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## THE VALUE SPREAD: A PUZZLE

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#### Abstract

The standard dynamic investment model fails to explain the value spread, which is the difference in the market equity-to-capital ratio between extreme book-to-market deciles. Even when the model manages to fit the valuation ratios across some testing assets, the implied expected return errors are large. In contrast to the model's superior in-sample fit of expected returns, recursive estimation reveals its poor out-of-sample performance. Time series instability and industry heterogeneity of the model parameters are the likely culprits. In all, we conclude that the dynamic investment framework is not yet useful for valuation and expected return estimation in practice.

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## 1 Introduction

Following the major breakthroughs of Cochrane $(1991,1996)$ and Berk, Green, and Naik $(1999)$, the investment-based asset pricing literature has experienced a period of rapid growth. Cochrane lays the foundation for adapting the dynamic investment framework of Jorgenson (1963), Tobin (1969), and Lucas and Prescott (1971) to study asset pricing issues. ${ }^{1}$ Berk et al. construct a real options model and show that it can reproduce the size and book-to-market effects in cross-sectional returns.

One strand of the literature has followed Berk, Green, and Naik (1999) in constructing dynamic models to match quantitatively stylized facts in the cross-section of returns. ${ }^{2}$ Another strand of the literature has followed Cochrane $(1991,1996)$ in conducting econometric evaluation of the dynamic investment framework in explaining cross-sectional expected returns. ${ }^{3}$ In particular, building on Cochrane (1991), Liu, Whited, and Zhang (2009) use generalized method of moments to match average stock returns to average levered investment returns derived from a dynamic investment
model. Their model captures the average stock returns of portfolios formed on earnings surprises,

[^0]book-to-market, and capital investment substantially better than traditional asset pricing models. Outside asset pricing, a prominent literature has also built on the dynamic investment framework to study important issues in corporate finance (e.g., Hennessy and Whited (2005, 2007)). ${ }^{4}$

We criticize this burgeoning literature. In particular, we quantify a major shortcoming of the dynamic investment framework. The framework has implications not only for the cross-section of expected stock returns, but also for the cross-section of equity valuation ratios. Under constant returns to scale, the market value of equity-to-capital ratio should equal the present value of marginal benefit of investment minus the debt-to-capital ratio. We use generalized method of moments (GMM) to estimate this valuation equation along with investment Euler equation at the portfolio level.

Our key finding is that the standard dynamic investment model fails miserably to explain the value spread, which we define as the equity-to-capital ratio of the high book-to-market decile minus the equity-to-capital ratio of the low book-to-market decile. The value spread is -5.61 in the data. Albeit going in the right direction, the model only generates a value spread of -1.94 . The error of -3.67 , which is more than 2.5 standard errors from zero, accounts for more than $65 \%$ of the value spread in the data. Figure 1 plots the average predicted valuation ratios against the average realized valuation ratios for the book-to-market deciles. If the model's performance is perfect, all the observations should exactly lie on the 45 -degree line. However, the model systematically underpredicts the valuation ratio of the low decile but overpredicts the valuation ratio of the high decile. Both extreme deciles have economically large errors that are also statistically significant.

We emphasize the need to match valuation ratios and expected returns simultaneously. The model parameters are in principle "deep" structural parameters, which should be invariant to changes in optimizing behavior and economic policy in the sense of Lucas (1976), and which should

[^1]Figure 1 : Average Predicted Equity Valuation Ratios versus Average Realized Equity Valuation Ratios, the Book-to-Market Deciles

in turn be invariant to whichever moments we choose to fit. However, we show that tension exists between matching valuation ratios and matching expected returns. For example, the model captures the valuation ratios across the asset growth deciles, but the resulting expected return errors are even larger in magnitude than those from the CAPM and the Fama-French (1993) three-factor model.

To examine the sources of the tension, we uncover evidence suggesting that the model parameters vary both over time and across industries. When we estimate the investment-based expected stock return model of Liu, Whited, and Zhang (2009) recursively, we find that the parameter estimates are far from stable. In contrast to the superior in-sample fit documented by Liu et al., this instability makes out-of-sample expected return errors as large as, if not larger than, the alphas from the CAPM and the Fama-French model. The model parameters also vary across industries. Industries with high valuation ratios tend to have higher estimates of the adjustment cost parameter and the capital's share in output than industries with low valuation ratios. When we use the parameter estimates from the quantity moments to construct industry costs of equity, we find that
they are even more imprecise than those from the CAPM and the Fama-French model.

Valuation is a vast topic of immense practical importance. A huge literature on accounting-based valuation has built on the dynamic dividend model and the residual income model (e.g., Williams (1938), Feltham and Ohlson (1995), and Ohlson (1995); also see Lundholm and Sloan (2007) and Penman (2010) for textbook treatments on financial statement analysis and equity valuation). A closely related literature has tested the accounting-based valuation models and applied them to estimate costs of equity (e.g., Frankel and Lee (1998), Dechow, Hutton, and Sloan (1999), and Gebhardt, Lee, and Swaminathan (2001)). Even though these models are conceptually sound, their implementation often involves ad hoc assumptions that seem to at least leave some room for an alternative approach. By modeling firms' optimal investment behavior, the dynamic investment framework provides an economics-based paradigm that links equity valuation to accounting information. However, our first stab at applying this framework for the purpose of valuation has proven disastrous.

In asset pricing, unlike the cross-section of expected stock returns that has received the lion's share of attention, the cross-section of equity valuation ratios seems largely uncharted. In particular, many studies have tried to explain the value premium, which is the average return spread between extreme book-to-market deciles (see footnotes 1 and 2 ). However, none of these studies examine the value spread. A few asset pricing studies do examine valuation issues, but mostly on the aggregate stock market (e.g., Pástor and Veronesi (2003, 2006, 2009), Merz and Yashiv (2007), and Israelsen (2010)). We stress that the cross-section of equity valuation ratios provides more stringent tests than the cross-section of expected stock returns. As such, cross-sectional valuation should be taken seriously as a new dimension of the data to improve current dynamic investment models.

The rest of the paper is organized as follows. Section 2 delineates the model. Section 3 discusses econometric and data issues. Section 4 presents the estimation results. Finally, Section 5 concludes.

## 2 The Model

To sharpen our critique on the existing literature, we adopt the investment-based asset pricing model of Liu, Whited, and Zhang (2009). Time is discrete and the horizon infinite. Taking the prices of costlessly adjustable inputs as given, firms choose these inputs each period to maximize operating profits (revenues minus the expenditures on these inputs). Taking these operating profits as given, firms choose investment and debt to maximize the market value of equity.

The operating-profit function for firm $i$ at time $t$ is given by $\Pi\left(K_{i t}, X_{i t}\right)$, in which $K_{i t}$ is capital and $X_{i t}$ is a vector of exogenous aggregate and firm-specific shocks. We assume that the firm has a Cobb-Douglas production function with constant returns to scale. This assumption implies that $\Pi\left(K_{i t}, X_{i t}\right)=K_{i t} \partial \Pi\left(K_{i t}, X_{i t}\right) / \partial K_{i t}$, and that the marginal product of capital, $\partial \Pi\left(K_{i t}, X_{i t}\right) / \partial K_{i t}=\eta Y_{i t} / K_{i t}$, in which $\eta$ is the capital's share in output and $Y_{i t}$ is sales.

Capital depreciates at an exogenous rate of $\delta_{i t}$, which is firm-specific and time-varying:

$$
\begin{equation*}
K_{i t+1}=I_{i t}+\left(1-\delta_{i t}\right) K_{i t}, \tag{1}
\end{equation*}
$$

and $I_{i t}$ is investment. Investing incurs adjustment costs, denoted $C\left(I_{i t}, K_{i t}\right)$, which is increasing and convex in $I_{i t}$, is decreasing in $K_{i t}$, and is of constant returns to scale in $I_{i t}$ and $K_{i t}$. We assume that the adjustment cost function is quadratic:

$$
\begin{equation*}
C\left(I_{i t}, K_{i t}\right)=\frac{a}{2}\left(\frac{I_{i t}}{K_{i t}}\right)^{2} K_{i t}, \tag{2}
\end{equation*}
$$

in which $a>0$ is the adjustment cost parameter.

Firms can finance investment with debt. At the beginning of time $t$, firm $i$ issues an amount of debt, denoted $B_{i t+1}$, which must be repaid at the beginning of $t+1$. Let $r_{i t}^{B}$ denote the gross corporate bond return on $B_{i t}$. Taxable corporate profits equal operating profits minus depreciation, adjustment costs, and interest expense, that is, $\Pi\left(K_{i t}, X_{i t}\right)-\delta_{i t} K_{i t}-C\left(I_{i t}, K_{i t}\right)-\left(r_{i t}^{B}-1\right) B_{i t}$.

Letting $\tau_{t}$ be the corporate tax rate, we can define the payout of firm $i$ as:

$$
\begin{equation*}
D_{i t} \equiv\left(1-\tau_{t}\right)\left[\Pi\left(K_{i t}, X_{i t}\right)-C\left(K_{i t}, K_{i t}\right)\right]-I_{i t}+B_{i t+1}-r_{i t}^{B} B_{i t}+\tau \delta_{i t} K_{i t}+\tau_{t}\left(r_{i t}^{B}-1\right) B_{i t}, \tag{3}
\end{equation*}
$$

in which $\tau_{t} \delta_{i t} K_{i t}$ is the depreciation tax shield and $\tau_{t}\left(r_{i t}^{B}-1\right) B_{i t}$ is the interest tax shield.
Firm $i$ takes the stochastic discount factor, denoted $M_{t+1}$, from time $t$ to $t+1$ as given when maximizing its cum-dividend market value of equity:

$$
\begin{equation*}
V_{i t} \equiv \max _{\left\{I_{i t+s}, K_{i t+s+1}, B_{i t+s+1}\right\}_{s=0}^{\infty}} E_{t}\left[\sum_{s=0}^{\infty} M_{t+s} D_{i t+s}\right], \tag{4}
\end{equation*}
$$

subject to the transversality condition given by $\lim _{T \rightarrow \infty} E_{t}\left[M_{t+T} B_{i t+T+1}\right]=0$.
Optimal investment behavior gives rise to investment Euler equations, which says that marginal cost of investment equals marginal benefit of investment:

$$
1+\left(1-\tau_{t}\right) a\left(\frac{I_{i t}}{K_{i t}}\right)=E_{t}\left[M_{t+1}\left(\begin{array}{c}
\left(1-\tau_{t+1}\right)\left[\eta Y_{i t+1}\right.  \tag{5}\\
\left.K_{i t+1}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1} \\
+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]
\end{array}\right)\right] .
$$

The left-hand side of the equation is the marginal cost of investment. In the right-hand side, $\left(1-\tau_{t+1}\right) \eta\left(Y_{i t+1} / K_{i t+1}\right)$ is the marginal after-tax profit from an additional unit of capital, $\left(1-\tau_{t+1}\right)(a / 2)\left(I_{i t+1} / K_{i t+1}\right)^{2}$ is the marginal after-tax reduction in adjustment costs due to economy of scale in capital adjustment, $\delta_{i t+1} \tau_{t+1}$ is the marginal depreciation tax shield, and $\left(1-\delta_{i t+1}\right)[1+$ $\left.\left(1-\tau_{t+1}\right) a\left(I_{i t+1} / K_{i t+1}\right)\right]$ is the marginal continuation value of the extra unit of capital net of depreciation. As such, equation (5) says that the present value of the marginal benefit of investment at time $t+1$, discounted back to time $t$, should equal the marginal cost of investment at time $t$.

