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INCOME DIFFERENCES AND PRICES OF TRADABLES

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ABSTRACT

I study the positive relationship between prices of tradable goods and per-capita income. I present a heterogeneous-firm model of international trade with non-homothetic consumer preferences that outperforms existing frameworks in accounting for observed cross-country variation in prices along three key dimensions. The model yields a testable prediction that relates prices to macroeconomic variables. I use the prediction to provide an unbiased estimate of the price elasticity with respect to per-capita income from unique data featuring prices of 180 identical goods sold online in eighteen countries. The estimated elasticity is 0.08 and the result is robust across a variety of specifications.

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1 Introduction

A large empirical literature has documented strong and persistent deviations from the law of one price for tradable goods. Evidence stems from studies that examine cross-country price levels of aggregate categories of goods, to tracking individual products across markets.[1] Moreover, tradable-goods prices are systematically positively related to countries' per-capita incomes. Hsieh and Klenow (2007) demonstrate that this relationship is particularly pronounced among tradable consumption goods. Since these goods comprise consumption bundles of individuals and their prices directly affect consumer welfare, it is of central importance to understand the underlying mechanisms that affect the behavior of tradable consumption-good prices across countries.

In this paper, I argue that variable mark-ups are a key contributor toward the observed positive relationship between price and per-capita income. To begin, I introduce non-homothetic consumer preferences over varieties into a general equilibrium model of international trade. Due to the presence of trade frictions, monopolistically competitive firms with varying productivity levels are able to supply their products at destination-specific prices. With non-homothetic preferences, different levels of consumer income result in non-constant shares of expenditure on different varieties, thus yielding varying price elasticities of demand for a given positively-consumed variety across destinations. In particular, rich countries' consumers are less responsive to price changes than those of poor ones, so firms find it optimal to price identical varieties higher in more affluent markets.

The model's prediction that prices of tradable goods are positively linked to per-capita income is shared by two alternative frameworks in the international-trade literature. First, Alessandria and Kaboski (2011) argue that high-wage earners have a high opportunity cost of searching for goods, which allows firms to charge high prices for identical goods in rich countries. While their model delivers a positive relationship between price and per-capita income, it ignores the effect of competition, or market size, on prices. On the contrary, the model introduced in the present paper predicts that firms suffer competitive pressures in larger markets and extract lower mark-ups there.

Second, within a Lancaster (1979) framework, Hummels and Lugovskyy (2009) argue that agents who experience an increase in income raise their consumption, which makes them more finicky and therefore more willing to pay a higher price in order to get closer to consuming their ideal variety. Moreover, a rise in per-capita income raises total income, thus attracting more firms to enter the market, which increases competition and depresses prices. Hence, the model predicts that, after controlling for total income, prices rise in per-capita income. But, according to the model, the (positive) elasticity of price with respect to per-capita income falls short of the (negative) elasticity of price with respect to total income. In contrast, the model introduced in

[1] Hsieh and Klenow (2007) and Alessandria and Kaboski (2011) employ prices of aggregate good categories, Schott (2004), Hummels and Klenow (2005), and Hummels and Lugovskyy (2009) study unit values from disaggregate trade data, Crucini et al. (2005a), Crucini et al. (2005b), and Crucini and Shintani (2008) use retail prices of products with identical characteristics, Goldberg and Verboven (2001) and Goldberg and Verboven (2005) track car prices in Europe, Ghosh and Wolf (1994) and Haskel and Wolf (2001) examine prices of the Economist magazine and IKEA products, respectively.

the present paper predicts that the positive effect of per-capita income on the price of a variety dominates the negative effect of total income on prices.

In order to test the predictions of the three models, I construct a unique database that features prices of 180 identical apparel products sold online in eighteen European countries. Tracking identical goods enables me to directly measure price discrimination on the basis of varying demand elasticities in the absence of product-quality differences, which may otherwise account for the positive relationship between tradable-goods prices and per-capita income, as Schott (2004) argues. Moreover, focusing on prices of goods sold online allows me to suppress Balassa-Samuelson effects—which refer to the contributions of non-tradable (distribution) channels to tradable-goods prices—emphasized by Burstein et al. (2003) and Crucini and Yilmazkuday (2009).

The empirical results suggest that the non-homothetic model is the only one among the alternative frameworks that can account for the observed price variation with respect to per-capita income, size, and total income of destinations. In particular, after controlling for the cost to ship goods to different markets, I find that prices rise in the per-capita income and fall in the population size of the destination. This result is in line with empirical findings for the US concrete industry, reported by Syverson (2004). Furthermore, after controlling for shipping costs, I find that prices rise in destinations' per-capita incomes with a higher elasticity than they fall in markets' total incomes. Hummels and Lugovskyy (2009) document a similar stylized fact using unit-value data to approximate prices of varieties across a large set of countries.

Given the success of the non-homothetic model in accounting for the behavior of prices across countries, I use it to estimate the elasticity of price with respect to per-capita income from the dataset described above. Estimating the elasticity is straightforward because the model yields a testable prediction that relates prices to measurable macroeconomic variables. In particular, for a pair of countries, the relative price of an identical item varies with destinations' relative per-capita incomes, trade barriers, and import shares. Using this prediction, I provide an unbiased estimate of the elasticity of price with respect to per-capita income. The benchmark estimate amounts to roughly 0.08 and lies within a tight range of 0.0684 to 0.0839 across a variety of empirical exercises. Thus, doubling a country's per-capita income yields an eight-percent rise in prices of tradables.

To summarize, in this paper, I outline a parsimonious and highly tractable heterogeneous-firm model of international trade that relates prices of tradable goods to per-capita income differences. I present direct support for the model's mechanism, which builds on non-homothetic consumer preferences, from a unique database that features prices of identical products sold via the Internet. Finally, I use the model's testable prediction to estimate a key parameter in the international-trade and development literatures—the elasticity of price with respect to per-capita income. Overall, the paper contributes toward the understanding of the positive relationship between per-capita income and the price level of tradable consumption goods, which is not only key in determining relative investment and growth patterns across countries, as argued by Hsieh and Klenow (2007), but is also central in measuring consumer welfare, as emphasized by Broda and Romalis (2009).

The paper is organized as follows: Section 2 outlines the model; Section 3 derives predictions of the model and relates them to the existing literature; Section 4 empirically tests the model against its alternatives and proceeds to estimate price elasticities from unique data; Section 5 concludes.

2 Model

2.1 Consumer Problem

I consider a world that consists of a finite number of countries, I, engaged in trade of varieties of a final good. Let i represent an exporter and j an importer.

I assume that country j is populated by identical consumers of measure L_j who have preferences over varieties of a good. Varieties originating from different countries enter symmetrically in a consumer's utility function according to the following rule

$$U_j^c = \sum_{i=1}^{I} \int_{\omega \in \Omega_{ij}} \log(q_{ij}^c(\omega) + \bar{q}) d\omega,$$

where $q_{ij}^c(\omega)$ is individual consumption of variety ω from country i in j and $\bar{q} > 0$ is a (non-country-specific) constant. To ensure that the utility function is well defined, I assume that, for all j, $\Omega_j \equiv \sum_{i=1}^{I} \Omega_{ij} \subseteq \bar{\Omega}$, where $\bar{\Omega}$ is a compact set containing all potentially-produced varieties.

Notice that the preference relation described above is non-homothetic. Moreover, marginal utility from each variety, $(q_{ij}^c(\omega)+\bar{q})^{-1}$, is bounded at any level of consumption. Hence, a consumer may not have positive demand for all varieties.

Let y_j denote consumer income in j. Then, demand for variety ω from i consumed in a positive amount in j, $q_{ij}(\omega) > 0$, is given by[2]

$$q_{ij}(\omega) = L_j \left\{ \frac{y_j + \bar{q} P_j}{N_j p_{ij}(\omega)} - \bar{q} \right\}. \tag{1}$$

In the expression above, N_j is the total measure of varieties consumed in j,

$$N_j = \sum_{i=1}^{I} N_{ij}, \tag{2}$$

where N_{ij} is the measure of the set Ω_{ij}, which contains varieties originating from i.

Furthermore, P_j is an aggregate price statistic summarized by

$$P_j = \sum_{i=1}^{I} \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) d\omega. \tag{3}$$

[2]See Appendix A.1 for derivation.

2.2 Firm Problem

The environment is static. Each variety is produced by a single firm using constant-returns-to-scale technology. Labor is the only factor of production. Following Melitz (2003), I assume that firms differ in their productivity, ϕ, and country of origin, i.

In every country i, there exists a pool of potential entrants who pay a one-time cost, $f_e > 0$, which entitles them to a single productivity draw from a distribution, $G(\phi)$, with support $[b_i, \infty)$. A measure J_i of firms that are able to cover their marginal cost of production enter. However, only a subset of productive entrants, N_{ij}, produce and sell to market j. These firms are able to charge a low enough price so as to generate non-negative demand in expression (1), while making non-negative profits. Thus, a subset of entrants immediately exit. Hence, in equilibrium, the expected profit of an entrant is zero. Aggregate profit rebates to each consumer are therefore also zero. Assuming that each consumer has a unit labor endowment—which, when supplied (inelastically) to the local labor market earns a wage rate of w_j—per-capita income necessarily equals w_j.

Having described the market structure, I proceed to set up an operating firm's maximization problem. Let the production function of a firm with productivity draw ϕ be $x(\phi) = \phi l$, where l is the amount of labor used toward the production of final output. Moreover, assume that each firm from country i wishing to sell to destination j faces an iceberg transportation cost incurred in terms of labor units, $\tau_{ij} \geq 1$, with $\tau_{ii} = 1$ $(\forall i)$. An operating firm must choose the price of its good p, accounting for the demand for its product q. I consider a symmetric equilibrium where all firms of type ϕ from i choose identical optimal pricing rules. Thus, I can index each variety by the productivity and the country of origin of its producer, which allows me to rewrite individual consumer and country demand as follows

$$q_{ij}^{c}(\phi) = \frac{w_j + \bar{q}P_j}{N_j p_{ij}(\phi)} - \bar{q}, \tag{4}$$

$$q_{ij}(\phi) = L_j \left\{ \frac{w_j + \bar{q}P_j}{N_j p_{ij}(\phi)} - \bar{q} \right\}. \tag{5}$$

Using demand from (5), the profit maximization problem of a firm with productivity ϕ from country i that is considering to sell to destination j becomes

$$\pi_{ij}(\phi) = \max_{p_{ij} \geq 0} \quad p_{ij} L_j \left\{ \frac{w_j + \bar{q}P_j}{N_j p_{ij}} - \bar{q} \right\} - \frac{\tau_{ij} w_i}{\phi} L_j \left\{ \frac{w_j + \bar{q}P_j}{N_j p_{ij}} - \bar{q} \right\}. \tag{6}$$

To solve this problem, each firm takes as given the measure of competitors N_j and the aggregate price statistic P_j. Taking first-order conditions, the resulting optimal price that a firm charges for its variety which is supplied in a positive amount is given by

$$p_{ij}(\phi) = \left(\frac{\tau_{ij} w_i}{\phi} \frac{w_j + \bar{q}P_j}{N_j \bar{q}} \right)^{\frac{1}{2}}. \tag{7}$$

2.3 Productivity Thresholds and Firm Mark-Ups

As noted earlier, in this model, not all firms serve all destinations. In particular, for any pair of source and destination countries, i and j, only firms originating from country i with productivity draws $\phi \geq \phi_{ij}^*$ sell to market j, where ϕ_{ij}^* is a productivity threshold defined by[3]

$$\phi_{ij}^* = \sup_{\phi \geq b_i} \{\pi_{ij}(\phi) = 0\}.$$

Thus, a productivity threshold is the productivity draw of a firm that is indifferent between serving a market or not, namely one whose variety's price barely covers the firm's marginal cost of production and delivery,

$$p_{ij}\left(\phi_{ij}^*\right) = \frac{\tau_{ij} w_i}{\phi_{ij}^*}. \tag{8}$$

The price a firm would charge for its variety, however, is limited by the variety's demand, which diminishes as the variety's price rises. In particular, it is the case that consumers in destination j are indifferent between buying the variety of type ϕ_{ij}^* or not. To see this, from (5), notice that consumers' demand is exactly zero for the variety whose price satisfies

$$p_{ij}\left(\phi_{ij}^*\right) = \frac{w_j + \bar{q} P_j}{N_j \bar{q}}. \tag{9}$$

Combining expressions (8) and (9) yields a simple characterization of the threshold

$$\phi_{ij}^* = \frac{\tau_{ij} w_i N_j \bar{q}}{w_j + \bar{q} P_j}. \tag{10}$$

Substituting (10) in (7), the optimal pricing rule of a firm with productivity draw $\phi \geq \phi_{ij}^*$ becomes

$$p_{ij}(\phi) = \underbrace{\left(\frac{\phi}{\phi_{ij}^*}\right)^{\frac{1}{2}}}_{\text{mark-up}} \underbrace{\frac{\tau_{ij} w_i}{\phi}}_{\text{marginal cost}}. \tag{11}$$

Expression (11) shows that mark-ups vary along two dimensions in this model. First, more productive firms charge higher mark-ups over marginal cost. This prediction is in line with the behavior of Slovenian manufacturers, as documented by Loecker and Warzynski (2009). Second, firms' prices and mark-ups vary systematically with market characteristics, which are summarized by the threshold firms must surpass in order to serve a destination. The thresholds are, in turn, equilibrium objects. Consequently, I proceed to characterize the equilibrium of the model.

[3] I restrict f_e to ensure that $b_i \leq \phi_{ij}^* (\forall i, j)$.

