil production at the lease-level, not the well-level, because individual wells are not flow-metered. I am therefore only able to identify the production from those wells that are drilled on leases on which there exist no other producing wells and there is no subsequent drilling: this is the case for 162 of the 1,150 drilled wells. For these wells, I tabulate the total production of each for the three years subsequent to drilling: the median well produces 8,625 barrels (bbl), and the mean produces 15,794 bbl. 4.3% of the wells are dry holes and produce nothing; the maximum production is 164,544 bbl.

Figure 2 displays the average monthly production profile of a drilled well in the sample. Production begins immediately subsequent to drilling, and depletion of the oil pool results in a fairly steep production decline so that a typical well's monthly production falls to one-half of its initial level only 7 months into the well's life. In addition, firms do not appear to alter production rates or delay the start of production due to oil price changes; the shape of the production profile is consistent throughout the data, including the 1998-1999 period when the price of oil was very low. This profile is consistent with a production technology in which production rates are constrained by geologic characteristics of the oil reservoir such as its pressure, the remaining volume of oil near the well, and rock permeability. It is also consistent with low operating expenses, so that the probability that the oil price will fall below the point at which revenues equal operating costs is extremely low. Thus, the option value represented by the ability to adjust a well's production rate in response to price changes is negligible, implying that drilling and production do not need to be modeled as a compound option.

2.3 Expected oil prices

I obtained data on the expected oil price from the prices of NYMEX crude oil futures contracts. With risk neutral traders and efficient aggregation of information by the market, the futures price is in theory the best predictor of the future price of oil. In practice, while futures prices have been found to provide slightly more precise predictions than the current spot price (i.e., a no-change forecast) during the 1993-2003 period I study here (Chernenko *et al.* 2004), the improvement is not statistically significant. Moreover, when data through 2007 are used, spot prices actually slightly outperform futures prices, though again the difference is not statistically significant (Alquist and Kilian 2010). Given the slightly superior performance of NYMEX futures during the sample period of this paper and the fact that a majority of producers claim to use futures prices in making their own price projections (SPEE 1995), I will use futures prices as the measure of firms' expected price of oil. In a secondary specification, I explore how the use of spot prices impacts the results.

I focus on the prices of futures contracts with 18 months to maturity.[5] This maturity is the longest time horizon for which NYMEX futures are traded regularly (on 84% of all possible trading days over 1993-2003). In addition, the typical production profile of drilled infill wells suggests that 18 months might be a reasonable forecast horizon for a firm to use when evaluating a drilling prospect, since approximately one-half the well's total expected production is likely to be exhausted at this time.[6] In an alternative specification, I use the 12-month contract.

Futures prices are generally consistent with mean-reverting expectations about the future price of oil, as shown in figure 3. When the front-month (nearest delivery month) oil price exceeds approximately $20/bbl (real $2003), the price of an 18-month futures contract tends to

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[5] In reality, it is rare that a NYMEX futures contract has a time to maturity of exactly 18 months (548 days) since the available contracts that can be traded have maturities that are either one full month or one full quarter apart. On any given trading date, I therefore treat contracts with a time to maturity that is within 46 days of 18 months as having a maturity of 18 months. When more than one such contract is traded on any given trading date, I average the prices across the contracts.

[6] This half-life is derived by fitting a hyperbolic curve to the average production data (figure 2) and extrapolating production beyond 3 years. Based on this curve and a 9.9% real discount rate (see section 5.1), half of a typical well's expected discounted production is exhausted in 19.2 months.

be lower than the front-month price, and the reverse holds when the front-month price is below $20/bbl. The prices of 12-month futures are usually near those of 18-month futures, indicating that the market generally believes that most mean reversion will occur within a year of the trading date.

2.4 Expected oil price volatility

I derive my primary measure of firms' expected future price volatility from the volatility implied by NYMEX futures options prices. Across numerous commodity and financial contracts, implied volatility has been found to be a better predictor of future volatility than measures based on historic price volatility, including GARCH models (Poon and Granger 2003, Szakmary *et al.* 2003). Intuitively, if markets are efficient then options prices incorporate up-to-date information beyond that available from price histories alone, improving their predictive power.

The classic formula for the value of a commodity option contract is based on the Black-Scholes model (1973) and given by Black (1976). It requires as inputs the expected volatility of the commodity price, the option's time to maturity τ, the price of the futures contract with time to maturity τ, the option's strike price, and the riskless rate of interest.[7] Given an options price, Black's formula can be inverted to calculate the expected price volatility implied by the option.

Black's formula assumes that the term structure of volatility is constant over the life of the option; that is, spot price volatility equals future price volatility at any time to maturity.[8] Hilliard and Reis (1998), however, show that when futures prices exhibit mean reversion, the expected volatility of futures prices declines as time to maturity increases. In figure 3, for example, it is apparent that the NYMEX front-month contract is, on average, more volatile than the 18-month contract. With mean reversion, an 18-month futures option price gives the average implied volatility of futures price contracts with maturities between the front-month and 18

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[7] I use the interest rate on one-year treasury bills to measure the riskless rate of interest.

[8] The Black (1976) formula also assumes that the options are European and that volatility is not stochastic. As discussed in appendix 1, however, these assumptions are likely to be minor in importance.

months. This average implied volatility will be greater than the implied volatility of an 18-month futures price contract, and my analysis requires the latter of these two variables, not the former.

To address this issue, I first use the realized volatility of futures prices to estimate the average term structure of volatility: the function by which the volatility of the future price of oil declines as time to maturity increases. I then use liquidly traded short-term futures options to generate a time series of the implied volatility of a one-month futures option contract. Because a one month time horizon is short, this time series is equivalent to the time series of the implied volatility of the one-month futures price contract. Finally, I combine the one-month futures price volatilities with the estimated term structure to generate the desired time series of the implied volatility of an 18-month futures price contract. In appendix 1, I discuss this procedure in more detail and provide evidence that the term structure of volatility is stable over time.[9] The time series of implied 18-month futures price volatilities is given in figure 1 alongside the time series of 18-month futures prices (both series are averages of daily observations within each month).

In secondary empirical specifications, I construct volatility forecasts using historic futures price volatility rather than implied volatility derived from futures options. These specifications address the possibility that oil production firms' volatility forecasts differ from those of the market. One possible forecast is a no-change forecast; that is, the expected future volatility of the NYMEX futures price is its recent historic volatility. Figure 4a compares the historic volatility of the futures price, measured over rolling windows of one month and one year, to the implied volatility series. The one-month forecast is clearly noisy relative to implied volatility, reflecting small sample variation in volatility calculated using only one month of price data. The one-year forecast is considerably smoother than the one-month forecast and sometimes

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[9] An alternative procedure to that used here would use the term structure of the implied volatility of futures options directly to derive the implied volatility of 18-month futures prices. This approach would use the fact that the volatility of a τ-month futures price is equal to the volatility of a τ-month futures option plus τ times the derivative of the futures option term structure (with respect to τ) at τ. The use of the derivative implies that this approach requires a very precise estimate of the term structure of futures options' implied volatility. Thin markets for futures options beyond 6 months render this procedure impractical. For example, 18-month futures options are traded, on average, only 18 days each year from 1993-2003.

deviates substantially from implied volatility. Historic volatility is relatively high in 1997, low in 1998, and does not reflect the implied volatility spikes in 1999 and September 2001.

I have also forecast volatility using a GARCH(1,1) model. For each date in the dataset, I estimate the GARCH parameters using a four-year rolling window of daily 18-month futures prices.[10] At each date, I then use the estimated GARCH model to forecast volatility over the upcoming month. The average forecasted volatility over this month is then used as the measure of firms' expected price volatility. Figure 4b plots this GARCH volatility forecast against the implied volatility from futures options. The GARCH forecast aligns more closely with the implied volatilities than do the measures of historic volatility, though these time series do still differ substantially at various points, most notably 1997-1998 and late 2001.

2.5 Drilling costs

The primary source for information on drilling costs is RigData, a firm that publishes reports on the onshore U.S. drilling industry and collects data on daily rental rates ("dayrates") for drilling rigs from surveys of drilling companies.[11] Rig rental comprises the single largest line-item in the overall cost of a well, and industry sources have suggested that at typical dayrates rig rental accounts for one-third of a well's total cost.[12] Because I observe dayrates but not other components of drilling costs, I will assume that non-rig costs are constant in real terms and equal to twice the rig rental cost at the average sample dayrate. This constant cost assumption seems reasonable over the 1993-2003 sample. Prices for steel, which factor into prices for drill pipe,

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[10] In the GARCH model, the mean price equation is a seventh-order autoregression; this number of lags is necessary to eliminate serial correlation in the price residuals. A GARCH(1,1) process is then sufficient to eliminate conditional heteroscedasticity in the residuals (the p-value for rejecting a null hypothesis of no conditional heteroscedasticity is 0.423).

[11] The oil production firms that hold leases, make drilling decisions, and are the focus of this study do not actually own the drilling rigs that physically drill their wells. Rigs are instead owned by independent drilling companies that contract out their drilling services. See Kellogg (2009) for further information regarding the contracting process between production firms and drilling companies.

[12] This one-third figure was suggested by RigData and substantiated by information from the Petroleum Services Association of Canada's (PSAC's) Well Cost Study (summers of 2000 through 2004). This study provides a break-out of the costs of drilling representative wells across Canada during the summer months. For the non-Arctic, non-offshore areas that most closely resemble conditions in Texas, rig rental costs averaged 35.2% of total costs.

bits, and well casing, were fairly stable over this time, nominally increasing by an average of 1.8% per year according to data from the Bureau of Labor Statistics. Other substantial components of cost, such as site preparation, construction, and general equipment rental (pumps, for example), should be based primarily on prices for non-specialized labor and capital inputs and therefore also be stable in real terms.[13] As for the assumption that these non-rig costs constitute two-thirds of total drilling costs on average, I explore the use of alternative ratios as robustness tests when estimating the model.

The RigData dayrate dataset is quarterly and continuously reported from 1993 onwards. Because I conduct my analysis at a monthly level, I generate monthly dayrate data by assigning each quarterly reported dayrate to the central month of each quarter and then linearly interpolating dayrates for the intervening months. The alternative approach of simply treating dayrates as constant within each quarter has only a minor effect on the estimated results.

Because drilling rigs are pieces of capital that are specific to the oil and gas industry, rig rental rates are positively correlated with oil and gas prices and, accordingly, vary over the sample frame. For a well of average depth (5,825 feet in the sample), the dayrate ranges from $5,015 to $12,056, with an average of $7,163.[14] Given an average drilling time of 19.2 days, the average rig rental cost for a well is therefore $137,528 and average non-rig costs, estimated to be twice this amount, are $275,057 (all figures in real 2003 US$).

For each month in the sample, I calculate the total drilling cost of an average well as the sum of 19.2 days times the prevailing dayrate for that month (in real terms) with average non-rig costs. The time series of drilling costs for an average well is plotted alongside oil futures prices in figure 5. The positive correlation between these two series is readily apparent.

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[13] Evidence in support of this claim is available from the 2002, 2003, and 2004 PSAC Well Cost Studies, during which time the specifications for the representative wells were essentially unchanged. These data indicate that non-rig drilling costs changed, on average, by only -0.2% in 2003 and +3.1% in 2004. Rig-related drilling costs, however, increased by 9.8% in 2003 and 30.9% in 2004, following increases in the price of oil.

[14] RigData reports dayrates separately for rigs drilling wells between zero and 5,999 feet deep and for rigs servicing wells between 6,000 and 9,999 feet deep. The dayrates used in this study are the average of these two depth classes for the Gulf Coast / South Texas region.

3. Descriptive results

Figure 1 plots the three time series of primary data: drilling activity, oil futures prices, and implied oil price volatility from futures options. Several features of the plot are worth noting. First, drilling activity rises and falls with the oil price. In particular, the oil price crash of 1998-1999 that was driven by the Asian financial crisis (Kilian 2009) is associated with a sharp reduction in drilling activity. Second, following the 1998-1999 price crash, oil prices rapidly recovered and by the beginning of 2000 actually surpassed their pre-1998 levels. However, oil drilling did not enjoy a similar recovery: activity did increase once prices began to rise in the summer of 1999 but recovered only to approximately two-thirds of its pre-1998 level. Why did drilling activity not reach its earlier level despite such a high oil price? The third line on the graph—implied volatility—suggests that an increase in uncertainty following the 1998 price crash may have caused producers to delay the exercise of their drilling options. Implied volatility increases sharply at the end of 1998 and remains at an elevated level for the remainder of the sample; this high level of volatility is associated with the period in which expected oil prices were high yet drilling activity was low. Moreover, several positive shocks to volatility subsequent to 1999, such as the volatility spike following 11 September, 2001, appear to be associated with reductions in drilling activity.

