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ABSTRACT

We use an equilibrium unemployment model with rigid wages and diminishing marginal returns to labor to analyze the cyclicality of optimal unemployment insurance (UI). To guide our analysis, we represent the labor market equilibrium with a labor supply-labor demand diagram in which labor market tightness acts as a price. A simple formula resolves the trade-off between insurance and job-search incentives. The formula relates the optimal replacement rate of UI to usual statistics—risk aversion and microelasticity of unemployment with respect to UI—and to a macroelasticity capturing the general-equilibrium effect of UI on unemployment. We calibrate the formula using empirical estimates of those sufficient statistics. Optimal UI is countercyclical because the trade-off tilts towards insurance in recessions. Indeed in recessions, the social value of search is low because jobs are rationed. Rationing implies that (i) aggregate search efforts cannot reduce unemployment much and (ii) individual search efforts create a negative externality by reducing the job-finding rate of others as in a rat race.

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1 Introduction

Increasing unemployment insurance (UI) helps workers smooth consumption when they become unemployed, but it also increases unemployment by discouraging job search. Optimal UI equalizes the marginal benefit of smoothing consumption with the marginal cost of increasing unemployment. Most models of UI assume that unemployment depends solely on search efforts [for example, Baily, 1978; Chetty, 2006a; Hopenhayn and Nicolini, 1997; Shavell and Weiss, 1979]. Yet the reality is more complex. During the Great Depression, unemployed workers queued for jobs at factory gates. In a queue, a jobseeker does increase his job-finding probability by searching more and pushing his way up the queue; but because the number of available jobs is limited, those in front of him in the queue fall behind and face a lower job-finding probability. Overall, unemployment depends not only on search efforts but also on the aggregate demand for labor.

This paper models the dependence of unemployment on search efforts and aggregate labor demand to understand more generally the properties of optimal UI. Our main finding is that optimal UI increases in recessions. The reason is that jobs are rationed in recessions. Rationing implies that firms are reluctant to hire workers irrespective of search efforts. Therefore, reducing UI would not reduce unemployment much, even though it would increase search efforts. Furthermore, reducing UI would exacerbate a negative externality similar to that imposed by jobseekers on each other in the queues of the Great Depression.

We begin in Section 2 by deriving a formula for the optimal replacement rate—the generosity of unemployment benefits as a fraction of the income of employed workers— in a static model. A critical feature of the model, and its departure from the standard Baily [1978] model, is that the job-finding rate of jobseekers responds to UI. This feature captures the general-equilibrium adjustment of the job-finding rate after a change in UI once the response of both jobseekers and firms is taken into account. The formula, expressed with sufficient statistics, does not require much structure on the primitives of the model. As in the Baily [1978] formula, a first term captures the trade-off between the need for insurance, measured by a coefficient of risk aversion, and the need for job search incentives, measured by an elasticity of unemployment with respect to UI. But we replace the microelasticity used in the Baily formula by a macroelasticity to measure the budgetary costs of

UI in general equilibrium. The microelasticity ϵ^m is the elasticity of the probability of unemployment for a worker whose individual benefits change, and the macroelasticity ϵ^M is the elasticity of aggregate unemployment when benefits changes for all workers. Formally, ϵ^m takes the job-finding rate as given, whereas ϵ^M accounts for the general-equilibrium adjustment of the job-finding rate after a change in UI. Moreover, our formula adds to the Baily formula a second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$. This wedge captures the welfare effect of the employment change arising from the general-equilibrium adjustment of the job-finding rate.

Our optimal UI formula has two advantages. First, it is very general. It applies to models that are more realistic than the simple model of Section 2. Following the approach of Gruber [1997] and Chetty [2006a], we show that the formula applies when workers partially insure themselves against unemployment. The formula also applies in a dynamic model in which jobs are continuously created and destroyed. Second, the formula can be calibrated using available empirical estimates of the sufficient statistics. Our formula shows that the optimal replacement rate falls between 40% and 60% for a range of realistic empirical estimates.

In Section 3, we characterize the optimal replacement rate over the business cycle by applying our formula to the labor market model of Michaillat [2012a]. The model builds on the equilibrium unemployment framework of Pissarides [2000] to capture two critical elements of the business cycle: (i) unemployment fluctuations, and (ii) job rationing—the property that the labor market does not converge to full employment in recessions even when search efforts are arbitrarily large. Feature (i) arises from technology shocks and real wage rigidity. Feature (ii) arises from the combination of wage rigidity and diminishing marginal returns to labor. Because of (i), the model accommodates periods of high unemployment and periods of low unemployment. Because of (ii), the model is consistent with the queues for jobs formed by unemployed workers in recessions.[1]

Our analysis rests on a new representation of the labor market equilibrium in a labor supply-labor demand diagram. Because search costs are sunk when a worker and a firm meet, a surplus arises from their match. Any wage sharing the surplus could be an equilibrium wage. Hence wages cannot equalize labor supply to labor demand. Instead labor market tightness—the ratio of

[1] Michaillat [2012a] shows that standard equilibrium unemployment models do not have job rationing. When job-search efforts are arbitrarily large, these models always converge to full employment, even in recessions.

vacancies to aggregate search effort—equalizes supply and demand. This property allows us to represent the equilibrium in a labor supply-labor demand diagram in which labor market tightness acts as a price, as depicted in Figure 1(a). The representation is general. If labor demand is perfectly elastic, unemployment depends solely on search efforts as in Baily [1978] and Chetty [2006a]. At the polar opposite if labor demand is perfectly inelastic, unemployment is independent of search efforts as in a rat race. Figure 1(b) illustrates these two special cases.

We first prove that the macroelasticity ϵ^M is smaller than the microelasticity ϵ^m, creating a wedge $\epsilon^m/\epsilon^M - 1 > 0$. The wedge arises because when the number of jobs available is limited, searching more to increase one's probability of finding a job mechanically decreases others' probability of finding one of the few jobs available. Because of this wedge, our formula calls for a higher replacement rate than the Baily formula. As jobseekers search taking the job-finding rate as given, without internalizing their influence on the job-finding rate of others, they impose a negative *rat-race externality*. A higher replacement rate corrects the externality by discouraging job search.

Next, we prove that the wedge $\epsilon^m/\epsilon^M - 1$ is countercyclical and the macroelasticity procyclical. Recessions are periods of acute job shortage during which job search and matching frictions have little influence on labor market outcomes. The search efforts of jobseekers have little influence on aggregate unemployment and the rat-race externality is exacerbated. Thus the macroelasticity is small and the wedge between microelasticity and macroelasticity is large.

Finally, we use our formula to prove that the optimal replacement rate is countercyclical. In recessions the macroelasticity falls. A higher UI only increases unemployment negligibly. Hence the marginal budgetary cost of UI is small. In recessions the wedge $\epsilon^m/\epsilon^M - 1$, which measures the welfare cost of the rat-race externality, increases. Hence the marginal benefit of UI from correcting the externality is high.

The property that $\epsilon^m/\epsilon^M > 1$ distinguishes our model from standard models of equilibrium unemployment: $\epsilon^m/\epsilon^M < 1$ in the canonical model with Nash bargaining; and $\epsilon^m/\epsilon^M = 1$ if bargaining is replaced by rigid wages. Crepon, Duflo, Gurgand, Rathelot and Zamora [2012] provide recent compelling empirical evidence that $\epsilon^m/\epsilon^M > 1$ using a large randomized experiment. They find that $\epsilon^m/\epsilon^M = 1.58$ on average. As predicted by our model, they also find that ϵ^m/ϵ^M is countercyclical. The optimal replacement rate based on their estimates is strongly countercyclical and increase by 10 to 15 percentage points from expansions to recessions.

In Section 4, we connect our results to the Baily [1978] formula for optimal UI and the Hosios [1990] condition for efficiency in equilibrium unemployment models. If the government can jointly optimize UI and labor market tightness, both the Baily formula and the Hosios condition apply at the optimum. In our framework, wages paid by firms and tightness are directly related by a labor demand equation. Thus, if payroll tax incidence were fully on firms, the government could control tightness by controlling wages using a payroll tax or subsidy. However, if we realistically assume that the government cannot control tightness, we obtain our generalized optimal UI formula that adds to the Baily formula a term measuring the deviation from the generalized Hosios condition. If the deviation from the Hosios condition is positive, optimal UI is above the Baily formula.

In Section 5, we show numerically that optimal UI remains countercyclical when the government adjusts the duration instead of the level of benefits. When unemployment benefits never expire, the optimal replacement rate is strongly countercyclical: it increases from 45% to 59% when the unemployment rate increases from 4% to 10%. When the government adjusts the duration of unemployment benefits, as in the United States, the optimal duration is strongly countercyclical: it increases from less than 6 weeks when unemployment is 4%; to 26 weeks when unemployment is 5.9%; and to over 100 weeks when unemployment reaches 10%.

We conclude in Section 6. Proofs, derivations, and extensions are collected in the Appendix.

2 Optimal Unemployment Insurance Formula

This section presents a generic static model of the labor market. Since workers are risk averse, cannot insure themselves against unemployment, and cannot be monitored during job search, the government must trade off the provision of unemployment insurance with the provision of job-search incentives. In addition, the job-finding rate of jobseekers responds to UI in equilibrium. We derive a formula solving the government's problem. The formula expresses the optimal replacement rate of UI as a function of four sufficient statistics: two standard statistics (risk aversion and microelasticity of unemployment with respect to UI) and two new statistics (macroelasticity of unemployment with respect to UI and elasticity of search with respect to job-finding rate).

2.1 Model

Labor market. There is a measure 1 of workers. Initially, $u \in (0, 1)$ workers are unemployed and $1 - u$ workers are employed. Unemployed workers search for a job with effort e, which is not observable. A jobseeker finds a job at a rate f per unit of effort; thus, a jobseeker searching with effort e finds a job with probability $e \cdot f$. A fraction $e \cdot f$ of the u unemployed workers find jobs, so the number of new hires is $h = u \cdot e \cdot f$. The new hires join incumbents in firms so the employment level after matching is

$$n = 1 - u + u \cdot e \cdot f.$$

Workers. Firms pay a wage w. To finance unemployment benefits $b \cdot w$, the government imposes a labor tax t. As in the public finance literature, the incidence of the tax is entirely on the worker's side, so w does not respond to t. Workers cannot save, borrow, or insure themselves against unemployment in other ways. Employed workers consume their post-tax labor income $c^e = w \cdot (1 - t)$ and unemployed workers consume unemployment benefits $c^u = b \cdot w$. The consumption gain from work $\Delta c \equiv c^e - c^u$ and the utility gain from work $\Delta v \equiv v(c^e) - v(c^u)$ measure the generosity of UI. Given job-finding rate f and utility gain from work Δv, a jobseeker chooses effort e to maximize expected utility

$$v(c^u) + e \cdot f \cdot \Delta v - k(e), \tag{1}$$

where consumption utility $v(c)$ is an increasing and concave function and search disutility $k(e)$ is an increasing and convex function. The optimal effort is a function $e(f, \Delta v)$ implicitly defined by the first-order condition

$$k'(e) = f \cdot \Delta v. \tag{2}$$

As $k(e)$ is convex, $e(f, \Delta v)$ increases with Δv and f. In other words, search efforts increase when UI becomes less generous. Search efforts also increase when jobs become easier to find.

Equilibrium. We represent the equilibrium in a labor supply-labor demand framework. The labor supply

$$n^s(f, \Delta v) \equiv 1 - u + u \cdot e(f, \Delta v) \cdot f \tag{3}$$

is the employment rate after matching when jobseekers search optimally. Labor supply increases with f and Δv as $e(f, \Delta v)$ increases with f and Δv. In other words, labor supply increases when jobs become easier to find, mechanically and through an increase in search efforts. Labor supply also increases when UI becomes less generous, through an increase in search efforts.

The job-finding rate f measures labor market conditions. These conditions affect labor supply but also labor demand from firms. In general equilibrium, labor supply equals labor demand, which determines labor market conditions and f. Naturally, the generosity Δv of the UI system influences search effort and labor supply. Hence, it will generally influence the equilibrium job-finding rate. In Section 3, we impose more structure on the labor market and derive a labor supply, labor demand, and equilibrium job-finding rate. For now, we write equilibrium job-finding rate as a function $f(\Delta v)$ of the UI system. Equilibrium employment $n(\Delta v)$ can be read off the labor supply at the equilibrium job-finding rate:

$$n(\Delta v) = n^s(f(\Delta v), \Delta v). \tag{4}$$

Our framework is quite general. It nests the Baily model as a special case. In the Baily model, employment is solely driven by search efforts. It is a partial-equilibrium model of unemployment in the sense that it fixes the job-finding rate f. At the polar opposite, our framework also nests the rat-race model as a special case. In the rat-race model, the number of jobs is fixed: u jobseekers queue in front of $o < u$ vacant jobs. The equilibrium job-finding rate $f = o/(u \cdot e)$ equates labor supply $1 - u + u \cdot e \cdot f$ with the fixed labor demand $1 - u + o$. The equilibrium job-finding rate responds to UI because it depends on aggregate search efforts, which respond to UI.

Unemployment insurance. The government chooses the consumption gain from work Δc and the consumption c^e of employed workers (this is equivalent to choosing the rate b of unemployment benefits and the labor tax rate t). This choice determines the consumption $c^u = c^e - \Delta c$ of unemployed workers. The goal of the government is to maximize welfare

$$n \cdot v(c^e) + (1-n) \cdot v(c^u) - u \cdot k(e). \tag{5}$$

The government balances its budget each period by financing outlays of unemployment benefits with a labor tax. Therefore, the government is subject to the budget constraint $(1-n) \cdot b \cdot w = n \cdot t \cdot w$, which we rewrite as

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w. \tag{6}$$

Given that the government chooses Δc and c^e and not Δv directly, it is convenient to redefine job-finding rate, labor supply, and employment as functions of $(\Delta c, c^e)$. We define $\Delta v = \Delta v(\Delta c, c^e) \equiv v(c^e) - v(c^e - \Delta c)$. Abusing notations slightly, we define $f(\Delta c, c^e) \equiv f(\Delta v(\Delta c, c^e))$, $n^s(f, \Delta c, c^e) \equiv n^s(f, \Delta v(\Delta c, c^e))$, and $n(\Delta c, c^e) \equiv n(\Delta v(\Delta c, c^e))$. The government accounts for the facts that (i) search effort e is chosen optimally by workers; (ii) job-finding rate $f = f(\Delta c, c^e)$ depends on UI; and (iii) employment n is read off the labor supply: $n = n^s(f, \Delta c, c^e)$.

2.2 Microelasticity and macroelasticity

To solve the government's problem, we need to characterize the partial-equilibrium response of jobseekers and the general-equilibrium response of the labor market to a change in UI. Unlike the partial-equilibrium response, the general-equilibrium response accounts for the adjustment of the job-finding rate after a change in UI. The responses are measured by two elasticities:

DEFINITION 1. The *microelasticity* of unemployment with respect to the consumption gain from work is

$$\epsilon^m \equiv \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n^s}{\partial \Delta c}\right|_{f, c^e}. \tag{7}$$

The *macroelasticity* of unemployment with respect to the consumption gain from work is

$$\epsilon^M \equiv \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n}{\partial \Delta c}\right|_{c^e}. \tag{8}$$

The elasticities are normalized to be positive. They are computed keeping the consumption c^e of employed workers constant; that is, c^e does not adjust to meet the budget constraint of the government. The microelasticity measures the percentage increase in unemployment $1-n$ when the net reward from work Δc decreases by 1%, taking into account jobseekers' reduction in search effort but ignoring the equilibrium adjustment of the job-finding rate f. It can be estimated by measuring the reduction in the job-finding probability of an individual unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant. The macroelasticity measures the percentage increase in unemployment when the net reward from work decreases by 1%, assuming that both search effort and job-finding rate adjust. It can be estimated by measuring the increase in aggregate unemployment following a general increase in unemployment benefits financed by deficit spending.

To relate microelasticity ϵ^m and macroelasticity ϵ^M, we introduce an elasticity that characterizes the response of jobseekers to a change in labor market conditions:

DEFINITION 2. The *discouraged-worker elasticity* ϵ^d is the elasticity of search effort with respect to the job-finding rate:

$$\epsilon^d \equiv \frac{f}{e} \cdot \left.\frac{\partial e}{\partial f}\right|_{\Delta v}.$$

If $\epsilon^d > 0$, workers search less when it becomes more difficult to find a job. In other words, $\epsilon^d > 0$ captures the discouragement of jobseekers when labor market conditions deteriorate. Lemma 1 shows that ϵ^m and ϵ^M admit a simple relationship:

LEMMA 1. *Microelasticity ϵ^m and macroelasticity ϵ^M are related by*

$$\epsilon^M = \epsilon^m + \frac{h}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \frac{\Delta c}{f} \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e}.$$

If the job-finding rate f is independent of the consumption gain from work Δc, $\epsilon^M = \epsilon^m$.

The elasticities ϵ^m and ϵ^M differ when the equilibrium job-finding rate responds to the consumption gain from work. The model of a rat race discussed above, in which the number of vacant jobs is fixed, illustrates the difference. In the rat-race model, $\epsilon^M = 0$ even though $\epsilon^m > 0$.

2.3 Formula

Proposition 1 provides a formula for the optimal replacement rate τ, defined as the amount c^u transferred to unemployed workers expressed as a fraction of the income c^e of employed workers. The replacement rate measures the generosity of the UI system. The formula can be expressed with four sufficient statistics. Two of the statistics are standard: coefficient of relative risk aversion and microelasticity of unemployment with respect to UI. Two of the statistics are new and appear because we use a general-equilibrium model of the labor market: the macroelasticity of unemployment with respect to UI, and the discouraged-worker elasticity.

PROPOSITION 1. *The optimal replacement rate $\tau \equiv c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right). \tag{9}$$

The Lagrange multiplier ϕ on the government's budget constraint satisfies the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right]. \tag{10}$$

If $n \approx 1$ and if the third and higher order terms of $v(c)$ are small, the formula simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right], \tag{11}$$

where $\rho \equiv -c^e \cdot v''(c^e)/v'(c^e)$ is the coefficient of relative risk aversion evaluated at c^e. If the job-finding rate is independent of the consumption gain from work, $\epsilon^m = \epsilon^M$, the second term in the right-hand side of (9) and (11) vanishes, and the formulas reduce to those in Baily [1978] and Chetty [2006a].

The formal proof of the proposition is relegated to the Appendix. We propose an informal version of the proof here, based on marginal deviations from the optimum. Let ϕ be the Lagrange multiplier on the budget constraint (6). Note that $h = u \cdot e \cdot f$, $n = 1 - u \cdot (1 - e \cdot f)$, and $c^u = c^e - \Delta c$. The Lagrangian of the government's problem is

$$\mathcal{L} = v(c^e) - u\left[1 - ef(\Delta c, c^e)\right]\left[v(c^e) - v(c^e - \Delta c)\right] - uk(e) + \phi\left[n(\Delta c, c^e)\,(w - \Delta c) - c^e + \Delta c\right],$$

where e maximizes (1).

First, consider changes $dc_e = dc/v'(c^e)$ and $dc^u = dc/v'(c^u)$. The changes have no first-order impact on $\Delta v = v(c^e) - v(c^u)$, and hence no impact on effort $e(f, \Delta v)$, job-finding rate $f(\Delta v)$, and employment $n(\Delta v)$. The effect on social welfare is $dSW = n \cdot v'(c^e) \cdot dc^e + (1-n) \cdot v'(c^u) \cdot dc^u = dc$. The effect on the expenditure of the government is $dX = -n \cdot dc^e - (1-n) \cdot dc^u = -dc \cdot \{[n/v'(c^e)] + [(1-n)/v'(c^u)]\}$. At the optimum, $d\mathcal{L} = dSW + \phi dX = 0$, which establishes the inverse Euler equation (10).

Next, consider a change $d\Delta c$, keeping c^e constant. We apply the envelope theorem as workers choose effort e optimally. The first-order condition $\partial \mathcal{L}/\partial \Delta c = 0$ implies that

$$0 = -v'(c^u) \cdot (1-n) + \Delta v \cdot u \cdot e \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e} + \phi \cdot \left[1 - n + (w - \Delta c) \cdot \left.\frac{\partial n}{\partial \Delta c}\right|_{c^e}\right],$$

which, dividing by $\phi \cdot (1-n)$, can be rearranged as

$$0 = \left[1 - \frac{v'(c^u)}{\phi}\right] + \frac{w - \Delta c}{\Delta c} \cdot \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n}{\partial \Delta c}\right|_{c^e} + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{\Delta c}{1-n} \cdot \frac{h}{f} \cdot \left.\frac{\partial f}{\partial \Delta c}\right|_{c^e}.$$

Lemma 1 shows that the last term, capturing the welfare effect of a general-equilibrium adjustment df, is proportional to the wedge $\epsilon^m - \epsilon^M$. Using the lemma and the definitions of ϵ^M and ϕ, we rewrite the first-order condition as

$$0 = n \cdot \left[1 - \frac{v'(c^u)}{v'(c^e)}\right] + \frac{w - \Delta c}{\Delta c} \cdot \epsilon^M + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1 + \epsilon^d} \cdot \left[\epsilon^M - \epsilon^m\right]. \tag{12}$$

Since $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c$, we rearrange (12) as (9), which completes the derivation.

The first term in (12) reflects the welfare effect of transferring resources from the unemployed to the employed by increasing Δc. As long as $c^u < c^e$ and workers are risk averse, this term is negative. The second term in (12) captures the desirable effect on the budget of reducing the unemployment rate by increasing Δc. This budgetary effect is due to workers' behavioral responses in general equilibrium and hence involves the macroelasticity ϵ^M. The third term in (12) captures the welfare effect due to the change in the job-finding rate f in general equilibrium. It is proportional to the elasticity wedge $\epsilon^m - \epsilon^M$. It is negative when $\epsilon^m > \epsilon^M$ because in that case, general-equilibrium effects destroy jobs and hence reduce welfare.

Equation (9) provides an exact formula while equation (11) provides a simpler formula using the approximation method of Chetty [2006a]. The first term in the optimal replacement rate (11) increases with the coefficient of relative risk aversion ρ, which measures the value of insurance. If micro- and macroelasticity are equal ($\epsilon^m = \epsilon^M$), our formulas reduce to the Baily formula. For instance the approximated formula (11) becomes $\tau/(1-\tau) \approx (\rho/\epsilon^m) \cdot (1-\tau)$. In the formula the trade-off between the need for insurance (captured by the coefficient of relative risk aversion ρ) and the need for incentives to search (captured by the microelasticity ϵ^m) appears transparently. In a model of equilibrium unemployment micro- and macroelasticity generally differ ($\epsilon^m \neq \epsilon^M$), and our formula presents two departures from the Baily formula.

The first term in the right-hand side of formulas (9) and (11) involves the macroelasticity ϵ^M and not the microelasticity ϵ^m, conventionally used to calibrate optimal benefits [Chetty, 2008; Gruber, 1997]. What matters for the government is the budgetary cost of UI from higher aggregate unemployment and higher outlays of unemployment benefits, and only ϵ^M captures this cost in an equilibrium unemployment framework. The optimal replacement rate naturally decreases with ϵ^M.

A second term, increasing with the ratio ϵ^m/ϵ^M, also appears in the right-hand side of formulas (9) and (11) when $\epsilon^m \neq \epsilon^M$. The term is a correction that accounts for the first-order welfare effects of the adjustment of employment that arises from the equilibrium adjustment of the job-finding rate f after a change in UI. Even in the absence of any concern for insurance—if workers are risk neutral—some unemployment insurance should be provided as long as the correction term is positive ($\epsilon^m/\epsilon^M > 1$).

Formula (11) is expressed in sufficient statistics, which means that the formula is robust to changes in the primitives of the model. Indeed the formula is valid for any utility over consumption with coefficient of relative risk aversion ρ; any job search behavior with discouraged-worker elasticity ϵ^d and microelasticity ϵ^m; and any labor demand yielding a macroelasticity ϵ^M. We now present estimates for these four sufficient statistics and show how to implement the formula.

2.4 Implementation of the formula

We combine our optimal UI formula with empirical estimates of the sufficient statistics to determine optimal UI. First, we extend the optimal UI formula (11) to account for the ability of workers to partially self-insure against unemployment shocks. This extension is necessary to obtain credible values for the optimal replacement rate because in the data, self-insurance is quantitatively important. Second, we identify studies that estimate empirically the needed sufficient statistics. Third, we bring those estimates into the formula to assess optimal UI.

Optimal UI with partial self-insurance. Let us assume that workers can partially self-insure with home production while unemployed. Formally, unemployed workers home produce an amount y at a utility cost $m(y)$, increasing and convex. They choose y to maximize $v(c^u + y) - m(y)$. Proposition 2 establishes that, as in Chetty [2006a], formulas (9) and (11) carry over with minor modifications in presence of self-insurance:

PROPOSITION 2. *If workers have access to home production when they are unemployed, the optimal replacement $\tau = c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right), \qquad (13)$$

where $c^h \equiv c^u + y$ is consumption while unemployed, and $\Delta v^h \equiv \min_y \left\{v(c^e) - \left[v(c^h) - m(y)\right]\right\}$ is the difference in utility between being employed and being unemployed. The Lagrange multiplier ϕ on the government's budget constraint satisfies the inverse Euler equation $1/\phi = [n/v'(c^e)] + \left[(1-n)/v'(c^h)\right]$. With the normalization $m(y) = 0$ at the optimum, if $n \approx 1$ and if the third and *higher order terms of $v(c)$ are small, the formula simplifies to*

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\xi) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\xi)\right] \cdot \left[\frac{1-\xi}{1-\tau}\right], \tag{14}$$

where $\xi = c^h/c^e$ is the consumption drop upon unemployment.

The proposition shows that the structure of the optimal UI formula does not change when workers have access to partial self-insurance. But if workers can partially self-insure, the optimal replacement rate tend to be lower than in the model without self-insurance because the insurance value $\left[v'(c^h)/v'(c^e)\right] - 1$ of UI is smaller as $c^h \geq c^u$.[2]

The proposition also expresses the optimal UI formula with five sufficient statistics: microelasticity ϵ^m, macroelasticity ϵ^M, discouraged-worker elasticity ϵ^d, coefficient of relative risk aversion ρ, and consumption drop upon unemployment $\xi = c^h/c^e$.[3] Naturally, the consumption drop depends on the replacement rate $\tau = c^u/c^e$. Without self-insurance, $\xi = \tau$. With partial self-insurance, around the current replacement rate $\hat{\tau}$, $\xi \approx \hat{\xi} + \epsilon^i \cdot (\tau - \hat{\tau})$, where $\hat{\xi}$ is the current consumption drop and $\epsilon^i \equiv -\partial c^h/\partial \Delta c|_{c^e}$ is the marginal effect of unemployment benefits on total unemployment consumption. The statistics ϵ^i measures the availability of self-insurance: without self-insurance, $\epsilon^i = 1$; with perfect self-insurance, $\epsilon^i = 0$. Using this linear expression for ξ, formula (14) expresses τ as a function of seven sufficient statistics: $\hat{\tau}$, $\hat{\xi}$, ϵ^i, ρ, ϵ^m, ϵ^M, and ϵ^d.

Empirical estimates of $\hat{\tau}$, $\hat{\xi}$, ϵ^i, ρ, ϵ^m, ϵ^M, and ϵ^d. In the US, weekly unemployment benefits replace between 50% and 70% of the last weekly pre-tax earnings of a worker [Pavoni and Violante, 2007]. Following Chetty [2008] we set the benefit rate to 50%. Since earnings are subject to a 7.65% payroll tax, we set the current replacement rate to $\hat{\tau} = 0.5/(1-0.0765) = 0.54$.

The ratio $\hat{\xi}$ captures the consumption drop upon unemployment in the current system. For food consumption φ, Gruber [1997] estimates $\left[\varphi^h - \varphi^e\right]/\varphi^e = -0.068$. As emphasized by Brown-

[2]In addition, the welfare effect of the equilibrium adjustment of the job-finding rate is dampened because $\Delta v^h \leq \Delta v = v(c^e) - v(c^u)$. Whether the optimal replacement rate increases or decreases as a result depends on the sign of the elasticity wedge $(\epsilon^m/\epsilon^M) - 1$.