2.4 Equilibrium of the World Economy

The subset of entrants from i who surpass the productivity threshold ϕ_{ij}^* serve destination j. These firms, denoted by N_{ij}, satisfy

$$N_{ij} = J_i[1 - G(\phi_{ij}^*)]. \tag{12}$$

Let $g(\phi)$ be the pdf corresponding to the productivity cdf $G(\phi)$. Then, the conditional density of firms operating in j is

$$\mu_{ij}(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi_{ij}^*)} & \text{if } \phi \geq \phi_{ij}^*, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

With these definitions in mind, the aggregate price statistic in (3) can be rewritten as

$$P_j = \sum_{i=1}^{I} N_{ij} \int_{\phi_{ij}^*}^{\infty} p_{ij}(\phi)\mu_{ij}(\phi)d\phi. \tag{14}$$

Using the above objects, total sales to country j by firms originating in country i become

$$T_{ij} = N_{ij} \int_{\phi_{ij}^*}^{\infty} p_{ij}(\phi)x_{ij}(\phi)\mu_{ij}(\phi)d\phi. \tag{15}$$

Furthermore, individual firm profits are the sum of profit flows from each destination a firm sells to. Hence, the average profits of firms originating from country i are

$$\pi_i = \sum_{j=1}^{I}[1 - G(\phi_{ij}^*)] \int_{\phi_{ij}^*}^{\infty} \pi_{ij}(\phi)\mu_{ij}(\phi)d\phi,$$

where potential profits from destination j are weighted by the probability that they are realized, $1 - G(\phi_{ij}^*)$. The average profit, in turn, barely covers the fixed cost of entry

$$w_i f_e = \sum_{j=1}^{I}[1 - G(\phi_{ij}^*)] \int_{\phi_{ij}^*}^{\infty} \pi_{ij}(\phi)\mu_{ij}(\phi)d\phi. \tag{16}$$

Finally, i's consumers' income, spent on final goods that are produced at home and abroad, is

$$w_i L_i = \sum_{j=1}^{I} T_{ji}. \tag{17}$$

Equilibrium. For $i, j = 1, ..., I$, given $\tau_{ij}, L_j, b_i, f_e, \bar{q}$, and a productivity distribution $G(\phi)$, an equilibrium is a set of total measures of firms serving j $\hat{N}_j$; productivity thresholds $\hat{\phi}_{ij}^*$; measures of firms from i serving j $\hat{N}_{ij}$; conditional densities of firms from i serving j $\hat{\mu}_{ij}(\phi)$; aggregate price statistics $\hat{P}_j$; total sales of firms from i serving j $\hat{T}_{ij}$; wage rates $\hat{w}_i$; measures of entrants $\hat{J}_i$; and, $\forall\phi \in [\phi^*_{ij}, \infty)$, per-consumer allocations $\hat{q}^c_{ij}(\phi)$, country allocations $\hat{q}_{ij}(\phi)$, firm pricing rules $\hat{p}_{ij}(\phi)$, firm production rules $\hat{x}_{ij}(\phi)$, and firm profits $\hat{\pi}_{ij}(\phi)$, such that: (i) $\hat{q}^c_{ij}(\phi)$ is given by (4) and solves the individual consumer's problem; (ii) $\hat{q}_{ij}(\phi)$ is given by (5) and satisfies a country's aggregate demand for a variety; (iii) $\hat{p}_{ij}(\phi)$ is given by (7) and solves the firm's problem; (iv) $\hat{x}_{ij}(\phi)$ satisfies goods' markets clearing $\hat{q}_{ij}(\phi) = \hat{x}_{ij}(\phi)$; (v) $\hat{\pi}_{ij}(\phi)$ is given by (6); (vi) $\hat{N}_j$, $\hat{\phi}^*_{ij}$, $\hat{N}_{ij}$, $\hat{\mu}_{ij}(\phi)$, $\hat{P}_j$, $\hat{T}_{ij}$, $\hat{w}_i$, $\hat{J}_i$ jointly satisfy (2), (10), (12), (13), (14), (15), (16), and (17).

3 Model Predictions

In this section, I derive the model's predictions regarding the behavior of firms within and across countries. I then discuss how the model relates to the existing literature.

In order to analytically solve the model and to derive stark predictions at the firm and aggregate levels, I follow Chaney (2008) and assume that firm productivities are drawn from a Pareto distribution with cdf $G(\phi) = 1 - b_i^\theta/\phi^\theta$, pdf $g(\phi) = \theta b_i^\theta/\phi^{\theta+1}$, and shape parameter $\theta > 0$. The support of the distribution is $[b_i, \infty)$, where b_i summarizes the level of technology in country i.[4] Moreover, varying levels of technology are related to per-capita income differences across countries. In particular, a relatively high b_i represents a more technologically-advanced country. Such a country is characterized by relatively more productive firms, whose marginal costs of production are low, and by richer consumers, who enjoy higher wages.[5] The sections below study how exporters respond to such market conditions.

3.1 Price Discrimination

In this section, I discuss the predictions of the model regarding the variation of prices with respect to three key country characteristics: per-capita income, size, and income. I relegate all the proofs to Appendix A.3.

3.1.1 Prices and Per-Capita Income

Expression (11) above demonstrated that firm mark-ups across markets depend crucially on the productivity thresholds of the destinations. Under the assumption that firm productivities are Pareto-distributed, I can characterize these thresholds via the following expression[6]

[4] All predictions derived in the remainder of the paper are identical if instead I set $b_i = b_j = b$ for all $i \neq j$ and let firms' production functions be $x_i(\phi) = A_i\phi l$, where A_i is country-specific total factor productivity.

[5] In Appendix A.2, I show that, in general equilibrium, a relative increase in b_i increases the relative wage in i.

[6] See Appendix A.2 for derivations.

$$\phi_{ij}^* = \frac{\bar{q}^{\frac{1}{\theta+1}}\tau_{ij}w_i}{[(\theta+1)f_e(1+2\theta)]^{\frac{1}{\theta+1}}}\left[\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{v\neq j}\frac{L_v b_v^\theta}{w_j\left(\tau_{vj}w_v\right)^\theta}\right]^{\frac{1}{\theta+1}}. \tag{18}$$

Consider an increase in the per-capita income of destination j, w_j, while keeping all other objects fixed. A rise in w_j lowers the threshold in (18), which raises the mark-up in (11). Intuitively, recall that the marginal utility of a variety is bounded at any level of consumption. Since a tiny amount of consumption of a variety does not give infinite increase in utility, the consumer spends her limited income on the subset of potentially-produced items whose prices do not exceed marginal valuations. An increase in an individual's income makes new varieties affordable and the consumer expands her consumption bundle. Hence, the model yields a positive link between countries' per-capita incomes and the set of purchased varieties, which is in line with empirical findings by Jackson (1984), Hunter and Markusen (1988), Hunter (1991), and Movshuk (2004).

A rise in per-capita income does not only expand an agent's consumption bundle, but it also results in an increase in consumption of each positively-consumed variety. To see this, substitute (10) and (11) into (4) to obtain the following expression for an individual's consumption of an item

$$q_{ij}^c(\phi) = \bar{q}\left[\left(\frac{\phi}{\phi_{ij}^*}\right)^{\frac{1}{2}} - 1\right]. \tag{19}$$

(19) falls in the cutoff productivity, so the quantity consumed rises in individual income. But, variations in consumption change elasticities of substitution and consequently affect prices of varieties. The elasticity of substitution for any two positively-consumed varieties in j, that are produced by firms with productivities ϕ_1 and ϕ_2, which originate from countries i and v respectively, is

$$\sigma_{q_{ij}^c(\phi_1),q_{vj}^c(\phi_2)} = 1 + \frac{\bar{q}}{2}\left[\frac{1}{q_{ij}^c(\phi_1)} + \frac{1}{q_{vj}^c(\phi_2)}\right].$$

As the consumer becomes richer, she consumes more of each variety, which drives down the elasticity of substitution between positively-consumed varieties. Prices of these varieties rise in response.

Another intuitive explanation of the price increase involves ordering varieties according to their "importance" to the consumer. As consumer income rises, new varieties produced by less productive firms are added to the consumption set. Conversely, if individual income were to fall, the new varieties are the first to be dropped from a consumer's bundle. Thus, the preference relation is "hierarchic"—a term introduced to the consumer-choice literature by Jackson (1984). The newly-added varieties are less important than the previously-consumed ones, which results in a fall in the demand elasticities of the latter. Hence, as income rises, prices of all previously-consumed varieties also rise.

To see this, let the (absolute value of the) price elasticity of demand for variety (ϕ, i) in j be

$$\epsilon_{ij}(\phi) = \left[1 - \left(\frac{\phi}{\phi_{ij}^{*}}\right)^{-\frac{1}{2}}\right]^{-1}. \tag{20}$$

If the per-capita income in market j rises, the productivity threshold falls. According to expression (20), the demand for a variety becomes less elastic. However, the elasticity of demand is reflected in the price of the item, which can be seen by combining expressions (11) and (20) to obtain

$$p_{ij}(\phi) = \frac{\tau_{ij} w_i}{\phi} \frac{1}{1 - [\epsilon_{ij}(\phi)]^{-1}}. \tag{21}$$

As consumer income rises, demand becomes less elastic, which allows firms to raise their prices.

Having described the behavior of prices within a country, it is easy to understand how prices of identical items vary across countries. Consider a firm with productivity draw ϕ, originating from country i and selling an identical variety to markets j and k, that is, $\phi \geq \max[\phi_{ij}^{*}, \phi_{ik}^{*}]$. Using expression (11), the relative price this firm charges across the two markets is given by

$$\frac{p_{ij}(\phi)}{p_{ik}(\phi)} = \frac{\tau_{ij}}{\tau_{ik}} \left(\frac{\phi_{ij}^{*}}{\phi_{ik}^{*}}\right)^{-\frac{1}{2}}. \tag{22}$$

Proposition 1 describes how the relative price relates to the countries' relative per-capita incomes.

Proposition 1. *If trade barriers obey the triangle inequality, $(\forall j, k, \upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$, then the relative price of a variety sold in two markets is strictly rising in the markets' relative per-capita incomes.*

Intuitively, consumers in rich countries are less responsive to price changes than consumers in poor ones. A firm exploits this opportunity, amid trade barriers that segment the markets, and charges a high mark-up in the affluent destination.

3.1.2 Prices, Market Size, and Market Income

Consider an increase in the population size of destination j, L_j, while keeping all other objects fixed. The productivity threshold in (18) rises, thus lowering the mark-up in expression (11). Intuitively, as the country becomes larger, it attracts more entrants. Hence, the market becomes more competitive, which forces a surviving firm to reduce the price of its variety there.

Furthermore, multiplying and dividing the first term in the bracket in (18) by w_j allows one to relate the threshold in j to country j's total income, $Y_j \equiv w_j L_j$,

$$\phi_{ij}^{*} = \frac{\bar{q}^{\frac{1}{\theta+1}} \tau_{ij} w_i}{[(\theta+1) f_e (1+2\theta)]^{\frac{1}{\theta+1}}} \left[\frac{Y_j b_j^{\theta}}{\tau_{jj}^{\theta} w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_{\upsilon} b_{\upsilon}^{\theta}}{w_j \left(\tau_{\upsilon j} w_{\upsilon}\right)^{\theta}}\right]^{\frac{1}{\theta+1}}. \tag{23}$$

Consider an increase in the income of destination j, Y_j, for a given level of per-capita income, w_j. The productivity threshold in (23) rises, thus lowering the mark-up in (11). Clearly, when one controls for a country's per-capita income, a rise in income must be driven by a rise in the country's population, which lowers the price of a variety, as discussed above. Conversely, keep the total income fixed, and consider a rise in per-capita income. The productivity threshold in (23) falls, thus raising the mark-up in (11).

Finally, using (23), one can verify that the sum of the elasticities of thresholds with respect to per-capita income, w_j, and total income, Y_j, is negative. Thus, in (11), the positive effects of per-capita income on prices dominate the negative effects caused by higher income.

Once again, having described the behavior of prices within a country, it is easy to understand how prices of identical items vary across countries. Proposition 2 describes how the relative price relates to the destinations' relative sizes.

Proposition 2. *For any two countries, j and k, $j \neq k$, if trade barriers obey the triangle inequality, $(\forall \upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$, and if the inequality for at least one $\upsilon \neq j$ is strict, then the relative price of a variety sold in markets j and k is strictly decreasing in the relative sizes of the markets.*

Proposition 2 ensures that the relative price of a variety across two markets falls in the relative sizes of the markets, as long as there is "some gravity" surrounding these markets. One example in which the necessary restriction holds is when the trade barriers for the two countries whose prices are being compared, j and k, satisfy $\tau_{kj}\tau_{jk} > \tau_{kk}$. In this case, the restriction requires that the cost to sell products within country k is strictly lower than the cost to export products to j and then import them back to k. This is guaranteed if international shipping costs are strictly positive.

A similar intuition applies to the relationship between relative prices and relative incomes of destinations, controlling for relative per-capita incomes. Proposition 3 summarizes the result.

Proposition 3. *For any two countries, j and k, $j \neq k$, if trade barriers obey the triangle inequality, $(\forall \upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$, and if the inequality for at least one $\upsilon \neq j$ is strict, then the relative price of a variety sold in markets j and k is strictly decreasing in the relative incomes of the markets, after controlling for the markets' relative per-capita incomes.*

Finally, Proposition 4 states the conditions under which the positive effect of relative per-capita income dominates the negative effect of relative income on relative prices. For a pair of countries, j and k, $j \neq k$, let $\zeta_{Y\,j,k}$ denote the elasticity of the relative price of any variety between j and k with respect to the relative incomes between j and k. Similarly, let $\zeta_{w\,j,k}$ denote the elasticity of the relative price between j and k with respect to the relative per-capita incomes of the markets.

Proposition 4. *If trade barriers obey the triangle inequality, $(\forall j, k, \upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$, then the relative price of a variety sold in two markets is strictly increasing in the relative per-capita incomes of the markets, after controlling for the markets' relative incomes. If in addition $\theta \geq 1$, then for any two countries, j and k, $j \neq k$, $\zeta_{Y\,j,k} + \zeta_{w\,j,k} > 0$.*

In the section that follows, I discuss how the four predictions relate to the existing literature.

3.2 Relation to Existing Literature

3.2.1 Per-Capita Income, Prices, and Demand Elasticities: Alternatives Models

In the model outlined in this paper, the price of a variety reflects the firm's marginal cost of production and delivery and the consumer's demand elasticity in a country, which can be seen from expression (21). Since the elasticity in expression (20) depends on the productivity threshold, it falls in the per-capita income and rises in the size and the income of the destination. The effects are a byproduct of the assumed non-homothetic preference relation.

