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ABSTRACT

This paper analyzes optimal unemployment insurance (UI) over the business cycle. We obtain an optimal UI formula that resolves the trade-off between insurance and job-search incentives in a broad class of models in which the job-finding rate depends on UI. Our formula generalizes the standard Baily-Chetty formula, only valid when the job-finding rate is a constant. The formula relates the optimal replacement rate of UI to the usual sufficient statistics (risk aversion, consumption-smoothing benefits of UI, and microelasticity of unemployment with respect to UI) and a new sufficient statistic (macroelasticity of unemployment with respect to UI). While the microelasticity accounts only for the response of job search to UI, the macroelasticity also accounts for the response of the job-finding rate to UI. We calibrate the formula using available empirical estimates of the sufficient statistics. The wedge between micro- and macroelasticity is positive and countercyclical in empirical studies, capturing negative job-search externalities that are more acute in recessions. An implication is that the Baily-Chetty formula underestimates optimal UI, especially in recessions. We show that the standard search-and-matching model with Nash bargaining generates a negative wedge between micro- and macroelasticity. To generate a wedge that is positive and countercyclical, we construct an alternative search-and-matching model with rigid wages and diminishing marginal returns to labor. Using our formula, we prove that optimal UI is countercyclical in this model. We also show that the calibrated model generates realistic fluctuations in unemployment and the elasticity wedge.

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In a seminal paper, Baily [1978] analyses the optimal provision of unemployment insurance (UI) when workers are risk averse, workers cannot insure themselves against unemployment, and workers' job-search effort is not observable. UI helps workers smooth consumption when they become unemployed, but it also increases unemployment by discouraging job search. The optimal UI equalizes the marginal benefit of smoothing consumption with the marginal cost of increasing unemployment. Optimal UI satisfies a very simple and robust formula in this model [Baily, 1978; Chetty, 2006a].

A key assumption of Baily's model is that the job-finding rate is a parameter that does not depend on UI. This assumption is too restrictive to study UI over the business cycle. The reason is that in the macroeconomic models used to study the labor market over the business cycle—the search-and-matching models—the job-finding rate does depend on UI. In this paper, we allow the job-finding rate to depend on UI and generalize the Baily formula accordingly. We apply the formula to a search-and-matching model that captures key empirical features of the labor market over the business cycle. We derive the implications of our formula for the cyclicality of optimal UI in this model.

We begin in Section 1 by deriving a formula for the optimal replacement rate—the generosity of unemployment benefits expressed as a fraction of the income of employed workers—in a model in which the job-finding rate depends on UI. The formula, expressed with estimable sufficient statistics, does not require much structure on the primitives of the model. As in the Baily formula, a first term captures the trade-off between the need for insurance, measured by the coefficient of risk aversion, and the need for job-search incentives, measured by the elasticity of unemployment with respect to UI. But to measure the budgetary costs of UI, we replace the microelasticity used in the Baily formula by a macroelasticity. The microelasticity accounts only for the response of job search to UI whereas the macroelasticity also accounts for the response of the job-finding rate to UI. Empirically, the microelasticity ϵ^m is the elasticity of the unemployment probability of a worker whose individual benefits change whereas the macroelasticity ϵ^M is the elasticity of aggregate unemployment when benefits change for all workers. Our formula also adds to the Baily formula a second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$. The wedge captures the welfare effect of the employment change following the response of the job-finding rate to UI.

Although we derive the formula in a static model in which workers cannot insure themselves, the formula also applies in more realistic models. Following the approach of Gruber [1997] and Chetty [2006a], we show that the formula also applies in a dynamic model in which jobs are continuously created and destroyed and workers can partially insure themselves against unemployment. Since the formula applies in presence of self-insurance and labor market flows, which are both important empirically, the formula can be combined with empirical estimates of the sufficient statistics to obtain illustrative optimal replacement rates. Available empirical evidence suggest that ϵ^m/ϵ^M is above 1 [Crépon et al., 2012;

Lalive, Landais and Zweimüller, 2012] and countercyclical [Crépon et al., 2012; Kroft and Notowidigdo, 2011]. Hence, the optimal replacement rate is above that given by the Baily formula and countercyclical.

From a theoretical perspective the empirical finding that $\epsilon^m/\epsilon^M > 1$ is surprising because it is not consistent with the behavior of the search-and-matching model with Nash bargaining, commonly used by macroeconomists to analyse optimal UI over the business cycle [for example, Mitman and Rabinovich, 2011]. Indeed, we show at the beginning of Section 2 that $\epsilon^m/\epsilon^M < 1$ in that model. The reason why $\epsilon^m < \epsilon^M$ is simple. Consider a reduction in UI. There is a microeffect: the reduction in unemployment from higher job-search effort, measured by ϵ^m. In addition, the Nash-bargained wage falls because the outside option of workers falls. This reduction in wage leads firms to hire even more, thus reinforcing the microeffect. Hence the macroeffect, measured by ϵ^M, is stronger than the microeffect.

To rationalize the empirical findings that ϵ^m/ϵ^M is above one and countercyclical, Section 2 develops a parsimonious macroeconomic model of UI. Our model builds on the framework of Michaillat [2012*a*]. That framework modifies the search-and-matching model with Nash bargaining by assuming that (i) the wage schedule is rigid instead of arising from Nash bargaining; and (ii) the production function has diminishing marginal returns to labor instead of constant returns to labor. Michaillat [2012*a*] shows that realistic unemployment fluctuations arise with assumption (i) and jobs are rationed in recessions with assumptions (i) and (ii)—that is, the labor market does not converge to full employment even when search efforts are arbitrarily large. We show that in this model, ϵ^m/ϵ^M is above one and countercyclical under assumptions (i) and (ii).

First, we prove that $\epsilon^m/\epsilon^M > 1$. Intuitively, the number of jobs available is limited because of diminishing marginal returns to labor. Hence, searching more to increase one's probability of finding a job mechanically decreases others' probability of finding one of the few jobs available. Since $\epsilon^m/\epsilon^M > 1$, our formula calls for a higher replacement rate than the Baily formula. The higher replacement rate discourages job search and thus corrects the negative *rat-race externality* imposed by jobseekers on others. This externality arises because jobseekers search taking the job-finding rate as given, without internalizing their negative influence on the job-finding rate of others. Since the replacement rate given by the Baily formula is also the replacement rate offered by small private insurers, our formula suggests that small private insurers would not provide enough insurance against unemployment.

Second, we prove that ϵ^m/ϵ^M is countercyclical, and also that the macroelasticity ϵ^M is procyclical. Intuitively, recessions are periods of acute job shortage during which job search and matching frictions have little influence on the labor market equilibrium. Therefore, aggregate search efforts have little influence on unemployment and the macroelasticity ϵ^M is small. Since the microelasticity ϵ^m remains broadly the same, the wedge ϵ^m/ϵ^M is large in recessions.

Combining the results on the cyclicality of ϵ^m/ϵ^M and ϵ^M with the optimal UI formula, we prove that the optimal replacement rate is countercyclical. In recessions, the macroelasticity decreases, implying that increasing UI only raises unemployment negligibly; thus the marginal budgetary cost of UI decreases. In recessions, the wedge ϵ^m/ϵ^M also increases, implying that the welfare cost of the rat-race externality increases; thus the marginal benefit of UI from correcting the externality increases. The lower marginal cost and higher marginal benefit imply that it is socially optimal to increase UI in recessions.

At the end of Section 2, we calibrate and simulate our model to show that the cyclical fluctuations of the elasticities accord with available empirical evidence. When the unemployment rate increases from 4% to 10%, ϵ^m increases slightly from 0.8 to 1, ϵ^M decreases from 0.6 to 0.2, and importantly, ϵ^m/ϵ^M increases from 1.3 to 5. As a consequence, the optimal replacement rate increases from 45% to 59%.

In Section 3, we derive an alternative optimal UI formula under the assumption that the government taxes profits. This assumption may not be realistic, but it is standard in macroeconomics and it allows us to connect our results with the literature. Our optimal UI formula adds to the Baily formula a term measuring the deviation from a generalized Hosios [1990] condition, which gives the efficient labor market tightness in our model. If the generalized Hosios condition holds, our formula coincides with the Baily formula even if micro- and macroelasticity differ; but if it does not hold and there is a wedge between micro- and macroelasticity, then our formula departs from the Baily formula. When the deviation from the Hosios condition is positive because labor market tightness is too low and when $\epsilon^m/\epsilon^M > 1$, the optimal replacement rate is above that given by the Baily formula. Using a standard approximation, we show that the optimal replacement rate remains countercyclical when the government taxes profits.

Section 4 concludes by showing that optimal UI remains countercyclical in three alternative settings. Simulations of a calibrated model suggest that optimal UI is countercyclical when the government adjusts the duration of unemployment benefits instead of their level: the optimal duration increases from less than 6 weeks when unemployment is 4%; to 26 weeks when unemployment is 5.9%; and to over 100 weeks when unemployment reaches 10%. Optimal UI is also countercyclical in a model in which business cycles are driven by aggregate demand shocks instead of technology shocks. Finally, optimal UI is countercyclical in a model in which the government can provide a wage subsidy to employers to attenuate employment fluctuations. All proofs and extensions are collected in the Appendix.

1 Optimal UI Formula

This section introduces a generic model of the labor market, which generalizes the model of Baily [1978] by allowing the job-finding rate to depend on UI. This extension is necessary to be able to apply our framework to search-and-matching models. Workers are risk averse and cannot insure themselves against unemployment. To improve welfare the government provides unemployment benefits, financed by a labor tax. Since jobseekers' search efforts are not observable, the government trades off the provisions of insurance and incentives to search. A simple formula resolves this trade-off. The formula relates the optimal replacement rate of UI to four sufficient statistics: two statistics present in the Baily formula (microelasticity of unemployment with respect to UI and risk aversion) and two new statistics arising because the job-finding rate depends on UI (macroelasticity of unemployment with respect to UI and elasticity of search with respect to job-finding rate). At the end of the section, we combine our formula with available empirical estimates of these sufficient statistics to illustrate the resulting replacement rates.

1.1 Model

Labor market. There is a measure 1 of workers. Initially, $u \in (0, 1)$ workers are unemployed and $1-u$ workers are employed. Unemployed workers search for a job with effort e, which is not observable. A jobseeker finds a job at a rate f per unit of effort; thus, a jobseeker searching with effort e finds a job with probability $e \cdot f$. A fraction $e \cdot f$ of the u unemployed workers find jobs so the number of new hires is $h = u \cdot e \cdot f$. After hiring, the number of employed workers is

$$n = 1 - u + u \cdot e \cdot f. \tag{1}$$

Workers. Firms pay a wage w. To finance unemployment benefits $B \cdot w$, the government imposes a labor tax T. As in the public finance literature, tax incidence is entirely on the worker's side so w does not respond to T. Workers cannot save, borrow, or insure themselves against unemployment in other ways. Employed workers consume their post-tax labor income $c^e = (1 - T) \cdot w$ and unemployed workers consume unemployment benefits $c^u = B \cdot w$. A worker's utility from consumption is $v(c)$, an increasing and concave function. We introduce two measures of the generosity of UI: the consumption gain from work $\Delta c \equiv c^e - c^u$ and the utility gain from work $\Delta v(\Delta c, c^e) \equiv v(c^e) - v(c^e - \Delta c)$, an increasing function of Δc. Given job-finding rate f and utility gain from work Δv, a jobseeker chooses effort e to maximize expected utility

$$v(c^u) + e \cdot f \cdot \Delta v - k(e), \tag{2}$$

where $k(e)$ is the disutility from search, an increasing and convex function. The optimal effort $e(f, \Delta v)$ satisfies the first-order condition

$$k'(e) = f \cdot \Delta v. \tag{3}$$

As $k(e)$ is convex, $e(f, \Delta v)$ increases with Δv and f. Workers search more when UI is less generous and when jobs are easier to find. We define labor supply $n^s(f, \Delta v)$ as the employment rate when workers search optimally:

$$n^s(f, \Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f \tag{4}$$

Labor supply increases with f and Δv as $e(f, \Delta v)$ increases with f and Δv. Labor supply is higher when UI is less generous because search efforts are higher. Labor supply is also higher when jobs are easier to find, mechanically and because search efforts are higher.

Job-finding rate. To capture the dependence of the job-finding rate on UI, we assume that the job-finding rate $f(\Delta v)$ is a function of the utility gain from work. Employment is also a function of the utility gain from work:

$$n(\Delta v) = n^s(f(\Delta v), \Delta v). \tag{5}$$

Our framework is quite general. It nests the Baily model, a rat-race model, and a broad class of search-and-matching models as special cases. To obtain the Baily model, we set the job-finding rate f as a parameter, independent of Δv. In the rat-race model, u jobseekers queue in front of a fixed number $o < u$ of vacant jobs. In equilibrium the job-finding rate equates the number $u \cdot e \cdot f$ of new hires with o; therefore, $f = o/(u \cdot e)$. To obtain the rat-race model, we set the job-finding rate as the function $f(\Delta v)$ implicitly defined by $f = o/\left[u \cdot e(f, \Delta v)\right]$. Finally, Section 2 determines the function $f(\Delta v)$ to obtain various search-and-matching models.

Unemployment insurance. The government chooses the consumption gain from work Δc and the consumption c^e of employed workers. This is equivalent to choosing directly the unemployment benefit rate B and the labor tax rate T. Given that the government chooses Δc and c^e and not Δv directly, it is convenient to redefine job-finding rate, labor supply, and employment as functions of $(\Delta c, c^e)$. Abusing notations slightly, we define $f(\Delta c, c^e) \equiv f(\Delta v(\Delta c, c^e))$, $n^s(f, \Delta c, c^e) \equiv n^s(f, \Delta v(\Delta c, c^e))$, and $n(\Delta c, c^e) \equiv n(\Delta v(\Delta c, c^e))$. The goal of the government is to maximize welfare

$$n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e). \tag{6}$$

The government accounts for the labor market structure given by (1), the fact that effort is chosen optimally by jobseekers, and the dependence of the job-finding rate on UI. The government balances its budget each period by financing outlays of unemployment benefits with the labor tax: $(1-n) \cdot B \cdot w = n \cdot T \cdot w$.

The budget constraint implies

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w. \tag{7}$$

1.2 Microelasticity and macroelasticity of unemployment

To characterize the solution of the government's problem, we need to define two elasticities:

DEFINITION 1. The *microelasticity* of unemployment with respect to consumption gain from work is

$$\epsilon^m \equiv \frac{\Delta c}{1-n} \cdot \left. \frac{\partial n^s}{\partial \Delta c} \right|_{f,c^e}. \tag{8}$$

The *macroelasticity* of unemployment with respect to consumption gain from work is

$$\epsilon^M \equiv \frac{\Delta c}{1-n} \cdot \left. \frac{\partial n}{\partial \Delta c} \right|_{c^e}. \tag{9}$$

Both elasticities are normalized to be positive. They are computed keeping the consumption c^e of employed workers constant; that is, c^e does not adjust to meet the budget constraint of the government. The microelasticity measures the percentage increase in unemployment $1-n$ when the net reward from work Δc decreases by 1%, taking into account jobseekers' reduction in search effort but ignoring the equilibrium adjustment of the job-finding rate f. It can be estimated by measuring the reduction in the job-finding probability of an individual unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant. The macroelasticity measures the percentage increase in unemployment when the net reward from work decreases by 1%, assuming that both search effort and job-finding rate adjust. It can be estimated by measuring the increase in aggregate unemployment following a general increase in unemployment benefits financed by deficit spending.

To relate microelasticity ϵ^m and macroelasticity ϵ^M, we introduce an elasticity that characterizes the response of jobseekers to a change in labor market conditions:

DEFINITION 2. The *discouraged-worker elasticity* ϵ^d is the elasticity of search effort with respect to the job-finding rate:

$$\epsilon^d \equiv \frac{f}{e} \cdot \left. \frac{\partial e}{\partial f} \right|_{\Delta v}.$$

If $\epsilon^d > 0$, workers search less when it becomes more difficult to find a job. In other words, $\epsilon^d > 0$ captures the discouragement of jobseekers when labor market conditions deteriorate. Lemma 1 shows that ϵ^m and ϵ^M admit a simple relationship:

LEMMA 1. *Microelasticity ϵ^m and macroelasticity ϵ^M are related by*

$$\epsilon^M = \epsilon^m + \frac{u \cdot e}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \Delta c \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e}.$$

If the job-finding rate f does not depend on the consumption gain from work Δc, $\epsilon^M = \epsilon^m$.

The formal proof is relegated to the Appendix. We propose an informal version of the proof here. Consider a cut in unemployment benefits $d\Delta c > 0$. Since Δv and f depend on Δc, the cut creates variations $d\Delta v$ and df. Using (4) and (5), we decompose the variation in employment as $dn = dn_{\Delta v} + dn_f$ where $dn_{\Delta v} = u \cdot f \cdot (\partial e/\partial \Delta v)\, d\Delta v$ is the variation keeping f constant and $dn_f = [u \cdot e + u \cdot f \cdot (\partial e/\partial f)]\, df = u \cdot e \cdot \left[1+\epsilon^d\right] df$ is the additional variation through a change in f. By definition, $\epsilon^M = [\Delta c/(1-n)] \cdot [dn/d\Delta c]$ and $\epsilon^m = [\Delta c/(1-n)] \cdot [dn_{\Delta v}/d\Delta c]$; therefore, we obtain the result of Lemma 1 when we multiply the decomposition $dn = dn_{\Delta v} + dn_f$ by $[\Delta c/(1-n)] \cdot [1/d\Delta c]$. If the job-finding rate is independent of UI, $df = 0$, $dn_f = 0$, and $\epsilon^M = \epsilon^m$. But if the job-finding rate responds to UI, $df \neq 0$, $dn_f \neq 0$, and $\epsilon^m \neq \epsilon^M$. The rat-race model illustrates the difference between ϵ^M and ϵ^m. In the rat-race model the number of jobs is fixed so $\epsilon^M = 0$, even though $\epsilon^m > 0$.

1.3 Formula

Proposition 1 provides a formula for the optimal replacement rate τ, defined as the ratio of unemployment benefits c^u over post-tax labor income c^e. The replacement rate measures the generosity of the UI system.

PROPOSITION 1. *The optimal replacement rate $\tau \equiv c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right). \tag{10}$$

The Lagrange multiplier ϕ on the government's budget constraint satisfies the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right]. \tag{11}$$

If $n \approx 1$ and if the third and higher order terms of $v(\cdot)$ are small, the formula simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right], \tag{12}$$

where $\rho \equiv -c^e \cdot v''(c^e)/v'(c^e)$ is the coefficient of relative risk aversion evaluated at c^e. If the job-finding rate is independent of the consumption gain from work, $\epsilon^m = \epsilon^M$, the second term in the right-hand side of (10) and (12) vanishes, and the formulas reduce to those in Baily [1978] and Chetty [2006a].

The formal proof is relegated to the Appendix. We propose an informal version of the proof here, based on marginal deviations from the optimum. The Lagrangian of the government's problem is

$$\mathcal{L} = v(c^e) - u\left[1 - ef(\Delta c, c^e)\right]\left[v(c^e) - v(c^e - \Delta c)\right] - uk(e) + \phi\left[n(\Delta c, c^e)\left(w - \Delta c\right) - c^e + \Delta c\right],$$

where e maximizes (2) and ϕ is the Lagrange multiplier on the budget constraint (7).

First, consider changes $dc^e = dc/v'(c^e)$ and $dc^u = dc/v'(c^u)$. The changes have no first-order impact on $\Delta v = v(c^e) - v(c^u)$, and hence no impact on effort $e(f, \Delta v)$, job-finding rate $f(\Delta v)$, and employment $n(\Delta v)$. The effect on social welfare is $dSW = n \cdot v'(c^e) \cdot dc^e + (1-n) \cdot v'(c^u) \cdot dc^u = dc$. The effect on the expenditure of the government is $dX = -n \cdot dc^e - (1-n) \cdot dc^u = -dc \cdot \{[n/v'(c^e)] + [(1-n)/v'(c^u)]\}$. At the optimum, $d\mathcal{L} = dSW + \phi dX = 0$, which establishes the inverse Euler equation (11).

Next, consider a change $d\Delta c$, keeping c^e constant. We apply the envelope theorem as workers choose effort e optimally. The first-order condition $\partial \mathcal{L}/\partial \Delta c = 0$ implies that

$$0 = -v'(c^u) \cdot (1-n) + \Delta v \cdot u \cdot e \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e} + \phi \cdot \left[1 - n + (w - \Delta c) \cdot \left.\frac{\partial n}{\partial \Delta c}\right|_{c^e}\right],$$

which, dividing by $\phi \cdot (1-n)$, can be rearranged as

$$0 = \left[1 - \frac{v'(c^u)}{\phi}\right] + \frac{w - \Delta c}{\Delta c} \cdot \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n}{\partial \Delta c}\right|_{c^e} + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{\Delta c}{1-n} \cdot \frac{h}{f} \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e}.$$

Lemma 1 shows that the last term, capturing the welfare effect of the variation df, is proportional to the wedge $\epsilon^m - \epsilon^M$. Using the lemma and the definitions of ϵ^M and ϕ, we rewrite the equation as

$$0 = n \cdot \left[1 - \frac{v'(c^u)}{v'(c^e)}\right] + \frac{w - \Delta c}{\Delta c} \cdot \epsilon^M + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1 + \epsilon^d} \cdot \left[\epsilon^M - \epsilon^m\right]. \tag{13}$$

The first term in (13) reflects the welfare effect of transferring resources from the unemployed to the employed by increasing Δc. As long as $c^u < c^e$ and workers are risk averse, this term is negative. The second term in (13) captures the desirable effect on the budget of reducing the unemployment rate by increasing Δc. This budgetary effect is due to the aggregate behavioral response of workers; hence, it involves the macroelasticity ϵ^M. The third term in (13) captures the welfare effect due to the variation in the job-finding rate f, which is proportional to the elasticity wedge $\epsilon^m - \epsilon^M$. Since $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c$, we rearrange (13) as (10), which completes the derivation.

If microelasticity ϵ^m and macroelasticity ϵ^M are equal, our formulas reduce to the Baily formula. For example, formula (12) becomes $\tau/(1-\tau) \approx (\rho/\epsilon^m) \cdot (1-\tau)$. The trade-off between the need for insurance, captured by the coefficient of relative risk aversion ρ, and the need for incentives to search, captured by the microelasticity ϵ^m, appears transparently.

In a model of equilibrium unemployment, micro- and macroelasticity generally differ and our formula presents two departures from the Baily formula. The first term in the right-hand side of formulas (10) and (12) involves the macroelasticity ϵ^M instead of the microelasticity ϵ^m conventionally used to calibrate optimal benefits [Chetty, 2008; Gruber, 1997]. ϵ^M captures the response of unemployment to UI when the response of the job-finding rate is accounted for while ϵ^m only captures the response of a jobseeker's job-finding probability to UI, keeping the job-finding rate constant. For government budgetary purposes, ϵ^M is the relevant parameter. A second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$ also appears in the right-hand side of formulas (10) and (12). The term accounts for the first-order welfare effect of the employment change that arises from the equilibrium adjustment of the job-finding rate after a change in UI. Even in the absence of any concern for insurance (if workers are risk neutral), some unemployment insurance should be provided as long as the second term is positive ($\epsilon^m/\epsilon^M > 1$). Below, we show that this term is a corrective tax on the externality created by job search on other jobseekers.

While equation (10) is an exact formula, equation (12) is a simpler formula obtained with the approximation method of Chetty [2006a]. Formula (12) is expressed with sufficient statistics, which means that the formula is robust to changes in the primitives of the model. Indeed the formula is valid for any utility over consumption with coefficient of relative risk aversion ρ; any search behavior with discouraged-worker elasticity ϵ^d and microelasticity ϵ^m; and any labor demand yielding a macroelasticity ϵ^M.

1.4 Implementing the formula using available sufficient statistics

We now combine our optimal UI formula with empirical estimates of the sufficient statistics to assess the optimal replacement rate of UI. But before that, we show that the formula also applies in a dynamic model in which jobs are continuously created and destroyed, and in which workers partially insure themselves through home production. As self-insurance and labor market flows are empirically important, this result ensures that the formula can be used to calibrate realistically the optimal replacement rate.

Formula in a dynamic model with partial self-insurance. We start by describing the dynamic model with partial self-insurance. The analysis focuses on the steady state of the model. At the end of period $t-1$, a fraction s of the n_{t-1} existing jobs is exogenously destroyed. Workers who lose their job become unemployed, and start searching for a new job at the beginning of period t. At the beginning of period t, $u_t = 1-(1-s)\cdot n_{t-1}$ unemployed workers look for a job and $h_t = [1-(1-s)\cdot n_{t-1}]\cdot e_t \cdot f_t$ jobseekers find a job. In steady state, employment n is constant so inflows to unemployment $s \cdot n$ equal outflows from unemployment $[1-(1-s)\cdot n]\cdot e\cdot f$ and

$$n=\frac{e\cdot f}{s+(1-s)\cdot e\cdot f}\equiv\tilde{n}(e\cdot f). \tag{14}$$

As in the static model, n is only a function of the product $e\cdot f$.

Workers unemployed in period t consume unemployment benefits and an amount y_t produced at home at a utility cost $m(y_t)$, an increasing and convex function. Workers choose y_t to maximize the utility when unemployed $v(c_t^e-\Delta c_t+y_t)-m(y_t)$. This choice minimizes the utility gain from work $v(c_t^e)-v(c_t^e-\Delta c_t+y_t)+m(y_t)$. Let $\Delta v^h(\Delta c,c^e)\equiv min_y\left\{v(c^e)-v(c^e-\Delta c+y)+m(y)\right\}$ be the optimal utility gain from work when unemployment insurance is $(\Delta c,c^e)$. As in the absence of home production, the utility gain from work is a function of Δc and c^e only.

There is no time discounting. In that case, given job-finding rate f and unemployment insurance $(c^e,\Delta c)$, the representative worker chooses search effort e to maximize expected per-period utility

$$v(c^e)-[1-\tilde{n}(e\cdot f)]\cdot\Delta v^h-[1-(1-s)\cdot\tilde{n}(e\cdot f)]\cdot k(e). \tag{15}$$

As in the static model, the optimal search effort $e(f,\Delta v^h)$ is a function of the job-finding rate and utility gain from work. We also define the labor supply $n^s(f,\Delta v^h)\equiv\tilde{n}(e(f,\Delta v^h)\cdot f)$.