[3]The proposition normalizes the cost of home production such that at the optimum, it is the same as the cost of holding a job in a firm. Hence at the optimum, $m(y) = 0$ and $\Delta v^h = v(c^e) - v(c^h)$.

ing and Crossley [2001], however, total consumption is more elastic than food consumption to a change in income. Using the estimates of Hamermesh [1982], we find that the elasticity of food consumption with respect to aggregate income for unemployed workers is 0.36, including food consumed at home and away from home.[4] Accordingly we expect that $1-\hat{\xi} = \left[c^e - c^h\right]/c^e = \left(\left[\varphi^e - \varphi^h\right]/\varphi^e\right)/0.36 = 0.068/0.36 = 0.19$ and $\hat{\xi} = 0.81$.

Gruber [1997] also estimates $-d\varphi^h/d\Delta c = 0.27$. Using again the estimates of Hamermesh [1982], we expect that $dc^h = d\varphi^h/0.36$ and hence $\epsilon^i = 0.27/0.36 = 0.75$. In words, increasing unemployment benefits by \$1 increases total consumption when unemployed by \$0.75.

Many studies estimate the coefficient of relative risk aversion [Chetty, 2004, 2006b]. We choose a coefficient of relative risk aversion $\rho = 1$, on the low side of available estimates. Naturally, the higher the coefficient of risk aversion, the more generous optimal UI.

Many studies estimate ϵ^m (see Krueger and Meyer [2002] for a survey). The ideal experiment to estimate ϵ^m is to compare individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics. Most studies evaluate the elasticity ϵ^s of the job-finding rate with respect to benefits. This elasticity approximately equals ϵ^m in normal circumstances.[5] In US administrative data from the 1980s, the classic study of Meyer [1990] finds an elasticity of 0.9 with few individual controls and 0.6 with more individual controls. In a larger US administrative dataset from the early 1980s, and using a regression kink design to better identify the elasticity, Landais [2012] finds an elasticity around 0.3.

The elasticity ϵ^M can be estimated by measuring the elasticity wedge ϵ^m/ϵ^M and using the estimates for ϵ^m described above.[6] The elasticity wedge captures the externalities of job search

[4] Hamermesh [1982] estimates that for unemployed workers the permanent-income elasticity of food consumption at home is 0.24 while that of food consumption away from home is 0.82. He also finds that in the consumption basket of an unemployed worker, the share of food consumption at home is 0.164 while that of food consumption away from home is 0.041. Therefore the aggregate income elasticity of food consumption is $0.24 \times [0.164/(0.164+0.41)] + 0.82 \times [0.041/(0.164+0.41)] = 0.36$.

[5] Equation (A49) in the Appendix gives the relationship between ϵ^m and ϵ^s in the steady-state of the dynamic model and shows that $\epsilon^m \approx \epsilon^s$ when $\tau/(1-\tau) \approx 1$ and $u \ll 1$.

[6] The ideal experiment to estimate ϵ^M is to offer higher unemployment benefits to all individuals in a randomly selected subset of labor markets and compare unemployment durations across treated and control labor markets. Hence, estimating the macroelasticity ϵ^M is inherently more difficult than estimating a microelasticity ϵ^m because it necessitates exogenous variations in benefits across comparable labor markets, instead of exogenous variations across comparable individuals within a single labor market. To circumvent the difficulty of directly estimating ϵ^M, it is easier

on other jobseekers. The most convincing study to date estimating such externalities is Crepon et al. [2012]. They analyze a large randomized field experiment in France in which some young educated jobseekers are treated by receiving job placement assistance. The experiment has a double-randomization design: (1) some areas are treated and some are not, (2) within treated areas some jobseekers are treated and some are not. We interpret the treatment as an increase in search effort from e^C for control jobseekers to e^T for treated jobseekers. We present their results for all workers finding long term employment. Compared to control jobseekers in the same area, treated jobseekers face a higher job-finding probability: $\left[e^T - e^C\right] \cdot f^T = 5.7\%$. But compared to control jobseekers in control areas, control jobseekers in treated areas face a lower job-finding probability: $e^C \cdot \left[f^T - f^C\right] = -2.1\%$. Therefore the increase in the job-finding probability of treated jobseekers in treated areas compared to control jobseekers in control areas is only $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right] = 5.7 - 2.1 = 3.6\%$.[7] By definition, the microelasticity ϵ^m is proportional to $\left[e^T - e^C\right] \cdot f^T$ and the macroelasticity ϵ^M is proportional $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right]$. These empirical results imply a wedge $\epsilon^m/\epsilon^M = 5.7/3.6 = 1.58$.[8]

There is little empirical work estimating the elasticity ϵ^d of job search effort with respect to the job-finding rate. Empirically, ϵ^d seems to be close to zero because labor market participation and other measures of search intensity are, if anything, slightly countercyclical even after controlling for changing characteristics of unemployed workers over the business cycle [Shimer, 2004].[9]

Optimal replacement rate τ. Table 1 presents the optimal replacement rate of UI based on empirical estimates of the sufficient statistics. Because of the uncertainty about the value of ϵ^m, ϵ^m/ϵ^M, and ϵ^d, we present the optimal replacement rate for a range of plausible estimates. We

to combine estimates of ϵ^m with estimates of ϵ^m/ϵ^M to recover an estimate of ϵ^M.

[7] Those estimates are from Table 10, column 1, panel B in Crepon et al. [2012].

[8] Using a large change in UI duration for a subset of workers in a subset of geographical areas in Austria, Lalive, Landais and Zweimuller [2012] also estimate significant search externalities that translate into a wedge $\epsilon^m/\epsilon^M \approx 1.33 > 1$ of similar magnitude as estimates in Crepon et al. [2012].

[9] The empirical finding that ϵ^d is small is consistent with the theoretical properties of ϵ^d in a dynamic model. In a dynamic model (as in Section 5), ϵ^d is equivalent to an uncompensated elasticity; therefore when the unemployment rate u is small, ϵ^d is close to zero. Indeed, Lemma A14 in the Appendix shows that $\epsilon^d \approx u \cdot \{k'(e)/\left[e \cdot k''(e)\right]\}$ in the dynamic model. Notice that the theoretical properties of ϵ^d are different in a static model. In a static model (as in this section), ϵ^d is equivalent to a Frisch elasticity; therefore, ϵ^d is not necessarily small. Indeed, Lemma A2 in the Appendix shows that $\epsilon^d = k'(e)/\left[e \cdot k''(e)\right] > 0$ in the static model. The advantage of our sufficient statistic approach is to set ϵ^d based on empirical evidence rather than the primitives of the model.

consider two possible values for ϵ^d: 0 and 0.5. Column (1) considers $\epsilon^m/\epsilon^M = 1$ as in the standard Baily model whereas column (2) considers $\epsilon^m/\epsilon^M = 1.58$ based on Crepon et al. [2012]. Columns (3) and (4) will be discussed in Section 3. To span the range of empirical estimates, panels A, B, and C consider the cases $\epsilon^m = 0.3$, $\epsilon^m = 0.6$, and $\epsilon^m = 0.9$.

Three results are noteworthy. First, and consistent with earlier simulations of the Baily model by Gruber [1997], optimal replacement rates in column (1) are fairly modest and actually below the current replacement $\hat{\tau} = 0.54$ even for a low value $\epsilon^m = 0.3$. Second, column (2) shows that introducing an elasticity wedge $\epsilon^m/\epsilon^M = 1.58$ increases the replacement rate across all rows by about 10 percentage points. The percentage point increases are somewhat higher for low values of ϵ^m. Thus, our extension of the Baily model has sizable effects on the optimal replacement rate for reasonable empirical estimates. Third, the elasticity ϵ^d has only modest effects on the optimal replacement rate, at least within the range of estimates considered.

3 Unemployment Insurance in Presence of Job Rationing

In this section, we specialize the generic model of Section 2 to obtain the equilibrium unemployment model of Michaillat [2012a]. The model captures two critical elements of the business cycle: unemployment fluctuations and job rationing in recessions. We propose a new representation of the labor market equilibrium using a labor supply-labor demand diagram in which labor market tightness acts as a price. The representation illustrates how the job-finding rate responds to a change in UI. After a change in UI, the labor supply shifts, which leads to an equilibrium adjustment in labor market tightness and to an adjustment in job-finding rate because the rate is a function of tightness. In the model, the macroelasticity ϵ^M is procyclical and the elasticity wedge $\epsilon^m/\epsilon^M - 1$ is positive and countercyclical. Hence, formula (9) implies that optimal UI is countercyclical. At the end of the section, we review empirical evidence that $\epsilon^m/\epsilon^M - 1$ is positive and countercyclical, as predicted by our model.

Table 1: Optimal replacement rate τ for different values of sufficient statistics

	Baily model $\epsilon^m/\epsilon^M = 1$	Average $\epsilon^m/\epsilon^M = 1.58$	Expansion $\epsilon^m/\epsilon^M = 1.2$	Recession $\epsilon^m/\epsilon^M = 2.5$
	(1)	(2)	(3)	(4)
	Panel A: $\epsilon^m = 0.3$			
$\epsilon^d = 0$	0.49	0.57	0.52	0.63
$\epsilon^d = 0.5$	0.49	0.56	0.52	0.62
	Panel B: $\epsilon^m = 0.6$			
$\epsilon^d = 0$	0.41	0.50	0.45	0.58
$\epsilon^d = 0.5$	0.41	0.49	0.44	0.56
	Panel C: $\epsilon^m = 0.9$			
$\epsilon^d = 0$	0.35	0.47	0.40	0.56
$\epsilon^d = 0.5$	0.35	0.45	0.39	0.53

Notes: The table presents the optimal replacement rate $\tau = c^u/c^e$ for a range of values for ϵ^m, ϵ^m/ϵ^M, and ϵ^d. The replacement rate is obtained by solving

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon^M} \cdot \frac{1-\xi}{\xi} + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \frac{\ln(1/\xi)}{1-\tau}.$$

This formula is (13) when $v(c) = \ln(c)$ (corresponding to a coefficient of relative risk aversion $\rho = 1$), $n \approx 1$, and we normalize $m(y) = 0$ at the optimum. (The approximated formula (14) generates comparable estimates.) In the formula, ϵ^m and ϵ^M are the micro- and macroelasticity of unemployment with respect to the consumption gain from work, ϵ^d is the elasticity of search effort with respect to the job-finding rate, $\xi = \hat{\xi} + \epsilon^i \cdot (\tau - \hat{\tau})$ is the consumption drop upon unemployment. As discussed in the text, we set the current replacement rate $\hat{\tau} = 0.54$, the current consumption drop $\hat{\xi} = 0.81$, the marginal consumption effect of unemployment benefits $\epsilon^i = 0.75$.

3.1 Matching process

Initially, $u \in (0,1)$ workers are unemployed and search for a job with effort e. Firms post o vacancies to recruit unemployed workers. The number h of matches made is given by a constant-returns matching function $h = h(e \cdot u, o)$ of aggregate search effort $e \cdot u$ and vacancies o, differentiable and increasing in both arguments, with the restriction that $h(e \cdot u, o) \leq u$.

Conditions on the labor market are summarized by labor market tightness $\theta \equiv o/(e \cdot u)$. A jobseeker finds a job at a rate $f = h(e \cdot u, o)/(e \cdot u) = h(1, \theta) \equiv f(\theta)$ per unit of search effort; a jobseeker searching with effort e finds a job with probability $e \cdot f$. It is easy for jobseekers to find jobs when the labor market is tight because the job-finding rate $f(\theta)$ increases with θ. A vacancy is filled with probability $q = h(e \cdot u, o)/o = h(1/\theta, 1) \equiv q(\theta)$. It is difficult for firms to find workers when the labor market is tight because the vacancy-filling rate $q(\theta)$ decreases with θ.

It costs $r \cdot a$ to post a vacancy, where $r > 0$ measures the resources spent on recruiting by firms and a is the level of technology.[10] We assume away randomness at the firm level: a worker is hired with certainty by opening $1/q(\theta)$ vacancies and spending $r \cdot a/q(\theta)$. When the labor market is tight, firms post many vacancies to fill a job, and recruiting is costly.

3.2 Firms

The representative firm takes labor n as input to produce a consumption good according to the production function $a \cdot g(n) = a \cdot n^{\alpha}$. $\alpha > 0$ measures the marginal returns to labor. $a > 0$ is the level of technology, which proxies for the position in the business cycle.

ASSUMPTION 1. The production function has diminishing marginal returns to labor: $\alpha < 1$.

The assumption yields a downward-sloping labor demand curve in a price θ-quantity n diagram, which has important macroeconomic implications. The assumption is motivated by the observation that, at business cycle frequency, some production inputs are slow to adjust.

Wages are set once worker and firm have matched. Since the costs of search are sunk at the time of matching, a surplus arise from each worker-firm match. Any wage sharing this surplus could be an equilibrium wage [Hall, 2005]. Given the indeterminacy of wages, we use the simple wage schedule of Blanchard and Galí [2010]:

ASSUMPTION 2. The wage schedule is rigid: $w = \omega \cdot a^{\gamma}$, where $\omega \in (0, +\infty)$ and $\gamma < 1$.

Wages are rigid in the sense that (i) they only partially adjust to a change in technology, and (ii) they do not respond to a change in UI. Rigidity (i) is measured by the parameter $\gamma < 1$. If $\gamma = 0$,

[10] For tractability, we follow Pissarides [2000] and assume that the cost of opening a vacancy is proportional to technology. An interpretation is that the recruiting technology itself is independent of technology, but that it uses labor as unique input [Shimer, 2010]. This interpretation is appealing since recruiting is a labor-intensive activity.

real wages do not respond to technology and are completely fixed over the cycle. Rigidity (i) generates unemployment fluctuations over the cycle [Hall, 2005]. Rigidity (ii) makes labor demand independent of UI and allows us to focus on the classical trade-off between insurance and incentive to search. Both rigidities are empirically grounded. First, many historical, ethnographic, and empirical studies document wage rigidity over the business cycle [for example, Bewley, 1999; Jacoby, 1984; Kramarz, 2001]. Second, empirical studies consistently find that reemployment wages do not respond to changes in unemployment benefits [Card, Chetty and Weber, 2007; Schmieder, von Wachter and Bender, 2012].

The firm sells its production on a perfectly competitive market. We normalize the price of the good to 1. The firm starts with $1-u$ workers. Given labor market tightness θ, technology a, and wage w, the firm chooses employment n to maximize real profit[11]

$$\pi = a \cdot g(n) - w \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)] \, .$$

The first-order condition implicitly defines labor demand $n^d(\theta, a)$, which satisfies

$$g'(n) = \frac{w}{a} + \frac{r}{q(\theta)}. \tag{15}$$

Under Assumption 1, g' decreases with n. q decreases with θ. Thus labor demand $n^d(\theta, a)$ decreases with θ. When the labor market is tight, it is expensive for firms to recruit, depressing labor demand. Under Assumption 2, w/a decreases with a so $n^d(\theta, a)$ increases with a. When technology is high, wages are relatively low, stimulating labor demand.

[11] We assume that technology is high enough for the firm to choose positive hiring: $n-(1-u) > 0$. The assumption requires $a > (\omega/\alpha) \cdot (1-u)^{(1-\alpha)/(1-\gamma)}$.

3.3 Equilibrium representation

We represent the equilibrium in the labor market with a labor supply-labor demand diagram in a price θ-quantity n plan. Figure 1(a) depicts a generic equilibrium. The labor supply is given by

$$n^s(f(\theta), \Delta v) = 1 - u + u \cdot e(f(\theta), \Delta v) \cdot f(\theta),$$

which gives the employment rate after matching when jobseekers search optimally for a given labor market tightness θ. The labor supply increases with θ because $f(\theta)$ increase with θ and $e(f, \Delta v)$ increase with f. The labor supply is concave in θ if and only if $(1-\eta) \cdot (1+\kappa)/\kappa < 1$, where $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $\kappa \equiv e \cdot k''(e)/k'(e)$.[12] We summarize the firm's demand for labor by a function $n^d(\theta, a)$ of labor market tightness θ. The labor demand $n^d(\theta, a)$ decreases with θ.

In presence of matching frictions, labor market tightness acts as a price equilibrating labor supply and labor demand. The wage cannot equilibrate supply and demand because the wage is set only once worker and firm have met. Equilibrium tightness $\theta(\Delta v, a)$ is implicitly defined by

$$n^s(f(\theta), \Delta v) = n^d(\theta, a). \tag{16}$$

If labor supply is above labor demand, a reduction in θ increases labor demand; it reduces labor supply by reducing the job-finding rate as well as optimal search effort; until labor supply and labor demand are equalized. Equilibrium employment $n(\Delta v, a)$ is given by the intersection of the labor supply with the labor demand. As in the generic model of Section 2, the equilibrium job-finding rate $f = f(\theta(\Delta v, a))$ is a function of the utility gain from work Δv.

Our framework encompasses as special cases the Baily model and the rat-race model, and the equilibrium for both models is represented in Figure 1(b). The Baily model is a partial-equilibrium model of unemployment in the sense that it fixes labor market tightness θ and job-finding rate $f(\theta)$. The Baily model is represented with a perfectly elastic labor demand that determines θ independently of UI. In the rat-race model, the number n of jobs is fixed. The rat-race model is

[12] See Lemma A10 in the Appendix. If jobseekers exert a constant search effort irrespective of labor market tightness ($\kappa = +\infty$), then the labor supply is concave for any parameter values.

represented with a perfectly inelastic labor demand that determines n independently of UI.

Figures 1(c) and 1(d) represent the equilibrium for the model with job rationing, characterized by Assumptions 1 and 2. The figures plot labor demand curves in expansions and recessions (panel (c) and (d)). They also plot labor supply curves for high and low unemployment benefits (dotted and solid line). Assumptions 1 and 2 determine the properties of the labor demand curve. Because of Assumption 1, the labor demand curve is downward sloping. Because of Assumption 2, the labor demand shifts inward when technology drops between panel (c) and panel (d).

Jobs are rationed in recessions in the sense that the labor market does not clear and some unemployment remains even as unemployed workers exert an arbitrarily large search effort. Job rationing appears Figure 1(d) because labor demand cuts the x-axis for employment strictly below 1. As labor demand intersects the x-axis below full employment, it is unprofitable for firms to hire some workers even if recruiting is costless at $\theta = 0$. Even if workers searched infinitely hard, shifting labor supply outwards such that $\theta \to 0$, firms would never hire all the workers: jobs are rationed. The mechanism creating job rationing is simple. After a negative technology shock the marginal product of labor falls but rigid wages adjust downwards only partially, so that the labor demand shifts inward (from panel (c) to panel (d)). If the adverse shock is sufficiently large, the marginal product of the least productive workers falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$.

3.4 Elasticity wedge over the business cycle

Formula (9) adds to the Baily formula a second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$. Proposition 3 establishes that $\epsilon^m/\epsilon^M > 1$ in our model with job rationing in which wages are rigid (Assumption 2) and the labor demand is downward sloping (Assumption 1):

PROPOSITION 3. *Under Assumption 2, the elasticity wedge ϵ^m/ϵ^M admits a simple expression:*

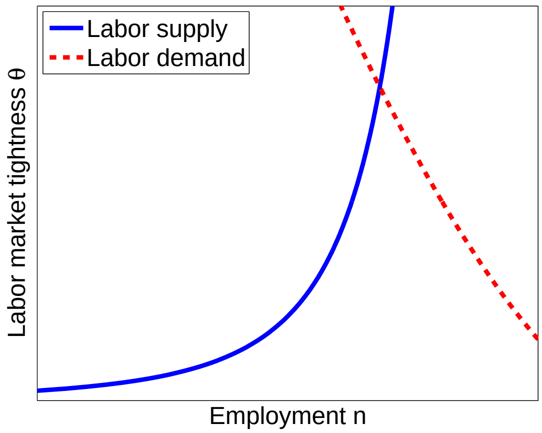
$$\frac{\epsilon^m}{\epsilon^M} = 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot \frac{1+\kappa}{\kappa} \cdot \frac{q(\theta)}{r} \cdot \frac{h}{n} \cdot n^{\alpha-1},$$

where $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $-\eta \equiv \theta \cdot q'(\theta)/q(\theta) < 0$ and $\kappa \equiv e \cdot k''(e)/k'(e)$. Under

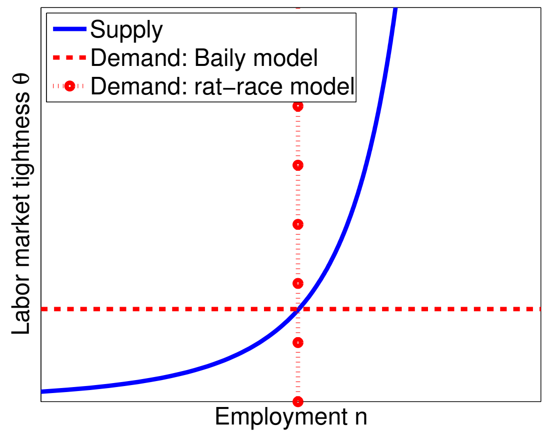
Assumption 1, macroelasticity is strictly smaller than microelasticity: $\epsilon^m/\epsilon^M > 1$.

Proposition 3, combined with formula (9), justifies the public provision of UI. If $\epsilon^m/\epsilon^M > 1$, small private insurers would underprovide UI because they maximize profits by using the Baily formula to determine how much insurance to provide to their clients. Small insurers solely take into account the microelasticity of unemployment, and do not internalize search externalities. In that case, the government would improve welfare by complementing the private provision of UI.

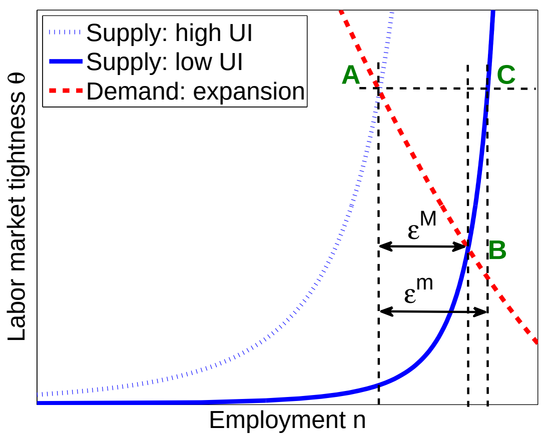
To understand why the microelasticity ϵ^m is larger than the macroelasticity ϵ^M, consider the cut in unemployment benefits $d\Delta c > 0$ depicted in Figure 1(c). The change creates variations $d\Delta v = v'(c^u)d\Delta c > 0$, dn, $d\theta$, df, and de so that all equilibrium conditions continue to be satisfied. The change in effort can be decomposed as $de = de_{\Delta v} + de_f$, where $de_{\Delta v} = (\partial e/\partial \Delta v|_f)\, d\Delta v$ is a partial-equilibrium variation in response to the change in UI, and de_f is a general-equilibrium adjustment following the change df in job-finding rate. Using the labor supply equation (3), we have $dn = dn_e + dn_f$ where $dn_e = u \cdot f \cdot de_{\Delta v}$ and $dn_f = [u \cdot e + u \cdot f \cdot (\partial e/\partial f \Delta v)]\, df$. Following a cut in benefits an individual jobseeker increases his search effort, increasing his own probability of finding a job by $dn_e > 0$. From the jobseeker's perspective, the job-finding rate f remains constant. The interval A–C in Figure 1(c) represents dn_e. When the jobseeker finds a job, however, he reduces the profitability of the marginal vacant job because (i) marginal productivity falls by diminishing marginal returns to labor, and (ii) the prevailing wage does not adjust to the drop in marginal productivity. Thus, firms reduce the number of vacancies posted to fill these less profitable jobs. Labor market tightness falls by $d\theta < 0$, reducing the job-finding rate of unemployed jobseekers by $df = f'(\theta)d\theta < 0$. $dn_f < 0$ is the corresponding reduction in employment, represented by interval C–B in Figure 1(c). The equilibrium increase dn in employment if benefits are reduced for all workers is smaller than the increase dn_e in the job-finding probability of an individual jobseeker whose benefits are reduced. The interval A–B in Figure 1(c) represents $dn < dn_e$. The difference between the microeffect dn_e and the macroeffect dn is $dn_f < 0$. Equation (8) says that $\epsilon^M = \epsilon^m + [\Delta c/(1-n)] \cdot dn_f/d\Delta c$. Since $dn_f < 0$, $\epsilon^M < \epsilon^m$.



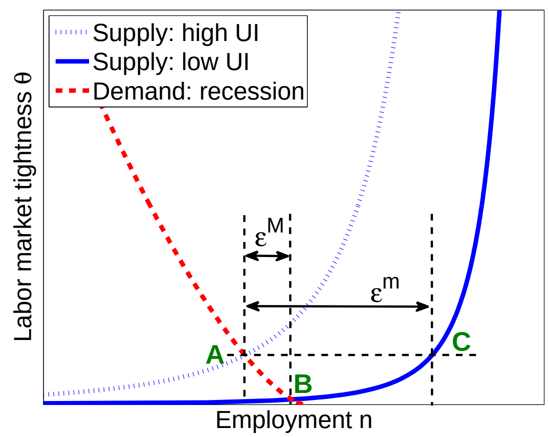
(a) Generic equilibrium



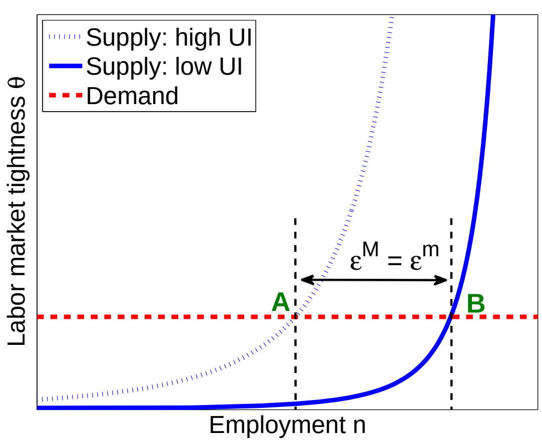
(b) Some special equilibria



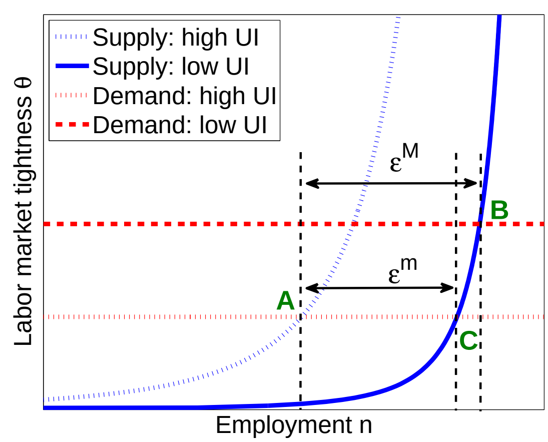
(c) Elasticities in expansion with job rationing



(d) Elasticities in recession with job rationing



(e) Elasticities in the Baily model



(f) Elasticities in the model with Nash bargaining

Figure 1: Labor market equilibria in a price θ-quantity n diagram

3.5 Optimal replacement rate over the business cycle

Our previous results do not require any assumptions on the functional forms of utility functions and matching function. They only involve the local elasticities η, ρ, and κ. But to characterize the cyclicality of the microelasticity, the macroelasticity, and the optimal replacement rate, we must control how the local elasticities fluctuate over the business cycle:

ASSUMPTION 3. The utility functions are isoelastic: $v(c) = \ln(c)$, $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. The matching function is Cobb-Douglas: $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^{\eta} \cdot o^{1-\eta}$.

The parameters $\omega_k > 0$ and $\omega_h > 0$ measure the cost of search and the effectiveness of matching.

To determine how the elasticities and the optimal replacement rate vary over the business cycle, we must also specify the level of initial unemployment associated with each technology level:

ASSUMPTION 4. For any technology level, initial unemployment u is such that in equilibrium $n - (1-u) = s \cdot n$ for $s \in (0,1)$.

