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Javier Bianchi
Enrique G. Mendoza

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ABSTRACT

We study overborrowing and financial crises in an equilibrium model of business cycles and asset prices with collateral constraints. Private agents in a decentralized competitive equilibrium do not internalize the effects of their individual borrowing plans on the market price of assets at which collateral is valued and on the wage costs relevant for working capital financing. Compared with a constrained social planner who internalizes these effects, they undervalue the benefits of an increase in net worth when the constraint binds and hence they borrow "too much" ex ante. Quantitatively, average debt and leverage ratios are only slightly larger in the competitive equilibrium, but the incidence and magnitude of financial crises is much larger. Excess asset returns, Sharpe ratios and the market price of risk are also much larger. A state-contingent tax on debt of about 1 percent on average supports the planner's allocations as a competitive equilibrium and increases social welfare.

Javier Bianchi
University of Maryland, College Park
bianchi@econ.umd.edu

Enrique G. Mendoza
Department of Economics
University of Maryland
College Park, MD 20742
and NBER
mendozae@econ.umd.edu

1 Introduction

A common argument in narratives of the causes of the 2008 global financial crisis is that economic agents "borrowed too much." The notion of "overborrowing," however, is often vaguely defined or presented as a value judgment on borrowing decisions, in light of the obvious fact that a prolonged credit boom ended in collapse. This lack of clarity makes it difficult to answer three key questions: First, is overborrowing a significant macroeconomic problem, in terms of playing a central role in driving macro dynamics during ordinary business cycles and financial crises? Second, are the social welfare implications of overborrowing significant so as to justify policy intervention? And third, are policy instruments like the so-called 'macro-prudential' taxes on debt effective to contain overborrowing and reduce financial fragility, and if so what quantitative features should these taxes have?

In this paper, we answer these questions using a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We provide a formal definition of overborrowing and use quantitative methods to determine how much overborrowing the model predicts and how it affects business cycles, financial crises, and social welfare. We also compute a state-contingent schedule of taxes on debt that can solve the overborrowing problem.

Our definition of overborrowing is in line with the one used in the academic literature (e.g. Lorenzoni, 2008, Korinek, 2009, Bianchi 2009): The difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner who faces the same frictions but internalizes the general-equilibrium effects of its borrowing decisions. In the model, the credit friction is in the form of a collateral constraint on debt that has two important features. First, it drives a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by a social planner. Second, when the constraint binds, it triggers Irving Fisher's classic debt-deflation financial amplification mechanism, which causes a financial crisis.

This paper contributes to the literature by providing a quantitative assessment of overborrowing in an equilibrium model of business cycles and asset prices. The model is similar to those examined by Mendoza and Smith (2006) and Mendoza (2010). These studies showed that cyclical dynamics in a competitive equilibrium lead to periods of expansion in which leverage ratios raise enough so that the collateral constraint becomes binding, triggering a Fisherian deflation that causes sharp declines in credit, asset prices, and macroeconomic aggregates. In this paper, we study instead the efficiency properties of the competitive equilibrium, by comparing its allocations with those attained by a benevolent social planner subject to the same credit frictions as agents in the competitive equilibrium. Thus, while previous studies focused on the amplification and asymmetry of the responses of macro variables to aggregate shocks induced by collateral constraints in competitive equilibria, we focus here on the differences between competitive equilibria and constrained social optima.

In the model, the collateral constraint limits private agents not to borrow more than a fraction of the market value of their collateral assets, which take the form of an asset in fixed aggregate supply (e.g. land). Private agents take the price of this asset as given, and hence a “credit externality” arises, because they do not internalize that, when the collateral constraint binds, fire-sales of assets cause a Fisherian debt-deflation spiral that causes asset prices to decline and the economy’s borrowing ability to shrink in an endogenous feedback loop. Moreover, when the constraint binds, production plans are also affected, because working capital financing is needed in order to pay for a fraction of labor costs, and working capital loans are also subject to the collateral constraint. As a result, when the credit constraint binds output falls because of a sudden increase in the effective cost of labor. This affects dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the credit externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions.

We conduct a quantitative analysis in a version of the model calibrated to U.S. data. The results show that the constrained-efficient allocations exhibit a substantially lower degree of financial fragility than those of the competitive equilibrium. This is reflected in lower incidence and severity of financial crises, lower volatility of consumption, employment, and output, and lower risk premia in asset returns. The decentralized equilibrium accounts for important regularities of actual financial crises, particularly the large drops in output, credit, consumption and asset prices. In contrast, the constrained-efficient allocations show only mild crises as a result of the prudential decisions of the social planner during normal business cycles. These allocations can be implemented as a decentralized equilibrium with a tax on debt of about 1 percent on average, and this tax also reduces sharply the probability and severity of financial crises.

The existing macro literature on credit externalities provides important background for our work. The externality we study is similar to those examined in the theoretical studies of Caballero and Krishnamurthy (2001), Lorenzoni (2008), and Korinek (2009). Our work is also related to the quantitative studies of Bianchi (2009) and Benigno, Chen, Otrok, Rebucci, and Young (2009) and (2010). These authors studied a credit externality at work in the model of emerging markets crises of Mendoza (2002), in which agents do not internalize the effect of their individual debt plans on the market price of nontradable goods relative to tradables, which influences their ability to borrow from abroad. Bianchi examined how this externality leads to excessive debt accumulation and showed that a tax on debt can restore constrained efficiency and reduce the vulnerability to financial crises. Benigno et al. studied how policy intervention via a subsidy on non-tradables during financial crisis can be welfare improving and studied how the credit externality interacts with labor supply effects.

Our analysis differs from the above quantitative studies in that we focus on asset prices as a key factor driving debt dynamics and the credit externality, instead of the relative price of nontradables. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Moreover, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent when the constraint is linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. In addition, because in our model working capital financing is subject to the collateral constraint, the credit externality distorts production plans and dividend rates, and thus again asset prices.

The quantitative studies by Nikolov (2009) and Jeanne and Korinek (2010) examine models in which assets serve as collateral. Nikolov found that tighter collateral requirements to prevent overborrowing may not be welfare-improving in a setup in which collateral requirements and consumption are linear functions that are not influenced by precautionary savings. In contrast, precautionary savings are critical determinants of optimal borrowing decisions in our model, because of the strong non-linear amplification effects caused by the collateral constraint, and for the same reason we find that debt taxes are welfare improving. Jeanne and Korinek construct quantitative estimates of a Pigouvian tax in a model similar to ours but in which output is a stochastic endowment.[1] In contrast, our model provides a transmission mechanism that links the credit externality with output dynamics, which produces large output drops when the constraint binds that feed back into larger declines in asset prices and in access to debt.

Our results also contrast with the findings of Uribe (2006). He found that an environment in which agents do not internalize an aggregate borrowing limit yields identical borrowing decisions to an environment in which the borrowing limit is internalized.[2] An essential difference in our analysis is that the social planner internalizes not only the borrowing limit but also the price effects that arise from borrowing decisions. Still, our results showing small differences in average debt ratios across competitive and constrained-efficient equilibria are in line with his findings.

The rest of the paper is organized as follows: Section 2 presents the analytical framework. Section 3 analyzes constrained efficiency. Section 4 presents the quantitative analysis. Section 5 provides conclusions.

2 Analytical Framework

We follow Mendoza (2010) in specifying the economic environment in terms of a representative firm-household who makes production and consumption decisions. Preferences are given by:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t - G(n_t))\right] \tag{1}$$

In this expression, $E(\cdot)$ is the expectations operator, β is the subjective discount factor, n_t is labor supply and c_t is consumption. The period utility function $u(\cdot)$ is assumed to have the

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[1] They also examined the existence of deterministic cycles in a non-stochastic version of the model.

[2] He provided analytical results for a canonical endowment economy model with a credit constraint where there is an exact equivalence between the two sets of allocations. In addition, he examined a model in which the exact equivalence of his first example does not hold, but still overborrowing is negligible.

constant-relative-risk-aversion (CRRA) form. The argument of $u(\cdot)$ is the composite commodity $c_t - G(n_t)$ defined by Greenwood et al. (1988). $G(n)$ is a convex, strictly increasing and continuously differentiable function that measures the disutility of labor supply. This formulation of preferences removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only.

The budget constraint faced by the representative firm-household is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = \varepsilon_t f(k_t, n_t) + q_t k_t + b_t \tag{2}$$

where b_t denotes holdings of one-period, non-state-contingent discount bonds at the beginning of date t, q_t is the market price of land, R_t is the real interest rate, k_t are individual land holdings, $f(k_t, n_t)$ is a constant-returns-to-scale production function, and ε_t is a productivity shock which has compact support and follows a finite-state, stationary Markov process.

The interest rate is assumed to be exogenous. This is equivalent to assuming that the economy is a price-taker in world credit markets, as in other studies of the U.S. financial crisis like those of Boz and Mendoza (2010), Corbae and Quintin (2009) and Howitt (2010). This assumption is adopted for simplicity, but is also in line with the evidence indicating that in the era of financial globalization even the U.S. risk-free rate has been significantly influenced by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. Warnock and Warnock (2009) provide econometric evidence of the significant effect of foreign capital inflows on U.S. T-bill rates since the mid 1980s. Mendoza and Quadrini (2009) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents.

A fraction θ of the wages bill $w_t n_t$, where w_t is the wage rate, has to be paid in advance using working capital loans. Since the labor market is competitive, the wage rate equals the marginal disutility of labor of the representative firm-household ($w_t = G'(n_t)$). Working capital loans are obtained at the beginning of each period and repaid at the end of the same period. The typical working capital model features an interest rate on working capital loans that is the same as that on one-period bonds, so that the effective marginal cost of labor becomes $w_t(1+\theta(R_t-1))$. We assume instead that the interest rate on within-period loans is zero, in line with some recent studies on the business cycle implications of working capital and credit frictions (e.g. Chen and Song (2009)). We follow this approach so as to show that the effects of working capital in our model hinge only on the need to provide collateral for working capital loans, as explained below, and not on the effect of interest rate fluctuations on effective labor costs.[3]

As in Mendoza (2010), agents face a collateral constraint that limits total debt, including both intertemporal debt and atemporal working capital loans, not to exceed a fraction κ of the market value of asset holdings (i.e. κ imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t}+\theta w_t n_t \leq \kappa q_t k_{t+1} \tag{3}$$

Following Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999), we interpret this constraint as resulting from an environment where limited enforcement prevents lenders to collect more than a fraction κ of the value of a defaulting debtor's assets, but we abstract from modeling the contractual relationship explicitly.

2.1 Private Optimality Conditions

In the competitive equilibrium, agents maximize (1) subject to (2) and (3) taking land prices and wages as given. This maximization problem yields the following Euler equations for bond and land holdings at each date t:

$$u'(t)=\beta R E_t\left[u'(t+1)\right]+\mu_t \tag{4}$$

$$q_t(u'(t)-\mu_t\kappa)=\beta E_t\left[u'(t+1)\left(\varepsilon_{t+1}F_k(k_{t+1},n_{t+1})+q_{t+1}\right)\right] \tag{5}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint.

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[3] Mendoza (2010) assumes that working capital requires collateral in a setup with the standard DSGE treatment of financing costs. However, interest rate effects on effective factors costs are negligible because calibration to actual data implies that $\theta = 0.26$ and $R = 1.086$, and hence working capital only adds 2.2 percent to the cost of labor. This is consistent with Oviedo's (2004) findings suggesting that the standard effects of working capital on factor costs are quantitatively small unless θ and/or R are set at very high levels.