Finally, the job-finding rate $f(\Delta v^h)$ is a function of the utility gain from work. The employment rate is $n(\Delta v^h)\equiv n^s(f(\Delta v^h),\Delta v^h)$. We define the elasticities ϵ^m, ϵ^M, and ϵ^d as in Definitions 1 and 2.

The government chooses Δc and c^e to maximize the per-period social welfare (15) subject to the per-period budget constraint (7). The government accounts for the structure (14) of the labor market, the dependence of the job-finding rate on UI, and the fact that effort and home production are chosen optimally by workers. The maximization problem has virtually the same structure as in the static model. As a consequence, the formulas of Proposition 1 remain valid:

PROPOSITION 2. *Consider a dynamic model in which workers have access to home production when they are unemployed. We normalize $k(\cdot)$ so that $k(e)=0$ at the optimum. Then in steady state with no time discounting, the optimal replacement rate $\tau\equiv c^u/c^e$ satisfies*

$$\frac{1}{n}\cdot\frac{\tau}{1-\tau}=\frac{n}{\epsilon^M}\cdot\left[\frac{v'(c^h)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v^h}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right), \tag{16}$$

where $c^h\equiv c^u+y$ is consumption while unemployed, and $\Delta v^h\equiv\min_y\left\{v(c^e)-\left[v(c^h)-m(y)\right]\right\}$ is the difference in utility between being employed and being unemployed. The Lagrange multiplier ϕ on the government's budget constraint satisfies the inverse Euler equation $1/\phi=[n/v'(c^e)]+\left[(1-n)/v'(c^h)\right]$.

Suppose that $n \approx 1$, that $m(\cdot)$ is normalized so that $m(y) = 0$ at the optimum, and that the third and higher order terms of $v(\cdot)$ are small. Then formula (16) simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\xi) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\xi)\right] \cdot \left[\frac{1-\xi}{1-\tau}\right], \tag{17}$$

where $\xi = c^h/c^e$ is the consumption drop upon unemployment.

The optimal UI formulas (10) and (12) carry over with minor modifications in a dynamic model in which workers have partial access to self-insurance. These results are similar to those in Chetty [2006a], who generalizes the analysis of Baily [1978]. The only difference is that when workers partially self-insure, the optimal replacement rate tends to be lower than without self-insurance because the insurance value $\left[v'(c^h)/v'(c^e)\right] - 1$ of UI is smaller as $c^h \geq c^u$.[1] The normalizations that the disutilities $k(e)$ from job search and $m(y)$ from home production are zero at the optimum imply that the welfare cost of unemployment is measured solely by the loss of consumption that it imposes.[2]

The optimal UI formula (17) is expressed with five sufficient statistics: microelasticity ϵ^m, macroelasticity ϵ^M, discouraged-worker elasticity ϵ^d, relative risk aversion ρ, and consumption drop upon unemployment $\xi = c^h/c^e$. Naturally, the consumption drop depends on the replacement rate $\tau = c^u/c^e$. Without self-insurance, $\xi = \tau$. With partial self-insurance, around the current replacement rate $\hat{\tau}$, $\xi \approx \hat{\xi} + \epsilon^i \cdot (\tau - \hat{\tau})$, where $\hat{\xi}$ is the current consumption drop and $\epsilon^i \equiv -\partial c^h/\partial \Delta c\big|_{c^e}$ is the marginal effect of unemployment benefits on consumption when unemployed. The statistics ϵ^i measures the availability of self-insurance: without self-insurance, $\epsilon^i = 1$; with perfect self-insurance, $\epsilon^i = 0$. Formula (17), combined with this expression for ξ, links τ to seven sufficient statistics: $\hat{\tau}$, $\hat{\xi}$, ϵ^i, ρ, ϵ^m, ϵ^M, and ϵ^d.

Empirical estimates of $\hat{\tau}$, $\hat{\xi}$, ϵ^i, ρ, ϵ^m, ϵ^M, and ϵ^d. In the US, weekly unemployment benefits replace between 50% and 70% of the last weekly pre-tax earnings of a worker [Pavoni and Violante, 2007]. Following Chetty [2008] we set the benefit rate to 50%. Since earnings are subject to a 7.65% payroll tax, we set the current replacement rate to $\hat{\tau} = 0.5/(1 - 0.0765) = 0.54$.

The ratio $\hat{\xi}$ captures the consumption drop upon unemployment in the current system. For food consumption φ, Gruber [1997] estimates $\left[\varphi^h - \varphi^e\right]/\varphi^e = -0.068$. As emphasized by Browning and Crossley [2001], total consumption is more elastic than food consumption to an income change. The estimates of Hamermesh [1982] imply that the elasticity of food consumption with respect to aggre-

[1] The welfare effect of the adjustment of the job-finding rate is dampened because $\Delta v^h \leq \Delta v = v(c^e) - v(c^u)$. Whether the optimal replacement rate increases or decreases as a result depends on the sign of $(\epsilon^m/\epsilon^M) - 1$.

[2] This is a neutrality assumption. In the model, job search and home production impose costs but we have omitted on-the-job labor costs. Unemployment may generate additional costs such as human capital loss or psychological cost, also omitted here.

gate income for unemployed workers is 0.36.[3] Accordingly we expect that $1-\hat{\xi}=\left[c^e-c^h\right]/c^e=\left(\left[\varphi^e-\varphi^h\right]/\varphi^e\right)/0.36=0.068/0.36=0.19$ and $\hat{\xi}=0.81$.

Gruber [1997] also estimates $-d\varphi^h/d\Delta c=0.27$. Using again the estimates of Hamermesh [1982], we find that $dc^h=d\varphi^h/0.36$ and hence $\epsilon^i=0.27/0.36=0.75$. In words, increasing unemployment benefits by $1 increases total consumption when unemployed by $0.75.

Many studies estimate the coefficient of relative risk aversion ρ. We choose $\rho=1$. This is on the low side of available estimates but is consistent with labor supply behavior [Chetty, 2004, 2006b]. Naturally the higher ρ, the more generous optimal UI.

There is little empirical work estimating the elasticity ϵ^d of job search effort with respect to the job-finding rate. Empirically, ϵ^d seems to be close to zero because labor market participation and other measures of search intensity are, if anything, slightly countercyclical even after controlling for changing characteristics of unemployed workers over the business cycle [Shimer, 2004].[4]

Many studies estimate the microelasticity ϵ^m (see Krueger and Meyer [2002] for a survey). The ideal experiment to estimate ϵ^m is to offer higher unemployment benefits to a randomly selected and small subset of individuals within a labor market and compare unemployment durations between these treated individuals and the other jobseekers. In practice, ϵ^m is estimated by comparing individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics. Most studies evaluate the elasticity ϵ^s of the job-finding rate with respect to benefits. This elasticity approximately equals ϵ^m in normal circumstances.[5] In US administrative data from the 1980s, the classic study of Meyer [1990] finds an elasticity of 0.9 with few individual controls and 0.6 with more individual controls. In a larger US administrative dataset from the early 1980s, and using a regression kink design to better identify the elasticity, Landais [2012] finds an elasticity around 0.3.

To investigate the cyclicality of ϵ^m, it is necessary to replicate the estimation across labor markets with different unemployment rates. The best empirical setting to do so is that of Schmieder, von Wachter and Bender [2012*b*]. They use sharp variations in the potential duration of unemployment benefits by age in Germany, population-wide administrative data, and a regression discontinuity approach to estimate the microelasticity of unemployment with respect to the potential duration of benefit entitlement. Their

[3]This estimate includes food consumed at home and away from home. Hamermesh [1982] estimates that for unemployed workers the permanent-income elasticity of food consumption at home is 0.24 while that of food consumption away from home is 0.82. He also finds that in the consumption basket of an unemployed worker, the share of food consumption at home is 0.164 while that of food consumption away from home is 0.041. Therefore the aggregate income elasticity of food consumption is $0.24\times[0.164/(0.164+0.41)]+0.82\times[0.041/(0.164+0.41)]=0.36$.

[4]The empirical finding that ϵ^d is small is consistent with the theoretical properties of ϵ^d in the dynamic model. Indeed, Lemma A14 in the Appendix shows that $\epsilon^d\approx u\cdot\{k'(e)/[e\cdot k''(e)]\}$, so ϵ^d is small when the unemployment rate u is small.

[5]Equation (A35) in the Appendix gives the relationship between ϵ^m and ϵ^s in the steady-state of the dynamic model and shows that $\epsilon^m\approx\epsilon^s$ when $\tau/(1-\tau)\approx 1$ and $u\ll 1$, which is the empirically relevant case.

estimates are broadly constant over the German business cycle, suggesting that ϵ^m is broadly acyclical.[6] Schmieder, von Wachter and Bender [2012*b*] however focus on the elasticity with respect to potential duration (instead of benefits) and in Germany (not the US). Landais [2012] focuses on the elasticity with respect to the benefit level and also finds that the regression kink design estimates of ϵ^m are fairly constant over the business cycle in the United States in the 1980s.[7]

Identifying empirically ϵ^M is inherently more difficult than estimating ϵ^m because it necessitates exogenous variations in benefits across comparable labor markets, instead of exogenous variations across comparable individuals within a single labor market. The ideal experiment to estimate ϵ^M is to offer higher unemployment benefits to all individuals in a randomly selected subset of labor markets and compare unemployment rates between these treated labor markets and the other labor markets. Although no study offers an ideal identification of ϵ^M, studies comparing individuals with different benefits across labor markets—for example across US states or within state over time—capture mainly macroelasticities [for example, Kroft and Notowidigdo, 2011; Moffitt, 1985]. Interestingly, the results of these studies suggest that macroelasticities strongly decline with state unemployment rates in the US. Combined with the finding that ϵ^m is acyclical, this finding suggests that the elasticity wedge ϵ^m/ϵ^M increases in recessions.

Alternatively, one can recover the macroelasticity ϵ^M and implement formula (17) by measuring the elasticity wedge ϵ^m/ϵ^M and using the estimates for the microelasticity ϵ^m described above. The elasticity wedge captures the externalities of job search on other jobseekers. Several papers have tried to directly estimate the sign and magnitude of these externalities. Early studies find that an increase in the search effort of some jobseekers, induced by a reduction in UI or by job training programs, has a negative effect on the job-finding probability of other jobseekers that implies a wedge $\epsilon^m/\epsilon^M > 1$ [Burgess and Profit, 2001; Ferracci, Jolivet and van den Berg, 2010; Gautier et al., 2012; Levine, 1993].[8] The estimates range from $\epsilon^m/\epsilon^M \approx 1.4$ in Denmark [Gautier et al., 2012] to $\epsilon^m/\epsilon^M \approx 2$ in the US [Levine, 1993].[9]

More recently, using a large change in UI duration for a subset of workers in a subset of geographical areas in Austria, Lalive, Landais and Zweimüller [2012] compellingly identify significant search

[6] Schmieder, von Wachter and Bender [2012*b*] estimate the effect of potential duration on the duration of both covered unemployment and total non-employment. They find that the elasticity of total non-employment is constant over the business cycle (Table 4, column 2), while the elasticity of the duration of covered unemployment is slightly countercyclical (Table 4, column 3). But the elasticity of the duration of covered unemployment is the sum of a mechanical effect (the truncation of benefit duration at a larger number of weeks of unemployment) and a behavioral response. They show that the countercyclicality is driven entirely by the mechanical effect (Table 4, columns 5 and 6). Their results can therefore be interpreted as evidence that behavioral responses are broadly constant over the business cycle.

[7] Note that elasticities in Schmieder, von Wachter and Bender [2012*b*] are smaller than our preferred elasticities in Meyer [1990] or Landais [2012] for at least two reasons. First, elasticities with respect to the benefit level are found to be larger than elasticities with respect to potential duration (see Landais [2012] for a side-by-side comparison in the US context). Second, elasticities are usually found to be larger in the US than in European countries where the baseline UI system is more generous.

[8] In contrast, Blundell et al. [2004] do not find any significant spillover effects of a job training program in the UK.

[9] See Table 3, column (1), in Gautier et al. [2012] and see Table 5, column (1), in the working-paper version of Levine [1993] that is available at http://dataspace.princeton.edu/jspui/handle/88435/dsp01wh246s14w.

externalities that translate into a wedge $\epsilon^m/\epsilon^M \approx 1.35 > 1$. Crépon et al. [2012] analyze a large randomized field experiment in France in which some young educated jobseekers are treated by receiving job placement assistance. The experiment has a double-randomization design: (1) some areas are treated and some are not, (2) within treated areas some jobseekers are treated and some are not. Interpreting the treatment as an increase in search effort from e^C for control jobseekers to e^T for treated jobseekers, their empirical results for long-term employment translate into a wedge $\epsilon^m/\epsilon^M = 1.58$.[10]

Crépon et al. [2012] also investigate the cyclicality of elasticity wedge ϵ^m/ϵ^M. They estimate that the wedge is larger in geographical areas and time periods with higher unemployment. For example, $\epsilon^m/\epsilon^M = 14.5/(14.5 - 7.6) = 2.10$ during the 2008-2009 recession in areas with high unemployment, compared with $\epsilon^m/\epsilon^M = 3.5/(3.5-0.9) = 1.35$ otherwise. The wedge for men in bad areas, bad periods is even higher: $\epsilon^m/\epsilon^M = 23.9/(23.9 - 14.6) = 2.57$.[11] As an important caveat, those estimates are not extremely precise and vary somewhat across specifications.

Optimal replacement rate τ. Table 1 illustrates the optimal replacement rate obtained with our formula and plausible empirical estimates of the sufficient statistics. Because the value of ϵ^m, ϵ^m/ϵ^M, and ϵ^d are uncertain, the table presents optimal replacement rates for a range of plausible estimates. We consider two values for ϵ^d: 0 and 0.5. Column (1) considers $\epsilon^m/\epsilon^M = 1$ as in the Baily model whereas column (2) considers $\epsilon^m/\epsilon^M = 1.5$ based on our discussion of empirical evidence above. Column (3) considers $\epsilon^m/\epsilon^M = 1.2$ and column (4) considers $\epsilon^m/\epsilon^M = 2.5$, in expansions and recessions. To span the range of available estimates, Panels A, B, and C consider the cases $\epsilon^m = 0.3$, $\epsilon^m = 0.6$, and $\epsilon^m = 0.9$.

Three results are noteworthy in columns (1) and (2). First, consistent with Gruber [1997], optimal replacement rates in the Baily model in column (1) are fairly low, and are actually below current replacement rates even for a low value $\epsilon^m = 0.3$. Second, introducing a wedge $\epsilon^m/\epsilon^M = 1.5$ as in column (2) increases the replacement rate across all rows by about 6-10 percentage points. The percentage point increases are somewhat higher for higher values of ϵ^m. Thus, our extension of the Baily model has sizable effects on the optimal replacement rate for reasonable empirical estimates. Third, the elasticity ϵ^d has only modest effects on the optimal replacement rate, at least within the range of estimates considered.

Columns (3) and (4) illustrate the consequences of the fluctuations of ϵ^m/ϵ^M over the business cy-

[10] Compared to control jobseekers in the same area, treated jobseekers face a higher job-finding probability: $\left[e^T - e^C\right] \cdot f^T = 5.7\%$. But compared to control jobseekers in control areas, control jobseekers in treated areas face a lower job-finding probability: $e^C \cdot \left[f^T - f^C\right] = -2.1\%$ (Table 10, column 1, panel B in Crépon et al. [2012]). Therefore the increase in the job-finding probability of treated jobseekers in treated areas compared to control jobseekers in control areas is only $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right] = 5.7 - 2.1 = 3.6\%$. By definition, the microelasticity ϵ^m is proportional to $\left[e^T - e^C\right] \cdot f^T$ and the macroelasticity ϵ^M is proportional to $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right]$, implying a wedge $\epsilon^m/\epsilon^M = 5.7/3.6 = 1.58$.

[11] Those estimates for all workers and long-term employment outcomes are reported in Table 11, panel A, column (2) in Crépon et al. [2012]. Estimates for men are in column (4).

Table 1: Optimal replacement rate τ for different values of sufficient statistics

	Baily model $\epsilon^m/\epsilon^M = 1$ (1)	Average $\epsilon^m/\epsilon^M = 1.5$ (2)	Expansion $\epsilon^m/\epsilon^M = 1.2$ (3)	Recession $\epsilon^m/\epsilon^M = 2.5$ (4)
	Panel A: $\epsilon^m = 0.3$			
$\epsilon^d = 0$	0.49	0.56	0.52	0.63
$\epsilon^d = 0.5$	0.49	0.55	0.52	0.62
	Panel B: $\epsilon^m = 0.6$			
$\epsilon^d = 0$	0.41	0.49	0.45	0.58
$\epsilon^d = 0.5$	0.41	0.48	0.44	0.56
	Panel C: $\epsilon^m = 0.9$			
$\epsilon^d = 0$	0.35	0.46	0.40	0.56
$\epsilon^d = 0.5$	0.35	0.44	0.39	0.53

Notes: The table presents the optimal replacement rate $\tau = c^u/c^e$ for a range of values for ϵ^m, ϵ^m/ϵ^M, and ϵ^d. The replacement rate is obtained by solving

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon^M} \cdot \frac{1-\xi}{\xi} + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \frac{\ln(1/\xi)}{1-\tau}.$$

This formula is (16) when $v(c) = \ln(c)$ (corresponding to a coefficient of relative risk aversion $\rho = 1$), $n \approx 1$, and we normalize $m(y) = 0$ at the optimum. (The approximated formula (17) generates comparable estimates.) In the formula, ϵ^m and ϵ^M are the micro- and macroelasticity of unemployment with respect to the consumption gain from work, ϵ^d is the elasticity of search effort with respect to the job-finding rate, $\xi = \hat{\xi} + \epsilon^i \cdot (\tau - \hat{\tau})$ is the consumption drop upon unemployment. As discussed in the text, we set the current replacement rate $\hat{\tau} = 0.54$, the current consumption drop $\hat{\xi} = 0.81$, the marginal consumption effect of unemployment benefits $\epsilon^i = 0.75$.

cle. We assume that all the other parameters remain constant over the business cycle.[12] The optimal replacement rate is countercyclical: it increases sharply from column (3), an expansion, to column (4), a recession. Two other results are interesting. In expansions, when $\epsilon^m/\epsilon^M \approx 1$, the optimal replacement rate is close to the replacement rate from the Baily formula in column (1). Hence, the Baily formula offers an excellent approximation of the optimal replacement rate in expansions. But in recessions, when $\epsilon^m/\epsilon^M = 2.5$, the optimal replacement rate increases by 15 to 20 percentage points. Hence, the Baily formula no longer offers a good approximation of the optimal replacement rate in recessions.

[12] As discussed above, it seems that the microelasticity is fairly constant over the business cycle. But the other parameters could vary over the business cycle. For instance, if the unemployed were more likely to deplete their savings in recessions, the consumption-smoothing benefits of UI would increase in recessions, pushing for countercyclical UI. Kroft and Notowidigdo [2011] find however fairly constant consumption smoothing benefits of UI over the business cycle, as assumed in Table 1. More empirical work on this important issue would be valuable [Chetty and Finkelstein, 2012].

2 A Macroeconomic Model of UI

Available empirical evidence suggest that $\epsilon^m/\epsilon^M > 1$. From a policy perspective, this finding is important because it implies that the Baily formula, conventionally used to calibrate optimal UI, underestimates the optimal replacement rate of UI. This finding is also important from a theoretical perspective because it is not consistent with the behavior of the standard macroeconomic model of UI. Indeed, the standard model is a search-and-matching model with Nash bargaining that predicts $\epsilon^m/\epsilon^M < 1$. To explain the finding that $\epsilon^m/\epsilon^M > 1$, we develop an alternative macroeconomic model of UI by building on the search-and-matching model of Michaillat [2012*a*]. In our model, ϵ^m/ϵ^M is not only above 1 but also countercyclical. In addition, the macroelasticity ϵ^M is procyclical. Applying formula (10) to our model, we show theoretically that the optimal replacement rate of UI is countercyclical.

2.1 The search-and-matching model with Nash bargaining

We begin by analyzing the implications for UI of the search-and-matching model with Nash bargaining.[13] This model is the canonical search-and-matching model and the standard model used by macroeconomists to analyse optimal UI. While the analysis usually abstracts from aggregate shocks, several recent papers use the model to study optimal UI over the business cycle.[14,15] For instance, the model of Mitman and Rabinovich [2011] is closely related to that presented below.[16] To be consistent with Section 1, we confine our analysis to a static model although our results hold in a dynamic model.

Matching frictions. Initially, $u \in (0, 1)$ workers are unemployed and search for a job with effort e. Firms post o vacancies to recruit unemployed workers. The number of matches is given by a constant-returns matching function $h(e \cdot u, o)$ of aggregate search effort $e \cdot u$ and vacancies o, differentiable and increasing in both arguments, with the restriction that $h(e \cdot u, o) \leq u$. Conditions on the labor market are summarized by labor market tightness $\theta \equiv o/(e \cdot u)$. A jobseeker finds a job at a rate $f(\theta) = h(e \cdot u, o)/(e \cdot u) = h(1, \theta)$ per unit of search effort. It is easy for jobseekers to find jobs when the labor market is tight because the job-finding rate $f(\theta)$ increases with θ. A vacancy is filled with probability $q(\theta) = h(e \cdot u, o)/o = h(1/\theta, 1)$. It is difficult for firms to find workers when the labor

[13] See Pissarides [2000] for an overview of search-and-matching models in which wages are determined by Nash bargaining.

[14] Analyses of optimal UI that abstract from aggregate shocks and hence business cycles include Fredriksson and Holmlund [2001], Cahuc and Lehmann [2000], Coles and Masters [2006], and Lentz [2009].

[15] A few papers, such as Kiley [2003], Sanchez [2008], Kroft and Notowidigdo [2011], and Andersen and Svarer [2011], study optimal UI over the business cycle in partial-equilibrium models in which the job-finding rate is a fixed parameter.

[16] Mitman and Rabinovich [2011] go beyond our analysis because they determine jointly the optimal level and the optimal expected duration of unemployment benefits. (Our theory focuses only on the optimal level of benefits.) However, since their analysis is more complex, they do not obtain theoretical results and must rely on simulation results. Another study of optimal UI over the business cycle in a search-and-matching model with Nash bargaining is Moyen and Stähler [2009].

market is tight because the vacancy-filling rate $q(\theta)$ decreases with θ. It costs $r \cdot a$ to post a vacancy, where $r > 0$ measures the resources spent on recruiting by firms and a is the level of technology. We assume away randomness at the firm level: a worker is hired with certainty by opening $1/q(\theta)$ vacancies and spending $r \cdot a/q(\theta)$. When the labor market is tight, $q(\theta)$ is low and recruiting is costly.

Firms. The representative firm takes labor n as input. It produces a consumption good according to the production function $a \cdot g(n) = a \cdot n^{\alpha}$, where $\alpha > 0$ measures the marginal returns to labor and $a > 0$ is the level of technology, which proxies for the position in the business cycle. The wage w is taken as given by the firm. The firm sells its production on a perfectly competitive market. We normalize the price of the good to 1. The firm starts with $1 - u$ workers. Given labor market tightness θ, technology a, and wage w, the firm chooses employment n to maximize real profit

$$\pi = a \cdot g(n) - w \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)] .$$

The first-order condition of this maximization is

$$g'(n) = \frac{w}{a} + \frac{r}{q(\theta)}. \tag{18}$$

The model makes the following assumption on the production function $g(n)$:

ASSUMPTION 1. The production function has constant marginal returns to labor: $\alpha = 1$.

Under this assumption, the first-order condition (18) becomes $1 = (w/a) + r/q(\theta)$.

Wage setting. Wages are set once worker and firm have matched. Since the costs of search are sunk at the time of matching, a surplus arise from each worker-firm match. Any wage sharing the surplus could be an equilibrium wage. The model makes the following assumption on the wage:

ASSUMPTION 2. The wage w is determined using the generalized Nash solution to the bargaining problem faced by firm-worker pairs. The bargaining power of workers is $\beta \in (0, 1)$.

The Nash bargaining solution allocates a fraction β of the surplus of the match to the worker and the rest to the firm. Assume a log utility function: $v(c) = \ln(c)$. Then, as showed in Appendix B, the bargained wage is

$$\frac{w}{a} = -\frac{\beta}{1 - \beta} \cdot \frac{1}{\Delta v} \cdot \frac{r}{q(\theta)}. \tag{19}$$

Equilibrium representation. To illustrate how the job-finding rate $f(\theta)$ responds to a change in UI, we propose a new representation of the labor market equilibrium using a labor supply-labor demand diagram in a price θ-quantity n plan. This labor supply-labor demand diagram is useful to illustrate the impact of a broad range of labor market policies and aggregate shocks. Section 4 uses the diagram to discuss the impact of aggregate demand shocks and shocks to disutility of search.[17]

In the search-and-matching model, the labor supply (4) becomes

$$n^s(f(\theta), \Delta v) = 1 - u + u \cdot e(f(\theta), \Delta v) \cdot f(\theta),$$

which gives the employment rate after matching when jobseekers search optimally for a given labor market tightness θ. The labor supply increases with θ because $f(\theta)$ increase with θ and $e(f, \Delta v)$ increase with f. The labor supply is concave in θ if and only if $(1 - \eta) \cdot (1 + \kappa)/\kappa < 1$, where $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $\kappa \equiv e \cdot k''(e)/k'(e)$.[18]

Combining the expression (19) for the bargained wage with the firm's first-order condition (18) yields the following the labor demand equation:

$$\frac{r}{q(\theta)} = \left[1 + \frac{\beta}{1-\beta} \cdot \frac{1}{\Delta v}\right]^{-1}. \tag{20}$$

This equation defines a perfectly elastic labor demand $\theta^d(\Delta v)$, which depends on the utility gain from work Δv but not on technology. Keeping Δv constant, there are no fluctuations in tightness over the business cycle.[19] In addition, labor demand $\theta^d(\Delta v)$ increases with Δv since $q(\theta)$ decreases with θ. When UI decreases, the outside option of jobseekers decreases, so the bargained wage decreases. As a consequence, it is more profitable for firms to hire workers, and firms are willing to hire workers for a higher labor market tightness.

