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THE NEW ECONOMICS OF EQUILIBRIUM SORTING AND ITS TRANSFORMATIONAL ROLE FOR POLICY EVALUATION

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ABSTRACT

Households "sort" across neighborhoods according to their wealth and their preferences for public goods, social characteristics, and commuting opportunities. The aggregation of these individual choices in markets and in other institutions influences the supply of amenities and local public goods. Pollution, congestion, and the quality of public education are examples. Over the past decade, advances in economic models of this sorting process have led to new framework that promises to alter the ways we conceptualize the policy evaluation process in the future. These "equilibrium sorting" models use the properties of market equilibria, together with information on household behavior, to infer structural parameters that characterize preference heterogeneity. The results can be used to develop theoretically consistent predictions for the welfare implications of future policy changes. Analysis is not confined to marginal effects or a partial equilibrium setting. Nor is it limited to prices and quantities. Sorting models can integrate descriptions of how non-market goods are generated, estimate how they affect decision making and, in turn, predict how they will be affected by future policies targeting prices or quantities. Conversely, sorting models can predict how equilibrium prices and quantities will be affected by policies which target product quality, information, or amenities generated by the sorting process. These capabilities are just beginning to be understood and used in applied research. This survey article aims to synthesize the state of knowledge on equilibrium sorting, the new possibilities for policy analysis, and the conceptual and empirical challenges that define the frontiers of the literature.

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1. *Introduction*

Economists use sorting as a metaphor for the way that market forces partition economic agents across segments of a market. Households "sort" across neighborhoods according to their wealth and their preferences for public goods, social characteristics, and commuting opportunities. Workers "sort" across jobs according to their qualifications and preferences for job attributes. In situations with other differentiated products such as automobiles, breakfast cereal, and computers, we expect that consumers who have similar preferences and face similar constraints will make similar choices. This sorting process reveals information about consumers, and firms have learned to exploit it to increase their profits. They design differentiated products and set prices to take advantage of what is known about consumer heterogeneity. Knowledge of consumer heterogeneity can also be used to evaluate past policies and design new ones. This is especially important for policy targeting public goods and externalities. The challenge for economists is to describe sorting behavior and learn from it. Our models need to reflect the information available to the agents involved, their constraints, and the implications of their collective actions for market and non-market outcomes.

Over the past decade, advances in economic models of sorting have led to a new framework for policy evaluation. These "equilibrium sorting" models use the properties of market equilibria, together with information on the behavior of economic agents, to infer structural parameters that characterize agent heterogeneity. The results can be used to develop theoretically consistent predictions for the welfare implications of future policy changes. Analysis is not confined to marginal effects or a partial equilibrium setting. Nor is it limited to prices and quantities. As heterogeneous agents sort, their collective behavior can influence the supply of amenities. These adjustments can be represented as part of the characterization of the equilibria and influence its properties. Pollution, congestion, and opportunities for social interaction provide examples. Sorting models can integrate descriptions of how these amenities are generated, estimate how they affect decision making and, in turn, predict how they will be affected by future policies targeting prices or quantities. Conversely, sorting models can predict how equilibrium prices and quantities will be affected by policies which target product quality, information, or amenities generated by the sorting process. These capabilities are just beginning to be understood and used in applied research.

Equilibrium sorting models build on the intellectual foundations of the literature on hedonic and discrete-choice models of differentiated product markets. They combine the information provided by an *equilibrium* hedonic price function (Sherwin Rosen 1974; Dennis Epple 1987; Ivar Ekeland, James J. Heckman, and Lars Nesheim 2004) with a formal description for the choice process that underlies market *sorting* of heterogeneous agents (Daniel McFadden 1974; Timothy Bresnahan 1987; Steven Berry, James Levinsohn, Ariel Pakes 1995). This *equilibrium sorting* framework can depict a mixture of discrete and continuous choices made by a population of heterogeneous agents, while recognizing that characteristics of the objects of choice may be determined endogenously (Epple and Holger Sieg 1999; Patrick Bayer and Christopher Timmins 2005, 2007).

What ideas distinguish the economics of equilibrium sorting from past strategies for modeling differentiated goods? First, in addition to characterizing sources of unobserved heterogeneity such as technology and preferences, they include a wide array of observable features that distinguish economic agents. These observable dimensions of agent heterogeneity can be used in descriptions of sorting behavior and are often especially important in characterizing the implications of polices. Through the sorting process, that heterogeneity is translated into endogenously determined attributes of the choice alternatives available to agents. In the housing market, for example, the attributes of the neighborhoods that a household chooses from may depend on where its primary earner works, while its preferences for school districts may depend upon the levels of education attained by the adult members of the household. As households with different incomes and levels of education decide where to live, they will influence the demographic compositions of neighborhoods. When households vote, their preferences will shape public policies that influence school quality, open space, and congestion. The supply of each of these amenities is thus determined endogenously–as an outcome of the sorting process. This creates an econometric problem for researchers simply interested in recovering consistent estimates of households' preferences.

Endogenous amenities will also influence market outcomes for private goods, such as housing prices and wage rates. This creates a second distinction between the economics of equilibrium sorting and earlier models of the demand for differentiated products. In particular, the sorting literature seeks to understand "general equilibrium" feedback effects between economic agents and their environments. For example, a shock to the housing market that induces a change in residential location patterns may lead to a redistribution of local amenities that induces more migration and housing development which continues until prices adjust and markets clear. Modeling these feedback effects is important for researchers interested in simulating the impacts of a counterfactual policy.

Third, the equilibrium sorting literature considers how public policies can be designed to exploit what we learn about forms of heterogeneity, endogeneity, and feedback. Some time ago, Alan S. Blinder and Harvey S. Rosen (1985) demonstrated how information about preference heterogeneity could, in principle, be used to design more efficient taxes on private goods. Emmanuel Saez (2010) recently used a similar logic to test whether workers respond to nonlinearities in the tax code. Equilibrium sorting models provide the means to implement both the original Blinder-Rosen idea and the Saez test and extend them to the consideration of policies that target public goods or other amenities that affect agents differently.

Applications of the new models have demonstrated that agent heterogeneity, endogenous attributes, and feedback effects can all have first-order policy implications (Sieg et al. 2004; V. Kerry Smith et al. 2004; Maria Marta Fererrya 2007; Timmins 2007; Randall L. Walsh 2007; Nicolai V. Kuminoff 2009; H. Allen Klaiber and Daniel J. Phaneuf 2010b; Constant I.Tra 2010). These studies investigate how sorting behavior in housing markets relates to air quality, school quality, open space, climate, and other amenities. One of the policy-relevant insights is that the properties of market equilibria can depend on feedback effects which occur through *non-market* transmission routes. For example, in Walsh (2007) households get utility from access to open space, which decreases as new houses are built in a closed community. This non-market feedback effect causes each household's location choice to depend on the choices made by other households. The demand side of a sorting equilibrium that clears this market is itself a Nash equilibrium that fits within the class of aggregative public goods games characterized by Richard Cornes and Roger Hartley (2007). Sorting in response to feedback leads to a surprising result in Walsh's policy simulation. Increasing the amount of land in public preserves can actually decrease the total amount of land in open space in the metro area. The mechanisms that produce this outcome mirror a counterintuitive result from Matthew J. Kotchen's (2006) theoretical model of Nash equilibria in green markets.

Housing markets provided the testing ground for equilibrium sorting models. The models were initially developed to deal with features of the homebuyer's location choice problem that were difficult to address using conventional methods. The resulting techniques have been used to study behavior in a wide range of differentiated product markets. Recent applications have considered network effects (Marc Rysman 2004; Shanjun Li 2006), location choices of firms (Katja Seim 2006), markets for education (Epple, Richard Romano, and Sieg 2006, 2010), social interactions in labor markets (Bayer, Steve Ross, and Giorgio Topa 2008), and the impact of congestion on recreation demand (Timmins and Jennifer Murdock 2007).

The potential for using equilibrium sorting models to conduct high-resolution policy analysis is exciting, but are their predictions reliable? Over the past decade the profession has become increasingly skeptical of structural modeling (Joshua D. Angrist and Jörn-Steffen Pischke 2010). This skepticism reinforces the need to understand how the features of structural models contribute to their identification of welfare measures and other policy implications (Michael Keane 2010). The equilibrium sorting literature has addressed some of the traditional criticisms of structural modeling by developing nonlinear and semiparametric estimators that allow functional forms and distributional assumptions to be selected based on data and theory rather than computational convenience. Nonetheless, to characterize the sorting process, modeling judgments must be made and these can influence policy implications. For example, different researchers have suggested competing specifications for preference functions and the form of agent heterogeneity which, in turn, differ in their implications for substitution possibilities and welfare estimates. Furthermore, in order to quantify the implications of a non-marginal policy change, an equilibrium selection rule may need to be formulated. Given the current debates about the assumptions being made in quasi-experimental versus structural models, questions about the relevance of the former for policy evaluation, and the recent developments in the structural sorting literature, the time is right to pause and identify what we have learned, the problem areas, and the puzzles that remain. This survey article aims to synthesize the state of knowledge on equilibrium sorting, the new possibilities for policy analysis, and the conceptual and empirical challenges that define the frontiers of the literature.

We concentrate on research in public and environmental economics, with particular attention to the market for housing. Models of household location choice build on theory and methods developed in related fields, especially industrial organization and labor economics. We highlight connections to work in those areas, without providing a comprehensive assessment. Our focus is on recent research. While we do not present a complete historical perspective, it should be noted that much of the work we cover was influenced by the seminal papers written by Charles M. Tiebout (1956), William Alonso (1964), John Krutilla (1967), Edwin S. Mills (1967), Richard F. Muth (1969), Wallace E. Oates (1969), Thomas C. Schelling (1969), and Edwin T Haefele (1971).

Our survey begins by describing the foundations of the new equilibrium sorting literature in section 2, from early median voter models of the demand for public goods to the modern discrete choice framework for describing how households sort over neighborhoods. Section 3 covers the evolution of sorting theory. This line of research has sought to characterize multi-community equilibria with peer effects, voting, and other forms of social interaction. The implied relationship between property values, housing characteristics, and local public goods can be described by a hedonic price function. Empirical models use properties of sorting equilibria to recover household preferences and estimate the demand for public goods. These models can be divided into two broad frameworks. Section 4 covers hedonic models, most of which take a reduced-form (and increasingly quasi-experimental) approach to estimation. Section 5 covers structural models of sorting behavior. In section 6 we contrast how hedonic and sorting models are used to evaluate public policy. Leading examples are provided (education, air pollution, and land use) and we conclude with an assessment of how the new sorting models can improve future evaluations. Finally, section 7 concludes by identifying current frontiers of the literature.

2. *Foundations and Motivation*

Empirical sorting models are motivated by a long-standing question. How can we estimate the demand for public goods that are not explicitly traded in formal markets? Early research sought to estimate demand by simply regressing expenditures for municipal services on the characteristics of voters. Tiebout (1956) recognized that households "vote with their feet". These migration patterns can bias reduced-form estimation. Mitigating the bias requires knowledge of the sorting process that underlies market equilibrium. This realization led to subsequent research on characterizing the properties of equilibria that result from heterogeneous households sorting themselves across differentiated communities. Formal models of the sorting process were developed using the characteristics approach to consumer theory (Kelvin J. Lancaster 1966; William M. Gorman 1980).

The remainder of this section describes the foundations of the new equilibrium sorting literature, from the early reduced form studies to the modern characteristics framework, and summarizes the features of the location choice problem that differentiate the resulting theory and econometrics from characteristics based models of the demand for differentiated products.

2.1 *A Reduced-Form Approach to Estimating the Demand for Public Goods*

Theodore C. Bergstrom and Robert P. Goodman (1973) were among the first to propose a strategy for estimating the demand for local public goods. They envisioned an urban landscape in which the level of public goods supplied by each community is determined by that community's median voter. Assuming the median voter also has the median level of income, the demand for a public good can be estimated by simply regressing actual public good expenditures (*A*) on the incomes (y_{med}) and marginal tax rates (τ_{med}) faced by the median household in each of several communities,

(1) $$\ln A = \beta_0 + \beta_1 \ln y_{med} + \beta_2 \ln \tau_{med} + \sum_k \beta_k d_{k,med} + u_{med} ,$$

where the d_k's describe the median household's demographic characteristics. The simplicity of Bergstrom and Goodman's estimator inspired numerous applications to community level data, as well as a microeconometric extension of the model to individual survey data (Bergstrom, Daniel L. Rubinfeld and Perry Shapiro 1982).

The problem with estimation of (1) is that it ignores the sorting process that underlies equilibrium in the market for housing. If households choose where to live based, in part, on their preferences for public goods, the community selection mechanism can bias estimation of the price and income elasticities. Gerald S. Goldstein and Mark V. Pauly (1981) labeled this problem "Tiebout bias" after Tiebout's (1956) conceptual model of local public goods provision.[1]

To illustrate Tiebout bias, we draw on an example from Rubinfeld, Shapiro and Judith Roberts (1987). Suppose household *i* maximizes its utility by locating in one of a discrete set of J communities based, in part, on its preferences for public goods,

$$j = \max_{j \in J} V\left(A_j, \tau_j, y_{ij}, d_i\right). \quad (2)$$

Then the estimating equation from Bergstrom and Goodman's (1973) model can be rewritten for an individual observation as

$$\ln A_{i,j} = \beta_0 + \beta_1 \ln y_{i,j} + \beta_2 \ln \tau_{i,j} + \sum_k \beta_k d_{k,i,j} + u_{i,j} \ . \quad (3)$$

Reformulating the problem in terms of individual behavior allows us to interpret the econometric error term as a function of unobserved preferences. In this context, preference based sorting presents a simultaneity problem. That is, household incomes and property taxes may be influenced by the sorting process in (2). A household's income will depend on its location choice if communities differ in the job opportunities they provide. A community's marginal tax rate will depend on the composition of its residents if tax rates are determined by voting. If income and taxes depend on location choices that are driven, in part, by unobserved preferences, $u_{i,j}$ will be correlated with $y_{i,j}$ and $\tau_{i,j}$, biasing OLS estimation of (3).

Rubinfeld, Shapiro, and Roberts (1987) propose a two-step selection model that has the potential to provide consistent estimates of the demand for a public good in the presence of Tiebout bias. While the logic behind their estimator is straightforward, there is a major hurdle to implementation—it requires instruments for the endogenous variables in (3). This requirement creates a challenge because the validity of any potential instrument depends on the ways in which the sorting of heterogeneous households influences the properties of equilibria. Put differently, to evaluate the validity of a potential instrument, one must provide a full specification of the sorting equilibrium. Thus, developing consistent estimates of the demand for a public good requires knowledge of the sorting process.

[1] Tiebout (1956) envisioned freely mobile households migrating across communities based on their preferences for the public goods provided by those communities. This type of sorting behavior poses a problem for OLS estimation of (1) regardless of whether the data describe the median household in each community or a random sample of households.

2.2 *A Model of Household Location Choice*

Equilibrium models of Tiebout sorting begin with a simple premise: the amount and character of housing and public goods varies across an urban landscape, and each household selects its preferred bundle of public and private goods given its income and the relative prices involved. Every household pays for its location choice through the price of housing. Working households may also pay indirectly through the wages they earn. In order to link a household's location choice to its preferences for an individual public good, the problem is formalized using the characteristics approach to consumer theory developed by Lancaster (1966) and Gorman (1980). The appendix provides a reference guide to the notation that we use.

Assume the urban landscape consists of $n = 1,....,N$ houses that can be divided into $j = 1,....,J$ communities. Each home can be defined by a bundle of housing characteristics and amenities. h_n is a vector of structural characteristics that fully describe the private good component of an individual home. For example, h_n could include the number of bedrooms, the number of bathrooms, square feet, and lot size. g_j denotes a vector of amenities conveyed to every household in community *j*. It may include local public goods such as school quality, urban and environmental services (such as crime rates, and air quality), and variables describing the demographic composition of the community (such as race, educational attainment). We will use the term "amenities" to refer to any of these non-market goods and services.

A household's utility depends on the characteristics of housing and amenities at its location and on its consumption of a composite numeraire private good, *b*. Households are heterogeneous. They differ in unobservable features of their preferences (α) and in observable factors such as their demographic characteristics (d). Let the population of households be indexed from $i = 1,....,I$. The utility obtained by household *i* from house *n* in community *j* can be represented as: $U(b, h_n, g_j; \alpha_i, d_i)$.[2]

Each household is assumed to choose a location and a quantity of *b* that maximize its

[2] A household may contain many members with different demographic characteristics and preferences, but it is treated as an indivisible economic agent.

utility subject to a budget constraint:

(4) $$\max_{n \in j, b} U\left(b, h_n, g_j; \alpha_i, d_i\right) \ \ subject\ to \ \ y_{i,j} = b + P_{n \in j}.$$

In the budget constraint, the price of the numeraire is normalized to one and $P_{n \in j}$ represents annualized expenditures on house *n* in community *j*; in other words, $P_{n \in j}$ is the after-tax cost of occupying a single home for one year.[3] $y_{i,j}$ is the household's total annual income. The *j* subscript on income recognizes that, in general, income is endogenous to location choice. In particular, wages will be endogenous if heterogeneous workers sort across a landscape with spatial variation in the composition of labor demand. Even if employment locations are fixed, income may be endogenous to the housing location decision because of commuting costs.

The specification in (4) implicitly removes three potential sources of "friction" from the problem. First, all households are assumed to share the same objective evaluation of housing characteristics and amenities. Second, households are assumed to be freely mobile within the geographic region defined as the choice set. Third, every household is assumed to face the same schedule of housing prices. These three assumptions—*full information*, *free mobility*, and *no discrimination*—are maintained in the literature with very few exceptions.

Equilibrium is achieved when every household occupies its utility-maximizing location and nobody wants to move, given housing prices, housing characteristics, wages, tax rates, and the levels of each amenity. The literature can be organized around this concept. Theoretical models investigate the existence and uniqueness of equilibria and their implications for equity and efficiency. Empirical models use the properties of equilibria to infer preferences for amenities from the observable characteristics of households and their location choices. Finally, because empirical models can be estimated in a way that characterizes equilibrium in the entire housing market, the estimation results can be used to predict market responses to policies that would make large scale changes to amenities.[4]

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[3] This interpretation is also an important part of the logic for Epple, Brett Gordon, and Sieg's (2010) new approach to estimating the supply of housing, discussed in section 7.3.

[4] In this context, a "prediction" simply describes what the model would imply for the equilibrium, given some counterfactual conditions. These predictions have been used to evaluate the types of responses that would be expected to follow specific policy changes. They have not been used to develop forecasts for future housing market conditions.

2.3 *Distinguishing Features of the Household's Location Choice Problem*

The structure of the location choice problem is quite similar to the structure of differentiated product models in other fields. At an abstract level, equation (4) simply depicts heterogeneous agents choosing among a set of differentiated objects to satisfy their idiosyncratic tastes. Because of this common foundation, the theory and methods that were initially developed to model households' location choices were built on work in related domains, especially industrial organization and labor economics.

Four features of the homebuyer's location choice problem presented new modeling challenges. First, location choices are differentiated by a mixture of public and private goods. Second, some of the public goods are endogenously determined by the sorting process. Third, sorting arises, in part, from the combination of heterogeneity in preferences and heterogeneity in the spatial landscape. Finally, with endogenous characteristics and heterogeneous preferences, there can be multiple equilibria. Before turning to the theory and econometrics of equilibrium sorting, we briefly explain these distinguishing features of the problem.

2.3.1 *A Mixture of Public and Private Characteristics*

Differentiated product models in industrial organization tend to focus on private goods defined by characteristics that are rivalrous and excludable such as automobiles (Berry, Levinsohn, and Pakes 1995), breakfast cereal (Aviv Nevo 2001), computers (Patrick Bajari and Lanier Benkard 2005), and laundry detergent (Igal Hendel and Nevo 2006). Recreation demand studies investigate how people choose among public goods such as lakes and parks, which may be differentiated by their opportunities for fishing and boating (Phaneuf and Smith 2005). These services are non-rival until a capacity level is reached, at which point their quality becomes a function of congestion (Timmins and Murdock 2007). In the homebuyer's location choice problem, some attributes are rival, some are not, and some depend on congestion.

As heterogeneous households sort themselves across the spatial landscape, their collective behavior can affect the supply of club goods and open access resources which, in turn, become part of the characterization of the equilibria and influence its properties. What are the implications of this process for the efficiency of equilibria? Does it impair our ability to identify the demand for characteristics? Can we exploit the complementarity between public and private characteristics to design more effective policies? The answers to these questions depend on our understanding of the mechanisms that determine the supply of endogenous characteristics.

2.3.2 *Endogenous Characteristics*

The recognition that characteristics of the object of choice may be endogenously determined through the market clearing process is perhaps the single most important feature to distinguish the equilibrium sorting literature from the broader literature on differentiated products. The level of an amenity can be determined through social choice, social interaction, or feedback effects. Consider the case of social choice. Expenditures on local public goods are determined by voting. Residents of each community vote on property tax rates and on special assessments that aim to provide additional funding for schools, law enforcement, and other services. This is the Bergstrom and Goodman (1973) logic for their median voter model cited earlier.

While voting may determine expenditures on local public goods, their quality may be determined by social interactions. For example, the quality of a school is often judged by how its students perform on standardized tests. The best predictors of a student's performance tend to be the demographic characteristics of her parents (particularly income and education) and the performances of her peers (Eric A. Hanushek 2003). Thus, peer effects make the level of school quality a function of the demographic characteristics of the parents who live in that district.

Social interactions can also influence the demographic composition of a community and the services it provides. If people care about the ethnicity of their neighbors (or other characteristics such as age, race, and wealth) then the sorting process that determines community demographics will reflect these social interactions. For example, David M. Cutler, Edward L. Glaeser, and Jacob L. Vigdor (1999) conclude that patterns of racial segregation in 1990 are best explained by whites preferring to live in predominantly white communities.