The equilibrium is determined given initial unemployment u and technology a. Assumption 4 ensures that in equilibrium, the fraction $\left[n - (1-u)\right]/n$ of new hires in the workforce is constant over the cycle. The assumption replicates in our static model a feature of dynamic equilibrium unemployment models, which assume a constant job-destruction rate s independent of technology.[13]

Proposition 4 establishes that the wedge ϵ^m/ϵ^M is countercyclical and the macroelasticity ϵ^M is procyclical in a model with job rationing:

PROPOSITION 4. *Under Assumptions 1, 2, 3, and 4, the elasticity wedge ϵ^m/ϵ^M is countercyclical and the macroelasticity is procyclical:*

$$\left.\frac{\partial\left(\epsilon^m/\epsilon^M\right)}{\partial a}\right|_{\tau} < 0 \quad \textit{and} \quad \left.\frac{\partial \epsilon^M}{\partial a}\right|_{\tau} > 0.$$

The proposition says that the macroelasticity is large in expansions but small in recessions, as illustrated by comparing Figure 1(c) to Figure 1(d). This is because in recessions, jobs are acutely

[13] Pissarides [2000] and many others assume a constant job-destruction rate s and balanced labor market flows. When flows are balanced, firms hire each period as many workers as they lose. Therefore the fraction of new hires in the workforce is constant over the cycle.

rationed and search efforts have little influence on aggregate unemployment. The proposition also says that the wedge between micro- and macroelasticity is small in expansions but large in recessions. This is because when jobs are acutely rationed, searching more mechanically increases one's job-finding probability but it decreases others' job-finding probability as in a rat race.

Proposition 5 establishes that optimal UI is countercyclical in a model with job rationing:

PROPOSITION 5. *Assume that the optimal unemployment insurance formula (9) implicitly defines a unique function $\tau(a) \in (0,1)$, continuous and differentiable. Under Assumptions 1, 2, 3, and 4, if $n > 1/2$ and $(\alpha/\eta) \cdot s \cdot (1-\eta) \cdot (\kappa+1)/\kappa \leq 1$, then the optimal replacement rate $\tau(a)$ is countercyclical: $d\tau/da < 0$.*

The proposition says that the optimal replacement rate is more generous in recessions than in expansions. The formal proof, relegated in the Appendix, exploits the exact optimal UI formula, given by (9). But we can sketch the proof informally using the approximated optimal UI formula, given by (11). Proposition 4 shows that the macroelasticity ϵ^M decreases in recessions. Hence, the first term in (11) increases. In recessions the marginal budgetary cost of UI is small because a higher UI only increases unemployment negligibly. Proposition 4 also shows that the wedge ϵ^m/ϵ^M increases in recessions. Hence, the second term in (11) increases. The wedge measures the welfare cost of a negative rat-race externality imposed by unemployed workers on others. The externality arises because unemployed workers search taking the job-finding rate as given, and do not internalize their influence on the job-finding rate of others. UI corrects the externality by discouraging job search. In recessions the externality is acute so the marginal benefits of UI are high. Since both terms in formula (11) increase, τ must increase.

The formal proof is more complex because n enters formula (9). The results of Proposition 4 are not sufficient to prove the proposition. We need to prove that ϵ^M is sufficiently procyclical and that ϵ^m/ϵ^M is sufficiently countercyclical to compensate the fluctuations in n. To do so, we need two additional assumptions. The assumption $n > 1/2$ is needed because if technology a is so low that most workers become unemployed, it becomes optimal to reduce the replacement rate τ. Suppose all workers are unemployed ($n = 0,\ \theta = 0$). Providing more consumption to employed workers has no budgetary cost but it provides incentives for unemployed workers to search more, which could raise employment. Clearly, it is optimal to reduce the generosity of UI. In fact Lemma A8 in the Appendix establishes that when $a \to 0$ then $n \to 0$ and $\tau \to 0$. This result implies that for very low levels of technology and employment, the optimal replacement rate is bound to increase with technology. The assumption that $(\alpha/\eta) \cdot s \cdot (1-\eta) \cdot (\kappa+1)/\kappa \leq 1$ is needed to ensure that the labor supply is convex enough. As shown by comparing Figures 1(c) and 1(d), the convexity of labor supply in the (n, θ) plane drives the cyclicality of elasticities. The assumption is satisfied for any reasonable calibration because s, which stands for a job-destruction rate, is very small. The calibration in Table 2 implies $(\alpha/\eta) \cdot s \cdot (1-\eta) \cdot (\kappa+1)/\kappa = 0.0025 \ll 1$.

3.6 Empirical evidence in favor of job rationing

In this section we present recent empirical evidence that supports our model with job rationing. An implication of the model with job rationing that distinguishes it from standard models of equilibrium unemployment is that microelasticity ϵ^m is larger than macroelasticity ϵ^M(Proposition 3). By estimating whether $\epsilon^m/\epsilon^M - 1 > 0$ we can therefore assess whether the model provides a better description of the labor market than alternative models.

Before turning to empirical estimates of $\epsilon^m/\epsilon^M - 1$, we determine $\epsilon^m/\epsilon^M - 1$ in standard models of equilibrium unemployment. First, we show that $\epsilon^m/\epsilon^M - 1 < 0$ in the canonical model of equilibrium unemployment. The canonical model is characterized by two assumptions that replace Assumptions 1 and 2 in the model with job rationing:[14]

ASSUMPTION 5. The production function has constant marginal returns to labor: $\alpha = 1$.

ASSUMPTION 6. The wage w is determined using the generalized Nash solution to the bargaining problem faced by firm-worker pairs. The bargaining power of workers is $\beta \in (0, 1)$.

The Nash bargaining solution allocates a fraction β of the surplus of the match to the worker and the rest to the firm. If the utility function has constant relative risk aversion: $v(c) = (c^{1-\rho} - 1)/(1-$

[14] See Pissarides [2000] for a comprehensive treatment of the canonical model of equilibrium unemployment. The results presented in this section are derived formally in Appendix B.

ρ), the bargained wage is

$$\frac{w}{a} = -\frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)} \cdot \frac{r}{q(\theta)}.$$

Substituting the wage w in the labor demand equation (15) yields

$$\frac{r}{q(\theta)} = \left[1 - \frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)}\right]^{-1}. \tag{17}$$

Equilibrium labor market tightness θ, determined by (17), does not depend on technology a. Therefore, keeping the replacement rate τ constant, there are no fluctuations in tightness over the business cycle.[15] As tightness is strongly procyclical in the data, the Nash bargaining solution cannot account for the labor market fluctuations observed over the business cycle.

In the canonical model, $\epsilon^m/\epsilon^M < 1$.[16] Figure 1(f) provides some intuition. When UI falls, jobseekers search more. The labor supply shifts outwards, which increases employment by ϵ^m. In addition when UI falls, jobseekers face a worse outside option. The wage obtained by Nash bargaining falls, which raises labor demand and equilibrium labor market tightness. Employment rises further, and the total increase in employment is measured by ϵ^M. Clearly, $\epsilon^M > \epsilon^m$.

Second, $\epsilon^m/\epsilon^M = 1$ in the canonical model with rigid wages characterized by Assumptions 5 and 2.[17] This property is illustrated in Figure 1(e). It arises because labor demand is perfectly elastic and independent of UI, such that equilibrium tightness is independent of UI.

The empirical estimates of ϵ^m/ϵ^M obtained by Crepon et al. [2012] allow us to assess the validity of these three models. As discussed above, Crepon et al. [2012] find an average wedge $\epsilon^m/\epsilon^M = 1.58$. The wedge $\epsilon^m/\epsilon^M > 1$ is evidence of a negative rat-race externality: in the short run, treated jobseekers displace control jobseekers in queues for jobs. This compelling randomized experiment suggests that our model with job rationing describes well the labor market over the business cycle.

Crepon et al. [2012] also find additional evidence supporting our model. Consistent with Propo-

[15] Blanchard and Galí [2010] and others have proved similar theoretical results in a variety of settings.
[16] Proposition A1 in the Appendix establishes this result.
[17] Proposition 3 applied to the case $\alpha = 1$ establishes this result.

sition 4, they estimate that the wedge ϵ^m/ϵ^M is larger in geographical areas and time periods with higher unemployment. For example, the wedge is $\epsilon^m/\epsilon^M = 14.5/(14.5 - 7.6) = 2.10$ during the 2008-2009 recession in areas with high unemployment, compared with $\epsilon^m/\epsilon^M = 3.5/(3.5 - 0.9) = 1.35$ otherwise. The wedge for men in bad areas, bad periods is even higher: $\epsilon^m/\epsilon^M = 23.9/(23.9 - 14.6) = 2.57$.[18] Naturally, those estimates are not extremely precise and vary somewhat across specifications.

Columns (3) and (4) in Table 1 present the consequences of the cyclical fluctuations of ϵ^m/ϵ^M for the optimal replacement rate. Column (3) considers $\epsilon^m/\epsilon^M = 1.2$ and column (4) considers $\epsilon^m/\epsilon^M = 2.5$. Those numbers are illustrative of the range of estimates obtained by Crepon et al. [2012] in expansions and recessions. We assume that all the other parameters remain constant over the business cycle.[19] The optimal replacement rate is sharply countercyclical because it increases sharply from column (3), an expansion, to column (4), a recession. Two other results are interesting. In expansions, when $\epsilon^m/\epsilon^M \approx 1$, the optimal replacement rate is close to the replacement rate from the Baily formula in column (1). Hence in expansions, the Baily formula offers an excellent approximation of the optimal replacement rate in our model. But in recessions, when $\epsilon^m/\epsilon^M = 2.5$, the optimal replacement rate increases by 15 to 20 percentage points. Hence in recessions, the Baily formula no longer offers a good approximation of the optimal replacement rate in our model.

4 Connection with Baily Formula and Hosios Condition

This section connects our theory of optimal UI with the standard Baily [1978] formula for optimal UI and the Hosios [1990] condition for efficiency in equilibrium unemployment models. To establish the connection, we need more structure on the model than in Section 2 so we work with

[18] Those estimates for all workers and long-term employment outcomes are reported in Table 11, panel A, column (2) in Crepon et al. [2012]. Estimates for men are in column (4).

[19] These parameters could vary over the business cycle. Landais [2012] estimates that the microelasticity is fairly constant over the business cycle. But if the unemployed were more likely to deplete their savings in bad times, the consumption-smoothing benefits of UI would increase in bad times, magnifying the countercyclicality of optimal UI. There are unfortunately no precise empirical estimates of the consumption-smoothing benefits of UI over the business cycle in the literature (see Chetty and Finkelstein [2012] for a recent survey).

the equilibrium unemployment model of Section 3. We also need to assume that the government fully taxes profits and uses profits to finance the UI system. The government's budget constraint is $(1-n)\cdot b\cdot w = n\cdot t\cdot w + \pi$ where $\pi = a\cdot g(n) - [r\cdot a/q(\theta)]\cdot h - w\cdot n$ are the firm's profits. Equivalently, the budget constraint is

$$(1-n)\cdot c^u + n\cdot c^e = a\cdot g(n) - \frac{r\cdot a}{q(\theta)}\cdot [n-(1-u)]\,. \tag{18}$$

As we shall see, the assumption that the government is able to tax profits fully is not realistic, but offers a useful theoretical benchmark.

4.1 Jointly optimal unemployment insurance and labor market tightness

In this section the government chooses not only the consumption c^e of employed workers and the consumption gain from work Δc, but also labor market tightness θ. By choosing c^e, Δc, and θ, the government maximizes welfare (5) subject to the budget constraint (18), optimal job search by workers, and the definition of equilibrium employment $n = n^s(f(\theta), \Delta c, c^e)$. Proposition 6 characterizes the jointly optimal unemployment insurance and tightness:

PROPOSITION 6. *Optimal tightness θ, optimal consumptions c^e and c^u, and Lagrange multiplier ϕ on the resource constraint satisfy the inverse Euler equation* (10)*, the Baily formula*

$$\frac{w-\Delta c}{\Delta c} = \frac{n}{\epsilon^m}\cdot\left[\frac{v'(c^u)}{v'(c^e)} - 1\right], \tag{19}$$

and the generalized Hosios condition

$$\frac{\Delta v}{\phi} + (w-\Delta c)\cdot\left(1+\epsilon^d\right) - \frac{\eta}{1-\eta}\cdot\frac{r\cdot a}{q(\theta)} = 0, \tag{20}$$

where we define the implicit wage w as $w \equiv a\cdot g'(n) - r\cdot a/q(\theta)$.

If workers are risk neutral, the Baily formula implies that it is optimal to set $w = \Delta c$ and to provide no UI. In that case, (20) simplifies to $\Delta v/\phi = \eta/(1-\eta)\cdot r\cdot a/q(\theta)$. Risk neutrality also implies $\phi = 1$ and $\Delta v = w$. Hence, (20) further simplifies to $w = \eta/(1-\eta) \cdot r \cdot a/q(\theta)$, which is the standard Hosios condition for efficiency in an equilibrium unemployment model. Therefore, equation (20) can be interpreted as a *generalized Hosios condition* that determines the optimal wage level in presence of risk aversion and optimal UI.

With log utility, it is optimal for the government to set a wage that is proportional to technology in order to eliminate unemployment fluctuations completely over the business cycle. The optimal unemployment rate and labor market tightness do not depend on technology. While the optimal consumption levels are proportional to technology, the optimal replacement rate of UI does not depend on technology either.

The Baily formula (19) applies here because the job-finding rate $f(\theta)$ is kept constant when choosing the optimal Δc; therefore, the effects that arise from the general-equilibrium adjustment of the job-finding rate in formula (9) disappear. The generalized Hosios condition (20) determines the optimal labor market tightness θ. A marginal increase in θ has two effects. First, it increases the job-finding rate $f(\theta)$, which increases employment because $n = 1 - u + u \cdot e \cdot f(\theta)$. The employment increase leads to a welfare gain proportional to Δv (first term of (20)) and to a budget surplus proportional to $w - \Delta c$ (second term of (20)). Second, it reduces profits by increasing hiring costs $r \cdot a/q(\theta)$. The profit reduction creates a budget deficit proportional to hiring costs (third term of (20)). At the optimum, the marginal benefits from the increase in θ (first and second terms of (20)) equal the marginal cost (third term of (20)).

Proposition 6 applies when the government can control labor market tightness over the business cycle. Is this a realistic assumption? In our framework, wages paid by firms and tightness are directly related by the labor demand equation (15). Thus, if payroll tax incidence is fully on firms, tightness can be controlled by controlling wages using a payroll tax or subsidy. But in practice, it is improbable that the government can implement a wage subsidy at no cost.[20] Appendix D shows that our results carry over unchanged if there are costs to implement a wage subsidy such that the government does not eliminate entirely cyclical fluctuations in unemployment. More importantly, controlling wages may not allow the government to control tightness directly. For example, if firms

[20] Possible sources of cost include informational frictions or political constraints (for instance, trade unions may resist the reduction of the labor cost incurred by firms).

are constrained by aggregate demand in recessions, reducing wages with a subsidy would not lead firms to hire additional workers because they would not be able to sell the additional production. The subsidy would only be a transfer from the government to firm owners. Designing optimal UI in presence of aggregate demand constraints is left for future research.

4.2 Optimal unemployment insurance with taxation of profits

Here as in Sections 2 and 3, the government cannot control labor market tightness, which is determined endogenously to equilibrate labor supply and labor demand. But unlike in Sections 2 and 3, the government can fully tax profits and use them to finance UI. Hence, the government faces the same problem as in Sections 2 and 3 except that the budget constraint is given by (18) instead of (6). Proposition 7 provides a new formula for optimal UI when the government taxes profits:

PROPOSITION 7. *We characterize the optimal unemployment insurance with profit taxation.*

(i) The optimal consumptions c^e and c^u satisfy the inverse Euler equation (10) and the formula

$$\frac{w-\Delta c}{\Delta c} = \frac{n}{\epsilon^m}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right] + \frac{1}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(1-\frac{\epsilon^M}{\epsilon^m}\right)\cdot\left[\frac{\Delta v}{\phi}+(w-\Delta c)\cdot\left(1+\epsilon^d\right)-\frac{\eta}{1-\eta}\cdot\frac{r\cdot a}{q(\theta)}\right]. \quad (21)$$

(ii) A formula equivalent to (21) is

$$\frac{w-\Delta c}{\Delta c} = \frac{n}{\epsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right)+a\cdot g''(n)\cdot\frac{h}{\Delta c}. \quad (22)$$

(iii) If $n \approx 1$, $u \ll 1$, and the third and higher order terms of $v(c)$ are small, (22) simplifies to

$$\frac{w-\Delta c}{\Delta c} \approx \frac{\rho}{\epsilon^M}\cdot(1-\tau)+\left[\frac{\epsilon^m}{\epsilon^M}-1\right]\cdot\frac{1}{1+\epsilon^d}\cdot\left[1+\frac{\rho}{2}\cdot(1-\tau)\right]. \quad (23)$$

(iv) If $r \ll 1$, $(w-\Delta c)/\Delta c \approx \alpha\cdot\tau/(1-\tau)-(1-\alpha)$. Therefore if $n \approx 1$, $u \ll 1$, and $r \ll 1$, then $d\tau/da < 0$ and $d\left[1-(\Delta c/w)\right]/da < 0$.

Part (i) shows that the optimal replacement rate $\tau/(1-\tau)$ is the sum of an insurance term, which is the term in the standard Baily formula (19), and externality-correction term, which is proportional to the deviation from the generalized Hosios condition (20). When the generalized Hosios condition holds, the externality-correction term vanishes and the optimal replacement rate is given by the Baily formula. When wages are too high (and tightness is too low) relative to the generalized Hosios condition, the deviation from the generalized Hosios condition is positive. Since $\epsilon^m/\epsilon^M > 1$, the externality-correction term is positive. Therefore, optimal UI is more generous than in the Baily formula.

The optimal replacement rate is the sum of an insurance term and an externality-correction term because the derivative of the Lagrangian $\mathcal{L}$ of the government's problem with respect to Δc can be decomposed as $\partial\mathcal{L}/\partial\Delta c|_{c^e} = \partial\mathcal{L}/\partial\Delta c|_{\theta,c^e} + \partial\mathcal{L}/\partial\theta|_{\Delta c,c^e} \cdot \partial\theta/\partial\Delta c|_{c^e}$. Therefore, the first-order condition $\partial\mathcal{L}/\partial\Delta c|_{c^e} = 0$ in the current problem is a linear combination of the first-order conditions $\partial\mathcal{L}/\partial\Delta c|_{\theta,c^e} = 0$ and $\partial\mathcal{L}/\partial\theta|_{\Delta c,c^e} = 0$ in the joint optimization problem of Section 4.1. Hence, the optimal formula is also a linear combination of the Baily formula and the generalized Hosios condition. Moreover, the generalized Hosios condition is multiplied by the elasticity wedge $1-(\epsilon^M/\epsilon^m)$ because the factor $\partial\theta/\partial\Delta c|_{c^e}$ is proportional to the wedge from Lemma 1.

Part (ii) shows that the optimal UI formula is the same as formula (9) except for the addition of a last term $a \cdot g''(n) \cdot h/\Delta c$. (Lemma A1 in the Appendix shows that $(1/n) \cdot \tau/(1-\tau) = (w-\Delta c)/\Delta c$ when the government cannot tax profits.) The last term reflects the negative impact of UI on marginal profits: an increase in UI leads to an increase in tightness that increases hiring costs and reduces profits. As the last term is negative, the optimal $(w-\Delta c)/\Delta c$ is lower than in (9).

The formula proposed by Part (ii) is quite general. In particular, it remains valid even if wages are not rigid but respond to the utility gain from work Δv. The reason is that the wage does not appear in the government's budget constraint when the government taxes profits; therefore, the wage does not appear at all in the government's problem.[21] The influence of Δv on wages and equilibrium employment is simply captured by the macroelasticity ϵ^M.

Part (iii) proposes an approximated formula that links the optimal implicit tax on work $1-$

[21] On the contrary the wage does appear in the government's budget constraint (6), which applies when the government cannot tax profits.

$(\Delta c/w)$ to the usual sufficient statistics. The right-hand-side of the approximated formula is exactly the same as that in formula (11). The reason is that if $u \ll 1$, then $h \leq u \ll 1$ and the additional marginal-profit term is quantitatively negligible relative to the other terms. The left-hand-side of the approximated formula is different from that in (11) because of profit taxation. Indeed profits π are assumed to be taxed and equally redistributed, which increases both c^u and c^e by the same amount π and introduces a wedge between the replacement rate $\tau = c^u/c^e$ and the implicit tax on work $1 - (\Delta c/w)$. More precisely with no profits, the budget constraint (6) can be written as $c^u + n \cdot \Delta c = w \cdot n$, which implies that $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c$. With profits equally redistributed, the budget constraint becomes $c^u + n \cdot \Delta c = w \cdot n + \pi$, which implies that $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c + \pi/(n \cdot \Delta c)$. Lemma A1 in the Appendix shows that if $r \ll 1$ and $n \simeq 1$, $\alpha \cdot \tau/(1-\tau) \approx (1-\alpha) + (w - \Delta c)/\Delta c$. The effect of profit taxation on consumption levels is conceptually similar to the effect of self-insurance. If profits were equally distributed, they would constitute a cushion against unemployment, making τ higher in the right-hand-side of (23) and requiring a smaller UI program as measured by the wedge on reward to work $(w - \Delta c)/\Delta c$. This implication of profit taxation on UI is misleading for practical implementation if profits are not equally distributed among workers.

Part (iv) shows that for small u and r, although its level is different, the optimal replacement rate τ remains decreasing with technology a in the case with profit taxation. This in turn implies that the implicit tax on work $1 - (\Delta c/w)$ is also countercylical. That is, both the optimal replacement rate and the optimal implicit tax on work increase in recessions when technology falls.

Proposition 7 applies when the government can tax profits and redistribute them equally. It also applies if profits are uniformly distributed to workers. Are these realistic assumptions? If firms are owned by foreigners, if profits are impossible to tax directly, or if ownership is very concentrated such that profits are not uniformly distributed, those are not good assumptions. Empirically, profits are negligible in the income of unemployed workers.[22] Therefore, the model without profit taxation that we studied in Sections 2 and 3 is the most relevant in practice.

[22] We analyzed individual income tax statistics for 2004. We found that, while individuals and families reporting positive unemployment benefits had an average income (Adjusted Gross Income) equal to 78% of the population average, they had a capital income (the sum of interest payments, dividends, and realized capital gains) equal to only 17% of the population average.

5 Dynamic Model

In this section, we calibrate and simulate a dynamic model to study a variation of the UI system that could not be studied in a static environment: the adjustment of the duration of unemployment benefits instead of their level. The dynamic model is obtained by casting the static model into a dynamic environment. We make the model more realistic and hence more amenable to calibration by assuming that workers use home production to insure themselves partially against unemployment. We find numerically that optimal UI increases significantly in recessions whether the government adjusts the duration or the level of benefits.

5.1 Model

This section provides an overview of the dynamic model. The solution to the worker's, firm's, and government's problems, and the definition of the equilibrium are in Appendix C. Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. The labor market is similar to that in the static model. The only difference is that at the end of period $t-1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job become unemployed, and start searching for a new job at the beginning of period t. At the beginning of period t, $u_t = 1-(1-s)\cdot n_{t-1}$ unemployed workers look for a job.

Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses job-search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{(1-n_t^s)\cdot[v(c_t^u+y_t)-m(y_t)] + n_t^s \cdot v(c_t^e) - \left[1-(1-s)\cdot n_{t-1}^s\right]\cdot k(e_t)\right\},$$

subject to the law of motion of the employment probability in period t,

$$n_t^s = (1-s)\cdot n_{t-1}^s + \left[1-(1-s)\cdot n_{t-1}^s\right]\cdot e_t \cdot f(\theta_t).$$

$\mathbb{E}_0$ is the mathematical expectation conditioned on time-0 information, $\delta < 1$ is the discount factor.

The representative firm is owned by a risk-neutral entrepreneur. Given wage, labor market tightness, and technology $\{w_t, \theta_t, a_t\}_{t=0}^{+\infty}$ the firm chooses employment $\{n_t^d\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[n_t^d - (1-s) \cdot n_{t-1}^d \right] \right\},$$

where $n_t^d - (1-s) \cdot n_{t-1}^d \geq 0$ is the number of hires in period t.

Wages follow an exogenous process $\{w_t\}_{t=0}^{+\infty}$ defined by $w_t = \omega \cdot a_t^\gamma$. Labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$: $n_t \equiv n_t^d = n_t^s$.

Given technology $\{a_t\}_{t=0}^{+\infty}$, the government chooses consumption $\{c_t^u\}_{t=0}^{+\infty}$ of unemployed workers and consumption $\{c_t^e\}_{t=0}^{+\infty}$ of employed workers to maximize social welfare subject for all t to the budget constraint

$$n_t \cdot w_t = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u. \tag{24}$$

5.2 Formula

We consider the steady state of the dynamic model: all variables are constant, technology a is constant, and the unemployment insurance $(\Delta c, c^e)$ is constant. We assume that there is no time discounting: $\delta = 1$. In that case, given f, c^e, and Δc, the representative worker chooses home production y and search effort e to maximize his expected per-period utility. The optimal search effort is a function $e(f, \Delta v^h)$ of f and $\Delta v^h = \Delta v^h(\Delta c, c^e) = \min_y \{v(c^e) - [v(c^e - \Delta c + y) - m(y)]\}$. We define the labor supply as the steady-state employment rate of workers when they search optimally for a given job-finding rate f:

$$n^s(f, \Delta v) = \frac{e(f, \Delta v) \cdot f}{s + (1-s) \cdot e(f, \Delta v) \cdot f}.$$

In the dynamic model, the labor supply corresponds to a Beveridge curve because it captures equality of inflows to and outflows from unemployment. We define the general-equilibrium job-finding rate $f = f(\Delta v^h)$ and equilibrium employment $n(\Delta v^h) = n^s(f(\Delta v^h), \Delta v^h)$.

The government chooses Δc and c^e to maximize the per-period social welfare subject to the per-period budget constraint (24). Proposition 8 establishes that the formula giving the optimal replacement rate of UI is exactly the same as in the static model when there is no time discounting:

PROPOSITION 8. *Assume no time discounting: $\delta = 1$. Assume that at the optimum, $k(e) = 0$. Then in steady state, the optimal replacement rate $\tau \equiv c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right),$$

where the Lagrange multiplier ϕ on the government's budget constraint satisfies the inverse Euler equation $1/\phi = [n/v'(c^e)] + \left[(1-n)/v'(c^h)\right]$. If $n \approx 1$, if at the optimum $m(y) = 0$, and if the third and higher order terms of $v(c)$ are small, the formula simplifies to (14).

The proposition implies that the optimal UI formula in sufficient statistics used in Table 1 is also valid in a dynamic environment. We assume that the disutilities $k(e)$ from job search and $m(y)$ from home production are zero at the optimum to limit the welfare cost of unemployment to the loss of consumption that it imposes. We are aware that unemployment generates additional costs such as loss of human capital, but we leave the integration of these costs for future research.

5.3 Calibration

We calibrate all parameters of the model at a weekly frequency as shown in Table 2.[23] We calibrate as many parameters as possible directly from microevidence and macrodata for the US for the December 2000–June 2010 period. Following Michaillat [2012a] we set $\delta = 0.999$, $s = 0.0094$, $r = 0.32 \cdot \omega$. We use a Cobb-Douglas matching function $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^{\eta} \cdot o^{1-\eta}$ and set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001; Shimer, 2005]. As in Section 2.4, we set $\rho = 1$. We calibrate the wage flexibility γ based on estimates obtained in microdata. The flexibility of wages in newly created jobs mostly drives job creation. The best estimate of this flexibility using US data is provided by Haefke, Sonntag and Van Rens [2008]. Using panel data following production and supervisory workers over the 1984–2006 period, they

[23] This exercise is only illustrative of the magnitudes of the optimal policy, because our model abstracts from a number of relevant issues and there remains considerable uncertainty about the calibration of some parameters, such as the coefficient of relative risk aversion.