In presence of matching frictions, labor market tightness acts as a price equilibrating labor supply and labor demand. The wage cannot equilibrate supply and demand because the wage is set only once worker

[17] Michaillat [2012*b*] uses the diagram to discuss the effect of public employment on private employment and total employment over the business cycle.

[18] See Lemma A11 in the Appendix. If jobseekers exert a constant search effort irrespective of labor market tightness ($\kappa = +\infty$), then the labor supply is concave for any parameter values.

[19] In a number of variations of the search-and-matching model with Nash bargaining, Blanchard and Galí [2010] and others also prove that labor market tightness does not fluctuate over the business cycle. In some variations of the model, tightness does fluctuate; but under commonly used parameter calibrations, the fluctuations are tiny compared to the data [Shimer, 2005]. The calibration strategy of Hagedorn and Manovskii [2008], however, generates realistic fluctuations. This is the calibration strategy followed by Mitman and Rabinovich [2011] to study optimal UI over the business cycle.

and firm have met. Here, equilibrium tightness is determined by the perfectly elastic labor demand:

$$\theta(\Delta v) = \theta^d(\Delta v).$$

Equilibrium employment can be directly read off the labor supply curve: $n(\Delta v) = n^s(f(\theta(\Delta v)), \Delta v)$. Figure 1(a) depicts the equilibrium of the search-and-matching model with Nash bargaining.

Wedge between microelasticity and macroelasticity. Proposition 3 establishes the macroelasticity ϵ^M is greater than the microelasticity ϵ^m in the search-and-matching model with Nash bargaining:

PROPOSITION 3. *Under Assumptions 1 and 2,* $\epsilon^m/\epsilon^M < 1$.

Figure 1(a) provides intuition in a price θ-quantity n diagram. When UI falls, jobseekers search more. The labor supply shifts outwards, which increases employment by ϵ^m. In addition when UI falls, jobseekers face a worse outside option. The wage obtained by Nash bargaining falls, which raises labor demand and equilibrium labor market tightness. Employment rises further, and the total increase in employment is measured by ϵ^M. Clearly, $\epsilon^M > \epsilon^m$.

If, as Hall [2005], we abandon the Nash bargaining assumption and assume that wages are rigid instead, then $\epsilon^m = \epsilon^M$. This property is illustrated in Figure 1(b). It arises because labor demand is perfectly elastic and independent of UI, such that equilibrium tightness is independent of UI.

The property that $\epsilon^m < \epsilon^M$ is inconsistent with the empirical evidence discussed in Section 1.4. But the evidence is obtained by analyzing the short-run response of the labor market to a change in UI. If wages take time to adjust to a change in UI, the long-run response of the labor market may be different and ϵ^M could be larger than ϵ^m in the long run. Therefore, while the search-and-matching model with Nash bargaining may not be appropriate to study the effects of UI over the business cycle, it may well provide a good description of the effects of UI in the long run. Indeed the labor supply literature finds that macroelasticities tend to be larger than microelasticities because frictions attenuate labor supply responses in the short run [Chetty, 2012].

2.2 An alternative search-and-matching model

To obtain $\epsilon^m/\epsilon^M > 1$, we develop a macroeconomic model of UI that replaces Assumptions 1 and 2 with the assumptions on the production function and wage schedule made by Michaillat [2012*a*]:

ASSUMPTION 3. The production function has diminishing marginal returns to labor: $\alpha < 1$.

ASSUMPTION 4. The wage schedule is rigid: $w = \omega \cdot a^\gamma$, where $\omega \in (0, +\infty)$ and $\gamma < 1$.

Assumption 3 is motivated by the observation that, at business cycle frequency, some production inputs are slow to adjust. Assumption 4 exploits the indeterminacy, highlighted by Hall [2005], of the equilibrium wage in search-and-matching models. The wage schedule in Assumption 4 borrows from Blanchard and Galí [2010]. Wages are rigid in the sense that (i) they only partially adjust to a change in technology, and (ii) they do not respond to a change in UI. Rigidity (i) is measured by the parameter $\gamma < 1$; if $\gamma = 0$, real wages do not respond to technology and are completely fixed over the cycle. Both rigidities are empirically grounded. First, many historical, ethnographic, and empirical studies document wage rigidity over the business cycle [for example, Bewley, 1999; Jacoby, 1984; Kramarz, 2001]. Second, empirical studies consistently find that reemployment wages do not respond to changes in unemployment benefits [Card, Chetty and Weber, 2007; Schmieder, von Wachter and Bender, 2012*a*].

Equilibrium representation. The labor supply is the same as in the model with Nash bargaining. But the labor demand is different. The first-order condition (18) implicitly defines labor demand $n^d(\theta, a)$. Under Assumption 3, $g'(n)$ decreases with n, q decreases with θ, so labor demand $n^d(\theta, a)$ decreases with θ. When the labor market is tight, it is expensive for firms to recruit, depressing labor demand. Under Assumption 4, w/a decreases with a so $n^d(\theta, a)$ increases with a. When technology is high, wages are relatively low, stimulating labor demand.

Once again, in presence of matching frictions, labor market tightness acts as a price equilibrating labor supply and labor demand. Equilibrium tightness $\theta(\Delta v, a)$ is implicitly defined by

$$n^s(f(\theta), \Delta v) = n^d(\theta, a). \tag{21}$$

Equilibrium employment $n(\Delta v, a)$ is given by the intersection of the labor supply with the labor demand. As in the generic model of Section 1, the equilibrium job-finding rate $f(\theta(\Delta v, a))$ is a function of the utility gain from work Δv.

How does tightness equilibrate labor supply and labor demand? If labor demand is above labor supply, an increase in θ reduces labor demand by increasing the marginal recruiting cost; it increases labor supply by increasing the job-finding rate as well as optimal search effort; until labor supply and labor demand are equalized. In practice, the equilibrium is reached through posting of vacancies. For instance if labor demand is above labor supply at the current tightness, the number of vacancies posted by firms is not sufficient to hire the desired number of workers. Firms post more vacancies, increasing tightness. The job-finding rate rises so more jobseekers find a job and jobseekers search more. We observe a movement along the labor supply curve. At the same time, the vacancy-filling probability falls, hiring costs rise, and the employment desired by firms fall. We observe a movement along the labor demand curve. To conclude, firms close the gap between supply and demand by posting vacancies.

Figures 1(c) and 1(d) represent the equilibrium in our model. The figures plot labor demand curves in expansions (panel (c)) and recessions (panel (d)). They also plot labor supply curves for high and low unemployment benefits (dotted and solid line). Because of diminishing marginal returns to labor (Assumption 3), the labor demand curve is downward sloping. Because of wage rigidity (Assumption 4), the labor demand shifts inward when technology drops between panel (c) and panel (d).

Jobs are rationed in recessions in the sense that the labor market does not clear and some unemployment remains even as unemployed workers exert an arbitrarily large search effort. Job rationing appears Figure 1(d) because labor demand cuts the x-axis for employment strictly below 1. As labor demand intersects the x-axis below full employment, it is unprofitable for firms to hire some workers even if recruiting is costless at $\theta = 0$. Even if workers searched infinitely hard, shifting labor supply outwards such that $\theta \to 0$, firms would never hire all the workers. The mechanism creating job rationing is simple. After a negative technology shock the marginal product of labor falls but rigid wages adjust downwards only partially, so that the labor demand shifts inward (from panel (c) to panel (d)). If the adverse shock is sufficiently large, the marginal product of the least productive workers falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$.

2.3 Wedge between microelasticity and macroelasticity

Formula (10) adds to the Baily formula a second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$. Proposition 4 establishes that $\epsilon^m/\epsilon^M - 1 > 0$ in our model:

PROPOSITION 4. *Under Assumption 4, the elasticity wedge ϵ^m/ϵ^M admits a simple expression:*

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot \frac{1+\kappa}{\kappa} \cdot \frac{q(\theta)}{r} \cdot \frac{h}{n} \cdot n^{\alpha-1},$$

where $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $-\eta \equiv \theta \cdot q'(\theta)/q(\theta) < 0$ and $\kappa \equiv e \cdot k''(e)/k'(e)$. Under Assumption 3, the macroelasticity is strictly smaller than the microelasticity: $\epsilon^m/\epsilon^M > 1$.

Proposition 4, combined with formula (10), justifies the public provision of UI. Small private insurers maximize profits by using the Baily formula to determine how much insurance to provide their clients with. They solely take into account the microelasticity of unemployment and do not internalize search externalities. Since $\epsilon^m/\epsilon^M > 1$, the replacement rate given by the Baily formula is below the optimal replacement rate and small private insurers underprovide UI. Hence, the government would improve welfare by complementing the private provision of UI.

To understand why macroelasticity ϵ^M is smaller than microelasticity ϵ^m, consider the cut in unemployment benefits $d\Delta c > 0$. The change creates variations $d\Delta v > 0$, $d\theta$, df, and dn so that all equilibrium conditions continue to be satisfied. As in the discussion of Lemma 1, we decompose the increase in employment as $dn = dn_{\Delta v} + dn_f$ where $dn_{\Delta v} \equiv u \cdot f \cdot (\partial e/\partial \Delta v)\, d\Delta v$ and $dn_f \equiv [u \cdot e + u \cdot f \cdot (\partial e/\partial f)]\, df$. Jobseekers search more, increasing their job-finding probability at the current job-finding rate by $dn_{\Delta v} > 0$. In Figure 1(c), the cut in benefits shifts the labor supply curve outward and the interval A–C represents $dn_{\Delta v}$. But the job-finding rate $f(\theta)$ does not remain constant. If it did, labor market tightness θ and marginal recruiting cost $r/q(\theta)$ would remain constant. Wages are rigid (Assumption 4) so they respond neither to the change in benefits or to possible changes in equilibrium employment and tightness; therefore, wages remain constant. Consequently, the marginal cost of labor would remain constant. At the same time, firms would need to absorb the $dn_{\Delta v}$ additional jobseekers who would find a job. Since the production function has diminishing marginal returns to labor (Assumption 3), the marginal productivity of these additional workers would be lower. Firms would face the same marginal cost of labor but a lower marginal product of labor. This would not be optimal. Thus, firms reduce the number of vacancies that they post, reducing labor market tightness by $d\theta < 0$ and job-finding rate by $df < 0$. A lower job-finding rate mechanically reduces the number of new hires and also leads jobseekers to search less. The corresponding reduction in employment is $dn_f < 0$, represented by interval C–B in Figure 1(c). The increase $dn > 0$ in equilibrium employment, represented by the interval A–B in Figure 1(c), is therefore smaller than the increase $dn_{\Delta v}$ and $\epsilon^m/\epsilon^M = dn_{\Delta v}/dn > 1$.

2.4 Optimal replacement rate over the business cycle

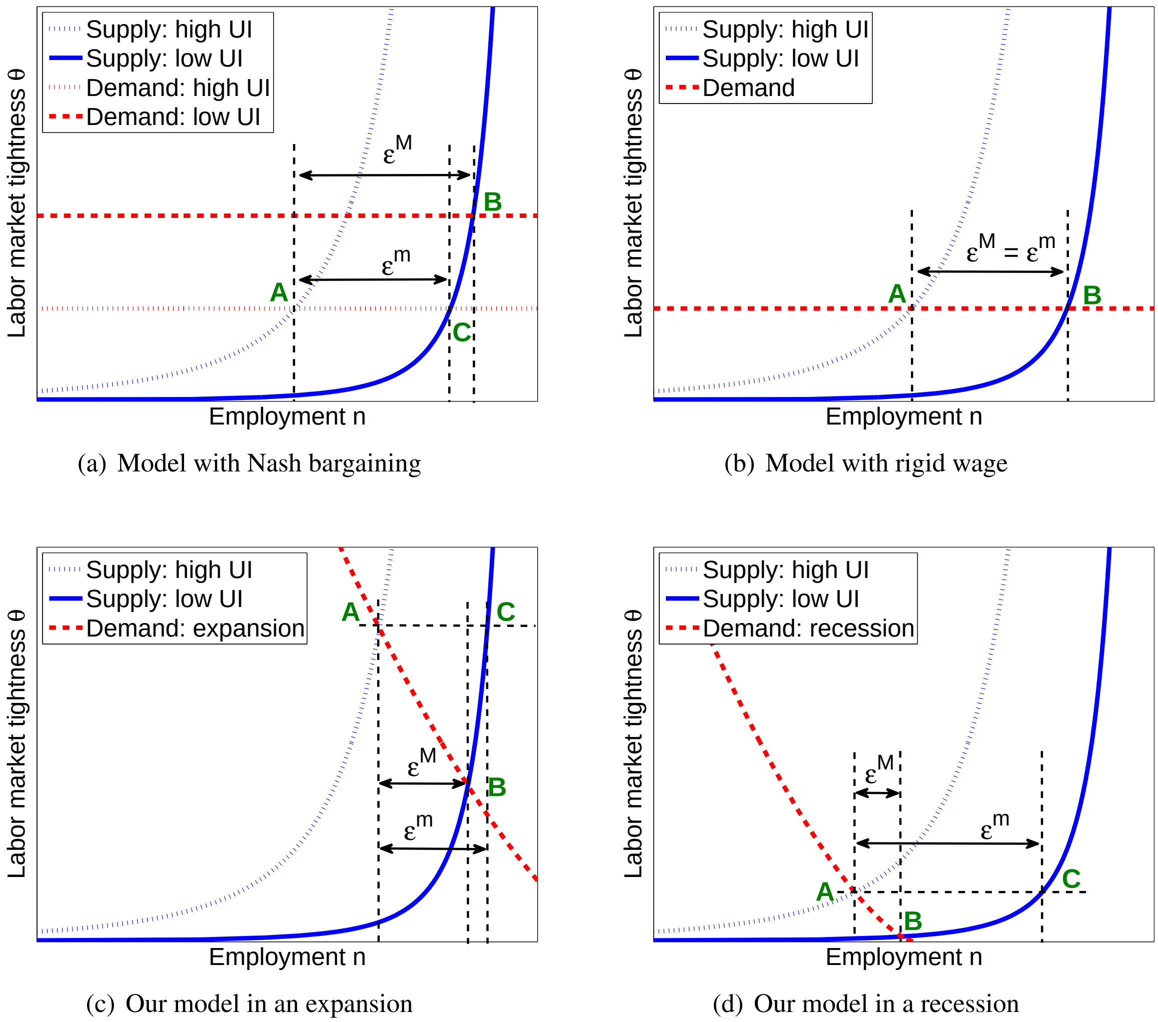
The previous results do not require any assumptions on the functional forms of utility functions and matching function. They only involve the local elasticities η, ρ, and κ. To characterize the cyclicality of the microelasticity, the macroelasticity, and the optimal replacement rate, we make an assumption that controls how the local elasticities fluctuate over the business cycle:

ASSUMPTION 5. The utility functions are isoelastic: $v(c) = \ln(c)$, $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. The matching function is Cobb-Douglas: $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^\eta \cdot o^{1-\eta}$.

The parameters $\omega_k > 0$ and $\omega_h > 0$ measure the cost of search and the effectiveness of matching.

To determine how the elasticities and the optimal replacement rate vary over the business cycle, we must also specify the level of initial unemployment associated with each technology level:

ASSUMPTION 6. For any technology level, initial unemployment u is such that in equilibrium $n - (1-u) = s \cdot n$ for $s \in (0,1)$.



(a) Model with Nash bargaining

(b) Model with rigid wage

(c) Our model in an expansion

(d) Our model in a recession

Figure 1: Equilibrium of various search-and-matching models in a price θ-quantity n diagram

The equilibrium is determined given initial unemployment u and technology a. Assumption 6 ensures that in equilibrium, the fraction $[n - (1 - u)] / n$ of new hires in the workforce is constant over the cycle. The assumption replicates in our static model a feature of dynamic search-and-matching models, which assume a constant job-destruction rate s independent of technology.[20]

Proposition 5 establishes the cyclicality of ϵ^m / ϵ^M and ϵ^M in our model with job rationing:

PROPOSITION 5. *Under Assumptions 3, 4, 5, and 6, the elasticity wedge ϵ^m / ϵ^M is countercyclical*

[20] Pissarides [2000] and many others assume a constant job-destruction rate s and balanced labor market flows. When flows are balanced, firms hire each period as many workers as they lose. Therefore the fraction of new hires in the workforce is constant over the cycle.

and the macroelasticity ϵ^M is procyclical:

$$\left.\frac{\partial\left(\epsilon^m/\epsilon^M\right)}{\partial a}\right|_\tau < 0 \quad \text{and} \quad \left.\frac{\partial \epsilon^M}{\partial a}\right|_\tau > 0.$$

The proposition says that the macroelasticity is larger in expansions than in recessions: in recessions, jobs are acutely rationed and search efforts have little influence on aggregate unemployment. The proposition also says that the wedge between micro- and macroelasticity is smaller in expansions than in recessions: when jobs are acutely rationed, searching more mechanically increases one's job-finding probability but decreases others' job-finding probability as in a rat race.

These results are illustrated in Figure 1. Figure 1(c) represents an expansion. Wages are low compared to technology so the labor demand is high. In equilibrium, labor market tightness is high and unemployment is low. The cut in unemployment benefits $d\Delta c > 0$ discussed after Proposition 4 shifts the labor supply curve outward. At the current job-finding rate, jobseekers search more, increasing their probability of finding a job by $dn_{\Delta v} > 0$, represented by the interval A–C. To avoid absorbing the $dn_{\Delta v}$ additional jobseekers who would find a job if the job-finding rate remained the same, firms reduce the number of vacancies posted by $dv < 0$. Since tightness is high, the matching process is congested by the large number of vacancies; therefore the reduction dv only has a small impact $(\partial h/\partial v)\, dv < 0$ on the number of matches.[21] It appears that vacancies only have a small effect on the number of matches because the labor supply is steep at the equilibrium point. As a result, firms have to reduce drastically the number of vacancies posted to avoid absorbing the additional jobseekers. At the new equilibrium, tightness is much lower but dn is close to $dn_{\Delta v}$. Therefore, ϵ^M is close to ϵ^m. In recessions, technology falls but the wages fall only partially because of wage rigidity (Assumption 4); therefore, the labor demand curve shifts inward from Figure 1(c) to Figure 1(d). In equilibrium, labor market tightness falls and unemployment rises. Since tightness is low, the matching process is congested by the large number of jobseekers. Vacancies have a large effect on the number of matches because the labor supply is flat at the equilibrium point. As a result, firms barely reduce the number of vacancies posted to avoid absorbing the $dn_{\Delta v}$ additional jobseekers. At the new equilibrium, tightness is barely lower but dn is much lower than $dn_{\Delta v}$. Therefore, ϵ^M is much lower than ϵ^m.

Proposition 6 establishes the cyclicality of optimal UI in our model with job rationing:

PROPOSITION 6. *Suppose that formula (10) defines the optimal replacement rate as an implicit function $\tau(a) \in (0,1)$ of technology $a \in (0,+\infty)$. Suppose Assumptions 3, 4, 5, and 6 hold, and that $n > 1/2$ and $(\alpha/\eta)\cdot s\cdot(1-\eta)\cdot(\kappa+1)/\kappa \leq 1$. Then the optimal replacement rate is countercyclical:*

[21] Formally, $(\partial h/\partial v)\, dv = (1-\eta)\cdot q(\theta) dv < 0$ where $q'(\theta) < 0$ because of the congestion effects.

for any $a < a^*$, $\tau(a) > \tau(a^*)$.

The proposition says that the optimal replacement rate is more generous in recessions than in expansions. The formal proof, relegated in the Appendix, exploits the exact optimal UI formula, given by (10). But we can sketch the proof informally using the approximated optimal UI formula, given by (12). Proposition 5 shows that the macroelasticity ϵ^M decreases in recessions. Hence, the first term in (12) increases. In recessions the marginal budgetary cost of UI is small because a higher UI only increases unemployment negligibly. Proposition 5 also shows that the wedge ϵ^m/ϵ^M increases in recessions. Hence, the second term in (12) increases. The wedge measures the welfare cost of a negative rat-race externality imposed by unemployed workers on others. The externality arises because unemployed workers search taking the job-finding rate as given, and do not internalize their influence on the others' job-finding rate. UI corrects the externality by discouraging search. In recessions the externality is acute so the marginal benefits of UI are high. As both terms in formula (12) increase, τ increases.

The formal proof is more complex because n enters formula (10). The results of Proposition 5 are not sufficient to prove the proposition. We need to prove that ϵ^M is sufficiently procyclical and that ϵ^m/ϵ^M is sufficiently countercyclical to compensate the fluctuations in n. To do so, we need two additional assumptions. The assumption $n > 1/2$ is needed because if technology a is so low that most workers become unemployed, it becomes optimal to reduce the replacement rate τ. Suppose all workers are unemployed ($n = 0,\ \theta = 0$). Providing more consumption to employed workers has no budgetary cost but it provides incentives for unemployed workers to search more, which could raise employment. Clearly, it is optimal to reduce the generosity of UI. In fact Lemma A9 in the Appendix establishes that when $a \to 0$ then $n \to 0$ and $\tau \to 0$. Hence, for very low levels of technology and employment, the optimal replacement rate is bound to increase with technology. The assumption that $(\alpha/\eta) \cdot s \cdot (1-\eta) \cdot (\kappa+1)/\kappa \leq 1$ is needed to ensure that the labor supply is convex enough. As shown by comparing Figures 1(c) and 1(d), the convexity of labor supply in the (n, θ) plane drives the cyclicality of elasticities. The assumption is satisfied for reasonable calibrations because s, which stands for a job-destruction rate, is tiny. The calibration of Table 2 implies $(\alpha/\eta) \cdot s \cdot (1-\eta) \cdot (\kappa+1)/\kappa = 0.0025 \ll 1$.

2.5 Quantitative analysis in a dynamic model with partial self-insurance

In this section, we calibrate and simulate a dynamic version of the model just presented to test whether it can realistically capture fluctuations in ϵ^m/ϵ^M and deliver optimal UI predictions consistent with our implementation presented in Table 1. For realism, we cast the static model into a dynamic environment and we add partial self-insurance through home production as we did in Section 1.4. Numerically, we obtain realistic fluctuations in ϵ^m/ϵ^M and we find that optimal UI increases significantly in recessions.

Model. The dynamic model is described in Appendix C. We only provide an overview here. Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. As in Section 1.4, there are job creations and job destructions on the labor market and workers can partially self-insure with home production while unemployed.

Given government policy and job-finding rate $\{c_t^e, c_t^u, f_t\}_{t=0}^{+\infty}$, the representative worker chooses job-search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \{(1-n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) - [1-(1-s)\cdot n_{t-1}] \cdot k(e_t)\},$$

subject to the law of motion for the employment probability n_t^s:

$$n_t^s = (1-s) \cdot n_{t-1}^s + \left[1-(1-s)\cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t).$$

$\mathbb{E}_0$ is the mathematical expectation conditioned on time-0 information, $\delta < 1$ is the discount factor.

The representative firm is owned by a risk-neutral entrepreneur. Given wage, labor market tightness, and technology $\{w_t, \theta_t, a_t\}_{t=0}^{+\infty}$ the firm chooses employment $\left\{n_t^d\right\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[n_t^d - (1-s)\cdot n_{t-1}^d\right]\right\},$$

where $n_t^d - (1-s) \cdot n_{t-1}^d \geq 0$ is the number of hires in period t.

Wages follow an exogenous process $\{w_t\}_{t=0}^{+\infty}$ defined by $w_t = \omega \cdot a_t^\gamma$. Labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\left\{n_t^d\right\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$: $n_t \equiv n_t^d = n_t^s$.

Given technology $\{a_t\}_{t=0}^{+\infty}$, the government chooses consumption $\{c_t^u\}_{t=0}^{+\infty}$ of unemployed workers and consumption $\{c_t^e\}_{t=0}^{+\infty}$ of employed workers to maximize social welfare subject to the period-by-period budget constraint

$$n_t \cdot w_t = n_t \cdot c_t^e + (1-n_t) \cdot c_t^u. \tag{22}$$

Calibration. We calibrate all parameters of the model at a weekly frequency as shown in Table 2.[22] We calibrate as many parameters as possible directly from microevidence and macrodata for the US for the December 2000–June 2010 period. Following Michaillat [2012*a*] we set $\delta = 0.999$, $s = 0.0094$, $r = 0.32 \cdot \omega$. We use a Cobb-Douglas matching function $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^\eta \cdot o^{1-\eta}$ and set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001; Shimer, 2005]. As in Table 1, we set

[22]This exercise is only illustrative of the magnitudes of the optimal policy, because our model abstracts from a number of relevant issues and there remains considerable uncertainty about the calibration of some parameters, such as the coefficient of relative risk aversion.

Table 2: Steady-state targets and parameter values used in simulations (weekly frequency)

	Steady-state target	Value	Source
$\hat{a}$	Technology	1	Normalization
$\hat{e}$	Effort	1	Normalization
$\hat{ls}$	Labor share	0.66	Convention
$\hat{u}$	Unemployment	5.9%	JOLTS, 2000–2010
$\hat{\theta}$	Labor market tightness	0.47	JOLTS, 2000–2010
$\hat{\tau}$	Replacement rate c^u/c^e	54%	Pavoni and Violante [2007], Chetty [2008]
$\hat{\xi}$	Consumption drop c^h/c^e	81%	Hamermesh [1982], Gruber [1997]
ϵ^i	Marginal consumption drop dc^h/dc^u	0.75	Hamermesh [1982], Gruber [1997]
ϵ^s	Elasticity of unemployment hazard rate	0.90	Meyer [1990]
	Parameter	**Value**	**Source**
δ	Discount factor	0.999	Corresponds to 5% annually
ρ	Coefficient of relative risk aversion	1	Chetty [2006b]
η	Unemployment-elasticity of matching	0.7	Petrongolo and Pissarides [2001]
γ	Real wage flexibility	0.5	Pissarides [2009], Haefke, Sonntag and van Rens [2008]
r	Recruiting cost	0.21	Barron, Berger and Black [1997], Silva and Toledo [2009]
s	Job-destruction rate	0.94%	JOLTS, 2000–2010
ω_h	Efficacy of matching	0.19	Matches steady-state targets
α	Marginal returns to labor	0.67	Matches steady-state targets
ω	Steady-state real wage	0.67	Matches steady-state targets
ω_m	Level of home-production cost	11.0	Matches steady-state targets
μ	Convexity of home-production cost	1.01	Matches steady-state targets
ω_k	Level of search disutility	0.20	Matches steady-state targets
κ	Convexity of search disutility	3.15	Matches steady-state targets

$\rho = 1$. We calibrate the wage flexibility γ based on estimates obtained in microdata. The flexibility of wages in newly created jobs mostly drives job creation. The best estimate of this flexibility using US data is provided by Haefke, Sonntag and van Rens [2008]. Using panel data following production and supervisory workers over the 1984–2006 period, they estimate an elasticity of total earnings of job movers with respect to productivity of 0.7. If the composition of jobs accepted by workers improves in expansions, 0.7 is an upper bound on the elasticity of wages in newly created jobs [Gertler and Trigari, 2009]. A lower bound on this elasticity is the elasticity of wages in existing jobs, estimated in the 0.1–0.45 range with US data [Pissarides, 2009]. We set $\gamma = 0.5$, in the range of plausible values.