The constant returns to scale assumption allows us to establish a link between the firm's equity value and its capital stock. Let $P_{i t} \equiv V_{i t}-D_{i t}$ denote the ex-dividend market value of equity, then:

$$
\begin{equation*}
P_{i t}+B_{i t+1}=q_{i t} K_{i t+1}, \tag{6}
\end{equation*}
$$

in which $q_{i t}$ is the marginal $q$, which is given by the right-hand side of equation (5). The investment

Euler equation then implies the following equity valuation equation:

$$
\begin{equation*}
\frac{P_{i t}}{K_{i t+1}}=E_{t}\left[M_{t+1}\binom{\left(1-\tau_{t+1}\right)\left[\eta \frac{Y_{i t+1}}{K_{i t+1}}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}}{+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]}\right]-\frac{B_{i t+1}}{K_{i t+1}} \tag{7}
\end{equation*}
$$

meaning that the market equity-to-capital ratio equals the present value of marginal benefit of investment minus the debt-to-capital ratio.

In addition, equation (5) implies that $E_{t}\left[M_{t+1} r_{i t+1}^{I}\right]=1$, in which $r_{i t+1}^{I}$ is the investment return:

$$
\begin{equation*}
r_{i t+1}^{I} \equiv \frac{\left(1-\tau_{t+1}\right)\left[\eta \frac{Y_{i t+1}}{K_{i t+1}}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]}{1+\left(1-\tau_{t}\right) a\left(\frac{I_{i t}}{K_{i t}}\right)} . \tag{8}
\end{equation*}
$$

Let $r_{i t+1}^{B a} \equiv r_{i t+1}^{B}-\left(r_{i t+1}^{B}-1\right) \tau_{t+1}$ be the after-tax corporate bond return, then $E_{t}\left[M_{t+1} r_{i t+1}^{B a}\right]=1$. Finally, let $r_{i t+1}^{S} \equiv\left(P_{i t+1}+D_{i t+1}\right) / P_{i t}$ be the stock return and $w_{i t} \equiv B_{i t+1} /\left(P_{i t}+B_{i t+1}\right)$ be the market leverage, then the investment return equals the weighted average of the stock return and the after-tax corporate bond return (see Liu, Whited, and Zhang (2009, Appendix A)):

$$
\begin{equation*}
r_{i t+1}^{I}=w_{i t} r_{i t+1}^{B a}+\left(1-w_{i t}\right) r_{i t+1}^{S} . \tag{9}
\end{equation*}
$$

## 3 Econometric Methodology and Sample Construction

We discuss econometric and data issues in this section.

### 3.1 Econometric Methods

## Explaining the Cross-Section of Stock Valuation Ratios

We use GMM to estimate the model parameters, $\eta$ and $a$, on the investment Euler equation moment:

$$
0=E\left[\begin{array}{r}
\left(1-\tau_{t+1}\right)\left[\eta \frac{Y_{i t+1}}{K_{i t+1}}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}  \tag{10}\\
1+\left(1-\tau_{t}\right) a\left(\frac{I_{i t}}{K_{i t}}\right)-\frac{+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]}{w_{i t} r_{i t+1}^{B a}+\left(1-w_{i t}\right) r_{i t+1}^{S}}
\end{array}\right],
$$

as well as the valuation equation moment:

$$
0=E\left[\begin{array}{r}
\left(1-\tau_{t+1}\right)\left[\eta \eta_{i t+1}+\frac{a}{K_{i t+1}}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}  \tag{11}\\
\frac{P_{i t}}{K_{i t+1}}+\frac{B_{i t+1}}{K_{i t+1}}-\frac{+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]}{w_{i t} r_{i t+1}^{B a}+\left(1-w_{i t}\right) r_{i t+1}^{S}}
\end{array}\right] .
$$

Two comments are in order. First, the valuation moment (11) is the unconditional version of the valuation equation (7). We evaluate the fit of the unconditional moment because our annual sample from 1963 to 2008 is not sufficiently long in the time series to allow conditional tests. Second, in both moment conditions we parameterize the stochastic discount factor $M_{t+1}$ as the inverse of the weighted average cost of capital. The rationale is that from equations (5) and (9), $E_{t}\left[M_{t+1} r_{i t+1}^{I}\right]=1$ and $r_{i t+1}^{I}$ equals the weighted average cost of capital. As such, when discounting investment payoffs, the weighted average cost of capital is an appropriate discount rate. Equivalently, the inverse of the weighted average cost of capital is an appropriate discount factor (e.g., Merz and Yashiv (2007). ${ }^{5}$

The combination of the moment conditions given by equations (10) and (11) in the cross section is a new dimension of the data not explored in the prior literature. Although the two moments are both derived from the investment Euler equation tested in, for example, Whited (1992) and Hall (2004), our test design aims to evaluate the model's ability to explain valuation ratios. In contrast, standard investment Euler equation tests use only investment and cash flow data, while assuming a constant discount rate. Merz and Yashiv (2007) and Israelsen (2010) evaluate different versions of the valuation equation using aggregate stock market data. Our focus is instead on the cross section, which arguably contains more information about stock valuation ratios.

[^2]To conduct a formal test of the moments, we define the investment Euler equation error as:

$$
e_{i}^{E}=E_{T}\left[1+\left(1-\tau_{t}\right) a\left(\frac{I_{i t}}{K_{i t}}\right)-\frac{\left(1-\tau_{t+1}\right)\left[\eta \frac{Y_{i t+1}}{K_{i t+1}}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}}{} \begin{array}{r}
+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]  \tag{12}\\
w_{i t} r_{i t+1}^{B a}+\left(1-w_{i t}\right) r_{i t+1}^{S}
\end{array}\right]
$$

and the valuation error as:

$$
e_{i}^{V}=E_{T}\left[\begin{array}{c}
\left(1-\tau_{t+1}\right)\left[\eta \frac{Y_{i t+1}}{K_{i t+1}}+\frac{a}{2}\left(\frac{I_{i t+1}}{K_{i t+1}}\right)^{2}\right]+\delta_{i t+1} \tau_{t+1}  \tag{13}\\
\frac{P_{i t}}{K_{i t+1}}+\frac{B_{i t+1}}{K_{i t+1}}-\frac{+\left(1-\delta_{i t+1}\right)\left[1+\left(1-\tau_{t+1}\right) a\left(\frac{I_{i t+1}}{K_{i t+1}}\right)\right]}{w_{i t} r_{i t+1}^{B a}+\left(1-w_{i t}\right) r_{i t+1}^{S}}
\end{array}\right]
$$

in which $E_{T}$ denotes the sample mean. We assume that both errors have a mean of zero.

We estimate the adjustment cost parameter $a$ and the capital's share $\eta$ using one-stage GMM to minimize the weighted average of $e_{i}^{E}$ and $e_{i}^{V}$. We use the identity weighting matrix in one-stage GMM to preserve the economic structure of the testing portfolios. The identity weighting matrix also provides more robust, albeit less efficient, estimates. When conducting inferences, we use a standard Bartlett kernel with a window length of five to calculate the optimal weighting matrix. To test whether all or a subset of the model errors are jointly zero, we use Hansen's (1982, lemma 4.1) $\chi^{2}$ test. We conduct the estimation and tests at the portfolio level. Forming portfolios smoothes investment data that tend to be lumpy at the firm level (e.g., Doms and Dunne (1998)). To the extent that the model with smooth investment behavior fails to explain stock valuation ratios even at the portfolio level, firm-level estimation will most likely work to reinforce our conclusion.

## Out-of-sample Tests with Expected Return Moments

Liu, Whited, and Zhang (2009) define the levered investment return as:

$$
\begin{equation*}
r_{i t+1}^{I w} \equiv \frac{r_{i t+1}^{I}-w_{i t} r_{i t+1}^{B a}}{1-w_{i t}} \tag{14}
\end{equation*}
$$

which, according to equation (9), should equal the stock return, $r_{i t+1}^{S}$, period by period and state by state. Liu et al. test whether the stock return equals the levered investment return on average:

$$
\begin{equation*}
E\left[r_{i t+1}^{S}-r_{i t+1}^{I w}\right]=0, \tag{15}
\end{equation*}
$$

and find that the model goes a long way in explaining the average returns of portfolios formed on book-to-market, earnings surprises, and corporate investment.

We conduct two sets of out-of-sample tests on the expected return moments explored in-sample by Liu, Whited, and Zhang (2009). In the first set of tests we estimate the structural parameters $a$ and $\eta$ using the investment Euler equation and the valuation equation moments. We use the parameter estimates and observed portfolio characteristics to construct the levered investment return, denoted $\widehat{r_{i t+1}^{I W}}$. We then calculate the out-of-sample expected return errors, denoted $\widehat{\alpha_{Q i}}$, as the expected stock returns minus the expected levered investment returns constructed in this way:

$$
\begin{equation*}
\widehat{\alpha_{Q} i} \equiv E_{T}\left[r_{i t+1}^{S}-\widehat{r_{i t+1}^{I w}}\right] . \tag{16}
\end{equation*}
$$

We also compare these errors in terms of the magnitude with the in-sample expected return errors, $\alpha_{Q i} \equiv E_{T}\left[r_{i t+1}^{S}-r_{i t+1}^{I w}\right]$, estimated using the Liu et al.'s methodology.