These conditions equate the marginal costs and benefits of savings in bonds and land respectively. When the collateral constraint binds, condition (4) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (5) equates the marginal cost of an extra unit of land investment with its marginal gain. The marginal cost nets out from the marginal utility of foregone current consumption a fraction κ of the shadow value of the credit constraint, because the additional unit of land holdings contributes to relax the borrowing limit.

Condition (5) yields the following forward solution for land prices:

$$q_t = E_t\left[\sum_{j=0}^{\infty}\left(\prod_{i=0}^{j} m_{t+1+i}\right) d_{t+j+1}\right], \quad m_{t+1+i} \equiv \frac{\beta u'(t+1+i)}{u'(t+i)-\mu_{t+i}\kappa}\ , \quad d_t \equiv \varepsilon_t F_k(k_t, n_t) \quad (6)$$

Thus, we obtain what seems a standard asset pricing condition stating that, at equilibrium, the date-t price of land is equal to the expected present value of the future stream of dividends discounted using the stochastic discount factors m_{t+1+i}, for $i = 0, ..., \infty$. The key difference with the standard asset pricing condition, however, is that the discount factors are adjusted to account for the shadow value of relaxing the credit constraint by purchasing an extra unit of land whenever the collateral constraint binds (at any date $t+i$ for $i = 0, ..., \infty$).

Combining (4), (5) and the definition of asset returns ($R^q_{t+1} \equiv \frac{d_{t+1}+q_{t+1}}{q_t}$), it follows that the expected excess return on land relative to bonds (i.e. the equity premium), $R^{ep}_{t+1} \equiv E_t(R^q_{t+1} - R)$, satisfies the following condition:

$$R^{ep}_{t+1} = \frac{\mu_t(1-\kappa)}{(u'(t)-\mu_t\kappa)E_t\left[m_{t+1}\right]} - \frac{cov_t(m_{t+1}, R^q_{t+1})}{E_t\left[m_{t+1}\right]}, \quad (7)$$

where $cov_t(m_{t+1}, R^q_{t+1})$ is the date-t conditional covariance between m_{t+1} and R^q_{t+1}.

Following Mendoza and Smith (2006), we characterize the first term in the right-hand-side of (7) as the direct effect of the collateral constraint on the equity premium, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell land, depressing its current price. This is a first-order effect. This effect vanishes when $\kappa = 1$, because when 100 percent of the value of land can be collateralized, the shadow value of relaxing the constraint by acquiring an extra unit of land equals the shadow value of relaxing it by reducing the debt by one unit. There is also a second-order indirect effect given by the fact that $cov_t(m_{t+1}, R^q_{t+1})$ is likely to become more negative when the constraint binds, because the collateral constraint makes it harder for agents to smooth consumption.

Given the definitions of the Sharpe ratio ($S_t \equiv \frac{R^{ep}_{t+1}}{\sigma_t(R^q_{t+1})}$) and the price of risk ($s_t \equiv \sigma_t(m_{t+1})/E_t m_{t+1}$), we can rewrite the expected excess return and the Sharpe ratio as:

$$R^{ep}_{t+1} = S_t \sigma_t(R^q_{t+1}), \qquad S_t = \frac{\mu_t(1-\kappa)}{(u'(t) - \mu_t \kappa) E_t \left[m_{t+1}\right] \sigma_t(R^q_{t+1})} - \rho_t(R^q_{t+1}, m_{t+1}) s_t \tag{8}$$

where $\sigma_t(R^q_{t+1})$ is the date-t conditional standard deviation of land returns and $\rho_t(R^q_{t+1}, m_{t+1})$ is the conditional correlation between R^q_{t+1} and m_{t+1}. Thus, the collateral constraint has direct and indirect effects on the Sharpe ratio analogous to those it has on the equity premium. The indirect effect reduces to the usual expression in terms of the product of the price of risk and the correlation between asset returns and the stochastic discount factor. The direct effect is normalized by the variance of land returns. These relationships will be useful later to study the quantitative effects of the credit externality on asset pricing.

Since $q_t E_t[R^q_{t+1}] \equiv E_t[d_{t+1} + q_{t+1}]$, we can rewrite the asset pricing condition in this way:

$$q_t = E_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} E_{t+i} R^q_{t+1+i} \right)^{-1} d_{t+j+1}, \tag{9}$$

Notice that (7) and (9) imply that a binding collateral constraint at date t implies an increase in expected excess land returns and a drop in asset prices at t. Moreover, since expected returns exceed the risk free rate whenever the collateral constraint is expected to bind at any future date, asset prices at t are affected by collateral constraint not just when the constraints binds at t, but whenever it is expected to bind at any future date.

The optimality condition for the allocation of labor is:

$$\varepsilon_t F_n(k_t, n_t) = w_t \left[1 + \theta \mu_t / u'(t)\right] \tag{10}$$

This condition equates the marginal product of labor with its effective marginal cost. The term $\theta\mu_t/u'(t)$ reflects the higher effective marginal financing cost of labor when a binding collateral constraint limits access to working capital loans.

2.2 Recursive Competitive Equilibrium

The competitive equilibrium is defined by stochastic sequences of allocations $\{c_t, k_{t+1}, b_{t+1}, n_t\}_{t=0}^{\infty}$ and prices $\{q_t, w_t\}_{t=0}^{\infty}$ such that: (A) the representative agent maximizes utility (1) subject to the sequence of budget and credit constraints given by (2) and (3) for $t = 0, ..., \infty$, taking as given $\{q_t, w_t\}_{t=0}^{t=\infty}$. (B) the markets of goods and land clear at each date t. Since land is in fixed supply $\bar{K}$, the market-clearing condition for land is $k_t = \bar{K}$. The market clearing condition in the goods market is $c_t + \frac{b_{t+1}}{R} = \varepsilon_t F(\bar{K}, n_t) + b_t$

We now characterize the competitive equilibrium in recursive form. The state variables for a particular individual's optimization problem at time t are the individual bond holdings (b), aggregate bond holdings (B), individual land holdings (k), and the TFP realization (ε). Aggregate land holdings are not carried as a state variable because land is in fixed supply. Denoting by $\Gamma(B, \varepsilon)$ the agents' perceived law of motion of aggregate bonds and $q(B, \varepsilon)$ and $w(B, \varepsilon)$ the pricing functions for land and labor respectively, the agents' recursive optimization problem is:

$$V(b, k, B, \varepsilon) = \max_{b',k',c,n} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} V(b', k', B', \varepsilon') \tag{11}$$

$$\begin{aligned} s.t. \quad q(B, \varepsilon)k' + c + \frac{b'}{R} &= \varepsilon f(k, n) + q(B, \varepsilon)k + b \\ B' &= \Gamma(B, \varepsilon) \\ -\frac{b'}{R} + \theta w(B, \varepsilon)n &\leq \kappa q(B, \varepsilon)k' \end{aligned}$$

The solution to this problem is given by the decision rules $\hat{b}'(b, k, B, \varepsilon)$, $\hat{k}'(b, k, B, \varepsilon)$, $\hat{c}(b, k, B, \varepsilon)$, and $\hat{n}(b, k, B, \varepsilon)$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}'(B, \bar{K}, B, \varepsilon)$. In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

Definition 1 ***(Recursive Competitive Equilibrium)***

A recursive competitive equilibrium is defined by an asset pricing function $q(B,\varepsilon)$*, a pricing function for labor* $w(B,\varepsilon)$*, a perceived law of motion for aggregate bond holdings* $\Gamma(B,\varepsilon)$*, and a set of decision rules* $\left\{\hat{b}'(b,k,B,\varepsilon),\hat{k}'(b,k,B,\varepsilon),\hat{c}(b,k,B,\varepsilon),\hat{n}(b,k,B,\varepsilon)\right\}$ *with associated value function* $V(b,k,B,\varepsilon)$ *such that:*

1. $\left\{\hat{b}'(b,k,B,\varepsilon),\hat{k}'(b,k,B,\varepsilon),\hat{c}(b,k,B,\varepsilon),\hat{n}(b,k,B,\varepsilon)\right\}$ *and* $V(b,k,B,\varepsilon)$ *solve the agents' recursive optimization problem, taking as given* $q(B,\varepsilon), w(B,\varepsilon)$ *and* $\Gamma(B,\varepsilon)$*.*

2. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion:* $\Gamma(B,\varepsilon)=\hat{b}'(B,\bar{K},B,\varepsilon)$*.*

3. *Wages satisfy* $w(B,\varepsilon) = G'(\hat{n}(B,\bar{K},B,\varepsilon))$ *and land prices satisfy* $q(B,\varepsilon) = E_{\varepsilon'|\varepsilon}\left\{ \frac{\beta u'\left(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right)\ \left[\varepsilon' f_k\left(\bar{K},\hat{n}(\Gamma(B,\varepsilon)\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right)+q(\Gamma(B,\varepsilon),\varepsilon')\right]}{u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right)-\kappa\max\left[0,u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right)-\beta R E_{\varepsilon'|\varepsilon}u'(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right]} \right\}$

4. *Goods and asset markets clear:*$\frac{\hat{b}'(B,\bar{K},B,\varepsilon)}{R}+\hat{c}(B,\bar{K},B,\varepsilon)=\varepsilon f(\bar{K},\hat{n}(B,\bar{K},B,\varepsilon))+B$*, and* $\hat{k}(B,\bar{K},B,\varepsilon)=\bar{K}$ *.*

3 Constrained-Efficient Equilibrium

3.1 Equilibrium without collateral constraint

We start studying the efficiency properties of the competitive equilibrium by briefly characterizing an efficient equilibrium in the absence of the collateral constraint (3). The allocations of this equilibrium can be represented as the solution to the following standard planning problem:

$$\begin{aligned} H(B,\varepsilon) &= \max_{b',c,n} u(c-G(n))+\beta E_{\varepsilon'|\varepsilon}H(B',\varepsilon') \qquad (12)\\ s.t. \qquad c+\frac{B'}{R} &= \varepsilon f(\bar{K},n)+B \end{aligned}$$

and subject also to either the natural debt limit, which is defined by $B' > \frac{\varepsilon^{\min} f(\bar{K},n^*(\varepsilon^{\min}))}{R-1}$, where $\varepsilon^{\min}$ is the lowest possible realization of TFP and $n^*(\varepsilon^{\min})$ is the optimal labor allocation that solves $\varepsilon^{\min} f_n(\bar{K},n)=G'(n)$, or to a tighter ad-hoc time- and state-invariant borrowing limit.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (see, for example, Mendoza and Smith, 2006 and Mendoza, 2010) is to compare the allocations of the competitive equilibrium with the Fisherian collateral constraint with those arising from the above benchmark case. Because of the Fisherian debt-deflation channel, the competitive equilibria with and without the collateral constraint differ in that in the former private agents borrow *less* (since the collateral constraint limits the amount they can borrow and they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint), and there is financial amplification of the effects of the underlying exogenous shocks (since binding collateral constraints produce large recessions and drops in asset prices). Compared with the constrained-efficient equilibrium we define next, however, we will show that the competitive equilibrium with collateral constraints displays overborrowing (i.e. agents borrow *more* than in the competitive equilibrium).

3.2 Recursive Constrained-Efficient Equilibrium

Consider a benevolent social planner who maximizes the agents' utility subject to the resource constraint, the collateral constraint and the same menu of assets of the competitive equilibrium.[4] The social planner is constrained to have the same "borrowing ability" (the same market-determined value of collateral assets, i.e. $\kappa q(B,\varepsilon)\bar{K}$) at every given state as agents in the decentralized equilibrium, but with the key difference that the planner internalizes the effects of its borrowing decisions on the market prices of assets and labor.[5]

The recursive problem of the social planner is defined as follows:

$$W(B,\varepsilon) = \max_{B',c,n} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} W(B',\varepsilon') \tag{13}$$

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[4] We refer to the social planner's equilibrium and constrained-efficient equilibrium interchangeably. Our focus is on second-best allocations, so when we refer to the social planner's choices it should be understood that we mean the constrained social planner.