We calibrate the remaining parameters by matching the steady-state value of variables in the model to the average value of their empirical counterparts. We normalize average technology and average effort to $\hat{a} = 1$ and $\hat{e} = 1$. We compute average labor market tightness using the seasonally-adjusted, monthly series of vacancy levels collected by the Bureau of Labor Statistics (BLS) in the Job Openings and Labor Turnover Survey (JOLTS) and of unemployment levels computed by the BLS from the Current

Population Survey (CPS). For the 2000–2010 period, $\hat{\theta} = \hat{v}/(\hat{e} \cdot \hat{u}) = 0.47$. Similarly, we find $\hat{u} = 5.9\%$, which implies $\hat{n} = 0.950$. As in Table 1, we set $\hat{\tau} = 54\%$.

To calibrate the matching efficacy ω_h we exploit the steady-state relationship $u \cdot e \cdot f(\theta) = s \cdot n = s \cdot (1-u)/(1-s)$. We find $\omega_h = [s/(1-s)] \cdot [(1-\hat{u})/(\hat{e} \cdot \hat{u})] \cdot \hat{\theta}^{\eta-1} = 0.19$. We target the conventional labor share of $\hat{ls} \equiv (\hat{w} \cdot \hat{n})/\hat{n}^{\alpha} = 0.66$. The firm's profit-maximization condition (equation (A14) in the Appendix) implies $\alpha = \hat{ls} \cdot \left([1 - \delta \cdot (1-s)] \cdot 0.32/q(\hat{\theta}) + 1\right) = 0.67$. The condition also allows us to recover $\omega = 0.67$, and $r = 0.32 \cdot \omega = 0.21$.

Next, we calibrate the home-production parameters $m(y) = \omega_m \cdot (y^{1+\mu} - \hat{y}^{1+\mu})/(1+\mu)$. Equation (A32) in Appendix C shows that μ is related to the statistics ϵ^i and ξ introduced earlier. As in Table 1, we set $\epsilon^i = 0.75$ and $\xi = 0.81$, which implies $\mu = 1.01$. The budget constraint (22) yields average home production $\hat{y} = 0.17$ and average unemployment consumption $\hat{c}^h = 0.53$. We set $\omega_m = 11.0$ for the worker's optimal choice of home production (equation (A17) in appendix) to hold for $\hat{c}^h$ and $\hat{y}$.

Finally, we calibrate the parameters of the search disutility $k(e) = \omega_k \cdot (e^{1+\kappa} - 1)/(1+\kappa)$. Equation (A34) in Appendix C shows that κ is related to the statistics ϵ^s and ξ introduced earlier. We follow Gruber [1997] and use the estimate $\epsilon^s = 0.9$ obtained by Meyer [1990]. We obtain $\kappa = 3.15$. We set $\omega_k = 0.20$ for the worker's optimal choice of effort (equation (A18) in the Appendix) to hold for $\hat{e} = 1$.

Simulations. To describe how the optimal replacement rate varies over the business cycle, we compare steady states parameterized by different technology levels.[23] The results are displayed in Figure 2. The figure shows that unemployment is higher in steady states with lower technology. It also shows that labor market tightness decreases with unemployment, shaping a Beveridge curve. Search efforts decrease in recessions, when UI becomes more generous and the job-finding rate falls. We obtain realistic fluctuations in unemployment for fairly modest changes in technology.

The figure also shows that when the unemployment rate increases from 4% to 10%, the microelasticity ϵ^m increases slightly from 0.8 to 1, whereas the macroelasticity ϵ^M decreases sharply from 0.6 to 0.2. Thus, the wedge ϵ^m/ϵ^M increases drastically from 1.3 to 5 when the unemployment rate increases from 4% to 10%. Since these numbers roughly have the same magnitude as those in Table 1, the calibrated model can deliver realistic variations of ϵ^m/ϵ^M over the business cycle. Finally, we find that optimal UI is strongly countercyclical. The optimal replacement rate increases from 45% to 59% when the unemployment rate increases from 4% to 10%. These replacement rates have the same magnitude as those in Table 1. In fact we find that in recessions, it is optimal to increase benefits not only relative to the consumption of employed workers, but also in absolute terms.

[23] In steady state, technology remains constant over time: $a_t = a$ for all t.

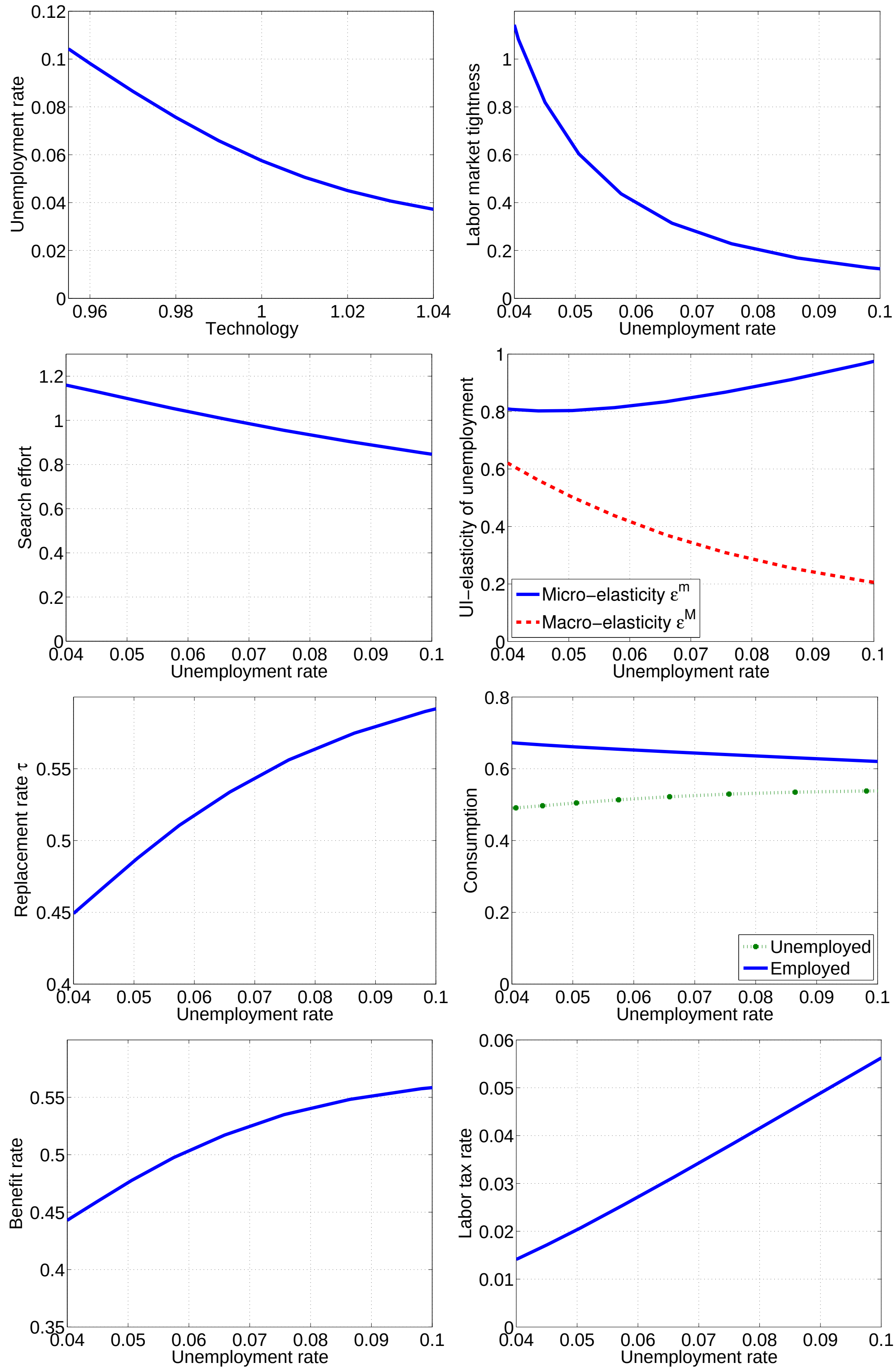


Figure 2: Elasticities and optimal unemployment insurance over the business cycle

Notes: Panels obtained by simulating the dynamic model with partial self-insurance. The model is calibrated in Table 2. The simulations are described in Appendix C. The elasticities ϵ^m and ϵ^M are given by equations (A36) and (A37) in Appendix C.

3 Taxation of Profits and Connection with Baily and Hosios

In Section 1, we derived a formula for optimal UI under the assumptions that profits are not redistributed to workers, and that they cannot be taxed by the government. We showed that our formula reduces to the standard Baily [1978] formula when microelasticity equals macroelasticity. In this section, we derive an alternative formula for optimal UI when profits can be taxed fully by the government. Equivalently, the formula applies if profits are uniformly distributed to workers.

Are these realistic assumptions? If profits cannot be fully taxed and firms are owned by foreigners or if ownership is very concentrated such that profits are not uniformly distributed, those are not good assumptions. Empirically, profits are negligible in the income of unemployed workers.[24] Therefore, the model without profit taxation that we studied in Sections 1 and 2 is the most relevant in practice.

Even though the assumption that the government is able to tax profits may not be realistic, this assumption is standard in the macroeconomic literature. Hence, it offers a useful theoretical benchmark which allows us to connect our theory of optimal UI with the standard Baily [1978] formula for optimal UI and the Hosios [1990] condition for efficiency in search-and-matching models. With profits taxation, the formula for optimal UI reduces to the standard Baily formula for optimal UI when a generalized Hosios condition holds. Using an approximation of the formula, we show that the result that optimal UI is countercyclical continue to hold when government can tax profits.

To establish the connection, we need more structure on the model than in Section 1 so we work with the search-and-matching model of Section 2. We assume that the government uses profits to finance the UI system. The government's budget constraint is $(1-n) \cdot B \cdot w = n \cdot T \cdot w + \pi$ where $\pi = a \cdot g(n) - [r \cdot a/q(\theta)] \cdot h - w \cdot n$ are the firm's profits. Equivalently, the budget constraint is

$$(1-n) \cdot c^u + n \cdot c^e = a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)]. \tag{23}$$

3.1 Jointly optimal unemployment insurance and labor market tightness

We begin by assuming that the government chooses not only the consumption c^e of employed workers and the consumption gain from work Δc, but also labor market tightness θ. The assumption that the government can control θ may not be realistic, but the results obtained under this assumption are a useful building block for the subsequent analysis.[25]

[24] We analyzed individual income tax statistics for 2004. While individuals and families reporting positive unemployment benefits had an average income (Adjusted Gross Income) equal to 78% of the population average, they had a capital income (the sum of interest payments, dividends, and realized capital gains) equal to only 17% of the population average.

[25] In our framework, wages paid by firms and tightness are directly related by the labor demand equation (18). If the government could control wages, then it could control tightness directly. But it seems unlikely that the government could

By choosing c^e, Δc, and θ, the government maximizes welfare (6) subject to the budget constraint (23), optimal search by jobseekers, and the definition of equilibrium employment $n = n^s(f(\theta), \Delta c, c^e)$. Proposition 7 characterizes the jointly optimal UI and tightness:

PROPOSITION 7. *Optimal tightness θ, optimal consumptions c^e and c^u, and Lagrange multiplier ϕ on the resource constraint satisfy the inverse Euler equation* (11)*, the Baily formula*

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right], \tag{24}$$

and the generalized Hosios condition

$$\frac{\Delta v}{\phi} + (w - \Delta c) \cdot \left(1 + \epsilon^d\right) - \frac{\eta}{1 - \eta} \cdot \frac{r \cdot a}{q(\theta)} = 0, \tag{25}$$

where we define the implicit wage w as $w \equiv a \cdot g'(n) - r \cdot a/q(\theta)$.

The Baily formula (24) applies here because the job-finding rate $f(\theta)$ is kept constant when choosing the optimal Δc; therefore, the effects that arise from the response of the job-finding rate in formula (10) disappear. The generalized Hosios condition (25) determines the optimal labor market tightness θ. A marginal increase in θ has two effects. First, it increases the job-finding rate $f(\theta)$, which increases employment through (1). The employment increase leads to a welfare gain proportional to Δv (first term of (25)) and to a budget gain proportional to $w - \Delta c$ (second term of (25)). Second, it reduces profits by increasing hiring costs $r \cdot a/q(\theta)$. The profit reduction creates a budget loss proportional to hiring costs (third term of (25)). At the optimum, the marginal benefits from the increase in θ (first and second terms of (25)) equal the marginal cost (third term of (25)).

If workers are risk neutral, the Baily formula implies that it is optimal to set $w = \Delta c$ and provide no UI. In that case, equation (25) simplifies to $\Delta v/\phi = [\eta/(1 - \eta)] \cdot r \cdot a/q(\theta)$. Risk neutrality also implies $\phi = 1$ and $\Delta v = w$. Hence, (25) further simplifies to $w = [\eta/(1 - \eta)] \cdot r \cdot a/q(\theta)$, the standard Hosios condition for efficiency in a search-and-matching model. Therefore, we interpret (25) as a *generalized Hosios condition* that determines the optimal wage level in presence of risk aversion and optimal UI.

3.2 Connection between Baily formula and Hosios condition

We assume again, as in Sections 1 and 2, that the government cannot control labor market tightness, which is determined endogenously to equilibrate labor supply and labor demand. But unlike in Sections 1

do so. See Okun [1981] for a useful discussion of this point from a macroeconomic perspective. In public economics, the standard assumption is that the incidence of payroll taxes is fully on workers; therefore payroll taxes are ineffective to manipulate the wage effectively paid by firms. See our discussion in section 4.3 below where we relax this assumption.

and 2, the government can tax profits and use them to finance UI. Hence, the government faces the same problem as in Sections 1 and 2 except that the budget constraint is given by (23) instead of (7). Proposition 8 provides the formula for optimal UI when the government taxes profits:

PROPOSITION 8. *We characterize the optimal unemployment insurance with profit taxation.*

(i) The optimal consumptions c^e and c^u satisfy the inverse Euler equation (11) *and the formula*

$$\frac{w-\Delta c}{\Delta c} = \frac{n}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m}\right) \cdot \left[\frac{\Delta v}{\phi} + (w-\Delta c) \cdot \left(1+\epsilon^d\right) - \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}\right]. \tag{26}$$

(ii) A formula equivalent to (26) *is*

$$\frac{w-\Delta c}{\Delta c} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) + a \cdot g''(n) \cdot \frac{h}{\Delta c}. \tag{27}$$

Part (i) shows that the optimal replacement rate $\tau/(1-\tau)$ is the sum of an insurance term, which is the term in the standard Baily formula (24), and an externality-correction term, which is proportional to the deviation from the generalized Hosios condition (25). When the generalized Hosios condition holds, the externality-correction term vanishes and the optimal replacement rate is given by the Baily formula. When wages are too high (and equivalently tightness is too low) relative to the generalized Hosios condition, the deviation from the generalized Hosios condition is positive. Since $\epsilon^m/\epsilon^M > 1$, the externality-correction term is positive. Therefore, optimal UI is more generous than in the Baily formula.

The optimal replacement rate is the sum of an insurance term and an externality-correction term because the derivative of the Lagrangian $\mathcal{L}$ of the government's problem with respect to Δc can be decomposed as $\partial\mathcal{L}/\partial\Delta c\big|_{c^e} = \partial\mathcal{L}/\partial\Delta c\big|_{\theta,c^e} + \partial\mathcal{L}/\partial\theta\big|_{\Delta c,c^e} \cdot \partial\theta/\partial\Delta c\big|_{c^e}$. Therefore, the first-order condition $\partial\mathcal{L}/\partial\Delta c\big|_{c^e} = 0$ in the current problem is a linear combination of the first-order conditions $\partial\mathcal{L}/\partial\Delta c\big|_{\theta,c^e} = 0$ and $\partial\mathcal{L}/\partial\theta\big|_{\Delta c,c^e} = 0$ in the joint optimization problem of section 3.1. Hence, the optimal formula is also a linear combination of the Baily formula and the generalized Hosios condition. Moreover, the generalized Hosios condition is multiplied by the elasticity wedge $1-(\epsilon^M/\epsilon^m)$ because the factor $\partial\theta/\partial\Delta c\big|_{c^e}$ is proportional to the wedge from Lemma 1.

Part (ii) shows that the optimal UI formula is the same as formula (10) except for the addition of a last term $a \cdot g''(n) \cdot h/\Delta c$. (Lemma A1 in the Appendix shows that $(1/n) \cdot \tau/(1-\tau) = (w-\Delta c)/\Delta c$ when the government cannot tax profits.) The last term reflects the negative impact of UI on marginal profits: an increase in UI leads to an increase in tightness that increases hiring costs and reduces profits. As the last term is negative, the optimal $(w-\Delta c)/\Delta c$ is lower than in (10).

The formula proposed by Part (ii) is quite general. In particular, it remains valid even if wages respond to the utility gain from work Δv. The reason is that when the government taxes profits, the wage does not appear in the government's budget constraint so it does not appear at all in the government's problem.[26] The influence of Δv on wages and equilibrium employment is simply captured by the macroelasticity ϵ^M. Hence this formula applies in a search-and-matching model with Nash bargaining, such as that used by Mitman and Rabinovich [2011].[27]

3.3 Optimal replacement rate over the business cycle

Proposition 9 provides an approximated formula for optimal UI when the government taxes profits:

PROPOSITION 9. *We characterize the optimal unemployment insurance with profit taxation.*

(i) If $n \approx 1$, $u \ll 1$, and the third and higher order terms of $v(\cdot)$ are small, (27) simplifies to

$$\frac{w - \Delta c}{\Delta c} \approx \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right]. \tag{28}$$

(ii) If $r \ll 1$, $(w - \Delta c)/\Delta c \approx \alpha \cdot \tau/(1-\tau) - (1-\alpha)$. Therefore if $n \approx 1$, $u \ll 1$, and $r \ll 1$, then $d\tau/da < 0$ and $d\left[1 - (\Delta c/w)\right]/da < 0$.

Part (i) proposes an approximated formula that links the optimal implicit tax on work $1 - (\Delta c/w)$ to the usual sufficient statistics. The right-hand-side of the approximated formula is exactly the same as in formula (12). The reason is that if $u \ll 1$, then $h \leq u \ll 1$ and the additional marginal-profit term is quantitatively negligible relative to the other terms. The left-hand-side of the approximated formula is different from that in (12) because of profit taxation. Indeed profits π are assumed to be taxed and equally redistributed, which increases both c^u and c^e by the same amount π and introduces a wedge between the replacement rate $\tau = c^u/c^e$ and the implicit tax on work $1 - (\Delta c/w)$. More precisely with no profits, the budget constraint (7) can be written as $c^u + n \cdot \Delta c = w \cdot n$, which implies that $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c$. With profits equally redistributed, the budget constraint becomes $c^u + n \cdot \Delta c = w \cdot n + \pi$, which implies that $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c + \pi/(n \cdot \Delta c)$. Lemma A1 in the Appendix shows that if $r \ll 1$ and $n \simeq 1$, $\alpha \cdot \tau/(1-\tau) \approx (1-\alpha) + (w - \Delta c)/\Delta c$. The effect of profit taxation on consumption levels is conceptually similar to the effect of self-insurance. If profits were equally distributed, they would constitute a cushion against unemployment, making τ higher in the

[26] In contrast, the wage does appear in the government's budget constraint (7) when the government cannot tax profits.

[27] It is conceivable that, in the long-run, wages respond to UI as in the Nash-bargaining model, leading to a higher wedge ϵ^m/ϵ^M. If this effect is acyclical, it would affect the level of optimal UI but not its cyclicality. Simulations incorporating both wage responses to UI (as in Mitman and Rabinovich [2011]) and wage rigidity (as in our model) are left for future research.

right-hand-side of (28) and requiring a smaller UI program as measured by the wedge on reward to work $(w - \Delta c)/\Delta c$. This implication of profit taxation on UI is misleading for practical implementation if profits are not equally distributed among workers.

Part (ii) shows that for small u and r, although its level is different, the optimal replacement rate τ remains decreasing with technology a in the case with profit taxation. This in turn implies that the implicit tax on work $1 - (\Delta c/w)$ is also countercylical. That is, both the optimal replacement rate and the optimal implicit tax on work increase in recessions when technology falls.

4 Robustness of the Countercyclicality of Optimal UI

This section examines the robustness of the result that optimal UI is countercyclical. It shows that optimal UI remains countercyclical (i) when the government adjusts the duration of unemployment benefits instead of their level; (ii) when business cycles are driven by aggregate demand shocks instead of technology shocks; and (iii) when the government uses wage subsidies to attenuate employment fluctuations.

4.1 Duration of unemployment benefits

In the baseline dynamic model, unemployment benefits never expire. In this section, unemployment benefits have a finite duration that the government adjusts over the cycle.[28] This model is more realistic because in practice, benefits have finite duration and the government modulates benefit duration over the cycle.[29] We follow Fredriksson and Holmlund [2001] and Mitman and Rabinovich [2011] and assume that eligible unemployed workers exhaust their benefits c_t^u with probability λ_t at the end of each period t. Ineligible unemployed workers receive social assistance $c_t^a < c_t^u$ until they find a job.

The replacement rates $\tau^{u,e} = c_t^u/c_t^e$ of unemployment benefits and $\tau^{a,e} = c_t^a/c_t^e$ of social assistance are constant over time. The government chooses the rate λ_t at which eligible workers become ineligible to maximize welfare subject to a budget constraint similar to (22). We solve the model numerically using the calibration in Table 2. We set the replacement rates at $\tau^{u,e} = 57\%$ and $\tau^{a,e} = 0.52 \cdot \tau^{u,e} = 30\%$ such that an expected duration of 26 weeks is optimal at the average unemployment rate of 5.9%.[30] The

[28] This section only provides an overview of the model, whose formal description and analysis is in Appendix D.1.

[29] US unemployment benefits have a maximum duration of 26 weeks in normal times. Under the Extended Benefits program, duration is extended by 13 weeks in states where unemployment is above 6.5% and by 20 weeks in states where unemployment is above 8%. Often, duration is further extended in severe recessions. For example in 2008, the Emergency Unemployment Compensation program further extended durations by 53 weeks when state unemployment was above 8.5%.

[30] We assume that social assistance only provides food stamps. According to Pavoni and Violante [2007], in the United States, the median monthly allotment of food stamps for a family of four was \$397 per month in 1996, and the median monthly wage for a worker with at most a high-school diploma was \$1,540. Thus the rate of social assistance is $397/1,540 = 26\%$. As the rate of unemployment benefits is 50%, $\tau^{a,e}/\tau^{u,e} = 0.26/0.5 = 0.52$.

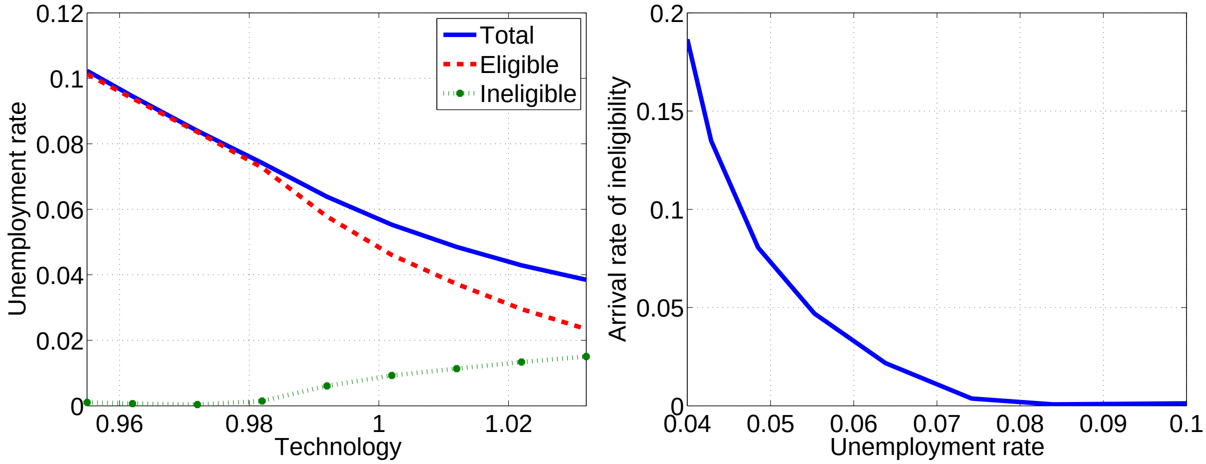


Figure 3: Optimal duration of unemployment insurance over the business cycle

Notes: Panels obtained with the dynamic model in which unemployment benefits have finite duration. The model is calibrated in Table 2. Appendix C describes the numerical simulations.

left panel in Figure 3 shows how unemployment and its composition varies with technology. When technology increases, total unemployment falls, the number of eligible jobseekers falls, but the number of ineligible jobseekers increases because the arrival rate of ineligibility increases drastically. The right panel shows that the optimal arrival rate of ineligibility λ is strongly procyclical. Accordingly the optimal expected duration of unemployment benefits $1/\lambda$ is strongly countercyclical. When unemployment is 4%, the optimal arrival rate of ineligibility is 18%, corresponding to an expected benefit duration of less than 6 weeks. When unemployment reaches 5.9% the optimal arrival rate falls to 3.9%, corresponding to an expected benefit duration of 26 weeks. When unemployment reaches 8.0%, the optimal arrival rate drops below 1.0%, corresponding to an expected benefit duration of 100 weeks. The optimal arrival rate is virtually zero when unemployment is above 9%.