The expected return errors, $\widehat{\alpha_{Q i}}$, are "out-of-sample" in the sense that the parameters are chosen to match the quantity moments (10) and (11), as opposed to expected return moments per se. The adjustment cost parameter and the capital's share are, in principle, "deep" technological parameters, which should be invariant to changes in optimizing behavior and economic policy in the sense of Lucas (1976). As such, the parameters uncovered from the quantity moments should be close to the parameters uncovered directly from the expected return moments. Any deviations and the resulting out-of-sample expected return errors would be evidence against the model.

In the second set of out-of-sample tests, we explore the same expected return moments (15) as in Liu, Whited, and Zhang (2009). However, instead of estimating the model only once using the full
sample, we estimate it recursively using a series of expanding windows of sample observations. In particular, in year $t$ we use all the sample observations up to year $t$ to estimate the model parameters using the expected return moments. We use the estimated parameters and observed portfolio characteristics dated year $t$ and $t+1$ to construct the levered investment return, denoted $\widetilde{r_{i t+1}}$, which also goes from $t$ to $t+1$. We then calculate the out-of-sample errors, denoted $\widetilde{\alpha_{Q i}}$, as the expected stock returns from year $t$ to $t+1$ minus the expected levered investment returns constructed in this way:

$$
\begin{equation*}
\widetilde{\alpha_{Q i}} \equiv E_{T}\left[r_{i t+1}^{S}-\widetilde{r_{i t+1}^{I w}}\right] . \tag{17}
\end{equation*}
$$

The same procedure is repeated every year up to year $T-1$ (with $T$ being the last year in the sample). We compare the magnitude of $\widetilde{\alpha_{Q i}}$ with that of in-sample errors.

We also examine the time-variation of the parameters estimated recursively. If the adjustment cost parameter, $a$, and the capital's share, $\eta$, are structural parameters that describe the nature of capital investment and production technologies, respectively, the estimates should be invariant over time. It is hard to imagine why the nature of the technologies should vary on an annual basis. As such, any time-variation in the parameters and the resulting deviations of out-of-sample errors from in-sample errors would be evidence against the model. Further, the recursive estimation forces an econometrician to face the same information restrictions, which an investor would face when using the model to estimate expected stock returns, and a corporate manager would face when using the model to estimate costs of equity capital in practice. As such, this out-of-sample test evaluates the model's performance in explaining expected stock returns in real time.

### 3.2 Data

The sample consists of common stocks on NYSE, Amex, and NASDAQ from 1963 to 2008. The firm level data are from the Center for Research in Security Prices (CRSP) monthly stock file and Standard and Poor's Compustat annual and quarterly files. We delete firm-year observations with missing data or for which total assets, net capital stock, or sales are either zero or negative. We
include only firms with fiscal year ending in the second half of the calendar year. We also exclude firms with primary standard industrial classifications between 4900 and 4999 (regulated firms) and between 6000 and 6999 (financial firms).

## Testing Portfolios

We use 30 testing portfolios: ten book-to-market $(B / M)$ deciles, ten asset growth deciles, and ten return-on-asset $(R O A)$ deciles. We use $B / M$ deciles because $B / M$ predicts returns. Forming portfolios on $B / M$ also produces a large cross-sectional spread in stock valuation ratio, which is the focus of our quantitative investigation. We use asset growth and $R O A$ deciles because factors based on these two characteristics, when combined with the market factor, provide a reasonable description of the cross-section of average stock returns (e.g., Chen, Novy-Marx, and Zhang (2010)).

Following Fama and French (1993), we sort all stocks on book-to-market at the end of June of year $t$ into ten deciles based on the NYSE breakpoints. Book-to-market is book equity for the fiscal year ending in calendar year $t-1$ divided by the market equity for December of year $t-1 .{ }^{6}$ We exclude observations with negative book equity. We calculate equal-weighted annual portfolio returns from July of year $t$ to June of year $t+1$, because equal-weighted returns are harder to explain than value-weighted returns. The sample period is from 1963 to 2008.

Following Cooper, Gulen and Schill (2008), at the end of June of year $t$, we sort all NYSE, Amex, and Nasdaq stocks into ten deciles based on asset growth for the fiscal year ending in the calendar year $t-1$. Asset growth is the annual change in total assets (Compustat annual item AT) divided by the lagged total assets. We calculate equal-weighted annual returns from July of year $t$ to June of year $t+1$, and rebalance the portfolios at the end of each June. The sample is from 1963 to 2008.
$R O A$ is the quarterly earnings (Compustat quarterly item IBQ) divided by one-quarter-lagged

[^3]assets (item ATQ). Following Chen, Novy-Marx, and Zhang (2010), we rank all NYSE, Amex, and Nasdaq stocks into ten deciles by their quarterly $R O A$ s at the beginning of each month. We use quarterly earnings in portfolio sorts in the months immediately after the most recent public earnings announcement month (item RDQ). We calculate equal-weighted portfolio returns for the current month, and rebalance the portfolios at the beginning of next month. The sample is from 1975 to 2008. The starting date is restricted by the availability of quarterly assets data.

## Variable Measurement

We largely follow Liu, Whited, and Zhang (2009) in measuring different variables and in aligning the timing of accounting variables with stock returns at the portfolio level. We make three adjustments. First, we equal-weight corporate bond returns for the testing portfolios to make the weighting scheme of bond returns consistent with that of stock returns. In contrast, Liu et al. value-weight bond returns. Second, we include in the sample all the firms with fiscal year ending in the second half of the calendar year. In contrast, Liu et al. only include firms with fiscal year ending in December. Our procedural change substantially enlarges the sample. Third, we measure capital stock, $K_{i t}$, as net property, plant, and equipment (Compustat annual item PPENT). In contrast, Liu et al. use gross property, plant, and equipment. Using net property, plant, and equipment is more consistent with the model because equation (1) implies that capital is net of depreciation.

Investment, $I_{i t}$, is capital expenditures (Compustat annual item CAPX) minus sales of property, plant, and equipment (item SPPE). The capital depreciation rate, $\delta_{i t}$, is the amount of depreciation (item DP) divided by the capital stock. Output, $Y_{i t}$, is sales (item SALE). Total debt, $B_{i t+1}$, is long-term debt (item DLTT) plus short term debt (item DLC). Market leverage, $w_{i t}$, is the ratio of total debt to the sum of total debt and the market value of equity. We measure the tax rate, $\tau_{t}$, as the statutory corporate income tax (from the Commerce Clearing House, annual publications). For the pre-tax corporate bond returns, $r_{i t+1}^{B}$, we follow Blume, Lim, and Mackinlay (1998) to impute the credit ratings for firms with no rating data from Compustat (item SPLTICRM), and assign the
corporate bond returns for a given credit rating (from Ibbotson Associates) to the firms with the same credit ratings (see Liu, Whited, and Zhang (2009) for details). The after-tax corporate bond returns, $r_{i t+1}^{B a}$, are computed from $r_{i t+1}^{B}$ using the average tax rate in years $t$ and $t+1$.

We aggregate firm-level characteristics to portfolio-level characteristics as follows. For example, $Y_{i t+1} / K_{i t+1}$ is the sum of sales in year $t+1$ for all the firms in portfolio $i$ formed in June of year $t$ divided by the sum of capital stocks at the beginning of year $t+1$ for the same set of firms. $I_{i t+1} / K_{i t+1}$ in the numerator of $r_{i t+1}^{I}$ is the sum of investment in year $t+1$ for all the firms in portfolio $i$ formed in June of year $t$ divided by the sum of capital stocks at the beginning of year $t+1$ for the same set of firms. Other characteristics are aggregated analogously.

For monthly rebalanced $R O A$ portfolios, we time-aggregate the monthly returns from July of year $t$ to June of $t+1$ to obtain annual returns. Constructing the annual portfolio characteristics is more complicated because the composition of a given portfolio changes from month to month. Consider the 12 low $R O A$ portfolios formed in each month from July of year $t$ to June of year $t+1$. For each month we calculate portfolio-level characteristics by aggregating firm-level characteristics over the firms in the low $R O A$ portfolio. Because the portfolio composition changes from month to month, these portfolio-level characteristics also change from month to month. As such, we average these portfolio characteristics over the 12 monthly low $R O A$ portfolios, and use these averages to construct the investment return. We repeat this procedure for the remaining nine $R O A$ deciles.

## Descriptive Statistics

Table 1 reports the average equity valuation ratios, $P_{i t} / K_{i t+1}$, for the testing portfolios. From Panel A, the valuation ratio decreases monotonically from 6.22 for the low- $B / M$ decile to 0.61 for the high- $B / M$ decile. The value spread, which we define as the market equity-to-capital ratio of the high- $B / M$ decile minus that of the low- $B / M$ decile, is -5.61 , and is more than 13 standard errors from zero. ${ }^{7}$ The value spread is large. The average valuation ratio across the $B / M$ deciles is 2.18 ,

[^4]meaning that the value spread is more than 2.5 times the magnitude of the average valuation ratio.

Panel B shows that, although sorting on asset growth produces a large equal-weighted return spread of $-17 \%$ per annum, the spread in the equity valuation ratios across the extreme deciles is only 0.77 . Albeit significant, the magnitude of the spread is small, and is only $36 \%$ of the average valuation ratios across the asset growth deciles, 2.15. From Panel C, sorting on $R O A$ produces a somewhat larger spread in the valuation ratio, but the valuation ratio is not monotonic across the deciles. The valuation ratio starts at 3.64 for the low decile, drops to 1.17 for decile three, and then climbs gradually to 1.99 for decile seven, and finally to 6.02 for the high- $R O A$ decile. The spread in the valuation ratio is about the same as the average valuation ratios across the $R O A$ deciles.

To lay the background for out-of-sample tests of the dynamic investment model on expected return moments, we also report the in-sample alphas from the CAPM and the Fama-French model. Panel A of Table 1 shows that the CAPM alpha of the high-minus-low $B / M$ decile is $14.52 \%$ per annum. The Gibbons, Ross, and Shanken (1989, GRS) test that all the CAPM alphas are jointly zero is strongly rejected (not tabulated). The Fama-French model produces a smaller alpha of $6.59 \%$ for the high-minus-low decile. The model is still rejected by the GRS test at the $1 \%$ level. From Panel B, the alphas for the high-minus-low asset growth decile are large: $-15.43 \%$ per annum for the CAPM and $-12.78 \%$ for the Fama-French model. The null hypothesis that the alphas are jointly zero across the deciles is strongly rejected for both models. From Panel C, the high-ROA decile earns a higher average return than the low- $R O A$ decile. The alphas for the high-minus-low decile are large: $9.85 \%$ for the CAPM and $10.55 \%$ for the Fama-French model.

