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THE EFFICIENCY GAINS FROM SOCIAL  
SECURITY BENEFIT - TAX LINKAGE

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ABSTRACT

This paper examines the efficiency gains from linking marginal Social Security benefits to marginal Social Security payroll taxes. In the U.S. the current combined employer-employee OASI payroll tax rate is 10.4 percent. Recent estimates suggest that the average marginal income tax rate is roughly 27 percent (Barro and Sahaskul (1983)). If marginal OASI payroll taxes provided no marginal Social Security benefits or were incorrectly perceived to provide no marginal benefits, the effective marginal federal government taxation of labor supply would average roughly 38 percent. Since the efficiency costs of distortionary taxation rise as roughly the square of the tax rate, the Social Security payroll tax may be more than doubling the dead weight loss of labor income taxation.

The findings of this paper suggest that there may be very significant efficiency gains available from tightening the connection between marginal Social Security taxes paid and marginal Social Security benefits received. Indeed, the simulated efficiency gains are very large in comparison with those obtained from analyses of the gains from structural tax reform. Restructuring Social Security to greatly enhance marginal benefit-tax linkage may be infeasible, at least in the short run. However, simply providing annual Social Security reports indicating how a worker's projected benefits are affected by his or her tax contributions could provide substantial increases in economic efficiency. Such efficiency gains are potentially as large as increasing GNP by 1 percent this year and every year in the future.

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The Efficiency Gains from Social Security

Benefit - Tax Linkage

by

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This paper examines the efficiency gains from linking marginal Social Security benefits to marginal Social Security payroll taxes. In the U.S. the current combined employer-employee OASI payroll tax rate is 10.4 percent. Recent estimates suggest that the average marginal income tax rate is roughly 27 percent (Barro and Sahaskul (1983)). If marginal OASI payroll taxes provided no marginal Social Security benefits or were incorrectly perceived to provide no marginal benefits, the effective marginal federal government taxation of labor supply would average roughly 38 percent. Since the efficiency costs of distortionary taxation rise as roughly the square of the tax rate, the Social Security payroll tax may be more than doubling the dead weight loss of labor income taxation.

In a fully funded Social Security System in which individual "tax" contributions were registered in individual accounts and paid out with market interest in old age, the government would simply be providing forced savings accounts for individuals, and, assuming no liquidity constraints, a dollar contributed to Social Security would be viewed as a dollar of saving, with no distortionary effect on labor supply. The linkage in this case, in present

value, of marginal benefits in return for marginal contributions ("taxes") is dollar for dollar. While individual accounts and tightly linking marginal benefits to marginal taxes have often alleged to be incompatible with an unfunded, "pay as you go" Social Security system, such is not the case. Marginal linkage can be equal to, greater than, or less than dollar for dollar in either a funded or an unfunded system. Consider, for example, a fully funded system in which uniform benefits are paid independent of individual tax contributions. In this case the marginal linkage is zero; full funding requires only that each cohort's old age benefits equal the cohort's accumulated tax contributions. It does not require that individual cohort members view their own tax payments as effectively identical to payments to a personal saving account.

In an unfunded system the government can establish marginal linkage by simply specifying a benefit formula which, at the margin, provides  $X$  dollars in present value of additional benefits for each dollar of additional tax contribution, where  $X$  can exceed, equal, or be less than one. The fact that one's marginal benefits and, indeed, one's total benefits are financed by members of the next generation is of no concern in formulating individual optimal intertemporal consumption and labor supply decisions.

Despite the fact that the U.S. Social Security System is essentially completely unfunded, marginal benefit-tax linkage in the U.S. appears to be significantly greater than one for one for some groups (e.g., older married males with low lifetime earnings and whose wives never worked); for other groups (e.g., low earning wives who will collect dependent and survivor benefits on their husbands' accounts) the marginal linkage is zero. Blinder, Gordon, and

Wise (1983) is the first systematic study of marginal benefit-tax linkage. This study shows that at least prior to 1977 the benefit formula provided a very significant return on marginal "tax" contribution to men in their early 60s. Some rough estimates of marginal linkage that are updated to include the 1983 legislated changes to Social Security are presented below. While these actuarial calculations are not highly complicated, they require a clear understanding of Social Security's benefit formula, including its method of wage indexing, its early retirement actuarial reduction provisions, its dependent and survivor benefit provisions, and its rules concerning the number and choice of years of earnings entering the calculation of AIME (average indexed monthly earnings). Given the surprised reaction to Blinder, Gordon, and Wise's findings by students of Social Security (including the authors), it appears extremely unlikely that typical American workers are aware of the marginal benefits they can expect under current law in exchange for their marginal taxes. Since the calculation of even a rough estimate of this linkage is difficult, since Social Security neither provides such information on a systematic basis nor will calculate such a number on request, and since Social Security legislation is subject to future changes, typical workers may simply, if incorrectly, assume that the marginal linkage is zero.

This paper studies the efficiency costs of zero linkage when dollar for dollar and, indeed, greater than dollar for dollar linkage is feasible. The analysis is based a fiscal policy simulation model (Auerbach and Kotlikoff (1983a, 1983b, 1983c, 1983d, 1985, 1986), Auerbach, Kotlikoff, and Skinner (1983)). The model assumes life cycle saving behavior and rational expectations

(actually perfect foresight since there is no uncertainty). The version of the model used here includes an unfunded Social Security System and the financing of government expenditures through alternative progressive or proportional income taxes. In addition, there is a special fiscal institution, the lump sum redistribution authority (LSRA), that can redistribute resources in a non distortionary manner across generations. This fiscal agency is used in the model to decompose welfare changes from benefit-tax linkage into those arising from pure efficiency gains and those associated with policy-induced intergenerational redistribution.

Calculations based on the model suggest very sizeable potential efficiency gains from running linked rather than unlinked Social Security programs. Indeed, the efficiency gains from dollar for dollar linkage exceed, in our model, the efficiency gains available from switching from proportional income to proportional consumption taxation. The results are illustrative and should not be viewed as providing estimates for the U.S. However, if American workers systematically underestimate actual marginal linkage, the results suggest that the efficiency of the U.S. fiscal structure could be greatly enhanced by providing, at a minimum, better information to workers about the marginal return on their payroll tax dollars, and at a maximum, by substantially increasing the extent of the marginal linkage. Assuming workers incorrectly believe that they receive nothing at the margin in return for Social Security taxes, then the model suggests that the efficiency gains from annual reporting of marginal benefit accrual could be as large as 1 percent of GNP on an annual basis, i.e., the possible efficiency gain is equivalent to a 1 percent larger

level of GNP this year and every year in the future.

The next section, II, contains rough estimates of marginal linkage in the U.S. system for young workers just entering the program. Section III describes how benefit-tax linkage alters effective labor income taxation. This description is conducted in the context of summarizing the basic equations of the simulation model and the method used to solve the model. Section IV presents the efficiency gains calculations, and Section V summarizes the paper's findings.

