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ASPECTS OF INVESTOR BEHAVIOR UNDER RISK

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ABSTRACT

The three sections of this paper support three related conclusions. First, asset demands with the familiar properties of wealth homogeneity and linearity in expected returns follow as close approximations from expected utility maximizing behavior under the assumptions of constant relative risk aversion and joint normally distributed asset returns. Second, although such asset demands exhibit a symmetric coefficient matrix with respect to the relevant vector of expected asset returns, symmetry is not a general property, and the available empirical evidence warrants rejecting it for both institutional and individual investors in the United States. Finally, in a manner analogous to the finite maximum exhibited by quadratic utility, a broad class of mean-variance utility functions also exhibits a form of wealth satiation which necessarily restricts its range of applicability.

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ASPECTS OF INVESTOR BEHAVIOR UNDER RISK

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A greatly enhanced understanding of the nature of economic uncertainty, and with it substantial insight into economic behavior in circumstances under which uncertainty is central to necessary decisions, stand as one of Kenneth Arrow's most significant contributions. His classic lectures on Aspects of the Theory of Risk-Bearing clarified key elements of the theory of choice under uncertainty, formalized crucial aspects of risk-averse behavior, and explored the implications of the relevant theory for such important economic activities as resource allocation and insurance. These lectures, together with many of Arrow's other papers on risk and uncertainty, have provided a foundation that is now standard in monetary and financial economics.

The object of this paper is to analyze several aspects of the asset demands characterizing investors' portfolio behavior under risk. Section I derives asset demand functions exhibiting wealth homogeneity and linearity in expected asset returns — two convenient properties that are often simply assumed, especially in the monetary economics literature. The main result here is that, among the numerous familiar sets of specific assumptions sufficient to derive mean-variance portfolio behavior from the more general theory of expected utility maximization, the assumptions of constant relative risk aversion and joint normally distributed asset return assessments are also jointly sufficient to derive asset demands with these properties, as close approximations, either in continuous time or in discrete time if the time unit is small.

Section II, however, provides empirical evidence that contradicts the plausibility of these assumptions — and, for that matter, a variety of others as well. In particular, a standard feature of asset demands, also often simply assumed in applied research, is that the responses of these demands to expected asset returns are symmetric. The evidence summarized here, based on the observed portfolio behavior of both institutional and individual investors in the United States, casts doubt on the hypothesis of symmetry and therefore also casts doubt on the set of more fundamental assumptions that imply symmetry in this sense.

Section III considers another aspect of investors' portfolio behavior implied by a familiar group of utility functions. It is well known that the quadratic utility function implies a wealth satiation level, or "bliss point." The analysis here shows that a number of other familiar utility functions similarly exhibit wealth satiation when investors' behavior is restricted only by the distribution of asset returns. This property imposes still another important caveat in applications to the study of investors' behavior based on such functions.

Section IV briefly summarizes the paper's principal conclusions.

I. The Derivation of Linear Homogeneous Asset Demand Functions

The asset demand functions used for both analytical and empirical research, especially in the monetary economics literature, are often assumed to exhibit the two convenient properties of wealth homogeneity and linearity in espected asset returns.[1] The convenience afforded by the tractability of the linear form is apparent enough, and the wealth homogeneity property in particular is often especially important in empirical applications to aggregate data.[2] Despite the frequent use of such return-linear and wealth-homogeneous asset demand functions, however, there exists (to the authors' knowledge) no readily available source setting forth sufficient conditions for the derivation, from underlying principles of expected utility maximization, of asset demands simultaneously exhibiting both of these properties.[3]

The purpose of this section is to show that, among the numerous familiar sets of specific assumptions sufficient to derive mean-variance portfolio behavior from more general expected utility maximization in continuous time, the assumptions of (a) constant relative risk aversion and (b) joint normally distributed asset return assessments are also jointly sufficient to derive, as approximations, asset demand functions with the two desirable (and frequently simply assumed) properties of wealth homogeneity and linearity in expected returns. Constant relative risk aversion and joint normally distributed asset return assessments are also sufficient to yield such asset demands as approximations in discrete time if the time unit is small.[4]

Analysis in Continuous Time

To begin with expected utility maximization, the investor's objective as of time t, given initial wealth W_t, is

$$\max_{\underline{\alpha}_t} E[U(\tilde{W}_{t+dt})] \tag{1}$$

subject to

$$\underline{\alpha}'_t \underline{1} = 1, \tag{2}$$

where $E(\cdot)$ is the expectation operator, $U(W_\tau)$ is utility as a function of wealth, and $\underline{\alpha}_t$ is a vector expressing the portfolio allocations in proportional form

$$\underline{\alpha}_t \equiv \frac{1}{W_t} \cdot \underline{A}_t \tag{3}$$

for vector $\underline{A}_t$ of asset holdings.

Assumption (a) noted above is that $U(W_\tau)$ is any power (or logarithmic) function such that the coefficient of relative risk aversion

$$\rho \equiv - W_\tau \cdot \frac{U''(W_\tau)}{U'(W_\tau)} \tag{4}$$

is constant.[5] Assumption (b) is that the investor perceives asset returns $\tilde{r}_{i\tau}$, $i = 1,\ldots,n$, to be generated as Wiener processes with respective means $r^e_{i\tau}$, standard deviations $\sigma_{i\tau}$ and correlations $\phi_{ij\tau}$, where the tilde sign indicates a random variable, and the time subscript generalizes the investor's assessments to permit variation over time. Given the assumption of Weiner processes for the asset yields, $\tilde{W}_{t+dt}$ is in turn generated by

$$\tilde{W}_{t+dt} = W_t \cdot \sum_i^n \alpha_{it}(1 + r^e_{it}dt + \sigma_{it}\tilde{z}_{it}\sqrt{dt}) \tag{5}$$

where $\tilde{z}_i$ is the unit normal random variable corresponding to each yield $\tilde{r}_i$.

Expanding $U(\tilde{W}_{t+dt})$ about W_t, for dt sufficiently small, and then taking the expectation yields a representation of the maximand in the form

$$E[U(\tilde{W}_{t+dt})] = \sum_{k=0}^{\infty} \frac{1}{k!} \cdot U^{(k)}(W_t) \cdot E[\tilde{W}_{t+dt} - W_t]^k \qquad (6)$$

where the notation $U^{(k)}(\cdot)$ indicates the k-th derivative of $U(\cdot)$. Substituting from (5) and omitting terms of higher than second order in dt yields

$$E[U(\tilde{W}_{t+dt})] = U(W_t) + U'(W_t) \cdot W_t \cdot \underline{\alpha}'_t \underline{r}^e_t dt + \frac{1}{2} U''(W_t) \cdot W_t^2 \cdot \underline{\alpha}'_t \Omega_t \underline{\alpha}_t dt \qquad (7)$$

where Ω_t is a variance-covariance matrix consisting of elements $\sigma_{it}\sigma_{jt}\phi_{ijt}$. Forming the Lagrangean for the maximization of (7) subject to (2), differentiating with respect to $\underline{\alpha}_t$, and equating the derivative to zero yields the first-order condition for the solution of (1) as

$$\underline{\alpha}^*_t = B_t \underline{r}^e_t + \underline{\pi}_t \qquad (8)$$

where the asterisk indicates an optimum. If there is no risk-free asset (because of price inflation, for example), B_t and $\underline{\pi}_t$ have the form[6]

$$B_t = -\frac{1}{\rho} [\Omega_t^{-1} - (\underline{1}'\Omega^{-1}\underline{1})^{-1} \Omega_t^{-1} \underline{1}\,\underline{1}' \Omega_t^{-1}] \qquad (9)$$

$$\underline{\pi}_t = (\underline{1}'\Omega_t^{-1}\underline{1})^{-1} \Omega_t^{-1} \underline{1}. \qquad (10)$$