Alternative explanations of varying demand elasticities, and therefore varying prices, exist. Lach (2007) hypothesizes that prices of consumption goods in Israel fell in the 1990s because there was a flow of immigrants with low search costs and high demand elasticities into the country during the period. Alessandria and Kaboski (2011) develop a formal model where high-wage earners have a high opportunity cost of searching for goods, which allows firms to charge high prices for identical goods in rich countries. While their model delivers a positive relationship between prices and per-capita income, it ignores the effect of competition, or market size, on prices. On the contrary, the present model yields a negative relationship between prices and market size.[7]

Hummels and Lugovskyy (2009) use a Lancaster (1979) model to argue that agents who experience an increase in income raise their consumption, which makes them more finicky and therefore more willing to pay a higher price in order to get closer to consuming their ideal variety. Moreover, in their model, a rise in per-capita income raises total income, thus attracting more firms to enter the market, which increases competition and depresses prices. Like the present framework, their model predicts that relative prices rise in relative per-capita incomes, after controlling for relative incomes. However, contrary to the present framework, their model predicts that the sum of the elasticities of relative prices with respect to relative per-capita income and income is negative.

In other related literature, Bekkers et al. (2011) identify a difference between the predictions of the non-homothetic and the ideal-variety model concerning the relationship between prices and income inequality *within* a country. The authors extend the two models to feature finite numbers of income groups within a country and they consider how increases in the mean-preserving spread in income, measured by changes in the Atkinson index, affect average prices in a country. The authors show that, in the ideal-variety model, a rise in income inequality raises prices, while the opposite relationship prevails in the non-homothetic model.

The authors confront the models with import unit-value data for 200 countries at the HS 6-digit level.[8] They find that, after controlling for per-capita income, unit values fall in the Atkinson

[7] Feenstra (2003) and Melitz and Ottaviano (2008) also develop frameworks that negatively link prices to market size. The models, however, do not feature per-capita income effects on prices.

[8] Bekkers et al. (2011) also examine the quality hypothesis, according to which richer countries pay higher prices

index of inequality, which suggests that prices are falling in within-country income inequality. This result provides further support for the non-homothetic model in explaining price variation.

Overall, the discussion in this section suggests that the three competing models that generate varying demand elasticities yield an identical relationship between prices and per-capita income, but they have different predictions about the behavior of prices in markets of different population sizes and income levels. In the empirical section that follows shortly, I test the predictions of the three models using unique data that features prices of identical goods across many countries.

3.2.2 Non-Homothetic Preferences and International Trade

The utility function that I employ in this paper represents a preference relation that is non-homothetic. Non-homothetic preferences have recently made a come-back in international trade (see Markusen (2010) for a comprehensive discussion on the usefulness of non-homothetic preferences in accounting for a variety of facts in international economics). Fieler (2010) incorporates non-homothetic preferences in the Ricardian model of Eaton and Kortum (2002) in order to explain the lack of trade between rich and poor countries. The author shows that, if income-elastic goods are produced with more variable technology, then rich countries, which are more technologically advanced, produce and consume a larger share of these goods. Moreover, since in that framework the set of consumed goods is fixed and identical across countries, consumption shares vary with per-capita income, which makes the model potentially useful for cross-sectoral analysis.

Unlike the paper above, the preference relation that I employ yields hierarchic demand due to bounded marginal utility of consumption. This feature of the utility function gives consumption sets that are expanding in per-capita income. Sauré (2009) argues that this mechanism has implications about trade patterns. The author uses the present utility function in the homogeneous monopolistic-competition framework of Krugman (1980) and derives a positive relationship between per-capita income and the extensive margin of imports.[9] The author's theoretical results are consistent with the empirical findings of Hummels and Klenow (2005) and Feenstra (2010).

Two additional functional forms that belong to the class of hierarchic demand systems have recently been introduced to the international trade literature of heterogeneity. First, Behrens et al. (2009) employ exponential (CARA) utility in a general equilibrium model of international trade with heterogeneous firms. They use the model to study the effects of the Canada-US trade liberalization on regional market aggregates such as wages, productivity, mark-ups, the mass of produced and consumed varieties, as well as welfare. While their model has desirable aggregate properties such as a gravity equation of trade under Pareto-distributed firm productivities, individual-firm

because they consume higher-quality goods. They show that the quality model, like the ideal-variety model, relates income inequality and prices in a positive manner. However, since the arguments in the present section are concerned with varying demand elasticities, I do not address the quality hypothesis.

[9] Young (1991) uses the same preference relation in a Ricardian framework to analyze the growth patterns of countries when firms engage in learning-by-doing.

prices and mark-ups are characterized via the Lambert-W function. This lack of tractability prevents the model from delivering structural equations that relate individual prices to observables.

Second, Melitz and Ottaviano (2008) use linear demand for varieties to study how mark-ups respond to changes in market size and trade policy. Their framework features a numéraire good that is produced with identical linear technology across countries and is freely traded. These assumptions imply wage equalization across countries and thus income effects on prices are absent from their model. In Appendix B, I drop the numéraire good and I characterize the general equilibrium of their model allowing for income effects. I obtain the following pricing rule for a firm with productivity ϕ from i selling to j (using the notation of the present paper for convenience)

$$p_{ij}(\phi) = \frac{\tau_{ij} w_i}{2} \left[\frac{1}{\phi} + \frac{1}{\bar{\phi}_{ij}} \right], \tag{24}$$

where $\bar{\phi}_{ij}$ is the minimum productivity level that firms from i must have in order to sell to j and it is implicitly characterized by expression (b.8) in Appendix B.

The pricing rule in (24) is identical to the one obtained by Melitz and Ottaviano (2008), except that, in the general equilibrium model, wages differ across countries and affect marginal costs of production as well as productivity cutoffs. In this model, firms add a mark-up to their marginal cost of production, and the mark-up varies with the trade barriers and productivity cutoffs characterizing different destinations. One can verify that the price of a variety rises in the per-capita income and falls in the population size of the destination. However, a testable prediction relating prices to measurable variables in the data cannot be derived since thresholds are not explicit functions of parameters and wages.

On the contrary, the non-homothetic utility function that I employ throughout the paper allows me to obtain a testable prediction that links relative prices to *measured* macroeconomic variables, one of which is per-capita income. I propose this particular utility function because it is tractable and because it allows me to relate the model's prediction to observed data. In the empirical section of the paper, I derive the testable prediction about the behavior of prices across countries. I then combine the expression with a unique price dataset to obtain unbiased estimates of a key parameter in international trade—the elasticity of price with respect to per-capita income.

4 Empirical Analysis

The three models of varying demand elasticities discussed above predict that individual goods' prices should be higher in richer countries. The positive relationship between prices and per-capita income predicted by the models is in line with a series of empirical findings. However, existing studies do not provide direct support for the underlying mechanisms that operate in the models, since they cannot measure the effect differing demand elasticities have on firms' mark-ups. Schott (2004), Crucini et al. (2005a), Crucini et al. (2005b), Hummels and Klenow (2005), Hsieh and Klenow (2007), Crucini and Shintani (2008), Hummels and Lugovskyy (2009), and Alessandria and Kaboski (2011) study prices of aggregate good categories, unit values from trade data, or prices of goods with similar characteristics across countries. Since goods are not identical, prices potentially reflect variable product quality, which is higher in richer countries, as Schott (2004) argues. Ghosh and Wolf (1994), Haskel and Wolf (2001), Goldberg and Verboven (2001), and Goldberg and Verboven (2005) track individual goods across countries. But, prices are collected from retail locations and they potentially reflect local distribution costs, which are higher in richer countries, as Burstein et al. (2003) and Crucini and Yilmazkuday (2009) argue.

For these reasons, I present a unique database that features prices of *identical* goods sold *online*, which allows me to establish a link between demand elasticities and mark-ups across countries.

4.1 Description of Data

I collect price data from the online catalogues of a Spanish apparel manufacturer called Mango. Mango specializes in the production of clothing for middle-income female consumers, although in 2009, they also established a men's line. The company opened its first store in Barcelona in 1984. Currently, Mango has 1220 stores in 91 countries. Mango's financial statement dated 2007 shows that total annual sales amounted to 1.956 billion USD, out of which 76 percent was generated from exports. Mango is the second largest textile exporter in Spain and it employs 7800 people.

More importantly, Mango operates a large-scale online store at http://shop.mango.com.[10] Each participating country has a website and customers from one country cannot buy products from another country's website due to shipping restrictions. Thus, a customer with a shipping address in Germany can only have items delivered to her if purchased from the German Mango website.

There are two unique features of the dataset that allow me to study price discrimination empirically. First, products sold in each market are identical, so quality differences are not responsible for the variation in prices across markets. Second, all products are sold online and prices do not reflect Balassa-Samuelson effects. I expand on each of these points below.

The online catalogue constitutes an identical basket of nearly one hundred products offered in all participating countries each season.[11] Each item in the catalogue has a unique name and an 8-digit code reported in every country. All items are stored and ship out of a single warehouse located in Palau de Plegamans, Spain, regardless of the shipping destination. Thus, prices do not reflect destination-specific production and storage costs. Upon receiving an online order, Mango

[10]Recently, some of Mango's competitors have begun to operate similar stores online. These companies include Zara (http://www.zara.com)—Spain's largest apparel exporter, H & M (http://www.hm.com/entrance.ahtml)—Sweden's largest apparel exporter, and Miss Sixty (http://www.misssixty.com/Index.aspx)—a division of Italy's Sixty Spa. At the time the study was conducted (in 2008), Mango's online store had the widest coverage of countries and items, which is necessary for empirical analysis, and it allowed one to collect prices of items in every country. As of 2011, Mango has expanded its online store to a larger set of countries.

[11]Often items sold online do not appear in stores and vice verse.

ships the items via DHL Express.[12] So, in addition to mark-ups, prices may reflect shipping costs.

The shipping and handling policy of Mango is such that no explicit fee is paid on purchases above a minimum value, which differs across countries. All other purchases incur an explicit shipping and handling fee. Many items sold by Mango classify for free shipping. However, it is not always the case that the same product ships at no fee to different destinations, since the minimum price requirement as well as the actual Euro-denominated price of the product often differ across markets. Thus, it is necessary to control for shipping costs in the analysis.

Information on the actual cost of shipping and handling that Mango incurs is not publicly available. In addition, the shipping and handling fees that Mango reports online may reflect variable mark-ups rather than true costs of shipping. So, it is not desirable to use this information to measure Mango's shipping costs. Instead, I use the structure of the model and the pricing rule reported by DHL, the company that Mango uses for shipping, in order to estimate trade barriers. I discuss estimation details in the next subsection.

Finally, prices reported on all EU-member websites are inclusive of sales taxes, or VAT. According to the European Commission, a company headquartered in an EU country, selling products online, and dispatching its products from its domestic location to another EU market, faces the following tax rule: (a) Add destination-specific VAT if annual sales per destination exceed a threshold value; (b) Choose between domestic and destination-specific VAT if annual sales per destination are below a threshold value.[13] Mango's sales data per destination are not publicly available. However, the European Commission reports the VAT rates on clothing for each member country, and Spain's rate is the third lowest in the sample. Thus, it is reasonable to assume that Mango would choose to apply the Spanish tax rate, if possible. So, in the benchmark analysis, I use prices collected from each country's website under the assumption that they reflect Spanish VAT. I conduct robustness exercises that account for destination-specific sales taxes in Appendices C and D.

Table 1: Per-Capita Income and Average Price of Items, 18 Countries

Country	Austria	Belgium	Estonia	Finland	France	Germany
Mean Price	66.10	63.51	64.74	67.06	62.30	65.80
Country	Greece	Hungary	Ireland	Italy	Norway	Portugal
Mean Price	57.29	65.25	73.29	64.13	74.10	51.71
Country	Slovakia	Slovenia	Spain	Sweden	Switzerland	United Kingdom
Mean Price	72.19	64.10	51.57	72.50	73.22	63.44

corr(mean price, per-capita income)=0.5248**

** significance at 5%-level

Data Sources: Prices for 180 goods from March/September 2008 online catalogues of clothing manufacturer Mango. Exchange rates for March/September 2008 from ECB. Nominal per-capita GDP for 2007 from WDI.

[12] I conducted a controlled experiment and collected DHL tracking codes for an identical item sent to all destinations and verified that the shipping and production origin are identical, regardless of destination.

[13] See http://ec.europa.eu/taxation_customs/taxation/vat/how_vat_works/vat_on_services/index_en.htm

I conclude the discussion with a summary of the price data. Table 1 reports the mean product price in Euro in each of the eighteen countries used in the analysis. The cross-country correlation between per-capita income and the average price is 0.52. Norway—the richest country in the sample—experiences the highest average price, which is 1.44 times higher than the lowest average price observed in the home country—Spain.

4.2 Empirical Tests of Three Models

4.2.1 Ingredients

To test the predictions of the models, I use prices of 180 goods across eighteen markets. I consider products from the Summer and Winter 2008 catalogues, which became available online in March and September of 2008, respectively. By pooling the data, I minimize the possibility of seasonal bias. Prices are recorded in local currency. I convert them to Euro using the European Central Bank's average exchange rate for March/September of 2008—the months when the catalogues were posted online. On the website, Mango claims that it adjusts online prices periodically to account for valuation changes, which supports my choice of exchange rates. For robustness purposes, I also report results that I obtain using exchange rates for February/August and January/July of 2008 to capture the fact that Mango may have set prices one or two months in advance.[14]

I now discuss the approximation of shipping costs. Mango ships its products via DHL, which offers a menu of prices for repeated shipments.[15] Thus, the actual cost that Mango incurs cannot be inferred. But, DHL prices all shipments according to regions. For an exporter, the main determinant of the price to ship to a region is distance, and regions are ranked according to numbers, with 1 being the cheapest region.