[5] We could also allow the social planner to manipulate the borrowing ability state by state. Allowing for this possibility would increase the welfare losses resulting from the externality but the macroeconomic effects remain very similar. For example, when the collateral constraint binds, the social planner may want to increase consumption in order to reduce the marginal utility of consumption and prop up the price of land. In addition, it would like to commit to increase future savings in order to reduce future excess expected returns and raise the price as well. Even with this constrained notion of efficiency, we will show that the competitive equilibrium is significantly suboptimal.

$$
\begin{aligned}
s.t. \quad c + \frac{B'}{R} &= \varepsilon f(\bar{K}, n) + B \\
-\frac{B'}{R} + \theta w(B, \varepsilon) n &\leq \kappa q(B, \varepsilon) \bar{K}
\end{aligned}
$$

where $q(B, \varepsilon)$ is the equilibrium pricing function of the competitive equilibrium and wages continue to satisfy $w(B, \varepsilon) = G'(n)$. Using the envelope theorem, this optimization problem yields the following first-order conditions for bonds and labor:

$$
u'(t) = \beta R E_t \left[u'(t+1) + \mu_{t+1} \psi_{t+1} \right] + \mu_t, \qquad \psi_{t+1} \equiv \kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}} \tag{14}
$$

$$
\varepsilon_t F_n(\bar{K}, n_t) = G'(n_t) \left[1 + \theta \mu_t / u'(t) \right] \tag{15}
$$

The key difference between the competitive equilibrium and the constrained-efficient allocations follows from examining the above Euler equation for bond holdings. In particular, the term $\mu_{t+1}\psi_{t+1}$ represents the additional marginal benefit of savings for the social planner, because it takes into account how an extra unit of bond holdings alters the tightness of the credit constraint through its effects on the prices of land and labor. Note that, since $\frac{\partial q_{t+1}}{\partial b_{t+1}} > 0$ and $\frac{\partial w_{t+1}}{\partial b_{t+1}} \geq 0$, ψ_{t+1} is the difference of two opposing effects and hence its sign is in principle ambiguous. The term $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ is positive, because an increase in net worth increases demand for land and land is in fixed supply. The term $\frac{\partial w_{t+1}}{\partial b_{t+1}}$ is positive, because the effective cost of hiring labor increases when the collateral constraint binds, reducing labor demand and pushing wages up. The value of ψ_{t+1}, however, is positive in all our quantitative experiments with baseline parameter values and variations around them. In particular, we will show in the quantitative analysis that $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ is large and positive when the credit constraint binds.

Definition 2 ***(Recursive Constrained-Efficient Equilibrium)***
The recursive constrained-efficient equilibrium is given by a set of decision rules $\left\{ \hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon) \right\}$ *with associated value function* $W(B, \varepsilon)$*, and wages* $w(B, \varepsilon)$ *such that:*

1. $\left\{ \hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon) \right\}$ *and* $W(B, \varepsilon)$ *solve the planner's recursive optimization problem, taking as given* $w(B, \varepsilon)$ *and the competitive equilibrium's asset pricing function* $q(B, \varepsilon)$.

2. Wages satisfy $w(B,\varepsilon) = G'(\hat{n}(B,\varepsilon))$.[6]

3.3 Comparison of Equilibria & 'Macro-prudential' Taxes

Using a simple variational argument, we can show that the allocations of the competitive equilibrium are inefficient, in the sense that they violate the conditions that support the constrained-efficient equilibrium. In particular, private agents undervalue net worth in periods during which the collateral constraint binds. To see this, consider first the marginal utility of an increase in individual bond holdings. By the envelope theorem, in the competitive equilibrium this can be written as $\frac{\partial V}{\partial b} = u'(t)$. For the constrained-efficient economy, however, the marginal benefit of an increase in bond holdings takes into account the fact that prices are affected by the increase in bond holdings, and is therefore given by $\frac{\partial W}{\partial b} = u'(t) + \psi_t \mu_t$. If the collateral constraint does not bind, $\mu_t = 0$ and the two expressions coincide. If the collateral constraint binds, the social benefits of a higher level of bonds include an extra term given by $\psi_t \mu_t$, because one more unit of aggregate bonds increases the inter-period ability to borrow by ψ_t which has a marginal value of μ_t.

The above argument explains why bond holdings are valued differently by the planner and the private agents "ex post," when the collateral constraint binds. Since both the planner and the agents are forward looking, it follows that those differences in valuation lead to differences in the private and social benefits of debt accumulation "ex ante," when the constraint is not binding. Consider the marginal cost of increasing the level of debt at date t evaluated at the competitive equilibrium in a state in which the constraint is not binding. This cost is given solely by the discounted expected marginal utility from the implied reduction in consumption next period $\beta R E\left[u'(t+1)\right]$. In contrast, the social planner internalizes the effect by which the larger debt reduces tomorrow's borrowing ability by ψ_{t+1}, and hence the marginal cost of borrowing at period t that is not internalized by private agents is given by $\beta R E_t\left[\mu_{t+1}\left(\kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}}\right)\right]$.

We can now show that the planner can implement the constrained-efficient allocations as a competitive equilibrium in the decentralized economy. In particular, the planner can

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[6] This condition implies that the social planner does not internalize the direct effects of current labor on wages. We have also investigated the possibility of having the planner internalize the effects of labor on wages but results are very similar. We keep this formulation because the social planner's equilibrium can be decentralized in a simpler way.

do this by constructing a schedule of state-contingent taxes or subsidies on bond purchases (τ_t) and land dividends (δ_t), with the cost (revenues) financed (rebated) as lump-sum taxes (transfers). The tax on bonds ensures that the planner's optimal plans for consumption and bond holdings are consistent with the Euler equation for bonds in the competitive equilibrium. The tax/subsidy on land dividends ensures that these optimal plans and the pricing function $q(B, \varepsilon)$ are consistent with the private agents' Euler equation for land holdings. The Euler equations of the competitive equilibrium with these taxes become:

$$u'(t) = \beta R(1+\tau_t)E\left[u'(t+1)\right] + \mu_t \tag{16}$$

$$q_t(u'(t) - \mu_t\kappa) = \beta E\left[u'(t+1)\left(\varepsilon_{t+1}f_k(k_{t+1}, n_{t+1})(1+\delta_{t+1}) + q_{t+1}\right)\right] \tag{17}$$

By combining the planner's Euler equation for bonds (eq. (14)) with the above Euler equation for land we can derive the expected excess return on land paid in the land market when the social planner's allocations are implemented with the taxes on land and debt. In this case, after-tax returns on land and bonds are defined as $\tilde{R}^q_{t+1} \equiv \frac{d_{t+1}(1+\delta_{t+1})+q_{t+1}}{q_t}$ and $\tilde{R}_{t+1} \equiv R(1+\tau_t)$ respectively, and the after-tax expected equity premium reduces to the same expression as in the decentralized equilibrium:

$$\tilde{R}^{ep}_{t+1} = \frac{\mu_t(1-\kappa)}{E_t\left[(u'(t) - \mu_t\kappa)m_{t+1}\right]} - \frac{Cov_t(m_{t+1}, \tilde{R}^q_{t+1})}{E_t\left[m_{t+1}\right]} \tag{18}$$

This excess return also has a corresponding interpretation in terms of the Sharpe ratio, the price of risk, and the correlation between land returns and the pricing kernel as in the case of the competitive equilibrium without taxes.

It follows from comparing expressions R^{ep}_{t+1} and $\tilde{R}^{ep}_{t+1}$ that differences in the after-tax expected equity premia with and without taxes are determined by differences in the direct and indirect effects of the credit constraint in the two environments. As shown in the next Section, these effects are stronger in the decentralized equilibrium without taxes, in which tax policy is not correcting the inefficiencies of the credit externality. Intuitively, higher leverage and debt in this environment implies that the constraint binds more often, which strengthens the direct effect. In addition, lower net worth implies that the stochastic discount factor covaries more strongly with the excess return on land, which strengthens the indirect effect. Notice also that dividends in the constrained-efficient allocations are discounted at a rate which depends positively on the tax on debt. This premium is required by the social planner so that the excess returns reflect the social costs of borrowing.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to annual frequency using data from the U.S. economy. The functional forms for preferences and technology are the following:

$$u(c - G(n)) = \frac{\left[c - \varkappa \frac{n^{1+\omega}}{1+\omega}\right]^{1-\sigma} - 1}{1-\sigma} \qquad \omega > 0, \sigma > 1 \tag{19}$$

$$f(k, n) = \varepsilon k^{\alpha_K} n^{\alpha_n}, \qquad \alpha_n, \alpha_n \geq 0 \quad \alpha_K + \alpha_n < 1 \tag{20}$$

The real interest rate is set to $R - 1 = 0.028$ per year, which is the ex-post average real interest rate on U.S. three-month T-bills during the period 1980-2005. We set $\sigma = 2$ and $\beta = 0.96$, standard values in quantitative DSGE models. The Frisch elasticity of labor supply $(1/\omega)$ is set equal to 1, in line with evidence by Kimball and Shapiro (2008). The parameter $\varkappa$ is inessential and is set so that mean hours are equal to 1, which requires $\varkappa = 0.64$. Aggregate land is normalized to $\bar{K} = 1$ without loss of generality and the share of labor in output α_n is equal to 0.64, the standard value.

We follow Schmitt-Grohe and Uribe (2007) in taking M1 money balances in possession of firms as a proxy for working capital. Based on the observations that about two-thirds of M1 are held by firms (Mulligan, 1997) and that M1 was on average about 14 percent of annual GDP over the period 1980 to 2009, we calibrate the working capital-GDP ratio to be $(2/3)0.14 = 0.093$. Given the 64 percent labor share in production, and assuming the collateral constraint does not bind, we obtain $\theta = 0.093/0.64 = 0.146$.

The values of κ, α_K and the TFP process are calibrated to match targets from U.S. data by simulating the model. We set α_K so as to match the average ratio of housing to GDP at current prices, which is equal to 1.35. The value of housing is taken from the *Flow of Funds* accounts, and is measured as real state tangible assets owned by households (reported in Table B.100, row 4). The model matches this ratio when we set $\alpha_K = 0.05$.[7]

TFP shocks follow a log-normal AR(1) process $log(\varepsilon_t) = \rho \log(\varepsilon_{t-1}) + \eta_t$. We construct a discrete approximation to this process with the quadrature procedure of Tauchen and Hussey (1991) using 15 points. The values of σ_ε^2 and ρ are set so that the autocorrelation and standard deviation of the output series produced by the model matches the corresponding moments for the cyclical component of U.S. GDP in the sample period 1947-2007 (which are 2.1 percent and 0.5 respectively). The estimation yields $\sigma_\varepsilon^2 = 0.014$ and $\rho = 0.53$.