## II. Benefit-Tax Linkage in the U.S. Social Security System

Many of the elements determining benefit-tax linkage in the U.S. Social Security System can be illustrated by considering the simple case of a twenty-year old in 1985 whose benefit formula is determined by the Social Security Act as amended in 1983. For simplicity assume that the worker never marries, retires at age 67 (the normal retirement age legislated for this worker), and dies at age 75. The worker's nominal wage rate is  $W_0$  at age 20 and grows through time at rate  $(1+m)(1+g)(1+\pi)-1$ , where  $m$  is an individual-specific growth factor,  $g$  is the assumed constant economy-wide growth in real labor earnings, and  $\pi$  is the assumed constant inflation rate.

The present value of the individual's labor earnings plus Social Security benefits (PVR) is given by:

$$\begin{aligned} \text{PVR} = & \sum_{s=0}^{47} \frac{W_0(1+m)^s(1+g)^s(1+\pi)^s(1-\tau_y(1-\frac{\tau_s}{2})-\tau_s)L_s}{(1+r)^s(1+\pi)^s} \\ & + \sum_{s=48}^{55} \frac{B(\text{AIME})(1+\pi)^{s-47}L_s}{(1+r)^s(1+\pi)^s}, \end{aligned} \quad (1)$$

where  $r$  is the real interest rate,  $(1+r)(1+\pi)-1$  is the nominal interest rate, and the function  $B(\text{AIME})$  gives the initial age 67 nominal benefit calculated on the basis of the worker's AIME. Since benefits, at least under current law, are indexed for inflation, nominal benefits rise at the rate of  $\pi$  each year. The income and OASI tax rates are given by  $\tau_y$  and  $\tau_s$ , respectively. The term  $(1-\tau_s/2)$  reflects the fact that the employer's half of the payroll tax is deductible from the income tax. Finally,  $L_s$  is the worker's supply of labor at



age s.

The function B(AIME) can be expressed as

$$B(AIME) = a + b(AIME \times 12), \quad (2)$$

where the constants a and b are determined by the workers' AIME bracket. There are three indexed AIME brackets. For workers in the lowest bracket b equals .90, b equals .32 for workers in the middle bracket, and it equals .15 for those whose AIME is in the highest bracket.

The formula for AIME is given in (3). It takes into account the fact that wage indexation occurs only up through age 60. Between ages 60 and 67 (unindexed) nominal earnings are entered into the average "indexed" monthly earnings calculation.

$$AIME = \frac{\sum_{s=13}^{42} W_0 (1+m)^s (1+g)^s (1+\pi)^s [(1+g)(1+\pi)]^{42-s} L_s + \sum_{s=43}^{47} W_0 (1+m)^s (1+g)^s (1+\pi)^s L_s}{38 \times 12} \quad (3)$$

This formula assumes that the worker's final 38 years are his or her 38 years of highest earnings. The term  $[(1+g)(1+\pi)]^{40-s}$  represents indexation for economy-wide nominal wage growth prior to age 60. Combining (1), (2), and (3) yields:

$$PVR = \sum_{s=0}^{47} \frac{W_0 (1+m)^s (1+g)^s (1 - \tau_y (1 - \tau_s/2) - \tau_s) L_s}{(1+r)^s} \quad (4)$$

$$+ \frac{aD}{\pi^{47}} + bD \left[ \frac{\sum_{s=13}^{40} W_0 (1+m)^s (1+g)^{40} (1+\pi)^{40} L_s + \sum_{s=41}^{47} W_0 (1+m)^s (1+g)^s (1+\pi)^s L_s}{38 \cdot (1+\pi)^{47}} \right],$$

where  $D = \sum_{s=48}^{55} \left(\frac{1}{1+r}\right)^s$ . Some minor additional manipulation indicates that the effective tax rate on labor supply prior to age 63 ( $s < 43$ ),  $\tau_{e,s}$  is:

$$\tau_{e,s} = \tau_y(1 - \tau_s/2) + \tau_s - \left[ \frac{b(1+g)^{40-s}(1+\pi)^{-5}(1+r)^s D}{38} \right], \quad 13 \leq s \leq 40, \quad (5)$$

and between ages 61 and 67 it is:

$$\tau_{e,s} = \tau_y(1 - \tau_s/2) + \tau_s - \left[ \frac{b(1+r)^s(1+\pi)^{s-47} D}{38} \right], \quad 41 \leq s \leq 47, \quad (6)$$

where the formula relating age to  $s$  is  $\text{age} = 20 + s$ . Prior to age 33 when  $s = 13$ , the benefit tax linkage is zero, and  $\tau_{e,s} = \tau_y(1 - \tau_s/2) + \tau_s$ . The bracketed terms on the right-hand side of (5) and (6) give the effect of benefit linkage in reducing the effective taxation of labor supply. Stated differently this is the marginal payroll tax offset due to linkage. If  $g = r = \pi = 0$  the offset equals  $bD/38$  independent of age. Positive values of  $\pi$  lower the offset and thus raise the effective tax rate at all ages prior to 67 because of the non-indexation of earnings between ages 60 and 67. Higher values of  $g$  raise the offset and thus lower effective tax rates prior to age 61, when indexation ceases, with the biggest reduction in effective tax rates occurring at young ages (prior to age 32). This reflects the fact that earnings in early years are given more weight in the AIME calculation the bigger the wage growth indexation factor. Larger values of  $r$  lower the offset factor and thus raise effective taxes; since  $D$  declines with  $r$ , when  $r$  rises the present value of the marginal Social Security benefits earned with marginal tax contributions declines. The decline is, naturally, larger at younger ages.

Table 1 presents values of the marginal payroll tax offsets arising from

benefit linkage for different ages, for different assumed values of  $g$ ,  $r$ , and  $\pi$ , and for the three bend point bracket values of  $b$ . To put these numbers in perspective, suppose that the value of the terms  $\tau_y(1-\tau_s/2)+\tau_s$  is .50 for a worker in the top ( $b=.15$ ) bend point bracket, .30 for a worker in the middle ( $b=.32$ ) bracket, and .20 for a worker in the first bracket ( $b=.90$ ). The offset factor for workers ending up in the first AIME bracket against which the .20 figure should be compared is quite substantial; assuming a real interest rate of .04, a growth rate of .03, and, and inflation rate of .05, and  $b=.90$ , the offset factors are .066 at age 32, .074 at age 45, .086 at age 60, and .159 at age 67. Hence, if .20 is the effective tax on labor supply in the absence of the benefit-tax offset, with the offsets it is reduced to .134 at age 32, .126 at age 45, .114 at age 60, and .041 at age 67. If the worker's spouse earns so little that he or she will collect dependent and survivor benefits based solely on the worker's covered earnings (i.e., the spouse has an offset of zero throughout his or her life), then the offsets of the hypothetical worker should be increased by a factor of roughly 2.<sup>1</sup> In this case the effective tax rate for the low earning ( $b=.90$ ) principal earning spouse is .068 at age 32, .052 at age 45, .028 at age 60, and -.118 at age 67!

