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THE IMPLICATIONS OF MEAN-VARIANCE OPTIMIZATION FOR FOUR QUESTIONS IN INTERNATIONAL MACROECONOMICS

Jeffrey A. Frankel

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ABSTRACT

The hypothesis that investors optimize with respect to the mean and variance of their end-of-period wealth has powerful implications for some standard questions of interest to international macroeconomists. The implications transcend the particular econometric technique used to estimate the return variance-covariance matrix.

(1) For conventional estimates of risk-aversion, substitutability between domestic and foreign securities is close to perfect in the sense that risk premiums are small in magnitude (a few basis points), and thus cannot explain much bias in forward rates. (2) Nevertheless, as long as risk-aversion is not zero, foreign exchange intervention still affects the level of the exchange rate. If interest rates are held constant, the effect is proportionate to the contemporaneous change in asset supplies, and is more-than-proportionate if the expectations of future asset supplies also change. (3) Current account deficits have effects that are comparable to, though smaller in magnitude than, the effects of equal-sized changes in asset supplies through intervention or government borrowing. (4) The perceived tendency for dollar depreciation to be associated with appreciation of the mark against the franc is not consistent with the implication of mean-variance optimization that the franc should be a closer substitute for the dollar than is the mark.

Jeffrey A. Frankel
Department of Economics
University of California
Berkeley, CA 94720

(415) 642-8084

1. INTRODUCTION

Many important questions in international finance depend critically on the nature of the exchange risk premium, defined as the expected rate of return on foreign assets relative to domestic assets. But the risk premium has proven to be a difficult variable to get a handle on empirically. In past work, in order to bring more structure to bear on the problem of the risk premium, I have made the assumption that investors diversify their portfolios so as to optimize with respect to the mean and variance of their wealth. But the contribution proved to be a negative one in that, using my econometric technique to impose the constraint of mean-variance optimization, I was unable to reject the hypothesis that investors were risk-neutral and that the risk premium therefore did not exist.[1] A failure to reject a null hypothesis could always be attributable to low power in the test. Furthermore, one criticism that could be made of this work is that it required that the supplies of dollar, mark, and other assets could be accurately measured, a questionable assumption.

We will see in this paper that the hypothesis of mean-variance optimization is sufficiently powerful to have important implications that transcend specific techniques of data computation or econometric estimation. The hypothesis turns out to imply that the exchange risk premium, even assuming it exists, is extremely small in magnitude. When the supply of foreign assets is increased by one percent of world wealth--a large number--the effect on the risk premium is only on the order of .02

percent, or 2 basis points. (By comparison, the forward discount on the dollar against the mark or yen was about 300 basis points as of early 1985.) This calculation assumes a coefficient of relative risk-aversion of 2 and a standard deviation for international returns of 10 percent. Even if these parameters are estimated at levels several times higher, the implied effects on the risk premium are still of a negligibly small magnitude. One implication, the answer to the first question considered below, is that the risk premium is relatively unimportant for evaluating tests of efficiency in the forward exchange market.

It does not follow, however, that imperfect substitutability between domestic and foreign bonds is unimportant for all questions of interest. Another way of stating the conclusion--the proposition that changes in asset supplies need have only small effects on the risk premium to be willingly absorbed into the market--is to say that small changes in the risk premium have large effects on asset demands and therefore on the exchange rate. We will see that the framework of optimal portfolio diversification is a useful one in which to examine some standard questions regarding the determination of exchange rates. Often it will be possible to get interesting quantitative results that are not sensitive to the precise techniques used for measuring asset supplies or estimating variances and covariances.

2. FOUR QUESTIONS

Question 1. Can risk premiums account for findings of a nonrandom component in the spot rate prediction errors of the forward rate?

Many authors have statistically rejected the hypothesis that the forward rate is a conditionally unbiased predictor of the future spot rate. Several, for example, have found statistically significant serial correlation in the forward rate's prediction errors. I found autoregressive coefficients on the order of .4 for the pound and the lire (against the dollar, July 1974 to April 1978) in Frankel [1980, p. 1094]. Hansen and Hodrick [1980, p. 840] regressed the prediction error against the lagged prediction errors, not just of the own currency, but of four other currencies as well (October 1974 to April 1978). The mark had an own-autoregression coefficient of minus .665. Many of the cross coefficients were significant as well; they had both positive and negative signs, but similarly large magnitudes.[2] Some authors have tested the hypothesis that the interest differential is an unbiased predictor of the future rate of currency depreciation, which given covered interest parity is equivalent to the first hypothesis, and have also rejected it. Cumby and Obstfeld [1984] found a statistically significant autocorrelation coefficient of .5 in the interest differential's forecast error for the yen.[3]

It is well-known that these are all tests of a joint null hypothesis:

(a) that there is no risk premium, so that the forward rate is equal to investors' expectations as to the future spot rate, and

(b) that the market is efficient, so that investors' expectations are equal to the mathematical expectation conditional on available information.

Some authors, such as Hansen and Hodrick [1980] and Cumby and Obstfeld [1981], interpret the test results as evidence of the importance of the risk premium, thus leaving the market efficiency part of the joint null hypothesis intact. Could nonzero autoregressive coefficients be attributable to risk premiums?

Let

Δs_{+1} ≡ the ex post percentage change in the spot price of foreign currency,

fd ≡ the forward discount on domestic currency (or, given covered interest parity, the domestic interest rate minus the foreign interest rate),

ε_{+1} ≡ the ex post expectational error $\Delta s_{+1} - \Delta s^e$, where Δs^e is the market's ex ante expectation of depreciation, and

rp ≡ the risk premium on foreign assets $\Delta s^e - fd$.

The prediction error can be decomposed into the expectational error and the risk premium:

$$(\Delta s_{+1} - fd) = rp + \varepsilon_{+1}. \tag{1}$$

Its autocovariance is given by

$$\text{(2)} \qquad \mathrm{Cov}\left(\Delta s_{+1} - fd,\ \Delta s - fd_{-1}\right) = \mathrm{Cov}\left(rp,\ \Delta s - fd_{-1}\right) + \mathrm{Cov}\left(\varepsilon_{+1},\ \Delta s - fd_{-1}\right).$$

Assuming rational expectations, the last covariance term is zero. Thus the autoregression coefficient is given by

$$\text{(3)} \qquad \mathrm{Autoregr}(\Delta s_{+1} - fd) = \frac{\mathrm{Cov}\left(rp,\ \Delta s - fd_{-1}\right)}{\mathrm{Var}\left(\Delta s - fd_{-1}\right)} .$$

The risk premium rp is not observable. But it can be assumed related to the relative supplies of countries' assets as in a portfolio balance equation for asset demands.[4]

$$\text{(4)} \qquad x = A + B(rp).$$

In inverted form

$$\text{(5)} \qquad rp = -B^{-1}A + B^{-1}x.$$

An increase in the supply of a country's assets requires an increase in the expected rate of appreciation of its currency (relative to the forward discount or interest differential) for them to be willingly held.[5]