4.2 Aggregate demand shocks

A limitation that the model shares with most search-and-matching models is that business cycles are generated by technology shocks only. This is implausible: aggregate demand shocks likely contribute to labor market fluctuations. To study optimal UI in a demand-generated business cycle, Appendix D.2 builds a simple model with nominal wage rigidity in which recessions are driven by aggregate demand shocks. Jobs are rationed in that model as well. Firms face a downward-sloping aggregate demand curve in the goods market. The larger the quantity produced by workers, the lower the market price for goods. When aggregate demand is low enough, the production of workers would sell at a price below the nominal wage if all workers were employed. In this situation, firms would not hire all workers in the labor force and some unemployment would remain even if recruiting were costless: jobs are rationed.

As showed by Figure A1 in the Appendix, we can use our labor supply-labor demand diagram to represent the labor market equilibrium in the model with demand-generated business cycles. The labor supply is the same, and the labor demand has the same properties, as in the model with technology-generated business cycles. The labor demand curve is downward sloping in the price θ–quantity n plane because higher employment n implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and it requires a lower tightness θ for firms to be willing to hire. When aggregate demand falls, prices fall and real wages rise, so the labor demand curve shifts inwards.

Since the labor market equilibria have a similar structure, it is not surprising that the results obtained in the model with technology-generated business cycles carry over to the model with demand-generated business cycles. The cyclicality of the wedge ϵ^m/ϵ^M and the cyclicality of the macroelasticity (Proposition 5), as well as the cyclicality of the optimal replacement rate (Proposition 6) remain the same once derivatives are taken with respect to aggregate demand instead of technology. Hence, optimal UI is more generous in recessions caused by low aggregate demand.

Other macroeconomic shocks could drive recessions. In Appendix D.3, we consider a preference shock that affects job-search disutility in the model. When it is unpleasant for unemployed workers to search, a recession arises because jobseekers reduce their effort, reducing labor supply and increasing unemployment. This shock can easily be represented using the labor supply-labor demand diagram. Unsurprisingly, simulations show that optimal UI is procyclical when business cycles are driven by preference shocks. However, this model is unrealistic because it has the property that labor market tightness is countercyclical, at odds with empirical evidence.[31]

4.3 Unemployment insurance combined with wage subsidies

Because of real wage rigidity, wages are high relative to technology in recessions, which raises unemployment. If the government could perfectly control wages paid by firms, it could completely eliminate unemployment fluctuations. As discussed in Section 3, it seems unlikely that the government be able to control wages at no cost. But changing the payroll tax imposed on employers to alter the wages effectively paid by firms, even if it is costly, could attenuate unemployment fluctuations. In this section we show that our results remain valid when the government attenuates unemployment fluctuations using wage subsidies. The formal proof is in Appendix D.4.

The government chooses unemployment benefit rate B, tax rate T imposed on the salary w^* received by employees, and a subsidy rate σ imposed on the salary w^* paid by employers. Firms pay a wage

[31] In future work, it would be valuable to consider fluctuations arising from shocks to recruiting costs r, which could proxy recessions arising from mis-match due to industrial shifts.

$w = (1-\sigma) \cdot w^*$, employed workers consume $c^e = (1-T) \cdot w^*$, and unemployed workers consume $c^u = B \cdot w^*$. Equivalently, we consider that the government chooses directly the wage w and consumptions c^e, and c^u. The government is subject to the budget constraint $(1-n) \cdot B \cdot w^* + n \cdot \sigma \cdot w^* = t \cdot n \cdot w^*$. This constraint can be rewritten exactly as the baseline budget constraint (7).

If wage subsidies were costless, it would be optimal to eliminate unemployment fluctuations and keep UI at a constant level. However, it is improbable that the government could implement a wage subsidy at no cost.[32] Formally, we represent these costs as an increasing convex cost function $\mathcal{C}(\sigma)$ included in the objective function of the government. In that case, it is not optimal to eliminate entirely cyclical fluctuations in unemployment because of the cost $\mathcal{C}(\sigma)$. Let w be the optimal wage chosen by the government given the cost of the subsidy.

Given w the government chooses Δc to maximize social welfare (6) subject to the budget constraint (7). This is exactly the problem faced by the government in the baseline model. Therefore the optimal UI formula (10) remains valid. Let $\tilde{w} \equiv w/a$ be the optimal wage w normalized by technology a. $\tilde{w}$ is the only source of fluctuations in the economy through the firm's profit-maximization condition (18). Since the government cannot stabilize unemployment completely, $\tilde{w}$ must fluctuate. Once we replace the derivatives with respect to a by derivatives with respect to $\tilde{w}$, the results on the cyclicality of the elasticities ϵ^m and ϵ^M (Proposition 5) and the result on the cyclicality of the optimal replacement rate (Proposition 6) remain valid. The sign of the derivatives naturally changes because an increase in $\tilde{w}$ has the same effect as a decrease in a: it raises unemployment and reduces labor market tightness. To conclude, the properties of optimal UI that we derive are robust to the presence of a wage subsidy that attenuates unemployment fluctuations, but does not fully eliminate them.

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[32]Possible sources of cost include informational frictions, political constraints (for instance, trade unions may resist the reduction of the labor cost incurred by firms), and adverse aggregate demand effects (shifting payroll taxes from employers to workers may reduce aggregate demand if workers have a higher marginal propensity to consume than firm owners).

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Appendix—For Online Publication

A Proofs

We begin by deriving a few preliminary results.

LEMMA A1.

(i) If the budget constraint faced by the government is (no taxation of profits)

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w,$$

then

$$\frac{w - \Delta c}{\Delta c} = \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

(ii) If the budget constraint faced by the government is (full taxation of profits)

$$n \cdot c^e + (1-n) \cdot c^u = a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)],$$

then

$$\alpha \cdot \frac{1}{n} \cdot \frac{\tau}{1-\tau} = (1-\alpha) + \frac{w - \Delta c}{\Delta c} + \left(1 - \alpha \cdot \frac{h}{n}\right) \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c}.$$

If $n \approx 1$ *and* $r \ll 1$*, then*

$$\alpha \cdot \frac{\tau}{1-\tau} \approx (1-\alpha) + \frac{w - \Delta c}{\Delta c}.$$

Proof. First, we prove Part (i). Recall that $\tau = c^u/c^e$ and $\Delta c = c^e - c^u$. The budget constraint implies that

$$c^u = n \cdot [w - \Delta c]$$
$$\frac{1}{n} \cdot \frac{c^u}{c^e - c^u} = \frac{w - \Delta c}{\Delta c}$$
$$\frac{1}{n} \cdot \frac{[c^u/c^e]}{1 - [c^u/c^e]} = \frac{w - \Delta c}{\Delta c}$$
$$\frac{w - \Delta c}{\Delta c} = \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

Second, we prove Part (ii). By definition of profits,

$$\pi = a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot h - n \cdot w,$$

Therefore, the budget constraint implies that

$$c^u = n \cdot (w - \Delta c) + \pi$$
$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{w - \Delta c}{\Delta c} + \frac{\pi}{n \cdot \Delta c}. \tag{A1}$$

We determine $\pi/(n \cdot \Delta c)$. The production function is isoelastic so $a \cdot g(n)/n = (1/\alpha) \cdot a \cdot g'(n)$. Furthermore, $a \cdot g'(n) = w + r \cdot a/q(\theta)$. So we use the definition of profits to write

$$\frac{\pi}{n} = \frac{1-\alpha}{\alpha} \cdot w + \left[\frac{1}{\alpha} - \frac{h}{n}\right] \cdot \frac{r \cdot a}{q(\theta)}$$

$$\frac{\pi}{\Delta c \cdot n} = \frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\alpha} \cdot \frac{w - \Delta c}{\Delta c} + \left[\frac{1}{\alpha} - \frac{h}{n}\right] \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c}. \tag{A2}$$

The result of the lemma follows from (A1) and (A2). □

LEMMA A2. *The derivatives of effort supply* $e(f, \Delta v)$ *satisfy*

$$\epsilon^d = \frac{f}{e} \cdot \left.\frac{\partial e}{\partial f}\right|_{\Delta v} = \frac{1}{\kappa}$$

$$\frac{\Delta v}{e} \cdot \left.\frac{\partial e}{\partial \Delta v}\right|_{f} = \frac{1}{\kappa}.$$

Proof. Obvious because the effort supply satisfies $k'(e) = f \cdot \Delta v$ and κ is the elasticity of $k'(e)$: $\kappa \equiv e \cdot [k''(e)/k'(e)]$. □

LEMMA A3. *The derivatives of the utility gain from work* $\Delta v(\Delta c, c^e)$ *satisfy*

$$\left.\frac{\partial \Delta v}{\partial \Delta c}\right|_{c^e} = v'(c^u)$$

$$\left.\frac{\partial \Delta v}{\partial c^e}\right|_{\Delta c} = v'(c^e) - v'(c^u).$$

Proof. Obvious because $\Delta v(\Delta c, c^e) = v(c^e) - v(c^e - \Delta c)$. □

A.1 Proof of Lemma 1

Using the definitions of ϵ^M and ϵ^m, we write

$$\epsilon^M = \frac{\Delta c}{1-n} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot n'(\Delta v)$$

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[u \cdot f \cdot \frac{\partial e}{\partial \Delta v}\right].$$

The expression of ϵ^m arises because $n^s(f, \Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f$, which implies that

$$\frac{\partial n^s}{\partial \Delta v} = u \cdot f \cdot \frac{\partial e}{\partial \Delta v}.$$

The employment rate is $n(\Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f(\Delta v)$. Using the definition of the discouraged-worker elasticity ϵ^d, we obtain

$$n'(\Delta v) = u \cdot \left[\frac{\partial e}{\partial \Delta v} \cdot f + \frac{\partial e}{\partial f} \cdot f'(\Delta v) \cdot f + e \cdot f'(\Delta v)\right] = u \cdot f \cdot \frac{\partial e}{\partial \Delta v} + u \cdot e \cdot f'(\Delta v) \cdot \left[\epsilon^d + 1\right].$$

Therefore, ϵ^M and ϵ^m are related by

$$\epsilon^M = \epsilon^m + \frac{u \cdot e}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \Delta c \cdot f'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c},$$

which is the result of Lemma 1 once we abuse notations by denoting $\partial f / \partial \Delta c \equiv f'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c)$.

A.2 Proof of Proposition 1

The Lagrangian of the government's problem is

$$\mathcal{L}(\Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\Delta v)) \cdot \Delta v - u \cdot k(e) + \phi \cdot [n(\Delta v) \cdot (w - \Delta c) - (c^e - \Delta c)],$$

where ϕ is the Lagrange multiplier on the budget constraint. Effort e is chosen by workers to maximize $e \cdot f(\Delta v) \cdot \Delta v - k(e)$, so we apply the envelope theorem. The first-order condition with respect to Δc is

$$-(1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial \Delta c} \cdot [\Delta v \cdot u \cdot e \cdot f'(\Delta v) + \phi \cdot n'(\Delta v) \cdot (w - \Delta c)] + \phi \cdot (1-n) = 0.$$

The first-order condition with respect to c^e is

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial c^e} \cdot [\Delta v \cdot u \cdot e \cdot f'(\Delta v) + \phi \cdot n'(\Delta v) \cdot (w - \Delta c)] - \phi = 0.$$

Lemma A3 implies $\partial \Delta v / \partial c^e = -\left[1 - (v'(c^e)/v'(c^u))\right] \cdot \partial \Delta v / \partial \Delta c$. We multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We obtain the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right].$$

We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$, rearrange the terms, and abuse notations as in the text by denoting $\partial f / \partial \Delta c \equiv f'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c)$ and $\partial n / \partial \Delta c \equiv n'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c)$. We obtain

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} \cdot (w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right] - \frac{1}{\phi} \cdot \frac{\Delta v}{1-n} \cdot u \cdot e \cdot \frac{\partial f}{\partial \Delta c}. \tag{A3}$$

The inverse Euler equation yields

$$\frac{v'(c^u)}{\phi} - 1 = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Lemma 1 implies

$$-\frac{\Delta v}{1-n} \cdot u \cdot e \cdot \frac{\partial f}{\partial \Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

By definition,

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^M.$$

Therefore we can rewrite (A3) as

$$\frac{w-\Delta c}{\Delta c}\cdot\epsilon^M = n\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\epsilon^m-\epsilon^M\right).$$

Using part (i) of Lemma A1 and dividing this equation by ϵ^M yields the formula

$$\frac{1}{n}\cdot\frac{\tau}{1-\tau}=\frac{n}{\epsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right).$$

Approximation. Assuming $n\approx 1$, we simplify the formula to

$$\frac{\tau}{1-\tau}\approx\frac{1}{\epsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{v'(c^e)}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right).$$

If the third and higher order terms of v are small ($v'''(c)\approx 0$), we approximate

$$\begin{aligned}\frac{\Delta v}{v'(c^e)\cdot\Delta c}&\approx 1-\frac{1}{2}\cdot\frac{v''(c^e)}{v'(c^e)}\cdot\frac{c^e}{c^e}\cdot[c^e-c^u]=1+\frac{1}{2}\cdot\rho\cdot(1-\tau)\\ \frac{v'(c^u)}{v'(c^e)}&\approx\frac{1}{v'(c^e)}\cdot\left[v'(c^e)-v''(c^e)\cdot c^e\cdot\frac{\Delta c}{c^e}\right]=1+\rho\cdot(1-\tau),\end{aligned}$$

where $\rho\equiv -c^e\cdot v''(c^e)/v'(c^e)$. Thus, the formula becomes

$$\frac{\tau}{1-\tau}\approx\frac{1}{\epsilon^M}\cdot\rho\cdot[1-\tau]+\frac{1}{1+\epsilon^d}\cdot\left[\frac{\epsilon^m}{\epsilon^M}-1\right]\cdot\left[1+\frac{\rho}{2}\cdot(1-\tau)\right].$$

A.3 Proof of Proposition 2

The government chooses Δc and c^e to maximize the per-period social welfare subject to the per-period budget constraint. The Lagrangian of the government's problem is

$$\begin{aligned}\mathcal{L}(\Delta c,c^e)=&\,v(c^e)-\left[1-\tilde{n}(e\cdot f(\Delta v^h))\right]\cdot\Delta v^h\\&-\left[1-(1-s)\cdot\tilde{n}(e\cdot f(\Delta v^h))\right]\cdot k(e)+\phi\cdot\left[n(\Delta v^h)\cdot(w-\Delta c)-(c^e-\Delta c)\right],\end{aligned}$$

where ϕ is the Lagrange multiplier on the budget constraint. We apply the envelope theorem because workers choose effort e to maximize per-period social welfare. We also exploit the following lemma:

LEMMA A4. *Let $\Delta v^h(\Delta c,c^e)=min_y\left\{v(c^e)-v(c^e-\Delta c+y)+m(y)\right\}$ be the utility gain from work when home production is optimal. The derivatives of $\Delta v^h(\Delta c,c^e)$ satisfy*

$$\begin{aligned}\frac{\partial\Delta v^h}{\partial\Delta c}&=v'(c^h)\\ \frac{\partial\Delta v^h}{\partial c^e}&=v'(c^e)-v'(c^h).\end{aligned}$$

Proof. Follows from the definition of Δv^h and the application of the envelop theorem. □

The first-order condition with respect to Δc is

$$-(1-n)\cdot v'(c^h) + \frac{\partial \Delta v^h}{\partial \Delta c}\cdot\left[\left[\Delta v^h + (1-s)\cdot k(e)\right]\cdot \tilde{n}'(ef)\cdot e\cdot f'(\Delta v^h) + \phi\cdot n'(\Delta v^h)\cdot(w-\Delta c)\right] + \phi\cdot(1-n) = 0. \tag{A4}$$

The first-order condition with respect to c^e is

$$nv'(c^e) + (1-n)v'(c^h) + \frac{\partial \Delta v^h}{\partial c^e}\left[\left[\Delta v^h + (1-s)k(e)\right]\tilde{n}'(ef)ef'(\Delta v^h) + \phi n'(\Delta v^h)\,(w-\Delta c)\right] - \phi = 0.$$

We can manipulate these two first-order conditions to obtain the inverse Euler equation. To obtain the optimal UI formula, we derive the pendant of Lemma 1 in the dynamic case.

LEMMA A5. *Microelasticity ϵ^m and macroelasticity ϵ^M are related by*

$$\epsilon^M = \epsilon^m + \frac{\tilde{n}'(e\cdot f)\cdot e}{1-n}\cdot\left(1+\epsilon^d\right)\cdot \Delta c\cdot f'(\Delta v^h)\cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e}.$$

Proof. Given the definitions of ϵ^m and the labor supply, we have

$$\epsilon^m = \frac{\Delta c}{1-n}\cdot \tilde{n}'(e\cdot f)\cdot f\cdot \frac{\partial e}{\partial \Delta v^h}\cdot\frac{\partial \Delta v^h}{\partial \Delta c}.$$

Given the definition of ϵ^d and the fact that the employment rate $n(\Delta v^h) = \tilde{n}(e(f,\Delta v^h)\cdot f(\Delta v^h)))$,

$$n'(\Delta v^h) = \tilde{n}'(e\cdot f)\cdot f\cdot\frac{\partial e}{\partial \Delta v^h} + \tilde{n}'(e\cdot f)\cdot e\cdot\left(1+\epsilon^d\right)\cdot f'(\Delta v^h).$$

We conclude the proof by multiplying this equation by $\Delta c/(1-n)$ and using the definition of ϵ^M. □

We come back to (A4), divide it by $\phi\cdot(1-n)$, normalize $k(e)=0$, rearrange terms, and abuse notations by denoting $\partial f/\partial\Delta c \equiv f'(\Delta v^h)\cdot\left(\partial \Delta v^h/\partial \Delta c\right)$ and $\partial n/\partial \Delta c\equiv n'(\Delta v^h)\cdot\left(\partial\Delta v^h/\partial\Delta c\right)$. We obtain

$$\frac{1}{1-n}\cdot\frac{\partial n}{\partial \Delta c}\cdot(w-\Delta c) = \left[\frac{v'(c^h)}{\phi}-1\right] - \frac{1}{\phi}\cdot\frac{\Delta v^h}{1-n}\cdot\tilde{n}'(e\cdot f)\cdot e\cdot\frac{\partial f}{\partial \Delta c} \tag{A5}$$

The inverse Euler equation yields

$$\frac{v'(c^h)}{\phi} - 1 = n\cdot\left[\frac{v'(c^h)}{v'(c^e)}-1\right].$$

Lemma A5 implies

$$-\frac{\Delta v^h}{1-n}\cdot\tilde{n}'(e\cdot f)\cdot e\cdot\frac{\partial f}{\partial \Delta c} = \frac{\Delta v^h}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\epsilon^m-\epsilon^M\right).$$

By definition,

$$\frac{1}{1-n}\cdot\frac{\partial n}{\partial \Delta c} = \frac{1}{\Delta c}\cdot\epsilon^M.$$

Therefore we can rewrite (A5) as

$$\frac{w - \Delta c}{\Delta c} \cdot \epsilon^M = n \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

Using part (i) of Lemma A1 and dividing this equation by ϵ^M, we obtain the formula

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

Approximation. Assuming $n \approx 1$, we simplify the formula to

$$\frac{\tau}{1-\tau} \approx \frac{1}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{v'(c^e)} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

If the third and higher order terms of v are small ($v'''(c) \approx 0$), we approximate

$$\frac{v'(c^h)}{v'(c^e)} \approx \frac{1}{v'(c^e)} \cdot \left[v'(c^e) - v''(c^e) \cdot \left(c^e - c^h\right)\right] = 1 + \rho \cdot (1 - \xi),$$

where $\rho \equiv -v''(c^e) \cdot c^e / v'(c^e)$ and $\xi = c^h / c^e$. We normalize $m(y) = 0$. Hence, $\Delta v^h = v(c^e) - v(c^h) + m(y) = v(c^e) - v(c^h)$. By linearization, we obtain

$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \frac{v(c^e) - v(c^e) - v'(c^e) \cdot (c^h - c^e) - v''(c^e)/2 \cdot (c^h - c^e)^2}{v'(c^e) \cdot (c^e - c^u)}$$

$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \left[\frac{c^e - c^h}{c^e - c^u}\right] - \frac{1}{2} \cdot \frac{v''(c^e) \cdot c^e}{v'(c^e)} \cdot \frac{c^e - c^h}{c^e} \cdot \left[\frac{c^e - c^h}{c^e - c^u}\right]$$

$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \left[\frac{1 - \xi}{1 - \tau}\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1 - \xi)\right].$$

The formula becomes

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1 - \xi) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1 - \xi)\right] \cdot \left[\frac{1 - \xi}{1 - \tau}\right].$$

A.4 Proof of Proposition 3

The equilibrium condition (20), obtained under Assumptions 1 and 2, implies that $d\theta/d\tau < 0$. As $v(c) = \ln(c)$, $\Delta v = -\ln(\tau)$, and $\theta'(\Delta v) > 0$. With $v(c) = \ln(c)$, Lemma 1 implies

$$\epsilon^M = \epsilon^m + \frac{1/\tau - 1}{1 - n} \cdot \frac{h}{f} \cdot \left(1 + \epsilon^d\right) \cdot f'(\theta) \cdot \theta'(\Delta v).$$

Since $\theta'(\Delta v) > 0$ and $\tau < 1$, we infer that $\epsilon^M > \epsilon^m > 0$.

A.5 Proof of Proposition 4

We define the following elasticities: $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$, $-\eta \equiv \theta \cdot q'(\theta)/q(\theta) < 0$, and $\kappa \equiv e \cdot k''(e)/k'(e)$. We abuse notations slightly and define equilibrium labor market tightness $\theta(\Delta c, c^e, a) \equiv$

$\theta(\Delta v(\Delta c, c^e), a)$ and equilibrium employment $n(\Delta c, c^e, a) \equiv n(\Delta v(\Delta c, c^e), a)$.

LEMMA A6. *The partial derivative of equilibrium tightness $\theta(\Delta c, c^e, a)$ with respect to Δc is*

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e,a} = \frac{\theta}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot \left(\epsilon^M - \epsilon^m\right).$$

Proof. The equilibrium job-finding rate is $f = f(\theta(\Delta c, c^e, a))$ so Lemma 1 implies that

$$\epsilon^M = \epsilon^m + \frac{h}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \frac{\Delta c}{f(\theta)} \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

The result follows because $\epsilon^d = 1/\kappa$ by Lemma A2 and $f'(\theta)/f(\theta) = (1-\eta)/\theta$ by definition. □

The labor demand equation (18) holds at any equilibrium such that

$$g'(n(\Delta c, c^e, a)) = \frac{w}{a} + \frac{r}{q(\theta(\Delta c, c^e, a))}.$$

We differentiate this relationship with respect to Δc, keeping c^e and a constant. We obtain

$$(\alpha - 1) \cdot \frac{g'(n)}{n} \cdot \frac{\partial n}{\partial \Delta c} = \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{\partial \theta}{\partial \Delta c}$$

because under Assumption 4, (w/a) does not depend on Δc. Lemma A6 implies

$$(\alpha - 1) \cdot g'(n) \cdot \frac{1-n}{n} \cdot \epsilon^M = \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(\epsilon^M - \epsilon^m\right)$$

$$-(1-\alpha) \cdot g'(n) = \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(1 - \frac{\epsilon^m}{\epsilon^M}\right)$$

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \left[(1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{1}{r} \cdot \frac{1-\eta}{\eta}\right] \cdot q(\theta) \cdot \left(\frac{h}{n}\right) \cdot n^{\alpha-1}.$$

Since $\theta > 0$, $h > 0$, $\eta \in (0,1)$, $\kappa > 0$, $\epsilon^m/\epsilon^M > 1$ if and only if $\alpha \in (0,1)$.

A.6 Some comparative statics

We now focus on log utility: $v(c) = \ln(c)$. It is natural to parameterize the equilibrium with (τ, a) instead of $(\Delta c, c^e, a)$ because $\Delta v = \ln(1/\tau)$. Technology a captures the position in the business cycle and the replacement rate τ captures the generosity of UI. We abuse notations slightly and define equilibrium labor market tightness $\theta(\tau, a) \equiv \theta(\ln(1/\tau), a)$, and equilibrium employment $n(\tau, a) \equiv n(\ln(1/\tau), a)$. .

LEMMA A7. *Under Assumptions 3 and 4, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium tightness $\theta(\tau, a)$ and equilibrium employment $n(\tau, a)$:*

$$\left.\frac{\partial \theta}{\partial a}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial a}\right|_{\tau} > 0.$$

Proof. If $v(c) = \ln(c)$, the labor market equilibrium (21) condition becomes

$$1 - u + u \cdot e(f(\theta(\tau, a)), \ln(1/\tau)) \cdot f(\theta(\tau, a)) = n^d(\theta(\tau, a), a).$$

We differentiate this condition with respect to a, keeping τ constant:

$$u \cdot \left[f \cdot \frac{\partial e}{\partial f} + e\right] \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial a} = \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial a} + \frac{\partial n^d}{\partial a}$$

$$\frac{\partial \theta}{\partial a} = \underbrace{\frac{\partial n^d}{\partial a}}_{+} \cdot \left[\underbrace{u}_{+} \cdot \left(\underbrace{f}_{+} \cdot \underbrace{\frac{\partial e}{\partial f}}_{+} + \underbrace{e}_{+}\right) \cdot \underbrace{f'(\theta)}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-}\right]^{-1}.$$

because under Assumptions 3 and 4, $\partial n^d/\partial \theta < 0$, and $\partial n^d/\partial a > 0$. So $\partial \theta/\partial a > 0$. We show that $\partial n/\partial a > 0$ using $n(\tau, a) = 1 - u + u \cdot e(f(\theta(\tau, a)), \ln(1/\tau)) \cdot f(\theta(\tau, a))$. □

A.7 Proof of Proposition 5

Under Assumption 4, we can apply Proposition 4. Under Assumptions 3, 5, and 6, Proposition 4 implies that $\epsilon^m/\epsilon^M = 1 + \chi \cdot q(\theta) \cdot n^{\alpha-1}$, where $\chi \equiv \alpha \cdot (1-\alpha) \cdot [(1-\eta)/\eta] \cdot [(1+\kappa)/\kappa] \cdot (s/r) > 0$ is constant. Under Assumptions 3 and 4, Lemma A7 implies that $\partial \theta/\partial a\big|_\tau > 0$ and $\partial n/\partial a\big|_\tau > 0$. Since $q'(\theta) < 0$ and $\alpha \leq 1$, we infer that $\partial \left[\epsilon^m/\epsilon^M\right]/\partial a\big|_\tau < 0$.