Household location choices can also generate feedback effects that affect the quality of environmental services. As people with strong preferences for open space move into a metropolitan area, the remaining open space gets developed for additional housing. Increases in pavement generate urban heat islands. Runoff of lawn fertilizer from the newly built subdivisions may cause water quality to decline.[5] The corresponding increase in automobile traffic may decrease air quality. As these feedback effects alter the metropolitan landscape, the environmental amenities that initially attracted households to the area may be degraded.

2.3.3 *Heterogeneity in Preferences and the Spatial Landscape*

Spatial stratification constrains the production and consumption of goods. To consume the amenities provided by a community, a household must move there. The scope for its residents to influence those amenities is defined, in part, by boundaries such as school districts, voting jurisdictions, and air basins. The characteristics of the resulting spatial equilibrium will reflect the distribution of preferences in the population of households. Tiebout (1956) reasoned that, with spatial variation in amenities and free mobility, the location choices that households make reveal their preferences. In his own words: "*There is no way in which the consumer can avoid revealing his preferences in a spatial economy. Spatial mobility provides the local public-goods counterpart to the private market's shopping trip*." A number of authors contributed to formalizing Tiebout's logic, as we discuss in the next section. In particular, Epple's research with a variety of collaborators provided the initial basis for understanding how preference heterogeneity influences spatial variation in the supply of amenities, yielding equilibria that can be used to recover information about those preferences.

Bayer and Timmins (2007) demonstrated that our ability to identify preferences can be enhanced by instrumental variables developed from our knowledge of the spatial distribution of endogenous and exogenous amenities. Information about preferences is needed to predict how new policies will affect features of the spatial equilibrium. Simon P. Anderson, André de Palma, and Jaques-François Thisse (1992) formalized one dimension of this argument, demonstrating that the structure of preference heterogeneity will determine the substitutability between differentiated objects of choice (in our case, communities).

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[5] For example, Phaneuf et al. (2008) link a model of building activity to a spatial model of a river system in order to assess the marginal damages of water pollution due to that construction.

2.3.4 *Multiplicity of Equilibria*

Because equilibrium sorting models provide a characterization of the equilibrium, we have the ability to predict how the supply of an endogenous amenity will adjust to a new policy that targets housing prices or quantities. Conversely, we can predict how equilibrium prices and quantities will be affected by policies which target amenities. To assess these effects, however, we must first solve for the new equilibrium that would follow a prospective policy shock. This task presents theoretical and computational challenges.

When households have heterogeneous preferences for multiple amenities there can be multiple equilibria. Analytical proofs of uniqueness can only be obtained by adding restrictions to the structure of preferences or by limiting the number of endogenous amenities. Without these restrictions, different equilibria may be compared on the basis of their stability, their implications for market efficiency, or their welfare implications for particular demographic groups. Which equilibrium emerges after a policy shock may depend on the market institutions that govern transitional dynamics. Research on the theory and methods associated with sorting dynamics is one of the current research frontiers.

3. *Equilibrium Sorting Theory*

The theoretical literature that followed Tiebout's early work focused on formalizing his conceptual model, proving the existence of an equilibrium in which no household would be better off by moving, and extending his framework to include peer effects and other forms of social interaction within communities. Part 1 of this section summarizes a series of articles that develop increasingly general depictions of sorting equilibria. Part 2 adds social interactions and discusses the implications for equity and efficiency. Part 3 describes the hedonic price function that characterizes the equilibrium relationship between property values, housing characteristics, and spatially differentiated amenities.

3.1 Equilibrium Stratification Patterns

For heuristic purposes, a household's choice process can be depicted as a two-stage problem where each household first determines the optimal quantities of housing and numeraire in each of a finite number of communities, and then chooses the particular community that maximizes its utility.[6] The first stage is

$$\max_{\bar{h},b} U\left(b,\bar{h},g_j,\alpha\right) \ \ subject\ to \ \ y = b + p_j \cdot \bar{h}\,. \tag{5}$$

Theoretical models usually treat housing as a homogeneous commodity that can be consumed in continuous quantities at a constant price. This is represented in (5) by defining p_j as the annualized per-unit price of housing in community *j* and $\bar{h}$ as the quantity of housing consumed. Note that the bar on $\bar{h}$ does not imply the quantity of housing consumed is fixed; the quantity consumed may vary across households according to their income, preferences, and chosen community. The bar superscript is meant to differentiate the concept of a homogeneous unit of housing from h, which continues to represent a vector of specific structural characteristics (e.g. bedrooms, bathrooms, square feet).[7]

Assuming households can purchase any quantity of housing at the market price in each community, housing is "optimized out" of the problem and preferences can be restated using the indirect utility function in (6).

$$V\left(g,p,\alpha,y\right) = U\left[g,\ \bar{h}\left(g,p,\alpha,y\right),\ y - p\cdot \bar{h}\left(g,p,\alpha,y\right),\ \alpha\right]. \tag{6}$$

Each household will choose the community that maximizes its well-being, given income and prices. A sorting equilibrium is achieved when every household has chosen its utility-maximizing community and nobody wants to move, given housing prices and the level of local public goods.

Bryan Ellickson (1971) first characterized the restrictions on preferences that would support the existence of sorting equilibria. Three features of his model form the basis for most of the subsequent studies. First, he assumed that provision of public goods in community *j* could be represented by a 1-dimensional measure, $\bar{g}_j$, an index that represents the composite quality of public goods in that community. Second, he assumed that households have homogeneous

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[6] We have dropped the *d* term describing the individual's observable demographic characteristics at this stage to simplify notation.

[7] This is one strategy for restricting the general specification used to estimate household preferences from data on housing transactions. In the "pure characteristics" approach to estimation, covered in section 5.1, housing expenditures are translated into a price index for homogenous housing using an approach developed by Sieg et al. (2002).

preferences $(\alpha_i = ... = \alpha_I)$ and therefore differ only in their income. Given the first two assumptions, he imposed the restriction that indifference curves in the $(\overline{g}, p)$ plane are strictly increasing in income. This, he reasoned, would support a sorting equilibrium in which households are perfectly stratified across communities by income. Figure 1 uses a two-community example to illustrate the idea. Household i is exactly indifferent between the two communities. Any household with lower income, such as household $i-1$, will always prefer the cheaper community because indifference curves cannot cross more than once. Conversely, any household with higher income, such as $i+1$, will always prefer the more expensive community.

Using the three restrictions from Ellickson's paper, Frank Westoff (1977) proved that a sorting equilibrium exists in a model where households in each community vote to determine public goods provision and community-specific tax rates. Epple, Radu Filimon, and Thomas Romer (1984, 1993) extended Westoff's model to include a market for housing that must clear within each community. Finally, Epple and Romer (1991) generalized the model further to allow voters to anticipate the consequences of their votes for housing prices and migration. While these models help to formalize Tiebout's theory, they do a poor job of reproducing the actual sorting behavior that we observe. To see why, let the J communities be ordered by their quality of public goods provision: $\overline{g}_1 < \overline{g}_2 < ... < \overline{g}_J$. The key restriction in the early theoretical literature—Ellickson's single-crossing restriction—implies that households are partitioned across communities by income, as illustrated in figure 2. In the figure, every household in community 1 has lower income than every household in community 2, and so on. This characterization is a poor approximation to reality. That is, actual community-specific income distributions overlap substantially.

One explanation for why households do not perfectly stratify by income is that they differ in their tastes for public goods. Recognizing this, Epple and Glen J. Platt (1998) extended the Epple-Romer model to allow households to differ in a single heterogeneous parameter that represents their preferences for composite provision of public goods relative to private goods. In this case, equilibrium is characterized by a more general version of Ellickson's single-crossing restriction. To formalize the restriction, equation (7) shows the slope of an "indirect indifference curve" in $(\overline{g}, p)$ space.

(7) $$M(\bar{g}, p, \alpha, y) = \left\langle \frac{dp}{d\bar{g}} \middle| V = \bar{V} \right\rangle = -\frac{\partial V(\bar{g}, p, \alpha, y)/\partial \bar{g}}{\partial V(\bar{g}, p, \alpha, y)/\partial p}.$$

Assuming *M* is monotonically increasing in $(y \mid \alpha)$ and $(\alpha \mid y)$, indifference curves in the $(\bar{g}, p)$ plane satisfy single crossing. This has an intuitive interpretation. Roy's Identity implies that $-\partial V(\cdot)/\partial p$ must equal the marginal utility of income, $\lambda = \partial V(\cdot)/\partial y$, times the Marshallian demand for housing, $\bar{h}(\bar{g}, p, \alpha, y)$.

(8) $$M(\cdot) = -\frac{\partial V(\cdot)/\partial \bar{g}}{\partial V(\cdot)/\partial p} = \frac{\partial V(\cdot)/\partial \bar{g}}{\lambda h(\cdot)} = \frac{1}{h(\cdot)}\left[\frac{\partial V(\cdot)/\partial \bar{g}}{\partial V(\cdot)/\partial y}\right].$$

The term in brackets in equation (8) is the Marshallian virtual price of public goods. Therefore, the single crossing restriction implies that the Marshallian virtual price, per unit of housing, is strictly increasing in income and in preferences for public goods relative to private goods.[8]

The single crossing condition implies that, in equilibrium, three properties characterize sorting by each household type: *boundary indifference, stratification,* and *increasing bundles* (Epple and Platt 1998). Without loss of generality, let the *J* communities be ordered according to the index of public goods, $\bar{g}_1 < ... < \bar{g}_J$. *Boundary indifference* requires a household on the "border" between two communities in (α, y) space to be exactly indifferent between those communities. Equation (9) defines the set of border individuals. It must hold for all $j = 1,..., J-1$.

(9) $$\left\{ (\alpha, y):\ V(\bar{g}_j, p_j, \alpha, y) = V(\bar{g}_{j+1}, p_{j+1}, \alpha, y) \right\}.$$

The *increasing bundles* property requires that for any two communities in the ordering, $(j, j+1)$ equation (10) must hold.

(10) $$y_{j+1}(\alpha) > y_j(\alpha) \ \Rightarrow\ p_{j+1} > p_j \quad \textit{and} \quad \bar{g}_{j+1} > \bar{g}_j.$$

That is, the ranking of communities by public goods provision must match the ranking by price.

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[8] This property is related to the Willig condition that is often applied together with weak complementarity to identify the Hicksian willingness to pay for changes in public goods. The Willig condition requires the willingness-to-pay per unit of the weak complement to be constant at all levels of income. See Smith and H. Spencer Banzhaf (2004, 2007) and Raymond B. Palmquist (2005) for details. David S. Bullock and Nicholas Minot (2006) have demonstrated that the Willig condition is sufficient but not necessary for identifying willingness to pay for changes in non-market goods with weak complementarity.

The third property, *stratification*, requires that households of each type are stratified across the J ordered locations by $(\alpha \mid y)$ and by $(y \mid \alpha)$, as defined in (11).

$$(y_{j-1} \mid \alpha) < (y_j \mid \alpha) < (y_{j+1} \mid \alpha) \quad \text{(11)} \quad \textit{and} \quad (\alpha_{j-1} \mid y) < (\alpha_j \mid y) < (\alpha_{j+1} \mid y).$$

Figure 3 illustrates the implied partition of households into communities in (y, α) space. Conditional on preferences, higher income households always choose to live in communities with more public goods. Likewise, conditional on income, households with stronger preferences always choose communities with more public goods. This two-dimensional stratification is consistent with Tiebout's (1956) reasoning and capable of explaining empirical income distributions.

While *boundary indifference, stratification,* and *increasing bundles* are necessary for a sorting equilibrium to exist, they are not sufficient. Any sorting equilibrium must also be characterized by a vector of housing prices and a vector of public goods such that no household could increase its utility by moving. The development of a general existence proof is complicated by preference heterogeneity. Epple and Platt (1998) rely on a Cobb-Douglas specification for utility and use a numerical example to demonstrate that a sorting equilibrium *may* exist. Sieg et al. (2004) reinforce their finding by demonstrating existence numerically using a constant elasticity-of-substitution (CES) specification for utility and Kuminoff (2009) allows households to differ in their relative preferences for multiple public goods.

Thus far, we have characterized the properties of equilibria that arise from the two-way interaction between households and the community-level provision of local public goods. Within a community, household interaction has been limited. Existing residents of a community are assumed to be indifferent to immigration unless the immigrants change housing prices or alter voting outcomes. This assumption overlooks the evidence that people care about the demographic characteristics of their neighbors (e.g. Cutler, Glaeser, and Vigdor 1999) and the evidence on public education indicates that peer group effects are among the main determinants of school quality (e.g. Hanushek 2003).

3.2 Social Interactions, Equity, and Efficiency

Charles A.M. de Bartolome (1990) was the first to build social interactions into a model of residential choice. He depicts two types of households (low skill and high skill) sorting across two communities according to their differentiated preferences for a single public good—school quality—which is increasing in both expenditures and the inherent skill of households.[9] This simple framework leads to two important insights that are robust to many subsequent extensions of the model: (i) social interactions can produce a multiplicity of sorting equilibria, and (ii) some of these equilibria are inefficient.

The nature of the equilibrium depends on the strength of the peer group effect. If peer effects have little or no impact on school quality, a single-crossing condition is sufficient to guarantee that the two types of households will be segregated by skill. As peer effects grow in importance, low skill households have a stronger incentive to move to the high skill community. This can lead to an "integrated" equilibrium in which both communities contain both household types, and each community provides a different level of education. Interestingly, the social interactions which underlie this equilibrium also cause it to be inefficient.

Social interactions generate externalities. In De Bartolome's (1990) model, migrating households do not internalize the effect of their location choices on the current residents of the destination communities. This is the underlying source of inefficiency. Raquel Fernandez and Richard Rogerson (1996) demonstrate that public policies which reduce the degree of stratification can be Pareto improving. They extend De Bartolome's model to depict I household types sorting among J communities under the added assumption that school quality in a given community can be measured by the average income of its residents. Given the usual single crossing condition, any stable sorting equilibrium must satisfy *boundary indifference*, *stratification*, and *increasing bundles*. To see why this is inefficient, consider a "boundary" household who is exactly indifferent between a wealthier community and a poorer community. A public policy that induces this household to move from the wealthy community to the poor community would raise average income in both communities which, in turn, raises the quality of

[9] While De Bartolome (1990) was the first to characterize the consequences of social interactions for sorting equilibria, Tiebout (1956) recognized their potential role in location choice. In a footnote, he observes that: "*Not only is the consumer-voter concerned with economic patterns, but he desires, for example, to associate with 'nice' people*."

education in both communities, making everyone better off. Following this logic, Fernandez and Rogerson (1996) demonstrate that school finance reforms which are most effective at inducing migration to poorer communities tend to be Pareto improving.

By influencing the production of education, social interactions and stratification in the housing market can have general equilibrium implications for efficiency and growth. Roland Bénabou (1993) demonstrates this result by adding a production sector to the economy with complementarity between high and low skill labor.[10] As before, peer effects in education give higher skill households an incentive to segregate themselves from lower skill households. Not only does this raise the cost of education in low skill communities; it also increases unemployment, which decreases production, exacerbating the inefficiency from stratification. These effects can be persistent. In a dynamic version of the model, Steven N. Durlauf (1996) demonstrates that short run stratification in the housing market can have long term consequences for inequality and economic growth. Parents who had the misfortune of having been born into a poor community may be unable to raise school quality enough for their children to obtain higher paying jobs. One potential solution to this poverty trap is to equalize expenditures on education across school districts. Bénabou (1996a) models the benefits and costs of this approach. The net benefits hinge on an intergenerational tradeoff between the short run cost of constraining expenditures on education and the long run benefit of reducing the inefficiency from stratification. Most of the key results from these "general equilibrium" sorting models are provided in Bénabou (1996b) who classifies the potential causes of stratification and their implications for equity and efficiency. He also observes that minor differences in preferences can create a "tipping" effect that leads to a high degree of equilibrium stratification by income.

The tipping effect also helps to explain the persistence of racial segregation. Over the past 50 years the black-white income gap has narrowed, the same is true for education, and survey data suggest that both races have become more willing to live in integrated communities. Yet racial segregation persists. Rajiv Sethi and Rohini Somathathan (2004) explain these results using a model of sorting behavior with social interactions in both income and race. In their model households prefer integrated communities to segregated ones but, if forced to choose

[10] He also moves to a representative agent framework where homogeneous households choose whether to be unemployed or to invest in education to obtain varying degrees of skill.

between two racially segregated communities, would prefer to live in the one occupied by their own race. A single crossing condition ensures that, all else constant, households will be stratified by income. Consider an initial equilibrium where households are effectively stratified by race due to a large black-white income gap, but would prefer to be integrated. As the income gap narrows, a rich black household living in the predominantly black community has *less* of an incentive to move to the predominantly white community because the white community has become less affluent in relative terms.

Throughout this literature, the single-crossing assumption on preferences is maintained in order to assist in characterizing the properties of the sorting equilibria. However, that assumption is not always necessary to guarantee that equilibria exist. If one is willing to alter some features of the model, it is possible to prove existence without requiring the single-crossing condition. For example, Thomas J. Nechyba (1997) proves existence after omitting social interactions and introducing some discreteness into the housing market. Specifically, he imposes exogenous community boundaries and endows households with fixed quantities of housing. Alternatively, Bayer and Timmins (2005) prove existence in a model with social interactions by smoothing the preference function. They add an idiosyncratic shock to utility and, in lieu of endogenous prices, they allow utility to depend on the share of households who choose to live in each community.[11] This endogenously determined share can be interpreted as a (negative) congestion effect or a (positive) agglomeration effect. Importantly, the equilibrium is shown to be unique in the case of congestion. In the case of agglomeration, whether the equilibrium is unique depends on the strength of preferences for the endogenous amenity.

3.3 The Equilibrium Hedonic Price Function

Hedonic price functions provide another useful way to characterize sorting equilibria. If utility is continuously differentiable, monotonic in the numeraire, and Lipschitz continuous, Bajari and Benkard (2005) prove that, in equilibrium, the price of a differentiated product will be a function of its characteristics. Thus, under mild restrictions on preferences, the equilibrium price of an individual home can be expressed as a function of its structural characteristics and the

[11] With an estimate of the supply curve for housing, it is a simple matter to go from this specification to one that includes the price of housing directly in utility. See Timmins (2007) for an example.

amenities it provides: $P_{n \in j} = P(h_n, g_j)$.

By explaining variation in housing prices within a community, the hedonic model relaxes the assumption that housing in each community can be treated as a homogeneous commodity sold at the constant price, p_j. Sieg et al. (2002) clarify the relationship between p_j and $P_{n \in j}$. They demonstrate that if utility is separable and homogenous of degree one in structural housing characteristics (h_n) then equilibrium housing expenditures will be separable in structural characteristics and amenities:

$$P_{n \in j} = P(h_n, g_j) = \bar{h}(h_n) \cdot p_j(g_j). \tag{12}$$

Thus, the hedonic price function can be factored into the product of a "quantity" index $\bar{h}(h_n)$, which depends on the vector of structural housing characteristics, and a price index that reflects the cost of consuming amenities, $p_j(g_j)$.

While empirical examples of hedonic modeling date back to Frederick V. Waugh's (1929) PhD Thesis, the first contributions to an underlying theory were made by A.D. Roy (1951) and Jan Tinbergen (1959). Their work focused on the market for labor. Roy (1951) argued that the equilibrium distribution of wages would reflect the underlying distributions of preferences and skills held by all of the producers and consumers in a market and Tinbergen (1959) provided the first analytical demonstration of this logic. However, their contributions to hedonic theory were not widely recognized until much later.

Hedonic models were first popularized by Zvi Griliches's (1961) work on using hedonic price functions to make quality adjustments to price indices for automobiles.[12] Rosen (1974) strengthened the economic foundations of the method by illustrating that the hedonic price function can be interpreted as an equilibrium relationship resulting from the interactions between all the buyers and sellers in a differentiated product market at a single point in time. He analyzed the properties of the price function in the special case where consumers are free to choose

[12] There is an important dichotomy in the literature on hedonic models that distinguishes Griliches (1961) and Jack E. Triplett's (1969) interests from those that followed the Rosen (1974) logic. Triplett's (1983) account of the development of the hedonic price function for price indexes as models without theory offers an interesting contrast to the post Rosen (1974) literature. His focus is the role of the price function in developing price and quantity indexes for quality differentiated goods rather than on the interpretation of its derivatives. See also W. Erwin Diewert (1993) for a discussion of the history of the early hedonic approach to price indexes.

continuous quantities of every product characteristic. Both Tiebout and Rosen recognized this as an extreme case of Tiebout's (1956) original model. In the special case with no economies of scale in producing public goods, Tiebout (1956) suggests that households would choose communities that *exactly* match their preferences, effectively making each household its own local government.[13] Likewise, Rosen (1974) observes that the hedonic price function reflects equilibrium stratification patterns that mirror those in Tiebout's work.[14]

The hedonic property value model provides a clear illustration of how the features of a sorting equilibrium can provide information about the demand for amenities. Consider a single amenity, g_1. Partially differentiating the equilibrium price function with respect to g_1 provides an estimate of the marginal price function for g_1:

$$P_1 = \frac{\partial P(h,g)}{\partial g_1}. \tag{13}$$

P_1 is the marginal contribution of g_1 to the price of housing given the current level of g_1 and levels of the other characteristics.

Because households are assumed to face a continuum of choices in Rosen's model, the first order conditions to the household's utility maximization problem for g_1 can be expressed as:

$$\frac{\partial P(h,g)}{\partial g_1} = \frac{\partial U/\partial g_1}{\partial U/\partial b} \equiv D(g_1; g_{-1}, h, \alpha, y). \tag{14}$$

The first equality in (14) implies that households will maximize their utility by choosing a housing location that provides them with a level for g_1 at which their marginal willingness-to-pay for an additional unit exactly equals its marginal implicit price. Assuming the marginal utility of income is constant, the second equality simply observes that as g_1 varies the marginal rate of substitution defines its inverse demand curve, conditional on all other amenities (g_{-1}) and

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[13] From page 421 of Tiebout (1956), "…the consumer-voters will move to that community which *exactly* satisfies their preferences. …this may reduce the solution of the problem to the trite one of making each person his own municipal government."