Alternatively, in the presence of a risk-free asset Ω_t is singular, so that it is necessary to partition the system of demands. The resulting solution, in which $\hat{\underline{\alpha}}_t$, $\hat{\underline{r}}^e_t$ and $\hat{\Omega}_t$ refer to the risky assets only, is

$$\hat{\underline{\alpha}}^*_t = \hat{B}_t \hat{\underline{r}}^e_t \tag{8'}$$

where

$$\hat{B}_t = -\frac{1}{\rho}\hat{\Omega}^{-1}_t \tag{9'}$$

and the optimum portfolio share for the risk-free asset is just $(1 - \hat{\underline{\alpha}}^{*}_t{}'\underline{1})$.[7]

It is apparent by inspection that the optimum portfolio allocations in both (8) and (8') exhibit the two properties of wealth homogeneity and linearity in expected returns. Moreover, since Ω_t (or $\hat{\Omega}_t$) is a variance-covariance matrix, the Jacobian B_t (or $\hat{B}_t$) indicates symmetrical asset substitutions associated with cross-yield effects.

Analysis in Discrete Time

In the discrete-time analog to the model developed above, the investor's single-period objective as of time t, given initial wealth W_t, is

$$\max_{\underline{\alpha}_t} E[U(\tilde{W}_{t+1})] \tag{11}$$

where

$$\tilde{W}_{t+1} = W_t \cdot \underline{\alpha}_t' (\underline{1} + \tilde{\underline{r}}_t) \tag{12}$$

and assessments of $\tilde{\underline{r}}_t$ (i.e., asset returns between time t and time t+1) are distributed as

$$\tilde{\underline{r}}_t \sim N(\underline{r}^e_t, \Omega_t). \tag{13}$$

Expanding $U(\tilde{W}_{t+1})$ about $E(\tilde{W}_{t+1})$ and then taking the expectation yields a representation of the maximand in the form

$$E[U(\tilde{W}_{t+1})] = \sum_{k=0}^{\infty} \frac{1}{k!} \cdot U^{(k)}[E(\tilde{W}_{t+1})] \cdot \{E[\tilde{W}_{t+1} - E(\tilde{W}_{t+1})]^k\}. \tag{14}$$

It follows from the moment generating function of the normal distribution that the term within brackets in (14) has value

$$E[\tilde{W}_{t+1} - E(\tilde{W}_{t+1})]^k = \frac{k!}{2^{(k/2)}\,(\frac{k}{2})!} \; [\mathrm{var}(\tilde{W}_{t+1})]^{(k/2)} \tag{15}$$

for k an even integer and

$$E[\tilde{W}_{t+1} - E(\tilde{W}_{t+1})]^k = 0 \tag{16}$$

for k an odd integer. Hence (14) simplifies to

$$E[U(\tilde{W}_{t+1})] = \sum_{m=0}^{\infty} \frac{1}{2^m m!} \cdot U^{(2m)}[E(\tilde{W}_{t+1})] \cdot [\mathrm{var}(\tilde{W}_{t+1})]^m. \tag{17}$$

Substituting from (12) and omitting terms of higher than second order yields

$$E[U(\tilde{W}_{t+1})] = U[E(\tilde{W}_{t+1})] + \frac{1}{2} U''[E(\tilde{W}_{t+1})] \cdot W_t^2 \cdot \underline{\alpha}_t' \Omega_t \underline{\alpha}_t \,. \tag{18}$$

Forming the Lagrangean for the maximization of (18) subject to (2), differentiating with respect to $\underline{\alpha}_t$, equating the derivative to zero, and again omitting terms of higher than second order yields the first-order condition for the solution of (11) if there is no risk-free asset as

$$\underline{\alpha}_t^* = B_t \underline{r}_t^e + \underline{\pi}_t \tag{8}$$

once again, where now

$$B_t = \left\{ \frac{-U'[E(W_{t+1})]}{W_t \cdot U''[E(\tilde{W}_{t+1})]} \right\} \cdot [\Omega_t^{-1} - (\underline{1}'\Omega_t^{-1}\underline{1})^{-1}\Omega_t^{-1}\underline{1}\underline{1}'\Omega_t^{-1}] \tag{19}$$

and $\underline{\pi}_t$ is again as in (10). Alternatively, in the presence of a risk-free asset the resulting solution is again (for $\hat{\underline{\alpha}}$, $\hat{B}_t$ and $\hat{\underline{r}}_t^e$ as defined above)

$$\hat{\underline{\alpha}}_t^* = \hat{B}_t \hat{\underline{r}}_t^e \tag{8'}$$

where

$$\hat{B}_t = \left\{ \frac{-U'[E(\tilde{W}_{t+1})]}{W_t \cdot U''[E(\tilde{W}_{t+1})]} \right\} \hat{\Omega}_t^{-1} \tag{19'}$$

and the optimum portfolio share for the risk-free asset is again just $(1-\hat{\underline{\alpha}}_t^{*\prime}\,\underline{1})$. If the time unit is sufficiently small to render W_t a good approximation to $E(\tilde{W}_{t+1})$ for purposes of the underlying expansion, then the scalar term within brackets in (19) and (19') reduces to the constant coefficient of relative risk aversion, and the discrete-time model yields the same linear homogeneous asset demand functions developed above.

Isomorphic Assumptions

Other combinations of assumptions, if they are isomorphic to constant relative risk aversion and joint normally distributed asset return assessments, also yield asset demand functions exhibiting both wealth homogeneity and linearity in expected returns, either exactly or as an approximation. For example, the negative exponential utility function with coefficient of absolute risk aversion inversely dependent on initial wealth yields results equivalent to those derived above.[8] Alternatively, the logarithmic utility function, in conjunction with the assumption of joint lognormally distributed returns, yields asset demand functions that are homogeneous in wealth and log-linear in expected returns, in either continuous or discrete time; but in this case yet a further (apparently reasonable) approximation is necessary, because a linear combination of lognormally distributed returns is not itself distributed lognormally.[9]

II. Evidence on the Symmetry Hypothesis[10]

Imposition of symmetry restrictions on coefficients describing responses to expected asset returns is a frequent practice in the empirical estimation of systems of asset demands. Wholly apart from the theoretical considerations laid out in Section I, a typical motive for imposing symmetry in such applied research is simply to reduce the number of independent coefficients to be estimated. In large systems of asset demands, the corresponding gain in degrees of freedom is substantial. As is true in the standard consumer demand paradigm, however, the coefficient matrix applicable to the vector of expected asset returns consists of a combination of symmetric Slutsky substitution effects and (in general) asymmetric Slutsky wealth effects.[11]

The analysis in Section I shows that in some specific cases the relevant wealth terms do exhibit symmetry. The linear homogeneous asset demands derived in Section I under constant relative risk aversion and joint normal asset return distributions provide a clear example. More generally, in terms of expected utility functions that reduce to exact mean-variance preference orderings, the symmetry restriction per se has corresponding behavioral implications. In particular, when such a mean-variance expected utility function has wealth as its argument, symmetry implies that investors exhibit constant absolute risk aversion.[12] When the argument is instead the portfolio rate of return, with wealth homogeneity as in Section I, symmetry implies constant relative risk aversion if the time unit is sufficiently small to render W_t a good approximation to $E(\tilde{W}_{t+1})$. In both cases the symmetry restriction implies that the Slutsky expected wealth (or portfolio rate of return) effects are identically equal to zero, leaving only a symmetric substitution matrix. By contrast, the symmetry property does not follow from (for example) the quadratic utility function, a form frequently encountered in the applied literature.