To control for the effect that Mango's shipping costs have on goods' prices, I construct distance-interval dummy variables with DHL's regional classification in mind. From CEPII, I obtain the distance between Spain and every European country featured in the Spanish DHL catalogue. I construct four non-overlapping distance intervals. The upper bound on each distance interval is the maximum distance between Spain and the destinations within a given DHL region. Then, I construct an $M \cdot (I-1)$−by−4 matrix c, where M and I are the numbers of goods and countries in the study (with Spain being the numéraire), respectively. In column i of matrix c, $i = 1, ..., 4$, I make an entry of one if the destination-good pair is associated with a country whose distance from Spain lies in the i-th interval, and zero otherwise.

Finally, I use per-capita GDP and total GDP (in current US dollars) as well as population size for the year 2007 from the World Development Indicators.

[14]Such pricing behavior is in line with findings by Bils and Klenow (2004) who report that, in the US, apparel prices adjust more frequently than median-good prices, which remain unchanged for four months out of the year.

[15]All information on DHL contained in this and subsequent paragraphs is available at http://www.dhl.es/en.html.

4.2.2 Econometric Model

To motivate the econometric specification, I revisit the predictions of the three models in more detail. First, in a two-country, two-tradable-good search framework, Alessandria and Kaboski (2011) show that firms add a mark-up to the marginal cost of production in each market. The mark-up is higher in the country where consumers enjoy higher per-capita income levels. Hence, the model predicts that the relative price of an item across two markets depends on good-specific characteristics (since mark-ups are additive) and relative per-capita incomes of the destinations.

Second, the ideal-variety framework of Hummels and Lugovskyy (2009) predicts that the relative price of a variety is higher in markets with relatively higher per-capita incomes and relatively lower total incomes. Moreover, the model predicts that the negative effect of total income overpowers the positive effect of per-capita income on the price. Finally, goods' characteristics affect relative prices because relative (net) mark-ups reflect marginal costs of production and delivery.

Contrary to the search framework, the non-homothetic model predicts that relative prices are higher in relatively richer (in per-capita terms) and lower in relatively larger (in terms of population size) markets. In addition, the model links prices to per-capita income and income of each destination. However, unlike the ideal-variety framework, the non-homothetic model predicts that the elasticity of price with respect to per-capita income exceeds the (absolute value of the) elasticity with respect to total income. Finally, (22) suggests that good-specific characteristics do not affect relative prices across different destinations in the non-homothetic model.

With the above discussion in mind, I propose the following econometric model

$$\log\left(\frac{p_{jm}}{p_{sm}}\right) = \hat{\gamma}_m + \hat{\gamma}_w \log\left(\frac{w_j}{w_s}\right) + \hat{\gamma}_x \log\left(\frac{x_j}{x_s}\right) + \hat{\gamma}_c c + \xi_{jm}, \quad x = L, Y. \tag{25}$$

In the above expression, p_{jm}/p_{sm} is the price of item m in country j, relative to the item's price in Spain. $\hat{\gamma}_m$ is item m's fixed-effect coefficient estimate. w_j/w_s is country j's per-capita income, relative to Spain's, and $\hat{\gamma}_w$ is the corresponding estimated coefficient. c is the shipping cost matrix and $\hat{\gamma}_c$ is the associated vector of coefficients, with typical element $\hat{\gamma}_{c_i}$, $i = 1, ..., 4$. ξ_{jm} is an error term. Finally, L and Y denote population size and total income, respectively.

When $x = L$, I test the predictions of the search model against those of the non-homothetic model. A positive and statistically significant estimate of $\hat{\gamma}_w$ is in line with the predictions of either model. However, a negative and statistically significant estimate of $\hat{\gamma}_L$ provides support for the non-homothetic model only.

When $x = Y$, I test the predictions of the ideal-variety model against those of the non-homothetic model. A positive and statistically significant estimate of $\hat{\gamma}_w$ and a negative and statistically significant estimate of $\hat{\gamma}_Y$ are in line with the predictions of either model. However, a positive sum of the estimates of $\hat{\gamma}_w$ and $\hat{\gamma}_Y$ provides support for the non-homothetic model only.

I estimate the coefficients in regression (25) using the OLS estimator. However, it is reasonable to fear that the errors of a given good may be correlated across destinations. For example, an item featured on the front page of the identical Mango catalogue that is offered across countries may be more popular than the remaining items everywhere. Hence, I cluster all errors by destination.[16]

4.2.3 Results

Table 2: Tests of Alternative Models, 18 Countries

Exchange Rate	$\hat{\gamma}_w$ (s.e.)	$\hat{\gamma}_x$ (s.e.)	$\hat{\gamma}_{c,1}$ (s.e.)	$\hat{\gamma}_{c,2}$ (s.e.)	$\hat{\gamma}_{c,3}$ (s.e.)	$\hat{\gamma}_{c,4}$ (s.e.)
Price, Per-capita Income, and Population Size ($x = L$)						
Mar/Sep	0.0646**	-0.0282**	-0.0105	0.2063***	0.2497***	0.2036***
	(0.0309)	(0.0126)	(0.0226)	(0.0180)	(0.0141)	(0.0383)
Feb/Aug	0.0699**	-0.0243**	-0.0046	0.2049***	0.2491***	0.2123***
	(0.0304)	(0.0117)	(0.0221)	(0.0173)	(0.0143)	(0.0384)
Jan/Jul	0.0635**	-0.0231*	-0.0055	0.2071***	0.2527***	0.2146***
	(0.0303)	(0.0120)	(0.0221)	(0.0172)	(0.0154)	(0.0388)
Price, Per-capita Income, and Income ($x = Y$)						
Mar/Sep	0.0928***	-0.0282**	-0.0105	0.2064***	0.2497***	0.2036***
	(0.0333)	(0.0126)	(0.0226)	(0.0180)	(0.0141)	(0.0383)
Feb/Aug	0.0943***	-0.0243**	-0.0046	0.2049***	0.2491***	0.2122***
	(0.0324)	(0.0118)	(0.0221)	(0.0173)	(0.0143)	(0.0384)
Jan/Jul	0.0867**	-0.0232*	-0.0054	0.2071***	0.2527***	0.2146***
	(0.0329)	(0.0120)	(0.0221)	(0.0172)	(0.0153)	(0.0388)

* significance at 10% level, ** significance at 5%-level, *** significance at 1%-level

Price regressions: N. Obs 3060, Country clusters 17 (Spain is numéraire), Fixed Effects 179 (relative to good 1)

Distance Intervals (in km): $[0, 501)$, $(501, 1367]$, $(1367, 1482]$, $(1482, 2953]$

Data Sources: Prices from Mar/Sep 2008 online catalogues of clothing manufacturer Mango. Exchange rates for Mar/Sep, Aug/Feb, and Jan/Jul 2008 from ECB. DHL Express shipping zones from DHL Spain Online. Distance from CEPII. Nominal per-capita GDP, nominal GDP, and population for 2007 from WDI.

The results from the test of the search versus the non-homothetic model are reported in the first half of Table 2. The coefficient estimates on per-capita income are positive and statistically significant, an observation supported by a large body of existing empirical work. The coefficients on country size are negative and statistically significant. These findings are in line with Syverson (2004), who documents the negative effects of competition on prices for the US concrete industry. The results support the non-homothetic over the search model, which predicts that $\hat{\gamma}_L$ is zero.

The dummy associated with the first region is not statistically different from zero. This result is due to the fact that Portugal is the only country that belongs to the first distance region, as it

[16]In principle, I could estimate the effects of per-capita income, size, and income on prices by interacting these variables with item dummies. Like Hummels and Lugovskyy (2009), I argue that it suffices to capture each item-specific effect by a single item dummy, since the items fall within one industry and are produced by one firm. Moreover, the richer specification would only apply to the ideal-variety model. The non-homothetic model postulates that item characteristics do not affect relative prices, while the search framework does not feature varieties.

is the only country that is classified in the first zone of shipping according to DHL Spain. The remainder of the regions display very high and statistically-significant positive coefficients, which suggests that prices increase sharply in the cost of shipping.

Another interesting finding is that among the 179 fixed effects, only (ten) six percent of the goods have coefficients that are statistically significant at the (ten-) five-percent level. Moreover, across all the specifications, the set of goods with statistically-significant fixed effects remains unchanged.[17] Hence, the majority of the goods observe a relative price ratio across countries that is not significantly different from the relative price ratio of the first good, whose dummy serves as a numéraire.[18] Such behavior is very symptomatic of the non-homothetic model, which predicts that relative prices of goods across countries do not reflect goods' characteristics.[19]

Overall, the empirical findings provide support for the non-homothetic over the search model. In particular, relative prices are systematically higher in richer countries and lower in larger countries.

The results from the test of the ideal-variety versus the non-homothetic model are reported in the second half of Table 2. The coefficient estimates on per-capita income are positive and statistically significant, while the coefficients on total income are negative and statistically significant. These findings are in line with Hummels and Lugovskyy (2009), who estimate a similar econometric model using unit-value data collected at the port of shipping to approximate prices of varieties from eleven source countries sold in two-hundred destinations over the 1990-2003 period.

However, the sum of the estimated coefficients on per-capita and total income in Table 2 is positive—an empirical finding that is also documented by Hummels and Lugovskyy (2009). This result supports the non-homothetic over the ideal-variety model, which predicts a negative sum.

4.2.4 Endogeneity and Robustness

A potential source of endogeneity in the empirical analysis above is omitted-variables bias. For example, in addition to distance, other destination-specific variables may be responsible for the DHL shipping costs that Mango incurs. If the omitted variables are reflected in the items' prices and if they co-vary with the per-capita income, size, or income of the destinations, then the estimates of $\hat{\gamma}_w$, $\hat{\gamma}_L$, or $\hat{\gamma}_Y$ will be biased (see Chapter 4 in Wooldridge (2002)).

International shipping costs typically reflect border effects (see Anderson and van Wincoop (2004) for a discussion). Consequently, for robustness purposes, I expand the benchmark specifica-

[17] Table 9 in the Supplementary Appendix reports the fixed effects for all three exchange-rate specifications. Fixed effects for the remaining regressions are suppressed due to space constraints and are available upon request.

[18] This finding provides support for the econometric specification that does not rely on interaction terms between good and country characteristics.

[19] The prediction is specific to the utility function employed in this paper and it is not shared by other hierarchic-demand models. The quadratic preference relation used by Melitz and Ottaviano (2008) yields an additive mark-up to a firm's marginal cost, which gives importance to good-specific characteristics in the determination of relative prices across countries. In addition, the Lambert-W function that characterizes a firm's mark-up over marginal cost in the exponential framework of Behrens et al. (2009) is not homogeneous in the firm's productivity. Thus, in their model, good-specific characteristics affect relative prices across destinations.

tion in (25) to accommodate a dummy variable, b, which takes on the value of one if the destination shares a border with Spain, and zero otherwise. Table 3 reports the results from the exercise. $\hat{\gamma}_b$ represents the coefficient estimate of the border effect and it is not statistically different from zero. In addition, the estimated coefficients on per-capita income, size, and income remain unchanged.

Table 3: Robust Tests of Alternative Models, 18 Countries

Exchange Rate	$\hat{\gamma}_w$ (s.e.)	$\hat{\gamma}_x$ (s.e.)	$\hat{\gamma}_{c,1}$ (s.e.)	$\hat{\gamma}_{c,2}$ (s.e.)	$\hat{\gamma}_{c,3}$ (s.e.)	$\hat{\gamma}_{c,4}$ (s.e.)	$\hat{\gamma}_b$ (s.e.)
	Price, Per-capita Income, and Market Size ($x = L$)						
Mar/Sep	0.0641**	-0.0268**	0.0127	0.2114***	0.2510***	0.2064***	-0.0215
	(0.0308)	(0.0126)	(0.0301)	(0.0212)	(0.0140)	(0.0388)	(0.0194)
Feb/Aug	0.0694**	-0.0228*	0.0227	0.2108***	0.2506***	0.2155***	-0.0253
	(0.0303)	(0.0119)	(0.0292)	(0.0193)	(0.0148)	(0.0391)	(0.0169)
Jan/Jul	0.0630**	-0.0215*	0.0236	0.2134***	0.2544***	0.2181***	-0.0269
	(0.0301)	(0.0122)	(0.0293)	(0.0190)	(0.0162)	(0.0396)	(0.0170)
	Price, Per-capita Income, and Income ($x = Y$)						
Mar/Sep	0.0910***	-0.0269**	0.0128	0.2114***	0.2511***	0.2064***	-0.0215
	(0.0331)	(0.0126)	(0.0301)	(0.0212)	(0.0140)	(0.0388)	(0.0194)
Feb/Aug	0.0922***	-0.0228*	0.0227	0.2108***	0.2506***	0.2155***	-0.0253
	(0.0324)	(0.0119)	(0.0291)	(0.0192)	(0.0148)	(0.0391)	(0.0169)
Jan/Jul	0.0845**	-0.0215*	0.0237	0.2135***	0.2544***	0.2181***	-0.0270
	(0.0330)	(0.0122)	(0.0293)	(0.0190)	(0.0161)	(0.0396)	(0.0170)

* significance at 10% level, ** significance at 5%-level, *** significance at 1%-level

Price regressions: N. Obs 3060, Country clusters 17 (Spain is numéraire), Fixed Effects 179 (relative to good 1)

Distance Intervals (in km): $[0, 501)$, $(501, 1367]$, $(1367, 1482]$, $(1482, 2953]$

Data Sources: Prices from Mar/Sep 2008 online catalogues of clothing manufacturer Mango. Exchange rates for Mar/Sep, Aug/Feb, and Jan/Jul 2008 from ECB. DHL Express shipping zones from DHL Spain Online. Distance and border from CEPII. Nominal per-capita GDP, nominal GDP, and population for 2007 from WDI.

Finally, as discussed in section 4.1, cross-country sales-tax variation may cause systematic price deviations. So, for robustness purposes, in Table 7 of Appendix C, I repeat the analysis by explicitly controlling for differences in sales taxes across destinations. Sales tax variation does not appear to be a source of bias in the analysis, since the estimated coefficient of a destination's gross sales tax, relative to Spain's tax, is not statistically different from zero in any of the specifications.