[14] From page 40 of Rosen (1974), "…a clear consequence of the model is that there are natural tendencies toward market segmentation, in the sense that consumers with similar value functions purchase products with similar specifications. …In fact, the above specification is very similar in spirit to Tiebout's (1956) analysis of the implicit market for neighborhoods, local public goods being the 'characteristics' in this case."

housing characteristics.

Figures 4 and 5 illustrate these first order condition for g_1. Figure 4 shows bid functions for housing in the g_1 dimension for two different households.[15] Each household will select the quantity of g_1 where its bid function is tangent to the hedonic price function. In the figure, the two households purchase houses that are identical except in their provision of the public good. Household 1 spends $\$_1$ on a house that provides $g_{1,1}$ units of the public good and household 2 spends $\$_2$ on a house with $g_{1,2}$.

The first order condition implies that, if markets are in equilibrium, evaluating P_1 at a household's chosen level of g_1 will return that household's marginal willingness-to-pay (MWTP) for *g*. Combining this information with the level of g_1 at a household's location identifies exactly one point on that household's inverse demand curve. In figure 5, household 1's inverse demand (D_1) intersects P_1 at the point where its MWTP exactly equals the marginal price for an extra unit of g_1. While MWTP is identified by the gradient of the price function, inverse demand curves are not. An infinite number of demand curves could pass through the points defined by $(MWTP_1, g_{1,1})$ and $(MWTP_2, g_{1,2})$. Thus, without additional assumptions about the nature of consumer preferences, the hedonic price function does not identify an individual household's demand for g_1 or its willingness-to-pay for non-marginal changes.

Moreover, if households are not free to choose continuous quantities of each amenity, the hedonic price function does not identify MWTP. There is a subtle but important distinction here between the price function itself and the information it conveys about preferences. The hedonic price function can be used to describe a sorting equilibrium, regardless of whether its characteristics are discrete or continuous (Bajari and Benkard 2005). However, if at least one characteristic is discrete, the first order condition in (14) will no longer characterize equilibrium behavior or reveal MWTP. This is important because many amenities do vary discretely across the urban landscape. School quality varies from school district to school district. Access to public pools, tennis courts and community centers may be limited to homeowners in a residential

[15] The bid functions express each household's willingness-to-pay for housing as a function of the amenity, given the household's preferences, income, levels of all the other characteristics, and the utility attained.

subdivision. Even for amenities which vary continuously in the natural environment, such as air quality and water quality, most of the variation may occur across natural boundaries, such as air basins and watersheds.

When amenities vary discretely across the spatial landscape, utility maximization is characterized by the set of inequalities in (15) rather than the first order conditions in (14).

$$(15) \qquad U\left(y_{i,j} - P_{n\in j}, h_n, g_j; \alpha_i\right) \geq U\left(y_{i,k} - P_{m\in k}, h_m, g_k; \alpha_i\right), \ \forall \ m,k .$$

The equation simply says that if household *i* chooses house *n* in community *j*, it is because that location provides it with at least as much utility as any other alternative in its budget set. Comparing a bundle of goods that was purchased with bundles that could have been purchased, but were not, can serve to identify bounds on a consumer's indifference curves (Paul A. Samuelson 1948). More precisely, assuming preferences are monotonic and convex, it is possible to recover bounds on the set of indifference curves that would be consistent with utility maximizing behavior (Hal R. Varian 1982). To identify a household's indifference curve within these bounds, however, the analyst must be willing to impose more structure on preferences (Kuminoff 2009).

Econometric approaches to demand estimation can be divided into two frameworks: hedonic models that exploit the first-order conditions in (14) and sorting models that exploit the inequalities in (15) together with the equilibrium price function. Both frameworks assume that households "pay" for amenities through the price of housing and then use data on housing prices and spatial variation in amenities to infer the demand. This strategy presents a fundamental identification problem. Since each household is typically observed making a single housing purchase it is possible to identify, at most, one point on that household's demand curve. In order to recover the entire demand curve, the analysis must add information about the structure of preferences. The next two sections describe how the empirical hedonic and sorting frameworks have exploited advances in how this information can be introduced.

4. *Hedonic Estimation*

The hedonic price function can be estimated using data on housing prices and characteristics from individual real estate transactions. However, it is impossible for the econometrician to observe every relevant structural characteristic and amenity. To reflect this, let $x \subset [g, h]$ represent the characteristics observed by both households and the analyst, and let $\xi \subset [g, h]$ represent characteristics that are observed by households but not by the analyst such that: $x \cup \xi = g \cup h$. Since hedonic models treat amenities the same as structural characteristics, there is no loss of generality in using x to represent the observable dimensions of both.[16] Using the new notation, the hedonic price function can be rewritten as the second term in (16).

(16) $$P(h, g) = P(x, \xi) = P(x, \mathrm{B}, e(\xi)).$$

The third term is an approximation characterized by the parameter vector B and the error term e, which is a function of unobserved characteristics. In order to focus on unobserved characteristics, other sources of error such as functional form misspecification and error in measuring x are assumed to be negligible. Finally, to keep the notation manageable, consider a single amenity: $g_1 = x_1 \in x$. This simplification does not affect any of the conclusions.

Rosen (1974) suggested a two-step procedure that would use the information in the first order condition to estimate the demand for a product characteristic—in this case x_1. The first step is to use micro data on housing transactions to estimate the reduced form housing price function in (16) and partially differentiate it to recover $\hat{P}_1$, the marginal price function for x_1. The first order condition in (14) implies that by evaluating $\hat{P}_1$ using each household's chosen level of x_1 it is possible to recover an estimate of the marginal willingness-to-pay (MWTP) for x_1. Combining this information with the level of x_1 at a household's location gives exactly one point on that household's demand curve, as illustrated in figure 5.

For the second step of his procedure, Rosen suggested regressing estimates for the MWTP on product characteristics and a set of exogenous demand shifters such as income and demographic characteristics. Equation (17) illustrates the idea, where Ω is a parameter vector, d is a vector of observable demographic characteristics including income, and ε is the residual.

[16] This modeling choice is important because empirical studies often attach measures of the amenities to information from housing transactions based on some feature of the house's location, such as a Census tract or a school attendance zone. Other unobserved amenities could easily be associated with the same spatial feature. The relative spatial scales at which observed and unobserved amenities influence the character of housing is particularly important. We return to this issue in discussing Joshua K. Abbott and Klaiber's (2010) instrumental variables strategy for addressing endogenous amenities in section 4.3.

(17) $\hat{P}_1 = f(x, d, \Omega, \varepsilon)$.

The logic is that if households' unobserved preferences for x_1 are highly correlated with their demographics, the regression in (17) would recover the inverse demand for x_1. It was later recognized that this logic makes two important assumptions.

First, identifying the inverse demand curve with data from a single housing market requires that there be some nonlinearity in the marginal implicit price function. James N. Brown and Harvey S. Rosen (1982) demonstrated the requirement for this condition for a case where the marginal implicit price function and the inverse demand curve are both linear. Their result is shown in (18), which is simply a linear representation of (17).

(18) $B_0 + B_1 x = \Omega_0 + \Omega_1 x + \Omega_2 d + \varepsilon$.

The regression will simply recover B_1, the parameter vector that characterizes the implicit marginal price function; there is no new information to identify the shape of the demand curve.[17] The second qualifying assumption arises because households choose prices and quantities simultaneously. As a result, x_1 will be endogenous in (18). This endogeneity problem was recognized by Epple (1987), Timothy J. Bartik (1987), and Shulamit Kahn and Kevin Lang (1988). Thus, to avoid biased estimates, instruments are required.

To recover the demand for an amenity from the hedonic price function, the analyst must provide two additional sets of information. First, preferences must be restricted in a way that makes it possible to identify the demand curve. Second, the data generating process must be restricted to assure that it is possible to estimate the demand curve consistently. Two general econometric strategies have been developed to address these issues.

4.1 *Identification Strategy #1: Multiple Markets*

The first identification strategy implements Rosen's two-step approach by estimating hedonic price functions for multiple housing markets (Brown and Rosen 1982). The identifying restriction on preferences is that they are highly correlated with income and demographic

[17] Robert Mendelsohn (1985) makes a related point in discussing a single market approach to estimating marginal willingness to pay factors.

characteristics. To develop this idea formally, suppose that preferences for structural characteristics of housing and amenities can be written as a constant function of observable demographic characteristics and unobserved household-specific tastes (ε) as in equation (19).

(19) $$\alpha = f(d,\varepsilon)$$

Thus, two households with the same demographic characteristics and idiosyncratic tastes will have the same value for α_i in the objective function defined in equation (4). The identifying restriction is that (19) is separable in d and ε, with the demographic sub-function constant across each of the Q markets.

(20) $$\alpha = f(d)+\varepsilon, \quad \text{with} \quad f_1(d)=f_2(d)=\ldots=f_Q(d).$$

Intuitively, this restriction makes it possible to obtain multiple observations on the demand curve of each household "type". For example, in a representative application by Palmquist (1984) one particular household type is 40 year old, white, married couples with two children. They are restricted to have the same demand for housing characteristics (up to ε) whether they live in Atlanta, Denver, Houston, Louisville, Miami, Oklahoma City, or Seattle. In each market, this type of household faces a different hedonic price schedule, and therefore will choose different implicit prices and qualities of housing characteristics, identifying 7 different points on the common portion of their demand curve.

The restriction in (20) is sufficient to identify individual demand curves, even if $\hat{P}_1$ is linear. The strategy is to estimate the hedonic price function separately in each housing market and then pool the resulting sets of estimates for the marginal implicit prices in a second step and regress them on housing characteristics and household demographics.[18]

(21.a) STEP 1: estimate $P(x,\mathrm{B}_q,e)$ for $q=1,\ldots,Q$ markets.

(21.b) STEP 2: estimate $\hat{P}_1(x,\mathrm{B}_q,e)=f(x,z,d,\Omega)+\varepsilon$, stacking data for Q markets.

[18] Of course a key step in these analyses is defining the separate markets. The challenges associated with this task are especially timely because recent quasi-experimental studies have assumed national housing markets (Kenneth Y. Chay and Michael Greenstone 2005; Greenstone and Justin Gallagher 2008) in order to develop research designs that allow the specification of instruments to mitigate the confounding effects of sorting behavior on hedonic estimation. A maintained assumption in these studies is that houses in metropolitan areas across the contiguous United States are linked through a single equilibrium price function. In contrast, a maintained assumption in the two-step approaches to demand estimation is that different metropolitan areas have different price functions.

In order for (21.b) to provide consistent estimates of the demand for x_1 the three restrictions shown in (22) must be satisfied.

(22.a) $E[\xi \mid x] = 0$.

(22.b) $E[z'x]$ *is full rank*.

(22.c) $E[\varepsilon \mid d, z] = 0$.

First, to estimate Β consistently, the unobserved characteristics reflected in the error term, $e = e(\xi)$, must be uncorrelated with the observed characteristics (22.a). Second, a set of instruments (z) for x must be introduced to address the endogeneity problem (22.b). Third, the residual in the inverse demand function (which is interpreted as household-specific tastes) must be uncorrelated with demographic characteristics and the instruments (22.c).

Empirical studies have invoked the restrictions in (20) and (22) to estimate demand functions for various amenities, using between two and thirteen distinct markets (for a review see Laura O. Taylor 2003). A common feature of these studies is that they restrict the hedonic price function to have an additively separable error term. This condition is important because it implies that unobserved housing characteristics do not influence the demand for x_1.

The multi-market approach is data intensive. To estimate the model, one must be able to define separate housing markets; obtain micro data on the characteristics of houses and households in these markets, and specify appropriate instruments for the housing characteristics. Alternatively, with restrictions on the structure of preferences, it is possible to identify the demand for a housing characteristic from data on a single market, without requiring instruments.

4.2 *Identification Strategy #2: Restricting the Structure of Preferences*

The multiple market identification strategy takes a reduced-form approach to estimating the inverse demand curve that requires preferences to be highly correlated with household demographics. The need for this assumption, along with the need for data on demographics, can be relaxed by specifying the utility function. This process starts by estimating the equilibrium price function as defined in (23.a).

(23.a) STEP 1: estimate $P(x, \mathrm{B}_q, e)$ for $q = 1$ market.

(23.b) STEP 2: solve for α using $\hat{P}_1(x, \mathrm{B}_q, e) = U_1(x, \xi, b; \alpha)$.

Next, with the shape of the utility function known, the first-order condition in (23.b) can be used to solve for the structural preference parameters. This allows the demand for x_1 to be calculated directly. Paul Driscoll, Brian Dietz, and Jeffrey Alwang (1994) provided the first illustration of this approach.

What can be learned about preferences from (23.b) depends on what is assumed about the structure of the utility function and preference heterogeneity. For example, in an application to air quality, Sudip Chattopadhyay (1999) uses Diewert's utility function to develop the first-order condition in (24).

$$\hat{P}_1 = \alpha_0 + \alpha_1 \sqrt{x} + \alpha_2 d + \varepsilon . \tag{24}$$

He constrains households to have homogeneous preferences and estimates constant values for α . More recently, Bajari and Matthew E. Kahn (2005) specify a log-linear utility function and estimate a semiparametric hedonic price function, yielding the expression in (25).

$$\hat{P}_1 = \frac{\alpha_1}{x_1} \Rightarrow \alpha_1 = x_1 \hat{P}_1 . \tag{25}$$

They allow households to have heterogeneous preferences for housing characteristics, and use the expression to solve for values of α <u>for each individual household</u>. That is, substituting the household's chosen level of x_1 and the other characteristics into $\hat{P}_1(x, \mathrm{B}_q, e)$ allows them to solve for α_i. As expected, a more rigid structure for the utility function allows more information about preference heterogeneity to be recovered.

If demographic data on households are available, the structural preference parameters estimated from (25) can be regressed on demographics. For example, Bajari and Kahn (2005) estimate the additively separable function in (26).

$$\alpha = f(d) + \varepsilon . \tag{26}$$

While the flexibility of the approach used by Bajari and Kahn (2005) is appealing, it has two limitations. First, it can globally identify only as many structural parameters as there are observable product characteristics. Second, in order to recover preferences for individual households, the mean independence assumption in (22.a) must be strengthened. These two restrictions are presented in equations (27.a) and (27.b).

(27.a) $U(x, \xi, b; \alpha)$ is fully specified with: $\dim(\alpha) \leq \dim(x)$

(27.b) x and ξ are independent

The independence assumption can be relaxed if there are instruments available for the endogenous characteristics (Bajari and Benkard 2005). Likewise, the dimensionality restriction can be relaxed to allow a more flexible functional form if one has the ability to observe multiple choice outcomes for the same household (Kelly Bishop and Timmins 2008).

Ekeland, Heckman, and Nesheim (2004) developed an alternative identification strategy that would relax the need to specify the utility function in the second stage. They demonstrate that the marginal implicit price function is generally nonlinear and will only be linear in extremely special cases. They demonstrate that this nonlinearity provides a way to identify the demand for x_1 provided the utility function satisfies some restrictions. Most importantly, a monotonic transformation of marginal utility must be additively separable in characteristics, demographics, and household-specific tastes so that it can be written as (28).

(28) $\partial U / \partial x = f_1(x) + f_2(d) + f_3(\varepsilon)$.

With this restriction, Rosen's suggested two-stage strategy can be used to estimate the demand for x_1 from data on a single housing market. The first step is to estimate the hedonic price function in (29.a).

(29.a) STEP 1: estimate $P(x, \mathrm{B}_q, e)$ for $q = 1$ market.

(29.b) STEP 2: estimate $\hat{P}_1(x, \mathrm{B}, e) = f_1(x, \Omega) + f_2(d, \Gamma) + \varepsilon$.

In the second step, the estimated marginal implicit prices are regressed on housing characteristics and demographics, using $E[f_1(x) | d]$ as an instrument for $f_1(x)$ in (29.b).[19] This is an interesting extension. The theorems that support it are proven for the case where x, d, and ε, are each 1-dimensional.

In theory, the requirement that x be 1-dimensional could be relaxed if the analyst were prepared to restrict the nature of preference heterogeneity. One approach would restrict preferences in a way that allows the bundle of housing characteristics and amenities to be

[19] Kahn and Lang (1988) first suggested this type of instrument in the context of a quadratic marginal price function.

consistently represented by a 1-dimensional index of overall housing "quality". This can be done if households have identical relative preferences for every pair of elements in the bundle; i.e. they must have identical values for the weights in the quality index. However, estimating Ekeland, Heckman, and Nesheim's model under this preference restriction would also require developing an econometric method for recovering the constant "weights" in the quality index.[20]

Heckman, Rosa L. Matzkin and Nesheim (2010) extend the analysis to situations where preferences are not strongly separable. While they focus on hedonic wage models, their theoretical results are general and serve to catalog the types of assumptions required to identify the MWTP for a locational amenity. Preferences are assumed to be quasi-linear, but ε is allowed to enter marginal utility in a nonadditive way. The key identifying assumptions amount to weak separability restrictions, generalizing (28) to allow $\partial U/\partial x = f(q(x,d),\varepsilon)$ or $\partial U/\partial x = f(q(x,\varepsilon),d)$. A variety of conditional identification strategies are also derived. To our knowledge none of these approaches have been considered in the context of housing markets.

4.3 *Addressing Endogenous Amenities*

The multi-market and structural approaches to hedonic estimation were originally developed under the assumption that observed and unobserved amenities are uncorrelated. This logic overlooks an important implication of the sorting process. When households sort themselves across the urban landscape, their behavior induces spatial correlation between endogenous and exogenous characteristics of that landscape. These characteristics can be local public goods, environmental services, or features of neighborhoods (including their demographic composition). If data are not available for some of these features, then conditions (22.a) and (27.b) may be violated, biasing the first-stage estimation of the price function. Suppose we want to estimate the demand for school quality, for example. If homebuyers also care about beach access then, all else equal, wealthier households will prefer to locate in beachfront communities. As the average income increases in those communities, so will school quality, through peer

[20] This issue is revisited during the discussion of sorting models in section 5.1. The sorting models developed by Epple and Sieg (1999) and Sieg et al. (2004) exploit a similar index restriction on preferences to recover households' (heterogeneous) preferences for overall housing quality and their (homogeneous) preferences for the individual elements of the quality index, while simultaneously controlling for unobserved public goods.

effects and voting outcomes. This sorting process may produce an equilibrium in which school quality is positively correlated with proximity to the coast. Thus, if proximity to the coast is omitted from the first-stage regression, its latent effect on property values may be confounded with the estimated effect of school quality.

One potential solution is to find instrumental variables. This task presents a similar challenge as the early reduced form studies of voting behavior (Rubinfeld, Shapiro, and Roberts 1987). That is, in the absence of a specification for the sorting equilibrium, instruments can only be evaluated on the basis of intuition and statistical testing. Elena G. Irwin and Nancy E. Bockstael (2001) provide an example of this approach. To estimate the implicit price of living marginally closer to privately owned open space, they construct instruments from data on attributes of those parcels that affect the costs of development (e.g. drainage, slope, and soil quality). Assuming these attributes are idiosyncratic to the parcel, they may be uncorrelated with other unobserved amenities. More recently, Abbott and Klaiber (2010) have argued that, in some cases, instruments may be developed by using attributes of the spatial landscape to construct a "panel-like" feature of a cross-section data set (e.g. defining housing subdivisions as a proxy for a neighborhood). This allows them to adapt the logic of Jerry A. Hausman and William E. Taylor (1981). In this case, the overidentifying restrictions could be used to test whether the instruments satisfy the conditional independence assumption. While the idea of developing instruments for hedonic estimation is interesting, there have been few applications to date.

The dominant strategy to address omitted variables is to add spatial fixed effects to the price function. Common examples include indicator variables for school districts, census tracts, and subdivisions. A collection of fixed effects for these jurisdictions can provide a "fuzzy" approximation to the spatial distribution of unobserved amenities. Kuminoff, Christopher F. Parmeter, and Jaren C. Pope (2010) document the growing popularity of this technique, and provide Monte Carlo evidence suggesting it is effective. Their positive findings rely on two features of the joint distribution of x and ε. First, x must vary within the regions indicated by the fixed effects. Second, all else constant, x and ε must be less correlated within regions than between regions.

One can "sharpen" the fixed effects strategy if there are obvious discontinuities in the way the amenity of interest is conveyed to homeowners. Sandra E. Black (1999) provided the first demonstration of this idea. She observed that public school quality shifts discretely as one crosses the boundary between two adjacent attendance zones, whereas other amenities are approximately unchanged (e.g. crime rates, air quality, access to the city center). Therefore, the average price effect of all the unobserved amenities that are common to houses on both sides of a boundary will be absorbed by a fixed effect for the "boundary zone". By limiting her analysis to sales that occurred within 0.15 miles of a boundary and including fixed effects for each boundary zone, Black sought to identify the MWTP for school quality in Boston from price differentials between structurally similar houses located on opposite sides of a boundary.

Bayer, Fernando Ferreira, and Robert McMillan (2007) refined Black's approach to control for sorting based on social interaction. If preferences for school quality are correlated with demographic characteristics, such as race or education, then similar "types" of households may tend to locate in the same attendance zones. Indeed, Bayer, Ferreira and McMillan observe that household demographics tend to shift discretely as one crosses an attendance zone boundary in the San Francisco Bay Area. As a result, adding measures of local demographic composition to the price function halved their estimate of the MWTP for school quality.

Despite its intuitive appeal, there have been few applications of the sharp fixed effects strategy to amenities other than school quality. The barrier to wider implementation is that discrete changes in amenities often coincide with geographic and political boundaries.[21] This makes it hard to control for unobserved variables in a sorting equilibrium. For example, air quality may change discretely from air basin to air basin, but the boundary between adjacent air basins may be a mountain range. Air quality changes as one crosses the range, but so might school quality, the incidence of violent crime, and other amenities. Faced with this difficulty, analysts have increasingly turned to capitalization models as a possible way to identify price function parameters.