A descriptive statistical analysis using a hazard model confirms that the negative relationship between drilling and expected oil price volatility that is apparent in figure 1 is in fact statistically significant. The unit of observation in this analysis is an individual drilling prospect and I model 7,787 such prospects: the 1,150 observed infill wells plus one prospect for each of the 6,637 sole-operated fields in which I observe no drilling activity. In doing so, I treat prospects that exist within the same field as independent of one another. While this treatment does not allow for the modeling of factors that might cause wells within the same field to be drilled at nearly the same time, the fact that most fields have zero or one well suggests that the impact of modeling all drilling decisions independently of one another may be minor.

I choose a hazard model, rather than a more conventional OLS regression of drilling activity on expected price and volatility, to capture the idea that drilling activity should decline over time as the set of available options is gradually reduced through drilling. In the simplest possible model, I model the hazard rate as an exponential function of the expected future price level and expected price volatility per (1) below.

$$\gamma(t) = \exp(\beta_0 + \beta_p \cdot Price_{t-3} + \beta_v \cdot Vol_{t-3}) \tag{1}$$

In estimating both this model and the structural model described below, I lag all covariates by three months, as industry participants have indicated that the engineering, permitting and rig contracting processes generally drive a three month wedge between the decision to drill and the commencement of drilling. For inference, I use a "sandwich" variance-covariance matrix estimator that allows arbitrary within-field correlation of the likelihood scores (Wooldridge 2002).[15] In practice, this estimator increases the estimated standard errors by about 25%, on average, relative to the standard BHHH estimator.

The results of estimating (1) are presented in column I of table 1. A \$1.00 increase in the expected future price of oil is associated with an increase in the likelihood of drilling of 4.1%, and a one percentage point increase in expected price volatility is associated with a decrease in the likelihood of drilling of 3.1%. Both of these point estimates are statistically significant at the 1% level. Columns II through IV of table 1 indicate that these correlations are robust to alternative specifications that allow for changes in drilling costs, unobserved prospect-specific heterogeneity, and a time trend. Column V, however, indicates that no statistically significant correlation between the drilling hazard and expected volatility is found when the specification includes an indicator variable for whether the date is greater than or equal to July 1998. Thus, the

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[15] Wooldridge (2002) shows that that this approach, which is analogous to clustering in linear regression models, still produces consistent estimates of the parameters even though serial and cross-well correlation within each field is not explicitly accounted for in the likelihood function. I also use this approach when estimating the structural model, discussed in sections 4 through 6. I have also estimated these models while clustering the standard errors on time rather than field to account for cross-sectional correlation of the likelihood scores. These estimated standard errors are generally similar to those obtained from the standard BHHH estimator.

observed negative correlation between drilling rates and expected volatility is largely accounted for by the substantial and persistent increase in volatility beginning in July 1998 and the coincident, persistent decrease in drilling.

Because these descriptive results, in the absence of an economic model, cannot speak to the optimality of firm decision-making or welfare, the remainder of this paper focuses on formulating and estimating a model of the infill drilling problem faced by oil production companies in Texas. The primary goal of this model is to relate the observed responses of firms to changes in uncertainty to the theoretically optimal response. In what follows, I also discuss the plausibility of alternative explanations for the persistent decrease in drilling subsequent to 1998.

4. A model of drilling investment under time-varying uncertainty

4.1 Model setup

Consider a risk-neutral, price-taking oil production firm that is deciding whether to drill some prospective well i at date t. Using geologic and engineering estimates, the firm generates an expectation regarding the monthly oil production from the well should it be drilled. The present value of the well's expected revenue is then equal to the sum, over the months of the well's productive life, of the product of the well's expected monthly production with the expected oil price each month, net of taxes and royalties, and discounted at the firm's discount factor δ. Rather than model this discounted sum explicitly, I model it instead as simply the product $r_i P_t$. Here, r_i represents the sum of the well's expected monthly production, net of taxes and royalties, and discounted so that it is in present value terms.[16] P_t represents the "average" oil price that will prevail over all barrels of oil expected to be produced by the well, so that the

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[16] A narrow view of r_i suggests that I am assuming that the ongoing production from any previously drilled wells in the same field as well i is unaffected by the drilling of well i. This assumption is incorrect if the new well is, at least to some extent, only accelerating the recovery of reserves from the field rather than exploiting new reserves that the existing well stock did not reach. However, the model can handle wells drilled with the purpose of acceleration by interpreting the expected productivity r_i as the expected production of the new well net of its expected impact on the production from the existing well stock (if any).

product r_iP_t is equal to the original discounted sum of monthly revenue. In the estimation that follows, I will use the 18-month futures price of oil as P_t. This simplification allow me to model the price level using only the single state variable P_t rather than a vector of state variables for the expected price in each month of the well's productive life.

I emphasize that r_iP_t is the firm's *expectation* of the value that will be obtained from drilling. Realized value may differ substantially from r_iP_t because the realized oil price may differ from P_t (though the firm could hedge this risk away) and because realized production may differ from r_i. Recall that some of the wells observed in the sample yielded zero oil production. Clearly, a dry hole was not the firms' expected outcome for these wells.

In month t, the well's drilling cost is equal to the sum of non-rig costs c_i with the product of the dayrate D_t and the number of days d_i required to drill the well.[17] Then, given an expected oil price P_t and a dayrate D_t, the expected profits π_{it} from drilling the well are given by the function π_i:

$$\pi_{it} = \pi_i(P_t, D_t) = r_iP_t - c_i - d_iD_t \qquad (1)$$

It will be useful for estimation to rearrange (1), defining the expected productivity of a well as the ratio of its expected production r_i to its drilling cost at the average dayrate. Denote this cost by $\overline{C}_i = c_i + d_i\overline{D}$ and let this ratio be denoted by x_i. Further, let $\tilde{c}$ denote $c_i / \overline{C}_i$ and let $\tilde{d}$ denote $d_i / \overline{C}_i$. Assuming that the ratio of non-rig costs to total costs at the average dayrate is constant across wells implies that both $\tilde{c}$ and $\tilde{d}$ are constant across wells (in the reference case model, I set $\tilde{c} = 2/3$ and $\tilde{d}\overline{D} = 1/3$ per the discussion in section 2.5). Then, expected profits π_{it} can be re-written as (2) below, in which all cross-well productivity heterogeneity relevant to the drilling timing decision is collapsed into the single variable x_i.

$$\pi_{it} = \pi_i(P_t, D_t) = \overline{C}_i(x_iP_t - \tilde{c} - \tilde{d}D_t) \qquad (2)$$

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[17] I assume that d_i does not vary over time. Learning-by-doing could cause d_i to decrease as more wells are drilled in the field (Kellogg 2009); however, since most of the observed sole-operated fields have only one new well during the sample, this effect is likely to be negligible. Technological progress might also decrease d_i over time; this possibility is part of the motivation for allowing for a time trend in an alternative specification.

I treat all firms as price takers, in the sense that they believe that their decisions do not impact P_t or D_t. This assumption almost certainly holds institutionally. The market for oil is global, and Texas as a whole constitutes only 1.3% of world oil production. With respect to oil producers' monopsony power in the market for drilling services, the largest firm in the dataset is responsible for only 2.2% of all wells drilled in Texas during the sample period, a quantity that seems insufficient for exertion of substantial market power.

Let the processes by which firms believe the price of oil and drilling costs evolve be first-order Markov and be given by (3) and (4) below. P_t denotes the oil price (18-month NYMEX future) in the current month t, and P_{t+1} is the price in month $t+1$. D_t and D_{t+1} represent the current and next month's dayrates.[18]

$$\ln P_{t+1} = \ln P_t + \mu(P_t, \sigma_t^2) - \sigma_t^2 / 2 + \sigma_t \varepsilon_{t+1} \tag{3}$$

$$\ln D_{t+1} = \ln D_t + \hat{\mu}(D_t, \hat{\sigma}_t^2) - \hat{\sigma}_t^2 / 2 + \hat{\sigma}_t \hat{\varepsilon}_{t+1} \tag{4}$$

The firm's current expectation of the volatility of the oil price is denoted by σ_t, and the price shock ε_{t+1} is an iid standard normal random variable that is realized subsequent to the firm's drilling decision in the current period. Because I do not observe expectations of dayrate volatility $\hat{\sigma}_t$, I assume that this volatility is a scalar multiple of the oil price volatility so that $\hat{\sigma}_t = \alpha \sigma_t$. The cost shock $\hat{\varepsilon}_{t+1}$ is drawn from a standard normal that has a correlation of ρ with ε_{t+1}.

$\mu(P_t, \sigma_t^2)$ and $\hat{\mu}(D_t, \hat{\sigma}_t^2)$ denote the expected price and drilling cost drifts as stationary functions of the current expected level and volatility of the oil price and dayrate. Dependence of these drifts on the price and dayrate levels allows for the mean reverting expectations exhibited by NYMEX futures prices (figure 3). I also allow the drifts to depend on volatility because, as pointed out by Pindyck (2004), an increase in volatility may increase the marginal value of storage and therefore raise near-term prices. In addition, a volatility increase may also affect investments related to oil production and consumption (via the real options mechanism

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[18] These transition functions are the discrete time analogue to geometric Brownian motion with drift (see Dixit and Pindyck 1994). Volatility is assumed to be constant within each time step.

considered here, for example), affecting expectations of future prices. The specification and estimation of $\mu(P_t, \sigma_t^2)$ and $\hat{\mu}(D_t, \hat{\sigma}_t^2)$ is discussed in section 5.1

4.2 Optimal drilling with time-varying volatility

The firm's problem at a given time t is to maximize the present value of the well V_{it} by optimally choosing the time at which to drill it. This optimal stopping problem is given by (5) below, in which Ω denotes a decision rule specifying whether the well should be drilled in each period $\tau \geq t$ as a function of P_τ and D_τ (conditional on the well not having been drilled already). I_τ denotes a binary variable indicating the outcome of this decision rule each period and δ denotes the firm's real discount factor.

$$V_{it} = \max_{\Omega} E\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} I_\tau \pi_i(P_\tau, D_\tau) \right] \tag{5}$$

In formulating (5), I assume that firms holding multiple drilling options treat them independently of one another. Given that I only observe zero or one well drilled in most fields in the sample, this assumption does not seem particularly strong. In those cases in which a firm holds multiple drilling options within the same field, it may be that the outcome from drilling one well may convey information regarding other prospects. That is, if the first well drilled by a firm in a field turns out to be highly productive, the firm may increase its estimate of x_i for its remaining prospects.[19] This contingent re-evaluation will result in temporal clustering of drilling activity in multi-well fields relative to what would be predicted by (5) alone.

Because drilling a well is irreversible and future prices and costs are uncertain, the decision rule for maximization of (5) is not simply to invest in the first period in which $\pi_{it} \geq 0$. The firm must trade off the value of drilling immediately against the option value of postponing the investment to a later date, at which time the expected oil price may be higher or the drilling

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[19] The process by which firms learn about the quality of fields through drilling is examined by Levitt (2009), which develops and estimates a dynamic learning model. That paper's approach cannot be used here because it requires data on oil production outcomes for all drilled wells and because the separate identification of learning effects and location-specific heterogeneity requires observations of different firms drilling wells in the same field (as well as an assumption of no cross-firm information spillovers).

cost lower. This trade-off is captured by re-stating the optimal stopping problem as the Bellman equation (6) below, in which $V_i(\cdot)$ represents the current maximized value of the drilling option as a function of the state variables P, D, and σ (from which I now remove the subscript t). V_i' represents this maximized value in the upcoming period.

$$V_i(P,D,\sigma) = \max\left\{\pi_i(P,D),\ \delta \cdot E[V_i'(P',D',\sigma')]\right\} \tag{6}$$

Equation (6) includes the firm's expected oil price volatility σ as a state variable even though it does not appear in the profit function $\pi_i(\cdot)$. Volatility impacts drilling decisions through its impact on the distribution of next period's expected oil price P' given the current expected price P. An increase in σ increases the variance of P' conditional on P, thereby increasing the value of holding the drilling option relative to the value of drilling immediately.