Table 2: Steady-state targets and parameter values used in simulations (weekly frequency)

	Steady-state target	Value	Source
$\hat{a}$	Technology	1	Normalization
$\hat{e}$	Effort	1	Normalization
$\hat{ls}$	Labor share	0.66	Convention
$\hat{u}$	Unemployment	5.9%	JOLTS, 2000–2010
$\hat{\theta}$	Labor market tightness	0.47	JOLTS, 2000–2010
$\hat{\tau}$	Replacement rate c^u/c^e	54%	Pavoni and Violante [2007], Chetty [2008]
$\hat{\xi}$	Consumption drop c^h/c^e	81%	Hamermesh [1982], Gruber [1997]
ϵ^i	Marginal consumption change dc^h/dc^u	0.75	Hamermesh [1982], Gruber [1997]
ϵ^s	Elasticity of unemployment hazard rate	0.90	Meyer [1990]
	Parameter	**Value**	**Source**
δ	Discount factor	0.999	Corresponds to 5% annually
ρ	Coefficient of relative risk aversion	1	Chetty [2006b]
η	Unemployment-elasticity of matching	0.7	Petrongolo and Pissarides [2001]
γ	Real wage flexibility	0.5	Pissarides [2009], Haefke et al. [2008]
r	Recruiting cost	0.21	Barron et al. [1997], Silva and Toledo [2009]
s	Job-destruction rate	0.94%	JOLTS, 2000–2010
ω_h	Efficacy of matching	0.19	Matches steady-state targets
α	Marginal returns to labor	0.67	Matches steady-state targets
ω	Steady-state real wage	0.67	Matches steady-state targets
ω_m	Level of home-production disutility	11.0	Matches steady-state targets
μ	Convexity of home-production disutility	1.01	Matches steady-state targets
ω_k	Level of search disutility	0.20	Matches steady-state targets
κ	Convexity of search disutility	3.15	Matches steady-state targets

estimate an elasticity of total earnings of job movers with respect to productivity of 0.7. If the composition of jobs accepted by workers improves in expansions, 0.7 is an upper bound on the elasticity of wages in newly created jobs [Gertler and Trigari, 2009]. A lower bound on this elasticity is the elasticity of wages in existing jobs, estimated in the 0.1–0.45 range with US data [Pissarides, 2009]. We set $\gamma = 0.5$, in the range of plausible values.

We calibrate the remaining parameters by matching the steady-state value of variables in the model to the average value of their empirical counterparts. We normalize average technology and average effort to $\hat{a} = 1$ and $\hat{e} = 1$. We compute average labor market tightness using the seasonally-adjusted, monthly series of vacancy levels collected by the Bureau of Labor Statistics (BLS) in the Job Openings and Labor Turnover Survey (JOLTS) and of unemployment levels computed by the BLS from the Current Population Survey (CPS). For the 2000–2010 period, $\hat{\theta} = \hat{v}/\left(\hat{e} \cdot \hat{u}\right) = 0.47$. Similarly, we find $\hat{u} = 5.9\%$, which implies $\hat{n} = 0.950$. As in Section 2.4, we set $\hat{\tau} = 54\%$.

To calibrate the matching efficacy ω_h we exploit the steady-state relationship $u \cdot e \cdot f(\theta) = s \cdot n = s \cdot (1-u)/(1-s)$. We find $\omega_h = [s/(1-s)] \cdot [(1-\hat{u})/\left(\hat{e} \cdot \hat{u}\right)] \cdot \hat{\theta}^{\eta-1} = 0.19$. We target the conventional labor share of $\hat{ls} \equiv \left(\hat{w} \cdot \hat{n}\right)/\hat{n}^{\alpha} = 0.66$. The firm's profit-maximization condition (equation (A18) in the Appendix) implies $\alpha = \hat{ls} \cdot \left([1 - \delta \cdot (1-s)] \cdot 0.32/q(\hat{\theta}) + 1\right) = 0.67$. The condition also allows us to recover $\omega = 0.67$, and $r = 0.32 \cdot \omega = 0.21$.

Next, we calibrate the parameters of the home-production disutility $m(y) = \omega_m \cdot \left(y^{1+\mu} - \hat{y}^{1+\mu}\right)/(1+\mu)$. Appendix C shows that μ is related to the statistics ϵ^i and ξ that we introduced in Section 2.4. As in Section 2.4, we set $\epsilon^i = 0.75$ and $\xi = 0.81$, which implies $\mu = 1.01$. The budget constraint (24) yields average home production $\hat{y} = 0.17$ and average unemployment consumption $\hat{c}^h = 0.53$. We set $\omega_m = 11.0$ for the worker's optimal choice of home production (equation (A21) in the Appendix) to hold for $\hat{c}^h$ and $\hat{y}$.

Finally, we calibrate the parameters of the search disutility $k(e) = \omega_k \cdot \left(e^{1+\kappa} - 1\right)/(1+\kappa)$. Appendix C shows that κ is related to the statistics ϵ^s and ξ that we introduced in Section 2.4. We follow Gruber [1997] and use the estimate $\epsilon^s = 0.9$ obtained by Meyer [1990]. As discussed in Section 2.4, 0.9 is on the high side of available estimates. We obtain $\kappa = 3.15$. We set $\omega_k = 0.20$ for the worker's optimal choice of effort (equation (A22) in the Appendix) to hold for $\hat{e} = 1$.

5.4 Baseline results

To describe how the optimal replacement rate varies over the business cycle, we compare steady states parameterized by different technology levels.[24] The results are displayed in Figure 2. The figure shows that unemployment is higher in steady states with lower technology. It also shows that labor market tightness decreases with unemployment, shaping a Beveridge curve. Search efforts

[24] In steady state, technology remains constant over time: $a_t = a$ for all t.

decrease in recessions, when UI becomes more generous and the job-finding rate falls. Home production decreases in recessions, when unemployment benefits become more generous.

The optimal UI is strongly countercyclical: the optimal replacement rate increases from 45% to 59% when the unemployment rate increases from 4% to 10%. Hence, the result of Proposition 5 holds in a realistic calibrated model: it is optimal to increase unemployment benefits relative to the consumption of employed workers in recessions. In fact, it is even optimal to increase unemployment benefits in absolute terms in recessions. (The gap between benefits and the consumption of unemployed workers is home production.)

The equilibrium described in Figure 2 is markedly different from the equilibrium that would arise if, as in Section 4, the government could jointly optimize over UI and labor market tightness. The equilibrium with jointly optimal UI and tightness, constructed in Appendix C, has a low unemployment rate of 4.3%, a high labor market tightness of 0.96, and a replacement rate $c^u/c^e = 55\%$ (leading to a ratio of actual consumptions $c^h/c^e = 70\%$). Labor market variables and replacement rate in the joint optimum are virtually constant when technology varies from 0.95–1.05.

5.5 Duration of unemployment benefits

In the baseline model, unemployment benefits never expire. In this section, unemployment benefits have a finite duration that the government adjusts over the cycle.[25] This model is more realistic because in practice, benefits have finite duration and the government modulates benefit duration over the cycle.[26] We follow Fredriksson and Holmlund [2001] and assume that eligible unemployed workers exhaust their benefits c_t^u with probability λ_t at the end of each period t. Ineligible unemployed workers receive social assistance $c_t^a < c_t^u$ until they find a job.

The replacement rates $\tau^{u,e} = c_t^u/c_t^e$ of unemployment benefits and $\tau^{a,e} = c_t^a/c_t^e$ of social assistance are constant over time. The government chooses the rate λ_t at which eligible workers

[25] This section only provides an overview of the model, whose formal description and analysis is in Appendix C.4.

[26] US unemployment benefits have a maximum duration of 26 weeks in normal times. Under the Extended Benefits program, duration is extended by 13 weeks in states where unemployment is above 6.5% and by 20 weeks in states where unemployment is above 8%. Often, duration is further extended in severe recessions. For example in 2008, the Emergency Unemployment Compensation program extended durations by an additional 53 weeks when state unemployment is above 8.5%.

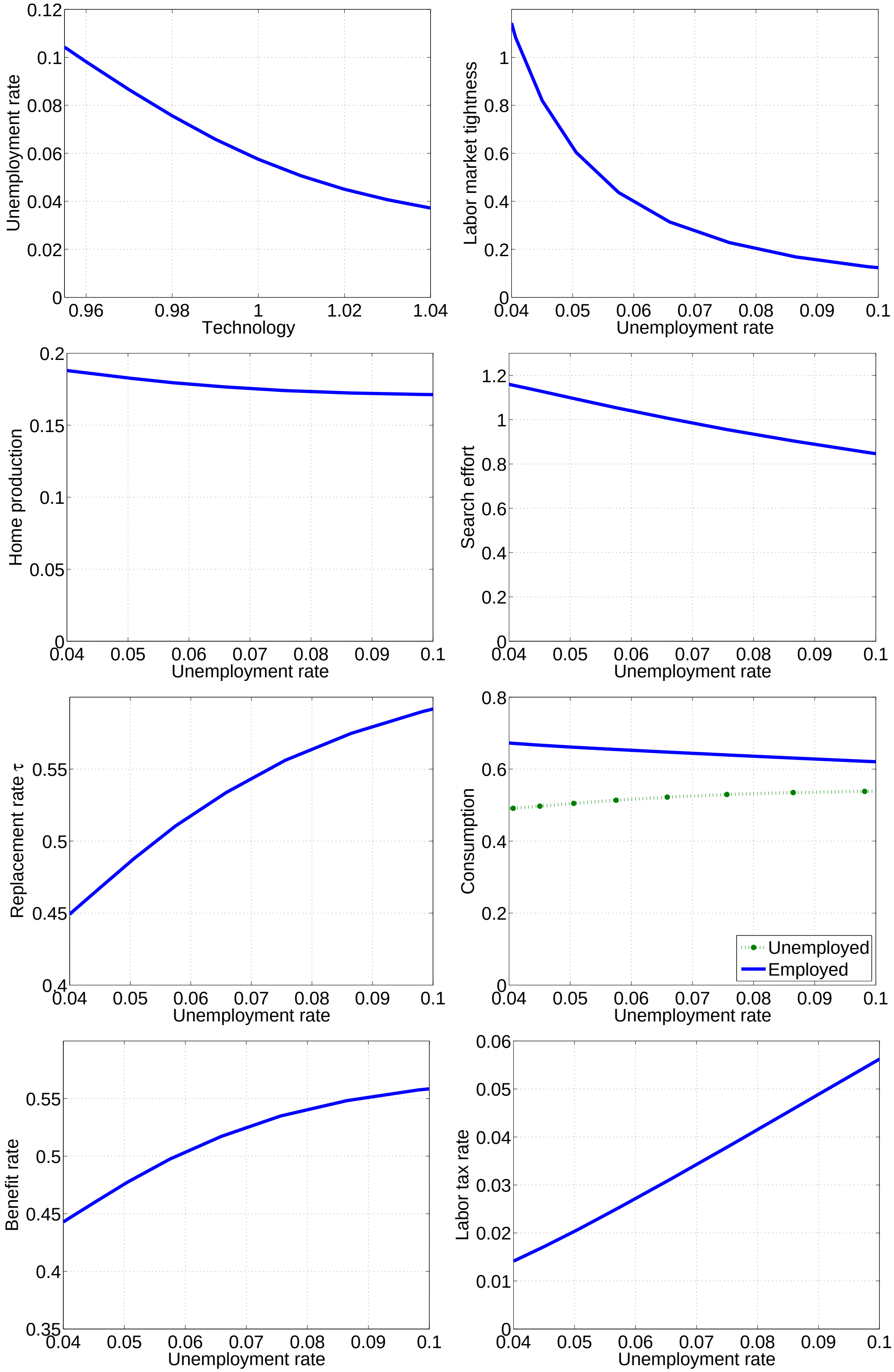


Figure 2: Optimal unemployment insurance over the business cycle

Notes: Panels obtained with the dynamic model in which unemployment benefits never expire. The model is calibrated in Table 2. The simulations are described in Appendix C.

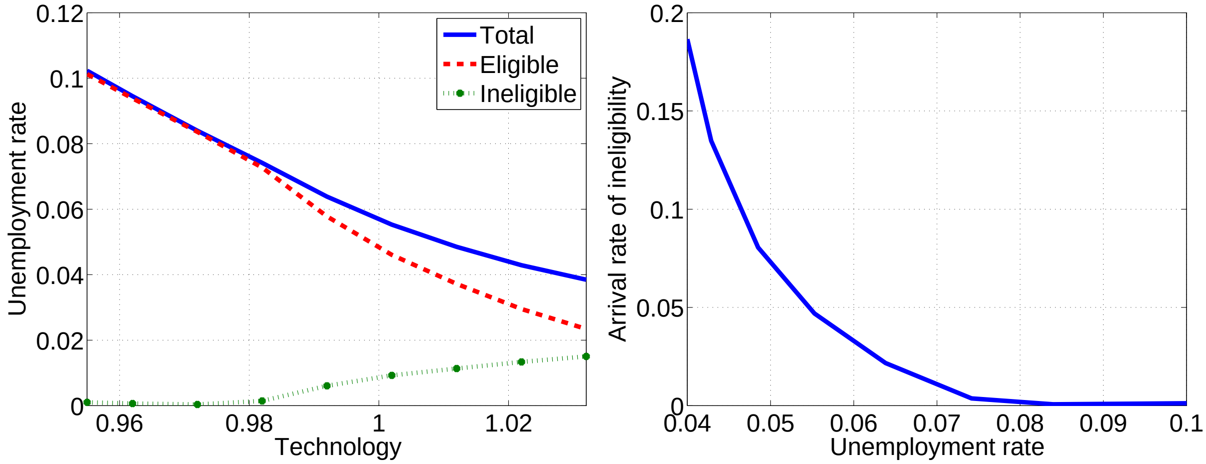


Figure 3: Optimal duration of unemployment insurance over the business cycle

Notes: Panels obtained with the dynamic model in which unemployment benefits have finite duration. The model is calibrated in Table 2. Appendix C describes the numerical simulations.

become ineligible to maximize welfare subject to a budget constraint similar to (24). We solve the model numerically using the calibration in Table 2. We set the replacement rates at $\tau^{u,e} = 57\%$ and $\tau^{a,e} = 0.52 \cdot \tau^{u,e} = 30\%$ such that an expected duration of 26 weeks is optimal at the average unemployment rate of 5.9%.[27] The left panel in Figure 3 shows how unemployment and its composition varies with technology. When technology increases, total unemployment falls, the number of eligible jobseekers falls, but the number of ineligible jobseekers increases because the arrival rate of ineligibility increases drastically. The right panel shows that the optimal arrival rate of ineligibility λ is strongly procyclical. Accordingly the optimal expected duration of unemployment benefits $1/\lambda$ is strongly countercyclical. When unemployment is 4%, the optimal arrival rate of ineligibility is 18%, corresponding to an expected benefit duration of less than 6 weeks. When unemployment reaches 5.9% the optimal arrival rate falls to 3.9%, corresponding to an expected benefit duration of 26 weeks. When unemployment reaches 8.0%, the optimal arrival rate drops below 1.0%, corresponding to an expected benefit duration of 100 weeks. The optimal arrival rate is virtually zero when unemployment is above 9%.

[27] Assume that social assistance only provides food stamps. According to Pavoni and Violante [2007], in the US, the median monthly allotment of food stamps for a family of four was \$397 per month in 1996, and the median monthly wage for a worker with at most a high-school diploma was \$1,540. Thus the rate of social assistance is $397/1,540 = 26\%$. As the rate of unemployment benefits is 50%, $\tau^{a,e}/\tau^{u,e} = 0.26/0.5 = 0.52$.

6 Concluding Remarks

A limitation that the model shares with most equilibrium unemployment models is that business cycles are generated by technology shocks only. This is implausible. For instance, aggregate demand shocks likely contribute to labor market fluctuations. To study optimal UI in a demand-generated business cycle, Appendix E builds a basic model in which recessions are driven by aggregate demand shocks amplified by nominal wage rigidity. Jobs are rationed in this model as well, albeit through a different mechanism. Firms face a downward-sloping aggregate demand curve in the goods market. The larger the quantity produced by workers, the lower the market price for goods. When aggregate demand is low enough, the production of workers would sell at a price below the nominal wage if all workers were employed. In this situation, firms would not hire all workers in the labor force even if recruiting were costless. Some unemployment would remain if jobseekers searched infinitely hard.

Our representation of the labor market is very general, and we can use it to represent the labor market of a model with demand-generated business cycle.[28] Figure A3 in the Appendix displays the labor supply-labor demand diagram for that model. The labor supply is the same, and the labor demand retains the same properties as in the model of technology-generated business cycle. The labor demand curve is downward sloping in the price θ–quantity n plane because higher employment n implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness θ for firms to be willing to hire. When aggregate demand falls, prices fall and real wages rise, so labor demand shifts inwards.

The labor market equilibria have similar structures in the two models, so it is not surprising that all the results derived in the model with technology-generated business cycle also apply in the model with demand-generated business cycles. The results on the cyclicality of the wedge ϵ^m/ϵ^M and the macroelasticity ϵ^M (Proposition 4), as well as the cyclicality of the optimal replacement rate

[28]To showcase the range of applications of our labor market representation, we entertain another source of business cycles in Appendix F. We study business cycles generated by a preference shock that affects job-search disutility. In recessions, it is unpleasant for unemployed workers to search. Jobseekers reduce their effort, reducing labor supply and increasing unemployment. Simulations suggest that optimal UI is procyclical in this model. But the model is unrealistic: it has the counterfactual property that labor market tightness is countercyclical. Michaillat [2012b] shows that our representation of the labor market is also useful to study other labor market policies over the business cycle.

(Proposition 5) remain valid once derivatives are taken with respect to aggregate demand instead of technology. Hence, the generosity of the optimal UI also increases in recessions caused by low aggregate demand.

Another limitation of the analysis is that additional mechanisms, absent from the model, could raise optimal UI further in recessions. We show that the trade-off between providing insurance and providing incentives to search tilts towards insurance in recessions. Additionally, if unemployed workers were more likely to exhaust their precautionary savings in recessions, the consumption-smoothing benefits of UI would also increase in recessions and it would be desirable to provide even more insurance. Or if aggregate demand were depressed in recessions and unemployment benefits stimulated aggregate demand, it would be desirable to provide even higher benefits in recessions. We leave the addition of these mechanisms to the framework for future research.

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Appendix — NOT FOR PUBLICATION

A Proofs

We begin by deriving a few preliminary results.

LEMMA A1.

(i) If the budget constraint faced by the government is

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w,$$

then

$$\frac{w - \Delta c}{\Delta c} = \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

(ii) If the budget constraint faced by the government is

$$n \cdot c^e + (1-n) \cdot c^u = a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)],$$

then

$$\alpha \cdot \frac{1}{n} \cdot \frac{\tau}{1-\tau} = (1-\alpha) + \frac{w - \Delta c}{\Delta c} + \left(1 - \alpha \cdot \frac{h}{n}\right) \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c}.$$

If $n \approx 1$ and $r \ll 1$, then

$$\alpha \cdot \frac{\tau}{1-\tau} \approx (1-\alpha) + \frac{w - \Delta c}{\Delta c}.$$

Proof. First, we prove Part (i). Recall that $\tau = c^u/c^e$ and $\Delta c = c^e - c^u$. The budget constraint implies that

$$c^u = n \cdot [w - \Delta c]$$
$$\frac{1}{n} \cdot \frac{c^u}{c^e - c^u} = \frac{w - \Delta c}{\Delta c}$$
$$\frac{1}{n} \cdot \frac{[c^u/c^e]}{1 - [c^u/c^e]} = \frac{w - \Delta c}{\Delta c}$$
$$\frac{w - \Delta c}{\Delta c} = \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

Second, we prove Part (ii). By definition of profits,

$$\pi = a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot h - n \cdot w,$$

Therefore, the budget constraint implies that

$$c^u = n \cdot (w - \Delta c) + \pi$$
$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{w - \Delta c}{\Delta c} + \frac{\pi}{n \cdot \Delta c}. \tag{A1}$$

We determine $\pi/(n \cdot \Delta c)$. The production function is isoelastic so $a \cdot g(n)/n = (1/\alpha) \cdot a \cdot g'(n)$. Furthermore, $a \cdot g'(n) = w + r \cdot a/q(\theta)$. So we use the definition of profits to write

$$\frac{\pi}{n} = \frac{1-\alpha}{\alpha} \cdot w + \left[\frac{1}{\alpha} - \frac{h}{n}\right] \cdot \frac{r \cdot a}{q(\theta)}$$
$$\frac{\pi}{\Delta c \cdot n} = \frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\alpha} \cdot \frac{w - \Delta c}{\Delta c} + \left[\frac{1}{\alpha} - \frac{h}{n}\right] \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c}. \tag{A2}$$

The result of the lemma follows from (A1) and (A2). □

LEMMA A2. *The derivatives of effort supply $e(f, \Delta v)$ satisfy*

$$\epsilon^d = \frac{f}{e} \cdot \left.\frac{\partial e}{\partial f}\right|_{\Delta v} = \frac{1}{\kappa}$$
$$\frac{\Delta v}{e} \cdot \left.\frac{\partial e}{\partial \Delta v}\right|_{f} = \frac{1}{\kappa}.$$

Proof. Obvious because the effort supply satisfies $k'(e) = f \cdot \Delta v$ and κ is the elasticity of $k'(e)$: $\kappa \equiv e \cdot [k''(e)/k'(e)]$. □

LEMMA A3. *The derivatives of the utility gain from work $\Delta v(\Delta c, c^e)$ satisfy*

$$\left.\frac{\partial \Delta v}{\partial \Delta c}\right|_{c^e} = v'(c^u)$$
$$\left.\frac{\partial \Delta v}{\partial c^e}\right|_{\Delta c} = v'(c^e) - v'(c^u).$$

Proof. Obvious because $\Delta v(\Delta c, c^e) = v(c^e) - v(c^e - \Delta c)$. □

A.1 Proof of Lemma 1

Equilibrium employment is defined as

$$n(\Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f(\Delta v)$$

Using the definition of the discouraged-worker elasticity, we obtain

$$n'(\Delta v) = u \cdot \left[\frac{\partial e}{\partial \Delta v} \cdot f + \frac{\partial e}{\partial f} \cdot f'(\Delta v) \cdot f + e \cdot f'(\Delta v)\right]$$
$$n'(\Delta v) = u \cdot f \cdot \frac{\partial e}{\partial \Delta v} + u \cdot e \cdot f'(\Delta v) \cdot \left[\epsilon^d + 1\right]$$

Using Definition 1 of ϵ^M and ϵ^m, we write

$$\epsilon^M = \frac{\Delta c}{1-n} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot n'(\Delta v)$$
$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[u \cdot f \cdot \frac{\partial e}{\partial \Delta v}\right].$$

The expression of ϵ^m results from the definition of labor supply

$$n^s(f, \Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f,$$

which implies that

$$\frac{\partial n^s}{\partial \Delta v} = u \cdot f \cdot \frac{\partial e}{\partial \Delta v}.$$

Therefore, ϵ^M and ϵ^m are related by

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \frac{h}{f} \cdot \left(1 + \epsilon^d\right) \cdot f'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c},$$

which is the result in Lemma 1 once we abuse notations by denoting $\partial f / \partial \Delta c = f'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c)$.

A.2 Proof of Proposition 1

Note that $h = u \cdot e \cdot f$ and $1 - n = u \cdot (1 - e \cdot f)$ in equilibrium. The Lagrangian of the government's problem is given by

$$\mathcal{L}(\Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\Delta v)) \cdot \Delta v - u \cdot k(e) + \phi \cdot \left[n(\Delta v) \cdot (w - \Delta c) - (c^e - \Delta c)\right],$$

where ϕ be the Lagrange multiplier on the budget constraint and Δv stands for $\Delta v(\Delta c, c^e)$. We exploit the envelope theorem because equilibrium effort e is chosen by workers to maximize $e \cdot f(\Delta v) \cdot \Delta v - k(e)$. The first-order condition with respect to Δc yields

$$-(1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[\Delta v \cdot u \cdot e \cdot f'(\Delta v) + \phi \cdot n'(\Delta v) \cdot (w - \Delta c)\right] + \phi \cdot (1-n) = 0.$$

The first-order condition with respect to c^e yields

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial c^e} \cdot [\Delta v \cdot u \cdot e \cdot f'(\Delta v) + \phi \cdot n'(\Delta v) \cdot (w - \Delta c)] - \phi = 0.$$

Using Lemma A3 we can rewrite $\partial \Delta v / \partial c^e = -[1 - (v'(c^e)/v'(c^u))] \cdot \partial \Delta v / \partial \Delta c$. We multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We obtain the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right].$$

We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$, rearrange the terms, and abuse notations as in the text by denoting $f'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c) = \partial f / \partial \Delta c$ and $n'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c) = \partial n / \partial \Delta c$. We obtain

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} \cdot (w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right] - \frac{1}{\phi} \cdot \frac{\Delta v}{1-n} \cdot u \cdot e \cdot \frac{\partial f}{\partial \Delta c} \tag{A3}$$

The inverse Euler equation yields

$$\frac{v'(c^u)}{\phi} - 1 = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Lemma 1 allows us to write

$$-\frac{\Delta v}{1-n} \cdot u \cdot e \cdot \frac{\partial f}{\partial \Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

By definition,

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^M.$$

Therefore we can rewrite (A3) as

$$\frac{w - \Delta c}{\Delta c} \cdot \epsilon^M = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

Using part (i) of Lemma A1 and dividing this equation by ϵ^M yields the optimal UI formula

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

Approximation. Assuming $n \approx 1$ allows us to simplify the optimal formula to

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{v'(c^e)} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

If the third and higher order terms of v are small ($v'''(c) \approx 0$), we approximate

$$\frac{\Delta v}{v'(c^e) \cdot \Delta c} \approx 1 - \frac{1}{2} \cdot \frac{v''(c^e)}{v'(c^e)} \cdot \frac{c^e}{c^e} \cdot [c^e - c^u] = 1 + \frac{1}{2} \cdot \rho \cdot (1 - \tau)$$
$$\frac{v'(c^u)}{v'(c^e)} \approx \frac{1}{v'(c^e)} \cdot \left[v'(c^e) - v''(c^e) \cdot c^e \cdot \frac{\Delta c}{c^e}\right] = 1 + \rho \cdot (1 - \tau),$$

where ρ is the coefficient of relative risk aversion measured at c^e. The optimal UI formula becomes

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon^M} \cdot \rho \cdot [1 - \tau] + \frac{1}{1 + \epsilon^d} \cdot \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1 - \tau)\right].$$

A.3 Proof of Proposition 2

Given c^e and $\Delta c = c^e - c^u$, unemployed workers choose effort e and home production y to maximize

$$[1 - e \cdot f] \cdot [v(c^e - \Delta c + y) - m(y)] + [e \cdot f] \cdot v(c^e) - k(e).$$

Unemployed workers can choose y and e sequentially. First, they choose y to maximize $v(c^e - \Delta c + y) - m(y)$. The first-order condition with respect to y is

$$m'(y) = v'(c^e - \Delta c + y). \tag{A4}$$

Let $\Delta v^h = \Delta v^h(\Delta c, c^e) = min_y \{v(c^e) - v(c^e - \Delta c + y) + m(y)\}$ be the utility gain from work. Then, unemployed workers choose e to maximize $e \cdot f \cdot \Delta v^h - k(e)$. The first-order condition with respect to e is $k'(e) = f \cdot \Delta v^h$. Finally, assume that the job-finding rate f is determined in general equilibrium as a function $f = f(\Delta v^h)$. Equilibrium employment is also a function $n = n(\Delta v^h)$.

The government chooses Δc and c^e to maximize social welfare. Let ϕ be the Lagrange multiplier on constraint (6). The Lagrangian of the government's problem is

$$\mathcal{L}(\Delta c, c^e) = v(c^e) - u \cdot \left[1 - e \cdot f(\Delta v^h)\right] \cdot \Delta v^h - u \cdot k(e) + \phi \cdot \left[n(\Delta v^h) \cdot (w - \Delta c) - (c^e - \Delta c)\right],$$

where Δv^h stands for $\Delta v^h(\Delta c, c^e)$. Search effort e is chosen optimally by workers given c^e and Δc, so we can apply the envelope theorem. The first-order condition with respect to Δc and c^e are

$$0 = -(1-n) \cdot v'(c^h) + \frac{\partial \Delta v^h}{\partial \Delta c} \cdot \left[\Delta v^h \cdot u \cdot e \cdot f'(\Delta v^h) + \phi \cdot n'(\Delta v^h) \cdot (w - \Delta c)\right] + \phi \cdot (1 - n)$$
$$0 = n \cdot v'(c^e) + (1-n) \cdot v'(c^h) + \frac{\partial \Delta v^h}{\partial c^e} \cdot \left[\Delta v^h \cdot u \cdot e \cdot f'(\Delta v^h) + \phi \cdot n'(\Delta v^h) \cdot (w - \Delta c)\right] - \phi.$$

Lemma A4 shows that Lemma A3 carries over to the extension with self-insurance when we replace Δv and c^u by Δv^h and c^h everywhere.