Table 1: Calibration

		Source / target
Interest rate	$R - 1 = 0.028$	U.S. data
Discount factor	$\beta = 0.96$	Standard DSGE value
Risk aversion	$\sigma = 2$	Standard DSGE value
Share of labor	$\alpha_n = 0.64$	U.S. data
Labor disutility coefficient	$\chi = 0.64$	Normalization
Frisch elasticity parameter	$\omega = 1$	Kimball and Shapiro (2008)
Supply of land	$\bar{K} = 1$	Normalization
Working capital coefficient	$\theta = 0.14$	Working Capital-GDP=9%
Collateral coefficient	$\kappa = 0.36$	Frequency of Crisis = 3%
Share of land	$\alpha_K = 0.05$	Housing-GDP ratio = 1.35
TFP process	$\sigma_\varepsilon = 0.014, \rho_\varepsilon = 0.53$	Std. dev. and autoc. of U.S. GDP

Finally, we set the value of κ so as to match the frequency of financial crises in U.S. data. Following Calvo, Izquierdo, and Loo-Kung (2006), we define a financial crisis as an event in which the credit constraint is binding and there is a decrease in credit of more than one standard deviation. Then, we set κ so that financial crises in the baseline model simulation occur about 3 percent of the time, which is consistent with the fact that the U.S. has experienced three major financial crises in the last hundred years.[8] This yields the value of $\kappa = 0.36$.

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[7] α_K represents the share of fixed assets in GDP, and not the standard share of capital income in GDP. There is little empirical evidence about the value of this parameter, with estimates that vary depending, for example, on whether we consider land used for residential or commercial purposes, or owned by government at different levels. We could also calibrate α_K using the fact that, in a deterministic steady state where the collateral constraint does not bind, the value-of-land-GDP ratio is equal to $\alpha_K/(R-1)$, which would imply $\alpha_K = 1.35(0.028) = 0.038$. This yields very similar results as $\alpha_K = 0.05$.

[8] The three crises correspond to the Great Depression, the Savings and Loans Crisis and the Subprime Mortgage Crisis (see Reinhart and Rogoff (2008)). While a century may be a short sample for estimating accurately the probability of a rare event in one country, Mendoza (2010) estimates a probability of about

We recognize that several of the parameter values are subject of debate (e.g. there is a fair amount of disagreement about the Frisch elasticity of labor supply), or relate to variables that do not have a clear analog in the data (as is the case with κ or θ). Hence, we will perform sensitivity analysis to examine the robustness of our results to changes in the model's key parameters.

4.2 Borrowing decisions

We start the quantitative analysis by exploring the effects of the credit externality on optimal borrowing plans. Since mean output is normalized to 1, all quantities can be interpreted as fractions of mean output.

Figure 1 shows the bond decision rules of private agents and the social planner as a function of initial bond holdings, both for a negative two-standard-deviations TFP shock. The Fisherian deflation mechanism generates non-monotonic policy functions, instead of the typical increasing policy functions. The point at which bond decision rules switch slope corresponds to the value of initial bond holdings at which the collateral constraint holds with equality but does not bind. To the right of this point, the collateral constraint does not bind and the bond decision rules display the usual upward-sloping shape. To the left of this point, the bond decision rules are *decreasing* in b, because a reduction in current bond holdings reduces the price of land and tightens the borrowing constraint, thus increasing b'.

We follow Bianchi (2009) in distinguishing three regions in the bond decision rules: a "constrained region", a "high-externality region" and a "low-externality region." The "constrained region" is given by the range of b in the horizontal axis with sufficiently high initial debt (i.e. low b) such that the collateral constraint binds in the constrained-efficient equilibrium. This is the range with $b \leq -0.385$. In this region, the collateral constraint binds in both constrained-efficient and competitive equilibria, because the credit externality implies that the constraint starts binding at higher values of b in the latter than in the former, as we show below.

By construction, *the total amount of debt* (i.e. the sum of bond holdings and working capital) in the constrained region is the same under the constrained-efficient allocations and

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3.6 percent for financial crises using a similar definition but applied to all emerging economies using data since 1980.

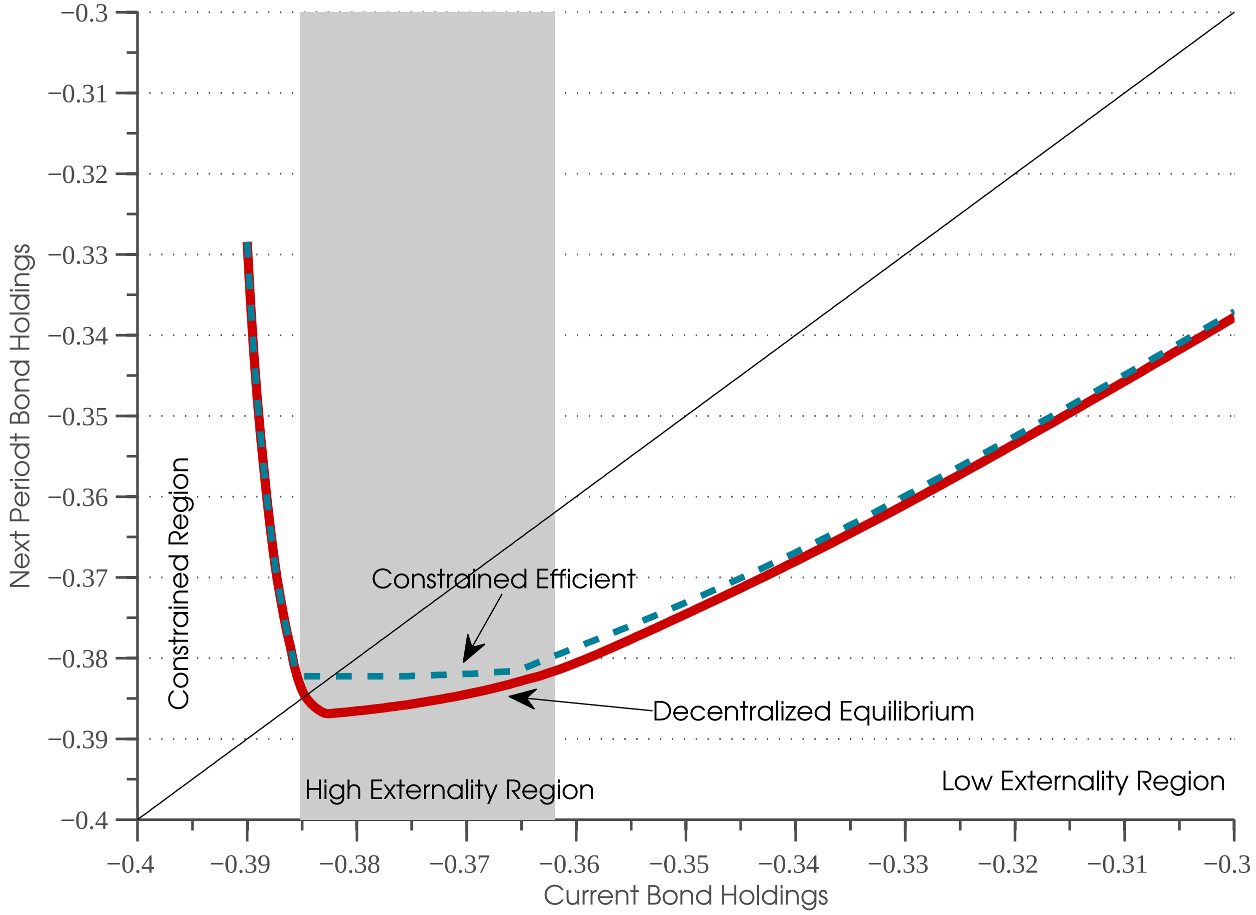

Figure 1: Bond Decision Rules for a Negative Two-standard-deviation TFP Shock

the competitive equilibrium. If working capital were not subject to the collateral constraint, the two *bond decision rules* would also be identical. But with working capital in the constraint the two can differ. This is because the effective cost of labor differs between the two equilibria, since the increase in the marginal financing cost of labor when the constraint binds, $\theta\mu_t/u'(t)$, is different. These differences, however, are extremely small in the numerical experiments, and thus the bond decision rules are approximately the same in the constrained region.[9]

The high-externality region is located to the right of the constrained region, and it includes the interval $-0.385 < b < -0.363$. Here, the social planner chooses uniformly higher bond positions (lower debt) than private agents, because of the different incentives driving

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[9] The choice of b' becomes slightly higher for the social planner as b gets closer to the upper bound of the constrained region, because the deleveraging that occurs around this point is small enough for the probability of a binding credit constraint next period to be strictly positive. As a result, for given allocations, conditions (14) and (4) imply that μ is lower in the constrained-efficient allocations.

the decisions of the two when the constrained region is near. In fact, private agents hit their debt limit at $b = -0.383$, while at that initial b the social planner still retains some borrowing capacity. Moreover, this region is characterized by "financial instability," in the sense that the levels of debt chosen for $t + 1$ are high enough so that a negative shock of standard size in that period can lead to a binding credit constraint that leads to large falls in consumption, output, land prices and credit. We will show later that this is also the region of the state space in which the planner uses actively taxes on debt to manage the inefficiencies of the competitive equilibrium.

The low-externality region is the interval for which $b \geq -0.363$. In this region, the probability of a binding constraint next period is zero. As a result, the social planner would not set a tax on debt, because negative shocks do not lead to a binding credit constraint in the following period. The bond decision rules still differ, however, because expected marginal utilities differ for the two equilibria.

The long-run probabilities with which the constrained-efficient (competitive) economy visits the three regions of the bond decision rules are 2 (4) percent for the constrained region, 69 (70) percent for the high-externality region, and 29 (27) percent for the low-externality region. Both economies spend more than 2/3rds of the time in the high-externality region, but the prudential actions of the social planner reduce the probability of entering in the constrained region by a half. Later we will show that this is reflected also in financial crises that are much less frequent and much less severe than in the competitive equilibrium.

The larger debt (i.e. lower bond) choices of private agents relative to the social planner, particularly in the high-externality region, constitute our first measure of the overborrowing effect at work in the competitive equilibrium. The social planner accumulates extra precautionary savings above and beyond what private individuals consider optimal in order to self-insure against the risk of financial crises. This effect is quantitatively small in terms of the difference between the two decision rules, but this does not mean that its macroeconomic effects and its normative implications are negligible, as we show below.

Overborrowing can also be assessed by comparing the long-run distributions of debt and leverage across the competitive and constrained-efficient equilibria. The fact that the planner accumulates more precautionary savings implies that its ergodic distribution concentrates less probability at higher leverage ratios than in the competitive equilibrium. Figure 2 shows the ergodic distributions and cumulative ergodic distributions of leverage ratios (measured as $\frac{-b_t+\theta w_t n_t}{q_t L_t}$) in the two economies. The maximum leverage ratio with positive long-run probability for the social planner is less than 38 percent, but in the competitive equilibrium this maximum reaches 50 percent. Moreover, the competitive equilibrium's leverage ratio exceeds the maximum leverage ratio of the planner with a long-run probability of 3 percent. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than 1 percent. Hence, overborrowing is relatively small again if measured by comparing differences in unconditional long-run averages of leverage ratios, even though leverage ratios in the competitive equilibrium exceed the maximum of those of the planner 3 percent of the time and by up to 12 percentage points.