Intuition suggests that the solution to (6) will be governed by the following "trigger rule": at any given P, D, and σ, there will exist a unique $x^*(P,D,\sigma)$ such that it will be optimal to drill prospect i if and only if $x_i \geq x^*(P,D,\sigma)$. Furthermore, x^* will be strictly decreasing in P and strictly increasing in D and σ. The following conditions on the stochastic processes governing the evolution of P, D, and σ (none of which is rejected by the data) are sufficient for this trigger rule to hold. S denotes the state space.

(i) $\delta E[P'|P,D,\sigma] < P \quad \forall P,D,\sigma \in S$ (oil prices cannot be expected to rise more quickly than the rate of interest)

(ii) $\dfrac{\partial E[P'|P,D,\sigma]}{\partial P} < \dfrac{1}{\delta}$, with the same holding for D and σ, $\forall P,D,\sigma \in S$ (the expected rates of change of each state variable cannot increase too quickly with the current state)

(iii) $\rho < 1$ (oil price shocks and dayrate shocks are not perfectly correlated)

(iv) The distribution of P' is stochastically increasing in P, with the same holding for D and σ

(v)　$\delta E[\pi(P',D',\sigma')\,|\,P,D,\sigma] < \pi(P,D,\sigma)$　$\forall P,D,\sigma \in S$　(the Hotelling condition necessary for drilling to be optimal: expected profits cannot rise more quickly than the rate of interest)

It is straightforward to show that conditions (i)-(iii) imply that $\pi(s) - E[\pi(s'\,|\,s)]$ is strictly increasing in P and x_i, and strictly decreasing in D and σ. Given this result and conditions (iv) and (v), a fixed point contraction mapping argument given in Dixit and Pindyck (1994) proves that the trigger $x^*(P,D,\sigma)$ must exist. There must also exist similar triggers $P^*(D,\sigma,x_i)$, $D^*(P,\sigma,x_i)$, and $\sigma^*(P,D,x_i)$, representing the minimum price, maximum drilling cost, and maximum volatility at which drilling is optimal as functions of the other variables. The existence of all four triggers implies that $x^*(P,D,\sigma)$ must be strictly decreasing in P and strictly increasing in D and σ.

Thus, an increase in expected volatility σ will cause a fully optimizing firm to increase the productivity trigger x^* necessary to justify investment, holding the expected price and dayrate constant. Consider such a firm for which the price volatility expectation σ is equal to the volatility implied by the futures options market, which I denote by σ^m. Figure 6 illustrates how the firm's critical productivity x^* will vary with P and σ^m for a well with an average drilling cost at the average sample dayrate. The relationship between x^* and P is shown at both low (10%) and high (30%) levels of expected price volatility σ^m. At both volatility levels, x^* decreases with price so that less productive wells may be drilled in relatively high price environments. Holding price constant, x^* is greater in the high volatility case than the low volatility case.

Now, however, suppose that firms have time-varying expectations about future volatility that coincide with those of NYMEX market participants but do not take these expectations into account when making drilling decisions, so that in terms of the model σ is effectively constant over time. In this case, the two lines in figure 6 will coincide. It is this difference in investment behavior between firms that respond to σ^m and those that do not that will provide identification in the empirical exercise described below. Note, however, that an observed lack of response to σ^m

could also reflect the possibility that, while firms properly take expected volatility into account when making investment decisions, they hold a belief that volatility σ is constant over time rather than equal to the time-varying σ^m. Thus, to the extent that the data imply differences between σ and σ^m, I will not be able to identify whether the differences are due to sub-optimal investment decision-making or to differences between firms' beliefs and those of the broader market.

I capture the extent to which firms optimally respond to the market's implied volatility σ^m by parameterizing firms' beliefs through a behavioral parameter β. First, define $\overline{\ln \sigma}$ to be the average log of the market volatility over the first year of the sample (12.3%), and let $\ln \sigma^d$ be the deviation of $\ln \sigma^m$ from $\overline{\ln \sigma}$. That is:

$$\ln \sigma^m = \overline{\ln \sigma} + \ln \sigma^d \tag{7}$$

I then relate the firm's expected volatility σ to σ^d via (8):

$$\ln \sigma = \overline{\ln \sigma} + \beta \ln \sigma^d \tag{8}$$

Through this formulation, the behavioral parameter β regulates the extent to which firms respond to changes in σ^m. A firm that behaves according to $\beta = 1$ is a firm that shares the market's beliefs regarding future price volatility and correctly optimizes its investment decisions according to those beliefs. Conversely, a firm with $\beta = 0$ does not respond to changes in σ^m because it either has beliefs that are orthogonal to σ^m or does not optimize its investment decisions correctly. The primary objective of the empirical work is to obtain an estimate of β and test whether the estimate is consistent with investment behavior that is optimal given beliefs that coincide with those of the market.

The final component of the model is the process by which firms believe σ^d evolves over time. My reference case specification models this process as a random walk per (9) below, in which γ denotes the volatility of the volatility process and η' is an iid standard normal random

variable.[20] I discuss alternatives to the random walk approach in the discussion of the estimation results in section 6.3.

$$\ln \sigma^{d'} = \ln \sigma^d - \gamma^2 / 2 + \gamma \eta'$$

(9)

5. Empirical Model and estimation

The parameter of primary interest is β, the behavioral parameter that reflects firms' sensitivity to the expected volatility of the price of oil. To obtain an estimate of β, I must also estimate the parameters α, ρ, and γ that govern the state transition processes as well as the oil price and dayrate drift functions $\mu(P_t, \sigma_t^2)$ and $\hat{\mu}(D_t, \hat{\sigma}_t^2)$. An estimate of the discount factor δ is also required. In what follows, I first discuss how I estimate these "secondary" parameters independently of the full model before turning to the estimation of β via a procedure based on the nested fixed point approach of Rust (1987).

5.1 Estimates of the discount factor and state transition processes

While the firms' discount factor δ can in principle be estimated as part of the nested fixed point routine, obtaining precise inference in practice is challenging. I adopt the standard approach in the literature by setting δ in advance. According to a 1995 survey by the Society of Petroleum Evaluation Engineers, the median nominal discount rate applied by firms to cash flows is 12.5%. Given average inflation over 1993-2003 of 2.36%, I set δ equal to the quotient 1.0236 / 1.125, approximately 0.910.

I assume that $\mu(P_t, \sigma_t^2)$, the expected drift of the log oil futures price, is the stationary linear function given by (10):

$$\mu(P_t, \sigma_t^2) = \kappa_{p0} + \kappa_{p1} P_t + \kappa_{p2} \sigma_t^2$$

(10)

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[20] A random walk process cannot be rejected using an augmented Dickey-Fuller test. With 12 lags, the p-value for rejecting the null of a unit-root process is 0.3182.

Per equation (3), consistent estimates of κ_{p0}, κ_{p1}, and κ_{p2} may be obtained via an OLS regression of $E[\ln P_{t+1}] - \ln P_t + \sigma_t^2/2$ on P_t and σ_t^2. Because the reference case specification uses 18-month future prices for P_t, I use 19-month future prices to measure $E[\ln P_{t+1}]$ in this regression. I estimate that $\kappa_{p0} = 0.0095$, $\kappa_{p1} = -0.00055$, and $\kappa_{p2} = 0.401$. These values are consistent with mean reversion to an oil price of \$19.56 per barrel (at the sample average volatility of 19.4%).

I similarly assume that $\hat{\mu}(D_t, \hat{\sigma}_t^2)$, the expected dayrate drift, is a linear function of the current dayrate, so that $\hat{\mu}(D_t, \hat{\sigma}_t^2) = \kappa_{d0} + \kappa_{d1}D_t + \kappa_{d2}\hat{\sigma}_t^2$. There does not exist a futures market for rig dayrates to facilitate estimation of the κ_d. Rather than attempt to estimate these parameters from a short time series of quarterly drilling cost observations, I instead assume that the parameters κ_{d0}, κ_{d1}, and κ_{d2} match those from the oil price drift equation, with κ_{d1} re-scaled by the ratio of the average dayrate to the average oil price.

To estimate α, the ratio of $\hat{\sigma}_t$ to σ_t in each period (this ratio is assumed to be constant), I first calculate $\xi_t = \ln P_t - \ln P_{t-1}$ and $\hat{\xi}_t = \ln D_t - \ln D_{t-1}$ in each period. α is then estimated by the ratio of the standard deviation of $\hat{\xi}_t$ to the standard deviation of ξ_t. I then estimate ρ to be the correlation coefficient between $\hat{\xi}_t$ and ξ_t. The estimate of α is 1.50, and the estimate of ρ is 0.395. Finally, I take γ, the volatility of the volatility process, to be the standard deviation of $\ln \sigma_t^m - \ln \sigma_{t-1}^m$. This value is 0.1136.

5.2 Primary empirical model and estimation

Given the state transition functions estimated above, the remaining unknowns in the econometric model are the behavioral parameter β and the unobserved expected productivity of each drilling prospect, the x_i. Because all firms face the same price, volatility, and dayrate processes, the trigger productivity x^* will be the same for all prospects in the data at any given time. If x_i is modeled as identical across prospects, then all firms would make the decision to drill at the same time, a prediction that conflicts with the spread of drilling activity over time evident in figure 1. Clearly, there must exist a distribution of x_i across prospects.

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It is therefore tempting, at first, to estimate a model in which x_i varies across prospects but for each individual prospect is constant over time. However, this model is also incapable of rationalizing the data. Given the trigger rule described in section 4, such a model implies that in each period t, all wells with productivity $x_i > x_t^*$ will be drilled. Now consider what would happen should x^* rise in period $t+1$, perhaps because the oil price fell or because volatility increased. In this case, only prospects with $x_i \geq x_{t+1}^*$ will be drilled. However, all such prospects will already have been drilled in period t since $x_{t+1}^* > x_t^*$. Thus, an implication of a model in which x_i does not vary over time is that there cannot be any drilling activity following an increase or no change in x^*. Such a model is clearly inconsistent with the drilling data. In 1999, for example, the expected price is considerably lower than it was in 1998 and the expected volatility is higher; however, drilling activity does not go to zero. Clearly, any firm that drilled a well during this period must have positively updated its x_i.

There exist numerous reasons why x_i may vary over time. The process by which geologists and engineers develop an estimate of a prospective well's production is inherently challenging and error-prone. They must make inferences about an oil reservoir buried thousands of feet below the earth's surface with very limited information: seismic surveys, production outcomes from previously drilled wells, and electromagnetic "logs" of the rock characteristics at nearby wells. Any individual geologist or engineer may change his or her views regarding a prospect as more time is spent studying the information, and different personnel may draw different conclusions from the same set of information (much like different econometricians may draw different inferences from the same data). Such re-evaluations of prospects, particularly if there is turnover amongst the firms' personnel, can drive substantial variation in the x_i over time. In addition, firms may sometimes "discover" new prospects in old fields in their analyses of their data. Observationally, such discoveries are equivalent to an increase in the x_i of what had been a low-quality prospect.

Prospect re-evaluation is not the only mechanism by which the x_i may vary over time. In multi-prospect fields, the results from the drilling of one well may yield information regarding

the quality of another prospect. Firms may also undertake costly information gathering by taking a seismic survey of their field. Finally, variance in the lag between the decision to drill and the actual commencement of drilling may arise due to delays in engineering design, permitting, or drilling contracting. These stochastic lags will lead to drilling at times not predicted by the model, observationally similar to variation over time in the x_i.