The symmetry property is therefore an empirically testable restriction. It does not necessarily hold for any reasonable but arbitrarily chosen form of expected utility maximizing behavior. Hence evidence indicating whether investors' behavior does or does not exhibit symmetry provides potentially useful information.

Evidence from Institutional Investors

Evidence based on the demands for two maturity classes of U.S. Treasury securities by institutional investors in the United States suggests that these investors' portfolio behavior does not exhibit symmetric responses to movements of asset returns. Table 1 summarizes this evidence for six major categories of institutional investors in the U.S. markets, including life insurance companies, other insurance companies, mutual savings banks, savings and loan associations, private pension funds, and state and local government retirement funds. The equations summarized in the table are estimated using quarterly Federal Reserve data (seasonally adjusted) for 1960-75. The data disaggregate the total financial asset holdings of each investor group into asset classes such as corporate bonds, U.S. Treasury securities, equities, commercial paper, mortgages, and currency and demand deposits. The data further disaggregate each group's holding of U.S. Treasury securities into four weighted maturity classes. The evidence in the table focuses on each of the six investor groups' demands for two distinct classes of Treasury securities: those with maturities ranging from about 1 1/2 to 5 years (S), and those with maturities over 10 years (L).[13]

As is typical in empirical models of financial asset demands, the specific form of asset demand functions estimated here rest on the assumption

TABLE 1

ESTIMATED INSTITUTIONAL ASSET DEMAND RESPONSES

		Unconstrained Estimates					Constrained Estimates				
Investor Category	Asset	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\bar{R}^2$	SE	DW	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\bar{R}^2$	SE	DW
Life Insurance Companies	S	.0270 (5.6)	-.0190 (-4.6)	.81	21	2.18	.0215 (5.1)		.80	22	2.14
	L	-.0025 (-0.3)	.0165 (2.0)	.92	39	2.32	-.0174 (-5.2)	.0174 (5.2)	.88	48	2.32
Other Insurance Companies	S	.1102 (3.8)	-.0637 (-2.6)	.67	52	1.76	.0827 (3.7)		.67	52	1.72
	L	-.0091 (-0.4)	.0128 (0.6)	.71	52	1.81	-.0363 (-1.9)	.0363 (1.9)	.68	54	1.84
Mutual Savings Banks	S	.1005 (4.2)	.0313 (0.8)	.52	57	2.01	.0782 (3.9)		.49	59	1.74
	L	.0406 (2.4)	-.0151 (-1.4)	.64	25	2.40	-.0035 (-0.4)	.0035 (0.4)	.62	25	2.19
Savings and Loan Associations	S	.0134 (0.9)	.0269 (1.4)	.76	46	1.99	-.0029 (-0.4)		.69	52	2.25
	L	.0063 (0.5)	.0079 (0.8)	.81	38	2.12	.0029 (0.4)	.0076 (0.7)	.77	41	2.05
Private Pension Funds	S	.0044 (0.1)	.0500 (1.2)	.57	161	1.64	.0660 (2.5)		.57	162	1.77
	L	.0263 (2.0)	-.0239 (1.8)	.67	52	2.20	-.0115 (-1.1)	.0115 (1.1)	.65	53	2.13
State-Local Retirement Funds	S	-.0071 (-0.9)	.0294 (2.4)	.36	26	2.15	.0074 (1.1)		.32	26	1.59
	L	-.0796 (-2.6)	.1498 (3.8)	.61	112	2.07	-.0074 (-1.1)	.0074 (1.1)	.47	130	2.22

that transactions costs preclude complete portfolio adjustment to desired asset holdings within one calendar quarter. The specific form of adjustment model used to describe this aspect of short-run portfolio behavior is the multivariate optimal marginal adjustment model

$$\Delta \underline{A}_t = \theta(\underline{A}^*_t - \underline{A}_{t-1}) + \underline{\alpha}^*_t \cdot \Delta W_t \qquad (20)$$

where $\underline{A}^*_t$ is the vector of equilibrium asset holdings corresponding to $\underline{\alpha}^*_t \cdot W_t$, for $\underline{\alpha}^*$ defined as in (8), and θ is a matrix of adjustment coefficients with column sums identically equal to an arbitrary scalar.[14] Substituting for $\underline{A}^*$ and $\underline{\alpha}^*$ from (3) and (8) yields

$$\Delta \underline{A}_t = \theta B \underline{r}^e_t \cdot W_t + \theta \underline{\pi} \cdot W_t - \theta \underline{A}_{t-1} + \underline{\pi} \cdot \Delta W_t + B \underline{r}^e_t \cdot \Delta W_t . \qquad (21)$$

For each of the six investor groups, only two asset demands are subjected to the symmetry test in the estimated equations.[15] In the data used here, however, investors' asset holdings are disaggregated into a minimum of nine categories, and selected yields on these other assets appear in the estimated demand equations. As a whole, therefore, the set of parameters in the estimated demand equations is underidentified either with or without the symmetry constraint. The subset of parameters relevant to the symmetry test is identified, however. Specifically, the null hypothesis corresponds to $\beta_{SL} = \beta_{LS}$ and $\sum_j \beta_{ij} = 0$ (i=S,L), for $\{\beta_{ij}\} = B$.[16] Moreover, because only this subset of the estimated parameters is identified, the system of equations may be estimated without using a nonlinear estimation technique.

The asset return series used in the symmetry test reported in Table 1 are the Federal Reserve yield series on "3-to-5-year" (r^e_S) and "long-term"

(r_L^e) U.S. Treasury securities. Hence for this test simple observed yields are taken as proxies for expected rates of return.[17] The cross-equation symmetry restriction involves the coefficients on the $r_S^e \cdot \Delta W$ and $r_L^e \cdot \Delta W$ terms in (21). Coefficients on $r_j^e \cdot \Delta W$ terms specified with these yields along with other yields are then used to form the within-equation row-sum constraints also implied by symmetry.

Table 1 shows the results of applying full-information instrumental variables estimation to (21). Although the undersized sample problem precludes such alternatives as full-information maximum likelihood or three-stage least squares, a full-information technique is nevertheless required to allow for contemporaneous error covariances in tests involving the two separate asset demands by each investor category.[18]

The left-hand side of Table 1 reports summary statistics and estimated β_{ij} coefficients for the 12 asset demand equations, (two for each of the six investor categories).[19] The estimated own-yield responses exhibit theoretically correct positive values in nine of the 12 cases, and the majority of these positive responses are statistically significant at the .05 level.