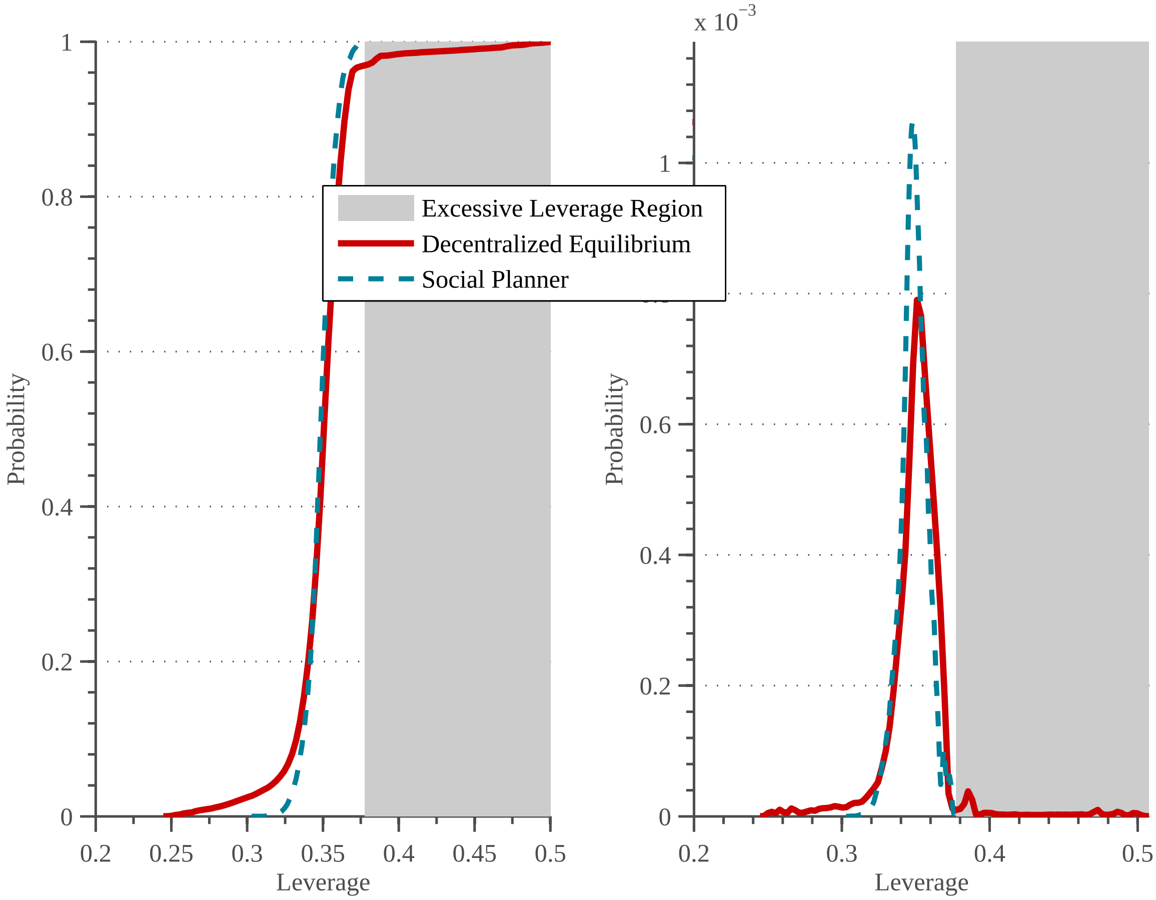

Figure 2: Ergodic Distribution of Leverage

(Leverage is calculated as debt-asset ratio at the beginning of the period.)

4.3 Asset Returns

Overborrowing has important quantitative implications for asset returns and their determinants. To study this issue, we report in Table 2 statistics that characterize the main properties of asset returns in the constrained-efficient and competitive equilibria. We also report statistics for a competitive equilibrium in which land in the collateral constraint is valued at a fixed price equal to the average price $\bar{q}$ (i.e. the credit constraint becomes $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} k_{t+1}$).[10] This fixed-price scenario allows us to compare the properties of asset returns in the competitive and social planner equilibria with a setup in which a collateral constraint exists but the Fisherian deflation channel is removed.

Table 2 lists expected excess returns after taxes, the direct and indirect (covariance) effects of the credit constraint on excess returns, the (log) standard deviation of returns, the price of risk, the Sharpe ratio and the mean debt tax. These moments are reported for the unconditional long-run distributions of each model economy, as well as for distributions conditional on the collateral constraint being binding and not binding.

The mean unconditional excess return is 1.09 percent in the competitive equilibrium v. only 0.17 percent in the constrained-efficient equilibrium and 0.86 percent in the fixed-price economy. The premium in the competitive equilibrium is large, about half as large as the risk-free rate. The fact that the other two economies produce lower premia indicates that the high equity premium of the competitive equilibrium is the combined result of the Fisherian deflation mechanism and the asset price and wage inefficiencies induced by the credit externality. Note also that the high premium produced by our model contrasts sharply with the findings of Heaton and Lucas (1996), who found that credit frictions without Fisherian deflation mechanisms do not produce large premia, unless transactions costs are very large.

The excess returns conditional on the collateral constraint not binding in the constrained-efficient or fixed-prices economies are in line with those obtained in classic asset pricing models that display the "equity premium puzzle." The equity premia we obtained in these two scenarios are driven only by the covariance effect, as in the classic models, and they

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[10] Because the asset is in fixed supply, these allocations would be the same if we use instead an ad-hoc borrowing limit such that $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} \bar{K}$. The price of land, however, would be lower since with the ad-hoc borrowing constraint land does not have collateral value.

are negligible: 0.03 percent in the fixed-price economy and 0.06 percent in the constrained-efficient economy. This is natural because, without the constraint binding and with the effects of the credit externality and the Fisherian deflation removed or weakened, the model is in the same class as those that display the equity premium puzzle.

Conditional on the collateral constraint being binding, mean excess returns in the competitive equilibrium are nearly 14 percent, 4.86 percent for the social planner, and 1.29 percent in the fixed-price economy. Interestingly, the lowest unconditional premium is the one for the constrained-efficient economy (0.17 percent), but conditional on the constraint binding, the lowest premium is the one for the fixed-price economy (1.29 percent). This is because on one hand the Fisherian deflation effect is still at work when the collateral constraint binds in the constrained-efficient economy, but not in the fixed-price economy, while on the other hand the constrained-efficient economy has a lower probability of hitting the collateral constraint (so that the higher premium when the constraint binds does not weigh heavily when computing the unconditional average). In turn, the probability of hitting the collateral constraint is higher for the fixed-price economy because the incentive to build precautionary savings is weaker when there is no Fisherian amplification, and this increases the probability of hitting the constraint.

The unconditional direct and covariance effects of the collateral constraint on excess returns are significantly stronger in the competitive equilibrium than in the constrained-efficient and fixed-price economies, and even more so if we compare them conditional on the constraint being binding. Again, the direct and covariance effects are larger in the competitive equilibrium because of the effects of the overborrowing externality and the Fisherian deflation mechanism.

In terms of the decomposition of excess returns based on condition (8), Table 2 shows that the unconditional average of the price of risk is about twice as large in the decentralized equilibrium than in the constrained-efficient and the fixed-price economies. The Sharpe ratio and the volatility of land returns are also much larger in the competitive equilibrium. In contrast, the correlations between land returns and the stochastic discount factor, not shown in the Table, are very similar under the three equilibria and very close to 1. Hence differences in excess returns cannot be attributed to differences in this correlation.

Table 2: Asset Prices Moments

	Excess Return	Direct Effect	Covariance Effect	s_t	$\sigma_t(R^q_{t+1})$	S_t	Tax on Debt
Decentralized Equilibrium							
Unconditional	1.09	0.87	0.22	5.22	3.05	0.79	0.00
Constrained	13.94	13.78	0.16	4.05	2.71	11.75	0.00
Unconstrained	0.23	0.00	0.23	0.23	3.08	0.05	0.00
Constrained-Efficient Equilibrium							
Unconditional	0.17	0.11	0.06	2.88	1.85	0.08	1.07
Constrained	4.86	4.80	0.06	3.02	2.07	2.38	0.09
Unconstrained	0.06	0.00	0.06	0.06	1.85	0.03	1.09
Fixed Price Equilibrium							
Unconditional	0.86	0.82	0.04	2.59	1.69	0.46	0.00
Constrained	1.29	1.23	0.05	2.81	1.84	0.69	0.00
Unconstrained	0.03	0.00	0.03	0.03	1.39	0.02	0.00

Note: The table reports average of the conditional excess return after taxes, the direct effect, the covariance effect, the price of risk s_t , the (log) volatility of the return of land denoted $\sigma_t(R^q_{t+1})$, the Sharpe ratio and the tax on debt calculated as $E_t \mu_{t+1} \psi_{t+1} / E_t u'(t+1)$. All numbers except the Sharpe ratios are in percentage

The tax on debt is listed in Table 2 because it affects the rate at which dividends are discounted when the planner implements the constrained-efficient equilibrium in a competitive economy. Intuitively, the tax represents the additional premium that the social planner imposes so as to equalize the social benefits of investing in bonds and land. The unconditional average of the tax is 1.07 percent, v. 0.09 when the constraint binds and 1.09 when it does not.

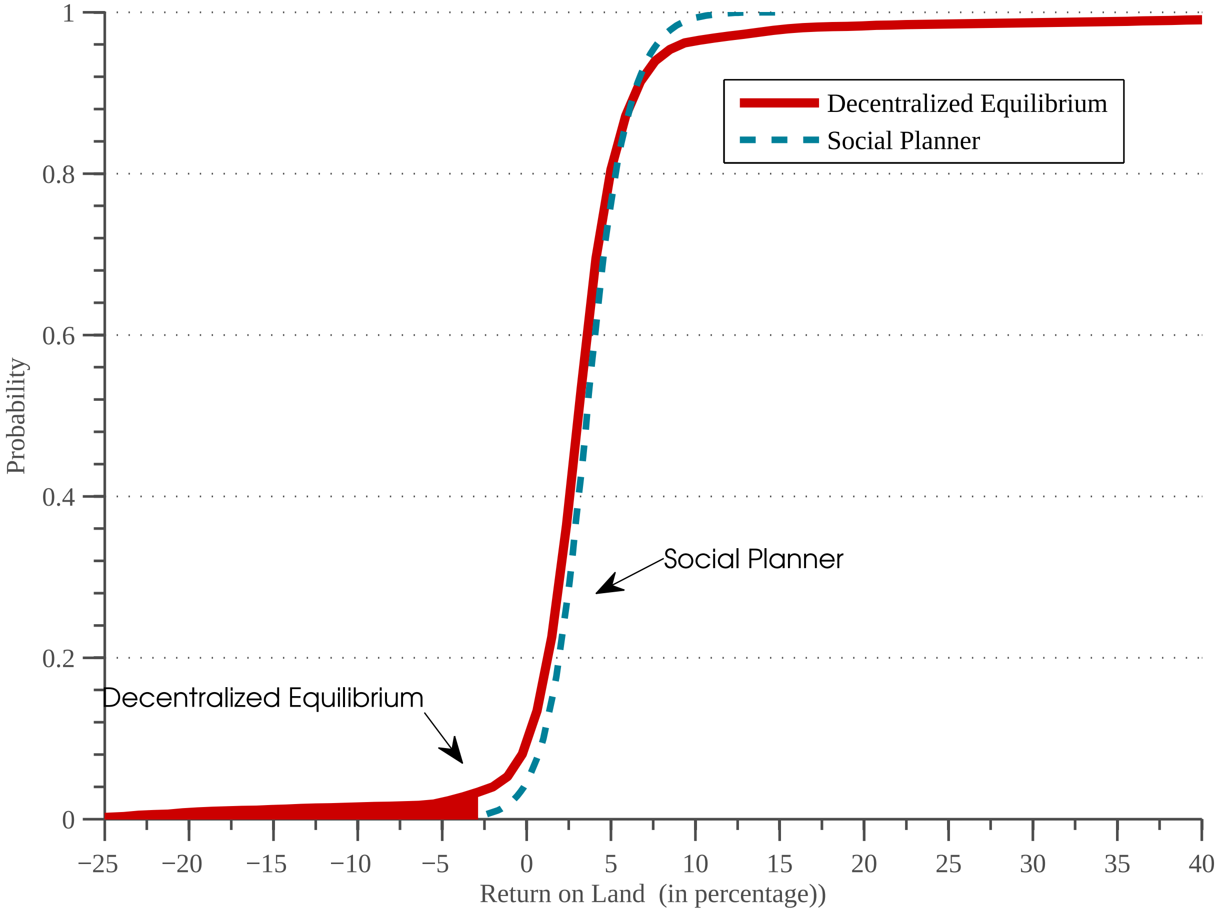

Figure 3: Ergodic Distribution of Land Returns

The tax on debt remains positive, albeit small, on average when the collateral constraint binds because the social planner wants to allocate its borrowing ability across bonds and working capital in a way that differs from the competitive equilibrium. If there is a positive probability that the credit constraint will bind again next period, the social planner allocates less debt capacity to bonds and more to working capital. As a result, a tax on debt is necessary in a subset of the constrained region. Note, however, that these states are not associated with financial crisis events in our simulations. They correspond to events in which the collateral constraint binds but the deleveraging that occurs is not strong enough for a crisis to occur.