To account for these changes in x_i, the econometric model must treat each prospect's expected productivity as x_{it}, an unobserved state variable that evolves over time. I therefore rewrite the original Bellman equation (6) as (11):

$$V_i(P,D,\sigma,x_i) = \max\left\{\pi(P,D,\sigma,x_i),\, \delta\cdot \mathrm{E}[V_i{}'(P',D',\sigma',x_i')]\right\} \qquad (11)$$

In modeling the state variable x_i, I abstract away from explicit modeling of the mechanisms above, such as firms' use of seismic surveys to gather information. A firm that undertakes such an action is in reality making an endogenous investment that should in principle be modeled dynamically in conjunction with the drilling model. This aspect of information acquisition is omitted from the model both to maintain tractability and because I lack data on the occurrence of seismic surveys. The present model can accommodate costly information gathering to the extent that the drilling of a well can be viewed as a compound investment: when prices rise or volatility falls so that the firm is ready to contemplate drilling, it first undertakes a seismic survey prior to drilling the well. I also continue to model each prospect independently, abstracting away from the process by which the drilling of a well in a field can influence the firm's beliefs about other prospects in the same field. This omission may result in un-modeled clustering of drilling behavior in fields with multiple wells drilled, motivating the use of a sandwich variance-covariance estimator (Wooldridge 2002).

In my reference case empirical specification, I treat log x_{it} as an iid normal variable across both prospects i and time t, and I estimate the mean μ and variance ζ of this distribution in addition to the behavioral parameter β. This approach agglomerates the possible sources of variation in the x_{it} into a single, parsimonious distribution. Separate identification of each source

of variation would require strong functional form assumptions and a substantially more complex model than that given here. In addition, for each source of variation discussed above, the shocks to the x_i are not due to exogenous arrival of new information but rather reassessments of old information, new prospect "discovery," costly and deliberate information acquisition, or variation in the lag between drilling decisions and actual drilling. Because the x_{it} incorporate these effects rather than exogenous information shocks, I model firms as believing that $x_{it+1} = x_{it}$.

Despite the emphasis of the above discussion on time-variance in x_{it}, there may exist some persistent cross sectional heterogeneity in the expected productivity of each prospect. I therefore also estimate a model in which log x_{it} is the sum of a time-invariant normally distributed random variable φ_i, with mean and standard deviation given by μ_1 and ζ_1, and an iid normal variable v_{it} with a zero mean and standard deviation ζ_2. In this specification, I estimate μ_1, ζ_1, and ζ_2 in addition to the behavioral parameter β.

Given the state transition processes discussed in section 5.1, the parameters governing the distribution of the x_{it}, the behavioral parameter β, and the realized monthly time series of future prices, rig dayrates, and implied volatilities (denoted by $\boldsymbol{P}$, $\boldsymbol{D}$, and $\boldsymbol{\sigma}$, respectively), the model's solution yields the likelihood that a given prospect will be drilled in any given month t conditional on not having been drilled already. This likelihood is simply the probability that x_{it} exceeds the trigger productivity x_t^*.[21] Starting from the initial period of January 1993, these conditional probabilities yield the probability that any given prospect will be drilled in each month t, as well as the probability that the prospect will not be drilled by the end of the sample.[22] These probabilities form the basis for the likelihood function. Let I_{it} denote an indicator variable that takes on a value of one if prospect i is drilled in month t and zero otherwise, T denote the

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[21] Unlike Rust (1987), the unobservable x_{it} is not additively separable to the reward function, implying that I cannot take advantage of the logit formulation of the likelihood. Instead, I directly model x_{it} as a state variable, and the model's solution then yields the trigger productivity each period. Details are provided in appendix 2.

[22] For example, the probability that the prospect will be drilled in February 1993 is the conditional probability that it is drilled in February 1993 multiplied by probability that it was not drilled in January 1993. The probability that it is drilled in March 1993 is then the conditional probability that it is drilled in March 1993 multiplied by probability that it was not drilled in February 1993 or earlier, and so on.

final month of the sample, N_t denote the number of wells actually drilled at t, and N_0 denote the number of prospects not drilled ($N_0 = 6,637$, the number of undrilled sole-operated fields).[23] The log-likelihood function is therefore:

$$\ell((N_1, N_2, ..., N_T), N_0 \mid \boldsymbol{P}, \boldsymbol{D}, \boldsymbol{\sigma}; \beta, \mu, \zeta) = \sum_{t=1}^{T} N_t \log \Pr(I_{it} = 1 \mid \boldsymbol{P}, \boldsymbol{D}, \boldsymbol{\sigma}; \beta, \mu, \zeta) \qquad (12)$$
$$+ N_0 \log \Pr(I_{it} = 0 \; \forall t \mid \boldsymbol{P}, \boldsymbol{D}, \boldsymbol{\sigma}; \beta, \mu, \zeta)$$

Estimation of β, μ, and ζ is carried out by maximizing this likelihood function using a nested fixed point routine. The outer loop searches over the unknown parameters while the inner loop solves the model and calculates the likelihood function at each guess. Details regarding this procedure, such as the discretization of the state space used to numerically solve the model, are provided in appendix 2. The specification with cross-sectional heterogeneity proceeds by integrating the likelihood over the distribution of φ_i.

6. Estimation results and discussion

6.1 Reference case estimation results

I begin by estimating the version of the model in which $\log x_{it}$ is assumed to be iid across prospects i and time t. As a baseline, column I of table 2 provides the estimation results when I impose the restriction that $\beta = 0$; that is, firms do not respond to changes in implied volatility. I find that a broad distribution of expected productivity x_{it} is needed to sufficiently smooth the model's simulated drilling activity such that it rationalizes the data. The estimated mean μ and variance ζ of $\log x_{it}$ are -0.653 and 3.094, respectively. Here, and throughout the presentation of the results, x_{it} is given in barrels of expected discounted production per $100,000 of drilling cost at the average rig dayrate. These estimates together imply that, in the model, the average prospect at any point in time is expected to produce only 62 barrels of oil per $100,000 of cost,

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[23] Throughout this section, I use "drilled" as shorthand for the drilling decision. As with the descriptive hazard model, I allow for a three-month lag between the drilling decision and the actual start of drilling. Thus, for example, the model's drilling probability for January 1993 is matched with drilling activity for April 1993. The final period of the sample is September 2003, which is matched with drilling activity for December 2003.

well below the productivity necessary to justify investment at any reasonable oil price.[24] This estimate reflects the presence of a large number of fields in the data (6,637) in which no drilling occurs. A large estimate of the variance ζ is therefore necessary to rationalize the observed drilling. For example, a prospect with average costs and a log x_{it} three standard deviations greater than the mean will be expected to produce 23,070 barrels of oil, sufficient to trigger drilling over a range of prices and implied volatilities in the sample.

In column II I allow β, the firms' sensitivity to implied volatility, to be a free parameter, and its point estimate is 1.039. This value is very close to one in both an economic and statistical sense (the standard error is 0.064), consistent with optimal investment responses to volatility expectations that match the implied volatility of NYMEX futures options.[25] Moreover, a likelihood ratio test strongly rejects, with a p-value less than 0.001, a null hypothesis that firms do not respond at all to implied volatility ($\beta = 0$).[26] The time series of predicted drilling under models I and II are given in figure 7, alongside actual drilling activity. The prediction from model II, allowing for a response to volatility, yields a better fit to the data, particularly during the 1999 low price period and the volatility spike following September 11th, 2001. More broadly, the model that does not allow a response to time-varying volatility under-predicts drilling in the early part of the sample and over-predicts drilling in the latter part. Allowing for a volatility response largely corrects these mis-predictions, though there remain sections of the time series, such as early 1997, that the model does not fit well.

Column III of table 2 presents estimates of the richer model in which x_{it} is permitted to have persistent cross-sectional heterogeneity. This relaxation has virtually no impact on the estimate of β. The estimated degree of persistent heterogeneity is low relative to the estimated

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[24] The 62 barrel per $100,000 figure is equal to $\exp(-0.653 + 3.094^2/2)$.

[25] Note that, in column II, the distribution of log x_{it} is estimated to have a lower mean and higher variance than in column I. This shift in parameters is necessary to rationalize non-zero drilling activity in early 1999 when oil prices were low and implied volatility was high: the increased variance allows simulated prospects to have an expected quality x_{it} greater than the high drilling cutoff x_t^* during this period.

[26] Rejection of the restricted estimate with a test size of 5% requires a difference in log likelihoods of 1.92. A likelihood ratio test does not take clustering of the likelihood scores on field into account so will therefore underestimate the true p-value.

variance of the time-varying shocks: the estimate of ζ_1 is 0.166 while that of ζ_2 is 6.177. Allowing for a deterministic time trend also does not significantly impact the estimate of β, as shown in column IV. The estimated time trend is slightly negative and indicates a productivity decrease of about 0.9% per year. This result is consistent with the presence of some cross-sectional heterogeneity that causes the most promising prospects to be drilled first.

Why might these estimates of firms' responses to changes in expected volatility accord so well with theory? Given the small size of the majority of these firms, it seems unlikely that they are formally solving Bellman equations. However, they may have developed decision heuristics that roughly mimic an optimal decision-making process. Moreover, the firms have a strong financial incentive to get their decision-making at least approximately right. Consider a firm that has a drilling prospect of average cost that is expected to produce 17,000 bbl, faces an average dayrate (so that the drilling cost is $412,585), and faces an expected oil price of $30/bbl. Suppose further that the expected oil price volatility is relatively high: 34%. If the firm is myopic in that its β equals zero, it will drill the well and realize an expected profit of $97,414. However, if the firm makes its decision optimally, it will postpone drilling, preserving the full value of the prospect: $148,000 (this value is taken directly from the model's value function). In this example, optimal decision-making in the presence of time-varying volatility yields a 52% increase in value over behavior consistent with an expectation that volatility is constant at 12.3%.

6.2 Identification and the July 1998 step change

The above results indicate that, in periods of high expected oil price volatility, drilling activity falls in a way that is commensurate with the predictions of real options theory. This section considers potential alternative explanations for this measured empirical response.

I first examine the extent to which the empirical results above can be explained by the correlation of implied volatility with the downward step change in drilling activity that began in July 1998. To do so, I model a permanent shock to the x_{it} that begins in July 1998 and is common across all prospects. In this approach, the shock proxies for an unobserved factor that may have

affected the likelihood of drilling subsequent to July 1998. The results of estimating the model while allowing for this step change are given in column V of table 2. The estimated shock is large, decreasing log x_{it} by 0.454, although it is not statistically significant. The point estimate of β is not significantly affected, changing from 1.039 in the reference case to 1.060 here.

The standard error on the estimate of β in specification V is 0.040, strongly rejecting a null of $\beta = 0$ in a simple Wald test. However, a likelihood ratio test against the restricted model of column VI in which $\beta = 0$ yields a p-value of only 0.036, suggesting the presence of non-concavities in the likelihood function when the July 1998 shock is included in the model. In fact, there is a second local maximum, shown in column VII, at which $\beta = 0.416$. The log likelihood at this maximum is lower than that at $\beta = 1.060$ by only 0.2, and $\beta = 0.416$ therefore cannot be rejected by the data. This last result suggests that the persistent step changes in volatility and drilling activity following 1998 played a non-trivial role in the identification of the robust response to volatility in the reference case.

Is it possible that unobserved and unmodeled factors took effect after 1998 and caused the subsequent decrease in drilling activity observed in the data? I investigate here two candidate explanations: (a) that there was a discontinuous decrease in the expected productivity of drilling prospects (the literal interpretation of the model estimated in columns V through VII of table 2); and (b) that the low price period beginning in 1998 caused production firms to lay off engineering and management staff that they could not subsequently re-hire, restricting their ability to carry out drilling programs when prices recovered.

Proposition (a), that there was a sudden decrease in prospect quality in 1998, seems unlikely. Prospect quality is a function of the geologic characteristics of oil reservoirs in Texas, and there is no obvious reason why firms' beliefs about these characteristics would sharply decrease, across many fields and firms, at precisely the same time that oil price volatility rose. Moreover, a fall in perceived prospect quality should be manifest in the realized production data from drilled wells. Figure 8 displays a scatter plot of the log of the ratio of oil production to drilling time for the 162 drilled wells that I could match to oil production data. This plot provides

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no evidence in support of a drop in prospect quality beginning in July 1998, though the production realizations are sufficiently noisy that they do not rule out such a drop either.