We use equation (5) in (3)

$$\text{(6)} \qquad \mathrm{Autoregr}\left(\Delta s_{+1} - fd\right) = B^{-1} \frac{\mathrm{Cov}\left(x,\ \Delta s - fd_{-1}\right)}{\mathrm{Var}\left(\Delta s - fd_{-1}\right)} .$$

Table 1 reports in column (3) estimates of the ratio appearing in equation (6), interpretable as the coefficient in a regression of portfolio shares against lagged prediction errors, in each case pretending for the

Table 1

	(1)	(2)	(3)	(4)
	$Cov(x_t, e_t)$	$Var(e_t)$	(1)/(2)	$B^{-1} = \rho Var$ per annum basis
Pound	-1.464×10^{-5}	79.5×10^{-5}	-.01842	.0198
Franc	-1.413×10^{-5}	98.7×10^{-5}	-.01432	.0237
Mark	$+4.972 \times 10^{-5}$	112.9×10^{-5}	+.04404	.0271
Canadian dollar	$+1.280 \times 10^{-5}$	18.9×10^{-5}	+.06772	.0045
Yen	-7.325×10^{-5}	98.8×10^{-5}	-.07414	.0237

$x_t \equiv$ share of portfolio allocated to each currency

$e_t \equiv s_t - s_{t-1} - fd_{t-1} \equiv$ forward rate prediction error

$\rho \equiv$ coefficient of relative risk aversion $\equiv 2$

Note: The data sources and computations are described at length in Appendix 4 to Frankel (1982). The sample period is August 1973 to August 1980.

moment that the entire portfolio consists only of dollars and the other currency in question. Without any way of bounding the parameter B^{-1}, we have no way of judging whether observed autoregression coefficients are too large to be explained by the risk premium. We must wait for further information from mean-variance optimization theory.

Question 2. Can foreign exchange intervention have a meaningful effect on the exchange rate?

One reason why there has been so much interest in estimating investors' degree of substitutability among countries' assets (as in the studies cited in footnote 5) is that it has been thought to determine the effectiveness of foreign exchange intervention, particularly intervention sterilized so as to have no effect on money supplies. The argument is that as the parameter B in equation (4) goes to infinity, B^{-1} in equation (5) goes to zero, so that asset supplies have no effect on relative expected rates of return nor, presumably, on anything else. But the step from relative expected rates of return to the spot exchange rate is not immediate.

The portfolio share allocated to foreign assets is defined as

$$x \equiv SF/W \tag{7}$$

where S ≡ the level of the spot exchange rate,

F ≡ the supply of foreign assets,

D ≡ the supply of domestic assets, which we will take to be dollars in the empirical work,

and W ≡ total wealth, SF + D.

The portfolio share allocated to domestic assets is given by

$$1 - x = D/W. \tag{8}$$

We take the ratio to get an equation of exchange rate determination:

$$S = \frac{D}{F}\frac{x}{1-x} \,. \tag{9}$$

Equation (9) says that the exchange rate is determined by the relative supply of domestic vs. foreign assets and the relative demand for the two assets. Taking logs,

$$s = \ell + \log(x) - \log(1-x), \tag{10}$$

where $s \equiv \log S$

and $\ell \equiv \log(D/F)$.

We want to know the effect of an unanticipated change in asset supplies ℓ on the exchange rate. If expected rates of return and therefore the asset demand shares x by equation (4) are unchanged, then the effect is simply proportionate to the change in asset supplies. Given actual magnitudes of intervention, this would not be a particularly large effect, even if we are talking about nonsterilized intervention. But the effect could be larger if the change in current asset supplies alters market expectations as to future asset supplies, and alters the expected rate of return and therefore x. Differentiating (10):

$$\frac{ds}{d\ell} = 1 + \frac{1}{x}\frac{dx(rp)}{d\ell} + \frac{1}{1-x}\frac{dx(rp)}{d\ell} \,. \tag{11}$$

The risk premium was defined to be the domestic-foreign interest differential minus expected depreciation Δs^e. The kind of foreign exchange intervention we will examine is one that leaves interest rates unchanged. This is equivalent to sterilized intervention, which holds the money supply constant, only in the special case (as in IS-LM) where money demand depends on the interest rate but not on the supply of bonds, and where the price level does not respond to the exchange rate.[6] If one believes that the interest rate is determined by the supply of money relative to bonds, then we are not talking about sterilized intervention, but rather intervention that changes the supply of money and bonds proportionately. The mean-variance optimization model has little to say about the allocation of the portfolio between non-interest-bearing money and bonds, so we choose to leave these considerations outside our model and to assume simply that monetary policy is whatever it must be to keep interest rates constant.

Allowing only Δs^e to move in equation (4),

(12) $$\frac{dx}{d\ell} = B\,\frac{d\Delta s^e}{d\ell}\ .$$

Equation (11) thus becomes

(13) $$\frac{ds}{d\ell} = 1 + \left(\frac{1}{x} + \frac{1}{1-x}\right) B\,\frac{d\Delta s^e}{d\ell}\ .$$

Define

(14) $$\lambda \equiv \left(\frac{1}{x} + \frac{1}{1-x}\right) B$$

(while recognizing that λ will be different at different points in time as x changes). Then equation (13) can be made to look more familiar:

$$ds = d\ell + \lambda d\left(s^{e}_{+1} - s\right). \tag{15}$$

If ℓ were interpreted as the relative supply of domestic money, then equation (15) would be the standard monetarist equation of exchange rate determination, where λ would be interpreted as the semielasticity of money demand.[7]

As in that literature, we can solve for s as a function of ℓ and s^{e}_{+1}, substitute for s^{e}_{+1} the rational expectation of next period's ℓ_{+1} and s^{e}_{+2}, substitute similarly for s^{e}_{+2}, and so forth. The result is an expression for this period's exchange rate as a present discounted sum of the entire future path of expected asset supplies:

$$ds = \frac{1}{1+\lambda} \sum_{\tau=0} \left(\frac{\lambda}{1+\lambda}\right)^{\tau} dE(\ell_{+\tau}) \tag{16}$$

where $dE\left(\ell_{+\tau}\right)$ is the revision in the expected value of the asset supply variable τ periods into the future.

The effect of a current change in asset supplies on the rationally expected future path depends on the time series properties of ℓ. For example, if ℓ follows a random walk, then $dE\left(\ell_{+\tau}\right) = d\ell$ and $ds = d\ell$, i.e., the exchange rate simply changes in proportion to the current asset supply (as we could have guessed directly from (15)). At the opposite extreme, if ℓ follows a random walk <u>on trends</u>, then $dE\left(\ell_{+\tau}\right) = \tau\, d\ell$, and $ds = (1+\lambda)d\ell$ (also as we could have guessed directly from (15)). Table 2 shows estimates of the time series properties of ℓ.