## 4 Estimation Results

We show that the dynamic investment model fails to explain the value spread in Section 4.1. We conduct out-of-sample tests on expected returns in Section 4.2. To understand the driving forces equity-to-capital ratio arises more naturally from the dynamic investment model (see the valuation equation (7)).
behind our key results, we also conduct industry-specific estimation of the model in Section 4.3.

### 4.1 Explaining the Cross-Section of Stock Valuation Ratios

Table 2 reports the results from matching the investment Euler equation and the valuation equation moments. The estimates of the adjustment cost parameter, $a$, range from 15.84 to 19.55 , and the estimates of the capital's share, $\eta$, range from 0.22 to 0.41 . Both estimates are significantly different from zero. The table also reports that none of $\chi^{2}$ statistics can reject the null hypothesis that the investment Euler equation and the valuation equation are satisfied across a given set of testing portfolios, either separately or jointly. As such, we only focus on the average absolute value of model errors (m.a.e.) as the overall performance measure.

The model seems better able to fit the investment Euler equation than the valuation equation. The m.a.e. for the investment Euler equation moments is around 0.08 for all three sets of testing portfolios. In contrast, the m.a.e. for the valuation equation moments is 0.95 for the $B / M$ deciles, which is about $43 \%$ of the average valuation ratio across these deciles. The valuation m.a.e. is 0.87 for the $R O A$ deciles, which is about $35 \%$ of the average valuation ratio across these deciles. The m.a.e. is low for the asset growth deciles, 0.17 , meaning that the model fits their valuation ratios well.

Table 2 only reports overall model performance. To provide a more complete picture, we report in Table 3 the valuation errors of each individual testing portfolios. The predicted valuation ratios are constructed using the parameter estimates in Table 2. We also report $t$-statistics calculated from the one-stage GMM, testing that a given individual portfolio's valuation error equals zero.

From Panel A of Table 3, the model systematically underpredicts the valuation ratio of the low$B / M$ decile and overpredicts the valuation ratio of the high- $B / M$ decile. The low- $B / M$ decile has a positive valuation error of 2.73 , which is more than 3.5 standard errors from zero. The high- $B / M$ decile has a negative valuation error of -0.94 , which is more than two standard errors from zero. As a result, the high-minus-low $B / M$ portfolio has a large error of -3.67 , which is more than 2.5 standard errors from zero. In terms of economic magnitude, Table 1 shows that the value spread
is -5.61 . As such, the model leaves more than $65 \%$ of the value spread unexplained.

The spread in the valuation ratio is only 0.77 between the extreme asset growth deciles, albeit significant (see Table 1). From Panel B of Table 3, the model does a good job in matching the valuation ratios for these deciles. The error for the high-minus-low decile is -0.41 , which is within 1.7 standard errors of zero. Panel C shows that the valuation errors of individual $R O A$ deciles have similar magnitude as those of individual $B / M$ deciles. Both the extreme deciles have large valuation errors, 1.55 for the low- $R O A$ decile and 2.22 for the high- $R O A$ decile. However, the high-minus-low $R O A$ decile has an error of only 0.66 , which is within one standard error of zero. In economic terms, the error amounts to only $28 \%$ of the valuation ratio spread between the extreme deciles.

### 4.2 Out-of-sample Tests with Expected Return Moments In-sample Fit

We follow Liu, Whited, and Zhang (2009) in calculating the in-sample expected return errors from the dynamic investment model. In particular, we estimate the parameters using one-stage GMM with the identity weighting matrix to minimize the in-sample expected return errors defined from the moment condition (15). In untabulated results, the adjustment cost parameter, $a$, is estimated to be $12.12,1.89$, and 2.47 , and the capital's share, $\eta$, is $0.25,0.15$, and 0.18 , with the $B / M$, asset growth, and $R O A$ deciles as the testing assets, respectively. The model is not rejected by the $\chi^{2}$ test. In addition, the mean absolute expected return error is $2.05 \%$ per annum for the $B / M$ deciles, $2.79 \%$ for the asset growth deciles, and $1.16 \%$ for the $R O A$ deciles. These errors are mostly smaller in magnitude than those from the CAPM and the Fama-French model (see Table 1).

The first two rows in each panel in Table 4 report the expected return errors from the dynamic investment model for each individual portfolios. The alphas from the investment model do not vary systematically with the characteristics used to form the testing portfolios. In particular, the alphas of the high-minus-low $B / M$, asset growth, and $R O A$ deciles are $-0.59 \%, 0.25 \%$, and $0.24 \%$ per annum, all of which are within 0.5 standard errors of zero. These alphas are substantially
smaller in magnitude than those from the CAPM and the Fama-French model in Table 1. Figure 2 plots the average predicted stock returns against the average realized stock returns for the testing deciles. Consistent with the superior in-sample fit documented in Liu, Whited, and Zhang (2009), the scatter plots are all largely aligned with the 45-degree line.

## Out-of-sample Fit with Parameters Estimated from Quantity Moments

The third and fourth rows in each panel of Table 4 report the out-of-sample alphas $\left(\widehat{\alpha_{Q}}\right)$ for each individual deciles when the parameters are estimated using the investment Euler equation and the valuation equation moments. The out-of-sample alphas are calculated from equation (16), in which we construct levered investment returns using parameters from fitting the two quantity moments.

In contrast to the small in-sample alphas, the out-of-sample alphas, $\widehat{\alpha_{Q}}$, are large. From Panel A, $\widehat{\alpha_{Q}}$ for the high-minus-low $B / M$ decile is $6.74 \%$ per annum, which is more than four standard errors from zero. This error is higher than that from the Fama-French model, $6.59 \%$, although lower than that from the CAPM, $14.52 \%$. More important, the out-of-sample alphas, ranging from $4.15 \%$ to $11.55 \%$, have an average magnitude of $8.34 \%$. This out-of-sample m.a.e. is higher than both that from the CAPM, $4.32 \%$, and that from the Fama-French model, $1.44 \%$. The scatter plots in Panel A of Figure 3 also show that the dynamic investment model in out-of-sample systematically underpredicts the average stock returns of the $B / M$ deciles.

Panel B of Table 4 reports even worse out-of-sample performance for explaining the expected returns of the asset growth deciles. Although the parameters from Panel B of Table 2 fits the valuation ratios of these deciles well (see Panel B of Table 3), the parameters imply very large expected return errors. In particular, $\widehat{\alpha_{Q}}$ for the high-minus-low asset growth decile is $24.77 \%$ per annum, which is more than 9.5 standard errors from zero. The magnitude of this error is higher than the CAPM alpha, $-15.43 \%$, and the Fama-French alpha, $-12.78 \%$. In addition, the average magnitude of the out-of-sample alphas from the dynamic investment model is $7.36 \%$, which is higher than those from the CAPM, $4.74 \%$, and from the Fama-French model, $2.62 \%$. The scatter plots in

Panel B of Figure 3 show further that in out-of-sample the dynamic investment model somewhat underpredicts the average returns of the high asset growth deciles but greatly overpredicts those of the low asset growth deciles. As a result, the error for the high-minus-low portfolio is very large.

The dynamic investment model has reasonable out-of-sample performance for the $R O A$ deciles. From the third and fourth rows in Panel C of Table 4, the high-minus-low $R O A$ decile has an out-of-sample error of $-2.81 \%$ per annum, which is within 0.5 standard errors of zero. The average magnitude of the errors across the deciles is $1.69 \%$. Both metrics compare favorably with those from the CAPM and the Fama-French model. Panel C of Figure 3 illustrates the good fit graphically.

The poor out-of-sample performance means tension between matching the quantity moments and matching the expected return moments. This issue is especially important for the asset growth deciles. The model fits the quantity moments well with the adjustment cost parameter, $a=15.84$, and the capital's share, $\eta=0.28$. Separately, the model also fits the expected return moments well, but with $a=1.89$ and $\eta=0.15$. What causes the two sets of estimates to diverge?

Comparing the valuation equation (7) and the investment return equation (8) provides some clues. In untabulated results, we show that the low asset growth decile has a slightly higher average sales-to-capital, $Y_{i t+1} / K_{i t+1}$, than the high asset growth decile, but the difference is insignificant. More important, the low asset growth decile also has a lower average investment-to-capital next period, $I_{i t+1} / K_{i t+1}$, than the high asset growth decile: $16.3 \%$ versus $25.1 \%$. The spread is more than 12 standard errors from zero. The valuation equation (7) says that the cross-sectional variation of $I_{i t+1} / K_{i t+1}$ goes in the right direction in explaining the valuation ratios across the asset growth deciles. But the investment return equation (8) says that it goes in the wrong direction in explaining the expected returns. Matching the valuation ratios requires a high estimate of $a$ because $I_{i t+1} / K_{i t+1}$ is the only characteristic going in the right direction. But the high estimate of $a$ decreases the model's ability to match the expected returns, which would require a lower estimate of $a$.

## Out-of-Sample Fit with Parameters Estimated Recursively

The last two rows in each panel of Table 4 report the out-of-sample alphas ( $\left.\widetilde{\alpha_{Q}}\right)$ when we recursively estimate the model parameters using the expected return moments (15). The errors are large, indicating poor out-of-sample performance of the dynamic investment model.

From Panel A, $\widetilde{\alpha_{Q}}$ increases almost monotonically from $-8.52 \%$ per annum for the low- $B / M$ decile to $6.77 \%$ for the high- $B / M$ decile. The error of $15.29 \%$ for the high-minus-low decile is more than 2.5 standard errors from zero. The average magnitude of the errors across the ten deciles is $4.23 \%$. Both the high-minus-low alpha and the m.a.e. have the same order of magnitude as those from the CAPM, which are in turn larger than those from the Fama-French model. The scatter plots in Panel A of Figure 4 are largely horizontal, meaning that the dynamic investment model fails to explain the average stock returns from the $B / M$ deciles on the out-of-sample basis.