Figure 3 shows the long-run distribution of land returns for the competitive equilibrium and the social planner. There is a key difference in these two distributions in that the distribution of the competitive equilibrium features a fatter left tail. The 99th percentile is -17.5 percent in the decentralized equilibrium v. -1.6 percent in the constrained-efficient equilibrium. The left tail in the former correspond to states in which a negative TFP shock hits when agents have a relatively high level of debt. Intuitively, as a negative TFP shock hits, expected dividends decrease which puts downward pressure on asset returns. In addition, if the collateral constraint becomes binding, fire-sales lead to a further drop in asset prices.[11]

4.4 Incidence and Magnitude of Financial Crises

We show now that overborrowing in the competitive equilibrium increases the incidence and severity of financial crises. To demonstrate this result we construct an event analysis of financial crisis with simulated data. The simulated data are obtained by performing long (100,000-period) stochastic time-series simulations of the competitive, constrained-efficient and fixed-price model economies, removing the first 1,000 periods. A financial crisis episode is defined as a period in which the credit constraint binds and this causes a decrease in credit that exceeds one standard deviation of the first-difference of credit in the corresponding ergodic distribution.

The first important result of this exercise is that the incidence of financial crises is significantly higher in the competitive equilibrium. The competitive economy experiences financial crises with a long-run probability of 3.0 percent. In the constrained-efficient economy, by contrast, financial crises occur only with 0.9 percent probability. Thus, the credit externality increases the frequency of financial crises by a factor of 3.33.[12]

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[11] Note that the decentralized equilibrium also has a fatter right-tail distribution. These high returns correspond to periods with positive TFP shocks which were preceded by low asset prices due to fire sales.

[12] We could also define crises in the constrained-efficient equilibrium by using the value of the credit threshold obtained in the competitive equilibrium. However, with this criterion we would obtain an even lower probability of crises, because credit declines equal to at least one standard deviation of the first-

The second important result is that financial crises are more severe in the competitive equilibrium. This is illustrated in the event analysis plots shown in Figure 4. The event windows are for total credit, consumption, labor, output, and land prices, all expressed as deviations from long-run averages, and for the tax on debt necessary to implement the constrained-efficient allocations as a competitive equilibrium. These event dynamics are shown for the decentralized, constrained-efficient, and fixed-price economies.

We construct comparable event windows for the three scenarios following this procedure: First we identify financial crisis events in the competitive equilibrium, and isolate five-year event windows centered in the period in which the crisis takes place. That is, each event window includes five years, the two years before the crisis, the year of the crisis, and the two years after. Second, we calculate the median TFP shock across all of these event windows in each year $t-2$ to $t+2$, and the median initial debt at $t-2$. This determines an initial value for bonds and a five-year sequence of TFP realizations. Third, we feed this sequence of shocks and initial value of bonds to the decisions rules of each model economy and compute the corresponding endogenous variables plotted in Figure 4. By proceeding in this way, we ensure that the event dynamics for the three equilibria are simulated using the same initial state and the same sequence of shocks.

The features of financial crises at date 0 in the competitive economy are very much in line with the results in Mendoza (2010): The debt-deflation mechanism produces financial crises characterized by sharp declines in credit, consumption, asset prices and output. The novelty here is in showing how different the dynamics are in the constrained-efficient economy.

The five macro variables illustrated in the event windows show similar dynamics in the two years before the financial crisis. When the crisis hits, however, the collapses observed in the competitive equilibrium are much larger. Credit falls about 20 percentage points more, and two years after the crisis the credit stock of the competitive equilibrium remains 10 percentage points below that of the social planner. Consumption, asset prices, and output also fall much more sharply in the competitive equilibrium than in the planner's equilibrium. The declines in consumption and asset prices are particularly larger (-16 percent v. -5

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difference of credit in the decentralized equilibrium are zero-probability events in the constrained efficient equilibrium.

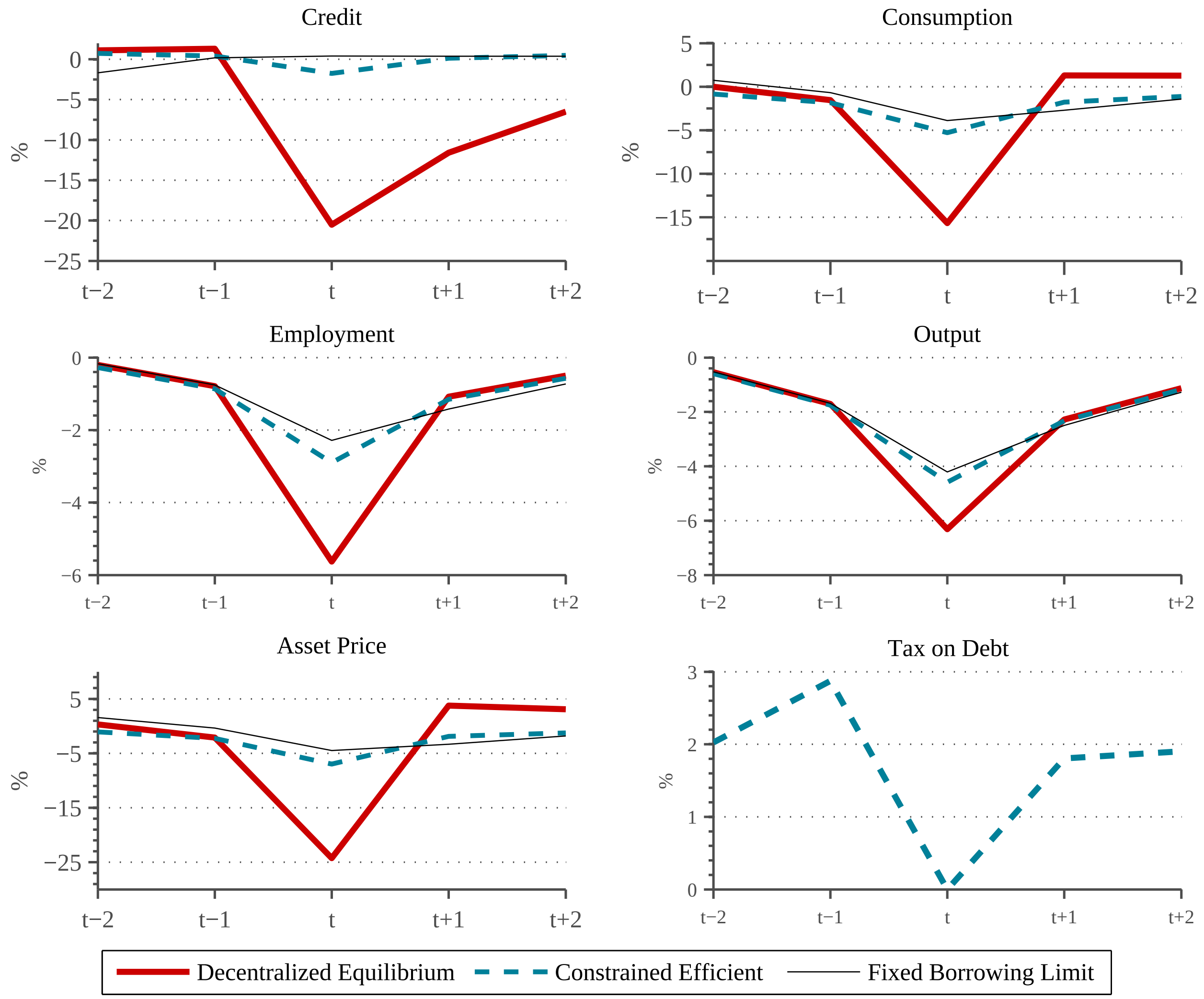

Figure 4: Event Analysis: percentage differences relative to unconditional averages

percent for consumption and -24 percent v. -7 percent for land prices). The asset price collapse also plays an important role in explaining the more pronounced decline in credit in the competitive equilibrium, because it reflects the outcome of the Fisherian deflation mechanism. Output falls by 2 percentage points more because of the higher shadow cost of hiring labor due to the tighter binding credit constraint. Labor falls almost 3 percentage points more.

The sequence of taxes on debt necessary to decentralize the constrained-efficient allocations shows strictly positive taxes of about 2.7 percent at $t-2$ and $t-1$ to mitigate the magnitude of the financial crisis if bad shocks occur. At date t the taxes fall to zero, and they increase again at $t+1$ and $t+2$ to about 1 and 2 percent respectively. The latter occurs because this close to the crisis the economy still remains financially fragile, i.e there is still a strictly positive probability of agents becoming credit constrained next period.

The fixed-price economy displays very little amplification given that the economy is free from the Fisherian deflation mechanism. Credit increases slightly at date t in order to smooth consumption and remains steady in the following periods. The fact that land is valued at the average price, and not the market price, contributes to mitigate the drop in the price of land since it remains relatively more attractive as a source of collateral.

To gain more intuition on why land prices drop more because of the credit externality, we plot in Figure 5 the projected conditional sequences of future dividends and returns on land up to 30 periods ahead of a financial crisis that occurs at date t (conditional on information available at date t). These are the sequences used to compute the present values of dividends that determine the equilibrium land price at t. The expected land returns sharply increase for both competitive and constrained-efficient equilibria, but significantly more for the former (peaking at about 40 percent) than the latter (peaking at 10 percent). On the other hand, expected dividends do not differ significantly, and therefore we conclude that the sharp change in the pricing kernel reflected in the surge in projected land returns is what drives the large differences in the drop of asset prices.

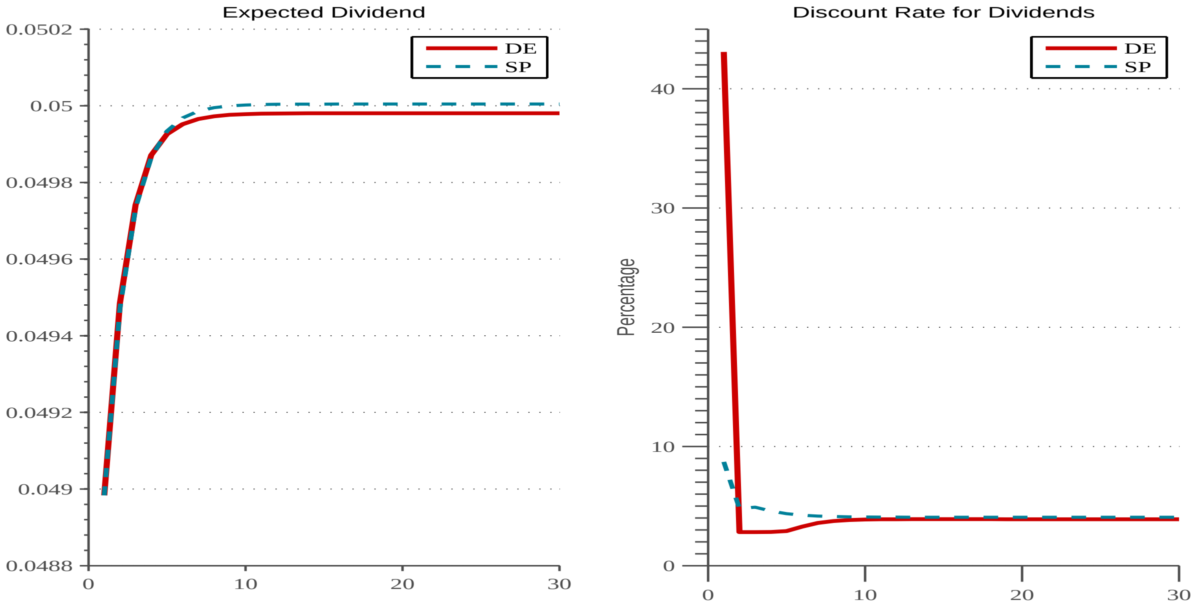

Figure 5: Forecast of expected returns on land and dividends.