Table 2: Time Series Properties of Changes in Asset Supplies

(Log Dollar Relative to Foreign)

Sample: August 1973-August 1980 (74 observations)

	AR(1) Representation		Best ARMA Representation				
	Coefficient	R^2					R^2
United Kingdom	.353** (.102)	.10		MA(1) = .268** (.111)	MA(2) = .304** (.112)	AR(12) = .392** (.110)	.25
France	.170* (.108)	.03	C = -.015 (.097)	AR(1) = .151 (.097)		AR(12) = .497** (.087)	.35
Germany	.140 (.107)	.02	C = .002 (.003)	AR(1) = .147 (.114)		AR(9) = .265** (.113)	.10
Canada	-.012 (.109)	-.01	C = -.002 (.004)			AR(12) = .338** (.114)	.11
Japan	.008 (.109)	-.10					

*significant at the 90 percent level
**significant at the 95 percent level

(Standard errors reported in parentheses.)

In each case, the process is not stationary on levels; first differencing is required. But after an innovation in the supply of domestic assets, the effect on expected future changes damps out fairly quickly. In no case is the specification of a random walk on trends appropriate. In different cases, different ARMA processes seem most appropriate. To simplify and standardize, we will consider ARMA (1,1,0); in the case of the United Kingdom there is a significant autoregressive parameter on changes of .35. Thus a 1 percent innovation raises the rationally expected relative asset supply by an estimated (1 + .35) percent in the following period, $(1 + .35 + .35^2)$ percent in the next, and so forth, gradually damping out. (The magnitude and significance levels of the first-order autoregressive parameter are lower for France and Germany. They are close to zero for Canada and Japan, suggesting that asset supplies for these countries follow a simple random walk on levels.)

Without any way of estimating B and therefore λ, we have no way of measuring the effect on the exchange rate of expected future changes in asset supplies. Again we must await the further enlightenment of mean-variance optimization theory.

Question 3. Can the current account have a meaningful effect on the exchange rate?

It has been recognized for some time that in models of exchange rate determination it is not proper to measure the supply of foreign assets as the cumulation of the current account (corrected for foreign exchange intervention),[8] except in the special case when foreign investors do not hold domestic assets and so are irrelevant to the determination of the exchange rate. But the current account will still have an effect on the aggregate world demand for foreign vs. domestic assets, and therefore on the exchange rate, if foreign residents have different behavior from domestic residents.[9]

We specify three separate asset-demand functions of the nature of (4): one for American residents, one for European residents, and one for all other residents, respectively. (We could as easily specify more):

(17)
$$x_a \equiv \frac{SF_a}{W_a} = A_a + B(rp)$$
$$x_e \equiv \frac{SF_e}{W_e} = A_e + B(rp)$$
$$x_r \equiv \frac{SF_r}{W_r} = A_r + B(rp).$$

The subscripts indicate the nationality of the investors. We can aggregate the three equations by multiplying each by its region's share in world wealth; $w_a \equiv \frac{W_a}{W}$, $w_e \equiv \frac{W_e}{W}$ and $w_r \equiv \frac{W_r}{W} = 1 - w_a - w_e$, respectively; and then taking the sum:

(18)
$$x \equiv \frac{SF}{W} = A_r + (A_e - A_r)w_e - (A_r - A_a)w_a + B(rp).$$

We see that the total world demand for F assets depends not only on their expected relative rate of return rp, but also on the distribution of wealth across countries. Assuming that European residents hold the highest proportion of their portfolios in F assets, followed by rest-of-world residents (whom we have here assumed not to have their own currency) and American residents:

(19) $$A_e > A_r > A_a.$$

Then a European current account surplus that increases w_e will raise the total world demand for F assets. So will an American current account deficit that decreases w_a.

Equation (10) provides the link from the portfolio share given by equation (18) to the level of the exchange rate. To see the effect of a change in w_e, we differentiate:

(20) $$\frac{ds}{dw_e} = \left(\frac{1}{x} + \frac{1}{1-x}\right)(A_e - A_r).$$

(We do not consider any effects on expectations regarding the future.) As we would expect, a European current account surplus that transfers wealth to Europeans from the rest of the world will cause the dollar to depreciate if European residents have a higher propensity to hold F assets. What determines the relative size of A_e, A_r and A_a? What are the magnitudes of the effects on s? These questions too will be answered below.

Question 4. If the dollar depreciates (because of an actual or expected increase in the supply of dollars), will the mark tend to appreciate against the franc?

A proposition that circulates widely in European policy circles is that the value of the dollar (against an average of other currencies) affects the cross exchange rates within the European Monetary System. When the dollar was weak in the late 1970s, both the mark and the franc appreciated against the dollar, but the mark was the stronger and the French repeatedly found it necessary to devalue, or in the days of the Snake to drop out of the arrangement altogether. Since the dollar began its great appreciation in 1980, the mark has at times been weaker than the franc, and the French have not found it necessary to devalue as often. The fear in Europe is that when and if the dollar makes its long-awaited fall, the change will take the form of a portfolio shift from dollars into marks, once again putting downward pressure on the franc's value against the mark. Giavazzi and Giovannini [1984] examine this problem from several alternative viewpoints. Here we pursue one of their suggestions, that the relationship between a dollar depreciation and the franc/mark cross rate depends on the nature of investor substitutability among the three currencies in a model of optimal portfolio diversification.

We return to the version of the model in which residents of all countries exhibit uniform behavior, but we add a third asset

$$x_M \equiv \frac{S_M M}{W} = A_M + B_{MM}(\Delta s_M^e) - B_{MF}(\Delta s_F^e)$$

(21)

$$x_F \equiv \frac{S_F F}{W} = A_F - B_{FM}(\Delta s_M^e) + B_{FF}(\Delta s_F^e).$$

Here x_M is the share of the portfolio allocated to mark assets, S_M is the spot dollar/mark exchange rate, M is the supply of mark assets, and similarly for franc assets F. B_{MM} indicates the degree of substitutability between dollars and marks, and B_{FF} the degree of substitutability between dollars and francs. We continue to assume interest rates constant and so we subsume them in the constant terms. The cross rate in francs per mark is given by the ratio of the two exchange rates. In log form it is

$$s_{FM} = f - m + \log(x_M) - \log(x_F),$$

where f is the log of F and m the log of M.