[21] This confounding can be resolved if an exogenous event alters the way the amenity influences households' location choices without affecting unobserved variables. However, by altering features of the equilibrium, such events may also confound our ability to translate the hedonic gradient into a measure of MWTP. See Pope (2008a) for discussion and an application to a new law mandating the disclosure of information about flood zone boundaries.

4.4 *The Capitalization Model*

Capitalization studies use data before and after an exogenous change in a feature of the equilibrium to measure its effect on property values. The power of this technique is the ability to simultaneously measure a change in asset values and demonstrate that the change was caused by some event. Capitalization models are routinely used by expert witnesses in litigation over private property externalities (Robert Simons 2006). They are also increasingly used to estimate the MWTP for amenities.

The idea for using panel data to measure how changes in quality characteristics influence property values dates back at least to Martin J. Bailey, Muth, and Hugh O. Nourse's (1963) work on constructing price indices for real estate. Palmquist's (1982) study of highway noise was the first application to a spatially delineated amenity. He tracked the capitalization of changes in decibel readings at houses that were sold before and after the construction of an interstate highway. Chay and Greenstone (2005) refined the framework to reflect insights from the quasi-experimental literature on policy evaluation. Their work set the stage for contemporary applications.

Chay and Greenstone's conceptual model bridges the capitalization and hedonic literatures. It integrates a quasi-experimental version of the identification strategy from the capitalization literature with the welfare interpretation of Rosen's hedonic model. They begin by assuming a linear-in-parameters price function such as

(30) $$P = x\mathrm{B}_1 + e,$$

where the subscript on B_1 is now used to indicate the time period. Invoking the first order conditions from Rosen (1974), they interpret each element of B_1 as the marginal willingness to pay (MWTP) for the corresponding element of x in period 1. The problem is that B_1 is not identified when e is correlated with x.

Now suppose that prices and characteristics are measured again at a later point in time. First-differencing the data produces a new estimator

(31) $$\Delta P = \Delta x \phi + \Delta \varepsilon ,$$

where ΔP and Δx represent the changes in prices and characteristics that occurred between the two periods. If the bias from omitted variables is purged by differencing the data, (31) provides an unbiased estimator for ϕ. Alternatively, if one suspects that $E[\Delta\varepsilon \mid \Delta x] \neq 0$, instrumental variables may be used to develop a consistent estimator for ϕ. For example, Chay and Greenstone (2005) use a discontinuity induced by the rules implementing the Clean Air Act as an instrument for changes in air quality between 1970 and 1980. Their instrument is an indicator for whether a county was designated as a "non-attainment area" in 1975 due to ambient concentrations of particulate matter that exceeded the primary standard. They suggest that changes in housing prices between 1970 and 1980 are likely orthogonal to the transitory elements that led to the Environmental Protection Agency's nonattainment designations.[22] In a subsequent application, Bayer, Nathanial Keohane, and Timmins (2009) use information on the atmospheric transport of pollutants to develop an instrument for changes in air quality at one location (e.g. Raleigh, NC) as a function of changes in emissions at distant locations (e.g. Nashville, TN).

Suppose the analyst develops a credible strategy to identify ϕ. What does ϕ measure? Interpreted literally, ϕ measures the average rate of change in property values associated with the changes that occurred in housing characteristics and amenities. Chay and Greenstone observe that these capitalization effects will equal MWTP if the gradient of the hedonic price function in (30) is constant over time (i.e. $\mathrm{B}_1 = \mathrm{B}_2$ implies $\phi = \mathrm{B}_1 = \mathrm{B}_2$). Several studies have implicitly or explicitly invoked this result to estimate the MWTP for amenities such as cancer risk (Lucas Davis 2004), air quality (Chay and Greenstone 2005; Bayer, Keohane, and Timmins 2009), hazardous waste (Greenstone and Gallagher 2008), crime (Leigh Linden and Jonah E. Rockoff 2008; Pope 2008b), and investment in education (Stephanie Riegg Cellini, Ferreira, and Jesse Rothstein 2010). In all of these studies, the ability to interpret ϕ as a parameter of the hedonic price function rests on the maintained assumption that its gradient is constant over the duration of the study. This assumption has been made for periods between 10 and 20 years, for study areas ranging from a single county to the contiguous United States.

[22] Chay and Greenstone do not have data on the actual status of each county. They estimate nonattainment status from data on monitor readings and standards for total suspended particulates. The actual definition of nonattainment depends on the number of exceedances in a given time period and varies with each criteria air pollutant.

Because the hedonic price function is an equilibrium outcome generated by interactions between all of the buyers and sellers in a market, assumptions about its evolution implicitly restrict preferences and technology. These restrictions go beyond the assumptions of the multi-market and structural hedonic models outlined earlier.[23] Recall that hedonic models based on Rosen (1974) describe market equilibria at a single point in time.[24] They have no problem admitting preference structures and institutions that would link changes in amenities to changes in the shape of the price function. The shape of the price function could also change over time as a result of concomitant changes in the distribution of income, preferences, technology, or the prices of goods collected in the numeraire.

Intuitively, one might expect the gradient of the hedonic price function to be approximately invariant to changes in amenities that are "small" or "localized" in the sense that they occur over a small portion of the market (Palmquist 1992).[25] On the other hand, if social interactions are important, then small or localized changes in one amenity may trigger tipping effects that produce large changes in other features of equilibria (Bénabou 1996b, Sethi and Somathathan 2004). Our present knowledge of the transitional dynamics between equilibria is extremely limited. This is an important area for future research. In the context of the capitalization model, one should test whether the shape of the price function changes over time, whenever it is possible to identify single-period price functions (Kuminoff and Pope 2010). More broadly, the new class of empirical sorting models offers the potential to help us uncover the structure of preferences.

[23] Kuminoff and Pope (2010) characterize these general restrictions and formalize their testable implications for linear price functions. In a "sharp" fixed effects application to school quality, they find that the shape of the price function changes over time. As a result, their estimates for MWTP are three times as large as capitalization effects.
[24] Consistent with their different conceptual models, empirical hedonic studies typically use data from much shorter intervals than capitalization studies. For example, Bayer, Ferreira, and McMillan (2007) use data on self-reported housing values from 1990; Black (1999) uses data on housing sales from 1993-1995; Irwin and Bockstael (1999) use data from 1995-1999.
[25] Klaiber and Smith (2009) investigate how the size of such changes influence the ability of capitalization models to approximate general equilibrium welfare measures.

5. *Equilibrium Sorting Estimation*

The new empirical sorting models use the properties of equilibria to identify and estimate households' preferences for amenities. This framework depicts a mixture of discrete location choices and continuous consumption decisions made by a population of urban residents, while recognizing that the amenities which differentiate communities may be determined endogenously by voting, peer effects, feedback, or other forms of social interaction. It is natural to expect that, in equilibrium, the price of housing and the level of each endogenous amenity will be correlated with omitted characteristics of communities. Fortunately, the properties of market equilibria can also suggest natural choices for instrumental variables.

Our coverage of empirical sorting models begins by describing the three predominant frameworks developed by Epple and Sieg (1999), Bayer, McMillan, and Kim Rueben (2004), and Ferreyra (2007). Most applications to date are linked to one of these frameworks. Each framework differs in how it defines the choice process, how it depicts household preferences, how it uses the properties of equilibria to develop instrumental variables, and how it approaches econometric estimation. After assessing the strengths and weakness of each estimation framework, we discuss calibration as an alternative strategy. The section concludes with a brief summary of the tradeoffs that have arisen with efforts to describe sorting equilibria using quasi-experimental research designs.

5.1 *The Pure Characteristics Sorting Model*

Epple and Sieg (1999) provided the first illustration of how the properties of a sorting equilibrium can be used to recover households' preferences for amenities. Their structural model parallels the theoretical literature on equilibrium stratification discussed in section 3.1 (Ellickson 1971; Westoff 1977; Epple, Filimon, and Romer 1993; and especially Epple and Platt 1998). This includes treating housing as a homogeneous commodity that can be consumed at a constant (community-specific) price. Under this assumption, the price of housing reflects the cost of consuming the amenities provided by each community.

In practice housing is not homogenous. Its structural characteristics (bedrooms, bathrooms, square feet) vary within and between communities, and these differences will be reflected in observable sale prices. This can be addressed if we are prepared to assume that the structural characteristics of housing enter the direct utility function through a sub-function that is homogeneous of degree one and separable from the effect of amenities and the numeraire. As discussed earlier in the context of (12), this separability restriction implies the equilibrium locus of housing expenditures defined by a hedonic price function can be expressed as the product of a price index and a quantity index: $P_{n\in j} = \overline{h}(h_n)\cdot p(g_j)$. By condensing all the information about the structural characteristics of a house into a single number, the quantity index provides an empirical analog to the concept of a homogeneous unit of housing.[26] Taking logs produces the hedonic property value model in (32).

$$\ln(P_{n\in j}) = \ln[\overline{h}(h_n)] + \ln[p(g_j)] \ . \tag{32}$$

Given a parametric form for the quantity index and micro data on housing transaction prices and their structural characteristics, the price of housing in each community can be estimated as a community-specific fixed effect: $p_1,..., p_J$.

The index of structural characteristics is optimized out of the indirect utility function under the standard hedonic assumption that households are free to choose continuous levels of the structural characteristics of housing, no matter where they choose to live. Thus, Epple and Sieg (1999) depict a discrete-continuous optimization problem where households choose one of a discrete number of communities and then, conditional on that selection, a continuous quantity of housing within that community.

Equation (33) shows their CES indirect utility function for household *i* in community *j*. The first term represents utility from community-specific amenities. $\overline{g}$ is a linear index of *K* amenities, all but one of which are observable. The K^{th} amenity ($g_{K,j} = \overline{\xi}_j$) represents an index of all the community-specific attributes that are not observed by the econometrician.

$$V_{i,j} = \left\{ \alpha_i (\overline{g}_j)^\rho + \left[\exp\left(\frac{y_i^{1-\nu} - 1}{1-\nu} \right) \exp\left(- \frac{\beta (p_j^{\eta+1}) - 1}{1+\eta} \right) \right]^\rho \right\}^{\frac{1}{\rho}} , \tag{33}$$

[26] Another interpretation of their analysis is that it offers a sufficient set of conditions for defining the price indexes for goods with difficult to measure quality attributes. As a result, it helps to connect the equilibrium descriptions of hedonic price functions with the price index descriptions often associated with Triplett (1983).

$$\text{where} \quad \bar{g}_j = \gamma_1 g_{1,j} + \gamma_2 g_{2,j} \cdots + \gamma_{K-1} g_{K-1,j} + \gamma_K \bar{\xi}_j, \qquad F(\alpha, y) \sim \text{lognormal}.$$

Notice that the weights in the public good index are constant $(\gamma_1, \ldots, \gamma_K)$. This requires households to agree on the ranking of communities by $\bar{g}$. However, households can differ in the strength of their preferences for amenities relative to private goods through the α_i term. Its joint distribution with income is assumed to be lognormal.

The second term in the CES function represents utility from private goods. Applying Roy's Identity yields the demand function for the private good component of housing:

$$\bar{h}_{i,j} = \beta p_j^{\eta} y_i^{\nu}. \tag{34}$$

The price and income elasticities of the demand for housing are represented by η and ν, β is a housing demand intercept, and ρ reflects substitution between public and private goods.[27] Given the expected signs for the housing demand parameters, the single crossing condition implies $\rho < 0$. This condition offers an opportunity to test the underlying theory.

Unlike most discrete choice models of consumer behavior, Epple and Sieg's indirect utility function does not include idiosyncratic shocks for each choice ($\varepsilon_{i,1}, \ldots, \varepsilon_{i,J}$). Without idiosyncratic shocks to preferences, all households must agree that communities differ only in the *K* characteristics that comprise the $\bar{g}_j$ index. Berry and Pakes (2007) have labeled this property the "pure characteristics" approach to modeling the demand for a differentiated product in order to distinguish models that invoke it from the more common "random utility models" that assume idiosyncratic shocks.

In their application to the Boston Metropolitan Area, Epple and Sieg (1999) define each community as a school district. Then, using data on housing prices, household income, school quality, and crime rates in 92 school districts, they estimate the parameters of the model in two stages. First, they use the *stratification* property from (11) to express the quantiles of the income distributions in each community as a function of the structural parameters. A subset of these parameters can be identified by minimizing the distance between predicted and observed income quantiles. Then, in the second stage, they use the *increasing bundles* and *boundary indifference*

[27] Specifically, the elasticity of substitution is defined as $\sigma = 1/(1-\rho)$.

properties (9)-(10) to develop a nonlinear instrumental variables estimator to identify the remaining parameters, including a composite unobserved public good in each community $(\bar{\xi}_1,...,\bar{\xi}_J)$.

Instruments are needed in the second stage because equilibrium prices (*p*) and amenities ($\bar{g}$) may be correlated with unobserved amenities ($\bar{\xi}$). Epple and Sieg develop instruments based on functions of the income rank of each community. The relevance of these instruments follows directly from the underlying theory. The *increasing bundles* property in (10) implies that p_j, $\bar{g}_j$, and $y_j(\alpha)$ will all follow the same ranking across communities. For the instruments to be valid, however, we must be willing to assume that unobserved amenities are of second order importance in the sorting process. In other words, while $\bar{\xi}$ may affect the *level* of income in each community, it must not affect the *ranking* of communities by income.

Sieg et al. (2004) refine Epple and Sieg's (1999) estimator by adding moment conditions based on the distribution of housing prices and using a GMM approach to estimate all the parameters of the model simultaneously. They use the model to estimate preferences for air quality, school quality, and crime rates in 93 school districts in the Los Angeles Air Basin. Other applications of the Epple-Sieg framework have focused on open space and access to recreation opportunities (JunJie Wu and Seong-Hoon Cho 2003, Walsh 2007). All of these applications have found that $\rho < 0$, suggesting that observed sorting behavior is consistent with the single crossing restriction on preferences.

One of the key limitations of the framework developed by Epple and Sieg (1999) and Sieg et al. (2004) is the maintained assumption that all households agree on a common ranking of communities by their provision of amenities. Relaxing this assumption is important because it is reasonable to expect households to evaluate the components of a vector of amenities goods quite differently. For example, households with school age children may be more concerned about school quality while retirees may place more emphasis on climate and other environmental services. Epple, Michael Peress and Sieg (2010) suggest one way to allow for these differences by specifying types of households that can be identified from observable attributes. For each type there can be a separate function describing how location specific amenities and public goods are measured. This formulation would allow households with children to evaluate local public schools differently than retirees or single individual households. Greater flexibility is admitted by Bayer, McMillan, and Rueben's (2004) random utility model.

5.2 *The Random Utility Sorting Model*

Bayer, McMillan, and Reuben (2004) develop a smoother, probabilistic approach to estimating household preferences. Following McFadden (1978), they begin by defining the object of choice as an individual house.[28] Then, like Epple and Sieg (1999), they assume each house is located within a community that provides a bundle of amenities. The resulting model is extremely flexible in its treatment of preference heterogeneity. Households are allowed to differ in their relative preferences for multiple housing characteristics and amenities. In addition, the random utility specification allows each household to have an idiosyncratic "taste" for every choice alternative.

Equation (35) depicts household *i*'s utility from occupying house *n* in community *j*. For notational simplicity, *i* superscripts are used to index households and subscripts are used to index a partition of the structural preference parameters into four vectors that correspond to preferences for housing characteristics, amenities, commuting distance to the household's job location, and the price of housing, $\alpha_i = \left[\alpha_h^i, \alpha_g^i, \alpha_c^i, \alpha_p^i\right]$.

(35) $$V_{i,n\in j} = \alpha_h^i h_n + \alpha_g^i g_j + \alpha_c^i c_{i,j} + \alpha_p^i p_n + \overline{\xi}_n + \varepsilon_n^i,$$

$$\text{where} \quad \alpha_a^i = \alpha_{0,a} + \sum_{r=1}^{R} \alpha_{r,a} d_r^i, \quad \text{and} \quad \varepsilon_j^i \sim \text{iid type I extreme value.}$$

A household's preferences depend on its demographics. This is modeled by decomposing preferences for each attribute into a common component, α_0, and a set of terms that interact with a vector of demographic characteristics describing the household, d^i. Finally, the idiosyncratic tastes are assumed to be identically and independently distributed following a type I extreme value distribution.

Because idiosyncratic tastes follow the type I extreme value distribution, the probability of choosing a house (or a type of house) can be expressed as a function of the structural

[28] Subsequent versions of this paper and related research have aggregated the choice alternative into classes of houses. See Bayer, Ferreira and McMillan (2007) and Klaiber and Phaneuf (2010b) as examples.

parameters (McFadden 1974). This statistical relationship forms the basis for estimation. Bayer, McMillan, and Reuben adapt the instrumental variables estimator from Berry, Levinsohn and Pakes (1995) to find values for the heterogeneous preference parameters that minimize the difference between predicted and observed location choices. The house-specific unobserved characteristics $\left(\bar{\xi}\right)$ are treated as a structural error term to be recovered during the estimation.

In their application to the San Francisco Metropolitan Area, Bayer, McMillan, and Reuben (2004) define each community as a census block group. They combine Census data on the prices and characteristics of houses within each community with data on a diverse set of community-specific amenities including crime rates, school quality, elevation, % white, % black, and average income. During the estimation, the endogeneity of housing prices and amenities is handled by developing two separate sets of instruments.

To address the correlation between observed and unobserved amenities, Bayer, McMillan, and Reuben adapt the amenity discontinuity strategy from Black's (1999) hedonic analysis of public school quality. First, they limit their sample to houses located less than a quarter mile from a school attendance zone boundary. Then they add fixed effects for each boundary to absorb the average effect of the omitted variables near each boundary. They argue that this allows them to isolate the variation in school quality that occurs across boundaries, as well as variation in the demographic composition of neighborhoods.[29]

The instrument for price in community *j* is calculated as a function of housing characteristics and exogenous amenities in all other communities. This approach builds on Berry, Levinsohn, and Pakes (1995), who suggested that the equilibrium price of a differentiated product should depend on the characteristics of its closest substitutes. Their suggestion translates naturally to the market for housing (Bayer and Timmins 2007). Bayer and Timmins demonstrate that, in equilibrium, the level of an endogenous attribute in community *j* will be a function of the exogenous attributes of every other location. This functional relationship ensures that exogenous attributes will be relevant instruments. The validity of these instruments follows from the maintained assumption that the utility from locating in community *j* is not directly affected by the exogenous attributes of any other community. This logic can be used to develop an

[29] The notion that race and other demographic characteristics may also vary across school district boundaries is broadly consistent with Sethi and Somathathan's (2004) model of racial segregation.

instrument for price or an endogenous amenity.

Subsequent studies have adapted the basic model and identification logic from Bayer, McMillan, and Reuben (2004) to estimate preferences for school quality, the demographic composition of a community, and open space (Bayer, Ferreira, and McMillan 2007; Klaiber and Phaneuf 2010b).

5.3 *The General Equilibrium Sorting Model*

Ferreyra (2007) developed a third structural estimator for recovering household preferences. Her framework is loosely based on the theoretical models developed by Nechyba (1997, 1999, 2000) and emphasizes school quality over all other amenities. The novel feature of her approach is that, rather than developing instruments for the endogenous components of the equilibrium, she models the mechanisms that underlie their endogeneity. That is, in addition to specifying household preferences, she specifies a production function for education where school quality is jointly determined by expenditures and peer effects, and households vote on the rate at which to tax themselves to fund public schools. She uses these relationships to estimate parameters describing household utility and the production of education. With this "general equilibrium" approach to estimation, each draw on the structural parameters can be used to predict housing choices, school choices, voting outcomes, tax rates, and expenditures on public education.

Compared to Epple and Sieg (1999) and Bayer, McMillan, and Reuben (2004), her specification for utility is quite simple. Households are assumed to have identical Cobb-Douglas preferences over school quality (*s*), an index of all other community attributes ($\bar{g}$), and the composite numeraire (*b*),

$$U_{i,j} = \alpha_s \ln s_{i,j} + \alpha_{\bar{g}} \ln \bar{g}_j + \left(1 - \alpha_s - \alpha_{\bar{g}}\right) \ln b_{i,j} + \varepsilon_{i,j}. \tag{36}$$

The only explicit source of preference heterogeneity is an idiosyncratic shock to individual preferences for each location, $\varepsilon_{i,j}$, which is assumed to follow the usual *iid* Type I extreme value distribution. However, households may differ in their *perceptions* about the quality of public schools relative to private Catholic schools. Equation (37) depicts the quality of public and private schools in community *j* as perceived by household *i*,

(37) $$s_{i,j} = q^{\phi} A^{1-\phi} \quad \text{for a public school, and}$$

$$s_{i,j} = \alpha_C^i q^{\phi} A^{1-\phi} \quad \text{for a private Catholic school.}$$

In this Cobb-Douglas production function ϕ is a technology parameter to be estimated, A is expenditures per student, and peer effects are denoted by q, which is set equal to the average income of households with children attending the school. Perceptions of school quality, α_C^i, are assumed to be uniformly distributed, with one distribution for Catholic households and another distribution for non-Catholics.

Amenities other than school quality play a less important role in Ferreyra's (2007) model. The composite index of non-school amenities, $\bar{g}_j$, is recovered through a preliminary hedonic regression of housing expenditures on their structural characteristics, community amenities, and a set of fixed effects for school districts. This regression is essentially the same as the procedure suggested by Sieg et al. (2002) and illustrated in (32). The difference is that Ferreyra discards her estimates for the fixed effects and then defines $\bar{g}_j$ as the average predicted price for the bundle of remaining explanatory variables.

Households in her model make three choices: (i) where to live, (ii) whether to send their children to public or private schools, and (iii) how to vote on the local property tax rate. The property tax rate (t_j) in each community is determined by majority rule and the state income tax rate (τ_j) is set to balance a state-level budget constraint. Finally, the household's expenditures must satisfy their individual budget constraint:

(38) $$(1 - \tau_j) y_i + w_i = b_{i,j} + (1 + t_j) p_j + T .$$

A household's disposable income is equal to their after tax income plus their non-taxable income, w_i. This is spent on consumption of the numeraire, housing, and possibly tuition for a private school, *T*.