The estimated coefficient matrix is inconsistent with symmetry, however. The right-hand side of Table 1 reports the corresponding constrained symmetric estimates. For five of the six investor categories, the null hypothesis of symmetry can be rejected at the .05 level.[20] the sixth category (savings and loan associations), symmetry can be rejected at the .10 level. As a whole, therefore, the results indicate that the observed portfolio behavior of U.S. institutional investors does not exhibit symmetry, and hence does not conform to the type of risk aversion implied by symmetry.

Evidence from Individual Investors

Evidence from the portfolio behavior of U.S. households also casts doubt on the assumption of symmetric responses of asset demands to expected asset returns, although less strongly so than in the case of institutional investors. Table 2 presents summary results, based on analogous quarterly data for 1960-80, for the estimation of the U.S. household sector's aggregate demands for three broad classes of financial assets that differ from one another according to the risks associated with holding them: Short-term debt (S) includes all assets bearing real returns that are risky, over a single year or calendar quarter, only because of uncertainty about inflation. Long-term debt (L) is risky because of uncertainty not only about inflation but also about changes in asset prices directly reflecting changes in market interest rates. Equity (E) is risky because of uncertainty about inflation and about changes in stock prices.

The pre-tax nominal return associated with the short-term debt category here is a weighted average of zero (for money), the Federal Reserve average rate on time and saving deposits (for other deposits bearing regulated yields), and the four-to-six month prime commercial paper rate (for all other instruments maturing in one year or less), weighted in each quarter according to the composition of the U.S. household sector's aggregate portfolio. The pre-tax nominal return on long-term debt is the Moody's Baa corporate bond yield plus the fitted value, from a simple univariate autoregressive process, of annualized percentage capital gains or losses approximated by applying the standard consol formula to changes in the Baa yield.[21] For equity the pre-tax nominal return is the dividend-price yield on the Standard and Poor's 500 index plus the fitted value, from an analogous autoregressive process, of annualized percentage capital gains

TABLE 2

ESTIMATED HOUSEHOLD ASSET DEMAND RESPONSES

Unconstrained Estimates

Asset	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
S	-.0192 (-1.7)	.00283 (1.3)	.00575 (2.7)	.78	11.71	1.53
L	.00201 (0.6)	-.000231 (-0.3)	-.00117 (-1.8)	.16	10.41	1.49
E	.0172 (2.2)	-.00260 (-1.8)	-.00458 (-3.0)	.25	3.43	1.81

Constrained Symmetric Estimates

Asset	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
S	-.0135 (-2.5)			.78	11.74	1.52
L	.00266 (2.0)	-.000299 (-0.8)		.16	10.42	1.48
E	.0108 (2.6)	-.00237 (-2.4)	-.00847 (-2.7)	.18	3.58	1.73

or losses on that index.[22] For each asset, the return used for $\underline{r}^e$ in (8) is the corresponding after-tax real return, calculated by applying the household sector's average effective marginal tax rates in each year for interest, dividends and capital gains to the respective components of the pre-tax nominal returns, and then subtracting the annualized percentage change in the consumer price index.[23]

Because there is substantial evidence that individual investors do not fully rebalance their portfolios within a time span as short as one quarter-year, it is again appropriate not to estimate (8) directly but to embed it within some model of portfolio adjustment out of equilibrium. The most familiar such model in the asset demand literature is the multivariate partial adjustment form

$$\Delta\underline{A}_t = \Theta(\underline{A}^*_t - \underline{A}_{t-1}) \tag{22}$$

where $\underline{A}^*$ is the vector of equilibrium asset holdings as before, and Θ is now a matrix of adjustment coefficients with columns satisfying "adding up" constraints analogous to those applying to B. Substituting for $\underline{A}^*$ from (3) and (8) yields

$$\Delta\underline{A}_t = \Theta B\underline{r}^e_t \cdot W_t + \Theta\underline{\pi}\cdot W_t - \Theta\underline{A}_{t-1} . \tag{23}$$

Table 2 shows the results (B estimates and summary statistics only) of applying nonlinear maximum likelihood estimation to (23), using data for $\underline{r}^e$ as described above and Federal Reserve data on actual household sector asset holdings for $\underline{A}$ (and hence W).[24] These data are constructed for each of the three assets by decrementing backward from the reported 1980 yearend value using the corresponding seasonally adjusted quarterly flows.[25] In addition, for equities (the only one of the three assets for which the asset stock data are at market value), quarterly valuation changes are included without seasonal adjustment. The data for W include the three financial assets only, in part to avoid inadequacies in the available data describing holdings of and returns on nonfinancial assets, and in part simply to limit the scope of the analysis. The data for W also omit the household sector's outstanding liabilities, since the great bulk of household borrowing is tied to the ownership of nonfinancial assets.[26]

Because each term in (23) has the dimension of nominal dollars, care is necessary to avoid spurious correlations due to common time trends. For purposes of estimation, therefore, the data for $\underline{A}$ (and hence W) are rendered in real per capita values, using the consumer price index and the total U.S. population series. In addition, both $\Delta\underline{A}_t$ and W_t exclude the current period's capital gains or losses (although the vector of lagged asset stocks $\underline{A}_{t-1}$ reflects previous periods' gains and losses), so that the estimated form focuses strictly on the household sector's aggregate net purchases or sales of each asset associated with the sector's net saving. Defining the asset flows in this way is equivalent to assuming that investors do not respond within the quarter to that quarter's changes in their holdings due to changing market valuations, but do respond to market valuations as of the beginning of each quarter.

The upper panel of Table 2 reports summary statistics and estimated β_{ij} values for each of the three asset demand equations, estimated in this way with no further constraints.[27] These β_{ij} estimates clearly bear little apparent relation to any asset demand response matrix that makes sense in theoretical terms, however, in that all three estimated on-diagonal "own" responses are negative. More to the point here, despite the absence of any contradiction in signs among the three pairs of off-diagonal responses, the data are inconsistent with symmetry. The lower panel of the table reports analogous summary statistics and estimated β_{ij} values for the same three equations estimated by exploiting the nonlinear maximum likelihood procedure to impose the set of three constraints that here comprise symmetry. The value of the test statistic for these three restrictions is $\chi^2(3) = 8.0$, which warrants rejecting the restrictions at the .05 level.

Because the after-tax real returns on all three classes of financial assets were serially correlated during the 1960-80 sample, the unconditional variation of the observed returns used for $\underline{r}^e$ in the estimation of these results presumably overstates the uncertainty that investors actually associated with their expectations of asset returns, over each coming calendar quarter, throughout this period.[28] An alternative (and presumarly superior) way of conducting such an analysis, therefore, is to construct some representation of investors' perceptions of these asset returns and risks that takes more careful account of what information investors did or did not have at any particular time.

As of the beginning of each calendar quarter, investors presumably know the stated interest rates on short-term debt instruments, the current prices and the coupon rates on long-term debt instruments, the current prices and (approximately) the dividends on equities, and the relevant tax rates. The three uncertain elements that they must forecast in order to form expectations for the coming quarter of the after-tax real returns on the three broad classes of assets considered here are therefore inflation, the capital gain or loss on long-term debt, and the capital gain or loss on equity.