Let us assume that there is a depreciation of the dollar that comes as the result of an increase in the supply of dollar assets, d in log form. Then

$$\frac{d\ s_{FM}}{d\ (d)} = \frac{1}{x_M}\frac{d\ x_M}{d(d)} - \frac{1}{x_F}\frac{d\ x_F}{d(d)}\ . \tag{22}$$

From (22) and (21),

$$\begin{aligned} \frac{d\ s_{FM}}{d\ (d)} &= \frac{1}{x_M}\left[B_{MM}\frac{d(\Delta s_M^e)}{d\ (d)} - B_{MF}\frac{d(\Delta s_F^e)}{d\ (d)}\right] \\ &\quad - \frac{1}{x_F}\left[-B_{FM}\frac{d(\Delta s_M^e)}{d\ (d)} + B_{FF}\frac{d(\Delta s_F^e)}{d\ (d)}\right] \\ &= \left[\frac{B_{MM}}{x_M} + \frac{B_{FM}}{x_F}\right]\frac{d(\Delta s_M^e)}{d\ (d)} - \left[\frac{B_{MF}}{x_M} + \frac{B_{FF}}{x_F}\right]\frac{d(\Delta s_F^e)}{d\ (d)}\ . \end{aligned} \tag{23}$$

If the increase in the supply of dollars is thought to be a one-time change, then $\frac{d(\Delta s_M^e)}{d\ (d)}$ and $\frac{d(\Delta s_F^e)}{d\ (d)}$ will each be zero. $\frac{d(s_{FM})}{d\ (d)}$ will also be zero, i.e., there is no effect on the franc/mark cross rate because the dollar/mark and dollar/franc rates each simply increase in proportion to the change in the supply of dollars.

But if the current increase induces expectations of a further increase in the next period, as seems from table 2 to be rational in some cases, then $\frac{d(\Delta s_M^e)}{d\ (d)}$ and $\frac{d(\Delta s_F^e)}{d\ (d)}$ will be positive. Consider for the moment the special case in which the effect on expected future appreciation of the two European currencies is equal:

$$\frac{d(\Delta s_M^e)}{d\ (d)} = \frac{d(\Delta s_F^e)}{d\ (d)} .$$

Then from (23) the condition for a positive effect on the franc/mark rate is:

$$(24) \qquad \Phi \equiv \frac{B_{MM}-B_{MF}}{x_M} + \frac{B_{FM}-B_{FF}}{x_F} > 0.$$

If the mark is a good substitute for the dollar, relative to the substitutability between the franc and the dollar (B_{MM} is high relative to B_{FF}), then the depreciation of the dollar is indeed more likely to be associated with an appreciation of the mark against the franc.

It is likely that Δs_M^e and Δs_F^e will not rise by the same amount. Which will rise more? If expectations regarding $s_{M_{+1}}$ and $s_{F_{+1}}$ are rational, then we can answer the question the same way as we did for s_M and s_F : in the case in which $\frac{d(\Delta s^e_{M+1})}{d\ (d)} = \frac{d(\Delta s^e_{F+1})}{d\ (d)}$, Δs_M^e would rise more than Δs_F^e if condition (24) holds. It would then follow from (23) that $\frac{d\ s_{FM}}{d\ (d)}$ would be even greater than it would be if $\frac{d(\Delta s_M^e)}{(d)}$ and $\frac{d(\Delta s_F^e)}{d\ (d)}$ were equal. One could extend the reasoning to expectations of further increases in the supply of dollar assets two periods into the future, and more. For each period, the inequality (24) remains the knife-edge condition that implies an increase in the current franc/dollar rate.[10]

Is (24) likely to hold? It is now time to turn to the mean-variance optimization theory, to help answer this and the preceding questions.

3. ANSWERS FROM THE THEORY OF MEAN-VARIANCE OPTIMIZATION

In equation (4) we assumed that asset demands as a share of the portfolio can be expressed as a linear function of the risk premium:

$$x = A + B(rp). \tag{4}$$

In Krugman [1981] and Frankel [1983] (or Frankel [1982] for the n-asset case) it is shown that under certain assumptions, notably that investors optimize with respect to the mean and variance of their end-of-period wealth, (4) is the correct form for the asset demand function, with

$$\begin{aligned} A &= \alpha\left(1 - \frac{1}{\rho}\right) + \frac{1}{2\rho} \\ B &= [\rho\ \Omega]^{-1} \end{aligned} \tag{25}$$

where ρ = the coefficient of relative risk-aversion

α = the weight of foreign goods in consumers' (Cobb-Douglas) utility function

Ω = the variance of the relative rate of return $r^* - r^\$$ (or in the n-asset case the variance-covariance matrix).[11]

Intuitively, the more important to an investor is risk diversification (the larger is ρ or Ω), the less will he or she shift the portfolio in response to a given change in expected returns (the smaller is B).

Looking at the inverted form of equation (4) we see that

$$\Delta rp = \rho\ \Omega(\Delta x).$$

For the moment, we will consider only portfolios of two assets at a time (dollars and one foreign currency at a time), to get an idea of magnitudes. The variance of the forward rate's prediction error was reported in column (2) of table 1. The coefficient of risk-aversion is generally considered to be in the neighborhood of 2.[12] Twice the variance is reported in the fourth column of table 1, multiplied by 12 so that it can be used to estimate effects on risk premiums or interest rates on a regular per annum basis. The numbers are quite small. They imply, for example, that a 1 percent increase in the share of the portfolio consisting of dollar assets would drive up the expected rate of return on dollars relative to pounds by only .02 percent per annum, or 2 basis points! We see that the risk premium cannot vary much. Since the case for a time-varying risk premium is the only case made, it follows that the magnitude of the risk premium is very small.[13]

Answer 1: It seems unlikely that the risk premium can explain significant autocorrelation in forward rate prediction errors.

In equation (6) we saw that the amount of autoregression in prediction errors which is explainable by the risk premium depends on B^{-1}, which we have now seen to be given by $\rho\ \Omega$. For each foreign currency, if one wishes to think about a simple two-currency portfolio as in tables 1 to 3, then B^{-1} is a scalar that could be estimated by twice the variance of $(\Delta s - fd_{-1})$. The variance cancels out the denominator in equation (6), and we are left with simply ρ times the numerator $Cov(x, \Delta s - fd_{-1})$, which was reported in column (1) of table 1. Even if that covariance is multiplied by 2 or higher estimates of the coefficient of risk aversion, the estimates are extremely small. The small magnitudes suggest that in studies where statistically significant autocorrelation is found, a risk premium is unlikely to be the explanation. At least, that seems to be the implication of the mean-variance optimization theory of portfolio balance.

Three points remain to be made. First, the argument is relatively invariant to measurement errors in the asset supplies. The "true" asset supplies would have to behave very differently from the measured asset supplies to have a correlation with the prediction errors which is very much greater in magnitude than those reported in the first column of table 1.