The results for the asset growth deciles and the $R O A$ deciles are similar. From Panel B , the high-minus-low asset growth decile has an error of $-20.09 \%$ per annum, which is more than 2.5 standard errors from zero. The m.a.e. across the asset growth deciles is also large, $5.06 \%$. Both errors are even higher in magnitude than those from the CAPM and the Fama-French model. From Panel C, the high-minus-low $R O A$ decile has an error of $-14.51 \%$, albeit insignificant. The m.a.e. across the $R O A$ deciles is $3.29 \%$. Both metrics are comparable with those from the CAPM and the Fama-French model. The scatter plots in Panels B and C of Figure 4 further illustrate the poor out-of-sample fit of the dynamic investment model.

What explains the large deviation of the out-of-sample fit from the in-sample fit? If the parameter estimates from the recursive estimation are identical to those from the full-sample estimation, the out-of-sample alphas must be equal to the in-sample alphas. As such, their differences must be caused by the time series instability of the parameter estimates from the recursive estimation. Table 5 reports the time series of the parameter estimates using a series of expanding windows ending from 1988 to 2007. (We choose to end the first expanding window in 1988, so that we have
in total 20 annual observations for each parameter.)

In all three sets of testing deciles, the parameter estimates are unstable over time. From Panel A, the adjustment cost parameter, $a$, increases almost monotonically from 3.68 in 1988 to 11.58 in 2007 for the $B / M$ deciles. The average estimate of $a$ from 1988 to 2007 is only 6.94 . The capital's share, $\eta$, goes up gradually from 0.17 in 1988 to 0.21 in 1998 , and further to 0.25 in 2007 . The full-sample estimates $(a=12.12, \eta=0.25$, both in 2008) are at the high end for both parameters. For the asset growth deciles, the $\eta$ estimates are relatively stable, ranging from 0.12 to 0.15 . But the $a$ estimate goes up almost monotonically from 0.18 in 1988 to about two in 2007 . The $R O A$ deciles show even more time series instability of the adjustment cost parameter. The estimate starts at 9.29 in 1988, jumps to above 19 in 1997 and 1998, before dropping down to 5.60 in 2003 and further to 2.76 in 2007. The full-sample estimate of $a$ is only 2.47 in 2008 .

The parameter instability is not specific to the expected return moments. In Panel B of Table 5, we find similar time-variation in the parameter estimates from the two quantity moments. In view of such parameter instability, the model's poor out-of-sample performance is perhaps not surprising.

### 4.3 Industry-Specific Estimation

We have shown that the standard dynamic investment model fails to explain the value spread (the equity-to-capital ratio of the high $-B / M$ decile minus the equity-to-capital ratio of the low- $B / M$ decile). Also, even if the model fits the valuation ratios across some testing assets such as the asset growth deciles, the parameter estimates imply large expected return errors. Finally, we show that the parameter estimates are unstable over time, indicating model misspecification.

We examine another source of misspecification based on industry heterogeneity of the model parameters. Because industries differ in their production and investment technologies, the capital's share and the adjustment cost parameters can vary across industries. However, the standard dynamic investment model in Section 2 is built on the simplifying assumption that all firms have the same technology regardless of their industry classifications.

Our test design is straightforward. We adopt the 48 industry classifications per Fama and French (1997). (Even though utilities and financial firms are excluded from the other tests, we include them for the industry analysis because they belong to Fama and French's industry classifications.) To estimate the adjustment cost parameter and the capital's share for each industry, we use GMM to fit the investment Euler equation and the valuation equation moments. Because we have two moments and two parameters for each industry, the estimation is exactly identified and the two moments fit perfectly. To provide an economic metric with which we can interpret the parameter estimates, we also construct industry costs of equity as the levered investment returns by combining the estimates and observed industry characteristics per equation (14). And we compare these costs of equity estimates with those from the CAPM and the Fama-French model.

## Industry Heterogeneity of Technology Parameters

Panel A of Table 6 reports substantial heterogeneity in the $a$ and $\eta$ parameters. Across the 48 industries, the median estimate of $a$ is 21.05 , the mean is 30.73 , and the standard deviation is 35.54 . The range for $a$ is large: the minimum is 1.14 for the transportation industry (Trans), and the maximum is 206.37 for the trading industry (Fin). The cross-industry distribution of $a$ is heavily positively skewed (skewness $=3.39$ ). Most estimates are between one and 50. In addition to the trading industry, two other outliers are the banking industry $(a=149.23)$ and the insurance industry ( $a=84.72$ ).

The capital's share, $\eta$, also varies across the 48 industries. The median estimate is 0.23 , which is close to the mean of 0.28 , and the cross-sectional standard deviation is 0.17 . The minimum estimate is 0.08 for the retail industry, and the maximum is one for the banking industry. The cross-industry distribution of $\eta$ is also positively skewed (skewness $=2.00$ ). Most estimates are between 0.1 and 0.4. In addition to the banking industry, two other outliers are the trading industry $(\eta=0.75)$ and the pharmaceutical industry $(\eta=0.55)$.

Figure 5 plots the estimates of $a$ and $\eta$ against average valuation ratios across the industries. From Panel A, industries with high valuation ratios tend to have higher estimates of $a$ than indus-
tries with low valuation ratios. For example, the insurance industry has the highest valuation ratio, 8.76, and a high adjustment cost estimate of 84.72 . The utility industry has the lowest valuation ratio, 0.54 , and a low adjustment cost estimate of 1.31 . Across the 48 industries, the correlation between average valuation ratios and $a$ is 0.62 . Eliminating the two outliers (the banking and trading industries) increases this correlation to 0.92 . Panel B of Figure 5 also shows a positive correlation between valuation ratios and the estimates of $\eta$, but the relation is flatter than that between valuation ratios and the estimates of $a$. Overall, the evidence is consistent with the valuation equation (7). All else equal, high estimates of $a$ and $\eta$ are required to match high valuation ratios.

## Industry Costs of Equity

Fama and French (1997) report a shortcoming of existing asset pricing models: Estimates of industry costs of equity are imprecise. Standard errors of more than $3 \%$ per annum are typical for both the CAPM and the Fama-French model. The first six columns of Table 6 replicate Fama and French's evidence in our sample. In total, 19 industries have costs of equity from the CAPM, and 34 industries have costs of equity from the Fama-French model, with standard errors larger than $3 \%$.

We estimate industry costs of equity from the dynamic investment model. For a given industry we plug the $a$ and $\eta$ estimates in Panel A of Table 6 and observed industry characteristics into the levered investment return equation (14). We identify the resulting levered investment returns as the industry's costs of equity, and report the time series mean (CE) and standard error (Ste).

Because these costs of equity are constructed from economic fundamentals such as sales-tocapital and investment-to-capital, our prior is that their estimates could be more precise than those from the CAPM and the Fama-French model. However, the last column of Table 6 shows otherwise. Industry costs of equity from the dynamic investment model are even more imprecise than those from the factor models. In total, 35 industries have costs of equity with standard errors large than $3 \%$ per annum. 25 industries have standard errors larger than $4 \%$, while the corresponding number is only 14 in the Fama-French model. Across the 48 industries, the average standard
errors for industry costs of equity are $2.85 \%$ from the CAPM, $3.61 \%$ from the Fama-French model, and $7.12 \%$ from the dynamic investment model. As such, the dynamic investment model makes the imprecision issue of the costs of equity even worse.

## 5 Conclusion

The cross-section of expected stock returns has been one of the most intensely studied topics in asset pricing research since Fama and French (1992, 1993). In contrast, the cross-section of equity valuation ratios seems virtually untouched. Why do some firms have higher equity valuation ratios than other firms? We have tried to address this question using the dynamic investment model. Under constant returns to scale, the market value of equity-to-capital ratio should equal the present value of marginal benefit of investment minus the debt-to-capital ratio. We use GMM to estimate this valuation equation along with investment Euler equation.

Our key finding is that unlike the cross-section of expected stock returns, the dynamic investment model fails to explain the value spread, which is the market equity-to-capital ratio of the high book-to-market decile minus the market equity-to-capital ratio of the low book-to-market decile. The value spread, therefore, is a puzzle. The evidence, combined with the poor out-of-sample fit of expected returns, suggests that the structural investment-based approach is not yet useful for valuation and expected return estimation in practice.

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Table 1: Descriptive Statistics
$P / K$ is the average equity-to-capital ratio. $r^{S}$ is the average stock return in annual percent. $\alpha$ is the intercept from the monthly CAPM regression in annual percent. $\alpha_{F F}$ is the intercept from the monthly Fama-French (1993) three-factor regression in annual percent. The data for the risk-free rate and the factor returns are from Kenneth French's Web site. The H-L portfolio is long in the high and short in the low portfolio. The heteroscedasticity-and-autocorrelation-consistent $t$ statistics $([t])$ are reported beneath the corresponding errors. m.a.e. is the mean absolute expected return error in annual percent for a given set of ten testing portfolios.