Table 3 presents an alternative set of results based on a procedure that infers investors' risk perceptions by representing investors as forming expectations of these three uncertain return elements, at each point in time, by estimating a linear vector autoregression model giving

TABLE 3

HOUSEHOLD ASSET DEMAND RESPONSES ESTIMATED FROM FORECASTED RETURNS

Unconstrained Estimates

Asset	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
S	.00923 (0.6)	-.0000482 (-0.0)	.00190 (1.1)	.79	11.49	1.66
L	-.00515 (-0.9)	.0000231 (0.0)	-.000338 (-0.5)	.19	10.24	1.61
E	-.00408 (-0.4)	.0000251 (0.0)	-.00157 (-1.4)	.16	3.68	1.68

Constrained Symmetric Estimates

Asset	$\beta_{\cdot S}$	$\beta_{\cdot L}$	$\beta_{\cdot E}$	$\bar{R}^2$	SE	DW
S	-.00255 (-2.5)			.80	11.36	1.65
L	.000645 (1.8)	-.000294 (-1.2)		.20	10.17	1.58
E	.00191 (2.8)	-.000351 (-1.4)	-.00156 (-3.3)	.17	3.66	1.69

the best linear projection of these elements from past values. In other words, at the beginning of each period investors estimate the three-variable vector autoregression using all then-available data (through the immediately preceding period), and then use the estimated model to project inflation and the respective capital gains on long-term debt and equity for the period immediately ahead. After that period elapses, investors incorporate into the sample the new observation on the three random variables, re-estimate the vector autoregression, and use the updated model to project the relevant unknowns for the subsequent period.

This inherently backward-looking forecast procedure enjoys the advantages and suffers the shortcomings of expecting the immediate future to be like the immediate past, so that the degree of success achieved by the resulting one-period-ahead forecasts naturally varies according to the extent of the serial correlation in the series being forecast. The first-order serial correlation coefficients of the realizations of the three random variables (again based on quarterly movements during 1960-80) are .90 for price inflation, .44 for long-term debt capital gains, and .31 for equity capital gains. The simple correlation coefficients between the realizations and the corresponding forecasts derived from this continual updating procedure are .88 for inflation, .42 for long-term debt capital gains, and .23 for equity capital gains.[29] The simple correlation coefficients between the realizations of after-tax real returns and the corresponding forecasts are .83 for short-term debt, .51 for long-term debt, and .30 for equity.

Table 3 reports estimation results, analogous to those shown in Table 2, for the same system of three asset demands estimated using these continually updated return forecasts for $\underline{r}^e$. Here too, the results are hardly satisfactory in theoretical terms. The unconstrained estimates, shown in the top panel of the table, still indicate a negative estimated response of the demand for equity to the estimated return on equity. More to the point here, two of the three pairs of off-diagonal estimated responses have opposite signs.

The lower panel of Table 3 reports the corresponding results for the same estimation subject to the further constraint that matrix B be symmetric. Although imposition of the symmetry restriction is not strictly inconsistent with the data in a statistical sense (the test statistic value is $\chi^2(3) = 2.65$), the constrained estimates are even less plausible than their unconstrained counterparts. Here the estimated responses of all three asset demands to their respective "own" expected returns are negative, as they were in Table 2. Moreover, all three asset pairs are now not substitutes but complements. Although asset complementarity is plausible enough in general, in this context there is nothing in the unconditional variance-covariance structure of the three assets' returns, or in the conditional variance-covariance structure that results from the continually updated forecast procedure, to suggest complementarity among any of these three asset pairs.

For individual as well as institutional investors, therefore, the available evidence suggests that asset demands exhibiting symmetry do not describe the observed portfolio behavior. Given the connection between symmetric asset demands and the specific assumptions underlying the maximization of expected utility, these results therefore cast doubt on the validity of standard assumptions often used — either explicitly or implicitly — to characterize the behavior of risk averse investors.

III. The "Bliss Point" Problem

In both theoretical and empirical analyses of investor behavior under risk, specific utility functions are frequently assumed to represent investors' preferences. The most analytically tractable and therefore most widely used utility functions are those that reduce to preference orderings over the mean and variance of wealth (or portfolio rate of return) under uncertainty. Because quadratic utility reduces in a straightforward manner to such a mean-variance function for all probability distributions of end-of-period wealth, it in particular is often applied to represent investors' utility.[30] The existence of a "bliss" (or wealth satiation) point in quadratic utility is widely acknowledged. In this case utility has a finite maximum with a corresponding satiation level of end-of-period wealth.

The possible existence of a different bliss point has also been shown in mean-variance models. A sufficient condition for this other bliss point to exist is that a riskless asset is not available and indifference curves are convex in variance-mean space.[31] The untenable implication of this second bliss point is that a satiation level of beginning-of-period wealth exists. In other words, there exist levels of initial wealth such that an investor maximizes utility by disposing some of his wealth before selecting his portfolio.

The existence and implications of initial wealth satiation have been frequently misinterpreted. In particular, initial wealth satiation is usually interpreted as being the same as end-of-period wealth satiation in quadratic utility.[32] These bliss points are in fact distinct, however. Indeed, in the quadratic utility case, initial wealth satiation occurs at a lower level of expected utility than end-of-period wealth satiation.

Hence those researchers who have placed importance on restricting the range of application of quadratic utility because of end-of-period wealth satiation should logically have restricted its application still further because of initial wealth satiation. Moreover, initial wealth satiation limits the usefulness not only of quadratic utility but also of many other common mean-variance utility functions.

Two specific examples serve both to show the existence of initial wealth satiation and to examine its consequences.[33] These examples involve quadratic utility and negative exponential utility with joint normally distributed asset returns. Before considering these two cases, however, it is useful to define initial wealth satiation in more precise terms. Initial wealth satiation is attained at initial wealth W_t^* if all levels of initial wealth $W_t < W_t^*$ yield lower mean-variance utility, and if, given $W_t > W_t^*$, an investor will maximize utility by disposing of an amount of initial wealth equal to $W_t - W_t^*$. In other words, at sufficiently high levels of initial wealth, marginal mean-variance utility is negative with respect to increments of initial wealth. The implication of this bliss point is therefore highly untenable, in that it is inconsistent with a generally accepted norm of rational behavior.

Quadratic Utility

Perhaps the most interesting example of initial wealth satiation involves quadratic utility

$$U(\tilde{W}_{t+1}) = \tilde{W}_{t+1} - b \cdot \tilde{W}_{t+1}^2 \tag{24}$$

where b is a positive scalar. While this utility function has been severely criticized for displaying increasing absolute risk aversion, it nevertheless is the only von Neumann-Morgenstern utility function that reduces to an exact mean-variance preference ordering for all probability distributions of end-of-period wealth.[34] The quadratic utility function also possesses a global maximum at

$$\tilde{W}^{*}_{t+1} = \frac{1}{2b} \tag{25}$$

thereby implying the existence of a satiation level of end-of-period wealth.