We now focus on the cyclicality of ϵ^M. First, we determine an expression for ϵ^m. By definition

$$\begin{aligned}
\epsilon^m &= \frac{\Delta c}{1-n} \cdot \frac{h}{e} \cdot \frac{\partial e}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \\
\epsilon^m &= \frac{\Delta c}{\Delta v} \cdot \frac{s \cdot n}{1-n} \cdot \frac{1}{\kappa} \cdot v'(c^u) \\
\epsilon^m &= \frac{s}{\kappa} \cdot \frac{(1/\tau) - 1}{\ln(1/\tau)} \cdot \frac{n}{1-n},
\end{aligned} \tag{A6}$$

where we used Assumption 6, the assumption that $v(c) = \ln(c)$, and the result from Lemma A2. We infer that

$$\epsilon^M = \epsilon^m \cdot \frac{\epsilon^M}{\epsilon^m} = \frac{s}{\kappa} \cdot \frac{(1/\tau) - 1}{\ln(1/\tau)} \cdot \frac{n}{1-n} \cdot \frac{\epsilon^M}{\epsilon^m}. \tag{A7}$$

Under Assumption 5, the elasticity κ is constant. According to Lemma A7, valid under Assumptions 3 and 4, $\partial n/\partial a\big|_\tau > 0$. We showed that $\partial \left[\epsilon^M/\epsilon^m\right]/\partial a\big|_\tau > 0$. We conclude that $\partial \epsilon^M/\partial a\big|_\tau > 0$.

A.8 Proof of Proposition 6

Under Assumption 5, using the result from Lemma A2 that $\epsilon^d = 1/\kappa$, formula (10) becomes

$$\begin{aligned}
\frac{1}{n} \cdot \frac{\tau}{1-\tau} &= \frac{n}{\epsilon^M} \cdot \frac{1-\tau}{\tau} + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{1-\tau} \cdot [(1-n) \cdot \tau + n] \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) \\
1 &= \frac{n^2}{\epsilon^M} \cdot \left(\frac{1-\tau}{\tau}\right)^2 + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{\tau} \cdot n \cdot [(1-n) \cdot \tau + n] \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).
\end{aligned} \tag{A8}$$

Next, we express $\left(\epsilon^m/\epsilon^M - 1\right)$ and ϵ^M as a function of n and τ.

LEMMA A8. *Under Assumptions 3, 4, 5 and 6 there exists $Z_0(\tau) > 0$ such that in equilibrium,*

$$Z(n,\tau) \equiv \frac{\epsilon^m}{\epsilon^M} - 1 = Z_0(\tau) \cdot n^{-\Omega} > 0, \tag{A9}$$

where the constant Ω is defined by

$$\Omega = (1-\alpha) + \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{s} > 0.$$

Proof. Under Assumption 4, we can use Proposition 4. Under Assumption 6, it says that

$$\frac{\epsilon^m}{\epsilon^M} - 1 = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot n^{\alpha-1}. \tag{A10}$$

We can write n as a function of θ and τ:

$$n = 1 - u + u \cdot e(f(\theta), \ln(1/\tau)) \cdot f(\theta).$$

Differentiating this equation with respect to θ, keeping τ constant, and using Lemma A2 under Assumption 6, we obtain

$$\begin{aligned}
\left.\frac{\partial n}{\partial \theta}\right|_\tau &= \frac{h}{f} \cdot f'(\theta) + \frac{h}{e} \cdot \frac{\partial e}{\partial f} \cdot f'(\theta) = \left(1+\epsilon^d\right) \cdot (1-\eta) \cdot \frac{h}{\theta} = \frac{\kappa+1}{\kappa} \cdot (1-\eta) \cdot \frac{h}{\theta} \\
\frac{\theta}{n} \cdot \left.\frac{\partial n}{\partial \theta}\right|_\tau &= \frac{\kappa+1}{\kappa} \cdot (1-\eta) \cdot s \\
\frac{n}{\theta} \cdot \left.\frac{\partial \theta}{\partial n}\right|_\tau &= \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1}{s}.
\end{aligned}$$

Combining (A10) with this relationship between n and θ yields

$$\left.\frac{\partial \ln\left(\epsilon^m/\epsilon^M - 1\right)}{\partial \ln(n)}\right|_\tau = -\left[(1-\alpha) + \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{s}\right] \equiv -\Omega,$$

where $\Omega > 0$ is constant under Assumption 5. We obtain (A9) by integrating this relationship. □

Using Lemmas A8 and (A7), we write ϵ^M as a function of n and τ

$$\frac{1}{\epsilon^M} = \frac{1-n}{n} \cdot \frac{\kappa}{s} \cdot \ln\left(\frac{1}{\tau}\right) \cdot \frac{\tau}{1-\tau} \cdot \left[1 + Z(n,\tau)\right]. \tag{A11}$$

Using Lemma A8 and (A11), we rewrite formula (A8) as

$$1 = n \cdot (1-n) \cdot \frac{\kappa}{s} \cdot \left[1 + Z(n,\tau)\right] \cdot \ln\left(\frac{1}{\tau}\right) \cdot \frac{1-\tau}{\tau} + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{\tau} \cdot n \cdot \left[(1-n) \cdot \tau + n\right] \cdot Z(n,\tau).$$

Let $S \equiv s/(\kappa+1) \in (0,1)$. We rearrange the terms to obtain

$$\frac{s}{\kappa} \cdot \frac{\tau}{\ln\left(\frac{1}{\tau}\right)} = n \cdot (1-n)(1-\tau) + n \cdot Z(n,\tau) \cdot \left[\tau \cdot S + (1-\tau) - n \cdot (1-\tau) \cdot (1-S)\right]. \tag{A12}$$

Let us define

$$F(\tau) \equiv \frac{s}{\kappa} \cdot \frac{\tau}{\ln(1/\tau)}$$
$$G(n,\tau) \equiv n \cdot (1-n) \cdot (1-\tau) + n \cdot Z(n,\tau) \cdot [\tau \cdot S + (1-\tau) - n \cdot (1-\tau) \cdot (1-S)].$$

Furthermore, we define $Q(\tau, a) \equiv G(n(\tau, a), \tau)$. We rewrite the optimal UI formula as $F(\tau) = Q(\tau, a)$. We assume that for any $a > 0$, $F(\tau)$ and $Q(\tau, a)$ cross only once at $\tau(a) \in (0, 1)$. The implicit function $\tau(a)$ characterizes the optimal replacement rate for technology a.

LEMMA A9. *Under Assumptions 3 and 4,* $\lim_{a\to 0} n(a, \tau(a)) = 0$ *and* $\lim_{a\to 0} \tau(a) = 0$.

Proof. Under Assumptions 3 and 4, the labor demand equation (18) implies that for any $a > 0$, $\alpha \cdot n(a, \tau(a))^{\alpha-1} \geq \omega \cdot a^{\gamma-1}$ and $0 \leq n(a, \tau(a)) \leq N(a) \equiv [(\alpha/\omega) \cdot a^{1-\gamma}]^{1/(1-\alpha)}$. Since $\gamma < 1$ and $0 < \alpha < 1$, $\lim_{a\to 0} N(a) = 0$. The squeeze theorem implies that $\lim_{a\to 0} n(a, \tau(a)) = 0$.

By definition, $q(\theta) \leq 1$. Therefore for any n and any τ,

$$n \cdot Z(n,\tau) = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot n^{\alpha} \leq (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n^{\alpha}.$$

The optimal UI formula is $F(\tau(a)) = Q(\tau(a), a)$. Using the definition of Q, we infer

$$F(\tau(a)) \leq n(a,\tau(a)) \cdot [1 - n(a,\tau(a))] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n(a,\tau(a))^{\alpha}.$$

We showed that $\lim_{a\to 0} n(a, \tau(a)) = 0$. So there exists $a_0 > 0$ such that for all $a < a_0$, $n(a, \tau(a)) < 1/2$. For any $a > 0$, $0 \leq n(a, \tau(a)) \leq N(a)$. Thus for any $a < a_0$,

$$0 \leq F(\tau(a)) \leq N(a) \cdot [1 - N(a)] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot N(a)^{\alpha}.$$

Under Assumptions 3 and 4, the limit of the right-hand-side term when $a \to 0$ is 0 because $\lim_{a\to 0} N(a) = 0$. Using the squeeze theorem, we infer that $\lim_{a\to 0} F(\tau(a)) = 0$. We conclude that $\lim_{a\to 0} \tau(a) = 0$ using the continuity of F on $(0, 1)$. □

Lemma A9 establishes that when employment converges to 0 because technology decreases to 0, then the optimal replacement rate converges to 0. This result implies that for very low levels of technology and employment, the optimal replacement rate is bound to increase with technology.

LEMMA A10. *If* $n > 1/2$ *and* $\Omega \geq 1$ *then* $\partial G/\partial n < 0$.

Proof. We differentiate $G(n, \tau)$ with respect to n, keeping τ constant.

$$\frac{\partial G}{\partial n} = -\left\{(2 \cdot n - 1) \cdot (1-\tau) + Z(n,\tau) \cdot [(2-\Omega) \cdot (1-S) \cdot (1-\tau) \cdot n - (1-\Omega) \cdot [\tau \cdot S + (1-\tau)]]\right\}.$$

If $n > 1/2$, the first term $(2 \cdot n - 1) \cdot (1-\tau) > 0$ since $\tau < 1$. If $\Omega \geq 1$, the second term is nonnegative. To see this, note that $(1-S) \cdot n < 1$ and rewrite the second term as

$$Z(n,\tau) \cdot [(\Omega - 1) \cdot [\tau \cdot S + (1-\tau) \cdot \{1 - (1-S) \cdot n\}] + (1-S) \cdot (1-\tau) \cdot n] \geq 0.$$

If $\Omega \in [0, 1)$, the second term may be negative. □

At technology a, the optimal replacement rate $\tau(a)$ satisfies $F(\tau(a)) = Q(\tau(a), a)$. We consider a change in technology from a to $a^* > a$. Using Lemma A7 under Assumption 5, we know that $n(\tau(a), a^*) > n(\tau(a), a)$. Using Lemma A10 for $n > 1/2$ and $\tau \in (0,1)$, $G(n(\tau(a), a^*), \tau(a)) < G(n(\tau(a), a), \tau(a))$ such that $Q(\tau(a), a^*) < Q(\tau(a), a) = F(\tau(a))$. Since $F(\tau)$ and $Q(\tau, a)$ cross only once for $\tau \in (0, 1)$, $\lim_{\tau\to 0} F(\tau) = 0$, and $\lim_{\tau\to 0} Q(\tau, a) > 0$, $F(\tau)$ necessarily crosses $Q(\tau, a)$ from below. Therefore, it must be that $\tau(a) > \tau(a^*)$.

A.9 Interpretation of the assumptions of Proposition 6

LEMMA A11. *The labor supply $n^s(f(\theta), \Delta v)$ is concave in θ if and only if $(1-\eta)\cdot(1+\kappa)/\kappa < 1$.*

Proof. By definition, $n^s(f(\theta), \Delta v) = 1 - u + u \cdot e(f(\theta), \Delta v) \cdot f(\theta)$. Thus,

$$\frac{\partial n^s}{\partial \theta} = \left(1 + \epsilon^d\right) \cdot \frac{n^s - (1-u)}{f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot (1-\eta) \cdot \frac{n^s - (1-u)}{\theta}.$$

Lemma A2 implies that $\epsilon^d = 1/\kappa$. Hence,

$$\frac{\partial^2 n^s}{\partial \theta^2} = \frac{1+\kappa}{\kappa} \cdot (1-\eta) \cdot \frac{n^s - (1-u)}{\theta^2} \cdot \left[\frac{1+\kappa}{\kappa} \cdot (1-\eta) - 1\right].$$

Since $n^s \geq 1 - u$, $\partial^2 n^s / \partial \theta^2 < 0$ if and only if $(1-\eta)\cdot(1+\kappa)/\kappa < 1$. □

A.10 Proof of Proposition 7

The Lagrangian of the government's problem is given by

$$\mathcal{L}(\theta, \Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\theta)) \cdot \Delta v - u \cdot k(e)$$
$$+ \phi \cdot \left\{ a \cdot g(n^s(f(\theta), \Delta v)) - \frac{r \cdot a}{q(\theta)} \cdot [n^s(f(\theta), \Delta v) - (1-u)] - c^e + [1 - n^s(f(\theta), \Delta v)] \cdot \Delta c \right\},$$

where ϕ is the Lagrange multiplier on the resource constraint. We use the envelope theorem as workers choose e optimally. The first-order condition with respect to Δc is

$$-(1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[a \cdot g'(n) - \frac{r \cdot a}{q(\theta)} - \Delta c\right] + \phi \cdot (1-n) = 0.$$

The firm's profit-maximization condition ensures that

$$w = a \cdot g'(n) - \frac{r \cdot a}{q(\theta)}$$

and we rewrite the first-order condition with respect to Δc as

$$-(1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot (w - \Delta c) + \phi \cdot (1-n) = 0.$$

Similarly, the first-order condition with respect to c^e is

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial c^e} \cdot (w - \Delta c) - \phi = 0.$$

Using Lemma A3 we can rewrite $\partial \Delta v / \partial c^e = -\left[1 - (v'(c^e)/v'(c^u))\right] \cdot \partial \Delta v / \partial \Delta c$. To obtain the inverse Euler equation, we multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$ and rearrange the terms to obtain

$$\frac{1}{1-n} \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot (w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right].$$

By definition,

$$\frac{1}{1-n} \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^m.$$

Therefore, we can rewrite the first-order condition as

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Last, the first-order condition with respect to θ is

$$0 = (1-\eta) \cdot \frac{h}{\theta} \cdot \Delta v + \phi \cdot [w - \Delta c] \cdot \frac{\partial n^s}{\partial f} \cdot f'(\theta) - \phi \cdot \eta \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{h}{\theta}.$$

Since the labor supply $n^s(f, \Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f$, we have

$$\frac{\partial n^s}{\partial f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot \frac{h}{f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot \frac{h}{\theta} \cdot (1 - \eta).$$

We divide the first-order condition by $\phi \cdot (h/\theta) \cdot (1-\eta)$ and obtain

$$\frac{\Delta v}{\phi} + [w - \Delta c] \cdot \left(1 + \epsilon^d\right) = \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}.$$

A.11 Proof of Proposition 8

The Lagrangian of the government's problem is given by

$$\begin{aligned}\mathcal{L}(\Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\theta)) \cdot \Delta v(\Delta c, c^e) - u \cdot k(e) \\ + \phi \cdot \left\{a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)] - c^e + (1-n) \cdot \Delta c\right\},\end{aligned}$$

where ϕ is the Lagrange multiplier on the resource constraint, θ stands for $\theta(\Delta v(\Delta c, c^e))$, and n stands for $n(\Delta v(\Delta c, c^e))$. We exploit the envelope theorem as workers choose effort e optimally. The first-order condition with respect to Δc is

$$-(1-n)v'(c^u) + \frac{\partial \Delta v}{\partial \Delta c}\left[(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\Delta v + \phi n'(\Delta v)(w - \Delta c) - \phi\eta\frac{ra}{q(\theta)}\frac{h}{\theta}\theta'(\Delta v)\right] + \phi(1-n) = 0.$$

Similarly, the first-order condition with respect to c^e is

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial c^e} \left[(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\Delta v + \phi n'(\Delta v)(w - \Delta c) - \phi\eta \frac{ra}{q(\theta)}\frac{h}{\theta}\theta'(\Delta v) \right] - \phi = 0.$$

Lemma A3 implies $\partial \Delta v / \partial c^e = -\left[1 - (v'(c^e)/v'(c^u))\right] \cdot \partial \Delta v / \partial \Delta c$. To obtain the inverse Euler equation, we multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$ and rearrange the terms to obtain

$$\frac{1}{1-n}\frac{\partial n}{\partial \Delta c}(w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right] - \frac{1}{1-n}(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\frac{\partial \Delta v}{\partial \Delta c} \cdot \left[\frac{\Delta v}{\phi} - \frac{\eta}{1-\eta}\frac{ra}{q(\theta)}\right].$$

The inverse Euler equation yields

$$\frac{v'(c^u)}{\phi} - 1 = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Lemma 1 implies

$$-\frac{1}{1-n} \cdot (1-\eta) \cdot \frac{h}{\theta} \cdot \theta'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

By definition,

$$\frac{1}{1-n} \cdot n'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^M.$$

Therefore, we can rewrite the first-order condition as

$$\frac{w - \Delta c}{\Delta c} \cdot \epsilon^M = n\left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right)\left[\frac{\Delta v}{\phi} - \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}\right]. \tag{A13}$$

Adding $[(w - \Delta c)/\Delta c] \cdot \left[\epsilon^m - \epsilon^M\right]$ on both sides and dividing by ϵ^m, we obtain the formula of Part (i):

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m}\right) \cdot \left[\frac{\Delta v}{\phi} + (w - \Delta c) \cdot (1+\epsilon^d) - \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}\right].$$

We can rewrite (A13) as

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) - \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

Proposition 4 implies that under Assumption 4, the elasticity wedge satisfies

$$\frac{\epsilon^m}{\epsilon^M} - 1 = -g''(n) \cdot \frac{1-\eta}{\eta} \cdot \frac{1}{1+\epsilon^d} \cdot \frac{q(\theta)}{r} \cdot h.$$

Combining the last two equations, we obtain the formula of Part (ii):

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) + a \cdot g''(n) \cdot \frac{h}{\Delta c}.$$

A.12 Proof of Proposition 9

To obtain the approximated formula in Part (i), we apply the methodology developed in the proof of Proposition 1 to formula (27). In addition, we need to handle the term $a \cdot g''(n) \cdot (h/\Delta c)$. If $u \ll 1$, then $h < u \ll 1$, and $a \cdot g''(n) \cdot (h/\Delta c) \approx 0$. Therefore, we neglect this term in the formula.

We turn to Part (ii). The result that $(w - \Delta c)/\Delta c \approx$ if $r \ll 1$ derives directly from Lemma A1. Using this result, the approximated formula of Part (i) becomes

$$\alpha \cdot \frac{\tau}{1-\tau} \approx (1-\alpha) + \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right].$$

By applying the methodology developed in the proof of Proposition 6 to this formula, it is easy to show that $d\tau/da < 0$. Since $(w/\Delta c) \approx \alpha \cdot \tau/(1-\tau) + \alpha$, it is clear that $d(\Delta c/w)/da > 0$ and $d\left[1 - (\Delta c/w)\right]/da < 0$.

B Nash-Bargained Wages

Consider the search-and-matching model with Nash bargaining of Section 2. This section solves the Nash bargaining problem faced by a worker-firm pair. Let $\mathcal{E}$ denote the value of being employed and $\mathcal{U}$ the value of being unemployed after matching. These values satisfy

$$\mathcal{E} = \ln\left((1-T) \cdot w\right)$$
$$\mathcal{U} = \ln\left(B \cdot w\right)$$
$$\mathcal{W} \equiv \mathcal{E} - \mathcal{U} = \ln\left((1-T) \cdot w\right) - \ln\left(B \cdot w\right) = \Delta v,$$

where $\mathcal{W}$ is the worker's surplus from a relationship with a firm. When worker and firm bargain, they take tax rate T and unemployment benefits $B \cdot w$ as given. In the term $(1-T) \cdot w$, w is the outcome of bargaining, but in the term $B \cdot w$, w is the equilibrium wage, taken as given. Therefore when the worker evaluates the marginal utility $d\mathcal{W}$ of an increase dw in the bargained wage, he only considers the change of the post-tax earnings $(1-T) \cdot w$. Accordingly, $d\mathcal{W}/dw = 1/w$.

In equilibrium the firm's surplus from an established relationship is simply given by the hiring cost since a firm can immediately replace a worker at that cost during the matching period: $\mathcal{F} = r \cdot a/q(\theta)$. Since the firm's utility is simply its profits, a wage w brings a utility $-w$ to the firm and $d\mathcal{F}/dw = -1$.

The generalized Nash solution to the bargaining problem is the wage w that maximizes

$$\mathcal{W}(w)^{\beta} \cdot \mathcal{F}(w)^{1-\beta},$$

where β is the worker's bargaining power. The first-order condition of the maximization problem implies that the worker's surplus each period is related to the firm's surplus by

$$\frac{\beta}{1-\beta} \cdot \mathcal{F} = w \cdot \mathcal{W}.$$

Using the previous expressions for $\mathcal{W}$, $\mathcal{F}$, and $d\mathcal{W}/dw$, we obtain the relationship between equilibrium variables imposed by Nash bargaining over wages:

$$\frac{w}{a} = -\frac{\beta}{1-\beta} \cdot \frac{1}{\Delta v} \cdot \frac{r}{q(\theta)}.$$

C Dynamic Model

This section studies the dynamic model used in the quantitative analysis of Section 2. We denote $c_t^h \equiv c_t^u + y_t$ the total consumption of unemployed workers (consumption of market and home-produced goods), $\Delta v_t^h \equiv v(c_t^e) - [v(c_t^u) - m(y_t)]$ the utility gain from work, and $\Delta c_t \equiv c_t^e - c_t^u$ the consumption gain from work. Unemployment is $u_t = 1-(1-s)\cdot n_{t-1}$ and the number of hires is $h_t = n_t-(1-s)\cdot n_{t-1}$.

Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. Together with initial employment n_{-1} in the representative firm, the history of technology realizations $a^t \equiv (a_0, a_1, \ldots, a_t)$ describes the state of the economy in period t. The unemployment insurance plan $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ is measurable with respect to (a^t, n_{-1}), and is taken as given by firms and workers. We assume that the government commits to the policy plan. The time-t element of the worker's choice and firm's choice must therefore be measurable with respect to (a^t, n_{-1}).

C.1 Equilibrium with unemployment insurance

Firms. Given labor market tightness, wage, and technology $\{\theta_t, w_t, a_t\}_{t=0}^{+\infty}$ the representative firm chooses employment $\{n_t^d\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[n_t^d - (1-s)\cdot n_{t-1}^d\right] \right\}.$$

The first-order condition with respect to n_t^d implies

$$a_t \cdot g'(n_t^d) = w_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right]. \tag{A14}$$

Workers. Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{(1-n_t^s)\cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) - \left[1-(1-s)\cdot n_{t-1}^s\right] \cdot k(e_t)\right\}, \tag{A15}$$

subject to the law of motion of the employment probability in period t,

$$n_t^s = (1-s)\cdot n_{t-1}^s + \left[1-(1-s)\cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t). \tag{A16}$$

The Lagrangian of the worker's problem is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ - \left[1-(1-s)\cdot n_{t-1}^s\right] \cdot k(e_t) + (1-n_t^s)\cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e)$$
$$+ A_t \cdot \left[\left[1-(1-s)\cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t) + (1-s)\cdot n_{t-1}^s - n_t^s\right] \Bigg\},$$

where $\{A_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ is a sequence of Lagrange multipliers. The first-order condition with respect to home production y_t is

$$m'(y_t) = v'(c_t^h). \tag{A17}$$

The first-order condition with respect to effort e_t is

$$k'(e_t) = f(\theta_t) \cdot A_t.$$

The first-order condition with respect to employment probability n_t^s is

$$A_t = \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1})\right] + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[A_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}))\right].$$

Combining both conditions, we find that the optimal search effort satisfies

$$\frac{k'(e_t)}{f(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{k'(e_{t+1})}{f(\theta_{t+1})} - e_{t+1} \cdot k'(e_{t+1}) + k(e_{t+1})\right] = \Delta v_t^h. \tag{A18}$$

Wage. In a labor market with matching frictions, the wage cannot equalize labor supply and demand. Since the wage is not determined by a market-clearing condition, we impose instead that the wage follows a stochastic process $\{w_t\}_{t=0}^{+\infty}$ defined for all $t \geq 0$ by

$$w_t = \omega \cdot a_t^{\gamma}. \tag{A19}$$

As in Hall [2005], we also require that the wage neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match.

Labor market equilibrium. Instead of the wage, labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\left\{n_t^d\right\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which defines employment $\{n_t\}_{t=0}^{+\infty}$:

$$n_t^d = n_t^s \equiv n_t. \tag{A20}$$

Equilibrium. Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$, an *equilibrium with unemployment insurance* is a collection of stochastic processes $\{y_t, e_t, n_t, \theta_t, w_t\}_{t=0}^{+\infty}$ that satisfy equations (A14),(A16), (A17), (A18), and (A19).

C.2 Optimal unemployment insurance

The government chooses a government policy $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A15) over all equilibria with unemployment insurance subject to the budget constraint

$$n_t \cdot w_t = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u. \tag{A21}$$

The constraint arises because the government must balance its budget each period. An *equilibrium with optimal unemployment insurance* is a solution to the problem of the government.