Second, when we equated the variance Ω perceived by investors with $\mathrm{Var}(\Delta s - fd_{-1})$, we implicitly assumed that they have no recent information relevant to forming expectations. Ω should be conditional on currently available information, and therefore should be smaller in magnitude than the unconditional variance. But there is wide agreement that recent information is of little help in predicting changes in the exchange rate, that the conditional variance is almost as large as the unconditional variance. (For example, Frenkel and Mussa [1980].) In any case, the direction of the bias is to overstate the numbers in the first column of table 1, representing autocorrelation attributable to a risk premium. A finding of an autocorrelation coefficient significantly greater in magnitude than those numbers would suggest an even more decisive rejection of rational expectations.

The third point is that, in theory, it should be a portfolio demand for $n > 2$ currencies that depends on the risk premiums; in inverted form, each risk premium should depend on n currency supplies, not just two. Equation (26) generalizes nicely if we consider market efficiency tests of the sort run by Hansen and Hodrick [1980] and Cumby and Obstfeld [1984]: for a given currency the current prediction error is regressed against the lagged values of the prediction errors in all of the currencies, not just the own currency. The results of such tests are reported in table 3. For the period 1973 to 1980 (which is as far as I have constructed asset supply data for), F tests reject the null hypothesis of no autoregression in the prediction errors in the case of several equations.

The matrix of regression coefficients that are estimated in table 3 can be represented, using boldface to represent five-currency matrices and vectors,

(27)

$$\left[\mathbf{Varcov}(\boldsymbol{\Delta s} - \mathbf{fd}_{-1})\right]^{-1} \mathbf{Cov}(\boldsymbol{\Delta s}_{+1} - \mathbf{fd},\ \boldsymbol{\Delta s} - \mathbf{fd}_{-1})$$

$$= \left[\mathbf{Varcov}(\boldsymbol{\Delta s} - \mathbf{fd}_{-1})\right]^{-1} \mathbf{Cov}(\mathbf{rp},\ \boldsymbol{\Delta s} - \mathbf{fd}_{-1})$$

$$= \left[\mathbf{Varcov}(\boldsymbol{\Delta s} - \mathbf{fd}_{-1})\right]^{-1} \mathbf{Cov}(\mathbf{B}^{-1}\mathbf{x},\ \boldsymbol{\Delta s} - \mathbf{fd}_{-1})$$

$$= \left[\mathbf{Varcov}(\boldsymbol{\Delta s} - \mathbf{fd}_{-1})\right]^{-1} \mathbf{Cov}(\rho\ \boldsymbol{\Omega}\ \mathbf{x},\ \boldsymbol{\Delta s} - \mathbf{fd}_{-1})$$

$$= \rho\ \mathbf{Cov}(\mathbf{x},\ \boldsymbol{\Delta s} - \mathbf{fd}_{-1}).$$

Table 3a: Autoregression of Prediction Errors

Sample: August 1973 to August 1980 (85 observations)

Dependent Variable $s_{t+1}-s_t-fd_t$	Independent Variables $s_t-s_{t-1}-fd_{t-1}$: Pound	Franc	Mark	Canadian Dollar	Yen	R^2	F	Dependent Variable Statistics: Mean	Standard Deviation
Pound	.249 (.133)	-.117 (.176)	-.100 (.164)	-.458* (.226)	-.028 (.118)	.10	2.12	.003	.028
Franc	-.009 (.146)	-.144 (.194)	-.037 (.180)	-.671* (.248)	.060 (.130)	.12	2.74*	.002	.032
Mark	.106 (.154)	-.392 (.203)	.065 (.189)	-.708* (.260)	.151 (.136)	.15	3.66*	.001	.034
Canadian Dollar	.047 (.067)	-.130 (.088)	.039 (.082)	-.050 (.113)	.069 (.059)	.04	0.86	-.001	.014
Yen	.093 (.150)	-.221 (.198)	-.073 (.185)	-.269 (.255)	.227 (.133)	.08	1.65	.002	.032

*significant at the 95 percent level
(Standard errors are reported in parentheses.)

Table 3b: Autoregression of Prediction Errors

Sample: August 1973 to December 1984 (137 observations)

Dependent Variable $s_{t+1}-s_t-fd_t$	Independent Variables $s_t-s_{t-1}-fd_{t-1}$: Pound	Franc	Mark	Canadian Dollar	Yen	R^2	F	Dependent Variable Statistics: Mean	Standard Deviation
Pound	.143 (.106)	-.138 (.162)	.150 (.157)	-.415* (.199)	-.024 (.096)	.03	1.14	-.004	.030
Franc	-.070 (.117)	-.027 (.179)	.056 (.174)	-.300 (.220)	.029 (.106)	.01	0.26	-.003	.033
Mark	.023 (.120)	-.170 (.183)	.147 (.177)	-.384 (.225)	.031 (.109)	.00	0.14	-.005	.033
Canadian Dollar	-.006 (.047)	-.169* (.072)	.127 (.070)	-.065 (.089)	.032 (.043)	.04	1.28	-.001	.013
Yen	.060 (.117)	-.133 (.178)	.105 (.173)	-.254 (2.20)	.105 (.106)	.02	0.74	-.002	.033

*significant at the 95 percent level
(Standard errors are reported in parentheses.)

Table 4

$\text{Cov.}\ (x_t\ ,\ s_t - s_{t-1} - fd_{t-1}) \times 10^5$

August 1973 - August 1980 (85 observations)

x_t	$s_t - s_{t-1} - fd_{t-1}$				
	Pound	Franc	Mark	Canadian dollar	Yen
Pound	-1.464	-0.121	-5.874	1.412	-8.799
Franc	-2.317	-1.413	-0.039	+0.025	-0.012
Mark	+6.003	+5.195	+4.972	-2.131	-4.331
Canadian dollar	-4.237	-0.715	-0.164	+1.280	+0.956
Yen	+40.00	+13.28	+7.082	-5.043	-7.325

The theory implies that the autoregression coefficient of prediction errors is bounded by (the corresponding covariance above) × (the coefficient of relative risk-aversion) × 10^{-5} .

We have used, first, the rational expectations assumption that the component of $\Delta s_{+1} - fd$ that is expectational error is uncorrelated with **rp**; second, that **rp** is governed by mean-variance optimization; and third, that we can approximate the conditional variance-covariance matrix Ω with the unconditional one so as to cancel out its inverse, as in the upper bound argument made above. The entire matrix $\mathrm{Cov}(x, \Delta s - fd_{-1})$ (the generalization of column (1) in table 1) is reported in table 4. Again we can assume $\rho = 2$. The magnitude of the autoregression parameters in table 3 is always far greater in magnitude than twice the numbers in table 4 (which are shown times 10^5), in other words are greater than can be explained by the risk premium under the assumption of mean-variance optimization.

Answer 2: An unexpected increase in the current supply of domestic relative to foreign assets can have a meaningful effect on the current exchange rate, especially when account is taken of effects on expected future asset supplies.