To examine the plausibility of proposition (b), I obtained data from the Bureau of Labor Statistics on the employment of petroleum engineers and geologists in Texas. These data are given in table 3. While the data do indicate a decrease in employment from 1998 to 1999, as expected, employment quickly rebounds with the oil price and in fact surpasses 1998 employment by 2001. These data therefore suggest that staffing constraints were unlikely to play a role in determining the low level of drilling activity following 1998. I cannot, however, rule out the possibility that the employees hired in 2000 and 2001 were of lower quality than those whose employment terminated after 1998. This quality decrease would need to be substantial, however, to explain the data: the estimated magnitude of the drop in $\log(x_{it})$ in the restricted model of column VI in table II is -0.359.

6.3 Alternative specifications

Alternative measures of expected volatility

The analysis thus far has used implied volatility from the NYMEX futures options market as the measure of firms' oil price volatility expectations. Table 4, column II reports results in which expected volatility is instead measured by the historic volatility of futures prices over a 12-month rolling window. The use of historic volatility yields a worse fit to the drilling data than does implied volatility, as evidenced by the decrease in the log likelihood relative to the implied volatility results in column I. Moreover, the estimate of the behavioral parameter β is only 0.288 and not statistically significant, indicating that firms do not respond as strongly to historic volatility as they do to volatility signals that are reflected in the NYMEX futures options market.

Column III of table 4 uses the GARCH(1,1) model to forecast future volatility. This model yields an estimate of β of 0.551 that is statistically significant at the 5% level, though the fit of the model is still substantially worse than when implied volatility is used (the decrease in the log likelihood is equal to 6.5). The reduced fit reflects the fact that, while GARCH provides a

closer match to implied volatility than does historic volatility, the two series still diverge substantially at several points in time (figure 4b). This result, as well that obtained from the direct use of historic volatility, suggests an explanation for why some previous empirical studies (Hurn and Wright 1994, Moel and Tufano 2002) have not found strong evidence that time-varying volatility significantly affects investment. These studies measure firms' volatility expectations using historic volatility, which may only be a noisy measure of firms' true beliefs because it does not reflect up-to-date information regarding volatility shocks.

Thus far, the analysis has modeled firms as believing that volatility follows a random walk per equation (9). Alternatively, column IV of table 4 models firms as believing that the volatility process is mean-reverting. I estimate firms' expected mean reversion rates using the GARCH model. At each month in the sample, this model provides volatility forecasts for both the current month (the forecast used in generating the time series in figure 4(b) and the estimates in column III) and the subsequent month. These two series of predictions are consistent with mean reversion: when the current GARCH volatility is relatively high, the upcoming-month forecast predicts a fall in volatility, and the reverse holds when the current GARCH volatility is relatively low. Using these predictions, I estimate that the expected rate of change in the log of expected volatility is given by 0.0544 minus 0.0021 times the current volatility (in annualized percent). This estimate implies that, if the current expected volatility is 10%, the expected volatility next month is 10.3%. In contrast, if the current expected volatility is 30%, the expected volatility next month is 29.7%.

When firms have volatility beliefs that are consistent with mean reversion, they will believe that changes in volatility will not be persistent and therefore not respond as strongly to such changes. Thus, the estimate of this model in column IV yields a relatively high value of β of 1.304 because a value of one will not yield sufficient sensitivity to volatility to match the data.[27] A null hypothesis that $\beta = 1$ is rejected at the 5% level.

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[27] To be clear, this estimate uses implied volatility, not GARCH volatility, as the measure of expected volatility over the current month. The GARCH model is used only to generate a forecast of expected mean reversion.

Alternative forecast horizons

Column V of table 4 considers a model in which firms respond to the 12-month oil futures price and volatility rather than the 18-month price and volatility. I replace the price series P_t with the NYMEX 12-month futures contract and the implied price volatility σ^m with that of 12-month futures prices. Re-estimating the full model (including auxiliary parameters such as the expected rate of price drift) yields an estimate of β of 1.044, very near the reference case estimate of 1.039. This result reflects the closeness of the 12-month price and volatility series to the 18-month series (figures 3 and A1).

Column VI considers a model in which firms respond to the current price and volatility of oil rather than 18 month futures and volatilities. I replace the price series P_t with the NYMEX front-month futures contract and the market's implied 18-month price volatility σ^m with that of front-month futures options. Because firms' use of current prices as expected prices is consistent with a no-change forecast for the price of oil, I set the price and cost drift functions $\mu(\cdot)$ and $\hat{\mu}(\cdot)$ to zero. The estimate of β from this model is 1.226, with a relatively large standard error of 0.549. The increase in the estimate of β relative to the reference case model likely reflects the zero price drift assumption associated with the use of front-month prices. A relatively high volatility state in this model is not associated with an expectation that prices will increase in the future, as was the case in the reference case model using 18-month futures. A higher estimate of β is therefore required in order to offset this change in the model and fit the data.

Alternative discount rate and drilling cost assumptions

The estimates heretofore have been based on an assumed 12.5% nominal discount rate, taken from a 1995 survey by the Society of Petroleum Evaluation Engineers. Columns II and III of table 5 examine the use of alternative discount rates. A 14.5% discount rate yields an estimate of β of 1.149 while a 10.5% discount rate yields $\beta = 0.890$. These changes to the estimated β are in line with real options theory's predictions. As the discount rate increases, firms value the future less, option value decreases, and firms become less responsive to changes in expected

volatility. Thus, to fit the empirical volatility response, the volatility sensitivity parameter β must increase when the assumed discount rate increases.

Finally, columns IV and V of table 5 examine the estimates' sensitivity to the assumption that rig costs constitute one-third of total drilling costs, on average. Assuming a value of 20% or 50% does not substantially alter the estimate of β.

7. Conclusions

The importance of irreversibility and uncertainty in investment decision-making has been recognized since Marschak (1949) and Arrow (1968). Theoretical work has since derived optimal timing rules for irreversible investments and demonstrated that firms should defer projects when uncertainty is relatively high. These concepts have taken a prominent role in industrial organization and the macroeconomic modeling of aggregate investment. However, there has been a shortage of empirical evidence regarding the extent to which firms actually take option value into account when making irreversible investments.

This paper tests the sensitivity of firms' investment decisions to changes in the uncertainty of their economic environment by assembling a new, detailed dataset that combines information on well-level oil drilling with expected oil price volatility data from the NYMEX market. I develop and estimate a dynamic model of firms' drilling investment timing problem, taking advantage of industry features that make a single-agent approach appropriate. I find not only that firms reduce their drilling activity when expected volatility rises but also that the magnitude of this reduction is consistent with the optimal response prescribed by theory. This result provides micro-empirical support for the frequent use of real options models in economic research. It is also consistent with the existence of a strong incentive for firms to behave optimally. I find that the cost of failing to respond to changes in volatility can be substantial, potentially exceeding 50% of a drilling prospect's value at in-sample oil price and volatility realizations.

I also show that a forward-looking measure of expected price volatility derived from futures options is a more powerful determinant of drilling behavior than are backward-looking measures based on historic volatility. The relative strength of the implied volatility measure is consistent with the hypothesis that participants in the NYMEX commodity market and physical industry participants share common beliefs about future price uncertainty. This result thereby provides support for the use of data from financial markets as measures of firms' expectations in applied work. It is also well-aligned with other research regarding the predictive power of option-based implied volatility and supports the intuition that options prices incorporate up-to-date information about uncertainty shocks that cannot be conveyed by price histories alone.

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Figure 1: Time series of monthly drilling activity, oil futures prices, and implied volatility from futures options prices
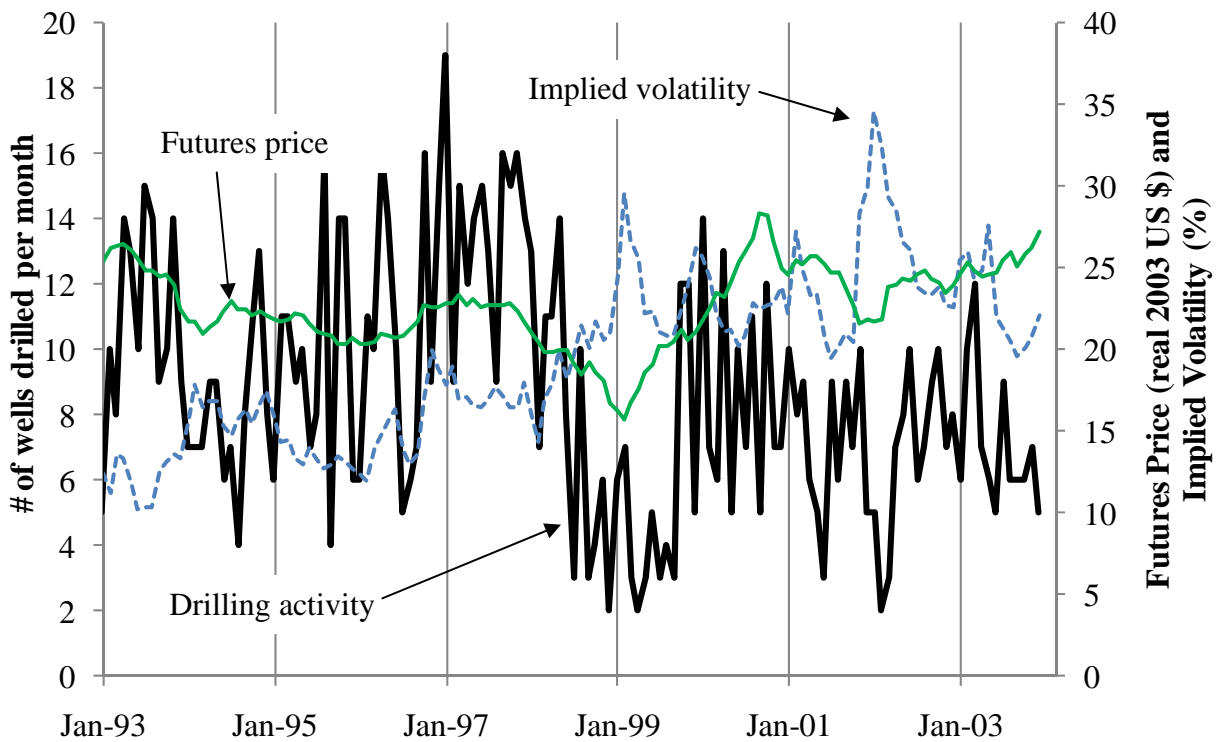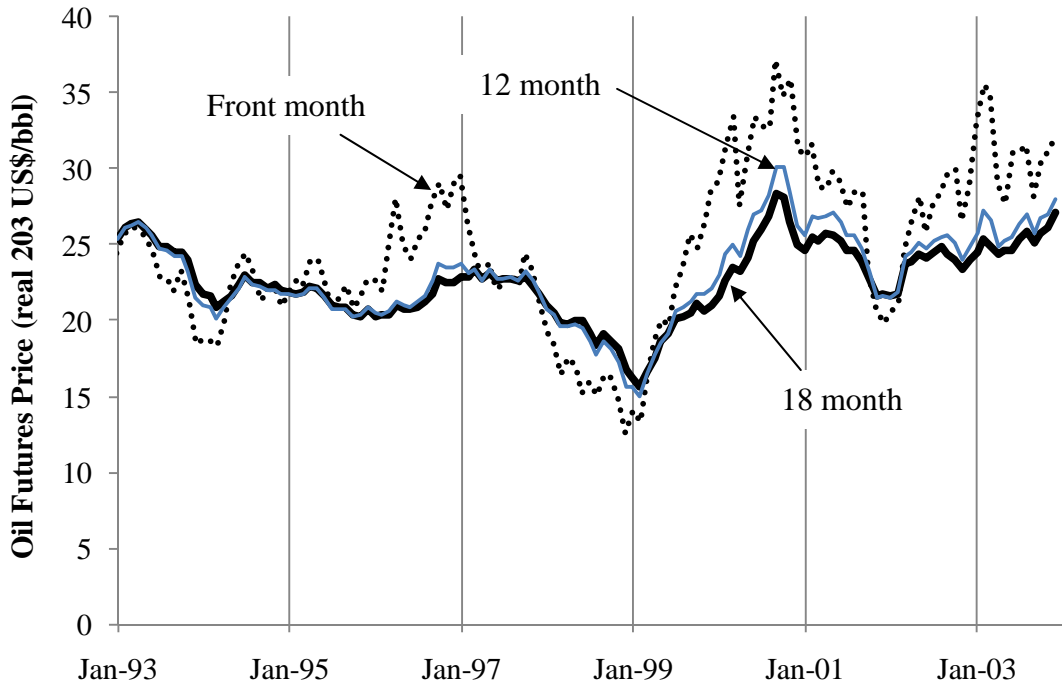

Notes: Oil futures prices are 18-month ahead prices from the New York Mercantile Exchange (NYMEX). Implied volatility is calculated from futures options prices per the discussion in section 2.4. Drilling activity corresponds only to infill oil wells drilled in sole-operated fields.