The large deleveraging that occurs when a financial crisis occurs in the competitive equilibrium implies that, after the recovery, the expected land returns drop significantly. Expected dividends remain slightly smaller than the long-run average because of the persistence of the TFP shock. In the long-run, expected dividends are higher for the social planner because the marginal productivity of land drops less during financial crises as a result of the lower amount of debt. As explained above, the expected returns on land are higher for the social planner since it requires a larger premium to compensate for the credit externality.

4.5 Long-Run Business Cycles

Table 3 reports the long-run business cycle moments of the competitive, constrained-efficient and fixed-price equilibria, which are computed using each economy's ergodic distribution. The credit externality at work in the competitive equilibrium produces higher business cycle variability in output and labor, and especially in consumption, compared with the constrained-efficient and fixed-price economies. The high variability of consumption and credit are consistent with the results in Bianchi (2009), but we find in addition that the credit externality produces a moderate increase in the variability of labor and a substantial increase in the variability of land prices and leverage.

It may seem puzzling that we can obtain non-trivial differences in long-run business cycle moments even though financial crises are a low probability event in the competitive equilibrium. To explain this result, it is useful to go back to Figure 1. This plot shows that even during normal business cycles the optimal plans of the competitive and constrained-efficient equilibria differ, and this is particularly the case in the high-externality region. Because the economy spends about 70 percent of the time in this region, where private agents borrow more and are more exposed to the risk of financial crises, long-run business cycle moments differ.

The business cycle moments for consumption, output and labor in the constrained-efficient economy are about the same as those of the fixed-price economy. This occurs even though the constrained-efficient economy is subject to the Fisherian deflation mechanism and the fixed-price economy is not. The reason for this is because the social planner accumulates extra precautionary savings, which compensate for the sudden change in the borrowing ability when the credit constraint binds. The constraint binds less often and when it does it has weak effects on macro variables. On the other hand, the constrained-efficient economy does display lower variability in leverage and land prices that the fixed-price economy, and this occurs because the social planner internalizes how a drop in the price tightens the collateral constraint.

The output correlations of leverage, credit, and land prices also differ significantly across the model economies. The GDP correlations of leverage and credit are significantly higher in the competitive equilibrium, while the correlation between the price of land and GDP is lower. The model without credit frictions would have a natural tendency to produce countercyclical credit and leverage because consumption-smoothing agents want to save in good times and borrow in bad times. This effect still dominates in the constrained-efficient and fixed-price economies, but in the competitive equilibrium the collateral constraint and the Fisherian deflation hamper consumption smoothing enough to produce procyclical credit and a higher GDP-leverage correlation. Similarly, the GDP-land price correlation is nearly perfect when the Fisherian deflation mechanism is weakened (constrained-efficient case) or removed (fixed-price case), but falls to about 0.8 in the competitive equilibrium.

In terms of the first-order autocorrelations, the competitive equilibrium displays lower autocorrelations in all its variables compared to both constrained-efficient and fixed-price equilibria. This occurs because crises in the competitive equilibrium are characterized by deep but not very prolonged recessions

Table 3 also shows the cyclical moments for the debt tax that decentralizes the constrained-efficient equilibrium as a competitive equilibrium. The tax fluctuates about 2/3rds as much as GDP and is negatively correlated with GDP. This may seem counterintuitive given the conventional wisdom that macroprudential taxes should be high in good times and low in bad times. Notice, however, that in our model this is the case with regard to fluctuations in leverage: The tax is high when leverage is building up and low when the economy is deleveraging, but since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation.

Table 3: Long Run Moments

	Standard Deviation			Correlation with GDP			Autocorrelation		
	DE	SP	FP	DE	SP	FP	DE	SP	FP
Output	2.10	1.98	1.97	1.00	1.00	1.00	0.50	0.51	0.51
Consumption	2.71	1.87	1.85	0.86	0.99	0.99	0.23	0.56	0.57
Employment	1.25	1.02	0.98	0.97	1.00	1.00	0.42	0.50	0.51
Leverage	6.20	2.61	3.76	-0.49	-0.83	-0.91	0.12	0.72	0.69
Total Credit	3.55	0.95	0.76	0.27	-0.35	-0.42	0.58	0.77	0.81
Land Price	3.95	2.24	3.48	0.79	0.97	0.97	0.16	0.56	0.60
Working capital	2.48	2.04	1.97	0.97	1.00	1.00	0.42	0.50	0.51
Tax on Debt		1.38			-0.75			0.40	

Note: 'DE' represents the decentralized equilibrium,'SP' represents the social planner, 'FP' represents an economy with land valued at a fixed price equal to the average of the price of land in the competitive equilibrium.

4.6 Welfare Effects

We move next to explore the welfare implications of the credit externality. To this end, we calculate welfare costs as compensating consumption variations for each state of nature that make agents indifferent between the allocations of the competitive equilibrium and the constrained-efficient allocations. Formally, for a given initial state (B, ε) at date 0, the welfare cost is computed as the value of γ such that the following condition holds:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1+\gamma) - G(n_t^{DE})) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(n_t^{SP})) \tag{21}$$

where the superscript DE denotes allocations in the decentralized competitive equilibrium and the superscript SP denotes the social planner's allocations. Note that these welfare costs reflect also the welfare gains that would be obtained by introducing the social planner's optimal debt and dividend tax policies, which by construction implement the constrained-efficient allocations as a competitive equilibrium.

The welfare losses of the DE arise from two sources. The first source is the higher variability of consumption, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs due to the effect of the credit friction on working capital. Without the working capital constraint, the marginal disutility of labor equals the marginal product of labor. With the working capital constraint, however, the shadow cost of employing labor rises when the constraint binds, and this drives a wedge between the marginal product of labor and its marginal disutility. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss.

Figure 6 plots the welfare costs of the credit externality as a function of b for a negative, two-standard-deviations TFP shock. These welfare costs approximate a bell shape skewed to the left. This is due to the differences in the optimal plans of the social planner vis-a-vis private agents in the decentralized equilibrium. Recall than in the constrained region, the current allocations of the decentralized equilibrium essentially coincide with those of the constrained-efficient economy, as described in Figure 1. Therefore, in this region the welfare gains from implementing the constrained-efficient allocations only arise from how future allocations will differ. On the other hand, in the high-externality region, the constrained- efficient allocations differ sharply from those of the decentralized equilibrium, and this generally enlarges the welfare losses caused by the credit externality. Notice that, since the constrained-efficient allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the social planner, but these are far outweighed by less vulnerability to sharp decreases in future consumption during financial crises. Finally, as the level of debt is decreased further and the economy enters the low-externality region, financial crises are unlikely and the welfare costs of the inefficiency decrease.

The unconditional average welfare cost over the entire ergodic distributions of bonds and TFP is just about 0.05 percentage points of permanent consumption. This contrasts with Bianchi (2009) who found welfare costs as large as 0.3. Note, however, that our results are in line with his if we express the welfare costs as a fraction of the variability of consumption. Consumption was more volatile in his setup because he examined a calibration to data for emerging economies, which are more volatile than the United States.

The fact that welfare losses from the externality are small although the differences in consumption variability are large is related to the well-known Lucas result that models with CRRA utility, trend-stationary income, and no idiosyncratic uncertainty produce low welfare costs from consumption fluctuations. Moreover, the efficiency loss in the supply-side when

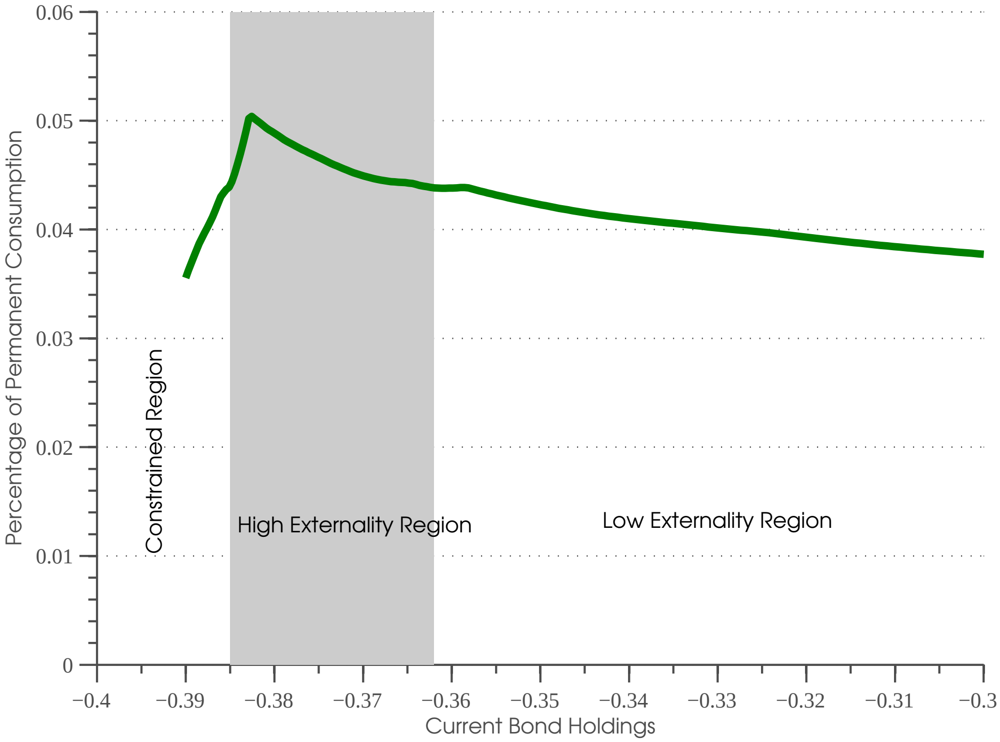

Figure 6: Welfare Costs of the Credit Externality for a two-standard-deviations TFP Shock

the constraint binds produces low welfare costs on average because those losses have a low probability in the ergodic distribution.

4.7 Sensitivity Analysis

We examine now how the quantitative effects of the credit externality change as we vary the values of the model's key parameters. Table 4 shows the main model statistics for different values of σ, κ, ω and θ. The Table shows the unconditional averages of the tax on debt and the welfare loss, the covariance effect on excess returns, the probability of financial crises, and the impact effects of a financial crisis on key macroeconomic variables. In all of these experiments, only the parameter listed in the first column changes and the rest of the parameters remain at their baseline calibration values.