Expected quadratic utility immediately follows from (24) and may be written as

$$E[U(\tilde{W}_{t+1})] = E(\tilde{W}_{t+1}) - b \cdot E(\tilde{W}_{t+1})^2 - b \cdot \mathrm{var}(\tilde{W}_{t+1}). \tag{26}$$

In selecting an optimal portfolio, an investor maximizes (26) subject to the contraint

$$\underline{A}'_t \, \underline{1} = W_t. \tag{27}$$

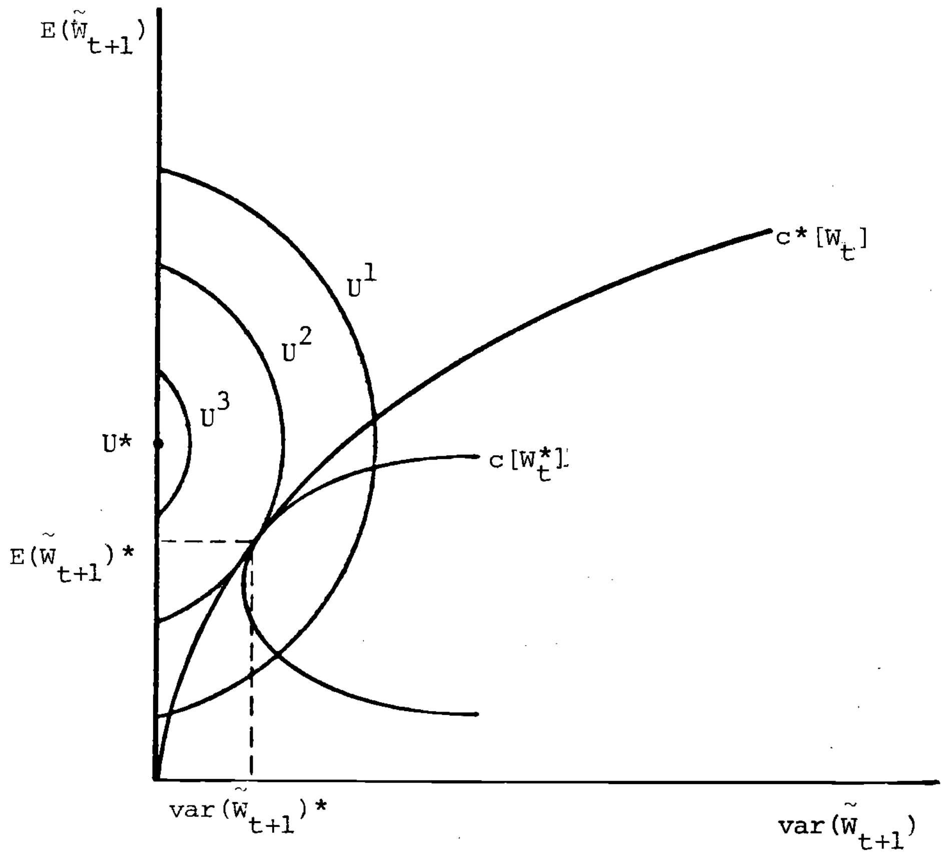
Equivalently, in the case considered here in which no risk-free asset exists, the optimal portfolio is the one that maximizes expected utility subject to the efficiency locus

$$\mathrm{var}(\tilde{W}_{t+1}) = \frac{W_t^2(\underline{r}^{e\prime}_t \Omega_t^{-1} \underline{r}^e_t) - 2W_t(\underline{r}^{e\prime}_t \Omega_t^{-1} \underline{1}) \cdot E(\tilde{W}_{t+1}) + (\underline{1}'\Omega_t^{-1}\underline{1}) \cdot E(\tilde{W}_{t+1})^2}{(\underline{1}'\Omega_t^{-1}\underline{1}) \cdot (\underline{r}^{e\prime}_t \Omega_t^{-1} \underline{r}^e_t) - (\underline{r}^{e\prime}_t \Omega_t^{-1} \underline{1})^2} \tag{28}$$

which is a parabola in variance-mean space dependent on the level of initial wealth and on the parameters of the joint probability distribution of asset returns.[35] Figure 1 displays an efficiency locus $c[W^*_t]$ with initial wealth equal to W^*_t. The well-known result that investors with convex indifference curves will always select efficient portfolios is readily apparent from the parabolic curvature of the efficiency locus. With quadratic utility,

FIGURE 1

QUADRATIC UTILITY



maximum mean-variance utility is obtained at $[E(\tilde{W}_{t+1})^*, \mathrm{var}(\tilde{W}_{t+1})^*]$, with expected utility U^2, as illustrated in the figure.

To find the conditions that lead to initial wealth satiation, the level of invested wealth is then varied to form a family of efficiency loci representing sets of feasible portfolios. The boundary of the set of all possible portfolios, denoted as $c^*[W_t]$ in Figure 1, is given by the envelope of the efficiency loci, expressed as[36]

$$\mathrm{var}(\tilde{W}_{t+1}) = E(\tilde{W}_{t+1})^2 \cdot (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e})^{-1}. \tag{29}$$

Each point on this boundary corresponds to a unique level of invested wealth.

To demonstrate that expected quadratic utility has a point of initial wealth satiation, a finite solution to the maximization of (26) subject to (29) must be found. The first- and second-order conditions associated with this problem are

$$1 - 2b \cdot (1 + (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e})^{-1}) \cdot E(\tilde{W}_{t+1}) = 0 \tag{30}$$

$$-2b \cdot (1 + (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e})^{-1}) < 0. \tag{31}$$

These conditions are jointly satisfied for the unique level of invested wealth

$$W_t^* = (1/2b) \cdot \underline{1}'(\Omega_t + \underline{r}_t^{e}\underline{r}_t^{e'})\underline{r}_t^{e} . \tag{32}$$

Consequently, a satiation level of initial wealth exists at W_t^*, and all initial wealth above this level will be divested.

Figure 1 illustrates the existence of initial wealth satiation in the quadratic utility case. The maximum possible level of expected utility is

$$U^* = \frac{1}{4b} \tag{33}$$

which occurs at the center of the set of concentric indifference curves. The level of expected utility associated with initial wealth W_t^* is

$$U^{**} = (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e}) \cdot [4b \cdot (1+ \underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e})]^{-1} \tag{34}$$

which is always less than that of the unconstrained maximum ($U^* > U^{**}$). It is therefore initial wealth satiation, not end-of-period wealth satiation, that effectively places the upper limit on the level of expected quadratic utility. Moreover, restrictions insuring $\tilde{W}_{t+1} < \tilde{W}_{t+1}^*$ do not necessarily preclude $W_t > W_t^*$. Initial wealth must instead be restricted to be less than W_t^* in order to circumvent the effective bliss point problem in the quadratic utility model.

Negative Exponential Utility with Joint-Normally Distributed Asset Returns

An expected utility model that also enjoys widespread use is derived from the combined assumption of negative exponential utility

$$U(\tilde{W}_{t+1}) = -\exp(-b \cdot \tilde{W}_{t+1}) \tag{35}$$

and joint normally distributed asset returns. One of the attractive features of this specification is that absolute risk aversion is nondecreasing. This model also exhibits increasing relative risk aversion.[37]

The expected utility model consistent with these assumptions can be shown to be maximized when the form

$$U[E(\tilde{W}_{t+1}), V(\tilde{W}_{t+1})] = E(\tilde{W}_{t+1}) - (b/2) \cdot \mathrm{var}(\tilde{W}_{t+1}) \tag{36}$$

is maximized. To obtain the satiation level of initial wealth, expected utility (36) may be maximized with respect to $\underline{A}_t$ subject to (29), with the constrained optimum yielding first- and second-order conditions

$$1 - b \cdot (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e})^{-1} \cdot E(\tilde{W}_{t+1}) = 0 \tag{37}$$

$$-b \cdot (\underline{r}_t^{e'} \Omega_t^{-1} \underline{r}_t^{e}) < 0. \tag{38}$$