Ferreyra estimates all of the parameters of her model using a combination of school district data and Census data from the metropolitan areas of New York, Chicago, Philadelphia, Detroit, Boston, St. Louis, and Pittsburg. A minimum distance estimator is used to match predicted and observed levels of several variables describing these metropolitan areas and the school districts located within them: (i) average household income, (ii) average housing price, (iii) average expenditures per student in public schools, (iv) the share of households with children in public schools, and (v) the share of households with children in Catholic schools.

5.4 *Strengths and Weaknesses of Empirical Sorting Models*

The three empirical sorting frameworks developed by Epple and Sieg (1999), Bayer, McMillan, and Reuben (2004), and Ferreyra (2007) (henceforth *ES*, *BMR*, and *F*) differ in the way they define the set of choices faced by each household, the way they specify the shape of the preference function, and they way they develop instruments to control for endogenous amenities. These modeling decisions are important because they influence what can be learned about preferences for amenities from observed behavior. Estimates for the distribution of structural preference parameters will, in turn, shape predictions for how households, market prices, and endogenous amenities would react to a prospective policy shock. Table 1 summarizes the key modeling decisions that differentiate these three models from each other and from the hedonic approaches to demand estimation discussed earlier.

5.4.1 *The Choice Set*

In a discrete choice framework, estimates for the structural preference parameters can be sensitive to the scope of the choice set and to the composition of choices within that set (Kenneth E. Train 2003; Kuminoff 2009). With this in mind, an advantage of the empirical sorting models developed by *BMR* and *F* is that they are sufficiently flexible to recognize that the choice of a home may be related to choices in other markets. In Ferreyra's model, households may be forced to choose between inexpensive housing in communities with weak public schools and expensive housing in communities with strong public schools. Faced with this tradeoff, some households may choose to purchase a large home in a less expensive community and then pay to send their children to a private school. Similarly, *BMR* recognize that a household's choice of a home may depend on where its primary earner works. They treat each worker's job location as fixed and use this information to calculate the commuting distance to each potential house location. Thus, every working household in their model faces a unique choice set.[30]

Of the three empirical sorting models, *BMR* is the only one to acknowledge the connection between housing and labor market choices. However, their random utility model also has a serious disadvantage. It lacks a budget constraint. Elsewhere in the discrete choice literature, it is common to assume that every alternative in the choice set is also in every consumer's budget set. This assumption seems plausible in markets for breakfast cereal (Nevo 2001), laundry detergent (Hendel and Nevo 2006) and even personal computers (Bajari and Benkard 2005). It is not plausible in the market for housing. Few households are sufficiently wealthy to be able to afford every home in a large metropolitan area. The models developed by *ES* and *F* are consistent with this observation. In Epple and Sieg's framework, the identifying assumption is that each household is able to purchase at least some of the houses in its current community and in the next most expensive community. Likewise, Ferreyra's general equilibrium model includes an explicit budget constraint for each household.

In principle, one could easily incorporate a budget constraint into the BMR framework. For example, replacing p_n in the indirect utility function with $\ln(y_i - p_n)$ would set the probability of occupying a house equal to zero if its annualized price exceeded the household's income. The challenge with implementing this approach would be measuring the dimensions of a household's wealth that constrain its purchase of a home. Ideally, this measure would reflect past savings, current income, and expected future earnings. Such detailed information is presently unavailable.

5.4.2 *Preference Heterogeneity*

To compare the depiction of preference heterogeneity in the three models consider an example where communities differ in two observable amenities, air quality and school quality. In all three models the utility function contains a separable, linear sub-function of amenities. Let γ represent the relative preferences for these amenities. Using this notation, the three sub-functions can be represented as follows:

[30] Bayer and Timmins (2007) demonstrate that variation in the choice set also increases the power of instruments based on substitute attributes.

$$
(39) \quad
\begin{array}{ll}
(ES) & \gamma_1 \cdot school_j + \gamma_2 \cdot air_j + \bar{\xi}_j \\
(F) & \gamma_{i,1} \cdot school_j + \gamma_2 \cdot \left(\gamma_3 \cdot air_j + \bar{\xi}_j\right) + \varepsilon_{i,j} \cdot \\
(BMR) & \gamma_{i,1} \cdot school_j + \gamma_{i,2} \cdot air_j + \bar{\xi}_j + \varepsilon_{i,j}
\end{array}
$$

$\bar{\xi}_j$ measures the mean effect of unobserved amenities at location *j*, while $\varepsilon_{i,j}$ is an idiosyncratic deviation from that mean.

In the *ES* specification, relative preferences are constant. In other words every household is required to have the same relative preferences for every amenity, observed and unobserved. Everybody trades air quality and school quality at the same rate. The *F* specification relaxes this depiction of preference heterogeneity by allowing households to differ in their relative preferences for school quality. However, households still have the same relative preferences for air quality and $\bar{\xi}_j$. The *BMR* specification relaxes this restriction to provide the most general depiction of preference heterogeneity.

The depiction of preference heterogeneity determines the way in which households perceive communities to be differentiated. When households share the same relative preferences for each pair of amenities in the *ES* specification, every household must also agree on a common ranking of communities by the overall quality of amenities they provide. The notion that a set of differentiated communities can be unanimously ranked by quality is analogous to Lancaster's (1979) description of "vertical" product differentiation. In contrast, when households differ in their relative preferences for amenities, as in *F* and *BMR*, they may also differ in the way they rank communities—a situation analogous to Lancaster's description of "horizontal" product differentiation. The distinction between vertical and horizontal differentiation is important because the two concepts differ in their implications for substitution patterns (Anderson, de Palma, and Thisse 1992). When households agree on the ranking of communities by overall amenity provision in the vertically differentiated case, they must also agree on the opportunities for spatial substitution. For example, if asked to identify the two closest substitutes for community *j*, every household in the *ES* specification will select the two adjacent communities in the ranking by overall amenity provision. Horizontal differentiation allows more diversity in perceived substitution possibilities. Since households in the *F* and *BMR* specifications can differ in how they rank communities, they may disagree on which communities represent the closest substitutes for *j*. The idiosyncratic preference shocks guarantee that each location is a substitute for every other location.

ES, *F*, and *BMR* also differ in how they restrict substitution between amenities and private goods. All three models assume that different amenities are perfectly substitutable. This conclusion follows from the additive form of the indices in (39). The CES specification used by *ES* allows the relationship between amenities and private goods to range from perfect substitutes to perfect complements. In contrast, the linear utility functions in *BMR* and *F* restrict amenities to be perfect substitutes for the private good numeraire. This restriction is quite common in random utility models of the choice among differentiated goods. However, it is not a requirement for consistent estimation. Preference specifications that relax the assumption of perfect substitutability would be desirable in future research.

Finally, the idiosyncratic preference shocks in the *F* and *BMR* specifications deserve some comment. It seems intuitively plausible that households will differ in their preferences for individual locations in ways the econometrician cannot observe. A household may be attached to a particular community because its family and friends live there for example. Failing to account for these effects may cause the *ES* specification to be biased. However, the extra flexibility provided by adding the *J* idiosyncratic shocks, $\varepsilon_{i,1},...,\varepsilon_{i,J}$, comes at a considerable cost because the model is only identified by the maintained assumption that these idiosyncratic preference shocks are drawn from a series of independent, identically distributed type I extreme value distributions. Thus, the vertical/horizontal modeling choice presents a bias/variance tradeoff. For example, suppose that horizontal differentiation is the "true" form of preference heterogeneity. By restricting relative preferences, vertical differentiation would bias the *ES* estimator for preferences, as well as the conclusions that would be drawn about demand curves and welfare measures. Horizontal differentiation eliminates the restriction that causes bias, but the added dimensionality of preferences increases the scope for untested distributional assumptions to drive the estimation results.

5.4.3 *Instruments*

Compared to the traditional hedonic literature on demand estimation, a major advantage of the new empirical sorting models is the way that properties of the equilibrium can be used to develop internally consistent instruments for endogenous amenities. *ES*, *BMR*, and *F* each suggest a different IV strategy. *F* directly models the production process for the endogenous amenity, *ES* develops instruments based on functions of the community income rank, and *BMR* develops instruments based on functions of the exogenous attributes of substitute locations.

The "income rank" and "substitute attribute" instruments are closely related. They both rely on a similar assumption about the role of unobserved amenities in household location choice. Recall that the validity of the income rank instruments rests on the assumption that unobserved amenities do not affect the ranking of communities by income.[31] This assumption is most likely to hold when unobserved amenities play a minor role in determining where people choose to live. As the importance of unobserved amenities increases, the strength of the substitute attribute instruments decreases. This is because the power of the substitute attribute instruments depends on the extent to which observable exogenous amenities drive household location choice. Thus, the relevance of the substitute attribute instruments used by *BMR* is linked to the validity of the income rank instruments used by *ES*.

The advantage of Ferreyra's production function approach to addressing endogenous amenities is that it does not require any assumption about the relative importance of unobserved amenities. She avoids the need to construct an instrumental variable by modeling the mechanisms through which a positive draw on $\overline{g}_j$ or $\varepsilon_{i,j}$ will raise the level of school quality in community *j* or the price of housing. This presents an ideal basis for general equilibrium simulation of policies that affect public school quality. However, her approach requires the analyst to assume a specific form for the amenity's production function. Furthermore, she implicitly assumes that unobserved amenities do not vary systematically across school districts. To the extent that unobserved amenities are spatially correlated across districts, they will be

[31] The "validity" of an instrument is defined by its relationship to the econometric error term. An "invalid" instrument is one that is correlated with the error and, therefore, does not support consistency of the estimator. An "irrelevant" instrument is one that is uncorrelated with the instrumented variable. A weak instrument has low relevance.

absorbed by the district fixed effects during the first-stage estimation of $\bar{g}_j$. This will confound the effects of school quality and unobserved amenities during the "general equilibrium" stage of the estimation.

Finally, it is important to note that instrumental variables need not arise from the structure of the equilibrium. In principle, any source of exogenous variation in an endogenous amenity can serve as an instrument, just as in the hedonic literature. Of course this is easier said than done. It can be quite difficult to find a truly exogenous source of variation in an amenity. Moreover, the quasi-experimental designs that attempt to exploit this variation can have other drawbacks. This is illustrated by *BMR* and Bayer, Ferreira, and McMillan (2007), both of which use Black's (1999) boundary discontinuity design to attempt to isolate exogenous variation in school quality. While the boundary discontinuity approach has intuitive appeal as an identification strategy, it involves discarding the data on all houses located more than a short distance from a boundary. Since this includes the vast majority of houses in the study region, the resulting estimates for preferences apply to a small subset of households. This selected sample limits the opportunities for policy evaluation.

5.5 *Calibration as an Alternative to Estimation*

Tiebout envisioned a world in which households could solve the local public goods problem on their own. That is, heterogeneous households would sort among heterogeneous communities to find the best match between the local public goods offered and their own demands. The process determining the financing and production of these local public goods was not a market. Diversity and costless mobility created his supermarket of alternatives.[32] The early analytical literature on sorting sought to isolate the restrictions required to characterize an equilibrium and then compare it with the observable features of "real world" communities. Informal comparison of model outcomes to actual communities led to revision of the models.

[32] In this context private good choices would take care of themselves. They conditioned what would make each community more or less desirable.

Perfect income stratification, for example, was implied in the earliest models but did not accord with what was observed. That assumption was relaxed in subsequent work.

Often predictions require specific assumptions about preference and production functions and even numerical values for key parameters. If our objective is to characterize distributions of outcomes that depend on features of that structure, are we better imposing them as maintained assumptions, estimating the parameters to be used in counterfactual simulations, or calibrating the model and then using simulation and sensitivity analysis to evaluate both the calibration and the policies of interest?

There are several different approaches to model calibration and studies associated with each. Table 1 provides examples of calibration studies that relate to some of the hedonic and sorting frameworks covered earlier. We will not be comprehensive here. Instead, we review two examples—Fernandez and Rogerson (1998) and Epple and Ferreyra (2008)—as a way to discuss the issues that distinguish calibration.

Epple and Ferreyra's (2008) approach to the calibration process is in the spirit of the early analytical sorting models. A stylized model of the urban landscape is used to prove the existence of sorting equilibria before and after a specific policy reform. Then the predictions of the model are characterized. The policy reforms that are modeled include a change in financing of public education and a change in expenditures, holding per pupil spending constant. Six specific analytical predictions are derived. Predictions for the ordering of prices (and measures for education), stratification of income, and demographic composition are evaluated based on what can be observed when the reform (in a more detailed format) was actually implemented. Thus in their application the calibration process is largely associated with imposing restrictions on properties of preference functions and the distribution of income that are motivated by qualitative comparisons to summary data. Given those restrictions, they are able to make testable predictions about policy outcomes without having to first choose (or estimate) specific parameter values for parametric functions.

Epple and Ferreyra then evaluate the importance of general equilibrium effects for a reform in public school financing and tax policies in Michigan. The reform lowered property tax burdens and reallocated revenue available to school districts. In particular, it increased revenue for low revenue districts and capped revenues of high revenue districts. Their analytical model predicts reforms will lead to limited changes in demographic composition across districts and neighborhoods, capitalization of tax and expenditure changes, and changes in the level and distribution of income that also contribute to changes in housing values. Using data from the Detroit metropolitan area, before and after the reform, they find clear support for their predictions. As a result, it emphasizes the importance of taking account of general equilibrium effects especially when they reflect preference heterogeneity and the diversity in household adjustments that condition outcomes when policy alters a complex mix of tax, public production processes, and other factors that can influence decisions made inside and outside markets.

Relative to Epple and Ferreyra (2008), the approach to calibration taken by Fernandez and Rogerson (1998) is closer to structural estimation. They develop a two-community, two-period, overlapping generations model where educational quality in period one contributes to the second period/generation's income. Households sort among the communities and vote on property taxes that determine educational quality and the next generation's income. There is an idiosyncratic shock so the relationship is not exactly known to agents in the first period. Households share a common form for their utility function. The initial income distribution is heterogeneous and the realized second generation income arises as a draw from a log-normal distribution whose mean depends on school quality in the first period.

The interaction between households is described as a three stage game. In the first stage, first generation agents simultaneously choose one of the communities. In the second stage, given a community decision, property tax rates are selected based on majority rule. Agents know community composition when voting. Aggregate tax receipts determine education quality. In the third stage, individuals make housing and consumption choices. The model can be solved with backward induction focusing on a boundary indifference condition that defines the fraction of the population in each community.[33] A single crossing condition assures stratification and allows them to characterize the equilibrium.

With this background, we turn to the calibration. Power functions were specified for preferences and for the contribution of "children" (the second generation in the model) to a household's well-being, along with a constant elasticity of housing supply. Parameter values

[33] Associated with this fraction is a measure of utility associated with the lowest income person in the high income community and the highest income person in the low income community.

were selected so that the steady state solution to the model reproduced a few features of the economy:

a) Average share of aggregate housing expenditures to aggregate consumption (including housing) over 1960 to 1990;
b) Average share of primary and secondary public education expenditures to aggregate consumption; and
c) Four elasticities: price elasticities of demand and supply for housing; the elasticity of mean earnings to quality of education; and the elasticity of community public education expenditures to *mean* community income.

This last elasticity is important because the model does not predict how mean community income will affect an individual's preferred tax rule. Income distributions for families in 1980 were used in the matching.

The calibration process involves selecting parameter values for the preference, education production, and housing supply functions, and for the distribution for the idiosyncratic effects that are implied by the model's assumptions using the steady state solution values for these parameters (see Table 1 in Fernandez and Rogerson 1998). The fitting criterion is presumably an equally weighted distance function. This is akin to what would enter a minimum distance estimator. In this case, however, there is only one "observed" value for each parameter, taken as a best estimate from the literature.

Two types of assessments are undertaken—sensitivity analyses to alternative parameter values, as the "true" values used in calibration are varied (one at a time) and an assessment of other outcome measures that could have been used as a fitting criteria (such as the implied rate of return to education investments) but were not. Both the elasticity of earnings to education quality and of public education expenditures to income are found to be important to the model's results. A key feature of this calibration strategy is that outcomes that were not using during the calibration process were later used to judge the "fit" of the integrated model.

Together, the Fernandez-Rogerson (1998) and Epple-Ferreyra (2008) examples illustrate how the process of model calibration can advance our understanding of equilibrium sorting in ways that complement structural estimation. In some cases, calibration allows the analyst to make useful predictions about policy outcomes in the absence of complete access to the exhaustive data sets that are required to point identify structural parameters. Epple and Ferreyra illustrate this. If the analyst needs specific values for unidentified model parameters, those values could be chosen from previous empirical studies, as demonstrated by Fernandez and Rogerson. However, this strategy carries a major caveat. One must first be confident that the features of the two studies are sufficiently similar to transfer a parameter between them.

Calibration studies have also demonstrated the benefits of running an "external" validity check on a structural model. If the model can predict observable outcomes that were not used during the calibration, then comparing predicted and observed outcomes can provide a useful way to judge the quality of the model. The ability to do model validation is not unique to calibration studies. Validity checks can, and should, be used to evaluate new structural estimators when the data allow.[34] One possibility would be to compare an out-of-sample prediction for how markets would respond to a new policy with data on the actual responses that occurred.

5.6 *Research Design Tradeoffs and Economics*

When it comes to comparing alternative approaches to estimate and test economic hypotheses much of the literature today would suggest it is fashionable to be extreme and dull to be otherwise. Nonetheless sometimes there are not cut and dry answers in determining which empirical methods are most effective at accomplishing these tasks. There are tradeoffs. Estimation of a spatial sorting model can present the analyst with a tradeoff between the credibility of their identification strategy and the credibility of the assumptions needed to support a policy-relevant interpretation of identified parameters.

To illustrate the issues involved, consider Thomas J. Holmes's (1998) study of the effects of right-to-work laws on the geographic distribution of economic activity. He uses a boundary discontinuity design, similar to Black (1999), to evaluate how right-to-work laws influence firms' decisions. By selecting counties of adjoining states with different pairs of rules he addresses the influence of these rules on their respective levels of manufacturing activity and employment. To identify this effect, several assumptions are made. The economy is a line

[34] Keane (2010) makes this point in the context of structural estimation in general.

segment. Entrepreneurs are uniformly distributed in space. Workers are homogeneous and perfectly mobile. The wage rate is constant. Productivity is independent of location, and so on. The testable implication of this framework (which is spelled out in more detail in his paper) is that differences in right-to-work laws should affect the distribution of economic activity. Holmes is clear about the fact that his model has no direct welfare implications. More manufacturing activity in a county does not imply its residents are necessarily better off. Thus, Holmes has enhanced the credibility of his identification strategy by adding assumptions that limit the opportunities for using his results to evaluate policy outcomes. An analogous tradeoff arises in the context of housing markets.

Equilibrium sorting theory suggests some broad guidelines for research design. For example, maintaining the assumption of free mobility would imply that the geographic extent of the market should be limited. Likewise, the individual outcomes described by models of sorting behavior and hedonic equilibria can be measured more directly with micro data than with aggregate data. With a static definition for the equilibrium that holds individual income and preferences fixed, one would prefer to have a fixed number of cross-sectional observations drawn from a shorter temporal interval. Based on this logic, many hedonic and sorting applications define "the housing market" to be a single metropolitan area observed over a few years where the spatial unit of observation, a housing sale, matches someone's actual location decision. This definition may provide a reasonably good match to the theory but, unfortunately, there may be a problem. The problem arises when the definition of a market is incompatible with the analyst's preferred instrument for an endogenous amenity. Variation in the instrument may only be observable over a larger spatial area, a longer time period, or a lower level of resolution. Thus, to use the instrument, the analyst needs to manipulate, at least implicitly, the definition for "the market" to strengthen the assumptions needed to interpret parameter estimates in terms of the underlying economic model. Unfortunately, the implications of these choices to enhance research design are often not discussed as clearly as in Holmes (1998).

The tradeoff is especially apparent in measuring capitalization effects. Studies that have sought to measure the capitalization of environmental amenities have assumed that the shape of the hedonic price function is unaffected by large changes in the amenity of interest (Davis 2004), extended the free mobility assumption to apply to the contiguous United States (Chay and

Greenstone 2005), and redefined the unit of observation to be the median value of houses located in a Census tract, as stated by the owners of those homes (Greenstone and Gallagher 2008). By adding these economic assumptions, the analysts are able to employ novel identification strategies that provide some of the most convincing estimates for capitalization effects in the literature. It is less clear how to interpret these effects, due to the economic implications of the assumptions maintained to realize these well defined instruments.

Can we assume that equilibrium outcomes are constant over time, over space, and over large exogenous changes to non-market goods? The answer will likely be context specific.[35] This is a somewhat different point than Keane's (2010) argument in his work on related questions. For him and others involved in these discussions of structural versus quasi-experimental research, all methods require assumptions. A choice cannot be made between them merely because one is alleged to make virtually none. Our point is that judgments about research design may not be independent of the interpretation of the results. In some settings they can be made to enhance the ability of instruments to purge estimates of the effects of unobservables and behavioral sorting. However in other settings they are not neutral to the economic interpretation of what is measured.

6. *Using Hedonic and Sorting Models for Policy Evaluation*

Most policy evaluations attempt to translate a proposed change in product quality, product quantity, or market institutions into an equivalent price change for an average producer or consumer, holding fixed all other features of the model. For example, we might characterize Arnold C. Harberger's (1964) approximations in these terms. He translates tax changes into equivalent price changes and then into measures of excess burden. Similarly, the conventional strategies used to estimate the value of new goods (or to incorporate them into price indexes) treat new commodities as if they have always been part of the choice set, but that prior to their "introduction", they were simply above everyone's choke price (see E. Rothbarth (1941) and

[35] Klaiber and Smith (2009) demonstrate how to use a calibrated model of sorting behavior to explore the ability of a quasi-experiment to define treatment and control groups that serve to identify the underlying economic parameters of interest.