|  | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H-L | m.a.e. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Panel A: The $B / M$ deciles |  |  |  |  |  |  |  |  |  |  |  |
| $P / K$ | 6.22 | 3.68 | 2.68 | 2.20 | 1.77 | 1.47 | 1.24 | 1.07 | 0.89 | 0.61 | -5.61 |  |
| $r^{S}$ | 9.27 | 11.70 | 13.24 | 14.60 | 16.17 | 16.65 | 18.08 | 18.10 | 19.58 | 23.99 | 14.72 |  |
| $\alpha$ | -4.14 | $-1.36$ | 0.47 | 1.85 | 3.27 | 4.01 | 5.21 | 5.59 | 6.91 | 10.38 | 14.52 | 4.32 |
| [t] | -2.54 | $-0.93$ | 0.29 | 1.15 | 2.10 | 2.77 | 3.29 | 3.15 | 3.77 | 4.40 | 7.02 |  |
| $\alpha_{F F}$ | -2.25 | $-1.29$ | -0.88 | -0.46 | 0.16 | 0.37 | 1.47 | 1.22 | 2.00 | 4.34 | 6.59 | 1.44 |
| [t] | -2.22 | -1.69 | $-1.20$ | -0.49 | 0.24 | 0.49 | 1.74 | 1.40 | 2.43 | 3.66 | 5.41 |  |
|  | Panel B: The asset growth deciles |  |  |  |  |  |  |  |  |  |  |  |
| $P / K$ | 1.80 | 1.65 | 1.58 | 1.76 | 1.83 | 2.28 | 2.47 | 2.82 | 2.72 | 2.57 | 0.77 |  |
| $r^{S}$ | 24.25 | 21.44 | 18.69 | 18.21 | 16.50 | 16.35 | 15.87 | 14.01 | 13.06 | 7.26 | $-17.00$ |  |
| $\alpha$ | 9.25 | 8.10 | 5.81 | 5.58 | 4.23 | 3.67 | 3.28 | 1.25 | -0.01 | -6.18 | -15.43 | 4.74 |
| [t] | 3.10 | 3.68 | 3.53 | 4.32 | 3.03 | 2.90 | 2.51 | 0.77 | -0.01 | $-3.20$ | -6.91 |  |
| $\alpha_{F F}$ | 5.35 | 4.20 | 1.76 | 2.28 | 0.84 | 1.10 | 0.59 | $-0.82$ | $-1.83$ | $-7.44$ | $-12.78$ | 2.62 |
| [t] | 2.90 | 3.38 | 2.20 | 3.24 |  |  |  |  | $-1.89$ | $-7.03$ | -5.75 |  |
|  | Panel C: The $R O A$ deciles |  |  |  |  |  |  |  |  |  |  |  |
| $P / K$ | 3.64 | 1.53 | 1.17 | 1.27 | 1.47 | 1.63 | 1.99 | 2.50 | 3.50 | 6.02 | 2.38 |  |
| $r^{S}$ | 15.74 | 13.89 | 13.67 | 14.89 | 17.66 | 19.34 | 20.20 | 21.30 | 22.01 | 25.69 | 9.95 |  |
| $\alpha$ | -0.11 | -1.01 | $-0.21$ | 1.44 | 3.90 | 5.52 | 6.33 | 6.94 | 7.40 | 9.75 | 9.85 | 4.26 |
| [t] | -0.02 | -0.33 | -0.09 | 0.74 | 2.28 | 3.38 | 4.38 | 4.41 | 4.83 | 5.73 | 2.50 |  |
| $\alpha_{F F}$ | -2.13 | $-3.69$ | $-4.83$ | -3.01 | -0.39 | 1.56 | 2.70 | 3.61 | 5.31 | 8.42 | 10.55 | 3.56 |
| [t] | -0.67 | $-1.66$ | $-2.87$ | $-2.58$ | $-0.36$ | 1.48 | 2.81 | 3.38 | 4.98 | 7.40 | 2.75 |  |

## Table 2 : Parameter Estimates and Tests of Overidentification

Results are from one-stage GMM with an identity weighting matrix on the investment Euler equation and the valuation equation moments given by equations (10) and (11). $a$ is the adjustment cost parameter, and $\eta$ is the capital's share in output. The $t$-statistics denoted $[t]$ test that a given estimate equals zero. $\chi^{2}$, d.f., and p -val are the statistic, the degrees of freedom, and the $p$-value testing that a given set of valuation errors is jointly zero. m.a.e. is the mean absolute moment error for a given set of deciles.

|  | Panel A: $B / M$ | Panel B: Asset growth | Panel C: $R O A$ |
| :---: | :---: | :---: | :---: |
| $a$ | 18.68 | 15.84 | 19.55 |
| $[t]$ | 6.03 | 5.32 | 4.33 |
| $\eta$ | 0.22 | 0.28 | 0.41 |
| $[t]$ | 3.97 | 4.49 | 3.85 |
| Tests based on the valuation moments |  |  |  |
| $\chi^{2}$ | 7.56 | 6.93 | 5.93 |
| d.f. | 8 | 8 | 8 |
| p-val | 0.48 | 0.54 | 0.65 |
| m.a.e. | 0.95 | 0.17 | 0.87 |
| Tests based on the investment Euler equation moments |  |  |  |
| $\chi^{2}$ | 2.00 | 5.04 | 0.29 |
| d.f. | 8 | 8 | 8 |
| p-val | 0.98 | 0.75 | 1.00 |
| m.a.e. | 0.08 | 0.09 | 0.08 |
| Tests based on both valuation and investment Euler equation moments |  |  |  |
| $\chi^{2}$ | 7.95 | 7.78 | 6.12 |
| d.f. | 18 | 18 | 18 |
| p-val | 0.98 | 0.98 | 1.00 |

Table 3 : Valuation Errors
Results are from one-stage GMM with an identity weighting matrix on the investment Euler equation and the valuation equation moments given by equations (10) and (11). The valuation equation error, $e_{i}^{V}$, is defined in equation (13).

|  | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H-L |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Panel A: The $B / M$ deciles |  |  |  |  |  |  |  |  |  |  |
| $e_{i}^{V}$ | 2.73 | 0.70 | 0.07 | -0.30 | -0.72 | -0.91 | -1.03 | $-1.07$ | -1.01 | -0.94 | -3.67 |
| $[t]$ | 3.76 | 1.94 | 0.18 | -0.72 | -1.46 | -1.72 | -1.82 | -1.90 | -1.91 | -2.05 | -2.62 |
|  | Panel B: The asset growth deciles |  |  |  |  |  |  |  |  |  |  |
| $e_{i}^{V}$ | 0.24 | -0.01 | $-0.23$ | -0.21 | $-0.21$ | -0.01 | 0.20 | 0.33 | 0.06 | $-0.17$ | $-0.41$ |
| $[t]$ | 1.56 | -0.09 | -1.63 | -1.35 | -2.17 | -0.14 | 1.99 | 1.75 | 0.38 | $-0.73$ | -1.68 |
|  | Panel C: The $R O A$ deciles |  |  |  |  |  |  |  |  |  |  |
| $e_{i}^{V}$ | 1.55 | -0.50 | $-0.70$ | -0.69 | -0.71 | $-0.87$ | $-0.80$ | $-0.53$ | 0.08 | 2.22 | 0.66 |
| [t] | 1.47 | -0.97 | $-1.17$ | -1.11 | -1.18 | -1.34 | -1.17 | -0.88 | 0.18 | 3.21 | 0.74 |

Table 4 : In-sample and Out-of-sample Expected Return Errors, the Dynamic Investment Model
In-sample expected return errors, $\alpha_{Q}$, are from one-stage GMM with an identity weighting matrix on the expected return moments given by equation (15) per Liu, Whited, and Zhang (2009). To calculate the out-of-sample expected return errors, $\widehat{\alpha_{Q}}$, we first estimate the model parameters using the investment Euler equation and the valuation equation moments given by equations (10) and (11). We then use the parameter estimates and observed portfolio characteristics to construct the levered investment return. Finally, we calculate $\widehat{\alpha_{Q}}$ as the average stock returns minus the average levered investment returns as in equation (16). To calculate the recursive out-of-sample expected return errors, $\widetilde{\alpha_{Q}}$, we estimate the model parameters recursively using an expanding window. At year $t$ we use all the sample observations up to year $t$ to estimate the parameters using the expected return moments. We then use the parameter estimates and observed portfolio characteristics dated year $t$ and $t+1$ to construct the levered investment return. Finally, we calculate the return errors as the stock returns from year $t$ to $t+1$ minus the levered investment returns constructed in this way, and report $\widetilde{\alpha_{Q}}$ as the average return errors.

|  | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H-L |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Panel A: The $B / M$ deciles |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{Q}$ | -2.35 | -2.19 | -1.79 | 1.10 | 1.43 | 2.00 | 3.84 | 0.56 | 2.29 | -2.94 | -0.59 |
| [t] | -1.23 | -1.81 | -1.18 | 1.01 | 1.04 | 1.53 | 1.92 | 0.31 | 0.99 | -1.69 | -0.41 |
| $\widehat{\alpha_{Q}}$ | 4.15 | 5.26 | 5.67 | 8.59 | 8.83 | 9.34 | 10.84 | 8.30 | 11.55 | 10.90 | 6.74 |
| [t] | 0.76 | 0.89 | 0.94 | 1.26 | 1.28 | 1.27 | 1.37 | 1.09 | 1.28 | 0.81 | 4.36 |
| $\widetilde{\alpha_{Q}}$ | -8.52 | -4.76 | $-1.80$ | 0.46 | 1.56 | 2.17 | 5.02 | 4.14 | 7.11 | 6.77 | 15.29 |
| [t] | -2.19 | -1.34 | $-0.52$ | 0.12 | 0.53 | 0.64 | 1.37 | 1.00 | 1.38 | 1.12 | 2.54 |
|  | Panel B: The asset growth deciles |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{Q}$ | -5.10 | -0.28 | 1.47 | 4.81 | 4.44 | 2.91 | 2.83 | $-1.00$ | $-0.21$ | -4.85 | 0.25 |
| [t] | -1.79 | -0.26 | 0.99 | 2.00 | 1.85 | 2.00 | 2.35 | $-1.13$ | -0.21 | -2.22 | 0.13 |
| $\widehat{\alpha_{Q}}$ | -20.25 | -10.46 | -5.84 | 1.58 | 3.34 | 5.56 | 8.87 | 6.20 | 6.99 | 4.52 | 24.77 |
| [t] | -2.31 | -2.09 | $-2.22$ | 0.67 | 1.30 | 1.76 | 2.38 | 1.98 | 1.87 | 1.32 | 9.61 |
| $\widetilde{\alpha_{Q}}$ | 11.54 | 8.37 | 5.05 | 4.63 | 3.99 | 0.42 | $-1.35$ | $-3.88$ | $-2.78$ | -8.55 | -20.09 |
| [t] | 1.35 | 1.82 | 1.31 | 1.29 | 1.07 | 0.16 | -0.48 | $-1.51$ | $-1.00$ | -2.92 | -2.60 |
|  | Panel C: The ROA deciles |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{Q}$ | 1.24 | 0.75 | -2.31 | -1.11 | -0.65 | 1.41 | 0.73 | 0.57 | $-1.33$ | 1.49 | 0.24 |
| [t] | 0.29 | 0.57 | -0.85 | $-0.36$ | -0.68 | 0.80 | 0.43 | 0.68 | -0.92 | 1.13 | 0.06 |
| $\widehat{\alpha_{Q}}$ | 4.28 | 5.43 | 0.66 | 0.07 | -1.21 | 0.86 | 0.66 | 0.51 | $-1.77$ | 1.47 | -2.81 |
| [t] | 0.47 | 0.60 | 0.06 | 0.01 | -0.12 | 0.08 | 0.07 | 0.06 | $-0.20$ | 0.18 | -0.34 |
| $\widetilde{\alpha_{Q}}$ | 15.76 | 8.20 | 0.34 | -1.17 | 0.20 | -0.24 | -0.41 | $-2.07$ | -3.29 | 1.25 | -14.51 |
| [t] | 1.63 | 1.28 | 0.05 | $-0.26$ | 0.05 | -0.07 | -0.11 | -0.66 | $-1.31$ | 0.38 | -1.74 |

## Table 5: Time Series of Parameter Estimates from Recursively Estimating the Dynamic Investment Model

We estimate the model parameters recursively using a series of expanding windows based on the expected return moments (15) in Panel A and on the two quantity moments (10) and (11) in Panel B. $a$ is the adjustment cost parameter, and $\eta$ is the capital's share parameter. The expanding windows start from 1963 for the $B / M$ and the asset growth deciles and from 1975 for the $R O A$ deciles. At year $t=1988, \ldots, 2007$, we use all the sample observations up to year $t$ to estimate the parameters. The last row of the table reports the full-sample estimation using all the sample observations ending in 2008.