One percent increases in the supply of domestic assets in themselves raise the exchange rate by only 1 percent.[14] But the ARIMA estimates reported in table 2 show that a given 1 percent innovation in the asset supply generates rational expectations of future asset supply increases. We saw in equation (16) that the effect on the exchange rate will be more than 1 percent, and of a magnitude depending on the semi-elasticity of asset demand λ. Now with equation (14) we see that λ is given by

(28) $$\lambda = \left(\frac{1}{x} + \frac{1}{1-x}\right)(\rho\ \Omega)^{-1}.$$

The share of the portfolio allocated to the dollar versus the other five currencies averaged .49 during the sample period 1973-1980. Thus $\left(\frac{1}{x} + \frac{1}{1-x}\right)$ is about 4.00. The third column of table 5 reports the estimates for λ implied by the variance estimates. The coefficient for the United Kingdom is typical: 2,516 on a monthly basis, or 210 if divided by 12 to allow interpretation of λ as a semielasticity of asset demand with respect to per annum rates of return. The numbers are very high as estimates of semielasticities of demand, but this is a direct implication of the theory of mean-variance optimization.

As a preliminary experiment, consider what happens if the 1 percent increase in asset supplies raises the expected permanent rate of asset supply growth by as little as 0.1 percent per annum. Our estimate of λ implies that the exchange rate will increase by 21.0 percent! This is how sensitive asset demands are to expected returns.

In the time series processes estimated in table 2, the effect of an innovation on future asset supply changes is not in fact permanent but dies out fairly quickly. Adopting the simple ARIMA (1,1,0) specification, we saw that a 1 percent innovation in the relative supply of dollars/pounds raises the relative supply expected next period by (1 + .353) percent, the following period by $(1 + .353 + .353^2)$ percent, and so on. We use our estimate of λ, the semielasticity of asset demand with respect to expected return, to take the correct present discounted sum of future asset supplies. The total effect on today's dollar/pound exchange rate is estimated at 1.545 percent. The effects for the franc or mark are somewhat smaller,

Table 5a: Effect of an innovation in asset supplies ℓ on the spot exchange rate s, assuming semi-elasticity of asset demand determined by mean-variance optimization

	$Var(s_t-s_{t-1}-fd_{t-1})$*	$B = [\frac{1}{2} Var(s_t-s_{t-1}-fd_{t-1})]^{-1}$	Semi-elasticity $\lambda = (\frac{1}{x}+\frac{1}{1-x})B$: λ for $x=\frac{1}{2}$, Monthly	λ for $x=\frac{1}{2}$, $\div$ 12	$\bar{x}_t$	λ for $x=\bar{x}_t$, Monthly
Pound	.000795	628.93	2515.7	209.6	.145	3,708.5
Franc	.000987	506.59	2026.3	168.9	.043	11,274.5
Mark	.001129	442.87	1771.5	147.6	.121	3,217.2
Canadian Dollar	.000189	2,645.50	10,582.0	881.8	.057	43,766.8
Yen	.000988	506.07	2,024.3	168.7	.144	3,008.3

*from Table 1

Table 5b: Effect of an innovation in asset supplies ℓ on the spot exchange rate s, assuming semi-elasticity of asset demand determined by mean-variance optimization

Effect Δs_t of an innovation $\Delta \ell_t = .01$

$$\Delta s_t = \frac{1}{1+\lambda} \sum_{\tau=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^{\tau} \Delta E_t \ell_{t+\tau} \text{ , where } \Delta E_t \ell_{t+\tau} = \sum_{\chi=0}^{\tau} R^{\chi}$$

	R = Autoregr($\Delta \ell$)*	Δs_t for $x = \frac{1}{2}$	Δs_t for $x = \bar{x}_t$
Pound	.353	.0154526	.0154537
Franc	.170	.0120470	.0120480
Mark	.140	.0116268	.0116273
Canadian Dollar	-.0	.01	.01
Yen	.008	.0100806	.0100806

*R from Table 2

because of more rapid damping out of asset supply changes. In the cases of the Canadian dollar and yen, the asset supply changes are essentially white noise, i.e., ℓ_t follows a random walk, so the effect of a 1 percent innovation is simply 1 percent: the spot rate follows a random walk in lock-step synchronization with the asset supplies.

The above calculations use semielasticities λ that are based on $x = \frac{1}{2}$, under the artificial assumption that in each case the currency in question constitutes the entire nondollar portfolio. In reality each foreign currency considered alone constitutes a smaller part of the portfolio. As a consequence, the λs are considerably larger, but, as table 5 shows, the effect on the exchange rate is virtually the same. The present-discounted sum is far less sensitive to λ than it is to the asset supply autoregression parameter R.

Answer 3: The effect of a current account surplus on the exchange rate _is_ meaningful in size, but is nevertheless less than the effect of a government budget deficit that creates an equal quantity of domestic assets.

Equation (18) says that a European current account surplus that redistributes 1 percent of world wealth to European residents raises x, the demand for European assets, by $(A_e - A_r)$ percent and thus raises the exchange rate by $(\frac{1}{x} - \frac{1}{1-x})(A_e - A_r)$ percent. The theory of mean-variance optimzation, as embodied in equation (25), tells us that

$$A_e = \alpha_e\left(1 - \frac{1}{\rho}\right) + \frac{1}{2\rho} \quad \text{and} \quad A_r = \alpha_r\left(1 - \frac{1}{\rho}\right) + \frac{1}{2\rho} ,$$

where α_e is the share of European consumption allocated to European goods and α_r is the share of rest-of-world consumption allocated to European goods, and we have assumed that everyone has the same coefficient of risk-aversion ρ.

The first point to note belongs to Krugman [1981]: for the wealth redistribution to have a positive effect, it is not sufficient that $\alpha_e > \alpha_r$, that residents of each country consume relatively more of their own goods. It is necessary that we also have $\rho > 1$. In the case of risk-neutrality $(\rho = 0)$, residents of each country would prefer the assets of the other country due to Jensen's inequality. The "risk premium," as defined above would be, not zero, but fixed at $\Omega\left(\alpha - \frac{1}{2}\right)$ if consumption patterns were uniform across residents.[15] In the knife-edge case of the "Bernouilli investor" who has logarithmic utility (the limit as ρ goes to 1), $A_e = A_r = A_a$ and the distribution of wealth has no effect. We will continue to consider the more realistic case of $\rho = 2$, where $A_i = \frac{1}{2}\alpha_i - \frac{1}{4}$. The current account does have an unambiguous effect in the expected direction, assuming $\alpha_e > \alpha_r > \alpha_a$.