LEMMA A4. *Let* $\Delta v^h(\Delta c, c^e) = min_y \{v(c^e) - v(c^e - \Delta c + y) + m(y)\}$ *be the utility gain from* *work when home production is optimal. The derivatives of* $\Delta v^h(\Delta c, c^e)$ *satisfy*

$$\frac{\partial \Delta v^h}{\partial \Delta c} = v'(c^h)$$
$$\frac{\partial \Delta v^h}{\partial c^e} = v'(c^e) - v'(c^h).$$

Proof. Follows from the definition of Δv^h and the application of the envelop theorem. □

As in the proof of Proposition 1, we can show that ϕ satisfies the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^h)}\right].$$

Lemma A1 remains valid, and Lemma 1 remains valid by replacing Δv by Δv^h, so we can rewrite the first-order condition with respect to Δc as in the baseline model. The optimal UI formula is now given by

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

Approximation. Assuming $n \approx 1$ allows us to simplify the optimal UI formula to

$$\frac{\tau}{1-\tau} = \frac{1}{\epsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1\right] + \frac{1}{v'(c^e)} \cdot \frac{\Delta v^h}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

If the third and higher order terms of v are small ($v'''(c) \approx 0$), we approximate

$$\frac{v'(c^h)}{v'(c^e)} \approx \frac{1}{v'(c^e)} \cdot \left[v'(c^e) - v''(c^e) \cdot \left(c^e - c^h\right)\right] = 1 + \rho \cdot (1 - \xi),$$

where $\rho = -v''(c^e) \cdot c^e / v'(c^e)$ and $\xi = c^h / c^e$. We normalize $m(y) = 0$ at the optimum. Hence, at the optimum, $\Delta v^h = v(c^e) - v(c^h) + m(y) = v(c^e) - v(c^h)$. We can write

$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \frac{v(c^e) - v(c^h)}{v'(c^e) \cdot \Delta c}$$
$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \frac{v(c^e) - v(c^e) - v'(c^e) \cdot (c^h - c^e) - v''(c^e)/2 \cdot (c^h - c^e)^2}{v'(c^e) \cdot (c^e - c^u)}$$
$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \left[\frac{c^e - c^h}{c^e - c^u}\right] - \frac{1}{2} \cdot \frac{v''(c^e) \cdot c^e}{v'(c^e)} \cdot \frac{c^e - c^h}{c^e} \cdot \left[\frac{c^e - c^h}{c^e - c^u}\right]$$
$$\frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \approx \left[\frac{1-\xi}{1-\tau}\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1 - \xi)\right].$$

The optimal UI formula becomes

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\epsilon^M} \cdot (1-\xi) + \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] \cdot \frac{1}{1+\epsilon^d} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\xi)\right] \cdot \left[\frac{1-\xi}{1-\tau}\right].$$

A.4 Proof of Proposition 3

We define the following elasticities: $1-\eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$, $-\eta \equiv \theta \cdot q'(\theta)/q(\theta) < 0$, and $\kappa \equiv e \cdot k''(e)/k'(e)$. We abuse notations slightly and define equilibrium labor market tightness $\theta(\Delta c, c^e, a) \equiv \theta(\Delta v(\Delta c, c^e), a)$ and equilibrium employment $n(\Delta c, c^e, a) \equiv n(\Delta v(\Delta c, c^e), a)$.

LEMMA A5. *The partial derivative of equilibrium tightness $\theta(\Delta c, c^e, a)$ with respect to Δc is*

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e,a} = \frac{\theta}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot \left(\epsilon^M - \epsilon^m\right).$$

Proof. The equilibrium job-finding rate is $f = f(\theta(\Delta c, c^e, a))$ so Lemma 1 implies that

$$\epsilon^M = \epsilon^m + \frac{h}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \frac{\Delta c}{f(\theta)} \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

The result follows because $\epsilon^d = 1/\kappa$ by Lemma A2 and $f'(\theta)/f(\theta) = (1-\eta)/\theta$ by definition. □

The labor demand equation (15) holds at any equilibrium such that

$$g'(n(\Delta c, c^e, a)) = \frac{w}{a} + \frac{r}{q(\theta(\Delta c, c^e, a))}.$$

We differentiate this relationship with respect to Δc, keeping c^e and a constant. We obtain

$$(\alpha - 1) \cdot \frac{g'(n)}{n} \cdot \frac{\partial n}{\partial \Delta c} = \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{\partial \theta}{\partial \Delta c}$$

because under Assumption 2, (w/a) does not depend on Δc. Lemma A5 and the definition of the macroelasticity ϵ^M imply

$$(\alpha - 1) \cdot g'(n) \cdot \frac{1-n}{n} \cdot \epsilon^M = \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(\epsilon^M - \epsilon^m\right)$$

$$-(1-\alpha) \cdot g'(n) = \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(1 - \frac{\epsilon^m}{\epsilon^M}\right)$$

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \left[(1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{1}{r} \cdot \frac{1-\eta}{\eta}\right] \cdot q(\theta) \cdot \left(\frac{h}{n}\right) \cdot n^{\alpha-1}.$$

Since $\theta > 0$, $h > 0$, $\eta \in (0,1)$, $\kappa > 0$, $\epsilon^m/\epsilon^M > 1$ if and only if $\alpha \in (0,1)$.

A.5 Some comparative statics

We now focus on log utility: $v(c) = \ln(c)$. It is natural to parameterize the equilibrium with (τ, a) instead of $(\Delta c, c^e, a)$ because $\Delta v = \ln(1/\tau)$. Technology a captures the position in the business cycle and the replacement rate τ captures the generosity of UI. We abuse notations slightly and define equilibrium labor market tightness $\theta(\tau, a) \equiv \theta(\ln(1/\tau), a)$, and equilibrium employment $n(\tau, a) \equiv n(\ln(1/\tau), a)$. .

LEMMA A6. *Under Assumptions 1 and 2, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium tightness $\theta(\tau, a)$ and equilibrium employment $n(\tau, a)$:*

$$\left.\frac{\partial \theta}{\partial a}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial a}\right|_{\tau} > 0.$$

Proof. If $v(c) = \ln(c)$, the labor market equilibrium (16) condition becomes

$$1 - u + u \cdot e(f(\theta(\tau, a)), \ln(1/\tau)) \cdot f(\theta(\tau, a)) = n^d(\theta(\tau, a), a).$$

We differentiate this condition with respect to a, keeping τ constant:

$$u \cdot \left[f \cdot \frac{\partial e}{\partial f} + e \right] \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial a} = \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial a} + \frac{\partial n^d}{\partial a}$$

$$\frac{\partial \theta}{\partial a} = \underbrace{\frac{\partial n^d}{\partial a}}_{+} \cdot \left[\underbrace{u}_{+} \cdot \left(\underbrace{f}_{+} \cdot \underbrace{\frac{\partial e}{\partial f}}_{+} + \underbrace{e}_{+} \right) \cdot \underbrace{f'(\theta)}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-} \right]^{-1}.$$

because under Assumptions 1 and 2, $\partial n^d / \partial \theta < 0$, and $\partial n^d / \partial a > 0$. So $\partial \theta / \partial a > 0$. We show that $\partial n / \partial a > 0$ using $n(\tau, a) = 1 - u + u \cdot e(f(\theta(\tau, a)), \ln(1/\tau)) \cdot f(\theta(\tau, a))$. □

A.6 Proof of Proposition 4

Under Assumption 2, we can apply Proposition 3. Under Assumptions 1, 3, and 4, Proposition 3 implies that $\epsilon^m / \epsilon^M = 1 + \chi \cdot q(\theta) \cdot n^{\alpha - 1}$, where $\chi \equiv \alpha \cdot (1 - \alpha) \cdot [(1 - \eta)/\eta] \cdot [(1 + \kappa)/\kappa] \cdot (s/r) > 0$ is constant. Under Assumptions 1 and 2, Lemma A6 implies that $\partial \theta / \partial a|_{\tau} > 0$ and $\partial n / \partial a|_{\tau} > 0$. Since $q'(\theta) < 0$ and $\alpha \leq 1$, we infer that $\partial \left[\epsilon^m / \epsilon^M \right] / \partial a|_{\tau} < 0$.

We now focus on the cyclicality of ϵ^M. First, we determine an expression for ϵ^m. By definition

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \frac{h}{e} \cdot \frac{\partial e}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c}$$
$$\epsilon^m = \frac{\Delta c}{\Delta v} \cdot \frac{s \cdot n}{1-n} \cdot \frac{1}{\kappa} \cdot v'(c^u)$$
$$\epsilon^m = \frac{s}{\kappa} \cdot \frac{(1/\tau)-1}{\ln(1/\tau)} \cdot \frac{n}{1-n}, \tag{A5}$$

where we used Assumption 4, the assumption that $v(c) = \ln(c)$, and the result from Lemma A2. We infer that

$$\epsilon^M = \epsilon^m \cdot \frac{\epsilon^M}{\epsilon^m} = \frac{s}{\kappa} \cdot \frac{(1/\tau)-1}{\ln(1/\tau)} \cdot \frac{n}{1-n} \cdot \frac{\epsilon^M}{\epsilon^m}. \tag{A6}$$

Under Assumption 3, the elasticity κ is constant. According to Lemma A6, valid under Assumptions 1 and 2, $\partial n/\partial a|_\tau > 0$. We showed that $\partial\left[\epsilon^M/\epsilon^m\right]/\partial a|_\tau > 0$. We conclude that $\partial \epsilon^M/\partial a|_\tau > 0$.

A.7 Proof of Proposition 5

Under Assumption 3, using the result from Lemma A2 that $\epsilon^d = 1/\kappa$, formula (9) becomes

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \frac{1-\tau}{\tau} + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{1-\tau} \cdot [(1-n)\cdot\tau + n] \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right)$$
$$1 = \frac{n^2}{\epsilon^M} \cdot \left(\frac{1-\tau}{\tau}\right)^2 + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{\tau} \cdot n \cdot [(1-n)\cdot\tau + n] \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right). \tag{A7}$$

Next, we express $\left(\epsilon^m/\epsilon^M - 1\right)$ and ϵ^M as a function of n and τ.

LEMMA A7. *Under Assumptions 1, 2, 3 and 4 there exists $Z_0(\tau) > 0$ such that in equilibrium,*

$$Z(n,\tau) \equiv \frac{\epsilon^m}{\epsilon^M} - 1 = Z_0(\tau) \cdot n^{-\Omega} > 0, \tag{A8}$$

where the constant Ω is defined by

$$\Omega = (1-\alpha) + \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{s} > 0.$$

Proof. Under Assumption 2, we can use Proposition 3. Under Assumption 4, it says that

$$\frac{\epsilon^m}{\epsilon^M} - 1 = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot n^{\alpha-1}. \tag{A9}$$

We can write n as a function of θ and τ:

$$n = 1 - u + u \cdot e(f(\theta), \ln(1/\tau)) \cdot f(\theta).$$

Differentiating this equation with respect to θ, keeping τ constant, and using Lemma A2 under Assumption 4, we obtain

$$\begin{aligned}
\left.\frac{\partial n}{\partial \theta}\right|_{\tau} &= \frac{h}{f} \cdot f'(\theta) + \frac{h}{e} \cdot \frac{\partial e}{\partial f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot (1-\eta) \cdot \frac{h}{\theta} = \frac{\kappa+1}{\kappa} \cdot (1-\eta) \cdot \frac{h}{\theta} \\
\frac{\theta}{n} \cdot \left.\frac{\partial n}{\partial \theta}\right|_{\tau} &= \frac{\kappa+1}{\kappa} \cdot (1-\eta) \cdot s \\
\frac{n}{\theta} \cdot \left.\frac{\partial \theta}{\partial n}\right|_{\tau} &= \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1}{s}.
\end{aligned}$$

Combining (A9) with this relationship between n and θ yields

$$\left.\frac{\partial \ln\left(\epsilon^m/\epsilon^M - 1\right)}{\partial \ln(n)}\right|_{\tau} = -\left[(1-\alpha) + \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{s}\right] \equiv -\Omega,$$

where $\Omega > 0$ is constant under Assumption 3. We obtain (A8) by integrating this relationship. □

Using Lemmas A7 and (A6), we write ϵ^M as a function of n and τ

$$\frac{1}{\epsilon^M} = \frac{1-n}{n} \cdot \frac{\kappa}{s} \cdot \ln\left(\frac{1}{\tau}\right) \cdot \frac{\tau}{1-\tau} \cdot \left[1 + Z(n,\tau)\right]. \tag{A10}$$

Using Lemma A7 and (A10), we rewrite formula (A7) as

$$1 = n \cdot (1-n) \cdot \frac{\kappa}{s} \cdot \left[1 + Z(n,\tau)\right] \cdot \ln\left(\frac{1}{\tau}\right) \cdot \frac{1-\tau}{\tau} + \frac{\kappa}{\kappa+1} \cdot \frac{\ln(1/\tau)}{\tau} \cdot n \cdot \left[(1-n) \cdot \tau + n\right] \cdot Z(n,\tau).$$

Let $S \equiv s/(\kappa+1) \in (0,1)$. We rearrange the terms to obtain

$$\frac{s}{\kappa} \cdot \frac{\tau}{\ln\left(\frac{1}{\tau}\right)} = n \cdot (1-n)(1-\tau) + n \cdot Z(n,\tau) \cdot \left[\tau \cdot S + (1-\tau) - n \cdot (1-\tau) \cdot (1-S)\right]. \tag{A11}$$

Let us define

$$\begin{aligned}
F(\tau) &\equiv \frac{s}{\kappa} \cdot \frac{\tau}{\ln(1/\tau)} \\
G(n,\tau) &\equiv n \cdot (1-n) \cdot (1-\tau) + n \cdot Z(n,\tau) \cdot \left[\tau \cdot S + (1-\tau) - n \cdot (1-\tau) \cdot (1-S)\right].
\end{aligned}$$

Furthermore, we define $Q(\tau, a) \equiv G(n(\tau, a), \tau)$. We rewrite the optimal UI formula as $F(\tau) = Q(\tau, a)$. We assume that for any $a > 0$, $F(\tau)$ and $Q(\tau, a)$ cross only once at $\tau(a) \in (0,1)$. The implicit function $\tau(a)$ characterizes the optimal replacement rate for technology a.

LEMMA A8. *Under Assumptions 1 and 2,* $\lim_{a\to 0} n(a, \tau(a)) = 0$ *and* $\lim_{a\to 0} \tau(a) = 0$.

Proof. Under Assumptions 1 and 2, the labor demand equation (15) implies that for any $a > 0$, $\alpha \cdot n(a, \tau(a))^{\alpha-1} \geq \omega \cdot a^{\gamma-1}$ and $0 \leq n(a, \tau(a)) \leq N(a) \equiv [(\alpha/\omega) \cdot a^{1-\gamma}]^{1/(1-\alpha)}$. Since $\gamma < 1$ and $0 < \alpha < 1$, $\lim_{a\to 0} N(a) = 0$. The squeeze theorem implies that $\lim_{a\to 0} n(a, \tau(a)) = 0$.

By definition, $q(\theta) \leq 1$. Therefore for any n and any τ,

$$n \cdot Z(n, \tau) = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot n^{\alpha} \leq (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n^{\alpha}.$$

The optimal UI formula is $F(\tau(a)) = Q(\tau(a), a)$. Using the definition of Q, we infer

$$F(\tau(a)) \leq n(a, \tau(a)) \cdot [1 - n(a, \tau(a))] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n(a, \tau(a))^{\alpha}.$$

We showed that $\lim_{a\to 0} n(a, \tau(a)) = 0$. So there exists $a_0 > 0$ such that for all $a < a_0$, $n(a, \tau(a)) < 1/2$. For any $a > 0$, $0 \leq n(a, \tau(a)) \leq N(a)$. Thus for any $a < a_0$,

$$0 \leq F(\tau(a)) \leq N(a) \cdot [1 - N(a)] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot N(a)^{\alpha}.$$

Under Assumptions 1 and 2, the limit of the right-hand-side term when $a \to 0$ is 0 because $\lim_{a\to 0} N(a) = 0$. Using the squeeze theorem, we infer that $\lim_{a\to 0} F(\tau(a)) = 0$. We conclude that $\lim_{a\to 0} \tau(a) = 0$ using the continuity of F on $(0, 1)$. □

Lemma A8 establishes that when employment converges to 0 because technology decreases to 0, then the optimal replacement rate converges to 0. This result implies that for very low levels of technology and employment, the optimal replacement rate is bound to increase with technology.

LEMMA A9. *If* $n > 1/2$ *and* $\Omega \geq 1$ *then* $\partial G/\partial n < 0$.

Proof. We differentiate $G(n, \tau)$ with respect to n, keeping τ constant.

$$\frac{\partial G}{\partial n} = -\left\{(2 \cdot n - 1) \cdot (1-\tau) + Z(n, \tau) \cdot [(2-\Omega) \cdot (1-S) \cdot (1-\tau) \cdot n - (1-\Omega) \cdot [\tau \cdot S + (1-\tau)]]\right\}.$$

If $n > 1/2$, the first term $(2 \cdot n - 1) \cdot (1-\tau) > 0$ since $\tau < 1$. If $\Omega \geq 1$, the second term is nonnegative. To see this, note that $(1-S) \cdot n < 1$ and rewrite the second term as

$$Z(n, \tau) \cdot [(\Omega - 1) \cdot [\tau \cdot S + (1-\tau) \cdot \{1 - (1-S) \cdot n\}] + (1-S) \cdot (1-\tau) \cdot n] \geq 0.$$

If $\Omega \in [0, 1)$, the second term may be negative. □

At technology a, the optimal replacement rate $\tau(a)$ satisfies $F(\tau(a)) = Q(\tau(a), a)$. We consider a marginal change in technology from a to $a^* > a$. Using Lemma A6 under Assumption 3, we know that $n(\tau(a), a^*) > n(\tau(a), a)$. Using Lemma A9 for $n > 1/2$ and $\tau \in (0, 1)$,

$G(n(\tau(a), a^*), \tau(a)) < G(n(\tau(a), a), \tau(a))$ such that $Q(\tau(a), a^*) < Q(\tau(a), a) = F(\tau(a))$. Since $F(\tau)$ and $Q(\tau, a)$ cross only once for $\tau \in (0, 1)$, $\lim_{\tau \to 0} F(\tau) = 0$, and $\lim_{\tau \to 0} Q(\tau, a) > 0$, it must be that $F(\tau)$ crosses $Q(\tau, a)$ from below. Thus is must be that $\tau(a) > \tau(a^*)$ and $d\tau/da < 0$.

A.8 Interpretation of the assumptions of Proposition 5

LEMMA A10. *The labor supply $n^s(f(\theta), \Delta v)$ is concave in θ if and only if $(1-\eta) \cdot (1+\kappa)/\kappa < 1$.*

Proof. By definition, $n^s(f(\theta), \Delta v) = 1 - u + u \cdot e(f(\theta), \Delta v) \cdot f(\theta)$. Thus,

$$\frac{\partial n^s}{\partial \theta} = \left(1 + \epsilon^d\right) \cdot \frac{n^s - (1-u)}{f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot (1 - \eta) \cdot \frac{n^s - (1-u)}{\theta}.$$

Lemma A2 imply that $\epsilon^d = 1/\kappa$. Hence,

$$\frac{\partial^2 n^s}{\partial \theta^2} = \frac{1+\kappa}{\kappa} \cdot (1-\eta) \cdot \frac{n^s - (1-u)}{\theta^2} \cdot \left[\frac{1+\kappa}{\kappa} \cdot (1-\eta) - 1\right].$$

Since $n^s \geq 1 - u$, $\partial^2 n^s / \partial \theta^2 < 0$ if and only if $(1-\eta) \cdot (1+\kappa)/\kappa < 1$. □

A.9 Proof of Proposition 6

The Lagrangian of the government's problem is given by

$$\mathcal{L}(\theta, \Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\theta)) \cdot \Delta v - u \cdot k(e)$$
$$+ \phi \cdot \left\{ a \cdot g(n^s(f(\theta), \Delta v)) - \frac{r \cdot a}{q(\theta)} \cdot [n^s(f(\theta), \Delta v) - (1-u)] - c^e + [1 - n^s(f(\theta), \Delta v)] \cdot \Delta c \right\}$$

ϕ is the Lagrange multiplier on the resource constraint and Δv stands for $\Delta v(\Delta c, c^e)$. We use the envelope theorem as workers choose e optimally. The first-order condition with respect to Δc is

$$-(1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[a \cdot g'(n) - \frac{r \cdot a}{q(\theta)} - \Delta c\right] + \phi \cdot (1-n) = 0.$$

The firm's profit-maximization condition ensures that

$$w = a \cdot g'(n) - \frac{r \cdot a}{q(\theta)}$$

and we rewrite the first-order condition with respect to Δc as

$$-(1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot (w - \Delta c) + \phi \cdot (1-n) = 0.$$

Similarly, the first-order condition with respect to c^e yields

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \phi \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial c^e} \cdot (w - \Delta c) - \phi = 0.$$

Using Lemma A3 we can rewrite $\partial \Delta v / \partial c^e = -\left[1 - (v'(c^e)/v'(c^u))\right] \cdot \partial \Delta v / \partial \Delta c$. We multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We obtain the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right].$$

We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$ and rearrange the terms to obtain

$$\frac{1}{1-n} \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} \cdot (w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right].$$

By definition,

$$\frac{1}{1-n} \cdot \frac{\partial n^s}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^m.$$

Therefore, we can rewrite the first-order condition as

$$\frac{w - \Delta c}{\Delta c} = \frac{n}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Last, we consider the first-order condition with respect to θ, keeping Δc and c^e constant. We have

$$0 = (1-\eta) \cdot \frac{h}{\theta} \cdot \Delta v + \phi \cdot [w - \Delta c] \cdot \frac{\partial n^s}{\partial f} \cdot f'(\theta) - \phi \cdot \eta \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{h}{\theta}.$$

Since the labor supply $n^s(f, \Delta v) = 1 - u + u \cdot e(f, \Delta v) \cdot f$, we have

$$\frac{\partial n^s}{\partial f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot \frac{h}{f} \cdot f'(\theta) = \left(1 + \epsilon^d\right) \cdot \frac{h}{\theta} \cdot (1 - \eta).$$

We divide the first-order condition by $\phi \cdot (h/\theta) \cdot (1-\eta)$ and obtain

$$\frac{\Delta v}{\phi} + [w - \Delta c] \cdot \left(1 + \epsilon^d\right) = \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}.$$

A.10 Proof of Proposition 7

The Lagrangian of the government's problem is given by

$$\mathcal{L}(\Delta c, c^e) = v(c^e) - u \cdot (1 - e \cdot f(\theta)) \cdot \Delta v(\Delta c, c^e) - u \cdot k(e) \\ + \phi \cdot \left\{ a \cdot g(n) - \frac{r \cdot a}{q(\theta)} \cdot [n - (1-u)] - c^e + (1-n) \cdot \Delta c \right\},$$

where ϕ be the Lagrange multiplier on the resource constraint, θ stands for $\theta(\Delta v(\Delta c, c^e))$, and n stands for $n(\Delta v(\Delta c, c^e))$. We exploit the envelope theorem as workers choose effort e optimally. The first-order condition with respect to Δc is

$$-(1-n)v'(c^u) + \frac{\partial \Delta v}{\partial \Delta c}\left[(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\Delta v + \phi n'(\Delta v)(w - \Delta c) - \phi\eta\frac{ra}{q(\theta)}\frac{h}{\theta}\theta'(\Delta v)\right] + \phi(1-n) = 0.$$

Similarly, the first-order condition with respect to c^e yields

$$n \cdot v'(c^e) + (1-n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial c^e}\left[(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\Delta v + \phi n'(\Delta v)(w - \Delta c) - \phi\eta\frac{ra}{q(\theta)}\frac{h}{\theta}\theta'(\Delta v)\right] - \phi = 0.$$

Using Lemma A3 we can rewrite $\partial \Delta v / \partial c^e = -\left[1 - (v'(c^e)/v'(c^u))\right] \cdot \partial \Delta v / \partial \Delta c$. We multiply the first-order condition with respect to Δc by $1 - [v'(c^e)/v'(c^u)]$ and add it to the first-order condition with respect to c^e. We obtain the inverse Euler equation

$$\frac{1}{\phi} = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right].$$

We come back to the first-order condition with respect to Δc. We divide it by $\phi \cdot (1-n)$ and rearrange the terms to obtain

$$\frac{1}{1-n}\frac{\partial n}{\partial \Delta c}(w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right] - \frac{1}{1-n}(1-\eta)\frac{h}{\theta}\theta'(\Delta v)\frac{\partial \Delta v}{\partial \Delta c} \cdot \left[\frac{\Delta v}{\phi} - \frac{\eta}{1-\eta}\frac{ra}{q(\theta)}\right].$$

The inverse Euler equation yields

$$\frac{v'(c^u)}{\phi} - 1 = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Lemma 1 allows us to write

$$-\frac{1}{1-n} \cdot (1-\eta) \cdot \frac{h}{\theta} \cdot \theta'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

By definition,

$$\frac{1}{1-n} \cdot n'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^M.$$

Therefore, we can rewrite the first-order condition as

$$\frac{w-\Delta c}{\Delta c}\cdot\epsilon^M = n\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\epsilon^m-\epsilon^M\right)\left[\frac{\Delta v}{\phi}-\frac{\eta}{1-\eta}\cdot\frac{r\cdot a}{q(\theta)}\right]. \tag{A12}$$

Adding $[(w-\Delta c)/\Delta c]\cdot\left[\epsilon^m-\epsilon^M\right]$ on both sides and dividing by ϵ^m yields

$$\frac{w-\Delta c}{\Delta c}=\frac{n}{\epsilon^m}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{1}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(1-\frac{\epsilon^M}{\epsilon^m}\right)\cdot\left[\frac{\Delta v}{\phi}+(w-\Delta c)\cdot(1+\epsilon^d)-\frac{\eta}{1-\eta}\cdot\frac{r\cdot a}{q(\theta)}\right].$$

The optimal UI formula when the government taxes profits is (A12). We rewrite it as

$$\frac{w-\Delta c}{\Delta c}=\frac{n}{\epsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right)-\frac{\eta}{1-\eta}\cdot\frac{r\cdot a}{q(\theta)}\cdot\frac{1}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right).$$

Proposition 3 implies that under Assumption 2, the ratio of elasticities satisfies

$$\frac{\epsilon^m}{\epsilon^M}-1=-g''(n)\cdot\frac{1-\eta}{\eta}\cdot\frac{1}{1+\epsilon^d}\cdot\frac{q(\theta)}{r}\cdot h.$$

Thus, we can rewrite the optimal UI formula as

$$\frac{w-\Delta c}{\Delta c}=\frac{n}{\epsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{1}{\phi}\cdot\frac{\Delta v}{\Delta c}\cdot\frac{1}{1+\epsilon^d}\cdot\left(\frac{\epsilon^m}{\epsilon^M}-1\right)+a\cdot g''(n)\cdot\frac{h}{\Delta c}.$$

A.11 Proof of Proposition 8

We consider the steady state of the dynamic model: all variables are constant, technology a is constant, and the unemployment insurance $(\Delta c, c^e)$ is constant. In steady state, employment is constant and is given by the Beveridge curve

$$n^*(e,f)=\frac{e\cdot f}{s+(1-s)\cdot e\cdot f}. \tag{A13}$$

We assume that there is no time discounting: $\delta=1$. For now, we abstract from self-insurance. In that case, given f, c^e, and Δc, the representative worker chooses search effort e to maximize expected per-period utility

$$v(c^e)-[1-n^*(e,f)]\cdot\Delta v(\Delta c,c^e)-[1-(1-s)\cdot n^*(e,f)]\cdot k(e).$$

The optimal search effort is a function $e(f,\Delta v)$ of Δv and f. We define the labor supply

$$n^s(f,\Delta v)\equiv n^*(e(f,\Delta v),f),$$

the general-equilibrium job-finding rate $f \equiv f(\Delta v)$, and equilibrium employment

$$n(\Delta v) \equiv n^s(f(\Delta v), \Delta v).$$

The government chooses Δc and c^e to maximize the per-period social welfare subject to the per-period budget constraint. The Lagrangian of the government's problem is

$$\begin{aligned}\mathcal{L}(\Delta c, c^e) = v(c^e) - (1 - n^*(e, f(\Delta v))) \cdot \Delta v \\ - [1 - (1 - s) \cdot n^*(e, f(\Delta v))] \cdot k(e) + \phi \cdot [n(\Delta v) \cdot (w - \Delta c) - (c^e - \Delta c)]\end{aligned}$$

where ϕ be the Lagrange multiplier on the budget constraint and Δv stands for $\Delta v(\Delta c, c^e) = v(c^e) - v(c^e - \Delta c)$. We exploit the envelope theorem because equilibrium effort e is chosen by workers to maximize per-period social welfare. The first-order condition with respect to Δc is

$$\begin{aligned}-(1 - n) \cdot v'(c^u) + \frac{\partial \Delta v}{\partial \Delta c} \cdot \left[(\Delta v + (1 - s) \cdot k(e)) \cdot \frac{\partial n^*}{\partial f} \cdot f'(\Delta v) + \phi \cdot n'(\Delta v) \cdot (w - \Delta c)\right] \\ + \phi \cdot (1 - n) = 0. \qquad \text{(A14)}\end{aligned}$$

The first-order condition with respect to c^e yields

$$nv'(c^e) + (1 - n)v'(c^u) + \frac{\partial \Delta v}{\partial c^e} \left[(\Delta v + (1 - s)k(e)) \frac{\partial n^*}{\partial f} f'(\Delta v) + \phi n'(\Delta v) (w - \Delta c)\right] - \phi = 0.$$

We can manipulate these two first-order conditions to obtain the inverse Euler equation (10). To obtain the optimal UI formula, we derive the pendant of Lemma 1 in the dynamic case.