Diewert (1993) for historical discussion and Hausman (1997) for an example.) In the context of non-market goods, Smith and Banzhaf (2007) suggest that the concept of weak complementarity can be interpreted as allowing the definition of price equivalents for quality changes.[36] In general, when product attributes (other than price) are exogenous, changes in those attributes can be translated into price equivalents, given assumptions about preferences or technology.

When we consider markets where heterogeneous consumers sort over differentiated goods with endogenous characteristics it becomes more difficult to define price equivalents for policy changes. Moreover, the concept of price equivalents may be insufficient to quantify the outcomes that matter to households and policymakers.

Consider a prospective policy that aims to change one amenity from its current level, g_j, to some new level g_j^*. The size of the change may vary across space. This is certainly true for national policies that define minimum standards for amenities, such as the No Child Left Behind Act, the Clean Air Act, and the "Superfund" program for cleanup of hazardous waste sites. It may also apply to local policies. For example, a local property tax assessment to fund the preservation of open space may affect households differently depending on where they live relative to preserved parcels.

There are several questions one may want to ask about the possible outcomes of such a policy. First, how will it affect market prices in targeted locations and non-targeted locations? Second, will the policy induce some households to move and, if so, how will the resulting migration patterns affect the levels of endogenous amenities at each location? Third, how much would households in each location be willing to pay for these outcomes? Finally, how are the gains and losses from the policy distributed across households? Are they disproportionately borne by specific demographic groups? All of these questions are equally relevant if we instead consider a policy that alters the dimensions of the choice set, or a policy that alters market institutions such as the existing schedule of taxes and subsidies.

This section contrasts how the evaluation task could be undertaken with hedonic and sorting models. Parts 1 and 2 explain in general terms how these models can (and have) been used. Part 3 covers three leading examples from the empirical literature (education, air quality,

[36] They are equivalent (for an individual) in terms of Hicksian welfare measures to changes in the non-market good.

and land use) where moving from hedonic analysis to sorting models has expanded the scope of policy evaluations. Finally, part 4 concludes with our opinion on the types of questions that the current generation of sorting models is capable of addressing.

6.1 *Hedonic Analysis of Public Policy*

Sherwin Rosen's 1974 paper currently has over 1700 citations in the *Social Science Citation Index*. However, empirical studies that use his model to evaluate policy are typically limited to the first stage of his procedure—estimating the hedonic price function. Under the assumptions of Rosen's model, the price function can reveal each household's marginal willingness to pay for an amenity, as defined in (14).

Rather than report the entire distribution of MWTP, the analyst typically emphasizes a single summary statistic, such as the mean or median. Mean or median MWTP is then used to construct a "back of the envelope" approximation to a statistic with some policy relevance. For example, Linden and Rockoff (2008) use hedonic MWTP to estimate the "victimization cost" of a sexual offense, Davis (2004) estimates the statistical value of a case of pediatric leukemia, Chay and Greenstone (2005) estimate the willingness to pay for large reductions in particulate matter, and Greenstone and Gallagher (2008) estimate the willingness to pay for cleanups of federal Superfund sites.

There are at least four important caveats to using hedonic approximations as a basis for evaluating policy. First, the assumptions enabling us to translate the hedonic gradient into MWTP are quite strong. In particular, markets must be perfectly competitive and households must be free to select an amenity bundle from a continuous joint distribution. The direction of the bias from violating these assumptions is indeterminate. Second, measures of MWTP cannot be used to calculate measures of compensating or equivalent surplus unless (a) demand curves are perfectly elastic over the range of the change, or (b) all households are identical. Third, these calculations assume the ways households adjust their behavior in response to the policy are offsetting so the hedonic price function does not change. Finally, hedonic approximations do not provide a consistent basis for predicting how a new policy (that is different from existing policies) will affect market outcomes. For a sufficiently large shock to an amenity, households may respond by changing their location, which will affect equilibrium housing prices, community demographics, other endogenous amenities, and feed back into welfare measures. For example, Banzhaf and Walsh (2008) find that when industrial facilities which emit toxic chemicals move into a neighborhood, some residents move out. When these facilities move out, households with different demographic characteristics move in. The price adjustments needed to clear the housing market following these shocks will affect the welfare of homeowners and renters. These outcomes cannot be predicted from the hedonic price function.[37]

Given these caveats, it is natural to ask whether hedonic MWTP might be used to construct an upper or lower bound on the benefits of a policy. Unfortunately, the answer, in general, is no. Suzanne Scotchmer (1985) demonstrated that the information contained in marginal implicit prices is insufficient to predict how markets will adjust to a future change. Bartik (1988) and Yoshitsugu Kanemoto (1988) investigated the possibility of using the information in marginal implicit prices to calculate ex ante bounds on ex post welfare measures. While they both report positive results, their conclusions are contradictory. The reason is that they differ in how they define the initial equilibrium and the possibilities for adjustment. Under a restrictive set of conditions, Kanemoto proves that ex ante welfare measures will overstate the benefits from an amenity improvement. The requirements for his proof include homogeneous households, only two types of houses, and that the improvement is funded from tax revenue. These restrictions are relaxed in Bartik's model. He argues that ex ante welfare measures will understate the benefits from an improvement when heterogeneous households face a diverse set of housing opportunities. This contrast highlights how the idiosyncratic features of an application may determine the direction of the difference between ex ante and ex post welfare measures.[38] It is also worth noting that both studies assume non-targeted amenities are strictly exogenous. With endogenous amenities it seems likely that "anything goes".

[37] It is tempting to think the hedonic price function could (at the very least) be used to predict the price that would be charged for a new choice alternative. However, the price function simply describes the current market equilibrium. Introducing a new choice alternative may change the entire price schedule. Bajari and Benkard (2005) discuss this same point in the context of markets for differentiated private goods.

[38] Heckman, Matzkin and Nesheim (2010) reinforce this point in distinguishing the restrictions for point identification of parameters from those required to measure welfare effects. They repeat the Kanemoto conclusion for a case of quasilinear preferences that admits a diverse group of alternative specifications (see their eq 4.1). They consider how a change in the hedonic price function for specific households who experience the change in the amenity directly would influence conclusions as compared to using the price function to evaluate welfare for those who might sort in response to a change in the amenity. The later are included in Bartik's assessment.

As noted at the outset, very few empirical studies have implemented Rosen's second stage to estimate the demand for an amenity. The barrier to reduced-form estimation has been the need for data from multiple markets on housing transactions, household demographics, and instruments. The traditional concern with structural estimation is that restrictions on preferences are arbitrary. It is encouraging that the recent microeconometric innovations by Ekeland, Heckman, and Nesheim (2004), Heckman, Matzkin, and Nesheim (2010), Bajari and Kahn (2005), and Bajari and Benkard (2005) have reduced the barriers to second-stage estimation. The first two papers provide preliminary evidence that data from multiple markets may be unnecessary to implement the reduced-form approach. Bajari and Kahn relax some of the rigidity of the structural approach by allowing individual households to differ in their tastes for each characteristic, and Bajari and Benkard (2005) relax the need for continuous choice sets and perfect competition. These methodological advances offer the potential to develop estimates of supply and demand curves for amenities in future research.

6.2 *Using Equilibrium Sorting Models for Policy Evaluation*

A structural model that provides estimates of household preferences allows the Hicksian compensating surplus to be measured for a specified change in amenities. Equation (40) defines this partial equilibrium measure of willingness to pay (WTP_{PE}) for a prospective change in the targeted amenity.

$$V\left(g_{1j}, g_{1,j}, p_j; \alpha_i, y_i\right) = V\left(g^*_{1j}, g_{-1,j}, p_j; \alpha_i, y_i - WTP_{PE}\right). \tag{40}$$

Baseline and new levels of the targeted amenity in location *j* are denoted by g_{1j} and g^*_{1j}. This is a partial equilibrium measure of the change in welfare in the sense that prices, income, and the levels of other amenities (g_{-1j}) are held fixed. The distribution of WTP_{PE} could be calculated from estimates for the joint distribution of income and preferences derived from a structural hedonic model or an equilibrium sorting model.

Sorting models extend the scope for evaluation by allowing the analyst to predict how markets might adjust to the proposed change. In the context of local public goods, the models allow relocation and price adjustment as housing demand and supply are equalized in each location. Moreover, depending on how the model represents the production of amenities, their supply may also be derived as part of the new equilibrium. In the simplest case this connection can be "technical". For example, the amenities associated with open space may be expressed as a function of the share of protected and vacant land in the neighborhood (Walsh 2007) or the production of school quality may be related to student teacher ratios that adjust as households with different numbers of school-age children move into the neighborhood (Klaiber and Smith 2010). Alternatively, expenditures of local public goods may be determined by voting (Ferreyra 2007). In this case migration may alter the median voters in some communities, changing voting outcomes, and so forth.

A "general equilibrium" measure of individual willingness to pay consistent with the preceding examples can be defined as

(41) $$V\left(g_{1j}, g_{1,j}, p_j; \alpha_i, y_i\right) = V\left(g^*_{1k}, g^*_{-1k}, p^*_k; \alpha_i, y_i - WTP_{GE}\right).$$

A key distinction from (40) is that willingness to pay is now evaluated using the household's new location, *k*, which may or may not be the same as its initial location. Following convention we label this welfare measure as "general equilibrium" in recognition of its allowance for market adjustment.[39] In fact, "multiple market" might be a more apt label. The measure of WTP_{GE} in (41) recognizes the interconnectedness between the supply and demand for public and private goods in each of the *J* communities. However unlike some computable general equilibrium models we are holding incomes fixed, as well as the price of the composite numeraire.

As noted earlier, policy evaluations need not focus on direct measures of welfare. Given a description for the ex post equilibrium, one has the ability to evaluate changes in any policy relevant features of the market (e.g. housing prices, demographics, provision of public goods, enrollment in public versus private schools).

There are many interesting aspects of how sorting models are actually used to define the new equilibrium that might follow a policy change. We have selected three classes of issues to

[39] Equation (41) is most directly related to the pure characteristics sorting models. The formulation would be a bit different for the random utility models. The reason is that its equilibrium condition does not predict the selection of a specific location. Instead it provides probabilities that each choice location will be selected. Equilibrium is defined by a vector of prices at which the expected housing demand in each location equals supply. This logic translates into measures of expected willingness to pay. In our view this is not an important variation on the central logic of the sorting model. The use of expectations smoothes the computation of equilibria and welfare measures, similarly to Berry, Levinsohn and Pakes (1995).

discuss: (a) the information used to close the model; (b) the assumptions about sources of "friction" in the market; and (c) the potential for multiple equilibria.

6.2.1 *Closing the Model*

Solving for a new equilibrium typically requires more information than was used during the estimation. Recall that empirical models are estimated using data for intervals over which the quantities of households, houses, and each amenity are assumed to be fixed. To solve for a new equilibrium, the analyst must consider how the supply of each might change.

Given some additional supply side information, numerical methods are used to solve for equilibrium housing prices, amenities, and location choices that simultaneously satisfy the following three conditions.[40]

(42.a) $$V\left(g_k^* p_k^*, y_i, \alpha_i\right) > V\left(g_m^*, p_m^*, y_i, \alpha_i\right) \quad \forall \quad i, m.$$

(42.b) $$H_k^S\left(p_k^*\right) = H_k^D\left(p_k^*\right) \quad \forall \quad k.$$

(42.c) $$g_k^* = f\left[g_k, R_k(\alpha, y, d)\right] \quad \forall \quad k.$$

The first condition simply states that each household must select the location that maximizes its utility. Condition b requires the supply of housing to equal demand in each location. In the pure characteristics model, for example, housing demand is calculated by aggregating over the individual demand curves in (34). Supply is treated differently depending on the application. Smith et al. (2004) restrict supply to be perfectly inelastic, Sieg et al. (2004) calibrate the supply curve using a range of elasticities, and Walsh (2007) estimates it from the data. Finally, condition c expresses the amenities in community k as a function of their baseline levels and $R_k(\alpha, y, d)$, the joint distribution of income, preferences, and demographic characteristics describing the population of households relevant to the supply of g_k^*. For example, the production of school quality in Ferreyra's (2007) general equilibrium model depends on social interactions and voting outcomes, as defined in (37) and (38).

[40] The details of numerical solution procedures vary with the model and application. For the pure characteristics model see Sieg et al. (2004), Walsh (2007), and Kuminoff and Abdul S. Jarrah (2010). For random utility models see Bayer and Timmins (2005), Timmins (2007), and Klaiber and Phaneuf (2010b). See Ferreyra (2007) for details on her general equilibrium model.

The population of households in the model is usually treated as fixed. In overlapping generation models such as Fernandez and Rogerson (1998) the distribution of income may differ from generation to generation, but the size of the population itself does not change. In other words, there is assumed to be no immigration or emigration from the study region. This assumption influences predictions for capitalization. Several studies have reported situations where housing prices decrease in communities that experience improvements (e.g. Sieg et al. 2004). Intuitively, because the model is closed, a policy that improves amenities in every location can still make some locations *relatively* less attractive.

Finally, an assumption is required about who collects the capital gains (or losses) from housing transactions. To date most applications have treated households as renters, assuming that changes in property values accrue to absentee landlords.[41] The distinction between owners and renters is especially important for welfare calculations. All else equal, homeowners cannot be made worse off from quality improvements. However, a renter with the same preferences and income may be worse off if their rent increases by more than their willingness-to-pay for the improvement. Distinguishing between owners and renters is an important consideration for future research.

6.2.2 *Potential Sources of Friction*

The first generation of "general equilibrium" applications has continued to maintain the free mobility assumption that is embedded throughout the empirical hedonic and sorting literatures. There are no explicit physical or wage-related costs of moving to a new location. Furthermore, the applications to date have abstracted from transitional dynamics. Numerical simulations are implemented as if markets adjust instantaneously. One could argue this approach is consistent with interpreting predictions for market outcomes as features of long run equilibria. On the other hand, the models do not suggest a procedure for discounting. It is certainly clear that the lack of friction in the current models has the potential to influence their findings. Several recent studies have underscored the need to address this issue.

[41] An exception is Daniel G. Hallstrom and Smith (2003) who develop a pure characteristics model where all gains or losses accrue to the occupants of those houses in the baseline equilibrium. The prospective gains or losses influence their decision for whether to relocate.

Kuminoff (2009) used an analytical version of the pure characteristics model to illustrate how the free mobility assumption creates a false sense of precision in estimates for preference parameters and welfare measures. The direction and magnitude of biases will depend on baseline equilibrium conditions as well as latent distributions of preferences and moving costs. Bayer, Keohane, and Timmins (2009) were the first to propose a solution. They developed a discrete-choice analog to the first-stage estimation of a hedonic price function that controls for the average cost of moving between metropolitan areas. Controlling for moving costs had a dramatic impact on their estimates of MWTP for reductions in air pollution (as measured by particulate matter). Ferreira (2010) reports similar findings in an investigation of the implicit moving costs conveyed by property tax regulations in California. Finally, a recent study by Epple, Romano, and Sieg (2010) includes moving costs in a new overlapping generation model with voting in multiple jurisdictions. After calibrating the model to Boston, they demonstrate that moving costs play an important role in determining the evolution of community demographics and tax revenue.

Extending the current generation of models to include moving costs and transitional dynamics raises new issues that are just beginning to be explored. We discuss these research frontiers in section 7.1.

6.2.3 *Multiple Equilibria*

Our earlier summary of theoretical properties of sorting equilibria focused on the types of conditions used to guarantee existence and uniqueness. Uniqueness proofs in particular have relied on strong restrictions on the dimensionality of preference heterogeneity and the vector of endogenous outcomes. Frankly, the situations where equilibria are known to be unique are the least interesting for policy evaluation. Analysts have addressed this reality by conducting sensitivity analysis and proposing informal decision rules for choosing among equilibria.

At present, there are no guarantees that equilibria are unique in *any* of the empirical sorting models covered in section 5. Of the three frameworks in that section, the pure characteristics model imposes the strongest restrictions on preference heterogeneity. It has been used in several applications to solve for new equilibria following simulated policy changes (e.g.

Smith et al. 2004; Walsh 2007; Klaiber and Smith 2010). None of these studies have reported multiple equilibria. It may be the case that the model's "vertical" ordering of households, combined with the market clearing condition for housing, is sufficient to ensure uniqueness. However this conjecture has not been proven. Nor has it been tested in a setting with multiple endogenous amenities.

Multiple equilibria have been found to arise when vertical differentiation is relaxed. Kuminoff and Jarrah (2010) recover several different equilibria from a relatively simple pure characteristics model where households have horizontally differentiated preferences over a vector of exogenous characteristics describing individual houses. Timmins (2007) notes the possibility of multiple equilibria for his random utility model of location choice in Brazil, which includes an agglomeration (or congestion) effect. Likewise Ferreyra (2007) confirms the possibility of multiple equilibria in her general equilibrium model.[42]

The possibility of multiple equilibria is relevant for policy evaluation, regardless of whether the analyst considers a "small" change or a "large" change. If one parameter vector can support multiple equilibria, the reverse may also be true. Multiple parameter vectors may each be equally capable of explaining the data that are used to estimate or calibrate a model. Different parameter vectors may generate different predictions for benefit-cost ratios and other metrics used to quantify policy outcomes.

How might one choose among a set of candidate equilibria for the purposes of policy evaluation? One approach is to define decision rules that eliminate equilibria that are judged to be less plausible. For example, Timmins (2007) solves for a new equilibrium using the iterative procedure developed by Bayer and Timmins (2005). Their algorithm has a unique solution, conditional on its starting value. Different starting values can lead to different equilibria. Timmins addresses this issue by defining the starting value as the baseline equilibrium. The resulting differences between the baseline and new equilibria are modest. His approach to the problem eliminates consideration of other potential equilibria that would be less plausible in the sense that they differ dramatically from the baseline. Kuminoff and Jarrah (2010) take a similar approach. They discard equilibria where housing prices decline in communities that experience

[42] In particular, see footnote 10 in Timmins (2007) and footnote 13 in Ferreyra (2007).

unambiguous improvements in amenities. Such an outcome would run counter to findings in empirical capitalization studies.

Another approach would be to attempt to characterize all of the possible equilibria. If key policy outcomes are invariant to the choice among these equilibria, there is no need to develop decision rules. This approach would be most applicable to situations where a strong prediction for the *direction* of change in a market outcome would be informative. It would also be consistent with developing a partial identification approach to using equilibrium sorting models for policy evaluation, following the logic of Charles F. Manski (2007).[43] Finding ways to address the implications of multiple equilibria will become increasingly important as the literature continues to evolve toward more flexible modeling frameworks, where multiple equilibria appear to be the reality. Recently, Bajari, Han Hong, John Krainer, and Denis Nekipelov (2006) developed an algorithm for recovering all of the equilibria in certain types of games considered in industrial organization. Their work could provide a starting point for the equilibrium sorting literature.

6.3. *Examples from the Empirical Literature*

Sorting models alter what might be termed the "landscape for policy evaluation" so that it more closely matches the ways decisions are made. To illustrate our point consider the typical way a hedonic model allows evaluation of a non-market service such as education quality or air quality. Some measure of the service that varies spatially and thus can be associated with housing consumption is included in the hedonic specification as a determinant of housing prices. In the case of education it is a test score. In the case of air quality it is the measure of ozone or particulate matter that is matched to the house.

Quasi experimental analyses focus on dealing with omitted variables that might interfere with the ability to identify the estimated contribution of an amenity to housing values. Bayer, McMillan, and Reuben (2007) find that the estimated effect of a one standard deviation increase of test scores (about a fourteen percent increase) declines by 75 percent when boundary fixed effects for attendance zones are included in their hedonic regression. When neighborhood

[43] We return to a broader discussion of the prospects for using partially identified sorting models for policy evaluation in our coverage of research frontiers in section 7.5.

demographics are included as well the estimates drop by another fifty percent. The overall decline is from $124 to $17 per month in housing costs for the improved scores.[44]

Chay and Greenstone (2005) make a similar comparison, noting that their quasi-experimental estimates for the elasticities of housing values with respect to changes in particulate concentrations are 4 to 5 times larger (in absolute magnitude) than measures implied by the Smith and Ju-Chin Huang's (1995) meta summary of past hedonic studies of the same parameter.

In reality, a policy intervention does not alter exogenously school quality, environmental quality, or any other spatially delineated service. It might change the allocation of resources to schools (Epple and Ferreyra 2008) or it might change families' ability to access private schools with different types of vouchers (Ferreyra 2009). A land use policy might create a park or purchase private land and put it in protected status. In the case of air quality pollution may exogenously change, but an exogenous change is not what people experience. <u>They move</u>. In the case of sorting models of education their moves induce social interactions that <u>change</u> the resulting school quality. In both examples, changes in housing values may affect non-movers.

For example, the policy evaluated by Fernandez and Rogerson (1998) involved switching to a system with no local financing of education. Their measure of the steady state welfare went down! Tax rates were between the levels realized in the decentralized two community situation, a point the authors note. The produced level of education was below the average of what is realized in the two community case. In the case of air pollution considered by Sieg et al. (2004), the improvement experienced by the households was different than the improvement to their original community.

Perhaps the most direct example of the effects can be seen by comparing hedonic and sorting models for evaluating land use policy. Virginia McConnell and Margaret Wells (2005) report a detailed review of hedonic estimates for the price effects associated with proximity to "open space". For models measuring the effects of open space using fixed effects (i.e. defining the impact based on whether a house is within a distance zone) the effects vary with type of open

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[44] The estimates for marginal willingness to pay implied by their sorting model for the same study area were approximately $20 per month for the same one standard deviation change. A key advantage they emphasize for this model concerns their ability to account for heterogeneity in the MWTP estimates. As a result, they estimate that MWTP would be $13 larger for households with a college degree compared to some college or less and $14 lower for black households compared to white.

space from two percent of the house price (for urban parks) to sixteen percent (for natural areas). The same phenomenon could be represented with a continuous measure of distance but the results are generally smaller (when evaluated at mean distance). This comparison does not hold the study area fixed, but that is not our point. Consider Klaiber and Phaneuf's (2010b) sorting model of the Twin Cities. The land use policies they consider increase non-park open space in different locations. There is no simple distance measure that is the focus of policy. Rather it is an outcome of policy and the ways in which households adjust to it. Hedonic models and capitalization models necessarily embed the policy and its effects in a reduced form description of outcomes. Each has a measure for the spatially delineated attribute that is realized as an outcome of market responses to a policy.