While the offsets are much smaller for workers in the  $b=.32$  and  $b=.15$  brackets, they are still important. At age 45 they reduce the effective tax rate for the  $b=.32$  worker from an assumed .30 to .274 for a single worker and to roughly .248 for a worker whose spouse will collect as a dependent and possibly as a survivor. For the  $b=.15$  age 45 worker, the assumed effective tax is lowered from .50 to .488 if the worker is single and to .476 if the worker is

married and has a low earning dependent/survivor spouse. While this reduction in effective marginal rates may seem small, the simulation findings presented below suggest that, starting from a marginal tax of 30 percent or greater, a reduction in the effective marginal tax by as little as one or two percentage points can significantly improve economic efficiency. It should also be pointed out that for high earners, whose earnings exceed the taxable maximum and who, correspondingly receive the maximum benefit, marginal linkage is zero.

At young ages the offsets in Table 1 are fairly sensitive to assumed values of  $r$  and  $g$ . For example, lowering  $r$  to .02 from .04 with  $g=.03$  and  $\pi=.05$  more than doubles the offset factors at age 32. As suggested by equations (5) and (6) the offsets are less sensitive to the assumed inflation rate. When  $r=.04$  and  $g=.03$  doubling  $\pi$  from .05 to .10 lowers the  $b=.90$  offset by only 2 percentage points at age 32.

A final point indicated by the table is that at least when  $r>g$ , which seems the historically relevant case (see Kotlikoff and Summers (1981)), the offsets rise significantly with age. In the case  $r=.04$ ,  $g=.03$ ,  $\pi=.05$  the offsets are zero prior to age 32 and then jumps to .066 when  $b=.90$ , to .023 when  $b=.32$ , and to .011 when  $b=.15$ . The offsets more than double between age 32 and 67. This feature would appear to reinforce the distortion arising under the capital income tax that leads workers to substitute leisure when young for leisure when old.

The simulation model described in the following sections abstracts from age-related changes in the marginal-tax offsets with the exception of those associated with transitional changes in the Social Security tax rate. Given the

apparently very poorly understood general level of these offsets, let alone their patterns with age, and given the added complexities of modeling age-varying offsets, the simulation analysis is confined to describing the efficiency gains from benefit-tax linkage that results in age-invariant offsets. Presumably, if in light of the potentially large efficiency gains from benefit-tax linkage, Congress were to legislate changes in Social Security that made explicit benefit-tax linkage (see Boskin, Kotlikoff, and Shoven (1982) for an example of a proposed benefit-tax linked restructuring of Social Security)), such legislation would also likely eliminate the anomalous pattern of offsets with age exhibited by the current system.

### III. Simulating the Efficiency Gains from Benefit - Tax Linkage

The simulation model used to study the effects of social security benefit-tax linkage is a general equilibrium life-cycle growth model with fifty-five overlapping generations of households. There is a single production sector, and three government sectors, one responsible for general fiscal policy that levies an income tax, a separate, self-financing social security system, and a self-financing lump sum redistribution authority (LSRA). The LSRA is used to isolate efficiency effects of various policy changes from the coincident intergenerational transfers of resources. For given values of parameters characterizing the tastes of households, the technology of firms, and the policies of government, numerical solution of the model yields a description of the path of the economy over time and of the behavior of individual households of different generations. Households are assumed to have perfect foresight; that is, their current decisions are based on expectations about the future that are correct.

We next describe, in turn, the household, production, and government sectors, paying particular attention to the way Social Security enters the model.

#### Households

At any given time, the household sector comprises fifty-five overlapping generations of adults. Each year one generation dies and another takes its place. It is useful to think of these "new" adults as being twenty-one years old with an expected age of death of seventy-five. As with other aspects of uncertainty found in the real world, lifetime uncertainty is not con-

sidered in the model.

Individual tastes are assumed to be identical, with differences in behavior being generated entirely by differences in economic opportunities. Hence, one can describe the aggregate behavior of members of a cohort by the behavior of a representative member. The version of the model examined here does not include children and explicit family structure. The rate of population growth is fixed at a constant annual rate, denoted  $n$ .

Households in the model make lifetime decisions about consumption and leisure based on the life cycle model of behavior, leaving no bequests and receiving no inheritances. Each household is assumed to have preferences that can be represented by a utility function with current and future values of consumption and leisure as arguments. Leisure is measured as a fraction of the maximum amount of time an individual could work in a given year, taking on a value between zero and one.

We restrict preferences by requiring that this utility function be time-separable and of the nested, constant elasticity of substitution form. Time separability means that lifetime utility can be expressed as a function of individual functions of leisure and consumption in each period:

$$(7) \quad U(c, l) = U[u_1(c_1, l_1), \dots, u_{55}(c_{55}, l_{55})]$$

where  $c_t$  and  $l_t$  are consumption and leisure in year  $t$ . It is also assumed here that the functions  $u_t(\ )$  do not vary over time, so that  $u_t(\ ) = u(\ )$ . The nested CES form further restricts both functions,  $u(\ )$  and  $U(\ )$ . The annual function takes the form:

$$(8) \quad u_t = [c_t^{(1-1/\rho)} + \mu_t^{(1-1/\rho)}]^{1/(1-1/\rho)},$$

while the lifetime function is written:

$$(9) \quad U = \left(\frac{1}{1-1/\gamma}\right) \sum_{t=0}^T (1+\delta)^{-(t-1)} u_t^{(1-1/\gamma)},$$

where  $\rho$ ,  $\mu$ ,  $\gamma$ , and  $\delta$  are taste parameters, and  $T$  is the age of death. Age zero in the model corresponds to actual age 20. Each is associated with a different aspect of individual tastes. Variation in these values produces a wide range of alternative behavioral responses.<sup>2</sup>

In the absence of social security, the household's budget constraint depends only on current and future values of after-tax interest rates and wage rates. The requirement that the present value of lifetime consumption not exceed the present value of lifetime earnings is, in this case:

$$(10) \quad \sum_{t=0}^T \left\{ \prod_{s=0}^{t-1} (1+r_s(1-\bar{\tau}_s))^{-1} \right\} [w_t(1-\bar{\tau}_t)e_t(1-l_t) - C_t] \geq 0,$$

where  $r_t$  is the annual interest rate at age  $t$ ,  $w_t$  is the standardized wage rate at age  $t$  (the wage rate of a new adult),  $\bar{\tau}_t$  is the average income tax rate that applies at age  $t$ , and  $e_t$  is an adjustment factor to allow for the fact that the household may earn more or less per hour at age  $t$  because of differences in skill levels among households of different ages. One may think of the vector  $e$ , composed of values of  $e_t$  for all  $t$ , as the household's "human capital" profile, reflecting its change in earning capacity over time. It is taken as fixed from the household's viewpoint.