## Table 6 : Industry Analysis

$P / K$ is the time series average of a given industry's equity-to-capital ratio. $a$ is the adjustment cost parameter, and $t_{a}$ is its $t$-statistic. $\eta$ is the capital's share, and $t_{\eta}$ is its $t$-statistic. We estimate $a$ and $\eta$ for each industry using the investment Euler equation moment (10) and the valuation equation moment (11). We report the average stock returns (Average) and their standard errors (Ste) for each individual industry portfolios. We use the CAPM and the Fama-French (1993) model to estimate each industry's cost of equity (CE) and its time series standard error (Ste). For each industry, we perform annual CAPM and the Fama-French three-factor regressions, and define CE as the average of the fitted component in a given regression minus the regression's intercept. To estimate the cost of equity from the investment-based model (Structural), we plug the $a$ and $\eta$ estimates (from matching the quantity moments) and observed portfolio characteristics into the levered investment return equation (14). We report $C E$ as the average of the levered investment returns as well as its time series standard error (Ste). We use the 48 industry classifications as in Fama and French (1997). The industries's short names are as follows. Agric is agriculture. Food is food products. Soda is candy and soda. Beer is alcoholic beverages. Smoke is tobacco products. Toys is recreational products. Fun is entertainment. Books is printing and publishing. Hshld is consumer goods. Clths is apparel. Hlth is healthcare. MedEq is medical equipment. Drugs is pharmaceutical products. Chems is chemicals. Rubbr is rubber and plastic products. Txtls is textiles. BldMt is construction materials. Cnstr is construction. Steel is steel works. FabPr is fabricated products. Mach is machinery. ElcEq is electrical equipment. Misc is miscellaneous. Autos is automobiles and trucks. Aero is aircraft. Ships is shipbuilding and railroad equipment. Guns is defense. Gold is precious metals. Mines is nonmetallic mining. Enrgy is petroleum and natural gas. Util is utilities. Telcm is telecommunications. PerSv is personal services. BusSv is business services. Comps is computers. Chips is electronic equipment. LabEq is measuring and control equipment. Paper is business supplies. Boxes is shipping containers. Trans is transportation. Whlsl is wholesale. Rtail is retail. Meals is restaurants, hotel, and motel. Banks is banking. Insur is insurance. RlEst is real estate. Fin is trading. Utilities and financial firms are included only for the industry analysis. Due to data limitations, the sample for the healthcare industry is from 1971 to 2008 , and the sample for the medical equipment industry and the real estate industry is from 1964 to 2008. The insurance industry does not have observations for 1963 and 1969. The sample for all the other industries is from 1963 to 2008 ( 45 years). To alleviate the impact of extreme outliers (mostly from financials), we winsorize all the industry-year observations of investment-to-capital, sales-to-capital, and market leverage ratios at the 0.5 th and 99.5 th percentiles.

| Panel A: Valuation and technology parameters |  |  |  |  |  | Panel B: Industry costs of equity |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Industry | $P / K$ | $a$ | $\eta$ | $t_{a}$ | $t_{\eta}$ | Realized returns |  | CAPM |  | Fama-French |  | Structural |  |
|  |  |  |  |  |  | Average | Ste | CE | Ste | CE | Ste | CE | Ste |
| Agric | 2.72 | 27.68 | 0.10 | 3.83 | 2.81 | 12.11 | 3.66 | 11.21 | 2.68 | 14.80 | 3.29 | 15.01 | 5.28 |
| Food | 3.07 | 28.79 | 0.18 | 4.77 | 3.45 | 14.80 | 2.57 | 10.59 | 2.28 | 15.63 | 2.78 | 12.09 | 2.07 |
| Soda | 3.56 | 24.91 | 0.34 | 5.25 | 5.22 | 19.75 | 4.30 | 12.97 | 3.03 | 21.65 | 3.65 | 14.98 | 3.77 |
| Beer | 4.54 | 41.92 | 0.44 | 2.93 | 2.55 | 15.01 | 2.98 | 10.50 | 1.96 | 9.89 | 1.95 | 18.39 | 7.78 |
| Smoke | 5.02 | 50.98 | 0.44 | 4.24 | 3.15 | 22.27 | 3.49 | 9.18 | 1.52 | 13.89 | 1.79 | 18.93 | 5.20 |
| Toys | 5.37 | 34.37 | 0.19 | 4.59 | 2.18 | 13.18 | 4.92 | 13.67 | 3.03 | 19.79 | 4.57 | 7.84 | 4.38 |
| Fun | 2.14 | 12.68 | 0.32 | 6.00 | 2.88 | 18.13 | 4.04 | 11.96 | 2.82 | 19.94 | 3.76 | 16.16 | 5.74 |
| Books | 4.62 | 43.91 | 0.27 | 5.03 | 2.70 | 13.67 | 3.71 | 13.19 | 3.14 | 19.11 | 3.50 | 14.15 | 6.69 |
| Hshld | 4.76 | 33.07 | 0.28 | 6.42 | 4.36 | 13.97 | 3.37 | 12.65 | 3.11 | 17.73 | 3.72 | 9.16 | 2.67 |
| Clths | 4.84 | 38.28 | 0.13 | 8.29 | 3.29 | 16.66 | 4.18 | 13.67 | 3.79 | 21.77 | 4.71 | 10.83 | 3.98 |
| Hlth | 1.70 | 13.28 | 0.18 | 3.58 | 1.71 | 22.23 | 6.26 | 13.11 | 2.73 | 20.88 | 6.01 | 10.53 | 6.88 |
| MedEq | 7.36 | 47.82 | 0.38 | 6.10 | 2.62 | 17.65 | 3.74 | 12.10 | 2.87 | 12.65 | 3.38 | 10.90 | 3.16 |
| Drugs | 8.11 | 60.74 | 0.55 | 6.09 | 3.00 | 18.20 | 4.56 | 11.33 | 2.25 | 4.59 | 2.99 | 13.15 | 2.84 |
| Chems | 1.88 | 13.25 | 0.26 | 4.67 | 6.84 | 13.69 | 2.42 | 11.12 | 2.64 | 15.18 | 2.76 | 11.64 | 2.63 |
| Rubbr | 2.17 | 19.10 | 0.20 | 5.38 | 3.85 | 17.31 | 3.72 | 13.00 | 2.81 | 18.14 | 3.72 | 17.37 | 5.22 |
| Txtls | 1.53 | 13.74 | 0.12 | 4.64 | 4.91 | 13.75 | 4.05 | 13.39 | 3.38 | 22.98 | 4.22 | 8.64 | 3.57 |
| BldMt | 1.83 | 16.66 | 0.20 | 4.66 | 4.66 | 13.98 | 3.14 | 12.42 | 2.93 | 19.92 | 3.21 | 10.20 | 2.86 |
| Cnstr | 3.48 | 36.08 | 0.15 | 3.06 | 2.95 | 20.92 | 4.33 | 13.59 | 3.68 | 23.41 | 4.29 | 25.65 | 9.12 |
| Steel | 0.99 | 5.93 | 0.16 | 2.92 | 5.43 | 14.74 | 3.38 | 10.70 | 1.90 | 17.15 | 2.40 | 12.26 | 2.20 |
| FabPr | 1.92 | 15.37 | 0.13 | 6.43 | 3.08 | 15.45 | 3.55 | 12.33 | 3.07 | 15.26 | 3.37 | 16.25 | 6.61 |
| Mach | 3.06 | 26.44 | 0.23 | 5.48 | 4.17 | 16.06 | 2.74 | 11.71 | 2.59 | 14.65 | 2.82 | 14.18 | 3.64 |
| ElcEq | 4.16 | 35.74 | 0.32 | 4.94 | 3.69 | 17.20 | 3.44 | 12.43 | 3.21 | 14.37 | 3.95 | 22.20 | 11.19 |
| Autos | 1.35 | 10.41 | 0.17 | 4.62 | 4.67 | 14.62 | 3.92 | 13.16 | 3.39 | 22.37 | 4.10 | 0.00 | 7.03 |
| Aero | 2.65 | 20.40 | 0.18 | 3.51 | 4.34 | 20.23 | 4.13 | 13.30 | 3.47 | 20.47 | 4.17 | 20.41 | 4.98 |