LEMMA A11. *Microelasticity ϵ^m and macroelasticity ϵ^M are related by*

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1 - n} \cdot \left(1 + \epsilon^d\right) \cdot \frac{\partial n^*}{\partial f} \cdot f'(\Delta v) \cdot \left.\frac{\partial \Delta v}{\partial \Delta c}\right|_{c^e}.$$

Proof. Given that $n(\Delta v) = n^*(e(f, \Delta v), f(\Delta v)))$,

$$\begin{aligned}n'(\Delta v) &= \frac{\partial n^*}{\partial e} \cdot \frac{\partial e}{\partial \Delta v} + \frac{\partial n^*}{\partial f} \cdot f'(\Delta v) + \frac{\partial n^*}{\partial e} \cdot \frac{\partial e}{\partial f} \cdot f'(\Delta v) \\ n'(\Delta v) &= \frac{\partial n^*}{\partial e} \cdot \frac{\partial e}{\partial \Delta v} + \frac{\partial n^*}{\partial f} \cdot \left(1 + \epsilon^d\right) \cdot f'(\Delta v). \qquad \text{(A15)}\end{aligned}$$

We replaced $\partial n^* / \partial e$ by $\partial n^* / \partial f$ using the property that

$$\frac{\partial n^*}{\partial e} = \frac{\partial n^*}{\partial f} \cdot \frac{f}{e},$$

which arises because e and f influence n^* only through $e \cdot f$. Given the definition of n^s, we have

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \frac{\partial n^*}{\partial e} \cdot \frac{\partial e}{\partial \Delta v} \cdot \frac{\partial \Delta v}{\partial \Delta c}.$$

Multiplying (A15) by $\Delta c/(1-n)$ and using the definition of ϵ^M, we obtain

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \frac{\partial n^*}{\partial f} \cdot \left(1+\epsilon^d\right) \cdot f'(\Delta v) \cdot \frac{\partial \Delta v}{\partial \Delta c}.$$

□

We come back to first-order condition (A14). We divide it by $\phi \cdot (1-n)$, normalize $k(e) = 0$ at the optimum, rearrange the terms, and abuse notations by denoting $f'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c) = \partial f / \partial \Delta c$ and $n'(\Delta v) \cdot (\partial \Delta v / \partial \Delta c) = \partial n / \partial \Delta c$. We obtain

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} \cdot (w - \Delta c) = \left[\frac{v'(c^u)}{\phi} - 1\right] - \frac{1}{\phi} \cdot \frac{\Delta v}{1-n} \cdot \frac{\partial n^*}{\partial f} \cdot \frac{\partial f}{\partial \Delta c} \tag{A16}$$

The inverse Euler equation yields

$$\frac{v'(c^u)}{\phi} - 1 = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right].$$

Lemma A11 allows us to write

$$-\frac{\Delta v}{1-n} \cdot \frac{\partial n^*}{\partial f} \cdot \frac{\partial f}{\partial \Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

By definition,

$$\frac{1}{1-n} \cdot \frac{\partial n}{\partial \Delta c} = \frac{1}{\Delta c} \cdot \epsilon^M.$$

Therefore we can rewrite (A16) as

$$\frac{w - \Delta c}{\Delta c} \cdot \epsilon^M = n \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\epsilon^m - \epsilon^M\right).$$

Using part (i) of Lemma A1 and dividing this equation by ϵ^M yields the optimal UI formula

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\epsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{1}{\phi} \cdot \frac{\Delta v}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right).$$

Following the argument in the proof of Proposition 2, we can show that this formula carries over if workers have access to home production by replacing $v'(c^u)$ and Δv by $v'(c^h)$ and Δv^h, which proves the first part of the proposition. We can also approximate the formula and express it terms of sufficient statistics as in Proposition 2, which proves the second part of the proposition.

B The Canonical Equilibrium Unemployment Model

This section studies the canonical model of Pissarides [2000], characterized by Assumptions 5 and 6. Utility is $v(c) = \left(c^{1-\rho} - 1\right)/(1-\rho)$, where ρ is the coefficient of relative risk aversion. We determine the outcome of the bargaining problem faced by a firm-worker pair.

$\mathcal{E}$ denotes the value of being employed, and $\mathcal{U}$ the value of being unemployed. Both values are evaluated after the matching process. They satisfy

$$\mathcal{E} = v\left((1-t)\cdot w\right),$$
$$\mathcal{U} = v\left(b\cdot w\right).$$

Combining both conditions yields the worker's surplus $\mathcal{W}$ from a relationship with a firm:

$$\mathcal{W} = \mathcal{E} - \mathcal{U} = \left[v\left((1-t)\cdot w\right) - v\left(b\cdot w\right)\right].$$

When worker and firm bargain, they take the tax rate t and unemployment benefits $b\cdot w$ as given because in the term $b\cdot w$ of $\mathcal{W}$, w is the equilibrium wage. In the term $(1-t)\cdot w$ of $\mathcal{W}$, w is the outcome of bargaining between firm and worker. Therefore when the worker evaluates the marginal utility $d\mathcal{W}$ of an increase dw in the wage bargained with the firm, he only considers the marginal change of the post-tax earnings $(1-t)\cdot w$. Accordingly,

$$\frac{d\mathcal{W}}{dw} = (1-t)\cdot v'\left((1-t)\cdot w\right) = (1-t)^{1-\rho}\cdot v'\left(w\right).$$

In equilibrium the firm's surplus from an established relationship is simply given by the hiring cost since a firm can immediately replace a worker at that cost during the matching period: $\mathcal{F} = r\cdot a/q(\theta)$. Since the firm's utility is simply its profits, a wage w brings a utility $-w$ to the firm (or its owners) and $d\mathcal{F}/dw = -1$.

The generalized Nash solution to the bargaining problem faced by a firm-worker pair is the wage w that maximizes

$$\mathcal{W}(w)^{\beta}\cdot\mathcal{F}(w)^{1-\beta},$$

where β is the worker's bargaining power. The first-order condition of the maximization problem implies that the worker's surplus each period is related to the firm's surplus by

$$\frac{\beta}{1-\beta}\cdot\frac{d\mathcal{W}}{dw}\cdot\mathcal{F} = \mathcal{W}.$$

Using the previous expressions for $\mathcal{W}$, $\mathcal{F}$, and $d\mathcal{W}/dw$, we obtain the relationship between equilibrium variables imposed by Nash bargaining over wages. Note that $\tau = c^u/c^e = b/(1-t)$.

$$\frac{\beta}{1-\beta} \cdot \frac{r \cdot a}{q(\theta)} \cdot (1-t)^{1-\rho} \cdot v'(w) = [v((1-t)\cdot w) - v(b \cdot w)]$$

$$\frac{w}{a} = -\frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)} \cdot \frac{r}{q(\theta)}.$$

Combining the expression for w/a with the labor demand equation (15) yields

$$\frac{r}{q(\theta)} = \left[1 - \frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)}\right]^{-1}. \tag{A17}$$

Tightness θ does not depend on technology a. Keeping τ constant, there are no fluctuations in the labor market with Nash bargaining.

Next, Proposition A1 establishes that under log utility, the macroelasticity ϵ^M is greater than the microelasticity ϵ^m in the canonical model

PROPOSITION A1. *Under Assumptions 5 and 6, and if $\rho = 1$, then $\epsilon^m/\epsilon^M < 1$.*

Proof. The equilibrium condition (A17), obtained under Assumptions 5 and 6, implies that $d\theta/d\tau < 0$. If $\rho = 1$, $v(c) = \ln(c)$, $\Delta v = -\ln(\tau)$, and $\theta'(d\Delta v) > 0$. With log utility, Lemma 1 implies

$$\epsilon^M = \epsilon^m + \frac{1/\tau - 1}{1-n} \cdot \frac{h}{f} \cdot \left(1 + \epsilon^d\right) \cdot f'(\theta) \cdot \theta'(d\Delta v).$$

Since $\theta'(\Delta v) > 0$ and $\tau < 1$, Lemma 1 implies that $\epsilon^M > \epsilon^m > 0$. Therefore, $\epsilon^m/\epsilon^M < 1$. $\square$

C The Dynamic Model

This section describes and studies the dynamic model of Section 5. We denote $c_t^h \equiv c_t^u + y_t$ the total consumption of unemployed workers (consumption of market and home-produced goods), $\Delta v_t^h \equiv v(c_t^e) - [v(c_t^u) - m(y_t)]$ the utility gain from work, and $\Delta c_t \equiv c_t^e - c_t^u$ the consumption gain from work. Unemployment is $u_t = 1-(1-s)\cdot n_{t-1}$ and the number of hires is $h_t = n_t - (1-s)\cdot n_{t-1}$.

Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. Together with initial employment n_{-1} in the representative firm, the history of technology realizations $a^t \equiv (a_0, a_1, \ldots, a_t)$ describes the state of the economy in period t. The unemployment insurance plan $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ is measurable with respect to (a^t, n_{-1}), and is taken as given by firms and workers. We assume that the government can fully commit to the policy plan. The time-t element of the worker's choice and firm's choice must therefore be measurable with respect to (a^t, n_{-1}).

C.1 Equilibrium with unemployment insurance

Firms. Given labor market tightness, wage, and technology $\{\theta_t, w_t, a_t\}_{t=0}^{+\infty}$ the representative firm chooses employment $\{n_t^d\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[n_t^d - (1-s) \cdot n_{t-1}^d \right] \right\}.$$

The first-order condition with respect to n_t^d implies

$$a_t \cdot g'(n_t^d) = w_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right]. \tag{A18}$$

Workers. Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ (1 - n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) - \left[1 - (1-s) \cdot n_{t-1}^s \right] \cdot k(e_t) \right\}, \tag{A19}$$

subject to the law of motion of the employment probability in period t,

$$n_t^s = (1-s) \cdot n_{t-1}^s + \left[1 - (1-s) \cdot n_{t-1}^s \right] \cdot e_t \cdot f(\theta_t). \tag{A20}$$

The Lagrangian of the worker's problem is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Big\{ - \left[1 - (1-s) \cdot n_{t-1}^s \right] \cdot k(e_t) + (1 - n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e)$$
$$+ A_t \cdot \left[\left[1 - (1-s) \cdot n_{t-1}^s \right] \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1}^s - n_t^s \right] \Big\},$$

where $\{A_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ is a sequence of Lagrange multipliers. The first-order condition with respect to home production y_t is

$$m'(y_t) = v'(c_t^h). \tag{A21}$$

The first-order condition with respect to effort e_t is

$$k'(e_t) = f(\theta_t) \cdot A_t.$$

The first-order condition with respect to employment probability n_t^s is

$$A_t = \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1}) \right] + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[A_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1})) \right].$$

Combining both conditions, we find that the optimal search effort satisfies

$$\frac{k'(e_t)}{f(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t\left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right] + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[e_{t+1} \cdot k'(e_{t+1}) - k(e_{t+1})\right] = \Delta v_t^h. \quad \text{(A22)}$$

Wage. In a labor market with matching frictions, the wage cannot equalize labor supply and demand. Since the wage is not determined by a market-clearing condition, we impose instead that the wage follows a stochastic process $\{w_t\}_{t=0}^{+\infty}$ defined for all $t \geq 0$ by

$$w_t = \omega \cdot a_t^{\gamma}. \quad \text{(A23)}$$

As in Hall [2005], we also require that the wage neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match.

Labor market equilibrium. Instead of the wage, labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\left\{n_t^d\right\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which defines employment $\{n_t\}_{t=0}^{+\infty}$:

$$n_t^d = n_t^s \equiv n_t. \quad \text{(A24)}$$

Equilibrium. Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$, an *equilibrium with unemployment insurance* is a collection of stochastic processes $\{y_t, e_t, n_t, \theta_t, w_t\}_{t=0}^{+\infty}$ that satisfy equations (A18),(A20), (A21), (A22), and (A23).

C.2 Optimal unemployment insurance

The government chooses a government policy $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A19) over all equilibria with unemployment insurance subject to the budget constraint

$$n_t \cdot w_t = n_t \cdot c_t^e + (1-n_t) \cdot c_t^u. \quad \text{(A25)}$$

The constraint arises because the government must balance its budget each period. An *equilibrium with optimal unemployment insurance* is a solution to the problem of the government.

We solve the problem of the government. The maximization of the government is over a collection $\{c_t^e(a^t), c_t^u(a^t), y_t(a^t), e_t(a^t), n_t(a^t), \theta_t(a^t), \ \forall a^t\}_{t=0}^{+\infty}$. We form the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ (1-n_t) \cdot \left[v(c_t^h) - m(y_t) \right] + n_t \cdot v(c_t^e) - [1-(1-s) \cdot n_{t-1}] \cdot k(e_t) \\ & + A_t \left[n_t \cdot w_t - n_t \cdot c_t^e - (1-n_t) \cdot c_t^u \right] \\ & + B_t \left[\left[v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] \right] - \frac{k'(e_t)}{f(\theta_t)} \right] + B_{t-1} \cdot (1-s) \left[\frac{k'(e_t)}{f(\theta_t)} + k(e_t) - e_t \cdot k'(e_t) \right] \\ & + Q_t \left[m'(y_t) - v'(c_t^h) \right] + C_t \left[a_t \cdot g'(n_t) - w_t - \frac{r \cdot a_t}{q(\theta_t)} \right] + C_{t-1} \cdot (1-s) \left[\frac{r \cdot a_t}{q(\theta_t)} \right] \\ & + D_t \left[(1-(1-s) \cdot n_{t-1}) \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1} - n_t \right] \Bigg\}\end{aligned}$$

where $\{A_t(a^t), B_t(a^t), Q_t(a^t), C_t(a^t), D_t(a^t), \ \forall a^t\}_{t=0}^{+\infty}$ are Lagrange multipliers. We define $B_{-1} \equiv 0$ and $C_{-1} \equiv 0$. The first-order conditions with respect to $y_t(a^t)$ for $t \geq 0$ are

$$0 = (1-n_t) \cdot \left[v'(c_t^h) - m'(y_t) \right] - B_t \cdot \left[v'(c_t^h) - m'(y_t) \right] + Q_t \cdot \left[m''(y_t) - v''(c_t^h) \right]$$

Using the optimal home production condition (A21), we obtain $0 = Q_t \cdot \left[m''(y_t) - v''(c_t^h) \right]$. Since $m'' > 0$ and $v'' < 0$,

$$0 = Q_t. \tag{A26}$$

The first-order conditions with respect to $c_t^e(a^t)$ for $t \geq 0$ are

$$A_t = v'(c_t^e) \cdot \left(1 + \frac{B_t}{n_t} \right). \tag{A27}$$

Using (A26), the first-order conditions with respect to $c_t^u(a^t)$ for $t \geq 0$ are

$$\begin{aligned} 0 &= -(1-n_t) \cdot A_t + (1-n_t) \cdot v'(c_t^h) - B_t \cdot v'(c_t^h) - Q_t \cdot v''(c_t^h) \\ A_t &= v'(c_t^h) \cdot \left[1 - \frac{B_t}{(1-n_t)} \right]. \end{aligned} \tag{A28}$$

The first-order conditions with respect to $e_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned} 0 &= -u_t \cdot k'(e_t) - B_t \cdot \frac{k''(e_t)}{f(\theta_t)} + (1-s) \cdot B_{t-1} \cdot \left\{ \frac{k''(e_t)}{f(\theta_t)} - e_t \cdot k''(e_t) \right\} + D_t \cdot u_t \cdot f(\theta_t) \\ 0 &= u_t + \kappa_t \cdot (1-s) \cdot B_{t-1} \cdot \left(1 - \frac{u_t}{h_t} \right) + \kappa_t \cdot \frac{u_t}{h_t} \cdot B_t - \frac{D_t \cdot h_t}{e_t \cdot k'(e_t)} \end{aligned} \tag{A29}$$

where $\kappa_t \equiv e_t \cdot k''(e_t)/k'(e_t)$. The first-order conditions with respect to $\theta_t(a^t)$ for $t \geq 0$ are

$$0 = (1-\eta) \cdot B_t \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta) \cdot (1-s) \cdot B_{t-1} \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} - C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + D_t \cdot u_t \cdot (1-\eta) \cdot e_t \cdot q(\theta_t)$$

$$0 = \frac{k'(e_t)}{\theta_t} \left[B_t - (1-s) \cdot B_{t-1}\right] - \frac{\eta}{1-\eta} \cdot r \cdot a_t \cdot \left[C_t - (1-s) \cdot C_{t-1}\right] + D_t \cdot h_t \cdot q(\theta_t) \quad \text{(A30)}$$

The first-order conditions with respect to $n_t(a^t)$ for $t \geq 0$ are

$$D_t = \Delta v_t^h + \delta(1-s)\mathbb{E}_t \left[k(e_{t+1}) + D_{t+1} \left(1 - e_{t+1} f(\theta_{t+1})\right)\right] + C_t a_t g''(n_t) + A_t \left[w_t - \Delta c_t\right]. \quad \text{(A31)}$$

The equilibrium with optimal unemployment insurance is a collection of 11 stochastic processes $\{c_t^e, c_t^u, y_t, e_t, n_t, \theta_t, A_t, B_t, C_t, D_t, Q_t\}_{t=0}^{+\infty}$ that satisfy 11 equations {(A25), (A18), (A20), (A21), (A22), (A26), (A27), (A28), (A29), (A30), (A31)}.

In steady state there are no aggregate shocks: $a_t = a$ for all t. The equilibrium is constant and characterized by a collection of 11 variables $\{c^e, c^u, y, n, \theta, e, A, B, C, D, Q\}$ determined by 11 equations {(A25), (A18), (A20), (A21), (A22), (A26), (A27), (A28), (A29), (A30), (A31)}. We combine a few of the first-order conditions and constraints of the government's problem to express some Lagrange multipliers in a simple form. These relationships are useful to solve for the steady state numerically. Combining (A27) and (A28), we obtain expressions for the Lagrange multipliers A and B as a function of equilibrium variables:

$$A = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^h)}\right]^{-1} \quad \text{(A32)}$$

$$B = n \cdot (1-n) \cdot \left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^h)}\right] \cdot A. \quad \text{(A33)}$$

Since $e/h = 1/(u \cdot f(\theta))$ in steady state, (A29) becomes

$$D = \frac{k'(e)}{f(\theta) \cdot u} \cdot \left[u + \kappa \cdot (1-s) \cdot B\right] + \frac{k''(e) \cdot e}{f(\theta)} \cdot \frac{s}{h} \cdot B$$

$$D = \frac{k'(e)}{f(\theta)} \cdot \left[1 + \frac{B}{n} \cdot \frac{\kappa}{u}\right], \quad \text{(A34)}$$

where $\kappa \equiv e \cdot k''(e)/k'(e)$. Using this expression, equation (A30) becomes:

$$0 = h \cdot D \cdot f(\theta) + k'(e) \cdot s \cdot B - \frac{\eta}{1-\eta} \cdot r \cdot a \cdot \theta \cdot s \cdot C$$

$$C = \frac{1-\eta}{\eta} \cdot \frac{k'(e)}{r \cdot a \cdot \theta} \cdot \left[n + B \cdot \left(\frac{\kappa}{u} + 1\right)\right]. \quad \text{(A35)}$$

C.3 Calibration

This section derives the relationships between the convexity μ and κ of the disutility from home production and from job search, and statistics estimated in the literature. The relationships are used to calibrate the dynamic model of Section 5.

Disutility from home production. We relate the convexity μ of the disutility $m(y) = \omega_m \cdot [y^{1+\mu} - \hat{y}^{1+\mu}] / (1+\mu)$ of home production to the statistics ϵ^i and ξ. Recall that $\xi = c^h/c^e$ and $\epsilon^i = dc^h/dc^u$ (where dc^u is a marginal change in benefits for one unemployed worker). Differentiating the optimal choice of home production (A4) for a marginal change dc^u,

$$v''(c^h) \cdot dc^h = m''(y) \cdot (dc^h - dc^u)$$
$$\epsilon^i = \frac{dc^h}{dc^u} = \left[1 - \frac{v''(c^h)}{m''(y)}\right]^{-1}.$$

Since m' is isoelastic, $\rho = -c \cdot v''(c)/v'(c)$, and $y = c^h - c^u = c^h \cdot (1 - \tau/\xi)$, we obtain

$$m''(y) = \mu \cdot \frac{m'(y)}{y} = \frac{\mu}{1 - \tau/\xi} \cdot \frac{v'(c^h)}{c^h} = -\frac{\mu}{1 - \tau/\xi} \cdot \frac{1}{\rho} \cdot v''(c^h).$$

Combining these two equations gives us an expression for μ as a function of ξ and ϵ^i:

$$\mu = \rho \cdot \left(1 - \frac{\tau}{\xi}\right) \cdot \frac{\epsilon^i}{1 - \epsilon^i}.$$

Disutility from job search. We relate the convexity κ of the disutility $k(e) = \omega_k \cdot (e^{1+\kappa} - 1) / (1+\kappa)$ from search to the statistics ϵ^s and ξ. Recall that $\epsilon^s = (c^u/\zeta) \cdot (d\zeta/dc^u)$, where dc^u is a marginal change in benefits for one unemployed worker, and $d\zeta = f \cdot de$ is the marginal response of the hazard rate for the worker due to the response of search de (we consider a change in benefits for one worker only, so the job-finding rate f is not affected by the policy experiment).

LEMMA A12. *Let $e(f, \Delta v^h)$ be the effort supply implicitly defined by (A22) in steady state:*

$$[1 - \delta \cdot (1 - s)] \cdot \frac{k'(e)}{f} + \delta \cdot (1 - s) \cdot [e \cdot k'(e) - k(e)] = \Delta v^h. \tag{A36}$$

At $e = 1$, the partial derivative of the effort supply satisfies

$$\frac{\Delta v^h}{e} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_f = \frac{1}{\kappa}.$$

Proof. We differentiate equation (A36) with respect to Δv^h, keeping f constant:

$$1 = \left\{[1-\delta\cdot(1-s)]\cdot\frac{k''(e)}{f} + \delta\cdot(1-s)\cdot[e\cdot k''(e)+k'(e)-k'(e)]\right\}\cdot\frac{\partial e}{\partial \Delta v^h}$$
$$1 = k''(e)\cdot\left\{\frac{1-\delta\cdot(1-s)}{f} + \delta\cdot(1-s)\cdot e\right\}\cdot\frac{\partial e}{\partial \Delta v^h}$$

At $e=1$, $k(e)=0$ and

$$\left[\frac{1-\delta\cdot(1-s)}{f} + \delta\cdot(1-s)\cdot e\right]\cdot k'(e) = \Delta v^h.$$

Therefore, given that $\kappa = e\cdot k''(e)/k'(e)$, we obtain

$$1 = e\cdot\frac{k''(e)}{k'(e)}\cdot\frac{\Delta v^h}{e}\frac{\partial e}{\partial \Delta v^h}$$
$$\frac{1}{\kappa} = \frac{\Delta v^h}{e}\cdot\frac{\partial e}{\partial \Delta v^h}.$$

□

LEMMA A13. *Let* $\Delta v^h(c^e,c^u) = min_y\left\{v(c^e) - v(c^u+y) + m(y)\right\}$ *be the utility gain from work when home production is optimal. At home production* y *such that* $m(y)=0$*, when* $c^h \approx c^e$*,*

$$\frac{c^u}{\Delta v^h}\cdot\left.\frac{\partial \Delta v^h}{\partial c^u}\right|_{c^e} \approx -\frac{\tau}{1-\xi}.$$

Proof. From Lemma A4, $\partial\Delta v^h/\partial c^u|_{c^e} = -v'(c^h)$. If $m(y)=0$ and if the second and higher order terms of $v(c)$ are small,

$$\Delta v^h = v(c^e) - v(c^h) \approx v'(c^h)\cdot(c^e - c^h) = v'(c^h)\cdot c^e\cdot(1-\xi).$$

To conclude,

$$\frac{c^u}{\Delta v^h}\cdot\left.\frac{\partial \Delta v^h}{\partial c^u}\right|_{c^e} \approx -\frac{c^u}{c^e\cdot(1-\xi)}\cdot\frac{v'(c^h)}{v'(c^h)} = -\frac{\tau}{1-\xi}.$$

□

On average, $\hat{e}=1$ so using Lemma A12,

$$\left.\frac{\partial\ln(\zeta)}{\partial\ln(\Delta v^h)}\right|_f = \left.\frac{\partial\ln(e)}{\partial\ln(\Delta v^h)}\right|_f = \frac{1}{\kappa}.$$

On average, $m(\hat{y}) = 0$ so using Lemma A13,

$$\left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{c^e} = -\frac{\tau}{1-\xi}$$

Combining these results implies We conclude that κ is related to ϵ^s by

$$\epsilon^s = -\left.\frac{\partial \ln(\zeta)}{\partial \ln(c^u)}\right|_{c^e,f} = -\left.\frac{\partial \ln(\zeta)}{\partial \ln(\Delta v^h)}\right|_{f} \cdot \left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{c^e} = \frac{\tau}{1-\xi} \cdot \frac{1}{\kappa}$$

$$\kappa = \frac{\tau}{(1-\xi)} \cdot \frac{1}{\epsilon^s}. \tag{A37}$$

C.4 Duration of unemployment benefits

This section describes and studies a dynamic model in which unemployment benefits have finite duration, as in Fredriksson and Holmlund [2001]. We introduce three superscripts: e for Employed worker; u for unemployed worker eligible for Unemployment benefits; a for unemployed worker whose benefits expired and who only receive social Assistance. The consumptions of market good are c_t^e, c_t^u, and c_t^a; the consumptions of home good for unemployed workers are y_t^u and y_t^a; the search efforts of unemployed workers are e_t^u and e_t^a. We define the following utility gains: $\Delta v_t^{u,e} \equiv v(c_t^e) - [v(c_t^u + y_t^u) - m(y_t^u)]$, $\Delta v_t^{a,e} \equiv v(c_t^e) - [v(c_t^a + y_t^a) - m(y_t^a)]$ $\Delta v_t^{a,u} \equiv \Delta v_t^{a,e} - \Delta v_t^{u,e}$.