What is the magnitude of the effect of a current account surplus equal to 1 percent of world wealth? Whatever the value of ρ, because α_e, α_r and α_a are (weakly) between zero and one the effect on the risk premium is less than (or equal to) the effect of a government budget deficit (or of creation of domestically-denominated assets in other ways such as foreign exchange intervention) equal to 1 percent of world wealth. Let us consider the example $\alpha_e = 1.0$, $\alpha_r = 0.5$ and $\alpha_a = 0.0$.[16] Then a European current account surplus equal to 1 percent of world wealth has an effect that is $(\alpha_e - \alpha_r) = \frac{1}{2}$ as great as a budget deficit equal to 1 percent of world wealth. If the United States is at the same time running a current account deficit of the same magnitude, then it too has an effect that is $(\alpha_r - \alpha_a) = \frac{1}{2}$ as great. Together the two current account imbalances have the same effect as the equivalent budget deficit. This example is a limiting case. If Americans consume nonzero German goods and vice versa, the effect of the current account deficits will be smaller. The result that the current account should have a smaller effect than an equal budget deficit was noted by Dornbusch [1983, p. 25].

The effect on the exchange rate of the 1 percent European current account surplus, using a value of $x = \frac{1}{2}$ in equation (18) is 4.00 $(A_e - A_r)$, which is 2.00 percent using our upper bound values of domestic consumption. The effect together with an equal U.S. current account deficit is 4.00 percent. The effect can be greater if there are expectations of continuing current account imbalances in the future, just as with the equivalent changes in asset supplies considered above.[17]

Answer 4: A depreciation of the dollar would be associated with an appreciation of the franc against the mark rather than the other way around.

We showed above that the inequality (23) is the condition necessary for a depreciation of the dollar, whether originating in an increase in the supply of dollar assets or in the expected future supply of dollar assets, to be associated with an appreciation of the mark against the franc. Now that we know the matrix of substitutability coefficients is proportionate to the inverse of the variance-covariance matrix $(\mathbf{B} = [\rho\,\Omega]^{-1})$, the condition becomes:

$$(29) \qquad \rho\,\Phi \equiv \frac{\Omega^{-1}_{MM} - \Omega^{-1}_{MF}}{x_M} + \frac{\Omega^{-1}_{FM} - \Omega^{-1}_{FF}}{x_F} > 0.$$

The conditional variance-covariance matrix estimated in Frankel [1982, p. 261] gives values for the relevant terms, shown here in table 6. As of 1980, x_M was .12 and x_F .05. Thus our estimate of $\rho\,\Phi$ is a negative number, -1644.2; the condition (23) fails; the dollar depreciation would be associated with a depreciation of the mark against the franc. This is the same result stated by Giavazzi and Giovannini [1984]: the portfolio-optimization theory gives the reverse answer from the relationship generally suspected between the dollar and the franc/mark cross rate.

As with most of the questions answered above, this conclusion would follow even if the variance-covariance matrix and asset shares were computed in somewhat different ways. It is in part a consequence of the fact that the variance of the mark/dollar rate is greater than the variance of the franc/dollar rate, which implies that a well-diversified investor insists on holding both marks and dollars. Similar findings were obtained in the unconditional variances reported in column (1) of table 5, in the unconditional variance-covariance matrix of real returns (i.e. allowing inflation rates to be stochastic) estimated in Kouri and de Macedo [1978], and in the conditional variance-covariance matrix of real returns estimated in Frankel and Engel [1984]. The conclusion is also in part a consequence of the fact that francs constitute a considerably smaller proportion of the world portfolio than marks, which implies that a shift of the portfolio equally into marks and francs would drive up the price of the franc farther than the price of the mark.

Table 6

Mark-Franc Substitutability

B with $\rho = 2$ quoted on a per annum basis			Ω^{-1} on a monthly basis		
B_{MM}	=	28.3	Ω^{-1}_{MM}	=	679.2
B_{MF}	=	-16.8	Ω^{-1}_{MF}	=	-403.2
B_{FM}	=	-16.8	Ω^{-1}_{FM}	=	-403.2
B_{FF}	=	84.2	Ω^{-1}_{FF}	=	2,020.8

How then do we explain the observed tendency of the franc to depreciate relative to the mark when the dollar declines in value? A possible answer is that marks and dollars are in fact relatively close substitutes, the theory of portfolio optimization notwithstanding. Consider two lines quoted by Giavazzi and Giovannini (from the International Monetary Fund, Occasional Paper No. 19, May 1983, p. 5): "When the U.S. dollar is relatively weak, mobile international capital seeks alternative locations, and particularly tends to move to the deutschemark, and vice versa. With the United Kingdom not actively participating in the EMS, currencies other than the deutschemark play only a limited role as alternative reserve and investment currencies." Given the existence of capital controls in France, there is little reason for foreign residents to hold francs unless they need transactions balances for business conducted in those countries. The absence of substantial capital controls in Germany, Switzerland, the United States, Canada, the United Kingdom and Japan means that these six countries' currencies are the world's investment assets. The dollar may be a closer substitute for the mark and the others than the dollar is for the French franc. It would then easily follow that a portfolio shift out of dollars would tend to be a portfolio shift into marks rather than into francs, and would cause the mark to appreciate against the other EMS currencies.

4. CONCLUSION

This paper has tried to show that the hypothesis of mean-variance optimization has powerful implications for some standard questions of interest to macroeconomists, implications that transcend the particular methods used for measuring asset supplies and estimating the variance-covariance matrix. The most striking conclusion is the small magnitude implied for the risk premium. An increase in the supply of dollar assets equal to 1 percent of world wealth raises the risk premium that must be paid on dollar assets by a mere 2 basis points per annum. One corollary is that a time-varying risk premium does not seem a promising explanation for any empirical findings of serial correlation or other conditional bias in the prediction errors made by the forward rate. Another corollary is that foreign exchange intervention has a negligible effect on the risk premium.

However, it does not follow that foreign exchange intervention or other changes in asset supplies need have a negligible effect on the level of the spot rate. A one-time permanent intervention undertaken so as to leave interest rates unchanged will affect the exchange rate in the same proportion as the change in asset supplies. An intervention that changes expectations of future policy in the same way as a typical innovation in asset supplies, will change the exchange rate more than proportionately. The hypothesis of mean-variance optimization implies that the current exchange rate is very sensitive to expectations of future changes. If the expected permanent rate of growth of the domestic asset supply is raised as little as 0.1 percent per annum, estimates from mean-variance optimization imply an increase in the current exchange rate of more than 20 percent.

We have also seen that, under the assumption that the coefficient of relative risk-aversion is greater than 1, current account surpluses can have the traditional effects on the exchange rate. The implied effects are of the same order of magnitude as the effects of changes in asset supplies, though they must be slightly smaller to the extent that residents of each country consume goods of the other country.

Finally, we have seen that the hypothesis of mean-variance optimization implies that a depreciation of the dollar should give rise to a stronger portfolio shift into francs than into marks; this would tend to appreciate the franc against the mark, which is the opposite of the correlation generally believed to hold.