In addition to this overall budget constraint, one must impose the requirement that labor supply can never be negative. This is represented by the



inequality constraints:

$$(11) \quad \lambda_t \leq 1 \quad \text{for all } t$$

Accounting for Social Security

The social security system in the model assesses payroll taxes on individual households and gives them retirement benefits. Because it, like the actual U.S. system, is an unfunded, pay-as-you-go scheme, it will not generally give each generation, as a whole, an actuarially fair return on its contributions. Instead, each generation's benefits are financed by the tax payments of those younger generations still working. In the long run, this will result in retirees, as a group, receiving a rate of return equal to the growth rate of covered earnings, rather than the after-tax interest rate. This rate, in turn, will equal the sum of the growth rates of real wages and the labor force.<sup>3</sup>

Social security affects household behavior in the model through its appearance in the lifetime budget constraint. Expression (10) becomes:

$$(12) \quad PVB + \sum_{t=0}^T \sum_{s=0}^{t-1} \{ \Pi (1+r_s (1-\bar{\tau}_s)^{-1}) [W_t (1-\bar{\tau}_t - \theta_t) (1-l) ]_{\bar{C}} \}_t \geq 0,$$

where PVB equals the present value of lifetime Social Security benefits, and  $\theta_t$  equals payroll taxes paid at age  $t$ . In an unfunded system, as in a funded system, the government is free to specify a formulae that relates Social Security benefits to lifetime labor earnings. The fact that, as a long run proposition, the return paid by Social Security on tax contributions equals the economy's growth rate places some restrictions on the generosity of the benefit

formula, at least in the long run. It does not, however, restrict the design of the benefit formula at the margin. Table 1 illustrates for the actual U.S. system that, at the margin, benefit formulae can be designed such that a dollar of tax payments generates, at the margin, more or less than a dollar in present value of benefits. In this paper we consider the following linear formula relating the present value of benefits ( $PVB_i$ ) received by generation  $i$  to the present value of its Social Security taxes ( $PVT_i$ ).

$$(13) \quad PVB_i = \alpha_i + \lambda_i PVT_i ,$$

Since

$$(14) \quad PVT_i = \sum_{t=0}^T \left\{ \prod_{s=0}^{t-1} (1+r_s(1-\bar{\tau}_s))^{-1} \right\} \theta_t \cdot w_t (1-l_t)$$

the payroll tax offset factor at age  $t$  simply equals  $\lambda_i \theta_t$ . Hence, if the social tax rate is constant over a worker's lifetime, his or her offset is constant at each age. While not capturing the rise in offsets with age (assuming a constant tax rate) exhibited in Table 1, this formulation (13) is quite convenient for simulating the efficiency gains from benefit-tax linkage.

With this formula the effective marginal labor income tax rate on a worker age  $s$  in year  $t$  is:  $\tau_{s,t} + \theta_t(1-\lambda_i)$ , where  $\tau_{s,t}$  is the age  $s$  year  $t$  marginal income tax rate, and  $\theta_t$  is the year  $t$  Social Security tax rate. Note that  $\lambda_i=0$  is the case of no linkage,  $\lambda_i=1$  is the case, in our model, in which the payroll tax offset exactly offsets the payroll tax, and  $\lambda_i>1$  is the case in which the payroll tax offset is sufficiently large to lower the effective tax on

labor income below the marginal income tax. We examine each of these cases below. Another case examined here is  $\alpha_i = 0$  and  $\lambda_i = PVB_i/PVT_i$ . Note that in the steady state  $PVB < PVT$   $\alpha$  is negative if  $\lambda$  exceeds  $PVB/PVT$ , it is positive if  $\lambda$  is less than  $PVB/PVT$ .

Figure 1 describing a one period consumption-leisure choice model with Social Security the only government policy may help clarify how the choice of  $\lambda$  can influence economic efficiency. The budget constraint linking consumption to labor earnings and Social Security benefits in a one period model is:

$$(15) \quad C = W(1-\theta)(1-l) + B$$

where  $C$  is consumption,  $W$  is the wage rate,  $l$  is leisure,  $\theta$  is the payroll tax rate, and  $B$  is the Social Security benefit. Now let  $B = \alpha + \lambda(\theta W(1-l))$ . In the figure budget line  $AA$  is the case of no Social Security and permits a maximum utility of  $U_0$ . Budget line  $ADD$  is the case in which a fixed benefit,  $\alpha$ , equal to the distance  $DA$ , is provided independent of the amount of payroll taxes paid, i.e.,  $\lambda=0$ . The utility level obtained in this case is  $U_1$ , and the reduction in private resources associated with the benefit tax program is  $AE$ ; i.e., the Social Security program depicted in Figure 1 is not actuarially fair since  $AE = \theta W(1-l) - B > 0$ .

The budget line  $AF$  also leaves the government collecting  $AE$  in net resources from the private sector, but permits utility of  $U_2$ . The difference in utility  $U_2 - U_1$  is the efficiency gain from switching from an unlinked benefit formula in which  $\alpha > 0$  and  $\lambda = 0$ , to a linked formula in which  $\alpha = 0$  and  $\lambda > 0$ .

In this case  $\lambda$  is set such that  $W(1-l_2)\theta(1-\lambda) = AE$ . Since  $l_2$  will, in general, depend on  $\lambda$ , this expression will, in general, be a non-linear equation in  $\lambda$ . Given the solution to this equation,  $\lambda^*$ , the government simply announces it will pay  $\lambda^*$  dollars in benefits for every dollar paid in taxes. Optimizing behavior on the part of atomistic workers leads each to supply  $l_2$  units of leisure providing the government with sufficient tax receipts to pay benefits equal to the preannounced  $\lambda^*$  times these tax receipts with  $AE$  left over for its other expenditure needs.

The budget line  $EE$  represents the first best method of collecting  $AE$ . In this case the government announced that  $\alpha = -AE$  and sets  $\lambda = 1$ . This is, of course, identical to levying a lump sum tax, which may seem infeasible. Suppose, however, the government disguises this lump sum tax by announcing a benefit formula with the following characteristics. There is a minimum benefit  $\alpha^* > 0$  plus an additional benefit equal to  $\lambda$  times the amount of taxes paid in excess of  $\alpha^{**}$ . If  $\lambda$  is set equal to one, and  $\alpha^*$  and  $\alpha^{**}$  are chosen such that  $\alpha^* - \alpha^{**} = \alpha = -AE$ , then the budget line is  $AGGE$  which, in terms of the worker's marginal choice behavior, is effectively equivalent to the budget line  $EE$ . Certainly "a minimum benefit coupled with higher benefits for tax payments in excess of a threshold value" sounds more feasible politically although it is really not different from a lump sum tax.