The results of the sensitivity analysis reported in Table 4 can be understood more easily by referring to the externality term derived in Section 2: The wedge between the social and private marginal costs of debt that separate competitive and constrained-efficient equilibria, $\beta R E_t\left[\mu_{t+1}\left(\kappa \bar{K}\frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1}\frac{\partial w_{t+1}}{\partial b_{t+1}}\right)\right]$. For given β and R, the magnitude of the externality is given by the expected product of two terms: the shadow value of relaxing the credit constraint, μ_{t+1}, and the associated price effects $\kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}}$ that determine the effects of the externality on the ability to borrow when the constraint binds. As explained earlier, the price effects are driven mostly by $\frac{\partial q_{t+1}}{\partial b_{t+1}}$, because of the documented large asset price declines when the collateral constraint binds. It follows therefore, that the quantitative implications of the credit externality must depend mainly on the parameters that affect $\mu_{t+1}$and $\frac{\partial q_{t+1}}{\partial b_{t+1}}$, as well as those that affect the probability of hitting the constraint.

The coefficient of relative risk aversion σ plays a key role because it affects both $\mu_{t+1}$and $\frac{\partial q_{t+1}}{\partial b_{t+1}}$. A high σ implies a low intertemporal elasticity of substitution in consumption, and therefore a high value from relaxing the constraint since a binding constraint hinders the ability to smooth consumption across time. A high σ also makes the stochastic discount factors more sensitive to changes in consumption, and therefore makes the price of land react more to changes in bond holdings. Accordingly, rising σ from 2 to 2.5 rises the welfare costs of the credit externality by a factor of 5, and widens the differences in the covariance effects across the competitive and constrained-efficient equilibria from 0.16 to 0.31 percent. Stronger precautionary savings reduce the probability of crises in the competitive equilibrium, and financial crises become a zero-probability event in the constrained-efficient equilibrium. Conversely, reducing σ to 1 makes the externality extremely small, measured either by differences in the incidence or severity of financial crises.[13]

The collateral coefficient κ also plays an important role because it alters the effect of land price changes on the borrowing ability. A higher κ implies that, for a given price response, the change in the collateral value becomes larger. Thus, this effect makes the externality term larger. On the other hand, a higher κ has two additional effects that go in the opposite direction. First, a higher κ implies that the direct effect of the collateral constraint on the land price is weaker, leading to a lower fall in the price of land during fire sales. Second, a higher κ makes the constraint less likely to bind, reducing the externality. The effects of changes in κ are clearly non-monotonic. If κ is equal to zero, there is no effect of prices on the borrowing-ability. At the same time, for high enough values of κ, the constraint

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[13] Notice that the probability of a crisis in the competitive equilibrium becomes 10 percent, more than three times larger than the target employed in the baseline calibration.

never binds. In both cases, the externality does not play any role. Quantitatively, Table 4 shows that small changes in κ are positively associated with the size of the inefficiency. In particular, an increase in κ from the baseline value of 0.36 to 0.40 increases the welfare cost of the inefficiency by a factor of 6 and financial crises again become a zero-probability event in the constrained-efficient equilibrium.

The above results have interesting policy implications. In particular, they suggest that while increasing credit access by rising κ may increase welfare relative to a more financially constrained environment, rising κ can also strengthen the effects of credit externalities and hence make optimal debt taxes or macro-prudential regulation more desirable (since the welfare cost of the externality also rises).

A high Frisch elasticity of labor supply ($(1/\omega) = 1.2$) implies that output drops more when a negative shock hits. If the credit constraint binds, this implies that consumption falls more, which increases the marginal utility of consumption and raises the return rate at which future dividends are discounted.[14] Hence, a higher elasticity of labor supply is associated with higher effects from the credit externality, captured especially by larger differences in the severity of financial crises, a higher probability of crises, and a larger welfare cost of the credit externality.

The fraction of wages that have to be paid in advance θ plays a subtle role. On one hand, a larger θ increases the shadow value of relaxing the credit constraint, since this implies a larger rise in the effective cost of hiring labor when the constraint binds. On the other hand, a larger θ implies, caeteris paribus, a weaker effect on borrowing ability, since the reduction of wages that occurs when the collateral constraint binds has a positive effect on the ability to borrow. Quantitatively, increasing (decreasing) θ by 5 percent increases (decreases) slightly the effects that reflect the size of the externality.

Changes in the volatility and autocorrelation of TFP do not have significant effects. Increasing the variability of TFP implies that financial crises are more likely to be triggered by a large shock. This results in larger amplification and a higher benefit from internalizing price effects. In general equilibrium, however, precautionary savings increase too, resulting

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[14] The increase in leisure mitigates the decrease in the stochastic discount factor but does not compensate for the fall in consumption

Table 4: Sensitivity Analysis: average tax, welfare, covariance effect and responses on impact during financial crises

							Average Effects on Impact during Financial Crises							
	Average Tax	Welfare Loss	Covariance Effect		Crisis Probability		Consumption		Credit		Land Price		GDP	
			DE	SP	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP
benchmark	1.1	0.05	0.22	0.06	3.0	0.9	-15.7	-5.3	-20.5	-1.7	-24.2	-7.0	-6.3	-4.6
$\sigma = 1$	0.6	0.001	0.03	0.03	0.1	0.1	-3.8	-3.3	-2.2	-1.4	-2.8	-2.4	-2.6	-2.6
$\sigma = 1.5$	1.0	0.01	0.08	0.04	0.1	0.0	-6.5	-3.1	-4.0	-0.2	-6.1	-2.5	-3.7	-2.9
$\sigma = 2.5$	1.2	0.24	0.37	0.08	2.0	0.0	-15.9	-4.2	-19.7	0.64	-30.3	-7.0	-5.8	-4.0
$\kappa = 0.32$	1.0	0.02	0.14	0.06	4.6	2.1	-8.8	-3.7	-9.4	0.4	-13.2	-5.1	-4.7	-3.6
$\kappa = 0.4$	1.2	0.29	0.34	0.06	2.2	0.0	-17.2	-4.2	-21.6	1.7	-27.9	-5.7	-5.7	-4.0
$1/\omega = 0.83$	1.04	0.03	0.16	0.05	3.7	1.1	-8.6	-3.4	-8.5	0.8	-12.3	-4.3	-4.4	-3.3
$1/\omega = 1.2$	1.06	0.12	0.27	0.06	3.9	2.3	-18.5	-4.9	-25.9	-1.0	-29.8	-6.3	-6.7	-4.6
$\theta = 0.13$	1.06	0.03	0.18	0.06	1.7	1.6	-15.3	-5.2	-20.0	-1.6	-23.7	-6.9	-6.2	-4.5
$\theta = 0.15$	1.05	0.04	0.21	0.06	2.8	1.2	-17.5	-5.4	-24.0	-2.0	-27.5	-7.1	-6.6	-4.6
$\sigma_\varepsilon = 0.010$	1.08	0.06	0.19	0.04	3.23	0.00	-13.9	-4.3	-19.3	-2.5	-21.9	-5.9	-4.9	-3.4
$\sigma_\varepsilon = 0.018$	1.01	0.05	0.26	0.08	2.51	0.16	-17.3	-6.9	-21.4	-1.9	-26.7	-9.0	-7.5	-5.9
$\rho_\varepsilon = 0.43$	1.06	0.05	0.22	0.05	2.78	0.00	-15.6	-4.9	-20.1	-0.9	-24.1	-6.4	-6.3	-4.5
$\rho_\varepsilon = 0.63$	1.05	0.05	0.23	0.06	2.74	1.17	-16.1	-5.2	-21.7	-2.0	-25.1	-6.9	-6.3	-4.5

Note: 'DE' represents the decentralized equilibrium,'SP' represents the social planner. The average tax on debt corresponds to the average value of the state contingent tax on debt to decentralize the constrained-efficient allocations. The covariance effect represents the unconditional average of the covariance effect. Consumption, credit, land prices and output are responses of these variables on impact during a financial crisis (see section 4.4 for a definition of the event analysis).

in a lower probability of financial crises for both equilibria. Therefore, the overall effects on the externality of a change in the variability of TFP depend on the relative change in the probability of financial crisis in both equilibria and the change in the severity of these episodes.

An increase in the autocorrelation of TFP leads to more frequent financial crises for given bond decision rules. Again, in general equilibrium, precautionary savings increase making ambiguous the effect on the externality.

In terms of the optimal debt on tax, the results of the sensitivity analysis produce an important finding: The average debt tax of about 1.1 percent is largely robust to the parameter variations we considered. Except for the scenario that approximates logarithmic utility ($\sigma = 1$), in all other scenarios included in Table 4 the mean tax ranges between 1.01 and 1.2 percent.

Overall, the results of the sensitivity analysis show that parameter changes that weaken the model's financial amplification mechanism also weaken the magnitude of the externality. This results in smaller average taxes, smaller welfare costs and smaller differences in the incidence and severity of financial crises. The coefficient of risk aversion is particularly important also because it influences directly the price elasticity of asset demand, and hence it determines how much asset prices can be affected by the credit externality. This parameter plays a role akin to that of to the elasticity of substitution in consumption of tradables and non-tradables in Bianchi (2009), because in his model this elasticity drives the response of the price at which the collateral is valued. Accordingly, he found that the credit externality has significant effects only if the elasticity is sufficiently low.

5 Conclusion

This paper examined the positive and normative effects of a credit externality in a dynamic stochastic general equilibrium model in which a collateral constraint limits access to debt and working capital loans to a fraction of the market value of an asset in fixed supply (e.g. land). We compared the allocations and welfare attained by private agents in a competitive equilibrium in which agents face this constraint taking prices as given, with those attained by a constrained social planner that faces the same borrowing limits but takes into account how current borrowing choices affect future asset prices and wages. This planner internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs.

The novelty of our analysis is in that it quantifies the effects of the credit externality in a setup in which the credit friction has effects on both aggregate demand and supply. The effects on demand are well-known from models with credit constraints: consumption drops as access to debt becomes constrained, and this induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and labor demand are affected by the collateral constraint because firms buy labor using working capital loans that are limited by the collateral constraint, and hence when the constraint binds the effective cost of labor rises, so the demand for labor and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the macro/finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing constrained-efficient v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to U.S. data. This analysis showed that, even though the credit externality results in only slightly larger average ratios of debt and leverage to output compared with the constrained-efficient allocations (i.e. overborrowing is not large), the credit externality does produce financial crises that are significantly more severe and more frequent than in the constrained-efficient equilibrium, and produces higher long-run business cycle variability. There are also important asset pricing implications. In particular, the credit externality and its associated higher macroeconomic volatility in the competitive equilibrium produce excess asset returns, Sharpe ratios, and market price of risk that are much larger than in the constrained-efficient equilibrium. We also found that the degree of risk aversion plays a key role in our results, because it is a key determinant of the response of asset prices to volatility in dividends and stochastic discount factors. For the credit externality to be important, these price responses need to be nontrivial, and we found that they are nontrivial already at commonly used risk aversion parameters, and larger at larger risk aversion coefficients that are still in the range of existing estimates.

This analysis has important policy implications. In particular, the social planner can decentralize the constrained-efficient allocations as a competitive equilibrium by introducing an optimal schedule of state-contingent taxes on debt and dividends. By doing so, it can neutralize the adverse effects of the credit externality and produce an increase in social welfare. In our calibrated model, the tax on debt necessary to attain this outcome is about 1 percent on average. The tax is higher when the economy is building up leverage and becoming vulnerable to a financial crisis, but before a crisis actually occurs, so as to induce private agents to value more the accumulation of precautionary savings than they do in the competitive equilibrium without taxes.

These findings are relevant for the ongoing debate on the design of new financial regulation to prevent financial crises, which emphasizes the need for "macro-prudential" regulation. Our results lend support to this approach by showing that credit externalities associated with fire-sales of assets have large adverse macroeconomic effects. At the same time, however, we acknowledge that designing and implementing policies like the tax we studied in this paper for actual use in financial markets remains a very challenging task.

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