We solve the problem of the government. The maximization of the government is over a collection

$\{c_t^e(a^t), c_t^u(a^t), y_t(a^t), e_t(a^t), n_t(a^t), \theta_t(a^t), \ \forall a^t\}_{t=0}^{+\infty}$. We form the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ (1-n_t) \cdot \left[v(c_t^h) - m(y_t) \right] + n_t \cdot v(c_t^e) - [1-(1-s)\cdot n_{t-1}] \cdot k(e_t) \\ &+ A_t \left[n_t \cdot w_t - n_t \cdot c_t^e - (1-n_t) \cdot c_t^u \right] \\ &+ B_t \left[\left[v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] \right] - \frac{k'(e_t)}{f(\theta_t)} \right] + B_{t-1} \cdot (1-s) \left[\frac{k'(e_t)}{f(\theta_t)} + k(e_t) - e_t \cdot k'(e_t) \right] \\ &+ Q_t \left[m'(y_t) - v'(c_t^h) \right] + C_t \left[a_t \cdot g'(n_t) - w_t - \frac{r \cdot a_t}{q(\theta_t)} \right] + C_{t-1} \cdot (1-s) \left[\frac{r \cdot a_t}{q(\theta_t)} \right] \\ &+ D_t \left[(1-(1-s)\cdot n_{t-1}) \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1} - n_t \right] \Bigg\}\end{aligned}$$

where $\{A_t(a^t), B_t(a^t), Q_t(a^t), C_t(a^t), D_t(a^t), \ \forall a^t\}_{t=0}^{+\infty}$ are Lagrange multipliers. We define $B_{-1} \equiv 0$ and $C_{-1} \equiv 0$. The first-order conditions with respect to $y_t(a^t)$ for $t \geq 0$ are

$$0 = (1-n_t) \cdot \left[v'(c_t^h) - m'(y_t) \right] - B_t \cdot \left[v'(c_t^h) - m'(y_t) \right] + Q_t \cdot \left[m''(y_t) - v''(c_t^h) \right]$$

Using the optimal home production condition (A17), we obtain $0 = Q_t \cdot \left[m''(y_t) - v''(c_t^h) \right]$. Since $m'' > 0$ and $v'' < 0$,

$$0 = Q_t. \tag{A22}$$

The first-order conditions with respect to $c_t^e(a^t)$ for $t \geq 0$ are

$$A_t = v'(c_t^e) \cdot \left(1 + \frac{B_t}{n_t} \right). \tag{A23}$$

Using (A22), the first-order conditions with respect to $c_t^u(a^t)$ for $t \geq 0$ are

$$\begin{aligned} 0 &= -(1-n_t) \cdot A_t + (1-n_t) \cdot v'(c_t^h) - B_t \cdot v'(c_t^h) - Q_t \cdot v''(c_t^h) \\ A_t &= v'(c_t^h) \cdot \left[1 - \frac{B_t}{(1-n_t)} \right]. \end{aligned} \tag{A24}$$

The first-order conditions with respect to $e_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned} 0 &= -u_t \cdot k'(e_t) - B_t \cdot \frac{k''(e_t)}{f(\theta_t)} + (1-s) \cdot B_{t-1} \cdot \left\{ \frac{k''(e_t)}{f(\theta_t)} - e_t \cdot k''(e_t) \right\} + D_t \cdot u_t \cdot f(\theta_t) \\ 0 &= u_t + \kappa_t \cdot (1-s) \cdot B_{t-1} \cdot \left(1 - \frac{u_t}{h_t} \right) + \kappa_t \cdot \frac{u_t}{h_t} \cdot B_t - \frac{D_t \cdot h_t}{e_t \cdot k'(e_t)} \end{aligned} \tag{A25}$$

where $\kappa_t \equiv e_t \cdot k''(e_t)/k'(e_t)$. The first-order conditions with respect to $\theta_t(a^t)$ for $t \geq 0$ are

$$0 = (1-\eta) \cdot B_t \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta) \cdot (1-s) \cdot B_{t-1} \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)}$$
$$- C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + D_t \cdot u_t \cdot (1-\eta) \cdot e_t \cdot q(\theta_t)$$
$$0 = \frac{k'(e_t)}{\theta_t} [B_t - (1-s) \cdot B_{t-1}] - \frac{\eta}{1-\eta} \cdot r \cdot a_t \cdot [C_t - (1-s) \cdot C_{t-1}] + D_t \cdot h_t \cdot q(\theta_t) \qquad \text{(A26)}$$

The first-order conditions with respect to $n_t(a^t)$ for $t \geq 0$ are

$$D_t = \Delta v_t^h + \delta(1-s)\mathbb{E}_t [k(e_{t+1}) + D_{t+1} (1 - e_{t+1} f(\theta_{t+1})] + C_t a_t g''(n_t) + A_t [w_t - \Delta c_t] . \qquad \text{(A27)}$$

The equilibrium with optimal unemployment insurance is a collection of 11 stochastic processes $\{c_t^e, c_t^u, y_t, e_t, n_t, \theta_t, A_t, B_t, C_t, D_t, Q_t\}_{t=0}^{+\infty}$ that satisfy 11 equations {(A21), (A14), (A16), (A17), (A18), (A22), (A23), (A24), (A25), (A26), (A27)}.

In steady state there are no aggregate shocks: $a_t = a$ for all t. The equilibrium is constant and characterized by a collection of 11 variables $\{c^e, c^u, y, n, \theta, e, A, B, C, D, Q\}$ determined by 11 equations {(A21), (A14), (A16), (A17), (A18), (A22), (A23), (A24), (A25), (A26), (A27)}. We combine a few of the first-order conditions and constraints of the government's problem to express some Lagrange multipliers in a simple form. These relationships are useful to solve for the steady state numerically. Combining (A23) and (A24), we obtain expressions for the Lagrange multipliers A and B as a function of equilibrium variables:

$$A = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^h)}\right]^{-1} \qquad \text{(A28)}$$
$$B = n \cdot (1-n) \cdot \left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^h)}\right] \cdot A. \qquad \text{(A29)}$$

Since $e/h = 1/(u \cdot f(\theta))$ in steady state, (A25) becomes

$$D = \frac{k'(e)}{f(\theta) \cdot u} \cdot [u + \kappa \cdot (1-s) \cdot B] + \frac{k''(e) \cdot e}{f(\theta)} \cdot \frac{s}{h} \cdot B$$
$$D = \frac{k'(e)}{f(\theta)} \cdot \left[1 + \frac{B}{n} \cdot \frac{\kappa}{u}\right], \qquad \text{(A30)}$$

where $\kappa \equiv e \cdot k''(e)/k'(e)$. Using this expression, equation (A26) becomes:

$$0 = h \cdot D \cdot f(\theta) + k'(e) \cdot s \cdot B - \frac{\eta}{1-\eta} \cdot r \cdot a \cdot \theta \cdot s \cdot C$$
$$C = \frac{1-\eta}{\eta} \cdot \frac{k'(e)}{r \cdot a \cdot \theta} \cdot \left[n + B \cdot \left(\frac{\kappa}{u} + 1\right)\right]. \qquad \text{(A31)}$$

C.3 Calibration

Cost of home production. We relate the convexity μ of the cost $m(y) = \omega_m \cdot [y^{1+\mu} - \hat{y}^{1+\mu}]/(1+\mu)$ of home production to the statistics $\xi = c^h/c^e$ and $\epsilon^i = dc^h/dc^u$ (where dc^u is a marginal change in benefits for one unemployed worker). Unemployed workers choose y to maximize $v(c^e - \Delta c + y) - m(y)$. The first-order condition is

$$m'(y) = v'(c^e - \Delta c + y).$$

Differentiating this condition for a marginal change dc^u, we obtain

$$v''(c^h) \cdot dc^h = m''(y) \cdot (dc^h - dc^u)$$
$$\epsilon^i = \frac{dc^h}{dc^u} = \left[1 - \frac{v''(c^h)}{m''(y)}\right]^{-1}.$$

Since m' is isoelastic, $\rho = -c \cdot v''(c)/v'(c)$, and $y = c^h - c^u = c^h \cdot (1 - \tau/\xi)$, we obtain

$$m''(y) = \mu \cdot \frac{m'(y)}{y} = \frac{\mu}{1-\tau/\xi} \cdot \frac{v'(c^h)}{c^h} = -\frac{\mu}{1-\tau/\xi} \cdot \frac{1}{\rho} \cdot v''(c^h).$$

Combining these two equations gives us an expression for μ as a function of ξ and ϵ^i:

$$\mu = \rho \cdot \left(1 - \frac{\tau}{\xi}\right) \cdot \frac{\epsilon^i}{1-\epsilon^i}. \tag{A32}$$

Disutility from job search. We relate the convexity κ of the disutility $k(e) = \omega_k \cdot (e^{1+\kappa} - 1)/(1+\kappa)$ from search to the statistics ϵ^s and ξ. Recall that $\epsilon^s = (c^u/\zeta) \cdot (d\zeta/dc^u)$, where dc^u is a marginal change in benefits for one unemployed worker, and $d\zeta = f \cdot de$ is the marginal response of the hazard rate for the worker due to the response of search de (we consider a change in benefits for one worker only, so the job-finding rate f is not affected by the policy experiment).

LEMMA A12. *Let $e(f, \Delta v^h)$ be the effort supply implicitly defined by (A18) in steady state:*

$$[1 - \delta \cdot (1-s)] \cdot \frac{k'(e)}{f} + \delta \cdot (1-s) \cdot [e \cdot k'(e) - k(e)] = \Delta v^h. \tag{A33}$$

At $e = 1$, the partial derivative of the effort supply satisfies

$$\frac{\Delta v^h}{e} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_f = \frac{1}{\kappa}.$$

Proof. We differentiate equation (A33) with respect to Δv^h, keeping f constant:

$$1 = \left\{[1 - \delta \cdot (1-s)] \cdot \frac{k''(e)}{f} + \delta \cdot (1-s) \cdot [e \cdot k''(e) + k'(e) - k'(e)]\right\} \cdot \frac{\partial e}{\partial \Delta v^h}$$
$$1 = k''(e) \cdot \left\{\frac{1 - \delta \cdot (1-s)}{f} + \delta \cdot (1-s) \cdot e\right\} \cdot \frac{\partial e}{\partial \Delta v^h}$$

At $e = 1$, $k(e) = 0$ and

$$\left[\frac{1 - \delta \cdot (1-s)}{f} + \delta \cdot (1-s) \cdot e\right] \cdot k'(e) = \Delta v^h.$$

Therefore, given that $\kappa = e \cdot k''(e)/k'(e)$, we obtain

$$1 = e \cdot \frac{k''(e)}{k'(e)} \cdot \frac{\Delta v^h}{e} \frac{\partial e}{\partial \Delta v^h}$$
$$\frac{1}{\kappa} = \frac{\Delta v^h}{e} \cdot \frac{\partial e}{\partial \Delta v^h}.$$

□

LEMMA A13. *Let $\Delta v^h(c^e, c^u) = min_y \{v(c^e) - v(c^u + y) + m(y)\}$ be the utility gain from work when home production is optimal. At home production y such that $m(y) = 0$, when $c^h \approx c^e$,*

$$\frac{c^u}{\Delta v^h} \cdot \left.\frac{\partial \Delta v^h}{\partial c^u}\right|_{c^e} \approx -\frac{\tau}{1-\xi}.$$

Proof. From Lemma A4, $\partial \Delta v^h / \partial c^u \big|_{c^e} = -v'(c^h)$. If $m(y) = 0$ and if the second and higher order terms of $v(c)$ are small,

$$\Delta v^h = v(c^e) - v(c^h) \approx v'(c^h) \cdot (c^e - c^h) = v'(c^h) \cdot c^e \cdot (1 - \xi).$$

To conclude,

$$\frac{c^u}{\Delta v^h} \cdot \left.\frac{\partial \Delta v^h}{\partial c^u}\right|_{c^e} \approx -\frac{c^u}{c^e \cdot (1-\xi)} \cdot \frac{v'(c^h)}{v'(c^h)} = -\frac{\tau}{1-\xi}.$$

□

On average $\hat{e} = 1$, so Lemma A12 implies

$$\left.\frac{\partial \ln(\zeta)}{\partial \ln(\Delta v^h)}\right|_f = \left.\frac{\partial \ln(e)}{\partial \ln(\Delta v^h)}\right|_f = \frac{1}{\kappa}.$$

On average $m(\hat{y}) = 0$, so Lemma A13 implies

$$\left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{c^e} = -\frac{\tau}{1-\xi}$$

Combining these results, we conclude that κ is related to ϵ^s by

$$\epsilon^s = -\left.\frac{\partial \ln(\zeta)}{\partial \ln(c^u)}\right|_{c^e,f} = -\left.\frac{\partial \ln(\zeta)}{\partial \ln(\Delta v^h)}\right|_f \cdot \left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{c^e} = \frac{\tau}{1-\xi} \cdot \frac{1}{\kappa}$$
$$\kappa = \frac{\tau}{(1-\xi)} \cdot \frac{1}{\epsilon^s}. \tag{A34}$$

C.4 Elasticities

Relationship between ϵ^m and ϵ^s. We relate the microelasticity ϵ^m to the elasticity ϵ^s estimated by Meyer [1990] and others. ϵ^s captures the response of search effort to a change in UI benefits. The relationship allows us to find empirical estimates of ϵ^m.

We use the notations introduced in the proof of Proposition 2. Since $e = e(f, \Delta v^h)$,

$$\epsilon^s = -\left.\frac{\partial \ln(e \cdot f)}{\partial \ln(c^u)}\right|_{f,c^e} = -\left.\frac{\partial \ln(e)}{\partial \ln(c^u)}\right|_{f,c^e} = -\left.\frac{\partial \ln(e)}{\partial \ln(\Delta v^h)}\right|_f \cdot \left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{f,c^e}$$

$$\epsilon^s = \frac{c^u}{e} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_f \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e}.$$

Since $n^s(f, \Delta c, c^e) = \tilde{n}(e(f, \Delta v^h(\Delta c, c^e)) \cdot f)$,

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \tilde{n}'(e \cdot f) \cdot f \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_f \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e}$$

$$\epsilon^m = \frac{\Delta c}{e} \cdot \frac{u \cdot n}{1-n} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_f \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e},$$

because it is clear from (14) that $\tilde{n}'(e \cdot f) \cdot f = [1 - (1-s) \cdot n] \cdot n/e = (u \cdot n)/e$. Since $\Delta c/c^u = (1-\tau)/\tau$,

$$\epsilon^m = \frac{1-\tau}{\tau} \cdot \frac{n \cdot u}{1-n} \cdot \epsilon^s \tag{A35}$$

In normal circumstances, $\tau \approx 0.5$, $n \approx 1$, and $u \approx (1-n)$ so $\epsilon^m \approx \epsilon^s$.

Magnitude of ϵ^d. Lemma A14 shows that the discouraged-worker elasticity ϵ^d is commensurable to unemployment in the dynamic model.

LEMMA A14. *Let $e(f, \Delta v^h)$ be the effort supply in steady state implicitly defined by (A33). The discouraged-worker elasticity satisfies*

$$\epsilon^d = \frac{f}{e} \cdot \left.\frac{\partial e}{\partial f}\right|_{\Delta v^h} = \frac{1}{\kappa} \cdot \frac{1 - \delta \cdot (1-s)}{1 - \delta \cdot (1-s) \cdot [(1-n)/u]}.$$

If $\delta \approx 1$, then $\epsilon^d \approx u/\kappa$.

Proof. We differentiate equation (A33) with respect to f, keeping Δv^h constant:

$$0 = \left\{[1 - \delta \cdot (1-s)] \cdot \frac{k''(e)}{f} + \delta \cdot (1-s) \cdot [e \cdot k''(e) + k'(e) - k'(e)]\right\} \cdot \frac{\partial e}{\partial f} - [1 - \delta \cdot (1-s)] \cdot \frac{k'(e)}{f^2}$$

$$\frac{f}{e} \cdot \frac{\partial e}{\partial f} = e \cdot \frac{k''(e)}{k'(e)} \cdot \frac{1 - \delta \cdot (1-s)}{1 - \delta \cdot (1-s) \cdot (1 - e \cdot f)}$$

Given that $\kappa = e \cdot k''(e)/k'(e)$ and $1 - e \cdot f = 1 - (s \cdot n)/u = (1-n)/u$, we obtain

$$\frac{f}{e} \cdot \frac{\partial e}{\partial f} = \frac{1}{\kappa} \cdot \frac{1 - \delta \cdot (1-s)}{1 - \delta \cdot (1-s) \cdot [(1-n)/u]}.$$

If $\delta \approx 1$, $1 - \delta \cdot (1-s) \approx s$ and $1 - \delta \cdot (1-s) \cdot [(1-n)/u] \approx [u - (1-s) \cdot (1-n)]/u = s/u$. Thus, $(f/e) \cdot (\partial e/\partial f) \approx u/\kappa$. □

Fluctuations of ϵ^m and ϵ^M over the business cycle. Combining (A35) and (A34), we find an expression for the microelasticity ϵ^m:

$$\epsilon^m = \frac{1-\tau}{1-\xi} \cdot \frac{n \cdot u}{1-n} \cdot \frac{1}{\kappa}. \tag{A36}$$

Next, we calculate an expression for the ratio ϵ^m/ϵ^M. The procedure is the same as that of Proposition 4 but for two steps. First, we replace the labor demand equation (18) by the labor demand in the steady-state of the dynamic model

$$g'(n) = \frac{w}{a} + [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)},$$

which derives from (A14). Second, we use a lemma that replaces Lemma A6 in a dynamic environment:

LEMMA A15. *The derivative of equilibrium tightness $\theta(\Delta c, c^e)$ with respect to Δc is*

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} = \frac{\theta}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{u \cdot n} \cdot \left(\epsilon^M - \epsilon^m\right).$$

If $\delta \approx 1$, we have the following approximation

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} \approx \frac{\theta}{\Delta c} \cdot \frac{\kappa}{\kappa+u} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{u \cdot n} \cdot \left(\epsilon^M - \epsilon^m\right).$$

Proof. The equilibrium job-finding rate is $f = f(\theta(\Delta v(\Delta c, c^e)))$ so Lemma A5 implies that

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \frac{\partial \tilde{n}}{\partial f} \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

It is clear from (14) that $\tilde{n}'(e \cdot f) \cdot e = (u \cdot n)/f$. By definition, $f'(\theta)/f(\theta) = (1-\eta)/\theta$. Thus,

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot (1+\epsilon^d) \cdot \frac{u \cdot n}{\theta} \cdot (1-\eta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

Using the expression for ϵ^d given by Lemma A14, we infer that when $\delta \approx 1$,

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \frac{u+\kappa}{\kappa} \cdot \frac{u \cdot n}{\theta} \cdot (1-\eta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

We obtain expressions for $\partial\theta/\partial\Delta c$ by rearranging the terms in these equations. □

To conclude, the ratio ϵ^m/ϵ^M admits a simple expression in the steady state of the dynamic model:

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot (1+\epsilon^d) \cdot \frac{q(\theta)}{[1-\delta \cdot (1-s)] \cdot r} \cdot u \cdot n^{\alpha-1}, \tag{A37}$$

where ϵ^d is a function of u and n given by Lemma A14.

D Robustness

D.1 Duration of unemployment benefits

This section studies a dynamic model in which unemployment benefits have finite duration. We introduce three superscripts: e for Employed worker; u for unemployed worker eligible for Unemployment benefits; a for unemployed worker whose benefits expired and who only receive social Assistance. The consumptions of market good are c_t^e, c_t^u, and c_t^a; the consumptions of home good for unemployed workers are y_t^u and y_t^a; the search efforts of unemployed workers are e_t^u and e_t^a. We define the following utility gains: $\Delta v_t^{u,e} \equiv v(c_t^e) - [v(c_t^u + y_t^u) - m(y_t^u)]$, $\Delta v_t^{a,e} \equiv v(c_t^e) - [v(c_t^a + y_t^a) - m(y_t^a)]$ $\Delta v_t^{a,u} \equiv \Delta v_t^{a,e} - \Delta v_t^{u,e}$.

Labor market. At the beginning of period t there are x_t^u eligible jobseekers exerting effort e_t^u, and x_t^a ineligible jobseekers exerting effort e_t^a. The number of matches h_t made is given by $h_t = h(e_t^a \cdot x_t^a + e_t^u \cdot x_t^u, o_t)$, where $e_t^a \cdot x_t^a + e_t^u \cdot x_t^u$ is aggregate search effort and o_t is vacancy. We define labor market tightness as $\theta_t \equiv o_t / \left(e_t^a \cdot x_t^a + e_t^u \cdot x_t^u\right)$. After matching, z_t^u eligible workers and z_t^a ineligible workers are unemployed. At the end of period t, a fraction λ_t of the z_t^u eligible unemployed workers become ineligible. The stocks of workers are related by

$$z_t^u = x_t^u \cdot [1 - e_t^u \cdot f(\theta_t)] \tag{A38}$$
$$z_t^a = x_t^a \cdot [1 - e_t^a \cdot f(\theta_t)] \tag{A39}$$
$$x_t^u = z_{t-1}^u \cdot (1 - \lambda_{t-1}) + s \cdot n_{t-1} \tag{A40}$$
$$x_t^a = z_{t-1}^a + \lambda_{t-1} \cdot z_{t-1}^u \tag{A41}$$

Firms. The problem of the firm is as in the baseline model. Optimal hiring satisfies (A14).

Workers. Given government policy $\{c_t^e, c_t^u, c_t^a, \lambda_t\}_{t=0}^{+\infty}$ and tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses efforts and home productions $\{e_t^u, e_t^a, y_t^u, y_t^a\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ v(c_t^e) - [x_t^u \cdot k(e_t^u) + x_t^a \cdot k(e_t^a) + z_t^u \cdot \Delta v_t^{u,e} + z_t^a \cdot \Delta v_t^{a,e}] \right\}, \tag{A42}$$

subject to the laws of motion (A38), (A39) (A40), and (A41) of the unemployment probabilities $\{x_t^u, x_t^a, z_t^u, z_t^a\}_{t=0}^{+\infty}$. Given $\{\theta_t\}_{t=0}^{+\infty}$, a choice of efforts $\{e_t^u, e_t^a\}_{t=0}^{+\infty}$ determines labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which is the employment rate in period t given by

$$n_t^s = 1 - (z_t^a + z_t^u). \tag{A43}$$

We form the Lagrangian of the worker's problem with multipliers A_t, B_t, C_t, D_t assigned to the laws of motion (A38), (A39) (A40),and (A41). The first-order conditions with respect to home productions y_t^u and y_t^a are

$$m'(y_t^u) = v'(c_t^u + y_t^u) \tag{A44}$$
$$m'(y_t^a) = v'(c_t^a + y_t^a). \tag{A45}$$

The first-order conditions with respect to efforts e_t^u and e_t^a are

$$k'(e_t^u) = f(\theta_t) \cdot A_t$$
$$k'(e_t^a) = f(\theta_t) \cdot B_t.$$

The first-order conditions with respect to unemployment probabilities x_t^u and x_t^a are

$$C_t = k(e_t^u) + A_t \cdot (1 - e_t^u f(\theta_t))$$
$$D_t = k(e_t^a) + B_t \cdot (1 - e_t^a f(\theta_t)) \,.$$

The first-order conditions with respect to probabilities z_t^u and z_t^a are

$$A_t = \Delta v_t^{u,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[C_{t+1} \right] + \lambda_t \cdot \delta \cdot \mathbb{E}_t \left[D_{t+1} - C_{t+1} \right]$$
$$B_t = \Delta v_t^{a,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[D_{t+1} \right] + s \cdot \delta \cdot \mathbb{E}_t \left[D_{t+1} - C_{t+1} \right].$$

Combining these equations we have

$$\frac{\Delta k_t'}{f(\theta_t)} = \Delta v_t^{a,u} + (1 - \lambda_t) \cdot \delta \cdot \mathbb{E}_t \left[D_{t+1} - C_{t+1} \right]$$
$$\mathbb{E}_t \left[D_{t+1} - C_{t+1} \right] = \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1} \right],$$

where $\Delta k_t = k(e_t^a) - k(e_t^u)$, $\Delta k_t' = k'(e_t^a) - k'(e_t^u)$, and $\Delta K_t = e_t^a \cdot k'(e_t^a) - e_t^u \cdot k'(e_t^u)$. Combining the equations once more yields

$$\frac{k'(e_t^u)}{f(\theta_t)} + (1-s) \cdot \delta {\cdot} \mathbb{E}_t \left[e_{t+1}^u \cdot k'(e_{t+1}^u) - k(e_{t+1}^u) - \frac{k'(e_{t+1}^u)}{f(\theta_{t+1})} \right]$$
$$= \Delta v_t^{u,e} + \lambda_t \cdot \delta \cdot \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1} \right] \quad \text{(A46)}$$
$$\frac{k'(e_t^a)}{f(\theta_t)} + (1-s) \cdot \delta {\cdot} \mathbb{E}_t \left[e_{t+1}^a \cdot k'(e_{t+1}^a) - k(e_{t+1}^a) - \frac{k'(e_{t+1}^a)}{f(\theta_{t+1})} \right]$$
$$= \Delta v_t^{a,e} + s \cdot \delta \cdot \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1} \right]. \quad \text{(A47)}$$

Labor market equilibrium. As in the baseline model, tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$ such that (A20) holds, defining employment $\{n_t\}_{t=0}^{+\infty}$.

Equilibrium with unemployment insurance. Given government policy $\{\lambda_t,\, c_t^e,\, c_t^u,\, c_t^a\}_{t=0}^{+\infty}$, an *equilibrium with unemployment insurance* is a collection of stochastic processes $\{y_t^u,\, y_t^a,\, e_t^u, e_t^a,\, n_t,\, \theta_t\}_{t=0}^{+\infty}$ that satisfy equations (A38), (A39), (A40), (A41), (A14), (A43), (A44), (A45), (A46),and (A47).

Steady state. In steady state there are no aggregate shocks: $a_t = a$ for all t. The stocks of workers are constant over time. We can recombine the laws of motion of employment and unemployment probabilities to express $\{z_u, x_u, z_a, x_a, n\}$ as a function of $\{\lambda, \theta, e^a, e^u\}$. These steady-state relationships are useful to solve steady-state equilibria numerically.

In steady state the outflows into and outflows from social assistance are equal.

$$x_a \cdot e^a \cdot f(\theta) = \lambda \cdot x_u \cdot [1 - e^u \cdot f(\theta)]$$
$$x_a = x_u \cdot \lambda \cdot \frac{1 - e^u \cdot f(\theta)}{e^a \cdot f(\theta)}.$$

The outflows from and inflows into employment are equal.

$$s \cdot n = x_a \cdot e^a \cdot f(\theta) + x_u \cdot e^u \cdot f(\theta)$$
$$n = \frac{1}{s} \cdot x_u \cdot \left[e^u \cdot f(\theta) \cdot (1-\lambda) + \lambda\right].$$

We write the stock of unemployment at the beginning of the period in two different ways.

$$1 - (1-s) \cdot n = x_a + x_u$$
$$1 - \frac{1-s}{s} \cdot x_u \cdot \left[e^u \cdot f(\theta) \cdot (1-\lambda) + \lambda\right] = x_u \left[1 + \lambda \cdot \frac{1 - e^u \cdot f(\theta)}{e^a \cdot f(\theta)}\right].$$

Combining our previous results, we get the following relationships:

$$x_u = \left[1 + \lambda \cdot \left[1 - e^u \cdot f(\theta)\right] \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s}\right] + \frac{1-s}{s} \cdot e^u \cdot f(\theta)\right]^{-1}$$
$$x_a = \left[1 + \frac{1-s}{s} \cdot e^a \cdot f(\theta) \cdot \left\{1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right\}\right]^{-1}$$
$$z_u = \left[1 + \lambda \cdot \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s}\right] + \frac{1}{s} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right]^{-1}$$
$$z_a = \left[1 + \left[\frac{1}{e^a \cdot f(\theta)} - 1\right]^{-1} \cdot \frac{1}{s} \cdot \left\{1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right\}\right]^{-1}$$
$$n = \left[1 + s \cdot \left[\frac{1}{e^a \cdot f(\theta)} - 1\right] + \frac{s}{(1-\lambda) \cdot e^u \cdot f(\theta) + \lambda} \cdot \left[1 - \frac{e^u}{e^a}\right]\right]^{-1}.$$

Optimal unemployment insurance. We assume that the generosity of the system of transfers is constant: there exists $\tau^{u,e} \in (0,1), \tau^{a,e} \in (0,1)$ such that for all t, $c_t^u = \tau^{u,e} \cdot c_t^e$ and $c_t^a = \tau^{a,e} \cdot c_t^e$. The government chooses a government policy $\{\lambda_t, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A42) over all equilibria with unemployment insurance subject to the budget constraint

$$n_t \cdot w_t = c_t^e \cdot \left[n_t + z_t^u \cdot \tau^{u,e} + z_t^a \cdot \tau^{a,e}\right]. \tag{A48}$$

The constraint arises because the government must balance its budget each period. An *equilibrium with optimal unemployment insurance* is a solution to the problem of the government.