The addition of an income tax to this one period model raises the possibility of choosing  $\lambda > 1$  in order also to eliminate the distortion from the income tax. If  $\tau_y$  is the income tax, setting  $\lambda = (\theta + \tau_y)/\theta$ , leaves the effective tax on labor supply equal to zero. Figure 1 can now be reinterpreted to repre-

sent both the case of payroll and income taxation. The slopes of DD and GG in this case are equal to  $W(1-\tau_y-\theta)$ , and the budget frontier AGGE results from the government announcing a minimum benefit of  $\alpha^*$  plus  $\lambda = (\theta+\tau_y)/\theta$  in additional benefits for each dollar paid in taxes above  $\alpha^{**}$ .

Unlike the non-convex budget constraint AGGD, the U.S. system may look, at least for some workers, more like the convex budget set AHID, where along the segment AH (corresponding to the first AIME bracket) the effective tax rate is negative, it is positive, but small along HI (the second AIME bracket), and it is substantial along ID (the third AIME bracket). In considering the simulation findings it may be useful to keep in mind the values of  $\lambda$  that would arise in the U.S. if the actual U.S. benefit formula were replaced by benefit formula (13), with  $\alpha_i$  set equal to zero and  $\lambda_i$  set equal to  $PVT_i/PVB_i$ . Table 2 based on Table 7 of Pellechio and Goodfellow (1983) provides estimates of these values of  $\lambda$  for different family types age 25 in 1983. The figures are based on current law and incorporate the 1983 Social Security Actuaries' intermediate interest rate and other assumptions. These assumptions involve a real interest rate of roughly 2 percent, a 4 percent inflation rate, and a 1.5 percent rate of real earnings growth. Unlike the calculations of Table 1, the Table 2 figures include OASI dependent and survivor benefits. Table 2 indicates values of  $PVB/PVT$  in excess of .9 for one earner couples in each of the 1983 earnings categories. For two earner couples the values of  $\lambda = PVB/PVT$  exceeds .9 for the earnings levels below \$25,000. These high values of  $PVB/PVT$  are paid, in part, by single workers, particularly those with high earnings levels. For a single 25 year old male earning \$35,700 (the 1983 taxable maximum), the ratio of  $PVB$  to  $PVT$  is .41.

Determination of Equilibrium Transition Values of Social Security's Benefit Formula Parameters and Tax Rates

In the simulations conducted in this study we examine (1) the case of setting  $\alpha_i = 0$  for all  $i$ , and  $\lambda_i = PVB_i/PVT_i$ , and (2) the case of setting  $\alpha_i = PVB_i - \lambda PVT_i$ , where  $\lambda$  is set equal to either 1 or 4 for all generations. The baseline from which these benefits formulae are evaluated is the economy's initial steady state in which  $\alpha = PVB$ , and  $\lambda = 0$ . At the time a new program of benefit-linkage is announced there are, of course, initial Social Security beneficiaries in the model. These initial steady state Social Security recipients who exceed age 45 (65 in real time) at the time of the new policy are grandfathered in under the old Social Security programs; i.e., they are permitted to continue receiving the same benefits they were collecting prior to the change in the benefit formula.

Since Social Security is financed on a pay as you go basis, aggregate Social Security benefits at any point in time,  $t$ , must equal aggregate Social Security taxes:

$$(16) \quad \theta_t \sum_{a=0}^{45} \frac{W_t e_a (1-l_{t,a})}{(1+n)^a} = \sum_{a=46}^{55} \frac{B_{t,a}}{(1+n)^a},$$

where  $n$  is the population growth rate,  $\theta_t$  is the year  $t$  Social Security tax rate,  $a$  indexes age,  $l_{t,a}$  is the leisure of a worker age  $a$  at time  $t$ , and  $B_{t,a}$  is the benefit received by a Social Security beneficiary age  $a$  in year  $t$ . For simplicity let us assume that each beneficiary's benefit remains constant between ages 46 through 55, i.e.,  $\beta_{t,46} = \beta_{t+s,46+s}$ ,  $0 \leq s \leq 9$ . Now (16) can be written as:

$$(17) \quad \theta_t \sum_{a=0}^{45} \frac{W_t e_a (1-l_{t,a})}{(1+n)^a} = \sum_{a=46}^{55} \frac{B_{t+46-a,46}}{(1+n)^a}$$

For each worker the present value of his or her benefits is related to the present value of taxes by the formula:

$$(18) \quad PVB_t = B_{t+46,46} \sum_{j=46}^{55} \left[ \frac{1}{\prod_{s=0}^j (1+r_{t+s} (1-\bar{\tau}_{t+s}))} \right]$$

$$= \alpha_t + \lambda_t \sum_{j=0}^{45} \frac{\theta_{t+j} W_{t+j} e_j (1-l_{t+j,j})}{\prod_{s=0}^j (1+r_{t+s} (1-\bar{\tau}_{t,s}))},$$

where  $PVB_t$ ,  $\alpha_t$ , and  $\lambda_t$  are respectively the values of the present value of benefits, of  $\alpha$ , and of  $\lambda$  for the generation born in year  $t$ .  $\bar{\tau}_{t,s}$  is the average tax rate paid by the generation age  $s$  in year  $t$ . Substituting for  $B_{t+46,46}$  from (18) into (17) gives a sequence of equations of the form (19).

$$(19) \quad \theta_t \sum_{a=0}^{45} \frac{W_t e_a (1-l_{t,a})}{(1+n)^a} =$$

$$\sum_{a=46}^{55} \left( \frac{1}{1+n} \right)^a \left[ \alpha_{t-a} + \lambda_{t-a} \left( \sum_{j=0}^{45} \frac{\theta_{t-a+j} W_{t-a+j} e_j (1-l_{t-a+j,j})}{\prod_{s=0}^j (1+r_{t-a+s} (1-\bar{\tau}_{t-a+s,s}))} \right) \right] \sum_{j=46}^{55} \frac{1}{\prod_{s=0}^j (1+r_{t-a+s} (1-\bar{\tau}_{t-a+s,s}))}$$

Suppose the time path of the Social Security tax rates, the values of  $\theta_t$ , is given. Also assume that either the sequences of  $\alpha_t$  or  $\lambda_t$  are set exogeneously according to the policy experiments (1) and (2) described above. If the time paths of  $W_t$ ,  $r_t$ ,  $\bar{\tau}_{t,a}$ , and  $l_{t,a}$  (which depends on  $\lambda_{t-a}$ ) were also given, this sequence of equations could be used to solve for the endogenous sequence of

either  $\alpha_t$  or  $\lambda_t$ . In the simulation model this sequence of equations plus other equations determining  $W_t$ ,  $r_t$ ,  $\bar{\tau}_{t,a}$ , and  $l_{t,a}$  are solved simultaneously. Actually, the values of the time path of the Social Security tax rates, the  $\theta_t$ , are also endogenously determined. The time path of tax rates is set equal to that which would be required to finance monthly benefits for each successive generation equal to 60 percent of its average indexed monthly earnings (AIME). This choice for setting the time path of Social Security tax rates ensures that the general scale of the system is not affected by the particular formula chosen that links individual benefits to individual taxes. Equation (20) indicates the determination of the time path of tax rates,  $\theta_t$ :

$$(20) \quad \theta_t \sum_{a=0}^{45} \frac{W_t e_a (1-l_{t,a})}{(1+n)^a} = .6 \times \sum_{a=46}^{55} \frac{12 \times \text{AIME}_{t,a}}{(1+n)^a},$$

where  $\text{AIME}_{t,a}$  is the level of AIME over the 45 year work span for the cohort that is age  $a$  in year  $t$ .