These conditions are satisfied for the unique level of initial wealth

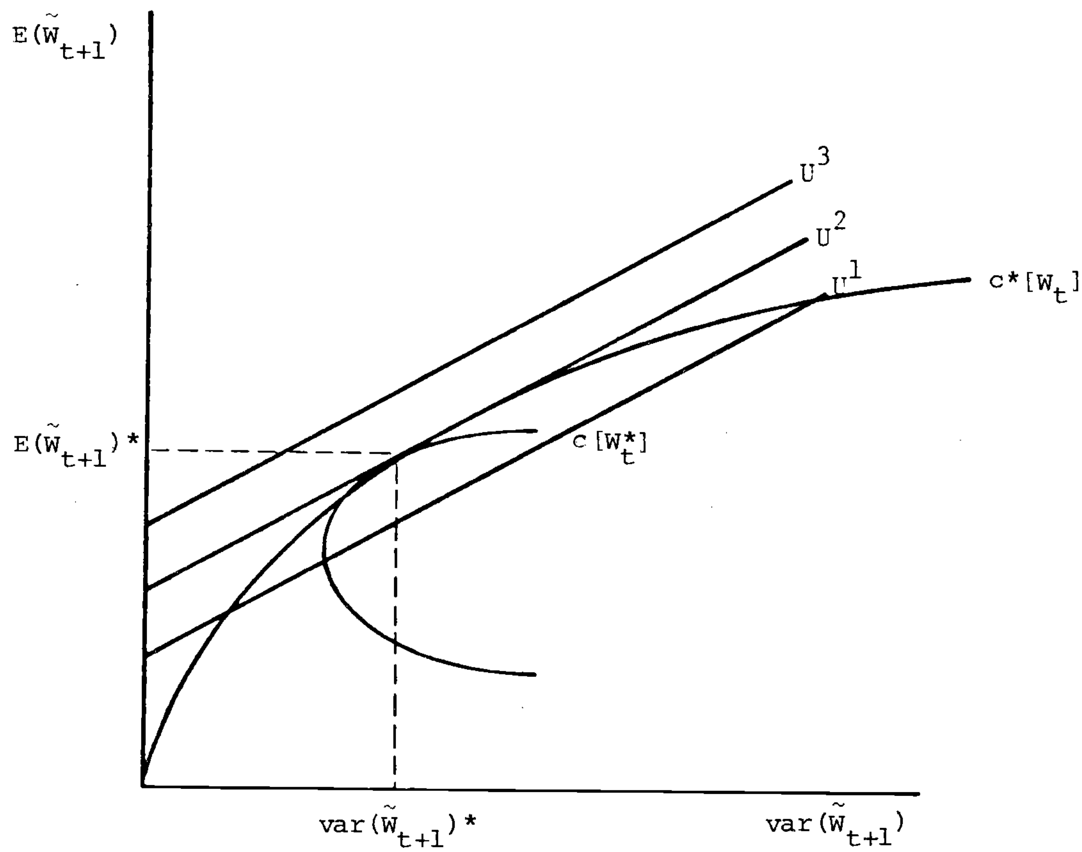
$$W_t^* = (1/b) \cdot (\underline{1}' \Omega_t^{-1} \underline{r}_t^{e}). \tag{39}$$

Figure 2 illustrates the initial wealth satiation point inherent in this expected-utility model. The envelope of the efficiency loci and indifference curves are labled as in Figure 1. This further example serves to highlight the important fact that initial wealth satiation is an issue completely unrelated to whether the utility function possesses an unconstrained maximum, since $U(\tilde{W}_{t+1})$ in (35) is monotonically increasing in $\tilde{W}_{t+1}$.

These results suggest that other mean-variance utility models with convex indifference curves in variance-mean space are also consistent with initial wealth satiation. Analogous satiation points also occur when utility is specified over portfolio rate of return instead of end-of-period wealth. Initial wealth satiation does not, however, occur either when the utility function is logarithmic with lognormally distributed end-of-period wealth, or when mean-variance utility is viewed as an arbitrarily close approximation to expected utility with constant relative risk aversion.[38] The presence of initial wealth satiation points in many common mean-variance utility functions does nevertheless limit the usefulness of these specific models.

FIGURE 2

NEGATIVE EXPONENTIAL UTILITY WITH JOINT-NORMALLY DISTRIBUTED ASSET RETURNS



IV. Summary of Conclusions

Following the lead of Kenneth Arrow's significant contribution to the theory of behavior under uncertainty, the development of the theory of portfolio behavior has led to a greater understanding of the combined effects of uncertainty and risk aversion on many aspects of individual and institutional financial behavior. The focus of this paper is on aspects of this theory involving the properties of investors' asset demands, including in particular specific characteristics of asset demands that in the monetary economics literature are often simply assumed and in the financial literature are often ignored altogether in the consideration of equilibrium asset returns.

The three sections of this paper support three related conclusions. First, asset demands with the familiar properties of wealth homogeneity and linearity in expected returns follow as close approximations from expected utility maximizing behavior under the assumptions of constant relative risk aversion and joint normally distributed asset returns. Second, although such asset demands exhibit a symmetric coefficient matrix with respect to the relevant vector of expected asset returns, symmetry is not a general property, and the available empirical evidence warrants rejecting it for both institutional and individual investors in the United States. Finally, in a manner analogous to the finite maximum exhibited by quadratic utility, a broad class of mean-variance utility functions also exhibits a form of wealth satiation which necessarily restricts its range of applicability.

Footnotes

*Harvard University and University of Washington, respectively. The authors are grateful to the late John Lintner for many helpful conversations that importantly influenced this research, as well as to the National Science Foundation and the Alfred P. Sloan Foundation for research support.

1. Brainard and Tobin (1968) and the voluminous work following their lead provide numerous examples in both abstract and empirical work.

2. Friedman (1956) and deLeeuw (1965) in particular provide useful discussions of the importance of the homogeneity property. For an alternative view, however, see Goldfeld (1966, 1969).

3. Cass and Stiglitz (1970) showed that constant relative risk aversion implies wealth homogeneity (and vice versa), but they did not consider the form of dependence on expected returns in this context. A large literature has investigated the conditions under which, in the presence of a risk-free asset, the ex post demands for risky assets that emerge from the market clearing process are linear in expected returns and linear homogeneous with respect to the total amount invested in risky assets only; see, for example, Sharpe (1964), Lintner (1965), Hakansson (1970), Cass and Stiglitz (1970) and Merton (1971). Nevertheless, these results do not apply to the ex ante demand relations that are usually the focus of analysis in the monetary economics literature, as exemplified by Tobin (1958). Moreover, these results do not carry over in general to cases in which there is no risk-free asset; and even when there is a risk-free asset the homogeneity is not with respect to total wealth (as is usually assumed in the monetary economics literature) and does not apply to the demand for the risk-free asset.

4. The rationale for mean-variance analysis provided by Samuelson (1970) and Tsiang (1972) suggests that mean-variance analysis per se is only an approximation that depends on (among other factors) a small time unit.

5. Friend and Blume (1975) who proceeded along the lines followed here (as did Ross [1975]), offered empirical evidence supporting the assumption of constant relative risk aversion. See also, more recently, Friend and Hasbrouck (1982).