In sum, the empirical results in this section suggest that the non-homothetic model outlined in this paper has the ability to account for the observed cross-country variation in prices across three key dimensions: per-capita income, size, and total income of destinations.

4.3 Estimating Price Elasticities

Section 4.2 above empirically tests three theories that aim to explain the positive relationship between per-capita income and prices of tradables. However, the analysis carried out thus far is less informative about the magnitude of the elasticity of price with respect to per-capita income. For example, in Table 2, the estimate of $\hat{\gamma}_w$ that is obtained when controlling for market size ($x = L$) is roughly two-thirds of the estimate that is obtained when controlling for total income ($x = Y$). Since the degree to which prices rise in per-capita income is crucial to quantitatively assess nations' growth patterns and consumers' welfare, as Hsieh and Klenow (2007) and Broda and Romalis (2009) argue, respectively, it is important to obtain an unbiased estimate of the price elasticity. Given that the non-homothetic model is qualitatively in line with the pricing behavior observed in the data, I use it to accomplish this task. First, I derive a testable prediction of the model that relates prices to measurable macroeconomic variables. Second, I use the prediction, together with the dataset described above, to estimate the elasticity of price with respect to per-capita income.

4.3.1 A Testable Prediction

To derive the model's testable prediction about price variation, substitute the thresholds from (18) for destinations j and k into the relative-price expression in (22) to obtain

$$\frac{p_{ij}(\phi)}{p_{ik}(\phi)} = \underbrace{\left(\frac{\tau_{ij}}{\tau_{ik}}\right)^{\frac{1}{2}}}_{\text{barriers}} \underbrace{\left(\frac{w_j}{w_k}\right)^{\frac{1}{2(\theta+1)}}}_{\text{pc income}} \underbrace{\left(\frac{\sum_v L_v b_v^\theta (\tau_{vj} w_v)^{-\theta}}{\sum_v L_v b_v^\theta (\tau_{vk} w_v)^{-\theta}}\right)^{-\frac{1}{2(\theta+1)}}}_{\text{general equilibrium object}}. \tag{26}$$

The first term emphasizes the role of trade barriers, while the third represents an equilibrium object, where the contributions of each destination's per-capita income and size are marginal since they are contained within a summation term. Hence, identification of the price elasticity with respect to per-capita income must come from the second term, which is simply a ratio of per-capita incomes of two destinations. The problem with taking this expression to the data, however, is the fact that the lower bound on each country's productivity distribution, b_v, is not observable.

To solve the problem, make use of predicted trade shares, which are observable statistics. First, multiply (19) by the destination's size L_j to obtain the quantity sold in j by a firm with productivity ϕ from i. To derive i's total sales in j, substitute this quantity, as well as the price from (11), the conditional density from (13) under the Pareto parametrization, and the measure of exporters from (12) using the equilibrium measure of entrants $J_i = L_i/[(\theta+1)f_e]$ derived in Appendix A.2, into expression (15). Then, using expression (18) and the fact that $\tau_{ij}w_i/\phi^*_{ij} = \tau_{jj}w_j/\phi^*_{jj}$ ($\forall i \neq j$) (which is apparent in expression (10)), the import share of i-goods in j can be defined as

$$\lambda_{ij} \equiv \frac{T_{ij}}{\sum_{v=1}^{I} T_{vj}} = \frac{L_i b_i^\theta (\tau_{ij} w_i)^{-\theta}}{\sum_{v=1}^{I} L_v b_v^\theta (\tau_{vj} w_v)^{-\theta}}. \tag{27}$$

Finally, substitute (27) into (26) to obtain the following testable prediction that relates relative prices to measurable variables

$$\frac{p_{ij}(\phi)}{p_{ik}(\phi)} = \underbrace{\left(\frac{\tau_{ij}}{\tau_{ik}}\right)^{\frac{2\theta+1}{2(\theta+1)}}}_{\text{barriers}} \underbrace{\left(\frac{w_j}{w_k}\right)^{\frac{1}{2(\theta+1)}}}_{\text{pc income}} \underbrace{\left(\frac{\lambda_{ij}}{\lambda_{ik}}\right)^{\frac{1}{2(\theta+1)}}}_{\text{market share}}.$$

Below, I rewrite this expression in logs and I multiply and divide logged trade barriers by θ for reasons that will become apparent shortly,

$$\log\left(\frac{p_{ij}(\phi)}{p_{ik}(\phi)}\right) = \underbrace{\frac{2\theta+1}{2(\theta+1)\theta}}_{\beta_\tau} \theta \log\left(\frac{\tau_{ij}}{\tau_{ik}}\right) + \underbrace{\frac{1}{2(\theta+1)}}_{\beta_w} \log\left(\frac{w_j}{w_k}\right) + \underbrace{\frac{1}{2(\theta+1)}}_{\beta_\lambda} \log\left(\frac{\lambda_{ij}}{\lambda_{ik}}\right). \tag{28}$$

The model predicts that, after controlling for relative import shares and scaled relative trade barriers, the elasticity of relative prices with respect to relative per-capita incomes is $\beta_w \equiv 1/[2(\theta+1)]$. In the proceeding subsections, I estimate this elasticity from Mango's price data. First, I discuss the additional data ingredients necessary for estimation. Then, I proceed to describe the econometric model employed. Finally, I report the empirical results and I conduct robustness exercises.

4.3.2 Ingredients

In order to estimate the elasticity of the relative price with respect to relative per-capita income, I need four variables. The two key variables, namely prices and per-capita income, were described in Section 4.2. I discuss the remaining variables next.

Given Mango's line of work, to measure trade shares λ_{ij}, I focus on the industry titled "Textiles, textile products, leather and footwear" in the Stats.OECD database. I let the denominator in the trade-share expression be Gross Output + Imports (from countries in the sample) - Exports (to countries in the sample), which represents a country's total expenditure on goods of the particular industry. The numerator in (27) is country j's imports from country i in the industry.

While trade shares are observable, trade barriers are not. Since I assume that trade barriers are of the iceberg form, I can estimate them from the model's gravity equation of trade. To derive gravity between an exporter i and an importer j, simply take the log of the ratio of the import share λ_{ij} to the domestic expenditure share of the importer λ_{jj}, which from expression (27) is

$$\log\left(\frac{\lambda_{ij}}{\lambda_{jj}}\right) = S_j - S_i - \theta \log \tau_{ij}. \tag{29}$$

In the expression above, S_j (S_i) represents a fixed effect for importer j (exporter i) and is given by $S_j = \theta \log(w_j) - \log(L_j) - \theta \log(b_j)(\forall j)$. Intuitively, this variable captures all the characteristics of a country that affect its trade flows.

I follow the trade literature to estimate the gravity equation. I identify S_j from a country-specific dummy. To estimate logged trade barriers scaled by θ, which are necessary in order to carry out the empirical analysis in (28), I apply the methodology of Eaton and Kortum (2002).[20] This strategy involves the use of dummies to capture the effects that typical geographic barriers, in addition to the country-specific effects described above, have on trade flows. Motivated by DHL's pricing rule, in the benchmark analysis, I let trade barriers be given by

$$\log \tau_{ij} = d_k + \delta_{ij}, \tag{30}$$

where d_k, $k = 1, ..., 4$, quantifies the effect of the distance between i and j lying in the k-th interval.[21] δ_{ij} is an error term, assumed to be a random variable distributed according to $N(0, \Sigma)$, where Σ is a diagonal square matrix with a typical entry of σ^2 along the diagonal.

I apply least squares to estimate scaled logged bilateral trade barriers for all the countries in the sample using equations (29) and (30). The R-squared of the regression can be found in Table 4 in section 4.3.4 below.[22] Although I only need the trade barriers that Spain faces for the estimation of price elasticities that follows, I use the full sample of countries to estimate the trade barriers in order to be able to separately identify the contributions of scaled trade barriers and country-specific characteristics to observed bilateral trade flows. In sum, the methodology that I use to estimate bilateral trade barriers has two advantages: First, it exploits DHL pricing data, which is particularly relevant in the present study. Second, by employing the gravity approach, I obtain estimates of bilateral trade barriers that are consistent with observed bilateral trade flows.

4.3.3 Econometric Model

Using Spain as numéraire, dropping country-of-origin subscripts since Mango is from Spain, and discretizing the set of varieties, the econometric model corresponding to (28) can be written as

$$\log\left(\frac{p_{jm}}{p_{sm}}\right) = \hat{\beta}_\tau \log\left(\frac{\hat{t}_j}{\hat{t}_s}\right) + \hat{\beta}_w \log\left(\frac{w_j}{w_s}\right) + \hat{\beta}_\lambda \log\left(\frac{\lambda_j}{\lambda_s}\right) + \psi_{jm}. \tag{31}$$

In the above expression, p_{jm}/p_{sm} is the Euro-denominated price of item m in country j, relative to Spain. $\log\left(\hat{t}_j/\hat{t}_s\right) \equiv \theta \log\left(\hat{\tau}_{sj}/\hat{\tau}_{ss}\right) = \theta \log \hat{\tau}_{sj}$ represents the difference between the scaled logged trade barriers from Spain to destination j and the scaled logged trade barriers from Spain to itself. By the assumptions of the model, iceberg costs are expressed relative to domestic shipping costs, which are normalized to unity. Hence, $\hat{\tau}_{ss} = 1$, which implies that $\theta \log \hat{\tau}_{ss} = 0$. The

[20] A variety of specifications for trade barriers exist (see Anderson and van Wincoop (2004)). For example, it is common to use data on tariffs and freight charges in order to approximate trade barriers. Unfortunately, this approach is not applicable in the present study. First, all the countries in the sample are members of the European Economic Area, so bilateral tariffs are zero. Second, Mango's actual shipping costs are not publicly available.

[21] I use the set of distance intervals described in Section 4.2.

[22] Coefficient estimates from the regression are available upon request.

"hat" appearing in $\theta \log \hat{\tau}_{sj}$ is to remind the reader that scaled logged trade barriers are estimates obtained from the gravity regression in the previous section. w_j/w_s is the per-capita income of country j, relative to Spain. λ_j/λ_s is the import share of country j from Spain, relative to Spain's domestic expenditure share. ψ_{jm} is an error term which I discuss below.

I use OLS to estimate the coefficients in (31). Notice that good characteristics do not appear as explicit controls in the regression. This means that good-specific effects (if present) may be captured by the error terms. In this case, I would need to estimate the variance-covariance matrix of the error terms using good clusters. As discussed earlier, error terms of a given good may also be correlated across destinations. Consequently, I estimate the variance-covariance matrix of the errors using Cameron et al.'s (2006) two-way clustering method (by product and by destination).

Finally, recall that, in expression (31), I use trade-barrier estimates that I obtain from textile-industry trade data and the gravity equation. Hence, I implicitly assume that Mango faces the same trade costs as the average firm in the textile industry. Having made this assumption, I expect that the estimate of $\hat{\beta}_\tau$ will not be statistically different from zero, while the estimate of $\hat{\beta}_\lambda$ will be negative. To see this, derive the difference between the log of j's import share from Spain and the log of Spain's domestic expenditure share using (27), under the assumption that $\tau_{ss} = 1$,

$$\log \lambda_{sj} - \log \lambda_{ss} = -\theta \log \tau_{sj} + \log \left[\frac{\sum_{\upsilon=1}^{I} L_\upsilon b_\upsilon^\theta (\tau_{\upsilon s} w_\upsilon)^{-\theta}}{\sum_{\upsilon=1}^{I} L_\upsilon b_\upsilon^\theta (\tau_{\upsilon j} w_\upsilon)^{-\theta}} \right].$$

The difference in logged expenditure shares is equal to the sum of $-\theta \log \tau_{sj}$ and a general-equilibrium object, where each destination's contribution is marginal. However, the estimate of $\theta \log \tau_{sj}$ is exactly the variable that bares the coefficient $\hat{\beta}_\tau$ in (31). Thus, by controlling for trade shares, one implicitly controls for the effect trade barriers have on prices.

Table 4: Coefficients from Benchmark Estimation, 18 Countries

Exchange Rate	$\hat{\beta}_w$ (s.e.)	$\hat{\beta}_\lambda$ (s.e.)	$\hat{\beta}_\tau$ (s.e.)	First Stage Gravity R^2
Mar/Sep	0.0795***	-0.0523*	0.0124	0.6752
	(0.0300)	(0.0289)	(0.0215)	
Feb/Aug	0.0836***	-0.0493*	0.0148	0.6752
	(0.0290)	(0.0284)	(0.0215)	
Jan/Jul	0.0784***	-0.0496*	0.0146	0.6752
	(0.0288)	(0.0284)	(0.0215)	

* significance at 10% level, *** significance at 1%-level

Price regressions: N. Obs 3060, N. clustervars 2, Country clusters 17 (Spain is numéraire), Good clusters 180.

Distance Intervals (in km): $[0, 501)$, $(501, 1367]$, $(1367, 1482]$, $(1482, 2953]$

Data Sources: Prices from Mar/Sep 2008 online catalogues of clothing manufacturer Mango. Exchange rates for Mar/Sep, Aug/Feb, and Jan/Jul 2008 from ECB. DHL Express shipping zones from DHL Spain Online. Distance from CEPII. Bilateral trade and gross output for "Textiles, textile products, leather and footwear" for 2006 from Stats.OECD.

4.3.4 Results

The results from the benchmark estimation are reported in Table 4. The benchmark estimated elasticity of price with respect to per-capita income, $\hat{\beta}_w$, ranges between 0.0784 and 0.0836. This means that doubling the per-capita income of a country results in a roughly eight-percent rise in the price of a typical good. As expected, the estimates of $\hat{\beta}_\lambda$ are negative and significant, while the estimates of $\hat{\beta}_\tau$ do not appear to be different from zero.

4.3.5 Measurement Error and Robustness

An important measurement-error issue needs to be addressed. In the econometric model in (31), I use estimates of trade barriers obtained from bilateral trade data. A classical error-in-variables problem may arise if trade barriers are measured with error, and the estimates of $\hat{\beta}_\tau$ may be biased toward zero, as discussed in Levi (1973). The direction of the bias in the estimate of any remaining coefficient depends on the covariance between the variable whose coefficient is being estimated and the variable measured with error. In the present study, it would be worrisome if trade barriers co-varied with destinations' per-capita incomes, because estimates of $\hat{\beta}_w$ would be biased if the trade barriers were measured with error.