Having described the inclusion of the Social Security System, we turn to a brief description of the production sector as well as the model's treatment of the government's non-Social Security fiscal policy.

### Firm Behavior

The model has a single production sector that is assumed to behave competitively, using capital and labor subject to a constant-returns-to-scale production function. Capital is assumed to be homogeneous and nondepreciating, while labor differs only in its efficiency. That is, all forms of labor are perfect substitutes, but individuals of different ages supply different amounts



of some standard measure of labor input per unit of leisure foregone. This amount is the term  $e_t$  for age cohort  $t$ , introduced above.

The production function is assumed to be of the Cobb-Douglas form:

$$(21) \quad Y_t = AK_t^\beta L_t^{1-\beta}$$

where  $Y_t$ ,  $K_t$ , and  $L_t$  are output, capital, and labor at time  $t$ ,  $A$  is a scaling constant, and  $\beta$  is a parameter measuring the factor share of capital in production.  $A$  is assumed to be constant over time, thereby ruling out the possibility of technological change.<sup>4</sup>

Competitive behavior on the part of firms insures that the marginal products of labor and capital are set equal to their respective factor returns,  $w$  and  $r$ .

#### Government Behavior

The government in this model raises income taxes to pay for its own consumption of goods and services which is assumed to grow at the same rate as the population. In addition, there is a separate social security system, already described, plus a self-financing redistribution branch, the LSRA, that is included for the sole purpose of disentangling the distributive and efficiency effects of various policies.

In financing its consumption, the government in this analysis uses either a proportional or a progressive income tax. In the case of progressive income tax marginal rates are linearly related to income. We assume that the main fiscal authority's budget is balanced in each year, so that there is no

national debt and government spending equals income tax collections.

When the LSRA is used to redistribute intergenerationally, it The LSRA assesses a lump sum tax (which may be negative) on each generations, subject to the constraint that the present value of these lump sum taxes equals zero. The lump sum taxes levied on those generations alive at the initiation of the new Social Security benefit-tax linkage policy are choosen to maintain the utility levels of these generations equal to what they would have been absent the new policy regime. The lump sum taxes on all generations at the start of the transition and thereafter are set so as to raise or lower the utility levels of all these generations by an identical amount. Since the model is solved in full general equilibrium the LSRA lump sum taxes are endogeneous and their equilibrium values are determined in the course of solving for the economy's equilibrium transition path. Auerbach, Kotlikoff, and Skinner (1983) provide a more extensive description of the LSRA. The Appendix describes the general method of solving the model and also describes the parameterization of the model.

#### IV. Simulation Results

Table 3 reports the efficiency gains from switching from an unlinked ( $\lambda=0$ ) Social Security benefit formula to three alternative benefit-tax linked formulae. The three formulae have alternative values of  $\lambda$  equal to either 1, 4, or the realized ratio of the present value of Social Security benefits to the present value of Social Security taxes. Two alternative methods of financing government consumption are considered. The first is a 30 percent proportional

income tax; the second is a progressive income tax in which the marginal tax rate,  $\tau_m$ , is a linear function of income:

$$(22) \quad \tau_m = .25 + .4Y$$

Recall that the LSRA in these simulations levies cohort-specific lump sum taxes such that (1) the utility levels of all generation's initially alive is not altered and (2) that the utility of all generations born at the time the new policy is introduced and thereafter is increased (in the case of a positive efficiency gain) or decreased (in the case of an efficiency decline) by a uniform amount. Thus the LSRA is charged with maximizing the minimum welfare of generations born after the new policy is announced. This maximin policy results in a uniform utility level for all further generations.

The efficiency gain is measured as the percentage increase in the present value of full time earnings required to raise the welfare of individuals living in the initial ( $\lambda = 0$ ) steady state to that uniform welfare level received by all new generations under one of the three benefit-tax linked formulae. In the case that  $\lambda = PVB/PVT$ , and government consumption is financed by an income tax, the efficiency gain is 1.3 percent of full lifetime resources. Since a new generation is born each year, the efficiency gain is equivalent, in present value, to an annual stream equal to 1.3 percent of full lifetime earnings. The present value of actual lifetime earnings in the initial steady state is somewhat more than one half of full lifetime earnings; the efficiency gain is over 2.4 percent of the present value of actual lifetime earnings (or

lifetime consumption since the two are equal). Measured as a percent of GNP, the efficiency gain is equivalent to permanently increasing GNP by .78 percent. To put the 1.3 percent figure in further perspective, one can compare it to the comparable efficiency gain associated with a switch from a proportional income tax to a proportional consumption tax. The gain from such a policy is 5.3 percent of full lifetime resources.<sup>5</sup> Hence, the gain from proportional benefit-tax linkage ( $\lambda = PVB/PVT$ ) is about one fourth of that available from switching to a consumption tax.

The final steady state value of  $\lambda$  in this simulation equals .13, and the final steady state payroll tax rate is 9.8 percent. Since the final steady state income tax rate is .29, proportional benefit-tax linkage lowers the effective tax rate from an initial steady state value of 39.8 percent to a final steady state value of 37.4 percent. Note that this simulated value of  $\lambda$  is much smaller than those reported in Table 2, reflecting in part the model's higher real interest rate.

Setting  $\lambda = 1$  or  $\lambda = 4$  produces strikingly large efficiency gains, 7.6 percent and 12.8 percent, respectively! The effective tax rate in the former case is reduced from 39.8 percent to 26.9 percent. In the latter case the effective tax is lowered from 39.8 percent to -7.1 percent; i.e., the benefit-tax linkage when  $\lambda = 4$  is sufficiently large to more than fully offset both the payroll tax and the income tax, leaving a net effective subsidy to labor supply in the final steady state.

As one would expect the efficiency gains from benefit tax linkage are larger still if a progressive rather than a proportional income tax is being

used to finance the same level of government consumption as under the proportional income tax. In the initial steady state the marginal tax rates associated with the progressive tax rate schedule (22) are 40 percent at age zero (age 20), 50 percent at age 24 (age 45), 31 percent at age 50 (age 70), and 25 percent at age 55 (age 75). The efficiency gains reported in Table 2 from benefit-tax linkage in the presence of this progressive income tax are 2 percent for  $\lambda = PVB/PVT$ , 15.1 percent for  $\lambda = 1$ , and 26.5 percent for  $\lambda = 4$ !. Measured as a percent of GNP these figures are 1.2 percent, 9.1 percent, and 16.0 percent.