Labor market. At the beginning of period t there are x_t^u eligible jobseekers exerting effort e_t^u, and x_t^a ineligible jobseekers exerting effort e_t^a. The number of matches h_t made is given by $h_t = h(e_t^a \cdot x_t^a + e_t^u \cdot x_t^u, o_t)$, where $e_t^a \cdot x_t^a + e_t^u \cdot x_t^u$ is aggregate search effort and o_t is vacancy. We define labor market tightness as $\theta_t \equiv o_t / \left(e_t^a \cdot x_t^a + e_t^u \cdot x_t^u\right)$. After matching, z_t^u eligible workers and z_t^a ineligible workers are unemployed. At the end of period t, a fraction λ_t of the z_t^u eligible unemployed workers become ineligible. The stocks of workers are related by

$$z_t^u = x_t^u \cdot [1 - e_t^u \cdot f(\theta_t)] \tag{A38}$$
$$z_t^a = x_t^a \cdot [1 - e_t^a \cdot f(\theta_t)] \tag{A39}$$
$$x_t^u = z_{t-1}^u \cdot (1 - \lambda_{t-1}) + s \cdot n_{t-1} \tag{A40}$$
$$x_t^a = z_{t-1}^a + \lambda_{t-1} \cdot z_{t-1}^u \tag{A41}$$

Firms. The problem of the firm is as in the baseline model. Optimal hiring satisfies (A18).

Workers. Given government policy $\{c_t^e, c_t^u, c_t^a, \lambda_t\}_{t=0}^{+\infty}$ and tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses efforts and home productions $\{e_t^u, e_t^a, y_t^u, y_t^a\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{v(c_t^e) - [x_t^u \cdot k(e_t^u) + x_t^a \cdot k(e_t^a) + z_t^u \cdot \Delta v_t^{u,e} + z_t^a \cdot \Delta v_t^{a,e}]\right\}, \tag{A42}$$

subject to the laws of motion (A38), (A39) (A40), and (A41) of the unemployment probabilities $\{x_t^u, x_t^a, z_t^u, z_t^a\}_{t=0}^{+\infty}$. Given $\{\theta_t\}_{t=0}^{+\infty}$, a choice of efforts $\{e_t^u, e_t^a\}_{t=0}^{+\infty}$ determines labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which is the employment rate in period t given by

$$n_t^s = 1 - (z_t^a + z_t^u). \tag{A43}$$

We form the Lagrangian of the worker's problem with multipliers A_t, B_t, C_t, D_t assigned to the laws of motion (A38), (A39) (A40),and (A41). The first-order conditions with respect to home productions y_t^u and y_t^a are

$$m'(y_t^u) = v'(c_t^u + y_t^u) \tag{A44}$$
$$m'(y_t^a) = v'(c_t^a + y_t^a). \tag{A45}$$

The first-order conditions with respect to efforts e_t^u and e_t^a are

$$k'(e_t^u) = f(\theta_t) \cdot A_t$$
$$k'(e_t^a) = f(\theta_t) \cdot B_t.$$

The first-order conditions with respect to unemployment probabilities x_t^u and x_t^a are

$$C_t = k(e_t^u) + A_t \cdot (1 - e_t^u f(\theta_t))$$
$$D_t = k(e_t^a) + B_t \cdot (1 - e_t^a f(\theta_t)).$$

The first-order conditions with respect to probabilities z_t^u and z_t^a are

$$A_t = \Delta v_t^{u,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t[C_{t+1}] + \lambda_t \cdot \delta \cdot \mathbb{E}_t[D_{t+1} - C_{t+1}]$$
$$B_t = \Delta v_t^{a,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t[D_{t+1}] + s \cdot \delta \cdot \mathbb{E}_t[D_{t+1} - C_{t+1}].$$

Combining these equations we have

$$\frac{\Delta k_t'}{f(\theta_t)} = \Delta v_t^{a,u} + (1-\lambda_t) \cdot \delta \cdot \mathbb{E}_t[D_{t+1} - C_{t+1}]$$
$$\mathbb{E}_t[D_{t+1} - C_{t+1}] = \mathbb{E}_t\left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1}\right],$$

where $\Delta k_t = k(e_t^a) - k(e_t^u)$, $\Delta k_t' = k'(e_t^a) - k'(e_t^u)$, and $\Delta K_t = e_t^a \cdot k'(e_t^a) - e_t^u \cdot k'(e_t^u)$. Combining the equations once more yields

$$\frac{k'(e^u_t)}{f(\theta_t)} + (1-s)\cdot\delta\cdot\mathbb{E}_t\left[e^u_{t+1}\cdot k'(e^u_{t+1}) - k(e^u_{t+1}) - \frac{k'(e^u_{t+1})}{f(\theta_{t+1})}\right]$$
$$= \Delta v^{u,e}_t + \lambda_t\cdot\delta\cdot\mathbb{E}_t\left[\frac{\Delta k'_{t+1}}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1}\right] \tag{A46}$$
$$\frac{k'(e^a_t)}{f(\theta_t)} + (1-s)\cdot\delta\cdot\mathbb{E}_t\left[e^a_{t+1}\cdot k'(e^a_{t+1}) - k(e^a_{t+1}) - \frac{k'(e^a_{t+1})}{f(\theta_{t+1})}\right]$$
$$= \Delta v^{a,e}_t + s\cdot\delta\cdot\mathbb{E}_t\left[\frac{\Delta k'_{t+1}}{f(\theta_{t+1})} + \Delta k_{t+1} - \Delta K_{t+1}\right]. \tag{A47}$$

Labor market equilibrium. As in the baseline model, tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n^d_t\}_{t=0}^{+\infty}$ to labor supply $\{n^s_t\}_{t=0}^{+\infty}$ such that (A24) holds, defining employment $\{n_t\}_{t=0}^{+\infty}$.

Equilibrium with unemployment insurance. Given government policy $\{\lambda_t,\, c^e_t,\, c^u_t,\, c^a_t\}_{t=0}^{+\infty}$, an *equilibrium with unemployment insurance* is a collection of stochastic processes $\{y^u_t,\, y^a_t,\, e^u_t, e^a_t,\, n_t,\, \theta_t\}_{t=0}^{+\infty}$ that satisfy equations (A38), (A39), (A40), (A41), (A18), (A43), (A44), (A45), (A46),and (A47).

Steady state. In steady state there are no aggregate shocks: $a_t = a$ for all t. The stocks of workers are constant over time. We can recombine the laws of motion of employment and unemployment probabilities to express $\{z_u, x_u, z_a, x_a, n\}$ as a function of $\{\lambda, \theta, e^a, e^u\}$. These steady-state relationships are useful to solve steady-state equilibria numerically.

In steady state the outflows into and outflows from social assistance are equal.

$$x_a\cdot e^a\cdot f(\theta) = \lambda\cdot x_u\cdot[1 - e^u\cdot f(\theta)]$$
$$x_a = x_u\cdot\lambda\cdot\frac{1-e^u\cdot f(\theta)}{e^a\cdot f(\theta)}.$$

The outflows from and inflows into employment are equal.

$$s\cdot n = x_a\cdot e^a\cdot f(\theta) + x_u\cdot e^u\cdot f(\theta)$$
$$n = \frac{1}{s}\cdot x_u\cdot[e^u\cdot f(\theta)\cdot(1-\lambda) + \lambda].$$

We write the stock of unemployment at the beginning of the period in two different ways.

$$1-(1-s)\cdot n = x_a + x_u$$
$$1 - \frac{1-s}{s}\cdot x_u\cdot[e^u\cdot f(\theta)\cdot(1-\lambda)+\lambda] = x_u\left[1 + \lambda\cdot\frac{1-e^u\cdot f(\theta)}{e^a\cdot f(\theta)}\right].$$

Combining our previous results, we get the following relationships:

$$x_u = \left[1 + \lambda \cdot [1 - e^u \cdot f(\theta)] \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s}\right] + \frac{1-s}{s} \cdot e^u \cdot f(\theta)\right]^{-1}$$

$$x_a = \left[1 + \frac{1-s}{s} \cdot e^a \cdot f(\theta) \cdot \left\{1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right\}\right]^{-1}$$

$$z_u = \left[1 + \lambda \cdot \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s}\right] + \frac{1}{s} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right]^{-1}$$

$$z_a = \left[1 + \left[\frac{1}{e^a \cdot f(\theta)} - 1\right]^{-1} \cdot \frac{1}{s} \cdot \left\{1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1\right]^{-1}\right\}\right]^{-1}$$

$$n = \left[1 + s \cdot \left[\frac{1}{e^a \cdot f(\theta)} - 1\right] + \frac{s}{(1-\lambda) \cdot e^u \cdot f(\theta) + \lambda} \cdot \left[1 - \frac{e^u}{e^a}\right]\right]^{-1}.$$

Optimal unemployment insurance. We assume that the generosity of the system of transfers is constant: there exists $\tau^{u,e} \in (0,1), \tau^{a,e} \in (0,1)$ such that for all t, $c_t^u = \tau^{u,e} \cdot c_t^e$ and $c_t^a = \tau^{a,e} \cdot c_t^e$. The government chooses a government policy $\{\lambda_t, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A42) over all equilibria with unemployment insurance subject to the budget constraint

$$n_t \cdot w_t = c_t^e \cdot \left[n_t + z_t^u \cdot \tau^{u,e} + z_t^a \cdot \tau^{a,e}\right]. \tag{A48}$$

The constraint arises because the government must balance its budget each period. An *equilibrium with optimal unemployment insurance* is a solution to the problem of the government.

To determine numerically the optimal arrival rate $\lambda(a)$ in a steady state with technology a, we perform a grid search over a range of arrival rates $\{\lambda_i\}$ and pick the λ_i that maximizes social welfare. Once we have picked λ, consumption c^e is given by budget constraint (A48). We repeat the computation for a sequence of technology $\{a_j\}$ to plot the graphs in Figure 3.

C.5 Elasticities

This section derives a few results about some key elasticities in the dynamic model.

Relationship between ϵ^m and ϵ^s. We relate the microelasticity ϵ^m to the elasticity ϵ^s estimated by Meyer [1990] and others. ϵ^s captures the response of search effort to a change in UI benefits. The relationship allows us to find empirical estimates of ϵ^m.

We use the notations introduced in the proof of Proposition 8. Since $e = e(f, \Delta v^h)$,

$$\epsilon^s = -\left.\frac{\partial \ln(e \cdot f)}{\partial \ln(c^u)}\right|_{f,c^e} = -\left.\frac{\partial \ln(e)}{\partial \ln(c^u)}\right|_{f,c^e} = -\left.\frac{\partial \ln(e)}{\partial \ln(\Delta v^h)}\right|_{f} \cdot \left.\frac{\partial \ln(\Delta v^h)}{\partial \ln(c^u)}\right|_{f,c^e}$$

$$\epsilon^s = \frac{c^u}{e} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_{f} \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e}.$$

Since $n^s(f, \Delta c, c^e) = n^*(e(f, \Delta v^h(\Delta c, c^e)), f)$,

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n^s}{\partial \Delta c}\right|_{c^e}$$

$$\epsilon^m = \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n^*}{\partial e}\right|_{f} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_{f} \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e}$$

$$\epsilon^m = \frac{\Delta c}{e} \cdot \frac{u \cdot n}{1-n} \cdot \left.\frac{\partial e}{\partial \Delta v^h}\right|_{f} \cdot \left.\frac{\partial \Delta v^h}{\partial \Delta c}\right|_{c^e},$$

because it is clear from (A13) that $\partial n^*/\partial e|_f = [1-(1-s)\cdot n]\cdot n/e = (u \cdot n)/e$. Since $\tau = c^u/c^e$, $\Delta c/c^u = (1-\tau)/\tau$. To conclude,

$$\epsilon^m = \frac{1-\tau}{\tau} \cdot \frac{n \cdot u}{1-n} \cdot \epsilon^s \tag{A49}$$

In normal circumstances, $\tau \approx 0.5$, $n \approx 1$, and $u \approx (1-n)$ so $\epsilon^m \approx \epsilon^s$.

Magnitude of ϵ^d. Lemma A14 shows that the discouraged-worker elasticity ϵ^d is commensurable to unemployment in the dynamic model.

LEMMA A14. *Let $e(f, \Delta v^h)$ be the effort supply in steady state implicitly defined by* (A36). *The discouraged-worker elasticity satisfies*

$$\epsilon^d = \frac{f}{e} \cdot \left.\frac{\partial e}{\partial f}\right|_{\Delta v^h} = \frac{1}{\kappa} \cdot \frac{1-\delta\cdot(1-s)}{1-\delta\cdot(1-s)\cdot[(1-n)/u]}.$$

If $\delta \approx 1$, then $\epsilon^d \approx u/\kappa$.

Proof. We differentiate equation (A36) with respect to f, keeping Δv^h constant:

$$0 = \left\{[1-\delta\cdot(1-s)]\cdot\frac{k''(e)}{f} + \delta\cdot(1-s)\cdot[e\cdot k''(e) + k'(e) - k'(e)]\right\}\cdot\frac{\partial e}{\partial f} - [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f^2}$$

$$\frac{f}{e}\cdot\frac{\partial e}{\partial f} = e\cdot\frac{k''(e)}{k'(e)}\cdot\frac{1-\delta\cdot(1-s)}{1-\delta\cdot(1-s)\cdot(1-e\cdot f)}$$

Given that $\kappa = e \cdot k''(e)/k'(e)$ and $1 - e \cdot f = 1 - (s \cdot n)/u = (1-n)/u$, we obtain

$$\frac{f}{e} \cdot \frac{\partial e}{\partial f} = \frac{1}{\kappa} \cdot \frac{1 - \delta \cdot (1-s)}{1 - \delta \cdot (1-s) \cdot [(1-n)/u]}.$$

If $\delta \approx 1$, $1 - \delta \cdot (1-s) \approx s$ and $1 - \delta \cdot (1-s) \cdot [(1-n)/u] \approx [u - (1-s) \cdot (1-n)]/u = s/u$. Thus, $(f/e) \cdot (\partial e/\partial f) \approx u/\kappa$. □

Fluctuations of ϵ^m and ϵ^M over the business cycle. Combining (A49) and (A37), we find an expression for the microelasticity ϵ^m:

$$\epsilon^m = \frac{1-\tau}{1-\xi} \cdot \frac{n \cdot u}{1-n} \cdot \frac{1}{\kappa}.$$

Next, we calculate an expression for the ratio ϵ^m/ϵ^M. The procedure is the same as that of Proposition 3 but for two steps. First, we replace the labor demand equation (15) by the labor demand in the steady-state of the dynamic model

$$g'(n) = \frac{w}{a} + [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)},$$

which derives from (A18). Second, we use a lemma that replaces Lemma A5 in a dynamic environment:

LEMMA A15. *The derivative of equilibrium tightness $\theta(\Delta c, c^e)$ with respect to Δc is*

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} = \frac{\theta}{\Delta c} \cdot \frac{1}{1+\epsilon^d} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{u \cdot n} \cdot \left(\epsilon^M - \epsilon^m\right).$$

If $\delta \approx 1$, we have the following approximation

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} \approx \frac{\theta}{\Delta c} \cdot \frac{\kappa}{\kappa + u} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{u \cdot n} \cdot \left(\epsilon^M - \epsilon^m\right).$$

Proof. The equilibrium job-finding rate is $f = f(\theta(\Delta v(\Delta c, c^e)))$ so Lemma A11 implies that

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \left(1+\epsilon^d\right) \cdot \frac{\partial n^*}{\partial f} \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

It is clear from (A13) that $\partial n^*/\partial f|_e = (u \cdot n)/f$. By definition, $f'(\theta)/f(\theta) = (1-\eta)/\theta$. Thus,

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot (1+\epsilon^d) \cdot \frac{u \cdot n}{\theta} \cdot (1-\eta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

Using the expression for ϵ^d given by Lemma A14, we infer that when $\delta \approx 1$,

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \frac{u+\kappa}{\kappa} \cdot \frac{u \cdot n}{\theta} \cdot (1-\eta) \cdot \frac{\partial \theta}{\partial \Delta c}.$$

We obtain expressions for $\partial\theta/\partial\Delta c$ by rearranging the terms in these equations. □

To conclude, the ratio ϵ^m/ϵ^M admits a simple expression in the steady state of the dynamic model:

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot (1+\epsilon^d) \cdot \frac{q(\theta)}{[1-\delta \cdot (1-s)] \cdot r} \cdot u \cdot n^{\alpha-1},$$

where ϵ^d is a function of u and n given by Lemma A14. For $\delta \approx 1$,

$$\frac{\epsilon^m}{\epsilon^M} \approx 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot \frac{u+\kappa}{\kappa} \cdot \frac{q(\theta)}{s \cdot r} \cdot u \cdot n^{\alpha-1}.$$

To describe how the elasticities vary over the business cycle, we compare steady states parameterized by different technology levels. The results are displayed in Figure A1. The figure shows that when the unemployment rate increases from 4% to 10%, the microelasticity ϵ^m increases slightly from 0.8 to 1, whereas the macroelasticity ϵ^M decreases sharply from 0.6 to 0.2. As a consequence, the elasticity wedge ϵ^m/ϵ^M increases drastically from 1.3 to 5 when the unemployment rate increases from 4% to 10%.

C.6 Taxation of profits

In this section, we assume that the government fully taxes profits, and that the profits contribute to financing the UI system. In addition, the government taxes or subsidizes labor income, and subsidizes unemployed workers. The government must balance its budget each period, so it chooses consumptions $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ subject to the resource constraint

$$a_t \cdot g(n_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}] = n_t \cdot c_t^e + (1-n_t) \cdot c_t^u. \tag{A50}$$

This resource constraint replaces the budget constraint (A25) in the baseline model.

The Lagrangian of the government problem is modified accordingly. The first-order conditions with respect to $\theta_t(a^t)$ for $t \geq 0$ become

$$0 = \frac{k'(e_t)}{\theta_t} [B_t - (1-s) \cdot B_{t-1}] - \frac{\eta}{1-\eta} \cdot r \cdot a_t \cdot [C_t - (1-s) \cdot C_{t-1} + A_t \cdot h_t] + q(\theta_t) \cdot h_t \cdot D_t.$$

This first-order condition replaces the first-order condition (A30) in the baseline model. The first-

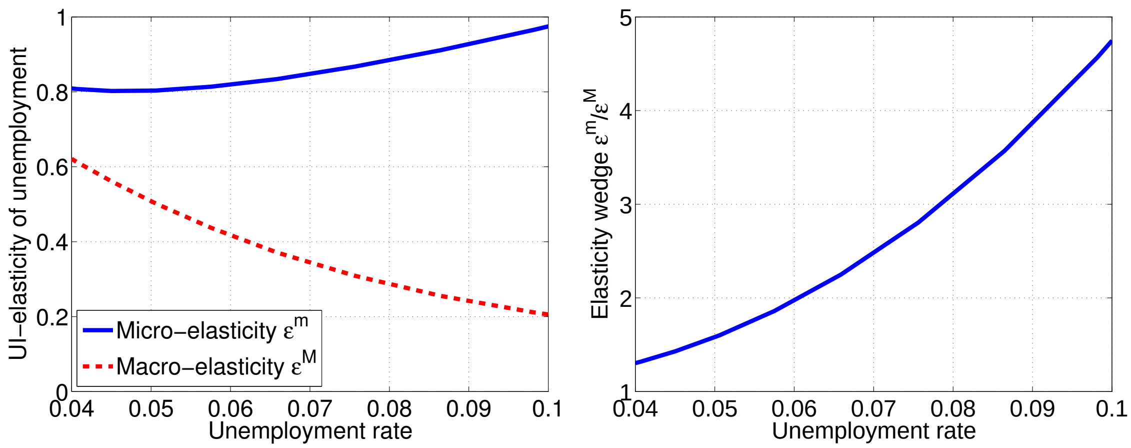

Figure A1: Microelasticity and macroelasticity over the business cycle

Notes: The simulations are based on the dynamic model calibrated in Table 2. The microelasticity is given by

$$\epsilon^m = \frac{1-\tau}{1-\xi} \cdot \frac{n \cdot u}{1-n} \cdot \frac{1}{\kappa}.$$

The elasticity wedge is given by

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \alpha \cdot (1-\alpha) \cdot \frac{1-\eta}{\eta} \cdot (1+\epsilon^d) \cdot \frac{q(\theta)}{[1-\delta \cdot (1-s)] \cdot r} \cdot u \cdot n^{\alpha-1},$$

where the discouraged-worker elasticity ϵ^d satisfies

$$\epsilon^d = \frac{1}{\kappa} \cdot \frac{1-\delta \cdot (1-s)}{1-\delta \cdot (1-s) \cdot [(1-n)/u]}.$$

order conditions with respect to $n_t(a^t)$ for $t \geq 0$ become

$$D_t = \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1}) + D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1})\right] + C_t \cdot a_t \cdot g''(n_t) + A_t \cdot [w_t - \Delta c_t] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[(A_{t+1} - A_t) \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right].$$

This first-order condition replaces the first-order condition (A31) in the baseline model. The equilibrium with optimal unemployment insurance is modified accordingly.

Figure A2 compares the equilibrium with optimal unemployment insurance with and without profit taxation. The solid blue lines represent the equilibrium without profit taxation; they are identical to the lines plotted in Figure 2. The red dashed lines represent the equilibrium with profit taxation. Unemployment and labor market tightness behave similarly in both equilibria. The cyclical behavior of the optimal replacement rate is also similar with and without profit taxation: the replacement rate is countercyclical and increases by 15 percentage points when unemployment increases from 4% to 10%. When the government is able to tax profits, the optimal replacement rate τ is higher by 10 percentage points because the government has more funding to finance UI.

Accordingly, our main measure of the generosity of UI, $\tau/(1-\tau)$, is higher with profit taxation. On the other hand, another measure of the generosity of UI, $(w-\Delta c)/\Delta c$, is lower with profit taxation. This numerical result is consistent with Proposition 7, which shows with profit taxation, the right-hand-side of the formula for $(w-\Delta c)/\Delta c$ includes an additional negative term.

The main difference between the equilibria with and without profit taxation appears in the labor tax rate and benefit rate required to implement the replacement rate. With profit taxation, the government raises so much revenue that it can afford to subsidize labor and pay generous unemployment benefits: the labor tax rate lies between -50% and -40% and the benefit rate lies between 80% and 100%. Without profit taxation, the labor tax rate is much higher and the benefit rate is much lower: the tax rate lies between 2% and 6% and the benefit rate lies between 40% and 60%.

C.7 Jointly optimal unemployment insurance and labor market tightness

We assume that the government controls directly tightness $\{\theta_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. This can be done through the labor demand equation (A18), by choosing the wage $\{w_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. Therefore, we drop from the government's problem constraints (A18) and variables $\{w_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$, and let the government choose directly variables $\{\theta_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. The government taxes profits and use them to finance the UI system; thus, the government is subject to the resource constraint (A50).

The optimization of the government is over a collection $\{c_t^e(a^t)$, $c_t^u(a^t)$, $y_t(a^t)$, $e_t(a^t)$, $n_t(a^t)$, $\theta_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. We form the Lagrangian

$$\begin{aligned}\mathcal{L} =&\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{(1-n_t)\cdot\left[v(c_t^h)-m(y_t)\right]+n_t\cdot v(c_t^e)-\left[1-(1-s)n_{t-1}\right]\cdot k(e_t)\\
&+A_t\left[a_t\cdot g(n_t)-\frac{r\cdot a_t}{q(\theta_t)}\cdot\left[n_t-(1-s)\cdot n_{t-1}\right]-n_t\cdot c_t^e-(1-n_t)\cdot c_t^u\right]\\
&+B_t\left[\left[v(c_t^e)-\left[v(c_t^h)-m(y_t)\right]\right]-\frac{k'(e_t)}{f(\theta_t)}\right]+B_{t-1}\cdot(1-s)\left[\frac{k'(e_t)}{f(\theta_t)}+k(e_t)-e_t\cdot k'(e_t)\right]\\
&+Q_t\left[m'(y_t)-v'(c_t^h)\right]\\
&+D_t\left[(1-(1-s)\cdot n_{t-1})\cdot e_t\cdot f(\theta_t)+(1-s)\cdot n_{t-1}-n_t\right]\Bigg\}\end{aligned}$$

where $\{A_t(a^t), B_t(a^t), Q_t(a^t), D_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ are Lagrange multipliers. We define $B_{-1}\equiv 0$. The first-order conditions with respect to $y_t(a^t)$, $c_t^e(a^t)$, $c_t^u(a^t)$, and $e_t(a^t)$ remain (A26), (A27), (A28), and (A29). The first-order conditions with respect to $\theta_t(a^t)$ for $t\geq 0$ are

$$\begin{aligned}0 =& -A_t\cdot\eta\cdot\frac{r\cdot a_t}{f(\theta_t)}\cdot h_t+(1-\eta)\cdot B_t\cdot\frac{k'(e_t)}{\theta_t\cdot f(\theta_t)}-(1-\eta)\cdot(1-s)\cdot B_{t-1}\cdot\frac{k'(e_t)}{\theta_t\cdot f(\theta_t)}\\
&+D_t\cdot u_t\cdot(1-\eta)\cdot e_t\cdot q(\theta_t)\\
0 =& \frac{k'(e_t)}{\theta_t}\left[B_t-(1-s)\cdot B_{t-1}\right]-\frac{\eta}{1-\eta}\cdot r\cdot a_t\cdot A_t\cdot h_t+D_t\cdot h_t\cdot q(\theta_t).\end{aligned}\tag{A51}$$

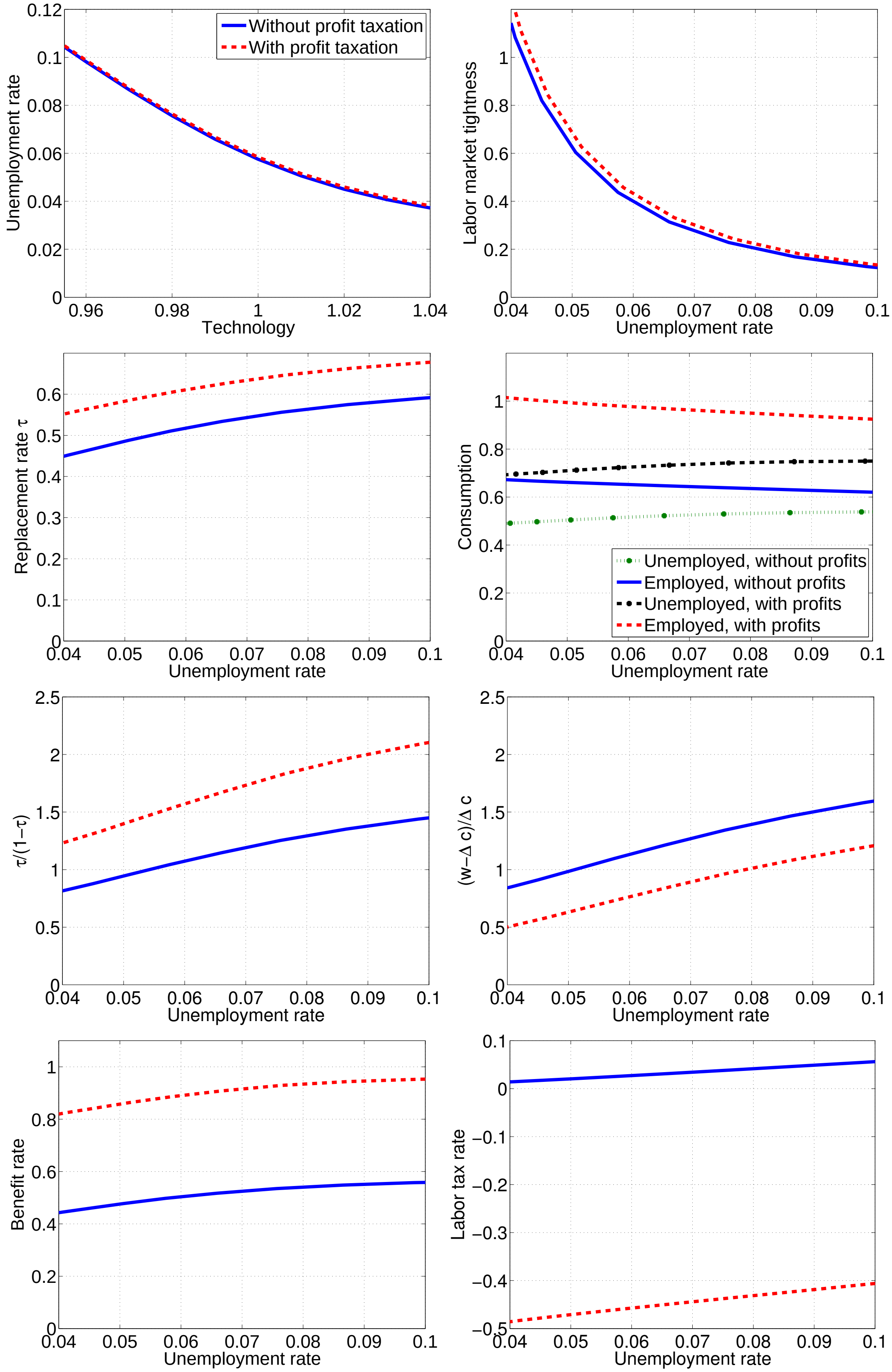

Figure A2: Optimal unemployment insurance, with and without taxation of profits

Notes: The simulations, described in the Appendix, are based on the dynamic model calibrated in Table 2.