I tackle the issue of measurement error in several steps. First, I expand the trade-barrier equation in (30) to accommodate border effects. I introduce a dummy variable that takes on the value of one if the pair of countries shares a border and zero otherwise. Second, I let trade barriers depend not only on geographic characteristics such as border and distance, but also on exporter-specific fixed effects. This specification is motivated by Waugh (2010), who convincingly argues that trade barriers co-vary with exporter characteristics such as per-capita income, rather than importer attributes. In this case, the estimate of $\hat{\beta}_w$ arising from (31) would be consistent.

Third, I consider the possibility of trade-barrier misspecification. In particular, if the true trade costs that Mango faces differ from the trade barriers that the average firm in the textile industry is subject to (perhaps because the average firm does not ship via DHL), then it is not appropriate to use trade barriers obtained from gravity in the price-elasticity estimation.

To allow for the possibility that Mango faces unique trade barriers, I consider a specification that includes per-capita income and trade shares, but instead of relying on estimated trade barriers from gravity, I use c, the matrix of distance dummies from Section 4.2. This way, I directly control for the effects that distance has on Mango's prices through DHL's pricing policy, without imposing the assumption that the textile industry faces the same type of trade barriers. For robustness, I further augment the matrix c by the vector of border dummies, b, from Section 4.2.

Table 5 reports the results from the robustness exercises that use the two different gravity-based trade-barrier estimates. The estimates of the elasticity of price with respect to per-capita income lie within a very narrow range around the benchmark estimates, which suggests that the measurement error in trade barriers is not systematically correlated with destination per-capita income. Hence, the price elasticity estimates are consistent. However, it is reasonable to conclude that trade barriers are measured with error. Notice that the R-squared of the gravity equation of trade does not change significantly with the inclusion of border effects, but it rises considerably once I account for exporter-specific fixed effects. This is expected, since allowing for exporter-specific fixed effects provides more degrees of freedom in order to fit bilateral trade flows.

Table 5: Coefficients From Estimation Using Gravity-Based Trade-Barriers, 18 Countries

Exchange Rate Specification	$\hat{\beta}_w$ (s.e.)	$\hat{\beta}_\lambda$ (s.e.)	$\hat{\beta}_\tau$ (s.e.)	First Stage Gravity R^2
Mar/Sep	0.0799***	-0.0472*	0.0164	0.6979
Distance/Border	(0.0295)	(0.0289)	(0.0219)	
Feb/Aug	0.0839***	-0.0442	0.0188	0.6979
Distance/Border	(0.0283)	(0.0289)	(0.0222)	
Jan/Jul	0.0787***	-0.0444	0.0187	0.6979
Distance/Border	(0.0281)	(0.0290)	(0.0223)	
Mar/Sep	0.0779***	-0.0487	0.0164	0.8713
Waugh (2010)	(0.0298)	(0.0334)	(0.0272)	
Feb/Aug	0.0817***	-0.0454	0.0192	0.8713
Waugh (2010)	(0.0286)	(0.0337)	(0.0278)	
Jan/Jul	0.0765***	-0.0453	0.0194	0.8713
Waugh (2010)	(0.0284)	(0.0338)	(0.0280)	

* significance at 10% level, *** significance at 1%-level

Price regressions: N. Obs 3060, N. clustervars 2, Country clusters 17 (Spain is numéraire), Good clusters 180.

Distance Intervals (in km): $[0, 501)$, $(501, 1367]$, $(1367, 1482]$, $(1482, 2953]$

Data Sources: Prices from Mar/Sep 2008 online catalogues of clothing manufacturer Mango. Exchange rates for Mar/Sep, Aug/Feb, and Jan/Jul 2008 from ECB. DHL Express shipping zones from DHL Spain Online. Distance and border from CEPII. Bilateral trade and gross output for "Textiles, textile products, leather and footwear" for 2006 from Stats.OECD.

Table 6 reports the results from the exercises that allow for Mango-specific shipping costs. The estimates of $\hat{\beta}_w$ remain within a very close range from the benchmark estimates. Moreover, it cannot be concluded that Mango's shipping costs differ from the industry's, since the coefficient estimates on the dummy variables associated with the various distance intervals and the border effect are not statistically different from zero.

Finally, in Appendix D, I introduce destination-specific sales taxes into the model and I derive an augmented testable prediction that accounts for cross-country tax variation. The estimates of $\hat{\beta}_w$ remain unchanged, while the coefficients on taxes are not statistically different from zero, which suggests that sales-tax variation is not responsible for the systematic price variation.

In sum, across the various empirical exercises, the estimates of the elasticity of price with respect to per-capita income range between 0.0684 and 0.0839, or roughly seven to eight percent. The mean and the median among the eighteen estimates amount to 0.0766 and 0.0781, respectively, or roughly eight percent. Thus, it is reasonable to conclude that doubling a country's per-capita income results in an eight-percent rise in the price level of tradable consumption goods.

Table 6: Coefficients From Estimation With Mango-Specific Shipping Costs, 18 Countries

Exchange Rate Specification	$\hat{\beta}_w$ (s.e.)	$\hat{\beta}_\lambda$ (s.e.)	$\hat{\beta}_{c,1}$ (s.e.)	$\hat{\beta}_{c,2}$ (s.e.)	$\hat{\beta}_{c,3}$ (s.e.)	$\hat{\beta}_{c,4}$ (s.e.)	$\hat{\beta}_b$ (s.e.)
Mar/Sep	0.0703***	-0.0562*	-0.0643	0.0232	0.0733	0.0557	—
Distance	(0.0266)	(0.0314)	(0.0542)	(0.0996)	(0.1162)	(0.1192)	—
Feb/Aug	0.0752***	-0.0539*	-0.0586	0.0296	0.0792	0.0663	—
Distance	(0.0264)	(0.0315)	(0.0541)	(0.1017)	(0.1167)	(0.1208)	—
Jan/Jul	0.0687***	-0.0530*	-0.0597	0.0343	0.0849	0.0692	—
Distance	(0.0261)	(0.0317)	(0.0542)	(0.1027)	(0.1179)	(0.1222)	—
Mar/Sep	0.0701***	-0.0553*	-0.0565	0.0276	0.0765	0.0590	-0.0064
Distance/Border	(0.0266)	(0.0336)	(0.0785)	(0.1141)	(0.1239)	(0.1279)	(0.0307)
Feb/Aug	0.0750***	-0.0527	-0.0488	0.0351	0.0833	0.0704	-0.0080
Distance/Border	(0.0265)	(0.0342)	(0.0788)	(0.1169)	(0.1259)	(0.1309)	(0.0276)
Jan/Jul	0.0684***	-0.0518	-0.0486	0.0406	0.0895	0.0739	-0.0091
Distance/Border	(0.0262)	(0.0345)	(0.0792)	(0.1181)	(0.1274)	(0.1328)	(0.0276)

* significance at 10% level, *** significance at 1%-level

Price regressions: N. Obs 3060, N. clustervars 2, Country clusters 17 (Spain is numéraire), Good clusters 180.

Distance Intervals (in km): $[0, 501)$, $(501, 1367]$, $(1367, 1482]$, $(1482, 2953]$

Data Sources: Prices from Mar/Sep 2008 online catalogues of clothing manufacturer Mango. Exchange rates for Mar/Sep, Aug/Feb, and Jan/Jul 2008 from ECB. DHL Express shipping zones from DHL Spain Online. Distance and border from CEPII. Bilateral trade and gross output for "Textiles, textile products, leather and footwear" for 2006 from Stats.OECD.

5 Conclusion

In this paper, I argue that firms' variable mark-ups represent a key contributor to the empirically documented regularity that final tradable goods' prices are systematically positively related to countries' per-capita incomes. I outline a parsimonious and highly tractable heterogeneous-firm model of international trade that relates prices of tradable goods to per-capita income differences. I present direct support for the model's mechanism, which builds on non-homothetic consumer preferences, from a unique database that features prices of identical apparel products sold via the Internet. Finally, I use the model's testable prediction to estimate a key parameter in the international trade literature—the elasticity of price with respect to per-capita income.

On a broader scale, this paper emphasizes the role that income differences play in shaping cross-country price variations in tradable consumption goods as well as in determining aggregate consumption patterns. Since tradable goods account for an ever increasing portion of consumption bundles of individuals, their prices directly affect consumer welfare. Hence, having obtained an understanding of one of the key mechanisms that affect the behavior of prices across countries, we can further pursue the measurement of welfare of consumers in an integrated world economy.

References

Alessandria, G. and J. P. Kaboski (2011): "Pricing-to-Market and the Failure of Absolute PPP," *American Economic Journal: Macroeconomics*, 3, 91–127.

Alvarez, F. and R. J. Lucas (2007): "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, 54, 1726–1768.

Anderson, J. E. and E. van Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, 42, 691–751.

Behrens, K., G. Mion, Y. Murata, and J. Sdekum (2009): "Trade, Wages and Productivity," CEP Discussion Papers dp0942, Centre for Economic Performance, LSE.

Bekkers, E., J. Francois, and M. Manchin (2011): "Import Prices, Income, and Inequality," *Johannes Kepler University of Linz, unpublished manuscript.*

Bils, M. and P. J. Klenow (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947–985.

Broda, C. and J. Romalis (2009): "The Welfare Implications of Rising Price Dispersion," *mimeo.*

Burstein, A., J. Neves, and S. Rebelo (2003): "Distribution costs and real exchange rate dynamics during exchange-rate-based stabilizations," *Journal of Monetary Economics*, 50, 1189–1214.

Cameron, A. C., J. B. Gelbach, and D. L. Miller (2006): "Robust Inference with Multiway Clustering," NBER Technical Working Papers 0327, National Bureau of Economic Research.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98 (4), 1707–21.

Crucini, M. and M. Shintani (2008): "Persistence in law of one price deviations: Evidence from micro-data," *Journal of Monetary Economics.*

Crucini, M., C. Telmer, and M. Zachariadis (2005a): "Price Dispersion: The Role of Borders, Distance and Location," *Carnegie Mellon University, unpublished manuscript.*

——— (2005b): "Understanding European Real Exchange Rates," *American Economic Review*, 95 (3), 724–738.

Crucini, M. J. and H. Yilmazkuday (2009): "A Model of International Cities: Implications for Real Exchange Rates," NBER Working Papers 14834, National Bureau of Economic Research.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70 (5), 1741–1779.

Feenstra, R. (2003): "A homothetic utility function for monopolistic competition models, without constant price elasticity," *Economics Letters*, 78 (1), 79–86.

——— (2010): *Product Variety and the Gains from International Trade*, MIT Press.

Fieler, A. C. (2010): "Non-Homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation," *Econometrica, forthcoming.*

Ghosh, A. and H. Wolf (1994): "Pricing in International Markets: Lessons From The Economist," *NBER Working Paper.*

Goldberg, P. and F. Verboven (2001): "The Evolution of Price Dispersion in the European Car Market," *The Review of Economic Studies*, 68 (4), 811–848.

——— (2005): "Market integration and convergence to the Law of One Price: evidence from the European car market," *Journal of International Economics*, 65 (1), 49–73.

Haskel, J. and H. Wolf (2001): "The Law of One Price-A Case Study," *Scandinavian Journal of Economics*, 103 (4), 545–558.

Hsieh, C. and P. Klenow (2007): "Relative prices and relative prosperity," *American Economic Review*, 97, 562–585.

Hummels, D. and P. J. Klenow (2005): "The Variety and Quality of a Nation's Exports," *The American Economic Review*, 95, 704–723.

Hummels, D. and V. Lugovskyy (2009): "International Pricing in a Generalized Model of Ideal Variety," *Journal of Money, Credit and Banking*, 41, 3–33.

Hunter, L. (1991): "The contribution of nonhomothetic preferences to trade ," *Journal of International Economics*, 30, 345–358.

Hunter, L. and J. Markusen (1988): "Per Capita Income as a Determinant of Trade ," *Empirical Methods for International Economics*, by R. Feenstra, (MIT Press).

Jackson, L. (1984): "Hierarchic Demand and the Engel Curve for Variety." *Review of Economics & Statistics*, 66 (1), 8–15.

Krugman, P. (1980): "Scale Economies, Product Differentiation, and the Pattern of Trade," *The American Economic Review*, 70, pp. 950–959.

Lach, S. (2007): "Immigration and Prices," *Journal of Political Economy*, 115, 548–587.

Lancaster, K. (1979): *Variety, Equity, and Efficiency*, Columbia University Press.

Levi, M. D. (1973): "Errors in the Variables Bias in the Presence of Correctly Measured Variables," *Econometrica*, 41, 985–86.

Loecker, J. D. and F. Warzynski (2009): "Markups and firm-level export status," Working Paper 15198, National Bureau of Economic Research.

Markusen, J. R. (2010): "Putting Per-Capita Income Back into Trade Theory," Working Paper 15903, National Bureau of Economic Research.

Mas-Colell, A., M. Whinston, and J. Green (1995): *Microeconomic Theory*, Oxford University Press.

Melitz, M. and G. Ottaviano (2008): "Market Size, Trade, and Productivity," *Review of Economic Studies*, 75 (1), 295–316.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71 (6), 1695–1725.

Movshuk, O. (2004): "International Differences in Consumer Preferences and Trade: Evidence from Multicountry, Multiproduct Data," *Toyama University, unpublished mimeo.*

Sauré, P. (2009): "Bounded Love of Variety and Patterns of Trade," Working Paper 2009-10, Swiss National Bank.

Schott, P. (2004): "Across-Product Versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, 119 (2), 647–678.

Syverson, C. (2004): "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 112, 1181–1222.

Waugh, M. E. (2010): "International Trade and Income Differences," *American Economic Review*, 100, 2093–2124.

Wooldridge, J. M. (2002): *Econometric Analysis of Cross Section and Panel Data*, MIT Press.