Table 4 contains information about the stock of capital and the supply of labor for the six economies referenced in Table 1. The first thing to notice from this table is that the initial steady state capital stock and labor supply under the progressive tax regime are significantly smaller (30 percent and 9 percent, respectively) than under the proportional tax regime. A second feature is that when  $\lambda = 1$  and  $\lambda = 4$  benefit-tax linkage significantly increases the supply of labor, particularly at the early stages of the transitions. The linkage, coupled with the LSRA's tax-transfer policy, leads to substantial long run increases in the capital stock when  $\lambda = 1$  or  $\lambda = 4$ . In the case of the progressive income tax when  $\lambda = 4$ , the capital stock increases by a factor of 2.7! In viewing these numbers it should be understood that the parameterization of the model is fairly conservative with respect to the extent of substitution possibilities between consumption and leisure both at a point in time and over time. The significant substitution effects underlying the results of Table 3 appear to reflect the substantial changes in the relative price of leisure that occurs when  $\lambda$  is set to 1 and especially when  $\lambda$  is set to 4.

Table 5 shows for the case of  $\lambda = 1$  how the LSRA affects the results. Note that without the LSRA, as with the LSRA, the economy's transition path involves a pareto improvement. The reduced long run welfare gain with no LSRA relative to that with the LSRA (1.5 percent rather than 7.6 percent) reflects the improved welfare of those generations who are initially alive at the time the  $\lambda = 1$  benefit-tax linkage policy is implemented. The capital stock is also larger with the LSRA since the LSRA must tax initial generations to lower their welfare to the value it would have attained in the absence of the new policy. These taxes lower the consumption of such early generations, accounting for the larger accumulated saving.

V. Conclusions

The findings of this paper suggest that there may be very significant efficiency gains available from tightening the connection between marginal Social Security taxes paid and marginal Social Security benefits received. Indeed, the simulated efficiency gains are very large in comparison with those obtained from analyses of the gains from structural tax reform. Restructuring Social Security to greatly enhance marginal benefit-tax linkage may be infeasible, at least in the short run. However, the results suggest that simply providing annual reports under the current system indicating how a worker's projected benefits are affected by his or her tax contributions could provide a considerable increase in economic efficiency - perhaps as large as 1 percent of GNP on an annual basis.

Footnotes

1. Suppose, as is typically the case, that the non-working or low earning spouse is the wife. The wife is entitled to collect dependent benefits equal to half of her husband's benefit while her husband is alive and 100 percent of the husband's benefit when he is dead. Since women live longer than men and wives are typically 2-3 years younger than their husband, the factor of 2 seems appropriate.

2. The parameter  $\rho$ , the elasticity of substitution between concurrent leisure and consumption, determines how responsive an individual's annual labor supply response is to that year's wage rate. The term  $\mu$  represents the intensity of preference by the household for leisure relative to consumption. The greater is  $\mu$ , the less labor the household will supply in order to obtain consumption goods, preferring a greater amount of leisure instead. The term  $\delta$  is the rate of time preference. The remaining taste parameter,  $\gamma$ , equals the household's elasticity of substitution between consumption in different years, between leisure in different years, and between consumption and leisure in different years. The size of  $\gamma$  determines in part the responsiveness of households to changes in the incentive to save.

3. For example, consider the steady state of a two-period model in which the elderly generation receives benefits equal to the taxes of the younger generation. In this case, the benefits of the elderly will equal  $\theta W_y L_y P_y$ , the product of the steady state Social Security tax rate,  $\theta$ , times the steady state earnings of the younger generation which equals the steady state wage,  $W_y$ , times the labor supply per young person,  $L_y$ , times the population of young workers,  $P_y$ . The taxes previously paid by the elderly were  $\theta W_y \cdot L_y P_0 / (1+g)$ , where  $P_0$  is the population of elderly retirees, and  $g$  is the growth rate of wages. If the growth rate of population is  $n$ , the elderly receive a return of  $(1+g)(1+n)$  on their taxes. Put another way, the present value of Social Security benefits equals  $(1+g)(1+n)/(1+r(1-\tau))$  per dollar of payroll taxes.

4. It is generally impossible to include such change without also assuming continuous changes in tastes; otherwise the result would be either an increasing or decreasing trend in labor force participation, leading in the long run to an absurd result.

5. This value is much larger than that reported in Auerbach, Kotlikoff, and Skinner (1983) which ignores Social Security. The larger value found here reflects the fact that the consumption tax implicitly levies a lump sum tax on Social Security benefits as well as private assets, when Social Security is included in the model.



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Appendix: Solution Method and Parameterization of Model

Solution for Equilibrium under Perfect Foresight

The calculation of the equilibrium path of the economy, given a particular parameterization, proceeds in three stages. In the first stage the model solves for long run steady state of the economy before the assumed change in fiscal policy begins. If the long run steady state is independent of the transition path, it next solves for the long run steady state to which the economy eventually converges after the policy takes effect, and finally it solves for the transition path that the economy takes between these two steady states over time. The last year, 150 of the transition path, also provides the final steady state equilibrium.

The perfect foresight assumption is important only in this third stage, since in either of the long run steady states economic variables are constant from one year to the next; any plausible assumption about expectations formation would lead to individuals having correct foresight in such situations. The transition begins when new information about the policy change becomes available. One should visualize this information as an unanticipated change in fiscal policy regimes.

The iteration techniques used in each of the three stages of the solution are basically the same, although the actual procedure is more complicated when solving for the transition path because economic variables are changing over time.

### The Steady States

Solution for the equilibrium of the economy in the initial steady state amounts to solving a complicated system of nonlinear equations based on the behavior of households, firms, and the government derived from the optimization behavior outlined above. The solution is obtained using an iterative technique often referred to in the literature as the Gauss-Seidel method.

The algorithm starts with initial guesses of a subset of the endogenous variables and momentarily treats these variables as exogenous in some of the equations of the system where they appear. This simplification makes the resulting system easier to solve for the endogenous variables plus the variables for which initial guesses were made. When the solution for these "guessed" variables equals the guesses themselves, a true solution to the full system has been found. Otherwise, the "solution" is not consistent with equal values of the guessed endogenous variables in all equations, and new guesses are tried, typically a combination of the two sets of values from the previous iteration. Typically, ten to twenty iterations are required to achieve convergence to a solution for the initial steady state. The solution for the final steady state is identical to that for the initial steady state.

### The Transition

Solution for the economy's equilibrium transition path proceeds in a manner similar to the approach used to calculate the initial and final steady states. There are several complications, however. First, because the economy undergoes a transition with conditions changing over time, it is necessary to

solve explicitly for behavior in each year. Moreover, because households and firms are assumed to take account of future prices in determining their behavior, it is necessary to solve simultaneously for equilibrium in all transition years.