**Figure 2: Average monthly production profile from a drilled well**
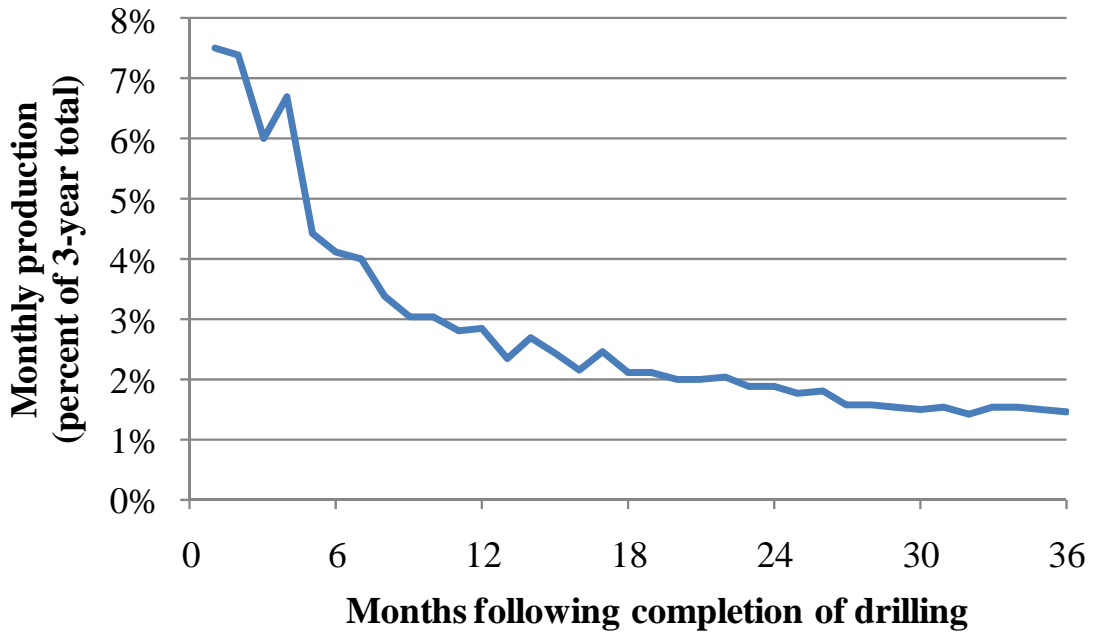
Notes: Production data are from the subset of observed drilled wells that are the only active producing well on their respective lease for the first 36 months subsequent to drilling. This subset amounts to 162 of the observed 1,150 drilled wells from 1993-2003.
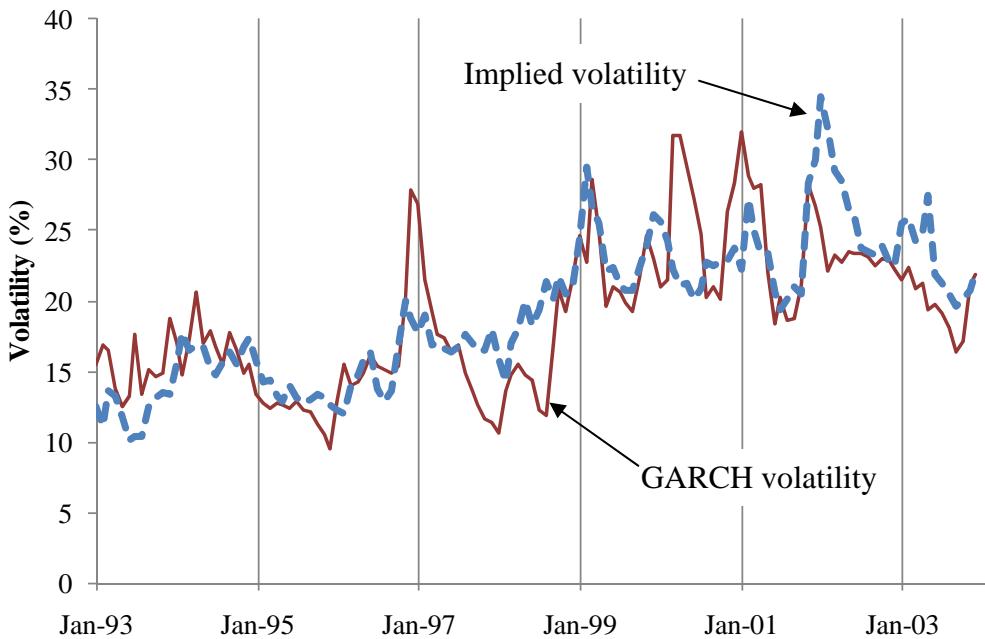
Figure 3: NYMEX front-month, 12 month, and 18 month oil futures prices

Figure 4a: Comparison of implied volatility to the historic volatility of the future price, calculated with one month and one year rolling windows
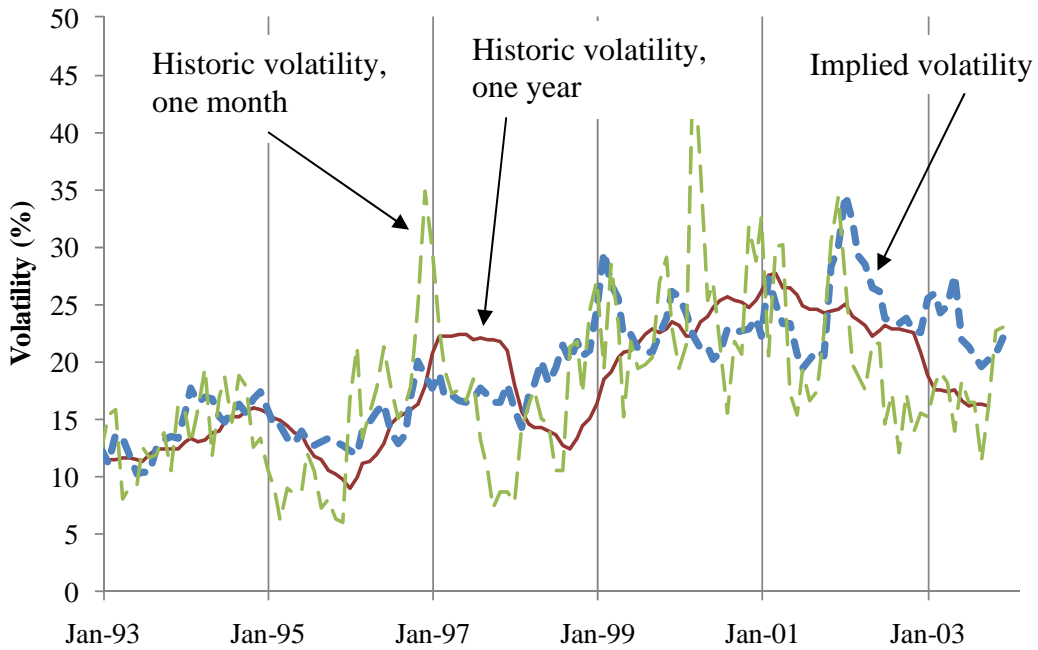

**Figure 4b: Comparison of implied volatility to a GARCH(1,1) forecast**

Notes: Implied volatility is calculated from futures options prices per the discussion in section 2.4. Historic volatility at any point in time is the standard deviation of the return on the 18-month futures price within a one month or one year rolling window. The GARCH(1,1) model is estimated at each date using a 4-year rolling window of 18-month futures prices.

**Figure 5: Drilling costs and oil futures prices**
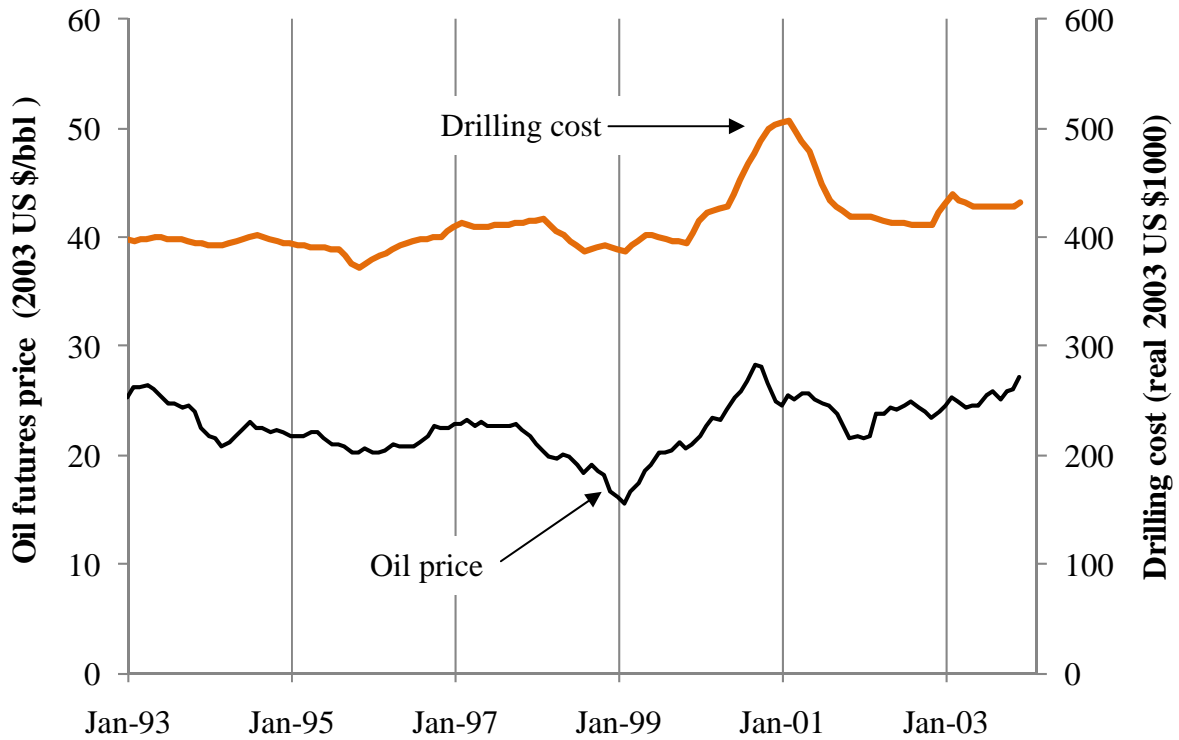
Notes: Oil futures prices are 18-month ahead prices from the New York Mercantile Exchange (NYMEX). Drilling costs are those for an average well that requires 19.2 days of drilling time. These costs are based on daily rig rental rates obtained from RigData, as discussed in section 2.5.