In the case of each of these answers, the suspicion may arise that perhaps the hypothesis of mean-variance optimization is steering us wrong. No final position is taken here on that issue. But if the degrees of substitutability that have been discussed here seem impossibly high, it should be recalled that the working hypothesis in many models, both theoretical and econometric, has long been _perfect_ substitutability. The hitherto-persuasive argument against perfect substitutability has been that it requires either risk-neutrality or the absence of any "outside assets," both highly implausible assumptions.[18] But we now see that conventional estimates of risk-aversion and return variances imply a degree of substitutability so high that for some purposes it might as well be infinite.

FOOTNOTES

[1]Frankel [1982b] fails to reject the null hypothesis that investors are risk neutral and the risk premium is nonexistent. The portfolio consists of six assets, goods prices are assumed to be nonstochastic when expressed in the currency of the producing country, and investor behavior is allowed to differ depending on country of residence. Frankel [1983] and Frankel and Engel [1984] use similar econometric techniques but test the mean-variance optimization hypothesis itself, and are based on different assumptions: the former assumes a two-asset portfolio while the latter allows goods prices to be stochastic in any currency, but assumes that investor behavior is uniform worldwide.

[2]Among the many other authors testing unbiasedness in the forward market--most of them rejecting the null hypothesis--are Frenkel [1976], Cornell [1977], Tryon [1979], Levich [1979], Bilson [1981], Longworth [1981], Hsieh [1982], Baillie, Lippens and McMahon [1983], Huang [1984], Park [1984], Gregory and McCurdy [1984], and Hodrick and Srivastava [1985]. Many of them test whether the forward discount overestimates or underestimates the tendency of the spot rate to regress toward a mean, and find the former. The proposition that this finding of "excessive speculation" (to use Bilson's term) can be explained by a risk premium can be challenged on the grounds of empirical magnitudes, much like the finding of autocorrelation.

[3]Dooley and Shafer [1983] and Cumby and Obstfeld [1981] also find serial correlation in the difference between the interest differential and the ex post rate of currency depreciation.

[4]One can easily interpret equations (4) and (5) and similar equations appearing below in an n-asset framework: x is a vector of portfolio shares allocated to the various assets, rp is a vector of expected rates of return on the various assets relative to a reference asset, A is a vector of intercept terms and B is a matrix of substitutability coefficients.

[5]One approach to testing whether the systematic component of the prediction error is indeed a risk premium has been to run regressions on an equation derived from (1) and (5):

$$(\Delta s_{+1} - fd) = -B^{-1}A + B^{-1}x + \varepsilon_{+1}. \tag{5'}$$

Frankel [1982a], Rogoff [1984], and Booth, Clinton, Côté and Longworth [1985] found no evidence of a relationship between prediction errors and asset supplies as in equation (5'); but Park [1984] and Loopesko [1984] did find such a relationship.

[6]Note that this special case is much less plausible in the long run than in the short run. Henderson [1984] describes a central bank policy of market intervention so as to keep both the exchange rate and the interest rate constant as a "rates constant" policy.

[7]See Frenkel [1983, p. 11], Frenkel and Mussa [1980], or the Bilson, Frenkel and Hodrick papers in Frenkel and Johnson [1978].

[8]It should be measured, rather, as the cumulation of the foreign government deficit (corrected for foreign exchange intervention), analogously to the supply of domestic assets.

[9]See Kouri and de Macedo [1978].

[10] The reasoning suggests an inductive proof that (23) is sufficient to imply a positive effect on s_{FM}. But some restriction on the effect of the supply of dollars on the entire expected future path is also needed for such a proof.

[11] The derivation assumes that goods prices are nonstochastic when expressed in the currency of the country producing them, so that the exchange rate is the only source of uncertainty. Frankel and Engel [1984] follow Kouri and de Macedo [1978] and Dornbusch [1982] in allowing goods prices to be stochastic like exchange rates; the effect is to modify A in equation (25), but to leave qualitatively unchanged the answers to most of the questions considered here.

[12] De Macedo [1980] and Krugman [1981] refer to the 'Samuelson presumption' that $\rho = 2$. Newberry and Stiglitz [1981] summarize the evidence.

[13] The constant term component of rp in equation (4),

$$-B^{-1}A = -\rho\ \Omega\ \left[\alpha\left(1 - \frac{1}{\rho}\right) + \frac{1}{2\rho}\right] = \Omega\left(\alpha - \frac{1}{2}\right) - \rho\ \Omega\ \alpha.$$

From the variance estimates in tables 1 or 3 it follows that the constant term is on the order of only .01 percent per annum. More generally, empirical studies have found no significant unconditional bias in the forward rate: for example, Cornell [1977] and Frankel [1980]. Thus Hansen and Hodrick [1980] and others refer only to a time-varying risk premium.

[14] For purposes of comparison, U.S. and European central banks (mostly the latter) are estimated to have sold about $5 billion in the foreign exchange market in early 1985 (Wall Street Journal, March 4 and

11, and Associated Press, March 8). This would represent an increase in the supply of dollar assets of about 0.3 percent. (Gross U.S. federal debt outstanding was $1,577 billion at end-1984, from Economic Indicators; this includes monetized debt but excludes dollar debt issued by foreign governments.) It also represents a decrease in the supply of foreign assets of the same magnitude, so that the total effect on the relative asset supply is larger than 0.3 percent.

[15]This follows from footnote 13. In the case of divergent consumption patterns, rp is undefined (overdetermined) under risk-neutrality, because residents of different countries try to set it at mutually inconsistent levels, much like risk-neutral speculators who have divergent expectations, or agents in any market who have infinitely elastic demands at divergent prices. But in the limit as ρ goes to zero, rp goes to $\Omega[\alpha_e - \frac{1}{2} + (\alpha_e - \alpha_r)w_e - (\alpha_r - \alpha_a)w_a]$.

[16]Shares of expenditure on German goods can be calculated at .99 for German residents, .01 for U.S. residents and .47 for rest-of-world residents (under the highly artificial assumption that all goods were produced in either Germany or the United States) [Frankel 1983, p. 322].

[17]If world wealth is measured as the sum of the outstanding stocks of government debt, whether monetized or not, of the seven largest countries ($3,057 billion as of 1982, or $2,465 if only central government debts are included; OECD Economic Outlook July 1984, p. 29, and Historical Statistics 1960-82, p. 14), then the recent U.S. current account deficits of $100 billion are redistributing several percent of world wealth. World wealth would of course be smaller if only monetized debt were considered

an outside asset. On the other hand, if equities and real estate were included in the portfolio, world wealth would be higher. Equities alone are valued at $2,945 billion for the total of twenty countries, as of December 31, 1984 (Capital International, January 1985). The effect of the U.S. current account deficit, as of the budget deficit, would diminish proportionately.

[18]This proposition is demonstrated in Frankel [1979]. The absence of outside assets would have to extend beyond government debt, to money and real assets.

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