This is done in the following way. The simulation model provides the economy with 150 years to reach a new steady state. After 150 years, the model constrains all prices, tax rates, and shadow wages to be constant. If the final steady state has already been calculated, it is used to provide the values of these variables. Otherwise, they are solved for together with those of the years 1 through 150. The choice of 150 years is arbitrary, but is intended to provide enough time so that the economy settles down by itself well before it is "forced" to in year 150. Thus, the constraint on the number of years in the transition to be no more than 150 is not binding. The same path would result if 140 or 160 years were assumed, but not if a substantially shorter period, such as 30 years, were used, for in that time the economy typically is still adjusting.

As with the steady states, a Gauss-Seidel iteration algorithm is employed, but here the problem is 150 times larger since the years are solved for all at once. Aside from this greater complexity, a final difference in solving for the transition path as opposed to the initial steady state is that individuals alive at the time the policy is adopted must be treated differently. While individuals born after the transition begins know the economic conditions that will confront them, those born before the beginning of the transition behave up to the time of the change in government policy as if the old steady

state would continue forever. At the time of the announcement of a new policy to be instituted either immediately or in the near future, existing cohorts are "born again" (see Falwell 1984, Orwell 1984): they behave like members of a new generation, but with a shorter life expectancy and with initial assets resulting from prior accumulation.

#### Parameterization of the Model

To solve the model, it is necessary to choose values for the preference parameters,  $\mu$ ,  $\delta$ ,  $\rho$ ,  $\gamma$ , the capital share in production,  $\beta$ , the production scaling constant,  $A$ , and the human capital vector,  $e$ . Some of these parameters have been precisely estimated in several empirical studies. For the others, however, this is not true, and for certain parameters indirect methods must be used to obtain values. For all simulations presented below, we use the following set of parameters:

$$\mu=1.5$$

$$\delta=.015$$

$$\rho=.8$$

$$\gamma=.25$$

$$\beta=.25$$

$A$  is scaled so that the wage of a 21 year old equals 1 when there is no social security and the income tax is 15 percent. The vector  $e$  is based on wage profiles estimated by Welch (1979). Further details are provided in Auerbach, Kotlikoff, and Skinner (1983).

Table 1

Marginal Payroll Tax Offsets

<u>Age</u>	<u>r</u>	<u>g</u>	<u>II</u>	<u>D</u>	<u>b</u>		
					<u>0.90</u>	<u>0.32</u>	<u>0.15</u>
20	0.02	0.03	0.05	2.89	0.000	0.000	0.000
30	0.04	0.03	0.05	1.07	0.000	0.000	0.000
32	0.02	0.03	0.05	2.89	0.141	0.050	0.024
32	0.04	0.03	0.05	1.07	0.066	0.023	0.011
32	0.06	0.03	0.05	0.40	0.031	0.011	0.005
32	0.04	0.02	0.05	1.07	0.050	0.018	0.008
32	0.04	0.03	0.10	1.07	0.047	0.017	0.008
45	0.02	0.03	0.05	2.89	0.124	0.044	0.021
45	0.04	0.03	0.05	1.07	0.074	0.026	0.012
45	0.06	0.03	0.05	0.40	0.045	0.016	0.008
45	0.04	0.02	0.05	1.07	0.064	0.023	0.011
45	0.04	0.03	0.10	1.07	0.054	0.019	0.009
60	0.02	0.03	0.05	2.89	0.107	0.038	0.018
60	0.04	0.03	0.05	1.07	0.086	0.031	0.014
60	0.06	0.03	0.05	0.40	0.070	0.025	0.012
60	0.04	0.02	0.05	1.07	0.086	0.031	0.014
60	0.04	0.03	0.10	1.07	0.062	0.022	0.010
67	0.02	0.03	0.05	2.89	0.173	0.062	0.029
67	0.04	0.03	0.05	1.07	0.159	0.057	0.027
67	0.06	0.03	0.05	0.40	0.147	0.052	0.025
67	0.04	0.02	0.05	1.07	0.159	0.057	0.027
67	0.04	0.03	0.10	1.07	0.159	0.057	0.027

TABLE 2

Ratio of Present Expected Value of Social Security  
Benefits to Present Expected Value of Payroll Taxes

Total Family Earnings in 1983	Family Type			
	One Earner Couple (Husband works)	Two Equal Earner Couple	Single Male	Single Female
\$10,000	1.58	1.30	.73	1.03
15,000	1.38	1.08	.63	.90
20,000	1.28	.94	.58	.83
25,000	1.11	.87	.50	.72
30,000	1.00	.82	.45	.64
35,700	.92	.78	.41	.59

Source: Pellechio and Goodfellows, "Individual Gains and Losses from Social Security Before and After the 1983 Social Security Amendments," Table 7.



Table 3

Efficiency Gains from Linking Social Security Benefits to Payroll Taxes

<u>Tax Regime</u>	$\lambda = \frac{PVB}{PVT}$	<u><math>\lambda = 1</math></u>	<u><math>\lambda = 4</math></u>
Proportional Income Tax	1.3%	7.6%	12.8%
Progressive Income Tax	2.0%	15.1%	26.5%

Table 4

## Steady State and Transitional Values of Capital and Labor

<u>Capital Stock</u>	<u>Proportional Income Tax</u>			<u>Progressive Income Tax</u>		
	$\lambda = \text{PVB/PVT}$	$\lambda=1$	$\lambda=4$	$\lambda=\text{PVB/PVT}$	$\lambda=1$	$\lambda=4$
<u>Year</u>						
0	56.2	56.2	56.2	39.4	39.4	39.4
5	56.7	58.7	61.9	39.7	42.5	46.6
10	57.2	61.7	68.4	40.1	46.5	55.7
50	58.9	75.0	97.5	41.7	67.2	108.2
150	58.7	74.4	97.5	42.5	66.9	106.9
 <u>Labor Supply</u>						
<u>Year</u>						
0	18.4	18.4	18.4	16.8	16.8	16.8
5	18.6	20.1	23.2	17.1	18.9	22.3
10	18.6	20.0	22.9	17.1	18.7	22.0
50	18.5	19.5	22.4	17.0	17.8	21.5
150	18.5	19.4	22.3	16.3	17.9	21.0

Table 5

## Efficiency Gains from Social Security Benefit/Payroll Tax Linkage

$\lambda = 1$   
LSRA versus NO LSRA

<u>Generation Born in Year</u>	<u>Welfare Gain</u>	
	<u>No LSRA</u>	<u>LSRA</u>
-55	0	0
-25	.3	0
-10	.9	0
0	1.4	0
1	1.5	7.6
10	1.7	7.6
25	1.7	7.6
50	1.6	7.6
100	1.5	7.6
150	1.5	7.6

## Capital Stock

<u>Transition Year</u>	<u>No LSRA</u>	<u>LSRA</u>
0	56.2	56.2
10	58.6	61.7
50	60.7	75.0
100	60.5	74.5
150	60.4	74.4