Figure 6: Illustration of the impact of the expected oil price and price volatility on the "trigger" expected production required so that drilling is optimal
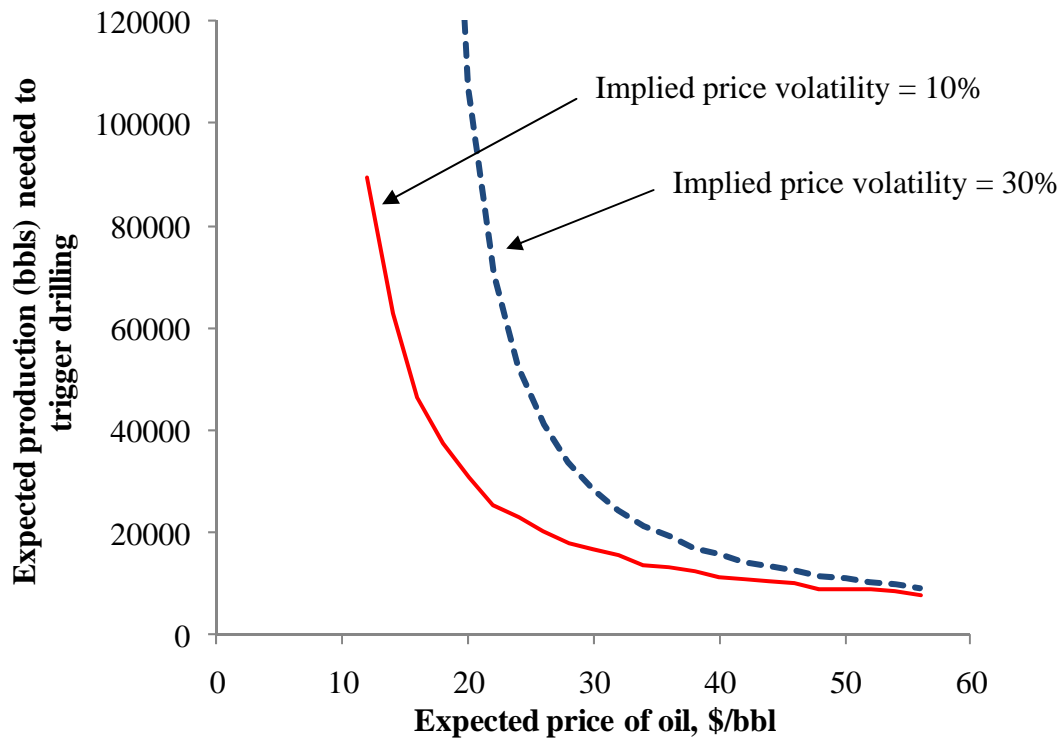

Notes: The relationships shown are for a well of average cost facing an average dayrate, so that the drilling cost is $413,000. The model used to generate these curves uses the state transition parameters estimated in section 5.1 of the text.

Figure 7: Predicted drilling from the estimated model vs. actual drilling
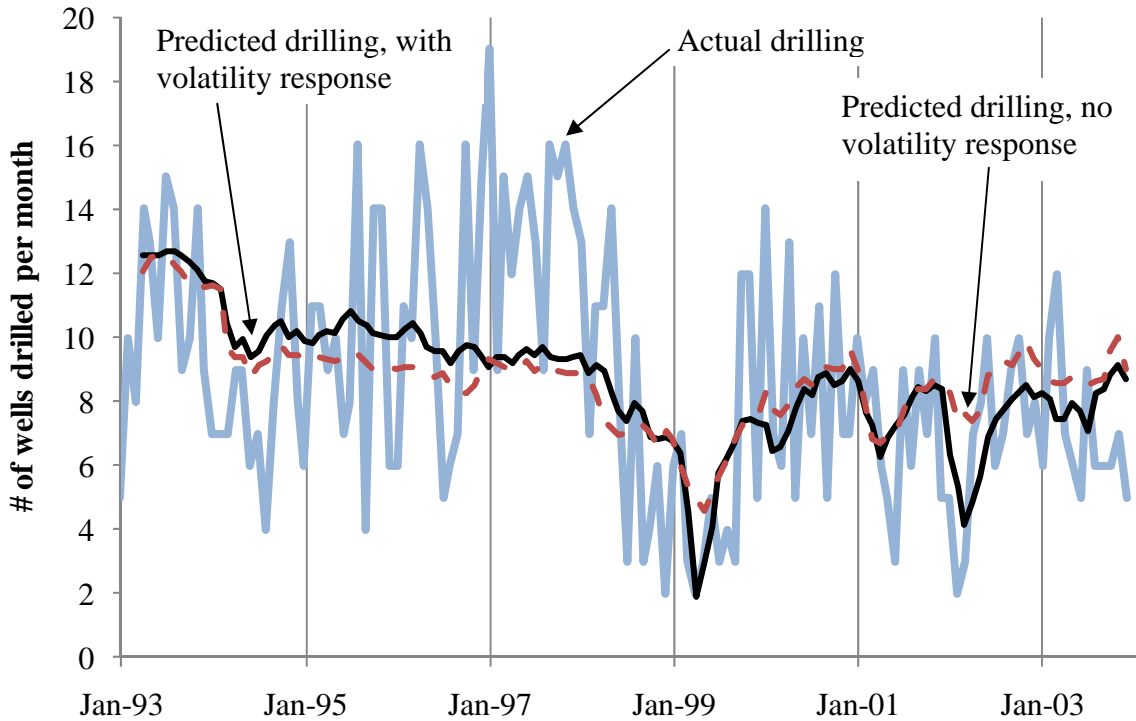
Notes: Predicted drilling with no volatility response refers to the time series of predicted drilling activity from the model with the parameters of table 2, column I, in which the behavioral parameter β is restricted to zero. Predicted drilling with a volatility response refers to table 2, column II, in which β is estimated to be 1.039.

Figure 8: Scatter plot of realized oil production from drilled wells, relative to the number of days required to drill each well
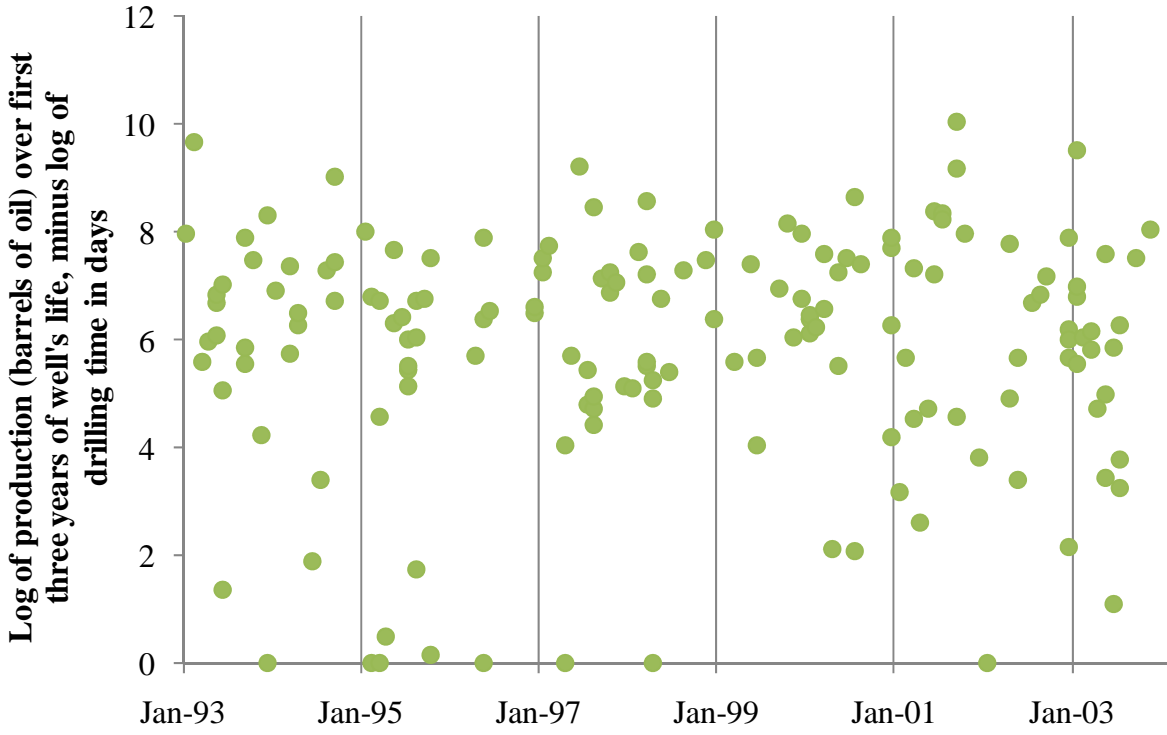
Notes: Production data are from the subset of observed drilled wells that are the only active producing well on their respective lease for the first 36 months subsequent to drilling. This subset amounts to 162 of the observed 1,150 drilled wells from 1993-2003. Dry holes (wells with zero production) are plotted as having a log(production/drilling time) of zero

Table 1: Hazard model results for the probability of drilling

Coefficient on covariate:	I Basic exponential hazard	II Include drilling cost	III Prospect-specific heterogeneity	IV Drilling cost and time trend	V Drilling cost and July 1998 dummy
Oil futures price ($/bbl)	1.041***	1.053***	1.041***	1.053***	1.056***
	(0.016)	(0.023)	(0.016)	(0.023)	(0.024)
Implied volatility of future price (%)	0.969***	0.973***	0.969***	0.964***	1.005
	(0.008)	(0.009)	(0.008)	(0.013)	(0.013)
Drilling Cost ($100,000)	-	0.842	-	0.804	0.949
	-	(0.163)	-	(0.159)	(0.189)
Linear time trend (in years)	-	-	-	1.020	-
	-	-	-	(0.021)	-
Dummy for date ≥ July 1998	-	-	-	-	0.654***
	-	-	-	-	(0.094)
Unobserved heterogeneity (gamma distribution)	N	N	Y	N	N
Log likelihood	-3979.0	-3978.5	-3979.0	-3978.0	-3972.0

Reported coefficients are hazard ratios: the multiplicative effect on the hazard rate of a one unit increase in the covariate

All estimates use prices of futures and options that are 18 months from maturity

Standard errors are estimated using a sandwich estimator that allows for correlation of the likelihood scores across wells within the same field

*,**,*** indicate significance at the 10%, 5%, and 1% level for a two-tailed test that the coefficient is different from one

All covariates are lagged by three months

Table 2: Results from estimation of dynamic model

Parameter:	I — Beta restricted to zero	II — Reference case model	III — Heterogeneous prospects	IV — Time trend	V — July 1998 step change	VI — July 1998 step, Beta restricted to zero	VII — July 1998 step, alternative local maximum
β (sensitivity to volatility)	-	1.039 (0.064)	1.039 (0.066)	1.042 (0.069)	1.060 (0.040)	-	0.416 (0.289)
μ (mean of $\log(x_{it})$)	-0.653 (2.668)	-9.675 (6.666)	-	-10.640 (7.915)	-18.173 (12.432)	-3.686 (4.838)	-3.656 (4.518)
ζ (std. dev. of $\log(x_{it})$)	3.094 (0.884)	6.176 (2.224)	-	6.511 (2.667)	9.051 (4.176)	4.145 (1.624)	4.144 (1.519)
μ_1 (mean of $\log(\phi_i)$)	-	-	-9.681 (7.177)	-	-	-	-
ζ_1 (std. dev. of $\log(\phi_i)$)	-	-	0.166 (12.805)	-	-	-	-
ζ_2 (std. dev. of $\log(v_{it})$)	-	-	6.177 (2.292)	-	-	-	-
Time trend (years)	-	-	-	-0.009 (0.034)	-	-	-
Dummy for date ≥ July 1998	-	-	-	-	-0.454 (0.438)	-0.359 (0.200)	-0.292 (0.182)
Log likelihood	-8681.1	-8670.8	-8670.8	-8670.7	-8668.5	-8670.7	-8668.7

All estimates use prices of futures and options that are 18 months from maturity

x_{it}, ϕ_i, and v_i are expressed in expected oil production (in bbl) divided by the cost of drilling (in \$100,000) at the average sample day rate

Standard errors are estimated using a sandwich estimator that allows for correlation of the likelihood scores across wells within the same field

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Table 3: Employment of petroleum engineers in Texas

Year	Employment	Year	Employment
1997	4600	2003	5670
1998	5280	2004	7240
1999	4670	2005	7610
2000	4830	2006	7000
2001	5470	2007	8240
2002	5110	2008	10640

Source: Bureau of Labor Statistics, occupation code 19-4041
for 1999-2008 and 22111 for 1997 and 1998

Table 4: Alternative specifications of volatility beliefs and decision time horizons

Parameter:	I Reference case model (table 2, column II)	II Historic volatility of futures prices, 12 month window	III GARCH volatility	IV Mean-reverting volatility	V 12-month futures and implied volatility	VI Front month futures and implied volatility
β (sensitivity to volatility)	1.039 (0.064)	0.288 (0.304)	0.551 (0.258)	1.304 (0.143)	1.044 (0.109)	1.226 (0.549)
μ (mean of $\log(x_{it})$)	-9.675 (6.666)	-0.995 (2.828)	-2.068 (3.269)	-8.389 (5.426)	-10.737 (6.524)	-3.736 (4.100)
ζ (std. dev. of $\log(x_{it})$)	6.176 (2.224)	3.214 (0.939)	3.603 (1.092)	5.768 (1.809)	6.511 (2.174)	4.224 (1.361)
Log likelihood	-8670.8	-8680.2	-8677.3	-8671.2	-8670.6	-8671.2

x_{it} is expressed in expected oil production (in bbl) divided by the cost of drilling (in \$100,000) at the average sample dayrate