Young, A. (1991): "Learning by Doing and the Dynamic Effects of International Trade," *The Quarterly Journal of Economics*, 106, 369–405.

A Appendix: Consumer Problem and Equilibrium

A.1 Deriving Consumer Demand

The maximization problem of a consumer in j, potentially buying varieties from $i = 1, ..., I$ is

$$\max_{\{q_{ij}^c(\omega)\}_{i=1}^I \geq 0} \quad \sum_{i=1}^I \int_{\omega \in \Omega_{ij}} \log(q_{ij}^c(\omega) + \bar{q}) d\omega \quad \text{s.t.} \quad \nu_j \left[\sum_{i=1}^I \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) q_{ij}^c(\omega) d\omega \leq y_j \right],$$

where ν_j is the Lagrange multiplier. The FOCs yield ($\forall q_{ij}^c(\omega) > 0$)

$$\nu_j p_{ij}(\omega) = \frac{1}{q_{ij}^c(\omega) + \bar{q}}. \tag{a.1}$$

Let $\Omega_j \equiv \sum_{i=1}^I \Omega_{ij}$ be the set of all positively-consumed varieties in country j. Letting N_{ij} be the measure of set Ω_{ij}, the measure of Ω_j, N_j, is given by $N_j = \sum_{i=1}^I N_{ij}$.

For any pair of varieties $\omega_{ij}, \omega'_{vj} \in \Omega_j$, (a.1) gives

$$p_{ij}(\omega)(q_{ij}^c(\omega) + \bar{q}) = p_{vj}(\omega') q_{vj}^c(\omega') + p_{vj}(\omega') \bar{q}.$$

Integrating over all $\omega'_{\upsilon j} \in \Omega_j$, keeping in mind that the measure of $\Omega_{\upsilon j}$ is $N_{\upsilon j}$, yields the consumer's demand for any variety $\omega_{ij} \in \Omega_j$

$$\begin{aligned}
\int_{\Omega_j} \left[p_{ij}(\omega)\left(q_{ij}^c(\omega)+\bar{q}\right)\right] d\omega' &= \int_{\Omega_j} \left[p_{\upsilon j}(\omega') q_{\upsilon j}^c(\omega') + p_{\upsilon j}(\omega')\bar{q}\right] d\omega', \\
\Rightarrow \left[p_{ij}(\omega)\left(q_{ij}^c(\omega)+\bar{q}\right)\right] \sum_{\upsilon=1}^{I} \int_{\Omega_{\upsilon j}} 1 d\omega' &= \sum_{\upsilon=1}^{I} \int_{\Omega_{\upsilon j}} \left[p_{\upsilon j}(\omega') q_{\upsilon j}^c(\omega') + p_{\upsilon j}(\omega')\bar{q}\right] d\omega', \\
\Rightarrow \left[p_{ij}(\omega)\left(q_{ij}^c(\omega)+\bar{q}\right)\right] \sum_{\upsilon=1}^{I} N_{\upsilon j} &= y_j + \sum_{\upsilon=1}^{I} \int_{\Omega_{\upsilon j}} p_{\upsilon j}(\omega')\bar{q} d\omega', \\
\Rightarrow \left[p_{ij}(\omega)\left(q_{ij}^c(\omega)+\bar{q}\right)\right] N_j &= y_j + \bar{q}P_j, \\
\Rightarrow q_{ij}^c(\omega) &= \frac{y_j + \bar{q}P_j}{N_j p_{ij}(\omega)} - \bar{q},
\end{aligned}$$

where $P_j \equiv \sum_{\upsilon=1}^{I} \int_{\Omega_{\upsilon j}} p_{\upsilon j}(\omega') d\omega'$ is an aggregate price statistic.

The total demand for variety ω from i by consumers in j becomes

$$q_{ij}(\omega) = L_j \left[\frac{y_j + \bar{q}P_j}{N_j p_{ij}(\omega)} - \bar{q}\right].$$

A.2 Equilibrium: Characterization, Existence, and Uniqueness

In this section, I rely on the Pareto distribution of firm productivities and characterize the equilibrium objects of the model. I express all objects in terms of wages and I derive a set of equations that solve for the wage rates of all countries simultaneously. I use υ as a counter throughout.

Using the optimal price (11), the measure of firms (12), and the conditional density (13) under the Pareto distribution in (14) yields

$$P_j = \sum_{\upsilon=1}^{I} J_\upsilon \left(\frac{b_\upsilon}{\phi_{\upsilon j}^*}\right)^\theta \int_{\phi_{\upsilon j}^*}^{\infty} \frac{\tau_{\upsilon j} w_\upsilon}{\left(\phi \phi_{\upsilon j}^*\right)^{\frac{1}{2}}} \frac{\theta \left(\phi_{\upsilon j}^*\right)^\theta}{\phi^{\theta+1}} d\phi = \sum_{\upsilon=1}^{I} J_\upsilon \left(\frac{b_\upsilon}{\phi_{\upsilon j}^*}\right)^\theta \frac{\tau_{\upsilon j} w_\upsilon}{\phi_{\upsilon j}^*} \frac{\theta}{\theta+0.5}. \tag{a.2}$$

Then, using (2), (10), and (12) into (a.2) gives

$$P_j = \frac{2\theta w_j}{\bar{q}}. \tag{a.3}$$

Moreover, using (a.3) and (10) into (2) yields

$$N_j = \left[\left(\frac{(1+2\theta) w_j}{\bar{q}}\right)^\theta \sum_{\upsilon=1}^{I} \frac{J_\upsilon b_\upsilon^\theta}{\left(\tau_{\upsilon j} w_\upsilon\right)^\theta}\right]^{\frac{1}{\theta+1}}. \tag{a.4}$$

Substituting (a.3) and (a.4) into (10) gives the following expression for the cutoff productivity

$$\phi_{ij}^* = \tau_{ij} w_i \left[\frac{\bar{q} \sum_{\upsilon=1}^{I} J_\upsilon b_\upsilon^\theta (\tau_{\upsilon j} w_\upsilon)^{-\theta}}{(1+2\theta) w_j} \right]^{\frac{1}{\theta+1}}. \tag{a.5}$$

In order to solve the model, it is necessary to jointly determine the wages, w_i, and the measures of entrants, J_i, $\forall i$. The system of equilibrium equations consists of the free entry condition, (16), and the income/spending equality, (17), for each country.

Free entry requires that average profits cover the fixed cost of entry, so

$$w_i f_e = \sum_{\upsilon=1}^{I} \left(\frac{b_i}{\phi_{i\upsilon}^*} \right)^\theta \frac{\bar{q} \tau_{i\upsilon} w_i L_\upsilon}{\phi_{i\upsilon}^* (\theta+1)(2\theta+1)}. \tag{a.6}$$

The income/spending identity requires that country i's consumers spend their entire income on imported and domestically-produced varieties, so

$$w_i L_i = \sum_{\upsilon=1}^{I} J_i \left(\frac{b_i}{\phi_{i\upsilon}^*} \right)^\theta \frac{\bar{q} \tau_{i\upsilon} w_i L_\upsilon}{\phi_{i\upsilon}^* (2\theta+1)}. \tag{a.7}$$

Expressions (a.6) and (a.7) yield

$$J_i = L_i [(\theta+1) f_e]^{-1}. \tag{a.8}$$

Substituting (a.8) into (a.5) yields expression (18) for the cutoff productivity in the text, where the terms in the summation that are particular to country j are emphasized for expositional purposes.

To characterize wages, use the definition for import shares (27) and trade balance $\sum_j T_{ij} = \sum_j T_{ji}$ in the definition of income/spending (17) to express income as $w_i L_i = \sum_j T_{ij} = \sum_j w_j L_j \lambda_{ij}$. Finally, in this expression, substitute out import shares using (27) to obtain

$$\frac{w_i^{\theta+1}}{b_i^\theta} = \sum_{j=1}^{I} \left(\frac{L_j w_j}{{\tau_{ij}}^\theta \sum_{\upsilon=1}^{I} L_\upsilon b_\upsilon^\theta (\tau_{\upsilon j} w_\upsilon)^{-\theta}} \right). \tag{a.9}$$

(a.9) implicitly solves for the wage rate w_i for each country i as a function of the remaining countries' wages. Rearrange (a.9) and use it to define

$$Z_i(w) \equiv \frac{b_i^\theta}{w_i^{\theta+1}} \sum_{j=1}^{I} \left(\frac{L_j w_j}{{\tau_{ij}}^\theta \sum_{\upsilon=1}^{I} L_\upsilon b_\upsilon^\theta (\tau_{\upsilon j} w_\upsilon)^{-\theta}} \right) - 1.$$

$Z_i(w)$ is the i-th contribution to the system of I equations that characterizes the equilibrium wage vector. Equilibrium wages satisfy $Z_i(w) = 0$ ($\forall i$). It is straightforward to show that there exists a unique equilibrium wage vector that satisfies the system equality, after setting one w_i

to be a numéraire (see Alvarez and Lucas (2007)). The idea is to treat the system above as an aggregate excess demand function of an exchange economy. For existence, it suffices to verify that the system satisfies properties 1-5 listed in Proposition 17.B.2 of Mas-Colell et al. (1995), p. 581. Existence follows from Proposition 17.C.1 of Mas-Colell et al. (1995), p. 585, which is essentially a reference to Kakutani's fixed point theorem. For uniqueness, notice that the system has the gross substitution property (differential version of Definition 17.F.2 in Mas-Colell et al. (1995), p. 612), $\forall i, k, k \neq i, \partial Z_i(w)/\partial w_k > 0$, and the result follows from Proposition 17.F.3 of Mas-Colell et al. (1995), p. 613.

When gross substitution holds, comparative static exercises with respect to wages are straightforward. Let $B \equiv \{\tau_{ij}, L_j, b_i, \theta\}_{i,j=1,\dots,I}$ denote the set of relevant parameters. Then the equilibrium system can be written as $Z(w; B)$. Let w^* be the unique wage vector corresponding to B^*; $Z(w^*; B^*) = 0$. WLOG, consider a positive productivity shock in country I, namely a rise in b_I. To determine the effect on wages, I need to characterize $Dw(B^*)$. By Implicit Function Theorem,

$$Dw(B^*) = -[D_w Z(w^*; B^*)]^{-1} D_B Z(w^*; B^*).$$

Since the system has the gross substitution property, Proposition 17.G.3 in Mas-Colell et al. (1995), p. 618, ensures that $[D_w Z(w^*; B^*)]^{-1}$ has all its entries negative. Moreover, differentiation shows that $D_B Z(w^*; B^*) db_I << 0$ for the first $I-1$ countries (and therefore the sign is positive for country I). Then, $Dw(B^*) db_I << 0$ for the first $I-1$ countries. Hence, a positive productivity shock in I lowers the wages of all countries relative to I; or, it raises I's relative wage. A more detailed proof is beyond the scope of the paper and is available upon request.

A.3 Comparative Statics and Proofs of Propositions

In this section, I show how productivity thresholds vary with respect to three market characteristics: per-capita income, size, and total income. I maintain the ceteris paribus assumption and I consider changes in one market characteristic at a time, holding all other objects fixed.

The analysis follows Hummels and Lugovskyy (2009), who choose the parameters of their model so that wages across countries are identical and fixed, and per-capita income differences reflect labor-efficiency differences. I conduct a similar exercise, however, I fix efficiency levels and consider changes in per-capita incomes that occur due to changes in wages.

Since prices are inversely related to thresholds, it is sufficient to show how thresholds change with market characteristics and take the opposite sign.

Differentiating (18) with respect to w_j, while keeping all other objects fixed, yields

$$\frac{\partial \phi_{ij}^*}{\partial w_j} = -\frac{\bar{q}^{\frac{1}{\theta+1}} \tau_{ij} w_i}{[(\theta+1) f_e (1+2\theta)]^{\frac{1}{\theta+1}}} \left[\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{v \neq j} \frac{L_v b_v^\theta}{w_j (\tau_{vj} w_v)^\theta} \right]^{-\frac{\theta}{\theta+1}} \left[\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \frac{1}{\theta+1} \sum_{v \neq j} \frac{L_v b_v^\theta}{w_j^2 (\tau_{vj} w_v)^\theta} \right] < 0.$$

Differentiating (18) with respect to L_j, while keeping all other objects fixed, yields

$$\frac{\partial \phi_{ij}^*}{\partial L_j} = \frac{\bar{q}^{\frac{1}{\theta+1}} \tau_{ij} w_i}{[(\theta+1) f_e (1+2\theta)]^{\frac{1}{\theta+1}}} \left[\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j \left(\tau_{\upsilon j} w_\upsilon\right)^\theta} \right]^{-\frac{\theta}{\theta+1}} \frac{1}{\theta+1} \frac{b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} > 0.$$

To compare elasticities of prices with respect to per-capita and total income, refer to (23). First, differentiating (23) with respect to Y_j, while keeping all other objects (including w_j) fixed, yields

$$\frac{\partial \phi_{ij}^*}{\partial Y_j} = \frac{\bar{q}^{\frac{1}{\theta+1}} \tau_{ij} w_i}{[(\theta+1) f_e (1+2\theta)]^{\frac{1}{\theta+1}}} \left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j \left(\tau_{\upsilon j} w_\upsilon\right)^\theta} \right]^{-\frac{\theta}{\theta+1}} \frac{1}{\theta+1} \frac{b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} > 0. \tag{a.10}$$

Using (a.10) and (23), the elasticity of ϕ_{ij}^* with respect to Y_j is

$$\frac{\partial \phi_{ij}^*}{\partial Y_j} \frac{Y_j}{\phi_{ij}^*} = \frac{\frac{1}{\theta+1} \left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} \right]}{\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta} \right]} > 0. \tag{a.11}$$

Differentiating (23) with respect to w_j, while keeping all other objects (including Y_j) fixed, yields

$$\begin{aligned} \frac{\partial \phi_{ij}^*}{\partial w_j} = & - \frac{\bar{q}^{\frac{1}{\theta+1}} \tau_{ij} w_i}{[(\theta+1) f_e (1+2\theta)]^{\frac{1}{\theta+1}}} \left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j \left(\tau_{\upsilon j} w_\upsilon\right)^\theta} \right]^{-\frac{\theta}{\theta+1}} \frac{1}{\theta+1} \cdots \\ & \cdots \left[(\theta+2) \frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+3}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j^2 \left(\tau_{\upsilon j} w_\upsilon\right)^\theta} \right] < 0. \end{aligned} \tag{a.12}$$

Using (a.12), (23), and (a.11), the elasticity of ϕ_{ij}^* with respect to w_j is

$$\frac{\partial \phi_{ij}^*}{\partial w_j} \frac{w_j}{\phi_{ij}^*} = - \frac{\frac{1}{\theta+1} \left[(\theta+2) \frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta} \right]}{\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta} \right]} = -(\theta+2) \frac{\partial \phi_{ij}^*}{\partial Y_j} \frac{Y_j}{\phi_{ij}^*} - \frac{\frac{1}{\theta+1} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta} \right]}.$$

Clearly, the sum of the elasticity of the threshold with respect to per-capita income and the elasticity of the threshold with respect to income is negative.