| Industry | $P / K$ | $a$ | $\eta$ | $t_{a}$ | $t_{\eta}$ | Realized returns |  | CAPM |  | Fama-French |  | Structural |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | Average | Ste | CE | Ste | CE | Ste | CE | Ste |
| Ships | 2.73 | 27.31 | 0.11 | 3.50 | 1.63 | 12.26 | 4.01 | 11.27 | 2.66 | 19.21 | 3.18 | 12.96 | 6.37 |
| Guns | 2.65 | 21.57 | 0.21 | 3.27 | 5.00 | 30.42 | 12.43 | 15.36 | 4.57 | 33.23 | 6.94 | 13.92 | 4.40 |
| Gold | 4.12 | 27.36 | 0.29 | 3.74 | 1.12 | 15.99 | 6.14 | 10.89 | 2.34 | 13.72 | 3.43 | 15.69 | 7.60 |
| Mines | 1.58 | 9.07 | 0.39 | 5.66 | 4.86 | 18.58 | 4.09 | 11.18 | 2.24 | 16.08 | 2.68 | 17.81 | 4.50 |
| Coal | 0.88 | 4.95 | 0.20 | 5.40 | 4.09 | 20.05 | 7.86 | 4.36 | 0.76 | 21.08 | 4.16 | 12.65 | 3.73 |
| Oil | 1.20 | 4.74 | 0.22 | 4.44 | 4.84 | 18.10 | 3.77 | 9.75 | 1.61 | 11.48 | 1.94 | 13.44 | 1.74 |
| Util | 0.54 | 1.31 | 0.29 | 0.80 | 9.33 | 12.11 | 1.89 | 8.71 | 1.39 | 13.09 | 1.62 | 9.62 | 1.39 |
| Telcm | 1.25 | 7.95 | 0.54 | 3.51 | 5.73 | 19.03 | 3.51 | 13.12 | 3.37 | 14.12 | 3.40 | 14.83 | 3.20 |
| PerSv | 1.42 | 7.46 | 0.18 | 5.35 | 2.54 | 16.63 | 4.91 | 14.81 | 4.00 | 23.34 | 5.20 | 10.46 | 6.29 |
| BusSv | 4.45 | 28.94 | 0.39 | 3.63 | 3.05 | 17.48 | 3.98 | 14.28 | 3.79 | 16.82 | 4.51 | 15.30 | 5.76 |
| Comps | 6.30 | 31.08 | 0.43 | 3.85 | 3.38 | 15.86 | 4.25 | 13.76 | 3.58 | 12.29 | 4.75 | 8.67 | 3.37 |
| Chips | 3.55 | 20.03 | 0.39 | 3.27 | 2.38 | 20.69 | 5.14 | 14.70 | 4.00 | 12.73 | 4.98 | 16.38 | 3.59 |
| LabEq | 6.20 | 42.82 | 0.33 | 4.98 | 2.65 | 19.24 | 4.92 | 13.11 | 3.24 | 9.23 | 5.11 | 12.90 | 3.40 |
| Paper | 1.44 | 10.87 | 0.20 | 3.13 | 5.72 | 12.06 | 2.60 | 11.06 | 2.25 | 17.00 | 2.64 | 9.87 | 2.70 |
| Boxes | 1.86 | 14.81 | 0.22 | 8.60 | 5.77 | 16.15 | 3.42 | 11.35 | 2.26 | 16.77 | 2.71 | 11.40 | 1.97 |
| Trans | 0.63 | 1.14 | 0.19 | 1.26 | 6.13 | 16.49 | 3.55 | 13.14 | 3.05 | 20.46 | 3.65 | 11.70 | 1.13 |
| Whlsl | 2.89 | 36.51 | 0.10 | 3.90 | 4.84 | 18.96 | 3.77 | 13.06 | 3.00 | 19.51 | 3.99 | 24.17 | 6.94 |
| Rtail | 2.29 | 17.85 | 0.08 | 5.35 | 3.38 | 16.90 | 4.00 | 13.66 | 3.32 | 20.66 | 3.61 | 9.56 | 2.00 |
| Meals | 1.49 | 10.29 | 0.18 | 5.71 | 2.48 | 13.32 | 3.95 | 11.88 | 2.67 | 19.91 | 3.61 | 5.71 | 2.67 |
| Banks | 7.49 | 149.23 | 1.00 | 5.22 | 2.61 | 17.62 | 3.87 | 12.06 | 2.71 | 19.88 | 2.97 | 111.90 | 55.30 |
| Insur | 8.76 | 84.72 | 0.34 | 4.43 | 2.26 | 24.47 | 5.87 | 12.13 | 2.39 | 21.81 | 3.06 | 86.62 | 47.87 |
| RIEst | 1.10 | 16.80 | 0.35 | 4.27 | 2.96 | 15.11 | 4.42 | 11.82 | 2.71 | 21.18 | 3.52 | 27.23 | 15.82 |
| Fin | 4.45 | 206.37 | 0.75 | 3.60 | 2.63 | 20.59 | 3.66 | 13.57 | 3.37 | 15.77 | 3.38 | 41.52 | 25.07 |
| Other | 2.38 | 20.54 | 0.33 | 3.21 | 2.57 | 17.79 | 3.90 | 12.71 | 3.03 | 16.71 | 3.00 | 17.37 | 5.34 |
| Average | 3.22 | 30.73 | 0.28 |  |  | 17.11 | 4.18 | 12.19 | 2.85 | 17.55 | 3.61 | 17.97 | 7.12 |

Figure 2: Average Predicted Stock Returns versus Average Realized Stock Returns, In-sample Fit of Expected Return Moments, the Dynamic Investment Model

Panel A: The $B / M$ deciles


Panel B: The asset growth deciles
Panel C: The $R O A$ deciles



Figure 3 : Average Predicted Stock Returns versus Average Realized Stock Returns, Out-of-sample Fit of Expected Return Moments Using Parameters Estimated from the Quantity Moments, the Dynamic Investment Model

Panel A: The $B / M$ deciles


Panel B: The asset growth deciles


Panel C: The $R O A$ deciles


Figure 4 : Average Predicted Stock Returns versus Average Realized Stock Returns, Out-of-sample Fit of Expected Return Moments Using Parameters Estimated Recursively from the Expected Return Moments, the Dynamic Investment Model

Panel A: The $B / M$ deciles


Panel B: The asset growth deciles


Panel C: The ROA deciles


Figure 5 : The Cross-sectional Relations between Technology Parameters and Average Market Equity-to-Capital Ratios across the 48 Industries

We estimate the adjustment cost parameter, $a$, and the capital's share parameter, $\eta$, by using GMM to fit the investment Euler equation moment (10) and the valuation equation moment (11) for each industry. We use Fama and French's (1997) 48 industry classifications. $P / K$ is the average equity-to-capital ratio for a given industry. Due to data limitations, the sample for the healthcare industry is from 1971 to 2008, and the sample for the medical equipment industry and the real estate industry is from 1964 to 2008. The sample for all the other industries is from 1963 to 2008 ( 45 years). To alleviate the impact of extreme outliers for the banking, insurance, real estate, and trading industries, we implement a $0.5 \%-99.5 \%$ winsorization for industry-level investment-tocapital, sales-to-capital, and market leverage ratios for these four industries.



[^0]:    ${ }^{1}$ The root of investment-based asset pricing can be traced to Fisher (1930), who uses an intertemporal general equilibrium model to reconcile two prior explanations of the interest rate: one based on consumer preferences and the other based on production opportunity. Subsequent writings that have further elaborated the opportunity-based determination of the interest rate include Hirshleifer $(1965,1970)$ and Fama and Miller (1972).
    ${ }^{2}$ Gomes, Kogan, and Zhang (2003) build a general equilibrium model to link expected stock returns to firm characteristics such as size and book-to-market. Carlson, Fisher, and Giammarino (2004) show that operating leverage helps explain the value premium in a real options model. Carlson, Fisher, and Giammarino (2006) show that a related model also helps explain the negative relation between equity issues and subsequent stock returns. Zhang (2005) and Cooper (2006) develop dynamic investment models to show that value stocks can be riskier than growth stocks due to costly reversibility and time-varying price of risk. Bazdrech, Belo, and Lin (2009) introduce labor adjustment costs into investment-based asset pricing. Li, Livdan, and Zhang (2009) introduce financing costs of raising equity into a dynamic investment model and show it can reproduce the negative relation between equity issuance and average stock returns. Livdan, Sapriza, and Zhang (2009) embed collateral constraints into a dynamic investment model to examine the relation between financing constraints and stock returns. Garlappi and Yan (2010) introduce financial distress into the dynamic investment model to study its relation with stock returns. Gomes and Schmid (2010) model defaultable bonds and examine the relation between leverage and stock returns. Tuzel (2010) studies the relation between corporate real estate holdings and stock returns.
    ${ }^{3}$ Gomes, Yaron, and Zhang (2006) introduce financing constraints into the Cochrane (1996) framework and show these constraints matter for the cross-section of returns. Lyandres, Sun, and Zhang (2008) show that adding an investment factor into the capital asset pricing model (CAPM) and the Fama-French (1993) three-factor model helps reduce the magnitude of the underperformance following equity issues. Xing (2008) shows that an investment growth factor helps explain the book-to-market effect. Belo (2010) uses marginal rate of transformation from firms' first-order conditions as a stochastic discount factor in asset pricing tests. Chen, Novy-Marx, and Zhang (2010) propose a new factor model consisting of the market factor, an investment factor, and a return-on-assets factor, and show that it helps explain several anomalies that are otherwise hard to explain using the CAPM and the Fama-French model. Jermann (2010) studies the equity premium as implied by firms' first-order conditions. Wu, Zhang, and Zhang (2010) show that the negative relation between capital investment and average returns also helps explain the accrual anomaly.

[^1]:    ${ }^{4}$ Hennessy (2004) incorporates debt into a dynamic investment model and quantifies the impact of debt overhang on optimal investment. Hennessy and Whited (2005) and Strebulaev (2007) develop dynamic trade-off models of capital structure. Hennessy and Whited (2007) apply simulated method of moments on a dynamic investment model to infer the magnitude of financing costs. Albuquerque and Wang (2008) introduce the separation of ownership and control into a dynamic investment model to study asset pricing implications of imperfect investor protection. Bolton, Chen, and Wang (2009) develop a dynamic model of corporate investment and risk management for a financially constrained firm. DeMarzo, Fishman, He, and Wang (2009) introduce dynamic agency into the $q$-theory of investment.

[^2]:    ${ }^{5}$ In consumption-based asset pricing studies, the stochastic discount factor is often unique because of the complete markets assumption. However, in valuation studies, it makes more sense to use firm-specific discount rates because risk is likely to differ across firms (see, for example, Lundholm and Sloan (2007, chapter 9) and Penman (2010, chapter 18)). Also, we implicitly assume that firms take their discount rates as given when maximizing the market value of equity. As such, the same investment Euler equation and the valuation equation hold even with firm-specific discount rates.

[^3]:    ${ }^{6}$ Book equity is stockholder equity plus balance sheet deferred taxes (Compustat annual item TXDB if available) and investment tax credit (item ITCB if available) plus post-retirement benefit liabilities (item PRBA if available) minus the book value of preferred stock. Depending on data availability, we use redemption (item PSTKRV), liquidation (item PSTKL), or par value (item PSTK), to represent the book value of preferred stock. Stockholder equity is equal to Moody's book equity (from Kenneth French's Web site), the book value of common equity (item CEQ) plus the par value of preferred stock, or the book value of assets (item AT) minus total liabilities (item LT).

[^4]:    ${ }^{7}$ Albeit related, our definition of the value spread differs from that of Cohen, Polk, and Vuolteenaho (2003), who define the value spread as the $\log$ book-to-market equity of the high- $B / M$ decile minus the log book-to-market equity