Klaiber and Phaneuf demonstrate that a 2.5 percent increase in protected open space (with specific locations designated) in three different areas (inner city, urban fringe and outside the city), yields different measures for willingness to pay depending on whether we consider the effects of movement (and changes in property values) and who is impacted (all households versus those in the areas where land is designated as open space). Table 2, drawn from their results, illustrates the differences. Their counterfactual policy generates the increase in open space by converting privately owned agricultural and undeveloped parcels to publicly owned (non-park) open space. The stock of housing is held fixed. Of course new houses may be built in the longer run.

Walsh (2007) demonstrates that policies designed to preserve open space may have unintended consequences that mitigate their short-term accomplishments. An increase in publicly owned open space may stimulate future urban development if an increase in the demand for housing in the improved areas raises the price of land by enough to induce the owners of vacant parcels to sell their land to developers.

Finally, if we consider an educational policy, such as the decision to cut teachers in response to budget deficits, there can be an adjustment in educational quality that arises from sorting. If school quality is a function of teachers and class sizes then the realized quality will depend on how households react to the policy. Klaiber and Smith (2010) answer this question, embedding an education production function in a simple model of sorting across school districts in Maricopa County, Arizona. They find that differential retention polices across districts results in declines in educational quality for all but one of the districts. Parents move, class sizes increase at the schools without teacher cuts, and all school districts "share" in the losses that arise from teacher cuts at a few of them.

6.4. *Are Sorting Models Ready for "Prime Time" Policy Evaluation?*

Sorting models have many potential uses in policy analysis. For example, Executive Order 12866 mandates the development of benefit-cost analyses for all major federal policies. Economic analysis can also help policymakers to understand the consequence of local governmental decisions, land use changes, and exogenous shocks to a region such as a plant closing, a hurricane, or a forest fire. In what follows we discuss three potential uses of sorting models: regulatory analysis, local government policy, and assessment of the economic effects of shocks to a region.

6.4.1 *Regulatory Analysis*

While there have been only a few sorting models that have considered specific changes in federal regulations, the consistency across models has been quite good and where there are differences they seem to be readily explained. Consider the case of air pollution. Sieg et al. (2004) (SSBW) and Tra (2010) both consider the benefits from the 1990 Clean Air Act Amendments for the Los Angeles area. Table 3 compares their results for MWTP as well as the partial and general equilibrium willingness to pay for the air quality improvements. Despite some differences in their regional scope, structure of preferences, and approach to measuring consumer surplus, their findings are relatively consistent.[45]

We also compute an approximate version of the Chay-Greenstone (2005) (CG) elasticity of housing expenditures to air pollution (designated as *e* here) by treating the general equilibrium WTP as the largest increase in annual housing expenditures a household would make for the improvement in air quality (particulate matter in the case of Chay and Greenstone and ozone for the other two studies). The CG estimates range from .20 to .35 in absolute magnitude

[45] Sieg et al (2004) (SSBW) include Los Angeles, Orange, Riverside, San Bernardino and Ventura counties. Tra (2010) omitted Ventura. SSBW also considered different community definitions taking advantage of the breakup of the LA school district. Tra's model also includes more controls for other local attributes than SSBW.

(depending on model specification). Most of the results in SSBW match this range (with only the results for Ventura County larger) and Tra's estimates are at the high end of the scale. This comparison suggests consistency in structural and quasi-experimental measures of the average of the estimates for the marginal response to the policy. However, their implications for the benefits of the policy are quite different. The capitalization statistics measured by CG do not make it possible to evaluate how benefits differ across the metropolitan landscape as a result of changes in housing prices, changes in the hedonic price function, and changes in location choices. SSBW and Tra find that these effects matter for evaluating the distribution of benefits.

Much of the analysis used for evaluation of environmental rules is based on a single consensus estimate from the literature. This estimate is then adjusted with simple functions for cost of living or income differences (over time or regions). The consistency in the estimates across sorting studies should not be taken to imply that such transfers would be the logical next step in using the models. In fact that strategy misses the rationale and potential policy strength of a sorting model. Ideally, specific regional sorting models would be used for areas with significant pollution problems or large population areas, similar to the way that EPA currently develops specialized, high resolution, air diffusion models for such areas. The sorting models would then supplement a national benefit assessment. In addition, the analyst could consider distributional consequences, the effects of policy on community demographics, and interaction effects with other spatially delineated policies. In fact, there is a pilot example of this logic in the literature. Smith et al. (2004) illustrated how the SSBW model could be used with the special projections developed as part of the EPA's Prospective Study (1999) for the South Coast Air Basin. Sorting models, developed for different regions are surely ready for this type of supplementary analysis to the larger national assessments.

6.4.2 *Local Government Policy*

The ability to account consistently for how heterogeneous households choose among a diverse choice set and how their subsequent actions contribute to equilibria is one of the strengths of the sorting framework. Indeed, the diverse features of households and how they factor into the differences in outcomes across communities are what many policy makers are interested in learning about. Economists, especially those working on benefit cost analysis, often overlook the importance of these "details". It is not simply who will gain and lose, but will integration of neighborhoods increase or decrease? Will the efforts to improve air quality complement or undermine efforts to improve local public education? The former effect could be due to higher income households' willingness to support public education and the later due to housing price increases pushing out the lower income households where policy seeks to enhance educational outcomes.

Outside of the environmental domain direct benefit measures are often not of central importance. Ferreyra's summary of the outcomes of two voucher policies identifies over forty outcome measures—approximately one-fifth relate to the distribution of economic measures of the gains. The rest describe effects such as the changes in educational quality, which households go to particular schools, expenditures for education, and tax rates. In contrast, hedonic estimates of the effects of school quality or neighborhood demographics on housing prices cannot hope to offer these details. These details might be incidental to a benefit cost analysis but they are not incidental to a policy maker's assessment.

6.4.3 *Shocks to the System*

Shortly before we submitted this draft, the social science directorate of the National Science Foundation invited proposals to design research that would investigate the economic effects of disasters. One need only perform a Google search on "Hurricane Katrina economics" to conclude that economists are good after evaluating shocks after the fact (for example see Vigdor 2008). How about an ex ante assessment? Sorting models may help to bound the equilibrium outcomes for some shocks. Suppose a sorting model includes a spatial amenity such as a forest. How can one measure the effects of proximity to the forest and its size on land values in the surrounding neighborhoods? Burn it down! That is, simulate the effect of removing it. This does not capture the disamenity created by the scorched earth—but it does offer a gauge of the new equilibrium and with calibration of the model and ways to address the possibility of multiple equilibria we might be able to do more.

7. *Research Frontiers*

Recent research has begun to extend sorting models in several ways. We conclude our review by considering five that are especially promising.[46] (1) *Dynamics and Forward Looking Agents*. For most households the purchase of a home is the largest purchase that they will ever make. Until the recent collapse of the housing market it was a decision that often reflected an expectation that the home, as an asset, would appreciate in value. Expectations about the future would be important in these judgments. Households are likely to be forward looking with respect to trends in neighborhood attributes, the appreciation of housing, and even their own evolving preferences. (2) *Agent Based Models*. While this class of models has evolved largely outside of the economics literature, there may be scope for its "bottom up" approach to describing interactions between agents to influence the way sorting models treat heterogeneity and the transitional dynamics between equilibria. (3) *Housing Supply*. With an eye toward "closing the loop" (i.e., developing a full equilibrium model of housing supply and demand) analysts have begun to focus attention on the behavior of builders. Such models must inevitably consider the production of housing and the formation of choice alternatives. This extension also provides a more direct route to understanding restrictions on land use and the ways policy influences community development. (4) *Labor Supply*. Working households make interrelated location choices in the housing and labor markets. Recent work has suggested that modeling both choices simultaneously may improve our understanding of preferences for amenities and the evolution of neighborhoods. (5) *Model Validation*. Several empirical sorting models are equally capable of explaining observed behavior. Differences in their maintained assumptions lead to differences in their predictions for the effects of prospective policies. The time is right to adapt strategies for evaluating model validity from the broader literature on structural econometrics.

7.1. *Dynamics and Forward Looking Agents*

Epple, Romano and Sieg (2010) extend the vertical class of sorting models to include overlapping generations in a multi-jurisdictional model of equilibrium voting. In particular, they

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[46] Consistent with a discussion of research frontiers, most of the papers we consider in this section are, at the time of writing, unpublished working papers.

introduce two important forms of heterogeneity that inject dynamics into the household's residential location decision—moving costs and age/family structure. Their model assumes that over the household's life-cycle, its preferences for education and housing services will evolve. With positive moving costs, forward looking agents will recognize these costs when making location choices early in life (i.e. child rearing years) as well as in their retirement years. Families with low moving costs will choose to move to areas with low education expenditures once their children depart, while those with strong preferences for public goods may choose to stay. These dynamics also affect the voting process, and with it the equilibrium levels of public goods. In other words, their model makes education expenditures and tax rates endogenous while accounting for evolving preferences over time.

Focusing on the class of stationary equilibria, Epple, Romano, and Sieg calibrate their model to the Boston metropolitan area. They find parameters that make the model predict expenditure shares from the Consumer Expenditure Survey (e.g. housing) and the fraction of people who move to a new location when they enter the "old" life stage. Mobility costs are found to play an important role in the evolution of communities. In a world without moving costs, more of the old would move to low education communities. Interestingly, by becoming older those communities would also become wealthier, raising their tax base and reducing the disparity between low and high education communities.

As we noted earlier, with the sole exception of Hallstrom and Smith (2003), the sorting literature has treated all households as renters.[47] This strategy ignores an important dimension of the home-buying decision. Households will purchase a house in a neighborhood where they expect to receive a return on their investment that is consistent with comparable alternatives. Until recently, these returns were regarded as relatively secure. Increasing property values provided capital gains for homeowners. By contrast they increased the costs for renters. While the literature has yet to model the choice between owning and renting, Bayer et al. (2010) allow households to be forward looking with respect to the appreciation of their house values. Moreover, their model accounts for moving costs. Because moving costs prevent repeated re-optimization, they cause homebuyers to be forward looking with respect to the evolution of

[47] The Hallstrom-Smith analysis considers the effects of housing price changes due to exogenous policies and GE welfare measures.

neighborhood characteristics (e.g., crime, pollution, and race). This has important consequences for recovering household preferences.

Consider a simple example with two houses—the first is located in a high pollution, high crime, but improving neighborhood. The second is located in a low pollution, low crime, but deteriorating neighborhood. The forward looking household may be willing to pay a premium to live in the first neighborhood. In a static model where its decision is defined in terms of current neighborhood attributes, an estimate for this household's marginal value of reducing crime or pollution will be biased downward. The opposite logic applies to the household choosing to live in the declining neighborhood. Likewise, the static model overstates the value of persistent amenities.

The challenge in modeling dynamics arises from the size of the state space. Even in the conservatively parameterized model used in Bayer et al. (2010), individuals choose between houses in 225 neighborhoods described by five attributes (housing price, violent crime, ground-level ozone, and percentage white). Suppose each of these attributes is allowed to take on one of ten values. The result is 10^{1125} potential points in the state space that might be visited in the future. This is impractical using traditional computational methods for estimating dynamic decision processes (John Rust 1987). Models of dynamic demand for consumer durables in industrial organization have dealt with a similar state-space problem by assuming that the logit inclusive value of the choice set is sufficient to describe its value, and that this inclusive value evolves according to some statistical process, such as first-order autoregressive (Oleg Melnikov 2001; Juan Esteban Carranza 2007; Gautam Gowrisankaran and Marc Rysman 2007; Pasquale Schiraldi 2007). This reduces the size of the state space to one.

Bayer et al. (2010) avoid this assumption by instead employing a variation of V. Joseph Hotz and Robert A. Miller's (1993) two-step approach to dynamic optimization. Estimates of choice-specific value functions (net of moving costs) are first recovered directly from the observed decisions of movers. These value function estimates are then used in a second-stage to recover estimates of moving costs from the move-stay decision. By measuring the financial component of moving costs, with 6% of the sale price paid to a realtor, this stage of the estimation procedure also yields an estimate of the marginal utility of wealth. Combining the estimates of choice-specific value functions and moving costs within a Bellman representation, estimates of flow utilities associated with each neighborhood and wealth group are recovered in a third stage. These are decomposed to recover utility parameters in a fourth and final stage. Comparing the results with a static sorting model suggests that failure to recognize forward-looking behavior biases estimates of preferences for reducing violent crime and ground-level ozone by 34% and 20% respectively. Conversely, the static estimate of whites' willingness-to-pay for same racial composition (which exhibits tremendous persistence over time) is biased upward by 71%.

Whereas calibration in Epple, Romano, and Sieg (2010) is predicated on the calculation of a dynamic sorting equilibrium, Bayer et al. (2010) does not yield equilibrium predictions. Their strategy avoids the challenges posed by a large choice set and technical feasibility of solving the dynamic programming problem. However, it cannot solve for a new price vector when there are exogenous changes and thus it is unable to replicate the type of policy analysis performed by Epple, Romano, and Sieg.

An important task for future research is to develop computationally feasible methods to calculate dynamic sorting equilibria while simultaneously preserving the richness of dynamic decision making. One possibility, described in Peter Arcidiacono et al. (2010) and related work by Ulrich Dorazelski and Kenneth Judd (2010), is a continuous time model. Viewing the sorting equilibrium as a high-dimensional multi-agent game, that game becomes hard to solve due to so many players making simultaneous moves. As the number of players grows the size of the state space quickly becomes prohibitive.[48] Modeling the problem in continuous time lets the researcher treat decisions as if they occur sequentially. In other words, a random process determines if an agent is eligible to make a decision at each point in time, but no two players are allowed to move at the same time. The number of alternatives underlying the individual's expectation then grows multiplicatively. Continued progress along these lines may make it feasible to calculate counterfactuals for dynamic sorting equilibria.

[48] In particular, in order to determine their optimal policy, players have to form an expectation over all the potential actions that every other agent might take. The number of alternatives underlying this expectation grows exponentially in the number of agents and choices.

7.2. *Agent Based Models*

Agent based models (ABM's) have largely been developed outside of the economics discipline, but are often used in the geographic sciences to answer questions similar to those posed in the sorting literature. Irwin (2010) summarizes key features of this literature, highlighting the relationship to structural economic models. ABM's take a "bottom-up" approach, modeling interactions between heterogeneous agents by specifying a set of behavioral rules and protocols—in other words, an entire institutional infrastructure in which agents interact. Simulated interactions between agents yield aggregate outcomes that can be matched to observed macro-level data for purposes of model calibration. These models generally allow for detailed spatial heterogeneity and dynamics, where an outcome at one location can influence the evolution of state variables at another.

Whereas sorting models make predictions about equilibrium outcomes, the features of agent-based models make them better suited to investigating the transitional dynamics *between* equilibria. This may prove to be useful if re-equilibration is a slow process relative to the time-horizon of the policy-maker. A lack of equilibrium restrictions allows ABM's to more easily incorporate rich spatial and agent heterogeneity along with multiple feedback effects. These can be particularly important when studying the impacts of non-marginal policy changes. For example, an ABM's could be used to characterize the "basin of attraction" for combinations of parameter values and policy variables that lead to desirable or undesirable tipping effects.

The current generation of agent based models is limited in the sense that they rarely invoke the assumptions of optimizing behavior that underlie the equilibrium sorting literature. Agents need not be rational from a static or dynamic perspective. Moreover, ABM's do not typically require the collective actions of individuals to satisfy basic market clearing conditions. Thus, an ABM simulation may produce a much wider set of potential outcomes than a structural model that imposes the discipline required by economic constraints. *Some* flexibility seems desirable, but assuming all behavior is random with simple survival rules determining overall outcomes and a limited role for prices will produce more noise in the transitional dynamics than a model that reflects basic economic principles about the functioning of markets. A blend of systematic rational responses at the agent level and less discipline imposed by static (or dynamic) market equilibrium conditions would seem a promising path for future research.

Irwin (2010) describes a number of recent efforts in the ABM literature to begin to incorporate conditions imposed by market equilibrium. In the future, ABM's and structural sorting models may move closer together. In particular, there is scope for the "bottom-up" modeling strategies from ABM's to influence the way in which structural sorting models treat interactions between agents. This could lead, in particular, to more realistic descriptions of heterogeneity and richer predictions about transitional dynamics, both of which are highly stylized or completely absent from the current equilibrium sorting literature.

7.3. *Housing Supply*

Recent work has sought to develop a more realistic portrayal of supply-side behavior in housing markets. Accurately depicting the decisions of builders is crucial, particularly in applications where results are driven by the timing and location of new construction. Consider, for example, models of suburban sprawl and open space preservation. Epple, Gordon, and Sieg (2010), citing Edward L. Glaeser, Joseph Gyorko, and Raven Saks (2005), note that the elasticity of housing supply is of crucial importance in determining how cities respond to macroeconomic shocks. Are these shocks transmitted to housing prices and wages, or do they simply result in more sprawl?

It is challenging to estimate a housing supply function. We observe transactions that reflect newly constructed houses and sales of pre-existing structures. The former reflect past decisions of builders, due to a relatively long production cycle, and the later reflect more recent decisions of current owners. In this case there is a selection effect. We observe what current owners choose to offer given their expectations for prices, not what would be offered under different expectations. Builders' decisions are analogous, but the timing of the expectations relates to an earlier period when construction must have begun in order to have the home ready for sale in the current period.[49] Given a constant returns to scale production function for housing, and variation in land prices, Epple, Gordon and Sieg (2010) demonstrate that the

[49] Of course, houses are also built for buyers. We are ignoring this issue here and considering houses built without specific buyers identified at the time of the construction decision.

production function for housing and the supply of housing services can be recovered while treating both housing prices and quantities as latent variables. Nonparametric estimates are derived by specifying supply per unit of land.[50]

While an important first step, there are several reasons to consider extensions. First Epple at al. (1984,1993) noted that the conditions for an equilibrium in multi-community models require ruling out arbitrarily small communities. If the choice alternatives in a sorting model were subdivisions (with homeowner associations voting on some services) then we would want to reconsider the constant return to scale assumption. Equally important, Klaiber and Phaneuf (2010a) suggest that builder size, as measured by the number of houses built, does matter. Their analysis is the first static, random utility version of supply designed for a sorting model. They highlight three challenges: (i) identifying builders and their attributes using housing transactions alone; (ii) understanding and modeling differences in the choice set for builders versus those demanding home/community combinations and; (iii) recovering estimates of the alternative specific constants with limited sample coverage of elements in the choice set. This can cause the estimates to be sensitive to outliers. Several strategies are considered for addressing this issue including quantile regression.

Murphy (2010) extends the static approach to treat builders as forward-looking agents, with a focus on their decisions about *when* to build. He notes that, while housing prices and quantities exhibit a high degree of volatility, they tend to move in cycles that exhibit strong serial correlation. In Murphy's model, a developer who owns a single plot of land decides each period whether to develop or wait, given his expectations about the evolution of future prices. If he chooses to develop, he decides on a level of quality. Using data from the California Bay Area in the 1990's, Murphy finds that many builders chose to build as prices begin a long (and predictable) run-up. There are two explanations for this seemingly odd behavior: (1) builders are static optimizers who react to the first sign of a price increase (an assumption that would be hard to justify on institutional grounds) or (2) there are pro-cyclical components of costs (e.g. fixed costs of permitting) and other building costs (e.g. difficulties in securing contractors) to which forward looking builders respond in their timing decisions. Murphy constructs a model

[50] This strategy is reminiscent of an early argument in the hedonic literature by Parsons (1990) to normalize housing prices per unit of land.

consistent with the second explanation and uses it to recover estimates of these costs. He finds that pro-cyclical building costs can explain why it is in builders' interests to smooth out construction patterns over time. Moreover, these costs are correlated with high rates of home ownership in the cross-section, suggesting a political economy story in which existing residents play a role in limiting housing supply (John M. Quigley 2006; Kahn 2007; François Ortalo-Magne and Andrea Prat 2005).

There is considerable scope for further research on the supply side of sorting equilibria in the housing market. For example, the literature has yet to consider how the presence of large developers (who are capable of exercising market power) might alter supply relationships. One could also develop a formal political economy model in which existing residents vote on development restrictions that constrain forward looking developers.

7.4. *Labor Supply*

For working households, there are two dimensions of location choice—the choice of a house and the choice of a job. Representing both choices as part of a “dual-market” sorting model could improve our understanding of preferences for amenities and the evolution of neighborhoods. This suggestion is underscored by Paul W. Rhode and Koleman S. Strumpf’s (2003) historical assessment of Tiebout sorting. As moving costs declined between 1850 and 1990, they find that U.S. counties and municipalities became *less* stratified by public goods provision and household demographics; the opposite of what we would predict in a traditional static model of Tiebout sorting (i.e. holding job locations fixed). Furthermore, the American Housing Survey consistently reports “convenient to job” as the reason most frequently cited by households for choosing to live in their current neighborhood.

Rosen (1979) outlined the conceptual logic for dual-market sorting. Because households are free to adjust their behavior in both markets, we should expect both wage rates and house prices to reflect spatial variation in amenities. Intuitively, locations providing fewer amenities must offer higher wages and lower housing prices to induce households to locate there. As more households locate in high-amenity areas, the supply of labor and the demand for housing will increase, decreasing wages and increasing rents. Thus, households “pay” for the amenities in their area through both rents and wages. Jennifer Roback (1982) formalized Rosen's intuition. In her model of interregional sorting, a household's implicit expenditures on amenities are jointly determined by a hedonic wage function and a hedonic price function. Empirical applications have reported that wage differentials reflect a substantial share of the total expenditures on amenities (Glenn C. Blomquist, Mark C. Berger, and John P. Hoehn 1988, Philip E. Graves and Donald M. Waldman 1991, Kahn 1995; and David Albouy 2009).[51]

Most of the literature following Rosen (1979) and Roback (1982) has yet to incorporate the insights from the sorting literature. Households are typically assumed to *freely sort across the nation* based on their *homogeneous preferences* for *exogenous amenities*. Each metropolitan area offers a different (price, wage, amenity) bundle, but there is assumed to be *no heterogeneity in the spatial landscape* within a metro area. A collection of recent studies has just begun to make progress toward relaxing these assumptions, building a bridge between the interregional hedonic and equilibrium sorting literatures. This is an exciting area for continued research.