6. Matrix B is singular, of course, so that the asset demand system (8), in conjunction with a given vector of asset supplies, will be capable of determining all relative yields and all absolute yields but one. See Brainard and Tobin (1968) and Smith (1975) for discussions of empirical implementation of such asset demand systems in the specific context of this singularity.

7. In the case including a risk-free asset, vector $\hat{\underline{r}}_t^e$ expresses the mean risky returns in excess of the risk-free return. See Roley (1977) for a detailed treatment of the distinctions based on the presence or absence of a risk-free asset.

8. For given initial wealth, this assumption is equivalent to expressing utility as a function of portfolio rate of return, with constant absolute risk aversion; see Melton (1976).

9. See Lintner (1975) for a comprehensive treatment of portfolio behavior based on the logarithmic utility function.

10. This section is based on Roley (1983) and Friedman (1984 and forthcoming); see these papers for further details about the data and estimation procedures used.

11. Others have recognized the similarity between systems of demand equations derived from consumer and portfolio theories; see, for example, Royama and Hamada (1967) and Bierwag and Grove (1968).

12. With a mean-variance expected utility function, $U[E(\tilde{W}_{t+1}), var(\tilde{W}_{t+1})]$, a necessary and sufficient condition for a symmetric coefficient matrix on expected asset returns is $\partial^2U/\partial E^2=\partial^2U/\partial E\partial var = 0$. This condition in turn implies constant absolute risk aversion.

13. The weighted maturity class data are defined in terms of four "definite" areas and three "borderline" areas. The definite areas corresponding to the two maturity classes examined here are 2 to 4 years and over 12 years to maturity. Securities with maturities in the borderline areas — in this case securities with 1 to 2 years and 8 to 12 years to maturity — are allocated to the definite classifications according to a weighting scheme.

14. The basic notion behind the optimal marginal adjustment model is that investors can allocate new investable flows ΔW_t less expensively than they can re-allocate assets already in their portfolios, and that such flows will be allocated according to desired asset proportions; see Friedman (1977).

15. The estimated model corresponds to that reported in Roley (1981). Additional asset demands are included in expanded versions of the model; see Roley (1982).

16. The <u>columns</u> of B must sum to zero regardless of whether the B matrix is symmetric. The <u>rows</u> of B are required to sum to zero only when symmetry is imposed.

17. Alternative measures of expected returns are considered below in the context of symmetry tests based on household sector portfolio behavior.

18. The technique used is a modified version of a technique suggested by Fair and Parke (1980). Under this procedure, the covariances of the errors between equations in an individual investor category are in general nonzero, but the error covariances between equations of different categories are constrained to equal zero.

19. The standard errors reported in the table are in millions of dollars.

20. The statistic presented by Gallant and Jorgenson (1979) is used to test the symmetry restriction. Under the null hypothesis, this test statistic is asymptotically distributed as chi-square with three degrees of freedom.

21. The equation is

$$cg_{Lt} = \underset{(-1.2)}{-1.63} + \underset{(5.0)}{0.567}\, cg_{L,t-1} - \underset{(-2.8)}{0.366} cg_{L,t-2} + \underset{(2.9)}{0.387} cg_{L,t-3} - \underset{(-0.0)}{.000615}\, cg_{L,t-4}$$

$\bar{R}^2 = .28$ SE = 11.25 DW = 1.99

where the standard error is in per cent per annum.

22. The equation is

$$cg_{Et} = \underset{(2.1)}{5.85} + \underset{(3.5)}{0.393}\, cg_{E,t-1} - \underset{(-2.2)}{0.268}\, cg_{E,t-2} - \underset{(-0.0)}{0.00331}\, cg_{E,t-3} + \underset{(0.1)}{0.017}\, cg_{t-4}$$

$\bar{R}^2 = .12$ SE - 23.18 DW = 2.00

where the standard error is again in per cent per annum.

23. The marginal tax rates applied to interest and dividends are values estimated by Estrella and Fuhrer (1983), on the basis of internal Revenue Service data, to reflect the marginal tax bracket of the average recipient of these two respective kinds of income in each year. The marginal tax rate applied to capital gains is an analogous estimate, including allowances for deferral and loss offset features, due to Feldstein et al. (1983). Preliminary experimentation with the respective price deflators for gross national product and personal consumption expenditures indicated that the results presented below are not very sensitive to the choice of specific inflation measure.

24. The nonlinear maximum likelihood procedure facilitates not only the direct estimation of asymptotic t-statistics on the elements of B but also the imposition of constraints as discussed below.

25. The purpose of this procedure is to generate series of seasonally adjusted end-of-quarter asset stocks without any gaps or inconsistencies due to splicing of data series. (The Federal Reserve System does not construct such series.)

26. Out of \$1,494 billion of household sector liabilities outstanding at yearend 1980, \$971 billion consisted of mortgage debt and \$385 billion of installment and other consumer credit.

27. The standard errors reported here have the dimension of thousands of constant 1967 dollars per capita.

28. The simple first-order serial correlation coefficients are .86 for short-term debt, .51 for long-term debt, and .33 for equity.

29. In comparing these "fit" correlations to the corresponding serial correlations, it is helpful to recall that investors did not know the 1960-80 serial correlation properties of these variables until after this period had ended. The forecasting procedure applied here uses only information that investors had at the time they needed to make each quarter's forecast.

30. In fact, Borch (1969) proved that quadratic utility function is the only von Neumann-Morgenstern (1944) utility function that induces mean-variance preferences for all probability distributions of end-of-period wealth.

31. Bierwag and Grove (1966) demonstrated that convexity of the indifference curves is a sufficient condition. Jones and Roley (1981) generalized this result and showed that some utility functions with concave mean-variance indifference curves also have bliss points.

32. Borch (1969) and Hakansson (1972), for example, interpreted the result of Bierwag and Grove (1966) as implying that indifference curves in standard deviation-mean space are concentric circles with the point of highest utility represented by a single point at the center. This bliss point corresponds to <u>end</u>-of-period wealth satiation in quadratic utility. Bierwag and Grove (1966), however, did not examine the case in which indifference curves in standard deviation-mean space have this representation. Instead, they assumed convex indifference curves in variance-mean space, and showed that this assumption implies a preference ordering in asset space represented by concentric circles. The center of these circles represents the point of <u>initial</u> wealth satiation.

33. See Jones and Roley (1981) for a more general analysis.

34. See Arrow (1965) for a discussion of the adverse risk aversion properties of quadratic utility.

35. Following Markowitz (1952), the efficiency locus may be derived from the problem

$$\underset{\underline{A}_t}{\text{minimize}}\ \underline{A}'_t \Omega_t \underline{A}_t \text{ subject to } \underline{A}'_t \underline{r}^e_t = E(\tilde{W}_{t+1}) \text{ and } \underline{A}'_t \underline{1} = W_t.$$

36. The envelope of the efficiency loci may be derived from the problem

$$\underset{\underline{A}_t}{\text{minimize}}\ \underline{A}'_t \Omega_t \underline{A}_t \text{ subject to } \underline{A}'_t \underline{r}^e_t = E(\tilde{W}_{t+1}).$$

37. Arrow (1965) argued, on both theoretical and empirical grounds, that relative risk aversion is an increasing function of wealth.

38. This latter result is due to Jones (1979). The additional cases mentioned here are examined by Roley (1977) and Jones and Roley (1981).

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