To determine numerically the optimal arrival rate $\lambda(a)$ in a steady state with technology a, we perform a grid search over a range of arrival rates $\{\lambda_i\}$ and pick the λ_i that maximizes social welfare. Once we have picked λ, consumption c^e is given by budget constraint (A48). We repeat the computation for a sequence of technology $\{a_j\}$ to plot the graphs in Figure 3.

D.2 Recessions caused by aggregate demand shocks

This section characterizes optimal UI in a model in which recessions are caused by the combination of low aggregate demand and nominal wage rigidity. After a negative demand shock, prices fall. The fall in prices, combined with nominal wage rigidity, increases the real wage and the marginal cost of labor, which reduces hiring and increases unemployment.[33]

Wage. Assume that nominal wages are rigid. The real wage w follows a simple wage rule

$$w = \frac{\mu}{p}, \tag{A49}$$

where p is the aggregate price level and μ is a parameter. The rule says that the wage is constant in nominal terms: $w \cdot p = \mu$.

Firms. The production function is linear: $g(n) = n$. The firm starts with $1 - u$ workers. Given real wage w and labor market tightness θ, the firm chooses employment n maximizes real profits

$$\pi = n - w \cdot n - \frac{r}{q(\theta)} \cdot [n - (1 - u)] .$$

The first-order condition implies

$$1 = w + \frac{r}{q(\theta)}. \tag{A50}$$

Money. The firm's production is sold in a perfectly competitive goods market. The firm takes the market price p as given. The firm's production n at a given price p determines the aggregate supply of goods. The aggregate demand for goods market takes the simple form m/p, borrowed from the quantity theory of money, where the parameter m characterizes the level of aggregate demand. Fluctuations in m drive the business cycle. In equilibrium, the price clears the goods market:

$$\frac{m}{p} = n. \tag{A51}$$

Equilibrium. The equilibrium price is $p = m/n$ so the equilibrium real wage is

$$w = \frac{\mu}{p} = \frac{\mu}{m} \cdot n.$$

When aggregate demand m falls, the real wage w tends to rise. Inserting the equilibrium real wage into the labor demand equation (A50) yields a labor demand curve

$$n^d(\theta, m) = \frac{m}{\mu} \cdot \left[1 - \frac{r}{q(\theta)}\right]. \tag{A52}$$

The labor supply $n^s(f(\theta), \Delta v)$ retains the same structure as in the model in the text. Equating labor demand with labor supply curve defines implicitly equilibrium labor market tightness $\theta(\Delta v, m)$ and employment $n(\Delta v, m)$ as a function of aggregate demand m and utility gain from work Δv. The labor market equilibrium, depicted in Figure A1, shares the same structure as the equilibrium in the text, depicted in Figure 1.

Jobs are also rationed in recessions. Higher employment implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness for

[33]The model loosely captures one story of the Great Depression: contractionary monetary policy lead to deflation, which raised real wages above trend in presence of nominal wage rigidity, which in turn depressed employment.

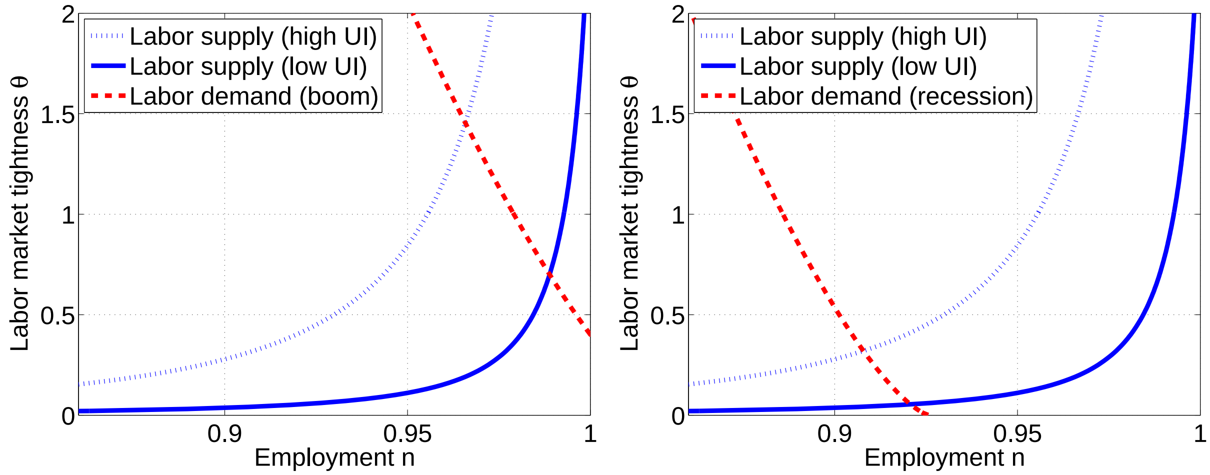

Figure A1: Labor market equilibrium in presence of demand shocks

firms to be willing to hire: the aggregate labor demand curve is downward sloping in a price θ-quantity n plane. If demand is low enough ($m < \mu$), the labor demand falls below zero for $n < 1$: jobs are rationed.

Business cycle fluctuations. We focus on the case with log utility: $v(c) = \ln(c)$. Since $\Delta v = \ln(1/\tau)$, we parameterize the equilibrium of the model with (τ, m). We have the following comparative statics for equilibrium variables:

$$\left.\frac{\partial \theta}{\partial m}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial m}\right|_{\tau} > 0.$$

The proof is identical to that of Lemma A7 because, even if the labor demand is different here from the labor demand in the text, it remains true that $\partial n^d/\partial \theta < 0,\ \partial n^d/\partial m > 0$.

Optimal UI formula. Real wages respond to UI. In equilibrium, UI affects search effort, tightness, employment, price, and eventually, because of nominal wage rigidity, real wage. The optimal UI formula must account for the impact of UI on the government's budget through wages. For instance if UI raises wages, then UI has an additional beneficial effect because it increases the tax base. Of course the wage increase is partly at the cost of firm's profits. For consistency, we account for profits: we assume that the government taxes profits and uses them to finance UI. Thus, the government faces budget constraint (23). Even if wages respond to Δv as here, the appropriate optimal UI formula remains (27). The influence of Δv on w and on equilibrium employment is simply captured by the macroelasticity ϵ^M. Note that $g'' = 0$. Also note from Lemma A1 that with $g(n) = n$, $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c + \{[1 - (h/n)]/\Delta c\} \cdot r \cdot a/q(\theta)$. If $n \approx 1$ and $r \ll 1$, then $\tau/(1-\tau) \approx (w - \Delta c)/\Delta c$. Note from Lemma A2 that $\epsilon^d = 1/\kappa$. Assume that $n \approx 1$ and that the third and higher order terms of v are small. The formula simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right]. \qquad \text{(A53)}$$

Elasticities. We study how microelasticity ϵ^m and macroelasticity ϵ^M fluctuate over the business cycle to determine whether the optimal replacement rate is procyclical or countercyclical. We first examine the elasticity wedge ϵ^m/ϵ^M. We differentiate the labor demand equation (A52):

$$\left.\frac{\partial n}{\partial \Delta c}\right|_{c^e} = -\frac{m}{\mu}\cdot\eta\cdot\frac{r}{q(\theta)}\cdot\frac{1}{\theta}\cdot\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e}.$$

Using Lemma A6 (which remains valid here) and the definition of the elasticity ϵ^M, we obtain

$$(1-n)\cdot\epsilon^M = -\frac{m}{\mu}\cdot\frac{r}{q(\theta)}\cdot\frac{\kappa}{\kappa+1}\cdot\frac{1-n}{h}\cdot\frac{\eta}{1-\eta}\cdot\left(\epsilon^M-\epsilon^m\right)$$
$$\left[\frac{\epsilon^m}{\epsilon^M}-1\right] = \frac{\mu}{m}\cdot\frac{q(\theta)}{r}\cdot\frac{\kappa+1}{\kappa}\cdot h\cdot\frac{1-\eta}{\eta}.$$

Under Assumption 6 we can write

$$\left[\frac{\epsilon^m}{\epsilon^M}-1\right] = \aleph\cdot\frac{q(\theta)}{r}\cdot n\cdot\frac{\mu}{m},$$

where, under Assumption 5, $\aleph$ is a constant defined by

$$\aleph \equiv \frac{1-\eta}{\eta}\cdot\frac{\kappa+1}{\kappa}\cdot s > 0.$$

Finally, using the labor demand equation (A52),

$$\left[\frac{\epsilon^m}{\epsilon^M}-1\right] = \aleph\cdot\left[\frac{q(\theta)}{r}-1\right] > 0.$$

There is an elasticity wedge $(\epsilon^m/\epsilon^M)-1>0$, as in the text. The wedge widens in recessions: since $\left.\partial\theta/\partial m\right|_\tau > 0$ and q is a decreasing function, $\left.\partial\left[\epsilon^m/\epsilon^M\right]/\partial m\right|_\tau < 0$.

We turn to the macroelasticity ϵ^M. The expression (A6) for ϵ^m remains valid so, since $\left.\partial n/\partial m\right|_\tau > 0$, $\left.\partial\epsilon^m/\partial m\right|_\tau > 0$. We can conclude that $\left.\partial\epsilon^M/\partial m\right|_\tau > 0$ because $\epsilon^M = \epsilon^m/(\epsilon^m/\epsilon^M)$.

Optimal replacement rate over the business cycle. Using optimal UI formula (A53) and the results that $\left.\partial\left[\epsilon^m/\epsilon^M\right]/\partial m\right|_\tau < 0$ and $\left.\partial\epsilon^M/\partial m\right|_\tau > 0$, we infer that $d\tau/dm < 0$. Therefore, the optimal replacement rate is also countercyclical in a model in which recessions are driven by aggregate demand shocks.

D.3 Recessions caused by preference shocks

This section characterizes optimal UI in a model in which recessions are caused by preference shocks that affect the disutility from job search.

Workers. A worker's utility is $v(c)-\psi\cdot k(e)$, where ψ is a parameter that characterizes the disutility of search. Fluctuations in ψ drive the business cycle. Given job-finding rate f and consumptions c^e and c^u, a jobseeker chooses effort e to maximize expected utility $v(c^u)+e\cdot f\cdot[v(c^e)-v(c^u)]-\psi\cdot k(e)$. The optimal search effort satisfies the first-order condition

$$k'(e) = f\cdot\frac{\Delta v}{\psi}, \tag{A54}$$

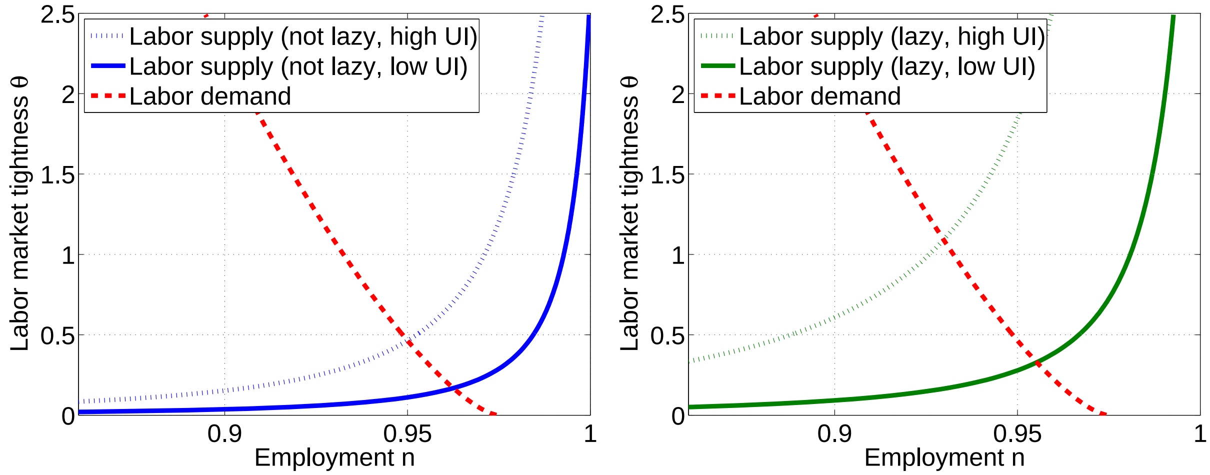

Figure A2: Labor market equilibrium in presence of preference shocks

where $\Delta v = v(c^e) - v(c^u)$ is the utility gain from work. As the disutility from search $k(e)$ is convex, the effort supply $e(f, \Delta v, \psi)$ increases with f and Δv, but decreases with ψ.

Equilibrium. The labor market equilibrium is depicted in Figure A2. It shares the same structure as the labor market equilibrium in the text, depicted in Figure 1. The only difference is the response of the labor market to a macroeconomic shock. When ψ increases, search becomes more costly, effort supply $e(f(\theta), \Delta v, \psi)$ diminishes for a given θ, and the labor supply curve $n^s(f(\theta), \Delta v, \psi) = 1 - u + u \cdot e(f(\theta), \Delta v, \psi) \cdot f(\theta)$ shifts left. Equilibrium employment falls, unemployment increases, and labor market tightness increases. Periods with higher disutility from search ψ are recessions because they are periods with higher unemployment. But these periods are unrealistic because they combine high unemployment with hight labor market tightness. In reality tightness falls when unemployment increases.

Business cycle fluctuations. We focus on the case with log utility: $v(c) = \ln(c)$. Since $\Delta v = \ln(1/\tau)$, we parameterize the equilibrium of the model with (τ, ψ). We have the following comparative statics for equilibrium variables:

$$\left.\frac{\partial \theta}{\partial \psi}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial \psi}\right|_{\tau} < 0.$$

The proof exploits the labor market equilibrium condition

$$n^d(\theta(\tau, \psi)) = 1 - u + u \cdot e(f(\theta(\tau, \psi)), \ln(1/\tau), \psi) \cdot f(\theta(\tau, \psi)).$$

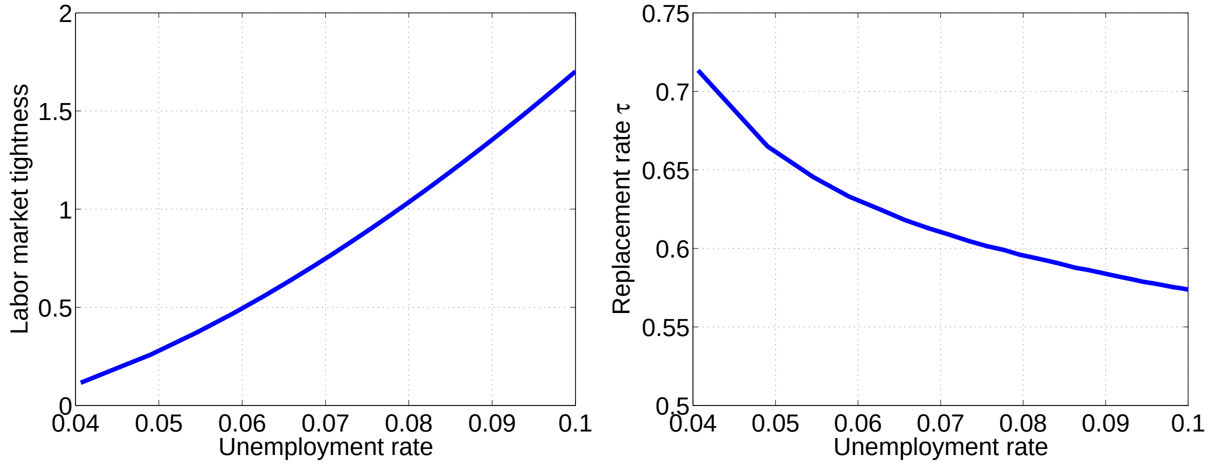

Figure A3: Optimal unemployment insurance over a business cycle driven by preference shocks

We differentiate this condition with respect to ψ, keeping τ constant:

$$\frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial \psi} = u \cdot f \cdot \frac{\partial e}{\partial \psi} + u \cdot \left[f \cdot \frac{\partial e}{\partial f} + e \right] \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \psi}$$

$$\frac{\partial \theta}{\partial \psi} = -u \cdot f \cdot \underbrace{\frac{\partial e}{\partial \psi}}_{-} \cdot \left[\underbrace{u}_{+} \cdot \left(\underbrace{f}_{+} \cdot \underbrace{\frac{\partial e}{\partial f}}_{+} + \underbrace{e}_{+} \right) \cdot \underbrace{f'(\theta)}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-} \right]^{-1}.$$

because under Assumptions 3 and 4, $\partial n^d / \partial \theta < 0$. So $\partial \theta / \partial \psi \big|_\tau > 0$. We show that $\partial n / \partial \psi \big|_\tau < 0$ using $n(\tau, \psi) = n^d(\theta(\tau, \psi))$.

Optimal unemployment insurance formula. Formulas (10) and (12) remain valid in this model.

Elasticities. We study how microelasticity ϵ^m and macroelasticity ϵ^M fluctuate over the business cycle to determine whether the optimal replacement rate is procyclical or countercyclical. Proposition 4 remains valid and under Assumptions 4, 5, and 6, the elasticity wedge is

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \chi \cdot q(\theta) \cdot n^{\alpha - 1},$$

where $\chi = \alpha \cdot (1 - \alpha) \cdot [(1 - \eta)/\eta] \cdot [(1 + \kappa)/\kappa] \cdot (s/r)$ is constant. The labor demand equation (18) implies

$$q(\theta) \cdot n^{\alpha - 1} = \frac{w}{\alpha} \cdot q(\theta) + \frac{r}{\alpha}.$$

When ψ increases in recessions, the wage w remains constant but $q(\theta)$ decreases (because θ increases) so the right-hand side of the equation decreases. Hence $q(\theta) \cdot n^{\alpha - 1}$ decreases. The elasticity wedge ϵ^m / ϵ^M therefore decreases. While the elasticity wedge was countercyclical in the model in the text, the wedge is procyclical here. In general, we cannot conclude on the cyclicality of ϵ^M and of the optimal replacement rate. Hence, we resort to simulations to study the cyclicality of the optimal replacement rate.

Simulations. The simulation results are displayed in Figure A3. All computations are based on the dynamic model calibrated in Table 2 (the calibration does not need to change even if the source of shock is different). The optimal replacement rate is procyclical: it increases from 58% to 71% when the unemployment rate decreases from 10% to 4%. But this model of the business cycle is unrealistic because labor market tightness increases sharply in recessions.

D.4 Optimal unemployment insurance and wage subsidy

We start by describing the labor market equilibrium under technology a, when the replacement rate is $\tau = c^e/c^u$ and the normalized wage is $\tilde{w} = w/a$. The firm's first-order condition (18) can be written as

$$g'(n) = \tilde{w} + \frac{r}{q(\theta)},$$

which implicitly defines a labor demand $n^d(\theta, \tilde{w})$. Under Assumption 5, $v(c) = \ln(c)$. The equilibrium condition (21) becomes

$$n^s(f(\theta), \ln(1/\tau)) = n^d(\theta, \tilde{w}),$$

which implicitly defines equilibrium labor market tightness $\theta(\tau, \tilde{w})$. Furthermore, we define equilibrium employment $n(\tau, \tilde{w}) \equiv n^s(f(\theta(\tau, \tilde{w})), \ln(1/\tau))$. Lemma A16 establishes how equilibrium variables respond to a change in the wage $\tilde{w}$:

LEMMA A16. *Under Assumptions 3 and 4, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium tightness $\theta(\tau, \tilde{w})$ and equilibrium employment $n(\tau, \tilde{w})$:*

$$\left.\frac{\partial \theta}{\partial \tilde{w}}\right|_{\tau} < 0, \quad \left.\frac{\partial n}{\partial \tilde{w}}\right|_{\tau} < 0.$$

Proof. Similar to the proof of Lemma A7. □

The government chooses unemployment benefit rate B, tax rate T imposed on the salary w^* received by employees, and subsidy rate σ imposed on the salary w^* paid by employers. Effectively, firms pay a wage $w = (1-\sigma) \cdot w^*$, employed workers consume $c^e = (1-T) \cdot w^*$, and unemployed workers consume $c^u = B \cdot w^*$. The government is subject to the budget constraint

$$(1-n) \cdot B \cdot w^* + n \cdot \sigma \cdot w^* = T \cdot n \cdot w^*$$
$$(1-n) \cdot B \cdot w^* + n \cdot w^* - n \cdot T \cdot w^* = n \cdot w^* - n \cdot \sigma \cdot w^*$$
$$(1-n) \cdot c^u + n \cdot c^e = n \cdot w.$$

The budget constraint remains the same as in the baseline model even though the labor tax is collected from workers and partly redistributed to firms as a wage subsidy. The budget constraint defines a function that gives the consumption of employed workers in equilibrium: $c^e(\tau, \tilde{w}, a) \equiv a \cdot \tilde{c}^e(\tau, \tilde{w})$ where

$$\tilde{c}^e(\tau, \tilde{w}) \equiv \frac{n(\tau, \tilde{w})}{n + [1 - n(\tau, \tilde{w})] \cdot \tau} \cdot \tilde{w}.$$

In equilibrium, the expected utility of a worker is

$$\ln\left(c^e(\tau, \tilde{w}, a)\right) + [1 - n(\tau, \tilde{w})] \cdot \ln(\tau) - u \cdot k\left(e(\tau, \tilde{w})\right) = \ln(a) + SW\left(\tau, \tilde{w}\right),$$

where we define the function

$$SW(\tau,\tilde{w}) \equiv \ln\left(\tilde{c}^e(\tau,\tilde{w})\right) + [1-n(\tau,\tilde{w})]\cdot\ln(\tau) - u\cdot k\left(e(\tau,\tilde{w})\right).$$

In Section 2, we maximized $SW(\tau,\tilde{w})$ over $\tau \in (0,1)$ for $\tilde{w} = \tilde{w}(a) \equiv \omega \cdot a^{\gamma-1}$ (because we made Assumption 4). The result from Proposition 6 in Section 2 tell us something about the properties of SW. Let $\tau^*(\tilde{w})$ be the function implicitly defined by

$$\frac{\partial SW(\tau,\tilde{w})}{\partial \tau} = 0.$$

Furthermore, we define the replacement rate $\tau(a) \equiv \tau^*(\tilde{w}(a))$. Under some conditions, Proposition 6 shows that $d\tau/da < 0$. Since $d\tilde{w}/da < 0$ and

$$\frac{d\tau}{da} = \frac{d\tau^*}{d\tilde{w}}\cdot\frac{d\tilde{w}}{da},$$

we infer that $d\tau^*/d\tilde{w} > 0$ (under the assumptions of Proposition 6).

Let us consider the problem of the government when the government chooses optimally both the wage $\tilde{w}$ and the replacement rate τ. To capture the various costs of implementing a wage subsidy, we assume that setting a wage $\tilde{w}$ when the technology is a imposes a welfare cost $\mathcal{C}(\tilde{w},a) > 0$. If the salary is a function $w^*(a)$ of a, a possible welfare cost could be an increasing convex function $\mathcal{C}(\sigma)$ of the subsidy rate σ. The reason is that $\sigma = [w - w^*(a)]/w^*(a) = [a\cdot\tilde{w} - w^*(a)]/w^*(a)$ so σ is only a function of $\tilde{w}$ and a. A critical assumption is that the welfare cost $\mathcal{C}$ does not depend on the replacement rate τ. The government chooses jointly τ and $\tilde{w}$ to maximize

$$\ln(a) + SW(\tau,\tilde{w}) - \mathcal{C}(\tilde{w},a).$$

The first-order condition with respect to τ is

$$\left.\frac{\partial SW(\tau,\tilde{w})}{\partial \tau}\right|_{\tilde{w}=\tilde{w}^\dagger} = 0$$

where $\tilde{w}^\dagger$ is the optimal wage. Therefore the optimal replacement rate is $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$, where $\tau^*(\cdot)$ is the function defined above. Our study of the government problem in Section 2 tell us that $\tau^*(\tilde{w})$ has the property that $d\tau^*/d\tilde{w} > 0$.

Note that the optimal wage $\tilde{w}^\dagger(a)$ is defined implicitly by the first-order condition

$$\left.\frac{\partial SW(\tau,\tilde{w})}{\partial \tilde{w}}\right|_{\tau=\tau^*(\tilde{w})} - \left.\frac{\partial \mathcal{C}}{\partial \tilde{w}}\right|_{a} = 0.$$

Assume that the replacement rate $\tau^\dagger$ is fixed. There is a technology shock from a to a' such that employment decreases after the optimal wage is adjusted from $\tilde{w}^\dagger(a)$ to $\tilde{w}^\dagger(a')$. Lemma A16 implies that $\tilde{w}^\dagger(a) < \tilde{w}^\dagger(a')$. Since the optimal replacement rate is solely a function of the optimal wage: $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$ with $d\tau^*/d\tilde{w} > 0$, $\tau^\dagger$ must increase. Therefore after an adverse shock that increases unemployment, the optimal replacement rate increases. The substantive conclusion of Proposition 6 is robust to the presence of wage subsidies: optimal UI is more generous when unemployment is high.