Bayer, Keohane, and Timmins (2009) relax the national free mobility assumption. They find that indirectly controlling for the cost of moving between metropolitan areas triples their estimates of the willingness-to-pay for a small improvement in air quality. While their empirical model tracks changes that occur over time, they do not model forward looking behavior. Other analysts have sought to adapt the recent advances in modeling dynamics. John Kennan and James R. Walker (2010) model interstate migration decisions with the goal of describing the role of expected future income prospects. This exercise is complicated by the need to account for the possibility of repeat and return migration decisions. In other words, they model optimal *sequences* of migration decisions, instead of a one-time decision to move or not move. Relying on numerical discrete approximation techniques to calculate the value function (Rust 1994), Kennan and Walker define the choice set as U.S. states, described only by time-invariant population and climate. Moreover, to shrink the state space, they restrict individuals' information sets to include only wages seen in recently visited locations. In a similar model of family location decisions accounting for marital status, Ahu Gemici (2008) restricts the choice set to nine US census regions. While both papers take an important step toward modeling

[51] Roback's model is most commonly used to calculate "quality-of-life" indices. These indices rank metro areas by the implicit cost of consuming their bundle of amenities.

forward looking behavior, their choice sets and data are not practical for analyzing heterogeneity in policy outcomes at a high level of spatial resolution.

Bishop (2009) adapts the two-step approach described by Arcidiacono and Miller (2008) to the Kennan-Walker interstate sorting context and proposes a forward looking model for analyzing outcomes that vary across metropolitan areas. Arcidiacono and Miller's (2008) approach reduces Kennan and Walker's computational burden by treating the choice-specific value function as a sequence of flow utilities, conditional choice probabilities (CCP's), and the present discounted value of the value function realized at some future state.[52] A limited memory assumption can then be used to difference away that future value function.[53] This strategy converts a complex dynamic decision problem to a simple discrete choice over combinations of differenced flow utilities and CCP's. Utility parameters are then estimated with simple discrete choice techniques (e.g. multinomial logit). As a result, the researcher is able to include any number of time-varying attributes without encountering the curse of dimensionality.

Finally, Kuminoff (2009) returns to a static setting to characterize dual-market sorting at an even higher level of spatial resolution. He focuses on the San Francisco and Sacramento metro areas because they contain diverse housing communities within several cities, which he treats as distinct job locations. Kuminoff's dual-market framework extends the structural model from Epple and Sieg (1999) in two ways. First, wage income and leisure time are both endogenous to location choice. Working households with heterogeneous job skills are assumed to select a job-house combination based on the wages they can earn, their preferences for the amenities provided by housing communities, and the required commute time. The model also relaxes vertical differentiation to allow households to differ in their relative preferences for leisure time and multiple amenities. Thus, in a dual-market locational equilibrium, working households are simultaneously sorted among housing and labor markets according to their heterogeneous preferences and skills. Opportunities for adjustment in both markets make the

[52] Hotz and Miller (1993) originally described the two-step method for solving complex dynamic programming problems by using conditional choice probabilities to approximate value functions.

[53] This assumption exploits the idea that an individual will have the same present discounted value of future utility starting from some point in the future regardless of the path taken to get there if her memory (i.e., the extent to which current utility is dependent upon previous decisions, aside from their effect on the current state) is limited. For example, if an individual only remembers her wage draw from the last location visited in a sequence of migration decisions, the value function associated with being in a particular state in period $t+2$ will not be a function of the location choice taken in period t.

implicit cost of consuming amenities depend on housing prices, wage rates, and commute times.

7.5. *Model Validation*

All of the structural and quasi-experimental models we have discussed make assumptions about sorting behavior. Stronger assumptions may allow the analyst to draw stronger conclusions, but some credibility is lost in the process. Charles F. Manski (2007) defines this tradeoff as the law of decreasing credibility: *The credibility of inference decreases with the strength of the assumptions maintained.* Over the past decade, there has been considerable progress toward relaxing the least credible assumptions maintained in early sorting models, such as homogenous preferences and exogenous amenities. That said, the current structural estimators still rely on parametric assumptions for utility functions (CES, quasi-linear), assumptions for the statistical distributions used to characterize sources of unobserved heterogeneity (log-normal, Type I extreme value), and assumptions eliminating sources of market friction. One of the advantages of structural estimation is that it allows us to keep track of these assumptions. Future research should do more to assess how they contribute to policy evaluations.

One approach would be to refine the current estimators using tools developed in the econometric literature on partial identification (see Manski 2007 and Elie Tamer 2010 for summaries). The general idea is to evaluate the sensitivity of outcome measures to the least credible assumptions needed to obtain point estimates for model parameters. Bajari and Benkard (2005) provide an example of how this logic can be applied to a structural discrete choice model. They use set-identification of preferences in a pure characteristics model of computer demand to place bounds on their estimates for price elasticities. The bounds describe the highest and the lowest elasticity that would be consistent with observed purchases under *any* shape assumption on the distribution of heterogeneous preference parameters. If one could relax some of their remaining assumptions, the bounds would grow wider.[54] In the context of neighborhood sorting, bounds on welfare measures, capitalization effects, or migration patterns may or may not be informative for policymakers. It would depend on the data and the question at hand. The point is that moving from point identification to partial identification would allow us to provide a more

[54] For example, their bounds are still conditioned by the assumption that consumer behavior can be described by a quasi-linear preference function with constant marginal utility for each computer attribute.

transparent characterization of the reflection problems that arise in the literature.

The next step would be to look for ways to narrow the set of candidate models. We covered three distinct structural frameworks (Epple and Sieg 1999, Bayer, McMillan and Reuben 2004; Ferreyra 2007). Each model can explain the pattern of prices, amenities, and location choices that we observe in a given metro area. Yet because the models embed different sets of maintained assumptions, they are likely to differ in their predictions for the outcomes that would follow a prospective policy change. Which, if any, of the current models can we trust to answer certain types of questions?

Keane (2010) emphasizes the need to perform validation exercises as part of the process of developing a new structural model or choosing between competing models. His insights are especially relevant for the current sorting literature. Fernandez and Rogerson (1998) provide an example of how validation can be done. They use their calibrated model to predict outcomes (such as the rate of return to investment in education) that were not used as fitting criteria during the calibration process. These predictions are compared against the range of direct estimates for the rate of return reported elsewhere in the literature. Similar exercises could be use to evaluate the predictive power of the new structural estimators. Another possibility would be to compare a sorting model's prediction for how markets would respond to a shock with any available quasi-experimental evidence on how the same market did respond to an actual shock. These and other approaches to model validation may prove useful as the equilibrium sorting literature continues to move forward.

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TABLE 1
FEATURES OF EMPIRICAL HEDONIC AND SORTING MODELS USED FOR POLICY EVALUATION

		(1) Multi-Market Hedonic	(2) Structural Hedonic	(3) Nonparametric Hedonic	(4) Pure Characteristics Sorting	(5) Random Utility Sorting	(6) General Equilibrium Sorting
Selected references	econometric methodology	Rosen (1974) Brown-Rosen (1982) Epple; Bartik (1987) Kahn-Lang (1988)	Driscoll et al. (1994)	Ekeland et al (2004) Heckman et al (2010)	Epple-Sieg (1999) Sieg et al. (2002, 2004)	Bayer et al. (2004) Bayer-Timmins (2005,2007)	Ferreyra (2007)
	calibrated models		Klaiber-Smith (2009) Kuminoff-Jarrah (2010)		Epple-Platt (1998)		Nechyba (2000)
	estimated models	Palmquist (1984)	Chattopadhyay (1999) Bajari-Kahn (2005)		Smith et al. (2004) Walsh (2007)	Bayer et al. (2007) Timmins (2007) Klaiber-Phaneuf (2010)	Ferreyra (2007)
Definition for the choice set	choice	house	house	house	house	house | job	house, school, vote
	budget constraint	yes	yes	yes	yes	no	yes
	amenities may vary discretely	no	no	no	yes	yes	yes

TABLE 1 (CONTINUED)
FEATURES OF EMPIRICAL HEDONIC AND SORTING MODELS USED FOR POLICY EVALUATION

		(1) Multi-Market Hedonic	(2) Structural Hedonic	(3) Nonparametric Hedonic	(4) Pure Characteristics Sorting	(5) Random Utility Sorting	(6) General Equilibrium Sorting
Restrictions on preferences	must specify utility function	no	yes	no, given weak separability of marginal utility	yes	yes	yes
	tastes for amenities	fully explained by demographics	horizontally differentiated	single amenity or vertical structure	vertically differentiated	horizontally differentiated	horizontally differentiated
	key distributional assumption	none	none	normalization of distribution for idiosyncratic error	lognormal bivariate distribution for income, preferences	iid type I EV shocks for every location	iid type I EV shocks for every location
Strategy for addressing endogenous amenities	instruments arising from model structure	none	none	none	income rank	substitute attributes	production function
	instruments used in applications to date	none	none	no known applications	income rank	substitute attributes + amenity discontinuity	production function
Issues for evaluating implications of large scale shocks	can solve for new market equilibrium	no	yes	no	yes	yes	yes
	data describe entire market	no	no	no	yes	no	yes
	# substitutes per location				2	*J-1*	*J-1*

TABLE 2

KLAIBER AND PHANEUF SORTING MODEL FOR OPEN SPACE

Increase in 2.5% Open Space	Average WTP_{PE}		Average WTP_{GE}	
	All	Impacted	All	Impacted
Inner City	242	193	255	184
Urban Fringe	569	863	425	592
Outside City	601	786	510	546

Note: The table reports predictions for annual average willingness to pay for a 2.5% increase in publicly owned (non-park) open space in the Minneapolis/St. Paul area. In this counterfactual policy simulation the open space is obtained from conversion of privately owned agricultural land and undeveloped vacant land. Partial equilibrium (PE) measures are evaluated at each household's current location and price of housing. General equilibrium (GE) measures are evaluated at new locations and prices. Not every community experiences an increase in open space. Communities that do are "impacted".

TABLE 3
SORTING ESTIMATES FOR MARGINAL AND TOTAL WILLINGNESS FOR AIR QUALITY IMPROVEMENTS

Geographic Aggregate (County)	Sieg et al. (2004)					Tra (2010)				
	Δq	*MWTP*	*e*	WTP_{PE}	WTP_{GE}	Δq	*MWTP*	*e*	WTP_{PE}	WTP_{GE}
Los Angeles	.208	50	.277	1472	1556	.21	50	.321	589	746
Orange	.180	44	.268	901	1391	.18	42	.278	595	748
Riverside	.207	25	.114	834	372	.16	51	.451	628	805
San Bernardino	.163	24	.135	738	367	.16	57	.400	489	633
Ventura	.062	26	.419	164	725	-	-		-	-

Note: The two studies describe the annualized marginal willingness to pay (MWTP), partial equilibrium willingness to pay (WTP_{PE}) and general equilibrium willingness to pay (WTP_{GE}) for reductions in ozone concentrations (Δq) in the Los Angeles metropolitan area. Sieg et al. use a pure characteristics sorting model. Tra uses a random utility sorting model. We used their predictions for WTP_{GE} to calculate the elasticity of housing expenditures to changes in air quality (*e*).

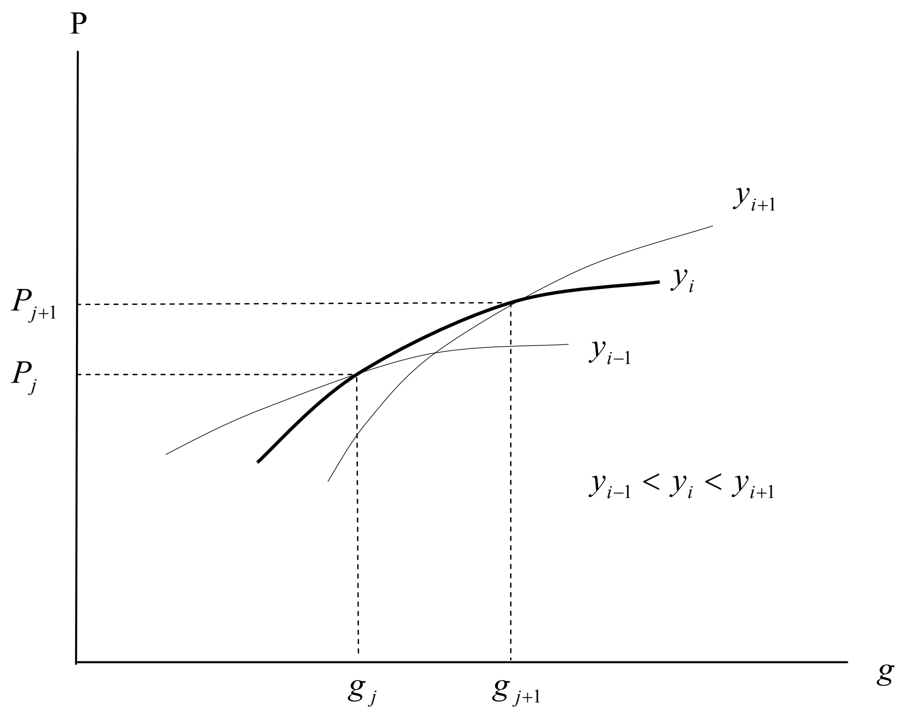

Figure 1. The Single Crossing Condition: Indifference Curves for Three Households

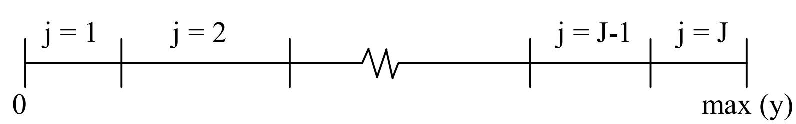

Figure 2. Partition of Households across Communities by Income

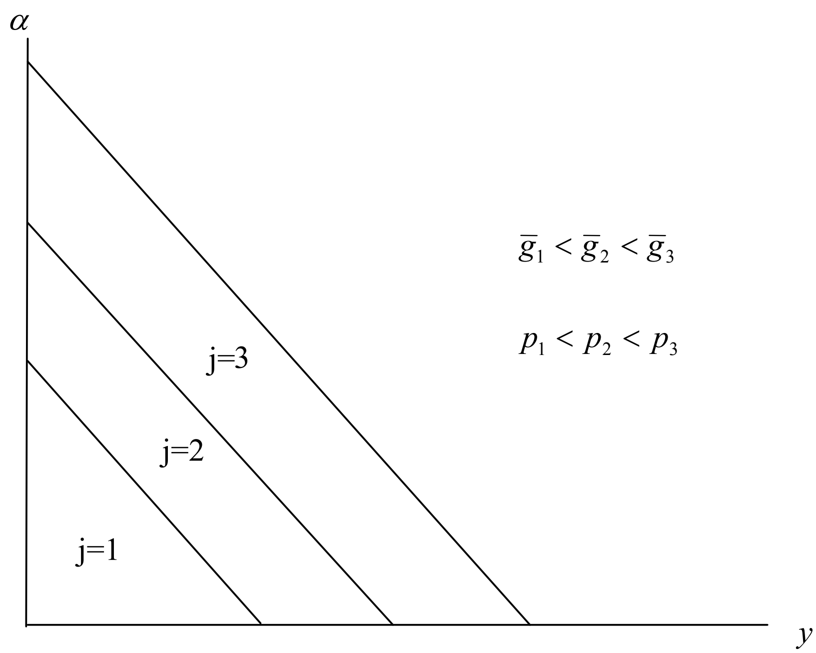

Figure 3. Partition of Households into Communities by Preferences and Income

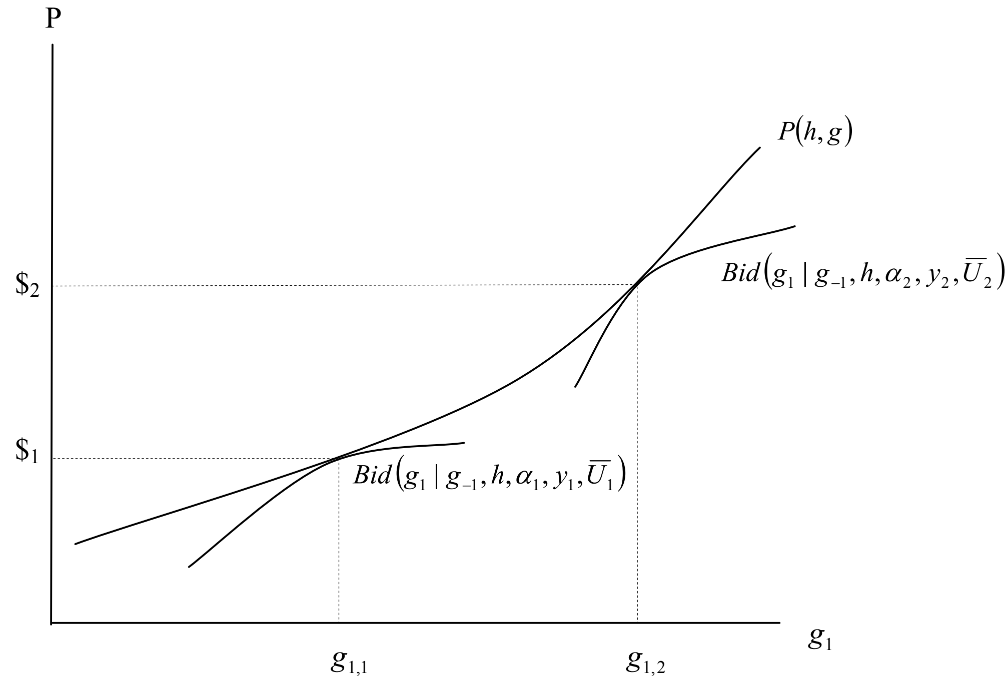

Figure 4. Bid Functions for Housing as a Function of g_1 in Hedonic Equilibrium

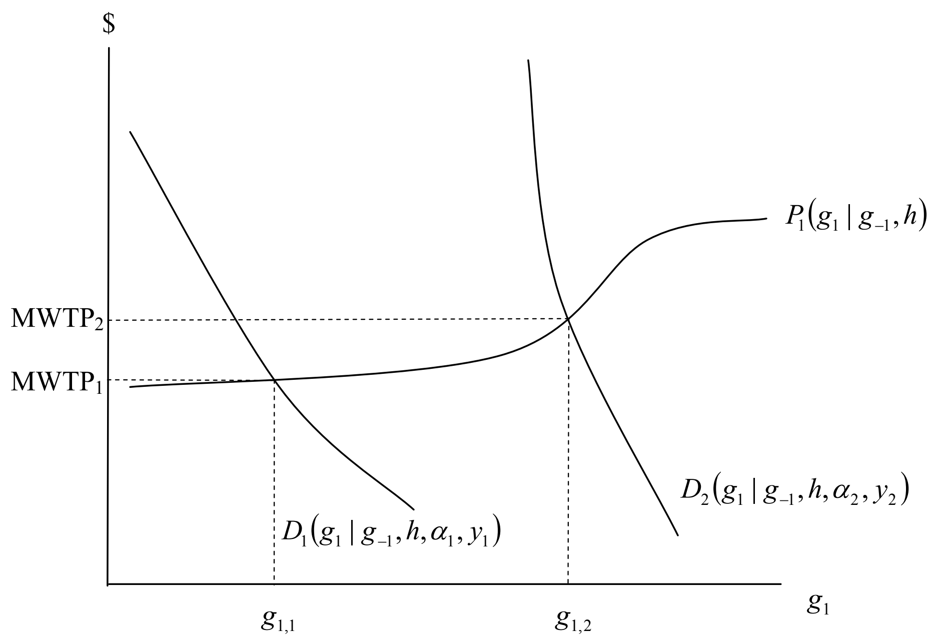

Figure 5. Implicit Price Function for g_1 and Demand Curves for Two Households

APPENDIX TABLE 1
GUIDE TO NOTATION

The Spatial Landscape	
$q = 1,\ldots,Q$	Index of housing markets
Locational Characteristics: indexed by the j subscript	
$j = 1,\ldots,J$	Index of locations (depending on the model, j = house, community, or metro area)
h	Vector of housing characteristics (e.g. bedrooms, bathrooms, pool)
g	Vector of public goods (e.g. air quality, school quality, open space)
$x \subset [g,h]$	Subset of housing characteristics and public goods observed by the analyst
ξ	Vector of characteristics not observed by the analyst
$\bar{h} = f(h)$	Index of housing characteristics
$\bar{g} = f(g)$	Index of public goods
$\bar{\xi} = f(\xi)$	Index of unobserved characteristics
P	Annualized expenditures on a single home
$\hat{P}_1 = f(x,\xi)$	Marginal price function for x_1 estimated from a hedonic model
p	Annualized "per-unit" price for a homogeneous unit of housing (i.e. the rental rate)
b	Composite numeraire private good. Its price is assumed to be normalized to unity
Household Characteristics: indexed by the i subscript	
$i = 1,\ldots,I$	Index of households
$y = \hat{y} + w$	Total annual income
$\hat{y}$	Annual exogenous non-wage income
w	Annual wage income
$\alpha = f(d,\varepsilon)$	Preferences for locational characteristics
d	Vector of household demographics (e.g. education, age, race, income)
ε	Idiosyncratic preferences for locational characteristics
Reduced Form Estimation	
B	Parameter vector estimated from the hedonic price function
ϕ	Parameter vector estimated from a capitalization regression
z	Instruments for x
Ω	Parameter vector estimated from the inverse demand function