Standard errors are estimated using a sandwich estimator that allows for correlation of the likelihood scores across wells within the same field

Table 5: Alternative specifications: discount rates and drilling costs

Parameter:	I Reference case model (table 2, column II)	II 14.5% nominal discount rate	III 10.5% nominal discount rate	IV Rig costs average 20% of total drilling cost	V Rig costs average 50% of total drilling cost
β (sensitivity to volatility)	1.039 (0.064)	1.149 (0.051)	0.890 (0.094)	1.046 (0.056)	1.044 (0.055)
μ (mean of $\log(x_{it})$)	-9.675 (6.666)	-10.522 (6.756)	-9.011 (6.497)	-10.410 (6.707)	-10.449 (6.410)
ζ (std. dev. of $\log(x_{it})$)	6.176 (2.224)	6.451 (2.251)	5.960 (2.171)	6.418 (2.235)	6.442 (2.136)
Log likelihood	-8670.8	-8671.2	-8670.8	-8671.4	-8670.9

All estimates use prices of futures and options that are 18 months from maturity

x_{it} is expressed in expected oil production (in bbl) divided by the cost of drilling (in \$100,000) at the average sample dayrate

Standard errors are estimated using a sandwich estimator that allows for correlation of the likelihood scores across wells within the same field

Appendix 1: Construction of the time series of implied futures price volatility

This appendix describes how I construct a time series of the implied volatility of the 18-month NYMEX oil futures contract. First, I discuss the estimation of the term structure of the volatility of oil futures. Second, I discuss how I use futures options to construct a time series of the implied volatility of the one-month oil futures contract. Finally, I discuss the use of the estimated term structure in converting this time series from one-month to 18-month volatilities.

Let $F_{t,\tau}$ denote the price of a NYMEX futures contract traded at date t with time to maturity τ measured in months.[28] For each t and τ, I calculate the realized volatility at t of the τ-month futures contract as the standard deviation of $\ln(F_{s,\tau} / F_{s-1,\tau})$ for all dates s within the 6 months prior and subsequent to t.[29] Let this volatility be denoted by $\sigma_{t,\tau}$. I then estimate the term structure of futures price volatility by regressing the log of $\sigma_{t,\tau}$ on fixed effects for each τ and t:[30]

$$\ln \sigma_{t,\tau} = \eta_\tau + \delta_t + \varepsilon_{\tau,t} \qquad\qquad (A1)$$

The fixed effects η_τ represent the estimated term structure while the δ_t control for the level of volatility on each date t. Given estimates of these fixed effects, the predicted volatility of a τ-month futures price is given by $A \cdot \exp(\eta_\tau)$, where $A = \exp(\delta_t + v^2 / 2)$ and v^2 is the variance of the estimated residuals. Thus, for a fixed trade date t, varying τ will trace out the term structure of volatility. Figure A1 verifies that the term structure of volatility is stable over the sample by plotting two estimates of the term structure: one using data from 1999-2003 and another using data prior to 1999. The constant term A for each estimate is set so that the one-month future price volatility is 31%, approximately equal to the average one-month volatility over 1993-2003. The plots overlay each other closely, particularly through 18 months, indicating that the term

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[28] Time to maturity in months is equal to the time to maturity in days divided by 365.25, multiplied by 12, and rounded to the nearest whole number.

[29] Observations $F_{s,\tau}$ for which date s - 1 is missing (for example, if s - 1 is a Sunday) are excluded.

[30] I use the log of $\sigma_{t,\tau}$ as the dependent variable rather than the level because the levels regression does not yield an estimated term structure that is stable over time. In levels, the term structure is has a steeper slope during 1999-2003 than in the earlier part of the data.

structure of volatility is stable over the sample despite the substantial increase in the overall level of volatility after 1999.

Given the estimated term structure (the η_τ), all that is needed to compute expected 18-month futures price volatilities is a time-series of short-run (one month) expected futures price volatilities. I derive this time series from the implied volatility of short-term futures options with a time to maturity between 60 and 180 days. The implied volatility of options with a shorter time to maturity are noisy, potentially reflecting low option values and integer problems (options prices must be in whole cents), while options with a longer time to maturity are thinly traded.

For each trade date and time to maturity within the 60 to 180 day window, I use the Black (1976) model to find the implied volatilities of the call and put options that are nearest to at-the-money.[31] I then estimate the implied volatility term structure by regressing the log of each option's implied volatility on its time to maturity τ (in days), a call/put dummy, and trade date fixed effects δ_t.[32] I then use this estimated term structure (the estimated coefficient on τ) to extrapolate implied volatility back to a 30 day maturity.

As a validation check on the this procedure, I compare the average, over 1993-2003, of the estimated implied volatilities of 30-day futures options to the average realized volatility of one-month futures prices over the same timeframe. These two averages should be approximately equal given the short one month time to maturity. The former series has an average volatility of 30.74% while the average of the latter is 31.07%. The closeness of these two numbers (derived from two completely different data sets) supports the argument that implied volatilities from one-month futures options can be used as implied volatilities of one-month futures prices.

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[31] The Black (1976) model assumes that the options are European rather than American and that volatility is constant. Neither of these assumptions holds here; however, their effects are likely to be minor and they save considerable computational complexity. Hilliard and Reis (1998) demonstrate that the American premium is no more than 2% of the European option price for volatilities similar to those considered here. Stochastic volatility acts in the opposite direction, causing the Black (1976) model to slightly over-price at-the-money options (this effect is particularly small for the relatively short maturities considered here); see Hull and White (1987), Wiggins (1987), and Poon and Granger (2003). The argument that these assumptions are of minor effect is supported by the close agreement between the average realized and average implied volatility over the 1993-2003 sample.

[32] Inspection of the residuals indicates that a linear term structure specification is appropriate. Moreover, when a squared time to maturity term is added, it is not statistically significant (p-value = 0.465).

Finally, I convert the time series of implied volatilities of one-month futures prices to implied volatilities of 18-month futures prices using the estimated term structure of futures price volatility (the η_τ). This conversion amounts to multiplying the one-month volatility at each trade date t by $\exp(\eta_{18} - \eta_1)$.

Figure A1: Estimated term structures of futures price volatility
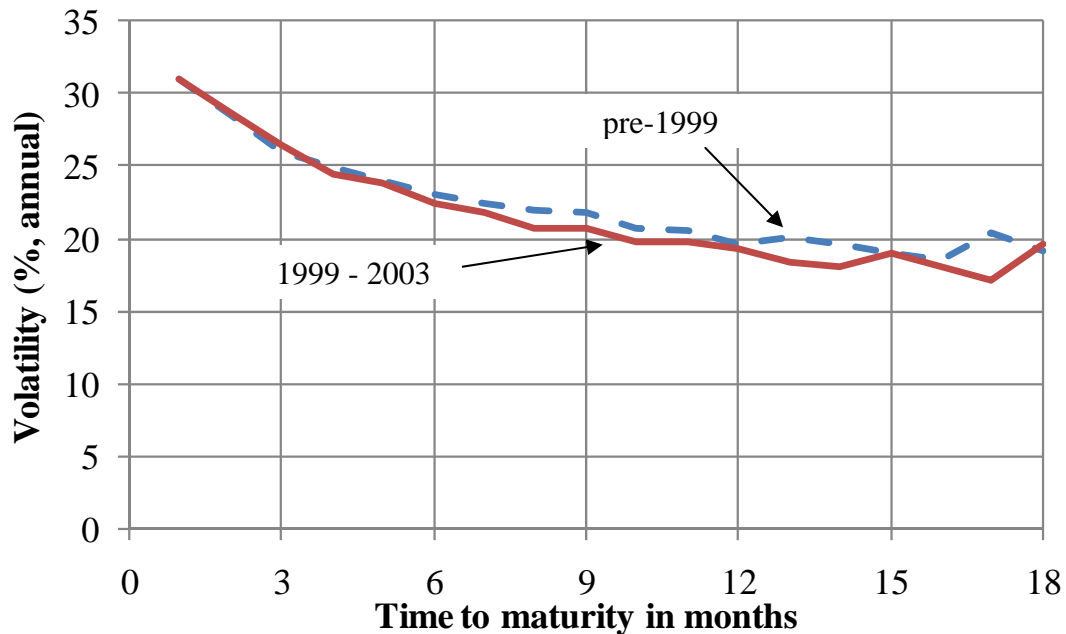
Notes: The figure displays two term structures, one estimated using data from before 1999, the other using data from 1999-2003. Volatility of a one-month future is set to 31.0% for both term structures.

Appendix 2: Numerical solution and estimation methods

A2.1 Value function iteration

I solve the value function (11) on a grid of points in (P,D,σ,x) space (in logs) using standard value function iteration. An important factor in defining the grid is that, while the price, dayrate, and volatility states that are realized in the data are bounded, the stochastic processes for these variables (equations 3, 4 and 9) imply that agents place nonzero probabilities on realizations outside of these bounds. Thus, the value function must be solved for states extending beyond the boundaries of the data. The state space I use extends from one-fifth of the lowest realized price and dayrate to five times the highest price and dayrate, and from one-half the lowest realized volatility to twice the highest volatility. With this state space, further extensions do not substantially affect the estimated parameters or the value function within the range of realized observations.

I found that a relatively dense grid was required to accurately capture the effects of stochastic volatility. The grid I used has 1,875,000 points: 50 price states by 50 dayrate states by 15 volatility states by 50 productivity states. Starting from this density, the estimated results are insensitive to variations in the number of grid points. For example, increasing the number of price and dayrate states to 55 does not change any estimated parameter by more than 2%. Sensitivity is substantial, however, at sparser state spaces.

In the full estimation routine, the initial value function used for each guess of parameters is the value function from the previous guess. For the first parameter guess, the initial value function is zero in all states. The convergence criterion is a tolerance of 10^{-6} on the sup norm of the value function (the value function used in the computations is in units of \$412,585, the average drilling cost at the average dayrate). Increasing the tolerance to 10^{-7} has essentially no affect on the parameter estimates or value function.

With the value function solved, I can then find, for any given P, D, and σ, the critical productivity x^* such that drilling is optimal iff $x_i \geq x^*$. Because the P, D, and σ realizations do

not coincide with the grid states used in the model, I use linear interpolation to find x^*. At each x_i grid point, I calculate the value function at the realized $P, D,$ and σ by linearly interpolating the value function between the states immediately above and below the $P, D,$ and σ. I then find the smallest x_i grid point such that the value of waiting exceeds the realized profits from drilling immediately and the largest x_i such that it is optimal to drill immediately (these two values of x_i will be adjacent grid points). Interpolation gives x^* as the productivity level for which the firm is indifferent: the value of waiting equals the value of drilling immediately. As described in the text, the realized time series of **$P, D,$** and **σ** can then be combined with a parameterized distribution on the x_{it} to yield the probability that a given prospect will be drilled each period.

A2.2 Estimation

I search for the parameters $\beta, \mu,$ and $\log \zeta$ that maximize the log-likelihood function (12) via a gradient-based search that uses the BFGS method for computing the Hessian at each step (I take the logarithm of ζ to allow for negative values in the parameter search). I accelerate the search by conducting it in two stages. First, holding β fixed, I search for the μ and $\log \zeta$ that maximize the likelihood. This stage is fast because changing μ and ζ does not require re-solving the model. The outer-most loop then searches for β. The stopping criterion is a tolerance on the likelihood function (scaled down by a factor of 10,000) of 10^{-10} for the μ and ζ loop and 10^{-8} for the β loop.

To compute the standard errors of the parameter estimates, I obtain the likelihood score of each observation (drilling prospect - month) numerically. With respect to each parameter θ_k, I calculate the derivative of the log likelihood for observation j as $\dfrac{\ell_j(\theta_k + \varepsilon_k) - \ell_j(\theta_k - \varepsilon_k)}{2\varepsilon_k}$. For the parameters β and μ, I use a value for ε_k of 0.001, and for $\log \zeta$ I use a value of 0.0001 because the likelihood function is particularly concave in this parameter. The standard errors are robust to values of ε_k that are an order of magnitude larger or smaller.