Proof of Proposition 1. Consider expression (22), which represents the price of variety ϕ from i in destination j relative to k, $k \neq j$. Since I can always relabel countries, without loss of generality, consider an increase in w_j, keeping w_k fixed. The goal is to show that $\partial(p_{ij}(\phi)/p_{ik}(\phi))/\partial w_j > 0$. From (22), it suffices to show that $\partial(\phi_{ij}^*/\phi_{ik}^*)/\partial w_j < 0$.

Using expression (18) for destination j and rewriting the sum in (18) for destination k so as to isolate the j-term yields

$$\frac{\phi^*_{ij}}{\phi^*_{ik}} = \frac{\tau_{ij}}{\tau_{ik}} \left[\frac{\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{L_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^\theta} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}} . \tag{a.13}$$

Differentiating (a.13) with respect to w_j yields

$$\frac{\partial(\phi^*_{ij}/\phi^*_{ik})}{\partial w_j} = \frac{\frac{\tau_{ij}}{\tau_{ik}} \frac{1}{\theta+1}}{\left[\frac{L_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^\theta} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}\right]^2} \left[\frac{\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{L_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^\theta} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}-1} \left[-\frac{L_j^2 b_j^{2\theta}}{\tau_{jj}^\theta w_k w_j^{2\theta+2} \tau_{jk}^\theta} \dots \right.$$
$$\dots - \frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2} w_k} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k} w_\upsilon)^\theta} - \frac{L_j b_j^\theta}{w_k w_j^{\theta+2} \tau_{jk}^\theta} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} w_\upsilon)^\theta} - \frac{1}{w_k w_j^2} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} w_\upsilon)^\theta} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k} w_\upsilon)^\theta} \dots$$
$$\left. \dots -\theta \frac{L_j b_j^\theta}{w_j^{\theta+2} w_k} \left\{ \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj} \tau_{\upsilon k} w_\upsilon)^\theta} - \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} \tau_{jk} w_\upsilon)^\theta} \right\} \right] . \tag{a.14}$$

A sufficient condition for (a.14) to be strictly negative is that the term in the curly bracket is non-negative. Since, by assumption $\tau_{jj} = 1$ $(\forall j)$, the term in the curly bracket is non-negative when trade barriers obey the triangle inequality, $(\forall j, k, \upsilon) \tau_{\upsilon j} \tau_{jk} \geq \tau_{\upsilon k}$. □

Proof of Proposition 2. Consider expression (22), which represents the price of variety ϕ from i in destination j relative to k, $k \neq j$. Since I can always relabel countries, without loss of generality, consider an increase in L_j, keeping L_k fixed. The goal is to show that $\partial(p_{ij}(\phi)/p_{ik}(\phi))/\partial L_j < 0$. From (22), it suffices to show that $\partial(\phi^*_{ij}/\phi^*_{ik})/\partial L_j > 0$.

Differentiating (a.13) with respect to L_j yields

$$\frac{\partial(\phi^*_{ij}/\phi^*_{ik})}{\partial L_j} = \frac{\frac{\tau_{ij}}{\tau_{ik}} \frac{1}{\theta+1}}{\left[\frac{L_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^\theta} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}\right]^2} \left[\frac{\frac{L_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{L_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^\theta} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}-1} \dots$$
$$\dots \frac{b_j^\theta}{w_j^{\theta+1} w_k} \left\{ \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj} \tau_{\upsilon k} w_\upsilon)^\theta} - \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} \tau_{jk} w_\upsilon)^\theta} \right\} . \tag{a.15}$$

A sufficient condition for (a.15) to be strictly positive is that the term in the curly bracket is strictly positive. Since, by assumption $\tau_{jj} = 1$ $(\forall j)$, the term in the curly bracket is strictly positive when the trade barriers for j and k obey the triangle inequality, $(\forall \upsilon) \tau_{\upsilon j} \tau_{jk} \geq \tau_{\upsilon k}$, and when the inequality for at least one $\upsilon \neq j$ is strict. □

Proof of Proposition 3. Consider expression (22), which represents the price of variety ϕ from i in destination j relative to k, $k \neq j$. Since I can always relabel countries, without loss of generality, consider an increase in Y_j, keeping w_j, and w_k and L_k (therefore also Y_k) fixed. The goal is to show that $\partial(p_{ij}(\phi)/p_{ik}(\phi))/\partial Y_j < 0$. From (22), it suffices to show that $\partial(\phi^*_{ij}/\phi^*_{ik})/\partial Y_j > 0$.

Using expression (23) for destination j and rewriting the sum in (23) for destination k so as to isolate the Y_j-term yields

$$\frac{\phi^*_{ij}}{\phi^*_{ik}} = \frac{\tau_{ij}}{\tau_{ik}} \left[\frac{\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{Y_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}}. \tag{a.16}$$

Differentiating (a.16) with respect to Y_j yields

$$\frac{\partial(\phi^*_{ij}/\phi^*_{ik})}{\partial Y_j} = \frac{\frac{\tau_{ij}}{\tau_{ik}} \frac{1}{\theta+1}}{\left[\frac{Y_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta} \right]^2} \left[\frac{\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{Y_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}-1} \cdots$$
$$\cdots \frac{b_j^\theta}{w_j^{\theta+2} w_k} \left\{ \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj} \tau_{\upsilon k} w_\upsilon)^\theta} - \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} \tau_{jk} w_\upsilon)^\theta} \right\}. \tag{a.17}$$

A sufficient condition for (a.17) to be strictly positive is that the term in the curly bracket is strictly positive. Since, by assumption $\tau_{jj} = 1$ $(\forall j)$, the term in the curly bracket is strictly positive when the trade barriers for j and k obey the triangle inequality, $(\forall \upsilon) \tau_{\upsilon j} \tau_{jk} \geq \tau_{\upsilon k}$, and when the inequality for at least one $\upsilon \neq j$ is strict. □

Proof of Proposition 4. Consider expression (22), which represents the price of variety ϕ from i in destination j relative to k, $k \neq j$. Since I can always relabel countries, without loss of generality, consider an increase in w_j, keeping Y_j, and w_k and L_k (therefore also Y_k) fixed. The goal is to show that $\partial(p_{ij}(\phi)/p_{ik}(\phi))/\partial w_j > 0$. From (22), it suffices to show that $\partial(\phi^*_{ij}/\phi^*_{ik})/\partial w_j < 0$.

Differentiating (a.16) with respect to w_j yields

$$\frac{\partial(\phi^*_{ij}/\phi^*_{ik})}{\partial w_j} = \frac{\frac{\tau_{ij}}{\tau_{ik}} \frac{1}{\theta+1}}{\left[\frac{Y_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta} \right]^2} \left[\frac{\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_j (\tau_{\upsilon j} w_\upsilon)^\theta}}{\frac{Y_j b_j^\theta}{w_k \tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{w_k (\tau_{\upsilon k} w_\upsilon)^\theta}} \right]^{\frac{1}{\theta+1}-1} \cdots$$
$$\cdots \left[-\frac{Y_j^2 b_j^{2\theta}}{\tau_{jj}^\theta w_k w_j^{2\theta+4} \tau_{jk}^\theta} - 2\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+3} w_k} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k} w_\upsilon)^\theta} - \frac{1}{w_k w_j^2} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} w_\upsilon)^\theta} \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k} w_\upsilon)^\theta} \cdots \right.$$
$$\left. \cdots -\theta \frac{Y_j b_j^\theta}{w_j^{\theta+3} w_k} \left\{ \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj} \tau_{\upsilon k} w_\upsilon)^\theta} - \sum_{\upsilon \neq j} \frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j} \tau_{jk} w_\upsilon)^\theta} \right\} \right]. \tag{a.18}$$

A sufficient condition for (a.18) to be strictly negative is that the term in the curly bracket is non-negative. Since, by assumption $\tau_{jj} = 1$ $(\forall j)$, the term in the curly bracket is non-negative when trade barriers obey the triangle inequality, $(\forall j,k,\upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$.

Finally, from (22), notice that, in order to compute the sum of the elasticities of relative prices with respect to relative incomes and relative per-capita incomes, $\zeta_{Y\,j,k} + \zeta_{w\,j,k}$, it is sufficient to compute the negative of the sum of the elasticities of relative thresholds with respect to the same variables. To derive the elasticity of relative thresholds with respect to relative incomes, multiply (a.17) by $Y_j/(\phi^*_{ij}/\phi^*_{ik})$, where ϕ^*_{ij}/ϕ^*_{ik} is given in (a.16), which yields

$$\chi_{Y\,j,k} \equiv \frac{Y_j}{\left[\frac{\phi^*_{ij}}{\phi^*_{ik}}\right]}\frac{\partial\left[\frac{\phi^*_{ij}}{\phi^*_{ik}}\right]}{\partial Y_j} = \frac{\frac{Y_j b_j^\theta}{w_j^{\theta+2}w_k}\left\{\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj}\tau_{\upsilon k}w_\upsilon)^\theta} - \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{\left(\tau_{\upsilon j}\tau_{jk}w_\upsilon\right)^\theta}\right\}}{(\theta+1)\left[\frac{Y_j b_j^\theta}{w_k\tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_k(\tau_{\upsilon k}w_\upsilon)^\theta}\right]\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_j(\tau_{\upsilon j}w_\upsilon)^\theta}\right]}. \quad \text{(a.19)}$$

Similarly, to derive the elasticity of relative thresholds with respect to relative per-capita incomes, multiply (a.18) by $w_j/(\phi^*_{ij}/\phi^*_{ik})$, where ϕ^*_{ij}/ϕ^*_{ik} is given in (a.16), and use (a.19) to obtain

$$\chi_{w\,j,k} \equiv \frac{w_j}{\left[\frac{\phi^*_{ij}}{\phi^*_{ik}}\right]}\frac{\partial\left[\frac{\phi^*_{ij}}{\phi^*_{ik}}\right]}{\partial w_j} = -\theta\frac{\frac{Y_j b_j^\theta}{w_j^{\theta+2}w_k}\left\{\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{jj}\tau_{\upsilon k}w_\upsilon)^\theta} - \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{\left(\tau_{\upsilon j}\tau_{jk}w_\upsilon\right)^\theta}\right\}}{(\theta+1)\left[\frac{Y_j b_j^\theta}{w_k\tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_k(\tau_{\upsilon k}w_\upsilon)^\theta}\right]\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_j(\tau_{\upsilon j}w_\upsilon)^\theta}\right]}\dots$$

$$\dots - \frac{\left[\frac{Y_j^2 b_j^{2\theta}}{\tau_{jj}^\theta w_k w_j^{2\theta+3}\tau_{jk}^\theta} + 2\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}w_k}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta} + \frac{1}{w_k w_j}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j}w_\upsilon)^\theta}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta}\right]}{(\theta+1)\left[\frac{Y_j b_j^\theta}{w_k\tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_k(\tau_{\upsilon k}w_\upsilon)^\theta}\right]\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_j(\tau_{\upsilon j}w_\upsilon)^\theta}\right]}$$

$$= -\theta\chi_{Y\,j,k} - \frac{\left[\frac{Y_j^2 b_j^{2\theta}}{\tau_{jj}^\theta w_k w_j^{2\theta+3}\tau_{jk}^\theta} + 2\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}w_k}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta} + \frac{1}{w_k w_j}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j}w_\upsilon)^\theta}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta}\right]}{(\theta+1)\left[\frac{Y_j b_j^\theta}{w_k\tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_k(\tau_{\upsilon k}w_\upsilon)^\theta}\right]\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_j(\tau_{\upsilon j}w_\upsilon)^\theta}\right]}. \quad \text{(a.20)}$$

Finally, using (a.20), the sum of the two elasticities is

$$\chi_{w_{j,k}} + \chi_{Y_{j,k}} = -(\theta-1)\chi_{Y\,j,k} - \frac{\left[\frac{Y_j^2 b_j^{2\theta}}{\tau_{jj}^\theta w_k w_j^{2\theta+3}\tau_{jk}^\theta} + 2\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}w_k}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta} + \frac{1}{w_k w_j}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon j}w_\upsilon)^\theta}\sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{(\tau_{\upsilon k}w_\upsilon)^\theta}\right]}{(\theta+1)\left[\frac{Y_j b_j^\theta}{w_k\tau_{jk}^\theta w_j^{\theta+1}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_k(\tau_{\upsilon k}w_\upsilon)^\theta}\right]\left[\frac{Y_j b_j^\theta}{\tau_{jj}^\theta w_j^{\theta+2}} + \sum_{\upsilon\neq j}\frac{L_\upsilon b_\upsilon^\theta}{w_j(\tau_{\upsilon j}w_\upsilon)^\theta}\right]}.$$

By assumption, $\tau_{jj} = 1$ $(\forall j)$ and trade barriers obey the triangle inequality, $(\forall j,k,\upsilon)\tau_{\upsilon j}\tau_{jk} \geq \tau_{\upsilon k}$. Hence, $\chi_{Y\,j,k} \geq 0$. A sufficient condition for the sum to be strictly negative is that $\theta \geq 1$. □