The first-order conditions with respect to $n_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned} D_t &= \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1}) + D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}) \right] \\ &\quad + A_t \cdot \left[a_t \cdot g'(n_t) - \frac{r \cdot a_t}{q(\theta_t)} - \Delta c_t \right] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[A_{t+1} \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right] \\ D_t &= \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1}) + D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1})) \right] \\ &\quad + A_t \cdot [w_t - \Delta c_t] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[(A_{t+1} - A_t) \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right], \end{aligned} \tag{A52}$$

where we define

$$w_t \equiv a_t \cdot g'(n_t) - \left\{ \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right] \right\}.$$

The *equilibrium with jointly optimal unemployment insurance and wage* is a collection of 10 stochastic processes $\{c_t^e, c_t^u, y_t, e_t, n_t, \theta_t, A_t, B_t, D_t, Q_t\}_{t=0}^{+\infty}$ that satisfy 10 equations {(A25), (A20), (A21), (A22), (A26), (A27), (A28), (A29), (A51), (A52)}.

In steady state there are no aggregate shocks: $a_t = a$ for all t. The equilibrium is constant and characterized by a collection of 10 variables $\{c^e, c^u, y, n, \theta, e, A, B, D, Q\}$ determined by 10 equations {(A25), (A18), (A20), (A21), (A22), (A26), (A27), (A28), (A29), (A51), (A52)}. We rearrange these equations to obtain a compact characterization of the steady state. The Lagrange multipliers A, B, and D remain given by (A32), (A33), and (A34). Using the two expressions for B and D, equation (A51) becomes

$$\begin{aligned} 0 &= q(\theta) \cdot h \cdot D + \frac{k'(e)}{\theta} \cdot s \cdot B - \frac{\eta}{1-\eta} \cdot r \cdot a \cdot s \cdot A \cdot n \\ 1 &= \frac{1-\eta}{\eta} \cdot \frac{k'(e)}{r \cdot a \cdot \theta} \cdot \left[\frac{1}{A} + (1-n) \cdot \left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^h)} \right] \cdot \left(\frac{\kappa}{u} + 1 \right) \right]. \end{aligned}$$

This is the first optimality condition of the government's problem. The second optimality condition is obtained by manipulating (A52).

$$D \cdot [1 - \delta \cdot (1-s) \cdot (1 - e \cdot f(\theta))] = \Delta v^h + \delta \cdot (1-s) \cdot k(e) + A \cdot [w - \Delta c].$$

D Optimal Unemployment Insurance and Wage Subsidy

We start by describing the labor market equilibrium under technology a, when the replacement rate is $\tau = c^e/c^u$ and the normalized wage is $\tilde{w} = w/a$. Under Assumption 3, $v(c) = \ln(c)$. The firm's profit-maximization condition (15) can be rewritten as

$$g'(n) = \tilde{w} + \frac{r}{q(\theta)},$$

which implicitly defines a labor demand $n^d(\theta, \tilde{w})$ under Assumption 1. The equilibrium condition (16) can be rewritten as

$$n^s(f(\theta), \ln(1/\tau)) = n^d(\theta, \tilde{w}),$$

which implicitly defines equilibrium labor market tightness $\theta(\tau, \tilde{w})$. Furthermore, we define equilibrium employment $n(\tau, \tilde{w}) \equiv n^s(f(\theta(\tau, \tilde{w})), \ln(1/\tau))$. Lemma A16 establishes how equilibrium variables respond to a change in the wage $\tilde{w}$:

LEMMA A16. *Under Assumptions 1 and 2, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium tightness $\theta(\tau, \tilde{w})$ and equilibrium employment $n(\tau, \tilde{w})$:*

$$\left.\frac{\partial \theta}{\partial \tilde{w}}\right|_{\tau} < 0, \quad \left.\frac{\partial n}{\partial \tilde{w}}\right|_{\tau} < 0.$$

Proof. Similar to the proof of Lemma A6. □

The government chooses a rate b of unemployment benefits, a tax rate t imposed on the salary w^* received by employees, and a subsidy rate σ imposed on the salary w^* paid by employers. Effectively, firms pay a wage $w = (1-\sigma) \cdot w^*$, employed workers consume $c^e = (1-t) \cdot w^*$, and unemployed workers consume $c^u = b \cdot w^*$. The government is subject to the budget constraint

$$(1-n) \cdot b \cdot w^* + n \cdot \sigma \cdot w^* = t \cdot n \cdot w^*$$
$$(1-n) \cdot b \cdot w^* + n \cdot w^* - n \cdot t \cdot w^* = n \cdot w^* - n \cdot \sigma \cdot w^*$$
$$(1-n) \cdot c^u + n \cdot c^e = n \cdot w.$$

The budget constraint remains the same as in the baseline model even though the labor tax is collected from workers and partly redistributed to firms as a wage subsidy. The budget constraint defines a function that gives the consumption of employed workers in equilibrium: $c^e(\tau, \tilde{w}, a) \equiv a \cdot \tilde{c}^e(\tau, \tilde{w})$ where

$$\tilde{c}^e(\tau, \tilde{w}) \equiv \frac{n(\tau, \tilde{w})}{n + [1 - n(\tau, \tilde{w})] \cdot \tau} \cdot \tilde{w}.$$

In equilibrium, the expected utility of a worker is

$$\ln\left(c^e(\tau, \tilde{w}, a)\right) + [1 - n(\tau, \tilde{w})] \cdot \ln(\tau) - u \cdot k\left(e(\tau, \tilde{w})\right) = \ln(a) + SW\left(\tau, \tilde{w}\right),$$

where we define the function

$$SW\left(\tau, \tilde{w}\right) \equiv \ln\left(\tilde{c}^e(\tau, \tilde{w})\right) + [1 - n(\tau, \tilde{w})] \cdot \ln(\tau) - u \cdot k\left(e(\tau, \tilde{w})\right).$$

In Section 3, we maximized $SW\left(\tau, \tilde{w}\right)$ over $\tau \in (0, 1)$ for $\tilde{w} = \tilde{w}(a) \equiv \omega \cdot a^{\gamma-1}$ (because we made Assumption 2). The result from Proposition 5 in Section 3 tell us something about the properties of SW. Let $\tau^*\left(\tilde{w}\right)$ be the function implicitly defined by

$$\frac{\partial SW(\tau, \tilde{w})}{\partial \tau} = 0.$$

Furthermore, we define the replacement rate $\tau(a) \equiv \tau^*(\tilde{w}(a))$. Under some conditions, Proposition 5 shows that $d\tau/da < 0$. Since $d\tilde{w}/da < 0$ and

$$\frac{d\tau}{da} = \frac{d\tau^*}{d\tilde{w}} \cdot \frac{d\tilde{w}}{da},$$

we infer that $d\tau^*/d\tilde{w} > 0$ (under the assumptions of Proposition 5).

Let us consider the problem of the government when the government chooses optimally both the wage $\tilde{w}$ and the replacement rate τ. To capture the various costs of implementing a wage subsidy, we assume that setting a wage $\tilde{w}$ when the technology is a imposes a welfare cost $\mathcal{C}(\tilde{w}, a) > 0$. If the salary is a function $w^*(a)$ of a, a possible welfare cost could be an increasing convex function $\mathcal{C}(\sigma)$ of the subsidy rate σ. The reason is that $\sigma = [w - w^*(a)]/w^*(a) = [a \cdot \tilde{w} - w^*(a)]/w^*(a)$ so σ is only a function of $\tilde{w}$ and a. A critical assumption is that the welfare cost $\mathcal{C}$ does not depend on the replacement rate τ. The government chooses jointly τ and $\tilde{w}$ to maximize

$$\ln(a) + SW(\tau, \tilde{w}) - \mathcal{C}(\tilde{w}, a).$$

The first-order condition with respect to τ is

$$\left.\frac{\partial SW(\tau, \tilde{w})}{\partial \tau}\right|_{\tilde{w}=\tilde{w}^\dagger} = 0$$

where $\tilde{w}^\dagger$ is the optimal wage. Therefore the optimal replacement rate is $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$, where $\tau^*(\cdot)$ is the function defined above. Our study of the government problem in Section 3 tell us that $\tau^*(\tilde{w})$ has the property that $d\tau^*/d\tilde{w} > 0$.

Note that the optimal wage $\tilde{w}^\dagger(a)$ is defined implicitly by the first-order condition

$$\left.\frac{\partial SW(\tau, \tilde{w})}{\partial \tilde{w}}\right|_{\tau=\tau^*(\tilde{w})} - \left.\frac{\partial \mathcal{C}}{\partial \tilde{w}}\right|_{a} = 0.$$

Assume that the replacement rate $\tau^\dagger$ is fixed. There is a technology shock from a to a' such that employment decreases after the optimal wage is adjusted from $\tilde{w}^\dagger(a)$ to $\tilde{w}^\dagger(a')$. Lemma A16 implies that $\tilde{w}^\dagger(a) < \tilde{w}^\dagger(a')$. Since the optimal replacement rate is solely a function of the optimal wage: $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$ with $d\tau^*/d\tilde{w} > 0$, $\tau^\dagger$ must increase. Therefore after an adverse shock that increases unemployment, the optimal replacement rate increases. The substantive conclusion of Proposition 5 is robust to the presence of wage subsidies: optimal UI is more generous when unemployment is high.

E Recessions Caused by Aggregate Demand Shocks

This section characterizes optimal UI in a model in which recessions are caused by the combination of low aggregate demand and nominal wage rigidity. After a negative demand shock, prices fall. The fall in prices, combined with nominal wage rigidity, increases the real wage and the marginal cost of labor, which reduces hiring and increases unemployment.[29]

Wage. Assume that nominal wages are rigid. The real wage w follows a simple wage rule

$$w = \frac{\mu}{p}, \tag{A53}$$

where p is the aggregate price level and μ is a parameter. The rule says that the wage is constant in nominal terms: $w \cdot p = \mu$.

Firms. The production function is linear: $g(n) = n$. The firm starts with $1 - u$ workers. Given real wage w and labor market tightness θ, the firm chooses employment n maximizes real profits

$$\pi = n - w \cdot n - \frac{r}{q(\theta)} \cdot [n - (1 - u)].$$

The first-order condition implies

$$1 = w + \frac{r}{q(\theta)}. \tag{A54}$$

Money. The firm's production is sold in a perfectly competitive goods market. The firm takes the market price p as given. The firm's production n at a given price p determines the aggregate supply of goods. The aggregate demand for goods market takes the simple form m/p, borrowed from the quantity theory of money, where the parameter m characterizes the level of aggregate demand. Fluctuations in m drive the business cycle. In equilibrium, the price clears the goods market:

$$\frac{m}{p} = n. \tag{A55}$$

Equilibrium. The equilibrium price is $p = m/n$ so the equilibrium real wage is

$$w = \frac{\mu}{p} = \frac{\mu}{m} \cdot n.$$

When aggregate demand m falls, the real wage w tends to rise. Inserting the equilibrium real wage into the labor demand equation (A54) yields a labor demand curve

$$n^d(\theta, m) = \frac{m}{\mu} \cdot \left[1 - \frac{r}{q(\theta)}\right]. \tag{A56}$$

The labor supply $n^s(f(\theta), \Delta v)$ retains the same structure as in the model in the text. Equating labor demand with labor supply curve defines implicitly equilibrium labor market tightness $\theta(\Delta v, m)$

[29] The model loosely captures one story of the Great Depression: contractionary monetary policy lead to deflation, which raised real wages above trend in presence of nominal wage rigidity, which in turn depressed employment.

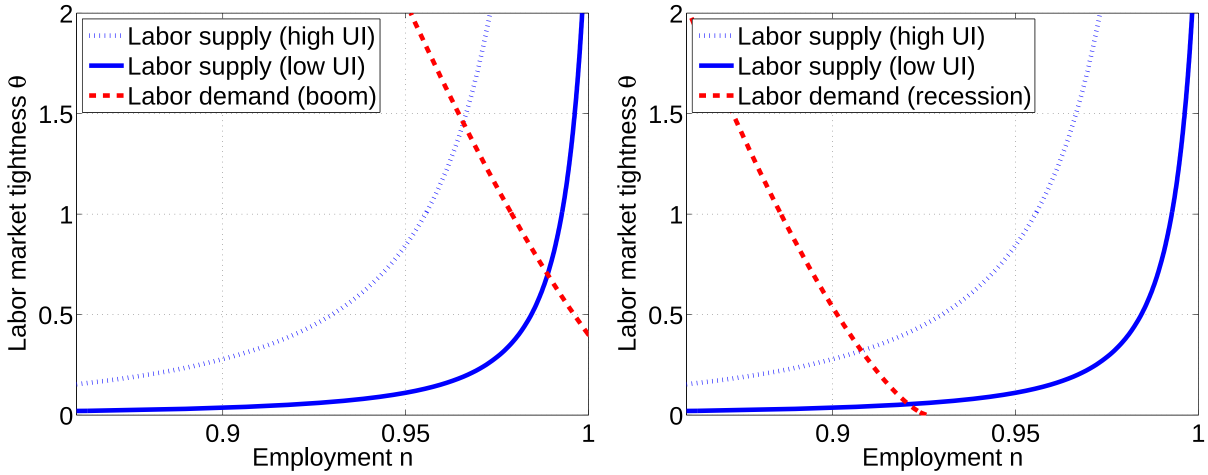

Figure A3: Labor market equilibrium in presence of demand shocks

and employment $n(\Delta v, m)$ as a function of aggregate demand m and utility gain from work Δv. The labor market equilibrium, depicted in Figure A3, shares the same structure as the equilibrium in the text, depicted in Figure 1.

Jobs are also rationed in recessions. Higher employment implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness for firms to be willing to hire: the aggregate labor demand curve is downward sloping in a price θ-quantity n plane. If demand is low enough ($m < \mu$), the labor demand falls below zero for $n < 1$: jobs are rationed.

Business cycle fluctuations. We focus on the case with log utility: $v(c) = \ln(c)$. Since $\Delta v = \ln(1/\tau)$, we parameterize the equilibrium of the model with (τ, m). We have the following comparative statics for equilibrium variables:

$$\left.\frac{\partial \theta}{\partial m}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial m}\right|_{\tau} > 0.$$

The proof is identical to that of Lemma A6 because, even if the labor demand is different here from the labor demand in the text, it remains true that $\partial n^d / \partial \theta < 0, \ \partial n^d / \partial m > 0$.

Optimal UI formula. Real wages respond to UI. In equilibrium, UI affects search effort, tightness, employment, price, and eventually, because of nominal wage rigidity, real wage. The optimal UI formula must account for the impact of UI on the government's budget through wages. For instance if UI raises wages, then UI has an additional beneficial effect because it increases the tax base. Of course the wage increase is partly at the cost of firm's profits. For consistency, we account for profits: we assume that the government taxes profits and uses them to finance UI. Thus, the government faces budget constraint (18). Even if wages respond to Δv as here, the appropriate optimal UI formula remains (22). The influence of Δv on w and on equilibrium employment is simply captured by the macroelasticity ϵ^M. Note that $g'' = 0$. Also note from Lemma A1 that with $g(n) = n$, $(1/n) \cdot \tau/(1-\tau) = (w - \Delta c)/\Delta c + \{[1 - (h/n)]/\Delta c\} \cdot r \cdot a/q(\theta)$. If $n \approx 1$ and $r \ll 1$, then $\tau/(1-\tau) \approx (w - \Delta c)/\Delta c$. Note from Lemma A2 that $\epsilon^d = 1/\kappa$. Assume that $n \approx 1$ and that the third and higher order terms of v are small. The formula simplifies to

$$\frac{\tau}{1-\tau} = \frac{\rho}{\epsilon^M} \cdot (1-\tau) + \frac{1}{1+\epsilon^d} \cdot \left(\frac{\epsilon^m}{\epsilon^M} - 1\right) \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right]. \tag{A57}$$

Elasticities. We study how microelasticity ϵ^m and macroelasticity ϵ^M fluctuate over the business cycle to determine whether the optimal replacement rate is procyclical or countercyclical. We first examine the elasticity wedge ϵ^m/ϵ^M. We differentiate the labor demand equation (A56):

$$\left.\frac{\partial n}{\partial \Delta c}\right|_{c^e} = -\frac{m}{\mu} \cdot \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e}.$$

Using Lemma A5 (which remains valid here) and the definition of the elasticity ϵ^M, we obtain

$$\begin{aligned}(1-n) \cdot \epsilon^M &= -\frac{m}{\mu} \cdot \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(\epsilon^M - \epsilon^m\right) \\ \left[\frac{\epsilon^m}{\epsilon^M} - 1\right] &= \frac{\mu}{m} \cdot \frac{q(\theta)}{r} \cdot \frac{\kappa+1}{\kappa} \cdot h \cdot \frac{1-\eta}{\eta}.\end{aligned}$$

Under Assumption 4 we can write

$$\left[\frac{\epsilon^m}{\epsilon^M} - 1\right] = \aleph \cdot \frac{q(\theta)}{r} \cdot n \cdot \frac{\mu}{m},$$

where, under Assumption 3, $\aleph$ is a constant defined by

$$\aleph \equiv \frac{1-\eta}{\eta} \cdot \frac{\kappa+1}{\kappa} \cdot s > 0.$$

Finally, using the labor demand equation (A56),

$$\left[\frac{\epsilon^m}{\epsilon^M} - 1\right] = \aleph \cdot \left[\frac{q(\theta)}{r} - 1\right] > 0.$$

There is an elasticity wedge $(\epsilon^m/\epsilon^M) - 1 > 0$, as in the text. The wedge widens in recessions: since $\partial\theta/\partial m|_\tau > 0$ and q is a decreasing function, $\partial\left[\epsilon^m/\epsilon^M\right]/\partial m|_\tau < 0$.

We turn to the macroelasticity ϵ^M. The expression (A5) for ϵ^m remains valid so, since $\partial n/\partial m|_\tau > 0$, $\partial\epsilon^m/\partial m|_\tau > 0$. We can conclude that $\partial\epsilon^M/\partial m|_\tau > 0$ because $\epsilon^M = \epsilon^m/(\epsilon^m/\epsilon^M)$.

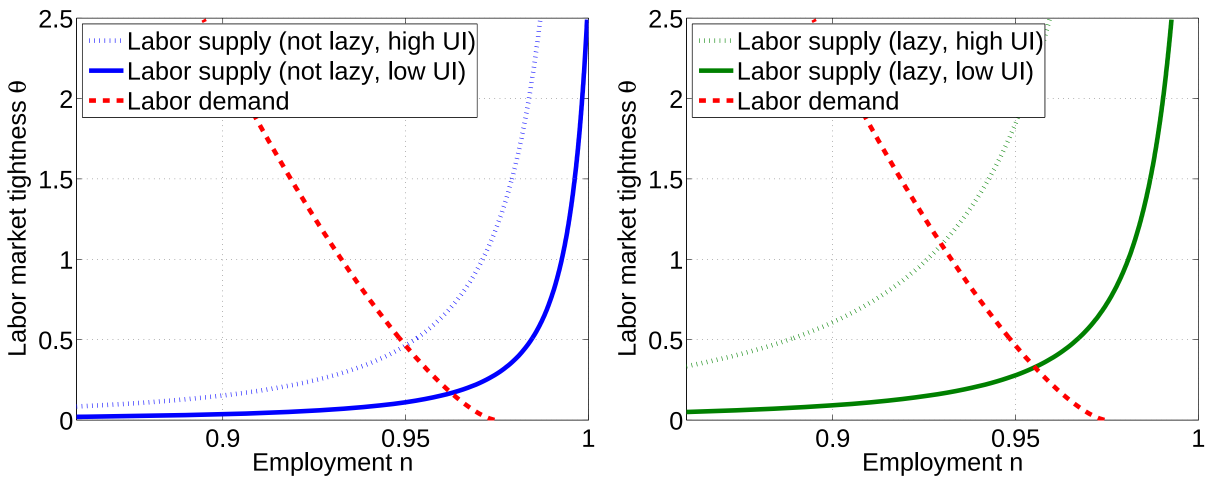

Figure A4: Labor market equilibrium in presence of preference shocks

Optimal replacement rate over the business cycle. Using optimal UI formula (A57) and the results that $\partial \left[\epsilon^m/\epsilon^M\right]/\partial m|_\tau < 0$ and $\partial \epsilon^M/\partial m|_\tau > 0$, we infer that $d\tau/dm < 0$. Therefore, the optimal replacement rate is also countercyclical in a model in which recessions are driven by aggregate demand shocks.

F Recessions Caused by Preference Shocks

This section characterizes optimal UI in a model in which recessions are caused by preference shocks that affect the disutility from job search.

Workers. A worker's utility is $v(c) - \psi \cdot k(e)$, where ψ is a parameter that characterizes the disutility of search. Fluctuations in ψ drive the business cycle. Given job-finding rate f and consumptions c^e and c^u, a jobseeker chooses effort e to maximize expected utility $v(c^u) + e \cdot f \cdot [v(c^e) - v(c^u)] - \psi \cdot k(e)$. The optimal search effort satisfies the first-order condition

$$k'(e) = f \cdot \frac{\Delta v}{\psi}, \tag{A58}$$

where $\Delta v = v(c^e) - v(c^u)$ is the utility gain from work. As the disutility from search $k(e)$ is convex, the effort supply $e(f, \Delta v, \psi)$ increases with f and Δv, but decreases with ψ.

Equilibrium. The labor market equilibrium is depicted in Figure A4. It shares the same structure as the labor market equilibrium in the text, depicted in Figure 1. The only difference is the response of the labor market to a macroeconomic shock. When ψ increases, search becomes more costly, effort supply $e(f(\theta), \Delta v, \psi)$ diminishes for a given θ, and the labor supply curve

$n^s(f(\theta), \Delta v, \psi) = 1 - u + u \cdot e(f(\theta), \Delta v, \psi) \cdot f(\theta)$ shifts left. Equilibrium employment falls, unemployment increases, and labor market tightness increases. Periods with higher disutility form search ψ are recessions because they are periods with higher unemployment. But these periods are unrealistic because they combine high unemployment with hight labor market tightness. In reality tightness falls when unemployment increases.

Business cycle fluctuations. We focus on the case with log utility: $v(c) = \ln(c)$. Since $\Delta v = \ln(1/\tau)$, we parameterize the equilibrium of the model with (τ, ψ). We have the following comparative statics for equilibrium variables:

$$\left.\frac{\partial \theta}{\partial \psi}\right|_{\tau} > 0, \quad \left.\frac{\partial n}{\partial \psi}\right|_{\tau} < 0.$$

The proof exploits the labor market equilibrium condition

$$n^d(\theta(\tau, \psi)) = 1 - u + u \cdot e(f(\theta(\tau, \psi)), \ln(1/\tau), \psi) \cdot f(\theta(\tau, \psi)).$$

We differentiate this condition with respect to ψ, keeping τ constant:

$$\frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial \psi} = u \cdot f \cdot \frac{\partial e}{\partial \psi} + u \cdot \left[f \cdot \frac{\partial e}{\partial f} + e\right] \cdot f'(\theta) \cdot \frac{\partial \theta}{\partial \psi}$$

$$\frac{\partial \theta}{\partial \psi} = -u \cdot f \cdot \underbrace{\frac{\partial e}{\partial \psi}}_{-} \cdot \left[\underbrace{u}_{+} \cdot \left(\underbrace{f}_{+} \cdot \underbrace{\frac{\partial e}{\partial f}}_{+} + \underbrace{e}_{+}\right) \cdot \underbrace{f'(\theta)}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-}\right]^{-1}.$$

because under Assumptions 1 and 2, $\partial n^d/\partial \theta < 0$. So $\partial \theta/\partial \psi|_{\tau} > 0$. We show that $\partial n/\partial \psi|_{\tau} < 0$ using $n(\tau, \psi) = n^d(\theta(\tau, \psi))$.

Optimal unemployment insurance formula. Clearly, the optimal UI formulas (9) and (11) remain valid in this model.

Elasticities. We study how microelasticity ϵ^m and macroelasticity ϵ^M fluctuate over the business cycle to determine whether the optimal replacement rate is procyclical or countercyclical. Proposition 3 remains valid and under Assumptions 2, 3, and 4, the elasticity wedge is

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \chi \cdot q(\theta) \cdot n^{\alpha - 1},$$

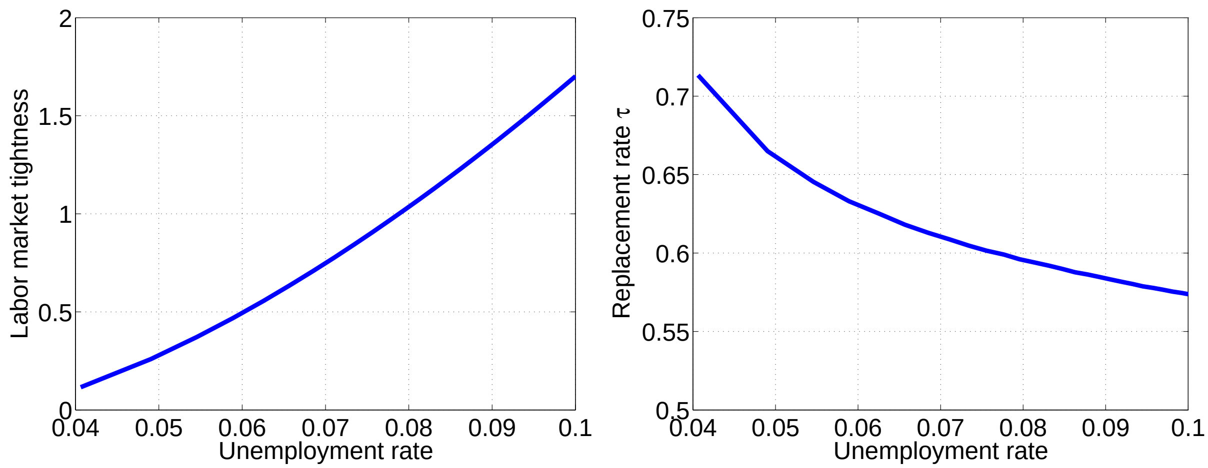

Figure A5: Optimal unemployment insurance over a business cycle driven by preference shocks

where $\chi = \alpha \cdot (1-\alpha) \cdot [(1-\eta)/\eta] \cdot [(1+\kappa)/\kappa] \cdot (s/r)$ is constant. The labor demand equation (15) implies

$$q(\theta) \cdot n^{\alpha-1} = \frac{w}{\alpha} \cdot q(\theta) + \frac{r}{\alpha}.$$

When ψ increases in recessions, the wage w remains constant but $q(\theta)$ decreases (because θ increases) so the right-hand side of the equation decreases. Hence $q(\theta) \cdot n^{\alpha-1}$ decreases. The elasticity wedge ϵ^m/ϵ^M therefore decreases. While the elasticity wedge was countercyclical in the model in the text, the wedge is procyclical here. In general, we cannot conclude on the cyclical behavior of ϵ^M; therefore we cannot conclude on the cyclical behavior of the optimal replacement rate.

Simulations. We resort to simulations to study the optimal replacement rate over the business cycle. The results from the simulations are displayed in Figure A5. All computations are based on the dynamic model calibrated in Table 2 (the calibration does not need to change even if the source of shock is different). The optimal replacement rate is procyclical: it increases from 58% to 71% when the unemployment rate decreases from 10% to 4%. But this model of the business cycle is unrealistic because labor market tightness increases sharply in recessions.